14.12 Game Theory – Midterm I

Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Each question is 33 points. Good luck!

1. Consider the following game.

1\2	L	M	R
T	3,2	4,0	1,1
M	2,0	3,3	0,0
B	1,1	0,2	2,3

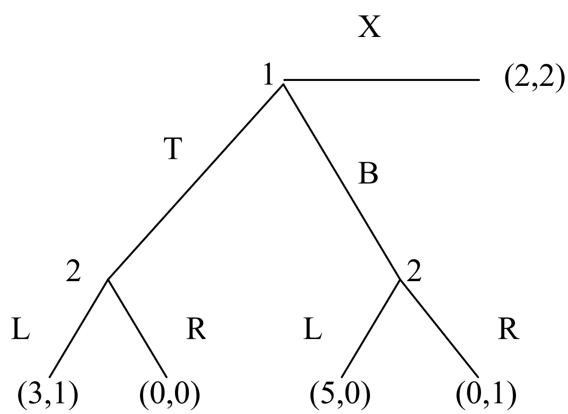
   (a) Iteratively eliminate all the strictly dominated strategies.

   (b) State the rationality/knowledge assumptions corresponding to each elimination.

   (c) What are the rationalizable strategies?

   (d) Find all the Nash equilibria. (Don't forget the mixed-strategy equilibrium!)

2. Consider the following extensive form game.



   (a) Find the normal form representation of this game.

   (b) Find all pure strategy Nash equilibria.

   (c) Which of these equilibria are subgame perfect?

3. Consider two agents $\{1,2\}$ owning one dollar which they can use only after they divide it. Each player's utility of getting x dollar at t is $\delta^t x$ for $\delta \in (0,1)$. Given any $n > 0$, consider the following n-period symmetric, random bargaining model. Given any date $t \in \{0,1,\ldots,n-1\}$, we toss a fair coin; if it comes Head (which comes with probability 1/2), we select player 1; if it comes Tail, we select player 2. The selected player makes an offer $(x,y) \in [0,1]^2$ such that $x + y \leq 1$. Knowing what has been offered, the other player accepts or rejects the offer. If the offer (x,y) is accepted, the game ends, yielding payoff vector $(\delta^t x, \delta^t y)$. If the offer is rejected, we proceed to the next date,

when the same procedure is repeated, except for $t = n - 1$, after which the game ends, yielding (0,0). The coin tosses at different dates are stochastically independent. And everything described up to here is common knowledge.

(a) Compute the subgame perfect equilibrium for $n = 1$. What is the value of playing this game for a player? (That is, compute the expected utility of each player before the coin-toss, given that they will play the subgame-perfect equilibrium.)

(b) Compute the subgame perfect equilibrium for $n = 2$. Compute the expected utility of each player before the <u>first</u> coin-toss, given that they will play the subgame-perfect equilibrium.

(c) What is the subgame perfect equilibrium for $n \geq 3$.