High-Accuracy Foil Optics for X-ray Astronomy

by

Olivier Mongrard

Diplôme d’Ingénieur
Ecole Nationale Supérieure de l’Aéronautique et de l’Espace
Toulouse, France (1999)

Submitted to the Department of Aeronautics and Astronautics
in partial fulfillment of the requirements for the degree of

Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2001

© Massachusetts Institute of Technology 2001. All rights reserved.
High-Accuracy Foil Optics for X-ray Astronomy

by

Olivier Mongrard

Submitted to the Department of Aeronautics and Astronautics on August 15, 2001, in partial fulfillment of the requirements for the degree of Master of Science

Abstract

This thesis will describe the project of high-accuracy foil x-ray optics that I have been working on in the past two years at the Space Nanotechnology Laboratory (SNL).

Achieving arcsecond angular resolution in a grazing incidence foil optic X-ray telescope, such as the segmented mirror approach being considered for the Constellation-X Spectroscopy X-Ray Telescope (SXT), requires accurate placement of individual foils. We have developed a method for mounting large numbers of nested, segmented foil optics with sub-micron accuracy using lithographically defined and etched silicon alignment micro-structures. A system of assembly tooling, incorporating the silicon micro-structures, or “microcombs”, is used to position the foils which are then bonded to a flight structure. The advantage of this procedure is that the flight structure has relaxed tolerance requirements while the high accuracy assembly tooling can be reused. We have designed and built a system to experimentally test this alignment and mounting technique. An assembly station is used to test the ability of the alignment micro-structures to provide sub-micron positioning. Properties of the alignment micro-structures are reported.

This thesis also reports our progress in developing a low-cost method for shaping thin-foil glass optics. Such optics might help reducing the errors associated with the shape of the foil substrate and serve as components for X-ray mirrors in missions such as Constellation-X. This method is based on novel thermal shaping techniques that achieve the desired shape with high accuracy, avoiding the need for replication. To demonstrate this method we have produced 200 micron-thick glass sheets with sub-micron flatness.

Finally, the last part of this thesis centers on one crucial question concerning the use of glass optics as substrates for X-ray mirrors aboard satellites: can they survive to the acoustic loads occurring during launch? Both analytical calculations and experiments have been performed to answer this question.

Thesis Supervisor: Mark Schattenburg
Title: Principal Research Scientist
To my mother and my grandfather
Acknowledgments

I would like to thank all the individuals who helped me not only with this thesis, but with all aspects of my graduate life at M.I.T.

I am very grateful to my advisor, Dr. Mark Schattenburg for offering me the opportunity to work in the MIT Space Nanotechnology Laboratory. He has provided me the freedom and support necessary to pursue my research interests and develop scientifically.

I thank my officemates Carl Chen, Paul Konkola and Glen Monnelly for both the work achieved together and for the great moments shared outside of work. It has been a pleasure working with you.

I thank Dave Breslau, Dr. Mike McGuirk, and Dr. Ralf Heilmann for furnishing their technical expertise; Euclid Moon for his experience and patience while training me to use the Atomic Force Microscope. “Merci à” Ed Murphy for his always cheerful attitude and for lending a hand when needed. I also wish to thank Jean Farewell, Kelley Fischer, Ray Scuzzarella, Robert Laliberte, Fred Miller, Mark Mondol and Mike Grossman for all their assistance and help in the various aspects of my work.

I am grateful to François Martel and my professors in SUPAERO, Prof. Christian Colongo, Prof. Bénédicte Escudier and Prof. Edith Roques, thanks to whom I spent two great years in Boston studying at M.I.T.

I close, on a more personal note, by acknowledging my friend Alexandre Doin for cheering me up in the difficult moments and for the good week-ends spent together and last but not least, my wife, Laurence for following me to this side of the Atlantic far from her friends and relatives and for fighting through the adversity of being unemployed in a foreign environment.

Fundings from the National Aeronautics and Space Administration and the Columbia University have made this thesis possible.
# Contents

1 Introduction ................................. 25
   1.1 X-rays formation and origins ......................... 27
   1.2 X-ray absorption and its consequences .................. 32
   1.3 X-rays observation techniques ......................... 37
      1.3.1 The need for focusing X-rays ....................... 37
      1.3.2 Grazing incidence optics ............................ 38
      1.3.3 X-ray imaging systems ............................... 41
   1.4 Major X-ray missions ............................... 45
   1.5 A look ahead .................................. 51
   1.6 Description of our concept ........................... 56

2 High-accuracy assembly .......................... 61
   2.1 Precision alignment concept ......................... 61
   2.2 Assembly truss and metrology frame .................... 65
      2.2.1 Foil alignment ................................ 65
      2.2.2 Alignment tolerance and implications on design .... 67
   2.3 Silicon microcombs ................................ 68
      2.3.1 Design requirements: generalities ................. 70
      2.3.2 Leaf spring design requirements .................... 71
      2.3.3 Leaf spring analytical model ....................... 73
      2.3.4 Leaf spring finite element model ................... 79
      2.3.5 Fabrication ................................ 82
   2.4 Tests of the metrology frame ....................... 82
2.4.1 Experimental setup ........................................ 86
2.4.2 Autocollimator drift ....................................... 86
2.4.3 Early results and associated problems .................... 87
2.4.4 Results ...................................................... 90
2.4.5 Degradation of reference precision ........................ 91

2.5 Impact of the spring microcomb actuation on the metrology frame accuracy ................................................. 92
   2.5.1 Impact of spring microcomb translation .................. 94
   2.5.2 Force applied by the spring microcomb ................. 95
   2.5.3 Impact of a layer of epoxy between the substrate surface and the front of the reference microcomb ............ 101

3 Thin-foils optics shaping ........................................ 105
   3.1 ERAF vs Thermally Formed Glass ......................... 106
   3.2 Microsheet slumping ....................................... 108
      3.2.1 Foil metrology ......................................... 108
      3.2.2 Slumping procedure .................................... 109
      3.2.3 Slumping to flat plate .................................. 111
      3.2.4 Slumping to pin chuck .................................. 113
   3.3 Microsheet bending ....................................... 121

4 Flight integration ............................................. 125
   4.1 Launch environment overview .............................. 125
      4.1.1 Loads on the spacecraft in the launch environment .... 125
      4.1.2 Spacecraft mechanical design ......................... 128
      4.1.3 Vibration tests ......................................... 129
      4.1.4 Conclusion .............................................. 130
   4.2 Flight frame design ....................................... 130
   4.3 Microsheets natural resonant frequencies ............... 134
      4.3.1 Cylindrical inflection of rectangular plates ........ 134
      4.3.2 Finite element model ................................... 137
4.4 Microsheets deformation and stress .......................... 138
4.5 Weight/Strength tradeoff ........................................ 145
4.6 Acoustic tests on glass microsheets ............................ 148
  4.6.1 Acoustic test chamber setup ................................. 148
  4.6.2 Microsheet glueing ........................................... 151
  4.6.3 Acoustic chamber calibration ............................... 153
  4.6.4 Acoustic tests procedures .................................. 155
  4.6.5 Acoustic tests results ...................................... 160

5 Conclusion ............................................................. 161

A Material properties ................................................. 165

B Friction Tests ......................................................... 167
  B.1 Methodology ....................................................... 167
  B.2 Setup .............................................................. 167
  B.3 Results ............................................................ 168

