Optimization of Outrigger Structural Systems

by

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B.Sc., Civil and Environmental Engineering (1999)

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ABSTRACT

Outriggers serve as efficient structural systems for tall buildings because they reduce the core moment and drift while allowing fewer perimeter columns. The location and number of outriggers are important criteria to be considered during design of a building. Available information about optimizing outrigger location and number is limited.

To examine this issue in more detail, a method of analysis for outrigger structural systems was chosen and used on an 80-story building to determine the optimum location and number of outriggers. Analysis showed that the optimum location of outriggers is equal interval spacing up the height of the building. Also, it was determined that the optimum number of outriggers has an upper limit of four.

Thesis Supervisor: Jerome Connor
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1 INTRODUCTION

The concept of outrigger structural systems has existed for many years. The outrigger idea stems from ancient sailing ships (Figure 1). Wind forces in the sails of the ship were resisted with the help of outrigger supports. Today, tall buildings can be constructed similar to mast systems of ancient sailing ships. The central core is like the mast, the outriggers are like the spreaders, and the columns like the stays hung from the spreaders (Kowalczyk 140).

A typical tall building outrigger system is diagrammed in Figure 2. Much like on ships, the function of outriggers is “to reduce the overturning moment in the core that would otherwise act as a pure cantilever, and to transfer the reduced moment to columns outside the core by way of a tension-compression couple, which takes advantage of the increased moment arm between these columns” (Kowalczyk 141). In 1962 Barbacki designed and used the first outrigger structural system for the 47-story Place Victoria building (Karnam 477).
Why Outriggers?

As land value continues to increase, buildings are being built taller and taller with larger aspect ratios (height-to-width). This leads to excessive deflection and larger overturning moments. To further complicate matters, owners are demanding fewer exterior columns, which places the responsibility of resisting lateral loads solely on the core (Taranath, Optimum 345).

To meet these demands tall buildings often consist of a central core with large column-free floor spaces between the core and exterior support columns. This arrangement allows for multi-facet use of the floor space, but at the same time it uncouples the two primary structural systems that resist overturning forces (core and perimeter columns). The uncoupling of the core and perimeter columns reduces the overall resisting moment to the sum of the individual systems. Outriggers are used to couple these systems and hence increase the overall resisting moment to be greater than the sum of their individual systems (Kowalczyk 141). Furthermore, outriggers when used efficiently can significantly reduce steel quantity and therefore the cost of the building. See Figure 3 for an example of an outrigger structural system.

Figure 3 - Example Outrigger Structural Systems (Smith, Tall 356)
Outrigger Benefits

The use of one or more levels of outriggers can minimize the problems and restrictions found in core only structural systems. The Council on Tall Buildings and Urban Habitat has given some guidance as to the benefits and drawbacks of outrigger structural systems. Outriggers can provide the following benefits to a building’s overall design:

- "Core overturning moments and their associated induced deformation can be reduced through the “reverse” moment applied to the core at each outrigger intersection.
- Significant reduction and possibly the complete elimination of uplift and net tension forces throughout the columns and the foundation system.
- The exterior column spacing is not driven by structural considerations and can easily mesh with aesthetic and functional considerations.
- Exterior framing can consist of “simple” beam and column framing without the need for rigid-frame-type connections, resulting in economies.
- For rectangular buildings, outriggers can engage the middle columns on the long faces of the building under the application of wind loads in the more critical direction." (Kowalczyk 141-2)

Outrigger Drawbacks

The major drawback of outrigger structural systems is their potential interference with rentable floor space. Each level of outrigger trusses usually takes up one or two floors of a building. The outriggers render that level impossible to rent because no one wants to rent a floor with trusses running through the window. A solution to this problem is to locate a mechanical floor at these levels. Therefore, for a one-outrigger structural system a mechanical floor must be located around the mid-height of the building. This is a problem because even for tall buildings a mechanical floor is more cost effective in the basement and is preferred in the leasing market (Banavalkar 875). To help minimize this drawback the following guidelines have been developed.

- “Locating outriggers in mechanical and interstitial levels.
- Locating outriggers in the natural sloping lines of the building profile.
- Incorporating multilevel single diagonal outriggers to minimize the member’s interference on any single level.


- Skewing and offsetting outriggers in order to mesh with the functional layout of the floor space” (Kowalczyk 143-4)

Objective

The objective of this thesis is to compile all known research about the optimization of outrigger locations and number and verify their conclusions for an 80-story tall building.

Organization

This thesis has three main sections. First, a simple model for the preliminary analysis and design of outrigger-braced systems is presented. Second, the performance of outrigger-braced systems is examined. And finally, an 80-story tall building is modeled using information obtained from the previous two sections.
A SIMPLE MODEL FOR THE PRELIMINARY ANALYSIS AND DESIGN OF OUTRIGGER-BRACED SYSTEMS

Behavior of Outrigger-Braced Structures

Early structural systems employed the use of a central core surrounding stairwells and elevator banks to resist lateral wind loads. But, as the aspect ratio of a building increases it becomes increasingly more expensive to stiffen the central core system to limit deflection. At this point a structural system must be added to the structure that will act in combination with the central core to increase its stiffness (Taranath, Optimum 345).

A building’s stiffness and gravitational load can be more effectively used to resist horizontal loads through the use of outriggers. Stiff outriggers are able to transform rotations at the core into vertical deflections at the perimeter columns. The columns resistance to axial deformation causes a restoring moment at the outrigger levels. This is where the outrigger structural system gains its efficiency. Axial deformation in the columns is able to introduce a more significant increase in stiffness for a building structure than bending deformation solely for a freestanding core system (Wargon 265). Building stiffness can be increased by 25 to 30 percent (Taranath, Steel 449). This increased stiffness reduces the lateral deflection and the base moment (Figure 4) that would be experienced by a freestanding core (Moudarres, Stiffening 225).
Deflection of outrigger braced structure

Deflection of Moment in core with structure without outriggers

Moment in core without outrigger bracing

Leeward columns in compression

Windward columns in tension

Figure 4 – a) Outrigger structure displaced under horizontal loading b) resultant deflections c) resultant core moments (Smith, Tall 357)

Although an outrigger structural system "is effective in increasing the structure's flexural stiffness, it does not increase its resistance to shear, which has to be carried mainly by the core" (Taranath, Steel 447).

Method of Analysis

Computer programs are capable of analyzing entire buildings. But there is still a need for simplified methods of analysis. Simplified calculations can a) check the accuracy of the computer, b) enable rapid design, and c) give an idea of the structural behavior of the building (Cheong-Siat-Moy 85).

Below a simplified calculation method for the analysis of an outrigger structural system is given. Due to the simplifying assumptions made for this analysis, the method should be restricted to the preliminary design of the building. For this thesis the simplified method of analysis provides a convenient procedure for determining the optimum location and number of outriggers.

This following approximate method is a summary of the method presented in Tall Building Structures: Analysis and Design by Bryan Stafford Smith and Alex Coull (356-364). This method is a compatibility formulation in which the rotation of the core at the outrigger level is set equal to the rotation of the outrigger.
Assumptions for Analysis (356-358)

The following assumptions are made to simplify the analysis.

1. “The structure is linearly elastic.
2. Only axial forces are induced in the columns.
3. The outriggers are rigidly attached to the core and the core is rigidly attached to the foundation.
4. The sectional properties of the core, columns, and outriggers are uniform throughout their height.” (356)

Although assumption four is unrealistic for an actual building this assumption will still allow accurate results for preliminary analysis. During preliminary design the drift at the top, the overturning moment, and column axial forces are primarily affected by the properties of the building near its base. Thus, using uniform sectional properties as found near the base of the building is adequate precision for preliminary design (358).

Compatibility Analysis of a Two-Outrigger Structure (358-361)

A structural system with two outriggers is used to demonstrate the general method of analysis. A freestanding core is statically determinate and each addition of an outrigger adds a unit of redundancy. Thus, for our example a two-outrigger system is twice redundant. The degree of redundancy is equal to the number of compatibility equations required for a solution. The compatibility equations say that for each outrigger level the rotation of the core is equal to the rotation of the outrigger. “The rotation of the core is expressed in terms of its bending deformation, and that of the outrigger in terms of the axial deformations of the columns and the bending of the outrigger” (359). Figure 5 is a representative model of a two-outrigger system subjected to uniform lateral loading.
The outrigger restraining moments for each outrigger (Figure 5c and Figure 5d) are subtracted from the external-load moment diagram (Figure 5b) to obtain the moment diagram for the core (Figure 5e). The restraining moments produced by the outriggers begin at their level and run uniformly down to the base.

The Moment-area method defines the core rotations at the outriggers level 1 and 2 as,

\[
\theta_i = \frac{1}{EI} \int_{x_1}^{x_2} \left( \frac{wx^2}{2} - M_1 \right) dx + \frac{1}{EI} \int_{x_1}^{x_2} \left( \frac{wx^2}{2} - M_1 - M_2 \right) dx
\]

Eq. 1

and
\[ \theta_2 = \frac{1}{EI} \int_0^H \left( \frac{wx^2}{2} - M_1 - M_2 \right) dx \]  

Eq. 2

where \( EI = \) flexural rigidity, \( H = \) total height of the core, \( w = \) uniform lateral loading, \( x_1 \) and \( x_2 \) are heights of outriggers 1 and 2 from the top of the building, and \( M_1 \) and \( M_2 \) are the restraining moments for each outrigger.

The rotation of the outriggers at the level where they are connected to the core includes two components. The first component is the differential axial deformation of the columns and the second, “the outrigger bending under the action of the column forces at their outbound ends” (359).

The rotation of the level 1 outrigger at the core connection is

\[ \theta_1 = \frac{2M_1(H-x_1)}{d^2(EA)_c} + \frac{2M_2(H-x_2)}{d^2(EA)_c} + \frac{M_2d}{12(EI)_o} \]  

Eq. 3

and for the level 2 outrigger

\[ \theta_2 = \frac{2(M_1 + M_2)(H-x_2)}{d^2(EA)_c} + \frac{M_2d}{12(EI)_o} \]  

Eq. 4

where \((EA)_c\) = the axial rigidity of the column, \(d/2\) = the horizontal distance form the centroid of the core, \((EI)_o\) = the effective flexural rigidity of the outrigger (Figure 6).

**Figure 6 -**

(a) Outrigger attached to edge of core

(b) equivalent outrigger beam attached to centroid of core (361)
The *effective* flexure rigidity allows for the wide-column effect of the core and can be calculated from the actual flexural rigidity of the outrigger by the following equation.

\[
(EI)_o = \left(1 + \frac{a}{b}\right)^3 \frac{1}{(EI')_o}
\]

Eq. 5

Setting the rotation of the core and the rotation of the outrigger at level 1 equal to each other (Equations 1 and 3) yields,

\[
\frac{2M_1(H-x_1)}{d^2(EA)_c} + \frac{2M_2(H-x_2)}{d^2(EA)_c} + \frac{M_1d}{12(EI)_o} = \frac{1}{EI} \int_1^2 \left(\frac{wx^2}{2} - M_1\right) dx + \frac{1}{EI} \int_1^2 \left(\frac{wx^2}{2} - M_1 - M_2\right) dx
\]

Eq. 6

Performing the same for Equations 2 and 4 yields,

\[
\frac{2(M_1 + M_2)(H-x_2)}{d^2(EA)_c} + \frac{M_2d}{12(EI)_o} = \frac{1}{EI} \int_1^2 \left(\frac{wx^2}{2} - M_1 - M_2\right) dx
\]

Eq. 7

Equations 6 and 7 can be rewritten to get

\[
M_1\left[S_1 + S(H-x_1)\right] + M_2S(H-x_2) = \frac{w}{6EI}(H^3 - x_1^3)
\]

Eq. 8

and

\[
M_1S(H-x_2) + M_2\left[S_1 + S(H-x_2)\right] = \frac{w}{6EI}(H^3 - x_2^3)
\]

Eq. 9
where $S$ and $S_1$ are

$$S = \frac{1}{EI} + \frac{2}{d^2(EA)} \quad \text{Eq. 10}$$

$$S_1 = \frac{d}{12(EI)} \quad \text{Eq. 11}$$

**Analysis of Forces (362)**

Solving Equations 8 and 9 simultaneously gives the restraining moment applied to the core by the outrigger at level 1

$$M_1 = \frac{w}{6EI} \left[ \frac{S_1(H^3 - x_1^3) + S(H - x_2)(x_2^3 - x_1^3)}{S_1^2 + S_1S(2H - x_1 - x_2) + S^2(H - x_2)(x_2 - x_1)} \right] \quad \text{Eq. 12}$$

and at level 2

$$M_2 = \frac{w}{6EI} \left[ \frac{S_1(H^3 - x_1^3) + S_1(H - x_1)(H^3 - x_1^3) - (H - x_2)(H^3 - x_1^3)}{S_1^2 + S_1S(2H - x_1 - x_2) + S^2(H - x_2)(x_2 - x_1)} \right] \quad \text{Eq. 13}$$

Knowing the outrigger restraining moments $M_1$ and $M_2$, it is now easy to solve for the resulting moment in the core (Figure 5e).

$$M_x = \frac{wx^2}{2} - M_1 - M_2 \quad \text{Eq. 14}$$

$M_1$ is included for $x > x_1$, and $M_2$ is included for $x > x_2$. 
Axial column force due to the outrigger rotations are

\[ \pm \frac{M_i}{2} \quad \text{for } x_1 < x < x_2 \]  
\[ (M_i + M_2)/d \quad \text{for } x \geq x_2 \]  

\text{Eq. 15} \quad \text{Eq. 16}

Finally, the maximum moment in the outriggers is

\[ M_1 \cdot b/d \quad \text{for level 1} \]  
\[ M_2 \cdot b/d \quad \text{for level 2} \]  

\text{Eq. 17} \quad \text{Eq. 18}

where \( b \) is the net length of the outriggers (Figure 6).

\textbf{Analysis of Horizontal Deflections (362)}

Use of the moment-area method leads to the horizontal deflections of the structure. This is done by applying the moment-area method to the bending moment diagram of the core. Below, only the equation for the top drift is given.

\[ \Delta_0 = \frac{wH^4}{8EI} - \frac{1}{2EI} \left[ M_1 (H^2 - x_1^2) + M_2 (H^2 - x_2^2) \right] \]  
\[ \text{Eq. 19} \]

The first term on the right is the deflection for the core only and the second term on the right is the reduction in deflection due to the outrigger restraining moments \( M_1 \) and \( M_2 \).
Generalized Solutions of Forces and Deflections (363)

The above example applied the simplified analysis for a two-outrigger system. This same method can be used for any number of outriggers. The following are generalized solutions for the restraining moments, moment in the core, and deflection at the top of the building.

Restraining moments,

\[
\begin{bmatrix}
M_1 \\
M_2 \\
\vdots \\
M_n
\end{bmatrix} = \frac{w}{6EI} \begin{bmatrix}
S_1 + S(H - X_1) & S(H - X_2) & \cdots & S(H - X_i) & \cdots & S(H - X_n) \\
S(H - X_2) & S_1 + S(H - X_2) & \cdots & S(H - X_i) & \cdots & S(H - X_n) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
S(H - X_n) & S(H - X_n) & \cdots & S_1 + S(H - X_n) & \cdots & S(H - X_n)
\end{bmatrix}^{-1} \begin{bmatrix}
H^3 - X_1^3 \\
H^3 - X_2^3 \\
\vdots \\
H^3 - X_n^3
\end{bmatrix}
\]

where \( n \) = number of levels of outriggers.

