Lecture 6
Applications of Rationalizability &Nash Equilibrium

14.12 Game Theory
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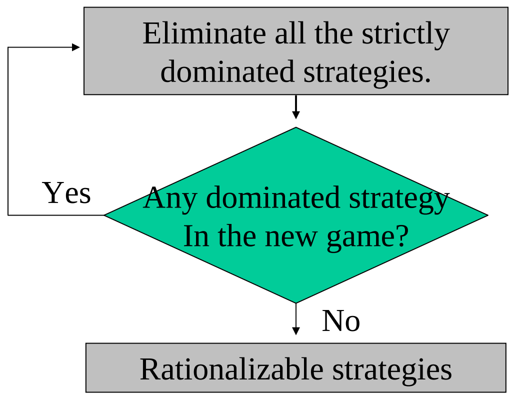
Road Map

1. Summary
2. Cournot Competition
3. Quiz
4. Simplified price competition
5. Two common games
6. Partnership Games
7. Mixed-strategy Nash Equilibrium

Rationalizability



Nash Equilibrium

Definition: A strategy-profile s* =(s_1*,…,s_n*) is a **Nash Equilibrium** iff, for each player i, and for each strategy s_i, we have

$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*) \geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*),$$

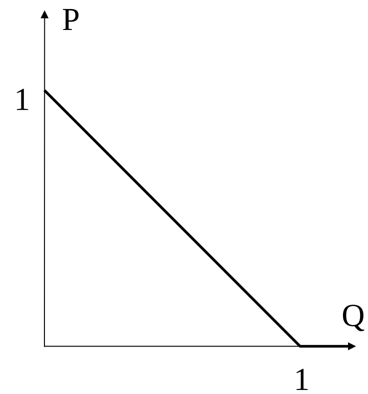
i.e., no player has any incentive to deviate if he knows what the others play.

Cournot Oligopoly

- N = {1,2,…,n} firms;
- Simultaneously, each firm i produces q_i units of a good at marginal cost c,
- and sells the good at price

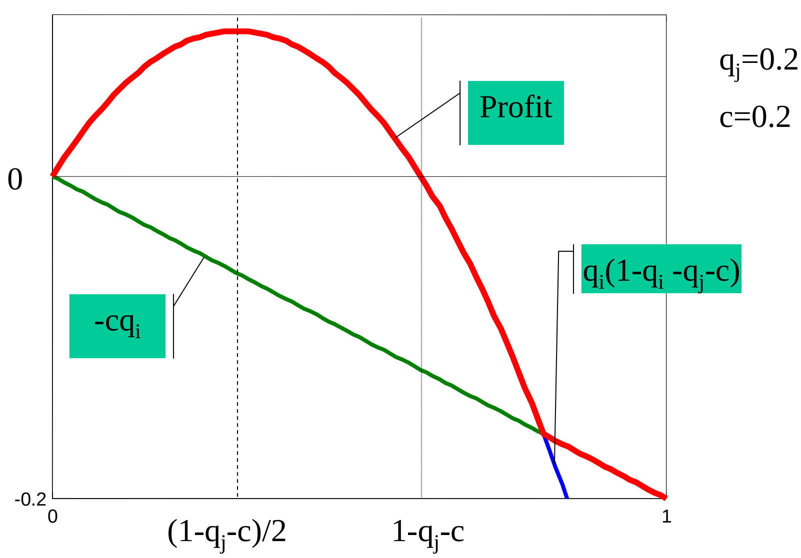
  $$P = \max\{0, 1-Q\}$$

  where $Q = q_1+\ldots+q_n$.
- Game = $(S_1,\ldots,S_n; \pi_1,\ldots,\pi_n)$ where $S_i = [0,\infty)$,



$$\pi_i(q_1,\ldots,q_n) = \begin{cases} q_i[1-(q_1+\ldots+q_n)-c] & \text{if } q_1+\ldots+q_n < 1, \\ -q_i c & \text{otherwise.} \end{cases}$$

