THEORY OF THE VALUE-ADDED TAX

by

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MISSING PAGE 47
Prof. Robert L. Bishop  
Chairman  
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Dear Professor Bishop:  

In partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics, I hereby submit the following thesis entitled  

"Theory of the Value-Added Tax".  

William H. Oakland
In the first chapter we analyze the nature of the value-added tax and its relationship to other concepts of taxation. Our analysis in this chapter is carried out under the assumptions of a classical full employment economy. We find that the value-added tax, except in its consumption form, is not a new tax, but is equivalent to a gross income tax or a net income tax—depending upon how depreciation is treated. Furthermore, unless saving is interest-elastic, any form of direct tax (including the value-added tax) has identical output effects.

In Chapter Two we attempt to determine the relative incidence effects of a profits tax and a value-added tax within the context of a classical model. If saving is not highly interest-elastic and/or the consumption good industry is not significantly more capital intensive than the capital-good industry, the distribution of income is more unequal under the value-added than under the profits tax. This conclusion holds for both the short-run and the long-run.

In Chapter Three we abandon the classical model in favor of a Keynesian unemployment model. Our aim is to discover the short-run stability implications of a shift from a corporate income tax to a value-added tax. We find that, as long as the value-added tax is only partially shifted by the firm, the short-run stability of the economy will be reduced. If the value-added tax is borne by the factors of production in proportion to their earnings, however, this conclusion is reversed.

Chapters Four and Five are devoted to an estimate of the impact of the tax substitution upon the timing and level of investment expenditures. Depending upon which set of shifting assumptions we choose to make investment may increase or decrease. The more likely case, however, is that investment will decrease. Finally, in the last chapter we summarize our findings and make some attempt to appraise the desirability of substituting a value-added tax for a corporate profits tax. Our conclusion is that this tax substitution should not be made.
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CHAPTER I

THE VALUE-ADDED TAX AND ITS RELATIONSHIP
TO OTHER FORMS OF TAXATION

INTRODUCTION

The existing literature on the value-added tax is excellent in its institutional content but is almost devoid of theoretical analysis. This is a bit surprising since the concept of value-added taxation is not new—it was first advanced in 1921 by T.S. Adams. However, a thorough theoretical analysis of a tax often follows its imposition; it is only recently that the value-added tax has been put into effect.

Before delving into theoretical matters, it would perhaps be wise to offer a brief description of the history and development of the concept of value-added taxation. The original interest in value-added taxation sprung from the desire for a general sales tax which would avoid the "cascade" feature of a general turnover tax. Under the latter, the amount of tax borne by any particular final good depends upon the number of intermediate stages of production which precede it. Since final products differ in the degree to which they are

vertically integrated, the turnover tax would create a distortion in the pattern of relative goods prices. This effect would be offset somewhat since there would be a tendency for firms to vertically integrate. Since some industries are vertically integrated more easily than others, this force could not fully offset the initial distortion in relative goods prices. A value-added tax on the other hand, is independent of the degree of vertical integration within an industry. Each firm is taxed only on that portion of its final product which is over and above what it has purchased from other value-added tax paying units. Hence, final goods will be taxed proportionately to their selling price; there will result no distortion of relative goods prices.¹

The value-added tax was also thought of as the best form of business taxation.² Government, it was argued, is a true partner in the productive process. Hence, a payment which is proportional to governmental services is necessary in order to prevent a distortion in the pattern of goods and services produced. The best index of a firm's use of government services, it was argued, is its value-added in production. Note, however, that if this is indeed the case, then the absence of a value-added tax would not distort the pattern of goods and services.

¹This is strictly true only if labor is supplied inelastically with respect to the real wage. Every tax system (except a lump-sum tax) will distort the relative price of leisure to other goods.

The primary interest in the value-added tax during the twenties and thirties was exhibited in Europe. Because it had the politically undesirable feature of requiring higher statutory rates than a turnover tax, the value-added tax was not enacted at that time.

Interest in the value-added tax was revived by the Shoup Mission to Japan in the early fifties. The Mission recommended the value-added tax as the primary source of revenue for the local governments.¹ The value-added tax was enacted but was later repealed before it could take effect. The primary reason for its rejection was that labor unions believed the tax to be regressive. An excellent treatment of the Japanese experience can be found in a series of articles by Broffenbrenner² and in an unpublished doctoral dissertation by Clara Sullivan.³ The latter has done the definitive study of the history and development of value-added taxation and an analysis of its administrative aspects. Let me say, however, that the tax has been enacted in France (1956) and in Michigan (1958). It is currently under study by the British government and Carl Shoup has undertaken


a study for the European Common Market. Finally, it has been proposed in the U. S. as at least a partial substitute for the corporate income tax.

Current theoretical interest in the value-added tax centers around its relationship to other well known forms of taxation. Depending upon how the base of the value-added tax is defined, it has been asserted that the value-added tax is equivalent to a consumption tax or a proportional income tax. The tax is also of interest to students of economic growth who believe that the value-added tax would be more stimulating to the rate of investment than either the corporate income tax or the personal income tax. That this may be the result in the case of the corporate income tax can be seen readily by comparing marginal tax rates on profits. If the two taxes are not shifted by the factors of production on which they are imposed, a corporate profits tax has a marginal rate of 52%, whereas a value-added tax (of equal tax yield) is likely to have a marginal rate of approximately 10%.

The purpose of this paper will be to study the value-added tax in a general equilibrium framework, using both dynamic analysis and comparative statics. Attention will be focused on the merits of the value-added tax on its own right and as a substitute for other well-known forms of taxation. To accomplish the former, the value-added tax will be compared with a system of lump-sum taxation--this being the ideal (if we are already at
a Pareto optimum) tax from an allocative point of view. Emphasis will also be put upon the distributional implications of value-added taxation; hence a general equilibrium incidence analysis will be undertaken. Attention will also be given to the effects of value-added taxation upon the short-run stability of the economy. Finally, we will study the effect of substituting a value-added tax for a corporate income tax upon the level and pattern of investment. Whenever possible existing empirical evidence will be employed in the analysis.

DEFINITIONS

The value-added tax is a tax upon the net sales of a firm minus an appropriate depreciation charge. To arrive at its tax base, a firm would deduct all of its intermediate purchases on current account, its indirect business taxes, and economic depreciation from total sales. In the aggregate, the tax base is equal to national income (or net national product if we assume away the existence of indirect business taxes). From this identity it has been deduced that the value-added tax is equivalent to a flat rate, no exemption, personal income tax. This equivalence was first noticed by Shoup who also suggested another form of value-added taxation.\(^1\) Under this other version no

\(^{1}\text{Carl Shoup, "Theory and Background of the Value-Added Tax," Proceedings of the National Tax Association, (Oct., 1955), pp. 6-19.}
depreciation is allowed on existing assets while all interfirm purchases, both on current account and on capital account, are deductible from sales. Adopting Shoup's terminology, the latter variant will be referred to as a value-added tax of the consumption type (CVA), and the former variant will be termed a value-added tax of the income type (IVA). We can go one step further and define a value-added tax of the gross product type (GVA) under which neither depreciation nor interfirm purchases on capital account are deductible from sales. As we shall see below, the effects of the value-added tax depend critically upon which of the three variants is imposed.

As Shoup has pointed out the tax base of the CVA is conceptually equivalent to total consumers' expenditure in the economy. Note that this equivalence is valid only in the absence of indirect business taxation and if all government expenditure is for capital goods. Otherwise the tax base of the CVA is equal to total expenditure upon consumers' goods (whether private or public) minus indirect business taxation. In what follows we will assume that indirect business taxes do not exist in our economy. Furthermore, it can be shown that the effects of the CVA are invariant to the inclusion or exclusion of government expenditure in the tax base (see p. 28). Hence for practical purposes the tax base of the CVA is equal to private consumer expenditure. On this basis, Shoup concluded that the CVA is equal to a flat rate consumption tax or
equivalently a retail sales tax. We shall see below that this conclusion is unwarranted.

The GVA has as its base an amount equal to gross national product (again assuming the absence of indirect business taxes). It is easy to show that the GVA is equivalent to a sales tax upon the final output of an economy (i.e. a retail sales tax and a sales tax upon capital goods--of equal rates).

The differences between the variants of the value-added tax, then, rest upon their treatment of depreciation. Under the CVA, instantaneous depreciation is granted to new plant and equipment and no depreciation is allowed to owners of existing assets. Under the IVA, the deduction of economic depreciation is allowed on all equipment, new or old. Finally, a GVA would not permit depreciation of any sort to be deducted.

Thus far we have not considered the definitional problems of imputed rents, imputed interest, or goods and services in kind. Indeed it would be impractical to include certain of these items in the tax base. However the value-added tax does not differ in this respect from other well known forms of taxation such as income taxes, sales taxes, or consumption taxes. We will simply assume these problems away so that we can isolate the differences between value-added taxation and other tax systems. Hence we will assume that all forms of income are taxed.
EQUIVALENCES AMONG THE VALUE-ADDED TAXES

In a classical full employment economy where the savings rate (or equivalently the investment rate) is independent of rate-of-return considerations and the supplies of factors of production are inelastic, each of our variants of the value-added tax has the same effect upon the pattern of goods and services produced. To show this let us assume an economy where government expenditures are financed entirely by a lump-sum tax. It will also be assumed that the government spends its entire tax revenue upon consumer's goods and that it balances its budget. Factors of production are supplied inelastically in any given period of time to two industries--a consumption good industry and a capital good industry. Goods prices and factor prices are established competitively. Finally, the level of savings is a function only of real disposable income. Under these assumptions our economy can be described by the following set of equations:

\[
\begin{align*}
(1.1) \quad & O_c = O_c(L_c, K_c) \\
(1.2) \quad & O_c = O_k(L_k, K_k) \\
(1.3) \quad & \frac{\partial O_c}{\partial L_c} P_c = w \\
(1.4) \quad & \frac{\partial O_k}{\partial L_k} P_k = w
\end{align*}
\]
(1.5) \[ \frac{\partial o_c}{\partial K_c} P_c = r \]

(1.6) \[ \frac{\partial o_k}{\partial K_k} P_k = r \]

(1.7) \[ L_c + L_k = \mathcal{L} \]

(1.8) \[ K_c + K_k = \mathcal{K} \]

(1.9) \[ Y = P_c O_c + P_k O_k \]

(1.10) \[ P_c O_c + P_k O_k = P_c (C + \mathcal{G}) + P_k I \]

(1.11) \[ \mathcal{G} = T \]

(1.12) \[ P_c T = \text{lump-sum tax formula} \]

(1.13) \[ C = C(Y_d / P_c) \]

(1.14) \[ Y_d = Y - P_c T \]

(1.15) \[ P_c (C + \mathcal{G}) = P_c O_c \]

(1.16) \[ \bar{M} = M(Y) \]

where

- \( O_c \) = output of consumer goods
- \( O_k \) = output of capital goods
\[ L_c = \text{labor employed by the consumer good industry} \]
\[ K_c = \text{capital employed by the consumer good industry} \]
\[ K_k = \text{capital employed by the capital good industry} \]
\[ L_k = \text{labor employed by the capital good industry} \]
\[ L = \text{total labor force} \]
\[ \mathcal{K} = \text{total capital stock} \]
\[ r = \text{dollar rental on a new machine} \]
\[ w = \text{dollar wage for one unit of labor} \]
\[ P_c = \text{price of consumer goods} \]
\[ P_k = \text{price of capital goods} \]
\[ Y = \text{gross national product (in dollars)} \]
\[ Y_d = \text{money disposable income} \]
\[ C = \text{consumer goods demanded by the private sector} \]
\[ I = \text{demand for new capital goods} \]
\[ G = \text{consumer goods demanded by government} \]
\[ T = \text{taxes in terms of consumption goods} \]
\[ M = \text{money demanded} \]
\[ \bar{M} = \text{money supply} \]

Equations (1.1) and (1.2) are the production functions for the two outputs. Equations (1.3) - (1.6) are the demand functions for the factors of production, while (1.7) and (1.8) are the equilibrium conditions for the factor markets. Equation (1.9) is the definition of gross national product which, by the use of Euler's equation, can be shown to be equal to \( wL + rK \). Equation (1.10) expresses the budget constraint for the economy, while
equation (1.11) is the requirement that the government balance its budget. Equation (1.12) states at what real level the lump-sum tax is to be set. Equation (1.13) is the consumption function which asserts that consumption is a function only of real disposable income. Equation (1.14) is simply the definition of money disposable income and (1.15) is the market equilibrium condition for the consumer goods sector. Together with the budget constraint, (1.15) also guarantees that the capital goods market is cleared. Equation (1.16) is simply the equilibrium condition in the money market. The form of the system of equations (1.1) - (1.15) suggests that the system can be dichotomized into real and monetary sectors. Hence the only function of money in this economy is to establish the absolute level of prices.

The above system of 16 equations and 16 unknowns will yield an equilibrium solution for all variables which we shall denote by barred variables. If we substitute a value-added tax of whichever type for the lump-sum tax the above system of equations will hold with the following exceptions:

\[
\begin{align*}
(1.3a) \quad & \frac{\partial \alpha^c}{\partial L_c} P_c = w(1+\tau) \\
(1.4a) \quad & \frac{\partial \alpha^k}{\partial L_k} P_k = w(1+\tau) \\
(1.5a) \quad & \frac{\partial \alpha^c}{\partial K_c} P_c = r(1+\tau)
\end{align*}
\]
\[ (1.6a) \quad \frac{\partial o_k}{\partial K_k} P_k = r(1+\tau) \]

\[ (1.12a) \quad T = \frac{\tau Y}{P_c} \quad \text{GVA} \]
\[ = \frac{(\tau Y - \tau P_k I)/P_c}{P_c} \quad \text{CVA} \]
\[ = \frac{(\tau Y - \tau P_k \delta K)/P_c}{P_c} \quad \text{IVA} \]

where \( \delta \) is the rate of depreciation permitted by the IVA.

If a double bar on a variable denotes the equilibrium solution of the above system under a value-added tax then I assert that the following equivalences hold

\[ \bar{w} = \bar{w}/(1+\tau) \]
\[ \bar{r} = \bar{r}/(1+\tau) \]

and all other variables remain as they were under the lump-sum tax.

Because we have assumed inelastic factor supplies we need only concentrate on the demand side of the model. On the demand side the crucial equation is the consumption function. If it can be shown that consumption remains constant then the pattern of goods and services produced will also have remained unchanged. Notice that taxes, \( P_c T \), enter only into the consumption function—and in an additive manner. Since we keep tax receipts constant, consumption must also be constant. Thus in a world where the rate of return is unimportant and factors are supplied inelastically any system of direct taxation will yield the same results.

Distributionally, however, this is not generally true; the distribution of income will vary greatly under different direct
tax regimes. For example, under the CVA the amount of income accruing to capitalists is

\[
\left(\frac{r}{1+\tau}\right)K + \tau P_k O_k
\]

where as under a GVA it is

\[
\left(\frac{r}{1+\tau}\right)K
\]

However, the equilibrium configuration of output and prices in our economy is assumed to be independent of the distribution of income. As long as we insist upon equal tax revenues any form of direct taxation yields the same result.

It can also be shown that a sales tax upon final output, ie. a GNP sales tax, will lead to the same pattern of output and relative prices as a direct tax of equal revenue yield (in real terms). For simplicity's sake let us assume that the money supply is adjusted so that the tax can be passed on in the form of higher prices. Under a sales tax of rate \( \tau \) we have the following set of equations:

\[
(1.1b) \quad O_c = O_c(L_c, K_c)
\]

\[
(1.2b) \quad O_k = O_k(L_k, K_k)
\]

\[
(1.3b) \quad \frac{\partial O_c}{\partial L_c} P_c = w
\]
(1.4b) \[ \frac{\partial O_k}{\partial L_k} p_k = w \]

(1.5b) \[ \frac{\partial O_c}{\partial K_c} p_c = r \]

(1.6b) \[ \frac{\partial O_k}{\partial K_k} p_k = r \]

(1.7b) \[ L_c + L_k = \bar{L} \]

(1.8b) \[ K_c + K_k = \bar{K} \]

(1.9b) \[ Y(1+\tau) = P_c(1+\tau)O_c + P_k(1+\tau)O_k \]

(1.10b) \[ P_c(1+\tau)O_c + P_k(1+\tau)O_k = P_c(1+\tau)(C+\bar{G}) + P_k(1+\tau)I \]

(1.11b) \[ \bar{G} = T \]

(1.12b) \[ (1+\tau)P_cT = \tau P_cO_c + \tau P_kO_k \]

(1.13b) \[ C = C\left(\frac{Y_d}{P_c(1+\tau)}\right) \]

(1.14b) \[ Y_d = Y \]

(1.15b) \[ P_c(1+\tau)(C+\bar{G}) = P_c(1+\tau)O_c \]

(1.16b) \[ \bar{M} = M(Y(1+\tau)) \]
Note that the dollar value of taxes is \((1+\tau)\) times that under the lump-sum regime because government expenditures must be maintained at the same real level. Also notice that in equation (1.13) the argument has shifted from \((Y-PcT)/Pc\) to \(Y/Pc(1+\tau)\). Thus in order for consumption to have remained constant it is necessary that

\[
\tau = PcT/(Y-PcT)
\]

That this tax rate will produce the correct revenue of \(Pc(1+\tau)T\) is easily verifiable. In the rest of the equations the \((1+\tau)\)'s cancel out and we are left with our original system (1.1)-(1.16) except for a larger money supply.

In the preceding section we dealt with an economy in which the rate of return to saving was of no consequence. We shall now turn our attention to an economy where the level of consumption (or equivalently the level of investment) depends upon the rate of return earned by a new capital good as well as upon the level of real disposable income. Specifically, it is assumed that consumption is negatively related to the rate of interest. This is by no means the most obvious assumption one can make about savings decisions with respect to the rate of interest. We shall treat the opposite case at the end of this section.

We assume, further, that investment is carried on up to the point where the marginal efficiency of capital is equal to the market rate of interest. Let \(\hat{r}(v)\) be the expected rental on a new machine \(v\) years in the future. Furthermore, let investors believe that the current market rate of interest will prevail
indefinitely into the future. Finally, let physical depreciation on a new machine occur exponentially at the rate $\delta$. In other words we assume that machines suffer radioactive decay; this implies that $v$ years in the future we will have $e^{-\delta v}$ of a machine left. Under these assumptions the demand price for new capital goods at any given point of time is given by:

$$
(1.17) \quad \hat{P}_k = \int_0^\infty r e^{-(i+\delta)v} dv
$$

where $\hat{P}_k$ is the demand price for capital and $i$ is the prevailing market rate of interest.

If the demand price (1.17) exceeds the supply price given by (1.1)-(1.8), investors will bid up the market rate of interest in order to secure more funds. In the preceding section this mechanism had no effect upon the supply of savings but in this formulation additional savings will be forthcoming. The rate of interest will be bid up until the demand price for new capital equals its supply price. I.e.,

$$
(1.18) \quad \hat{P}_k = P_k
$$

If we combine (1.17) and (1.18) we can solve for the market rate of interest under the lump-sum tax.

$$
(1.19) \quad i = (r - \delta P_k)/P_k
$$

For simplicity's sake we have assumed that $\hat{r}(v)=r(0)$, for all $v$, in calculating $i$. 
Now when we replace the lump-sum tax by a value-added tax the outcome will depend critically upon which variant of the value-added tax we choose. If we replace the lump-sum tax by an IVA, (1.17) becomes

\[ P_k = \int_0^\infty \left[r(1-\tau^I) + \tau^I \delta P\right] e^{-(i+\delta)v} \]

where \( \tau^I \) is the rate of IVA which would produce a yield equal to that of a lump-sum tax. If we combine (1.17a) and (1.18) and solve for the equilibrium market rate of interest we find

\[ i^I = \frac{(r-\delta P_k)(1-\tau^I)}{P_k} \]

Comparing (1.19a) with (1.19) it is clear that the market rate of interest under the IVA is less than that under the lump-sum tax. Since consumption is negatively related to the market rate of interest, there will be more consumption under an IVA than under a lump-sum tax or equivalently less investment.

If we replace the lump-sum tax by a GVA, (1.17) becomes

\[ P_k = \int_0^\infty r(1-\tau^G) e^{-(i+\delta)v} \]

where \( \tau^G \) is the rate of GVA. Solving for the equilibrium rate of interest we find

\[ i^G = \frac{(r-\delta P_k)(1-\tau^G) - \tau^G \delta P_k}{P_k} \]

which is clearly less than that obtained under the lump-sum tax. While it would also appear that the rate of interest is higher under the IVA than under the GVA, this must be shown since the rate of tax under the GVA is less than that under the IVA (the latter allows depreciation whereas the former does not). If we take the ratio of \( i^G \) to \( i^I \) at the point where the system was in
equilibrium under the lump-sum tax we find

\begin{equation}
\frac{i^G}{i^I} = \frac{1 - \tau^G Z}{1 - \tau^I}
\end{equation}

where

\[ Z = \frac{r_0}{\tau^o - \delta P^o_k} \]

Clearly \( i^G > i^I \) only if

\begin{equation}
\tau^G Z < \tau^I
\end{equation}

But to produce equal revenues it must be that

\begin{equation}
\tau^G = \tau^I [1 - \frac{\delta P^o K}{r_0 K + w_0 L}]
\end{equation}

Substituting from (1.22) into (1.21) we find

\begin{equation}
\tau^G \left( \frac{r_0}{\tau^o - \delta P^o_k} \right) < \tau^G \left( \frac{r_0 K + w_0 L}{\tau^o K + w_0 L - \delta P^o k} \right)
\end{equation}

or

\[ 0 < -\delta P^o k w_0 L \]

which is a contradiction. Hence \( i^I > i^G \). Q.E.D.