Moment in the core,

\[
M_s = \frac{wx^2}{2} - \sum_{i=1}^{n} M_i
\]  

Eq. 21

For the distance between the top of the building and the first outrigger, the second term of the above equation will be zero.

Top horizontal deflection,

\[
\Delta_0 = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - x_i^2)
\]  

Eq. 22
3 OUTRIGGER PERFORMANCE

General

The above simplified analysis can not only be used to determine the core moments and horizontal deflections but can also be used to determine the optimum location of outriggers as well as the optimum number of outriggers.

“The efficiency of this bracing system in increasing the lateral stiffness depends on the extent to which the axial rigidities of the columns are mobilized to resist overturning, in other words, how complete is the tube action. This is strongly affected by the location of the outrigger arms along the height, and also by their number and rigidity” (Rutenberg, Stability 1990).

Optimum Outrigger Levels

An optimum location for outriggers can be found by maximizing the drift reduction, which is the second term on the right of Equation 19. Again, the two-outrigger system will be used to demonstrate the process of determining the optimum location of outriggers. Differentiating the second term on the right side of Equation 19 with respect to $X_1$ then $X_2$ and solving simultaneously will yield the optimum location of outriggers. The following information was summarized from an article by Bryan Stafford Smith entitled “Parametric Study of Outrigger-Braced Tall Building Structures.”

Thus for example,

$$\left(H^2 - x_1^2\right) \frac{dM_1}{dx_1} + \left(H^2 - x_2^2\right) \frac{dM_2}{dx_1} - 2x_1M_1 = 0$$  Eq. 23

and

$$\left(H^2 - x_1^2\right) \frac{dM_1}{dx_2} + \left(H^2 - x_2^2\right) \frac{dM_2}{dx_2} - 2x_2M_2 = 0$$  Eq. 24
where $\frac{dM_1}{dx_1}$, $\frac{dM_2}{dx_1}$, $\frac{dM_1}{dx_2}$, and $\frac{dM_2}{dx_2}$ are the derivatives of $M_1$ and $M_2$ found from Equations 12 and 13 (2008).

If Equations 23 and 24 are written out in their entirety, it is found that the sectional properties are expressed in terms of $S$ and $S_1$ (Equations 10 and 11). It is possible to write Equation 23 and 24 in dimensionless structural ratios that are more meaningful. These structural ratios $\alpha$ and $\beta$, represent the core-to-column and the core-to-outtrigger inertial ratios.

\[
\alpha = \frac{2EI}{d^2(EA)_c} \quad \text{Eq. 25}
\]

\[
\beta = \frac{EI}{(EI)_0} \frac{d}{H} \quad \text{Eq. 26}
\]

Equations 23 and 24 can be further simplified by combining $\alpha$ and $\beta$ together to obtain one dimensionless term $\omega$.

\[
\omega = \frac{\beta}{12(1+\alpha)} \quad \text{Eq. 27}
\]

The term $\omega$ is a dimensionless structural term for a uniform outrigger structure with flexible outriggers (2008). Equations 23 and 24 have been solved by Bryan Stafford Smith for outrigger levels $x_1$ and $x_2$ with different values of $\omega$. The results can be seen in Figure 7b.

The optimum levels $X_1$ through $X_n$ can be found for an outrigger with $n$ levels through the simultaneous solution of the following matrix.
where $M_i$ is the restraining moment due to outrigger $i$, and $M_{ij}$ is the derivative of $M_i$ with respect to $X_j$ (2010).

$$M_{ij} = \frac{dM_i}{dX_j}$$

Figure 7 and Figure 8 demonstrate optimum outrigger levels for 1, 2, 3, and 4 levels of outriggers.
Figure 7 – a) Outrigger Optimum Levels in One-Outrigger Structure b) Outrigger Optimum Levels in Two-Outrigger Structure (2006)
Examination of the above graphs reveals some important points about optimum outrigger locations.
1. As the flexibility of the outrigger increases and other properties remain constant, the optimum levels are higher up the building.

2. For greater values of column system inertia, the building is more sensitive to the flexibility of the outrigger.

3. If the core-to-outrigger inertia ratio is held constant and the column stiffness is decreased, the optimum outrigger levels move down towards the optimum levels for the case of flexurally rigid outriggers (2010).

Outriggers serve two principal functions. First, they increase stiffness in the building, and second, they reduce the rotation of the core due to horizontal loads. Since the stiffness decreases as the outrigger is moved further from the base the optimum location for stiffness considerations would be near the base of the building. However, since the rotation for a cantilever beam with uniform distributed loading varies parabolically with a maximum value at the end and minimum value at the base, the outrigger location for rotation considerations should be placed at the top. Thus, the optimum location for one outrigger should be at about the mid-height of the building (Taranath, Steel 457).

Indeed, examination of Figure 7 shows that the optimum location for a one-outrigger system with a very stiff outrigger ($\omega = 0$) is about $0.455H$. This conclusion has been verified by Taranath in his article, “Optimum Belt Truss Locations for High-rise Structures.”

Optimum Number of Outriggers

It has been shown by Rutenburg and Smith that the efficiency of outrigger bracing diminishes with each additional outrigger (Rutenburg, Lateral 53) (Smith, Behavior 517). If the number of outriggers were taken to the limit ($\infty$) a fully composite beam would result between the columns and the core. An upper limit for the number of outriggers can be established by determining the number of outriggers that achieves around 95% fully composite behavior for drift reduction efficiency. Smith has determined that 95% drift reduction efficiency can be accomplished for four outriggers.
Outrigger Flexibility

For real building design the outriggers will have flexibility. Figure 7 and Figure 8 show that as ω increases (i.e., as the outrigger becomes more flexible), the outrigger must be placed further up the building to limit top drift. However, for many cases, outrigger flexibility is small and can be ignored for preliminary analysis (Rutenburg, *Lateral* 57).

“Efficiency” of Outrigger Structures

An efficient means of determining the effectiveness of an outrigger structural system is to compare its reduction of horizontal deflection and base moment to a corresponding system that incorporates fully composite behavior between the core and columns. Fully composite behavior “implies that, in overall flexure of the structure, the stresses in the vertical components are proportional to their distances from their common centroidal axis, with the structure having an overall flexural rigidity equal to” (Smith, *Tall* 368)

\[
(EI)_r = \frac{(EA_c d^2}{2} + EI
\]

Eq. 29

Moment Reduction Efficiency

The maximum possible moment reduction, \( M_c \), occurs when the core and the columns behave fully compositely. This is expressed as

\[
M_c = \left[ \frac{(EA_c d^2}{2} \right] \left[ \frac{wH^2}{2} \right]
\]

Eq. 30
The ratio of $M_e$ to the actual moment reduction is expressed as the moment reduction efficiency by

$$M\% = \left( \frac{\sum_{i=1}^{n} M_i}{M_e} \right) \times 100 \tag{Eq. 31}$$

Furthermore, “the moment reduction efficiency for buildings with up to four outriggers optimally located for maximum drift reduction is given as a function of the characteristic structural parameter $\omega$” (Figure 9) (2011).

![Figure 9 - Efficiency in Moment Reduction (2011)](image)

**Drift Reduction Efficiency**

The fully composite drift reduction is given as

$$\Delta_c = \frac{1}{EIS} \cdot \frac{wH^4}{8EI}$$
and the drift reduction efficiency is expressed as

$$\Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - X_i^2)$$

The structural parameter $\omega$ is again used to compare up to four outrigger levels for various drift reduction efficiencies (Figure 10).

Figure 10 - Efficiency in Drift Reduction (2012)
General

The method of analysis found in Chapter 2 will now be used for an 80-story building to verify by example the optimum location and number of outriggers predicted in Chapter 3. First, for a one-outrigger system the optimum location will be established and second, the optimum number of outriggers will be established by comparing outriggers at 1,2,3,4,5 and 6 levels. The following information about the geometry and wind loads holds true for all cases.

Geometry

A summary of the geometry of the building is as follows.

<table>
<thead>
<tr>
<th># stories</th>
<th>Total height (H)</th>
<th>Total width (d)</th>
<th>aspect ratio</th>
<th>Inter-story height (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>960 ft</td>
<td>120 ft</td>
<td>8 :1</td>
<td>12 ft</td>
</tr>
</tbody>
</table>

Table 1 – Building dimensions
Wind Loads

Wind loads used for the sample model come from the Massachusetts Building Code. It was assumed that the sample building would experience zone 3 exposure. Also, a uniform load was obtained by averaging the wind loading of each story (Figure 12).

<table>
<thead>
<tr>
<th>Wind Loading (feet)</th>
<th>Zone 3 Exposure</th>
<th>B (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-50</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>50-100</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>100-150</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>150-200</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>200-250</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>250-300</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>300-400</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>400-500</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>500-600</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>600-700</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>700-800</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>800-900</td>
<td>62</td>
<td></td>
</tr>
<tr>
<td>900-1000</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>42.08</td>
<td></td>
</tr>
</tbody>
</table>

Figure 12 - Wind Loadings

Optimum Location Example

The method of analysis described in Chapter 2 is used to analyze a 1-outrigger system for different location cases: top, ¼, predicted optimum, middle, and near the bottom of the building. For each case the building’s top deflection, deflection reduction, base moment, and base moment reduction will be compared. The optimum location will have the greatest deflection reduction. For all examples the core and outrigger inertias are kept equal. See Appendix 1 for the assumed core design.
Step-by-step Analysis of Outrigger Located at Predicted Optimum Location

For this case the outrigger is located at the predicted optimum location (Figure 7), 0.455H. A step-by-step process through the analysis will be shown below only for this case. For all other cases the reader is referred to Appendix 2 through 6 for the complete analysis. The knowns in this case are the same for all the cases except for \( x_1 \), which will vary for each case.

Givens:

\[
\begin{align*}
H &= 960 \text{ ft} \\
d &= 120 \text{ ft} \\
x_1 &= 432 \text{ ft} \\
w &= 5.05 \text{ k/ft} \\
E &= 29,000 \text{ ksi} \\
I_c &= 25,110 \text{ ft}^4 \\
I_o &= 785 \text{ ft}^4 \\
A_{eq} &= 1400 \text{ in}^2 \\
a &= 20 \text{ ft} \\
b &= 40 \text{ ft}
\end{align*}
\]

\( H \) = total height of building  
\( d \) = width of building  
\( x_1 \) = outrigger location from top of building  
\( w \) = uniform wind load (Figure 12)  
\( E \) = “Young’s” modulus for steel  
\( I_c \) = Core inertia (Appendix 1)  
\( I_o \) = Outrigger inertia  
\( A_{eq} \) = Equivalent area of the columns  
\( a \) = half the width of the core  
\( b \) = \( d/2 - a \)

Step 1: Determine axial rigidity of the columns \((EA)_c\) and effective flexural rigidity of the outrigger \((EI)_o\).

\[
(EA)_c = 40,600,000 \text{ kips}
\]

\[
(EI)_o = \left(1 + \frac{a}{b}\right)^3 (EI')_o
\]

\[
(EI)_o = 1.11 \times 10^{10} \text{ k-ft}^2
\]

Step 2: Determine \( S \) and \( S_1 \).
\[ S = \frac{1}{EI} + \frac{2}{d^2 (EA)} \]

\[ S = 1.30 \times 10^{11} \text{ l/(k-ft)}^{2} \]

\[ S_i = \frac{d}{12(EI)} \]

\[ S_i = 9.04 \times 10^{-10} \text{ l/(k-ft)} \]

Step 3: Determine restraining moment applied to core M1.

\[ M_1 = \frac{w}{6EI} \cdot [S_i + S(H - X_i)] \cdot [H^3 - X_i^3] \]

\[ M_1 = 833,146 \text{ kip-ft} \]

Step 4: Determine resulting moment in the core.

![Figure 13 - Resultant Core Moment Diagram](image-url)
\[ M_c = \frac{wx^2}{2} - M_i \]

\[ M_c = 1,304,040 \text{ kip-ft} \]

**Step 5:** Determine moment reduction efficiency.

\[
M\% = \frac{\sum_{i=1}^{n} M_i}{M_c} \times 100
\]

\[ M\% = 48.65\% \]

**Step 6:** Determine horizontal deflection at the top.

\[
\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \left[ M_i(H^2 - x_i^2) \right]
\]

\[ \Delta_o = 2.192 \text{ ft} \]

If the allowable deflection of the building is assumed to be H/400 the above deflection is okay.

**Step 7:** Determine drift reduction efficiency.

\[
\Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI}
\]

\[ \Delta_c = 3.763 \text{ ft} \]
\[
\Delta\% = \frac{\frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - X_i^2)}{\Delta_c}
\]

\[
\Delta\% = 77.60\%
\]

Step 8: Determine optimum location nondimensional parameters.

\[
\alpha = \frac{EI}{(EA_c)(d^2/2)}
\]

\[\alpha = 0.36\]

\[
\beta = \frac{EI}{(EI)_o} \frac{d}{H}
\]

\[\beta = 1.19\]

\[
\omega = \frac{\beta}{12(1+\alpha)}
\]

\[\omega = 0.073\]

When the nondimensional parameter \(\omega\) is used in Figure 9 and Figure 10 to verify the results of the analysis it is found that the error between the above analysis and the figures for drift reduction and moment reduction are 0.78\% and 1.46\% respectively.
Step 9: Analyze results

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>833,146 kip-ft</td>
<td>restraining moment</td>
</tr>
<tr>
<td>( M_c )</td>
<td>1,493,539 kip-ft</td>
<td>resulting moment in core</td>
</tr>
<tr>
<td>( M% )</td>
<td>48.65%</td>
<td>moment reduction efficiency</td>
</tr>
<tr>
<td>( \Delta_0 )</td>
<td>2.192 ft</td>
<td>deflection at top</td>
</tr>
<tr>
<td>( \Delta_{allowable} )</td>
<td>2.400 ft</td>
<td>allowable deflection</td>
</tr>
<tr>
<td>( \Delta% )</td>
<td>77.60%</td>
<td>deflection reduction efficiency</td>
</tr>
</tbody>
</table>

**Table 2 - Summary Table for One-outrigger Optimum Location**

Table 2 shows that the moment reduction efficiency is 48.65% and the deflection reduction efficiency is 77.60% for a one-outrigger system located at the predicted optimum location. Also, the deflection at the top is less than the allowable deflection (H/400).

![Figure 14 - Core Bending Moment vs. Outrigger Distance from Top](image)

Figure 14 compares three different core moment diagrams. \( M_{core} \) represents the bending moment for a 1-outrigger system located at 0.455H. \( M_{exter} \) represents the bending moment for a freestanding core without outrigger support. \( M_{comp} \) represents the bending moment for a fully composite outrigger system. A fully composite outrigger system is “the maximum possible moment reduction that would occur if the core and
column behaved fully compositely” (Smith, Parameter 2010). It can be seen from this figure that the 1-outrigger system ($M_{core}$) is about 48% effective when compared to a fully composite system ($M_{comp}$). Also, the one-outrigger system reduces the freestanding core moment ($M_{exter}$) by 833,147 k-ft. This is an approximate 35% decrease in base core moment.

