Lecture 11
Applications of SPE & Single deviation-principle

14.12 Game Theory

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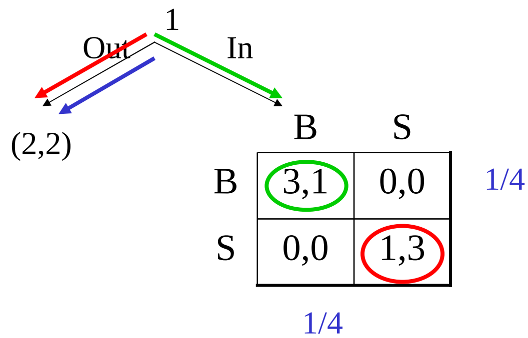
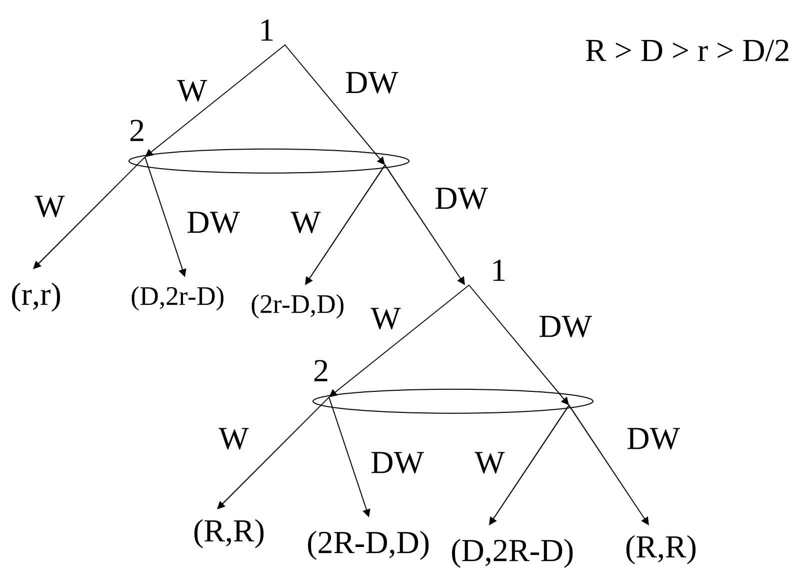
Road Map

1. Battle of the sexes with outside option
2. Bank Runs
3. Single-deviation principle
   1. Infinite-horizon bargaining
4. Quiz/Evaluations
5. Some repeated games

Battle of sexes with outside option



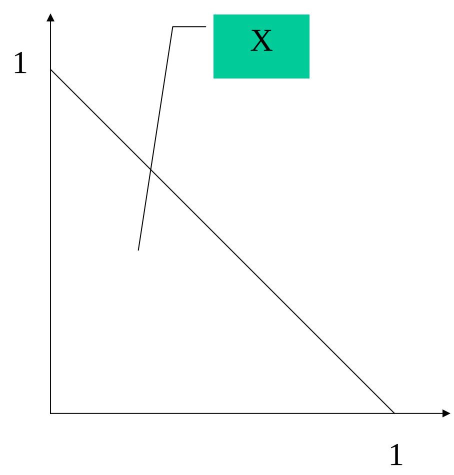
Bank Run



Single-Deviation principle

- Assume that the game is "continuous at infinity."
- $s = (s_1, s_2, \ldots, s_n)$ is a SPE
- ⬄ it passes the following test
- for each information set, where a player i moves,
  - fix the other players' strategies as in s,
  - fix the moves of i at other information sets as in s;
  - then i cannot improve her conditional payoff at the information set by deviating from s_i at the information set only.

Sequential Bargaining



- $N = \{1,2\}$
- X = feasible expected-utility pairs ($x,y \in X$)
- $U_i(x,t) = \delta_i^t x_i$
- $d = (0,0) \in X$ disagreement payoffs

Timeline – ∞ period

T = {1,2,…, n-1,n,…}

If t is odd,

- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

SPE of ∞-period bargaining

Theorem: At any t, proposer offers the other player $\delta/(1+\delta)$, keeping himself $1/(1+\delta)$, while the other player accept an offer iff he gets $\delta/(1+\delta)$.

Proof

- Single-deviation principle:
- Take any t; i offers, j accepts/rejects.
- At t+1, j will get $1/(1+\delta)$.
- Hence, it is a best response for j to accept an offer iff she gets at least $\delta/(1+\delta)$.
- Given this, i must offer $\delta/(1+\delta)$.

