14.12 Game Theory
October 29, 2005

Recitation 5 Problems

1. (From 2003 Midterm II) Consider an infinite horizon bargaining game with three players $N = \{1, 2, 3\}$. In each period t, one of the players is randomly selected to make an offer: Player 1 is selected with probability $\frac{1}{2}$, each one of Players 2 and 3 is selected with probability $\frac{1}{4}$. The selected Player i offers a division of the cake (x_t, y_t, z_t) where $x_t, y_t, z_t \geq 0$ and $x_t + y_t + z_t = 1$ (x_t denotes player 1's share y_t denotes player 2's share and z_t denotes player 3's share). The two other Players j and k observe i's offer (x_t, y_t, z_t), then j and k *simultaneously* accept or reject this offer. If both j and k accept then the division is carried out, if at least one of them rejects then the offer is rejected and they proceed to period $t + 1$.

   Players maximize discounted expected payoffs and have the common discount factor $\delta \in (0, 1)$. If no offer is ever accepted, then each player receives a payoff of zero. The selection of who makes an offer is i.i.d. across periods.

   (a) Conjecture an SPE where (as usual) players accept any division where they receive at least δ times their continuation payoff. Write down formally the strategy profile and verify that it is indeed an SPE by using the single deviation property.

2. Consider a modification of Problem 3 on Pset 1. In a college there are n students who can send data using AOL or the college network. Let $x_i \geq 0$ be the size of the data sent by student i. Each student i chooses x_i himself or herself. If the data is sent over AOL, the student receives a payoff of

$$u + \frac{\delta}{2}x_i - x_i^2$$

   If the student chooses the college network, the speed of the network is inversely proportional to the total size of the data, so that it takes $x_i t(x_1, .., x_n)$ minutes to send the message where

$$t(x_1, .., x_n) = x_1 + ... + x_n$$

   and the payoff of the student i is

$$x_i - x_i t(x_1, .., x_n)$$

   First, simultaneously, the students decide which network to use. Then, observing how many students are on each network, they simultaneously choose the amount of data to send.

(a) Find all the subgame-perfect equilibria in pure strategies.

3. (Gibbons Problem 2.13) Recall the static Betrand duopoly model (with homogeneous products): the firms name prices simultaneously; demand for firm i's product is $a - p_i$ if $p_i < p_j$, is 0 if $p_i > p_j$, and is $\frac{a-p_i}{2}$ if $p_i = p_j$; marginal costs are $c < a$. Consider the infinitely repeated game baed on this stage game. Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame-perfect Nash equilibrium if and only if $\delta \geq \frac{1}{2}$.

4. (From 2004 Midterm II) Below, there are pairs of stage games and strategy profiles. For each pair, check whether the strategy profile is a subgame-perfect equilibrium of the game in which the stage game is repeated infinitely many times. Each agent tries to maximize the discounted sum of his expected payoffs in the stage game, and the discount rate is $\delta = 0.99$. (Clearly explain your reasoning in each case.)

   (a) (11) **Stage Game:** There are $n > 2$ players. Each player, simultaneously, decides whether to contribute \$1 to a public good production project. The amount of public good produced is $y = (x_1 + \cdots + x_n)/2$, where $x_i \in \{0, 1\}$ is the level of contribution for player i. The payoff of a player i is $y - x_i$.

   Strategy profile: Each player contributes, choosing $x_i = 1$, if and only if the amount of public good produced at each previous date is greater than $n/4$; otherwise each chooses $x_i = 0$. (According to this strategy profile, each player contributes in the first period.)