![Figure 15 - Restraining Moment vs. Outrigger Distance from Top](image)

Figure 15 compares the restraining moment for a 1-outrigger system ($M_1$) to the restraining moment for a fully composite system ($M_{comp}$). $M_1$ in the above figure represents the restraining moment produced by one outrigger placed at 432ft from the top. It is shown in Table 2 that $M_1$ is approximately 48.65% of the maximum possible bending moment reduction ($M_{comp}$). As additional outriggers are added, the sum of their restraining moments will approximate the fully composite outrigger diagram ($M_{comp}$).
Optimum Location Summary

<table>
<thead>
<tr>
<th>Outrigger Location</th>
<th>$x_i$ (ft)</th>
<th>$M_i$ (k-ft)</th>
<th>$M_c$ (k-ft)</th>
<th>$M%$</th>
<th>$\Delta_o$ (ft)</th>
<th>$\Delta_{allow}$ (ft)</th>
<th>$\Delta%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>0</td>
<td>532,127</td>
<td>1,794,559</td>
<td>31.1%</td>
<td>2.77</td>
<td>2.40</td>
<td>62.1%</td>
</tr>
<tr>
<td>3/4</td>
<td>240</td>
<td>682,989</td>
<td>1,643,697</td>
<td>39.9%</td>
<td>2.30</td>
<td>2.40</td>
<td>74.8%</td>
</tr>
<tr>
<td>Optimum</td>
<td>432</td>
<td>833,146</td>
<td>1,493,539</td>
<td>48.7%</td>
<td>2.19</td>
<td>2.40</td>
<td>77.6%</td>
</tr>
<tr>
<td>Middle</td>
<td>480</td>
<td>872,122</td>
<td>1,454,563</td>
<td>50.9%</td>
<td>2.24</td>
<td>2.40</td>
<td>76.4%</td>
</tr>
<tr>
<td>Bottom</td>
<td>720</td>
<td>1,022,645</td>
<td>1,304,040</td>
<td>59.7%</td>
<td>3.15</td>
<td>2.40</td>
<td>52.3%</td>
</tr>
</tbody>
</table>

Table 3 - One-outrigger Location Summary

where: $M_i$ (k-ft) = restraining moment

$M_c$ (k-ft) = resulting moment in core at base

$M\%$ = moment reduction efficiency

$\Delta_o$ (ft) = deflection at top

$\Delta_{allow}$ (ft) = allowable deflection

Table 3 is a summary of the analysis of an 80-story building with one outrigger located at the top, 3/4, predicted optimum, middle, and near the bottom. From this table it is seen that the outrigger located nearest the bottom is most effective for moment reduction (greatest restraining moment). Also, an outrigger located at the optimum level has the greatest drift reduction efficiency. This is because, as seen from Equation 19, deflection reduction is a function of both the restraining moment and the outrigger location. Thus the optimum location for a single outrigger is near the middle of the building. The following graphs will help visualize these results.
Figure 16 shows that when the drift reduction efficiency is graphed versus outrigger distance from the top, the shape is parabolic. The greatest drift reduction is seen near the middle of the building with the least effective drift reduction being near the top and
bottom. These results are consistent with Figure 17 where the deflection at the top is graphed versus outrigger distance from the top.

Figure 18 - Moment Reduction Efficiency vs. Outrigger Distance from Top

Figure 19 - Core Bending Moment vs. Outrigger Distance from Top
Figure 18 and Figure 19 demonstrate that the moment reduction efficiency increases and the core bending moment decreases as the outrigger is located closer and closer to the bottom of the building.

![Graph showing moment reduction efficiency vs. deflection reduction efficiency]

**Figure 20 - Moment Reduction Efficiency vs. Deflection Reduction Efficiency**

Graphing moment reduction efficiency versus deflection reduction efficiency results in Figure 20. This figure illustrates that the predicted “Optimum” location has the greatest deflection reduction efficiency and the “Bottom” location has the greatest moment reduction efficiency. In the case of tall buildings usually the deflection at the top controls and is more significant than base moment.

As a result, the true optimum location for the 80-story building is the location that has the greatest deflection reduction efficiency and adequate moment reduction efficiency. This is true for the predicted optimum location 0.455H. However, it can be seen from Figure 20 that if, for architectural reasons, the middle location is desired, its deflection reduction efficiency is very near to the optimum location and therefore more than adequate to use. In fact, if this equal interval spacing is done for 1,2,3 or any number of outriggers, the results are very near the actual optimum locations. For example, if three outriggers are used the spacing location would be \(x_1 = 1/4H, x_2 = 2/4H, x_3 = 3/4H\). This is considered equal interval spacing. In the next section it will be shown
that for our 80-story example four outriggers is the reasonable upper limit of outriggers that should be used.

**Optimum Number Example**

The method of analysis described in Chapter 2 is now used for the 80-story building example to analyze an outrigger system with 1, 2, 3, 4, 5 and 6 outriggers. The purpose of this is to determine the upper limit of outriggers that should be used for buildings. For each case the building’s top deflection is kept constant. This was done to allow for better comparison between the deflection reduction efficiency and base moment reduction efficiency. The optimum upper limit will occur when, with each additional outrigger, the additional efficiency is minimal. For all examples the core and outrigger inertias are kept equal. See Appendix 1 for the assumed core design. Below a step-by-step analysis is done for an outrigger system with four outriggers. The other cases have their analysis in Appendix 7 through Appendix 12.

**Step-by-step Analysis of a 4-Outrigger Structural System**

For this case a 4-outrigger structural system is analyzed. The outriggers are spaced in equal intervals of 1/5H. A step by step analysis will be shown below only for this case.

**Givens:**

- \( H = 960 \text{ ft} \)  
  \( H = \text{total height of building} \)
- \( d = 120 \text{ ft} \)  
  \( d = \text{width of building} \)
- \( x_1 = 192 \text{ ft} \)  
  \( x_1 = \text{outrigger location from top of building} \)
- \( x_2 = 384 \text{ ft} \)  
  \( x_1 = \text{outrigger location from top of building} \)
- \( x_3 = 576 \text{ ft} \)  
  \( x_1 = \text{outrigger location from top of building} \)
- \( x_4 = 768 \text{ ft} \)  
  \( x_1 = \text{outrigger location from top of building} \)
- \( w = 5.05 \text{ k/ft} \)  
  \( w = \text{uniform wind load (Figure 12)} \)
- \( E = 29,000 \text{ ksi} \)  
  \( E = \text{“Young’s” modulus for steel} \)
\[ I_c = 25,110 \text{ ft}^4 \quad I_c = \text{Core inertia (Appendix 1)} \]
\[ I_o = 785 \text{ ft}^4 \quad I_o = \text{Outrigger inertia} \]
\[ A_{eq} = 640 \text{ in}^2 \quad A_{eq} = \text{Equivalent area of the columns} \]
\[ a = 20 \text{ ft} \quad a = \text{half the width of the core} \]
\[ b = 40 \text{ ft} \quad b = \frac{d}{2} - a \]

**Step 1:** Determine axial rigidity of the columns \((EA)_c\) and effective flexural rigidity of the outriggers \((EI)_o\).

\[
(EA)_c = 18,560,000 \text{ kips} \]
\[
(EI)_o = \frac{1 + \frac{a}{b}}{3} (EI')_o \]
\[
(EI)_o = 1.11 \times 10^{10} \text{ k-ft}^2 \]

**Step 2:** Determine \(S\) and \(S_1\).

\[
S = \frac{1}{EI} + \frac{2}{d^2 (EA)_c} \]
\[
S = 1.70 \times 10^{-11} \text{ 1/(k-ft}^2) \]
\[
S_1 = \frac{d}{12(EI)_o} \]
\[
S_1 = 9.04 \times 10^{-10} \text{ 1/(k-ft)} \]

**Step 3:** Determine restraining moments applied to core.

From Eq. 20 the following matrices can be completed.
<table>
<thead>
<tr>
<th>$S$ matrix</th>
<th>units = 1/(k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40E-08</td>
<td>9.80E-09</td>
</tr>
<tr>
<td>9.80E-09</td>
<td>1.07E-08</td>
</tr>
<tr>
<td>6.54E-09</td>
<td>6.54E-09</td>
</tr>
<tr>
<td>3.27E-09</td>
<td>3.27E-09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S^{-1}$ inverse</th>
<th>units = (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.04E+08</td>
<td>-1.66E+08</td>
</tr>
<tr>
<td>-1.66E+08</td>
<td>3.39E+08</td>
</tr>
<tr>
<td>-3.06E+07</td>
<td>-1.41E+08</td>
</tr>
<tr>
<td>-5.44E+06</td>
<td>-2.51E+07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H$ matrix</th>
<th>units = ft$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.78E+08</td>
<td></td>
</tr>
<tr>
<td>8.28E+08</td>
<td></td>
</tr>
<tr>
<td>6.94E+08</td>
<td></td>
</tr>
<tr>
<td>4.32E+08</td>
<td></td>
</tr>
</tbody>
</table>

$$M_i = \frac{w}{6EI} S^{-1} \text{Matrix} \cdot H\text{Matrix}$$

$M_1 = 141,311$ kip-ft

$M_2 = 212,262$ kip-ft

$M_3 = 296,475$ kip-ft

$M_4 = 321,371$ kip-ft
Step 4: Determine resulting moment in the core.

![Diagram of Outrigger Distance vs Bending Moment](image)

**Figure 21 - Resultant Core Moment Diagram**

\[ M_c = \frac{wx^2}{2} - M_1 - M_2 - M_3 - M_4 \]

\[ M_c = 1,355,268 \text{ kip-ft} \]

Step 5: Determine moment reduction efficiency.

\[ M\% = \frac{\sum_{i=1}^{n} M_i}{M_c} \times 100 \]

\[ M\% = 74.52\% \]

Step 6: Determine horizontal deflection at the top.

\[ \Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i \left(H^2 - x_i^2\right) \]
\[ \Delta_0 = 2.390 \text{ ft} \]

If the allowable deflection of the building is assumed to be \( H/400 \) the above deflection is okay. The above deflection should not be compared to the deflection determined in the last example due to different sectional properties.

Step 7: Determine drift reduction efficiency.

\[ \Delta_c = \frac{1}{8EI} \frac{wH^4}{EI} \]

\[ \Delta_c = 2.865 \]

\[ \Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - X_i^2) \]

\[ \Delta\% = 95.02\% \]

Step 8: Determine optimum location nondimensional parameters.

\[ \alpha = \frac{EI}{(EA)_c (d^2 / 2)} \]

\[ \alpha = 0.78 \]

\[ \beta = \frac{EI \frac{d}{(EA)c}}{H} \]

\[ \beta = 1.19 \]
\[ \omega = \frac{\beta}{12(1 + \alpha)} \]

\[ \omega = 0.055 \]

When the nondimensional parameter \( \omega \) is used in Figure 9 and Figure 10 to verify results of the analysis it is found that the error between the above analysis and the figures for drift reduction and moment reduction is 0.02\% and 0.65\% respectively.

Step 9: Analyze results

| \( M_1 \) = 141,311 kip-ft | - restraining moment |
| \( M_2 \) = 212,262 kip-ft | - restraining moment |
| \( M_3 \) = 296,475 kip-ft | - restraining moment |
| \( M_4 \) = 321,371 kip-ft | - restraining moment |
| \( M_e \) = 1,355,268 kip-ft | - resulting moment in core |
| \( M\% = 74.51\% \) | - moment reduction efficiency |
| \( \Delta_\delta = 2.390 \text{ ft} \) | - deflection at top |
| \( \Delta_{\text{allowable}} = 2.400 \text{ ft} \) | - allowable deflection |
| \( \Delta\% = 95.02\% \) | - deflection reduction efficiency |

**Table 4 - 4-Outrigger Structural System Summary**

Table 4 summarizes the analysis for an 80-story building with a 4-outrigger structural system. The moment reduction efficiency for this system is 74.51\%. This is a significant increase over the optimally located 1-outrigger structural system (48.65\%) examined in the previous section. Also, the deflection reduction efficiency for the 4-outrigger is 95.02\%. A 95.02\% deflection efficiency is important because, as discussed in Chapter 3 Optimum Location, the upper limit for the number of outriggers is established when 95\% drift reduction efficiency is achieved.
Figure 22 – Core Bending Moment vs. Outrigger Distance from Top

Figure 22 contains three different core moment diagrams. $M_{\text{core}}$ represents the bending moment diagram for a 4-outrigger system located at equal interval of $1/5H$. $M_{\text{exter}}$ represents the bending moment diagram for a freestanding core without outrigger support. $M_{\text{comp}}$ represents the bending moment diagram for a fully composite outrigger system. It is seen that with four outriggers the $M_{\text{core}}$ line is becoming more and more similar to the fully composite line. The 4-outrigger system reduces the freestanding core moment ($M_{\text{exter}}$) by 971418 k-ft. This is a 42% reduction in the core base moment.

Figure 23 - Restraining Moment vs. Outrigger Distance from Top
Figure 23 compares the sum of the restraining moments for a 4-outrigger system ($M_{sum}$) to the sum of the restraining moments of a fully composite outrigger system ($M_{comp\ rest}$). As mentioned previously as more outriggers are added the sum of the restraining moments will more closely approximate the fully composite outrigger line. $M_1$ through $M_4$ represent the individual restraining moment for that outrigger. For this example $M_{sum}$ is 74.51% of $M_{comp\ rest}$. 74.51% represents the moment reduction efficiency for the system.

**Optimum Number Summary**

<table>
<thead>
<tr>
<th>Outrigger Number</th>
<th>Spacing Interval</th>
<th>$M_1$ (k-ft)</th>
<th>$M_2$ (k-ft)</th>
<th>$M_3$ (k-ft)</th>
<th>$M_4$ (k-ft)</th>
<th>$M_5$ (k-ft)</th>
<th>$M_6$ (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2H</td>
<td>825,605</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1/3H</td>
<td>381,203</td>
<td>505,026</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1/4H</td>
<td>217,941</td>
<td>324,224</td>
<td>393,987</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1/5H</td>
<td>141,311</td>
<td>212,262</td>
<td>296,475</td>
<td>321,371</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1/6H</td>
<td>99,632</td>
<td>148,372</td>
<td>212,416</td>
<td>268,551</td>
<td>269,496</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1/7H</td>
<td>74,323</td>
<td>109,355</td>
<td>156,999</td>
<td>205,076</td>
<td>242,548</td>
<td>230,516</td>
</tr>
</tbody>
</table>

Table 5 - Summary of Outrigger Restraining Moments

Table 5 summarizes the restraining moments for a structural system with one to six outriggers.

