Mechanics of Graded Wrinkling

by

Shabnam Raayai Ardakani
B.S., Mechanical Engineering
Sharif University of Technology, Tehran, Iran (2011)

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2013

© Massachusetts Institute of Technology 2013. All rights reserved.
Mechanics of Graded Wrinkling

By

Shabnam Raayai-Ardakani

Submitted to the Department of Mechanical Engineering
on May 21, 2013, in partial fulfillment of
the requirements for the degree of
MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Abstract

The properties of a surface depend on the inherent material and the surface topography. Nature uses surface texture as a means to impact different surface behavior such as cleanliness, adhesion control, drag reduction, etc. As one way to mimic nature to obtain particular surface properties, different methods have been used to alter surface topography including surface wrinkling. Through buckling of a thin film of stiff material bonded to a substrate of a softer material, wrinkled patterns can be created by inducing compressive stress in the thin film. Using this same principle, changing the geometry of the surface or other means of creating a gradient in the stress distribution along the film, a gradient in the wrinkle topography can be created. The graded wrinkles possess varying amplitudes and wave lengths along the length of the film. In this work, the mechanics of graded wrinkling are first investigated through analytical modeling. Then, using finite element analysis, different aspects of graded wrinkling such as the wrinkle profile, stress and strain distributions are explored. Afterwards, different methods for creating wrinkled surfaces are introduced. In this work, the method of mechanical stretch and release is used for creating the wrinkled surfaces. PDMS sheets were prepared and coated with a stiff polymer using the method of initiated Chemical Vapor Deposition. The results of the graded wrinkling experiments are then presented and the trends are compared with the trends found through the finite element analysis.

Thesis Supervisor: Mary C. Boyce
Title: Ford Professor of Mechanical Engineering
ACKNOWLEDGEMENTS

I would like to express deepest appreciation to my advisor, Professor Mary C. Boyce for giving me the opportunity to work on this project. Her endless support and guidance made this work possible and her teachings and wisdom throughout the last two years helped me learn many new things and look at problems in different ways to find new and innovative solutions for them.

I would also like to thank Professor Karen Gleason for her guidance throughout the project and Doctor Jose Luis Yague who helped with the iCVD method and experiments. I would also like to thank the members of the Boyce group, especially Doctor Jie Yin, for their advice and help.

In the end I would like to thank my family, which without their support none of these would have been possible.

This research was supported by the King Fahad University of Petroleum and Minerals in Dhara, Saudi Arabia, through the center for clean water and clean energy at MIT and KFUPM.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE OF FIGURES</td>
<td>IV</td>
</tr>
<tr>
<td>TABLE OF TABLES</td>
<td>XII</td>
</tr>
<tr>
<td>CHAPTER 1 – SIGNIFICANCE OF SURFACE TEXTURE</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>NATURAL SURFACE TEXTURES AND THEIR SURFACE PROPERTIES</td>
<td>2</td>
</tr>
<tr>
<td>SHARK SCALES</td>
<td>2</td>
</tr>
<tr>
<td>LOTUS LEAVES</td>
<td>3</td>
</tr>
<tr>
<td>RICE LEAVES</td>
<td>4</td>
</tr>
<tr>
<td>BUTTERFLY WINGS</td>
<td>5</td>
</tr>
<tr>
<td>FISH (MICROPTERUS) SCALES</td>
<td>5</td>
</tr>
<tr>
<td>SURFACE TEXTURES AND PHYSICS BEHIND THE SURFACE PROPERTIES</td>
<td>6</td>
</tr>
<tr>
<td>WETTABILITY AND CONTACT ANGLES</td>
<td>6</td>
</tr>
<tr>
<td>ANTI-FOULING - SELF-CLEANING</td>
<td>9</td>
</tr>
<tr>
<td>DRAG REDUCTION</td>
<td>10</td>
</tr>
<tr>
<td>AIM OF THE THESIS</td>
<td>12</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>13</td>
</tr>
<tr>
<td>CHAPTER 2 – THEORY OF WRINKLING OF THIN COATING ON COMPLIANT SUBSTRATE</td>
<td>15</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>15</td>
</tr>
<tr>
<td>SCALING LAWS</td>
<td>16</td>
</tr>
</tbody>
</table>
TABLE OF FIGURES

Figure 1 - 1 – Moody Chart, as it is seen in the chart, in the laminar regime, the friction factor is only dependent on the $Re$, however in the turbulent regime, friction factor is dependent on $Re$ and the relative roughness..............................................................................................................1

Figure 1 - 2 – (a) Scales of a sand tiger shark, A) top view B)Apical view, (b) Scales of a shortfin mako shark, A) Frontal view of (about 55x) and B) top view (about 32x) (c) Scales of a smalltail shark (d) Scales of a wahle shark, A) lateral and epical views of a scale (about 75x), B) Scales together (about 35x) ........................................................................................................................................3

Figure 1 - 3 – Micro- and nano-scale structure on a Lotus leaf .........................................................................................4

Figure 1 - 4 – Hierarchical Structure of rice leaves – Arrows on indicate the preferred direction of the water on the leaves ........................................................................................................4

Figure 1 - 5 – Hierarchical structure of butterfly wings – Arrows indicte the preffered direction of the water movement ........................................................................................................5

Figure 1 - 6 – Surface Pattern of fish scales – (a) Sector-like arrays of 4 – 5 mm diameter scales, (b) Micro-papillae of length of 100 – 300 $\mu$m and width of 30 – 40 $\mu$m, (c) High magnification SEM shows the roughness of the surface of the papillae, d) Nano-structures on top of the papillae. ........................................................................................................................................6

Figure 1 - 7 – Schematic of the contact angle between a liquid droplet and smooth solid in air ........................................................................................................................................7

Figure 1 - 8 – Schematic of (a) Wenzel and (b) Cassie-Baxter interfaces ........................................................................9

Figure 1 - 9 – Schematic of the different definition of wettability based on the contact angles and examples of Wenzel and Cassie-Baxter droplets ........................................................................9

Figure 1 - 10 – An oil droplet on fish scale – (a) demonstrating the oleophilic behavior of fish scales in air- (b) Oleophobicity of oil on fish scales in water, showing a contact angle of 156.4° ± 3.0° ........................................................................................................................................9

Figure 1 - 11 – Schematic of self-cleaning properties of hydrophilic and hydrophobic surfaces ........................................................................................................................................10
In this curve, $s^+ = \frac{s u_t}{\nu}$ and $u_t = \tau_0 \rho$ where $\tau_0$ is the shear stress on a reference plate exposed to identical flow conditions, $s$ is the dimension of the riblet, $\rho$ is the density of the fluid, $\nu$ is the dynamic viscosity of the fluid and $\Delta \tau = \tau - \tau_0$ is the difference of the shear stresses between the test surface and the smooth reference surface.
Figure 2 - 12 - Non-dimensional critical load versus the ratio of the smallest width to the largest width in graded wrinkling $L = 50\ mm$ and $b_0 = 30\ mm$. The point with $\tan(\theta) = 0$ corresponds to the non-dimensional critical load in uniform wrinkling.......................................................... 36

Figure 3 - 1 - Schematic of the mirrored model created in ABAQUS for graded wrinkling – (a) Three dimensional view of the model (b) top view of the film. Geometric parameters are indicated on the figures. ........................................................................................................................ 39

Figure 3 - 2 - Schematic of a three-dimensional continuum quadratic element.................41

Figure 3 - 3 - Uniform wrinkle profile with outer surfaces of $\epsilon_{22}$ contours after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm. 43

Figure 3 - 4 - Non-dimensional wrinkle profile in uniform wrinkling after undergoing 4% of macroscopic strain.......................................................................................................................... 43

Figure 3 - 5 - Graded wrinkle profile with outer surfaces of $\epsilon_{22}$ contours ($\theta = 2.86\ ^\circ$) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm................................................................. 44

Figure 3 - 6 - Non-dimensional Graded wrinkle profile ($\theta = 2.86\ ^\circ$) after undergoing 4% of macroscopic strain.......................................................................................................................... 44

Figure 3 - 7 - Graded wrinkle pattern with outer surfaces of $\epsilon_{22}$ contours ($\theta = 5.7\ ^\circ$) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm................................................................. 44

Figure 3 - 8 - Non-dimensional Graded wrinkle profile ($\theta = 5.7\ ^\circ$) after undergoing 4% of macroscopic strain.......................................................................................................................... 45

Figure 3 - 9 - Graded Wrinkle profile with outer surfaces of $\epsilon_{22}$ contours ($\theta = 8.53\ ^\circ$) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm.......................... 45
Figure 3 - 10 - Non-dimensional graded wrinkle profile ($\theta = 8.53^\circ$) .................................................. 45

Figure 3 - 11 - Film Cross section on the symmetry plane ................................................................. 46

Figure 3 - 12 - Non-dimensional (a) Stress ($\sigma_{zz}E_f$) and (b) strain profile in the film in uniform wrinkling ................................................................................................................................. 47

Figure 3 - 13 - Non-dimensional (a) Stress ($\sigma_{zz}E_f$) and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 2.86^\circ$ ................................................................. 48

Figure 3 - 14 - Non-dimensional (a) Stress ($\sigma_{zz}E_f$) and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 5.7^\circ$ ........................................................................ 48

Figure 3 - 15 - Non-dimensional (a) Stress ($\sigma_{zz}E_f$) and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 8.53^\circ$ ........................................................................ 48

Figure 3 - 16 - (a) Intersections of the strain profiles for uniform wrinkling after undergoing 3.5% of macroscopic strain (b) Each of the waves found through the strain profile are identified on the uniform wrinkle profile as well ........................................................................................................... 49

Figure 3 - 17 - (a) Intersections of the strain profiles for graded wrinkling and the corresponding graded wrinkle profile of a trapezoidal geometry with $\theta = 2.86^\circ$ after undergoing 3.5% of macroscopic strain - The intersections help define the waves in graded wrinkling and knowing the positions. (b) Each of the waves found through the strain profile are identified on the graded wrinkle profile ................................................................................................................................. 49

Figure 3 - 18 - Evolution of wrinkle profile over time as the macroscopic strain is increasing .......... 50

Figure 3 - 19 - Evolution of graded wrinkle profiles of trapezoidal geometries with (a) $\theta = 2.86^\circ$, (b) $\theta = 5.7^\circ$ and (c) $\theta = 8.53^\circ$, over time as the macroscopic strain is increasing ......................................... 51

Figure 3 - 20 - Strain profiles in graded wrinkling of a trapezoidal geometry with $\tan \theta = 0.05$ under two different compressive macroscopic strain (a) 2.9% and (b) 3.2 %. In (a), as the 3rd wave is showing up, it has $\frac{\lambda}{t} \cong 20.25$ and then in (b) the 5th wave is has $\frac{\lambda}{t} \cong 20.29$. Also, $\frac{\lambda}{t}$ of the 3rd wave decreases by 0.1 % as the compressive strain is increased by 0.3% .................................................................................................................. 53
Figure 3 - 21 - Strain profiles in graded wrinkling of a trapezoidal geometry with \( \tan \theta = 0.1 \) under two different compressive macroscopic strain (a) 2.9\%, (b) 3.5\% and (c) 3.7\%. As the 3rd, 6th and 7th waves show up, they all have \( \frac{\lambda}{t} \approx 20.32 \). Also, \( \frac{\lambda}{t} \) of the 3rd wave decreases by 0.9\% as the macroscopic compressive strain is increased by 0.8%.

Figure 3 - 22 – Critical wrinkling wave length for different taper angles. Uniform wrinkling is presented as graded wrinkle with a taper angle of 0°.

Figure 3 - 23 – Trend in the amplitude of the wrinkles over the length of the film at 4\% macroscopic strain for (a) \( \tan \theta = 0.05 \), (b) \( \tan \theta = 0.1 \), (c) \( \tan \theta = 0.15 \).

Figure 3 - 24 – (a) Trend in the maximum amplitude in graded wrinkling over different macroscopic strains for \( \tan(\theta) = 0.05 \), (b) ratio of the amplitude over the maximum amplitude at that macroscopic strain, over a range of macroscopic strains for \( \tan \theta = 0.05 \).

Figure 3 - 25 – Eigenvalues from finite element analysis vs. the geometric parameter

\[ \left( \frac{2 \tan(\theta)}{\ln\left(\frac{2\lambda}{h}\right)} \right)^\frac{1}{2} \]

Figure 4 - 1 - Wrinkling using the thermal strain due to heating the substrate and cooling it. With the film constrained to the substrate, it undergoes compression and thus wrinkles.

Figure 4 - 2 – Wrinkling using the swelling of a laterally confined elastomer.

Figure 4 - 3 - Schematic of method creating wrinkled patterns using the Focused Ion Beam method.

Figure 4 - 4 – Wrinkling using the stretch and release method. After the substrate is stretched (having a non-zero strain energy), the film is deposited (adhered) to the substrate. At this stage the film has zero strain energy. Releasing the stretched substrate, puts the film under compression and results in a wrinkled pattern.

Figure 4 - 5 – Schematic of the experimental method used.
Figure 4 - 6 – The iCVD process and the vacuum reactor setup ............................................. 64

Figure 4 - 7 – (a) Geometry which the PDMS substrates are cut into. (b) Two of the samples used for creating graded wrinkling. The samples are made from PDMS. ............................................. 65

Figure 4 - 8 - Sample geometry with $\theta = 4.57^\circ$ and $\frac{b_1}{b_0} = \frac{2}{3}$ ............................................. 67

Figure 4 - 10 – Wrinkled pattern corresponding to geometry in figure 4 - 8 with $\theta = 4.57^\circ$ and $\frac{b_1}{b_0} = \frac{2}{3}$ and pre-strained to 20%(a) Top view and (b) three dimensional view of Area (1) with the largest width (c) Top view and (d) three dimensional view of area (2) with the medium width (e) Top view and (f) three dimensional view of area (3) ...................................................... 68

Figure 4 - 11 - Cross section of the wrinkled surface of geometry in figure 4 - 8 with $\theta = 4.57^\circ$ and $\frac{b_1}{b_0} = \frac{2}{3}$ and pre-strained to 20% (a) in area (1) with the largest width, the horizontal distance between the designated peak and trough ($\frac{\lambda}{2}$) is $12.09 \mu m$ and the vertical distance ($2A$) is equal to $5.468 \mu m$ (b) in area (2) with the medium width, the horizontal distance between the designated peak and trough ($\frac{\lambda}{2}$) is $11.96 \mu m$ and the vertical distance ($2A$) is equal to $6.455 \mu m$ (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough ($\frac{\lambda}{2}$) is $11.82 \mu m$ and the vertical distance ($2A$) is equal to $6.898 \mu m$ ................................................................................. 69

Figure 4 - 12 - Sample geometry – In this geometry $\theta = 2.3^\circ$ and $\frac{b_1}{b_0} = \frac{5}{6}$ ............................................. 70

Figure 4 - 13 - Sample geometry – In this geometry $\theta = 6.84^\circ$ and $\frac{b_1}{b_0} = \frac{1}{2}$ ............................................. 70

Figure 4 - 14 – Wrinkled pattern corresponding to the geometry in figure 4 – 11 with $\theta = 2.3^\circ$ and $\frac{b_1}{b_0} = \frac{5}{6}$ and pre-strained to 20%(a) Top view and (b) three dimensional view of area (1) with the largest width (c) Top view and (d) three dimensional view of area (2) with the medium width (e) Top view and (f) three dimensional view of area (3) with the smallest width ............................................. 71
Figure 4 - 15 – Cross section of the wrinkled surface of figure 4 - 11 with \( \theta = 2.3^\circ \) and \( \frac{b_1}{b_0} = \frac{5}{6} \) and pre-strained to 20\%(a) in area (1) with the largest width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 10.5 \( \mu m \) and the vertical distance (2A) is equal to 5.851 \( \mu m \) (b) in area (2) width the medium width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 10.76 \( \mu m \) and the vertical distance (2A) is equal to 5.78 \( \mu m \) (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 10.76 \( \mu m \) and the vertical distance (2A) is equal to 6.134 \( \mu m \) ............................... 72