C Conversion of angles into linear displacement for the assembly truss setup ......................................................... 171
  C.1 Definitions ........................................................ 171
  C.2 Calculation ........................................................ 173
  C.3 Result ............................................................. 173
List of Figures

1-1 Optical (left) and X-ray (right) views of the nearly edge-on galaxy NGC 253 .......................... 26

1-2 Schematic of some physical mechanisms producing X-rays. .............. 28

1-3 (Left) The X-ray spectrum of the Perseus cluster of galaxies as observed by the Einstein satellite. The continuum emission can be accounted for by the thermal bremsstrahlung of hot intracluster gas. The thermal nature of the radiation is confirmed by the observation of the emission lines of highly ionized iron at $\sim$7 keV. (Right) Assumed spectra of X-ray sources. ........................................ 29

1-4 Photoelectric absorption by an oxygen atom. ................................. 33

1-5 Normalized transmission coefficient in air as a function of incident photon energy for various propagation distances. ......................... 35

1-6 Attenuation of electromagnetic radiation in the atmosphere. Solid curves indicate the altitude at which the indicated fractional attenuation occurs for radiation of a given wavelength. From M. S. Longair, High Energy Astrophysics (Cambridge University Press, 1981), p. 90. 35

1-7 Attenuation length as a function of incident photon energy for silica. 36

1-8 Schematic of the Rossi X-ray timing explorer. The two main instruments onboard are the Proportional Counter Array which measures X-rays in the 2-60 keV range and the High-Energy X-ray Timing Experiment, a scintillation counter which measures X-rays in the 10-200 keV band. ........................................ 38

1-9 Grazing incidence and critical angle. ............................................. 39
1-10 Reflectivity as a function of the energy of the incident photons for several grazing angles. ........................................... 41
1-11 The sine condition defines the effective surface (heavy line) for an optical system and assures constant magnification for all rays. ............. 42
1-12 Kirkpatrick-Baez imaging system. ........................................... 43
1-13 A nested Wolter type I telescope. ........................................... 44
1-14 Wolter type I telescope. The paraboloid and the hyperboloid are coaxial, and confocal at $F_1$. ........................................... 44
1-15 A close-up view of the XMM mirrors shows the small thickness of the individual shells, leaving open a large fraction of the telescope entrance window, which would be blocked by thick mirrors, as used on several previous missions. The thickness of the mirror shells varies from 0.5 mm (inner shells) to 1 mm (outer shells). The distance between the shells, i.e. the free aperture for incoming radiation onto each shell, ranges from 1.8 mm (inner) to 4 mm (outer). Photo courtesy of D. de Chambure, XMM Project. ........................................... 49
1-16 Mirror effective areas for various X-ray telescopes. The shape of those curves is only determined by the grazing angle, and the mirror coating and does not take into account the efficiencies of the imaging detectors. 50
1-17 Thin foil coated with gold, similar to those used on Astro-E, standing on a table. ........................................... 53
1-18 An Astro-E mirror quadrant. About 170 thin foils are stacked up in this modular unit. ........................................... 53
1-19 An Astro-E assembled telescope. For a better illustration the four quadrants have been outlined in white. ........................................... 54
1-20 Packing densities of wide-field foil optic systems. ........................................... 56
1-21 Scale model of a wide-field Kirkpatrick-Baez optic module frame. An additional frame would focus the cross axis. Each frame holds 30 parabolic and 30 hyperbolic foils; only six are shown in the figure. .......................... 57
1-22 A mounting bar from Astro-E and a close-up view on some of its EDMed grooves. ........................................... 58

2-1 Assembly process. Foil mirrors are held loosely in the flight module by coarse combs. The flight module is then inserted inside the assembly truss that uses silicon microcombs to precisely align the foil mirrors. The foil mirrors are then bonded to the flight frame, and the assembly truss is removed and can be reused to align another flight module. ... 63

2-2 Assembly system and flight frame. The outer structure, including the base plate, reference flat, and top plate comprises the assembly truss. The reference flat and the reference microcombs constitute the metrology frame. Spring and reference microcombs slide in grooves in the baseplate of the assembly truss (see inset). ........................................... 64

2-3 Scanning electron microscope (SEM) images of our silicon spring (left) and reference (right) microcombs’ teeth. ........................................... 65

2-4 Foils are referenced by the spring microcombs against the reference microcombs which in turn are referenced against a flat reference surface. All the references are made by point contacts as illustrated in the top view of a foil being clipped in between a reference tooth and a spring tooth. ........................................... 66

2-5 Side view of a spring and reference microcomb installed in the assembly truss, shown alone (left) and mounting a thin glass mirror (right). ... 66

2-6 Close-up on a spring microcomb pushing a foil, loosely held by coarse combs, against a reference microcomb (not shown). The hook springs, holding the microcombs and their support bars against the baseplate, are also shown. ........................................... 69

2-7 CAD drawing of the base plate and its precision surfaces. .......... 69
Microcombs design requirements. The leaf spring of the spring microcomb ensures that foils of variable thickness can be pushed against the reference microcomb. The leaf spring must be strong enough to overcome friction and the contact point with the foils must be far enough from their rough edges.

Reference and spring microcombs final design.

Constraints applied in the finite element model used to determine the force required to locally displace the glass substrate by an amount d.

Illustration of the relationship between the two requirements on the leaf spring design, $F_{\text{min}}$ and $\delta_{\text{max}}$.

Cantilever with an end load.

Spring design for $F_{\text{min}} = 0.18$ N and $\delta_{\text{max}} = 20\mu$m.

Close-up on the region of the mesh where the stress is concentrated. On the thick lines, extra-meshing constraints were imposed such that the mesh density reflects the stress concentration.

Finite element analysis of stress profiles in the leaf spring.

Silicon microcomb fabrication process.

Top view of the silicon wafer at the end of fabrication step E. The dummy features are the parts of the silicon wafer that have not been etched and that are not microcombs.

Schematic of test operation. The repeatability is the standard deviation of the measured angles $\theta_A$ or $\theta_B$. The slot-to-slot accuracy is the difference between the angles $\theta_A$ or $\theta_B$ and the mean deviation of all slots. The reference flat measurement is the averaged value of $\theta_A$ or $\theta_B$.

Left: Autocollimator pointing at a rigid flat plate in the assembly truss. Right: Closeup of microcombs mounting a rigid plate used for testing purposes.

Microcombs configurations for early round of tests. Numbers identify microcomb pairs from each other.

Remaining oxide on the reference surfaces of the microcombs teeth.
2-22 Defect on the backside of microcomb 3 reference surface. 89

2-23 Close-up on a microcomb tooth reference surface before (left) and after (right) repeated use. 92

2-24 Fretting on the silicon base. 93

2-25 Autocollimator pointing at a rigid flat plate in the assembly truss. A differential screw translator enables accurate translation of the top spring microcomb. 94

2-26 Spring microcomb applying a force on the back of the plate. 96

2-27 Error in placement accuracy due to the force applied by the spring microcomb. 97

2-28 Comparison between the error in placement accuracy as predicted by Hertz theory for L=40 μm and experiments. 100

2-29 Spring microcomb applying a force on the back of a plate covered by a thin epoxy layer. 101

2-30 Comparison between the error in placement accuracy as predicted by Hertz theory (no epoxy) and experiments with an epoxy layer between the substrate and reflection surfaces. 102

3-1 Hartmann foil surface metrology measurement tool. A grid of light sources is reflected from the test optic and imaged on a CCD. The deviation of the grid image from that of a flat reference directly measures the test surface topography. 109

3-2 Slumping settings. The glass microsheet and the fused silica mandrel are enclosed in a Ni-coated copper box inside the oven. Six thermometers are placed inside the copper box above the glass to ensure that a good uniformity is attained. 110
3-3 Thermal profile used during slumping. The temperature of the oven is first raised over the annealing point of the glass and then slowly decreased until it reaches 400°C. From this point the temperature is brought back to the room temperature much faster. A “holding period” can also be used when the temperature of the oven is at its maximum. The temperature seen by the glass inside the copper box (TC1) is always slightly lower than the preset oven temperature.