<table>
<thead>
<tr>
<th>Outrigger Number</th>
<th>$M_c$ (k-ft)</th>
<th>$M%$</th>
<th>$\Delta_\alpha$ (ft)</th>
<th>$\Delta_{allow.}$ (ft)</th>
<th>$\Delta%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2,326,686</td>
<td>0.00%</td>
<td>2.39</td>
<td>2.40</td>
<td>0.00%</td>
</tr>
<tr>
<td>1</td>
<td>1,501,081</td>
<td>51.3%</td>
<td>2.39</td>
<td>2.40</td>
<td>77.0%</td>
</tr>
<tr>
<td>2</td>
<td>1,440,457</td>
<td>63.9%</td>
<td>2.39</td>
<td>2.40</td>
<td>89.4%</td>
</tr>
<tr>
<td>3</td>
<td>1,390,535</td>
<td>70.4%</td>
<td>2.39</td>
<td>2.40</td>
<td>93.2%</td>
</tr>
<tr>
<td>4</td>
<td>1,355,268</td>
<td>74.5%</td>
<td>2.39</td>
<td>2.40</td>
<td>95.0%</td>
</tr>
<tr>
<td>5</td>
<td>1,328,219</td>
<td>77.4%</td>
<td>2.39</td>
<td>2.40</td>
<td>96.1%</td>
</tr>
<tr>
<td>6</td>
<td>1,307,868</td>
<td>79.5%</td>
<td>2.39</td>
<td>2.40</td>
<td>96.7%</td>
</tr>
</tbody>
</table>

Table 6 - Summary of all Outrigger Analysis

where:  
$M_1$ (k-ft) = restraining moment  
$M_c$ (k-ft) = resulting moment in core  
$M\%$ = moment reduction efficiency
\[ \Delta_o \text{ (ft)} = \text{deflection at top} \]
\[ \Delta_{\text{allow.}} \text{ (ft)} = \text{allowable deflection} \]
\[ \Delta\% = \text{deflection reduction efficiency} \]

Table 6 summarizes the analysis of a 80-story building with one to six outriggers. The table shows that the moment reduction efficiency increases with increasing number of outriggers. Also, the drift reduction efficiency increases with increasing number of outriggers. The following graphs help visualize these results.

Figure 24 clearly shows that as outriggers are added to the structural system the drift reduction efficiency increases. But the drift reduction efficiency also experiences "diminishing returns." For each additional outrigger added the increase in drift reduction efficiency decreases. Indeed, after four outriggers the increase in drift reduction is minimal.
Figure 25 and Figure 26 show that increasing the number of outriggers will increase the moment reduction efficiency and decrease the base core bending moment. But, again the phenomenon of “diminishing returns” occurs. Figure 25 shows the moment reduction efficiency line asymptote more slowly but Figure 26 shows the graph quickly asymptote after two or three outriggers.
Figure 27 demonstrates precisely what has already been discussed. The figure illustrates that increasing the number of outriggers increases the moment and drift reduction efficiency. However, this increase diminishes quickly after four outriggers. A 4-outrigger system is almost as efficient as a five or six outrigger system. Therefore, the optimum upper limit of outriggers for the 80-story building example is four.

**Optimum Location/Number Summary**

In conclusion, it has been shown that for an 80-story building the optimum location for a one-outrigger system is 0.455H. However, for architectural reasons the outrigger can be placed at exactly 1/2H with minimal decrease in efficiency. This equal spacing of one outrigger can be used for any number of outriggers. Also, it has been shown that the optimum upper limit of outriggers for an 80-story building is four. Anything above four outriggers adds very little efficiency. For the 80-story building example it is recommended that four outriggers are used at equal spacing (i.e. 1/5H, 2/5H, 3/5H, 4/5H).
SUMMARY AND RECOMMENDATIONS

Summary

The objective of this thesis was to research all the information available about the optimization of outrigger structural systems. After extensive research into many different methods of analysis of outrigger structural systems, it was decided to use the method put forth by Bryan Stafford Smith in his book, *Tall Building Structures*. After an in-depth review, this analysis was used on an 80-story building to determine the optimum location and number of outriggers.

Analysis of the 80-story building showed that the optimum location of a 1-outrigger system was 0.455H. This location offered the greatest drift reduction efficiency (77.60%) and adequate moment reduction efficiency (48.65%). Furthermore, it was determined that locating the outrigger at exactly the mid-height of the building (equal interval spacing, 1/2H) offered little decrease in efficiency. This allowed equal interval spacing to be used for any number of outriggers desired.

Using equal interval spacing for 1, 2, 3, 4, 5 and 6 outriggers it was determined that an upper limit of four outriggers exists. Adding additional outriggers after four minimally increases the drift reduction and moment reduction efficiency minimally. Therefore, for the 80-story example building it was recommended that four outriggers be used at equal interval spacing (1/5H, 2/5H, 3/5H, and 4/5H).
Works Cited


Appendix 1 – Core Design

Calculation of Sectorial Properties

Determine, for the open-section shown:
1. the location of the shear center,
2. the principal sectorial coordinate diagram,
3. the sectorial moment of inertia, $I_s$, and
4. the St. Venant torsion constant $J$.

1. Location of Shear Center.

$$I_{s,x} = \int \omega' y'ds$$

For
- DC = $53,333 \text{ ft}^4$
- CB = $240,000 \text{ ft}^4$
- BA = $88,889 \text{ ft}^4$

For half the section $I_{s,x} = \frac{1}{2} \int \omega' y'ds = 382,222 \text{ ft}^4$

For the whole section $I_{s,x} = 2 \cdot \text{half} = 764,444 \text{ ft}^4$

The moment of inertia of the section about the X axis

$$I_{s,x} = 25,110 \text{ ft}^4$$

The distance of shear center from the pole $O'$ is

$$\alpha_s = \frac{I_{s,x}}{I_{s,x}} = 30.44 \text{ ft}$$

2. Principal Sectorial Coordinate Diagram.

$$\omega = \omega' - \alpha_s y$$

At A $\omega = 394.08 \text{ ft}^2$
At B $\omega = 191.12 \text{ ft}^2$
At C $\omega = -208.88 \text{ ft}^2$

From antisymmetry the respective values at G, F, and E have the same magnitude but are of opposite signs.
Appendix 1 – Core Design Continued

3. Sectorial Moment of Inertia \( I_n \):

\[
\int \omega^{-1} \, db 
\]

For

- DC: 290,879 ft\(^6\)
- CB: 268,245 ft\(^6\)
- BA: 593,642 ft\(^6\)

Total = 1,152,766 ft\(^6\) - for half section

\( I_n \) for the whole section = 2 \times \text{half section}

\( I_n = 2,305,532 \text{ ft}^6 \)

4. Torsion Constant J. For the open section core,

\[
J = \frac{1}{3} \sum bt^3
\]

\( J = 31.11 \text{ ft}^4 \)
Appendix 2 – Case L1 Calculations (x₁ = 0 ft)

Analysis of Outrigger Structural System

Case L1 - Outrigger located at the top

Givens:
Inter-story h = 12 ft
# stories = 80
Aspect ratio = 8 : 1
Total height (H) = 960 ft
Width of building (d) = 120 ft
Outrigger location x₁ = 0 ft
Uniform wind loading (w) = 5.05 kip/ft
Modulud of elasticity (E) = 29,000 ksi

Step 1: Determine axial rigidity of the column (EA), and effective flexural rigidity of the outrigger (EI)o.

\[ E = 29,000 \text{ ksi} \]
\[ A = 9.72 \text{ ft}^2 \]
\[ (EA)_c = \begin{cases} 4.06 \times 10^7 \text{ k} & (EA)_c = \text{axial rigidity of the column} \\ \end{cases} \]

Column Area
b = 3.118 ft
h = 3.118 ft

Equivalent column area
A = 1400 in²

\[ (EI)_o = (1+a/b)^3 (EI)_0 \]
\[ E = 29,000 \text{ ksi} \]
\[ I' = 785 \text{ ft}^4 \]
\[ (EI)_0 = 3.28 \times 10^9 \text{ k-ft}^2 \]
\[ (EI)_0 = \text{effective flexural rigidity of the outrigger} \]

Outrigger Inertia
b = 13.352 ft
h = 13.352 ft

a = 20 ft
b = 40 ft
\[ (EI)_o = 1.11 \times 10^{-10} \text{ k-ft}^3 \]

Step 2: Determine S and S₁.

\[ S = 1/(EI) + 2/(d^2(EA)_c) \]
\[ E = 29,000 \text{ ksi} \]
\[ I = 25,110 \text{ ft}^4 \]
\[ EI = 1.06 \times 10^8 \text{ k-ft}^2 \]
\[ EI = \text{flexural rigidity of core} \]

Core Inertia
b = 23.429 ft
h = 23.429 ft

\[ S = 1.30 \times 10^{-11} \text{ 1/(k-ft)} \]

\[ S₁ = d / (12(EI)_o) \]
\[ S₁ = 9.04 \times 10^{-10} \text{ 1/(k-ft)} \]
Appendix 2 – Case L1 Calculations Continued

Step 3: Determine restraining moment applied to core $M_i$.

Matrices

| S matrix units = 1/(k-ft) | $1.33E-08$ |
| H matrix units = ft$^2$ | $8.85E+08$ |

| $S^{-1}$ units = (k-ft) | $74942854$ |

$w/(6EI) = 8.03E-12$ 1/(ft$^3$)

$$M_i = \frac{w}{6EI} \cdot (S + S(H - X))^T \cdot (H^T - X^T)$$

$$M_i = 532,127 \text{ kip-ft}$$

Step 4: Determine resulting moment in the core.

$$M_c = \frac{wx^2}{2} - M_i$$

$$M_c = 1,794,559 \text{ k-ft}$$

Step 5: Determine moment reduction efficiency

$$M_c = \frac{1}{EIS} \frac{wH^2}{2}$$

$$M_c = 1,712,422 \text{ k-ft}$$

$$M \% = \sum_{i=1}^{n} \frac{M_i}{M_c} \times 100$$

$$\text{sum } M_i = 532,127 \text{ k-ft}$$

$$M\% = 31.07\%$$

Step 6: Determine horizontal deflection at the top.

$$\Delta = \frac{wH}{8EI} \frac{1}{2EI_{pl}} \sum_{i=1}^{n} M_i (H - x_i)$$

$$\Delta = 2.774 \text{ ft} \quad \text{NOT OK}$$

$$\Delta_{\text{allowable}} = H/400 = 2.400 \text{ ft}$$
Appendix 2 – Case L1 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI} \]

\[ \Delta_c = 3.763 \text{ ft} \]

\[ \Delta\% = \frac{\sum_{i=1}^{n} M_i(H_i^2 - X_i^2)}{\Delta_c} \]

\[ \Delta\text{sum M} = 2.338 \text{ ft} \]

\[ \Delta\% = 62.15\% \]

Step 8: Determine force in columns and moment in outriggers

Force in Columns due to outriggers
4,434 kip for \( x_1 < x < \) \( x \)

Maximum moment in the outriggers
177,376 k-ft level 1

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EIL}{(EA)_c(d^2/2)} \]

\[ \alpha = 0.36 \quad \alpha \text{ = the core-to-column rigidities} \]

\[ \beta = \frac{EIL}{(EI)_o} \frac{d}{H} \]

\[ \beta = 1.19 \quad \beta \text{ = the core-to-outrigger rigidities} \]

\[ \omega = \frac{\beta}{12(1+\alpha)} \]

\[ \omega = 0.073 \quad \omega \text{ = the characteristic structural parameter} \]

\text{for a uniform structure with flexible outriggers} \]

Step 10: Summary

\[ M_r = 532,127 \text{ kip-ft} \quad \text{- restraining moment} \]

\[ M_c = 1,794,559 \text{ kip-ft} \quad \text{- resulting moment in core} \]

\[ M\% = 31.07\% \quad \text{- moment reduction efficiency} \]

\[ \Delta_t = 2.774 \text{ ft} \quad \text{- deflection at top} \]

\[ \Delta_{\text{allowable}} = 2.400 \text{ ft} \quad \text{- allowable deflection} \]

\[ \Delta\% = 62.15\% \quad \text{- deflection reduction efficiency} \]
Appendix 2 – Case L1 Calculations Continued

![Graph 1: Bending Moment vs. Outrigger Distance from Top](image)

![Graph 2: Restraining Moment vs. Outrigger Distance from Top](image)
Appendix 3 – Case L2 Calculations ($x_1 = 240$ ft)

Analysis of Outrigger Structural System

Case L2 - Outrigger located at 3/4 height

Given:
Inter-story $h = 12$ ft
# stories = 80
Aspect ratio = 8 : 1
Total height ($H$) = 960 ft
Width of building ($d$) = 120 ft
Outrigger location $x_1 = 240$ ft
Uniform wind loading ($w$) = 5.05 kip/ft
Modulud of elasticity ($E$) = 29,000 ksi

Step 1: Determine axial rigidity of the column $(EA)_c$ and effective flexural rigidity of the outrigger $(EI)_o$.

$$
E = 29,000 \text{ ksi} \\
A = 9.72 \text{ ft}^2 \\
(\frac{EA}{})_c = 4.06E+07 \text{ k}
$$

Column Area

$b = 3.118 \text{ ft}$
$h = 3.118 \text{ ft}$

Equivalent column area

$A = 1400 \text{ in}^2$

$$(\frac{EI}{})_c = (1+a/b)^3 (\frac{EI}{})_o$$

$$
E = 29,000 \text{ ksi} \\
l' = 785 \text{ ft}^4 \\
(\frac{EI}{})_o = 3.28E+09 \text{ k-ft}^2 \\
(\frac{EI}{})_o = 29,000 \text{ ksi} \\
(\frac{EI}{})_o = 785 \text{ ft}^4 \\
(\frac{EI}{})_o = 3.28E+09 \text{ k-ft}^2 \\
$$

Outrigger Inertia

$b = 13.352 \text{ ft}$
$h = 13.352 \text{ ft}$

$a = 20 \text{ ft}$
$b = 40 \text{ ft}$

$$(\frac{EI}{})_o = 1.11E+10 \text{ k-ft}^2$$

Step 2: Determine $S$ and $S_1$.

$$
S = \frac{1}{(EI)} + 2/(d^2(\frac{EA}{})_c) \\
S = 29,000 \text{ ksi} \\
l = 25,110 \text{ ft}^4 \\
EI = 1.05E+11 \text{ k-ft}^2 \\
EI = \text{flexural rigidity of core} \\
$$

Core Inertia

$b = 23.429 \text{ ft}$
$h = 23.429 \text{ ft}$

$S = 1.30E-11 \text{ 1/(k-ft)^2}$

$S_1 = d / (12(\frac{EI}{})_o)$

$S_1 = 9.04E-10 \text{ 1/(k-ft)}$
Appendix 3 – Case L2 Calculations Continued

Step 3: Determine restraining moment applied to core $M_1$.

