Coverage Optimization Using a Single Satellite Orbital Figure-of-Merit

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Thank you.
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Abstract

A figure-of-merit for measuring the cost-effectiveness of satellite orbits is introduced and applied to various critically inclined daily repeat ground track orbits. The figure-of-merit measures orbit performance through the coverage time provided over a specified point or region on the ground. This thesis primarily focuses on coverage to a ground station, however coverage to a region is briefly examined. The selected repeat ground track orbits are optimized to maximize the coverage provided by varying the orbit’s argument of perigee and longitude of ascending node. Several known orbits were reproduced as a result of this coverage optimization. The figure-of-merit measures the cost of the orbit by the launch cost in ΔV to attain the mission orbit. The launch ΔV is calculated using a series of analytic formulas.

Trends in the figure-of-merit are investigated with respect to repeat ground track pattern, ground station location, orbit eccentricity, and minimum elevation angle. Using the proposed figure-of-merit, various repeat ground track orbits are examined and compared to draw conclusions on the cost-effectiveness of each orbit. This figure-of-merit has potential for use by satellite system designers to compare the cost-effectiveness of different orbits to determine the optimal orbit for a single satellite and by extension, a constellation of satellites.

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John L. Young III
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1 Introduction

1.1 Thesis Objective

Satellites in Geostationary orbit (GEO) are extremely useful by providing 24 hour coverage to large regions of the Earth. Cost-effective alternatives to the GEO satellite are desirable due to high satellite and launch costs, as well as limited space in the Geostationary Belt. Constellations of satellites in low-Earth orbit (LEO), medium-Earth orbit (MEO) or highly elliptical orbit (HEO) can be used effectively in communications systems and have potential for providing excellent performance with lower cost than Geostationary satellites.

A metric that can be used by satellite designers to compare the cost-effectiveness of different orbits would provide a valuable way to determine the optimal orbit for a single satellite and by extension, a constellation. This thesis investigates and expands the figure-of-merit for single-satellite orbits proposed by Captain John Draim, USN (ret) [1]. The figure-of-merit examines the cost-effectiveness of the satellite’s orbit applied from the standpoint of coverage, coupled with the launch cost of the satellite. The figure-of-merit was initially developed to apply to communications satellite systems, however it can be tailored to suit other applications. Orbits examined in this study are restricted to daily repeat ground track orbits which are critically inclined at either 63.4° or 116.6°.

1.2 Figure-of-Merit

The non-dimensional orbital figure-of-merit, $J$, examines the coverage time versus the launch cost in $\Delta V$. The premise behind this metric is that in general, one can achieve better coverage for satellites placed in higher orbits, but the cost in $\Delta V$ also increases. For example, a satellite in Geostationary orbit will provide continuous coverage to the ground, but the launch cost in $\Delta V$ to attain the orbit is high. Therefore, a lower orbit with less coverage, but also a lower $\Delta V$ might offer an acceptable alternative.

In this thesis, the figure-of-merit is defined as:

$$J = \max_{\Omega, \omega, M} \frac{T(a, e, i, \Omega, \omega, M)}{\Delta V(a, e, i)} g$$  \hspace{1cm} (1.1)
where $T$ is a measure of the average coverage time per day in seconds over the interval $[t_0, t]$ where $t - t_0$ is the repeat period. The orbit elements $a, e, i, \Omega, \omega,$ and $M$ correspond to time $t_0$. The quantity $\Delta V$ is the velocity increment in m/s required to attain the satellite’s mission orbit from a launch beginning at the Earth’s surface. The constant $g$ is the acceleration due to gravity on the Earth’s surface (9.81 m/s$^2$) and is used to non-dimensionalize the figure-of-merit. It is important to note that the figure-of-merit yields the same value regardless of the unit system used.

The coverage, $T$, which is shown as a function of orbit elements, will generally depend on additional parameters. This study focuses on coverage provided to a given location on Earth; thus $T$ depends implicitly on latitude, longitude, and on a minimum elevation parameter.

To first order, the $\Delta V$ can be written as a function of the orbit’s semimajor axis, eccentricity, and inclination. The remaining orbital elements, the right ascension of the ascending node, argument of perigee, and mean anomaly, can be adjusted without significant costs in $\Delta V$. However, variations in these elements ($\Omega, \omega,$ and $M$) can significantly affect the orbit’s coverage time. If a repeat ground track orbit is specified, the semimajor axis, eccentricity, and inclination must be fixed. The coverage can be maximized over the orbital elements $\Omega, \omega,$ and $M$ while maintaining the specified repeat ground track characteristics. Thus, in this thesis the figure-of-merit is considered to be a function of the three orbital elements $a, e,$ and $i$, which define the repeat ground track orbit as well as the ground station location.

The launch $\Delta V$ can be used as a measure of launch cost by examining the rocket equation. The rocket equation,

$$\Delta V = g \cdot I_{sp} \cdot \ln \left( \frac{m_i}{m_f} \right)$$

shows that $\Delta V$ is a function of the specific impulse, $I_{sp}$, and the ratio of the initial mass ($m_i$) to the final mass ($m_f$) of the launch vehicle. The acceleration due to gravity, $g$, plays a similar role in Equation 1.2 as it does in the figure-of-merit equation. In the rocket equation, $g$ is used to ensure that the units for the calculated velocity are correct while in the figure-of-merit expression, $g$ is used to non-dimensionalize the term. In the rocket equation, both $g$
and $I_{sp}$ are assumed to be constant. Although the function is logarithmic, in the ranges we are examining the $\Delta V$ can be assumed to vary linearly with the mass ratio. In particular, this assumption is true in cases of multi-staged vehicles. Thus, a higher $\Delta V$ results in a higher mass ratio, yielding a larger launch vehicle. The size of the launch vehicle can be assumed to be directly related to the dollar cost for a satellite launch.

1.3 Thesis Summary

Chapter 2 introduces previous work conducted in the orbital design aspects of analyzing satellite systems. Early work in satellite constellation design is also examined in this chapter. Chapter 3 introduces properties of repeat ground track orbits and describes the algorithm used to calculate them. The calculation of the coverage time per day, $\Delta V$, and the figure-of-merit are described in detail. Chapter 4 presents results from the coverage optimization and trends found in the $\Delta V$. The figure-of-merit is used to compare different daily repeat ground track orbits with each other. Trends within each repeat ground track type are also noted. Chapter 5 draws conclusions of the figure-of-merit, while Chapter 6 presents proposals for potential future work with the orbital figure-of-merit. Appendix A describes the code developed in the figure-of-merit calculation. The code is divided into three sections: (1) Coverage Calculation and Optimization, (2) Figure-of-Merit calculation, and (3) Supporting Functions. Appendix B lists results from the coverage optimization, $\Delta V$ calculation, and figure-of-merit analysis. The results are organized into tables based on the repeat ground track pattern and the specified minimum elevation angle. Within each table, the optimized argument of perigee and longitude of ascending node, maximum coverage time, $\Delta V$, and figure-of-merit for each orbit / ground station combination are given. Appendix C presents figure-of-merit contour plots which demonstrate how the figure-of-merit varies with orbit eccentricity and ground station location. Appendix D presents plots where the repeat ground track orbits considered in this thesis are compared with each other based on their figure-of-merit. Finally, Appendix E briefly investigates determining the coverage over a region rather than a single ground station. The methodology for calculating the coverage is explored and implemented. A figure-of-merit analysis for the six repeat ground track patterns is performed using the continental United States as the region of interest. A CD-ROM accompanies this thesis as Appendix F. The contents include the
Matlab code and additional coverage contour plots. A color version of this thesis is included to allow the reader to gain a better insight into some of the contour plots created.
2 Background

2.1 Chapter Overview

Significant research in satellite constellation design has been done dating back to the 1960s. A brief summary of this work is presented in this chapter. Several existing constellation performance metrics are examined.

2.2 Cost-Effectiveness Metrics

This section examines two metrics to compare the cost-effectiveness of different satellite systems that have been developed and used in previous studies. Extensive work has been done in comparing different satellite systems by Violet and Gumbert, and Shaw. Violet and Gumbert focused on analyzing mobile satellite phone systems by employing a cost per billable minute metric to compare the different systems. The cost per billable minute metric has been used for satellite broadband applications.

2.2.1 Cost Per Billable Minute

The cost per billable minute metric is used to measure the cost-effectiveness of satellite based mobile phone networks with respect to an expected market. The cost per billable minute represents what a company must charge its consumers to recover costs associated with designing, launching, operating and maintaining the system, given a specified internal rate of return. The system with the lowest cost per billable minute represents the option that is most cost-effective and has the highest chances of returning a profit. Michael Violet [2] and Cary Gumbert [3] examined different satellite constellations using the cost per billable minute metric with an internal rate of return of 30%.

Both Violet and Gumbert used this metric to compare five proposed satellite communications systems. These systems included a GEO, two MEO, and two LEO satellite constellations. Three of the systems were models of the FCC licensed systems: Iridium [4], Globalstar [5], and Odyssey [6, 7]. The remaining two were systems proposed by the Hughes Space and Communications Company [8] and by students in an MIT space systems engineering course [9]. In their analysis, they found that a 48 satellite LEO constellation
modeled after the Globalstar system provided the best cost per billable minute of the five systems analyzed. The system with the highest cost per billable minute is another LEO system modeled after the Iridium mobile communication system with 66 satellites. In a later analysis [10], they included the Ellipso constellation [11], a system proposed by Mobile Communications Holdings Inc. (MCHI) where elliptical orbits are used. The cost per billable minute for this system given a 30% market penetration level is approximately 60 cents versus 75 cents for the 48 satellite LEO constellation previously examined. The cost per billable minute for the other systems examined ranged from approximately $1.00 to $1.75 for a 30% market penetration level. In their work, Violet and Gumbert model the mobile communications market and define the market penetration level as the fraction of customers in the modeled market that subscribe to the communications system.

2.2.2 Cost Per Billable T1 Minute

The cost per billable T1 minute is similar to the cost per billable minute used in mobile communications systems. This metric applies to satellite based broadband systems intended to provide users with the capability for high speed data transmission. The T1 data rate (1.544 Mbps) is used as a benchmark to compare the data transfer among the different systems. By using five satellite systems as models, Kelic [12] presents a detailed development of the modified metric. The systems modeled were the Spaceway [13], Astrolink [14], CyberStar [15], Voicespan [16], and Teledesic [17] constellations. All of these systems are GEO constellations except for Teledesic, which was assumed to be a LEO network.

Both Shaw [18] and Kashitani [19] apply the cost per billable T1 minute metric in system analysis methods they formulated. Shaw develops a comprehensive methodology to analyze distributed satellite systems called the Generalized Information Network Analysis (GINA). In the GINA methodology, a “Cost per Function” (CPF) metric is defined based on the system’s mission. He applies his methodology to three different types of systems, the NAVSTAR Global Positioning System [20, 21], broadband satellite communications systems, and a space based radar system. In his case study of broadband networks, he uses the cost per billable T1 minute metric developed by Kelic as the CPF metric. In his
investigation, he compares three Ka-band systems using GINA. These networks are two GEO systems, Spaceway and Cyberstar, and the LEO Teledesic system.

Kashitani develops an analysis methodology specifically aimed at broadband satellite communications systems. He adopts the cost per billable T1 minute metric and analyzes five proposed Ku-band systems: HughesLink [22], HughesNet [23], SkyBridge [24,25], Virgo [26, 27], and a Boeing [28] proposal. These systems include two LEO systems, two MEO systems, and a HEO design. He concluded that the performance of the systems he analyzed varied based on the number of customers available to the system. For example, a MEO system has a better cost per billable T1 minute than a LEO system for a small number of customers, but as the number of customers increases, the LEO system becomes superior. His study also resulted in the creation of computer software to quickly analyze the cost per billable T1 minute given specified design variables [29].

2.3 Satellite Constellation Performance

Metrics also exist to measure the performance of satellite constellations. This thesis uses the average coverage time per day to measure satellite performance, however a metric that is often used is the revisit time.

2.3.1 Revisit Time

Revisit time is one metric commonly used to measure satellite constellation performance where continuous global coverage is not achieved. Revisit time is defined as the time a region or point on the ground is not in view of a satellite in the constellation. These coverage gaps can be averaged to determine the average revisit time. Another way of using the revisit time is by measuring the longest period that the desired region or point does not have satellite coverage. This is known as the maximum revisit time. Both of these performance measurements are useful, however Williams, Crossley, and Lang [30] note that when using conventional optimization methods, if one metric is minimized, the other is not. In their study they used a multiobjective genetic algorithm which attempted to minimize both average and maximum revisit time.
2.3.2 Coverage Time

While the revisit time measures the gaps between coverage, the coverage time metric measures the duration a specified point or region is in view of the satellite. This thesis makes use of the average coverage time per day to measure the satellite’s performance for use in the figure-of-merit, where the average coverage time per day is maximized for a given orbit. This thesis examines only daily repeat ground track orbits and therefore the average coverage time per day is equal to the total coverage time per day. If repeat ground track orbits with repeat periods greater than one day are considered, the coverage time would be divided by the repeat period to yield the average coverage time per day.

2.4 Constellation Design

Global communication systems are one of the major satellite applications dependent on constellations to provide continuous worldwide coverage. Though continuous global coverage is often desirable, partial coverage constellations are useful in a variety of missions. Among the major contributors in the field of constellation design are Lüders, Easton, Brescia, Walker, Beste, Ballard, Draim, Lang, and Hanson. Their work attempts to minimize the number of satellites needed to attain specified coverage parameters.

2.4.1 R.D. Lüders

Lüders [31] investigated continuous coverage constellations to regions bounded by specified latitudes. He examined two different cases as seen by the shaded regions in Figure 2.1. The region on the left is bounded by a minimum latitude and the poles while the region on the right is bounded by the equator and a maximum latitude. For the first case he examined, minimum latitude bounds of 0°, 30°, and 60° were used. The 0° minimum latitude case represents constellations that provide continuous global coverage. In the second case examined, maximum latitude bounds at 30°, 60°, and 90° were used.
His study looked at constellations restricted to circular orbits and satellites that were uniformly spaced in each orbital plane. In addition, the ascending nodes of the orbits were evenly distributed. Lüders determines the number of satellites required for full coverage of the regions based on their altitudes. He makes plots of the number of satellites as a function of their altitudes where the altitudes varied from less than 200 nm to 2000 nm. For complete global coverage, the constellation altitudes ranged from approximately 300 nm with 60 satellites to 2000 nm with approximately 14 satellites. A final observation made was that for continuous global coverage, orbits with inclinations of 90° performed better than inclined orbits.

2.4.2 R. L. Easton and R. Brescia

Some of the first work in minimizing the number of satellites to provide continuous worldwide coverage was carried out by Easton and Brescia [32]. Their analysis was limited to evenly spaced satellites in two orbital planes. The orbital configurations considered were an equatorial orbit and a polar orbit \((i = 90°)\) or two polar orbits \((i = 90°)\). They concluded that at least three satellites per plane were required for continuous coverage. In addition, they examined how an elevation angle requirement affected the altitude of the constellation’s orbit. For coverage with no constraints on the elevation angle, they concluded that the minimum altitude for a continuous coverage constellation was 6320 nm. Included in their analysis was minimum elevation angles of 5°, 10°, and 15°, where the altitudes of the satellites increased to 30,000 nm for the 15° elevation angle case.
2.4.3 John Walker

By placing satellites in five different orbital planes, Walker [33] was able to improve upon Easton and Brescia’s work. He found that only five satellites in constellation of circular orbits with altitudes of approximately 10924 nm (12 hour period) or 19365 nm (24 hour period) are necessary for continuous global coverage. His study also determined that the minimum number of satellites needed in constellations requiring double coverage is seven, where the orbits have 24 hour periods.

In later work, he defines a method to describe satellite constellations, known as the Walker delta pattern [34]. These Walker delta patterns continue to be used by designers of multi-satellite arrays. Three integer parameters T, P, and F, can fully define the Walker constellation by the total number of satellites (T), the number of orbital planes (P), and the relative spacing between satellites in adjacent planes (F). The phasing of the satellites in the constellation is determined by both T and F. When a satellite in a given plane is at the ascending node, the satellite in the adjacent plane to the east is located at an angle of \(360^\circ \times F/T\) from the ascending node. All satellites are evenly spaced within each orbital plane in a Walker constellation. An example of a Walker delta pattern is a constellation described by 5/5/1 where there are five satellites in five orbital planes. The relative spacing of the satellites between the orbital planes becomes \(72^\circ\) (360*1/5).

