Lecture 9

Applications of Backwards Induction

14.12 Game Theory

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Roadmap

- Stackelberg Competition
- Quiz
- Bargaining
- Pretrial Negotiation

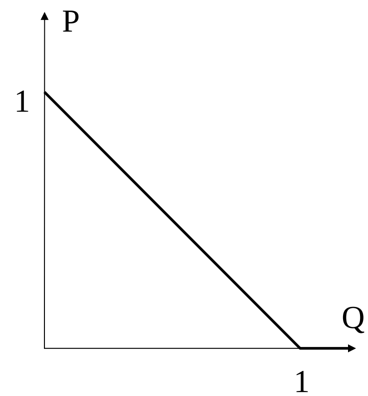
Stackelberg Duopoly

Game:

N = {1,2} firms w MC = 0;

1. Firm 1 produces q_1 units
2. Observing q_1, Firm 2 produces q_2 units
3. Each sells the good at price

   $P = \max\{0, 1-(q_1+q_2)\}$.



$\pi_i(q_1, q_2) = q_i[1-(q_1+q_2)]$ if $q_1 + q_2 < 1$,
0 otherwise.

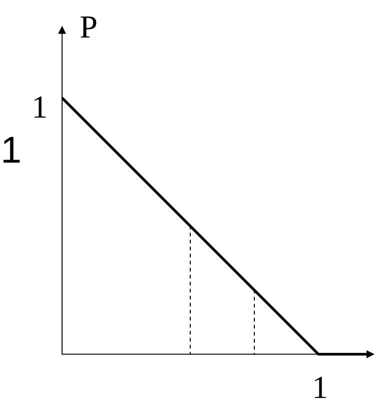
"Stackelberg equilibrium"

- If $q_1 > 1$, $q_2^*(q_1) = 0$.
- If $q_1 \leq 1$, $q_2^*(q_1) = (1-q_1)/2$.
- Given the function q_2^*, if $q_1 \leq 1$

  $\pi_1(q_1;q_2^*(q_1)) = q_1[1-(q_1+ (1-q_1)/2)]$

  $= q_1 (1-q_1)/2$;

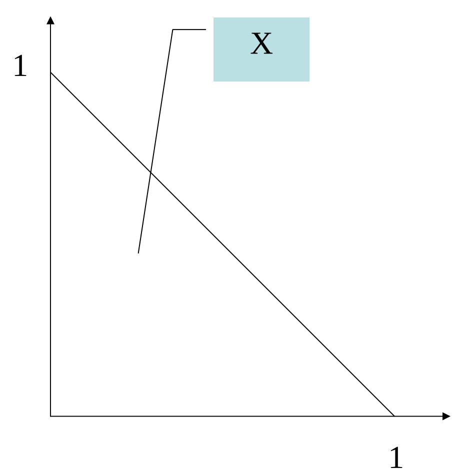
  0 otherwise.
- $q_1^* = ½$.
- $q_2^*(q_1^*) = ¼$.



Bargaining

- Buying a car, house, or shopping at a bazaar
- Wage Negotiations
- International Agreements
- Legislative Bargaining
- Litigation

Sequential Bargaining



- N = {1,2}
- X = feasible expected-utility pairs (x,y $\in$X)
- $U_i(x,t) = \delta^t x_i$
- (0,0) $\in$ X disagreement payoffs