Matrices

<table>
<thead>
<tr>
<th>$S$ matrix units = 1/(k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.02E-08$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S$ inverse units = (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9.77E+07$</td>
</tr>
</tbody>
</table>

$w/(6EI) = 8.03E-12 \text{ 1/ft}^2$

$$M_1 = \frac{w}{6EI} \left[ 6S + S(H - X_6)^{\frac{1}{4}} \left[ \frac{H^3}{3} - X_6^3 \right] \right]$$

$M_1 = 682,989 \text{ kip-ft}$

Step 4: Determine resulting moment in the core.

$$M_c = \frac{wx^2}{2} - M_1$$

$M_c = 1,643,697 \text{ k-ft}$

Step 5: Determine moment reduction efficiency

$$M_c = \frac{1}{EIS} \frac{wH^2}{2}$$

$M_c = 1,712,422 \text{ k-ft}$

$$M\% = \frac{\sum_{i=1}^{n} M_i}{M_c} \times 100$$

sum $M_i = 682,989 \text{ k-ft}$

$M\% = 39.88\%$

Step 6: Determine horizontal deflection at the top.

$$\Delta_o = \frac{wH}{8EI} \left[ 1 - \frac{1}{2EI_{cd}} \sum_{i=1}^{n} M_i \left( \frac{H^2}{2} - X_6^2 \right) \right]$$

$\Delta_o = 2.299 \text{ ft} \quad \text{OK}$

$$\Delta_{\text{allowable}} = \frac{H}{400}$$

$\Delta_{\text{allowable}} = 2.400 \text{ ft}$
Appendix 3 – Case L2 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{8EI} \frac{wH^4}{EIS} \]

\[ \Delta_c = 3.783 \text{ ft} \]

\[ \Delta\% = \frac{1}{2Ei} \sum \Delta \left( \frac{H^2 - X_i^2}{H^2} \right) \]

\[ \Delta\text{sumM} = 2.814 \text{ ft} \]

\[ \Delta\% = 74.78\% \]

Step 8: Determine force in columns and moment in outriggers

Force in Columns due to outriggers

5,692 kip for \( x_i < x < \)

Maximum moment in the outriggers

227,663 k-ft level 1

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)_{c}\left(d^2/2\right)} \]

\[ \alpha = 0.36 \]

\( \alpha \) = the core-to-column rigidities

\[ \beta = \frac{EI}{(EI)_{o}} \frac{d}{H} \]

\[ \beta = 1.19 \]

\( \beta \) = the core-to-outrigger rigidities

\[ \omega = \frac{\beta}{12(1+\alpha)} \]

\[ \omega = 0.073 \]

\( \omega \) = the characteristic structural parameter

for a uniform structure with flexible outriggers

Step 10: Summary

\( M_r = 682,989 \) kip-ft - restraining moment

\( M_c = 1,643,697 \) kip-ft - resulting moment in core

\( M\% = 39.88\% \) - moment reduction efficiency

\( \Delta_c = 2.299 \) ft - deflection at top

\( \Delta_{allowable} = 2.400 \) ft - allowable deflection

\( \Delta\% = 74.78\% \) - deflection reduction efficiency
Appendix 3 – Case L2 Calculations Continued
Appendix 4 – Case L3 Calculations \((x_1 = 432 \text{ ft})\)

Analysis of Outrigger Structural System

**Case L3 - Outrigger located at calculated optimum location**

**Givens:**
- Inter-story height \(h\) = 12 ft
- Number of stories \# stories = 80
- Aspect ratio = 8 : 1
- Total height (H) = 960 ft
- Width of building (d) = 120 ft
- Optimum location = 437 ft
- Actual Outrigger location \(x_1\) = 432 ft\
- Uniform wind loading \(w\) = 5.05 kip/ft\
- Modulus of elasticity \((E)\) = 29,000 ksi

**Step 1: Determine axial rigidity of the column \((EA)_c\) and effective flexural rigidity of the outrigger \((EI)_o\)**

\[
E = 29,000 \text{ ksi} \\
A = 9.72 \text{ ft}^2 \\
(\text{EA})_c = 4.06 \times 10^7 \text{ kips ft} \\
\text{Column Area} \\
b = 3.118 \text{ ft} \\
h = 3.118 \text{ ft} \\
\text{Equivalent column area} \\
A = 1400 \text{ in}^2 \\
(\text{El})_o = (1 + a/b)^3 (\text{El'})_o \\
E = 29,000 \text{ ksi} \\
I' = 785 \text{ ft}^4 \\
(\text{El'})_o = 3.28 \times 10^9 \text{ k-ft}^2 \\
(\text{EI})_o = 1.11 \times 10^10 \text{ k-ft}^2 \\
\text{Outrigger Inertia} \\
b = 13.352 \text{ ft} \\
h = 13.352 \text{ ft} \\
a = 20 \text{ ft} \\
b = 40 \text{ ft} \\
(\text{EI})_o = 1.11 \times 10^10 \text{ k-ft}^2 \\
\text{Step 2: Determine S and S_t.} \\
S = 1/(\text{EI}) + 2/(d^2(\text{EA})_c) \\
E = 29,000 \text{ ksi} \\
I = 25,110 \text{ ft}^4 \\
\text{EI} = 1.05 \times 10^11 \text{ k-ft}^2 \\
\text{Core Inertia} \\
b = 23.429 \text{ ft} \\
h = 23.429 \text{ ft} \\
S = 1.30 \times 10^{-11} 1/(\text{k-ft}^2) \\
S_t = d / (12(\text{EI})_o) \\
S_t = 9.04 \times 10^{-10} 1/(\text{k-ft})
Appendix 4 – Case L3 Calculations Continued

Step 3: Determine restraining moment applied to core $M_1$.

Matrices

<table>
<thead>
<tr>
<th></th>
<th>S matrix</th>
<th>units = 1/(k-ft)</th>
<th>H matrix</th>
<th>units = ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.75E-09</td>
<td></td>
<td></td>
<td>8.04E+08</td>
</tr>
<tr>
<td>S inverse</td>
<td>129101833</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$w/(6EI) = 8.03E-12$ 1/(ft³)

\[
M_1 = \frac{w}{6EI} \cdot [S_1 + S(H - X_1)]^{-1} \cdot [H^3 - X_1^3]
\]

$M_1 = 833,146$ kip-ft

Step 4: Determine resulting moment in the core.

\[
M_c = \frac{w x^2}{2} - M_1
\]

$M_c = 1,493,539$ kip-ft

Step 5: Determine moment reduction efficiency

\[
M_c = \frac{1}{EIS} \frac{wH^2}{2}
\]

$M_c = 1,712,422$ k-ft

\[
M\% = \frac{\sum M_i}{M_c} \times 100
\]

sum $M_i = 833,146$ k-ft

$M\% = 48.65\%$

Step 6: Determine horizontal deflection at the top.

\[
\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^n M_i \left(H^2 - X_i^2\right)
\]

$\Delta_o = 2.192$ ft \(\rightarrow\) OK

\[
\Delta_{allowable} = \frac{H}{400}
\]

$\Delta_{allowable} = 2.400$ ft
Appendix 4 – Case L3 Calculations Continued

Step 7: Determine drift reduction efficiency

\[
\Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI}
\]

\[
\Delta_c = 3.763 \text{ ft}
\]

\[
\Delta\% = \frac{1}{2EI} \sum \frac{M_i(H^2 - X_i^2)}{\Delta_c}
\]

\[
\Delta\% = 77.60\%
\]

\[
\Delta_c \text{ sum } M = 2.920 \text{ ft}
\]

Step 8: Determine force in columns and moment in outriggers

Force in Column due to outriggers

6,943 kip

for \( x_1 < x < \)

Maximum moment in the outriggers

277,715 k-ft

level 1

Step 9: Determine optimum location nondimensional parameters

\[
\alpha = \frac{EI}{(EA)_c \left(d^2 / 2\right)}
\]

\[
\alpha = 0.36 \quad \alpha = \text{the core-to-column rigidities}
\]

\[
\beta = \frac{EI}{(EI)_c} \frac{d}{H}
\]

\[
\beta = 1.19 \quad \beta = \text{the core-to-outrigger rigidities}
\]

\[
\omega = \frac{\beta}{12(1 + \alpha)}
\]

\[
\omega = 0.073 \quad \omega = \text{the characteristic structural parameter}
\]

for a uniform structure with flexible outriggers

Step 10: Summary

\[
M_1 = 833,146 \text{ kip-ft} \quad \text{- restraining moment}
\]

\[
M_c = 1,493,539 \text{ kip-ft} \quad \text{- resulting moment in core}
\]

\[
M\% = 48.65\% \quad \text{- moment reduction efficiency}
\]

\[
\Delta_0 = 2.192 \text{ ft} \quad \text{- deflection at top}
\]

\[
\Delta_{allowable} = 2.400 \text{ ft} \quad \text{- allowable deflection}
\]

\[
\Delta\% = 77.60\% \quad \text{- deflection reduction efficiency}
\]
Appendix 4 – Case L3 Calculations Continued
Appendix 5 – Case L4 Calculations ($x_1 = 480$ ft)

Analysis of Outrigger Structural System

Case L4 - Outrigger located at the middle

Givens:
- Inter-story $h =$ 12 ft
- # stories = 80
- Aspect ratio = 8 : 1
- Total height (H) = 960 ft
- Width of building (d) = 120 ft
- Outrigger location $x_1 =$ 480 ft $x_1 =$ distance from top to first outrigger
- Uniform wind loading (w) = 5.05 kip/ft
- Modulus of elasticity (E) = 29,000 ksi

Step 1: Determine axial rigidity of the column $(EA)_c$ and effective flexural rigidity of the outrigger $(EI)_o$

\[
E = 29,000 \text{ ksi} \\
A = 9.72 \text{ ft}^2 \\
(EA)_c = 4.06 \times 10^7 \text{ k} = (EA)_c \text{ = axial rigidity of the column}
\]

\[
\text{Column Area} \quad b = 3.118 \text{ ft} \\
h = 3.118 \text{ ft} \\
\text{Equivalent column area} \quad A = 1400 \text{ in}^2
\]

\[
(EI)_c = (1+a/b)^3 (EI')_o \\
E = 29,000 \text{ ksi} \\
I' = 785 \text{ ft}^4 \\
(EI')_o = 3.28 \times 10^9 \text{ k-ft}^2 \quad (EI')_o \text{ = actual rigidity of the outrigger}
\]

Outrigger Inertia
\[
\begin{align*}
a &= 20 \text{ ft} \\
b &= 40 \text{ ft} \\
(EI)_o &= 1.11 \times 10^{10} \text{ k-ft}^2
\end{align*}
\]

Step 2: Determine $S$ and $S_{1}$.

\[
S = \frac{1}{EI} + \frac{2}{(d^2)(EA)_c} \\
E = 29,000 \text{ ksi} \\
I = 25,110 \text{ ft}^4 \\
EI = 1.05 \times 10^{11} \text{ k-ft}^2 \quad EI = \text{flexural rigidity of core}
\]

Core Inertia
\[
\begin{align*}
b &= 23.429 \text{ ft} \\
h &= 23.429 \text{ ft}
\end{align*}
\]

\[
S = 1.30 \times 10^{-11} \frac{1}{(k-ft^2)}
\]

\[
S_{1} = \frac{d}{12(EI)_o} \\
S_{1} = 9.04 \times 10^{-10} \frac{1}{(k-ft)}
\]
Appendix 5 – Case L4 Calculations Continued

Step 3: Determine restraining moment applied to core \( M_i \).

Matrices

\[
\begin{array}{c|c}
S \text{ matrix units} & \frac{1}{\text{(k-ft)}} \\
7.12\text{E-09} & 7.74\text{E+08} \\
S \text{ inverse units} & \text{(k-ft)} \\
1.40\text{E+08} & \\
\end{array}
\]

\[
w/(6EI) = 8.03\text{E-12} \text{ 1/(ft}^3\text{)}
\]

\[
M_i = \frac{w}{6EI} \cdot \left[ S_i + S(H - X_i) \right] \cdot \left[ H^3 - X_i^3 \right]
\]

\[
M_i = 872,122 \text{ kip-ft}
\]

Step 4: Determine resulting moment in the core.

\[
M_e = \frac{wx^2}{2} - M_i
\]

\[
M_e = 1,454,563 \text{ kip-ft}
\]

Step 5: Determine moment reduction efficiency

\[
M_\% = \frac{\sum_{i=1}^{n} M_i}{M_c} \times 100
\]

\[
\text{sum } M_i = 872,122 \text{ k-ft}
\]

\[
M_\% = 50.93\%
\]

Step 6: Determine horizontal deflection at the top.

\[
\Delta_v = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H^2 - X_i^2 \right)
\]

\[
\Delta_v = 2.238 \text{ ft} \quad \text{OK}
\]

\[
\Delta_{\text{allowable}} = \frac{H}{400}
\]

\[
\Delta_{\text{allowable}} = 2.400 \text{ ft}
\]
Appendix 5 – Case L4 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{8EI} \frac{wH^4}{EIS} \]

\[ \Delta_c = 3.763 \text{ ft} \]

\[ \Delta_c = \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H_i^2 - X_i^2 \right) \]

\[ \Delta_c \text{ sum M} = 2.874 \text{ ft} \]

\[ \Delta_c \% = 76.39\% \]

Step 8: Determine force in columns and moment in outriggers

Force in Column due to outriggers
7,268 kip for \( x_1 < x < \)

Maximum moment in the outriggers
290,707 k-ft level 1

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA) \left( \frac{d^2}{2} \right)} \]

\[ \alpha = 0.36 \]

\( \alpha \) = the core-to-column rigidities

\[ \beta = \frac{EI}{(EI)_o} \frac{d}{H} \]

\[ \beta = 1.19 \]

\( \beta \) = the core-to-outrigger rigidities

\[ \omega = \frac{\beta}{12(1 + \alpha)} \]

\[ \omega = 0.073 \]

\( \omega \) = the characteristic structural parameter for a uniform structure with flexible outriggers

Step 10: Summary

\[ M_1 = 872,122 \text{ kip-ft} \]

\( M_1 \) = restraining moment

\[ M_2 = 1,454,563 \text{ kip-ft} \]

\( M_2 \) = resulting moment in core

\[ M c\% = 50.93\% \]

\( M c\% \) = moment reduction efficiency

\[ \Delta c = 2.238 \text{ ft} \]

\( \Delta c \) = deflection at top

\[ \Delta_{allowable} = 2.400 \text{ ft} \]

\( \Delta_{allowable} \) = allowable deflection

\[ \Delta_c \% = 76.39\% \]

\( \Delta_c \% \) = deflection reduction efficiency
Appendix 5 – Case L4 Calculations Continued

### Restraining Moment (k-ft)

<table>
<thead>
<tr>
<th>Restraining Moment (k-ft)</th>
<th>Outrigger Distance from Top (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500,000</td>
<td>500</td>
</tr>
<tr>
<td>1,000,000</td>
<td>600</td>
</tr>
<tr>
<td>1,500,000</td>
<td>700</td>
</tr>
<tr>
<td>2,000,000</td>
<td>800</td>
</tr>
<tr>
<td>2,500,000</td>
<td>900</td>
</tr>
</tbody>
</table>

### Bending Moment (k-ft)

<table>
<thead>
<tr>
<th>Bending Moment (k-ft)</th>
<th>Outrigger Distance from Top (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500,000</td>
<td>500</td>
</tr>
<tr>
<td>1,000,000</td>
<td>600</td>
</tr>
<tr>
<td>1,500,000</td>
<td>700</td>
</tr>
<tr>
<td>2,000,000</td>
<td>800</td>
</tr>
<tr>
<td>2,500,000</td>
<td>900</td>
</tr>
</tbody>
</table>

### Outrigger Distance from Top (ft)

<table>
<thead>
<tr>
<th>Outrigger Distance from Top (ft)</th>
<th>M core</th>
<th>M exter</th>
<th>M comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>1000</td>
<td>500</td>
<td>600</td>
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</tr>
<tr>
<td>1500</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>2000</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
<tr>
<td>2500</td>
<td>500</td>
<td>600</td>
<td>700</td>
</tr>
</tbody>
</table>
Appendix 6 – Case L5 Calculations ($x_1 = 720\text{ft}$)

Analysis of Outrigger Structural System

Case L5 - Outrigger located near the bottom

Given:
- Inter-story $h = 12 \text{ ft}$
- Stories $= 80$
- Aspect ratio $= 8 : 1$
- Total height ($H$) = 960 ft
- Width of building ($d$) = 120 ft
- Outrigger location $x_1 = 720 \text{ ft}$ $x_1 = \text{distance from top to first outrigger}$
- Uniform wind loading ($w$) = 5.05 kip/ft
- Modulus of elasticity ($E$) = 29,000 ksi

Step 1: Determine axial rigidity of the column ($EA_e$) and effective flexural rigidity of the outrigger ($EI_0$).