2.4.4 David Beste

Beste [35] looked at constellations designed to provide either continuous global coverage or continuous coverage limited to polar or high latitude regions. His analysis was aimed at communications applications and can be extended to Earth observation missions. For polar regions, he defines the number of satellites needed based on the Earth-centered half-cone-angle and a cutoff latitude limit where coverage is provided to regions above the specified latitude. The Earth-centered half-cone-angle is defined as the angle between the subsatellite point and the edge of the circle of coverage measured from the Earth’s center as shown in Figure 2.2. The circle of coverage is a function of the sensor’s maximum scan angle (\(\alpha_S\)) and the sensor’s range (\(R_S\)). If there are no constraints on the sensor, the circle of coverage and thus the Earth-centered half-cone-angle become a function of the satellite’s
altitude. Beste examines the effects of the sensor’s maximum scan angle and maximum range on the number of satellites required for coverage. He also investigates continuous triple coverage of the Earth, but does not develop an analytical expression for the number of satellites needed. Instead he uses an iterative approach to determine the number of satellites required for a triple coverage based on a specified Earth-centered half-cone-angle.

![Earth-Centered Half-Cone-Angle](image)

**Figure 2.2: Earth-Centered Half-Cone-Angle [35]**

### 2.4.5 Arthur Ballard

In his work, Ballard [36] examines the minimum number of satellites required for continuous worldwide coverage based on the number of satellites required to be in view. This can be applied to both communications missions as well as navigation applications such as the Global Positioning System constellation. He confirms Walker’s work that the minimum number of satellites in circular orbits is five assuming that only single satellite visibility is required. He included an additional constraint where a 10° minimum elevation angle was required for viewing and he discovered that the constellation of five satellites would have to have an altitude of approximately 19365 nm resulting in a 24 hour orbital period. He also looks at constellations where two, three, and four satellites must be
Ballard shows that constellations in “rosette” patterns provided better results than other patterns. He describes the “rosette” as uniformly distributed orbital planes containing satellites with common period circular orbits. Thus, when observing the traces of the orbits from the North Pole, the pattern resembles petals of a flower as seen in Figure 2.3. This example shows a rosette pattern with six satellites in six orbital planes with the ascending nodes that are separated by 60°. While this rosette pattern shows only one satellite per plane, Ballard examines constellations with multiple satellites in an orbital plane. The orbital inclination in each constellation is the same for all satellites and the satellites are phased such that their locations are proportional to the right ascension of the orbital plane.

Ballard’s rosette patterns are similar to Walker’s delta patterns, however Ballard defines the phasing differently than Walker. Ballard replaces F, the relative spacing between satellites in adjacent planes, with a harmonic factor \( m \), where \( m \) is not limited to integer values. A constellation with multiple satellites per plane is defined if the harmonic factor is a fraction. To determine the initial position of satellites in the constellation, the harmonic factor is multiplied by the right ascension of the ascending node (\( \alpha_i \) shown in Figure 2.3) of the orbital plane, yielding the initial angle between the satellite and the ascending node.

Figure 2.3: “Rosette” Constellation as Viewed From the North Pole [36]
2.4.6 John Draim

While most constellation designers studied patterns of circular orbits, Draim has examined the use of elliptical orbits to achieve global coverage. The work of Walker and Ballard indicate that the minimum number of satellites for complete Earth coverage is five. Draim introduces a four satellite common period elliptical orbit constellation that exhibits continuous global coverage [37]. This constellation is composed of satellites in supersynchronous orbits with periods of 26.49 hours that are inclined at 31.3°. Each orbit is moderately eccentric with an eccentricity of 0.263. Two of the orbits have apogees in the northern hemisphere, while the other two have apogees in the southern hemisphere. The orbital planes are each separated by 90° and the satellites are phased so that in orbits with the same argument of perigee, one satellite is at the apogee and the other is at perigee. The other two satellites are placed between the apogee and perigee with mean anomalies of 90° and 270°. A summary of the orbital elements in the Draim constellation is shown in Table 2.1.

Table 2.1: Orbital Elements in a Draim Constellation [37]

<table>
<thead>
<tr>
<th>Satellite</th>
<th>a'(km)</th>
<th>i</th>
<th>e</th>
<th>ω</th>
<th>Ω</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45033</td>
<td>31.3</td>
<td>0.263</td>
<td>-90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>45033</td>
<td>31.3</td>
<td>0.263</td>
<td>90</td>
<td>90</td>
<td>270</td>
</tr>
<tr>
<td>3</td>
<td>45033</td>
<td>31.3</td>
<td>0.263</td>
<td>-90</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>45033</td>
<td>31.3</td>
<td>0.263</td>
<td>90</td>
<td>270</td>
<td>90</td>
</tr>
</tbody>
</table>

Draim also describes a three satellite constellation to provide continuous hemispheric coverage [38]. He uses satellites with 24 hour periods that are inclined at 30° with an eccentricity of 0.28. While he notes that orbits with periods as low as 16.1 hours can provide continuous coverage to the northern hemisphere, by using well placed 24 hour period orbits, continuous coverage to the major landmasses in the southern hemisphere is also provided. The ground track and 24 hour coverage plot, shown in Figure 2.4, is taken from Draim’s work on the three satellite constellation. The shaded region in the plot indicates that all

---

1 Semimajor axis must be equal to or greater than value cited for continuous coverage
major landmasses except for Antarctica and the tip of South America receive continuous coverage using Draim's proposed constellation.

Figure 2.4: Ground Track and Coverage Plot for Draim’s Three Satellite Constellation [38]

Yet another constellation that Draim has designed is the Ellipso Mobile Satellite System [39, 40] for Mobile Communications Holdings, Inc. The Ellipso system is a hybrid constellation designed to provide continuous coverage to all areas north of 50° S latitude by using satellites in two sub-constellations: Ellipso-Borealis and Ellipso-Concordia. Concordia is composed of eight satellites in a circular equatorial orbit at an altitude of 8050 km. This constellation is designed to provide continuous coverage from 50° S to 15° N. Coverage to the northern hemisphere (15° N to 90° N), where a majority of the mobile communications market lies, is provided by Borealis. The Borealis constellation is composed of two elliptical 8:1 repeat ground track orbits, each with five satellites. The orbit is inclined at 116.6° and is sun-synchronous to provide coverage that is favored during the daylight hours, a time when the system demand is the greatest. The coverage of the two sub-constellations overlap in the lower to mid-latitudes in the northern hemisphere where the mobile communications market is expected to be the largest.
Another important orbit proposed by Draim is the “Communications Orbiting Broadband Repeating Array” or COBRA orbit [41, 42] that has a period of 8 hours. Draim first introduced the idea of using an 8-hour orbit in 1992 [43]. The COBRA orbit can be used with six or more satellites, providing continuous coverage to virtually the entire northern hemisphere. Although continuous global coverage is not achieved with this constellation, from a broadband communications standpoint, regions with the largest concentration of potential consumers are covered: North America, Europe, and Asia. Leaning elliptical orbits similar to the COBRA orbit are also discussed by Maas [44]. COBRA orbits and their properties are discussed in greater detail in Chapter 4.

Using the COBRA orbit, Draim introduces the concept of the COBRA “Teardrop” array. By using both a left leaning and right leaning COBRA orbit, the two can be combined in such a way that to a viewer on the ground, it appears that a single satellite is orbiting overhead. An example of the teardrop array is shown in Figure 2.5, where a teardrop is located over eastern Asia, the U.S., and Europe. The advantage of the teardrop concept is that it allows an antenna to track the satellite continuously without having to execute a large maneuver to change the antenna’s orientation.

Figure 2.5: Draim COBRA “Teardrop” Array [42]
2.4.7 Thomas Lang

While continuous global coverage is desirable, often it is not necessary for all applications. Lang has conducted extensive work in optimizing partial coverage satellite constellations. In his studies, he attempts to minimize revisit time for satellite constellations [30, 45, 46]. Other research was done to concentrate coverage to certain latitude regions rather than the entire globe [47]. His research also seeks to reduce satellite system costs by providing multiple options for continuous coverage constellations [48]. Lang asserts that the smallest number of satellites to provide global coverage might not be the best option for satellite system designers. By looking at alternatives, a constellation with more satellites could provide a low cost system.

2.4.8 John Hanson

Hanson [49] has focused on constellations that achieve near-continuous coverage. He analyzed satellite constellations to determine the optimum constellation design, where he defined optimum as the “minimum number of satellites at the minimum possible inclination with the smallest possible maximum time gap.” Hanson develops a method to design optimal satellite constellations and he shows that his designs often exhibit better performance than Walker constellations. His work only considered circular orbits, yet he showed that repeat ground track orbits provided better coverage than non-repeat ground track orbits. While repeat ground track orbits with repeat periods greater than one day might provide improved constellations, his work focused on daily repeat ground track orbits.
3 Methodology

3.1 Chapter Overview

This chapter examines the process used to calculate the orbital figure-of-merit. Repeat ground track orbits and the procedures for calculating them are described. The method used to calculate the coverage time per day over a point provided by a repeat ground track orbit is explained. Testing of the function designed to calculate the coverage is also given. A brief description of the optimization methods used to maximize the coverage time per day is presented. The method chosen to calculate the $\Delta V$ to attain the maximized orbit is examined. Finally, the algorithm used to combine the coverage time per day and the $\Delta V$ to form the figure-of-merit is discussed.

3.2 Repeat Ground Track Orbits

Repeat ground track orbits are used in a wide variety of applications and can be especially useful for communications satellites. This type of orbit is specifically designed to repeat the satellite ground trace in a desired period, $T$; thus there are a finite number of ascending equator crossing longitudes. The ground track is a closed curve and the latitude $\lambda$, and longitude $l$, can be written as a periodic functions as seen in Equation 3.1. If an initial latitude/longitude point on the ground track $(0, l_0)$ is specified, the repeat period is defined as the time between two successive passes over the point $(0, l_0)$.

$$\begin{align*}
\lambda(t) &= \lambda(t + T) \quad \text{for all values of } t \\
l(t) &= l(t + T)
\end{align*} \quad (3.1)$$

If the satellite is at an ascending node at time $t = 0$ where $l(0) = l_0$, based on the relationships in Equation 3.1, then $l(T) = l_0$. The longitude of ascending node $l_0$ can be written as a function of the right ascension of the ascending node and the right ascension of Greenwich:

$$\begin{align*}
\Omega(0) - \alpha_G(0) &= l_0 \quad (3.2) \\
\Omega(T) - \alpha_G(T) &= l_0 \quad (3.3)
\end{align*}$$
We can equate Equation 3.2 and Equation 3.3, to yield:

\[- \Omega(T) + \alpha_g(T) + \Omega(0) - \alpha_g(0) = 0 \quad (3.4)\]

The right ascension of the ascending node (\( \Omega \)) and the right ascension of Greenwich (\( \alpha_g \)) after one repeat period can be written as:

\[\Omega(T) \equiv \Omega(0) + \dot{\Omega} \cdot T \mod 2\pi \quad (3.5)\]

\[\alpha_g(T) \equiv \alpha_g(0) + \omega_e \cdot T \mod 2\pi \quad (3.6)\]

where \( \dot{\Omega} \) is the rate of change of the right ascension of the ascending node and \( \omega_e \) is the Earth’s rotation rate. Substituting \( \Omega(T) \) from Equation 3.5 and \( \alpha_g(T) \) from Equation 3.6 into Equation 3.4, the relationship for the repeat period \( T \) is obtained:

\[ (\omega_e - \dot{\Omega})T = 2\pi \cdot M \quad \text{for some integer } M \quad (3.7) \]

The rate of change of the right ascension of the ascending node, \( |\dot{\Omega}| \), is much smaller than \( \omega_e \), the Earth’s rotation rate, thus \( M \) must be a positive integer. Since the repeat period is approximately an integer multiple of a day, \( M \) is used to represent the approximate repeat period in days. Solving for the repeat period, the following expression is obtained:

\[ T = \frac{2\pi \cdot M}{(\omega_e - \dot{\Omega})} \quad (3.8) \]

The latitude is also a periodic function as shown in Equation 3.1. Therefore, both \( \lambda(0) = 0 \) and \( \lambda(T) = 0 \) are true. To ensure this holds, the repeat period must be an integer multiple of the satellite’s nodal period \( T_\Omega \):

\[ T = N \cdot T_\Omega \quad \text{for some integer } N \quad (3.9) \]

where the nodal period is defined as:

\[ T_\Omega = \frac{2\pi}{n + \dot{\omega} + \dot{M}_0} \quad (3.10) \]

Repeat ground track orbits are typically defined by a repeat pattern ratio of \( N:M \) where \( M \), shown in Equation 3.8, represents the approximate repeat period in days and \( N \), shown in Equation 3.9, denotes the number of orbits in one repeat period. For example, a 3:1 repeat ground track orbit completes three revolutions in one day. With \( N \) and \( M \) defined, the following relationship is true for all repeat ground track orbits:
The rate of change of the right ascension of the ascending node ($\dot{\Omega}$) is caused primarily by the Earth’s oblateness where a satellite’s orbital plane precesses in inertial space. This thesis examines only the secular $J_2$ effects on orbital elements, thus an analytic expression can be written to determine the average rate of change for orbit elements affected by the Earth’s shape [50, 51]. The rate of change of the right ascension of the ascending node is given by:

\[
\dot{\Omega} = -\frac{3 \cdot n \cdot r_E^2 \cdot J_2}{2 \cdot a^2 \cdot (1 - e^2)} \cos(i)
\]  

(3.12)

where $n$ is the satellite’s mean motion, $r_E$ is the Earth’s radius, $a$ is the semimajor axis, $e$ is the eccentricity, and $i$ is the inclination. The mean motion term $n$ is defined as the average angular rate of a satellite over one revolution [51]. The mean motion is a function of the Earth’s gravitational parameter and the satellite’s semimajor axis:

\[
n = \sqrt{\frac{\mu}{a^3}}
\]  

(3.13)

The time rate of change of the epoch mean anomaly, $\dot{M}_0$, is determined from the $J_2$ perturbation theory and is expressed in the following equation:

\[
\dot{M}_0 = -\frac{3 \cdot n \cdot r_E^2 \cdot J_2 \cdot \sqrt{1 - e^2}}{4 \cdot a^2 \cdot (1 - e^2)} \left[3 \cdot \sin^2(i) - 2\right]
\]  

(3.14)

The time rate of change in the argument of perigee is the third orbital element affected by the $J_2$ perturbations and is shown in Equation 3.15. Similar to the equations for $\dot{\Omega}$ and $\dot{M}_0$, $\dot{\omega}$ is a function of the mean motion, semimajor axis, Earth’s radius, eccentricity, and inclination.

\[
\dot{\omega} = \frac{3 \cdot n \cdot r_E^2 \cdot J_2}{4 \cdot a^2 \cdot (1 - e^2)} \left[4 - 5 \cdot \sin^2(i)\right]
\]  

(3.15)
Once the ascending nodes are fixed to points on the ground, other conditions must be met to achieve a repeat ground track orbit. The eccentricity, inclination and argument of perigee must be held constant. Changes in these elements result in motion in the ground track. Since perturbations in this study are limited to J_2 effects, the eccentricity and inclination are assumed to be constant. To fix the argument of perigee, the orbits are critically inclined at 63.4° or 116.6°. Critical inclinations results from setting the bracketed term in Equation 3.15 to zero and solving for the inclination. If the inclination is set to the critical values, the rate of change of the argument of perigee is eliminated. Setting an orbit to one of the critical inclinations is not the only method to fix the argument of perigee. Special “frozen” orbits can also be designed where both the argument of perigee and the eccentricity are held constant [52].

3.3 Repeat Ground Track Orbit Calculation

In the orbital figure-of-merit analysis, it was necessary to first determine the orbital elements for a specified repeat ground track orbit. Two methods were used to define repeat ground track orbits in this thesis. The first way to define the orbit is by specifying the repeat pattern (N:M), the eccentricity, and the inclination. With this information, the semimajor axis, perigee and apogee radii are determined. The second approach uses the repeat pattern (N:M), inclination, and perigee radius. This method results in the semimajor axis and eccentricity. With either of these methods, a repeat ground track orbit is defined where the semimajor axis, eccentricity and inclination are determined.