Finally, let us consider the case where the lump-sum tax is replaced by a CVA. Remembering that the CVA allows instantaneous depreciation, equation (1.17) becomes

\begin{equation}
P_k = r(1-\tau^F) e^{-(i+\delta)v} dv + \tau^F P_k
\end{equation}

where \( \tau^F \) is the rate of CVA. Combining (1.17c) and (1.18) we find
(1.19c) \[ r^* = \frac{r - \delta P_k}{P_k} \]

which is the same rate of interest as obtained under the lump-sum tax. Thus even when saving is interest elastic the CVA yields the same result as the lump-sum tax. It appears, therefore, that the CVA is an ideal tax—ideal being used in the allocative sense. If labor is supplied elastically, however, the two taxes are not equivalent since the real wage is reduced under the CVA. The CVA, nevertheless, more nearly approximates the results sought by H. G. Brown who argued that a proportional tax upon all factors of production (i.e. an IVA) was completely neutral.\(^1\) He was wrong, of course, because the proportional factor tax results in a distortion in the choice between future and current consumption and between work and leisure. The CVA eliminates the former but not the latter distortion.

In summary, of all the variants of the value-added tax only the CVA produces the same results as the lump-sum tax (except in the case where labor is supplied elastically). The GVA, and to a lesser extent, the IVA, discriminate in favor of current consumption at the expense of future consumption; hence they produce a lower rate of capital accumulation than would a lump-sum tax or a CVA.

The latter conclusion can be reached only if saving is positively related to the rate of interest. There is no a priori reason to suspect that this is the case. As a matter of fact a convincing argument can be made that saving will be less in the face of higher interest rates. Savers might be tempted to spread the gain from higher interest receipts between future and present consumption. To see this more clearly we can make use of the following simple diagram:

![Figure (1)](image)

Under, say, a GVA an individual can, if he does not save, enjoy $C_o$ of consumption in the present and $C_1$ of consumption in the future (after all taxes have been paid). Let us assume that he decides to do some saving and his equilibrium
point is point B on AC. Under a CVA, on the other hand, the individual is faced with the same alternatives as before except that he can now transform present income into future income along the line AC. It is reasonable to assume that the individual may choose as his new equilibrium a point along AC between the coordinates D and E. In this area the individual will enjoy more consumption in both time periods. Hence he will be on a higher utility curve—something which cannot definitely be said about any other point along AC. Of course we cannot tell if the individual will in fact choose a point in this triangle since we do not know his preference map. But it certainly represents a plausible pattern of behavior—much more plausible in fact than a point along AC to the right of E.

Thus, a priori reason is not enough to tell us whether saving is positively related to the interest rate. However, in order to remain within the classical model this assumption will be made throughout the paper. If saving is negatively related to the rate of interest most of our conclusions with respect to capital accumulation in the preceding section and in what follows need simply be reversed.

EQUIVALENCES WITH OTHER TYPES OF TAXES

That the IVA is equivalent to a flat rate income tax can be readily seen by the use of the following income identity:

\[
\text{sales} - \text{purchases on current account} + \text{net change in inventories} - \text{depreciation} = \text{wages} + \text{interest} + \text{rents} + \text{profits}
\]
First, note that we must deduct the net change in inventories from gross purchases because these are purchases on capital account and not on current account; to exclude them from the tax base would be to discriminate in favor of inventory investment as opposed to durable capital. Next, notice that the right hand side of our identity is what we usually define as the tax base of an income tax. The left hand side on the other hand is equal to the tax base of the IVA. Therefore we are justified in regarding the IVA as equivalent to an income tax.

Whereas an income tax serves as a wedge between a factor's take-home pay and his net pay, the IVA serves as a wedge between the gross wage paid by the firm and a factor's take-home pay--the result is identical.

There is one slight difference between an IVA and an income tax which is related to the concept of loss offsets. Under an income tax, a bondholder can, if the company goes into bankruptcy, write off the capital loss against other income; or he may even be permitted to carry the loss forward over the next few years. No such provision would exist under the IVA since the taxable unit is the firm. Insofar as lenders are influenced by risk and the latter is approximated by the expected variance of the yield of a bond, there will be an increase in the rate of interest demanded by lenders when the IVA is substituted for a proportional income tax.

One might be tempted, at this point, to assert that under an IVA there will be a substitution made of the factor intermediate
purchases for other factors of production. This is due to the fact that the latter is taxed whereas the former is not. While this assertion is true if we restrict the tax to a single firm or a single industry (or even several industries), it is not true if the tax is general (i.e., applicable to all firms within the economy). Because the tax is all pervasive, the price of intermediate goods will reflect the tax (intermediate goods are merely embodied primary factors). Consequently, the short-run neutrality of the IVA with respect to productive technique is maintained. Note, however, that the foregoing argument can only strictly be applied to a closed economy. In an open economy, there will be a substitution made of imported raw materials for other factors of production. In order to maintain neutrality in an open economy, a tax of equal rate must be levied on imports.

In his textbook, The Theory of Public Finance, Professor Musgrave asserts the equivalence between a flat rate income tax and a system of retail sales taxation and a sales tax on capital goods. This implies that the IVA is also equal to that system of taxation. We will set out to show that this equivalence does not hold. Specifically, there will be a higher level of investment under the sales tax regime than under the IVA.

In order for the two systems of taxation to be equivalent they must produce the same rate of return on new capital; this

---

implies a definite restriction on the size of the tax rate on new capital goods. Secondly, they must yield the same real level of consumption; this implies a restriction on the size of the tax rate on consumer goods and hence a second restriction on the size of the tax rate on capital goods. We will show that the two restrictions on the tax rate on capital goods are inconsistent.

Before we can begin the proof, we must make several institutional assumptions that will facilitate the analysis. We assume that when we change from an IVA to a sales tax the money supply is adjusted so that consumers bear the tax in the form of higher prices. This assumption will not change the results of the analysis in any way. We also assume that the IVA is paid by factor owners and not by the firm; in this sense we are dealing with an income tax. Finally, we assume that firms own no capital--they rent all of their equipment. Capital is owned entirely by a group to whom we shall refer to as rentiers. The rentiers pay all taxes on capital.

Under this set of assumptions, the economy can be described by the following sets of equations:

<table>
<thead>
<tr>
<th>IVA</th>
<th>SALES TAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.22) ( \partial c - )</td>
<td>(1.22) ( c = c(K_c, L_c) )</td>
</tr>
<tr>
<td>( 0_c = 0_c(K_c, L_c) )</td>
<td>( 0_c = 0_c(K_c, L_c) )</td>
</tr>
<tr>
<td>(1.23) ( \partial k - )</td>
<td>(1.23) ( k = k(K_k, L_k) )</td>
</tr>
<tr>
<td>( 0_k = 0_k(K_k, L_k) )</td>
<td>( 0_k = 0_k(K_k, L_k) )</td>
</tr>
<tr>
<td>(1.24) ( \frac{\partial o_k}{\partial L_c} = \frac{w}{P_c} )</td>
<td>(1.24) ( \frac{\partial o_c}{\partial L_c} = \frac{w}{P_c} )</td>
</tr>
</tbody>
</table>
\[
\frac{\partial \bar{O}_k}{\partial \bar{L}_k} = \frac{w}{P_k} \quad \frac{\partial \bar{O}_k}{\partial \bar{L}_k} = \frac{w}{P_k}
\]
\[
\frac{\partial \bar{O}_c}{\partial \bar{K}_c^c} = \frac{r}{P_c} \quad \frac{\partial \bar{O}_k}{\partial \bar{K}_k^c} = \frac{r}{P_c}
\]
\[
\frac{\partial \bar{O}_k}{\partial \bar{K}_k} = \frac{r}{P_k} \quad \frac{\partial \bar{O}_k}{\partial \bar{K}_k} = \frac{r}{P_k}
\]
\[
\bar{L}_c + \bar{L}_k = \bar{L}
\]
\[
\bar{K}_c + \bar{K}_k = \bar{K}
\]
\[
Y = P_c \bar{O}_c + P_k \bar{O}_k \quad Y = P_c(1+\tau^c)\bar{O}_c + P_k(1+\tau^k)\bar{O}_k
\]
\[
Y = P_c(C+\bar{G}) + P_k\bar{I} \quad Y = P_c(1+\tau^c)(C+\bar{G}) + P_k(1+\tau^k)\bar{I}
\]
\[
\bar{G} = T \quad \bar{G} = T
\]
\[
P_cT = \bar{I}Y - \bar{I}P_k\delta K \quad P_c(1+\tau^c)T = \tau^cP_c\bar{O}_c + \tau^kP_k\bar{O}_k
\]
\[
C = C(\frac{Y_d}{P_c}, i) \quad C = C(\frac{Y_d}{P_c(1+\tau^c)}, i)
\]
\[
Y_d = Y - TP_c \quad Y_d = Y - P_c(1+\tau^c)T
\]
\[
P_c(C+\bar{G}) = P_c\bar{O}_c \quad P_c(1+\tau^c)(C+\bar{G}) = P_c(1+\tau^c)\bar{O}_c
\]
\[
P_k = \int_{0}^{\infty} \left[ r(1-\tau^c) + \tau \delta P_k \right] e^{-(i+\delta)\nu} d\nu \quad P_k(1+\tau^k) = \int_{0}^{\infty} re^{-(i+\delta)\nu} d\nu
\]
Where $\tau^I$ is the rate of IVA, $\tau^C$ is the sales tax on consumer goods, and $\tau^K$ is the sales tax on capital goods.

If the two tax systems are to be equivalent, they must yield identical solutions for all variables except the definitional variables $Y$ and $Y_d$. Specifically, the level of consumption expenditure (in real terms) must be the same in both economies. In order to provide equal levels of consumption, the two systems must provide the same level of real disposable income and the same rate of interest. There is a case where these two variables might offset one another but we shall ignore it as being too improbable. We will now proceed with the proof under the assumption that the equilibrium solutions of the two systems are the same.

Proof:

Let $Y_d^V$ and $Y_d^S$ denote the equilibrium solution for money disposable incomes under the IVA and sales tax systems respectively. In order for the same level of real disposable income to result it must be that

$$\frac{Y_d^V(1+\tau^C)P_c}{Y_d^SP_c} = \frac{(Y_d^S-TP_c)(1+\tau^C)}{Y_d^S} = 1$$

this implies that

$$(i) \quad \tau^C = \frac{P_cT}{Y_d^S-P_cT}$$
In order to provide revenues of \( T(1+\tau^c)P_c \), \( \tau^k \) must satisfy

\[
(ii) \quad \tau^k P_k O_k + \tau^c P_c O_c = (1+\tau^c)P_c T
\]

Substituting for \( \tau^c \) from (i) we find that

\[
(iii) \quad \tau^c = \tau^k
\]

Now \( \tau^k \) must also be set so as to equalize the rate of return between the tax systems. Hence

\[
(iv) \quad \tau^k = \frac{1}{X\tau - 1} \quad \text{where} \quad X = \frac{r}{r-5P_k}
\]

It can be shown that

\[
(v) \quad \frac{1}{X\tau - 1} < \tau^c = \frac{TP_c}{Y_d - TP_c}
\]

Hence (iii) and (iv) are inconsistent. Furthermore from the above it can be deduced that

\[
\tau^k < \frac{TP_c}{Y_d - TP_c} < \tau^c
\]

if we insist on equal rates of return. But this implies that real disposable income is less under a sales tax than under an IVA. Hence,
Thus it appears that a sales tax system is more effective than an income tax in stimulating investment. It accomplishes this by reducing real disposable income to a greater extent than the more direct IVA. The reason for the latter is purely arithmetic. In order to bring about the equality of the rate of return under both tax systems it is necessary that the sales tax on consumer goods be set so high as to reduce real disposable income below that of the IVA. We shall see that the same phenomenon occurs when we compare the CVA with the consumption tax.

**CVA vs CONSUMPTION TAX**

That the tax base of the CVA is equal to that of a consumption tax can easily be shown through the use of the basic national income identity—consumption + gross investment = gross national product. Under both the CVA and the consumption tax, gross investment is excluded from the tax base. Notice, however, that when we introduce government the national income identity becomes—consumption + gross investment + government expenditures = gross national product. In order to maintain the equivalence between the tax bases, government expenditures must be exempted from the CVA or included in the base of the consumption tax. It can be shown that if the base of the consumption tax is expanded to include government purchases, precisely the same
tax rate would result as when government purchases are excluded from the tax base.

\[ TP_c(1+\tau_c) = \tau_c (C+\bar{G}) \]

is equivalent to

\[ TP_c = \tau_c PcC \]

Hence, altering the base of the consumption tax to include government expenditures changes nothing. We can validly claim the equivalence between the tax base of the CVA and the consumption tax.

Even though they have identical tax bases, the consumption tax and the CVA are not equivalent as has been usually asserted. To show this we make the same set of assumptions as we did when we compared the IVA with a system of sales taxes. As a matter of fact we can employ the same set of equations except the tax equations and the rate of return equations. Furthermore we must set \( \tau^k = 0 \) is the sales tax model. The tax equations now read:

(1.33a) \[ P_c T = \tau_c Y - \tau_c P_kO_k \]

(1.33b) \[ (1+\tau_c)P_cT = \tau_c PcC + \tau_c Pc\bar{G} \]

for the CVA and consumption tax respectively. Furthermore the rate of return equations now become:

(1.37a) \[ P_k = \int_{\tau_c}^{\infty} r(1-\tau_c)e^{-(i+\delta)\nu}dv + \tau P_k \]
\[(1.37b) \quad P_k = \int_0^\infty r e^{-(i+\delta)v} dv\]

for the CVA and consumption taxes. From our previous analysis of the CVA we know that (1.37a) collapses to:

\[(1.37a) \quad P_k = \int_0^\infty r e^{-(i+\delta)v} dv\]

Shoup and others have argued on the basis of the identity of (1.37b) and (1.37a) and the equivalence of tax bases, that the two taxes were equivalent.\(^1\) Their error was that they did not probe deeply enough into the general equilibrium system. For upon examination of the consumption functions of both tax models, we see that precisely the same phenomenon occurs as did in our comparison of a sales tax system with the IVJ. Namely, real disposable income will be less under the consumption tax regime than under the CVA. To prove this we need only to reproduce the requirement for equal real disposable incomes—

\[\tau^c = \frac{P_c T}{Y_d - P_c T}\]

But this rate of tax will not provide the government with enough revenue. For example let our comparison function be of the form:

\[C = \alpha(i) \frac{Y_d}{P_c (1+\tau^c)}\]

Then our tax receipts will be

\[\frac{TP_c}{Y_d - P_c T} \cdot P_c O_c = \alpha TP_c\]

which is adequate only if \(\alpha = 1\). Thus

\(^1\)Shoup, loc. cit.
and hence consumption is higher under a CVA than under a consumption tax. Again the rationale for this conclusion is simply a matter of arithmetic. Consumption taxes simply reduce disposable income to a greater extent than an equal yield CVA. An alternative explanation is that, under a consumption tax, an individual can reduce his tax burden by consuming less. Under a CVA, on the other hand, an individual does not affect his tax bill by consuming more or less; it depends only upon his income. It is true, however, that the more all individuals consume the higher the tax bill that they, as a group, must pay.

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1 This is because consumption taxes affect all of disposable income whereas the CVA affects only that portion which is spent on consumption. To see this, simply compare real disposable income under the two regimes. For the consumption tax this is given by

\[ Y_d^c = \frac{Y}{P_c(1+\tau^c)} = \frac{O_c}{1+\tau^c} + \left( \frac{P_k}{P_c} \right) \frac{O_k}{1+\tau^c} \]

Under a CVA we have

\[ Y_d^r = \frac{Y-P_cT}{P_c} = O_c(1-\tau^r) + \left( \frac{P_k}{P_c} \right) O_k \]

to provide equal revenues it must be that

\[ \tau^r O_c = \tau^c O_c \]

\[ \tau^r = \frac{\tau^c}{1+\tau^c} \]

Thus our expression for real disposable income under the CVA becomes

\[ Y_d^r = \frac{O_c}{(1+\tau)^c} + \left( \frac{P_k}{P_c} \right) O_k . \]
There is another major difference between a consumption tax and a CVA. Let us rewrite (1.37a) as

\[(1.37a) \quad P_k = r e^{-(i+\delta)\nu} dv + [\tau P_k - \tau^* r e^{-(i+\delta)\nu} dv] \]

As we have previously pointed out the term in brackets is equal to zero. But what is the term in brackets? It is the present-discounted value of taxes paid by owners of new capital—zero. If we had started our economy from scratch (zero capital stock) capitalists would never have paid any taxes. As a matter of fact if the economy is always growing (positive net investment) there would have been a negative flow of taxes from government to business (in present-value terms). In such a situation the CVA resembles a wage tax, since only wage income is taxed.

This result should not be surprising. Samuelson's "non-substitution" theorem tells us that if there is only one primary factor of production, the relative price of goods will reflect only their labor requirements, both direct and indirect. Consequently, if we have a tax system in which both the returns to labor and to capital are taxed, the prices of those goods which are relatively capital intensive will rise relative to those goods which are relatively

---

labor intensive. This is so because we tax the capital good input twice—once in the form of labor and again in the form of capital. Only a wage tax would not change the long-run equilibrium set of relative prices dictated by technology. The nice neutrality aspects of the CVA arise similarly through its exemption of the returns to capital from taxation. While it is true that the CVA would tax owners of existing capital, if it were imposed tomorrow, this would not affect long-run relative prices because these returns are pure economic rents.

The consumption tax, on the other hand, reaches all forms of income, whatever the source when that income is spent for consumption. The consumption tax would approximate a wage tax only if all wages were consumed and all returns to capital were saved.

**GVA vs GNP SALES TAX**

To show that a GVA is equal to a GNP sales tax we will again refer to the model used in the analysis of the IVA. As before we will have new tax equations and new rate of return equations. Furthermore, in the sales tax model we must equate $\tau^c$ and $\tau^k$. The new tax equations now read

\[(1.33c) \quad P_c T = \gamma^c Y\]

\[(1.33d) \quad P_c T(1+\tau^c) = \tau^c P_c O_c + \tau^k P_k O_k\]
for the GVA and GNP sales tax respectively. The rate of return equations now become

\[(1.37c) \quad P_k = \int_0^\infty r(1-\tau^\$)e^{-(i+\delta)v} dv\]

\[(1.37d) \quad P_k(l+\tau^k) = \int_0^\infty re^{-(i+\delta)v} dv\]

In order for the GNP sales tax to provide the same rate of return as the GVA it is necessary that

\[(1.40) \quad \tau^c = \tau^\$/\!(1-\tau^\$)\]

Now \(\tau^\$\) must satisfy

\[(1.41) \quad \tau^\$Y = \tau^\$Y_d = TP_c\]

or

\[(1.42) \quad \tau^\$ = \frac{TP_c}{Y_d^S}\]

If we substitute from (1.42) into (1.40) we obtain

\[(1.43) \quad \tau^c = \frac{TP_c}{Y_d^S - TP_c}\]

which is precisely the value of \(\tau^c\) necessary to make consumption the same under both tax systems. Hence it must be that the GNP sales tax and the GVA are equivalent.
MODEL WITH INTERMEDIATE GOODS

So far we have been concerned with an economy in which there exists only two productive sectors: a capital-good producing sector and a consumption-good producing sector. The results of this model can easily be generalized to an economy which, besides the two aforementioned sectors, has an intermediate-good producing sectors. Such an economy can be described by the following model:

\[ O_i = O_i(L_i, K_i, O_i^1, \ldots , O_i^n) \quad i = (c, k, l, \ldots , n) \]

\[ \frac{O_i}{f_i} = \frac{P_f}{P_i} \quad i = (c, k, l, \ldots , n) \]

\[ i \cdot f_i = f \quad \]

\[ Y = P_k O_k + P_c O_c \]

\[ Y = P_k I + P_c (C + G) \]

\[ \bar{G} = T \]

\[ P_c (C + \bar{G}) = P_c O_c \]

\[ C = C\left(\frac{Y_d}{P_c}, i\right) \]

\[ Y_d = Y - P_c T \]
As one can observe only the production equations have changed from our earlier system. If we consider the imposition of a value-added tax, of whatever type, the production equations remain unchanged (since labor and capital are supplied inelastically). The demand side has also remained unchanged, save for a larger money supply requirement. Hence all of our previous analysis also applies to this expanded system.

**CAPITAL GAINS AND THE VALUE-ADDED TAX**

We have yet to consider what will be the effect, upon the price of existing capital goods, of the substitution of a value-added tax for a lump-sum tax. As might be expected the price change varies, depending upon which variant of the value-added tax we choose to impose.

It can be easily shown that the price of a machine of age \( v \) is equal to \( e^{-\delta v} \) times the price of a new machine; this is true so long as the tax system treats new and old machines in the same manner. Hence if the price of a new machine rises when we substitute a value-added tax for a lump-sum tax, a capital gain will accrue to owners of
existing capital. The only way for the price of a new machine to increase is through an increased demand for capital goods (unless constant opportunity costs exist between the two sectors). As we have already shown, the demand for new capital goods falls when we substitute either a GVA or an IVA for a lump-sum tax. Hence there will be capital losses if the change is made. Similarly the demand for capital goods is lower under a GVA than under an IVA. Thus a shift from the former to the latter results in a capital gain.

We can extend this analysis to a profits tax. A profits tax results in the smallest demand for capital goods of all the aforementioned taxes. Consequently the imposition of a profits tax results in a capital loss.

When we compare a CVA with a lump-sum tax, on the other hand, we find that the CVA results in a capital loss despite the fact that the demand for new capital remains unchanged. This is because owners of existing assets are allowed no depreciation allowance at all. Consequently, a machine of age \( v \) will sell for \( (1-T)e^{-\delta v} \) times the price of a new machine. Now it is impossible to compare a CVA with an IVA, GVA, or profits tax unless we are capable of specifying the parameters of the system so that we can tell exactly how much the price of new capital goods changes.

We can also infer from our previous analysis that if we substitute a consumption tax for a lump-sum tax owners
of existing assets will enjoy a capital gain. Thus if we substitute any of the other taxes for a consumption tax capital losses will result.

Finally, our conclusions will be altered if we allow the money supply to be increased so that the tax is passed on in the form of higher prices. This, however, is purely an inflation effect and should be ignored.
CHAPTER II
INCIDENCE EFFECTS
OF VALUE-ADDED TAXATION

Of all the effects of value-added taxation, its incidence effects are perhaps the most important and generally the most elusive. By the term incidence we mean the changes in the distribution of welfare brought about by the introduction of a particular tax. Since a person's welfare is not measurable and hence not comparable, economists usually assume that real income serves as a satisfactory index of a person's well being and as a basis for making interpersonal welfare comparisons. In this chapter, we will be concerned with determining the effects of value-added taxation upon the distribution of real income within an economy.