$E = 29,000 \text{ ksi}$

$A = 9.72 \text{ ft}^2$

$(EA_e) = 40,600,000 \text{ k}

$\text{Column Area}$

$A = 1400 \text{ in}^2$

$(EI)_0 = (1+b/a)(EI)'$

$E = 29,000 \text{ ksi}$

$I' = 785 \text{ ft}^4$

$(EI)' = 3.28 \times 10^9 \text{ k-ft}^2$

$(EI)'_0$ = effective flexural rigidity of the outrigger

$Outrigger Inertia$

$b = 13.352 \text{ ft}$

$h = 23.429 \text{ ft}$

$a = 20 \text{ ft}$ $a = \text{half the width of core}$

$b = 40 \text{ ft}$ $b = d/2 - a$

$(EI)'_0 = 1.11 \times 10^{10} \text{ k-ft}^2$

Step 2: Determine $S$ and $S_1$.

$S = 1/(EI) + 2/(d^2(EA_e))$

$E = 29,000 \text{ ksi}$

$I = 25,110 \text{ ft}^4$

$EI = 1.05 \times 10^{11} \text{ k-ft}^2$ $EI = \text{flexural rigidity of core}$

$Core Inertia$

$b = 23.429 \text{ ft}$

$h = 23.429 \text{ ft}$

$S = 1.30 \times 10^{-11} \text{ 1/(k-ft)}$

$S_1 = d/(12(EI)_0)$

$S_1 = 9.04 \times 10^{-10} \text{ 1/(k-ft)}$
Appendix 6 – Case L5 Calculations Continued

Step 3: Determine restraining moment applied to core $M_1$.

Matrices

<table>
<thead>
<tr>
<th>S matrix units = $1/(k$-ft)</th>
<th>H matrix units = $ft^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.01E-09$</td>
<td>$5.11E+08$</td>
</tr>
</tbody>
</table>

$w/(6EI) = 8.03E-12 \ 1/(ft^3)$

\[
M_1 = \frac{w}{6EI} \left[ S + S(H - X_1) \right]^{-1} \left[ H^3 - X_1^3 \right]
\]

$M_1 = 1,022,645 \ \text{kip-ft}$

Step 4: Determine resulting moment in the core.

\[
M_e = \frac{wx^2}{2} - M_1
\]

$M_e = 1,304,040 \ \text{kip-ft}$

Step 5: Determine moment reduction efficiency

\[
M_e = \frac{1}{EIS} \frac{wH^2}{2}
\]

$M_e = 1,712,422 \ \text{k-ft}$

\[
M\% = \frac{\sum M_i - M_e}{\sum M_i} \times 100
\]

sum $M_i = 1,022,645 \ \text{k-ft}$

$M\% = 59.72\%$

Step 6: Determine horizontal deflection at the top.

\[
\Delta_y = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - X_i^2)
\]

$\Delta_y = 3.146 \ \text{ft} \quad \text{NOT OK}$

$\Delta_{\text{allowable}} = \frac{H}{400}$

$\Delta_{\text{allowable}} = 2.400 \ \text{ft}$
Appendix 6 – Case L5 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{EIS \frac{8EI}{wH^4}} \]

\[ \Delta_c = 3.763 \text{ ft} \]

\[ \Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H^2 - X_i^2 \right) \]

\[ \Delta\% = 52.25\% \]

Step 8: Determine force in columns and moment in outriggers

Force in Column due to outriggers

8,522 kip for \( x_1 < x < \)

Maximum moment in the outriggers

340,882 kip-ft level 1

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)_c \left( d^4 / 2 \right)} \]

\[ \alpha = 0.36 \quad \alpha = \text{the core-to-column rigidities} \]

\[ \beta = \frac{EI}{(EI)_o} \frac{d}{H} \]

\[ \beta = 1.19 \quad \beta = \text{the core-to-outrigger rigidities} \]

\[ \omega = \frac{\beta}{12(1 + \alpha)} \]

\[ \omega = 0.073 \quad \omega = \text{the characteristic structural parameter for a uniform structure with flexible outriggers} \]

Step 10: Summary

\( M_1 = 1,022,645 \text{ kip-ft} \) - restraining moment

\( M_c = 1,304,040 \text{ kip-ft} \) - resulting moment in core

\( M\% = 59.72\% \) - moment reduction efficiency

\( \Delta_c = 3.146 \text{ ft} \) - deflection at top

\( \Delta_{allowable} = 2.400 \text{ ft} \) - allowable deflection

\( \Delta\% = 52.25\% \) - deflection reduction efficiency
Appendix 6 – Case L5 Calculations Continued

![Graph 1: Restraining Moment vs. Outrigger Distance from Top](image1)

![Graph 2: Bending Moment vs. Outrigger Distance from Top](image2)
Appendix 7 – Case N1 Calculations: 1-Outrigger

Analysis of Outrigger Structural System

Case N1 - 1-outriggers placed at equal interval

Givens:
Inter-story h = 12 ft
# stories = 80
Aspect ratio = 8 : 1
Total height (H) = 960 ft
Width of building (d) = 120 ft
Outrigger location x_1 = 480 ft
Uniform wind loading (w) = 5.05 kip/ft
Modulus of elasticity (E) = 29,000 ksi

Step 1: Determine axial rigidity of the column (EA)_c and effective flexural rigidity of the outrigger (EI)_o.

\[
E = 29,000 \text{ ksi} \\
A = 7.81 \text{ ft}^2 \\
(\text{EA})_c = 32,625,000 \text{ k} \\
\text{(EA)}_c = \text{axial rigidity of the column}
\]

Column Area
\[
b = 3.118 \text{ ft} \\
h = 2.506 \text{ ft}
\]

Equivalent column area
\[
A = 1125 \text{ in}^2
\]

\[
(\text{EI})_o = (1+a/b)^3 (\text{EI})_o \\
E = 29,000 \text{ ksi} \\
I' = 785 \text{ ft}^4 \\
(\text{EI})_o = 3.28E+09 \text{ k-ft}^2 \\
\text{(EI)}_o = \text{effective flexural rigidity of the outrigger}
\]

Outrigger Inertia
\[
b = 13.352 \text{ ft} \\
h = 13.352 \text{ ft}
\]

\[
a = 20 \text{ ft} \\
b = 40 \text{ ft} \\
\text{(EI)}_o = 1.11E+10 \text{ k-ft}^2
\]

Step 2: Determine S and S_1.

\[
S = 1/(EI) + 2/(d^2(\text{EA})_c) \\
E = 29,000 \text{ ksi} \\
I = 25,110 \text{ ft}^4 \\
EI = 1.05E+11 \text{ k-ft}^2 \\
\text{EI} = \text{flexural rigidity of core}
\]

Core Inertia
\[
b = 23.429 \text{ ft} \\
h = 23.429 \text{ ft}
\]

\[
S = 1.37938E-11 1/(k-ft^2) \\
S_1 = d / (12(\text{EI})_o) \\
S_1 = 9.04218E-10 1/(k-ft)
\]
Appendix 7 – Case N1 Calculations Continued

Step 3: Determine restraining moment applied to core $M_1$.

Matrices

<table>
<thead>
<tr>
<th></th>
<th>$S$ matrix units = 1/(k-ft)</th>
<th>$H$ matrix units = ft$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.53E-09</td>
<td>7.74E+08</td>
</tr>
</tbody>
</table>

$S$ inverse units = (k-ft)

|             | 132886023                  |

$\frac{w}{(6EI)} = 8.03E-12$ 1/(ft$^3$)

$$M_1 = \frac{w}{6EI} \left[ S + S(H - X_i) \right] \cdot \left[ H^3 - X_i^3 \right]$$

$M_1 = 825,605$ kip-ft

Step 4: Determine resulting moment in the core.

$$M_c = \frac{wx^2}{2} - M_1$$

$M_c = 1,501,081$ kip-ft

Step 5: Determine moment reduction efficiency

$$M_\% = \frac{\sum_{i=1}^{n} M_i}{M_c} \times 100$$

$\text{sum } M_i = 825,605$ k-ft

$M_\% = 51.32\%$

Step 6: Determine horizontal deflection at the top.

$$\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H^3 - X_i^3 \right)$$

$\Delta_o = 2.391$ ft  ← OK

$$\Delta_{allowable} = \frac{H}{400}$$

$\Delta_{allowable} = 2.400$ ft
Appendix 7 – Case N1 Calculations Continued

Step 7: Determine drift reduction efficiency

$$\Delta_c = \frac{1}{EIS \cdot 8EI} \frac{wH^4}{EI}$$

\[\Delta_c = 3.535 \text{ ft}\]

$$\Delta% = \frac{1}{EI} \sum_{i=1}^{n} M_i \left( \frac{H^2 - X_i^2}{\Delta_c} \right)$$

\[\Delta_c \text{ sum M} = 2.721 \text{ ft}\]

\[\Delta% = 76.99\%\]

Step 8: Determine force in columns and moment in outriggers

Force in Column due to outriggers

6,880 kip  

for \(x_1 < x <\)

Maximum moment in the outriggers

275,202 k-ft  

level 1

Step 9: Determine optimum location nondimensional parameters

$$\alpha = \frac{EI}{(EA) \left( d^2 / 2 \right)}$$

\[\alpha = 0.45\] \(\alpha\) = the core-to-column rigidities

$$\beta = \frac{EI}{(EI)\cdot H} \frac{d}{H}$$

\[\beta = 1.19\] \(\beta\) = the core-to-outrigger rigidities

$$\omega = \frac{\beta}{12(1 + \alpha)}$$

\[\omega = 0.068\] \(\omega\) = the characteristic structural parameter for a uniform structure with flexible outriggers

Step 10: Summary

\(M_r = 825,605\) kip-ft  

- restraining moment

\(M_c = 1,501,081\) kip-ft  

- resulting moment in core

\(M\% = 51.32\%\)  

- moment reduction efficiency

\(\Delta_0 = 2.391\) ft  

- deflection at top

\(\Delta_{allowable} = 2.400\) ft  

- allowable deflection

\(\Delta% = 76.99\%\)  

- deflection reduction efficiency
Appendix 7 – Case N1 Calculations Continued
Appendix 8 – Case N2 Calculations: 2-Outriggers

Analysis of Outrigger Structural System

Case N2 - 2-outriggers placed at equal intervals

Givens:

Inter-story h = 12 ft
# stories = 80
Aspect ratio = 8 : 1
Total height (H) = 960 ft
Width of building (d) = 120 ft
Optimum location x₁ = 288 ft
Optimum location x₂ = 624 ft
Actual Outrigger location x₁ = 320 ft
Actual Outrigger location x₂ = 640 ft
Uniform wind loading (w) = 5.05 kip/ft
Modulus of elasticity (E) = 29,000 ksi

Step 1: Determine axial rigidity of the column (EA)₀ and effective flexural rigidity of the outrigger (EI)₀.

\[ E = 29,000 \text{ ksi} \]
\[ A = 5.14 \text{ ft}^2 \]
\[ (EA)_0 = 2.15 \times 10^7 \text{ k} \]
\[ (EI)_0 = \text{axial rigidity of the column} \]

Column Area
\[ b = 2.312 \text{ ft} \]
\[ h = 2.223 \text{ ft} \]

Equivalent column area
\[ A = 740 \text{ in}^2 \]
\[ (EI)_0 = (1+a/b)^3 (EI)'_0 \]
\[ (EI)'_0 = \text{effective flexural rigidity of the outrigger} \]

Outrigger Inertia
\[ b = 13.352 \text{ ft} \]
\[ h = 13.352 \text{ ft} \]

a = 20 ft \[ a = \text{half the width of core} \]
b = 40 ft \[ b = \frac{d}{2} - a \]
\[ (EI)'_0 = 3.28 \times 10^9 \text{ k-ft}^2 \]
\[ (EI)'_0 = \text{actual rigidity of the outrigger} \]

Step 2: Determine S and S₁.

\[ S = \frac{1}{EI} + \frac{2}{d^2 (EA)_0} \]
\[ E = 29,000 \text{ ksi} \]
\[ I = 25,110 \text{ ft}^4 \]
\[ EI = 1.05 \times 10^{11} \text{ k-ft}^2 \]
\[ EI = \text{flexural rigidity of core} \]

Core Inertia
\[ b = 23.429 \text{ ft} \]
\[ h = 23.429 \text{ ft} \]

\[ S = 1.60 \times 10^{-11} \text{ 1/(k-ft}^3) \]

\[ S_1 = \frac{d}{12(EI)_0} \]
\[ S_1 = 9.04 \times 10^{-10} \text{ 1/(k-ft)} \]
Appendix 8 – Case N2 Calculations Continued

Step 3: Determine restraining moments applied to core.

Matrices

\[
\begin{align*}
S_i & = \begin{bmatrix}
1.11E-08 & 5.12E-09 \\
5.12E-09 & 6.03E-09 \\
\end{bmatrix} \\
S_i^{-1} & = \begin{bmatrix}
1.47E+08 & -1.25E+08 \\
-1.25E+08 & 2.72E+08 \\
\end{bmatrix} \\
H & = \begin{bmatrix}
8.52E+08 \\
6.23E+08 \\
\end{bmatrix} \\
\end{align*}
\]

\[w/(6EI) = 8.025E-12 \text{ 1/(ft)}^{3}\]

\[M_i = \frac{wS_i^{-1}Matrix \cdot HMatrix}{6EI}\]

\[M_1 = 381,203 \text{ kip-ft}\]

\[M_2 = 505,026 \text{ kip-ft}\]

Step 4: Determine resulting moment in the core.

\[M_c = \frac{wL^2}{2} - M_1 - M_2\]

\[M_c = 1,440,457 \text{ kip-ft}\]

Step 5: Determine moment reduction efficiency

\[M_c = \frac{1}{\sum_{i=1}^{n} M_i/2EI} \cdot \text{sum } M_i\]

\[M_c = 1,386,052 \text{ k-ft}\]

\[M\% = \left(\frac{\text{sum } M_i\%}{M_c}\right) \times 100\]

\[\text{sum } M_i = 886,228 \text{ k-ft}\]

\[M\% = 63.94\%\]

Step 6: Determine horizontal deflection at the top.

\[\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i \left(H^2 - x_i^2\right)\]

\[\Delta_o = 2.390 \text{ ft \hspace{1cm} OK}\]

\[\Delta_{allowable} = \frac{H}{400}\]

\[\Delta_{allowable} = 2.400 \text{ ft}\]
Appendix 8 – Case N2 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI} \]

\[ \Delta_c = 3.046 \text{ ft} \]

\[ \Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H_i^2 - x_i^2 \right) \]

\[ \Delta_c \text{ sum} M = 2.722 \text{ ft} \]

\[ \Delta\% = 89.38\% \]

Step 8: Determine force in columns and moment in outriggers

Forces in Columns due to outriggers
- 3,177 kip for \( x_1 < x < x_2 \)
- 7,385 kip for \( x_2 < x \)

Maximum moment in the outriggers
- 127,068 k-ft level 1
- 158,342 k-ft level 2

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)_c d^2 / 2} \]

\[ \alpha = 0.68 \]

\( \alpha \) = the core-to-column rigidities

\[ \beta = \frac{EI}{(EI)_o H} \]

\[ \beta = 1.19 \]

\( \beta \) = the core-to-outrigger rigidities

\[ \omega = \frac{\beta}{12(1 + \alpha)} \]

\[ \omega = 0.059 \]