3-4 a) Hartmann topograph of a 50×50 mm² glass microsheet before thermal slumping, showing a ~100 μm warp. b) Topograph of sheet after slumping, showing ~5 μm warp. Note the 15x change of vertical scale between the two images.

3-5 Glass microsheet shaping process. a),b): Sheet is slumped against a flat fused silica mandrel. c),d): sheet is slumped against a pin chuck fused silica mandrel.

3-6 Fused silica pin chuck pattern.

3-7 Microscope image of a defect in the fused silica mandrel that has produced a crack in the glass during the thermal shaping process.

3-8 (Left) Hartmann topograph of a glass sheet after slumping on a fused silica pin chuck mandrel. (Right) Histogram of surface angular deviation showing a global resolution of ~18 arcsecond.

3-9 Schematic of the “bag test” used to anticipate the results of a slumping run. A plastic bag filled with air is used to apply a uniform pressure against the glass microsheet. If a dust particle prevents the sheet from flattening to the top of the pins the resulting wedge between the bottom of the glass and the mandrel surface will produce concentric interference fringes.

3-10 Atomic Force Microscopy data of two TiO₂-coated fused silica mandrels. For each mandrel, a 3D plot of the measured surface along with a height profile and the overall roughness are given.
3-11 Microscope view of sticking evidence. Looking through the glass, both the fused silica pins and the damages they left on the glass, because of sticking, can be seen. .................................................. 119

3-12 (Left) Hartmann topograph of a glass sheet after slumping on a TiO$_2$-coated fused silica pin chuck mandrel. (Right) Histogram of surface angular deviation showing a global resolution of ~10 arcsecond. .................. 120

3-13 Histogram of angular deviation for the pin chuck (solid line, rms = 4 arcseconds) and for the slumped microsheet from Figure 3-12 (dashed line, rms = 10 arcseconds). .................................................. 120

3-14 (Left) Surface error topograph and (Right) histogram of surface angular deviation of an initially flat foil bent to its KB optical shape using two points along its longer edges. .................. 122

3-15 (Left) Surface error topograph and (Right) histogram of surface angular deviation of an initially flat foil bent to its KB optical shape using three points along its longer edges. .................. 122

4-1 Payload Interface Random Vibration Specification. Extracted from Pegasus users guide. .................................................. 127

4-2 Payload Acoustic Environment. Extracted from Pegasus users guide. 127

4-3 Full-sized structural model of a flight frame. The coarse combs with their oversized grooves can be seen. .................................................. 132

4-4 Third flight frame design considered. The aperture loss due to the reinforcement cross is about 3%. .................................................. 132

4-5 Resonant modes of the flight frame first design. For each mode, both the corresponding resonant frequency and deformed shape are given. 133

4-6 Model used to analytically determined the natural resonant frequencies of a glass microsheet. The microsheet is idealized as a rectangular plate constrained along two edges. .................................................. 134
4-7 Resonant modes of a 140×100 mm², 400 μm thick glass microsheet. For each mode, both the corresponding resonant frequency and deformed shape are given.

4-8 Stress and displacement of a 0.2 mm thick glass microsheet loaded with a static pressure of 69.35 N/m² (Pegasus OverAll Pressure Sound Level).

4-9 Stress repartition around microsheet mounting fixture. The arrows are vectors representing the stress principal directions and intensities. The empty elements are the elements bonded to the flight frame.

4-10 Gravity sag of a 0.2 mm-thick foil fixed at eight points.

4-11 Deformation of a plate held almost vertical under the action of gravity. The angle between the gravity field and the plate is 0.82°. Two different thickness of the plate and two different mounting configurations are considered.

4-12 System setup of the acoustic test chamber.

4-13 Mechanical configuration inside the acoustic chamber. Both the structural part holding the microsheet, in the same way it would be held inside a flight frame, and the speaker used to generate sound are shown.

4-14 Cross-section view of a glass microsheet being bonded to a coarse comb. Cylindrical holes in the coarse combs allow a thin precision tip to dispense glue inside the oversized grooves.

4-15 Glass microsheet glued to a coarse comb. The holes through which the glue is dispensed can be seen.

4-16 Pressure in the acoustic chamber as a function of the frequency when a constant voltage is used to drive the speaker.

4-17 Voltage to be applied to the speaker to get a constant sound pressure level inside the chamber as a function of the frequency.

4-18 Determining phase angle from Lissajous patterns.

4-19 Acoustic chamber configuration. The top edge of the glass microsheet under test and the two microphones recording the acoustic pressure inside the chamber can be seen.
B-1 CAD drawing of the setup used to measure the coefficient of friction between the edges of the foils and the microcombs. 168

C-1 Problem geometry. 172
List of Tables

1.1 Assumed radiation mechanisms and spectra of X-ray sources. \( I=\text{intensity}, \) \( \nu=\text{frequency}, \) \( a=\text{dimensionless number}, \) \( h=\text{Planck's constant}, \) \( k=\text{Boltzmann's constant}, \) \( T=\text{temperature (in degrees Kelvin)}.
\)

1.2 Mirror parameters for early X-ray telescopes

1.3 Mirror parameters for recent X-ray telescopes

1.4 Mirror parameters for future X-ray telescopes.

1.5 Mirror parameters for X-ray telescopes using segmented foil optics.

1.6 Angular resolution error budgets for Astro-E and Constellation-X/SXT. The overall errors are computed as root-sum-squared. The first two terms are included twice, once for each reflection stage. The intrinsic errors are due to the conical shapes of the foils.

2.1 Force required to locally displace a 140 \( \times \) 100 mm\(^2\) glass sheet of thickness \( e \) by an amount \( d = 20 \ \mu m \). Two different configurations are considered. In the first one, six microcombs are used (Figure 2-10), whereas in the second one only four microcombs, one at each corner, are used.