For both methods, the algorithm for calculating the orbit stems from the calculation of the semimajor axis. The semimajor axis must be determined so that the satellite’s nodal period is calculated such that Equation 3.11 holds true.

As previously shown in Equation 3.10, the nodal period is a function of the mean motion, the epoch mean anomaly rate, and the argument of perigee rate. These three terms are functions of the semimajor axis along with several other constant terms. The repeat period, \( T \), is a function of the Earth’s rotation rate and the node rate. The node rate (Equation 3.12) is also a function of the semimajor axis and constants. With an initial estimate for the semimajor axis and a series of iterations, the repeat ground track orbit conditions can be satisfied.
The initial estimate for the semimajor axis \(a_0\) is computed by using the mean motion \(n\) defined in Equation 3.13. By rearranging Equation 3.13, the estimate of the semimajor axis can be determined given an initial value of the mean motion \(n_0\), as shown in Equation 3.17. The approximate value for mean motion is calculated by making use of the desired repeat ground track pattern and the Earth’s rotation rate:

\[
n_0 = \frac{N}{M} \omega_E \tag{3.16}
\]

\[
a_0 = \sqrt[3]{\frac{\mu}{n_0^2}} \tag{3.17}
\]

The value of the semimajor axis is then refined through a series of iterations. This is done by differencing the two periods to find the term \(dT_\Omega\). The goal is to drive \(dT_\Omega\) to zero by making corrections to the semimajor axis.

\[
dT_\Omega = \frac{T}{N} - T_\Omega \tag{3.18}
\]

With the term in Equation 3.18 computed, a correction term of \(da\) can be defined. This equation is derived by taking the derivative of the Keplerian period with respect to the semimajor axis:

\[
da = \frac{2}{3} \frac{dT_\Omega}{T_\Omega} a \tag{3.19}
\]

Once the correction term is found, it is then added to the initial semimajor axis and the process is repeated until convergence.

When the repeat pattern ratio, eccentricity, and inclination are specified, the process described above is used. The procedure for determining values of the semimajor axis and eccentricity given only the repeat pattern ratio, perigee radius, and inclination is similar. In order to determine the nodal rate, argument of perigee rate, and epoch mean anomaly rate, the eccentricity of the orbit must be given. By calculating the apogee radius from the specified perigee radius and the estimated semimajor axis, an approximation for the eccentricity can be computed. The orbital element rates are determined and the process continues...
continues as in the first method where iterations are carried out until the semimajor axis converges.

3.4 Coverage Function

In order to calculate the figure-of-merit, the average coverage time per day is needed. Coverage is defined as the total time in seconds when the satellite is above a specified horizon when viewed from the ground station.

The repeat period for orbits examined in this thesis is approximately one day. The exact repeat period is defined by Equation 3.8 which depends on \( a, e, \) and \( i \). The coverage time is calculated over this repeat period.

A Matlab function was written to calculate the coverage time per day over a specified ground station. This function uses the satellite’s orbital elements, the ground station’s location, and a minimum elevation angle parameter. Three major subroutines were used in writing the coverage function. These Matlab files can be found on the accompanying CD-ROM. The diagram in Figure 3.1 indicates how these subroutines are organized to calculate the coverage time. A detailed explanation of each subroutine and calculations used is given in the following sections.

![Diagram of Coverage Function Subroutines](image)
3.4.1 CoverageTime.m

The coverage time function is the main Matlab file used to calculate the coverage time per day over a ground station. The function begins by stepping forward in time and then propagating the orbital elements according to the time past the epoch. This is accomplished using the COEUpdate.m subroutine. The elevation angle relative to the minimum elevation angle parameter is calculated at the new time with the CalcDeltaElAngle.m function. The process continues until the satellite crosses the minimum elevation angle, which results in a sign change where the elevation below the minimum elevation angle is negative and positive when the elevation is above it. At this point, the routine makes use of the "fzero" function to determine the exact time of the beginning of a satellite pass. The Matlab "fzero" function numerically solves for a zero crossing given the function (CalcDeltaElAngle.m), an initial starting guess, and the fixed input parameters associated with the function. Once the starting time of the pass is found, the orbit is propagated forward to the point where the elevation crosses below the minimum elevation angle constraint. The "fzero" function is used again to locate the time that the pass ends. This process repeats until the time past the epoch exceeds the repeat period.

Special cases of coverage are taken into account in the coverage time calculation. Some of these cases are if there is 100% coverage or if the satellite is already above the minimum elevation angle constraint at the beginning of the coverage time calculation. Numerous test cases were conducted to ensure that accurate coverage times were being calculated.

To expedite the coverage calculation, the time step in the propagation was varied based on the elevation angle. As the elevation angle approached the minimum elevation angle constraint, the time step became smaller, with the smallest time step having a value of 30 seconds. As the elevation angle increased, the time step was also increased.

3.4.2 COEUpdate.m

This function propagates the mean orbital elements forward in time according to the $J_2$ secular perturbation theory. The semimajor axis, eccentricity, and inclination are assumed
to remain constant, while the argument of perigee, right ascension of the ascending node, and the mean anomaly are assumed to vary linearly with time as shown below.

\[
\Omega(t) = \Omega_0 + \dot{\Omega} \cdot \Delta t \quad (3.20)
\]

\[
\omega(t) = \omega_0 + \dot{\omega} \cdot \Delta t \quad (3.21)
\]

\[
M(t) = M_0 + M_0 \cdot \Delta t + n \cdot \Delta t \quad (3.22)
\]

\( \Omega_0, \omega_0, \) and \( M_0 \) are the values of the orbital elements at the epoch, \( \dot{\Omega}, \dot{\omega}, \) and \( \dot{M}_0 \) are the time rate of change of the elements, \( n \) is the satellite’s mean motion, and \( \Delta t \) is the time past the epoch. The rates used in the three preceding equations are determined from Equations 3.12, 3.14, and 3.15 and are independent of time. Since in the J\_2 secular theory the orbital element rates are assumed to be constant, they are only calculated once in an upper level routine.

### 3.4.3 CalcDeltaElAngle.m

The purpose of this subroutine is to calculate the elevation angle of the satellite with respect to the minimum elevation angle as a function of the time past the epoch. The elevation angle is defined as the angle between the local horizon and the range vector as shown in Figure 3.2. The range vector is the difference of the satellite’s position vector and the site vector \( \bar{p} = \bar{r}_{\text{sat}} - \bar{r}_{\text{site}} \), also illustrated in Figure 3.2.

The satellite’s position vector is written in the Earth-Centered Inertial (ECI) coordinate frame and is determined from the orbital elements at the given time. The COEToPosition.m function described in the following section is used to carry out this transformation.

The site vector is determined using the epoch date, the ground station’s altitude above the ellipsoidal Earth, geodetic latitude and longitude. The Matlab function ascAndDecl.m is used to compute the right ascension, declination, and radius of the ground station at epoch. The declination remains constant, but the right ascension of the ground station varies linearly with time. CalcDeltaElAngle.m propagates the ground station’s right ascension forward in time from the epoch as shown below:
\[ \alpha(t) = \alpha_0 + \omega_E \Delta t \] (3.23)

where \( \alpha_0 \) is the right ascension of the ground station at the epoch, \( \Delta t \) is the time past the epoch, \( \omega_E \) is the Earth’s rotation rate. Having this information, the site vector \( \vec{r}_{site} \) at a point in time past the epoch is written in the ECI coordinate frame as:

\[ \vec{r}_{site}(t) = r_{site} \cdot [\cos(\alpha(t)) \cdot \cos(\delta_0), \sin(\alpha(t)) \cdot \cos(\delta_0), \sin(\delta_0)] \] (3.24)

The local horizon is defined as the plane tangent to the Earth’s surface at the ground station. Instead of determining a vector in the direction of the local horizon, the local vertical is used. The site vector is measured from the Earth’s center, but the local vertical is perpendicular to the local horizon. The local vertical differs in direction from the site vector due to the Earth’s oblateness. To compute this vector, the right ascension of the ground station and its geodetic latitude (\( \phi \)) are used:

\[ \vec{N}(t) = [\cos(\alpha(t)) \cdot \cos(\phi), \cos(\alpha(t)) \cdot \sin(\phi), \sin(\phi)] \] (3.25)

The elevation angle, \( \varepsilon \), is the complement of the angle between the range vector and the local vertical vector. By using the dot product relationship in Equation 3.26, the elevation angle is determined

\[ \varepsilon(t) = \arcsin \left( \frac{\vec{\rho}(t) \cdot \vec{N}(t)}{||\vec{\rho}(t)||} \right) \] (3.26)

To find the elevation angle with respect to the minimum elevation angle parameter, the difference between the two is taken, thus the relationship that the function CoverageTime.m implements is developed.
3.4.4 COEtoPosition.m

The Matlab file COEtoPosition.m is a simple function used to transform the satellite's orbital elements to a position vector. Inputs to this function are the orbital elements at a particular time, while the output is the position vector corresponding to that time. A detailed explanation of an algorithm similar to COEtoPosition.m is presented by Vallado [51, pp. 149-152].

The procedure used in this calculation first determines the satellite's position in the nodal coordinate system, a satellite-based coordinate system. In the nodal coordinate system, the x-axis is aligned in the direction of the ascending node and the z-axis is perpendicular to the orbital plane. This position vector is then transformed into the Earth-Centered Inertial coordinate system using a rotation matrix that is a function of the right ascension of the ascending node and the inclination.
3.5 Function Testing

The coverage function was extensively tested to ensure that the computed coverage times were consistent with coverage times calculated using Satellite Tool Kit (STK) [53]. The Matlab coverage function agrees well with STK. Five test cases are shown in Table 3.1. The reason STK was not used in the figure-of-merit analysis is because the coverage is optimized by varying the orbital elements, making STK impractical in this application. Since the orbits were propagated forward for approximately one day and the only perturbations used were the J2 secular perturbations, the Matlab code was adequate in determining the coverage time. In addition, the Matlab code proved to be more efficient in optimizing the orbits over the free orbital elements because no interface was required between Matlab and STK.

Table 3.1 Coverage Function Test Cases (Epoch: 01 January 2000)

<table>
<thead>
<tr>
<th>Ground Station</th>
<th>(15° N, 0°)</th>
<th>(15° N, 0°)</th>
<th>(15° N, 0°)</th>
<th>(30° N, 0°)</th>
<th>(30° N, 0°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>16721.535</td>
<td>20260.888</td>
<td>42158.165</td>
<td>20265.721</td>
<td>26553.941</td>
</tr>
<tr>
<td>e</td>
<td>0.570725</td>
<td>0.6457</td>
<td>0.82973</td>
<td>0.4</td>
<td>0.72968</td>
</tr>
<tr>
<td>I (°)</td>
<td>63.4</td>
<td>63.4</td>
<td>63.4</td>
<td>63.4</td>
<td>63.4</td>
</tr>
<tr>
<td>w (°)</td>
<td>-160.813</td>
<td>-127.565</td>
<td>-64.83948</td>
<td>-116.327</td>
<td>-113.641</td>
</tr>
<tr>
<td>Ω (°)</td>
<td>-37.760</td>
<td>-60.142</td>
<td>45.31375</td>
<td>51.597</td>
<td>-69.267</td>
</tr>
<tr>
<td>Matlab Coverage (s / %)</td>
<td>15208.87</td>
<td>42584.26</td>
<td>67058.82</td>
<td>34444.74</td>
<td>58584.53</td>
</tr>
<tr>
<td>STK Coverage (s / %)</td>
<td>15208.78</td>
<td>42445.087</td>
<td>67058.252</td>
<td>34444.701</td>
<td>58205</td>
</tr>
<tr>
<td>Difference (s)</td>
<td>-0</td>
<td>0.327</td>
<td>-0</td>
<td>-0</td>
<td>0.652</td>
</tr>
</tbody>
</table>

3.6 Coverage Plots

The figure-of-merit maximizes the coverage for the prescribed orbit by varying three of the epoch orbital elements: Ω, w, and M. These elements are assumed to be free in that they can be adjusted without significant costs in ΔV. The Matlab code written to calculate the total coverage time is written as a function of the orbital elements that are allowed to vary. This function must be maximized over three variables, resulting in a time intensive calculation using a 550 MHz PC running Windows 2000.

By examining the properties of repeat ground track orbits, the coverage function can be reduced to a function of two variables, the longitude of ascending node and the argument of perigee. The Earth-fixed longitude of ascending node (L) replaces the right ascension of
the ascending node because it is not tied to a defined epoch. The longitude of ascending node is a function of the right ascension of the ascending node and the right ascension of Greenwich (\(\alpha_c\)) and determined by the following expression:

\[
L = \Omega(t) - \alpha_c(t)
\]  

(3.27)

By placing the satellite at the ascending node, the mean anomaly becomes a function of the argument of perigee. If the initial true anomaly (\(\theta\)) is set to the negative value of the argument of perigee, the satellite will start at an equator crossing. The mean anomaly can then be determined from the true anomaly, the orbit’s eccentricity, and the semimajor axis.

To further simplify the optimization, constraints were placed on the range of the longitude of ascending node. The argument of perigee varies between \(-180^\circ\) to \(180^\circ\), but the longitude of ascending node could be constrained due to the fact that repeat ground track patterns were the only orbits examined in this study. As a result, the ascending nodes repeat at longitude intervals of \(360^\circ/N\). Therefore, the range chosen for the longitude of ascending node was \(\pm 180^\circ/N\) about the longitude of the ground station.

Once the coverage time is written as a function of two variables, the maximization becomes manageable, yet a difficulty was encountered with the optimization: the possibility of multiple maxima in the function. Some orbit / ground station combinations were discovered to exhibit strange coverage behavior with multiple local maxima. The Matlab function “fmincon” was used to optimize the coverage function, but this optimization function does not guarantee that the global maximum will be found.

To ensure that the global maximum was found, a two step process was devised. The Matlab function Contour.m was written to evaluate the coverage over a coarse interval over the range of longitudes of ascending node and arguments of perigee. This function can be found on the accompanying CD-ROM. These data were then formed into a contour plot to show the variation of coverage with respect to the optimization variables, as well as to allow the optimizer to obtain a suitable starting point close to the global maximum of the function. This starting point was input into “fmincon” to determine the exact maximum coverage time. Examples of contour plots that were produced are shown in Chapter 4.
3.7 Nonlinear Optimization / Matlab “fmincon” function

The Matlab [54] function “fmincon” is a constrained nonlinear optimization function that employs sequential quadratic programming to determine the location of a local minimum. This function is found in the Matlab Optimization Toolbox.

Sequential quadratic programming [55] is a method commonly used to solve constrained nonlinear optimization problems. This method works well for continuous, smooth functions and is guaranteed to converge to a local minimum. This process minimizes a quadratic program sub-problem to determine a search direction. A quadratic program is a nonlinear optimization where all constraints are linear and the objective function is quadratic. Since the coverage function might not be quadratic, the quadratic sub-problem approximates a small region as a quadratic function using a 2nd order Taylor series expansion. This is assumed to be valid because if the region is small enough, deviations between the actual function and the Taylor series expansion will be small. A line search is then used to move closer towards the minimum value of the function. At this point, iterations are conducted with a new quadratic program sub-problem and line search until the solution converges.

Although the goal is to maximize the coverage time over a ground station, this can be converted to a minimization problem by multiplying the coverage time by -1. The only constraints applied to this problem are the bounds that are placed on the optimization variables: the longitude of ascending node and the argument of perigee.

3.8 Delta-V Calculation

There are several possible methods to compute the cost in ΔV required to attain a desired orbit. Two options considered were an analytic approach and the use of a launch trajectory software tool. The analytic approach chosen provides a good estimate of the ΔV necessary for launch.

