14.12 Game Theory
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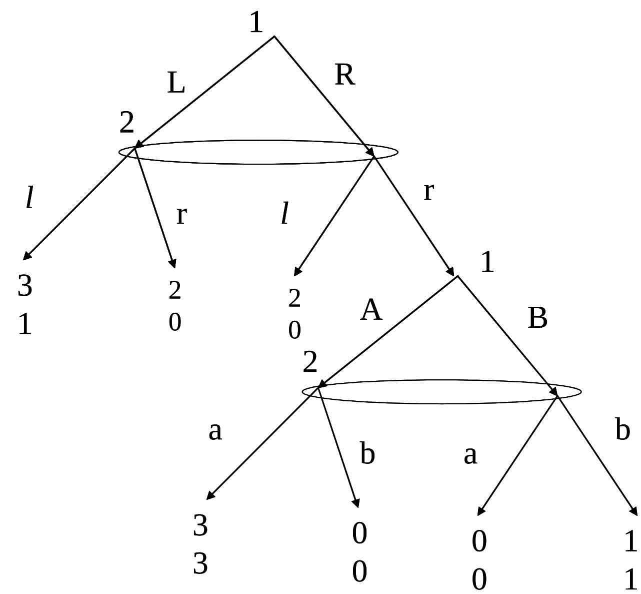
Midterm 2

November 14th, 2005 (80 minutes)
Each question is 25 points.
Good Luck!

1. Find all the pure-strategy subgame-perfect equilibria of the following game. (Ignore the equilibria in mixed strategies.)



2. For each strategy profile below, check whether the strategy profile is a subgame-perfect equilibrium of the game in which the following stage game is repeated infinitely many times and the discount factor is $\delta = 0.99$. (Show your reasoning clearly.) Stage game:

	H	M	L
H	6,6	0,8	0,7
M	8,0	3,3	0,7
L	7,0	7,0	2,2

   (a) There are two modes: Collusion and Punishment. At $t = 0$, we are in Collusion mode. In Collusion mode each player plays H. In Collusion mode, if both players play H, then we remain in the Collusion mode in the next period; otherwise we go to Punishment mode in the next period. In Punishment mode, each player plays M, and we automatically go to the Collusion mode in the following period.

   (b) The same as in (a), except that in Punishment mode, each player plays L.

3. This question is about collusion among Cournot oligopolists. Consider the infinitely repeated game with the following stage game and the discount factor δ.

   Stage Game (Linear Cournot Oligopoly**):** There are $n > 1$ firms, each with zero marginal cost. At a given date, each firm i simultaneously produces q_i units of a good, and they sell at price $P = 1 - (q_1 + q_2 + \cdots + q_n)$. (The payoff of i is $q_i P$.)

   Consider the following strategy profile.

   Strategy Profile: At $t = 0$, each firm produces $1/(2n)$. At any $t > 0$, the production levels depend of what firms produced at $t - 1$. If each firm produced $1/(2n)$ or each firm produced x at $t-1$, then each firm produces $1/(2n)$; otherwise, each firm produces x.

   Find conditions on x, n, and δ under which this strategy profile is a subgame-perfect equilibrium. [Hint: You just need to find two inequalities that x, n, and δ must satisfy.]

4. Two players, namely 1 and 2, are trying to divide a dollar, which they can consume only when they divide. If they agree on a division that gives x to player i at date t, then the payoff of player i is $\delta^t x$ for some $\delta \in (0, 1)$. The dates are $t = 0, 1, 2, \ldots$. If players do not agree by date n, the game automatically ends at the beginning of date n, and each player gets 0.

   - At each date $t < n$, we select one of the players randomly as a proposer. The probability that player 1 is selected is p, and the probability that player 2 is selected is $1 - p$, where $0 \leq p \leq 1$.
   - The proposer offers a division $(x, 1 - x)$, and the other player accepts or rejects the offer.
   - If the offer is accepted, then player 1 gets x and player 2 gets $1 - x$.
   - If the offer is rejected, we proceed to the next date.

   (a) (5 points) Compute the subgame-perfect equilibrium for $n = 1$. What are the expected payoffs of players in the subgame-perfect equilibrium at the beginning of the game?

   (b) (10 points) Compute the subgame-perfect equilibrium for $n = 2$. What are the expected payoffs of players in the subgame-perfect equilibrium at the beginning of the game? [Hint for part c: simplify the expressions for the expected payoffs!!!]

   (c) (10 points) For $n = \infty$, conjecture a subgame-perfect equilibrium and check that it is indeed a subgame-perfect equilibrium.

   (d) (Bonus: 10 points) Now imagine that players invest in their "bargaining power" before the bargaining starts. Players 1 and 2 invest x and y dollars, simultaneously. Then, they play the bargaining game for $n = \infty$ with $p = x/(x + y)$. Find the subgame-perfect equilibrium. [Hint: the derivative of $x/(x + y)$ with respect to x is $y/(x + y)^2$.]