2.2 Various contributions to \( F_{\text{min}} = F_{\text{shape}} + F_{\text{friction}} \), the force required to force foils into the desired shape. \( F_{\text{friction}} \) is obtained by multiplying the glass weight by the friction coefficient.
2.3 Finite Element Model results for various steps in the iterative design process. $h$ and $l$ are the parameters of the design problem. $\sigma_u$ is the maximum stress in the leaf spring when both $F_{\text{min}}$ and $\delta_{\text{max}}$ are applied while $\sigma_m$ is the maximum stress when the gap between the leaf spring and its base is closed. This gap is $70 \mu m$ wide in the first three designs and $60 \mu m$ wide in the final design. 80

2.4 Early results of measurements of the assembly system. 88

2.5 Results of measurements of the assembly system. 91

2.6 Standard deviation of Pitch errors in $\mu m$. 95

2.7 Total deflection of the system as a function of $L$ for $F=0.36 N$, calculated from Equation 2.21. 99

2.8 Surface distortion per unit force, which results in placement errors. 103

2.9 Surface distortion per spring displacement, which relates directly to thickness variation of substrate. 103

4.1 Natural resonant frequencies (in Hertz) of the three different flight frame designs studied. The frequencies marked with an asterisk are only affecting the reinforcing crosses and do not involve displacement in the other parts of the structure. 131

4.2 First four solutions to the equation $\cos(X) \cosh(X) = 1$ 136

4.3 First natural resonant frequencies (in Hertz) of a glass microsheet as given by Equation 4.10 for various values of the design parameters. 136

4.4 Natural resonant frequencies (in Hertz), obtained using Finite Element Methods, of a glass microsheet of various thickness and held in different configurations. The values in bold are problematic values. 137

4.5 Evaluation of the mass of the various components of a flight frame. 146

4.6 Properties of various design configurations for a "Super Chandra" mission concept. The mass of the flight frame module is based on an aluminum structure. An actual flight model would use composite materials and thus save about 33% in mass. 147
4.7 Subdivisions of the frequency spectrum and their associated sound pressure level. ............................... 159

4.8 Natural resonant frequencies, obtained using the experimental setup described in the previous section, of a 200 μm thick glass microsheet fixed at eight points. ................................................. 160

A.1 Mechanical properties of various materials used for this project. .... 165

B.1 Results from friction tests. .......................................................... 169
Chapter 1

Introduction

Astronomers have long studied the sky in order to determine the nature of celestial objects. In the past century, the use of powerful telescopes and advanced instrumentation has led to great strides in our understanding of the cosmos. We have learned that the Sun is only one of 100 billion stars that make up the Milky Way Galaxy and that this galaxy is only one of billions of galaxies in an expanding Universe. Moreover, astronomers have learned much about the life histories of stars, galaxies, and even of the universe itself.

X-ray astronomy is special in the sense that it reveals an invisible universe. Having evolved beneath an atmosphere that absorbs incoming X-rays, human beings are blind to the cosmic phenomena that produce the emanations. Rocket astronomy finally lifted the veil in 1949. X-ray astronomy came as a spinoff from the rocket weapons of World War II. For the first time, captured German V-2 rockets made it possible to place X-ray detectors above Earth's absorbing atmosphere that hid the X-ray sun. Until then, astronomers' observations were restricted to visible light. A previously unimagined universe was about to be revealed.

Figure 1-1 displays two different views of the galaxy NGC 253, one obtained with an optical telescope, the other with an X-ray telescope. The optical image shows the lovely spiral shape of the galaxy and the position of all the stars within it. The X-ray image, at first glance, seems like a mess but provides a gold mine of information for astronomers. What you see there are not individual stars but rather the extremely
hot gas in between the stars and forming a halo around the galaxy. We can deduce from such a picture that NGC 253 is a starburst galaxy. The rate at which stars are forming there is 50-100 times greater than in normal galaxies, such as our own. Since very massive stars use up their fuel and explode in only a few million years, and since there are so many of these stars, a star explodes in NGC 253 about every 10 years. The violent stellar explosions, or supernovae, heat the surrounding gas in the galaxy disk to about 6 million degrees. This gas has blasted out of the disk along the path of least resistance, perpendicular to the plane of the galaxy.

As it appears today, the invisible X-ray universe is very different from the one observed through unaided eyes: it is a cosmos of explosive high temperatures, intense gravitational fields, and rapid time variations. Once considered a tranquil abode, the cosmos has been transformed into a realm of extraordinary vigor and violence.

Many discoveries have come from X-ray astronomy, including accreting neutron-star binaries, hot gases in clusters of galaxies, stellar coronae, and black-hole candidates. X-rays have been detected from essentially every type of astronomical object: black holes, accreting white dwarfs and neutron stars, supernovae, coronal active stars, active galactic nuclei (e.g. quasars), and clusters of galaxies. There is also
a diffuse background of X-rays from the entire sky. The celestial regions of highest temperature and strongest gravitational fields typically emit intense high-energy radiation, X-rays and gamma rays. X-radiation arises from gases accreting onto compact stellar objects such as white dwarfs, neutron stars, and black holes. Similarly, the hot plasmas from the innermost regions of active galactic nuclei at the center of many galaxies emit intense high-energy radiation. The most likely explanation of active galactic nuclei is that the nucleus is a black hole 10 to 100 million times more massive than the sun. Surrounding matter is pulled by gravity into its deep potential well with the attendant emission of radiation at many wavelengths. The X-rays and gamma rays will come from the innermost and hottest regions close to the massive black hole. X-rays provide astronomers with detailed information about conditions in the environment of incredibly strong gravitational and magnetic fields near a compact object. Furthermore the spectra of the X-ray light can reveal the kind of gas that is emitting the radiation. So scientists can catalog the types and concentrations of elements among galaxies.

1.1 X-rays formation and origins

X-rays are one of the several types of radiation that comprise the electromagnetic spectrum. They are characterized by a high photon energy of 120 eV to about 120 keV. Within the X-ray region itself, investigators often distinguish between less energetic “soft” X-rays¹ (between 0.1 and 1 keV) and more powerful “hard” X-rays (up to 120 keV). Like all electromagnetic radiation, X-rays can also be described by their wavelength (λ in angströms): soft X-rays have long wavelengths (up to 100 Å) while hard X-rays have short wavelengths (less than 1 Å). To determine an X-ray photon’s wavelength from its energy, one uses the formula:

$$\lambda = \frac{12.4}{E}$$  \hspace{1cm} (1.1)

¹Soft X-rays are still hundreds to thousands of times more energetic than optical photons.
Figure 1-2: Schematic of some physical mechanisms producing X-rays.

where \( \lambda \) is expressed in Å and \( E \) in keV.

X-rays photons can be emitted by gases of very high temperature (~10 million degrees Kelvin) or by very energetic non-thermal particles. The mechanisms for producing X-rays are similar to those responsible for production of other energies of electromagnetic radiation. X-rays are produced in two main types of process; acceleration of charged particles (usually electrons) and when an electron changes from an atomic or ionic energy level to a lower one. Examples of the first type are bremsstrahlung and synchrotron radiation.

Bremsstrahlung radiation is present in all X-ray sources. It originates from the deceleration of electrons in coulomb collisions with other electrons and with ions and nuclei. It comes from the German, “brems” for braking, and “strahlung” for radiation. The most common situation is the emission from a hot gas as the electrons collide with the nuclei due to their random thermal motions. This is called “thermal bremsstrahlung”. Bremsstrahlung can also occur when a beam of particles decelerates on encountering an obstacle.

Thermal bremsstrahlung produces a characteristic spectrum. Each collision event can be regarded as producing a photon, and the energy of the photon corresponds approximately to the change in energy suffered during the collision. The electrons in a gas have a distribution of energies, with the mean proportional to the temperature. The distribution of photon energies produced by bremsstrahlung reflects the electron energy distribution, and has an average which is proportional to temperature. Thus,
a measurement of the spectrum can be used to determine the temperature of the gas.

Figure 1-3 (left) shows the spectrum of the Perseus cluster of galaxies as observed in the X-ray waveband. The continuum emission can be accounted for by the thermal bremsstrahlung of hot intracluster gas at a temperature corresponding to \( kT = 6.5 \text{ keV} \), i.e. \( T = 7.5 \times 10^7 \text{ K} \).

