14.12 Game Theory-Midterm I
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Instructions: This is an open book exam, you can use any written material. You have 1 hour and 20 minutes. Each question is 35 points where the bonus question 3(c) accounts for the extra 5 points. Good luck!

1. Consider the following two player game where Player 1 chooses one of the three rows and Player 2 chooses one of the three columns:

	C_1	C_2	C_3
R_1	2,-1	4,2	2,0
R_2	3,3	0,0	1,1
R_3	1,2	2,8	5,1

   (a) What are the strategies that survive IESDS?

   (b) At each step of the elimination what were your rationality and knowledge assumptions?

   (c) Find all Nash equilibria, including the mixed one.

2. Consider the following extensive form game with perfect information:

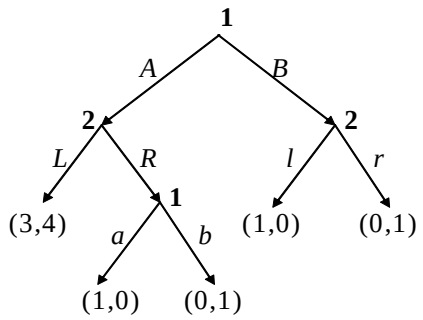

Figure 1:

   (a) Find out the backwards induction outcome.

   (b) At each step of the backwards induction, what were your sequential rationality and knowledge assumptions?

   (c) Determine the strategies of each player and write out the corresponding normal form game.

   (d) Find all four pure strategy Nash equilibria.

3. (Cournot quantity competition with prior technology choice) There are two competing firms $N = \{1, 2\}$ and two periods. Firm i has to invest $f(c_i) = (1 - c_i)^2$ in period 1, in order to adopt a technology where its marginal cost of production in period 2 will be $c_i \in [0, 1]$. Note that a technology with a lower marginal cost requires a higher initial investment. In period 2, the firms engage in Cournot quantity competition given their previously determined technologies.

   Formally, in period 1, each firm i simultaneously chooses its marginal cost of production $c_i \in [0, 1]$. In period 2, firms learn each other's marginal costs and then each firm i simultaneously determines its quantity level $q_i \geq 0$. The net profit of firm i is:

$$\pi_i(q_i, q_j, c_i) = q_i[P(q_i + q_j) - c_i] - f(c_i) \qquad i \neq j$$

   where market demand is given by $P(q_1 + q_2) = \max\{2 - (q_1 + q_2), 0\}$. (Note that the intercept of the demand is $a = 2$ not 1!) In parts (a) and (b), you are asked to compute the subgame perfect Nash equilibrium of this game.

   (a) Given $c_1, c_2 \in [0, 1]$, what are the equilibrium quantity choices $q_1(c_1, c_2)$ and $q_2(c_1, c_2)$ in period 2?

   (b) Given that in period 2 the firms will set quantities according to (a), what are the equilibrium levels of marginal costs c_1 and c_2 in period 1? (Hint: First write down the equilibrium profit levels $\pi_1(q_1(c_1, c_2), q_2(c_1, c_2), c_1)$ and $\pi_2(q_1(c_1, c_2), q_2(c_1, c_2), c_2)$ as functions of c_1 and c_2 only.)

   (c) (Extra credit, take a shot at this only if you have extra time) Consider a modification of the above game where initially firm 1 determines its marginal cost c_1, then firm 2 observes c_1 and determines c_2, and finally firm 1 observes c_2 and the firms engage in simultaneous quantity competition as in above. What are the subgame perfect Nash equilibrium strategies and the equilibrium levels of marginal costs and quantities in this case?