

# Lecture 7

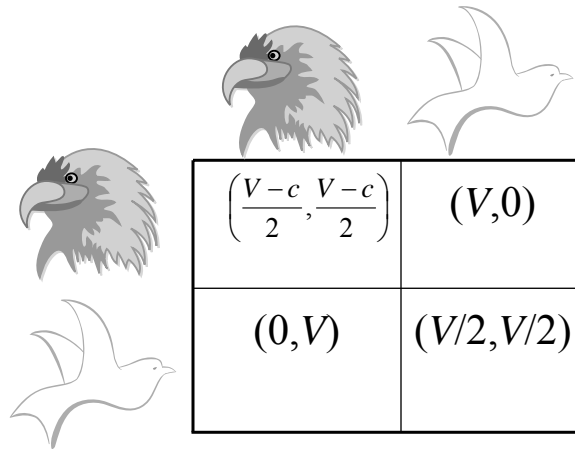
## Mixed-Strategy Nash Equilibrium

14.12 Game Theory  
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### Road Map

1. Some 2x2 games
2. Mixed-strategy Nash Equilibrium
3. Quiz
4. Applications and examples:
  1. Price competition with costly search
  2. Dove-Hawk
  3. Matching Penny

## Hawk-Dove game



The diagram shows a 2x2 payoff matrix for the Hawk-Dove game. The rows represent the strategies of the first player (Hawk or Dove), and the columns represent the strategies of the second player (Hawk or Dove). The payoffs are given as (Player 1, Player 2). The cost of a fight is denoted by  $c$ .

	$\left(\frac{V-c}{2}, \frac{V-c}{2}\right)$	$(V, 0)$
	$(0, V)$	$(V/2, V/2)$

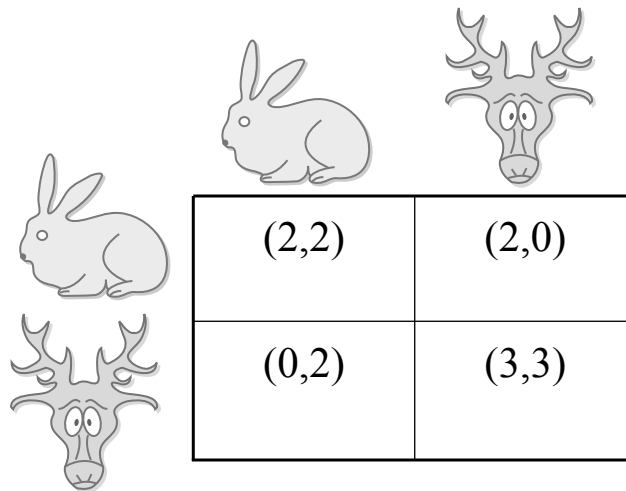
Figures by MIT OCW.

## Last week's Quiz Question





Without talking to anyone, each student is to write either INVEST or CONSUME on a piece of paper. TAs will randomly match students. Your grade:

You \ Your Partner	INVEST	CONSUME
INVEST	100	0
CONSUME	60	60

## Stag Hunt



A 2x2 payoff matrix for the Stag Hunt game. The rows represent the strategies of Player 1 (Rabbit or Stag) and the columns represent the strategies of Player 2 (Rabbit or Stag). The payoffs are shown in the cells of the matrix.

		
	(2,2)	(2,0)
	(0,2)	(3,3)

Figures by MIT OCW.

## Equilibrium in Mixed Strategies

What is a strategy?

- A complete contingent-plan of a player.
- What the others think the player might do under various contingency.

What do we mean by a mixed strategy?

- The player is randomly choosing his pure strategies.
- The other players are not certain about what he will do.





## Mixed-strategy Nash Equilibrium

Assume that players choose mixed strategies. A mixed strategy profile  $(\sigma_1^*, \dots, \sigma_n^*)$  is a Nash equilibrium iff for each player  $i$ , and each  $\sigma_i$ ,

$$\sum_s \sigma_1^*(s_1)u_i(s) \dots \sigma_n^*(s_n)u_i(s) \geq \sum_s \sigma_1^*(s_1)u_i(s) \dots \sigma_i(s_i)u_i(s) \dots \sigma_n^*(s_n)u_i(s).$$

Equivalently, for each  $i$  and  $\sigma_i^*(s_i) > 0$ ,  $s_i$  is at least as good a best response as any  $s_i'$  to  $\sigma_{-i}^*$ .

## Stag Hunt

			
	(2,2)	(2,0)	p
	(0,2)	(3,3)	1-p

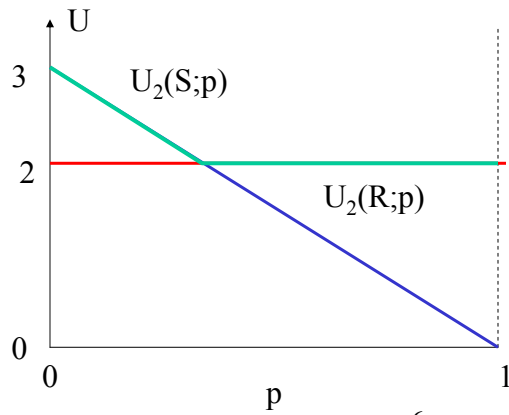
Assume: Player 2 thinks that, with probability  $p$ , Player 1 targets for Rabbit.

His payoff from targeting Rabbit:  $U_2(R;p) =$       His payoff from targeting Stag:  $U_2(S;p) =$

She is indifferent iff

Figures by MIT OCW.

## Mixed-strategy equilibrium in Stag-Hunt game



$$q^{BR}(p) = \begin{cases} 0 & \text{if } p < 1/3 \\ q \in [0,1] & \text{if } p = 1/3 \\ 1 & \text{if } p > 1/3 \end{cases}$$

## Best responses in Stag-Hunt game