Synchrotron radiation is associated with the acceleration suffered by electrons as they spiral around a magnetic field. The force felt by a charged particle in a magnetic field is perpendicular to the direction of the field and to the direction of the particle’s velocity. The net effect of this is to cause the particle to spiral around the direction of the field. Since circular motion represents acceleration (i.e., a change in velocity), the electrons radiate photons of a characteristic energy, corresponding to the radius of the circle. For non-relativistic motion, the radiation spectrum is simple and is called “cyclotron radiation”. The frequency of radiation is simply the gyration frequency,
which is given in terms of the magnetic field as

$$\nu = \frac{eB}{mc}$$  \hspace{1cm} (1.2)

where $B$ is the field strength, $e$ is the electric charge, $m$ is the particle (electron) mass, and $c$ is the speed of light. Cyclotron and synchrotron radiations are strongly polarized; detection of polarization is regarded as strong observational evidence for synchrotron or cyclotron radiation.

The situation becomes more complicated when the particle energy is relativistic (their speed approaches the speed of light). This is more common in astrophysical objects. In this case, the radiation is compressed into a small range of angles around the instantaneous velocity vector of the particle. This is referred to as 'beaming', and it results in a spreading of the energy spectrum in a way that depends on the momentum of the particle in the direction perpendicular to the field. In such a case, there is still a maximum photon energy that can be radiated, which is proportional to the field strength and inversely proportional to the particle momentum.

Synchrotron spectra typically have a power law shape, i.e., the flux proportional to photon energy to some power. This is due to the fact that the particle momenta also have a power law distribution. They are commonly observed in the radio region of the spectrum, but can extend to the X-rays and beyond. Clearly, both synchrotron and cyclotron emissions apply only to particle motion perpendicular to the direction of a magnetic field. Real gases must also have particle motions parallel to the field, and radiate ordinary thermal bremsstrahlung.

Recombination radiation is emitted by an initially free electron as it loses energy on recombination with an ion. The emission spectrum is a continuum for each bound state of the ion with a low-frequency cut-off, the recombination limit, corresponding to the minimum energy needed to remove the electron from the bound state. The shape of the spectrum depends on the free-electron energy distribution and on the energy-dependent capture cross section into the bound state.

If an already bound electron loses energy by falling to a lower ionic energy state
then a line spectrum is obtained. The spectral line shape depends on the spontaneous lifetime of the upper state, the velocity distribution of the emitting ions and on perturbations caused by collisions and by the influence of electromagnetic fields. The emitted radiation may also interact with other particles in the plasma\(^2\) which can distort the line profile. Thus the line profile can contain much information about the state of the plasma. Prior to emission the bound electron has to be excited to a higher energy state, which can take place by collisions with other particles, primarily electrons. This process results in a line spectrum which depends on the electron temperature and density; as the temperature increases ions can be raised to more energetic states and so will emit radiation of shorter wavelength. The \text{Ly}\alpha \text{ and Ly}\beta \text{ emission lines of highly ionized iron, Fe}^{+25}, \text{can be seen on Figure 1-3 (left).}

The relative strengths of the continuum and line emissions depends on how the plasma was formed; typically, for a plasma formed from a high-Z material, continuum emission dominates, while for a low-Z material line emission can be stronger.

The Compton Scattering process does not generate new photons, but scatters photons from lower to higher energies (or vice versa) in interactions with electrons of higher (or lower) energies. The non-relativistic version is called “Thomson scattering” and results in negligible change in photon energy. In the most widely discussed scenario, low energy photons (UV, optical, or below) scatter with ultrarelativistic electrons, making X-rays and/or gamma-rays. This should actually be called ‘inverse Compton scattering’, since it is the inverse to the process first described by Arthur Compton, but the distinction is often not made by astronomers. For scattering from a thermal plasma, the fractional energy transfer per scattering is:

\[
E = \frac{4kT}{mc^2}
\]  \hspace{1cm} (1.3)

where \(T\) is the electron temperature, \(m\) is the electron mass, \(k\) is the Boltzmann constant, and \(c\) is the speed of light. Thus, unless \(kT\) is much greater than \(mc^2 \sim 6 \times 10^9\) K (which is unlikely) many scatterings are required in order to shift an optical

\(^2\)\text{A plasma consists of matter brought to a highly ionized state by a very high temperature.}
or UV photon into the X-ray band.

The resulting spectra are referred to as “saturated” or “unsaturated” depending on whether sufficient scatterings have occurred to shift all the photons to the electron energies. In the former case, the photon spectrum will resemble the electron energy distribution. In the latter case, the photon spectrum is a power law spectrum extending from the UV/optical up to the electron characteristic energy. Unsaturated Compton spectra are currently considered one of the most likely mechanisms for making the hard (greater than 10 keV) X-rays observed from many classes of objects, including active galaxies and black hole binaries in our Galaxy.

We have seen that the energy of X-ray sources can tell a lot about the way the radiation is produced. Table 1.1 summarizes the basic relationships between the intensity and frequency of radiations that characterize the various spectra and Figure 1-3 (right) provides a graphical representation of the spectra considered.

<table>
<thead>
<tr>
<th>Radiation mechanism</th>
<th>Spectral proportionalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchrotron</td>
<td>$I(\nu) \propto \nu^{-a}$</td>
</tr>
<tr>
<td>Unsaturated Inverse Compton ('Power Law')</td>
<td>$I(\nu) \propto \exp(-h\nu/kT)$</td>
</tr>
<tr>
<td>Bremsstrahlung</td>
<td>$I(\nu) \propto \nu^3/[\exp(h\nu/kT) - 1]$</td>
</tr>
<tr>
<td>Blackbody</td>
<td>$I(\nu) \propto \nu^3/[\exp(h\nu/kT) - 1]$</td>
</tr>
</tbody>
</table>

Table 1.1: Assumed radiation mechanisms and spectra of X-ray sources. $I$=intensity, $\nu$=frequency, $a$=dimensionless number, $h$=Planck’s constant, $k$=Boltzmann’s constant, $T$=temperature (in degrees Kelvin).

### 1.2 X-ray absorption and its consequences

The ways in which X-rays interact with matter govern the nature of X-ray optics and explain why X-rays cannot penetrate the Earth’s atmosphere. X-rays interact by elastic and inelastic scattering and by photoelectric absorption, the relative amounts of each being material and wavelength dependent. It can been shown [35] that for soft X-rays (energies less than about 2 keV) scattering accounts for less than 1 % of
Photoelectric absorption occurs when an X-ray photon transfers its energy to a bound atomic electron, ionizing the atom. Figure 1-4 shows a schematic of this process. The cross section for this process has a series of jumps at absorption edges, where the X-ray has sufficient energy to remove an electron from a particular atomic energy level.

The macroscopic interaction of x-rays with matter is described by the complex refractive index

$$n = 1 - \delta - i\beta$$

(1.4)

where $\delta$ is called the refractive index decrement and $\beta$ is the absorption index. Both depend on the material and the wavelength of the incident X-rays. The macroscopic quantities $\delta$ and $\beta$ (the optical constants) are related to the atomic scattering factors $f_1$ and $f_2$ via

$$\delta = Kf_1, \beta = Kf_2$$

(1.5)

where $f_1$ and $f_2$ must be calculated from relativistic quantum dispersion theory [35] and

$$K = \frac{r_0 \lambda^2 N_A}{2\pi A \rho}$$

(1.6)

where $r_0(= e^2/4\pi\varepsilon_0 m_e c^2 = 2.8179 \times 10^{-15}m)$ is the classical electron radius, $N_A$ is Avagadro’s number and $A$ and $\rho$ are the atomic weight and density of the material.
For all media, $\delta$ and $\beta$ are small quantities at X-ray frequencies.