Figure 4 - 16 – Wrinkled pattern corresponding to figure 4 - 12 with \( \theta = 6.84^\circ \) and \( \frac{b_1}{b_0} = \frac{1}{2} \) and pre-strained to 20\%(a) Top view and (b) Three dimensional view of Area (1) with the largest width (c) Top view and (d) Three dimensional view of area (2) with the medium width (e) Top view and (f) Three dimensional view of area (3) with the smallest width.................................................. 73

Figure 4 - 17 – Cross section of the wrinkled surface of figure 4 - 12 with \( \theta = 6.84^\circ \) and \( \frac{b_1}{b_0} = \frac{1}{2} \) and pre-strained to 20\%(a) in area (1) with the largest width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 11.56 \( \mu m \) and the vertical distance (2A) is equal to 7.72 \( \mu m \) (b) in area (2) with the medium width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 12.22 \( \mu m \) and the vertical distance (2A) is equal to 6.714 \( \mu m \) (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough (\( \frac{1}{2} \)) is 13.55 \( \mu m \) and the vertical distance (2A) is equal to 6.242 \( \mu m \) .................................................................................................................. 74

Figure 4 - 18 – Free body diagram of a film on elstic substrate where the substrate is under loading. The loading is transferred to the film through the shear stress between the film and the substrate........................................................................................................................................ 78
Figure 4 - 19 – General form of the shear stress in the thin film and the tensile stress in the film. The tensile stress builds up in the film as a result of the shear stress. The relation between them is governed by the equilibrium equation as presented in figure 4 – 17................................. 78

Figure 5 - 1 – A trapezoidal geometry combination to create new surface textures .................. 83
Figure 5 - 2 – Anti-trapezoidal geometry combination to create new surface textures ............ 83
Figure 5 - 3 – Periodic patterns for creating periodic surface patterns ................................. 84
Table 4 - 1 – Data for wave length and amplitude of the wrinkles in the graded geometry with
with \( \theta = 4.57^\circ \) and \( \frac{b_1}{b_0} = \frac{2}{3} \) and pre-strained to 20% corresponding to figure 4 – 8 .................. 70

Table 4 - 2 - Data for wave length and amplitude of the wrinkles in the graded geometry with
with \( \theta = 2.3^\circ \) and \( \frac{b_1}{b_0} = \frac{5}{6} \) and pre-strained to 20% corresponding to figure 4 – 11 .................. 75

Table 4 - 3 - Data for wave length and amplitude of the wrinkles in the graded geometry with
with \( \theta = 6.84^\circ \) and \( \frac{b_1}{b_0} = \frac{1}{2} \) and pre-strained to 20% corresponding to figure 4 – 12 .................. 75

Table 4 - 4 - Data for the amplitude of the wrinkles in the graded geometry different taper angles
and different ratio of the smallest width to the largest width ................................................................. 76

Table 4 - 5 - Data for the wave length of the wrinkles in the graded geometry different taper
angles and different ratio of the smallest width to the largest width ...................................................... 76
INTRODUCTION

The properties of a surface depend on the material and the surface topography. Friction, drag, contact angles, surface tension, light reflectivity, etc. are examples of surface properties.

One basic example of this dependence is the friction factor in pipe flow as shown in the Moody charts. As the fluid flow transitions from laminar to turbulent, the friction factor which was only a function of the Reynolds number \( Re_D = \frac{pUD}{\mu} \), will become a function of both the Reynolds number and the surface roughness. Therefore, the lines on the chart turn from one line in the laminar regime into several lines in the turbulent regime based on the size of the roughness features of the pipes. The same idea can be found when dealing with other surface properties as well.

Figure 1 - 1 – Moody Chart, as it is seen in the chart, in the laminar regime, the friction factor is only dependent on the \( Re \), however in the turbulent regime, friction factor is dependent on \( Re \) and the relative roughness.
Nature uses the surface topography as a way to control the behaviors of the skins of different animals and plants. Plants and animals such as lotus leaves, rice leaves, shark skin, etc. have textures on their skin which helps them enhance their interactions with the environment around them by controlling friction, surface tension and other properties of those surfaces.

In this chapter, an overview of natural surface textures and the way they control surface properties is presented.

**NATURAL SURFACE TEXTURES AND THEIR SURFACE PROPERTIES**

Various plants and animals have different surface textures on their skins. The textures, which are at different length scales varying from nanometers up to hundreds of microns, give each of these surfaces different abilities, such as reducing drag, self-cleanliness, hydrophobicity and oleophobicity, etc. Some examples of such surfaces and their dimensions are presented below:

**SHARK SCALES**

A shark’s body is covered with scales of the size on the order of 100 of microns. The shape of the scales varies from one type of shark to another and even the way they are positioned on their body can be different among different types of sharks. Figures 1 – 2 shows the shape of the scales (also called denticles) for different types of sharks.

As shown in the figures, shark scales, even though different among different types of sharks, each consists of a certain number of riblets. These riblets can even have different lengths within one scale depending on the type of shark. The riblets are specially sized and spaced and they are parallel to the swimming direction of the shark.¹ The height of the riblets is seen to vary between 200 and 500 $\mu m$ and the spacing between the riblets vary between 100 and 300 $\mu m$.²
However, due to the surface texture of the scales, a shark is able to swim faster than other animals in water, as well as keep its skin clean from biological or chemical matter from adhering to it. The presence of the riblets can control the drag during swimming, as well as reducing the adhesion forces between alien matter and the shark’s body, making sharks one of the marine animals with a self-cleaning surface.

LOTUS LEAVES

The lotus flower (*Nelumbo nucifera*) is a plant which grows in muddy water. The fact that it grows out to be completely clean in that inhabitant makes it a symbol of purity. The self-cleaning ability of these leaves is due to the fact that the waxy hierarchical surface structure makes the leaf super-hydrophobic and low adhesion surface. The structure of the pattern of the lotus leaves are shown in figure 1 – 3. The lotus leaves consists of bumps which are on the micro-scale and hair-like structures which are on the nano-scale. With such a surface pattern, water droplets sitting on the leaves keep their spherical shape and tilting the leaves will result in the droplets rolling off the leaves and taking the contaminant particles with them.
The surface of rice leaves is covered by hierarchical micro-papillae of epicuticular wax and directed in longitudinal grooves. While this structure is similar to the structure of lotus leaves, lotus leaves can direct the water droplets in any direction while the presence of the grooves in rice leaves, creates an ability to control the direction of the droplet. Figure 1 - 4 demonstrates the grooves, micro- and nano-scale structures on the rice leaves and the directionality that they give to the wettability of the surface.
BUTTERFLY WINGS

Similar to rice leaves, butterfly wings are also covered with hierarchical structures which, in contrast with Lotus leaves, can direct water droplets in specific directions. SEM images of the wings of butterfly *Morpho aega* show that they are covered with rectangular scales of length of about 150 μm and width of about 70 μm. The scales overlap with each other to form a periodic hierarchy. Also, further magnification shows numerous separate ridging stripes of 184.3 ± 9.1 nm in width. These strips are in different lengths and are stacked stepwise along the outward radial direction.

![SEM images of butterfly wings](image)

**Figure 1-5** - Hierarchical structure of butterfly wings - Arrows indicate the preferred direction of the water movement

FISH (*MICROPTERUS*) SCALES

*Micropterus* is a genus of fish found in fresh water in North America and some of them grow as tall as 70 cm. Scales of these fish are composed of calcium phosphate, protein and a thin layer of mucus. Fish scales have diameters 4 – 5 mm and are arranged in sector-like arrays. The SEM images of the scales also show the presence of radially positioned micro-papillae of length of 100 – 300 μm and width of 30 – 40 μm. In addition to the papillae, atomic force microscopy has shown fine-scale roughness on top of the papillae. These hierarchical structures on the fish scales make the body of the fish to be hydrophilic and super-oleophilic in air. However, when submerged under the water, they show super-oleophobic behavior with contact angles of 156.4° ± 3.0°.
Figure 1 - 6 - Surface Pattern of fish scales – (a) Sector-like arrays of 4 – 5 mm diameter scales, (b) Micro-papillae of length of 100 – 300 µm and width of 30 – 40 µm, (c) High magnification SEM shows the roughness of the surface of the papillae, d) Nano-structures on top of the papillae. 11

SURFACE TEXTURES AND PHYSICS BEHIND THE SURFACE PROPERTIES

As was mentioned in the last section, the presence of micro- and nano-scale structures on the body of animals and plants plays an important role in imparting special surface properties such as self-cleanliness, hydrophobicity, oleophobicity, drag reduction, etc. There are different explanations associated with each of the structures and their abilities and some of these mechanisms are presented here.

WETTABILITY AND CONTACT ANGLES

One of the surface properties, which is common to nearly all the surfaces, is the requirement of control of the wettability of the surfaces with different liquids, mainly with water or oil. As was shown earlier, rice leaves, butterfly wings and Lotus leaves are super-hydrophobic while fish scales are super-oleophobic when the scales are submerged under water. In order to investigate these properties, scientists study the surface properties such as surface tension and the interfacial energy of the surfaces to be able to explain the phenomenon.
When a material of a liquid phase comes in contact with a solid in the presence of a third medium (gas or liquid), the materials will find an equilibrium state where the force equilibrium can be presented by the Young-Dupré equation which is for smooth surfaces.

\[
\cos(\theta_0) = \frac{\gamma_{LS} - \gamma_{SG}}{\gamma_{LG}}
\]

Equation 1 - 1

Where \(\gamma_{LS}\) is the surface tension between the liquid and solid, \(\gamma_{SG}\) is the surface tension between the solid and the gas and \(\gamma_{LG}\) is the surface tension between the liquid and the gas.

However, when roughness is added to the surface, the equilibrium state and the contact angles do not follow the Young-Dupré's equation and the presence of the roughness should be considered. As a result, Wenzel in 1936 and Cassie and Baxter in 1944 presented 2 different theories on the wetting behavior and the equilibrium states. Wenzel considers rough surfaces to have more contact area (actual area) available than the geometrical area of the surface (i.e. flat projected area). Therefore, introducing a new parameter called the “roughness factor” \(r\) as given below:

\[
r = \frac{\text{actual surface area}}{\text{flat projected surface area}}
\]

Equation 1 - 2

And therefore, the apparent contact angle of the rough surface will be different from the smooth surface and defined as below:
\[ \cos(\theta) = r \cos(\theta_0) \]  

Equation 1 - 3

Where \( \theta \) is the contact angle of the rough surface and \( \theta_0 \) is the contact angle of the same materials when the surface is smooth. \(^{12,13}\)

On the other hand, Cassie and Baxter assume that in presence of the roughness, the liquid does not wet all of the available surface and part of the area between the roughness and the liquid will be filled by air pockets and thus, underneath the liquid phase, there will be many more liquid/air/solid interfaces which follow the Young-Dupré equation. Therefore, Cassie and Baxter, construct equation 1 – 4 for the apparent contact angle. \(^{12,14}\)

\[ \cos(\theta) = f_1 \cos(\theta_1) + f_2 \cos(\theta_2) \]  

Equation 1 - 4

Where \( f_1 \) and \( \theta_1 \) are the fraction of the solid area in contact with medium 1 and \( f_2 \) and \( \theta_2 \) are the fraction of the solid area in contact with medium 2. \(^{12,14}\) Both Wenzel state and Cassie-Baxter state are possible states for such composite interfaces, however with different roughness sizes it is shown that the interface tries to find the stable form. Therefore the interfaces can jump between these two states depending on which state is energetically more preferable. \(^{12}\)

So, with these two definitions for the wettability and contact angles of rough surfaces, it can be seen that the presence of roughness in plants and animals is playing a role in creating such composite interfaces. In the lotus leaves, rice leaves and butterfly wings, air pockets are trapped between the nano- and micro-structures and thus in presence of water, they result in a composite interface causing these three surfaces to show super-hydrophobic properties and if tilted, the water droplets on them will roll off the surfaces. \(^{6}\)

As for the fish scales, when the fish scales under water come in contact with oil droplets, water fills the nanostructures and therefore creates a composite oil/water/solid interface and therefore a super-oleophobic surface in water. \(^{11}\)
Figure 1 - 8 – Schematic of (a) Wenzel and (b) Cassie-Baxter interfaces

Figure 1 - 9 – Schematic of the different definition of wettability based on the contact angles and examples of Wenzel and Cassie-Baxter droplets

Figure 1 - 10 – An oil droplet on fish scale – (a) demonstrating the oleophilic behavior of fish scales in air- (b) Oleophobicity of oil on fish scales in water, showing a contact angle of $156.4^\circ \pm 3.0^\circ$

**ANTI-FOULING - SELF-CLEANING**

The Presence of micro- and nano-structures on the body of different animals and plants gives them the ability to keep themselves clean. Lotus leaves, butterfly wings, rice leaves, shark skin, fish scales are all among the self-cleaning surfaces in nature, which as explained earlier, have textures on the micro- and nano-scales.

Super-hydrophobicity or super-oleophobicity of surfaces pushes the droplets of water or oil to have a shape close to a sphere. In such surfaces, when the surface is inclined, the water
droplet rolls off the surface and collects the contaminant particles as it is moving, thus making the surface clean. A schematic of this self-cleaning method is shown in figure 1 - 11. On the other hand, super-hydrophilic surfaces create an evenly distributed water layer by attracting water and thus disrupting the settlements of the contaminant particles and micro-organisms.

Another theory for explaining the fouling/self-cleaning surfaces is the attachment point theory. This theory implies that micro-organisms prefer to adhere to areas which are slightly bigger than their size to increase the protection and the contact area. Therefore, adding micro-structures which are smaller than the size of the corresponding micro-organisms will help reduce the number of attachment points and thus lower bio-adhesive strength between the surfaces and the organism. Also, when the micro-textures are smaller than the size of the species, they are not able to give protection against the hydrodynamic forces in the environment and contaminants are removed from the surfaces. This is thought to be the main mechanism behind the anti-fouling abilities of shark skin, considering both micro-textures and the high swimming speed of the shark.  

---

**DRAG REDUCTION**

As was shown earlier, shark skin is comprised of riblets which are specially sized and spaced and are positioned parallel to their swimming direction and thus create an anisotropic roughness on the surface. These riblets are the main reason for the low drag which shark exhibit
while swimming. It is shown that the presence of riblets can reduce the turbulent drag by constraining the natural vortices and thus reducing the momentum transfer and thus the wall-shear force. \(^1,16,17\)

There have been different investigations into the mechanisms of the drag reduction and possible ways to mimic this ability for adding riblets to surfaces. One of the explanations for this is that riblets aligned with the flow direction can rectify the direction of the turbulent flow to the main flow direction. Riblets' side-walls reduce the effect of the fluctuating cross-flow components. Reduction in the cross flow component of the velocity decreases the momentum transfer close to the wall, causing the shear stress to decrease. \(^16\)

Different sets of experiments have been conducted over the years to find the most effective riblet geometry to efficiently reduce drag on aircraft and marine structures. It has been shown that with the riblets present, the best length scale to be used in expressing the data is the riblet's length scale. Thus, different drag reduction curves have been constructed for different riblets shapes and dimensions. An example of one curve is shown in figure 1 - 12.

In curves such as the one in figure 1 - 12, the change in the shear stress on the surface compared to a smooth reference surface is plotted versus the change in the size of the topographical features. In the case of figure 1 - 12, the topographical feature is a riblet with size \(s\) and using the definitions for the non-dimensional length scales in turbulent flow it is non-dimensionalized as \(s^+ = \frac{s u_\text{r}}{v}\). As the figure shows, as \(s^+\) increases, the shear stress first decreases and then starts to increase, giving a way to find the most efficient size for the riblet features to decrease the shear stress.
Figure 1 - 12 - General structure of a drag reduction curve – In this curve, \( s^+ = \frac{s}{u_t} \) and \( u_t = \sqrt{\frac{\tau_0}{\rho}} \) where \( \tau_0 \) is the shear stress on a reference plate exposed to identical flow conditions, \( s \) is the dimension of the riblet, \( \rho \) is the density of the fluid, \( v \) is the dynamic viscosity of the fluid and \( \Delta \tau = \tau - \tau_0 \) is the difference of the shear stresses between the test surface and the smooth reference surface. 