Not only will we be concerned with the intratemporal distribution of income, but we will also take into account shifts in the intertemporal distribution of income. As we have seen in the preceding chapter, certain shifts in tax structure may alter the rate of capital accumulation--thereby changing the income available for distribution in future periods. Expressed differently--we must take into account changes in the size of the income "pie" as well as its division.

In order to determine the incidence effects of the value-added tax we must specify the manner in which it is to be introduced. Musgrave has suggested three potential
experiments that we can perform.¹ First, we can hold all other taxes and expenditures constant. Musgrave refers to this as "specific" incidence. Whereas there may be valid reasons for wanting to know the specific incidence of a tax, in the current context it is clearly inadequate. The effects of the value-added tax would be confounded with the effects of the resulting unemployment and/or demand readjustments.

Secondly, we can increase government expenditures by the amount of the yield of the value-added tax. This experiment would determine the "balanced-budget" incidence of the value-added tax. While this approach avoids some of the pitfalls of specific incidence it creates new ones which may perhaps be more troublesome. The effects of the value-added tax now become entangled with the distributional aspects of the increased government expenditure. Furthermore not all of the demand problems associated with specific incidence have been solved because of the employment and/or inflationary effects caused by the balanced-budget multiplier.

The third experiment suggested by Musgrave is termed differential incidence. Here we replace an existing tax with a value-added tax of equal yield while holding government expenditures constant. Clearly this last approach is the most appropriate for our purposes. It is true, however, that our conclusions depend critically upon which tax is

¹Musgrave, op. cit., pp. 211-17.
replaced. From a normative point of view the latter is not a drawback but an advantage since it permits us to choose the best tax structure among a multitude of alternatives. A given tax structure is rarely of interest on its own right but only of interest when compared to the existing structure. The value-added tax is no exception. It is commonly proposed as a substitute for the corporate profits tax. Hence, among the experiments we will perform is the substitution of a value-added tax for a profits tax. Other experiments include a comparison of the different variants of the value-added tax and a comparison of the CVA and a consumption tax.

The primary reason for the elusiveness of incidence analysis is that, in general, nothing can be said. The general assumptions of profit maximization and perfect competition are insufficient, in themselves, to provide unambiguous conclusions--other assumptions must be added. Even this may not be enough since our conclusions often depend upon the size of parameters about which little is known and little can be determined. Therefore, the conclusions of our incidence analysis may depend critically upon the set of assumptions we choose to make concerning the structure of the model and its parameter values.

In what follows, our basic assumptions will be the same as those of the preceding chapter: balanced budget, perfect
competition, inelastic factor supplies in the short run and two producing sectors. For analytical convenience, we will assume that our money supply is held fixed at that level where the price of consumer goods is equal to one. Thus, all of the money variables of the preceding chapter are now expressed in terms of consumption goods. More specific assumptions will be made as the occasion arises, but we shall strive for as much generality as possible.

SHORT-RUN INCIDENCE

A. Caeteris Paribus Shifts in Tax Structure

Even though we have assumed factor supplies to be inelastic in the short run, the question of short-run incidence is not uninteresting. The first question which comes to mind is how the various tax systems affect the functional distribution of income. One method of measuring these effects is to compare after-tax factor rewards as we make caeteris paribus shifts in tax structure. That is for any given set of prices and pattern of output, we calculate and compare net factor earnings under the different tax regimes. While it is generally true that all other things do not remain equal in the face of a shift in tax structure, this approach provides us with a useful first approximation. We shall see, in the preceding section, that this first approximation is valid under a wide variety of assumptions about relative price changes.

If we compare the net real wage (wage in terms of consumer goods) under the different tax regimes, we find that it will
be highest under a profits tax and lowest under the CVA and the consumption tax. The complete ranking of tax systems by net real wage is as follows:

(I) Profits tax > GVA > IVA > CVA = Consumption tax

To show this we make use of the following table.

**TABLE I**

Comparison of Net Wages and Net Rentals Under Alternative Tax Systems

<table>
<thead>
<tr>
<th>Tax System</th>
<th>Tax Formula</th>
<th>After-tax Real Wage</th>
<th>After-tax Real Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profits tax</td>
<td>$T = \tau^P(rK-5PK)$</td>
<td>$w$</td>
<td>$r(1-\tau^P)+\delta\tau^P$</td>
</tr>
<tr>
<td>GVA</td>
<td>$T = \tau^G(rK+wL)$</td>
<td>$w(1-\tau^G)$</td>
<td>$r(1-\tau^G)$</td>
</tr>
<tr>
<td>IVA</td>
<td>$T = \tau^I(rK+wL-5PK)$</td>
<td>$w(1-\tau^I)$</td>
<td>$r(1-\tau^I)+\delta\tau^I$</td>
</tr>
<tr>
<td>CVA</td>
<td>$T = \tau^C(rK+wL-PO_k)$</td>
<td>$w(1-\tau^C)$</td>
<td>$r(1-\tau^C)$</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>$T = \tau^C C$</td>
<td>$w\left(\frac{1}{1+\tau^C}\right)$</td>
<td>$r\left(\frac{1}{1+\tau^C}\right)$</td>
</tr>
</tbody>
</table>

where

$\tau^P$ = rate of profits tax

$\tau^G$ = rate of GVA

$\tau^I$ = rate of IVA

$\tau^C$ = rate of Consumption tax

$\delta$ = rate of Consumption tax

$r$ = rate of rental
\( \tau^c \) = rate of consumption tax
\( r \) = gross rental in terms of consumption goods
\( w \) = gross wage
\( P \) = price of capital goods in terms of consumption goods
Other variables as they have been previously defined.

This table was obtained by simply applying the tax rate to the before-tax wage and subtracting this amount from it. It is obvious from Table (I) that the profits tax results in the highest net wage. It is equally obvious that \( \tau^s \) is less than \( \tau^I \) and \( \tau^r \) since we have narrowed the tax base; hence the net wage is higher under the GVA than under the IVA or CVA. Furthermore, as long as net investment is positive, \( \tau^I < \tau^r \); thus the IVA results in a higher net wage than the CVA. All that remains to show is the ranking of the consumption tax. If we equate the tax formulae of the CVA and consumption tax we obtain:

\[
(2.1) \quad \tau^c C = \tau^r (rK + wL - PO_k) = \tau^r O_c
\]

also

\[
(2.2) \quad \tau^c C = \tau^c (O_c - T) = \tau^c O_c (1 - \tau^r)
\]

Substituting from (2.2) into (2.1) we find
Hence the consumption tax and the CVA have equivalent effects upon real labor income.

Contrary to what one may have expected, the ranking (I) is not exactly reversed when we consider net rentals. In the normal case, the ranking of tax systems by rental incomes will look as follows:

(II) \( \text{IVA} > \text{GVA} > \text{CVA} = \text{Consumption tax} > \text{Profits Tax} \)

By normal we mean that gross labor income (inclusive of tax) exceeds gross investment expenditures. If this condition does not hold the profits tax will exchange positions with the CVA and the consumption tax. To show how this ranking was derived, we will proceed in step-by-step fashion.

**GVA vs IVA**

Equating tax functions, we find

\[
(2.4) \quad \tau^G(rK+wL) = \tau^\text{I}[r-\delta P(K+wL)]
\]

If the IVA is to be ranked below the GVA, then

\[
(2.5) \quad r(1-\tau^\text{I}) + \delta P\tau^\text{I} < r(1-\tau^\text{I})
\]
Substituting for $r^I$ from (2.4) into (2.5) we find

$$r^I \left( \frac{r}{r^p} \right) < 1$$

which is a contradiction. Hence IVA > GVA:

**GVA vs CVA**

Equating tax functions, we obtain

$$\tau^G(rK+wL) = \tau^R(rK+wL-PO_k)$$

or

$$\tau^G = \gamma^r \tau^r$$

where $\gamma = \frac{rK+wL-PO_k}{rK+wL} < 1$

If the GVA is to be ranked below the CVA, then

$$r(1-\gamma^r) < r(1-\tau^r)$$

which is obviously false. Hence GVA > CVA.

**CVA vs Profits Tax**

Equating tax functions, we find

$$\tau^P(rK+wL-PO_k) = \tau^P(r-\delta P)_k$$

or

$$\tau^P = \left[ \frac{wL+rK-PO_k}{(r-\delta P)_k} \right] \tau^r$$
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only at the expense of existing capital because all tax payments and tax credits, under these taxes, affect only factors of production. Note, however, that we do not make both factors worse off when we shift from a profits tax to a CVA. Adding labor income to the tax base more than offsets the tax credit given to investors from the point of view of the net rental.

We encounter the same phenomenon when we shift from an IVA or GVA to a consumption tax--the net reward (in terms of consumption goods) to both factors is reduced. The reason here is straightforward. Because we measure factor returns in terms of consumption goods, the entire rate of consumption tax is applicable to them. Thus to produce an equivalent effect upon factor incomes, a direct tax system must have the same tax base as the consumption tax (ie. a CVA). The bases of the IVA and GVA are greater than that of the consumption tax; hence the tax rates of the former are lower than those necessary to reduce factor returns to that level produced by the consumption tax.

B. Prices and Outputs Allowed to Adjust

Our next step will be to examine how sensitive the rankings (I) and (II) are to our assumption of caeteris paribus tax shifts. We will find that they may be quite sensitive, depending upon the relative capital intensities of our two producing sectors. To show this, let us consider an economy which is in short-run equilibrium under a GVA. Let the
equilibrium real wage rate be given by \( \bar{w}^G \) and let \( \tau^G \) be the rate of GVA. The net wage is thus \( (1-\tau^G)\bar{w}^G \). If we replace the GVA by a profits tax and allow the economy to readjust, the new equilibrium net wage will be \( \bar{w}^P \). In our previous discussion we implicitly assumed that \( \bar{w}^P = \bar{w}^G \). Recall, however, that replacing a GVA with a profits tax results in a demand shift from capital goods to consumer goods (the rate of return on investment being higher under a GVA than under a profits tax). If the capital good industry is unambiguously less capital intensive than the consumer good industry (i.e., at any wage-rental ratio the capital goods industry employs a lower ratio of capital to labor than does the consumer good industry) and the production functions exhibit constant returns to scale, a shift from capital goods to consumer goods will be accompanied by a fall in the capital-labor ratio of both industries.\(^1\) Because the pro-

\[^1\text{This can be easily demonstrated by the use of a Bowley-Edgeworth box diagram.}\]

To say that the capital-good industry is unambiguously less capital-intensive than the consumer-good industry is to say that the contract curve (CC) always lies below the diagonal. Furthermore, our assumption of homogeneous production functions guarantees that as we move along the contract curve in the \( Q \) direction we increase the capital-labor ratio of both industries.
duction functions of both industries exhibit constant returns to scale, \( w^P \) must be less than \( w^G \). If, on the other hand, the capital good industry is relatively capital intensive, \( w^P \) must be greater than \( w^G \). In the former case the net wage increases by less than \( \tau^G w^G \) when we shift tax structure. In the latter case it increases by more than \( \tau^G w^G \). Only if constant opportunity costs exists will the net wage rise by exactly \( \tau^G w^G \).

Nothing of what we have said so far has shown the rankings (I) and (II) to be an incorrect index of changes in the functional distribution of income. As long as the net wage, in terms of consumption goods, does not fall, labor is better off under a profits tax than under a GVA. But can the net wage fall? We know from the properties of homogeneous production functions that as the gross wage falls the gross rental on capital must rise. The elasticity of the gross rental with respect to the gross wage is given by the relative share of labor in the consumer good industry.\(^1\)

\(^1\) To see this, consider the production function of the consumer good industry

\[
O_c = F_c(L_c, K_c) = L_c F_c(1, m_c) = L_c f(m_c)
\]

where

\[
m_c = \frac{K_c}{L_c}
\]

In equilibrium

\[
w = f - m_c f' \\
r = f'
\]

Hence

\[
w = f - m_c r
\]

Footnote continued
If we can show that a fall in the gross wage, such that \( w^P < (1-T_r)w^E \), must be accompanied by a rise in the rate of return on investment, then we have demonstrated the impossibility of reducing the net real wage when we shift from a GVA to a profits tax. This is because, under our assumptions, a shift from capital goods to consumer goods must be accompanied by a fall in the marginal propensity to save; the latter, in turn, can only be accomplished via a reduction in the rate of return to investment. To show this we make use of the following production possibility diagram.

(Footnote continued)

and \( \frac{\partial w}{\partial r} = m_c \)

Now \( r_1 = \frac{\partial w}{\partial r} \cdot \frac{r}{w} = m_c \frac{r}{w} = \frac{rK_c}{wL_c} \) Q.E.D.
Initially, under the GVA, we were in equilibrium at point $E^G$, with output of consumption goods of $\overline{OC}^G$ and investment $\overline{OI}^G$. $\overline{OG}$ represents the real level of government taxation; hence disposable income is given by $\overline{GY}^G$. When we replace the GVA by a profits tax, our equilibrium point shifts to $E^D$, with consumer good output of $\overline{OC}^D$ and investment $\overline{OI}^D$. Since the level of government taxation remains constant, disposable income has fallen to $\overline{GY}^D$. Now at $E^D$ we are consuming more than at $E^G$ and we are doing this with a smaller disposable income. If our consumption function is of the form $C = C(Y_d, i)$ with $\frac{\partial C}{\partial Y_d} > 0$ and $\frac{\partial C}{\partial i} < 0$, then the above could only have come about if $i^D < i^G$.

We can also demonstrate that, for a certain class of production functions, there may be no conflict between $\overline{WP} < \overline{YG}(1-T)$ and $i^P < i^G$. In this case it is possible for labor to be made worse off when we switch from a GVA to a profits tax. Our proof is given as follows:

Let our production functions be of the Cobb-Douglas form. Namely

\begin{equation}
(2.14) \quad O_c = X_c L_c (1-\beta)_K^c \beta^c
\end{equation}

\begin{equation}
(2.15) \quad O_k = X_k L_k (1-\sigma)_K^c c^c
\end{equation}

We can also express these as

\begin{equation}
(2.14a) \quad O_c = X_c L_c \beta^c
\end{equation}
(2.15a) \[ o_k = L_k x_k m_k^\sigma \]

where \[ m_i = \frac{K_i}{L_i} \quad i = (c,k) \]

In equilibrium the following conditions will hold:

(2.16) \[ r = x_c \beta m_c (\beta - 1) \]

(2.17) \[ r = P x_k \sigma m_k (\sigma - 1) \]

(2.18) \[ w = x_c (1 - \beta) m_c^\beta \]

(2.19) \[ w = P x_k (1 - \sigma) m_k^\sigma \]

Now we wish to establish that

(2.20) \[ \bar{w}^\xi = \bar{w}^p (1 - \tau^\xi) \]

implies that

(2.21) \[ i_p = \frac{r (1 - \tau^p)}{\bar{r}} - \beta > \frac{r (1 - \tau^\xi)}{\bar{r}} = \bar{c} = i^\xi \]

in the case where \( \beta > \sigma \)

---

\( ^1 \)From the form of (2.21) it appears as if depreciation is not being permitted under the profits tax. However, it is easy to see that as long as rentals make up the entire tax base it makes little difference whether we introduce a depreciation allowance or not. The tax rate can simply be adjusted so as to keep the two variants at equal revenue yield. I.e. the following relationship exists between tax rates: \( \tau^p = \left[ \frac{r}{r/(r-\delta p)} \right] \bar{r} \) where \( \tau^p \) is that variant of the profits tax which permits depreciation to be deducted.
If we substitute from (2.16) and (2.17), (2.21) becomes

\[(2.21a) \quad \left(\frac{m^p_k}{m^g_k}\right) (\sigma - 1) < \frac{1 - \tau^g}{1 - \tau^p}\]

from (2.16) through (2.18) it is easily seen that

\[(2.22) \quad \frac{m^p_c}{m^p_k} = \frac{B}{S} \quad B = \frac{\beta}{1 - \beta} \quad S = \frac{\sigma}{1 - \sigma}\]

Furthermore if we combine (2.18) and (2.20) we obtain

\[(2.20a) \quad m^p_c = m^g_c (1 - \tau^g) \frac{1}{\beta}\]

Together with (2.22) this yields

\[(2.23) \quad \frac{m^p_k}{m^g_k} = \frac{1}{\beta}\]

If we substitute from (2.23) into the inequality (2.21a) we obtain

\[(2.24) \quad (1 - \tau^P) > (1 - \tau^g) \frac{1 + \beta - \sigma}{\beta}\]

Equating tax functions and substituting we find

\[(2.25) \quad \tau^P = [(1 - \tau^g) \frac{1 - \beta}{\beta} + (1 - \tau^g)^{-1} \frac{m^p_c}{m} (\frac{1 - \beta}{\beta})] \tau^g\]

where \(m = K/L\) and by assumption \(m^p > m\). Substituting from (2.25) into (2.24) and using Taylor series approximations
we obtain

\[(2.26) \quad 1 - \frac{1+\beta-\sigma}{\beta} \tau^g < 1 - \left[ (1-(1-\beta) \frac{m_c^p}{m} - (1-\beta)) \right] \]

which reduces to

\[(2.27) \quad (1-\sigma) > (1+\tau^g) \frac{m_c^p}{m} (1-\beta) - (1-\beta) \tau^g \]

Now (2.27) is clearly a function of $m_c^p/m$. It can be shown that (2.27) is satisfied when $m_c^p/m$ is set at its minimum value and is violated when $m_c^p/m$ is at its maximum value. To see this we use the identity.

\[(2.28) \quad m = \varphi m_c + (1-\varphi)m_k \]

where $\varphi = L_c/L$ and serves as the weighting factor. Furthermore from (2.22) we find that we can rewrite (2.28) as:

\[(2.29) \quad \frac{m_c^p}{m} = \frac{1}{\varphi + (1-\varphi)S/B} \]

Clearly, for any given $\sigma$ and $\beta$, $\frac{m_c^p}{m}$ is at a maximum where $\varphi = 0$. I.e.

\[(2.30) \quad \frac{m_c^p}{m}_{\text{max}} = B/S \]

If we substitute from (2.30) into (2.27) we obtain
which is clearly untrue. On the other hand we can see from (2.29) that

\[
(2.31) \quad 1 - \frac{\beta}{\sigma} > \tau \left[ \frac{\beta}{\sigma} - \frac{1-\beta}{1-\sigma} \right]
\]

at \( \phi = 1 \). Plugging (2.31) into (2.27) we obtain

\[
(2.32) \quad 1 - \beta + \sigma > 1 + \frac{1-\beta}{\beta}
\]

which is clearly true. Thus, in the case of Cobb-Douglas production functions, whether or not the net real wage can fall when we shift from a GVA to a profits tax, depends upon, among other things, the proportion of output devoted to the production of consumer goods (indicated by the size of \( \phi \)). The larger the output of consumer goods, all other things constant, the less likely the net wage will fall when we shift from a GVA to a profits tax. Q.E.D.

What is the significance of these findings? They suggest that it may be possible to make labor worse off when a profits tax is substituted for a GVA. This is a bit surprising since intuitively it does not seem possible that a secondary force—the reallocation of factors of production—could undo the effects of the primary force—a change in tax structure. No increase in the size of the capital stock is
necessary to achieve this effect; the reallocation of existing capital stock can do the trick.

Under what conditions can this "pervasive" result be realized? There appears to be two primary factors at work. First, the saving rate must be highly sensitive to changes in the rate of return. If the interest-elasticity of saving were zero there would occur no demand shift from capital goods to consumer goods and hence no relative price change. A necessary condition, therefore, is that the saving rate be highly interest elastic. The latter is not a sufficient condition since, as we have already seen, the presence of constant opportunity costs between the two industries results in a constant gross wage—regardless of any demand shift. A constant gross wage necessarily means a higher net real wage under the profits tax than under the GVA. This suggests that the production functions of the two industries must differ significantly in capital intensity. Not only must the capital intensities differ but they must do so in the direction of a higher capital intensity in the consumer good industry.

From what we know about the parameters of the system, it appears unlikely that this "pervasive" result could ever be witnessed. First of all there is evidence that savings is not highly interest elastic. Furthermore, it is a subject of much debate as to whether saving is even positively related to the rate of interest. Nothing can be said about relative capital intensities since little is known about them.
We have yet to inquire as to what happens to the after-tax rental when we change from a GVA to a profits tax. Fortunately, the answer is quite clear. We know that

\[(2.33) \quad i^p = \frac{r^p(1-\tau^P)}{P^P} + \delta^P \cdot \delta < \gamma = \frac{r(1-\tau^g)}{P^g} - \delta\]

From this and the fact that \(P^P \leq P^g\) it is easy to establish that

\[(2.34) \quad r^P(1-\tau^P) + \delta^P \tau^P < r(1-\tau^g)\]

Thus the after tax rental always decreases when we shift from a GVA to a profits tax. It is conceivable, therefore, that if we make the aforementioned tax shift both factors might be made worse off. One might question this conclusion on the basis that with constant taxes and the identity

\[
\text{Gross National Product} = \text{Taxes} + \text{Net Wages} + \text{Net Rentals}
\]

it is impossible to make both factors worse off. The answer is that, because we measure our variables in terms of consumption goods, gross national product falls when we substitute a profits tax for a GVA. To see this simply refer to Figure (2) on p. 51. Under the GVA, gross national product, in terms of consumer goods, is given by \(OY^g\) whereas under the profits tax it is given by \(OY^P\).
The above analysis can be generalized to the cases of shifting from an IVA to a profits tax or an IVA to a GVA. We must, however, amend our conclusions somewhat when we consider the case of replacing a CVA with a profits tax. In this case it is possible that the after-tax rental increases. Unlike the GVA or IVA, the rate of return under the CVA is equal to the before-tax rental-minus-depreciation-divided by the price of capital goods \([(r/P -\delta)]\), whereas under the GVA or IVA it was equal to the after-tax rental divided by the price of capital goods [ie. \(r(1-\tau)/P -\delta\)]. Thus the inequality (2.33) becomes

\[
(2.33a) \quad i^P = \frac{r(1-\tau^P)}{p^P} + \delta^P - \delta < i^r = \frac{r^r}{p^r} - \delta
\]

from which we can establish

\[
(2.35) \quad r^P(1-\tau^P) + \tau^P\delta < r^r
\]

or

\[
[r^P(1-\tau^P) + \delta^P] (1-\tau^r) < r^r(1-\tau^r)
\]

Thus it is possible that

\[
(2.36) \quad r^P(1-\tau^P) + \delta^P > r^r(1-\tau^r)
\]
Let us next consider replacing a CVA with a GVA. If, as before, the consumer good industry is relatively capital-intensive industry, then there is one thing we can be sure of: the after-tax rental will rise. It will rise because not only is the tax rate reduced but the before-tax rental increases. As in our earlier analysis labor may or may not be made worse off. The crucial variable in this instance appears to be the relative size of the share of capital in the consumer good industry and the share of labor in the capital good industry. If the former is greater than the latter the after-tax wage may fall (in the case of Cobb-Douglas production functions). Conversely, if the capital good industry is relatively capital intensive then we are sure that labor will always be made better off. In this case it is quite possible to reduce or to increase the net rental. These conclusions can also be generalized to the case where the CVA is replaced by an IVA.

