Lecture 8
Backwards Induction

14.12 Game Theory

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Road Map

1. Bertrand competition with costly search
2. Quiz
3. Backward Induction
4. Agenda Setting
5. Stackelberg Competition
6. Sequential Bargaining
7. Pre-trial Negotiations

Bertrand Competition with costly search

- $N = \{F1,F2,B\}$; F1, F2 are firms; B is buyer
- B needs 1 unit of good, worth 6;
- Firms sell the good; Marginal cost = 0.
- Possible prices $P = \{3,5\}$.
- Buyer can check the prices with a small cost $c > 0$.

Game:

1. Each firm i chooses price p_i;
2. B decides whether to check the prices;
3. (Given) If he checks the prices, and $p_1 \neq p_2$, he buys the cheaper one; otherwise, he buys from any of the firm with probability ½.

Bertrand Competition with costly search

Check

F1 \ F2	High	Low
High	5/2 5/2 1-c	0 3 3-c
Low	3 0 3-c	3/2 3/2 3-c

Don't Check

F1 \ F2	High	Low
High	5/2 5/2 1	5/2 3/2 2
Low	3/2 5/2 2	3/2 3/2 3

Mixed-strategy equilibrium

- Symmetric equilibrium: Each firm charges "High" with probability q;
- Buyer Checks with probability r.
- $U(\text{check};q) = q^2 1 + (1-q^2)3 - c = 3 - 2q^2 - c;$
- $U(\text{Don't};q) = q1 + (1-q)3 = 3 - 2q;$
- Indifference: $2q(1-q) = c$; i.e.,
- $U(\text{high};q,r) = (1-r(1-q))5/2;$
- $U(\text{low};q,r) = qr3 + (1-qr)3/2$
- Indifference: $r = 2/(5-2q)$.

Dynamic Games of Perfect Information
&
Backward Induction

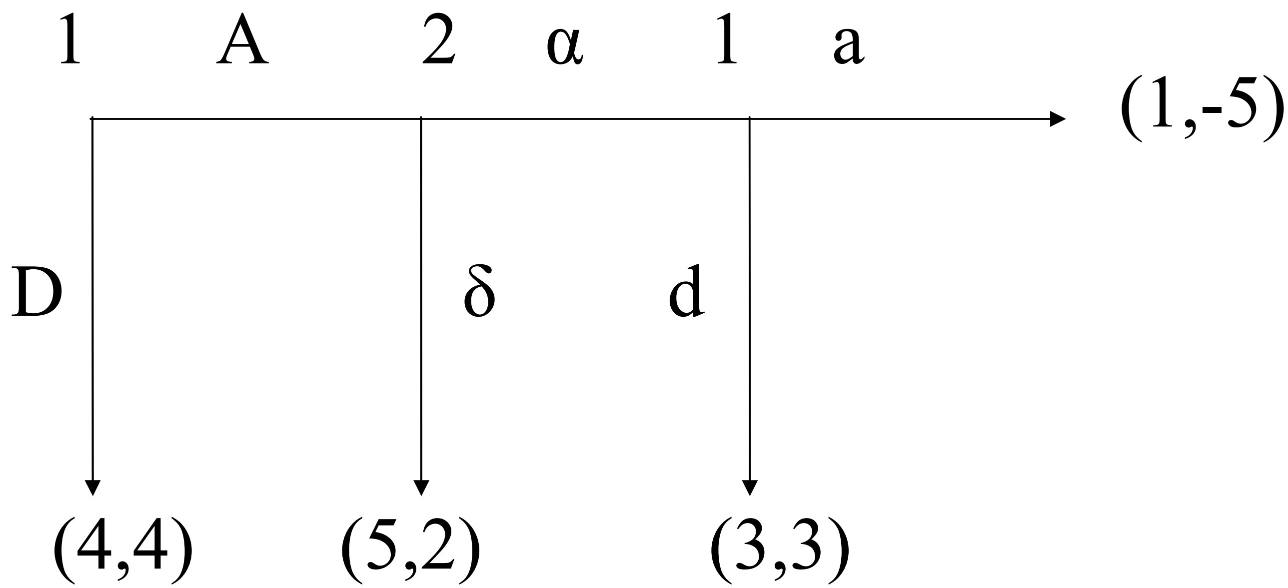
Definitions

Perfect-Information game is a game in which all the information sets are singleton.