AIM OF THE THESIS

As was demonstrated throughout this chapter, surface texture can play an important role in properties of different surfaces and their behavior when in contact with different materials. Therefore, there have been attempts in creating micro- and nano- patterns on surfaces. One promising method for creating structured surface topographies is using mechanics to generate wrinkled surfaces.

In this thesis, the process of wrinkling as a way of patterning surfaces and its different forms are investigated through analytical solutions and finite element analysis. Also, different experimental methods for creating wrinkled surfaces and the results of experiments conducted on graded geometries are presented. The ability to precisely tailor the wrinkling phenomenon by controlling the geometries, the materials and the strains, provides a means to fabricate textured surfaces with tunable properties such as drag force and wettability.
REFERENCES


CHAPTER 2 - THEORY OF WRINKLING OF THIN COATING ON COMPLIANT SUBSTRATE

INTRODUCTION

Wrinkling is a form of instability which happens when a thin film of one material bonded to a relatively softer compliant substrate, undergoes compressive forces. When the compliant substrate is not present, during compression, the film will buckle into half of a sine wave upon reaching a critical load; when attached to a compliant substrate, the film will buckle into several sine waves with shorter wave lengths compared to the isolated film.

This phenomenon of a higher mode of buckling, referred to as wrinkling, can be explained through energy minimization. Without the compliant matrix, as the film undergoes compressive load, it will depart from direct compression and will buckle at a critical load, because buckling will be energetically favorable over direct compression. From an energy point of view, among all possible sine waves, the film will choose to buckle into a half sine wave, since it is the lowest energy mode. However, in the presence of the compliant matrix, in addition to bending of the film, the substrate undergoes stretching. Therefore, the total energy is the energy in the coating and the energy in the substrate. Buckling will result in bending energy of the film (which is energetically favorable over direct compression) and also an additional stretching energy in the substrate. Therefore, the minimum state is one which minimizes the total energy. Thus, in order to reach the minimum energy state, instead of only half a sine wave, the film buckles into multiple sine waves of a given wave length, as also called wrinkling.

In the following chapter, the theory behind wrinkling and the different forms it will take is explored.
SCALING LAWS

Scaling laws for the dependence of the wrinkling wave length on the coating properties, coating thickness and substrate properties can be found through energy minimization by scaling the bending energy of the film and the stretching energy of the substrate. Calling the deflection of the film \( w \), the bending energy of the film per unit length is scaled as shown in equation 2-1 and the stretching energy per unit length can be scaled as shown in equation 2-2.

\[
U_f \propto E_f l_f \left( \frac{\partial^2 w}{\partial x_2^2} \right)^2 \\
U_s \propto \int E_s \varepsilon_s^2 dx_1 \, dx_3
\]

Equation 2 - 1

Equation 2 - 2
Where here $x_1, x_2$ and $x_3$ represent the directions along the width of the film, along the length of the plate and along the thickness of the film respectively.

![Figure 2-3](image1.png)  
**Figure 2-3** – Bending of the thin film as it undergoes wrinkling

![Figure 2-4](image2.png)  
**Figure 2-4** – Stretching of the substrate during wrinkling

Assuming that the deflection of the film is of the wrinkled form, having a sinusoidal profile (equation 2-3), curvature of the film and the local strain of the substrate can be scaled as in equations 2-4 and 2-5. Here it is assumed that the stretching of the substrate happens over a length of the order of the wave length of the sinusoidal wrinkles, $\lambda$, and the displacement in that direction can be scaled with the amplitude of the waves, $w_m$.

$$w(x_2) = w_m \sin\left(\frac{2\pi}{\lambda} x_2\right)$$  
*Equation 2-3*

$$\left(\frac{\partial^2 w}{\partial x_2^2}\right)^2 \propto \frac{w_m^2}{\lambda^4}$$  
*Equation 2-4*

$$\varepsilon_s \propto \frac{w_m}{\lambda}$$  
*Equation 2-5*
Therefore, using equations 2 - 1 and 2 - 2 and substituting from equations 2 - 4 and 2 - 5 assuming the local stretching of the substrate happens over a distance $\lambda$ in the $z$ direction, bending energy of the film and the stretching energy of the substrate can be found.

\[
U_f \propto E_f t^3 b \frac{w_m^2}{\lambda^4}
\]

Equation 2 - 6

\[
U_s \propto E_s \varepsilon_s^2 \times \text{Area} \propto E_s \left(\frac{w_m}{\lambda}\right)^2 b \lambda
\]

\[
\Rightarrow U_s \propto E_s \frac{w_m^2}{\lambda} b
\]

Equation 2 - 7

Then in order to obtain the most desired wave length (which minimizes the energy), the derivate of the total energy with respect to the assumed wave length is put equal to zero.

\[
\frac{\partial U}{\partial \lambda} = \frac{\partial (U_f + U_s)}{\partial \lambda} = 0
\]

Equation 2 - 8

\[
\lambda_{cr} \propto t \left(\frac{E_f}{E_s}\right)^{\frac{1}{3}}
\]

Equation 2 - 9

Hence, the ratio of the critical wave length at which wrinkling happens is found to be directly proportional to the film thickness and proportional to the ratio of the stiffness of the materials to the one third power. In addition to scaling methods, as it will be shown further into the chapter, the same power law dependence can be found through the solving the corresponding eigenvalue equations.

The scaling laws presented here are all based on the energy per unit length and therefore, the resulting expression is a general statement for any form of wrinkling over various geometries and thus as it will be shown later in this chapter and the proceeding chapters, the same proportionalities will be observed for different forms of wrinkling as well.
DIFFERENT FORMS OF WRINKLING

Depending on the geometry used, wrinkling can take different forms. Hence, we will focus on two categories: uniform wrinkling and graded wrinkling. Uniform wrinkling will refer to the case where the geometry of the thin wrinkled film is uniform with the same wave length and amplitude everywhere. However, grading the geometry can cause the wrinkle to change from uniform wrinkling into graded wrinkling, where the amplitude and/or the wave length of the wrinkles are not constant along the length of the film.

This trend can be explained by examining a uniform and a graded geometry under compression. When the geometry is uniform, the cross section of the film undergoing compression will be constant and thus the compressive stress will always be constant along the length of the film. But, graded geometries cause the stress and strain distribution along the length of the plate to be non-constant and position dependent; thus the plate will undergo graded wrinkling with wrinkles of different wave lengths and amplitudes along the length of the plate.

Figure 2 - 5 - Schematic of the geometry of film and substrate in (a) uniform wrinkling (b) graded wrinkling. (c) Free body diagram of the film in uniform wrinkling subject to compressive force $P_f$, (d) Free body diagram of film in graded wrinkling.
Here, in order to explore the concept of graded wrinkling, first the general equations for uniform wrinkling are presented as benchmarks. Then as the simplest graded geometry, a trapezoidal film (Figure 2 – 5 – b, d) with constant thickness (in the $x_3$ direction) is investigated through theory and in the later chapters, through finite element simulations and experiments.

**STRESS DISTRIBUTION**

Taking the case of thin film bonded to a substrate where the cross sectional area of the film and of the substrate is uniform as shown in figure 2 – 5 (a and c) and subject to compression results in a uniform strain $\varepsilon_{22}$ distribution in the length of the film. Thus, the film will wrinkle uniformly upon reaching a critical stress and all the wrinkles will have the same amplitude and wave lengths. This wrinkling instability will be analyzed in the next section.

Taking the case of a cross sectional geometry with a gradient will give the film a non-constant stress distribution inside the film and thus the wrinkles will not be uniform anymore. Having a thin film with constant thickness and a symmetric trapezoidal shape as shown in figure 2 – 5 (b and d), gives a gradient in the width of the film and thus the cross sectional area of the film. The cross section of the film is still a rectangle at every section, however, the width of the film changes along the length which gives rise to a gradient in the stress and strain distribution within the film.

When the geometry of the film is changed from a rectangle into a trapezoid, new parameters are introduced which can affect the process of wrinkling. Here, it is assumed that the film has geometry of a symmetric trapezoid on the $x_1 - x_2$ plane with a constant thickness of $t$ in the $x_3$ direction, and is undergoing compressive displacement in the $x_2$ direction. Thus the stress distribution along the film can be easily found based on the compressive force being applied.
For the case of a film of length $L$ cross section $b(x_2) \times t$, undergoing total change in length of $\delta$ in the $x_2$ direction, the stress at every point is defined as in equation 2-10, where $P_f$ is the total force on the film and $A$ is the cross sectional area of the film at that point.

$$\sigma_f = \frac{P_f}{A} = \frac{P_f}{bt}$$  \hspace{1cm} \text{Equation 2-10}

From Hooke's law, the stress at every point can be related to the strain at every point.

$$\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{1}{E_f} \frac{P_f}{bt}$$ \hspace{1cm} \text{Equation 2-11}

And, in order to find the total force in the film ($P_f$) corresponding to the total displacement $\delta$, we integrate equation 2-11 over the length of the film (0 to $L$):

$$\delta = \int_0^L \varepsilon_f dx_2 = \frac{P_f}{E_f} \int_0^L \frac{1}{bt} dx_2$$ \hspace{1cm} \text{Equation 2-12}

So, depending on how the cross sectional area changes over the length, the relationship between $P_f$ and $\delta$ will be different. For the case of a constant cross section (or rectangular film), $b$ and $t$ will be constant, therefore:

$$\delta = \frac{P_f}{bt} \frac{L}{E_f}$$ \hspace{1cm} \text{equation 2-13}

Substituting back from equation 2-10 and rearranging gives:

$$\frac{\sigma_f}{E_f} = \frac{\delta}{L}$$ \hspace{1cm} \text{Equation 2-14}

When the film has a trapezoidal shape, there is a gradient in the width of the film. Assuming that the width of the cross section decreases along the length of the film via $b(x_2) =$
\[ b_0 - 2 \tan \theta x_2 \text{ where } \theta \text{ is the taper angle as shown in figure 2 - 5 - d, using Equation 2 - 12, we have:} \]

\[
\sigma_f = \frac{P_f}{bt} = \frac{P_f}{t (b_0 - 2 \tan \theta x_2)} \quad \text{Equation 2 - 15}
\]

\[
\delta = \frac{P_f}{t E_f} \frac{1}{2 \tan(\theta)} \ln \left( \frac{b_0}{b_l} \right) \quad \text{Equation 2 - 16}
\]

Where \(b_l\) is the smallest width in the film. Substituting from equation 2 - 15 and rearranging gives:

\[
\frac{\sigma_f}{E_f} = \frac{2 \tan(\theta)}{\ln(\frac{b_0}{b_l})} \frac{\delta}{b(x_2)} \quad \text{Equation 2 - 17}
\]

And therefore, the stress distributions on the films one with a rectangular cross section and the other with a trapezoidal shape will clearly be different.

![Figure 2 - 6– General form of stress distribution in uniform geometry and in graded geometry (i.e. trapezoid)](image)

Therefore, with a non-constant stress distribution, new parameters will affect buckling of a thin film with a non-uniform geometry (in this case a trapezoidal geometry) on a compliant matrix.
UNIFORM WRINKLING - THEORY

Similar to buckling analysis, wrinkling analysis is done through the eigenvalue equations that result from the equilibrium equations for the film and the compliant matrix. The analysis can also be conducted through energy analysis of the system. However, here in order to find these relations, the equilibrium equations following eigenvalue analysis are used.

In order to find the equations, some simplifying assumptions are used:

- Film and substrate, both, are comprised of linear elastic material.
- The film is thin enough to be modeled as a plate. In this case where the loading is in one direction and thus the deflection of the film will be independent of one in-plane direction, it can even be simplified into a beam with rectangular cross section. Therefore, beam equations are used.
- The effect of the compliant substrate on the film can be represented as a series of distributed load on the bottom surface of the film.
- The distributed load, represented as $\sigma_{33}$, has the units of force per unit area and based on Newton’s third law, the same stress distribution (in the reverse direction) should act on the substrate. Therefore, for finding that, equilibrium equations for the substrate should be solved subject to appropriate boundary conditions.

With these assumptions, using a free body diagram (figures 2 - 7 and 2 - 8), force and momentum equilibrium can be constructed.

$$\frac{\partial V(x_2)}{\partial x_2} = -\sigma_{33} b$$  \hspace{1cm} \text{equation 2 - 18}

$$\frac{\partial M}{\partial x_2} = -P_f \frac{\partial w(x_2)}{\partial x_2} + V(x_2)$$  \hspace{1cm} \text{equation 2 - 19}
Figure 2 - 7 – Free body diagram of the film and the substrate. Film is modeled as a beam under compressive load \( P_f \) and distributed load due to the substrate. The substrate is modeled as a semi-infinite body under the same distributed load in the opposite direction.

Figure 2 - 8 – Free body diagram of an infinitesimal element of the film (i.e. beam)

Using beam theory (equation 2 - 20) combined with equations 2 - 18 and 2 - 19, the governing equation of the film can be found.

\[
M(x_2) = -E_f I_f \frac{\partial^2 w}{\partial x_2^2}
\]

Equation 2 - 20

\[
\frac{\partial^2}{\partial x_2^2} \left( E_f I_f \frac{\partial^2 w}{\partial x_2^2} \right) + P_f \frac{\partial^2 w}{\partial x_2^2} + \sigma_{33} b = 0
\]

Equation 2 - 21

It is shown that equation 2 - 21 admits solutions of sinusoidal form. Therefore, it is assumed that the film deflection has a form of equation 2 - 22.

\[
w_f(x_2) = w_f \sin \left( \frac{2\pi}{\lambda} x_2 \right)
\]

Equation 2 - 22

Now, in order to find the response of the substrate (i.e. \( \sigma_{33} b \)), the substrate can be modeled as a two dimensional solid (in the \( x_2 - x_3 \) plane). This is following the assumption that
the deflection of the film and thus the substrate is not dependent on the in plane direction (x₁). Therefore the stress state within this solid may be defined by an Airy stress function(φ) which satisfies the biharmonic equation.² (Equation 2 - 23)

\[ \nabla^2 \nabla^2 \phi = 0 \quad \text{Equation 2 - 23} \]

From the definition of the stress function, the stresses in the substrate can be found (equations 2 - 24, 2 - 25 and 2 - 26).

\[ \sigma_{33} = \frac{\partial^2 \phi}{\partial x_2^2} \quad \text{Equation 2 - 24} \]
\[ \sigma_{22} = \frac{\partial^2 \phi}{\partial x_3^2} \quad \text{Equation 2 - 25} \]
\[ \sigma_{23} = -\frac{\partial^2 \phi}{\partial x_2 \partial x_3} \quad \text{Equation 2 - 26} \]

Using the stresses and the linear elastic assumption for the material of the substrate, the strains in the substrate are found:

\[ \varepsilon_{22} = \frac{1}{E_s} \left( \sigma_{22} - \nu_s \sigma_{33} \right) = \frac{1}{E_s} \left( \frac{\partial^2 \phi}{\partial x_3^2} - \nu_s \frac{\partial^2 \phi}{\partial x_2^2} \right) \quad \text{Equation 2 - 27} \]
\[ \varepsilon_{33} = \frac{1}{E_s} \left( \sigma_{33} - \nu_s \sigma_{22} \right) = \frac{1}{E_s} \left( \frac{\partial^2 \phi}{\partial x_2^2} - \nu_s \frac{\partial^2 \phi}{\partial x_3^2} \right) \quad \text{Equation 2 - 28} \]

The substrate is very large, thus it is often called a half space; therefore, the displacement and tractions at the very far end of the substrate should asymptotically go to zero.

\[ \lim_{x_3 \to \infty} w_s = 0 \quad \text{Equation 2 - 29} \]
\[
\lim_{x_3 \to \infty} \sigma_{11}, \sigma_{33}, \sigma_{13} = 0 \quad \text{Equation 2 - 30}
\]

On the other hand, the displacement on the top of the substrate (where the substrate is in contact with the film) should be exactly the same as the deflection in the film.