So far the only way our original rankings could be altered is for the two sectorsto differ markedly in capital intensities and for saving to be interest elastic. When we compare the CVA with a consumption tax this is no longer true. Recall, that from our analysis of the preceding chapter, we found that there will be a higher level of consumption expenditures under a CVA than under a consumption tax. Even if constant opportunity costs prevail between the two industries this fact changes our original rankings. To
show this, let us refer to the tax constraint.

\[(2.37) \quad \tau^c \sigma^c = \tau^r \sigma^r\]

where the superscript \(c\) refers to the consumption tax system and the superscript \(r\) to the CVA system. What we have said above is that \(C^c < C^r\). We can rewrite (2.37) as

\[(2.38) \quad \tau^c \sigma^c = \tau^r (C^r + T) \]
\[= \tau^r (C^c + \gamma C^c + T) \]
\[= \tau^r \sigma^c (1 + \gamma + \tau^c) \]

whence

\[(2.39) \quad \tau^c = \frac{\tau^r}{1 - \tau^r} (1 + \gamma) \]

In our analysis of a caeteris paribus tax shift we found that

\[\tau^c = \frac{\tau^r}{1 - \tau^r} < (1 + \gamma) \frac{\tau^r}{1 - \tau^r} \]

Therefore, if constant opportunity costs exist, the consumption tax must be ranked below the CVA for both labor and capital.

If capital intensities differ, we have several cases to consider. First, if the consumption good industry is
more capital intensive than the capital good industry, the net rental will always be higher under a CVA than under a consumption tax. Labor, on the other hand, may or may not be worse off; the conditions required to make labor worse off are similar to those mentioned in the previous cases. Conversely, if the capital good industry is relatively capital intensive, the after-tax wage will always be higher under the GVA than under a consumption tax, whereas the after-tax rental may or may not be lower.

RECAPITUALATION

With the exception of the consumption tax, we have seen that the ranking of tax systems according to after-tax wages given by (I) is valid if constant opportunity costs prevail or if the capital-good industry is relatively capital intensive. Amending the ranking (I) to take into account the change of ranking of the consumption tax, we have

(III) Profits tax > GVA > IVA > CVA > Consumption tax

Similarly, if the consumer-good industry is relatively capital intensive or if constant opportunity costs exist, the ranking (II) is valid for after-tax rentals (again with the exception of the consumption tax). Our amended ranking in this case is given by:

(IV) IVA > GVA > CVA > Consumption tax > Profits tax

Furthermore, if saving is not highly interest-elastic and/or capital intensities do not differ markedly,(III) and
(IV) apply regardless of which industry is more capital intensive. Thus, it is probable that the rankings given to us by the caeteris paribus analysis are correct but the magnitudes of change indicated by it are probably too large.

EFFECTS UPON THE SIZE DISTRIBUTION OF INCOME

The determination of the effects of our different tax systems upon the functional distribution of income is not a goal in itself but only an intermediate step in determining the effects of a tax shift upon the size distribution of income. The usual approach to the latter is to first determine what changes occur in the functional distribution of income, and then, on the basis of evidence concerning factor ownership, infer what the effect will be on the size distribution of income.

A strict application of this procedure cannot be followed in our analysis, however. Such a procedure requires that there be a tradeoff between factor shares when we shift from one tax system to another. In other words it requires that the after-tax rental rise when the after-tax wage falls. This is clearly not the case when we consider substituting a CVA or consumption tax for another tax system: factor shares will often move in the same direction. The reason that this occurs in the case of the CVA is obvious. In our analysis of factor shares we did not take into account the allocation of the tax credit granted to new investment.
Since the tax credit accrues entirely to capital we need only amend ranking (IV), leaving the ranking for labor unchanged. No such procedure can solve our problem in the case of the consumption tax. It appears that little analysis is possible because of the absence of a "sacrifice" relationship here.

Our amended version of (IV) now reads

\[(IVa) \quad CVA > IVA > GVA > \text{Consumption tax} > \text{Profits tax}\]

The same conditions that were put on (IV) also apply here; the capital intensities must not differ markedly nor must saving be highly interest elastic. The rankings (III) and (IVa), with the exception of the consumption tax, indicate that a true tradeoff relation exists among our tax systems.

The usual assumption concerning factor ownership is that capital is relatively concentrated in the hands of a few whereas wages are relatively equally distributed. Under this assumption, (IVa) provides us with an index of relative income equality under our different tax regimes. The CVA leads to the most unequal distribution while the profits tax results in the most equal distribution of real income.

**LONG-RUN INCIDENCE**

In the long run the supply of capital is no longer inelastic. The stock of capital at any given point of time depends upon the past time path of savings. The latter is, in turn, a function of the time paths of real disposable
income and interest rates. Tax policy affects both disposable income and the rate of return to savings. However, as we have previously shown, any system of direct taxation affects real disposable income in the same way. If saving is interest inelastic, all systems of direct taxation will result in the same time path of capital accumulation. Consequently, gross factor rewards will exhibit the same time path for any system of direct taxation. In such a case a comparison of the effects upon net factor rewards of different forms of direct taxation can be made simply by looking at rankings (III) and (IV). That is, the long-run rankings are equal to the short-run rankings.

A consumption tax, on the other hand, will reduce real disposable income by a greater amount than will a direct tax of equivalent yield. Hence consumption will be lower and savings higher under a consumption tax than under a direct tax. As a result the consumption tax will increase the rate of capital accumulation relative to the direct tax. To show this, consider a one-sector economy where the government taxes and consumes a constant proportion, \( \tau \), of total output, \( Y \). Also let the labor force grow exponentially at the rate \( n \), and let a constant proportion, \( (1-s) \), of disposable income, \( Y_d \), be spent upon consumption by the private sector. Under any direct tax system, the economy can be described by the following set of equations:

\[
(2.40) \quad Y = F(K,L) = LF(m,l) = Lf(m)
\]
(2.41) \[ w = F_1(K, L) = f(m) - mf'(m) \]

(2.42) \[ r = F_K(K, L) = f'(m) \]

(2.43) \[ L = L_0 e^{nt} \]

(2.44) \[ \dot{K} = s(1-\tau) Lf(m) - \delta K \]

(2.45) \[ T = \tau Y \]

Equation (2.40) is the production function for our single output and is assumed to be homogeneous of the first degree. Equations (2.41) and (2.42) are the demand functions for the factors of production, while (2.43) and (2.44) are the supply functions.

Solow has shown that the balanced-growth capital-labor ratio \( \bar{m} \) is given by the solution to the following differential equation:

(2.46) \[ \ddot{m} = s(1-\tau)F(m, l) - (\delta + n)m = 0 \]

Under a consumption tax, on the other hand, the balanced growth capital-labor ratio is given by:

(2.47) \[ sF(m, l) - (\delta + n)m = 0 \]

which can be derived as follows:

\[ C = (1-s) Y \left( \frac{1}{1+\tau^c} \right) \]

\[ S = Y - C - T = Y - C(1+\tau^c) \]

or

\[ S = sY \]

Thus in such an economy there is no difference in the rate of capital accumulation when we shift from a no-government economy to a government economy when the latter is financed by a consumption tax. Graphically we can depict (2.46) and (2.47) as

\[ \text{Figure (3)} \]

Clearly, the gross wage will be higher and the gross rental lower at \( \bar{m}^c \) than at \( \bar{m}^d \). An interesting question is whether the after-tax wage is higher at \( \bar{m}^c \) than at \( \bar{m}^d \). The answer depends upon the elasticity of substitution of the function \( F \). We can expect to answer in the negative for those
cases where the elasticity of substitution is high and in the affirmative if the elasticity is low. Our answer will also depend upon the type of direct tax which is imposed. The higher the rate of tax on labor relative to that on capital, the more likely that the net wage at \( m^c \) exceeds that at \( m^d \).

We will compare the consumption tax with the CVA since the two are often compared. We will also assume that \( F \) is Cobb-Douglas because the latter is often proposed as empirically relevant. Under these assumptions the real net wage in balanced growth under the consumption tax is given by:

\[
(2.48) \quad \bar{w}^c = \left( \frac{1}{1+\tau^c} \right) (1-\beta) \bar{m}^c \frac{\beta}{\beta-1} \\
= \left( \frac{1}{1+\tau^c} \right) (1-\beta) \left( \frac{n+\delta}{s} \right)
\]

with \( \tau^c = \frac{\bar{t}}{1-s-\bar{t}} \)

For a CVA we have

\[
(2.49) \quad \bar{w}^r = (1-\tau^r)(1-\bar{t}) \quad (1-\beta) \left( \frac{n+\delta}{s} \right)
\]

with \( \tau^r = \frac{\bar{t}}{1-s+\bar{t}s} \)

It can be shown that as long as \( s \leq \beta \), \( \bar{w}^c > \bar{w}^r \). That is, as long as the rate of savings is less than or equal to
the gross share of capital, labor is made better off in the long run by substituting a consumption tax for a CVA. It should be noted that $s = \beta$ is the "golden rule" rate of savings; i.e., that rate of savings at which balanced growth per-capita consumption is at its maximum. Empirically, $s$ is less than $\beta$ in the U.S.

If we compare the real net wage in balanced growth under the profits tax ($w^P$) with $w^c$ we find that the former always exceeds the latter if $F$ is Cobb-Douglas. The balanced growth net wage under the profits tax is given by

$$w^P = (1-\tau) \left(1-\beta\right) \left[\frac{n+\delta}{s}\right]^\frac{1}{1-\beta}$$

It can easily be shown that $w^P > w^c$ so long as $\beta < (1-\beta)$; i.e. the gross share of capital is less than the gross share of labor.

What can we say about the return to capital in the above cases? In the long-run, the correct measure of the welfare of capitalists is not net rental but the rate of return earned on their investments. Since the rate of return on investment is equal to the gross rental minus depreciation for both the CVA and consumption tax, it is clear that capital will be worse off under a consumption tax than under a CVA. Furthermore, the rate of return will be higher under a consumption tax than under a profits tax. To show this simply compare rates of return in balanced growth;
\[
(2.51) \quad \bar{i}^c = \beta \left( \frac{n + \delta}{s} \right) - \delta
\]

\[
(2.52) \quad \bar{i}^p = \beta \left( \frac{n + \delta}{s} \right) \left( \frac{1 - \tau / \beta}{1 - \tau} \right) - \delta
\]

Since \(1 / \beta > 1\) it is clear that \(\bar{i}^c > \bar{i}^p\).

If one doesn't feel that the Cobb-Douglas form violently misrepresents reality and if the rate of return does not affect the desire to save, the following ranking of taxes will hold for the real net wage.

Profits tax > Consumption tax > CVA

The ranking for capital is given by

CVA > Consumption tax > Profits tax

Furthermore, the direct taxes can be ranked according to their short-run ranking. That is,

Profits tax > GVA > IVA > CVA

for labor, and

CVA > IVA > GVA > Profits tax

for capital. The latter two rankings hold for any production functions.

VARIABLES SAVINGS RATE

Thus far we have considered only the case where saving is interest-inelastic. We must now consider the case where the rate of capital accumulation is positively related to the rate of return of investment. Under a direct tax system, the balanced growth capital-labor ratio is given by the solution to
(2.53) \[ \dot{m} = s(i)(1-\tau)F(m,1) - (\delta+n)m = 0 \]

where

\[ i = g(m) \]

such that

\[ \frac{\partial g}{\partial m} < 0 \]

Clearly, the form of \( g \) depends upon the type of direct tax system which is in effect. For example, \( g \) will yield a higher \( i \) and consequently a higher \( s \) for any given \( m \) under a CVA than under a GVA. This can be depicted graphically as follows:

Figure (4)

Let curve A represent the system under a CVA and similarly let curve B represent the system under a GVA. Note, first of all, that the CVA yields a higher balanced-growth capital-labor ratio (\( \bar{m}_r \)) than does the GVA (\( \bar{m}_g \)). This is the consequence of the fact that the rate of capital accumulation is higher
under the CVA than under the GVA. Notice, also, that the two curves are identical up to the capital-labor ratio \( m_0 \); now \( m_0 \) is that capital-labor ratio which yields the maximum savings ratio under the GVA. This maximum is at most one and is reached at a larger capital-labor ratio under the CVA. \( m_1 \), on the other hand, is that capital-labor ratio which yields a zero savings rate under the GVA. Under the CVA a zero savings rate is achieved at an \( m > m_1 \).

We can conclude from the above that the higher the rate of return for a given capital-labor ratio the higher will be the balanced growth capital-labor ratio. We have already ranked our direct tax systems according to their effect upon the rate of return. The ranking is

\[ \text{CVA} > \text{IVA} > \text{GVA} > \text{Profits tax} \]

This ranking, along with the above, implies that the level of gross (before tax) wages will be higher and gross rentals lower in balanced growth, as we pass from a profits tax to a CVA tax. It is clear that the tax systems which reduce short-run after-tax wages to the greatest extent are precisely those which give rise to the largest gross wage in the long-run.

The question, then, immediately comes to mind as to whether tax systems which are relatively harsh on labor, in the short-run, can make labor better off in the long-run. Indeed, this will be an important consideration when deciding whether or not to replace a profits tax with a value-added tax.
If it can be shown that a temporary sacrifice on the part of labor will result in a higher level of take-home pay in the future, then we have a powerful argument against those who reject the value-added tax on distributional grounds. If, on the other hand, it can be shown that the lot of labor will not be improved, the proponents of value-added taxation will have to seek its justification on other grounds. Economic growth may be a goal of society but it may be hard sell at the expense of distributional considerations.\(^1\)

GVA vs PROFITS TAX

The first case that we will consider is a comparison of a GVA and a profits tax. In order to refresh the mind of the reader, the model to be examined is given below:

\[
Y = F(K,L) = LF(m,l) = Lf(m)
\]

\[
w = F_L(K,L) = f(m) - mf'(m)
\]

\[
r = F_K(K,L) = f'(m)
\]

\[
-\bar{w} = w(1-\tau+\gamma\tau)
\]

\[
\bar{r} = r(1-\tau)
\]

Equations (2.57), (2.58) and (2.61) are simply definitional equations, defining the net wage, net rental and rate of interest respectively. Equation (2.62) is our tax constraint which states that taxes shall be maintained at a constant proportion, $\tau$, of total output. $\gamma$ serves as a shift parameter. Under a GVA $\gamma = 0$ and under a profits tax $\gamma = 1$.

The system of equations (2.54) - (2.62) will yield a solution, given an initial capital stock, for any time period in the future. To arrive at the balanced growth solution, we merely set $K/k = n$ and solve. Doing this we find that the above system reduces to

\begin{align*}
(2.63) \quad \bar{w} - (1-\tau+\gamma\tau)(f(m)-m_t f'(m)) &= 0 \\
(2.64) \quad n + \delta - s(i)(1-\tau) \frac{f(m)}{m} &= 0 \\
(2.65) \quad i + \delta - f'(1-\tau) &= 0
\end{align*}
In order to determine whether shifting from a GVA to a profits tax will increase the net wage, we will evaluate the partial derivative of \( \bar{w} \) with respect to \( y \), in the range \( 0 \leq y \leq 1 \). If this partial derivative is positive then \( \bar{w} \) is higher under the profits tax than under the GVA, and vice versa.

In general we find that the sign of \( \frac{\partial \bar{w}}{\partial y} \) depends upon the elasticity of substitution and the elasticity of saving with respect to the interest rate. The higher the elasticity of substitution and the lower the elasticity of saving, the more likely that \( w^* \) is higher under the profits tax than under the GVA. In particular, if the production function is Cobb-Douglas and the rate of profits tax is less than a half, then \( \frac{\partial \bar{w}}{\partial y} > 0 \). These are sufficient but not necessary conditions. This result can be arrived at in another way without the restriction on the profits tax. When we replace a profits tax with a GVA it must be that the rate of interest is lower under the former than under the latter. I.e.

\[
(2.67) \quad r^P(1-\tau^P) - \delta < r^G(1-\bar{\tau}) - \delta
\]

or

\[
\frac{\beta_m^P(\beta-1)}{(1-\tau^P)} < \frac{\beta_m^G(\beta-1)}{(1-\bar{\tau})}
\]

This reduces to

\[
(2.68) \quad \frac{m^P}{m^G} > \left( \frac{1-\tau^P}{1-\bar{\tau}^P} \right)^{\frac{1}{1-\beta}}
\]
In order for the net wage to be higher under the GVA than under the profits tax, it must be that

\[ (2.69) \quad \frac{w}{w'} (1-\tau) > \frac{w'}{w} \]

or

\[ \frac{w_p}{w'G} < (1-\beta \tau_p) \]

Combining (2.68) and (2.69) we have

\[ (1-\tau_p)\beta < (1-\beta \tau_p) \]

but

\[ (1-\tau_p)\beta = 1 - \beta \tau_p + \frac{\beta(1-\beta)\tau_p^2}{2} - \frac{\beta(1-\beta)(2-\beta)\tau_p^3}{6} + \ldots \]

\[ = 1 - \beta \tau_p + \Delta \quad \Delta > 0 \]

Thus (2.68) and (2.69) are contradictory. This means that the net wage under the profits tax must always be higher than the net wage under the GVA when the production function is Cobb-Douglas.

**GVA vs. IVA**

The model to be examined in this case is the same as the preceding model except for the following changes:

\[ (2.57a) \quad \bar{w} = (1-\tau)w \]

\[ (2.58a) \quad \bar{r} = r(1-\tau) + \gamma \tau \delta \]
(2.62a) \( \tau(rK+wL) - \gamma \tau \delta K = \overline{\tau}(rK+wL) \)

Under the GVA, \( \gamma = 0 \), and under the IVA, \( \gamma = 1 \). In balanced growth this system can be reduced to:

(2.70) \( \bar{w} - (1-\tau)[f(m) - mf'(m)] = 0 \)

(2.71) \( n + \delta - S(i)(1-\tau) \frac{f(m)}{m} = 0 \)

(2.72) \( i + \delta - f'(1-\tau) - \gamma \tau \delta = 0 \)

(2.73) \( (\tau - \overline{\tau}) - \tau \gamma \frac{m}{f(m)} = 0 \)

If we take the partial derivative of this system with respect to \( \gamma \), in the range \( 0 \leq \gamma \leq 1 \), we find that \( \frac{\partial \bar{w}}{\partial \gamma} \) is strictly negative, regardless of the elasticity of substitution and the elasticity of the saving rate. Therefore, by permitting economic depreciation under a value-added tax we can only make labor worse off, both in the long-run and in the short-run. Finally, in the case of Cobb-Douglas production functions, we have established that the long-run net wage can be ranked as follows

Profits tax > GVA > IVA
CVA vs. IVA

This turns out to be the most difficult case to evaluate, even if the production function is Cobb-Douglas. The use of a shift parameter does not succeed here. The two taxes are identical in every respect, save one. The CVA permits depreciation to be taken instantaneously whereas the IVA allows economic depreciation. It has already been shown that the CVA results in a higher level of savings than an IVA. We will now show that if the production function is Cobb-Douglas and if the interest elasticity of saving is sufficiently high, the CVA will result in a higher balanced growth net wage than will the IVA.

Furthermore, the relative size of the net wage, under the two tax systems, is intimately connected with the rate of growth of the labor force. It will be shown that, if the labor force is stationary and the production function is Cobb-Douglas, then the net wage will be higher under a CVA than under an IVA.

When the labor force is not growing, balanced growth corresponds to a stationary state. In a stationary state, gross investment is equal to depreciation. Thus, if we switch from an IVA to a CVA the tax rate on wages would remain constant (initially).¹

¹Once the CVA has been introduced, however, the rate of return will rise. Thus, for a certain period, net investment will be positive before settling back to zero. In this new stationary state, the capital stock will be larger and hence depreciation will be larger. Because of the latter the tax rate on labor will have to adjust.
Under these conditions the tax constraint for both systems can be written as follows:

\[ \tau(rK+wL) - \tau\delta K = \overline{\tau}(rK+wL) \]

or

\[ (2.74) \quad \tau = \overline{\tau}\left(\frac{1}{1-\delta \frac{K}{Y}}\right) \]

From the form of (2.74) it can be observed that the tax rate on labor income is a function solely of the capital output ratio. If the CVA and IVA resulted in the same equilibrium capital-output ratio they would have exactly the same tax rate. However, we can express \( \frac{K}{Y} \) as

\[ (2.75) \quad \frac{K}{Y} = \frac{K}{L} \cdot \frac{L}{Y} = \frac{m}{f(m)} \]

Furthermore,

\[ \frac{\partial \left( \frac{K}{Y} \right)}{\partial m} = \frac{f(m) - mf'(m)}{[f(m)]^2} > 0 \]

We have already established that \( m \) will be higher under a CVA than under an IVA; hence the balanced-growth tax rate will be higher under a CVA than under an IVA.