Boltz [56] provides a complex method where one can calculate the ΔV from launch to final orbit by using the equations of motion. In his approach, he takes into account drag forces, mass losses, staging and launch vehicle heating. While this method allows for an accurate calculation of the ΔV, a simplified method is desirable.
Loftus, Teixeira, and Kirkpatrick [57] present a simplified approach for estimating the $\Delta V$ required for a launch vehicle. The Delta-$V$ ($\Delta V_{\text{design}}$) is given by:

$$ \Delta V_{\text{design}} = V_{\text{burnout}} + \Delta V_{\text{gravity}} + \Delta V_{\text{drag}} $$ (3.28)

In Equation 3.28, $V_{\text{burnout}}$ is the orbital velocity of the satellite at injection, $\Delta V_{\text{gravity}}$ are velocity losses due to gravity and $\Delta V_{\text{drag}}$ are velocity losses due to atmospheric drag. Accurately determining the losses due to gravity is a complex process involving the launch vehicle trajectory, but generally it accounts for a 750 to 1500 m/s velocity loss [57]. The losses due to drag are quoted as being small, approximately 3% of the total $\Delta V$ required. This method does not take into account the launch site location or the orbit’s inclination.

In this thesis, a similar approach to the method described by Equation 3.28 was taken to calculate the total $\Delta V$ to orbit. Equation 3.28 was modified and written as:

$$ \Delta V = \Delta V_{\text{Hohmann}} + \Delta V_{\text{losses}} $$ (3.29)

where $\Delta V_{\text{Hohmann}}$ is the change in velocity for a Hohmann transfer from the Earth’s surface to the final orbit. The $\Delta V_{\text{losses}}$ term accounts for losses due to gravity and drag. Typical values for losses range from 4000 - 6000 ft/s (1219 – 1829 m/s) [58] and in this study, the velocity losses due to gravity and drag were set to 5000 ft/s (1524 m/s).

The Hohmann transfer [50, 51] is a two impulse orbital maneuver used to transfer from an initial orbit to a final orbit, where the impulses are assumed to be instantaneous. In this case, the initial orbit is defined as a circular orbit with a semimajor axis equal to the Earth’s radius. If given the final orbit’s semimajor axis, eccentricity and inclination, the calculation of the $\Delta V_{\text{Hohmann}}$ term is straightforward. The $\Delta V_{\text{Hohmann}}$ term is found by summing the velocity impulses required to implement an elliptical transfer orbit from the launch site to the final orbit’s apogee (Equation 3.30).

$$ \Delta V_{\text{Hohmann}} = \Delta V_{\text{Launch}} + \Delta V_{\text{Final}} $$ (3.30)

The $\Delta V_{\text{Launch}}$ term is the difference between the velocity of the transfer orbit at perigee and launch vehicle’s initial velocity, as shown in Equation 3.31.
\Delta V_{\text{Launch}} = \left\| \vec{v}_{\text{Transfer}} - \vec{v}_{\text{Initial}} \right\| \quad (3.31)

To facilitate a launch to any inclination, the launch site was placed on the equator. The launch vehicle’s initial velocity vector is defined as the velocity at the Earth’s surface at the launch site. Using the assumption that the launch site is on the equator, the initial velocity is defined as a vector with the magnitude of the product of the Earth’s rotation rate \( \omega_E \) and the Earth’s radius \( r_E \):

\[
\vec{v}_{\text{Initial}} = [\omega_E \cdot r_E, 0, 0] \quad (3.32)
\]

The transfer orbit’s velocity vector at the launch site \( \vec{v}_{\text{Transfer}} \) is also needed in the \( \Delta V_{\text{Launch}} \) calculation. The vector \( \vec{v}_{\text{Transfer}} \) is a function of the final orbit’s inclination and can be written in the form:

\[
\vec{v}_{\text{Transfer}} = v_{\text{Transfer}} \cdot [\cos(i), 0, \sin(i)] \quad (3.33)
\]

where the scalar \( v_{\text{Transfer}} \) can be found using the vis-viva integral in Equation 3.34:

\[
v_{\text{Transfer}} = \sqrt{\frac{\mu}{\left( \frac{2}{r_{\text{Launch}}} + \frac{1}{a_{\text{Transfer}}} \right)}} \quad (3.34)
\]

The vectors for the initial velocity and velocity of the transfer orbit at perigee are used only to distinguish difference in the two directions of the velocity vectors. The vectors are referenced to the plane of the equator where the x-component points towards the direction of the velocity of a point on the Earth and the y-component points in the direction of the Earth’s center.

The semimajor axis \( a_{\text{Transfer}} \) of the transfer orbit is needed to calculate \( v_{\text{Transfer}} \) and is determined by using Equation 3.35.

\[
a_{\text{Transfer}} = \left( r_E + r_{\text{Final}} \right)/2 \quad (3.35)
\]

where \( r_{\text{Final}} \) is dependent on the semimajor axis and eccentricity of the final mission orbit as shown below:

\[
r_{\text{Final}} = a_{\text{Final}} \cdot (1 + e_{\text{Final}}) \quad (3.36)
\]
The other term in Equation 3.30 is $\Delta V_{\text{Final}}$ and is defined as the difference between the velocity at the apogee of the final orbit and the velocity at the apogee of the transfer orbit.

$$\Delta V_{\text{Final}} = v_{\text{Final}} - v_{\text{Transfer}}$$  \hfill (3.37)

The two velocities required in Equation 3.37 are found using the vis-viva integral.

$$v_{\text{Transfer}} = \sqrt{\frac{2\mu}{r_{\text{Final}} - a_{\text{Transfer}}}}$$  \hfill (3.38)

$$v_{\text{Final}} = \sqrt{\frac{2\mu}{r_{\text{Final}} - a_{\text{Final}}}}$$  \hfill (3.39)

With these equations the $\Delta V$ required for launch is written as a function of the three orbital elements, $a$, $e$, and $i$. These values are all known since the repeat ground track orbit is defined. A listing of the Matlab code to calculate the $\Delta V$ is found on the accompanying CD-ROM.

To verify the validity of this method, $\Delta V$ values for launch into low-Earth orbit published by Humble, Henry and Larson [59] were compared with the $\Delta V$ values calculated using the analytic approach. A comparison of the values is shown in Table 3.2.

It is important to note that in the $\Delta V$ calculation, the launch site was on the Equator, but the reference values shown are from different launch sites. The difference in the launch site locations can account for some of the errors in the actual $\Delta V$ values.

**Table 3.2 Comparison of Calculated Delta-V Values with Delta-V Examples**

<table>
<thead>
<tr>
<th>Launch Vehicle</th>
<th>Semimajor Axis (km)</th>
<th>Eccentricity</th>
<th>Inclination (degrees)</th>
<th>$\Delta V_{\text{reference}}$ (m/s)</th>
<th>$\Delta V_{\text{calculated}}$ (m/s)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariane 6548</td>
<td>6548</td>
<td>0</td>
<td>7.0</td>
<td>9138</td>
<td>9071</td>
<td>0.73</td>
</tr>
<tr>
<td>A-44L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atlas I</td>
<td>6756</td>
<td>0.0339</td>
<td>27.4</td>
<td>9243</td>
<td>9241</td>
<td>0.02</td>
</tr>
<tr>
<td>Delta 7925</td>
<td>6625</td>
<td>0.0108</td>
<td>33.9</td>
<td>8814</td>
<td>9196</td>
<td>4.33</td>
</tr>
<tr>
<td>Saturn V</td>
<td>6554</td>
<td>0</td>
<td>28.5</td>
<td>9267</td>
<td>9130</td>
<td>1.48</td>
</tr>
<tr>
<td>Titan IV/ Centaur</td>
<td>6688</td>
<td>0.02287</td>
<td>28.6</td>
<td>9207</td>
<td>9208</td>
<td>0.01</td>
</tr>
</tbody>
</table>
3.9 Figure-of-Merit Calculation

A series of Matlab functions were created to calculate the figure-of-merit by using the coverage and ΔV functions. These figure-of-merit functions can be found on the accompanying CD-ROM. The LoadCountourData.m function is used to determine the global maximum coverage time for the orbit. This function references both MaximumCoverage.m and FOMCoverage-Calc.m. Data from the created contour plots were used to locate the approximate location of the global maximum in the MaximumCoverage.m function. The FOMCoverageCalc.m function then uses that data as a starting point for “fmincon” to determine the exact location of the global maximum coverage.

MaxCoverageRPT*.m (where * represents N) collects and saves the exact maximum coverage values for all repeat ground track orbit / ground station combinations examined in this thesis. Appendix F provides the Matlab file MaxCoverageRPT1.m which examines all of the 1:1 repeat ground track cases. Additional Matlab scripts for the remaining repeat ground track orbits are also listed. These scripts differ from the listed code, MaxCoverageRPT1.m, by the called data files containing the coverage contour data for the correct N:1 repeat ground track orbits. The other difference is in the variable RPT, which is used to define N. RPT is changed to correspond to the specified repeat ground track pattern.

The final Matlab file used to calculate the figure-of-merit is FOMCompile.m. This script combines the maximum coverage for a repeat ground track orbit / ground station combination and calls DeltaVCalc.m to determine the ΔV associated with the orbit. The orbital elements corresponding to the maximum coverage point are saved along with the computed figure-of-merit.

The calculation of the figure-of-merit using these Matlab functions was a manual process where the coverage contour data was collected first, then the figure-of-merit scripts were run. The primary reason for not automating the figure-of-merit calculation is because the process was time intensive. However, the Matlab scripts developed can be easily modified to automate the figure-of-merit analysis.
4 Results

4.1 Chapter Overview

This chapter presents the results obtained from the coverage function, delta-V calculation, and the final figure-of-merit analysis. When examining the maximized coverage for different repeat ground track orbits, several known orbits were observed. These were the Tundra/Sirius, Molniya, Cobra, and Ellipso-Borealis orbits. In addition, general trends in the figure-of-merit are noted.

4.2 Coverage Results

4.2.1 Orbits Examined

This study examined six N:1 repeat ground track patterns where N included 1, 2, 3, 4 and 8 revolutions. All of the orbits considered were critically inclined at 63.4°, except the 8:1 repeat period where both critical inclinations of 63.4° and 116.6° were analyzed. The 8:1 case at an inclination of 116.6° allows for a sun-synchronous repeat ground track orbit [60, 61]. Three different eccentricities were chosen for each repeat period. Eccentricities of 0, 0.4 and the maximum eccentricity possible for an 800 km perigee altitude were selected. In the case of the 8:1 orbits, the maximum eccentricity was approximately 0.32. Because of this, an eccentricity of 0.2 replaced the 0.4 eccentricity point. The maximum eccentricity can be calculated as a function of the repeat cycle, inclination, and a minimum perigee radius. Ground station locations were varied from the equator to the North Pole in 15° latitude increments. Unless otherwise noted, all ground stations were located on the Prime Meridian.

The final variable that was introduced in the coverage calculation was the minimum elevation angle. Minimum elevation angles of 10° and 30° were chosen in the figure-of-merit analysis. The 10° elevation angle constraint is suited for commercial satellite telephony systems such as Iridium, Ellipso, and Globalstar. The 30° elevation angle constraint is relevant to higher frequency systems that are currently being considered for broadband applications.
4.2.2 Contour Plots

The possibility for multiple maxima over the range of the optimization variables presented a problem in determining the maximum coverage. An initial solution to this problem was to select random values for the argument of perigee and longitude of ascending node as starting points for the optimization function. The algorithm took 100 random samples in an attempt to find the global maximum. The validity of this procedure was suspect in that the variation in coverage over these two orbital elements was not understood.

To better understand how coverage varies with the argument of perigee and the longitude of the ascending node, coverage times were computed for a range of these variables. There were 130 points taken over both the argument of perigee and longitude of ascending node ranges, yielding a total of 16,900 data points. Once the data points were collected, a Matlab function was used to interpolate between the points to create a contour plot showing the coverage versus the Earth-fixed longitude of ascending node and the argument of perigee. The increment in the longitude of ascending node varied from 0.34° for an 8:1 repeat ground track orbit to 2.77° for a 1:1 repeat ground track orbit. The resolution in the argument of perigee remained constant at 2.77°. Plots for the circular repeat ground track orbits were simplified since the argument of perigee in these cases is undefined. The coverage plot for these cases was a plot of the coverage as a function of the longitude of ascending of node only.

A total of 252 coverage plots were created accounting for all combinations of orbit repeat patterns, ground station locations, orbit eccentricities, and minimum elevation angle parameters examined in the study. These contour plots can be found in the accompanying CD-ROM. With the contour plots, the approximate location of the global maximum coverage time provided by the orbit was clear. By focusing on this area, the Matlab “fmincon” was able to determine the precise location of the global maximum.

When examining the coverage behavior of the orbits, it was found that ground station locations at low latitudes often resulted in coverage contour plots with unusual features. An example of this unusual behavior is shown in Figure 4.1, a contour plot of a 3:1 repeat ground track orbit with an eccentricity of 0.4.
Figure 4.2 is a 3-dimensional contour plot of the same data and better shows the large number of local maxima over the given range.

The ground station for this case is located on the equator and the minimum elevation angle is $10^\circ$. The contour plots show the coverage as the percentage of time in view of the ground station over the total time for one repeat cycle. An examination of Figure 4.1 shows that there are multiple maxima, with some that fall in a narrow region. Due to the ground station’s location at $0^\circ$ latitude, there is symmetry in the contour plot and two global maxima result. These are shown in Figure 4.1, where one global maximum occurs in a region at approximately a longitude of ascending node $L = -30^\circ$ and an argument of perigee $\omega = -90^\circ$, while the other occurs at $L = 30^\circ$, $\omega = 90^\circ$. Since this region is extremely small, it is unlikely that this point would be found using the random point process described in the beginning of this section. The global maximum, while optimal, is not robust in that small deviations in either the argument of perigee or longitude of ascending node results in a large change in the coverage.

Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit | Ground Station Latitude: $0^\circ$

$i = 63.4^\circ$ | $e = 0.4$ | Min. Elevation Angle = $10^\circ$

Figure 4.1: Contour Plot of Coverage, Low Latitude Ground Station
The optimal orbit for this case was determined; Table 4.1 lists its orbital elements. The reason for the narrow regions of high coverage in this case can be explained from the ground track of the orbit shown in Figure 4.3. The expected maximum coverage point was with the apogee in the northern (or southern) hemisphere, centered over the longitude of the ground station. The ground track plot shows the apogee slightly to the east of the ground station longitude. This allows for a small segment near the descending node over South America to be in view of the ground station. If the ascending node is pushed slightly towards the west, this segment is no longer in view and the loss accounts for the decrease in the coverage time. If the ascending node is pushed further to the west, the contour plot shows that there is an initial decrease in coverage followed by an increase to a maximum coverage time with a longitude of ascending node of approximately 315°. This point corresponds to the expected global maximum where the apogee is centered on the longitude of the ground station.
Table 4.1: Orbital Elements for Optimized Orbit

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor Axis</td>
<td>20265.721 km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.4</td>
</tr>
<tr>
<td>Inclination</td>
<td>63.4°</td>
</tr>
<tr>
<td>Argument of Perigee</td>
<td>-88.59°</td>
</tr>
<tr>
<td>Longitude of Ascending Node</td>
<td>332.31°</td>
</tr>
<tr>
<td>Coverage Time per Repeat Cycle</td>
<td>24818 s</td>
</tr>
</tbody>
</table>

Figure 4.3: Ground Track Corresponding to Optimized Orbit for Coverage

While low latitude ground stations exhibited complex behavior, as the ground station’s latitude was increased, the coverage contours became straightforward and the behavior was as expected. Figure 4.4 illustrates the coverage variation for the same orbit as in Figure 4.1, however the ground station is now located at 75° N latitude. Figure 4.5 shows a 3-dimensional view of Figure 4.4. There is little dependence on the longitude of ascending node and the best argument of perigee is $\omega = -90°$. 
Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit | Ground Station Latitude: 75° N
\( i=63.4^\circ \) | \( e = 0.4 \) | Min. Elevation Angle = 10°

Figure 4.4: Contour Plot of Coverage- High Latitude Ground Station

Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit | Ground Station Latitude: 75° N
\( i=63.4^\circ \) | \( e = 0.4 \) | Min. Elevation Angle = 10°

Figure 4.5: 3-D Contour Plot of Coverage- High Latitude Ground Station
4.2.3 Optimal Orbits

In optimizing the coverage for the orbits, previously developed orbits resulted as the optimal solution for some repeat ground track cases that were examined. Orbits that were reproduced with remarkable similarity in the optimization routine were the Tundra/Sirius [62, 63, 64, 65, 66, 67], Molniya [68], COBRA [42], and Ellipso-Borealis [39, 40] orbits.