\[
w_s(x_2, x_3 = 0) = -w_f(x_2) \quad \text{Equation 2 - 31}
\]

Thus, it can be shown that a function of the form of equation 2 - 22 satisfies the biharmonic equation and two of the boundary conditions (equations 2 - 29 and 2 - 30).

\[
\phi = A \sin \left( \frac{2\pi}{\lambda} x_2 \right) (1 - Bx_3) \exp \left( -\frac{2\pi}{\lambda} x_3 \right) \quad \text{Equation 2 - 32}
\]

For finding \( B \), it is assumed that the shear stress between the film and the substrate on the top of the substrate is zero.\(^1\) Therefore:

\[
\sigma_{23}(x_3 = 0) = A \frac{2\pi}{\lambda} \cos \left( \frac{2\pi}{\lambda} x_2 \right) \left( B + \frac{2\pi}{\lambda} \right) = 0
\]

\[\Rightarrow B = -\frac{2\pi}{\lambda} \quad \text{Equation 2 - 33}\]

Now, in order to find \( A \), equation 2 - 31 is used. From kinematics it is known that the total displacement in the \( x_3 \) direction can be found by integrating the strain in that direction (when we take shear stress effects to be neglected). Thus:

\[
w_s(x_2, x_3 = 0) = -\int_0^\infty \varepsilon_{33} \, dx_3 = -\int_0^\infty \frac{1}{E_s} \left( \frac{\partial^2 \phi}{\partial x_3^2} - v_s \frac{\partial^2 \phi}{\partial x_2^2} \right) \, dx_3
\]

\[\Rightarrow w_s(x_2, x_3 = 0) = -\frac{2}{E_s 2\pi} A \sin \left( \frac{2\pi}{\lambda} x_2 \right) \quad \text{Equation 2 - 34}\]

Combining equations 2 - 34 and 2 - 22:
\[ \frac{2}{E_s} \frac{2\pi}{\lambda} A \sin \left( \frac{2\pi}{\lambda} x_2 \right) = w_f \sin \left( \frac{2\pi}{\lambda} x_2 \right) \]

\[ \Rightarrow A = \frac{E_s}{2} \frac{\lambda}{2\pi} w_f \] \text{ Equation 2 - 35}

Hence, the stress function can be rewritten as equation 2 - 36.

\[ \phi = \frac{E_s}{2} \frac{\lambda}{2\pi} w_f \sin \left( \frac{2\pi}{\lambda} x_2 \right) \left( 1 + \frac{2\pi}{\lambda} x_3 \right) \exp \left( -\frac{2\pi}{\lambda} x_3 \right) \] \text{ Equation 2 - 36}

In addition, \( \sigma_{33} \) is found:

\[ \sigma_{33}(x_2) = -\frac{E_s}{2} \frac{2\pi}{\lambda} w_f \sin \left( \frac{2\pi}{\lambda} x_2 \right) \] \text{ Equation 2 - 37}

Substituting \( \sigma_{33} \) from equation 2 - 37 as well as the admissible deflection (equation 2 - 22) back into equation 2 - 21, yields an expression which \( w_f \sin \left( \frac{2\pi}{\lambda} x_2 \right) \) cancels out from all the terms, reducing into the form of equation 2 - 38. Note that since the cross section of the film is constant, \( E_s I_s \) can be taken out of the derivative.

\[ E_s I_s \left( \frac{2\pi}{\lambda} \right)^4 - P_f \left( \frac{2\pi}{\lambda} \right)^2 + \frac{E_s}{2} \frac{2\pi}{\lambda} b = 0 \] \text{ Equation 2 - 38}

With the constant cross section, the stress distribution within the film will be the same along the length. Therefore, force can be represented as a constant stress \( (\sigma_f) \) times the area of the film.

\[ P_f = \sigma_f bt \] \text{ Equation 2 - 39}

Substituting for \( P_f \) from equation 2 - 39 and the moment of inertia of a rectangular cross section \( \left( \frac{1}{12} bt^3 \right) \), equation 2 - 38 can be simplified. Rearranging the simplified equation gives:
\[ \sigma_f = \frac{E_f}{12} \left( \frac{2\pi}{\lambda} \right)^2 + \frac{E_s}{2} \left( \frac{\lambda}{2\pi} \right) \]

Equation 2 - 40 shows that the stress is a function of \( \frac{2\pi t}{\lambda} \) as well as the material properties of both materials. In order to find the critical wave length and stress when wrinkling occurs, \( \sigma_f \) should be minimum with respect to \( \lambda \). Hence:

\[ \frac{\partial \sigma_f}{\partial \lambda} = 0 \]

\[ \Rightarrow \lambda_{cr} = \frac{2\pi E_f}{3E_s} \]

Equation 2 - 41

\[ \Rightarrow \varepsilon_{cr} = \frac{\sigma_{f,cr}}{E_f} = \frac{1}{4} \frac{3E_s}{E_f}^2 \]

Equation 2 - 42

Note that because of the constant stress, strain will also be constant (Hooke’s law).

At the critical point, the amplitude of the waves is zero and as the imposed stress or strain on the film is increased, the amplitude of the waves increases. In order to find the amplitude for strains larger than the critical strain, the “accordion model” is used. In this model, it is assumed that the length of the film is always constant. Thus, the length of the film prior to buckling, \( \lambda_0 \), is equal to the length of the entire buckled curve. 

As shown in figures 2 - 9 and 2 - 10, after buckling, the length along the \( x_2 \) direction reduces to \( \lambda \). Knowing the critical strain (\( \varepsilon_{cr} \)) and the imposed strain on the film (\( \varepsilon \)), \( \lambda \) can be determined to have a form of equation 2 - 43.
\[ \lambda = \lambda_0 (1 - \varepsilon + \varepsilon_{cr}) \]  

Equation 2 - 43

For finding the amplitude,

\[ \lambda_0 = s \]  

Equation 2 - 44

Also,

\[ s = \int_0^1 \frac{ds}{dx} \; dx = \int_0^1 \sqrt{1 + \left( \frac{dw}{dx} \right)^2} \; dx \approx \int_0^1 \left( 1 + 2 \left( \frac{dw}{dx} \right)^2 \right) \; dx \]  

Equation 2 - 45

Equation 2 - 38 is valid for \( \frac{dw}{dx} \ll 1 \). Therefore, substituting for \( w \) from Equation 2 - 22, \( \varepsilon_{cr} \) from Equation 2 - 42 and working through the algebra, the amplitude of the waves can be found as defined in equation 2 - 46.

\[ w_f = t \frac{\varepsilon}{\sqrt{\varepsilon_{cr}} - 1} \]  

Equation 2 - 46

Note that this is valid for small imposed displacement and for larger displacements extra factors are needed in order to be able to determine the wave length and amplitude of the waves.\(^4\)

On the other hand, changing the geometry of the film can create different forms of wrinkling which is introduced in the following sections.

**GRADED WRINKLING - THEORY**

In order to put the equilibrium and kinematic equations together for graded wrinkling, one can see that the equations are similar to those of the uniform case; while the effect of the gradients in geometry must now be considered. So, for the case of a trapezoidal geometry, the width of the film and its gradient will play a role in the corresponding eigenvalue equations.
Thus, up to equation 2 – 21, the equations remain the same. However, for the graded case, it is not possible to find an easy admissible deflection for the film. However, from the trend in the solution for the uniform wrinkling, we can assume that the response of the substrate in driving wrinkling has a general form of:

\[ \sigma_{33} b(x_2) = A \frac{\partial^2 w}{\partial x_2^2} \]  

Equation 2 - 47

Therefore, substituting Equation 2 - 46 into equation 2 – 20 gives:

\[
\frac{\partial^2}{\partial x_2^2} \left( E_f l_f \frac{\partial^2 w}{\partial x_2^2} \right) + P_f \frac{\partial^2 w}{\partial x_2^2} + A \frac{\partial^2 w}{\partial x_2^2} = 0 \\
\Rightarrow \frac{\partial^2}{\partial x_2^2} \left( \frac{b(x_2)}{b_0} \frac{\partial^2 w}{\partial x_2^2} \right) + \left( \frac{P_f + A}{E_f \frac{b_0 t_3}{12}} \right) \frac{\partial^2 w}{\partial x_2^2} = 0 
\]

Equation 2 - 48

Thus, equation 2 – 48 can be turned into an eigenvalue problem, where \( A \) represents the eigenvalues of the problem:

\[
\frac{\partial^2}{\partial x_2^2} \left( \frac{b(x_2)}{b_0} \frac{\partial^2 w}{\partial x_2^2} \right) + A^2 \frac{\partial^2 w}{\partial x_2^2} = 0 
\]

Equation 2 - 49

Now, in order to find \( A \), the equilibrium equation for the substrate should be used. However, for such a non-uniform geometry, it is not possible to use the biharmonic equation and the stress function as defined in the uniform case. Therefore, the equilibrium equations should be set up for the substrate.

In the uniform case, the substrate is assumed to have a response in the \( x_2 - x_3 \) plane and the effect of the width of the substrate would cancel out since it is constant everywhere. However, in the graded wrinkling, this assumption does not hold and therefore the equilibrium equations will not be the same as the ones in uniform wrinkling.
In order to set up the equilibrium equations, a free body diagram of a strip element of the substrate is used. This strip element has a varying width and therefore, the equilibrium equations for this element reduce to:

\[
\frac{\partial}{\partial x_2} (\sigma_{22} b(x_2)) + \frac{\partial}{\partial x_3} (\sigma_{23} b(x_2)) = 0 \quad \text{Equation 2 - 50}
\]

\[
\frac{\partial}{\partial x_2} (\sigma_{23} b(x_2)) + \frac{\partial}{\partial x_3} (\sigma_{33} b(x_2)) = 0 \quad \text{Equation 2 - 51}
\]

Afterwards, combining the derivatives of equations 2 - 50 and 2 - 51 gives:

\[
\frac{\partial^2}{\partial x_2^2} (\sigma_{22} b(x_2)) + \frac{\partial^2}{\partial x_2 \partial x_3} (\sigma_{23} b(x_2)) + \frac{\partial^2}{\partial x_3^2} (\sigma_{33} b(x_2)) = 0 \quad \text{Equation 2 - 52}
\]

Equation 2 - 52 is the same equation used for equilibrium in the uniform case; however due to the non-constant width, \(b(x_2)\) cannot come out of the derivatives and therefore, it is not possible to define the stress function in the same way as was done before. But a modified stress function can be defined, in which the derivatives of the function result in stress times the width rather than just the stresses. Therefore, the modified stress function is defined as:
\[ b(x_2)\sigma_{33} = \frac{\partial^2 \phi}{\partial x_2^2} \quad \text{Equation 2 - 53} \]
\[ b(x_2)\sigma_{22} = \frac{\partial^2 \phi}{\partial x_3^2} \quad \text{Equation 2 - 54} \]
\[ b(x_2)\sigma_{23} = -\frac{\partial^2 \phi}{\partial x_2 \partial x_3} \quad \text{Equation 2 - 55} \]

On the other hand, with no tractions on the sides of the substrate except the compressive load, it can be assumed that \( \sigma_{11} = 0 \) and therefore strains in the substrate can be written as:
\[ \varepsilon_{22} = \frac{1}{E_s} (\sigma_{22} - \nu_s \sigma_{33}) \quad \text{Equation 2 - 56} \]
\[ \varepsilon_{33} = \frac{1}{E_s} (\sigma_{33} - \nu_s \sigma_{22}) \quad \text{Equation 2 - 57} \]
\[ \varepsilon_{23} = \frac{1}{G_s} \sigma_{23} \quad \text{Equation 2 - 58} \]

Therefore, substituting from equations 2 - 53, 2 - 54 and 2 - 55 into equations 2 - 56, 2 - 57 and 2 - 58 and then putting into compatibility equations, we will have:
\[ \nabla^2 \nabla^2 \phi = 0 \quad \text{Equation 2 - 59} \]

Therefore, in order to find the effect of the substrate on the film, the biharmonic equation should be solved for the modified stress function. On the other hand, as mentioned earlier, it is not possible to find an admissible solution similar to the case of uniform wrinkling. Therefore, if the assumption in equation 2 - 47 and the definition in equation 2 - 53 are used together, on the top surface of the substrate we have:
\[ \frac{\partial^2}{\partial x_2^2} \phi = A \frac{\partial^2 w}{\partial x_2^2} \quad \text{Equation 2 - 60} \]
Knowing the value of $\sigma_{33}$ on top of the substrate, it is assumed that the expression can be expanded to the entire substrate by just multiplying by a function $f(x_2, x_3)$. This function should be set up in a way that has a value of 1 on the top surface ($x_3 = 0$) and decay away as $x_3$ goes toward infinity. Therefore, similar to the function used in the uniform case, the function used is defined as:

$$
\sigma_{33} b(x_2) = \left( A \frac{\partial^2 w}{\partial x_2^2} \right) (1 + Cx_3) \exp(-Cx_3) \quad \text{Equation 2 - 61}
$$

Note that equation 2 – 61 automatically satisfies the boundary conditions mentioned earlier in equation 2 – 29, 2 – 30 and no shear stress on the top of the substrate. Thus, the modified stress function can be found by integrating equation 2 – 61:

$$
\phi = \iint \left( A \frac{\partial^2 w}{\partial x_2^2} \right) (1 + Cx_3) \exp(-Cx_3) \, dx_2 \, dx_2 \quad \text{Equation 2 - 62}
$$

Now, in order to find $A$, the boundary condition in equation 2 – 31 can be used. On the other hand, the argument in the exponential should be dimensionless; therefore trying to follow similar assumptions the ones in uniform wrinkling, it is assumed that $C = k \Lambda \left( \frac{2 \tan(\theta)}{\ln\left( \frac{b_0}{b(x_2)} \right)} \right)^{\frac{3}{2}} \left( \frac{b_0}{b(x_2)} \right)^2$

($\Lambda$ is the eigenvalue from equation 2 – 49 and $k$ is constant) and the stress function can be rewritten as:

$$
\phi = \iint \left( A \frac{\partial^2 w}{\partial x_2^2} \right) \left( 1 + k \Lambda \left( \frac{2 \tan(\theta)}{\ln\left( \frac{b_0}{b(x_2)} \right)} \right)^{\frac{3}{2}} x_3 \right) \exp \left( -k \Lambda \left( \frac{2 \tan(\theta)}{\ln\left( \frac{b_0}{b(x_2)} \right)} \right)^{\frac{3}{2}} \left( \frac{b_0}{b(x_2)} \right)^2 x_3 \right) \, dx_2 \, dx_2 \quad \text{Equation 2 - 63}
$$

Thus using the boundary condition as in equation 2 – 31:
\[
\frac{2A}{\lambda k} \left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{3}{2}} \frac{\partial^2 w}{\partial x_2^2}
\] Equation 2 - 64

Also, based on the eigenvalue equation, an assumption, based on the form of the eigenvalue equation (equation 2 - 49), is made to be able to find \( A \):

\[
\frac{\partial^2 w}{\partial x_2^2} = -\frac{\Lambda^2 b_0}{b(x_2)} w
\] Equation 2 - 65

And thus, \( A \) can be found by putting equations 2 - 64 and 2 - 65 equal:

\[
A = -\frac{E_s k}{2\Lambda} \left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{3}{2}} b_0
\] Equation 2 - 66

Now, substituting back into equation 2 - 48 and rearranging:

\[
P = \frac{E_f t^3 b_0}{12} \Lambda^2 + \frac{E_s k}{2\Lambda} \left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{3}{2}} b_0
\] Equation 2 - 67

Therefore, the critical load can be found by minimizing the load with respect to the eigenvalue and thus the critical eigenvalue is found to be:

\[
\Lambda_{cr} = k^3 \frac{1}{t} \left( \frac{3E_s}{E_f} \right)^{\frac{1}{3}} \left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}}
\] Equation 2 - 68

Thus, as it is seen from the entire procedure and equation 2 - 68, the eigenvalue in this case is dependent on the geometry of the film which shows itself as the ratio of the smallest to the largest width as well as the gradient in the geometry (i.e. \( \tan(\theta) \)).
On the other hand, equation 2 – 68 is in agreement with the scaling laws presented in the beginning of the chapter. However, since the eigenvalue is dependent on the geometry gradients, graded wrinkling will not be solely dependent on the film thickness and material properties and geometry plays an important role in the characteristics of it.