We can write the net wage under both tax systems as

\[ (2.76) \quad \overline{w} = (1-\tau)(1-\beta)\overline{m}^\beta \]
Substituting from (2.74) for \( \tau \) we have

\[
(2.77) \quad \bar{w} = \left[ 1 - \frac{\tau}{1 - \delta \bar{m}(1 - \beta)} \right] (1 - \beta)\bar{m}^{\beta}
\]

which is a function of \( m \) alone. Since \( m \) is higher under a CVA than under an IVA all we need do is evaluate \( \frac{\partial \bar{w}}{\partial \bar{m}} \).

\[
\frac{\partial \bar{w}}{\partial \bar{m}} = \frac{[\bar{m}(\beta - 1) - \delta (1 - \beta)[1 - \delta \bar{m}(1 - \beta)]}{[1 - \delta \bar{m}(1 - \beta)]^2}
\]

\[
+ \frac{[\bar{m}(\beta - 1) - \delta (1 - \beta)^2 \delta \bar{m}(1 - \beta)]}{[1 - \delta \bar{m}(1 - \beta)]^2}
\]

The second term of this expression is clearly positive. A sufficient condition for the entire expression to be positive is that

\[
(2.78) \quad (1 - \tau)\beta f(\bar{m}) - \delta \bar{m} > 0
\]

But we know that in a stationary state

\[
(2.79) \quad (1 - \tau)\beta f(\bar{m}) - \delta \bar{m} = 0
\]

so that (2.78) is satisfied as long as \( \beta > s \). Another interpretation can be put on the inequality (2.78). The expression \( \beta \bar{m}(\beta - 1) \) is equal to the gross rental on capital. Under a CVA the net rental on capital is given by \( (1 - \tau)\beta \bar{m}(\beta - 1) \) which is less than \( (1 - \tau)\beta \bar{m}(\beta - 1) \) because \( \tau > \bar{\tau} \). The inequality
(2.78) is satisfied as long as the after-tax rental exceeds the depreciation on the equipment.

One we allow the labor force to grow, on the other hand the expression for the net wage under the CVA is given by

\[
\bar{w} = [1 - \frac{-1}{1 - (\delta + n)\bar{m}(1-\beta)}](1-\beta)\bar{m}^\beta
\]

For any given \( \bar{m} \), (2.80) yields a lower net wage than does (2.77). Furthermore, this difference increases the larger is the rate of growth of labor, \( n \). Therefore, the faster the rate of growth of labor the greater the difference between the rates of tax under the CVA and IVA, and the smaller the difference between the balanced growth capital-labor ratios of the two tax systems.

There is little we can say, in general, about the long-run outcome of shifting from a CVA to an IVA. The best we can do is give some indication of the increase in saving rate necessary to make labor better off in the long-run under the CVA than under the IVA. Before we make such estimates, however, we will compare the CVA with a GVA and a profits tax. If we find, for reasonable values of \( s \), that the CVA increases the net wage, then we can make inferences about the CVA vs. the IVA.

**CVA vs. PROFITS TAX**

Given a Cobb-Douglas production function, the relative size of the balanced growth net wage under these tax regimes
is a function of the interest elasticity of saving. Hence, we can compare the solution for the net wage given by each tax system and from this we can infer what size increase in the saving rate is necessary to make the CVA yield a higher net wage than the profits tax. The balanced growth solution for the net wage under the CVA is given by:

\[(2.81) \quad \overline{w^p} = (1-\beta) (\frac{-n+6}{\beta-1}) s^{\beta} \frac{\beta}{(1-\gamma)(1-s^r)} \]

whereas under the profits tax we have

\[(2.82) \quad \overline{w^p} = (1-\beta) \frac{n+6}{1-\gamma} s^p \frac{\beta}{1-\gamma} \]

where \(s^r\) and \(s^p\) are the rates of saving under the CVA and profits tax, respectively. Their ratio yields

\[(2.83) \quad \frac{\overline{w^p}}{\overline{w^r}} = \frac{s^p}{s^r} \frac{\beta}{1-\gamma} \frac{1-\gamma}{(1-s^r)(1-s^r)} \]

Suppose

\[s^p = (1-\gamma)s^r \quad (0 \leq \gamma \leq 1)\]

then

\[(2.84) \quad \frac{\overline{w^p}}{\overline{w^r}} = (1-\gamma) \frac{\beta}{1-\gamma} \frac{(1-s^r)(1-\gamma)}{(1-\gamma)(1-s^r)} \]

If we expand \((1-\gamma)\) in Taylor series and ignore all terms but first two, \((2.84)\) becomes
\[ (2.85) \quad \frac{\bar{w}^p}{w^r} = \left[ 1 - \left( \frac{\beta}{1-\beta} \right) \gamma \right] \frac{1-s^r(1-\tau)}{(1-\tau)(1-s^r)} \]

now \( (2.85) \) is less than 1 as

\[ (2.86) \quad \bar{\tau} - \left( \frac{\beta}{1-\beta} \right) \gamma [1-s^r(1-\tau)] < 0 \]

It is usually assumed that \( \beta/1-\beta \) is in the neighborhood of 1/3. Hence \( (2.86) \) becomes

\[ (2.87) \quad 3\bar{\tau} - \gamma + \gamma s^r(1-\tau) < 0 \]

The solution of \( (2.87) \) for \( \gamma \), given different values of \( s^r \) and \( \bar{\tau} \), are shown in the following table.

**TABLE 2.**

Solutions to the Inequality \( (2.87) \)

\[ \gamma \quad \frac{s^r}{s^p} - 1 = \frac{\gamma}{1-\gamma} \]

<table>
<thead>
<tr>
<th>( \bar{\tau} )</th>
<th>( s^r )</th>
<th>0</th>
<th>.1</th>
<th>.2</th>
<th>0</th>
<th>.1</th>
<th>.2</th>
</tr>
</thead>
<tbody>
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<td>.15</td>
<td>.17</td>
<td>.18</td>
<td>.18</td>
<td>.20</td>
<td>.23</td>
<td></td>
</tr>
<tr>
<td>.10</td>
<td>.30</td>
<td>.33</td>
<td>.38</td>
<td>.43</td>
<td>.50</td>
<td>.60</td>
<td></td>
</tr>
<tr>
<td>.15</td>
<td>.45</td>
<td>.50</td>
<td>.54</td>
<td>.82</td>
<td>1.00</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>.20</td>
<td>.60</td>
<td>.65</td>
<td>.71</td>
<td>1.50</td>
<td>1.88</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
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<td>.75</td>
<td>.80</td>
<td>.90</td>
<td>3.00</td>
<td>4.29</td>
<td>8.50</td>
<td></td>
</tr>
</tbody>
</table>

It appears that in the relevant range of values for the parameter \( (\bar{\tau} \geq .10, s^r \geq .20) \), a substantial increase in savings is necessary to make labor better off under the CVA. Also note that these are not the increases in saving rates which are necessary initially but the increases in saving
rate from balanced growth path to balanced growth path. Initially, (ie. when the tax shift is first made), the saving rate would have to increase by a much greater amount because the rate of return in balanced growth is much lower than when the tax change is made (ie. because the capital-labor ratio is higher in balanced growth).

To show this, consider the relative size of the rate of return (including depreciation) under the two tax systems in balanced growth:

\[
\frac{R^r}{R^P} = \left( -\frac{s^P}{s^r} \right) \left( \frac{1}{1-\tau/\beta} \right) = \frac{1-\gamma}{1-\tau/\beta}
\]

When we first made the tax switch we had

\[
\frac{R^r}{R^P} = \frac{1}{1-\tau/\beta}
\]

Thus

\[
\frac{R^r}{R^o} = (1-\gamma)
\]

The balanced growth rate of return is \(\gamma\)% lower than the initial rate of return. If saving is a linear and proportional function of the rate of return we would expect the saving rate of have fallen by \(\gamma\)%.

Ie.

\[
\frac{s^r}{s^o} = (1-\gamma)
\]

Thus the saving rate would have had to increase by nearly double of what we have calculated in Table (II), when the
tax change was first put into effect. It appears extremely remote that we could ever witness an increase in the saving rate of the magnitude which is necessary to make labor better off under a GVA than under a profits tax.

**CVA vs. GVA**

The balanced growth solution for net wages under the GVA is given by

\[ w^G = (1-\beta) \left[ \frac{n+\xi}{1-\tau} \right] \left[ \frac{1}{1-\beta} \right] s^G (1-\tau) \]

If we again make the assumption that \( s^G = (1-\gamma)s^R \) and take the ratio of \( w^G \) to \( w^R \) we find

\[ \frac{w^G}{w^R} = (1-\gamma) \left[ \frac{1}{1-\beta} \right] \frac{\left[ \frac{1}{1-s^R(1-\tau)} \right]}{l-s^R(1-\tau)} > 1 \]

as

\[ 3\tau s^R - \gamma[l-s^R(l-\tau)] > 0 \]

The solutions of this inequality for \( \gamma \), given different values of \( s^R \) and \( \tau \), are easily seen to be \( s^R \) times the solutions for \( \gamma \) in the preceding section. These rates of increase of \( s \) seem quite modest in comparison with those of the previous section. However, we must think in relative terms. In the preceding case we compared a profits tax to a CVA; we should expect larger increases in \( s \) in this case. Instead of comparing absolute changes in \( s \) we should compare the percentage increase in \( s \) with the percentage increase in
rates of return. If we assume, for simplicity's sake, that savings is related to the rate of return on investment gross of depreciation (i.e. i+δ), then the elasticity of the saving rate with respect to the rate of return is given by:

\[
\frac{S^r}{S^G} - 1 = \frac{\gamma}{1-\gamma} \cdot \frac{1-\tau}{\tau - \gamma}
\]

(2.93)

where \( R^r = i^r + \delta \) and \( R^G = i^G + \delta \).

Assume that this elasticity is equal to unity. Then (2.93) yields

\[
\gamma = (1+\tau)^{-1} - 1
\]

(2.94)

\[
\gamma = \frac{1}{2\tau}
\]

If we substitute this value of \( \gamma \) into the inequality (2.92) we find that the inequality holds as long as

\[
S^r > \frac{1}{7-\tau}
\]

Given that \( \tau \leq .25 \) this implies that the inequality is satisfied as \( S^r \geq .15 \). We would expect \( S^r \) to be as least as large as .15 since this is what it is presently equal to. Furthermore, the inequality is satisfied for even larger elasticities of saving rates if saving is a function of \( i \) and not of \( R \).
Empirical evidence seems to indicate that savings elasticities are appreciably less than one. This leads me to doubt that the lot of labor could be improved in the long run by substituting a CVA for a GVA.

VARIABLE SUPPLY OF LABOR

So far we have assumed that labor offers its services for whatever price it can get. If we abandon this assumption and assume that the supply of labor is a function of the real wage, our entire analysis may be turned on its head.

In the above analysis we have argued that the relatively capital oriented taxes such as the profits tax and the GVA tend to reduce the level of savings more than the relatively labor oriented taxes such as the IVA and GVA. It was argued that the latter taxes result in a higher rate of capital accumulation and hence a higher level of per-capita income. Once we allow labor to vary in supply, this may no longer be true. There appears to be two major forces counteracting this effect. First, the labor oriented taxes will result in a smaller volume of physical output than the capital oriented taxes because of a smaller labor force. Secondly, with less labor to cooperate with it, the rental on capital will be lower, thus inhibiting the desire to save and invest. These forces could be so strong as to make the labor oriented taxes result in a lower rate of capital accumulation than the capital oriented taxes. What the net effect will be is quite uncertain.
SUMMARY

We had set out to discover the effects of our various tax systems upon the distribution of income. We discovered that little can be said in general. In the short-run, certain of the direct effects of a tax upon a factor's income can be alleviated somewhat by shifts in the allocation of factors over the different producing sectors. This force is assumed to be too weak to overcome the ranking of the tax systems under the assumption that constant opportunity costs exist between industries. First, the saving rate does not appear to be highly sensitive to changes in the rate of interest and secondly factor intensities do not differ dramatically between consumption goods and capital goods.

In the long-run it was discovered that the tax systems which reduced the take-home wage most in the short run also give rise to the highest before-tax wage in the long run. The object of our analysis was to determine whether or not the take-home wage would also rise. If we assume a Cobb-Douglas production function then the answer is no. In particular it is doubtful that the substitution of a value-added tax, of any type, for a profits tax could prove beneficial to labor in the long-run. The proponents of a value-added tax must seek their justification elsewhere.
In our previous analysis, we assumed that the economy conformed to the pattern of behavior of a classical full employment model. In such an economy, the profit and utility maximizing conditions are always satisfied. Furthermore, in response to a change in underlying conditions, the economy moves instantaneously from one equilibrium to the next; the economy is constantly in full employment equilibrium.

Because of our assumption of inelastic factor supplies, the sole determinant of the rate of investment and hence of the entire future time path of the economy is the rate of saving. Saving serves as the 'engine' of the economy. Clearly then, the only way tax structure can affect such an economy is through its effects upon consumption behavior.

In what follows, we shall drop some of the assumptions of the classical model. Instead, because of factors such as wage and price rigidities, market imperfections, adjustment problems caused by the structure of time lags, the liquidity trap, etc., we assume that the economy is not continuously at full employment. We can also assume that either the economy does not automatically tend towards full employment or if it does that the process takes time.
In this economy, the profit and utility maximizing conditions do not hold at every point of time. Instead, they represent "target" levels towards which the participants in the economy are always striving. Thus, the conditions provided us by the classical model do have validity but only in the long-run sense; they provide the trend line around which the economy oscillates.

In this new and more realistic model of the economy, the tax structure can affect behavior other than by influencing the rate of saving. In the first place, the tax structure can exert significant influence upon the short-run stability of the economy. For example, it is often argued, and probably correctly, that a poll tax results in more cyclical instability than an income tax. Secondly, because the rate of investment is no longer constrained by the level of savings (except in the meaningless ex-post sense), the tax structure can directly affect the level of investment. In the classical model, investment was determined by the level of savings; thus the only way government could increase investment was by increasing savings. In the new model, on the other hand, a tax policy directed towards increasing savings might well result in a lower level of investment.

The balance of this paper will be devoted to exploring these two aspects of tax structure. Specifically, we will attempt to determine the effect of substituting a value-added
tax for a corporate profits tax upon the short-run stability of the economy and upon the timing and level of investment expenditures. In this chapter, we shall deal with the former whereas the latter will be discussed in the next two chapters.

AUTOMATIC STABILIZATION

In a recent article, Otto Eckstein asserts that the net effect of replacing a corporate profits tax with a value-added tax is a reduction in the built-in stability of the economic system.\(^1\) His reasoning proceeds as follows: corporate profits are perhaps the most volatile component of national income—being relatively high in prosperity and relatively low in recession. As a consequence, corporate profits taxes also vary significantly over the course of the business cycle—and in a countercyclical pattern. As a matter of fact, corporate profits taxes are responsible for most of the variation in total government receipts over the business cycle.

The base of the value-added tax, on the other hand, is much more stable over the course of the business cycle; corporate profits comprise only a small portion of the tax base. Therefore, government receipts would be more stable under the value-added tax than under the corporate profits

tax. On this basis, Eckstein argues that, to substitute a value-added tax for a corporate profits tax, would be to reduce the automatic stabilizing power of the tax system. In what follows, we will investigate this proposition.

Our first task is to define precisely what we mean by the built-in-flexibility of a tax system. The most obvious definition is the change in tax receipts which results from a given change in national income. I.e.

\[ S = \frac{\Delta T}{\Delta Y} \]

where \( S \) is an index of the built-in-flexibility of a tax structure. The larger \( S \), the greater the built-in-flexibility of the tax system. According to this definition, the built-in-flexibility of the profits tax is greater than that of the value-added tax. To see this we need simply calculate \( \Delta T/\Delta Y \) for each of these tax systems. For the value-added tax (IVA) we have

\[ T = \tau^V Y \]

hence

\[ (3.1) \quad \frac{\Delta T}{\Delta Y} = \tau^V \]

Under a profits tax we have

\[ T = \tau^P \pi \]

\[ (3.2) \quad \frac{\Delta T}{\Delta Y} = \tau^P \frac{\Delta \pi}{\Delta Y} \]
In order to provide equal revenue, $\tau^p$ must satisfy

$$\tau^p \pi = \tau^v Y$$

Thus equation (3.2) becomes

$$\frac{d \tau^T}{d Y} = \tau^v \frac{d \pi}{\pi} \cdot \frac{Y}{d \pi} = \epsilon \tau^V$$

where $\epsilon$ is the elasticity of profits with respect to output. As Eckstein has argued, and as we will see below, this elasticity is greater than one. Thus the built-in-flexibility of the profits tax is greater than that of the value-added tax.

Before we can relate the built-in-flexibility of a tax system to its automatic stabilizing power, we must first define what we mean by stability. As Brown has pointed out, there is no obvious definition.¹ This can be seen by the use of the following diagram

---

Curves A and B denote the cyclical behavior of the economy under two tax systems, A and B. Tax system B gives rise to cycles of greater amplitude but of lower periodicity than tax system A. Which system provides the most stability? Clearly we cannot answer without some notion of the social welfare function.

Suppose we sidestep this problem by assuming that the tax structure which yields the lowest sum of absolute deviations from the trend line is the most desirable. Can we go directly from the size of $\Delta T/\Delta Y$ to the degree of automatic stability of a tax structure? The answer is no. We must first relate the change in taxes to changes in expenditure. There is no reason to suppose that a dollar increase in taxes under a consumption tax will result in the same reduction in private expenditure as a dollar increase in a tax on savings. To state the problem more precisely we are interested in

$$\frac{\Delta E}{\Delta Y} = \frac{\Delta E}{\Delta T} \cdot \frac{\Delta T}{\Delta Y}$$

where $\Delta E$ is the change in expenditure. Even though a tax structure A yields a higher $\frac{\Delta T}{\Delta Y}$ than tax structure B, there is no reason to suppose that $\frac{\Delta E}{\Delta T}$ will be the same under A than under B. Thus, it is conceivable that the value-added tax would provide a lower $\frac{\Delta E}{\Delta Y}$ than would a profits tax.

Even if we have ascertained that $\Delta E/\Delta Y$ is lower under a profits tax than under a value-added tax there is no guarantee that the former provides greater automatic stability than the
latter. Phillips and others have shown that the stability of the economy is very sensitive to the structure of time lags in the economy.¹ There is no guarantee that the tax systems which result in the lowest $\frac{\Delta E}{\Delta Y}$ will also produce the greatest amount of stability. Furthermore, there is no guarantee that the lag structure will remain constant as we shift from one tax structure to another.

In order to properly determine the effect of shifting from a corporate profits tax to a value-added tax upon the stability of the economy, we would have to estimate an econometric model of the U. S., using the correct lag structure, and then make the heroic assumption that our parameters and time lags remain constant when we impose the value-added tax. This is a formidable task which we have neither the time nor the resources to undertake. Instead we shall have to be content with a much more modest approach. We will study and compare the automatic stabilizing power of our two tax structures within the framework of the static Keynesian model. While it is true that this approach sheds little, if any, light upon the actual stability effects of the tax substitution, it is of some interest from a purely theoretical viewpoint. More importantly, the results of the static model can be easily translated into an estimate of the impact multiplier of an autonomous disturbance under the two tax regimes. While impact

multipliers may tell us little about the overall stability of the economy, they provide us with a measure of the short-run sensitivity of the economy to random shocks. The smaller the impact multiplier the less the economy will deviate from its equilibrium position in the very short-run. This dampening effect may be quite important to policy-makers who could bring into play discretionary measures to offset the random shock, but could do this only with a short time lag. Our work in this section is based largely upon an article by E. Cary Brown entitled "The Static Theory of Automatic Fiscal Stabilization." 1

CONSUMPTION MODEL

In order to assess the impact of a change in tax regime upon the automatic stability of the economy, it is not enough to measure the change in the pattern of tax receipts which result, but we must also relate the latter to changes in the expenditure. To begin with let us assume that

\[(3.3) \quad C = C(Y_d)\]

where \(C\) is real consumption expenditures, and \(Y_d\), real disposable income. We assume that the supply of output, \(Y\), is infinitely elastic at current market prices so that \((3.3)\) holds in money terms as well as in real terms. Also assume for the moment that investment expenditure is exogenous and

\[1\]Brown, op. cit.
that government expenditure is constant. Under these assumptions, gross national product (we will abstract from depreciation) can be written as:

\[(3.4) \quad Y = C(Y_d) + A\]

where \(A\) represents all exogenous expenditure. Brown suggests that, as a measure of the automatic stability of the economy, we use:

\[(3.5) \quad F = \frac{1}{dA/dY} = 1 - C'(Y_d) \cdot \frac{dY_d}{dY}\]

\(F\) is that amount of change in autonomous expenditure required to raise income by one dollar. Clearly the larger is \(F\) the more stable is the system. \(F\) is simply the reciprocal of the familiar income multiplier.

Under any direct tax system, we can write (3.4) more explicitly as

\[(3.6) \quad Y = C(Y-T-S^C) + A\]

where \(T\) is the level of taxation and \(S^C\) represents corporate savings. We can also rewrite (3.5) as

\[(3.7) \quad F = 1 - C' \cdot [1 - \frac{dT}{dY} - \frac{dS^C}{d\pi} \cdot \frac{d\pi^n}{d\pi} \cdot \frac{d\pi}{dY}]\]

where \(\pi\) is corporate profits before tax and \(\pi^n\) profits after
tax. It will be assumed that before tax profits are an increasing function of output, such that \( \frac{d\pi}{dY} \leq 1 \). Furthermore, we assume that corporate saving is an increasing function of after-tax profits.

If the tax system is a corporate profits tax \( (T = \tau^p \pi) \), then equation (3.7) yields

\[
F^\pi = 1 - C' \cdot [(1-\tau^p)(1-\mu S) + \tau^p(1-\mu)]
\]

where \( \mu = \frac{d\pi}{dY} \) and \( S = \frac{dS^c}{d\pi^n} \).