\( \omega \) = the characteristic structural parameter for a uniform structure with flexible outriggers

Step 10: Summary

\( M_1 = 381,203 \) kip-ft - restraining moment
\( M_2 = 505,026 \) kip-ft - restraining moment
\( M_3 = 1,440,457 \) kip-ft - resulting moment in core
\( M\% = 63.94\% \) - moment reduction efficiency
\( \Delta_o = 2.390 \) ft - deflection at top
\( \Delta_{allowable} = 2.400 \) ft - allowable deflection
\( \Delta\% = 89.38\% \) - deflection reduction efficiency
Appendix 8 – Case N2 Calculations Continued
Appendix 9 – Case N3 Calculations: 3-Outriggers

Analysis of Outrigger Structural System

Case N3 - 3-outriggers placed at equal intervals

Givens:
- Inter-story h = 12 ft
- # stories = 80
- Aspect ratio = 8 : 1
- Total height (H) = 960 ft
- Width of building (d) = 120 ft
- Optimum location x1 = 230 ft
- Optimum location x2 = 490 ft
- Optimum location x3 = 710 ft
- Actual outrigger location x1 = 240 ft
- Actual outrigger location x2 = 480 ft
- Actual outrigger location x3 = 720 ft
- Uniform wind loading (w) = 5.05 kip/ft
- Modulus of elasticity (E) = 29,000 ksi

Step 1: Determine axial rigidity of the column (EA)0 and effective flexural rigidity of the outrigger (EI)n.

\[ E = 29,000 \text{ ksi} \]
\[ A = 4.65 \text{ ft}^2 \]
\[ (EA)_0 = 19,430,000 \text{ k} \]

Column Area
\[ b = 2.173 \text{ ft} \]
\[ h = 2.141 \text{ ft} \]

Equivalent column area
\[ A = 670 \text{ in}^2 \]

\[ (EI)_n = (1+a/b)^2 (EI)_0 \]
\[ E = 29,000 \text{ ksi} \]
\[ I' = 785 \text{ ft}^4 \]
\[ (ET)_n = 3.28E+09 \text{ k-ft}^2 \]

Outrigger Inertia
\[ b = 13.352 \text{ ft} \]
\[ h = 13.352 \text{ ft} \]

\[ a = 20 \text{ ft} \]
\[ b = d/2 - a \]

\[ (EI)_n = 1.1E+10 \text{ k-ft}^2 \]

Step 2: Determine S and S1.

\[ S = 1/(EI) + 2/(d^2(EA)_0) \]
\[ E = 29,000 \text{ ksi} \]
\[ I = 25,110 \text{ ft}^4 \]
\[ EI = 1.05E+11 \text{ k-ft}^2 \]

Core Inertia
\[ b = 23.429 \text{ ft} \]
\[ h = 23.429 \text{ ft} \]

\[ S = 1.67E-11 \text{ 1/(k-ft)} \]

\[ S_1 = d / (12(EI)_0) \]
\[ S_1 = 9.04E-10 \text{ 1/(k-ft)} \]
Appendix 9 – Case N3 Calculations Continued

Step 3: Determine restraining moments applied to core.

Matrices

<table>
<thead>
<tr>
<th>S matrix</th>
<th>units = 1/(k-ft)</th>
<th>H matrix</th>
<th>units = ft⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.29E-08</td>
<td>8.01E-09</td>
<td>4.00E-09</td>
<td>8.71E+08</td>
</tr>
<tr>
<td>8.01E-09</td>
<td>8.91E-09</td>
<td>4.00E-09</td>
<td>7.74E+08</td>
</tr>
<tr>
<td>4.00E-09</td>
<td>4.00E-09</td>
<td>4.91E-09</td>
<td>5.11E+08</td>
</tr>
</tbody>
</table>

\[
S_{\text{inverse}} \quad \text{units} = (k-ft) \\
1.76E+08 | -1.48E+08 | -2.30E+07 \\
-1.48E+08 | 3.02E+08 | -1.25E+08 \\
-2.30E+07 | -1.25E+08 | 3.25E+08 \\
\]

\[
w/(6EI) = 8.025E-12 \quad 1/(f^3) \\
M = \frac{w}{6EI} S^{-1} \cdot H \text{Matrix} \\
M_1 = 217,941 \text{ kip-ft} \\
M_2 = 324,224 \text{ kip-ft} \\
M_3 = 393,987 \text{ kip-ft} \\
\]

Step 4: Determine resulting moment in the core.

\[
M_c = M_1 - M_2 - M_3 \\
M_c = 1,390,535 \text{ kip-ft} \\
\]

Step 5: Determine moment reduction efficiency

\[
M_c = \frac{1}{EIS} \frac{wH^2}{2} \\
M_c = 1,329,880 \text{ k-ft} \\
\sum M_i = 936,151 \text{ k-ft} \\
M% = 70.39\% \\
\]

Step 6: Determine horizontal deflection at the top.

\[
\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - x_i^2) \\
\Delta_o = 2.368 \text{ ft} \quad \text{OK} \\
\Delta_{\text{allowable}} = \frac{H}{400} \\
\Delta_{\text{allowable}} = 2.400 \text{ ft} \\
\]
Appendix 9 – Case N3 Calculations Continued

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{8EI} \frac{wH^4}{EIS} \]

\[ \Delta_c = 2.922 \text{ ft} \]

\[ \Delta% = \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H_i - X_i^2 \right) \]

\[ \Delta_c \]

\[ \Delta_c \text{ sum M} = 2.724 \text{ ft} \]

\[ \Delta% = 93.22\% \]

Step 8: Determine force in columns and moment in outriggers

Forces in Column due to outriggers
- 1,816 kip for \( x_1 < x < x_2 \)
- 4,518 kip for \( x_2 < x < x_3 \)
- 7,601 kip for \( x_3 < x \)

Maximum moment in the outriggers
- 72,647 k-ft level 1
- 108,075 k-ft level 2
- 131,329 k-ft level 3

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)_{c} \left(d^2 / 2\right)} \]

\[ \alpha = 0.75 \quad \alpha = \text{the core-to-column rigidities} \]

\[ \beta = \frac{EI}{(EI)_{o}} \frac{d}{H} \]

\[ \beta = 1.19 \quad \beta = \text{the core-to-outrigger rigidities} \]

\[ \omega = \frac{\beta}{12(1 + \alpha)} \]

\[ \omega = 0.056 \quad \omega = \text{the characteristic structural parameter for a uniform structure with flexible outriggers} \]

Step 10: Summary

\[
\begin{align*}
M_1 &= 217,941 \text{kip-ft} & - & \text{restraining moment} \\
M_2 &= 324,224 \text{kip-ft} & - & \text{restraining moment} \\
M_3 &= 393,987 \text{kip-ft} & - & \text{restraining moment} \\
M_4 &= 1,390,535 \text{kip-ft} & - & \text{resulting moment in core} \\
M\% &= 70.39\% & - & \text{moment reduction efficiency} \\
\Delta_a &= 2.388 \text{ ft} & - & \text{deflection at top} \\
\Delta_{allowable} &= 2.400 \text{ ft} & - & \text{allowable deflection} \\
\Delta% &= 93.22\% & - & \text{deflection reduction efficiency}
\end{align*}
\]
Appendix 9 – Case N3 Calculations Continued

![Graph 1: Bending Moment vs. Outrigger Distance](image1)

![Graph 2: Restraining Moment vs. Outrigger Distance](image2)
Appendix 10 – Case N4 Calculations: 4-Outriggers

Analysis of Outrigger Structural System

Case N4 - 4-outriggers placed at equal intervals

Givens:

- Inter-story h = 12 ft
- # stories = 80
- Aspect ratio = 8 : 1
- Total height (H) = 960 ft
- Width of building (d) = 120 ft
- Optimum location x₁ = 182 ft
- Optimum location x₂ = 394 ft
- Optimum location x₃ = 588 ft
- Optimum location x₄ = 758 ft
- Actual outrigger location x₁ = 192 ft
- Actual outrigger location x₂ = 384 ft
- Actual outrigger location x₃ = 576 ft
- Actual outrigger location x₄ = 768 ft
- Uniform wind loading (w) = 5.05 kip/ft

Step 1: Determine axial rigidity of the column (EA)ₜ and effective flexural rigidity of the outrigger (EIₜ)

\[ E = 29,000 \text{ ksi} \]
\[ A = 4.44 \text{ ft}^2 \]
\[ (EA)_t = \frac{18,560,000}{k} \]

Column Area
\[ b = 2.124 \text{ ft} \]
\[ h = 2.092 \text{ ft} \]

Equivalent column area
\[ A = 640 \text{ in}^2 \]

\[ (EI)_t = \frac{(1+a/b)^3}{4} (EI)_0 \]
\[ (EI)_0 = 3.28E+09 \text{ k-ft}^2 \]

Outrigger Inertia
\[ b = 13.352 \text{ ft} \]
\[ h = 13.352 \text{ ft} \]
\[ a = 20 \text{ ft} \]
\[ b = 40 \text{ ft} \]

\[ (EI)_e = 1.11E+10 \text{ k-ft}^2 \]

Step 2: Determine S and S₁

\[ S = \frac{1}{E} + 2/d^2 (EA)_t \]
\[ E = 29,000 \text{ ksi} \]
\[ I = 25,110 \text{ ft}^4 \]
\[ EI = 1.05E+11 \text{ k-ft}^2 \]

Core Inertia
\[ b = 23.429 \text{ ft} \]
\[ h = 23.429 \text{ ft} \]

\[ S = 1.70E-11 1/(k-ft^3) \]

\[ S_1 = \frac{d}{12(EI)_0} \]
\[ S_1 = 9.04E-10 1/(k-ft) \]
Appendix 10 – Case N4 Calculations Continued

Step 3: Determine restraining moments applied to core.

Matrices

<table>
<thead>
<tr>
<th>S matrix</th>
<th>units = 1/(k-ft)</th>
<th>H matrix</th>
<th>units = ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40E-08</td>
<td>9.80E-09</td>
<td>6.54E-09</td>
<td>3.27E-09</td>
</tr>
<tr>
<td>9.80E-09</td>
<td>1.07E-08</td>
<td>6.54E-09</td>
<td>3.27E-09</td>
</tr>
<tr>
<td>6.54E-09</td>
<td>6.54E-09</td>
<td>7.44E-09</td>
<td>3.27E-09</td>
</tr>
<tr>
<td>3.27E-09</td>
<td>3.27E-09</td>
<td>3.27E-09</td>
<td>4.17E-09</td>
</tr>
</tbody>
</table>

\[
M_{i} = \frac{w}{6EI} S^{-1} \text{Matrix·HM matrix}
\]

<table>
<thead>
<tr>
<th>S inverse</th>
<th>units = (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.04E+08</td>
<td>-1.66E+08</td>
</tr>
<tr>
<td>-1.66E+08</td>
<td>3.39E+08</td>
</tr>
<tr>
<td>-3.06E+07</td>
<td>-1.41E+08</td>
</tr>
<tr>
<td>-5.44E+06</td>
<td>-2.51E+07</td>
</tr>
</tbody>
</table>

\[
w(6EI) = 8.025E-12 \text{ lb ft³}
\]

\[
M_{i} = 141,311 \text{ kip-ft}
\]

\[
M_{2} = 212,262 \text{ kip-ft}
\]

\[
M_{3} = 296,475 \text{ kip-ft}
\]

\[
M_{4} = 321,371 \text{ kip-ft}
\]

Step 4: Determine resulting moment in the core.

\[
M_{r} = \sum_{i=1}^{4} M_{i} - M_{2} - M_{3} - M_{4}
\]

\[
M_{r} = 1,355,268 \text{ kip-ft}
\]

Step 5: Determine moment reduction efficiency

\[
M_{c} = \frac{1}{2} \frac{wH^{2}}{EI} \sum_{i=1}^{n} M_{i}
\]

\[
M_{c} = 1,303,699 \text{ kip-ft}
\]

\[
\sum_{i=1}^{n} M_{i} \times 100
\]

\[
M\% = \frac{\sum_{i=1}^{n} M_{i}}{M_{r}} \times 100
\]

\[
\text{sum } M_{i} = 971,418 \text{ kip-ft}
\]

\[
M\% = 74.51\%
\]

Step 6: Determine horizontal deflection at the top.

\[
\Delta_{o} = \frac{wH^{4}}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_{i} (H^{2} - x_i^{2})
\]

\[
\Delta_{o} = 2.390 \text{ ft} \quad \text{OK}
\]

\[
\Delta_{allowable} = \frac{H}{400}
\]

\[
\Delta_{allowable} = 2.400 \text{ ft}
\]
Appendix 10 – Case N4 Calculations Continued

Step 7: Determine drift reduction efficiency

\[
\Delta_c = \frac{1}{8EI_s} \frac{wH^4}{8EI} \\
\Delta_c = 2.865 \text{ ft}
\]

\[
\Delta\% = \frac{\sum M_i (H^2 - X_i^2)}{2EI_c \Delta_c}
\]

\[
\Delta\% = 95.02\%
\]

\[
\Delta\text{sum M} = 2.722 \text{ ft}
\]

Step 8: Determine force in columns and moment in outriggers

Forces in Column due to outriggers

- 1,178 kip for \( x_1 < x < x_2 \)
- 2,946 kip for \( x_2 < x < x_3 \)
- 5,417 kip for \( x_3 < x < x_4 \)
- 8,095 kip for \( x_4 < x \)

Maximum moment in the outriggers

- 47,104 k-ft level 1
- 70,754 k-ft level 2
- 96,825 k-ft level 3
- 107,124 k-ft level 4

Step 9: Determine optimum location nondimensional parameters

\[
\alpha = \frac{EI}{(EA)_{n} \left(\frac{d^2}{2}\right)}
\]

\( \alpha = 0.78 \)

\( \alpha = \) the core-to-column rigidities

\[
\beta = \frac{E_1}{(EI)_n} \frac{d}{H}
\]

\( \beta = 1.19 \)

\( \beta = \) the core-to-outrigger rigidities

\[
\omega = \frac{\beta}{12(1 + \alpha)}
\]

\( \omega = 0.055 \)

\( \omega = \) the characteristic structural parameter for a uniform structure with flexible outriggers

Step 10: Summary

\[
M_1 = 141,311 \text{ kip-ft} - \text{restraining moment}
\]

\[
M_2 = 212,262 \text{ kip-ft} - \text{restraining moment}
\]

\[
M_3 = 296,475 \text{ kip-ft} - \text{restraining moment}
\]

\[
M_4 = 321,371 \text{ kip-ft} - \text{restraining moment}
\]

\[
M_5 = 1,355,268 \text{ kip-ft} - \text{resulting moment in core}
\]

\[
M\% = 74.51\% - \text{moment reduction efficiency}
\]

\[
\Delta_s = 2.390 \text{ ft} - \text{deflection at top}
\]

\[
\Delta_{allowable} = 2.400 \text{ ft} - \text{allowable deflection}
\]

\[
\Delta\% = 95.02\% - \text{deflection reduction efficiency}
\]
Appendix 10 – Case N4 Calculations Continued
Appendix 11 – Case N5 Calculations: 5-Outriggers

Analysis of Outrigger Structural System

Case N5 - 5-outriggers placed at equal intervals

Givens:
Inter-story h = 12 ft
# stories = 80
Aspect ratio = 8:1
Total height (H) = 960 ft
Width of building (d) = 120 ft
Actual outrigger location \( x_1 \) = 160 ft \( x_1 \) = distance from top to first outrigger
Actual outrigger location \( x_2 \) = 320 ft \( x_2 \) = distance from top to second outrigger
Actual outrigger location \( x_3 \) = 480 ft \( x_3 \) = distance from top to third outrigger
Actual outrigger location \( x_4 \) = 640 ft \( x_4 \) = distance from top to fourth outrigger
Actual outrigger location \( x_5 \) = 800 ft \( x_5 \) = distance from top to fourth outrigger
Uniform wind loading (w) = 29,000 ksi
Modulus of elasticity (E) = 5.05 kip/ft

Step 1: Determine axial rigidity of the column \((EA)_c\) and effective flexural rigidity of the outrigger \((EI)_o\).

\[ (EA)_c = \frac{E \cdot A}{(1 + \frac{a}{b})^3} \]

\[ (EI)_o = \frac{2.124 \text{ ft}^2 \cdot 625 \text{ in}^2}{3.28 \times 10^9 \text{ k-ft}^2} \]

\[ (EI)_o = 1.11 \times 10^{-10} \text{ k-ft}^2 \]

Outrigger Inertia
\[ b = 13.352 \text{ ft} \]
\[ h = 13.352 \text{ ft} \]

\( a = 20 \text{ ft} \quad a = \text{half the width of core} \)

\[ b = 40 \text{ ft} \quad b = d/2 - a \]

Step 2: Determine \( S \) and \( S_1 \).

\[ S = \frac{1}{EI} + \frac{2}{(d^2)(EA)_c} \]

Core Inertia
\[ b = 23.429 \text{ ft} \]
\[ h = 23.429 \text{ ft} \]

\[ S = 1.71995 \times 10^{-11} \text{ l/(k-ft)} \]

\[ S_1 = \frac{d}{12(EI)_o} \]

\[ S_1 = 9.042 \times 10^{-10} \text{ l/(k-ft)} \]
Appendix 11 – Case N5 Calculations Continued

Step 3: Determine restraining moments applied to core.

