Lecture 12
Repeated Games I

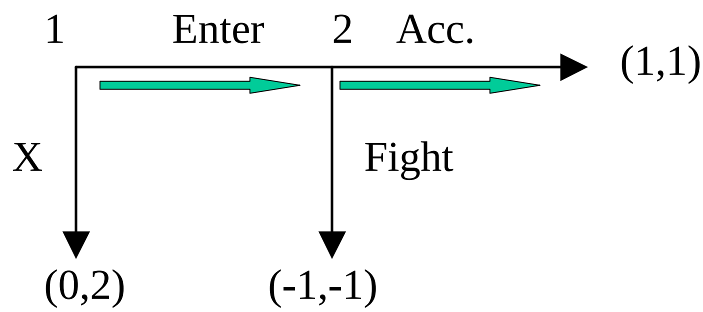
14.12 Game Theory

Muhamet Yildiz

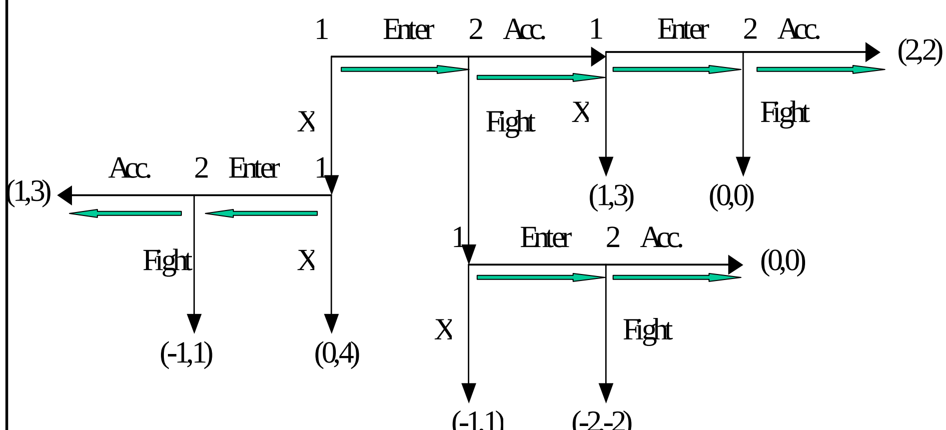
Road Map

1. Quiz
2. Finitely repeated games
    1. Entry-Deterrence/Chain-store paradox
    2. Repeated Prisoners' Dilemma
    3. A general result
    4. When there are multiple equilibria
3. Infinitely repeated games with observable actions
    1. Discounting / Present value
    2. Single-deviation principle
    3. Example

Entry deterrence

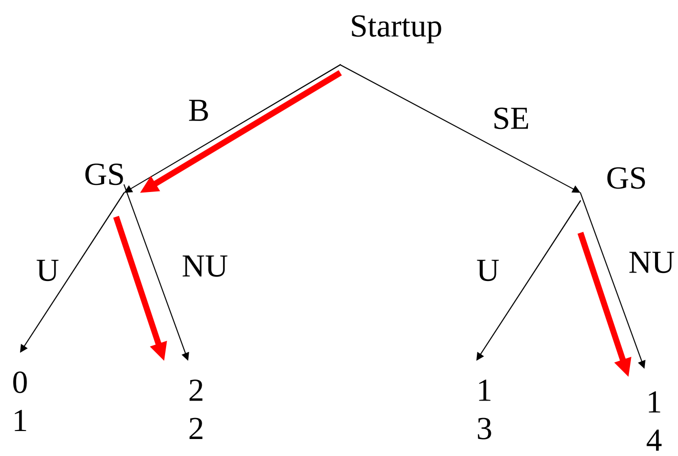


Entry deterrence, repeated twice



What would happen if there are n entrants?

Goliath Software v. Startups



What would happen if there are n startups?