4.2.3.1 Tundra/Sirius Orbit

The Sirius satellite system is a constellation of three satellites designed to provide 24 hour commercial digital-quality radio coverage to the United States. It is the first system to use a critically inclined 1:1 repeat ground track orbit, or Tundra orbit. Two coverage optimizations were performed using Tundra orbits. The first optimization was for a 1:1 repeat ground track orbit where the eccentricity was set to that of the Sirius-I orbit, while the second was for a 1:1 repeat ground track orbit with the maximum eccentricity possible for an 800 km perigee altitude. The minimum elevation angle for both optimizations was set to 10°. Since the Sirius constellation was designed to provide coverage to the U.S., the ground station for the optimizer was located at 36.5° N and 96.5° W, the approximate center of the U.S.

Table 4.2 lists the orbit elements for three orbits: the actual Sirius-1 satellite, the optimized Sirius orbit, and the overall optimal 1:1 repeat ground track orbit for a 10° minimum elevation angle. The arguments of perigee and longitudes of ascending node for the Sirius-1 orbit and the optimized Sirius orbit are nearly identical. When the ground tracks are plotted with Satellite Tool Kit (shown in Figure 4.6), there are only minor differences in the two ground tracks. Figure 4.6 also plots the ground track for the overall optimal orbit for the given ground station with a 10° minimum elevation angle. The optimal orbit exhibits the best coverage over the ground station for a 10° cutoff elevation, but other constraints are not taken into account. Due to design constraints, the Sirius orbit has a lower eccentricity, accounting for the differences between the two orbits. The major factor in the selected eccentricity is so the spacecraft can avoid the Van Allen Belts. All three orbits exhibit similarities in that the arguments of perigee are approximately 270° while the longitudes of ascending node are chosen to place the apogee over the target.
Table 4.2: Orbit Elements for Sirius-1 and Optimized Orbits for a Ground Station at (36.5° N, 96.5° W, 0 km), Minimum Elevation Angle: 10°

<table>
<thead>
<tr>
<th></th>
<th>Sirius-1 [69]</th>
<th>Optimized Sirius Orbit</th>
<th>Optimal Maximum Eccentricity Orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>42163.79</td>
<td>42163.40</td>
<td>42158.17</td>
</tr>
<tr>
<td>E</td>
<td>0.2635</td>
<td>0.2635</td>
<td>0.8297</td>
</tr>
<tr>
<td>i (degrees)</td>
<td>63.24</td>
<td>63.40</td>
<td>63.40</td>
</tr>
<tr>
<td>ω (degrees)</td>
<td>269.56</td>
<td>269.98</td>
<td>270.04</td>
</tr>
<tr>
<td>L (degrees)</td>
<td>294.00</td>
<td>293.33</td>
<td>346.15</td>
</tr>
</tbody>
</table>

Figure 4.6: STK Ground Track Comparison for Sirius-1 Orbit and Optimized Orbit

The effect of the minimum elevation angle on coverage was examined with the Sirius orbit and the optimal orbit through two sets of computations. The minimum elevation angle was increased from 10° to values of 20°, 30°, and 45°.

First, the orbital elements of the Sirius and optimal maximum eccentricity orbits (shown in Figure 4.6) were held constant and coverage times over the ground station were calculated for the four minimum elevation angles. These times are displayed in Table 4.3. For all minimum elevation angles except 45°, the optimal maximum eccentricity orbit provides more coverage to the ground station than the Sirius orbit.
Table 4.3: Comparison of Sirius and Optimal Orbit Coverage Times Based on Minimum Elevation Angle

<table>
<thead>
<tr>
<th>Minimum Elevation Angle</th>
<th>Sirius Orbit Coverage Time (s)</th>
<th>Optimal Orbit Coverage Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>67347.42</td>
<td>74599.65</td>
</tr>
<tr>
<td>20°</td>
<td>63599.37</td>
<td>68641.07</td>
</tr>
<tr>
<td>30°</td>
<td>59876.72</td>
<td>61452.76</td>
</tr>
<tr>
<td>45°</td>
<td>54210.24</td>
<td>41552.47</td>
</tr>
</tbody>
</table>

In the next set of computations, additional coverage optimizations were performed where the minimum elevation angle was varied to determine if the orbital elements remained constant for the maximum eccentricity orbit and the Sirius eccentricity orbit. The optimization results for the maximum eccentricity orbit are shown in Table 4.4, while the results for the Sirius eccentricity orbit are listed in Table 4.5.

The orbital elements of the repeat ground track orbit with the Sirius eccentricity remain constant as the minimum elevation angle increased, but the argument of perigee and longitude of ascending node for the maximum eccentricity orbit change significantly when the minimum elevation angle increases to 30°. Thus the optimal orbit shown in Figure 4.6 is optimal for minimum elevation angles of up to 20°, but not for higher cutoff elevations.

When comparing the coverage times with the Sirius eccentricity orbit, the maximum eccentricity orbits have better coverage except when the minimum elevation angle is 45°. The figure-of-merit values indicate that the optimized maximum eccentricity orbits are better than the orbit with the Sirius eccentricity. It is important to note that although the figure-of-merit values for the maximum eccentricity orbit are higher than the Sirius orbit, the orbital elements change from the 20° cutoff elevation to the 30° cutoff elevation. Thus, it appears that the Sirius orbit is better suited for a wide range of elevation angles than the maximum eccentricity orbit.

Table 4.4: 1:1 Repeat Ground Track Orbit Optimizations, $e = 0.8297$

<table>
<thead>
<tr>
<th>Minimum Elevation Angle</th>
<th>$\omega$ (°)</th>
<th>LAN (°)</th>
<th>Coverage Time (s)</th>
<th>Figure-of-Merit</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>270.04</td>
<td>346.15</td>
<td>74599.65</td>
<td>60.39</td>
</tr>
<tr>
<td>20°</td>
<td>270.01</td>
<td>346.05</td>
<td>68641.07</td>
<td>55.57</td>
</tr>
<tr>
<td>30°</td>
<td>249.24</td>
<td>24.89</td>
<td>61443.03</td>
<td>49.74</td>
</tr>
<tr>
<td>45°</td>
<td>248.93</td>
<td>42.76</td>
<td>51065.56</td>
<td>41.33</td>
</tr>
</tbody>
</table>
Table 4.5: 1:1 Repeat Ground Track Orbit Optimizations: $e = 0.2635$

<table>
<thead>
<tr>
<th>Minimum Elevation Angle</th>
<th>$\omega$ (°)</th>
<th>LAN (°)</th>
<th>Coverage Time (s)</th>
<th>Figure-of-Merit</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>269.98</td>
<td>293.33</td>
<td>67347.42</td>
<td>50.93</td>
</tr>
<tr>
<td>20°</td>
<td>270.26</td>
<td>293.43</td>
<td>63599.37</td>
<td>48.09</td>
</tr>
<tr>
<td>30°</td>
<td>270.41</td>
<td>293.46</td>
<td>59876.72</td>
<td>45.2916</td>
</tr>
<tr>
<td>45°</td>
<td>270.43</td>
<td>293.41</td>
<td>54210.24</td>
<td>40.99</td>
</tr>
</tbody>
</table>

Although the Sirius orbit does not provide the maximum coverage possible for a 1:1 repeat ground track orbit below a 30° minimum elevation angle, the orbit is a satisfactory choice since it provides good coverage for high viewing angles.

4.2.3.2 Molniya Orbit

Another well known orbit that was generated through coverage optimization was the Molniya orbit. The Molniya orbit is a 2:1 repeat ground track orbit that has been used since 1965 [68, 70, 71]. Leaning Molniya orbits with arguments of perigee between 280° and 288° have commonly been used in the past by the Soviet Union. The Soviet Union has employed leaning Molniya orbits in ballistic missile launch detection applications since the mid-1980s. These satellites had initial arguments of perigee between 316° and 319°, resulting in significant leans in the orbit [72]. The ground tracks of these orbits can be seen in Figure 4.7.

A Molniya orbit generated by Satellite Tool Kit was compared to that of the optimized 2:1 repeat ground track orbit set for a ground station at 45° N latitude. The only inputs for the STK generated orbit were the apogee longitude and the perigee altitude. The STK Molniya orbit has an argument of perigee of 270°.

Table 4.6 shows the element sets for the two orbits, while Figure 4.8 shows the ground tracks corresponding to the two orbits. In the STK generated orbit, the apogee was placed over the target’s longitude based on my intuition for maximum coverage. Contrary to my intuition, this is not the optimal coverage point given a 10° minimum elevation angle. In fact, the optimum coverage occurs when the ground station is located halfway between the two apogees. By examining the coverage contour for the 2:1 repeat ground track case at maximum eccentricity as shown in Figure 4.9, we can see how the coverage varies as a
function of the longitude of ascending node. Although Figure 4.9 shows the optimal coverage for a ground station located at 45° N, the contour plots for both 60° N and 75° N latitudes also show that the optimal coverage point is identical to the 45° N location. As the minimum elevation angle is increased, the STK generated Molniya orbit provides better coverage. When the minimum elevation angle is 45°, the optimized orbit in Figure 4.8 provides no coverage to a ground station at 45° N latitude. The optimized 2:1 maximum eccentricity repeat ground track orbit with a minimum elevation angle of 30° covering a ground station at 45° N latitude results in a leaning Molniya orbit, where the argument of perigee and the longitude of ascending node are different those listed in Table 4.6. However, for the 30° minimum elevation angle case covering ground stations at 60° N and 75° N, the orbit remains as shown in Figure 4.8.

Figure 4.7: Leaning Soviet Molniya Ground Tracks [72]
Table 4.6: Orbit Elements for an STK Generated Molniya Orbit and Optimized Orbit (10° Cutoff Elevation) for a Ground Station at (45° N, 0° W, 0 km)

<table>
<thead>
<tr>
<th></th>
<th>STK Generated Molniya</th>
<th>Optimized Orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>26553.94</td>
<td>26553.94</td>
</tr>
<tr>
<td>e</td>
<td>0.7297</td>
<td>0.7297</td>
</tr>
<tr>
<td>i (degrees)</td>
<td>63.40</td>
<td>63.40</td>
</tr>
<tr>
<td>ω (degrees)</td>
<td>270.00</td>
<td>270.00</td>
</tr>
<tr>
<td>L (degrees)</td>
<td>172.61</td>
<td>82.73</td>
</tr>
</tbody>
</table>

Figure 4.8: STK Ground Track Comparison for a Typical Molniya Orbit and Optimized Orbit
4.2.3.3 COBRA Orbit

The “Communications Orbiting Broadband Repeating Array” or COBRA orbit proposed by Draim [42], is a leaning 3:1 repeat ground track orbit with an orbit period of 8 hours. The “lean” in the orbit is caused by the argument of perigee. These orbits can be either right leaning or left leaning where right leaning orbits correspond to those that have arguments of perigee greater than 270°, while the left leaning orbits have arguments of perigee less than 270°. Table 4.7 shows the orbit elements for both Draim’s COBRA Orbit and an optimized orbit with a minimum cutoff elevation set to 10°. The ground station in this optimization run was placed at 30° N latitude. The coverage optimization produced a COBRA-like orbit; both the COBRA and the optimized orbits are leaning. Draim’s orbit leans further to the left with an argument of perigee of 226.45°, while the optimized orbit has an argument of perigee of 245.03°, but the optimized orbit still shows a significant lean. The difference in the arguments of perigee can be attributed to additional constraints that were applied to the COBRA orbit such as the requirement to remain clear of the GEO region. The
coverage contour plot for this orbit / ground station combination is shown in Figure 4.11, and it indicates that there are two optimal orbits that exist. These two orbits correspond to a right leaning and a left leaning orbit and both have the same amount of coverage.

Table 4.7: Orbit Elements for a Typical COBRA Orbit and Optimized Orbit for a Ground Station at (30° N, 0° W, 0 km)

<table>
<thead>
<tr>
<th></th>
<th>COBRA Orbit [42]</th>
<th>Optimized Orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>20260.85</td>
<td>20260.86</td>
</tr>
<tr>
<td>e</td>
<td>0.6458</td>
<td>0.6457</td>
</tr>
<tr>
<td>i (degrees)</td>
<td>63.40</td>
<td>63.40</td>
</tr>
<tr>
<td>ω (degrees)</td>
<td>226.45</td>
<td>245.03</td>
</tr>
<tr>
<td>L (degrees)</td>
<td>60.00</td>
<td>51.13</td>
</tr>
</tbody>
</table>

Figure 4.10: STK Ground Track Comparison for a Typical COBRA Orbit and Optimized Orbit
4.2.3.4 Ellipso-Borealis Orbit

The final orbit that appeared through the optimization routine was similar to the Ellipso-Borealis orbit. Ellipso aims to provide low cost global communication coverage through a hybrid constellation using two sub-constellations. One of these constellations is Ellipso-Borealis, series of elliptical 8:1 repeat ground track orbits at an inclination of 116.6°. The Ellipso-Borealis constellation is designed to provide coverage to northern latitudes.

An optimization of a retrograde 8:1 orbit for a ground station located at 30° N with a 10° cutoff elevation was performed. Table 4.8 shows orbit element sets of both the proposed Ellipso-Borealis constellation and that of the optimized orbit. As with the previous two cases, the orbit was optimized for a ground station located at 0° longitude. Therefore, the arguments of perigee are similar, however the longitudes of ascending nodes differ because the Borealis orbit was optimized for coverage over the northern hemisphere. Figure 4.13 illustrates the variation in coverage vs. the longitude of ascending node and the argument of perigee for this orbit. This contour plot shows that there are two areas of maximum

Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit | Ground Station Latitude: 30° N
\[ i = 63.4° \] \[ e = 0.6457 \] \[ \text{Min. Elevation Angle} = 10° \]

Figure 4.11: Contour Plot of Coverage
coverage, both of which have an argument of perigee near -90°, but the optimal point is located at a longitude of ascending node near 16°.

Table 4.8: Orbit Elements for a Typical Ellipso-Borealis Orbit and Optimized Orbit For a Ground Station at (30° N, 0° W, 0 km)

<table>
<thead>
<tr>
<th></th>
<th>Ellipso-Borealis [73]</th>
<th>Optimized Orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (km)</td>
<td>10559.25</td>
<td>10558.58</td>
</tr>
<tr>
<td>e</td>
<td>0.3453</td>
<td>0.3202</td>
</tr>
<tr>
<td>i (degrees)</td>
<td>116.57</td>
<td>116.6</td>
</tr>
<tr>
<td>ω (degrees)</td>
<td>270.00</td>
<td>269.96</td>
</tr>
<tr>
<td>L (degrees)</td>
<td>10.00</td>
<td>15.75</td>
</tr>
</tbody>
</table>

Figure 4.12: STK Ground Track Comparison for a Typical Ellipso-Borealis Orbit and Optimized Orbit
Coverage Percentage vs. Optimization Variables

8:1 Repeat Ground Track Orbit | Ground Station Latitude: 30° N
i = 116.6° | e = 0.3202 | Min. Elevation Angle = 10°

With a specified eccentricity, inclination, orbit repeat period, ground station location, and minimum elevation angle, the optimization over the longitude of ascending node and argument of perigee was shown to yield known orbits. This demonstrates that some of the known orbits presented are either optimal, or near optimal in terms of coverage.