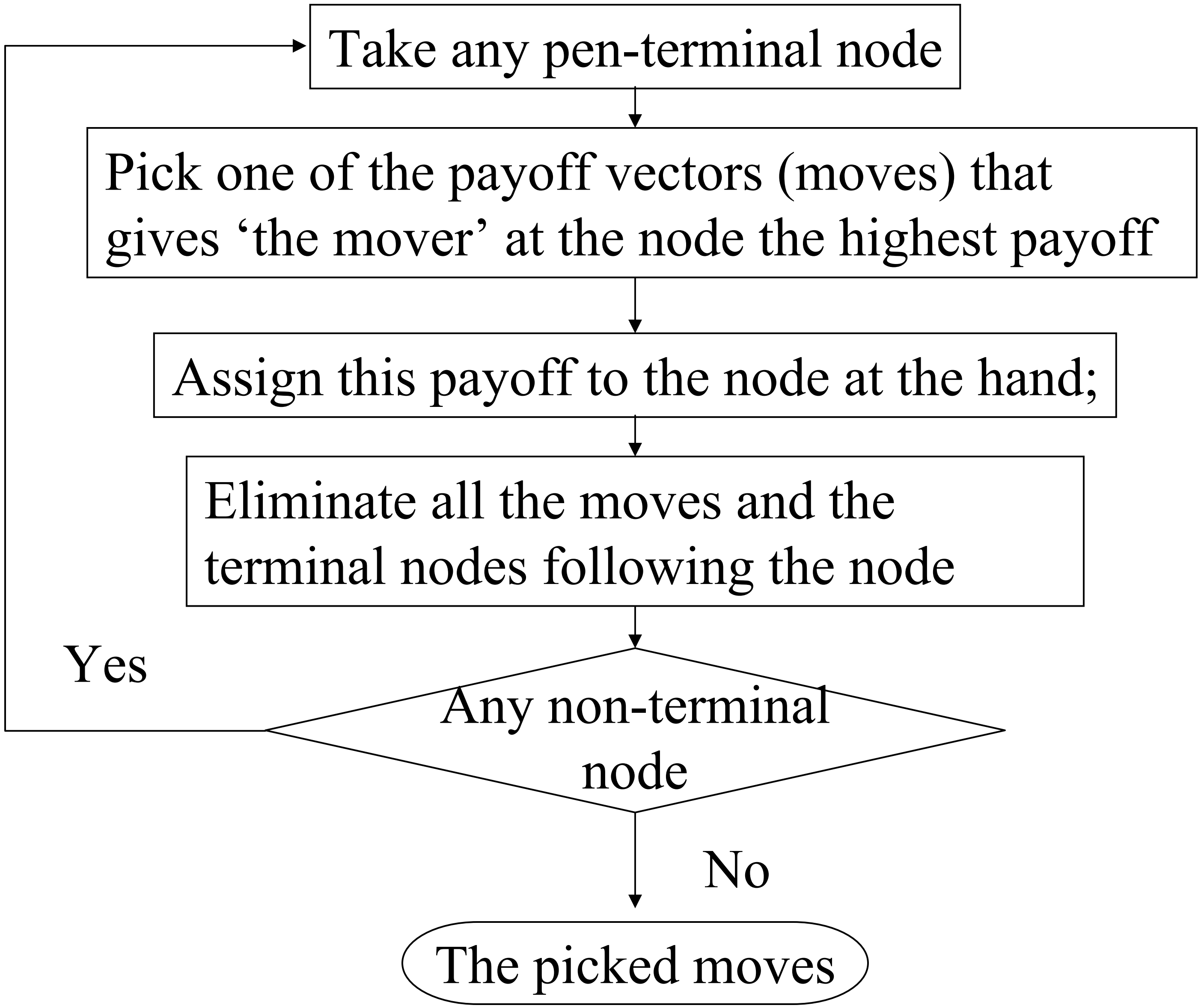
Sequential Rationality: A player is sequentially rational iff, at each node he is to move, he maximizes his expected utility conditional on that he is at the node – even if this node is precluded by his own strategy.