The amplitude of the electromagnetic wave after passing a distance $t$ through the material is

$$A = A_0 \exp\left(-\frac{2\pi \beta t}{\lambda}\right) \exp\left(i \frac{2\pi \delta t}{\lambda}\right)$$  \hspace{1cm} (1.7)

and the intensity is

$$I = |A|^2 = I_0 \exp\left(-\frac{4\pi \beta t}{\lambda}\right)$$  \hspace{1cm} (1.8)

where $A_0$ and $I_0$ are the incident amplitude and intensity. Thus the phase change is $2\pi \delta t/\lambda$ and the attenuation is $\exp(-\mu t)$ where

$$\mu = \frac{4\pi \beta}{\lambda}$$  \hspace{1cm} (1.9)

is the linear absorption coefficient.

As indicated in Figure 1-5, all X-ray radiations of energy less than 2 keV have been absorbed after only 10 cm of propagation in air and those of energy less than 7 keV after 3 m. In fact the Earth’s atmosphere is thick enough that virtually none are able to penetrate from outer space all the way to the Earth’s surface. X-rays in the 0.5-5 keV range, where most celestial source give off the bulk of their energy, can be stopped by a few sheets of paper; as seen on Figure 1-5 ninety percent of the photons in a beam of 3 keV X-rays are absorbed by traveling through only 10 cm of air. So contrary to radio, near-infrared and visible wavelength, which can reach the Earth’s surface as seen on Figure 1-6, X-rays ground-based observations are impossible.

To observe X-rays from the sky, the X-ray detectors must be flown above most of the Earth’s atmosphere. There are at present three methods of doing so, rocket flights, balloons and satellites. Rockets and balloons were used for the earlier experiments in X-ray astronomy. The largest drawback to rocket flights is their very short duration (a few minutes) and their limited field of view$^3$. They are not used anymore. Balloons are able to stay aloft for much longer period of time$^4$ but they can carry instruments

$^3$A rocket launched in the Northern hemisphere will not be able to see sources in the southern sky and vice-versa

$^4$Balloons can stay aloft for up to a couple months
Figure 1-5: Normalized transmission coefficient in air as a function of incident photon energy for various propagation distances.

Figure 1-6: Attenuation of electromagnetic radiation in the atmosphere. Solid curves indicate the altitude at which the indicated fractional attenuation occurs for radiation of a given wavelength. From M. S. Longair, High Energy Astrophysics (Cambridge University Press, 1981), p. 90.
only up to altitudes of 35 kilometers above sea level. Therefore as seen on Figure 1-6, X-rays with energies less than 35 keV cannot reach balloons. Balloons are limited to the study of hard X-rays and Gamma-rays. Satellites can be placed to an orbit well above the atmosphere and can remain functional for over ten years. Thus satellites are the essential tools for X-ray astronomy.

The interaction of X-rays with matter also makes traditional refractive optics difficult to implement in this frequency range. Because X-rays are so severely extinguished by most surfaces, any refracting lens would have to be sufficiently thin to transmit X-rays efficiently. Figure 1-7 shows the 1/e attenuation length, equal to the inverse of the linear absorption coefficient $\mu$, as a function of the incident photon energy for silica, a common material used in making ultraviolet lenses. An X-ray refractive lens would have to be thinner than a few microns, a challenge yet to be achieved. In any case, assuming that a lens could be made thin enough, because the refractive index $n$ of the lens is only slightly less than one, the focal length of the system would have to be extremely long to focus rays coming parallel to the optical axis. Such a system would be highly impractical for use on a satellite.
1.3 X-rays observation techniques

1.3.1 The need for focusing X-rays

Before the introduction of imaging optics into X-ray astronomy, the most sensitive X-ray instruments consisted of collimated detectors with large collecting areas. A large collecting area was required in order to obtain a sufficiently strong signal from the relatively weak X-ray sources, in the presence of a large background signal. Placing a collimator in front of a large-area detector restricted the size of the sky from which a signal was collected, and thus reduced the background signal when the detector was pointed at a source. For very bright X-ray sources, this approach is still adequate, and can still lead to major scientific advances. This was the rationale behind the Rossi X-Ray Timing Explorer, launched in December 1995, with its array of large area proportional counters and scintillation counters shown on Figure 1-8. For more detailed concerning proportional and scintillation counters please refer to [32, 25].

However there are practical limitations on how large an array of proportional counters or how restricted a collimator one can construct. Thus, a collimated detector cannot detect any of the many thousands of weak X-ray sources that comprise the background as seen by proportional counters.

One concept for increasing the ability to detect weaker sources is the use of an X-ray telescope to create an image of a portion of the X-ray sky. In much the same way as an optical telescope increases the ability of the human eye to see faint stars, an X-ray telescope can in principle concentrate the light from an X-ray star onto a small portion of an electronic “eye” or detector. If that electronic detector is able to determine the location of the arrival of the X-ray signal in two dimensions, then the effective background signal from the sky is reduced dramatically to just that amount coincident with the source location. Furthermore, such an imaging detector can view several X-ray sources at the same time. It can also create pictures of regions from which only diffuse X-ray emission emanates. Even if the appeal for an imaging X-ray system seems now obvious, the development of the technologies needed to construct an X-ray telescope required many years after the birth of X-ray astronomy.
Figure 1-8: Schematic of the Rossi X-ray timing explorer. The two main instruments onboard are the Proportional Counter Array which measures X-rays in the 2-60 keV range and the High-Energy X-ray Timing Experiment, a scintillation counter which measures X-rays in the 10-200 keV band.

X-ray imaging systems are still limited to the study of X-rays with energy less than approximately 10 keV. This is due in part to the time needed to develop efficient electronic detectors such as CCD spectrometers but primarily to the difficulty to bring a beam of X-rays to a focus.

1.3.2 Grazing incidence optics

The interaction of X-rays with matter makes the task of designing an X-ray imaging system difficult. Section 1.2 has already shown that refracting systems for X-rays were highly impractical. But as we shall see next, normal incidence mirrors, like those used for optical telescopes, are also ruled out.

Using the boundary conditions for an electromagnetic wave across any planar boundary [35, 45], the ratio of the reflected to the incident intensity at the interface,
i.e. the reflectivity, is found to be equal to\(^5\)

\[
R = \frac{\left| \cos \phi - \sqrt{n^2 - \sin^2 \phi} \right|^2}{\left| \cos \phi + \sqrt{n^2 - \sin^2 \phi} \right|^2}
\]  

(1.10)

where \(n\) is the complex refractive index as defined by (1.4) and \(\phi\) is the angle between the incident wave and the normal to the interface as shown on Figure 1-9.

For near-normal incidence, \(\phi \approx 0\), and using the expression of \(n\) in terms of \(\delta\) and \(\beta\) this leads to

\[
R_n = \frac{\delta^2 + \beta^2}{(2 - \delta)^2 + \beta^2}
\]  

(1.11)

At X-ray frequencies, both \(\delta\) and \(\beta\) are tiny, therefore

\[
R_n \approx \frac{\delta^2 + \beta^2}{4}
\]  

(1.12)

For a reflection of 1 keV photons on gold, we would get \(\delta \approx 2.10 \times 10^{-3}\) and \(\beta \approx 1.03 \times 10^{-3}\), which leads to \(R_n \approx 1.37 \times 10^{-6}\). This small reflectivity shows that X-rays do not reflect at normal incidence, they are absorbed. Therefore any attempt to produce an X-ray imaging telescope with normal incidence reflecting mirrors is vain.