Matrices

\[
\begin{array}{cccccc}
S \text{ matrix} & \text{units} = 1/(\text{k-ft}) & & & & \\
1.47E-08 & 1.10E-08 & 8.26E-09 & 5.50E-09 & 2.75E-09 & 8.81E+08 \\
1.10E-08 & 1.19E-08 & 8.26E-09 & 5.50E-09 & 2.75E-09 & 8.82E+08 \\
8.26E-09 & 8.26E-09 & 9.16E-09 & 5.50E-09 & 2.75E-09 & 7.74E+08 \\
5.50E-09 & 5.50E-09 & 5.50E-09 & 6.41E-09 & 2.75E-09 & 6.23E+08 \\
2.75E-09 & 2.75E-09 & 2.75E-09 & 2.75E-09 & 3.66E-09 & 3.73E+08 \\
\end{array}
\]

\[
\begin{array}{cccccc}
S \text{ inverse} & \text{units} = (\text{k-ft}) & & & & \\
2.29E+08 & -1.81E+08 & -3.75E+07 & -7.74E+06 & -1.53E+06 & \\
-1.81E+08 & 3.73E+08 & -1.52E+08 & -3.13E+07 & -6.20E+06 & \\
-3.75E+07 & -1.52E+08 & 3.79E+08 & -1.50E+08 & -2.98E+07 & \\
-7.74E+06 & -3.13E+07 & -1.50E+08 & 3.80E+08 & -1.44E+08 & \\
-1.53E+06 & -6.20E+06 & -2.98E+07 & -1.44E+08 & 4.10E+08 & \\
\end{array}
\]

\[ \frac{w}{6EI} = 8.0255E-12 \text{ 1/(ft)} \]

\[ M_j = \frac{w}{6EI} S^{-1} \text{Matrix : HMatrix} \]

\[ M_c = \frac{wH^2}{2} - M_1 - M_2 - M_3 - M_4 - M_5 \]

\[ M_c = 1,328,219 \text{ kip-ft} \]

Step 4: Determine resulting moment in the core.

Step 5: Determine moment reduction efficiency

\[ M_c^2 = \frac{1}{EIS} \]

\[ M_c = 1,290,086 \text{ k-ft} \]

\[ M% = \frac{\sum M_i}{M_c} \times 100 \]

\[ \text{sum} M_i = 998,467 \text{ k-ft} \]

\[ M% = 77.40% \]
Appendix 11 – Case N5 Calculations Continued

Step 6: Determine horizontal deflection at the top.

\[
\Delta_o = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i (H^2 - x_i^2)
\]

\[\Delta_o = 2.389 \text{ ft} \quad \text{OK} \]

\[\Delta_{allowable} = \frac{H}{400} \]

\[\Delta_{allowable} = 2.400 \text{ ft} \]

Step 7: Determine drift reduction efficiency

\[
\Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI}
\]

\[\Delta_c = 2.835 \text{ ft} \]

\[\Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i \frac{(H^2 - x_i^2)}{\Delta_o}
\]

\[\Delta_c \text{ sum } M = 2.723 \text{ ft} \]

\[\Delta\% = 96.06\% \]

Step 8: Determine force in columns and moment in outriggers

Forces in Column due to outriggers
- 830 kip for \(x_1 < x < x_2\)
- 2,067 kip for \(x_2 < x < x_3\)
- 3,837 kip for \(x_3 < x < x_4\)
- 6,075 kip for \(x_4 < x < x_5\)
- 8,321 kip for \(x_5 < x\)

Maximum moment in the outriggers
- 33,211 k-ft level 1
- 49,457 k-ft level 2
- 70,805 k-ft level 3
- 89,517 k-ft level 4
- 89,832 k-ft level 5
Appendix 11 – Case N5 Calculations Continued

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)(d^2/2)} \]

\[ \alpha = 0.80 \quad \alpha = \text{the core-to-column rigidities} \]

\[ \beta = \frac{EI}{(EI)_o} \frac{d}{H} \]

\[ \beta = 1.19 \quad \beta = \text{the core-to-outrigger rigidities} \]

\[ \omega = \frac{\beta}{12(1+\alpha)} \]

\[ \omega = 0.055 \quad \omega = \text{the characteristic structural parameter for a uniform structure with flexible outriggers} \]

Step 10: Summary

\[ M_1 = 99,632 \text{ kip-ft} \quad \text{- restraining moment} \]
\[ M_2 = 148,372 \text{ kip-ft} \quad \text{- restraining moment} \]
\[ M_3 = 212,416 \text{ kip-ft} \quad \text{- restraining moment} \]
\[ M_4 = 268,551 \text{ kip-ft} \quad \text{- restraining moment} \]
\[ M_5 = 269,496 \text{ kip-ft} \quad \text{- restraining moment} \]
\[ M_6 = 1,328,219 \text{ kip-ft} \quad \text{- resulting moment in core} \]
\[ M\% = 77.40\% \quad \text{- moment reduction efficiency} \]
\[ \Delta_o = 2.389 \text{ ft} \quad \text{- deflection at top} \]
\[ \Delta_{\text{allowable}} = 2.400 \text{ ft} \quad \text{- allowable deflection} \]
\[ \Delta\% = 96.06\% \quad \text{-deflection reduction efficiency} \]
Appendix 11 – Case N5 Calculations Continued
Appendix 12 – Case N6 Calculations: 6-Outriggers

Analysis of Outrigger Structural System

Case N6 - 6-outriggers placed at equal intervals

Givens:

- Inter-story height \( h \) = 12 ft
- Number of stories \( \# \) = 80
- Aspect ratio = 8 : 1
- Total height \( H \) = 960 ft
- Width of building \( d \) = 120 ft
- Actual outrigger location \( x_1 \) = 137 ft
- Actual outrigger location \( x_2 \) = 274 ft
- Actual outrigger location \( x_3 \) = 411 ft
- Actual outrigger location \( x_4 \) = 549 ft
- Actual outrigger location \( x_5 \) = 686 ft
- Actual outrigger location \( x_6 \) = 823 ft
- Uniform wind loading \( w \) = 29,000 ksf
- Modulus of elasticity \( E \) = 29,000 ksi

Step 1: Determine axial rigidity of the column \( (EA)_c \) and effective flexural rigidity of the outrigger \( (EI)_o \).

\[
E = 29,000 \text{ ksi} \\
A = 4.27 \text{ ft}^2 \\
(\text{EA})_c = \frac{17,838,000}{k} \quad \text{(EA)}_c = \text{axial rigidity of the column} \\
(\text{EI})_o = (1 + a/b)^3 (\text{EI})_o \\
\text{Column Area} \\
b = 2.124 \text{ ft} \\
h = 2.011 \text{ ft} \\
\text{Equivalent column area} \\
A = 615 \text{ in}^2 \\
(\text{EI})_o = 3.28E+09 \text{ k-ft}^2 \quad \text{(EI)}_o = \text{effective flexural rigidity of the outrigger} \\
(\text{EI})_o = 3.28E+09 \text{ k-ft}^2 \quad \text{(EI)}_o = \text{actual rigidity of the outrigger} \\
\text{Outrigger Inertia} \\
b = 13.352 \text{ ft} \\
h = 13.352 \text{ ft} \\
a = 20 \text{ ft} \quad a = \text{half the width of core} \\
b = 40 \text{ ft} \quad b = d/2 - a \\
(\text{EI})_o = 1.11E+10 \text{ k-ft}^2 \\
(\text{EI})_o = 1.11E+10 \text{ k-ft}^2 \\

Step 2: Determine \( S \) and \( S_1 \).

\[
S = \frac{1}{E} \left[ 2(\text{EI})_o \right] \\
E = 29,000 \text{ ksi} \\
I = 25,110 \text{ ft}^4 \\
\text{EI} = 1.05E+11 \text{ k-ft}^2 \quad \text{EI} = \text{flexural rigidity of core} \\
\text{Core Inertia} \\
b = 23.429 \text{ ft} \\
h = 23.429 \text{ ft} \\
S = 1.73241E-11 \frac{1}{(k-ft)} \\
S_1 = d / (12(\text{EI})_o) \\
S_1 = 9.04218E-10 \frac{1}{(k-ft)}
Appendix 12 – Case N6 Calculations Continued

Step 3: Determine restraining moments applied to core.

<table>
<thead>
<tr>
<th>Matrices</th>
<th>S matrix</th>
<th>units = 1/(k-ft)</th>
<th>H matrix</th>
<th>units = ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>8.82E+08</td>
<td>8.64E+08</td>
</tr>
<tr>
<td></td>
<td>1.52E-08</td>
<td>1.19E-08</td>
<td>1.13E-09</td>
<td>4.75E-09</td>
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<td>1.04E-08</td>
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<tr>
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<td>2.38E-09</td>
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<td>4.75E-09</td>
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<tr>
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<td>2.38E-09</td>
<td>2.38E-09</td>
<td>2.38E-09</td>
<td>2.38E-09</td>
</tr>
</tbody>
</table>

S⁻¹ matrix

<table>
<thead>
<tr>
<th>units = (k-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.51E+08</td>
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<tr>
<td>-1.94E+08</td>
</tr>
<tr>
<td>-4.41E+07</td>
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<tr>
<td>-1.00E+07</td>
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<tr>
<td>-2.27E+06</td>
</tr>
<tr>
<td>-4.91E+05</td>
</tr>
</tbody>
</table>

w/(6EI) = 8.0255E-12 1/(ft³)

\[ M_i = \frac{w}{6EI} S^{-1} \text{Matrix} \cdot \text{HMatrix} \]

- \( M_1 = 74,323 \) kip-ft
- \( M_2 = 109,355 \) kip-ft
- \( M_3 = 156,999 \) kip-ft
- \( M_4 = 205,076 \) kip-ft
- \( M_5 = 242,548 \) kip-ft
- \( M_6 = 230,516 \) kip-ft

Step 4: Determine resulting moment in the core.

\[ M_e = \frac{w^2}{2} - M_1 - M_2 - M_3 - M_4 - M_5 - M_6 \]

- \( M_e = 1,307,868 \) kip-ft

Step 5: Determine moment reduction efficiency

\[ M_c = \frac{1}{EIS} \frac{wH^2}{2} \]

- \( M_c = 1,280,807 \) k-ft

\[ M\% = \frac{\sum M_i}{M_e} \times 100 \]

- \( \sum M_i = 1,018,817 \) k-ft
- \( M\% = 79.54\% \)

100
Appendix 12 – Case N6 Calculations Continued

Step 6: Determine horizontal deflection at the top.

\[ \Delta_a = \frac{wH^4}{8EI} - \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H^2 - x_i^2 \right) \]

\[ \Delta_a = 2.390 \text{ ft} \quad \rightarrow \quad \text{OK} \]

\[ \Delta_{allowable} = \frac{H}{400} \]

\[ \Delta_{allowable} = 2.400 \text{ ft} \]

Step 7: Determine drift reduction efficiency

\[ \Delta_c = \frac{1}{EIS} \frac{wH^4}{8EI} \]

\[ \Delta_c = 2.814 \text{ ft} \]

\[ \Delta\% = \frac{1}{2EI} \sum_{i=1}^{n} M_i \left( H^2 - x_i^2 \right) \]

\[ \Delta\% = \frac{\Delta_a \text{ sum } M}{\Delta_c} = 2.722 \text{ ft} \]

\[ \Delta\% = 96.73\% \]

Step 8: Determine force in columns and moment in outriggers

Forces in Column due to outriggers:
- 619 kip for \( x_1 < x < x_2 \)
- 1,531 kip for \( x_2 < x < x_3 \)
- 2,839 kip for \( x_3 < x < x_4 \)
- 4,548 kip for \( x_4 < x < x_5 \)
- 6,569 kip for \( x_5 < x < x_6 \)
- 8,490 kip for \( x_6 < x \)

Maximum moment in the outriggers:
- 24,774 k-ft level 1
- 36,452 k-ft level 2
- 52,333 k-ft level 3
- 68,359 k-ft level 4
- 80,849 k-ft level 5
- 76,839 k-ft level 6
Appendix 12 – Case N6 Calculations Continued

Step 9: Determine optimum location nondimensional parameters

\[ \alpha = \frac{EI}{(EA)_{c}\left(d^2/2\right)} \]

\[ \alpha = 0.82 \quad \alpha = \text{the core-to-column rigidities} \]

\[ \beta = \frac{EI}{(EI)_{c}H} \]

\[ \beta = 1.19 \quad \beta = \text{the core-to-outrigger rigidities} \]

\[ \phi = \frac{\beta}{12(1+\alpha)} \]

\[ \phi = 0.054 \quad \phi = \text{the characteristic structural parameter for a uniform structure with flexible outriggers} \]

Step 10: Summary

\( M_1 = 74,323 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_2 = 109,355 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_3 = 156,999 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_4 = 205,076 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_5 = 242,548 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_6 = 230,516 \text{ kip-ft} \quad - \text{restraining moment} \)
\( M_7 = 1,307,868 \text{ kip-ft} \quad - \text{resulting moment in core} \)
\( M\% = 79.54\% \quad - \text{moment reduction efficiency} \)
\( \Delta_e = 2.390 \text{ ft} \quad - \text{deflection at top} \)
\( \Delta_{\text{allowable}} = 2.400 \text{ ft} \quad - \text{allowable deflection} \)
\( \Delta\% = 96.73\% \quad - \text{deflection reduction efficiency} \)
Appendix 12 – Case N6 Calculations Continued