In a finite game of perfect information, the "common knowledge" of sequential rationality in future gives **"Backward Induction"** outcome.

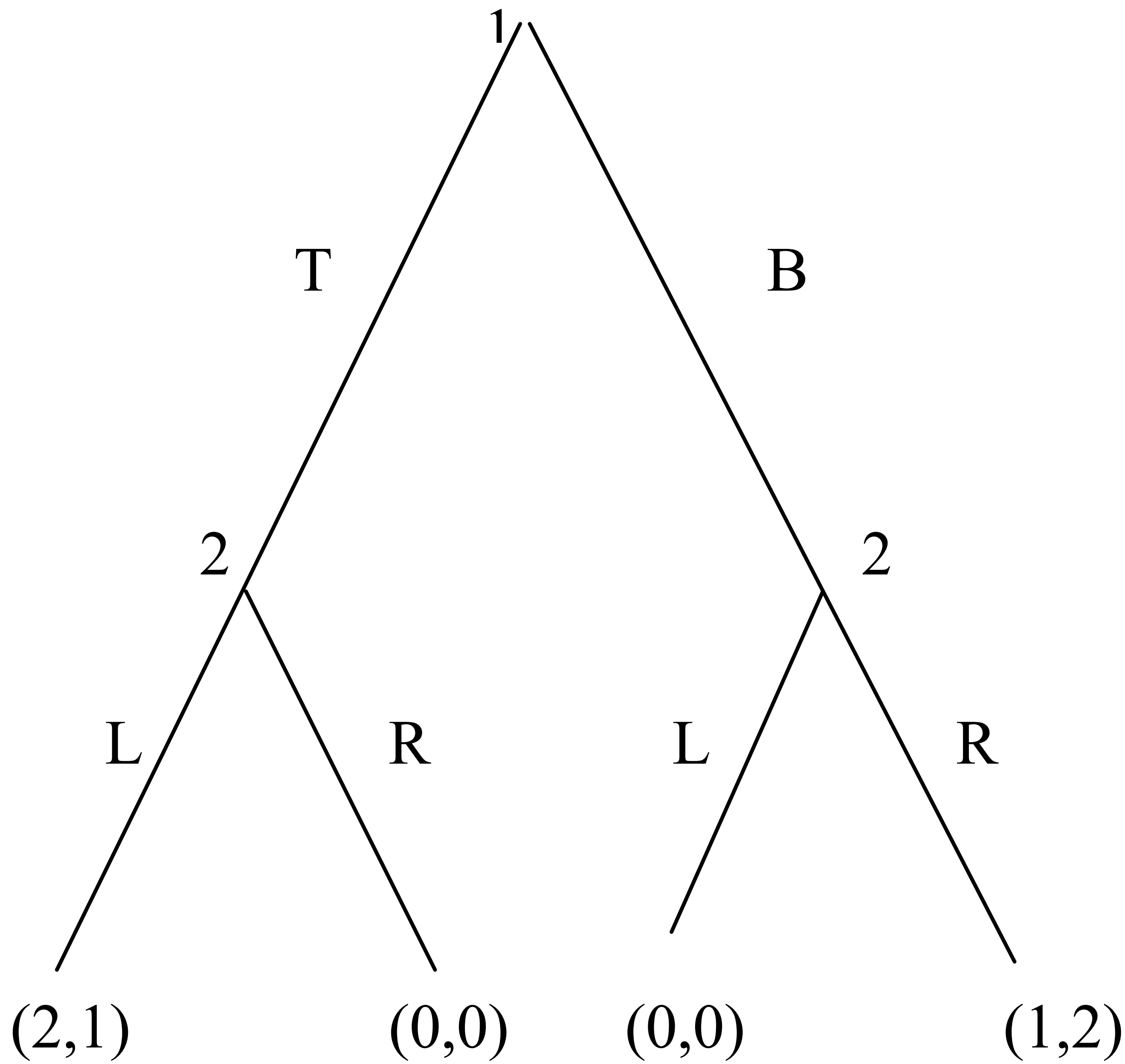
A game



Backward Induction



Battle of The Sexes with perfect information



Note

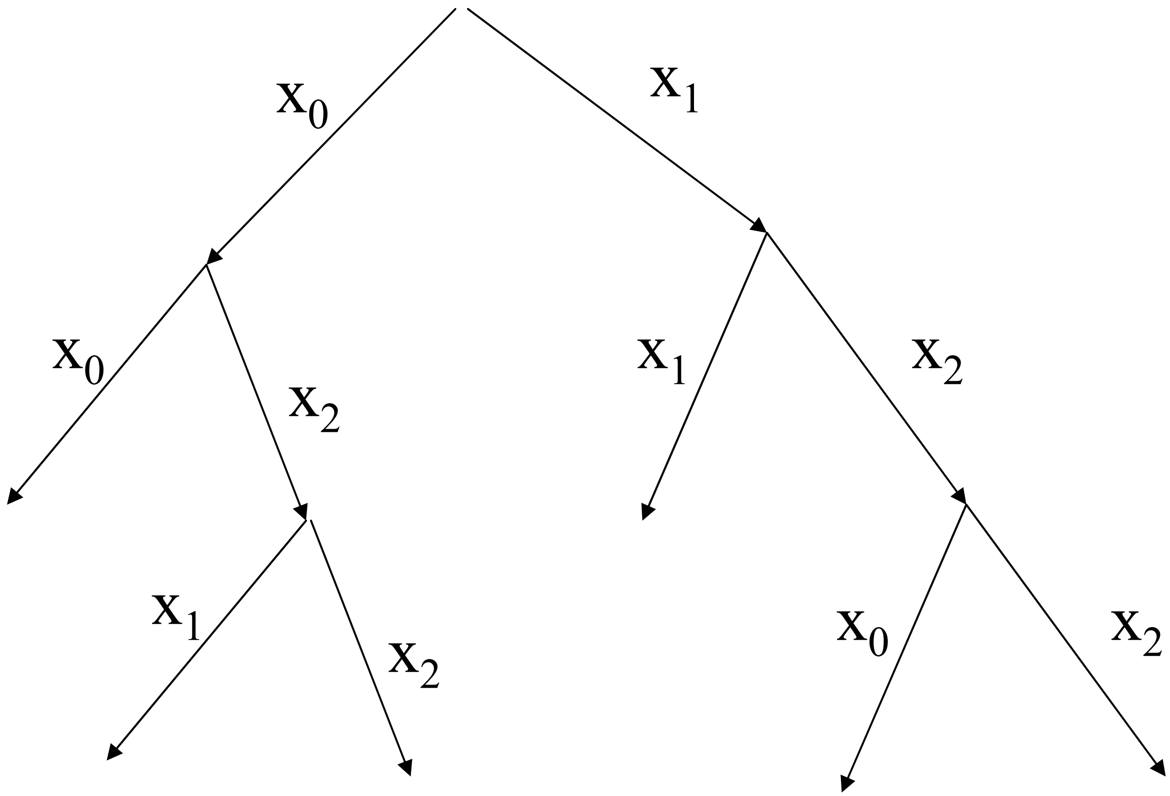
- There are Nash equilibria that are different from the Backward Induction outcome.
- Backward Induction always yields a Nash Equilibrium.
- Sequential rationality is stronger than rationality.

Agenda Setting

Voting with a fixed agenda

1. 2n+1 players
2. Alternatives: $x_0, x_1, \ldots, x_m$
3. Each player i has a fixed strict preference about alternatives: $x_{i0} >_i x_{i1} >_i \ldots >_i x_{im}$
4. There is a fixed binary agenda.
5. Assume: everything above is common knowledge

