Lecture 7
Mixed-Strategy Nash Equilibrium

14.12 Game Theory
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Road Map

1. Some 2x2 games
2. Mixed-strategy Nash Equilibrium
3. Quiz
4. Applications and examples:
   1. Price competition with costly search
   2. Dove-Hawk
   3. Matching Penny

Hawk-Dove game




	$\left(\frac{V-c}{2},\frac{V-c}{2}\right)$	$(V,0)$
	$(0,V)$	$(V/2,V/2)$

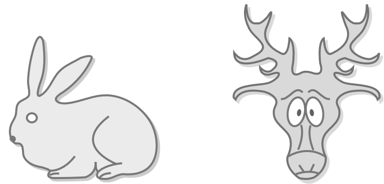
Figures by MIT OCW.

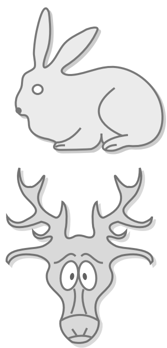
Last week's Quiz Question

Without talking to anyone, each student is to write either INVEST or CONSUME on a piece of paper. TAs will randomly match students. Your grade:

You \ Your Partner	INVEST	CONSUME
INVEST	100	0
CONSUME	60	60

Stag Hunt





	(2,2)	(2,0)
	(0,2)	(3,3)

Figures by MIT OCW.

Equilibrium in Mixed Strategies

What is a strategy?

- A complete contingent-plan of a player.
- What the others think the player might do under various contingency.

What do we mean by a mixed strategy?

- The player is randomly choosing his pure strategies.
- The other players are not certain about what he will do.

Mixed-strategy Nash Equilibrium

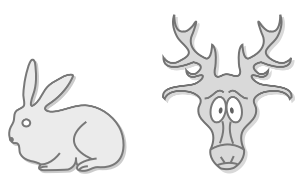
Assume that players choose mixed strategies. A mixed strategy profile $(\sigma_1^*,\ldots,\sigma_n^*)$ is a Nash equilibrium iff for each player i, and each σ_i,

$$\Sigma_s\ \sigma_1^*(s_1)u_i(s)\ \ldots\sigma_n^*(s_n)u_i(s) \geq$$

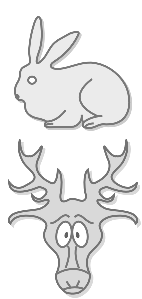
$$\Sigma_s\ \sigma_1^*(s_1)u_i(s)\ \ldots\ \sigma_i(s_i)u_i(s)\ \ldots\ \sigma_n^*(s_n)u_i(s).$$

Equivalently, for each i and $\sigma_i^*(s_i) > 0$, s_i is at least as good a best response as any s_i' to σ_{-i}^*.

Stag Hunt



Assume: Player 2 thinks that, with probability p, Player 1 targets for Rabbit.



(2,2)	(2,0)	p
(0,2)	(3,3)	1-p

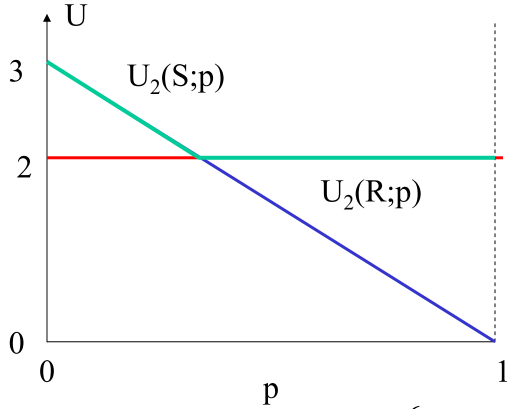
His payoff from targeting Rabbit: $U_2(R;p) =$.

His payoff from targeting Stag: $U_2(S;p) =$

She is indifferent iff

Figures by MIT OCW.

Mixed-strategy equilibrium in Stag-Hunt game



$$q^{BR}(p) = \begin{cases} 0 & \text{if } p < 1/3 \\ q \in [0,1] & \text{if } p = 1/3 \\ 1 & \text{if } p > 1/3 \end{cases}$$

Best responses in Stag-Hunt game