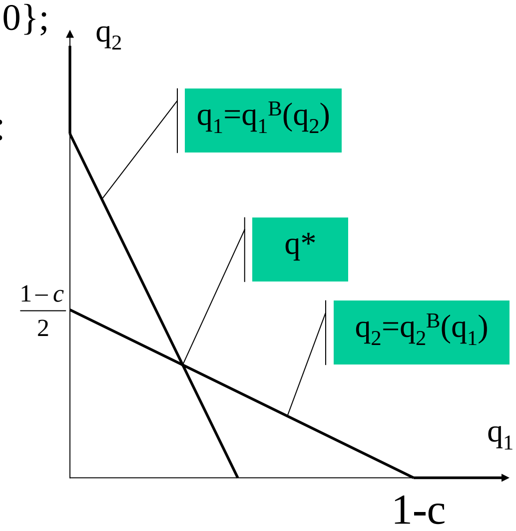
Cournot Duopoly -- profit



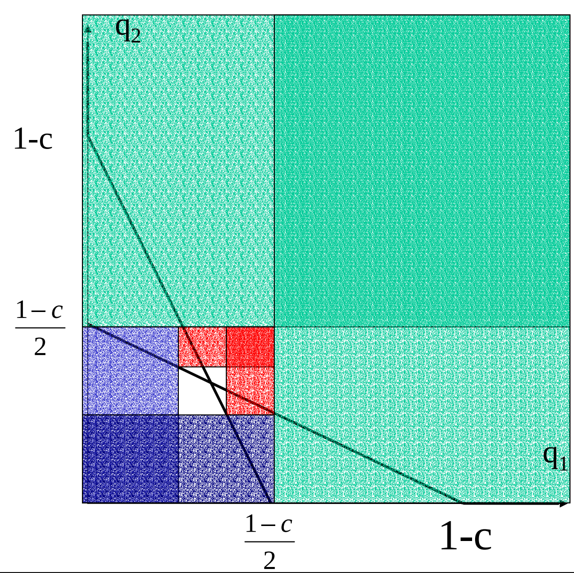
C-D – best responses

$q_i^B(q_j) = \max\{(1-q_j-c)/2,0\};$

- Nash Equilibrium q*:
  - $q_1^* = (1-q_2^*-c)/2;$
  - $q_2^* = (1-q_1^*-c)/2;$
- $q_1^* = q_2^* = (1-c)/3$



Rationalizability in Cournot Duopoly



Rationalizability in Cournot duopoly

- If i knows that $q_j \leq q$, then $q_i \geq (1-c-q)/2$.
- If i knows that $q_j \geq q$, then $q_i \leq (1-c-q)/2$.
- We know that $q_j \geq q^0 = 0$.
- Then, $q_i \leq q^1 = (1-c-q^0)/2 = (1-c)/2$ for each i;
- Then, $q_i \geq q^2 = (1-c-q^1)/2 = (1-c)(1-1/2)/2$ for each i;
- …
- Then, $q^n \leq q_i \leq q^{n+1}$ or $q^{n+1} \leq q_i \leq q^n$ where
  $q^{n+1} = (1-c-q^n)/2 = (1-c)(1-1/2+1/4-\ldots+(-1/2)^n)/2$.
- As $n \to \infty$, $q^n \to (1-c)/3$.

Rationalizability in Cournot oligopoly

1. n = 3 is not very helpful!!!
2. Everybody is rational
3. => $q_i \leq (1-c)/2$;
4. Everybody is rational and knows 2
5. => $q_i \geq 0$
6. Everybody is rational and knows 4
7. => $q_i \leq (1-c)/2$;
8. Everybody is rational and knows 6
9. => $q_i \geq 0$

Cournot Oligopoly --Equilibrium

- $q>1-c$ is strictly dominated, so $q \le 1-c$.
- $\pi_i(q_1,\ldots,q_n) = q_i[1-(q_1+\ldots+q_n)-c]$ for each i.
- FOC: $$\left.\frac{\partial \pi_i(q_1,\ldots,q_n)}{\partial q_i}\right|_{q=q^*} = \left.\frac{\partial [q_i(1-q_1-\cdots-q_n-c)]}{\partial q_i}\right|_{q=q^*}$$ $$=(1-q_1^*-\cdots-q_n^*-c)-q_i^*=0.$$
- That is, $$\begin{aligned} 2q_1^*+q_2^*+\cdots+q_n^* &= 1-c \\ q_1^*+2q_2^*+\cdots+q_n^* &= 1-c \\ &\vdots \\ q_1^*+q_2^*+\cdots+2q_n^* &= 1-c \end{aligned}$$
- Therefore, $q_1^*=\ldots=q_n^*=(1-c)/(n+1)$.

Simplified price-competition

- 2 firms in a market with 2 units of demand
- Firms simultaneously choose prices p_1 and p_2;
- A price can be low (4), medium (5) or high (6).
- The firm with the lower price sell 2 units;
- if the prices are equal, each sell 1 unit.

Simplified price-competition

Firm 1 \ Firm 2	High	Medium	Low
High	6,6	0,10	0,8
Medium	10,0	5,5	0,8
Low	8,0	8,0	4,4

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