14.12 Game Theory – Midterm II

Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Each question is 33 points. Good luck!

1. Consider the infinitely repeated game with observable actions where $\delta = 0.99$ and the following Prisoners' Dilemma game is repeated.

	C	D
C	5,5	-6,8
D	8,-6	0,0

   Check if any of the following strategy profiles is a subgame-perfect Nash equilibrium.

   (a) Each player's strategy is: Play always C.

   (b) Each player's strategy is: Play C in the first date; and at the second date and thereafter, play whatever the other player played in the previous date.

   (c) There are four modes; Cooperation Mode, Punishment Mode for player 1, Punishment Mode for player 2, and Non-cooperation Mode. In the first date they are in the Cooperation mode. In this mode, each player plays C. In the punishment mode for i, i plays C, while the other player plays D. In non-cooperation mode each player plays D. Once they are in non-cooperation mode they stay in non-cooperation mode forever. In any other mode, if both player stick to their strategy, they go to cooperation mode in the next date; if any player i deviates unilaterally (while the other player sticks with his strategy), they go to the punishment mode for i in the next date; and if both player deviate, in the next date they go to the non-cooperation mode.

2. Consider the following private-value auction of a single object, whose value for the seller is 0. there are two buyers, say 1 and 2. The value of the object for each buyer $i \in \{1, 2\}$ is v_i so that, if i buys the object paying the price p, his payoff is $v_i - p$; if he doesn't buy the object, his payoff is 0. We assume that v_1 and v_2 are independently and identically distributed uniformly on $[\underline{v}, 1]$ where $0 \leq \underline{v} < 1$.

   (a) We use sealed-bid first-price auction, where each buyer i simultaneously bids b_i, and the one who bids the highest bid buys the object paying his own bid. Compute the symmetric Bayesian Nash equilibrium in linear strategies, where $b_i = a + cv_i$. Compute the expected utility of a buyer for whom the value of the object is v.

   (b) Now assume that v_1 and v_2 are independently and identically distributed uniformly on $[0, 1]$. Now, in order to enter the auction, a player must pay an entry fee $\phi \in (0, 1)$. First, each buyer simultaneously decides whether to enter the auction. Then, we run the sealed-bid auction as in part (a); which players entered is now common knowledge. If only one player enters the auction any bid $b \geq 0$ is accepted. Compute the symmetric perfect Bayesian Nash equilibrium where the buyers use the linear strategies in the auction if both buyer enter the auction. Anticipating this equilibrium, which entry fee the seller must choose? [**Hint**: In the entry stage, there is a cutoff level such that a buyer enters the auction iff his valuation is at least as high as the cutoff level.]

3. Consider the entry deterrence game, where an Entrant decides whether to enter the market; if he enters the Incumbent decides whether to Fight or Accomodate. We consider a game where Incumbent's payoff from the Fight is private information, the entry deterrence game is repeated twice and the discount rate is $\delta = 0.9$. The payoff vectors for the stage game are (0,2) if the Entrant does not enter, $(-1, a)$ if he enters and the Incumbent Fights; and (1,1) if he enters and the Incumbent accomodates, where the first entry in each paranthesis is the payoff for the entrant. Here, a can be either -1 or 2, and is privately known by the Incumbent. Entrants believes that $a = -1$ with probability π; and everything described up to here is common knowledge.

   (a) Find the perfect Bayesian Equilibrium when $\pi = 0.4$.

   (b) Find the perfect Bayesian Equilibrium when $\pi = 0.9$.