In addition, putting \( \lambda_{cr} \) back into equation 2-67, the critical load in graded wrinkling can be found.

\[
P_{cr} = \frac{1}{4} \left( \frac{3E_s}{E_f} \right)^{\frac{2}{3}} k^2 \frac{2 \tan(\theta)}{\ln\left(\frac{b_0}{b_t}\right)} \frac{b_0}{b_t}
\]

equation 2 - 69

If \( b_t \to b_0 \), then equation 2 – 69 should reduce into equation 2 – 42; therefore, taking the limit of equation 2 – 69 and equating it with equation 2 – 42 found in the previous section, \( k \) can be found:

\[
k = \left( \frac{L}{b_{uniform}} \right)^{\frac{3}{2}} = \left( \frac{L}{b_0} \right)^{\frac{3}{2}}
\]

equation 2 - 70

Where \( b_{uniform} \) is the width in the uniform case and it is equal to \( b_0 \). Thus substituting \( k \) back into equation 2 – 69 gives:

\[
P_{cr} = \frac{1}{4} \left( \frac{3E_s}{E_f} \right)^{\frac{2}{3}} \frac{2 \tan(\theta)}{\ln\left(\frac{b_0}{b_t}\right)} \frac{L}{b_t}
\]

equation 2 - 71

Therefore, for a constant ratio of \( \frac{E_s}{E_f} \) in graded wrinkling is dependent on the taper angle and other geometrical parameters.
The assumptions made in this section are based on the form of the stress distribution that was found for a non-uniform geometry undergoing compression as well as the form of the eigenvalue equation. These assumptions can act as starting points for solving the eigenvalue equations more accurately and finding more accurate ways to define the critical parameters in all forms of wrinkling and not just the uniform ones.

In order to explore graded wrinkling and the effect of different variables on different characteristics of graded wrinkling, finite element analysis has been used. In the next chapter, the finite element model, procedures and results used for graded wrinkling will be discussed.
REFERENCES


CHAPTER 3 – FINITE ELEMENT ANALYSIS OF GRADED WRINKLING

INTRODUCTION

Graded wrinkling is a form of wrinkling which rises from the non-uniform compressive stress distribution in a film that is bonded to a relatively softer compliant matrix. As it was shown in the previous chapter, wrinkling in general is a buckling instability. The wave length and amplitude of the wrinkles are dependent on the thickness of the film and the ratio of the stiffness of the film to the substrate. In the case of graded wrinkling, the geometry gradients play a role in the form of the corresponding wrinkled surface. Therefore, in order to explore graded wrinkling in detail, finite element analysis is used. For that, finite element models were built and solved using the finite element software ABAQUS.

In the following chapter, the finite element model used, the procedures and the results of the analysis are presented. The analysis is performed as a parametric study over different geometries and different stiffness ratios. Finite element models are created using Python scripts for ABAQUS, in order to alter the geometrical parameters easily. The corresponding scripts can be found in the appendix.

FINITE ELEMENT MODEL

In order to model graded wrinkling, the simple case of a symmetric trapezoidal geometry is used. Due to the symmetry, one-half of the geometry (figure 3 – 1) was modeled to reduce the size and cost of the analysis. The general form of the model is shown in figure 3 - 1.
In this model, five geometrical features are taken into account:

- Shortest width of the film: \( b_l \)
- Largest width of the film: \( b_0 \)
- Length of the film: \( L \)
- Thickness of the film: \( t \)
- Thickness of the substrate: \( D \)

While based on the explanations in the previous chapter, the taper angle, representing the gradient in both geometry and stress, is one of the important parameters in graded wrinkling. Based on the geometric parameters introduced above, the taper angle can be defined based on its tangent:

\[
\tan(\theta) = \frac{1}{2} \frac{b_0 - b_l}{L} \tag{3-1}
\]

Even though the main focus in wrinkling is on the film and its geometry, the thickness of the substrate plays an important role as well. In order for the effect of the substrate to be local, the thickness of the substrate is chosen to be large enough compared to the thickness of the film, so that the bottom of the substrate does not feel anything from the displacements of the film in the \( x_3 \) direction. In this case \( \frac{D}{t} \) is chosen to be in the range of 150-160. As for the other...

---

Figure 3-1 – Schematic of the mirrored model created in ABAQUS for graded wrinkling – (a) Three dimensional view of the model (b) top view of the film. Geometric parameters are indicated on the figures.
geometrical features, in order to see the effect of the change in the angle, \( b_l \) and \( L \) are kept constant, while \( b_0 \) is changed in each of the models. In the following sections of this chapter \( \frac{b_l}{t} = 120, \frac{L}{t} = 200 \) are kept constant. Based on the critical wave length found in the previous chapter for uniform wrinkling, for two materials with a stiffness ratio of 100, \( \frac{\lambda_{cr}}{t} \equiv 20 \). Therefore, \( \frac{L}{t} \) is chosen to be a large multiple of \( \frac{\lambda_{cr}}{t} \), to not alter the wave length predictions for other cases.

**MATERIAL PROPERTIES**

There are two materials used for this analysis. The material for the film is set to be a linear elastic material, while the substrate is chosen to be a hyper-elastic material defined by a neo-Hookean model. The substrate material is chosen to be PDMS which is a hyper-elastic rubbery material. At low strains, PDMS is measured to have a Young’s modulus of about 450 kPa and Poisson’s ratio of close to 0.49 (due to the high bulk modulus of the rubbery materials, on the order of GPa). In order to input this material as a hyper-elastic material in ABAQUS, the compressible neo-Hookean model has been used.

In order to use the neo-Hookean constitutive model, the strain energy potential is defined as below:

\[
U_s = \frac{G_s}{2} (\overline{I}_1 - 3) + \frac{K_s}{2} (J - 1)^2
\]

Equation 3 - 2

Where \( G_s \) is the shear modulus of the substrate, \( K_s \) is the bulk modulus of the substrate, \( \overline{I}_1 \) is the first invariant in the isochoric left Cauchy-Green tensor after factoring out the volume ratio from the deformation gradient and \( J \) is the volume ratio. ABAQUS implements the model of equation 3 – 2 with different coefficients as shown in equation 3 – 3. Therefore, \( \frac{G_s}{2} \) and \( \frac{2}{K_s} \) are given to the software as \( C_{10} \) and \( D_1 \).
\[ U_3 = C_{10} \left( \bar{T}_1 - 3 \right) + \frac{1}{D_1} (J - 1)^2 \]  

Equation 3 - 3

The film material is set to be a linear elastic material with Young’s modulus of 100 times the Young’s modulus of PDMS and a Poisson ratio of close to 0.45.

**BOUNDARY CONDITIONS**

As for the boundary conditions, firstly since the bottom of the substrate should not have an upward movement, it is fixed to not move in the upward direction. As for the symmetry plane, it is set to have no movement in the \( x_1 \) direction to work as a symmetry plane. Lastly, the \( x_1 - x_3 \) plane which is at the largest width, is fixed to not move in the \( x_2 \) direction while the \( x_1 - x_3 \) planes which is at the shortest width is subjected to a \( x_2 \) displacement \( \delta \), to produce a macroscopic strain of 4%, taken over the entire length \( \left( \frac{\delta}{L} = 4\% \right) \).

**MESH**

As for discretization, a structured mesh was used for both film and substrate. In order to increase the accuracy of the calculations, the elements were chosen to be three dimensional, quadratic, continuum elements. An example of such an element is shown in figure 3 - 2.

In the thickness of the film, 4 nodes and 3 elements have been used to have enough accuracy to extract data later for stress, strain and wrinkle profiles.

![Figure 3 - 2 - Schematic of a three-dimensional continuum quadratic element](image)
The far end of the substrate is not of interest and, therefore, fewer nodes are used while more nodes are used close to the film.

In total close to 120,000 nodes and 2,7000 elements are used to discretize this model.

**BUCKLING AND POST BUCKLING ANALYSIS**

Analysis of wrinkling using finite elements is done through a two-step method. First, using the buckling mode, eigenvalues and eigen-modes of the wrinkled surfaces are found and recorded in a separate file. Then using a standard solver, the eigen-modes are used to introduce an imperfection to the same geometrical model in the buckling analysis. The imperfections are introduced as one percent of the whole deformation (eigen-mode) found in the buckling analysis. After that, the same analysis is performed using the standard solver to find the stresses, strains, etc.

The same procedure is used for all the finite element analyses conducted, whether for uniform or graded wrinkling.

**RESULTS FROM FINITE ELEMENT ANALYSIS**

**WRINKLE PROFILES**

In order to investigate the profile of graded wrinkles, a rectangular geometry is chosen as a benchmark. Then the rectangle is tapered into symmetric trapezoids with different taper angles. In the following cases, all the geometries undergo compression and the profiles are presented at the point where they have undergone 4% of compressive strain. As it can be seen in figures 3 - 3 and 3 - 4, for a simple rectangular geometry, the wrinkles are uniform, having the same amplitude and wave length everywhere along the length of the film.
Figure 3 - 3 – Uniform wrinkle profile with outer surfaces of $\varepsilon_{22}$ contours after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm.

Figure 3 - 4 – Non-dimensional wrinkle profile in uniform wrinkling after undergoing 4% of macroscopic strain

The first wave in figure 3 – 4 seems smaller than the rest of the wrinkles and it is due to edge effects in the finite element simulation. Therefore, having a length that is a large multiple of the critical wave length gives enough wrinkles to be able ignore the wrinkles that are affected by the edge effects.

However, in order to see the effect of the graded geometry, the rectangle is tapered into symmetric trapezoids of different taper angles. As shown in the following figures (figures 3 – 5 and 3 – 10), the tapered geometries change the profile of the wrinkles into a graded wrinkled profile with higher amplitude wrinkles at the smaller width (where film stress is the highest) and lowest amplitudes at the largest width (where film stress is the lowest).
Figure 3 - 5 – Graded wrinkle profile with outer surfaces of $\varepsilon_{22}$ contours ($\theta = 2.86^\circ$) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm.

Figure 3 - 6 – Non-dimensional Graded wrinkle profile ($\theta = 2.86^\circ$) after undergoing 4% of macroscopic strain

Figure 3 - 7 – Graded wrinkle pattern with outer surfaces of $\varepsilon_{22}$ contours ($\theta = 5.7^\circ$) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm.
Figure 3 - 8 - Non-dimensional Graded wrinkle profile (θ = 5.7°) after undergoing 4% of macroscopic strain.

Figure 3 - 9 - Graded Wrinkle profile with outer surfaces of ε_{22} contours (θ = 8.53°) after undergoing 4% of macroscopic strain; The length of the film is 50 mm and the thickness of the film is 0.25 mm.

Figure 3 - 10 - Non-dimensional graded wrinkle profile (θ = 8.53°) after undergoing 4% of macroscopic strain.
By tapering the geometry, as shown, the amplitude of the wrinkles turn out to be graded along the length of the film. As it is seen in figures 3 – 5 to 3 – 10 as the width of the film decreases along the length, the amplitude of the waves increase. From the previous chapter, it is known that along this direction, stress in the film increases. Thus, compared to the case of a rectangular geometry which creates a constant stress everywhere in the film, the non-constant stress distribution within the film in graded geometries plays an important role in creating the graded wrinkles.

**STRESS/STRAIN PROFILES**

As a way to understand the graded wrinkling, profiles of the stress and strain ($\sigma_{zz}$ and $e_{zz}$) within the film along two lines were also found. The first line is the line furthest away from the centerlines of the film ($x_3 = \frac{1}{2}$) and the second line is the one that is away from the centerline by a one sixth of the thickness ($x_3 = \frac{1}{6}$). As mentioned earlier, there are 4 nodes in the film cross section and thus it is possible to extract the data for these 2 lines.

![Figure 3 - 11 – Film Cross section on the symmetry plane](image)

As it was seen in the previous chapter, the profile of wrinkles in graded wrinkling is not as simple as it is in uniform wrinkling. Therefore, as a means to understanding this form of wrinkling, as well as characterizing it, stress and strain distributions within the film are investigated. First looking at the stress and strain distribution in the uniform wrinkling, it can
be seen that both stress and strain profiles have the similar sinusoidal shape as the wrinkle profile. (Figures 3 – 12, 3 – 13)

However, compared to stress and strain profiles in uniform wrinkling, in graded wrinkling, the strain, stress and wrinkle profiles have their troughs and peaks happen at the same point along the length. Therefore, with the shape of the strain profile in graded wrinkling, it is easier to find the wave lengths compared to the wrinkle profiles. In the following figures, the strain profiles and stress profiles within the film along the length of the film for different taper angles are shown.

On the other hand, knowing that wrinkling is due to the bending of the film, the strain profiles of the film along different lines at different distances from the centerline of the film are different. Thus, in order to find the wave length of each of the waves, the intersections of the two representative strain profiles are used. This way, the wave length for each wave is introduced as the distance between every other intersection of the two profiles.

Figure 3 - 12 – Non-dimensional (a) Stress and (b) strain profile in the film in uniform wrinkling.
Figure 3 - 13 - Non-dimensional (a) Stress $\frac{\sigma_{22}}{E_f}$ and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 2.86^\circ$

Figure 3 - 14 - Non-dimensional (a) Stress $\frac{\sigma_{22}}{E_f}$ and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 5.7^\circ$

Figure 3 - 15 - Non-dimensional (a) Stress $\frac{\sigma_{22}}{E_f}$ and (b) strain profile in the film in graded wrinkling of a trapezoidal geometry with $\theta = 8.53^\circ$
As shown in figure 3 - 16, by using the intersections of the two strain profiles along the length of the film at $x_3 = \frac{t}{6}$ and $x_3 = \frac{t}{2}$, each of the waves can be identified. Then using those points, the wave lengths can be found and the waves on the wrinkle profiles can be found as well.

Since strain profiles show the similar wrinkling trend as well as giving an easier method in defining the waves in wrinkling and specifically graded wrinkling, in the rest of the work, strain profiles are frequently used.

![Figure 3 - 16](image)

**Figure 3 - 16** – (a) Intersections of the strain profiles for uniform wrinkling after undergoing 3.5% of macroscopic strain (b) Each of the waves found through the strain profile are identified on the uniform wrinkle profile as well.

![Figure 3 - 17](image)

**Figure 3 - 17** – (a) Intersections of the strain profiles for graded wrinkling and the corresponding graded wrinkle profile of a trapezoidal geometry with $\theta = 2.86^\circ$ after undergoing 3.5% of macroscopic strain - The intersections help define the waves in graded wrinkling and knowing the positions. (b) Each of the waves found through the strain profile are identified on the graded wrinkle profile
WRINKLE EVOLUTION

In uniform wrinkling, as the film undergoes compression, when the stress in the film reaches the critical value, then wrinkles initiate over the length, all at the same time, and the amplitude of each wrinkle increases in the same manner as the compressive load increases. However, as mentioned earlier, a graded geometry creates a non-constant stress distribution in the film and thus the wrinkles will not initiate or evolve in the same manner as uniform wrinkling.

![Diagram](image.png)

Figure 3 - 18 - Evolution of wrinkle profile over time as the macroscopic strain is increasing

In order to investigate the evolution process, the wrinkle profiles, stress distributions and strain distributions along the length of film for every time frame at which the calculation were done, were extracted from the results of simulations. As it was shown, the stress and strain profiles for graded wrinkles show a better representation of the wave lengths and thus, they were used as a way to be able to define each of the waves.

For better visualization, all the profiles are presented in a 3D plot, showing how the profiles evolve with increasing compressive displacement.
The evolution plots for graded wrinkles (trapezoids with different taper angles) are shown in figure 3 -18. The main observations from these plots are listed as below:

- Graded wrinkling is a sequential process, meaning that waves appear in a progressive manner with increasing the applied macroscopic strain one after the other.
- The first wave initiates at the point where the stress is the highest (i.e. the shortest edge of the trapezoid)
- As the new waves appear with increasing the macroscopic strain, the amplitude of the older waves increase and their wave lengths decrease.