Optimized coverage times per day for each orbit examined with a 10° cutoff elevation are shown in Table 4.9 as a percentage of the ground track repeat period. Aside from the 1:1 case, as the eccentricity increases and the ground station latitude increases, the coverage time increases. The other trend to note in this table is that as the number of orbits per day (N) increases, the coverage time decreases. A table of optimized coverage times for the 30° minimum elevation angle case can be found in Appendix B. In addition, Appendix B lists the argument of perigee and longitude of ascending node for each optimized orbit examined in this thesis.
Table 4.9: Coverage Time Per Day (%) vs. Ground Station Latitude

(10° Minimum Elevation Angle)

<table>
<thead>
<tr>
<th>Repeat Type</th>
<th>Latitude</th>
<th>Eccentricity</th>
<th>0°</th>
<th>15° N</th>
<th>30° N</th>
<th>45° N</th>
<th>60° N</th>
<th>75° N</th>
<th>90° N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>e = 0</td>
<td>100.00</td>
<td>86.09</td>
<td>74.70</td>
<td>65.78</td>
<td>57.00</td>
<td>47.79</td>
<td>38.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.4</td>
<td>100.00</td>
<td>87.85</td>
<td>82.85</td>
<td>78.95</td>
<td>75.05</td>
<td>70.47</td>
<td>64.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.8297</td>
<td>67.4</td>
<td>77.84</td>
<td>84.32</td>
<td>88.47</td>
<td>90.61</td>
<td>90.88</td>
<td>90.96</td>
<td></td>
</tr>
<tr>
<td>2:1</td>
<td>e = 0</td>
<td>41.19</td>
<td>36.19</td>
<td>32.18</td>
<td>32.99</td>
<td>34.07</td>
<td>34.90</td>
<td>35.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.4</td>
<td>45.99</td>
<td>44.28</td>
<td>42.38</td>
<td>52.18</td>
<td>57.77</td>
<td>60.31</td>
<td>61.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.7297</td>
<td>48.05</td>
<td>47.58</td>
<td>68.02</td>
<td>76.83</td>
<td>80.66</td>
<td>82.20</td>
<td>82.63</td>
<td></td>
</tr>
<tr>
<td>3:1</td>
<td>e = 0</td>
<td>23.23</td>
<td>26.20</td>
<td>26.91</td>
<td>29.23</td>
<td>30.93</td>
<td>32.01</td>
<td>32.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.4</td>
<td>28.81</td>
<td>39.21</td>
<td>39.99</td>
<td>47.43</td>
<td>53.94</td>
<td>57.23</td>
<td>58.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.6457</td>
<td>31.77</td>
<td>39.46</td>
<td>50.94</td>
<td>59.02</td>
<td>69.36</td>
<td>73.59</td>
<td>74.80</td>
<td></td>
</tr>
<tr>
<td>4:1</td>
<td>e = 0</td>
<td>23.14</td>
<td>24.99</td>
<td>25.30</td>
<td>24.86</td>
<td>27.25</td>
<td>29.28</td>
<td>29.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.4</td>
<td>30.96</td>
<td>34.35</td>
<td>38.08</td>
<td>40.93</td>
<td>50.41</td>
<td>54.52</td>
<td>55.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.5707</td>
<td>32.99</td>
<td>36.42</td>
<td>42.87</td>
<td>48.16</td>
<td>61.20</td>
<td>66.01</td>
<td>67.38</td>
<td></td>
</tr>
<tr>
<td>8:1 (i=63.4°)</td>
<td>e = 0</td>
<td>10.95</td>
<td>12.67</td>
<td>14.77</td>
<td>15.54</td>
<td>15.95</td>
<td>18.22</td>
<td>19.88</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.2</td>
<td>13.29</td>
<td>16.37</td>
<td>20.37</td>
<td>23.12</td>
<td>25.43</td>
<td>31.17</td>
<td>32.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.3178</td>
<td>13.88</td>
<td>18.25</td>
<td>23.29</td>
<td>27.58</td>
<td>31.89</td>
<td>38.97</td>
<td>40.66</td>
<td></td>
</tr>
<tr>
<td>8:1 (i=116.6°)</td>
<td>e = 0</td>
<td>11.22</td>
<td>12.75</td>
<td>14.63</td>
<td>15.56</td>
<td>15.77</td>
<td>18.34</td>
<td>19.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.2</td>
<td>12.72</td>
<td>16.37</td>
<td>20.26</td>
<td>22.78</td>
<td>25.81</td>
<td>31.27</td>
<td>32.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e = 0.3202</td>
<td>13.25</td>
<td>18.43</td>
<td>23.52</td>
<td>27.60</td>
<td>31.86</td>
<td>39.24</td>
<td>40.92</td>
<td></td>
</tr>
</tbody>
</table>

4.3 Delta-V Results

Once the optimized orbit was determined, the launch ΔV was computed to complete the figure-of-merit calculation. The results for the ΔV values for each orbit are summarized in Table 4.10. The table shows that as expected, the circular 1:1 case has the highest ΔV. Also, if the eccentricity increases, the ΔV decreases for all repeat types. This is true even though the semimajor axis remains almost constant within each repeat type.
<table>
<thead>
<tr>
<th>Repeat Type</th>
<th>e = 0</th>
<th>e = 0.4</th>
<th>e = max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>13242.25</td>
<td>12810.11</td>
<td>12118.03</td>
</tr>
<tr>
<td>2:1</td>
<td>12826.92</td>
<td>12417.67</td>
<td>11877.16</td>
</tr>
<tr>
<td>3:1</td>
<td>12441.51</td>
<td>12058.80</td>
<td>11666.03</td>
</tr>
<tr>
<td>4:1</td>
<td>12091.95</td>
<td>11732.07</td>
<td>11470.03</td>
</tr>
<tr>
<td>8:1</td>
<td>10955.70</td>
<td>10860.89</td>
<td>10750.15</td>
</tr>
<tr>
<td>8:1 (i=116.6°)</td>
<td>11380.04</td>
<td>11285.72</td>
<td>11173.53</td>
</tr>
</tbody>
</table>

### 4.4 FOM Results

With the optimal orbit in terms of coverage and the ΔV corresponding to that orbit, the figure-of-merit values for each of the cases were calculated. Trends within each class of repeat ground track orbits were examined. The figure-of-merit values were also compared with each other to determine which provided the most cost-effective orbit.

#### 4.4.1 Contour Plots

Contour plots of figure-of-merit vs. the orbit’s eccentricity and ground station latitude were created to examine the behavior of each repeat ground track pattern. Aside from the 1:1 repeat ground track orbits, the general pattern seen is that the figure-of-merit increases as the eccentricity increases. This is not surprising in that eccentric orbits allows for concentrated coverage in the appropriate hemisphere. In addition, as the eccentricity increases, the launch ΔV decreases. A typical figure-of-merit plot is shown in Figure 4.14, a contour plot of the 3:1 repeat ground track orbit with a minimum elevation angle of 10°. The best figure-of-merit occurs around 90° N latitude at the maximum eccentricity. As the eccentricity increases, the figure-of-merit increases for all latitudes. The other trend that can be noted from Figure 4.14 is that for a given eccentricity, the figure-of-merit increases as the latitude increases.

As the minimum elevation angle increases from 10° to 30°, these patterns remain consistent, but the figure-of-merit values decrease and the region with the best figure-of-
merit is smaller. This can be seen by comparing Figure 4.14 and Figure 4.15, where Figure 4.15 is the 3:1 case with a minimum elevation angle of 30°.

The 1:1 repeat ground track case is the exception to the previously noted trends. Figure 4.16 shows the figure-of-merit contour of the 1:1 case with a 10° minimum elevation angle. This plot indicates that there are two regions of high figure-of-merit values. For the lower latitudes, the figure-of-merit is the best between eccentricities of 0.1 and 0.5, but for the mid to upper latitudes, the figure-of-merit is the best at the maximum eccentricity.

When the minimum elevation angle is increased to 30° in the 1:1 case, the figure-of-merit behavior at the lower latitudes changes significantly. As seen in Figure 4.17, for ground stations located below 10° N latitude, the optimal orbit has a low eccentricity, but for latitudes between 10° and 30°, the optimal orbit has an eccentricity around 0.4. If the region of interest is in the mid to upper latitudes, then the optimal orbit is still at the maximum eccentricity. Additional figure-of-merit contour plots can be found in Appendix C.
Figure of Merit vs. Eccentricity and Ground Station Latitude (3:1)
(10° Minimum Elevation Angle)

Figure 4.14: Figure-of-Merit Contour Plot

Figure of Merit vs. Eccentricity and Ground Station Latitude (3:1)
(30° Minimum Elevation Angle)

Figure 4.15: Figure-of-Merit Contour Plot
Figure 4.16: Figure-of-Merit Contour Plot

Figure 4.17: Figure-of-Merit Contour Plot
The data used to create each figure-of-merit plot was limited since calculating the figure-of-merit was a slow process. Each figure-of-merit contour plot was created using 21 total data points. The concern with the contour plots was that the limited amount of data might not accurately reflect the trends in the figure-of-merit. Shown below are two contour plots for the 1:1 repeat ground track case. Figure 4.18 shows the 1:1 case where 21 data points were used. These data points are shown by the stars (*). Additional data for this case was collected to fill in the region between the 0 eccentricity and 0.4 eccentricity points as well as the region between the 0.4 and the maximum eccentricity points. The eccentricities chosen were 0.2 and 0.6. A total of 35 data points were used to create the plot in Figure 4.19 to determine if the contour plots changed significantly with additional data. The new contour plot that was generated was slightly different than the initial one, however the trends remained consistent. The major difference between the two are the intensities of the maximum figure-of-merit values. Figure 4.18 shows that the overall maximum figure-of-merit occurs in the upper right corner (max eccentricity and 90° latitude). Figure 4.19 shows that the overall maximum figure-of-merit occurs around the 0.2 to 0.4 eccentricity range near the equator, but this is only slightly larger than the maximum figure-of-merit in Figure 4.18. This presents a problem for the 1:1 repeat ground track orbit figure-of-merit contour plots and warrants further investigation. To gain a better understanding of the figure-of-merit behavior for 1:1 repeat ground track orbits, more data points must be collected. For the other repeat ground track patterns, the trends in the figure-of-merit remain the same where the maximum figure-of-merit occurs at the maximum eccentricity and maximum latitude.
Figure 4.18: Figure-of-Merit Contour Plot (21 Data Points)

Figure 4.19: Figure-of-Merit Contour Plot (35 Data Points)
4.4.2 Repeat Ground Track Comparisons

Ultimately, the different repeat ground track orbits were compared with each other based on their eccentricity to determine which was the most cost-efficient in terms of the figure-of-merit.

In each of the figures below, the 1:1 repeat ground track orbit has a higher figure-of-merit than any of the other repeat ground track orbits examined in this paper. These figures show plots of figure-of-merit values vs. ground station latitudes for each repeat pattern. The minimum elevation angle for these plots is 10°. These three figures correspond to the 0, 0.4 and maximum eccentricity cases. For the 8:1 case, the maximum eccentricity is approximately 0.32, so the 0.2 eccentricity case was plotted in Figure 4.21.

Not shown in these figures is the calculated figure-of-merit for a geostationary satellite. The figure-of-merit for geostationary satellites is a step function where the figure-of-merit for a point is approximately 66.9 for latitudes ranging from 71.4° S to 71.4° N, and zero for latitudes north and south of 71.4° in the 10° minimum elevation case. In the 30° case minimum elevation angle case, the figure-of-merit is 66.9, but the latitude region for coverage is reduced to areas between 52.5° S and 52.5° N. When the ground station is located north or south of this region, the figure-of-merit for the GEO case becomes zero since no coverage is provided.

In both Figure 4.20 and Figure 4.21, the figure-of-merit values tend to increase as the ground station latitude increases. The exception to this trend is the 1:1 repeat ground track orbit. Another trend that can be seen in these plots is that as the ground station locations increase in latitude, the figure-of-merit values for the different ground track patterns move towards each other. Although the 1:1 repeat ground track is shown to be the best orbit, the 2, 3, and 4 to 1 repeat patterns have figure-of-merit values that are similar to each other, especially for the 0 and 0.4 eccentricity cases.

In most of the cases, the figure-of-merit for the 2:1 repeat ground track orbit was better than the 3:1 case, however, a close examination of Figure 4.22 reveals that at the ground station with a latitude of 15° N, the 3:1 case is in fact better. Even though this is a single point, it illustrates that selecting a 3:1 repeat ground track orbit could be a better choice, depending on the application.
When the minimum elevation angle was increased to 30°, there were slight changes in the plots. All of the figure-of-merit values decreased, but the general trends remained the same as in the cases with a minimum elevation angle of 10°. The plots for the 30° cutoff elevation cases can be found in Appendix D.

Uncertainties in the ΔV can effect the trends of the figure-of-merit, especially in the zero and 0.4 eccentricity cases. Looking at Figure 4.20 at 90° latitude, if there is a 10% error in the ΔV, the 4:1 case can have a larger figure-of-merit than the 1:1 case. However, even though errors in the ΔV can affect the figure-of-merit, the general trends remain consistent. When the figure-of-merit for two repeat ground track patterns are similar, further analysis must be done in selecting the appropriate orbit based on the spacecraft’s mission.

![Figure of Merit vs. Ground Station Latitude](image)

Figure 4.20: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0)
Figure 4.21: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0.4)

Figure 4.22: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e = max)
5 Conclusions

This thesis developed an orbital figure-of-merit for a single satellite by examining the performance of a satellite orbit versus its cost. The orbital figure-of-merit was used as an analysis tool to compare several repeat ground track orbits. Performance was measured by the coverage time per day provided to the ground and the cost by the $\Delta V$ to attain the mission orbit. The coverage was shown to be dependent on the orbital parameters, ground station location, and minimum cutoff elevation.

In some cases, the coverage for repeat ground track orbits for a single point was shown to be complex. In general, ground stations located at latitudes below 30° exhibited this complex coverage behavior with multiple coverage maxima. As the ground station latitude increased, the coverage behavior became simplified with most cases having only one maximum coverage time per day.

Through the coverage optimization and analysis, this study reproduced characteristics of several known orbits. These characteristics include the “leaning” orbit as seen in the COBRA orbit and the ascending node and perigee placement in the Sirius orbit. The Molniya and Ellipso-Borealis orbits were also reproduced. The optimization was performed by varying the longitude of ascending node and argument of perigee while the semimajor axis, inclination, and eccentricity were fixed. Based on this optimization, some of the known orbits appeared to be optimal in terms of coverage for their repeat pattern. In the Sirius case, the orbit appeared to be optimal for its eccentricity over a wide range of specified cutoff elevations. It was shown that a 1:1 repeat ground track orbit with a maximum eccentricity provided more coverage than the Sirius orbit where a 10° minimum elevation angle was specified.

Repeat ground track orbits with high eccentricities tended to produce orbits that provide more coverage than orbits with low eccentricities. Ground stations at higher latitudes were shown to have favorable coverage when compared to lower latitude ground station locations. In addition, with ground stations in the northern hemisphere, orbits with apogees in the northern hemisphere had better coverage characteristics. Repeat ground track
orbits with lower revolutions per day had higher coverage times per day than those with more revolutions per day.

The analysis of the launch cost in ΔV resulted in trends that were expected. As the orbit's semimajor axis increases, the cost in ΔV to attain that orbit increases. In addition, for a given semimajor axis, an increase in the orbit's eccentricity results in a lower ΔV cost.

The figure-of-merit for geostationary satellites is 66.9 and is larger than all of the critically inclined repeat ground track orbits examined for latitudes less than 71.4° when considering a 10° minimum elevation angle. The drawback of the geostationary case is that above this latitude, the figure-of-merit becomes zero since no coverage is provided. Similarly, in the 30° cutoff elevation case, the figure-of-merit for the GEO case is the largest (66.9) for all orbits examined in this thesis if ground stations latitudes are below 52.5°. No coverage is provided by the GEO satellite above this latitude. While geostationary satellites are good for low to mid latitudes, critically inclined repeat ground track orbits can provide coverage in the northern hemisphere up to the North Pole.

The initial expectation in this study was that 1:1 repeat ground track orbits would provide the most coverage, but the cost in ΔV would outweigh its advantages. The findings of this thesis indicated that the 1:1 repeat ground track orbit provided the most cost-effective solution to coverage over a given ground station. Though the belief that the 1:1 repeat pattern would not have the highest figure-of-merit was incorrect, it is important to note that the 2:1, 3:1, and 4:1 cases are roughly equivalent in figure-of-merit. In addition, even though the 2:1 and 3:1 repeat patterns do not provide as much coverage to one point on the Earth as the 1:1 case, when these satellites are not in view of the specified ground station, there are other points on the Earth receiving equal coverage. For example, the 3:1 repeat pattern provides the same percentage of coverage over three separate regions in the world, while the 1:1 pattern only provides coverage to one region.

A possible approach to consider global coverage aspects with the figure-of-merit is to sum the figure-of-merit values for a specified city in each separate region covered by the satellite. If coverage is desired for three cities with mid to high latitudes such as Tokyo (35° N), Oslo (59° N), and Vancouver (49° N), the combined figure-of-merit for a highly eccentric 3:1 repeat ground track orbit is approximately 129.84 in the 10° cutoff elevation angle case and 86.91 for a 30° cutoff elevation. The figure-of-merit in the 10° minimum
elevation angle case is almost double the figure-of-merit for the geostationary case. In addition, a single geostationary satellite can provide coverage to at most, two of the selected cities. As the minimum elevation angle increases to 30°, high latitude cities, in this example Oslo at 59° N, cannot be covered by a geostationary satellite and thus the figure-of-merit becomes 0. When considering coverage three widely spaced points on the globe, the figure-of-merit shows that a single satellite with a 3:1 repeat ground track orbit appears to be more cost-effective and perhaps better suited for coverage than the geostationary case.

