14.12 Game Theory – Midterm I

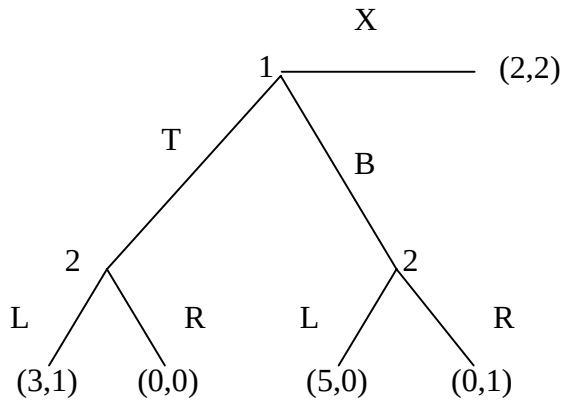
Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Each question is 33 points. Good luck!

1. Consider the following game.

1\2	L	M	R
T	3,2	4,0	1,1
M	2,0	3,3	0,0
B	1,1	0,2	2,3

- **a.** Iteratively eliminate all the strictly dominated strategies.
- **b.** State the rationality/knowledge assumptions corresponding to each elimination.
- **c.** What are the rationalizable strategies?
- **d.** Find all the Nash equilibria. (Don't forget the mixed-strategy equilibrium!)

2. Consider the following extensive form game.



- **a.** Find the normal form representation of this game.
- **b.** Find all pure strategy Nash equilibria.
- **c.** Which of these equilibria are subgame perfect?

3. Consider two agents $\{1,2\}$ owning one dollar which they can use only after they divide it. Each player's utility of getting x dollar at t is $\delta^t x$ for $\delta \in (0,1$. Given any $n > 0$, consider the following n-period symmetric, random bargaining model. Given any date $t \in \{0,1,\ldots,n-1\}$, we toss a fair coin; if it comes Head (which comes with probability 1/2), we select player 1; if it comes Tail, we select player 2. The selected player makes an offer $(x,y \in [0,1]^2$ such that $x+y \leq 1$. Knowing what has been offered, the other player accepts or rejects the offer. If the offer $(x,y$ is accepted, the game ends, yielding payoff vector $(\delta^t x, \delta^t y$. If the offer is rejected, we proceed to the next date, when the same procedure is repeated, except for $t = n-1$, after which the game ends, yielding (0,0). The coin tosses at different dates are stochastically independent. And everything described up to here is common knowledge.

- **a.** Compute the subgame perfect equilibrium for $n = 1$. What is the value of playing this game for a player? (That is, compute the expected utility of each player before the coin-toss, given that they will play the subgame-perfect equilibrium.)
- **b.** Compute the subgame perfect equilibrium for $n = 2$. Compute the expected utility of each

player before the first coin-toss, given that they will play the subgame-perfect equilibrium.

c. What is the subgame perfect equilibrium for $n \geq 3$.