14.12 Game Theory
Fall 2004

Midterm 1

10/13/04 in class

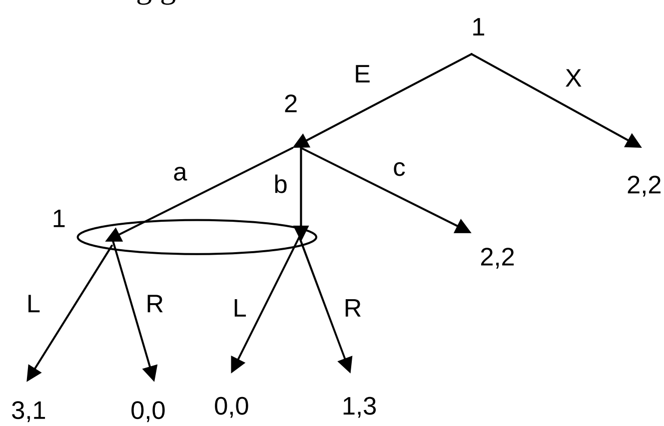
(You have 80 minutes. Good Luck!)

1. Consider the following game:

	A	B	C
a	3,0	0,3	0,x
b	0,3	3,0	0,x
c	x,0	x,0	x,x

   a. (15pts) Compute two Nash equilibria for x = 1.
   b. (10pts) For each equilibrium in part a, check if it remains a Nash equilibrium when x = 2.

2. Consider the following game:



   a. (10pts) Compute two subgame-perfect equilibria.
   b. (10pts) Represent this game in strategic form.
   c. (5pts) Find a Nash equilibrium that is not subgame-perfect.

3. Consider two players who own a dollar, which they need to share to consume. Each player cares about his and his opponent's consumption level. We will first assume that the players are malevolent. The payoff of a player is

$$u = x - \alpha y,$$

where x is his own consumption level, y is his opponent's consumption level, and α is a number in (0,1).

   a. (8pts) One of the players is to offer a division of the dollar, and the other player is to decide whether to accept or reject the offer. If the offer is accepted, then they divide the dollar as offered; otherwise the game ends, and each consumes 0. Use backwards induction to compute an equilibrium.
   b. (8pts) Assume that there are two periods. If the offer is rejected in the first period, the game proceeds to the second period, rather than ending. In the second period, they toss a fair coin, which comes up head or tail with

equal probabilities. If it comes up head, player 1 makes an offer, and player 2 accepts or rejects the offer. If it comes up tail, player 2 makes an offer and player 1 accepts or rejects the offer. If the offer in the second period is rejected, too, then the game ends, and each consumes 0. Use backwards induction to compute an equilibrium. (Assume that the payoff of a player from a division (x,y) is u, independent of when they divide the dollar.)

   c. (9pts) Repeat the analysis in parts a and b assuming that

$$u = x + \alpha y$$

   for some α in (0,1), i.e., the players are benevolent.

4. A seller is to sell a $100 bill through a "first-price auction." There are two buyers. Each buyer i simultaneously bids $b_i \in \{0,1,2,3,\ldots,99\}$. If a buyer bids a higher number than the other, then he wins the auction; otherwise the winner is determined by a coin toss. The winner gets the $100 bill and pays his own bid. All in all, the payoff of player i is $100 - b_i$ if $b_i > b_j$, $(100 - b_i)/2$ if $b_i = b_j$, and 0 if $b_i < b_j$. (Notice that the bid must be a non-negative integer that is less than 100.)
   a. (20pts) Compute the rationalizable strategies. State the rationality and knowledge assumptions needed for the first three rounds of elimination.
   b. (5pts) Compute the Nash equilibria in pure strategies.