Quiz

Simultaneously, each student bids a number in {1,2,3}. i gets

$$U_i = 20(2+2\min_j bid_j - bid_i)$$

bid \ min	1	2	3
1	60	-	-
2	40	80	-
3	20	60	100

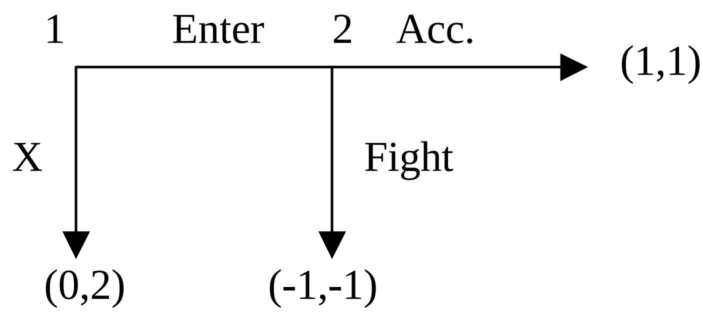
Table for the bidding game

$U_i = 20(2+2\min_j bid_j - bid_i)$

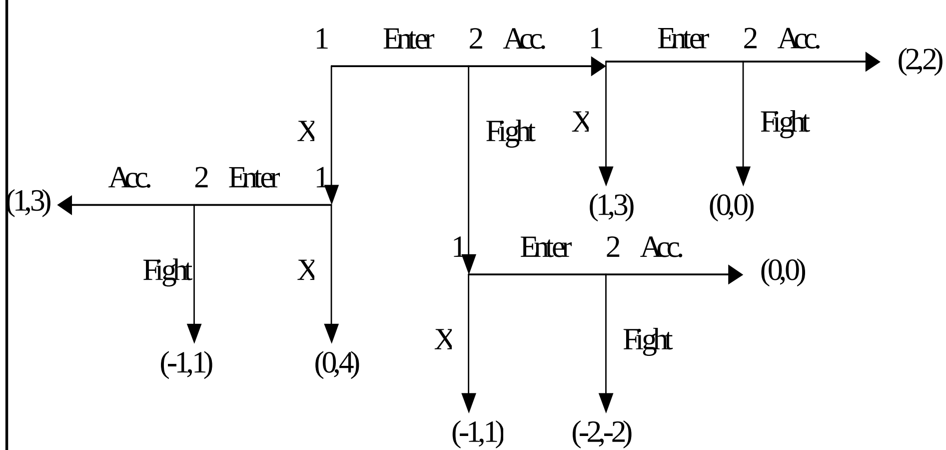
	min / bid	1	2	3
s^1	1	60	-	-
s^2	2	40	80	-
s^3	3	20	60	100

Repeated Games

Entry deterrence



Entry deterrence, repeated twice



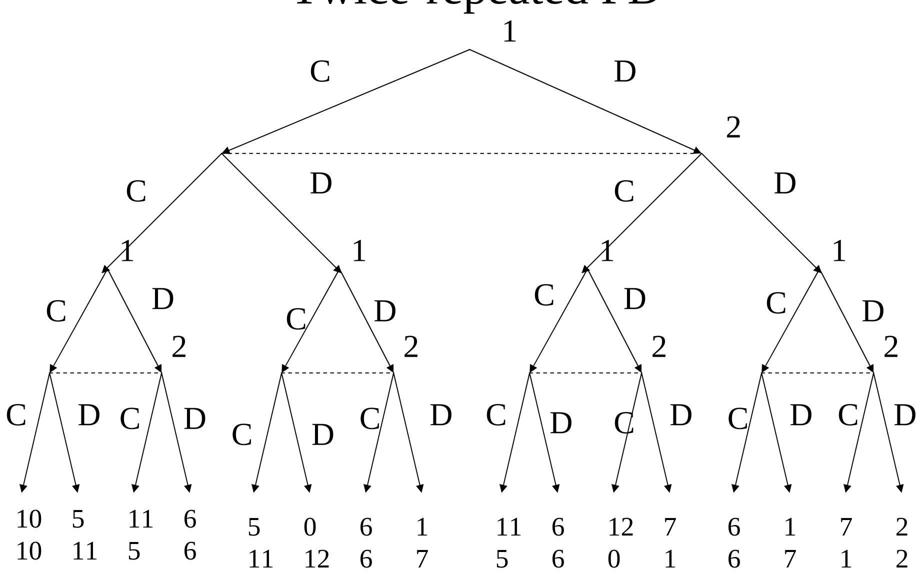
Prisoners' Dilemma, repeated twice, many times

- Two dates T = {0,1};
- At each date the prisoners' dilemma is played:

	C	D
C	5,5	0,6
D	6,0	1,1

- At the beginning of 1 players observe the strategies at 0. Payoffs= sum of stage payoffs.

Twice-repeated PD



What would happen if T = {0,1,2,…,n}?

A general result

- G = “stage game” = a finite game
- T = {0,1,…,n}
- At each t in T, G is played, and players remember which actions taken before t;
- Payoffs = Sum of payoffs in the stage game.
- Call this game G(T).

Theorem: If G has a unique subgame-perfect equilibrium s*, G(T) has a unique subgame-perfect equilibrium, in which s* is played at each stage.