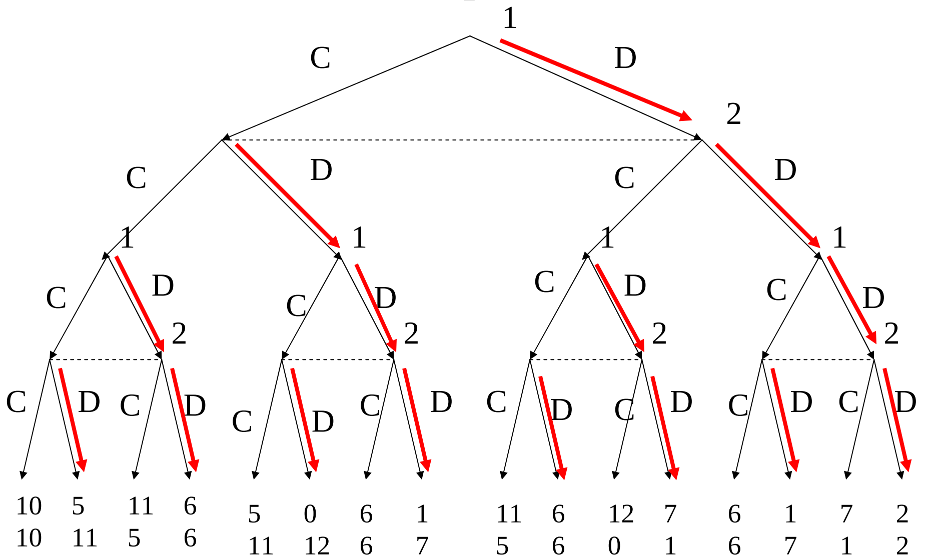
Prisoners' Dilemma, repeated twice, many times

- Two dates T = {0,1};
- At each date the prisoners' dilemma is played:

	C	D
C	5,5	0,6
D	6,0	1,1

- At the beginning of 1 players observe the strategies at 0. Payoffs= sum of stage payoffs.

Twice-repeated PD



What would happen if T = {0,1,2,…,n}?

A general result

- G = "stage game" = a finite game
- T = {0,1,…,n}
- At each t in T, G is played, and players remember which actions taken before t;
- Payoffs = Sum of payoffs in the stage game.
- Call this game G^T.

Theorem: If G has a unique subgame-perfect equilibrium s*, G^T has a unique subgame-perfect equilibrium, in which s* is played at each stage.

With multiple equilibria

T = {0,1}

1 \ 2	L	M	R
A	1,1	5,0	0,0
B	0,5	4,4	0,0
C	0,0	0,0	3,3

s* =
- At t = 0, play (B,M)
- At t = 1, play (C,R) if (B,M) at t = 0, play (A,L) otherwise.

	L	M	R
A	2,2	6,1	1,1
B	1,6	7,7	1,1
C	1,1	1,1	4,4

Can you see on the path of SPE?

T={0,1}

	L	M	R
A	1,1	5,0	0,0
B	0,5	4,4	0,0
C	0,0	0,0	3,3

- (B,M) (B,M) NO
- (B,M) (A,L) NO
- (B,L) (C,R) YES
- (C,L) (C,R) NO
- Take T={0,1,2}
- (C,L) (B,M) (C,R) YES

Infinitely repeated Games with observable actions

- T = {0,1,2,…,t,…}
- G = "stage game" = a finite game
- At each t in T, G is played, and players remember which actions taken before t;
- Payoffs = Discounted sum of payoffs in the stage game.
- Call this game G(T).