If, on the other hand, a GVA (=IVA) exists and the tax is borne by the factors of production in proportion to their earnings, \( F \) becomes

\[
F^g = 1 - C' \cdot (1-\tau^g)(1-\mu s)
\]

Our next step will be to evaluate \( F^g - F^\pi \). If this expression is positive then the value-added tax results in greater stability than the profits tax. Clearly if \( \tau^p = \tau^g \) then \( F^g > F^\pi \). In general, however, \( \tau^p > \tau^g \) since the taxes are of equal revenue. Specifically,

\[
\tau^g = \beta \tau^p \quad \beta = \frac{\pi}{Y} < 1
\]

substituting from (3.10) into (3.9) we find

\[
F^g = 1 - C' \cdot (1-\beta \tau^p)(1-\mu s)
\]
Now \( F^g - F^\pi > 0 \) if

\[
(3.12) \quad S > \frac{u - \beta}{(1-\beta)u}
\]

Clearly if \( \beta \geq \mu \), (3.12) is satisfied. It is easy to see that \( \beta > \mu \) if the elasticity of corporate profits with respect to income is less than one. I.e.,

\[
e = \frac{d\pi}{dY} \cdot \frac{Y}{\pi} = \frac{u}{\beta}
\]

Thus a necessary, but not sufficient, condition for Eckstein's argument to be correct is that \( e > 1 \). Rewriting (3.12) in terms of elasticities we have

\[
(3.13) \quad S > (1 - \frac{1}{e}) \frac{1}{(1-\beta)}
\]

What has been determined empirically about the sizes of the parameters in (3.13)? In the long-run, the share of profits in output has remained basically constant (i.e. \( e = 1 \)). In a static world, where adjustments occur instantaneously, it must be that the value-added tax provides more stability than the profits tax. The reason is obvious. Because the share in output of profits is constant, the built-in-flexibility of the profits tax is no higher than that of the value-added tax. On the other hand, retained earnings absorb a greater percentage of the change in taxes under the profits tax than under the value-added tax. Hence, disposable income will
change more under the profits tax than under the value-added tax.

That a static framework for the analysis of stability is entirely inadequate is quickly brought out by the above analysis. The static model is incapable of incorporating into the analysis the fact that, in the short-run, profits tend to vary substantially more than output. As a consequence, the difference in the built-in-flexibility of the two tax systems is ignored. Therefore, in the analysis that follows we shall ignore the static model and concentrate instead upon estimating and comparing the first quarter response of the economy to a change in exogenous expenditure. That is, \( F \) now becomes the reciprocal of the impact multiplier.

Returning to the inequality (3.13), we find that Lintner,\(^1\) and more recently Brittain,\(^2\) have made estimates of \( S \). The form of the equation used to estimate \( S \) is given by

\[
(3.14) \quad D_t = a + (1-S)X_{t-1} + bD_{t-1}
\]

where \( D \) is quarterly dividends and \( X \) a measure of the pool of funds which might be used for dividend purposes. In the Lintner formulation \( X \) was profits net of both taxes and depreciation. Brittain, on the other hand, argues that because


of the arbitrary nature of depreciation allowances, profits net of taxes but gross of depreciation should be employed. He argues, further, that it is the total supply of internal funds which influences dividends rather than book profits. Brittain estimated both of these models for the period 1942-1960 and found $S = .83$ for the Lintner formulation and $.85$ for his own formulation. Both parameter estimates were significant at the 1% level. It appears from these estimates that we may safely assume $S$ to be in the neighborhood of $5/6$. Plugging this value of $S$ into (3.13) we obtain

\[ \varepsilon < \frac{6}{1 + 5\beta} \]  

as a necessary and sufficient condition for the value-added tax to be more stable than the profits tax.

There has been no empirical estimates of $\varepsilon$ as such. However, C. L. Schultze has estimated $\mu$;\textsuperscript{1} from a knowledge of $\mu$ we can construct $\varepsilon$. Now $\mu$ is the product of two terms.

\[ \mu = \frac{\Delta \pi}{\Delta Y} \cdot \frac{\Delta Y_c}{\Delta Y} \]

where $Y_c$ is corporate product and $Y$ is GNP. Schultze estimated the following equation for corporate profits:

---

where $Y_{ck}$ is the capacity level of the output of the corporate sector. Schultze hypothesized that the short-run share of corporate profits in corporate product is the sum of the long-run share of profits plus a term which reflects deviations from optimum output levels. From (3.16) we can calculate $\Delta \pi/\Delta Y_c$.

\begin{equation}
\frac{\Delta \pi}{\Delta Y_c} = a + b \tag{3.17}
\end{equation}

In the post war period Schultze found $a = .28$ and $b = .20$. Thus $(a+b) = 1/2$.

Similarly, Schultze estimated the share of corporate product in GNP.

\begin{equation}
\frac{Y_c}{Y} = c + d \frac{Y - Y_k}{Y} \tag{3.18}
\end{equation}

where $Y - Y_k$ is the departure of GNP from normal. Schultze took as normal the trend level of the peaks of GNP over the post-war period. From (3.18) we can calculate

\begin{equation}
\frac{\Delta Y_c}{\Delta Y} = c + d \tag{3.19}
\end{equation}

The value of $c$ was estimated at 57% and the $d$ was estimated at 2%. Thus $c + d = 3/5$. Multiplying $(c+d)(a+b)$ we obtain
\[ \mu = \frac{\Delta P}{\Delta Y} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10} \]

We can also calculate \( \beta \) from Schultze's estimates. Remember that \( a \) stood for the long-run ratio of profits to corporate product while \( c \) represented the long-run ratio of corporate product to gross national product. Hence

\[ \beta = \frac{P}{Y} = \frac{P}{Y_c} \cdot \frac{Y_c}{Y} = a \cdot b = 0.15 \]

Thus \( \varepsilon = \frac{\mu}{\beta} = 2 \) and the inequality (3.15) becomes

\[ 2 < \frac{6}{1 + 0.75} \approx 3.44 \]

which, even after allowing for errors of estimation, appears to be true. It appears, therefore, that in this simplified model of the economy the value-added tax has a smaller impact multiplier (a larger \( F \)) than the profits tax.

Before leaving this simplified model we should inquire whether a CVA would promote greater stability than the GVA (which was discussed above). Recall that the difference between a CVA and GVA is that the former permits the deduction of investment expenditure whereas the latter does not. If the increase in exogenous expenditure is made by the government or by some other non-investment source, the value of \( F \) under the CVA is given by:

\[ (3.2\text{C}) \quad F_c = 1 - C' \cdot (1 - \tau_c) (1 - \mu S) \]
Now (3.20) is of the same form as F₇. Note, however, that τₖ > τ₇ because the base of the CVA is narrower than that of the GVA (by the amount of the investment credit). Thus Fₙ > F₇.

If, on the other hand, the increase in exogenous expenditure is caused by a rise in investment the form of (3.20) must change to take into account the increase in tax credit which will result. I.e.

\[(3.20a) \quad F_C^* = \frac{1 - C'(1-\tau^c)(1-\mu S)}{1 + C' \cdot \tau^c (1-S)} < F_C \]

If we compare Fₖ we find that

\[F_C^* > F_S \]

if

\[\frac{1-S}{(1-\mu S)i} \cdot (1-C') (1-\mu S) (1+\tau^c i - \tau^c) < 1\]

\[i = \frac{I}{Y}\]

Empirical evidence suggests that C' is in the neighborhood of .6¹ while the ratio of business investment expenditure to GNP has been about .08 (for the period 1950 - 1962).²

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We find that, as long as $\tau_c$ is less than one, this inequality is violated. Whether or not the CVA is more stable than the GVA, then, depends upon the source of the increase in exogenous expenditures. If the source is investment expenditure, the GVA has a smaller impact multiplier, whereas if the source is government expenditure the CVA is more stable.

Finally, if we consider the imposition of a flat-rate no exemption income tax we will find that the resulting $F$ is of the same form as a GVA except that a higher rate of tax is necessary under the former because it exempts retained earnings. Thus, the income tax is more stable than the GVA. Furthermore, it should be the case that the income tax is less stable than the CVA because gross business investment usually exceeds corporate savings; this implies a higher rate of tax under the CVA than under the income tax.

**MODEL WITH INDUCED INVESTMENT**

We will now consider a model where a portion of investment expenditures is a function of the level of output. There are many forms which this function could assume but we will concern ourselves with the "flexible accelerator" approach. Under this approach, investment is assumed to be equal to the difference between the actual and the desired capital stock, times a reaction coefficient which dictates the speed at which the adjustment is to take place. Furthermore, it is assumed that the desired capital stock is proportional to output in the current period. This can be expressed algebraically
\[ \dot{K} = I = b(K^d - \bar{K}) \]

with

\[ K^d = cY \]

Thus

\[ I = aY - b\bar{K} \quad a = b \cdot c \]

If we are at less than full employment this hypothesis is questionable, particularly with respect to plant and equipment expenditure. Nevertheless, there may still be bottlenecks which have to be resolved and inventories which have to keep pace with sales. In any case, most short-run models of investment include some capacity measure (which is itself an accelerator concept). Under this assumption we can write GNP as

\[(3.21) \quad Y = \theta + aY - b\bar{K} + A\]

Furthermore, under any direct tax system \( F \) becomes

\[(3.22) \quad F = 1 - C' \cdot (1 - \frac{dT}{dY} - \frac{dS^c}{dY}) - a\]

If we calculate \( F \) for the profits tax, income tax and GVA we find that the analysis of the preceding section holds. I.e.

\[ F^\pi < F^g < F^i \]

where
\[ F^\pi = 1 - C' \cdot [(1-\tau^P)(1-\mu S) + \tau^P(1-\mu)] - a \]

\[ F^g = 1 - C' \cdot [(1-\tau^g)(1-\mu S)] - a \]

\[ F^I = 1 - C' \cdot [(1-\tau^I)(1-\mu S)] - a \]

It is easy to see that the \( F \)'s of this section are simply the \( F \)'s of the preceding section minus \( a \). Thus, the rankings of the tax systems are unaffected. The reason is obvious. Because investment is a function of output alone, the relationship between a change in exogenous expenditure and investment is unaffected by tax structure. Under each of our tax systems the change in investment will be proportional to the change in output. If, on the other hand, investment were a function of internal funds, tax structure would be a crucial variable because each tax system affects internal funds in a different way. We shall consider this case in the next section.

When we consider a CVA, however, the form of \( F \) must change somewhat in order to take into account the increase in tax credit which accompanies the increase in induced investment. This force, however, is not so strong as to make the CVA less stable than the GVA. To show this, consider the expression for \( F \) under the CVA when the increase in exogenous expenditures occurs through some non-investment source.

\[ (3.23) \quad F^C = 1 - C' \cdot [(1-\tau^C)(1-\mu S) + \tau^C(1-S)a] - a \]
Comparing $F^c$ with $F^g$ we have:

\[(3.24) \quad F^c - F^g = C' \cdot [(\tau^c - \tau^g)(1-\mu S) - \tau^c(1-S)a] \]

with

\[\tau^c(Y-I) = \tau^g Y \]

or

\[\tau^g = \tau^c(1-a-i_a)\]

where

\[i_a = \frac{bK}{Y}\]

Rewriting (3.24) we find

\[(3.24a) \quad F^c - F^g = C' \cdot [\tau^c(i_a + a)(1-\mu S) - \tau^c a(1-S)]\]

Clearly for any $S$ such that $0 \leq S \leq 1$, $F^c - F^g > 0$.

Even though the introduction of induced investment reduces the stability of the CVA relative to the GVA it remains the case that $F^c > F^g$.

If the change in exogenous expenditures occurs through a change in autonomous investment, $F$ must be altered to include the increase in tax credit from both autonomous investment and induced investment. I.e.

\[F^*_c = \frac{F_c}{1 + C' \tau^c(1-S)} < F_c\]
Since $F_C$ was less than $F^c$ in the model where all investment was autonomous, it must also be less in this case. This is due to the fact that we get a change tax credit in this model which we did not get in the previous case.

**INVESTMENT AS A FUNCTION OF NET PROFITS**

Thus far we have shown that if investment is entirely exogenous or a function of output, it is not likely that the value-added tax provides less stability (in our sense) than the corporate profits tax. As we see below, however, these results crucially depend upon our assumption that, in the short-run, the value-added tax is borne by the factors of production in proportion to their earnings. Before we abandon this shifting assumption, however, we will first examine the case where a portion of investment expenditure is determined by short-run profits as well as by output. That is,

\[
I = I(\pi, Y) + I_a
\]

where $I_a$ is exogenous investment.

There has been considerable controversy over the form of (3.25). One school of thought argues that profits are important in the expectational sense; that is, they reflect expected rates of return on investment. Another school argues that profits are important for their cash flow effect; i.e. their effect upon the supply of internal funds. A good summary of this debate and of the state of knowledge in this
area can be found in a survey article by E. Kuh.\textsuperscript{1} In the present case, we will consider the effects of profits upon investment as a cash flow effect since most of the empirical effort has been made in this area.

Under the above assumptions about investment, we can write $F$ for any direct tax system as

\begin{equation}
F = 1 - C' \cdot \left[ 1 - \frac{dT}{dY} - \frac{ds}{dY} \right] - a - \eta \frac{d\pi}{d\pi}
\end{equation}

where $\eta = \frac{dI}{d\pi}$

Solving (3.26) for specific tax systems we find

\begin{equation}
F^\pi = 1 - C' \cdot \left[ (1-\tau)(1-\mu) + \tau(1-\mu) \right] - a - \eta(1-\tau)
\end{equation}

\begin{equation}
F^g = 1 - C' \cdot \left[ (1-\tau)(1-\mu) \right] - a - \eta(1-\tau)
\end{equation}

for the profits tax and the GVA, respectively. Hence,

\begin{equation}
F^g - F^\pi = C' \cdot \left[ (1-\tau)(1-\mu) + \tau(1-\mu) \right] + (\tau - \tau^p)\eta\mu
\end{equation}

Substituting $\tau^g = \frac{1}{\beta} \tau^p$, (3.29) becomes

\begin{equation}
F^g - F^\pi = C' \cdot (\beta - \mu) - (\eta - SC)\mu(1-\beta)
\end{equation}

Since we assume that \( \mu > \beta \), (3.30) is strictly negative if \( \eta > SC' \). If all retained earnings were invested, i.e. \( \eta = S \), the latter inequality holds since \( C' < 1 \). In general (3.30) will be strictly positive if

\[
(3.31) \quad S > \frac{n}{C'} + (1 - \frac{1}{\varepsilon}) \frac{1}{(1 - \beta)}
\]

We have already concluded that \( S > (1 - \frac{1}{\varepsilon}) \frac{1}{(1 - \beta)} \); hence the inequality (3.31) depends upon \( \frac{n}{C'} \). The larger the marginal propensity to invest out of net profits relative to the marginal propensity to consume, the less likely that \( F^g > F^n \). Now \( \eta \) is the product of two independent terms:

\[
\eta = \frac{dI}{dn^R} = \frac{dI}{dS^C} \cdot \frac{dS^C}{dn^R} = S \frac{dI}{dS^C}
\]

Given our earlier estimates of the parameters of (3.31) \( (\varepsilon = 2, \ C = .6, \ \beta = .15, \ S = .85) \), we find that (3.31) holds as

\[
(3.32) \quad \frac{dI}{dS^C} < .3
\]

In their recent book, John Meyer and Robert Glauber have estimated \( AI/AS_C \) at .165 for manufacturing industry as a whole.\(^1\)

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Their study covered the period 1950 - 1958. A complete specification of the model they employed is given by

\[ I = I(S^C, H, MP, r, I_{-2}) \]

where \( H \) is a measure of capacity (our accelerator), \( MP \) is the change in stock market prices—included as a measure of expectations, and \( r \) is the market rate of interest. The fit of the model was good—an \( R^2 \) of .93 was obtained.

On the basis of these findings, we might conclude that the inequality (3.32) is satisfied. Even after allowing for the statistical errors of estimation there appears to be good reason for believing that the value-added tax provides greater stability than the profits tax.

We have not yet told the whole story, however. In testing other models of investment, plausible as the above, Meyer and Glauber found that the coefficient of \( S^C \) could range from .13 all the way to .58. Simply by changing the lag structure of the model or by using alternative measures of capacity, we might completely reverse our conclusion. The model (3.33) was simply the one chosen by Meyer and Glauber as best representing reality. Thus the conclusion drawn above must be accepted with reservation.

Even more significant than the above, Meyer and Glauber

\[ \text{ibid., p. 143} \]
advanced an alternative hypothesis that investment behavior is radically different during an expansion than during a contraction. In the expansionary phase, the primary determinant of investment expenditure is the need to increase productive capacity. During such periods, either internal funds are sufficient to meet investment requirements, or if not, firms will seek external funds. The primary constraint on investment is the rate of increase of output. On the downside, on the other hand, firms are loathe to use external sources to finance investment. The primary constraint on investment in a recession is the internal supply of funds.

Because we shall treat this hypothesis much more fully in Chapter V, we won't comment on these arguments at the present. Instead, we shall simply note that the argument was strengthened by the fact that the coefficient of $S^c$ turned out statistically insignificant during expansionary phases of the business cycle. During the downswings, on the other hand, the coefficient of $S^c$ was estimated at .40. Thus it appears that the inequality (3.32) is satisfied during upswings but violated during recessionary periods. If we accept this hypothesis of investment, a direct answer to the question of the relative stability of the value-added tax versus the profits tax is not possible. In order to give an answer we must first know our initial conditions. However, the case in which we are most interested is the one where the economy is at full employment. We are interested in finding out which tax system would result in the smallest
deviation from full employment. On the basis of Meyer and Glauber's argument our answer is that the value-added tax provides greater stability than the corporate profits tax.

**SUMMARY**

We began the discussion with the observation that the corporate profits tax has greater built-in-flexibility than the value-added tax. That is, for a given change in output, the profits tax results in a greater change in tax collections than the value-added tax. In the analysis that followed we demonstrated that, in spite of having greater built-in-flexibility, the profits tax created less stability (i.e., a larger impact multiplier) than the value-added tax. The reason is, of course, that the value-added tax resulted in a smaller increase in expenditure. It is not enough to compare the built-in-flexibility of alternative tax structures in order to determine their relative stability. Instead, the change in tax receipts which result must be related to changes in expenditure. The bulk of the value-added tax (that portion which comes out of wages) affects disposable income directly, whereas the profits tax affects disposable income only indirectly (through dividends). The retained earnings mechanism serves as a buffer between the increase in profits tax and disposable income. Thus a dollar's change in tax under the profits tax results in much less of a change in disposable income than would a dollar's increase in tax under the value-added tax. Therefore, even though a value-added tax results in a smaller change in tax receipts than a
profits tax, disposable income changes less under the former than the latter. In what follows, we shall see that this conclusion depends crucially upon our assumption that the value-added tax is shifted backwards onto the factors of production.

**VALUE-ADDED TAX UNSHIFTED**

In the preceding sections we assumed that when we replace the corporate profits tax by a value-added tax, the latter is absorbed by the factors of production in proportion to their earnings. In other words, businessmen are able to shift, onto other factors of production, the entire burden of the value-added tax, except that portion which falls upon the return to capital. The result is, of course, a large increase in the rate of corporate profits. Notice that nothing has been said concerning the shifting behavior of corporations under the profits tax. However, nothing needs to be assumed; we can take corporate shifting behavior under the corporate profits tax as a datum. The analysis of the preceding sections is consistent with full, partial, or no shifting of the corporate profits tax.

Whereas the aforementioned pattern of shifting of the value-added tax is consistent with the classical full employment model with which we dealt in the first two chapters (particularly if the profits tax were not shifted), it has serious deficiencies within the Keynesian income model with
which we are presently dealing. In the first place, the prices of the factors of production are not, at least in the short-run, set according to the marginal productivity principles of the classical model. Instead, the level of wages, for example, is determined by such factors as the rate of unemployment, the rate of change of consumer prices and the level of profits (or the rate of return on capital). Secondly, money wages are extremely sticky in the downward direction. Thus firms could shift the value-added tax only by raising prices. Given our current political setting, this might prove quite difficult since firms could not point to a lower level of profits as their justification for raising prices. (Indeed, firms have found it difficult to raise prices even when profits are falling). The average level of profits would have remained the same because the two taxes are of equal yield.

Even if firms succeeded in raising goods prices, money wages might rise sufficiently so as to wipe out any gain in real profits. First, wages might rise in response to the higher levels of consumer prices. Furthermore, the higher level of corporate profits would provide labor unions with a potent bargaining weapon when making subsequent wage demands. George Perry has estimated short-run wage changes, using as independent variables the rate of return on capital, changes in the consumer price index, and the rate of unemployment.¹

His findings suggest that, in the very short-run, wage increases would absorb approximately fifty-percent of any increase in money profits. The derivation of this figure and the estimates of the model are given in the appendix to this chapter.

Thus, even if firms were able to pass the tax forward in the forms of higher prices, we would not be justified in assuming that factors of production bear the tax in proportion to their earnings. A different set of shifting assumptions appears to be called for in our analysis of the Keynesian model. The alternative model of shifting that we shall adopt is that firms bear the entire burden of the value-added tax. This is equivalent to assuming that, in the short-run, firms do not distinguish between the corporate profits tax and the value-added tax. While this may appear to be as unrealistic as our earlier assumption, there is considerable merit for this approach.

First, and most important, rigid money wages make it unlikely that part of the tax could be shifted directly onto wages in the form of a lower take-home pay. Secondly, it is also unlikely that firms would try to pass the tax forward in the form of higher prices; the pressure of public opinion and hence the threat of government action would present too much of an obstacle. Thirdly, in the aggregate, firms would enjoy the same average level of profits under the value-added tax as under the corporate profits tax; hence there would be no sense of urgency for shifting the tax. Finally, this shifting assumption is of the opposite extreme to our
earlier shifting assumption; combined with our earlier assumption, this approach enables us to put limits on the possible outcome of the shift in tax structure.

This shifting assumption is at best a short-run behavioristic assumption. In the long-run firms will seek their new optimal positions with respect to the value-added tax. What we argue, however, is that this new equilibrium point can be reached only through changes in investment behavior and not through short-run pricing behavior.

Our next step will be to reexamine the analysis of the preceding sections in the light of this new shifting assumption.

**CONSUMPTION MODEL**

We shall begin with the simple model in which all investment expenditure is exogenous. Let $F_g$ be the stability factor of a system where the value-added tax (GVA) is shifted. Similarly let $F_\pi$ and $F_G$ represent the stability factors for a profits tax and a non-shifted value-added tax, respectively. From our previous analysis we saw that

$$F_g = 1 - C' \cdot [(1-\tau_g)(1-S\mu)]$$

and

$$F_\pi = 1 - C' \cdot [(1-\tau_\pi)(1-S\mu) + (1-\mu)\tau_P]$$

Since

$$\pi^n = \pi - \tau_g Y$$

in the non-shifted value-added tax case, it can be deduced that
Clearly \( F_g < F \). That is, the non-shifted value-added tax provides less stability than the shifted value-added tax. Furthermore,

\[(3.35) \quad F_g - F^r = C' \cdot [(1-\tau_g)(1-S\mu) + S\tau_g(1-\mu)]
- C' \cdot [(1-\beta\tau_g)(1-\mu S) + S\beta\tau_g(1-\mu)]
\]

which is less than zero as

\[(3.36) \quad Z = (1-\mu S)(\beta-1) - (1-\mu)(\beta S-1) < 0
\]

Now \( Z \) is a function of \( S \), and the latter must fall into the range \( 0 \leq S \leq 1 \). Evaluated at \( S = 1 \), \( Z \) becomes

\[Z = (\beta-1) - (\beta-1) = 0\]

at \( S = 0 \), it becomes

\[Z = (\beta-1) - (1-\mu) < 0\]

because \( \mu > \beta \). Furthermore,

\[
\frac{dZ}{dS} = (1-\beta)\mu - (1-\mu)\beta > 0
\]
By the Mean Value Theorem we can deduce that $F^\pi > F_2^g$. Q.E.D.