In addition, it is possible to examine the evolution of the strain profiles. As mentioned in the last section, strain profiles show the same number of waves as the wrinkle profiles, while it is
easier to define each wave from the strain profiles. Therefore, the strain profiles show the same sequential trend as was seen using the wrinkle profiles. In order to investigate the sequential process, the strain profiles at different macroscopic compressive strains were used in the following sections.

During the sequential formation of graded wrinkling, as different locations reach the critical stress, they start to undergo wrinkling and waves show up at their critical wave lengths. The critical wave length of each of the waves can be found using the strain profiles at different times. Through figures 3–18 and 3–19, it can be seen that critical wave length of each of the waves that show up are the same. This trend is shown for trapezoids with different taper angles.

Another observation from figures 3–18 and 3–19 is that as the compressive strain on the film increases, the waves that showed up earlier keep growing into larger amplitudes, but at the same time their wave lengths keep decreasing. As is it shown in figures 3–18 and 3–19, \( \frac{A}{t} \) of the third wave decreases by 0.1 % as the compressive strain increases from 2.9% to 3.2% for the case of \( \tan(\theta) = 0.05 \) and by 0.9 % as the compressive strain increases from 2.9% to 3.7% for the case of \( \tan(\theta) = 0.1 \).

It should be noted that the number of the time steps at which the calculations are performed, is controlled by the software and thus, with the data in hand, the wave lengths found in figures 3–20 and 3–21 are the closest values which can be extracted from these data. In order to increase the accuracy of the data extraction, the number of the time increments should be increased. However, this causes the cost of the calculations to increase as well as.
Figure 3 - 20 - Strain profiles in graded wrinkling of a trapezoidal geometry with $\tan(\theta) = 0.05$ under two different compressive macroscopic strain (a) 2.9% and (b) 3.2%. In (a), as the 3rd wave is showing up, it has $\frac{\lambda}{t} = 20.25$ and then in (b) the 5th wave is has $\frac{\lambda}{t} = 20.29$. Also, $\frac{\lambda}{t}$ of the 3rd wave decreases by 0.1% as the compressive strain is increased by 0.3%.

Figure 3 - 21 - Strain profiles in graded wrinkling of a trapezoidal geometry with $\tan(\theta) = 0.1$ under two different compressive macroscopic strain (a) 2.9%, (b) 3.5% and (c) 3.7%. As the 3rd, 6th and 7th waves show up, they all have $\frac{\lambda}{t} = 20.32$. Also, $\frac{\lambda}{t}$ of the 3rd wave decreases by 0.9% as the macroscopic compressive strain is increased by 0.8%.
The critical wave length as defined for uniform wrinkling is the wave length at which wrinkling initiates and corresponds to the critical load (i.e. stress) defined as the load that buckles the film. The critical wave length in graded wrinkling follows the same definition. However, as seen graded wrinkling is a sequential process and it is easier to identify waves using the strain profiles than the actual wrinkle profiles. Therefore, the critical wave length will be defined using the strain profiles while the sequential process is also taken into account.

Firstly, since graded wrinkling is a sequential process, each of the waves will initiate at a different level of macroscopic strain. The smallest width of the trapezoidal film experiences the largest stress and thus the stress at that point reaches the critical stress, the film starts to undergo wrinkling. At this point, the first wave initiates. Therefore, the wave length of the first wave can be called the “initial critical wave length” and then wave lengths of the consequent waves are called 2nd critical wave length, 3rd critical wave length and so on, respectively.

Using the strain profiles at different time frames, it is seen that the critical wave length of each of the consequent waves are the same as the initial critical wave length. In figures 3 – 20 and 3 – 21 the strain profiles of a wrinkled film, at different time frames (i.e. different macroscopic strains) are shown, where the critical wave lengths of the waves that are just showing up are specified. As it is seen for a couple of different taper angles, the critical wave length stays constant as the film goes through the sequential wrinkling process, showing that the critical wave length is independent of position (i.e. $x_2$).

Another aspect of critical wave length in graded wrinkling is the fact that it is dependent on the geometry. As is it seen from figures 3 – 20 and 3 – 21 critical wave lengths of trapezoidal films with different taper angles are different. In order to bring geometry into play, the critical wave lengths found for different geometries have been plotted against the taper angle for
different geometries. As it is presented in figure 3 - 22 critical wave length of trapezoidal films with different taper angles increases with increase in taper angles. In other words, increase in the taper angle represents an increase in the stress gradient within the film. Thus it can be said that increase in the stress gradient within the film increases the critical wave length in graded wrinkling.

Another aspect of graded wrinkling is the non-uniform amplitude of the waves. Due to the non-constant stress distribution in the film, in graded wrinkling amplitude of the waves turn out to be non-uniform as well. As it was demonstrated in previous sections, in graded wrinkling, each of the waves shows up one by one and with the same critical wave length. Therefore, assuming that the length of the film does not change over the process, the trend in the amplitudes can be explained using the accordion model. When the initial wave shows up, wave length is $\lambda_0$ and amplitude is zero; then as the displacement of the film is increased, the wave length of the wave decreases to $\lambda < \lambda_0$ while the length of the curve (i.e. wrinkle curve) should be constant. Therefore, in order for the length to be constant, amplitude of the wave keeps growing. This happens while the consequent waves either have not shown up or just started to
go through the same process. Thus, at each instance of time, the waves that can be seen will have different amplitudes and wave lengths.

Figure 3 - 23 shows the trend in amplitude of the waves over the length of the film in graded wrinkling for different taper angles. Figure 3 - 24 shows the evolution of the amplitudes of the wrinkles as the macroscopic strain on the film increases in the case where $\tan(\theta) = 0.05$. It can be seen that as $\varepsilon_{\text{macro}}$ increases, the ratio of the amplitude over the maximum amplitude, at that macroscopic strain, increases for the wrinkles. In other words, the waves which initiated later than the first one will grow to have larger amplitudes and closer to the maximum amplitude.

Figure 3 - 23 – Trend in the amplitude of the wrinkles over the length of the film at 4% macroscopic strain for (a) $\tan(\theta) = 0.05$, (b) $\tan(\theta) = 0.1$, (c) $\tan(\theta) = 0.15$
Figure 3 - 24 – (a) Trend in the maximum amplitude in graded wrinkling over different macroscopic strains for $\tan(\theta) = 0.05$, (b) ratio of the amplitude over the maximum amplitude at that macroscopic strain, over a range of macroscopic strains for $\tan \theta = 0.05$

**EIGENVALUES FROM THE BUCKLING MODE**

In order to investigate the eigenvalue solution proposed in chapter 2 (equation 2 – 67), the results from buckling step of the finite element analysis is used. In that step, the critical eigenvalue of the buckling of different geometries with (with similar $L$ and $b_I$) were found. For all the cases, thickness of the film and material properties are the same. The only thing that changes between the cases is the geometrical features which show themselves in $\tan(\theta)$ and $\frac{b_I}{b_0}$. Therefore, the first or the critical eigenvalue found via the finite element analysis were plotted
against $\left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}}$, which represents the effect of the geometry change when $L$ and $b_0$ are constant. As it is seen in figure 3–23, the eigenvalues from the finite element show a linear dependence with $\left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}}$.

In this plot, $\left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}}$ for the uniform geometry is presented as the limit of this ratio as $b_1 \to b_0$; even though $\tan(\theta)$ and $\ln \left( \frac{b_0}{b_1} \right)$ are zero at that limit, $\frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)}$ has a finite limit. 

$\left( \lim_{b_1 \to b_0} \left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}} \right) = 0.7746$ Therefore, this figure confirms that the assumptions made in solving the eigenvalue problem are good starting points for finding the solution to this problem.

Figure 3 - 25 - Eigenvalues from finite element analysis vs. the geometric parameter $\left( \frac{2 \tan(\theta)}{\ln \left( \frac{b_0}{b_1} \right)} \right)^{\frac{1}{2}}$
REFERENCES


CHAPTER 4 - EXPERIMENTS

INTRODUCTION

In the previous chapters, the theory behind wrinkling (either uniform or graded) and the results and observations from the finite element modeling of both uniform and graded wrinkling were discussed. In chapter 2, the equations were kept dimensionless, so that only the relative ratio of the material properties and geometrical features be important and not the absolute values. Therefore, from the theoretical point of view, wrinkling can occur at different scales from hundreds of nanometers up to tens of centimeters or even larger.

Also in ABAQUS, the input values for geometry and for material properties are all scalable based on the dimension chosen for the analysis. Therefore, through finite element analysis it is possible to see that the relative dimensions and material properties play the important role.

In the following chapter, different experimental methods used for creating wrinkled surfaces are introduced. Later in the chapter, the experimental method used in this work for creating graded wrinkled surfaces and the results of the experiments are shown. The experiments (presented later on in this chapter) were performed in collaboration with Dr. Jose Yague and Professor Karen Gleason in the department of chemical engineering at MIT.

METHODS FOR CREATING WRINKLED SURFACES

There are different methods used for creating wrinkled surfaces. The methods use various mechanical, thermal or chemical techniques to create mismatched deformations in the film and the compliant substrate. This will cause the film to undergo compression and result in wrinkling.
In thermal methods, thin films of metals with thicknesses of the orders of tens of nanometer are deposited at elevated temperature on a thicker softer substrate (usually elastomers). After the film-substrate composite is cooled down, due to the mismatch in the thermal expansion coefficient of the two materials, a resulting compressive load is created in the film. Therefore, at a critical temperature, the film starts to buckle into a wrinkled form. Figure 4-1 shows the thermal procedure Bowden et. al used for creating wrinkle surfaces.

![Figure 4-1: Wrinkling using the thermal strain due to heating the substrate and cooling it. With the film constrained to the substrate, it undergoes compression and thus wrinkles.](Image)

Another method is using solvent swelling/shrinkage of a laterally confined elastomer which is based on a method developed by Southern and Thomas. Through this method, an elastomeric film is deposited on a stiff substrate. Since the film is bonded to the substrate, a corresponding solution is deposited on the film to make it swell. Since the film is confined, the swelling will cause an osmotic stress to be coupled with the confinement and thus results in a compressive stress in the film and therefore, the film will start to wrinkle.
Swelling is constrained by the rigid substrate. As a result, a compressive stress develops which drives wrinkle formation.

Figure 4 - 2 – Wrinkling using the swelling of a laterally confined elastomer

Another method is focused ion beam exposure (FIB). Through this method, elastomers such as PDMS are exposed to focused ion beams, creating a stiff skin on the elastomers. Also, it has been shown that upon formation, the stiff film tends to expand in the direction perpendicular to the direction of the irradiation. Thus, the mismatch between the deformation of the skin and the PDMS substrate causes the skin to undergo wrinkling. In addition, by controlling the FIB and the exposed area, it is possible to create different patterns from one dimensional ordered wrinkled pattern to two dimensional complex patterns.

Figure 4 - 3 – Schematic of method creating wrinkled patterns using the Focused Ion Beam method

Another method, which was also used to explore graded wrinkling, is mechanical stretching and releasing. In this method, the elastomer substrate is pre-stretched and then a thin coating of a stiff material is deposited on the pre-stretched substrate. After that upon
releasing the substrate, the thin coating will undergo compressive loading and after a critical strain, it starts to wrinkle. In this method, unlike the thermal expansion which only leads to a few percents of strain, the substrate can be pre-stretched from fractions of percentage up to a few hundreds of percent of pre-strain.\cite{6,7}

![Diagram showing wrinkle formation through stretching and releasing](image)

Figure 4 - Wrinkling using the stretch and release method. After the substrate is stretched (having a non-zero strain energy), the film is deposited (adhered) to the substrate. At this stage the film has zero strain energy. Releasing the stretched substrate, puts the film under compression and results in a wrinkled pattern.\cite{6}

In this work, in order to create wrinkled surfaces, specifically the one dimensional graded wrinkles, the stretching and releasing method is used. In the following chapter, the methods used and the resulting patterns and trends are presented.

**WRINKLING THROUGH MECHANICAL STRETCHING AND RELEASING**

As was mentioned in the earlier chapters, wrinkling is mainly dependent on the relative values of the material properties and geometrical features. Therefore, the stretch and release method can be used in either micro-scale or milli-scale to create wrinkled surfaces.

In order to create wrinkled surfaces in the micro-scale, in this work, PDMS sheets of thickness of 2 – 3 mm are used as the substrates and for the coating, the method of initiated Chemical Vapor Deposition is used to deposit a thin film of polymer on the PDMS substrate. This method was used for the first time by Yin et al. to obtain wrinkled patterns.\cite{8} With the iCVD

63
method, thin polymer coatings of hundreds of nanometers are deposited on the substrate and upon the release the thin films undergo compressive loads and thus wrinkle. The resulting wrinkles have wavelengths of the order of microns. Figure 4 - 5 shows a schematic of the procedure.

**Figure 4 - 5** - Schematic of the experimental method used

In the iCVD method, one or more vinyl monomers and an initiator are introduced in the vapor phase inside a vacuum reactor. In the reactor, the initiator is thermally decomposed by heated filaments to create radicals. After that, polymerization happens on the surface of the substrate through free radical mechanism. This method is a low-energy, one step and solvent free process. Thus any kind of substrate can be coated with this method without having any incompatibilities or unwanted modifications. 8, 9 A schematic of the iCVD process and the vacuum reactor used for it is presented in figure 4 - 6.

**Figure 4 - 6** - The iCVD process and the vacuum reactor setup

---

64
SAMPLE GEOMETRY AND MATERIALS

As was shown in previous chapters, one method to obtain graded wrinkles is to use a film with gradient in the width. Therefore, the symmetric trapezoid was used as the simplest case.

The method used is mechanical stretching and releasing. For the substrate, PDMS (Polydimethylsiloxane) is used. PDMS is a non–linear elastomeric material, and as mentioned it can be modeled as an almost incompressible Neo-Hookean material with a Young’s modulus of close to 450 kPa. The PDMS substrate is produced in sheets of about 2 mm thick and then cut into shapes as presented in figure 4 – 7 – a. Figure 4 – 7 – b shows two of the representative samples used in this work.

![Figure 4-7](image)

Figure 4 – 7 – (a) Geometry which the PDMS substrates are cut into. (b) Two of the samples used for creating graded wrinkling. The samples are made from PDMS.

The samples consist of a trapezoidal body that is connected to two rectangular grip areas to be used for the pre-stretching.

As for the coating (i.e. film), as mentioned, the iCVD method is used to deposit thin layers of stiff polymers on PDMS. In this work, a thin film of 300 nm of EGDA (ethyleneglycoldiacrylate) is deposited on the substrate. EGDA is a polymer which shows elastic behavior with Young’s modulus of $E_f \approx 775 \text{ MPa}$ and Poisson’s ratio of $\nu_f = 0.45$.\textsuperscript{8}
For creating wrinkled surfaces, PDMS substrates are prepared and cut into the geometries similar to the one shown in figure 4–10. Then the substrates are pre-stretched to 20% and placed in the vacuum reactor for coating, using the iCVD method. The substrates are coated with 300 nm of EGDA and after that, the load and the samples are released, putting the film under compression and thus the wrinkles are formed.

Through this process different samples were examined. The results from three different samples are shown below. These samples having the same length have varied taper angles as well as varied ratios of the smallest width to largest width by keeping the largest width constant and changing the smallest width.

The first sample, as shown in figure 4–10, has a taper angle of \( \theta = \tan^{-1} 0.08 = 4.57^\circ \) and the ratio of the smallest width to largest width is \( \frac{2}{3} \). With the sample lengths being in the order of millimeters while the thickness of the film in the nano-scale, the wave length and amplitude of the waves are on the micron scale. Therefore, in order to examine the wave lengths and amplitudes, in each sample, three representative areas are chosen and the wave lengths and amplitudes of the waves are measured in those areas. These areas, which represent "smallest width", "medium width" and "largest width" regions are chosen in arbitrary positions close to those areas. As it is shown on figure 4–10, the exact location of these points is not precisely determined. Hence, they give an estimate of the wave length and amplitudes in the representative region of the sample and enable comparing the wave lengths found at different locations along the length.