A binary agenda



A preference profile

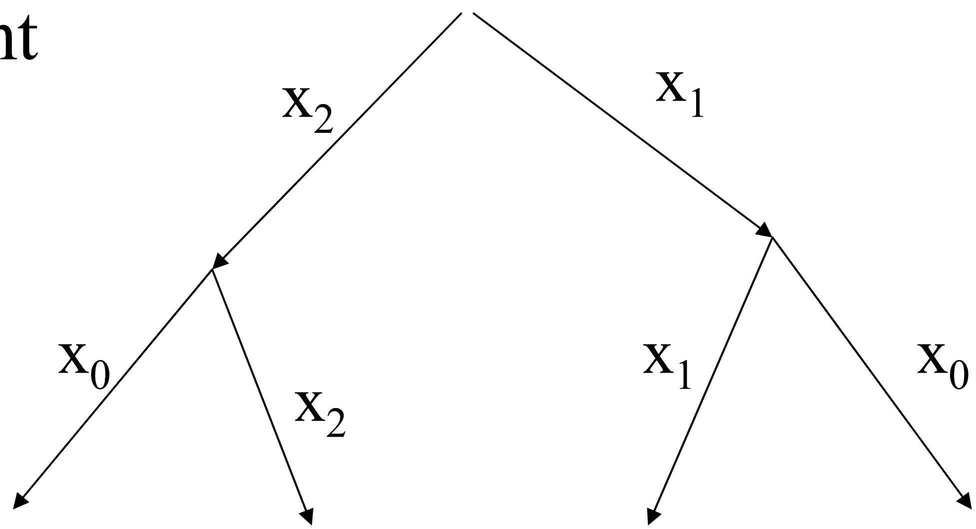
1	2	3
x_0	x_2	x_1
x_1	x_0	x_2
x_2	x_1	x_0

Naïve Voting

Sophisticated Voters

17th Amendment

- x_0 = status quo
- x_1 = 17th amendment
- x_2 = DePew Amendment



Preference profile

1	2	3
x_0	x_2	x_1
x_2	x_1	x_0
x_1	x_0	x_2

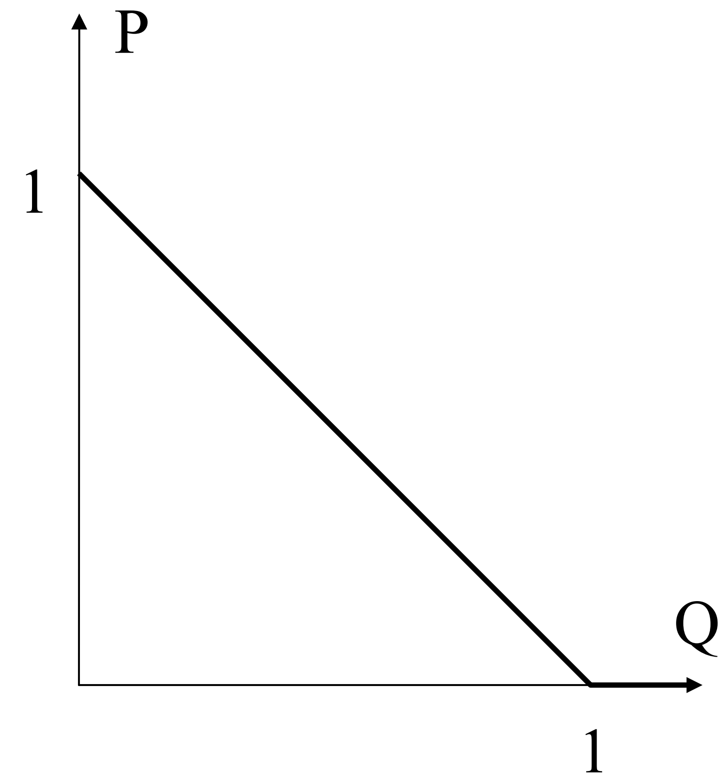
Stackelberg Duopoly

Game:

$N = \{1,2\}$ firms w $MC = 0$;

1. Firm 1 produces q_1 units
2. Observing q_1, Firm 2 produces q_2 units
3. Each sells the good at price

$$P = \max\{0, 1-(q_1+q_2)\}.$$



$$\pi_i(q_1, q_2) = \begin{cases} q_i[1-(q_1+q_2)] & \text{if } q_1 + q_2 < 1, \\ 0 & \text{otherwise.} \end{cases}$$