Thus, if the value-added tax is fully absorbed by business, it will provide less stability than the corporate profits tax. On the other hand, we have already shown that if the value-added tax is shifted backwards it will provide greater stability than the corporate profits tax. The reason for this turnabout is obvious. When the value-added tax is fully absorbed by firms, it falls completely upon profits. Therefore, it must have exactly the same effect upon expenditure as a profits tax. Recall, however, that the profits tax has greater built-in-flexibility than the value-added tax. (Tax receipts change to a larger extent in response to a change in output.) Since we are trying to isolate

$$\frac{\Delta E}{\Delta Y} = \frac{\Delta E}{\Delta T} \cdot \frac{\Delta T}{\Delta Y}$$

for each tax system, it must be that $\Delta E/\Delta Y$ is smaller under the profits tax than under the (non-shifted) value-added tax. In our analysis of the shifted value-added tax, on the other hand, a dollar's change in taxes had a larger effect upon expenditure under a value-added tax than under the profits tax (i.e. $\frac{\Delta E}{\Delta T}$ was larger in absolute value). This, of course, was due to the fact that the bulk of the value-added tax affected disposable income directly, whereas a good deal of the profits tax was absorbed by retained earnings.

The above analysis is based upon the assumption that all
investment expenditure is exogenous. However, we can generalize these results to the cases where investment is a function of output and/or profits. Adding these assumptions will only make the profits tax even more stable than the value-added tax.

**SUMMARY AND CONCLUSIONS**

We have seen, in this chapter, that in a world where all adjustments are instantaneous (or occur at least within one quarter), we can make certain statements concerning the relative stability of the value-added tax as compared with the corporate profits tax. It is not true, however, that the response to changes is so immediate. Instead it is true that the response will be made according to some distributed lag pattern. The stability implications of a tax structure are a product of the interactions of the distributed lag patterns of economic variables. This is the classic indictment of the value of static analysis which is directed towards stability questions. Stability discussions properly belong within the sphere of dynamic analysis. It has been shown that the stability of an economic system hinges crucially upon its lag structure. This is easily seen in the cases of the cobweb cycle and simple models of business cycles.

Specifically related to the area of automatic stabilization is the work of Phillips.¹ Phillips has shown that the stability implications of certain fiscal structures, such as

¹Phillips, *op. cit*
the personal income tax, were crucially dependent upon the lag structure of the government tax and private spending equations. He was able to demonstrate that certain automatic stabilizers may turn out to be destabilizing.

Before any meaningful discussion of automatic stability can be begun, therefore, a thorough knowledge of the lag structure of the economy is necessary. The most we can do with static models is to obtain impact multipliers. Indeed our $F$ is precisely the impact multiplier of changes in exogenous expenditure. While impact multipliers are important they are useful only in determining the level of next quarter's income. What happens to income in subsequent periods cannot be ascertained. We can only guess that a system with a smaller impact multiplier will also produce more overall stability.

Thus the conclusions of this chapter must be accepted with serious reservation. In the chapters that follow we will attempt to determine the effects of changing tax structures upon the time paths of several of the components of GNP. Ideally, this should be done in a general equilibrium context. However, the sheer magnitude of the task forces us to be content with a more partial analysis of these components. In any case, this approach has the merit of studying the dynamic effects of the tax structure shift, something which is impossible in our current static framework.

With the aforementioned reservations in mind, what can we conclude about the relative stability of a profits tax
and a value-added tax? We have seen that the relative stability hinges crucially upon how the value-added tax is shifted. The value-added tax probably cannot be shifted in the short-run so that its immediate effect will be to reduce the stability of the economy. In the long-run, however, firms will tend towards the position dictated by the classical full employment model. Firms will tend to substitute capital for labor because, at the current wage-profit ratio, the existing capital-labor ratio is too low. This will have the effect of increasing the sensitivity of before-tax profits to changes in output (ie. \( \mu \)). This in turn will tend to increase the stability of the economy. To see this, simply consider the expression for \( F \) under the non-shifted value-added tax:

\[
F_g^e = 1 - C' \cdot [(1-t^g)(1-S^g) + S^g(1-S^g)]
\]

Now

\[
\frac{dF_g^e}{d\mu} = C' \cdot [- (1-t^g)S - S^g\mu] > 0
\]

Thus in the long-run the system will be more stable than in the short-run. Whether or not this force can be so strong as to eventually make the value-added tax more stable than the profits tax cannot be known. It is, however, a possibility. If it does, we might safely argue that the non-shifting model has greater applicability in the short-run whereas, the shifted model has greater validity in the long-run.
APPENDIX

The equation used by Perry\(^1\) to estimate the quarterly rate of increase in wages is given by:

\[
\begin{align*}
\Delta w &= -4.313 + .367 C_{-1} + 14.711 U_{-1} + .424 R_{-1} + .796 \Delta R + \epsilon \\
R^2 &= .870
\end{align*}
\]

where \(w\) is the wage rate, \(C\) is the change in the cost of living index, \(U\) is the unemployment rate, \(R\) is the rate of return on capital, and \(\Delta\) indicates quarterly changes. Assuming the capital stock is held fixed we can rewrite (A.1) as

\[
\begin{align*}
\Delta w &= -4.313w + .367w_{-1} + 14.711 U_{-1}w + .424 \pi_{-1} \frac{W}{K} + .796 \pi \Delta \pi \frac{W}{K}
\end{align*}
\]

where \(\pi\) is the level of profits. Given initial conditions (A.2) yields the time path of the wage rate. When we impose the value-added tax and allow prices and profits to rise the initial conditions are changed and (A.2) will give rise to a new time path of the wage rate. What we seek is to isolate

\(^1\)Perry, op. cit
the difference between these time paths. If we denote the change in wage rate under our original initial conditions by \( \Delta w \) and let \( \Delta w \) represent the change in wage rate under a value-added tax, then we want to determine \( \Delta w - \Delta w \).

Assuming that the unemployment rate remains unchanged and that \( c_{-1} = 0 \) and \( \pi = 0 \) under our original system, we have

\[
(A.3) \quad \Delta w - \Delta w = 0.796 \Delta \pi \frac{w}{K}
\]

as the difference in the time path of wages for the first quarter. We can ignore the lagged variables in this calculation since they are as yet unaffected.

The difference in the wage bill for the first quarter is easily calculated as

\[
(A.4) \quad \Delta \text{wages} = (\Delta w - \Delta w)L = 0.796 \Delta \pi \frac{wL}{K}
\]

It has been determined empirically that

\[
(A.5) \quad \frac{wL}{NK} = 3
\]

Hence we can rewrite (A.4) as

\[
(A.6) \quad \Delta \text{wages} = 0.796 \Delta \pi 3R \\
= 2.4R \Delta \pi
\]

Assuming that \( R = 10 \), (A.6) becomes
(A.7) \[ \Delta \text{wages} = 0.24 \overline{\Delta \pi} \]

along with equation (A.7) we must also employ the following accounting identity

(A.8) \[ \overline{\Delta \pi} = (\Delta \pi)^* - \Delta \text{wages} \]

where \((\Delta \pi)^*\) is that increase in profits which would have resulted had there been no wage increase. Equation (A.7) and (A.8) together yield

(A.9) \[ \Delta \text{wages} = 0.2(\Delta \pi)^* \]

That is, approximately one-fifth of the potential increase in profits would occur to wages in the first quarter.

In the second quarter, the price rise (which we assume to be \(1\%\)) comes into play. If we solve equation (A.2) for the second quarter, we find

(A.10) \[ \Delta \text{wages}_{+1} = 0.26(\Delta \pi)^* \]

That is by the second quarter, wage increases have absorbed approximately \(46\%\) of the increase in profits which would have resulted from the price rise \((\Delta \text{wages} + \Delta \text{wages}_{+1})\). To see we simply rewrite equation (A.2) as

\[ \Delta \text{wages}_{+1} = 0.367cwL + 0.424[\overline{\pi - \pi}] \frac{wL}{K} + 0.796 \overline{\Delta \pi}_{+1} \]
Assuming the price rise to be equal to 10\% (\tau = 10\%) and making use of identities previously employed, this collapses to

\begin{equation}
(A.11) \quad \Delta \text{wages}_{+1} = 0.11 \pi + 0.10(\Delta \pi)^* - 0.24 \pi + 0.24 \pi_{+1}
\end{equation}

Now

\begin{equation}
(A.12) \quad \frac{\pi_{+1}}{\pi} = 1 - \Delta \text{wages}_{+1}
\end{equation}

thus

\begin{equation}
(A.13) \quad \Delta \text{wages}_{+1} = 0.11 \pi + 0.09(\Delta \pi)^*
\end{equation}

We assume that profits would have doubled had there been no wage increase; hence

\[ \pi^* = 2(\Delta \pi)^* \]

and

\[ \frac{\pi^*}{\pi} = \pi^* - 0.2(\Delta \pi)^* = 1.8(\Delta \pi)^* \]

thus from (A.13)

\[ \Delta \text{wages}_{+1} = 0.26(\Delta \pi)^* \quad Q. \ E. \ D. \]

In subsequent quarters the $\Delta \pi$ term and the $\pi_{-1}$ term cancel each other out so that this iteration need not go on any further. The 46\% figure is a slight underestimate since Perry's equation applies to before tax wages. Thus a value-added tax of 10\% would have to be added giving us a total of approximately 50\%.
CHAPTER IV

THE VALUE-ADDED TAX AND INVESTOR'S RISK

VALUE-ADDED TAX - SHIFTED

One of the more probable effects of shifting from a profits tax to a value-added tax is an increase in the variance of the rate of return on capital over the business cycle. The corporate profits tax, with its high marginal rate, serves as a substantial buffer between the gross and net profits of business. The value-added tax, contrarily, because its tax base is more stable than that of the profits tax and because its marginal rate is relatively low, provides must less of a cushion for net profits.

How does variance in the rate of return affect the level of investment? It can be argued that among the variables that affect investment decisions are included the expected rate of return on the project and some objective measure of the riskiness of the project. The expected variance in the rate of return can be regarded as one possible objective measure of the risk of an investment. Thus our investment function might be written as

\[ I = I(\bar{r}, o_r^2, ...) \]

where \( \bar{r} \) and \( o_r^2 \) are the first and second moments of the rate of return.
of return on investment.

Because it is unlikely that businessmen can successfully predict \( \sigma^2_r \) for any given project, it can be argued that businessmen use as an approximation, the variability of the rate of return which they have experienced in the past. Thus, if by changing tax structure we increase the variability of the rate of return on existing assets, businessmen will likely scale upwards the level of risk associated with a new investment project. If all other things remain equal this should decrease the level of aggregate investment expenditure.

Before we can estimate the change in the variance of the rate of return caused by the shift in tax structure, we must first decide how likely it is that other things will remain equal. This question is deeply embedded in the question of the short-run shifting of the value-added tax. If we employ our assumptions of the first two chapters, taxes are not shifted in the short-run because factor supplies are inelastic. In this case the corporate profits tax is completely borne by business and the value-added tax is borne proportionately (according to factor rewards) by the factors of production. Gross factor rewards are determined by the capital-labor ratio and the latter is fixed in the short-run. Under these assumptions, the net rate of return can be written

\[
(4.2) \quad r_p^n = (1 - \tau^p) r
\]
(4.3) \[ r^n_v = (1 - \tau^v)r \]

for the profits tax and value-added tax, respectively. For simplicity's sake we will assume that the value-added tax is restricted to the corporate sector so that

\[ \tau^v = \beta \tau^p \quad \beta = \frac{\pi}{Y} \]

\[ \pi = \text{gross corporate profits} \]
\[ Y = \text{net corporate product} \]

From (4.2) and (4.3) we can calculate the variances of the net rate of return under both tax systems

(4.4) \[ V(r^n_p) = (1 - \tau^p)^2 V(r) \]

(4.5) \[ V(r^n_v) = (1 - \tau^v)^2 V(r) \]

Because we assume that \( r \) is the same for both tax systems, in the short-run, it is obvious that \( V(r^n_p) < V(r^n_v) \). (Recall that \( \tau^v : 1 \) and \( \tau^p : 0.5 \)).

However, it is also obvious that the expected rate of return on investment, \( \bar{r} \), is higher under the value-added tax than under the profits tax. As a matter of fact,

\[ \frac{r^n_v}{r^n_p} = \frac{(1 - \tau^v)}{(1 - \tau^p)} = 0.5 \]
That is, the expected rate of return under the value-added tax should be nearly double that under the corporate profits tax.

Thus, under the above set of shifting assumptions, all other things have not remained equal. The net result of shifting from a profits tax to a value-added tax is to increase both the variance and the mean of the rate of return. Unless we know what the tradeoff is between risk (variance) and the yield (mean), we cannot say what will be the effect upon the level of investment of shifting tax structure.¹

VALUE-ADDED TAX NOT SHIFTED

It was argued, in the previous chapter, that in the short-run businessmen will not be able to shift the value-added tax when the latter is substituted for a profits tax. In other words, in the short-run, businessmen are forced to

¹Suppose yield and risk enter our investment function in the following way:

\[ I(\bar{r}, o^2_r) = I(\frac{\bar{r}}{o^2_r}) = I(x). \]

That is, the tradeoff between yield and risk is proportional. In such a case it is clear that investment will be higher under the profits tax than under the value-added tax.

Proof:

\[ x^P = \frac{\bar{r}}{(1-\tau^P)V(r)}, \quad x^V = \frac{\bar{r}}{(1-\tau^V)V(r)} \quad \therefore \frac{x^2}{x^P} = \frac{(1-\tau^P)}{(1-\tau^V)} < 1. \]

Generally, however, it is assumed that people are risk-aversers. That is, for a given percentage increase in risk people demand an even greater percentage increase in yield. If this is the case, then investment will be lower under the value-added tax than under the profits tax.
treat all taxes as costs--ie. to be deducted from profits. This would imply that two taxes of equal yield result in an equal average level of profits. It follows, therefore that if we substitute a value-added tax for a profits tax, we can write

\[ r^n_v = r - \tau_v \frac{Y}{K} \]  

\[ r^n_p = r - \tau_p r \]  

where we assume that \( r \) is the same under both tax regimes, \( Y \) is value-added by corporations, and \( K \) is the value of the capital stock. It should be clear that because the taxes are of equal yield \( r^n_v = r^n_p \). We should explain what is meant by equivalent tax yield. If we were to insist on equal tax yields for any given quarter or any given year, it would be the case that the time path of the rate of return under both tax systems would be identical. Thus there could be no difference in the variance of the rate of return between the two tax systems. Clearly, this is a meaningless definition because it would be impossible to implement. Instead, we define equivalent yields as equal tax yield over the course of a business cycle. Defined in this way, the tax systems result in a different time path of \( r^n \) even though \( r^n \) remains the same for both tax systems.

If we adopt the above approach, all that remains to be done is a calculation of the resulting variances of \( r^n \)
under both tax regimes. If we find (as we will) that the variance of $r^n_v$ is greater than that for $r^n_p$ we can conclude that investment will be lower under the value-added tax than under the profits tax.

**A. Measures of Variance**

The following procedure was followed to estimate the difference in the variance of $r^n$ which would result by replacing a value-added tax for a corporate profits tax. For the period 1947 to 1962 the following data were obtained from the *Survey of Current Business*:¹ (1) net profits of corporation before tax and before inventory valuation adjustment; (2) value-added by corporate business. The period (1947 - 1962) was broken up into five sub-periods corresponding to the post-war business cycles. They are given as follows: 1947 - 1 to 1949 - 4; 1950 - 1 to 1954 - 3; 1954 - 4 to 1958 - 1; 1958 - 2 to 1961 - 1; 1961 - 2 to 1962 - 2. For these sub-periods the following variances and covariances were calculated: $V(\pi)$, $V(Y)$, and $\text{Cov}(\pi,Y)$. $\pi$ is used as a proxy for $r$ because a good capital stock series was unavailable. This shouldn't make too much difference since the capital stock will not change significantly within a business cycle subperiod.

With the above information we can calculate the variance

in net profits under the corporate income tax.

\[(4.8) \quad V(\pi_p^n) = (1-\tau_p)^2 V(\pi)\]

Similarly, for the value-added tax we have

\[(4.9) \quad V(\pi_v^n) = V(\pi) + \tau_v^2 V(Y) - 2\tau_v^2 \text{Cov}(Y,\pi)\]

Taking the ratio of (4.9) to (4.8) we have

\[(4.10) \quad \frac{V(\pi_v^n)}{V(\pi_p^n)} = \left(\frac{1}{1-\tau_v}\right)^2 + \left(\frac{\tau_v}{1-\tau_v}\right)^2 \frac{V(Y)}{V(\pi)} - \frac{2\tau_v}{(1-\tau_v)^2} \frac{\text{Cov}(Y,\pi)}{V(\pi)}\]

All that remains to be calculated is \(\tau_v^V\) and \(\tau_p\). We assume \(\tau_p\) to be .5. Actually it was much lower than this early in the period and much higher than .5 during the Korean War, when excess profits taxes were in effect. The pitfalls of this assumption will be discussed in the next section.

The rate of value-added tax, \(\tau_v^v\) was calculated as follows:

\[\tau_v^V \cdot \Sigma_i Y_i = \tau_p \cdot \Sigma_i \pi_i \quad i = (1947, 48 - -, 62)\]

That is, tax yields were equated over the entire period and not merely over any single business cycle. This is perhaps the most realistic constraint on \(\tau_v^v\) since it is unlikely that the tax rate could be manipulated frequently. The values of the ratio (4.10) for each of the subperiods is listed in column one of Table (3).
Table 3.

RATIO OF $V(\pi^n_{V})$ to $V(\pi^n_{P})$

<table>
<thead>
<tr>
<th>Period</th>
<th>TIVA</th>
<th>GVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947-1--49-4</td>
<td>3.60</td>
<td>3.75</td>
</tr>
<tr>
<td>1950-1--54-3</td>
<td>4.41</td>
<td>4.59</td>
</tr>
<tr>
<td>1954-4--58-1</td>
<td>3.45</td>
<td>3.61</td>
</tr>
<tr>
<td>1958-2--61-1</td>
<td>2.60</td>
<td>2.70</td>
</tr>
<tr>
<td>1961-2--62-2</td>
<td>2.45</td>
<td>2.43</td>
</tr>
<tr>
<td>Weighted Average</td>
<td>3.53</td>
<td>3.86</td>
</tr>
</tbody>
</table>

As can be observed, the ratios clearly indicate that the value-added tax would result in much more volatility in after-tax profits than does the profits tax. If we take a weighted average of these subperiod ratios, using the number of quarters in a subperiod as the weighting factor, we obtain a ratio of 3.53. The variance in net profits would be on the average nearly 3 1/2 times as large under the value-added tax than under the profits tax. There is, however, substantial variation among the subperiod ratios. Because of the Korean War, period (2) should be eyed with suspicion. Various abnormal constraints were in operation during much of this period. This is evidenced by the lack of correlation between value-added and before-tax profits in this period. Hence period (2) is likely an overestimate.
Furthermore, period (5) has only five observations—all of them during an expansionary phase. This probably leads to an underestimate of the relative variances. In any case, the variation in after-tax profits, under the profits tax, appears to be about one-third of that under the value-added tax.

The model of the value-added tax, tested above, was that of an IVA since depreciation is excluded from the tax base. Under a GVA (depreciation included), the following expression holds for after-tax profits.

\[
\pi^n_v = \pi - \tau^v Y - \tau^v D
\]

where \(D\) stands for depreciation. Now,

\[
\frac{V(\pi^n_v)}{V(\pi^n_p)} = \frac{1}{(1-\tau^p)^2} + \frac{\tau^v}{(1-\tau^p)^2} \frac{V(Y)}{V(\pi)} + \frac{\tau^v}{(1-\tau^p)^2} \frac{V(D)}{V(\pi)} - \frac{2\tau^v}{(1-\tau^p)^2} \cdot \frac{\text{Cov}(D, \pi) - \text{Cov}(\pi, Y)}{V(\pi)} + \frac{2\tau^v}{(1-\tau^p)^2} \cdot \frac{\text{Cov}(D, Y)}{V(\pi)}
\]

The results of the calculation (4.12) appear in column (2) of Table (3). It appears that the GVA provides slightly more variability to after-tax profits than does the IVA. Table (3) reveals that the weighted average is 3.86 under the GVA as compared with 3.53 for the IVA. This result is to be expected because, by adding depreciation, we add an element to the tax base which shows little cyclical variability.
On the basis of the above analysis, there is reason to believe that investment would be lower, on the average, under a value-added tax than under a profits tax. Note, however, that this conclusion hinges critically upon the assumption that businessmen are forced to completely absorb the value-added tax in the short-run. As we have pointed out earlier, if the value-added tax is borne by the factors of production, no such conclusion can be reached without further information as to how businessmen regard risk.

B. Some Reservations

There were two major simplifying assumptions made in the above analysis: (1) it was assumed that the basic variables of the model, (\( \pi \), \( Y \), and \( D \)) were invariant to the tax system in existence; (2) that these variables would assume the same values if total tax collections by government differed from those actually realized. We will deal with these matters separately.

If a change in tax structure does affect the level of investment, as we have hypothesized, then our assumption of the invariance of \( \pi \), \( Y \), and \( D \) is wholly unwarranted. In so doing, we are dealing with a general equilibrium system with partial equilibrium methods. For example, simple income analysis tells us that if we change the level of investment we change the level of income (value-added) and more directly, we change the level of depreciation. The latter can be ignored because investment is only a small proportion of the
existing capital stock. Furthermore the change in investment brought about by the change in tax structure is only a fraction of investment.