The images of the wrinkled surfaces were obtained using a Zeta-20 optical profilometer, which gives a three dimensional image of the surface pattern as well as measuring different dimensions that can be captures in the image. As a result, for each of the areas in each sample, it
is possible to get images of the top view of the wrinkled pattern and also a three dimensional view of the same surface. A line in the two dimensional view and a surface in the three dimensional view show the cross section at which the wave length and amplitude were measured.

![Diagram](image)

Figure 4 - 8 - Sample geometry with $\theta = 4.57^\circ$ and $\frac{b_1}{b_0} = \frac{2}{3}$

Figures 4 - 9 and 4 - 10 show the results of the imaging of the three areas specified in figure 4 - 8. Figure 4 - 9 shows the two dimensional and three dimensional view of the wrinkle in the area with the smallest width as noted in figure 4 - 8 by area 1, the medium width noted as area 2 and the largest width noted as area 3. Figure 4 - 10 shows the wrinkle cross section as well as the peak and trough chosen for measuring the amplitude and wave length of the waves in each of the three areas.

The resulting amplitudes and wave lengths found from the figures 4 - 9 and 4 - 10 are presented in table 4 - 1. As it can be seen from the numbers, by increasing the width of the film, the amplitudes of the waves decrease while the wave lengths of the waves increase.
Figure 4 - 9 - Wrinkled pattern corresponding to geometry in figure 4 - 8 with $\theta = 4.57^\circ$ and $\frac{b_L}{b_0} = \frac{2}{3}$ and pre-strained to 20% (a) Top view and (b) three dimensional view of Area (1) with the largest width (c) Top view and (d) three dimensional view of area (2) with the medium width (e) Top view and (f) three dimensional view of area (3)
Figure 4 - Cross section of the wrinkled surface of geometry in figure 4 - 8 with $\theta = 4.57^\circ$ and $\frac{b_1}{b_2} = \frac{2}{3}$ and pre-strained to 20% (a) in area (1) with the largest width, the horizontal distance between the designated peak and trough ($c_1$) is 12.09 $\mu$m and the vertical distance ($2A$) is equal to 5.468 $\mu$m (b) in area (2) with the medium width, the horizontal distance between the designated peak and trough ($c_2$) is 11.96 $\mu$m and the vertical distance ($2A$) is equal to 6.455 $\mu$m (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough ($c_3$) is 11.82 $\mu$m and the vertical distance ($2A$) is equal to 6.898 $\mu$m.
Table 4 - 1 – Data for wave length and amplitude of the wrinkles in the graded geometry with 
with \( \theta = 4.57^\circ \) and \( \frac{b}{b_0} = \frac{2}{3} \) and pre-strained to 20% (corresponding to figure 4 – 8

<table>
<thead>
<tr>
<th>Area</th>
<th>( \lambda (\mu m) )</th>
<th>( A (\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Largest width)</td>
<td>24.18</td>
<td>2.734</td>
</tr>
<tr>
<td>2 (Medium width)</td>
<td>23.92</td>
<td>3.2275</td>
</tr>
<tr>
<td>3 (Smallest width)</td>
<td>23.64</td>
<td>3.449</td>
</tr>
</tbody>
</table>

The images from the other samples’ geometry are shown in figures 4 – 11 and 4 – 12, the results are shown in figures 4 – 13 to 4 – 16. Also, the final results for all the cases are presented in tables 4 – 2 and 4 – 3.

Figure 4 - 11 - Sample geometry – In this geometry
\( \theta = 2.3^\circ \) and \( \frac{b}{b_0} = \frac{5}{6} \)

Figure 4 - 12 - Sample geometry – In this geometry \( \theta = 6.84^\circ \) and \( \frac{b}{b_0} = \frac{1}{2} \)
Figure 4 - 13 – Wrinkled pattern corresponding to the geometry in figure 4 - 11 with $\theta = 2.3^\circ$ and $\frac{\alpha_h}{\alpha_0} = \frac{5}{6}$ and pre-strained to 20% (a) Top view and (b) three dimensional view of area (1) with the largest width (c) Top view and (d) three dimensional view of area (2) with the medium width (e) Top view and (f) three dimensional view of area (3) with the smallest width.
Figure 4 - 14 – Cross section of the wrinkled surface of figure 4 – 11 with $\theta = 2.3^\circ$ and $\frac{b_a}{b_a} = \frac{5}{6}$ and pre-stained to 20% (a) in area (1) with the largest width, the horizontal distance between the designated peak and trough ($c_1^2$) is 10.5 $\mu$m and the vertical distance ($2A$) is equal to 5.851 $\mu$m (b) in area (2) width the medium width, the horizontal distance between the designated peak and trough ($c_2^2$) is 10.76 $\mu$m and the vertical distance ($2A$) is equal to 5.78 $\mu$m (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough ($c_3^2$) is 10.76 $\mu$m and the vertical distance ($2A$) is equal to 6.134 $\mu$m.
Figure 4 - 15 – Wrinkled pattern corresponding to figure 4 – 12 with $\theta = 6.84^\circ$ and $\frac{b_t}{b_0} = \frac{1}{2}$ and pre-strained to 20% (a) Top view and (b) Three dimensional view of Area (1) with the largest width (c) Top view and (d) Three dimensional view of area (2) with the medium width (e) Top view and (f) Three dimensional view of area (3) with the smallest width
Figure 4 - 16 - Cross section of the wrinkled surface of figure 4 - 12 with $\theta = 6.84^\circ$ and $\frac{b_1}{b_0} = \frac{1}{2}$ and pre-strained to 20% (a) in area (1) with the largest width, the horizontal distance between the designated peak and trough $(\frac{1}{2})$ is 11.56 $\mu m$ and the vertical distance $(2A)$ is equal to 7.72 $\mu m$ (b) in area (2) with the medium width, the horizontal distance between the designated peak and trough $(\frac{1}{2})$ is 12.22 $\mu m$ and the vertical distance $(2A)$ is equal to 6.714 $\mu m$ (c) in area (3) with the smallest width, the horizontal distance between the designated peak and trough $(\frac{1}{2})$ is 13.55 $\mu m$ and the vertical distance $(2A)$ is equal to 6.242 $\mu m$
Table 4 - 2 - Data for wave length and amplitude of the wrinkles in the graded geometry with \( \theta = 2.3^\circ \) and \( \frac{b_t}{b_0} = \frac{5}{6} \) and pre-strained to 20% corresponding to figure 4 - 11

<table>
<thead>
<tr>
<th>Location on sample shown in figure 4 - 11</th>
<th>( \lambda(\mu m) )</th>
<th>( A(\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Largest width)</td>
<td>21</td>
<td>2.9255</td>
</tr>
<tr>
<td>2 (Medium width)</td>
<td>21.52</td>
<td>2.89</td>
</tr>
<tr>
<td>3 (Smallest width)</td>
<td>21.52</td>
<td>3.067</td>
</tr>
</tbody>
</table>

Table 4 - 3 - Data for wave length and amplitude of the wrinkles in the graded geometry with \( \theta = 6.84^\circ \) and \( \frac{b_t}{b_0} = \frac{1}{2} \) and pre-strained to 20% corresponding to figure 4 - 12

<table>
<thead>
<tr>
<th>Location on sample shown in figure 4 - 12</th>
<th>( \lambda(\mu m) )</th>
<th>( A(\mu m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Largest width)</td>
<td>27.1</td>
<td>3.121</td>
</tr>
<tr>
<td>2 (Medium width)</td>
<td>24.44</td>
<td>3.357</td>
</tr>
<tr>
<td>3 (Smallest width)</td>
<td>23.12</td>
<td>3.86</td>
</tr>
</tbody>
</table>

**DISCUSSION**

In the previous section, the results of experiments done on graded wrinkling were presented. As mentioned, the experiments considered three samples with different taper angles. Tables 4 - 1, 4 - 2 and 4 - 3, present results for each of the samples. Tables 4 - 4 and 4 - 5 show a summary of the results based on the taper angles and the ratio of the smallest to the largest width of the samples.

From the results different trends can be seen:

- In one sample, as the width of the sample increases, amplitudes of the waves decrease: It was also shown with the finite element analysis that in graded wrinkling as the width of the trapezoidal film increases, amplitudes of the waves decrease.
In one sample, as the width of the sample increases, wave lengths of the waves increase. Again through finite element analysis, it was shown that wave length of the waves at different locations of a sample are not the same and in a trapezoidal film, as the width of the film increases, wave length of the waves increase.

- Increase in the taper angle, increases the amplitude of the waves at a given macroscopic strain of the sample.
- Increase in the taper angle, increases the wave length of the waves.

Table 4 - 4 - Data for the amplitude of the wrinkles in the graded geometry different taper angles and different ratio of the smallest width to the largest width

<table>
<thead>
<tr>
<th>$\tan \theta$</th>
<th>$\frac{b_0}{b_1}$</th>
<th>$A , (\mu m)$ (Area 1)</th>
<th>$A , (\mu m)$ (Area 2)</th>
<th>$A , (\mu m)$ (Area 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>$\frac{5}{6}$</td>
<td>2.9255</td>
<td>2.89</td>
<td>3.067</td>
</tr>
<tr>
<td>0.08</td>
<td>$\frac{2}{3}$</td>
<td>2.734</td>
<td>3.2275</td>
<td>3.449</td>
</tr>
<tr>
<td>0.12</td>
<td>$\frac{1}{2}$</td>
<td>3.121</td>
<td>3.357</td>
<td>3.86</td>
</tr>
</tbody>
</table>

Table 4 - 5 - Data for the wave length of the wrinkles in the graded geometry different taper angles and different ratio of the smallest width to the largest width

<table>
<thead>
<tr>
<th>$\tan \theta$</th>
<th>$\frac{b_0}{b_1}$</th>
<th>$\lambda , (\mu m)$ (Area 1)</th>
<th>$\lambda , (\mu m)$ (Area 2)</th>
<th>$\lambda , (\mu m)$ (Area 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>$\frac{5}{6}$</td>
<td>21</td>
<td>21.52</td>
<td>21.52</td>
</tr>
<tr>
<td>0.08</td>
<td>$\frac{2}{3}$</td>
<td>24.18</td>
<td>23.92</td>
<td>23.64</td>
</tr>
<tr>
<td>0.12</td>
<td>$\frac{1}{2}$</td>
<td>27.1</td>
<td>24.44</td>
<td>23.12</td>
</tr>
</tbody>
</table>

The only sample which does not follow all the trends above is the sample with the smallest taper angle ($\tan(\theta) = 0.04$). In that sample, as we move toward the larger width, the amplitude seems to be nearly constant and varying around a value close to $2A \approx 5.8 \, \mu m$. Also,
looking at the wave lengths of the wave in this sample, again it can be seen that the wave lengths are nearly constant and again varying around the value of $\frac{A}{2} \approx 10.6 \mu m$. This can be explained by the fact that this taper angle is quite small and does not create a substantial stress gradient in the film, therefore the results of the wrinkling in this sample is more similar to uniform than graded wrinkling.

In addition, one needs to have in mind that the peaks and troughs picked for the measurements in each of the samples are not precise and their exact location on the films were not identified in the set of experiments. Therefore, this makes the comparison rather hard and less accurate. A more complete set of experiment is part of ongoing work.

**PATTERNING THE COATING ON A SUBSTRATE**

Another point to be made is that when the coating is patterned onto the substrate in trapezoidal or other shapes, it is not directly feeling stressed during the mechanical stretching method. Instead, the compressive loading is transferred to the film through the shear stress between the film and substrate through a shear lag load transfer mechanism. The schematic of the idea is shown in figure 4 - 17. As the substrate undergoes tensile loading (compression or tension) and the coating is deposited in a pre-stressed way, by release of the tension, the film will be loaded through the shear stress between the film and the substrate denoted by $r$.

The free body diagram shows how the shear stress in the interface builds up the compressive stress in the film. Therefore, in the film there will be a certain distance at the beginning and the end of the length of the film which feels the shear lag load transfer. These two areas over which there is compressive stress will not undergo wrinkling and there will be a “lag” region before wrinkles occur in such a patterned domain.
\[ \tau \cdot b \, dx = d \sigma_f \cdot b \, t \]
\[ \frac{d \sigma}{dx} = \frac{\tau}{t} 
\]

\[ dx \]
\[ d \sigma_f \]

Figure 4 - 17 – Free body diagram of a film on elastic substrate where the substrate is under loading. The loading is transferred to the film through the shear stress between the film and the substrate.

In our experiments, with the two rectangular areas at sides of the sample, it is possible to have enough grip areas as well as not let the load transfer area reach the main sample area (trapezoidal body).

\[ \tau \]
\[ \sigma_f \]
\[ L \]

Figure 4 - 18 – General form of the shear stress in the thin film and the tensile stress in the film. The tensile stress builds up in the film as a result of the shear stress. The relation between them is governed by the equilibrium equation as presented in figure 4 – 17.

In conclusion it can be said that graded wrinkling can be also seen in micro-scale samples. They seem to be following the same trend in amplitude and wave length as expected from finite element analysis.
REFERENCES


CHAPTER 5 – CONCLUSIONS AND FUTURE WORK

INTRODUCTION

In the previous chapters, the wrinkling phenomenon was studied using analytical models, finite element analysis and, finally, experiments. As seen, wrinkling of a coating on a compliant substrate is a controllable phenomenon which gives us the ability to create surface patterns with different desired geometrical features, either uniform or graded, with different wave lengths and amplitudes. If wrinkling is accomplished using the mechanical stretching method, it can even be tuned by changing the pre-strain of the substrate.

In this chapter, a summary of the conclusions on graded wrinkling as another method of giving different textures to the surfaces and a discussion on the future directions of this research is presented.

SUMMARY AND CONCLUSIONS

Wrinkling is the buckling instability of a thin film of relatively stiffer material bonded to a soft compliant matrix. Previously, different research has been conducted on uniform wrinkling through both analytical solutions and experiments to show different aspects of uniform wrinkling. In this work, graded wrinkling was introduced, which is the same buckling instability, but with altered geometry. Graded wrinkling can happen in any geometry with gradients in the geometrical features or other means of giving a stress gradient in the film. Here for simplicity, symmetric trapezoids were used which have a linear gradient in their width.

As shown with the analytical models, solving the eigenvalue equations for the graded wrinkling is not an easy task and the solutions presented were based on some assumptions. Those assumptions were made based on the different stress profiles discussed earlier in chapter
2 as well as the previous solutions presented by others. It was seen in chapter 3 that the assumptions were reasonable and that the analytical solution agreed well with the eigenvalues found from ABAQUS finite element based buckling solutions. Thus the analytical eigenvalue solutions can be a starting point for finding the analytical solutions for graded wrinkling in trapezoidal films.

Through finite element models, different aspects of graded wrinkling were discussed. Firstly, strain and stress distributions along the length of the film in graded wrinkling are graded as well. Due to the geometry gradient in the film and the substrate, the compressive strain and stress in the length of the film have a non-uniform distribution. Therefore, after buckling occurs, the stress and strain distribution along the length of the film will give a graded wrinkled profile.

Secondly, through finite element analysis, it was seen that graded wrinkling is a sequential process where wrinkles initiate one after the other depending on the stress distribution within the film. For the trapezoidal case, the first wave initiates at the smallest width where the stress is the highest and after that sequentially other locations start to wrinkle as they reach a critical stress.

The third point is that compared to the uniform wrinkling, through graded wrinkling it is possible to get wrinkling with different amplitudes and wave lengths in one film. By using the trapezoidal geometry and knowing the stress distribution in the film, it was concluded that at points where the stress is highest in the film, the wrinkles first initiate and the amplitude of the wrinkles are the highest, while for the wrinkles that initiate last, at points where the stress is the lowest in the film, the amplitude of the wrinkles are the smallest and the wave length of the wrinkles are the largest.

On the other hand, it is not an easy task to define wave lengths in the graded wrinkling. Knowing that the stress distribution in the wrinkled coating is mainly due to buckling, the stress
and strain distributions along different lines along the film give a way of cleanly defining the wave length for the waves in graded wrinkling.