The use of elliptical orbits allows satellite system designers to concentrate coverage over regions of interest. The eccentricity of the orbit also significantly affects the figure-of-merit. In general, better figure-of-merit values result when the orbit approaches the maximum eccentricity, especially for ground stations in higher latitudes. This is due to increased coverage time and decreased ΔV cost.

Aside from the 1:1 repeat ground track orbits, when the minimum elevation angle is increased, the trends in the figure-of-merit remain consistent. The higher eccentricity orbits have larger values for the figure-of-merit, especially at the higher latitudes. The effect of the minimum elevation angle does affect the figure-of-merit by reducing the coverage time per day of the orbit. This lowers the figure-of-merit when compared with a small cutoff elevation. When comparing the 10° minimum elevation angle cases to the 30° cases, the 1:1 repeat ground track orbit remains the most cost-effective repeat pattern.

The proposed figure-of-merit has interesting implications in orbit design. Through coverage optimization, the figure-of-merit analysis can provide valuable insight into potential coverage issues for a single satellite. As shown in Chapter 4, for low latitude ground stations, the orbit might be optimized for coverage, but slight changes in those optimal orbital elements can significantly change the coverage. Many considerations in addition to the coverage provided and the cost in ΔV must be taken into account in designing orbits. While the figure-of-merit developed and studied in this paper cannot be used solely to determine the best orbit for satellite systems, it provides a useful tool in the orbit design process.
6 Future Work

There are several areas for additional work and improvement on the figure-of-merit. These include changes in the calculation techniques in the figure-of-merit terms as well as an expansion to include a broader range of satellite applications.

A more thorough investigation into the correlation between the launch cost and ΔV is desirable. This thesis demonstrated the basis for using the ΔV as a cost measurement, but supporting data from existing launch vehicles is needed.

In terms of the coverage calculation, alternative optimization approaches could also be explored. In particular, a genetic algorithm or simulated annealing could be considered since these methods can be used to directly determine the global maximum. A method that allows for the computation of the global maximum would allow for a figure-of-merit calculation with significantly less human intervention.

High eccentricity orbits tend to evolve with time due to perturbations such as effects of the Sun and Moon. By using an orbit propagator with multiple perturbations, an accurate determination of the coverage can be accomplished. Specifically, the coverage calculation function can be coupled with the Draper Semianalytical Satellite Theory (DSST) orbit propagator.

The work presented thus far provides an analysis of the figure-of-merit as it relates to a single satellite. While the 1:1 repeat ground track orbit was shown to have the best figure-of-merit for a single satellite, it is possible that a constellation of different repeat ground track orbits could yield better results. One desired goal of the figure-of-merit is to use it as an analysis tool for comparing orbits or constellations. The figure-of-merit would have to be modified to account for multiple satellites in both the coverage term as well as the ΔV term.

Another area of improvement for the figure-of-merit is in the ΔV calculation. While the current method employed provides a good approximation for the actual ΔV, a more accurate value could change the figure-of-merit. An investigation into using a trajectory design software such as Boeing Autometric’s Ascent would be valuable. This software package optimizes the trajectory and a minimum ΔV could be quickly determined. Unsuccessful attempts were made to obtain Ascent to compare its ΔV calculation with the
analytic approach taken in this thesis. Another option in calculating the ΔV is to further explore the analytic method presented by Boltz [56]. Both of these techniques allow for an accurate computation of the ΔV, accounting for a wide variety of variables associated with the satellite launch.

The figure-of-merit is not limited to communications satellite missions. By modifying the definition of coverage, the figure-of-merit can be tailored to many different satellite applications. For example, the figure-of-merit could be used to analyze an astronomical science mission. The Spectroscopy and Photometry of the Intergalactic Medium's Diffuse Radiation (SPIDR) spacecraft [74] is designed to spend a specified portion of its lifetime above 6 Earth radii to make observations of deep space. Repeat ground tracks are to be used for the SPIDR mission, where the repeat periods are not limited to one day. If the coverage in the figure-of-merit is defined as the average time per day that the spacecraft is above the required altitude, the cost-effectiveness of different orbits can be compared. While the coverage in this thesis was shown to be dependent on the satellite’s six orbital elements, the elements that affect the coverage in the SPIDR application are the semimajor axis and eccentricity. However, if other constraints are placed on the spacecraft’s orbit, the remaining orbital elements (ω, Ω, and M) could play a role in the coverage. A possible constraint would be if the spacecraft must be in the Earth’s shadow while above the required altitude.

The Pegasus launch vehicle was selected to place SPIDR into a parking orbit, while a solid rocket motor would boost it into its mission orbit. Thus, specifics in calculating the ΔV can be identified such as the launch location and launch vehicle mass parameters. In this case, Boltz’s method or a trajectory design program would be an ideal choice in calculating the ΔV.

The SPIDR spacecraft mission is one example of applying the figure-of-merit to an astronomical missions. By modifying the coverage definition in the figure-of-merit, Earth observation, Earth science and satellite-based navigation missions are other potential applications where the figure-of-merit could be employed in the orbit design process.

The figure-of-merit need not be limited to Earth satellites. A network of communication satellites on Mars could be designed and analyzed using the figure-of-merit. Noreen, et al. [75] designed 42 Martian repeat ground track orbits. The objective of these
orbits is to provide a communications platform to relay data from a Martian rover or lander back to Earth. It would be possible to use the figure-of-merit to rank these orbits by measuring coverage by the average time per day the satellite is in view of the rover/lander of interest. A more complicated form of coverage could also be devised where the average coverage time per day to the Martian surface is coupled with the average time per day the satellite can to transmit data to the Earth. Regardless of the coverage definition, the basic idea behind the figure-of-merit of using performance versus cost can be applied to this application. The ΔV for this application would likely be defined as the total ΔV from Earth launch to Martian orbit insertion.

With these proposed changes, the figure-of-merit could be a robust analysis tool to compare different satellite orbits and constellations, where the analysis is not constrained to communications missions.
References


[29] Kashitani, Tatsuki, personal communications, 29 April 03. email: tatsuki@alum.mit.edu


Appendix A: Figure-of-Merit Analysis Code

The Matlab code used to calculate the figure-of-merit is described below. A listing of the code can be found in the accompanying CD-ROM. The files are grouped into four sections based on their function: Coverage Calculation Code, Delta-V Calculation Code, Figure-of-Merit Calculation Code, and Miscellaneous Supporting Functions. Figure A.1 illustrates how the functions work together to ultimately calculate the figure-of-merit for a specified orbit.

The coverage contour plot data for the orbit is collected using the coverage calculation functions. The Matlab script GetContourDataRPT*.m defines ground station latitude, minimum elevation angle and the repeat ground track orbit for a *:1 repeat type. By changing the variable N in this script, the repeat type can be varied. In addition, the file name where the data is stored must also be changed to represent the desired repeat ground track orbit.

With the information provided by GetContourDataRPT*.m, coverage time data is collected with the Contour.m function to ultimately create a coverage contour plot. The argument of perigee and longitude of ascending node are varied and the coverage time is calculated for each combination of $\omega$ and $L$. The coverage time is determined from the CoverageTime.m function. CoverageTime.m uses the sub-functions COEUpdate.m, Calc-DeltaElAngle.m, and COEtoPosition.m.

Once the contour data is collected for all repeat ground track orbits desired, it is saved into files for future use by the figure-of-merit calculation functions. The MaxCoverage-RPT*.m script is used to determine the exact argument of perigee and longitude of ascending node that yields the maximum coverage over the specified ground station for a *:1 repeat ground track orbit. By changing the variable RPT to the desired value of N, the repeat type can be varied. In this script, the repeat ground track orbit, minimum elevation angle, and ground station latitude are specified.

The LoadContourData.m function is used to determine the exact maximum coverage point for the specified orbit / ground station combination. The MaximumCoverage.m function takes the saved coverage contour plot data to determine the approximate location of
the global maximum coverage point. This approximate location is used as a starting point for the coverage maximization function “fmincon.” The “fmincon” function is called by the FOMCoverageCalc.m function and the orbital elements for the maximum coverage point are saved. The orbital elements, ground station location, minimum elevation angle, and maximum coverage time are saved into a file based on the repeat ground track type for use in the FOMCompile.m script.

The functions GetContourDataeORPT*.m, ContoureO.m, LoadContourDataeO.m, MaximumCoverageeO.m, and FOMCoverageCalceO.m were written to determine the contour plot data and maximum coverage times for circular repeat ground track orbits.

The Matlab FOMCompile.m script is the final file used in the figure-of-merit analysis. The figure-of-merit for all of the repeat ground track orbits examined is determined using this script. First, the ΔV is calculated using the DeltaVCalc.m function from the saved data files. With the maximum coverage time and the calculated ΔV, the figure-of-merit is calculated. Finally, the specified orbit data, maximum coverage time, ΔV, and figure-of-merit are saved into a Matlab file for analysis.
Figure A.1: Matlab Code Flow Chart
Coverage Calculation Code

GetContourDataeORPT1.m:
Calculates contour plot data for all 1:1 circular repeat ground track orbits examined.

GetContourDataRPT1.m:
Calculates contour plot data for all 1:1 repeat ground track orbits examined.

Contour.m:
Calculates the coverage times for a given LAN and argument of perigee to generate contour plot data.

Contoure0.m:
Calculates the coverage times for a given LAN and argument of perigee to generate contour plot data for specified circular orbits.

CoverageTime.m:
Calculates coverage time in seconds that a satellite is above the cutoff elevation over the specified time period.

CalcDeltaElAngle.m:
Calculates the difference in the elevation angle and the minimum cutoff elevation angle.

COEtoPosition.m:
Calculates ECI position coordinates given Kepler elements.

COEUpdate.m:
Propagates forward classical orbital elements according to the \( J_2 \) secular perturbation theory.

Delta-V Calculation Code

DeltaVCalc.m:
Delta-V is calculated as a Hohmann transfer from a circular orbit from the surface of the Earth to the final orbit.

Figure-of-Merit Calculation Code

FOMCompile.m:
Loads data from maximum coverage orbits to calculate the figure-of-merit.

MaxCoverageRPT*.m:
Calculates the exact maximum coverage time and the orbital elements associated with that optimal orbit.

LoadContourData.m:
Calculates the optimized orbit and coverage time associated with that orbit.

LoadContourDataeO.m:
Calculates the optimized circular orbit and coverage time associated with that orbit.

MaximumCoverage.m:
Determines approximate maximum coverage point from the contour plot data.

MaximumCoverageeO.m:
Determines approximate maximum coverage point from the contour plot data for circular orbits.
FOMCoverageCalc.m:
Returns the exact maximum coverage time and associated orbital elements.

FOMCoverageCalceO.m:
Returns the exact maximum coverage time and associated orbital elements for circular orbits.

**Miscellaneous Supporting Functions**

AscAndDecl.m:
Returns right ascension and declination given geodetic latitude and longitude in degrees, height in meters above the earth ellipsoid, and Julian date.

AscGreenwich.m:
Returns the right ascension of greenwich in radians for the given Julian date.

MeanAnomaly.m:
Returns mean anomaly, given true anomaly and eccentricity.

Julian.m:
Returns the Julian date equivalent of a Gregorian calendar date.

EpochMeanAnomalyRate.m:
Computes orbit epoch mean anomaly rate (J2 secular theory).

MeanMotion.m:
Returns mean motion given semimajor axis.

NodalPeriod.m:
Computes orbit nodal period.

NodeRate.m:
Computes orbit ascending node rate (J2 secular theory).

PerigeeRate.m:
Computes orbit argument of perigee rate (J2 secular theory).

RepeatGroundTrack.m:
Computes semimajor axis for prescribed repeat ground track.

RepeatSemimajorAxis.m:
Computes semimajor axis for prescribed repeat ground track.

TrueAnomaly.m:
Returns true anomaly, given mean anomaly and eccentricity.
Appendix B: Coverage, Delta-V and Figure-of-Merit Results

The tables listed show results from the figure-of-merit analysis. The tables are grouped by the six specified repeat ground track patterns and minimum elevation angles. The data is further broken down within each table based on the orbit’s eccentricity, and ground station latitude. All of the ground stations are located on the Prime Meridian (0° longitude). For each orbit / ground station combination the optimized longitude of ascending node (LAN) and argument of perigee (ω) are given along with the coverage time per day in seconds. The percent coverage per day is also listed. The ΔV to attain the orbit is given along with the calculated figure-of-merit (FOM) for the orbit.
Table B.1: 1:1 Repeat Ground Track Orbit Results (Minimum Elevation Angle: $10^\circ$, $i=63.4^\circ$)

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Table B.2: 1:1 Repeat Ground Track Orbit Results (Minimum Elevation Angle: 30°, i=63.4°)

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Table B.7: 4:1 Repeat Ground Track Orbit Results (Minimum Elevation Angle: 10°, i=63.4°)

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Table B.8: 4:1 Repeat Ground Track Orbit Results (Minimum Elevation Angle: 30°, i=63.4°)

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Table B.9: 8:1 Repeat Ground Track Orbit Results (Minimum Elevation Angle: 10°, i=63.4°)

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Appendix C: Figure-of-Merit Contour Plots

Figure-of-merit contour plots showing the figure-of-merit as a function of the orbit’s eccentricity and the ground station latitude are given below. There are twelve plots total, one for each of the repeat patterns examined in this thesis. For each repeat pattern, there is a plot for the 10° minimum elevation angle case and one for the 30° case.
Figure C.1: Figure-of-Merit Contour Plot- 1:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 10°
Figure C.2: Figure-of-Merit Contour Plot- 1:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 30°
Figure C.3: Figure-of-Merit Contour Plot - 2:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 10°
Figure C.4: Figure-of-Merit Contour Plot- 2:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 30°
Figure C.5: Figure-of-Merit Contour Plot- 3:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 10°
Figure C.6: Figure-of-Merit Contour Plot- 3:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 30°
Figure C.7: Figure-of-Merit Contour Plot- 4:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 10°
Figure C.8: Figure-of-Merit Contour Plot- 4:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 30°
Figure C.9: Figure-of-Merit Contour Plot- 8:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 10°
Figure C.10: Figure-of-Merit Contour Plot - 8:1 Repeat Ground Track Orbit, Minimum Elevation Angle: 30°
Figure C.11: Figure-of-Merit Contour Plot- 8:1 Repeat Ground Track Orbit, $i=116.6^\circ$, Minimum Elevation Angle: $10^\circ$
Figure C.12: Figure-of-Merit Contour Plot- 8:1 Repeat Ground Track Orbit, i=116.6°, Minimum Elevation Angle: 30°
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Appendix D: Repeat Ground Track Pattern Comparison

The figure-of-merit values for repeat ground track patterns examined in this thesis are compared with each other based on the orbit eccentricity and minimum elevation angle. Plots of the figure-of-merit vs. ground station latitude are given for the three orbit eccentricities examined. In the 0.4 eccentricity figure-of-merit plots, the 8:1 repeat ground track patterns have eccentricity values of 0.2.
Figure D.1: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0, Min. Elevation Angle = 10°)
Figure D.2: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0, Min. Elevation Angle = 30°)
Figure D.3: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0.4, Min. Elevation Angle = 10°)
Figure D.4: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=0.4, Min. Elevation Angle = 30°)
Figure D.5: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=max, Min. Elevation Angle = 10°)
Figure D.6: Figure-of-Merit Comparison for Varying Repeat Ground Track Patterns (e=\max, Min. Elevation Angle = 30°)
Appendix E: Coverage to a Region

The figure-of-merit examined in this thesis defined the coverage as the total amount of time a satellite was in view of a ground station over one day. This appendix briefly looks at using coverage provided to a region rather than a single point on the ground. A method to calculate this coverage is developed in Matlab and is implemented in the figure-of-merit analysis. The six daily repeat ground track types examined in this thesis are used in the regional coverage analysis. The region selected for this particular case is the continental United States.