“Stackelberg equilibrium”

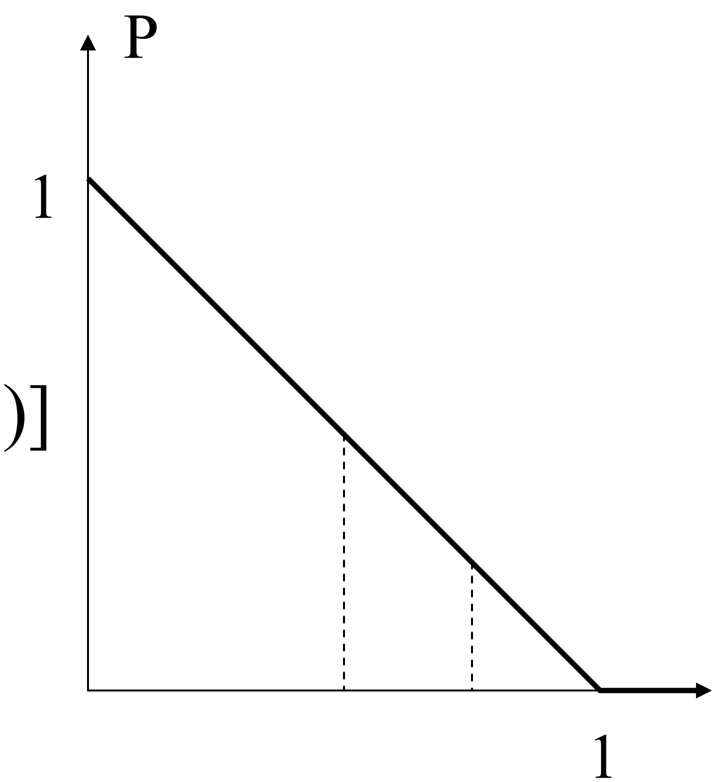
- If $q_1 > 1$, $q_2^*(q_1) = 0$.
- If $q_1 \leq 1$, $q_2^*(q_1) = (1-q_1)/2$.
- Given the function q_2^*, if $q_1 \leq 1$