Definitions

The *Present Value* of a given payoff stream $\pi = (\pi_0, \pi_1, \ldots, \pi_t, \ldots)$ is

$$PV(\pi;\delta) = \pi_0 + \delta\pi_1 + \ldots + \delta^t\pi_t + \ldots$$

The *Average Value* of a given payoff stream π is

$$(1-\delta)PV(\pi;\delta) = (1-\delta)(\pi_0 + \delta\pi_1 + \ldots + \delta^t\pi_t + \ldots)$$

The *Present Value* of a given payoff stream π *at* t is

$$PV_t(\pi;\delta) = \pi_t + \delta\pi_{t+1} + \ldots + \delta^s\pi_{t+s} + \ldots$$

A *history* is a sequence of past observed plays

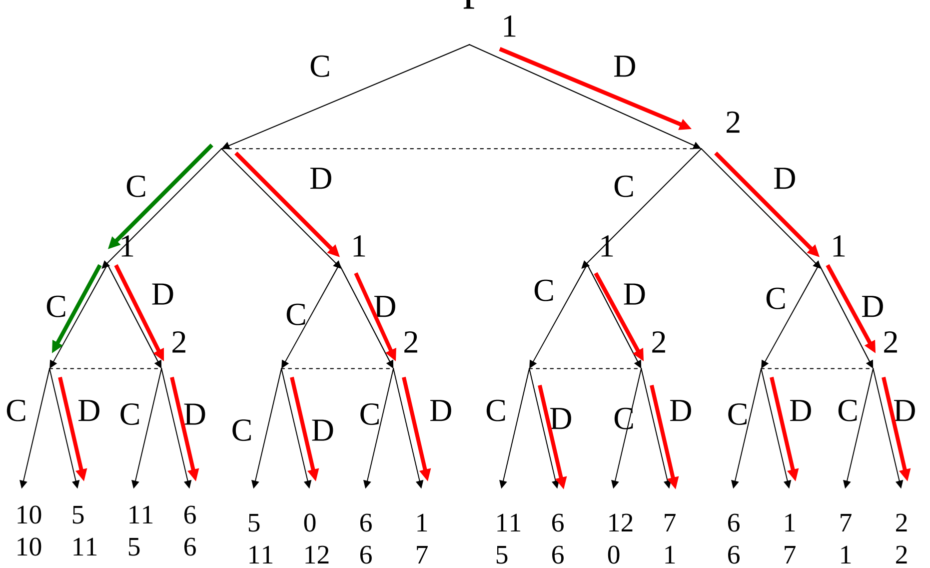
e.g. (C,D), (C,C), (D,D), (D,D) (C,C)

Single-deviation principle

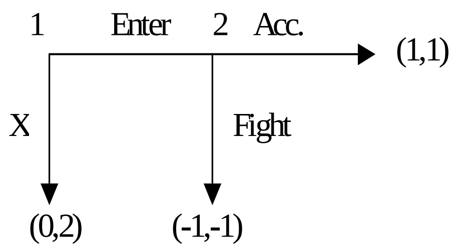
- $s = (s_1, s_2, \ldots, s_n)$ is a SPE
- ⇔ it passes the following test
- At each history and for each player i, assume
  - everybody will follow s from tomorrow on,
  - everybody except for i plays according to s today;

  then i does not have an incentive to deviate today.

Twice-repeated PD



Infinite-period entry deterrence



Strategy of Entrant:
Enter iff Accomodated before.

Strategy of Incumbent:
Accommodate iff accomodated before.

Incumbent:

- V(Acc.) = $V_A = 1/(1-\delta)$;
- V(Fight) = $V_F = 2/(1-\delta)$;
- Case 1: Accommodated before.
  - Fight => $-1 + \delta V_A$
  - Acc. => $1 + \delta V_A$.
- Case 2: Not Accommodated
  - Fight => $-1 + \delta V_F$
  - Acc. => $1 + \delta V_A$
  - Fight $\Leftrightarrow -1 + \delta V_F \geq 1 + \delta V_A$
  $\Leftrightarrow V_F - V_A = 1/(1-\delta) \geq 2/\delta$
  $\Leftrightarrow \delta \geq 2/3$.

Entrant:

- Accommodated
  - Enter => $1+V_{AE}$
  - X => $0 + V_{AE}$
- Not Acc.
  - Enter => $-1+V_{FE}$
  - X => $0 + V_{FE}$