\(^5\)This result is valid in the case where the incident electric field is perpendicular to the plane of incidence. In other cases, the expression is different but leads to the same conclusions.
Now let us study what would happen to the reflectivity if we consider rays that make a small angle $\theta$ with the interface. This is known as grazing incidence. We define

$$\theta = \frac{\pi}{2} - \phi$$

(1.13)

As $\theta$ is small we can, using small angle approximations $\cos \phi \approx \theta$ and $\sin^2 \phi \approx 1 - \theta^2$, rewrite Equation (1.10) as

$$R_g = \frac{\theta - \sqrt{n^2 + \theta^2 - 1}}{\theta + \sqrt{n^2 + \theta^2 - 1}}$$

(1.14)

Since $\delta, \beta \ll 1$, we can also neglect second order terms in $n^2$, leaving us with

$$n^2 \approx 1 - 2\delta + 2i\beta$$

(1.15)

Thus

$$R_g = \frac{\theta - \sqrt{\theta^2 - 2\delta + 2i\beta}}{\theta + \sqrt{\theta^2 - 2\delta + 2i\beta}}$$

(1.16)

For a lossless medium, i.e. $\beta = 0$, we can see that if $\theta \leq \sqrt{2\delta} \equiv \theta_c$ then the previous equation is the ratio of the modulus of a complex number over the modulus of its conjugate, i.e. 1. This means that X-rays will undergo total external reflection for angles smaller than the critical grazing angle $\theta_c$ (Figure 1-9).

As $\delta$ is proportional to the atomic number $Z$ of the material, high $Z$ materials have a bigger critical grazing angle and thus reflect X-rays more efficiently than low $Z$ materials. Figure 1-10 shows actual reflectivity curve for gold which is commonly used as a reflecting material. Note that for energy greater than 10 keV it is impossible to reflect X-rays for grazing angles greater than half a degree. In fact, $\theta_c$ becomes smaller at higher X-ray energies. Other reflecting materials used include nickel and iridium. The critical angle at 1 keV of all those materials is about a couple degrees.

We see that mirrors for focusing X-rays must be used at small angles of grazing incidence. This means that the optical designs look very different from conventional
1.3.3 X-ray imaging systems

A single spherical mirror reflecting at grazing incidence results in severe aberrations in the image. In fact, in order for any optical system to image an extended field and not only a single point, all geometrical paths through the system must give the same magnification. This requirement for constant magnification can be quantitatively expressed by the Abbe sine condition:

\[
\frac{\sin u}{\sin u'} = \frac{y'}{y} = \text{constant}
\]  

(1.17)

where \( u \) and \( u' \) are the aperture angles of rays on the side of the object and image, and \( y/y' \) is the magnification (Figure 1-11). The sine condition defines for each optical system the shape of the effective surface, which is defined as the intersection of each input ray with its corresponding output ray. For an object at a very large distance, the Abbe condition requires that the effective surface is a sphere around the object. A normal incidence mirror approaches such a sphere for small apertures and can produce
The sine condition defines the effective surface (heavy line) for an optical system and assures constant magnification for all rays. Usable image fields. But the surface of a grazing incidence mirror is perpendicular to the desired surface and thus represents the largest possible deviation from the sine condition. For this reason, single reflectors are not used for high-resolution X-ray optics.

Grazing incidence mirrors require at least two reflecting surfaces to produce any extended image. There are two main types of such compound systems; one invented by Kirkpatrick and Baez in 1948 and the other by Wolter in 1952.

Kirkpatrick and Baez used successive reflections from at least two 90°-crossed mirrors to form X-ray images. The simplest implementation, shown on Figure 1-12, uses two crossed cylindrical mirrors with equal curvature. This produces a real point image of a point object but the magnification is different in the two directions. In order to resolve this aberration and to significantly reduce coma four reflections are needed, two in each direction. Thus practical wide-field Kirkpatrick-Baez systems use two pairs of crossed mirrors. To completely eliminate coma and to ensure good image quality, aspheric surface mirrors are used instead of cylindrical ones. To increase the collecting area of such systems, many approximately parallel mirrors are used in each reflecting stage. This is referred as nesting (see Figure 1-13). Nesting is a crucial attribute to X-ray imaging systems used for astronomy. Because most celestial X-ray sources are weak and grazing incidence optics offer much less collecting area for the
same dimensions than conventional optics, maximizing the light gathering power of a mirror system is critical. Since the reflective surfaces of KB systems are only curved in one dimension, they can be made of optically flat glass plates bent to the proper curvature by mechanical stressing. This greatly contributes to make the construction of KB optics inexpensive. On the other hand, the coalignment of many reflecting surfaces to form an optimum image is a very challenging task. For this reason, and even though the first imaging telescope used for non-solar X-ray astronomy was a KB system, the most commonly X-ray optical system used to date is the one proposed by Wolter in 1952.

In Wolter optical systems, the focusing properties of conicoidal surfaces are used. These systems involve reflections from two confocal conical surfaces, a paraboloid followed by a hyperboloid. In fact, Wolter described three different imaging configurations, the Types I, II, and III. Because it has the simplest mechanical configuration and offers the possibility of nesting several telescopes inside one another (Figure 1-13), the Type I design is the most commonly used for X-ray telescopes. In Wolter type I design, an incoming ray hits first a paraboloid which will reflect rays parallel to the optical axis to a point at the focus. However, rays incident off-axis would not converge to the same point. Thus a simple paraboloid mirror would not satisfy the Abbe sine condition. The effective surface would not be a sphere but the paraboloid itself. To reduce coma, Wolter proposed to add a confocal and coaxial hyperboloid reflector following the paraboloid. For an hyperboloid, rays converging to a virtual
Figure 1-13: A nested Wolter type I telescope.

Figure 1-14: Wolter type I telescope. The paraboloid and the hyperboloid are coaxial, and confocal at $F_1$. 

44
point at one focus are reflected to the other focus. Figure 1-14 shows that an incoming ray parallel to the optical axis hits first the paraboloid at $P_1$ then converges toward $F_1$, the common focal point between the paraboloid and the hyperboloid, then hits the hyperboloid in $P_2$ and finally focuses at the second focal point of the hyperboloid $F_2$.

1.4 Major X-ray missions

The first hint that cosmic X-rays exist came in 1949, when radiation detectors aboard a rocket were briefly carried above the atmosphere where they detected X-rays coming from the sun. But it took more than a decade before a greatly improved detector discovered X-rays coming from sources beyond the solar system. In June 1962, another rocket flight detected the first extrasolar source, Scorpius X-1\(^6\), and the all-sky diffuse X-ray background. These early observations and those that followed in the interval 1962-1970 were focused on source detection and existence. They showed that X-rays stars and galaxies could be order of magnitude more luminous than their optical counterparts. By the end of the decade some 30-40 X-ray sources had been discovered in the course of a few hours of rocket flights.