$$\pi_1(q_1;q_2^*(q_1)) = q_1[1-(q_1+(1-q_1)/2)]$$

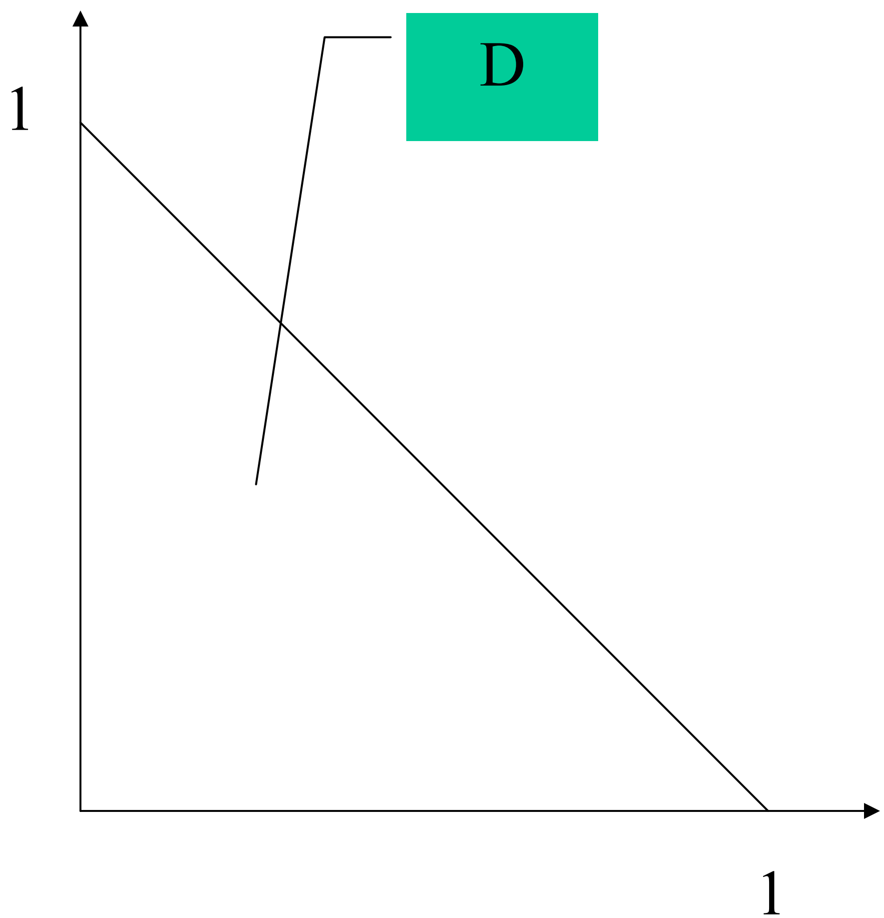
$$= q_1 (1-q_1)/2;$$

0 otherwise.

- $q_1^* = ½$.
- $q_2^*(q_1^*) = ¼$.



Sequential Bargaining



- $N = \{1,2\}$
- X = feasible expected-utility pairs ($x,y \in X$)
- $U_i(x,t) = \delta_i^t x_i$
- $d = (0,0) \in D$ disagreement payoffs

Timeline – 2n period

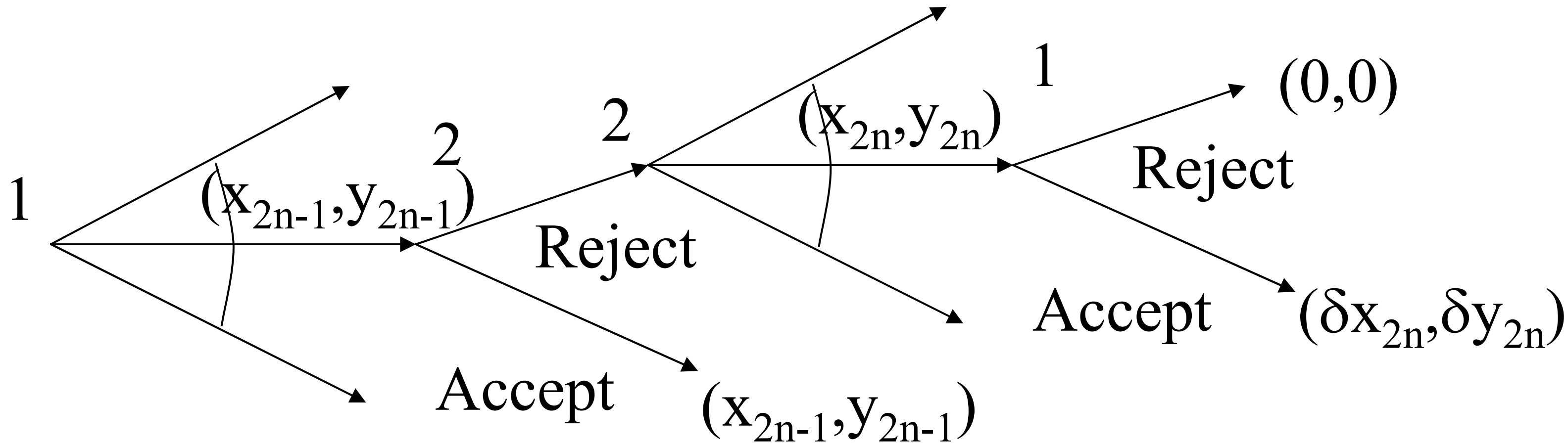
$T = \{1,2,\ldots,2n-1,2n\}$

If t is odd,

- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff (x_t,y_t),
- Otherwise, we proceed to date t+1, except at $t = 2n$, when the game end yielding $d = (0,0)$.