Timeline – 2n period

$T = \{1,2,\ldots,2n-1,2n\}$

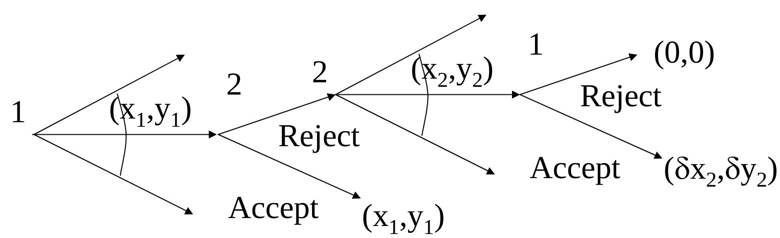
If t is odd,

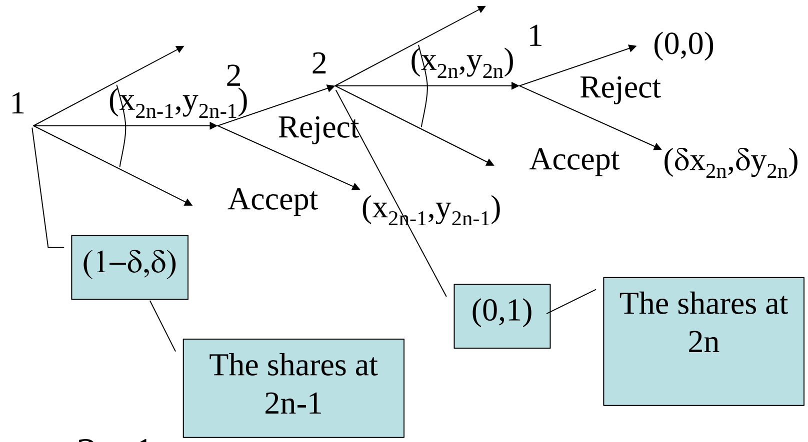
- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff (x_t,y_t),
- Otherwise, we proceed to date t+1, except at t = 2n, when the game end yielding (0,0).

n = 2





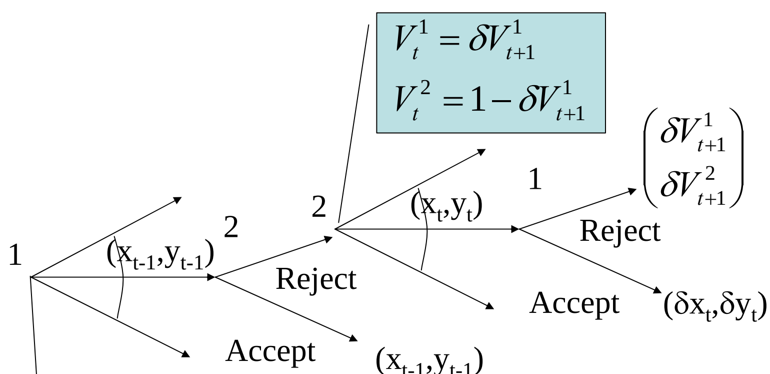
At t = 2n-1,

- Accept iff $x_{2n-1} \geq \delta$.
- Offer (1–δ,δ).

At t = 2n,

- Accept iff $y_{2n} \geq 0$.
- Offer (0,1).

The share of i at t+1 = V_{t+1}^{i}



$$V_{t-1}^{1} = 1-\delta V_t^2 = 1-\delta\left(1-\delta V_{t+1}^{1}\right) = 1-\delta+\delta^2 V_{t+1}^{1}$$

$$V_{t-1}^{2} = \delta V_{t+1}^{2}$$

Equilibrium Behavior

- The proposer makes an offer that makes the responder indifferent between accepting and rejecting the offer;
- The responder accepts the offer.
- The game end in the first round: Proposer makes an offer and the other party accepts it.

$$V^1_{2n-2k-1} = 1-\delta+\delta^2 \boxed{1-\delta+\delta^2 V^1_{2n-2k+3}}$$

$$= 1-\delta+\delta^2(1-\delta)+\delta^4 \boxed{1-\delta+\delta^2 V^1_{2n-2k+5}}$$

$$= 1-\delta+\delta^2(1-\delta)+\delta^4(1-\delta)+\delta^6 V^1_{2n-2k+5}$$

$$\vdots$$

$$= (1-\delta)(1+\delta^2+\delta^4+\cdots+\delta^{2k})$$

$$= \frac{1-\delta^{2k+1}}{1+\delta}$$

$$n \longrightarrow \infty$$

$t = 2n - 2k - 1$

$$x_t = \frac{1-\delta^{2k+1}}{1+\delta} = \frac{1-\delta^{2n-t}}{1+\delta} \xrightarrow{n\to\infty} \frac{1}{1+\delta}$$

Timeline – ∞ period

$T = \{1,2,\dots, n-1,n,\dots\}$

If t is odd,

- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff (x_t,y_t),
- Otherwise, we proceed to date t+1.

Pretrial Negotiation

Model

- Players:
  - Plaintiff
  - Defendant
- In court Defendant is to pay T to Plaintiff
- Cost of court
  - C_P; C_D; $C = C_P+C_D$
- Lawyer cost per day:
  - c_p; c_d; $c = c_p+c_d$

1. The players bargain using alternating offers on dates {1,2,…,2n} offering a settlement amount s, paid by D to P.
2. On 2n+1, they go to court.

Assume: players are risk neutral and no discounting.

Backwards Induction

Date	Proposer	Settlement
2n	P	
2n-1	D	
2n-2	P	
2n-3	D	
2n-4	P	
2n-5	D	
…		
2	P	
1	D	