E.1 Methodology

An accurate, computationally efficient method of calculating the coverage to a region is desirable in the figure-of-merit analysis. The technique developed to accomplish this is to determine if the region falls within the satellite’s footprint. A satellite’s footprint is dependent upon its altitude as well as any minimum elevation angle constraint. First, a collection of points on the boundary of the region is taken. With a specified orbit, the orbital elements are propagated forward over one day using the $J_2$ secular perturbation theory. At each time step of the propagation:

1. The satellite’s position vector is calculated from its orbital elements
2. The central Earth angle is calculated based on the satellite’s position vector.
3. The latitude and longitude of the subsatellite point are calculated.
4. The angular distance from the points on the region to the subsatellite point is calculated
5. If the angular distance for all the selected points in the is less than the central Earth angle, the total coverage time is increased by time step.

The central Earth angle and angular distance are shown in Figure E.1. This method assumes that the Earth is spherical when calculating the central Earth angle and the angular distance. While this method is not nearly as precise as the function developed for coverage to a single ground station, the coverage times were accurate to approximately 30 - 60 seconds when tested. The method developed for coverage to a region is similar to that used by Williams, et al. in their revisit time optimization study [30].
The region selected for this case is the continental U.S with 31 points selected on the boundary as seen in Figure E. 2. A larger number of points will yield a more reliable coverage time, however the computational time increases significantly with additional points in the region.
Figure E. 2: Selected Points on Boundary of Region (Continental U.S.)

E.2 Results

E.2.1 Coverage Plots

Coverage contour plots were created to determine the approximate location of the
global maximum coverage point for each orbit. The orbits examined were the same six daily
repeat ground track patterns examined in this thesis, with three eccentricity values: 0, 0.4 and
the maximum eccentricity for an 800 km perigee altitude. The minimum elevation angle
constraints of 10° and 30° were used.

To reduce the computation time for each contour plot, the range for argument of
perigee values in the contour plots was 0° to -180°. This was done since no maximum
coverage points for single ground station cases had an argument of perigee between 0° and
180°. The purpose of the reduction in the perigee range was to reduce the computation time.
The resolution in the contour plots is 5° for the argument of perigee and between 1° and 5°,
based on the orbit, for longitude of ascending node. In addition, the step size ranged from 15
seconds to 60 seconds, based on the repeat ground track type. The data for each contour plot was generated in approximately 3 hours using a 2.39 GHz PC running Windows XP.

The twenty four coverage contour plots created are shown below. For the most part, only one global maximum was present in the contour plots, however in some cases, multiple maxima exist. The 8:1 repeat ground track orbits provide little coverage over the entire continental U.S., especially in the 30° minimum elevation angle case. The best coverage is exhibited by the 1:1 repeat ground track orbits.
Coverage Percentage vs. Optimization Variables
1:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.4$
Min. Elevation Angle = $10^\circ$

Figure E.3: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
2:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.4$
Min. Elevation Angle = $10^\circ$

Figure E.4: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit | i = 63.4° | e = 0.4
Min. Elevation Angle = 10°

Figure E.5: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
4:1 Repeat Ground Track Orbit | i = 63.4° | e = 0.4
Min. Elevation Angle = 10°

Figure E.6: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit $|i| = 63.4^\circ$, $e = 0.4$
Min. Elevation Angle = $10^\circ$

Figure E.7: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit $|i| = 116.6^\circ$, $e = 0.4$
Min. Elevation Angle = $10^\circ$

Figure E.8: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
1:1 Repeat Ground Track Orbit $| i = 63.4^\circ | e = 0.8297$
Min. Elevation Angle = $10^\circ$

Figure E.9: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
2:1 Repeat Ground Track Orbit $| i = 63.4^\circ | e = 0.7297$
Min. Elevation Angle = $10^\circ$

Figure E.10: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit $|i| = 63.4^\circ$ | $e = 0.6457$
Min. Elevation Angle = $10^\circ$

Figure E.11: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
4:1 Repeat Ground Track Orbit $|i| = 63.4^\circ$ | $e = 0.5707$
Min. Elevation Angle = $10^\circ$

Figure E.12: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit $|i = 63.4^\circ| e = 0.3178$
Min. Elevation Angle = $10^\circ$

Figure E.13: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit $|i = 116.6^\circ| e = 0.3202$
Min. Elevation Angle = $10^\circ$

Figure E.14: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
1:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.4$
Min. Elevation Angle = $30^\circ$

Figure E.15: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
2:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.4$
Min. Elevation Angle = $30^\circ$

Figure E.16: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit $| i = 63.4^\circ | e = 0.4$
Min. Elevation Angle = $30^\circ$

Figure E.17: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
4:1 Repeat Ground Track Orbit $| i = 63.4^\circ | e = 0.4$
Min. Elevation Angle = $30^\circ$

Figure E.18: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.4$
Min. Elevation Angle $= 30^\circ$

Figure E.19: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit | $i = 116.6^\circ$ | $e = 0.4$
Min. Elevation Angle $= 30^\circ$

Figure E.20: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables

1:1 Repeat Ground Track Orbit $i = 63.4^\circ$, $e = 0.8297$
Min. Elevation Angle = $30^\circ$

Figure E.21: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables

2:1 Repeat Ground Track Orbit $i = 63.4^\circ$, $e = 0.7297$
Min. Elevation Angle = $30^\circ$

Figure E.22: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
3:1 Repeat Ground Track Orbit \( i = 63.4^\circ \) \( e = 0.6457 \)
Min. Elevation Angle = 30°

Figure E.23: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
4:1 Repeat Ground Track Orbit \( i = 63.4^\circ \) \( e = 0.5707 \)
Min. Elevation Angle = 30°

Figure E.24: Regional Coverage Contour Plot
Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit | $i = 63.4^\circ$ | $e = 0.3178$
Min. Elevation Angle = $30^\circ$

Figure E.25: Regional Coverage Contour Plot

Coverage Percentage vs. Optimization Variables
8:1 Repeat Ground Track Orbit | $i = 116.6^\circ$ | $e = 0.3202$
Min. Elevation Angle = $30^\circ$

Figure E.26: Regional Coverage Contour Plot
E.2.2 Figure-of-Merit Results

Using the generated contour plots, the approximate location of the global maximum coverage point was determined. Using this information, the exact global maximum was located using the Matlab “fmincon” function. The step size for the coverage function was reduced to 5 seconds to yield more accurate coverage times. The run time for the optimization was approximately 30 minutes for each case when run on a 2.39 GHz PC running Windows XP. The optimized orbit’s longitude of ascending node, argument of perigee, coverage time, ΔV, and figure-of-merit are listed below in Table E.1 for the 10° cutoff elevation case and Table E.2 for the 30° cutoff elevation case. All of the arguments of perigee are near -90°, however, none are exactly -90°. Thus, the optimized orbits have some degree of lean in them. In addition, since the coverage times for the 8:1 were very low, the resulting figure-of-merit values were also very low.

Table E.1: Coverage, Delta-V and Figure-of-Merit Results (Min. Elevation Angle 10°)

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<th>Ω (°)</th>
<th>Coverage (s)</th>
<th>Coverage (%)</th>
<th>Delta-V (m/s)</th>
<th>FOM</th>
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Table E.2: Coverage, Delta-V and Figure-of-Merit Results (Min. Elevation Angle $30^\circ$)

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<th>$\omega$ ($^\circ$)</th>
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<th>Coverage (%)</th>
<th>Delta-V (m/s)</th>
<th>FOM</th>
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<td>3.36</td>
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</table>

**E.2.3 Repeat Ground Track Comparison**

The orbits examined were compared against each other based on their repeat type and eccentricity. Figure E.27 plots the $10^\circ$ minimum elevation angle cases while Figure E.28 plots the $30^\circ$ cases. The 8:1 cases with inclinations of 116.6° are plotted slightly to the right of the 8:1 cases with inclinations of 63.4°. The trend in both cutoff elevation cases is that the 1:1 repeat ground track orbits have higher figure-of-merit values than the other repeat ground track types examined. As the number of orbits per day (N) increases, the figure-of-merit decreases. High eccentricity orbits for a given repeat pattern generally have higher figure-of-merit values. These trends are consistent with those noted when coverage was limited to a single point on the ground. An interesting observation in the $30^\circ$ minimum elevation angle case from Figure E.28 is that with the 1:1 repeat ground track pattern, the 0.4 eccentricity orbit has a higher figure-of-merit than the maximum eccentricity orbit. A potential future figure-of-merit analysis would be to run the optimization for an orbit with the Sirius eccentricity ($e = 0.2635$) and compare that with the other eccentricities examined.
Figure E.27: Repeat Ground Track Orbit Comparison (10° Minimum Elevation Angle)

Figure E.28: Repeat Ground Track Orbit Comparison (30° Minimum Elevation Angle)
E.3 Matlab Code

The code for the figure-of-merit analysis using coverage provided to a region is listed below. The listed code is divided into three sections: Figure-of-Merit Calculation Code, Contour Plot Data Generation Code, and Regional Coverage Calculation Code.

The figure-of-merit calculation code optimizes the each repeat ground track case for coverage to the region and then computes the figure-of-merit values. This data is compiled and saved into a Matlab file for analysis. Two primary Matlab function are used in this final analysis.

The contour plot data generation code collects the data for each repeat ground track orbit specified. There are two major functions which are used to collect this data: GetContourDataRegion.m and ContourRegionCentralAngle.m. The first file is used to specify the repeat ground track type, eccentricity, and minimum elevation angle. It also calls the second function, ContourRegionCentralAngle.m, which generates the data for each case. There are specialized functions, GetContourDataRegionc0.m and ContourRegionCentralAnglec0.m, which are used for circular orbits.

The regional coverage calculation code is composed of one main function and four sub-functions. CoverageTimeRegionCentralAngle.m is the primary function that propagates the orbit forward in time and determines when the entire region is in view of the ground. The function USPoints.m is used to define all of the points in the region. In this case, 31 points are used to define the boundary of the continental U.S. The function IJKtoLatLong.m is used to determine the location of the subsatellite point at each time step. This algorithm is taken from Vallado [51, pp.204-205]. The CalcCentralEAngle.m computes the central Earth angle based on the satellite's position vector. CalcDistance.m is used to calculate the angle between a selected ground point and the subsatellite point. The function CoverageTimeRegionCentralAnglec0.m is used for circular orbits.
Figure-of-Merit Calculation

RegionFOM.m
Loads data from maximum coverage orbits to calculate the figure-of-merit
RegionMaximizationScript.m
Calculates the maximum coverage time for all repeat ground track orbits
examined (e = 0.4 and maximum eccentricity orbits)
RegionMaximizationScripte0.m
Calculates the maximum coverage time for all repeat ground track orbits
examined (circular orbits)
RegionMaximization.m
Calculates the exact maximum coverage time and the orbital elements
associated with that optimal orbit.
RegionMaximizatione0.m
Calculates the exact maximum coverage time and the orbital elements
associated with that optimal orbit. (circular orbits only)
StartGuessPoint.m
Determines approximate maximum coverage point from the contour plot data.

Contour Plot Data Generation Code

GetContourDataRegion.m
Coverage contour data is collected for all e=0.4 and maximum eccentricity
repeat ground track orbits examined.
GetContourDataRegione0.m
Coverage contour data is collected for all circular repeat ground track
orbits examined.
ContourRegionCentralAngle.m
Collects coverage contour data for the specified repeat ground track orbit
ContourRegionCentralAnglee0.m
Collects coverage contour data for the specified circular repeat ground track
orbit

Regional Coverage Calculation Code

CoverageTimeRegionCentralAngle.m
Calculates the coverage time over the designated region for the specified orbit.
CoverageTimeRegionCentralAnglee0.m
Calculates the coverage time over the designated region for the specified orbit.
(circular orbits)
IJKtoLatLong.m
Calculates the latitude and longitude of the satellite's subsatellite point.
CalcCentralEAngle.m
Calculates the central Earth angle based on the satellite's position.
CalcDistance.m
Calculates the angular distance between the subsatellite point and a point on
the region.

USPoints.m

31 latitude and longitude points are listed which lie on the border of the continental United States.
Appendix F: Accompanying Computer Files

A CD-ROM was created to present additional data collected in this thesis. All coverage contour plots that were created in the figure-of-merit analysis are given. The coverage plots are given as jpeg images and labeled by their repeat type, eccentricity, ground station latitude, and minimum elevation angle. For example, the file RPT1e04Lat45Sig10.jpg is a coverage contour plot for a 1:1 repeat ground track orbit with an eccentricity of 0.4. The ground station is located at 45° N latitude on the Prime Meridian and the minimum elevation angle is set to 10°. All of the Matlab code generated is given in electronic format as .m files. Finally, a color version of this thesis is given in pdf format.

If the CD-ROM is missing from the thesis, the author can be reached via email at: John.Young@2001.usna.com.

CD-ROM File Listing

Filename: Young_SM_Thesis_Color.pdf
Directory: Root Directory
Description: Color version of the thesis “Coverage Optimization Using a Single Satellite Orbital Figure-of-Merit” in pdf format

Directory: \FOM-SinglePoint-Code\nDescription: Matlab code for figure-of-merit analysis for coverage to a single point. An explanation of each file is given in Appendix A.
Filenames:
- ascAndDecl.m
- ascGreenwich.m
- CalcDeltaELAngle.m
- COEtoPosition.m
- COEUpdate.m
- Contour.m
- Contour00.m
- CoverageTime.m
- deltaVCalc.m
- earthShape.m
- epochMeanAnomalyRate.m
- FOMCompile.m
- FOMCoverageCalc.m
- FOMCoverageCalc0.m
- GetContourData1to1.m
- GetContourData1to10ecc.m
- GetContourData2to1.m
- GetContourData2to10ecc.m
- GetContourData3to1.m
- GetContourData3to10ecc.m
- GetContourData4to1.m
- GetContourData4to10ecc.m
- GetContourData8to1.m
- GetContourData8to10ecc.m
- GetContourData8to1116.m
- GetContourData8to11160ecc.m
- Julian.m
- LoadContourData.m
- LoadContourData0.m
- MaxCoverageRPT1.m
- MaxCoverageRPT2.m
- MaxCoverageRPT4.m
- MaxCoverageRPT8.m
- MaxCoverageRPT8116.m
- MaximumCoverage.m
- MaximumCoverage0.m
- meanAnomaly.m
- meanMotion.m
- mu.m
- nodalPeriod.m
- nodeRate.m
- perigeeRate.m
- repeatGroundtrack.m
- repeatSemimajorAxis.m
- trueAnomaly.m
Directory: \FOM-Region-Code\nDescription: Matlab code for figure-of-merit analysis for coverage to a region. An explanation for each file is given in Appendix E.
Filenames:
CalcCentralEAngle.m IJKtoLatLong.m
CalcDistance.m RegionFOM.m
ContourRegionCentralAngle.m RegionMaximization.m
ContourRegionCentralAngle0.m RegionMaximization0.m
CoverageTimeRegionCentralAngle.m RegionMaximizationScript.m
CoverageTimeRegionCentralAngle0.m RegionMaximizationScript0.m
GetContourDataRegion.m StartGuessPoint.m
GetContourDataRegion0.m USPoints.m

Directory: \SinglePointContourPlots\RPT1\nDescription: Single point coverage contour plots for 1:1 repeat ground track orbits examined in this thesis. Files in jpeg format. (42 files)

Directory: \SinglePointContourPlots\RPT2\nDescription: Single point coverage contour plots for 2:1 repeat ground track orbits examined in this thesis. Files in jpeg format. (42 files)

Directory: \SinglePointContourPlots\RPT3\nDescription: Single point coverage contour plots for 3:1 repeat ground track orbits examined in this thesis. Files in jpeg format. (42 files)

Directory: \SinglePointContourPlots\RPT4\nDescription: Single point coverage contour plots for 4:1 repeat ground track orbits examined in this thesis. Files in jpeg format. (42 files)

Directory: \SinglePointContourPlots\RPT8\nDescription: Single point coverage contour plots for 8:1 repeat ground track orbits examined in this thesis. Files in jpeg format. (42 files)

Directory: \SinglePointContourPlots\RPT8i116\nDescription: Single point coverage contour plots for 8:1 repeat ground track orbits (i=116.6°) examined in this thesis. Files in jpeg format. (42 files)

Directory: \RegionContourPlots\nDescription: Regional coverage contour plots for repeat ground track orbits examined in this thesis. Files in jpeg format (36 files)