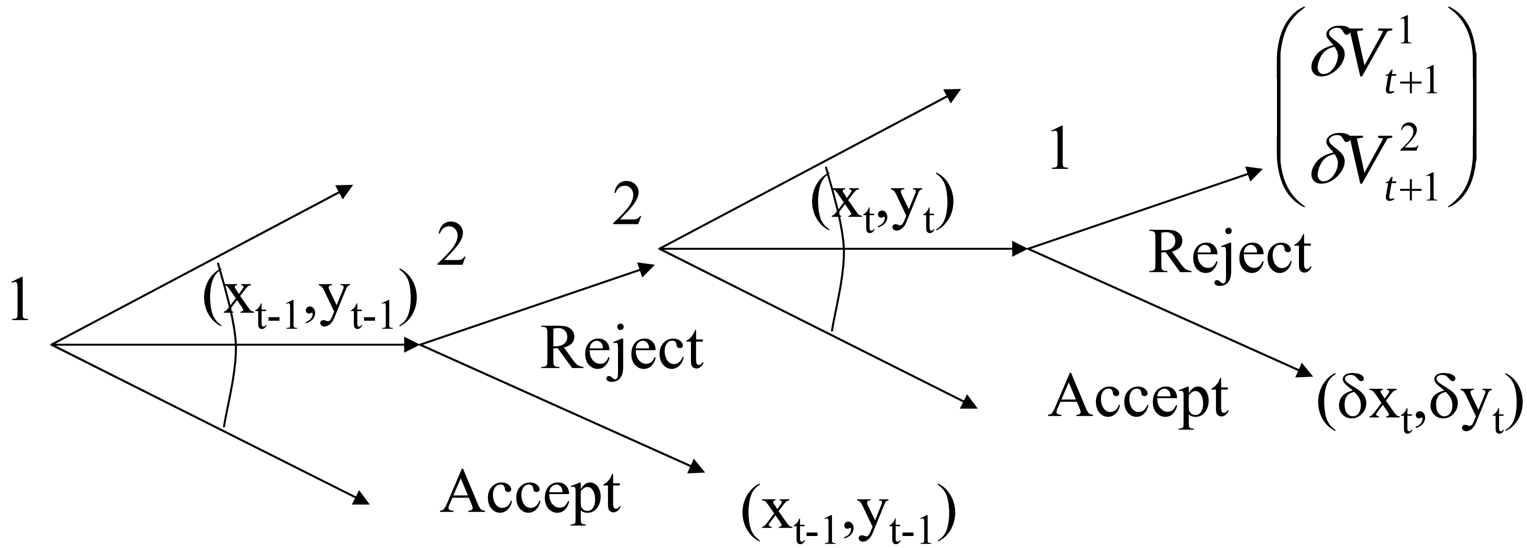
At $t = 2n-1$,

- Accept iff $x_{2n-1} \geq \delta$.
- Offer $(1-\delta,\delta)$.

At $t = 2n$,

- Accept iff $y_{2n} \geq 0$.
- Offer (0,1).

Continuation Value of i at t+1 = V_{t+1}^{i}



$$
\begin{aligned}
V_{2n-2k-1}^{1} &= 1-\delta+\delta^{2}V_{2n-2k+1}^{1} \\
&= 1-\delta+\delta^{2}\left(1-\delta\right)+\delta^{4}V_{2n-2k+3}^{1} \\
&= 1-\delta+\delta^{2}\left(1-\delta\right)+\delta^{4}\left(1-\delta\right)+\delta^{6}V_{2n-2k+5}^{1} \\
&\;\;\vdots \\
&= \left(1-\delta\right)\left(1+\delta^{2}+\delta^{4}+\cdots+\delta^{2k}\right) \\
&= \frac{1-\delta^{2k+1}}{1+\delta}
\end{aligned}
$$

Pretrial Negotiation

Model

- Players:
  - Plaintiff
  - Defendant
- In court Defendant is to pay T to Plaintiff
- Cost of court
  - C_P; C_D; $C = C_P + C_D$
- Lawyer cost per day:
  - c_p; c_d; $c = c_p + c_d$

1. The players bargain using alternating offers on dates {1,2,…,2n} offering a settlement amount s, paid by D to P.
2. On 2n+1, they go to court.

Assume: players are risk neutral and no discounting.

Backwards Induction

Date	Proposer	Settlement
2n	P	
2n-1	D	
2n-2	P	
2n-3	D	
2n-4	P	
2n-5	D	
…		
2	P	
1	D	

Graphically