We cannot treat so lightly the resulting change in value-added, however. Because profits are a function of value-added, a change in tax structure which affects investment will also affect profits. But this change in profits will result in a further change in investment, and so on until a new equilibrium (if there is one) is reached. Therefore, if we believe that the substitution of a value-added tax will lower investment, our analysis would tend to underestimate the change that would actually occur.

Given this criticism of our approach, there remains considerable justification for the method we employed. First, we are interested in isolating the behavior of net profits, under different tax structures, over a typical business cycle and are not concerned with the level of investment as such. Instead, all that we have to assume is that the basic relationship between profits and value-added is unaffected by the tax structure. Given this assumption and goal our analysis retains some validity. For what we are doing, is merely comparing the effects of two tax regimes upon the variance of net profits over a typical business cycle. Whether or not a cycle of the same amplitude and periodicity would occur under the two tax systems during the period under discussion is wholly irrelevant and cannot be determined by our analysis. Our goal is to merely compare the variance
in net profits over a given business cycle.

The second problem with which we have to deal is less troublesome than the first and has been partially resolved by the discussion above. The problem is that the corporate profits tax was not at uniform level throughout the period analyzed. Instead, it started out at about 30% in the late forties, went to approximately 60% during the Korean War and then settled down to 52% in the post-Korean War period. This is not a major problem, however. As we have mentioned above, we are merely trying to determine the effect of a 50% corporate income tax over a given business cycle. Such a procedure is no more illegitimate than assuming that a value-added tax was in effect during the period. Thus our conclusions of this chapter must be amended to read: If there would be no difference in the pattern of business cycle behavior under the two tax regimes, there is reason to believe that there would be a lower level of investment under the value-added tax because of increased variability in net profits.
CHAPTER V

THE VALUE-ADDED TAX, RETAINED EARNINGS, AND INVESTMENT

In the preceding chapter we concluded that the substitution of a value-added tax for a corporate profits tax would result in a net increase in the variance of the rate of return over the business cycle. We could not, however, determine or even put limits on the change in investment which would result. To do this it would be necessary to determine which shifting pattern would be followed by business. Even if we were able to determine the latter we do not have quantitative estimates of key parameters. Hence we can make only qualitative statements about the effects of the alternative tax structures on investment.

There is, however, another avenue through which investment can be affected by a change in tax structure. And this effect is much less subtle than in the preceding case. Investment, particularly in the short-run, is often postulated to be a function of the supply of internal funds. As we have already demonstrated, the tax substitution affects both the level and the pattern of short-run profits—and hence internal funds—over the business cycle. Unlike the case of increased variance in the rate of return, there is an abundance of empirical work in this area. Using these empirical findings, we can estimate the effects of the change
in tax structure upon investment spending.

Before we can proceed we must make clear our shifting assumptions. As before, we will treat two separate cases. The first is that businessmen will not distinguish between the taxes as long as they are of equivalent yield. This, of course, is the case where the value-added tax is paid entirely from profits. The second case is that with which we have dealt throughout this paper. That is, because factor supplies are inelastic in the short-run, no shifting is possible, and the value-added tax is borne by each factor in proportion to its reward.

The model of investment and its estimation which we shall employ in this chapter is the "residual-funds accelerator" hypothesis of John Meyer and Robert Glauber.¹ Their main arguments can be summarized as follows. Most short-run models of investment are based upon either simple accelerator or capacity-output relationships on one hand or simply cash flow models of investment on the other. Meyer and Glauber reject these simple relationships in favor of an eclectic view.

A discontinuity in investment behavior occurs at the point where full utilization of productive capacity is achieved. An accelerator or capacity-output relationship is suggested as the key factor in establishing short-run

¹Meyer and Glauber, op. cit
investment budgets when capacity is fully utilized and contrarily, the level of cash funds flowing into the firm from current operations is considered a prime determinant when capacity is less fully utilized.

There exists an optimal capital-output ratio, for a given technology, which results in lowest unit costs. During a business upswing investment demands stem primarily from the desire to maintain this optimal capital-output ratio. If the supply of internal funds is not sufficient to meet these investment demands firms will seek external sources of funds. In the downswing, on the other hand, firms will invest primarily in cost-saving devices and in increased capacity to meet subsequent boom demands. However, firms are loath to employ external financing in such periods, and thus will only invest up to the available supply of internal funds.

Thus this model of investment behavior is non-linear. There must be a different investment equation for periods of upswing than for periods of downswing. This suggests that, not only is the total amount of internal funds over and given period of significance for investment, but the timing of the flow of internal funds is also of importance. A dollar increase of internal funds is much more potent for raising investment during a downswing than during an upswing.

The estimates for this bifurcated model of investment are as follows:
\[ I_t = 490.7 + .409 F_{t-1} - 433.3 r_{t-3} + .877 I_{t-2} + \text{seasonal corrections} + \epsilon_t \quad R^2 = .900 \]
\[
\begin{align*}
(126) & \quad (199.3) & \quad (217) \\
\end{align*}
\]

**Upswings:**
\[ I_t = 747.7 + 2563.3 H_{t-1} + 19.2 MP_{t-1} - 935.0 r_{t-3} + .868 I_{t-2} + \text{seasonal corrections} + \epsilon_t \quad R^2 = .977 \]
\[
\begin{align*}
(700.6) & \quad (3.4) & \quad (274.1) \\
(0.094) & \\
\end{align*}
\]

where
- \( H \) = capacity measure
- \( MP \) = stock market prices
- \( r \) = market rate of interest
- \( F \) = internal funds = net profits + depreciation - dividends

It should be noted that this model has been estimated for the manufacturing sector only and was restricted to the period 1949-3--1958-4. Furthermore, the non-linearity of the model was attested to by the fact that the coefficient of internal funds was statistically insignificant in the upswing model.

**VALUE-ADDED TAX -- NOT SHIFTED**

The first test that we shall perform will be based upon the assumption that the value-added tax is not shifted. This implies that the basic variables of the model will remain unchanged as we change from a profits tax to a
value-added tax. For example, output and prices will remain unchanged and the expression for net profits at any time $t$ is given by

$$\pi^p_n = \pi - \tau^p \pi$$

$$\pi^v_n = \pi - \tau^v Y$$

for the profits tax and the value-added tax, respectively. We again make the assumption that a 50% profits tax existed throughout the period (see p. 139).

Under the above assumptions, we can calculate the time series of internal funds that would have arisen had there been a 50% corporate profits tax or a equal yield value-added tax in effect throughout the period. These figures are shown in Table (4).

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<thead>
<tr>
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TABLE 4. (cont d)

Source: SEC - FTC Quarterly Report on Manufacturing
Note: Column totals may not add due to rounding.

From these time series it can be observed that internal funds are generally larger under a value-added tax during expansion and larger during recessions under the profits tax. Indeed, if one use GNP as a measure of upswings (47-1--48-4, 50-2--53-3, 55-1--57-2, 58-4--59-4) and recessions, one finds that the value-added tax generates six billion dollars less internal funds during recessions than does the profits tax. If we use some other measure of upswings and downswings, specifically one related to capacity utilization (as did Meyer and Glauber), this difference is greatly expanded.

This effect upon the supply of internal funds should be expected because it has already been demonstrated that the variance in net profits is much larger under a value-added than under a profits tax.

Our next step is simply to simulate the model estimated by Meyer and Glauber using the new time series on internal funds. The results of this simulation are shown in columns one and two of Table (5).

TABLE 5.

MANUFACTURING INVESTMENT UNDER THE
PROFITS TAX AND VA TAX (millions 1954 dollars)
<table>
<thead>
<tr>
<th>Year</th>
<th>PROFITS TAX</th>
<th>VA TAX</th>
<th>VA TAX (Shifted)</th>
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</thead>
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<tr>
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<td>1996</td>
<td>1845</td>
<td>2545</td>
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<td>1950-1</td>
<td>1190</td>
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TABLE 5. (cont d)

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<thead>
<tr>
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<th>PROFITS TAX</th>
<th>VA TAX</th>
<th>VA TAX (Shifted)</th>
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<tr>
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<td>1776</td>
<td>4675</td>
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<tr>
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Comparing columns one and two it can be observed that, for the period considered, the value-added tax would yield roughly 11% less investment than would the profits tax. However, this 11% refers only to manufacturing investment. Because investment by manufacturing corporation comprises only about one-half of total private investment our figure must be reduced to approximately 5 1/2%. It is likely that two other major components of private investment, residential
construction of owner-occupied housing and investment by public utilities, would be unaffected by the change of tax structure. In the case of owner-occupied housing this is because neither tax affects the stream of returns of the investment. As for public utilities, the return earned on capital is so regulated that it would remain constant under either tax regime. Furthermore, it is doubtful whether public utilities rely significantly on internal finance, since they have easy access to the capital markets.

This leaves us with the final major component of private fixed investment—construction of residential rental property and commercial rental property. Though the manufacturing sector may finance a portion of this investment expenditure through retained earnings, the bulk of such investment is financed through financial institutions particularly—banks and insurance companies. It is doubtful, therefore, that we can safely apply the same structural equations to this form of investment as we used for manufacturing investment. For lack of a better alternative, we shall have to omit this component of investment from our discussion.

There is another qualification which must be made to the above estimates. Recall that lagged investment appears in our explanatory equations. Because of this feature, our results may be biased. This is due to the fact that the lagged investment variable makes our results quite sensitive to the timing of the tax substitution. If we make the substitution during a downswing, there will be a much
greater cumulative effect upon investment than if the substitution were made during an upswing. Recall that a value-added tax gives rise to a much smaller flow of internal funds during a recession than does a profits tax. Furthermore, the internal funds variable enters the model during a downswing. Since the estimation period begins during a downswing, we have exaggerated the effect on investment of the change in tax structure.

To get an estimate of the bias created by beginning the simulation during a downswing we need only simulate the model again—except this time we should begin the simulation during an upswing. The difference between the two outcomes is a measure of the bias. The results of this simulation are shown in Table (6).

TABLE 6.


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<tr>
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Whereas the difference in investment between the two tax regimes was originally 9.6 billion, it is now only 3.7 billion. The mere fact that we began our simulation during a downswing, then, was responsible for 5.9 billion or 62% of the total decrease in investment which results when we shift from a profits tax to value-added tax. As a percentage of cumulative investment, however, this factor will tend to zero as we take a long enough period. Thus, instead of an 11% decrease in manufacturing investment we should expect only a 4 1/2% (3.7/88.0) decrease when we shift from a profits tax to a value-added tax.

**VALUE-ADDED TAX NOT SHIFTED**

We will now consider the case where the value-added tax is borne proportionately by the factors of production according to factor payments. In this case we can write profits as:

$$\pi^n_v = \pi - \tau^v \pi$$
\[ \pi_p^n = \pi - \tau^p \pi \]

for the value-added tax and profits tax, respectively. It should be clear that, in this model, the flow of internal funds will be larger under the value-added tax than under the profits tax at any point in time. Consequently, investment will also be higher under the value-added tax at any point in time. All we have to do is calculate by how much investment would be increased. The results of this calculation are shown in column (3) of Table (5).

It can quickly be seen, comparing columns (1) and (3), that there will be huge increases in investment under the value-added tax. Towards the end of the period, investment is nearly three times as high under the value-added tax than for the profits tax. For the period as a whole, investment has increased by 65%.

The above calculations must be amended somewhat to take into account the fact that dividends will have increased in the face of higher profit levels. Firms will gradually increase their rate of dividend payments until their "target" payout ratio has been reached. What this "target" ratio is, has been open to some dispute. However, Brittain\(^1\) has made a convincing argument that dividends are not based upon net profits but upon cash flow (see p. 100). Using this model,

\(^1\)Brittain, *op. cit.*, p. 275
he has estimated the target payout ratio to be in the neighborhood of 30%. Thus, approximately one-third of the increase in net profits which result from the substitution of the value-added tax for the corporate profits tax will be distributed in the form of dividends. It should be clear, however, that there may be a substantial lag before dividends are fully adjusted to the higher level of profits. Even after allowing for increased dividends we must conclude that there will be larger increases in investment if we substitute a value-added tax for a profits tax.

CONCLUSION

We have observed, once again, that the effect of substituting a value-added tax for a corporate profits tax upon the rate of investment depends completely upon how the value-added tax is shifted. If, on one hand, businessmen are able to shift the value-added backwards onto the factors of production, the resulting higher level of profits will lead to large increases in investment. On the other hand, if firms completely absorb the value-added tax, there will be a reduction in investment because the value-added tax reduces the supply of internal funds during periods of recession.

In the short-run, it appears that the latter shifting assumption is more realistic. The reasons have been advanced before and need not be repeated here (see p. 115). Thus, the immediate effect of shifting from a profits tax to a value-added tax will be a reduction in investment.
SUMMARY AND CONCLUSION

We began this study with an inquiry into the nature of value-added taxation and its relationship to other well-known forms of taxation. This analysis was conducted within the context of a classical full employment model so that we could isolate the "optimal" equilibrium solutions of the economic variables under any given tax structure at a given point of time. That is, the assumptions of the classical model enable us to isolate the "target" values of economic variables which the economy is continually striving to achieve.

Our conclusions hinge crucially upon our assumptions concerning the supply of labor and the interest-elasticity of savings. The analysis of the first chapter was based upon the assumption that the supply of labor is inelastic with respect to the real wage. In most situations, however, it is easy to extend the analysis to the case where the labor force is a function of the real wage. With these reservations in mind we can summarize our findings.

There are three variants of the value-added tax which are of interest. These variants differ only with respect to their treatment of depreciation allowances. We can have a system of value-added taxation which permits no depreciation allowance (GVA), one which allows economic depreciation to
be deducted from the tax base (IVA), or one which allows depreciation to be taken instantaneously (CVA). Unless both labor supply and savings are inelastic with respect to their respective rewards, each of these variants will yield a different pattern of goods and services at a given point of time and over time; otherwise, they will have identical output effects. Specifically, if labor supply is inelastic and saving is positively related to the rate of interest, the CVA will yield a higher rate of capital accumulation than will the IVA, and the latter will result in more investment than the GVA. The reason is straightforward—the greater the depreciation allowance (in present value terms), the higher the rate of return on investment and consequently the greater the rate of interest paid to savers.

On the other hand, if labor supply is also an increasing function of its reward, we cannot, by a priori reasoning, determine the relative effect upon investment of our different versions of the value-added tax. This is because those variants which yield the highest rate of return to saving also yield the smallest labor force. Because the level of saving is an increasing function of both the rate of interest and the level of disposable income, and because the latter is, in turn, positively related to the size of the labor force, we cannot tell, without a knowledge of the parameters of the system, what will be the effect upon investment of changing from one variant of the value-added tax to another.
Furthermore, if saving is interest inelastic and labor supply is a positive function of the real wage, the relative effect upon investment of each of the variants is obvious. Investment will be highest under the GVA and lowest under the CVA. Similarly, we can calculate the investment effects for other combinations of assumptions about labor supply and saving.

Comparing the value-added tax with other well-known forms of taxation, we find that the value-added tax, except in its CVA formulation, is not a new concept of taxation. We have shown that the GVA is equivalent to a sales tax upon the final output of the economy (GNP). This equivalence holds for any set of assumptions about labor force and saving elasticities. Furthermore, the IVA is basically equivalent to a flat-rate-no exemption-income tax. The sole difference between these taxes is that the latter provides a loss offset against bankruptcy while the former does not.

The CVA, however, is not, as has been frequently asserted, equivalent to a flat rate consumption tax. The reason is because the CVA affects only that portion of disposable income which is spent on consumer goods, whereas the consumption tax affects all of disposable income — however it is spent. The net result of shifting from a value-added tax (CVA) to a consumption tax, therefore, is an increase in the rate of capital accumulation. Moreover, if the supply of labor is inelastic, the CVA is the only form of direct tax (other than a lump-sum tax) which is completely "neutral" (in the allocative sense) within a period of time and over time. The GVA and the IVA, on the other hand, distort the choice between future and present
consumption. If the labor supply is a function of the real wage this is no longer true. The CVA, as well as the GVA and IVA, affects the real wage; thus the CVA distorts the choice between work and leisure.

In the second chapter we turned our attention to the incidence effects of the value-added tax. Our primary objective was to determine the effect upon the real distribution of income of a shift in tax structure from a profits tax to a value-added tax. This particular comparison was made because it is often proposed that we replace the corporation profits tax, at least in part, by a value-added tax. As in the first chapter, the classical full employment model was employed along with the assumption of an inelastically supplied labor force.

The conclusion of this chapter can be summarized as follows. In the short run, it is likely that the after-tax real wage will fall and after-tax profits will rise. This result is to be expected because we shift from a tax which bears solely on profits to one under which wages and profits are taxed equally. There is a case, however, where this movement in after-tax wages is reversed. If the capital-good industry is substantially less capital intensive than the consumer-good industry and if the saving rate is highly sensitive to the rate of interest (in a positive manner), then both after-tax real profits and after-tax real wages may rise in the face of a shift from a profits tax to a value-added tax. However, neither of these conditions seem to characterize the United States economy.
Among the variants of the value-added tax, the CVA results in the lowest after-tax real wage and the GVA in the highest. An opposite conclusion was reached with respect to after-tax real profits. Assuming that profits are concentrated in the hands of a few and that wages are relatively equally distributed, then, in the short run, the distribution of real income is more unequal under a value-added tax than under a profits tax. Furthermore, the GVA will yield less income inequality than the IVA, and the latter will result in less inequality than the CVA.

We can generalize these results to the long run as well. If the saving rate is positively related to the rate of interest, however, factor rewards under the alternative tax structures will differ less in the long run than in the short run. Those tax systems which yield the highest after-tax profits and the lowest after-tax wages in the short run also give rise to the fastest rate of capital accumulation. A larger capital stock in the future implies higher wages and lower profits. This effect cannot go so far as to reverse the short-run incidence effects however, without increases in the rate of saving which are wholly unrealistic. We can conclude, therefore, that the substitution of a value-added tax for a profits tax will increase income inequality in both the short and the long run.

In the next three chapters our attention was devoted to an examination of the short-run effects of substituting a value-added tax for a corporate profits tax. Specifically, we analyzed the implications for the short-run stability of the economy. In addition, we examined the short-run effect of the tax
substitution upon the pattern and the level of investment expenditure. To perform these tasks properly, we had to abandon the assumption of the classical full employment model. Instead, we assumed that the economy is not always at full employment and that the profit maximizing conditions are not always satisfied. That this change in assumption is necessary to analyze the short-run stability of the economy is obvious. Moreover, because the primary constraint on investment in the classical world is the rate of saving, the classical model is wholly inadequate for a study of the short-run effects of the tax change upon investment. A Keynesian unemployment model is more suitable for this purpose since investment is no longer constrained by the level of saving (except in the definitional ex-post sense).

Our conclusions regarding the short-run stability of the economy hinge critically upon the way that the value-added tax is shifted. If the value-added tax is borne completely by profits, the short-run stability of the economy will be reduced as a result of the shift from a corporate income tax to a value-added tax. On the other hand, if the value-added tax is borne by each factor of production in proportion to his earnings, the stability of the economy is increased. There is reason to believe that, at least in the first instance, the first shifting assumption is more plausible. In the long run, however, businessmen will move towards the profit-maximizing points on their production functions, thus increasing the stability of the economy.
This force may go so far as to bring about the results of our second shifting assumption.

At this point it should be mentioned that we have utilized a very restrictive definition of short-run stability. When we speak of greater stability we simply mean a smaller impact multiplier. Ideally we would like to say something about the overall stabilizing power of our two tax systems. In order to do so, however, we would have to estimate the entire lag structure of the explanatory equations of the economy. Lack of time and resources prevented us from carrying out such an ambitious task. Instead, we had to be satisfied with our simpler approach.

Our next step was to examine the impact of the change in tax structure upon investment. Investment can be affected in two ways: first, the degree of riskiness associated with a given investment project will differ under the two tax systems; second, the level and timing of the flow of internal funds is changed when we change tax structure.

As before, our conclusions hinge crucially upon which set of shifting assumptions we adopt. If we assume that the value-added tax is not shifted, the level of risk associated with a given investment project is increased while the expected rate of return is unchanged. This force tends to make investment less attractive under a value-added tax than under the profits tax. Furthermore, if the model of investment behavior advanced by Meyer and Glauber is valid, investment may be reduced because of the resulting change in the time path of internal funds. According to their hypothesis, investment depends
upon the supply of internal funds only in recessionary periods. The value-added tax, because its base is relatively stable, results in a smaller flow of funds in a recession than does the profits tax. The total supply of internal funds over any given business cycle is the same under both tax systems, however—only the timing of these flows differ.

If we alter our shifting assumptions in such a way that each factor absorbs the value-added tax in proportion to its earnings our conclusions are radically different. In the first place, the supply of internal funds will be higher in any time period under the value-added tax than under the profits tax. Thus investment will be higher in recessions under the former than under the latter. Secondly, although the degree of risk associated with any particular investment is higher under the value-added tax, so is the expected rate of return. Since we do not know businessmen's indifference map with respect to yield and risk, we cannot say how this force will affect behavior.

In the text we have argued and have given some empirical evidence that, at least in the short-run, businessmen would not be able to shift the value-added tax. On this basis, we conclude that investment would decrease—at least initially.

What can we say about the desirability of replacing the corporate income tax with the value-added tax? Before we can make any statement it would be wise to point out that our analysis, in most part, has been carried out under extremely simplifying assumptions. Furthermore, our analysis encompassed only a few of the many effects that such a tax substitution
could have. For example, we have ignored the international trade aspects of the tax substitution. It has been argued that the value-added tax because it can be rebated to exporters, might improve our balance of payments position. In addition the tax substitution might increase the efficiency of the economy. The tax shift would result in a redistribution of income from those firms whose profits are low (and hence pay little profits tax) to those firms whose profits are high (and hence pay the bulk of the profits tax).

Nevertheless the tax substitution would not accomplish its primary goal—a higher level of investment. Furthermore, and most important, the change in tax structure would worsen the distribution of income (from the author's viewpoint). On this basis therefore, I feel that we should reject the proposal that the value-added tax be substituted for the corporate profits tax.
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