In addition, by examining the stress or strain profiles at different macroscopic strains it was seen that the critical wave length at which each of the waves initiate are nearly the same for all the waves in the film in the case of graded wrinkling and the critical wave length seems to be independent of location of the wave along the length of the film. However, it is dependent on the taper angle of the film and as it was shown critical wave length has an increasing trend with the increase in the taper angle in the film (i.e. a higher stress gradient gives a larger wave length).

Lastly, it was seen that amplitudes of the waves in graded wrinkling have a decreasing trend as the stress in the film decreases along the length. In trapezoidal geometry, as the width of the film increases, at any given macroscopic strain the amplitude of the waves decrease since the stress level in the film is lower. Also, after one wrinkle initiates, by increasing the macroscopic strain on the film, the amplitude of that wave starts to grow.

Another aspect of wrinkling is the tunability of the phenomenon. As was shown, amplitudes and wave lengths of the wrinkles are dependent on the stiffness ratio of the materials used, thickness of the film as well as the geometry gradients in the film. Therefore, by changing any of these 4 parameters, it is possible to tune the wrinkles to have wrinkles with different desired amplitudes and wave lengths. Also, through the analytical solutions it was shown, the parameters which define wrinkles can be introduced in dimensionless forms. Therefore, wrinkles can be created in different dimensions from hundreds of nano-meters up to a couple of centimeters and even larger.
FUTURE WORK

Wrinkling is a broad topic which is still under investigation. By using the graded wrinkling, it is possible to create different surface patterns by manipulating the geometry in different ways. This research can be further expanded in different directions.

In the analytical solutions, the next step is modifying the assumptions to find the best definitions for the eigenvalues, critical wave lengths and other aspects of wrinkling. It is desired to find an admissible function that would satisfy the boundary conditions and the governing equations.

Another direction is using finite elements to investigate other possible patterns by altering other geometrical features, such as the thickness of the film or using asymmetric geometries. Also, it is possible to create new patterns by putting the trapezoids in different arrangements. Examples of these patterns are shown in figures 5 - 1 and 5 – 2.

Figure 5 - 1 – A trapezoidal geometry combination to create new surface textures

Figure 5 - 2 – Anti-trapezoidal geometry combination to create new surface textures
In addition, it is desired to create repeating patterns with trapezoids using both finite element and through real experiments. Examples of such repeating patterns are shown in figure 5 - 3.

![Figure 5 - 3: Periodic patterns for creating periodic surface patterns](image)

Such patterns can be implemented in the finite elements by only modeling a representative volume element (RVE) and applying appropriate periodic boundary conditions to it.

Another direction for this research is to investigate the properties of such surfaces in contact with fluid and flow. Presence of the wrinkles on the surfaces can manipulate the boundary layers in flows and thus could be used as a possible way for drag control over different surfaces. In addition, it is possible to run different experiments to measure contact angles of different fluids on such wrinkled surfaces and investigate the possible ways to control the wettability of surfaces by manipulating the surface texture.

One other possible direction of this research is the use of wrinkling either uniform or graded in controlling the light reflectivity of surfaces. Presence of the wrinkles on surfaces,
uniform or non-uniform can give a way to control light transmission and reflection of different surfaces.
APPENDIX 1 - PYTHON SCRIPT FOR CREATING THE MODEL
FOR BUCKLING ANALYSIS

# Creating a THREE_D and DEFORMABLE_BODY of Trapezoid shape
# Shabnam Raayai
# September 2012
# The script should be run in the abaqus interface
# Sub = Substrate, F = Film

# Defining a Viewport for
myViewport = session.Viewport(name='Viewport for Model Sub-F', origin=(0, 0), width=150, height=100)

# creating a viewport
import sketch
import part

# Creating the Model
SubFModel = mdb.Model('SubF');

# Creating the sketch
SubFSketch = SubFModel.ConstrainedSketch(name='SubFSktech', sheetSize=200.00)

# Dimensions of the Trapezoid
# Long side
L = 30.00
# Short side
S = 25.0
# Height of the plate
H = 50.00
# Depth of the substrate and film together
D = 40.00
# Thickness of the plate
t = 0.25

# Mesh Type
Linear = False

# Material Type
HyperE = True

# Points for constructing the Trapezoid
xyCoords = [(0, H/2), (L/2, H/2), (S/2, -H/2), (0, -H/2), (0, H/2)]

# Drawing the Trapezoid, using a four loop and the point specified in the array above
# Dimensions L and S and H on x-y plane
for i in range(len(xyCoords)-1):
    SubFSketch.Line(point1=xyCoords[i], point2=xyCoords[i+1])

# Creating the Part as a THREE-D and DEFORMABLE_BODY
SubFPart = SubFModel.Part(name='SubFPart', dimensionality=THREE_D,
type=DEFORMABLE_BODY)

# Extrude the body with D
SubFPart.BaseSolidExtrude(skech=SubFSketch, depth=D)

# Creating the partitions for Substrate and film
# Film thickness = t
mdb.models['SubF'].parts['SubFPart'].PartitionCellByPlaneThreePoints( cells =
    mdb.models['SubF'].parts['SubFPart'].cells[0], point1 = (L/2,H/2,D-t), point2 = (-L/2,H/2,D-t),
    point3 = (S/2,-H/2,D-t))

# Material Definitions

# PDMS:
SubFModel.Material('PDMS')
if HyperE == True:
    SubFModel.materials['PDMS'].Hyperelastic(type = NEO_HOOKE , testData = OFF , table =
        ((75.503 , 0.00026667),))
else:
    SubFModel.materials['PDMS'].Elastic(table = ((450,0.49), ))

# FILM:
SubFModel.Material('FILM')
Ratio = 100
SubFModel.materials['FILM'].Elastic(table = ((450*Ratio,0.45), ))

# Defining all the edges of the trapizoid in different sets
# Name of the sets are combinations of the abbreviation of the name of the surfaces that they are in contact with
# R = Right , L = Left , T = Top , B = Bottom , FR = Front , BC = Back

# Edge Sets

# Edge on the Front Left of the Substrate
Sub_FR_L = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, H/2, D/4), ))
 mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_FR_L, name='Sub-FR-L')

# Edge on the Front Left of the Film
F_FR_L = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, H/2, D-t/2), ))
 mdb.models['SubF'].parts['SubFPart'].Set(edges = F_FR_L, name='F-FR-L')

# Edge on the Front Right side of the Substrate
Sub_FR_R = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/2, H/2, D/4), ))
 mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_FR_R, name='Sub-FR-R')

# Edge on the Front Right side of the Film
F_FR_R = edges= mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/2, H/2, D-t/2), ))
 mdb.models['SubF'].parts['SubFPart'].Set(edges = F_FR_R, name='F-FR-R')
# Edge on the Back Left of the Substrate
Sub_BCL = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, -H/2, D/4), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_BCL, name='Sub-BC-L')

# Edge on the Back Left of the Film
F_BCL = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, -H/2, D-t/2), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_BCL, name='F-BC-L')

# Edge on the Back Right of the Substrate
Sub_BCR = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((S/2, -H/2, D/4), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_BCR, name='Sub-BC-R')

# Edge on the Back Right of the Film
F_BCR = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((S/2, -H/2, D-t/2), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_BCR, name='F-BC-R')

# Edge on the Top Front of the Film
F_T_FR = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/4, H/2, D), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_T_FR, name='F-T-FR')

# Edge on the Top Back of the Film
F_T_BC = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/4, -H/2, D), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_T_BC, name='F-T-BC')

# Edge on the Bottom Front of the Substrate
Sub_B_FR = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/4, H/2, o), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_B_FR, name='Sub-B-FR')

# Edge on the Bottom Back of the Substrate
Sub_B_BC = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L/4, -H/2, o), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_B_BC, name='Sub-B-BC')

# Edge on the Top Right of the Film
F_T_R = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L+S)/4, o, D), )
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_T_R, name='F-T-R')

# Edge on the Top Left of the Film
F_T_L = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, o, D), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = F_T_L, name='F-T-L')

# Edge on the Bottom Right
Sub_B_R = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((L+S)/4, o, o), )
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_B_R, name='Sub-B-R')

# Edge on the Bottom Left
Sub_B_L = mdb.models['SubF'].parts['SubFPart'].edges.findAt(((0, o, o), ))
mdb.models['SubF'].parts['SubFPart'].Set(edges = Sub_B_L, name='Sub-B-L')

# Surface Sets
mdb.models['SubF'].parts['SubFPart'].Set(faces = mdb.models['SubF'].parts['SubFPart'].faces.findAt(((L/4, H/2, D/2), )), name='Sub-FR')

# Surface on the Front of the Film
# Defining regions for section assignment
# Cell[1] = Substrate
# Cell[0] = Film
Reg1 = (mdb.models['SubF'].parts['SubFPart'].cells[1],)
Reg2 = (mdb.models['SubF'].parts['SubFPart'].cells[0],)

# Section creation
SubFModel.HomogeneousSolidSection(name='Substrate', material='PDMS')
SubFModel.HomogeneousSolidSection(name='Film', material='FILM')

# Section Assignments
mdb.models['SubF'].parts['SubFPart'].SectionAssignment(region=Reg1, sectionName='Substrate')
mdb.models['SubF'].parts['SubFPart'].SectionAssignment(region=Reg2, sectionName='Film')

# View options
myViewport.setValues(displayedObject=SubFPart)
myViewport.partDisplay.setValues(renderStyle=SHADED)

# Creating the mesh
import mesh
# Setting mesh element shapes
mdb.models['SubF'].parts['SubFPart'].setMeshControls(elemShape=QUAD, regions=Reg1, technique=STRUCTURED)
mdb.models['SubF'].parts['SubFPart'].setMeshControls(elemShape=QUAD, regions=Reg2, technique=STRUCTURED)

# Types of mesh

89
Lin = C3D8
Quad = C3D20
# Choosing the mesh element type based on the problem definition
if Linear==True:
    Elcode = Lin
else:
    Elcode = Quad
# Defining mesh element types
mdb.models['SubF'].parts['SubFPart'].setElementType(elemTypes=(mesh.ElemType(elemCode=C3D20, elemLibrary=STANDARD), ), regions=(Reg1,))
mdb.models['SubF'].parts['SubFPart'].setElementType(elemTypes=(mesh.ElemType(elemCode=C3D20, elemLibrary=STANDARD), ), regions=(Reg2,))
# Seeding substrate thickness the biased method
D_N = 15
R = 50
mdb.models['SubF'].parts['SubFPart'].seedEdgeByBias(biasMethod = SINGLE, end2Edges = Sub_FR_L, ratio = R, number = D_N )
mdb.models['SubF'].parts['SubFPart'].seedEdgeByBias(biasMethod = SINGLE, end2Edges = Sub_FR_R, ratio = R, number = D_N )
mdb.models['SubF'].parts['SubFPart'].seedEdgeByBias(biasMethod = SINGLE, end2Edges = Sub_BC_L, ratio = R, number = D_N )
mdb.models['SubF'].parts['SubFPart'].seedEdgeByBias(biasMethod = SINGLE, end2Edges = Sub_BC_R, ratio = R, number = D_N )
# Seeding film thickness by number
# Node number = 4
t_N = 3
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_FR_L , number = t_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_FR_R , number = t_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_BC_L , number = t_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_BC_R , number = t_N)
# Seeding the top edges
# Node number = 45
SL_N = 15
H_N = 100
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_T_L , number = H_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = Sub_B_L , number = H_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_T_R , number = H_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = Sub_B_R , number = H_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_T_FR , number = SL_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = Sub_B_FR , number = SL_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = F_T_BC , number = SL_N)
mdb.models['SubF'].parts['SubFPart'].seedEdgeByNumber(edges = Sub_B_BC , number = SL_N)
# Generating the entire mesh
mdb.models['SubF'].parts['SubFPart'].generateMesh(regions = Reg1)
mdb.models['SubF'].parts['SubFPart'].generateMesh(regions = Reg2)
# Assembly
import assembly
# Dependant assmebeley is due to meshing the part and not the instance
mdb.models['SubF'].rootAssembly.Instance(name = 'SubFInstance', part = mdb.models['SubF'].parts['SubFPart'], dependent = ON)
#-----------------------------
# Buckling step
import step

mdb.models['SubF'].BuckleStep(name = 'Buckle', previous = 'Initial', numEigen = 4, eigensolver = SUBSPACE, maxIterations = 100000)

# Boundary Conditions (BC)
# Displacement boundary conditions
# Boundary Conditions will be applied to the long and short sides
# Regions where the boundary conditions will apply
RegSubFRbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-FR']
RegFFRbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-FR']
RegSubBCbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-BC']
RegFBCbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-BC']
RegSubLbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-L']
RegFLbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-L']
RegSubBbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-B']

# Actual boundary conditions
disp = 2
mdb.models['SubF'].DisplacementBC( name = 'Sub-FR-dispBC', createStepName = 'Buckle', region = RegSubFRbc , u2 = 0 , )
mdb.models['SubF'].DisplacementBC( name = 'F-FR-dispBC', createStepName = 'Buckle', region = RegFFRbc , u2 = 0)
mdb.models['SubF'].DisplacementBC( name = 'Sub-BC-dispBC', createStepName = 'Buckle', region = RegSubBCbc , u2 = disp )
mdb.models['SubF'].DisplacementBC( name = 'F-BC-dispBC', createStepName = 'Buckle', region = RegFBCbc , u2 = disp )
mdb.models['SubF'].DisplacementBC( name = 'Sub-B-dispBC', createStepName = 'Buckle', region = RegSubBbc , u3 = 0 )
# Symmetry Boundary Condition along the middle line
mdb.models['SubF'].DisplacementBC( name = 'Sub-L-dispBC', createStepName = 'Buckle', region = RegSubLbc , u1 = 0)
mdb.models['SubF'].DisplacementBC( name = 'F-L-dispBC', createStepName = 'Buckle', region = RegFLbc , u1 = 0 )
APPENDIX 2 - PYTHON SCRIPT FOR CREATING THE MODEL FOR POST-BUCKLING ANALYSIS

The script used for post buckling is the same as the one used for buckling (Appendix-1), with a slight difference in the way that the step and the boundary conditions are introduced. Therefore, in order to get the script for post-buckling, the parts “buckling step” and “boundary conditions” are replaced with the code below:

```python
# Post-Buckling step
import step
mdb.models['SubF'].StaticStep(name='Post-Buckle', previous='Initial', nlgeom=ON, maxNumInc=10000, initialInc=0.001)

# Boundary Conditions (BC)
# Displacement boundary conditions
# Boundary Conditions will be applied to the long and short sides
# Regions where the boundary conditions will apply
RegSubFRbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-FR']
RegFFRbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-FR']
RegSubBCbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-BC']
RegFBCbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-BC']
RegSubLbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-L']
RegFLbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['F-L']
RegSubBbc = mdb.models['SubF'].rootAssembly.instances['SubFInstance'].sets['Sub-B']

# Actual boundary conditions
disp = 2
mdb.models['SubF'].DisplacementBC(name='Sub-FR-dispBC', createStepName='Post-Buckle', region=RegSubFRbc, u2=0)
mdb.models['SubF'].DisplacementBC(name='F-FR-dispBC', createStepName='Post-Buckle', region=RegFFRbc, u2=0)
mdb.models['SubF'].DisplacementBC(name='Sub-BC-dispBC', createStepName='Post-Buckle', region=RegSubBCbc, u2=disp)
mdb.models['SubF'].DisplacementBC(name='F-BC-dispBC', createStepName='Post-Buckle', region=RegFBCbc, u2=disp)
mdb.models['SubF'].DisplacementBC(name='Sub-B-dispBC', createStepName='Post-Buckle', region=RegSubBbc, u3=0)
mdb.models['SubF'].DisplacementBC(name='Sub-L-dispBC', createStepName='Post-Buckle', region=RegSubLbc, u1=0)
mdb.models['SubF'].DisplacementBC(name='F-L-dispBC', createStepName='Post-Buckle', region=RegFLbc, u1=0)
```