In the interval 1970-78 the observational situation changed dramatically with the use of satellite borne X-ray detectors. *Uhuru*, the first X-ray satellite, was launched in December 1970. It was equipped with a relatively simple instrument, a sensitive X-ray detector similar to a Geiger counter enhanced by the addition of a collimator which enables a more accurate location of the sources. As the goal of the mission was to carry out the first sensitive all-sky survey in a few months, the spacecraft was spinned about its axis with a period of 12 minutes. In its two years of operating life, *Uhuru* produced a catalogue of 339 discrete sources and detected evidence of black holes and superdense neutron stars pulling out matter from companion stars, and vast expanses of hot gas in gigantic systems containing thousands of galaxies. This lead to two major break-throughs in the understanding of how X-rays are produced. Around

---

\(^6\)The source was named after the constellation it was discovered in.
compact objects the energy source is gravitational and in clusters of galaxies X-rays are generated via thermal bremsstrahlung emission from the surrounding hot gas. While the Uhuru observations were taking place, the technology of grazing incidence telescopes for X-ray astronomy was being developed.

The first large focusing X-ray telescope was the Apollo Telescope Mount aboard Skylab from May 1973 until February 1974. This pioneering telescope used two pairs of concentric mirrors to make stunning X-ray images of the sun. It set the stage for the development of the Einstein X-ray Observatory.

The launch of the Einstein satellite in 1978 lead to the next major advance in the subject. It was the first X-ray mission to use focusing optics with imaging detectors with an angular resolution of a few arcseconds and a field-of-view of tens of arcminutes. The sensitivity was several hundreds times greater than any previous X-ray astronomy mission. The imaging system was a nested Wolter type I grazing incidence telescope with a diameter of 0.6 m. Einstein was able to focus X-rays with energies between 0.1 and 4 keV (see Table 1.2). The deployment of such a powerful instrument has had a considerable impact on the field. The major one has been to bring X-ray astronomy into the mainstream of current astronomical research by showing that essentially all known astronomical objects are candidates for detailed X-ray studies. Einstein made the first X-ray images of shock waves from exploded stars, and images of hot gas in clusters of galaxies. It also accurately located over 7000 X-ray sources.

During the 1980s the European, Russian, and Japanese space agencies launched
successful X-ray astronomy missions, such as the European X-ray Observatory Satellite (EXOSAT), Granat (Russia), the Kvant module (Russia), Tenma (Japan), and Ginga (Japan). These missions were more modest in scale than the Einstein program and were directed toward in depth studies of known phenomena.

In 1990, the Roentgen Satellite (ROSAT), a joint project of Germany, the United States, and Great Britain, carried a telescope larger than the one aboard Einstein into orbit [3]. It has expanded the number of known X-ray sources to more than 60,000 and has proven to be especially valuable for investigating the multi-million degree gas present in the upper atmospheres of many stars.

The Advanced Satellite for Cosmology and Astrophysics (ASCA), a joint mission by Japan and the United States launched in 1993, was the first major imager satellite to use foil X-ray mirrors and CCD detectors. All previous imagers were using monolithic shells as mirrors, i.e. single undivided pieces. But ASCA was equipped with four identical conical foil X-ray mirrors. These were lightweight versions of similar mirrors flown earlier on the Broad Band X-ray Telescope experiment (BBXRT) aboard a shuttle. The foil mirror concept is, as we shall see later, a means of obtaining high throughput, broad-band, inexpensive X-ray optics. ASCA was especially designed to study the detailed distribution of X-rays with energy and thus provided important information about the elements that make up the hot X-ray emitting gas.

Two other important missions are the Rossi X-ray Timing Explorer (RXTE), a NASA mission launched in 1995, and BeppoSAX, a program of the Italian Space Agency launched in 1996. Both are still operational today. Although it does not have focusing X-ray mirrors, RXTE has the unique capability to study rapid time variability in the emission of cosmic X-ray sources over a wide band of energies. BeppoSAX is the first X-ray mission with a scientific payload covering more than three decades of energy, from 0.1 to 300 keV, with moderate imaging capability. It has proven to be useful for X-ray imaging the sources associated with Gamma-ray bursts.

The two main X-ray astronomy missions of the present decade, NASA’s Chandra X-ray Observatory and ESA’s X-ray Multi-Mirror (XMM), were both launched in
<table>
<thead>
<tr>
<th>mirrors configuration</th>
<th>ASCA</th>
<th>Chandra</th>
<th>XMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>120 nested/module</td>
<td>4 nested</td>
<td>58 nested/module</td>
</tr>
<tr>
<td>substrate material</td>
<td>conical</td>
<td>Wolter I</td>
<td>Wolter I</td>
</tr>
<tr>
<td>substrate thickness (mm)</td>
<td>Al foil &amp; epoxy</td>
<td>Zerodur</td>
<td>Solid Nickel</td>
</tr>
<tr>
<td>reflecting coating</td>
<td>0.125</td>
<td>13-76</td>
<td>0.47-1</td>
</tr>
<tr>
<td>aperture diameter (cm)</td>
<td>34.5/module</td>
<td>120</td>
<td>70/module</td>
</tr>
<tr>
<td>focal length (m)</td>
<td>3.5</td>
<td>10</td>
<td>7.5</td>
</tr>
<tr>
<td>high-energy cutoff (keV)</td>
<td>12</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>effective area at 1 keV (cm²)</td>
<td>1800 (4 modules)</td>
<td>1100</td>
<td>6000 (3 modules)</td>
</tr>
<tr>
<td>on-axis resolution (arcseconds)</td>
<td>&gt; 120</td>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>weight optic (kg)</td>
<td>40</td>
<td>950</td>
<td>700</td>
</tr>
</tbody>
</table>

Table 1.3: Mirror parameters for recent X-ray telescopes

1999. They both provide unprecedented improvements to the field of X-ray telescopes but in different areas. The strength of Chandra is its overall angular resolution while the one of XMM lies in its sensitivity. The complementarity of these two pioneering observatories is opening up a new golden age for high energy astrophysics. Chandra’s optics consist of a set of four nested paraboloid-hyperboloid mirror pairs arranged in the Wolter type I geometry (Figure 1-13). The main parameters of those mirrors are given in Table 1.3. Each mirror is carved out of a block of Zerodur and precisely figured and polished to the desired conicoidal shape. A thin layer of Iridium is then added to give the mirror its reflectivity. Iridium is used here instead of conventional Gold in order to increase the reflectivity in the high-energy regime (higher Z see Section 1.3.2). The finished mirror assembly has an unprecedented on-axis-angular resolution of 0.5 arcseconds, 5 times sharper than any other X-ray telescopes to date.

XMM is equipped with three X-ray telescopes, each composed of 58 nested Wolter type I mirrors (Figure 1-15). This high number of nested mirrors leads to a colossal increase in collecting area, as shown in Figure 1-16. The high throughput of XMM will enable astronomers to discover more X-ray sources than with any of the previous space observatories. In one day, XMM will see more sources in a small area than one of the earliest X-ray satellites Uhuru found across the whole sky during its three
Figure 1-15: A close-up view of the XMM mirrors shows the small thickness of the individual shells, leaving open a large fraction of the telescope entrance window, which would be blocked by thick mirrors, as used on several previous missions. The thickness of the mirror shells varies from 0.5 mm (inner shells) to 1 mm (outer shells). The distance between the shells, i.e. the free aperture for incoming radiation onto each shell, ranges from 1.8 mm (inner) to 4 mm (outer). Photo courtesy of D. de Chambure, XMM Project.
Figure 1-16: Mirror effective areas for various X-ray telescopes. The shape of those curves is only determined by the grazing angle, and the mirror coating and does not take into account the efficiencies of the imaging detectors.
years in operation. Unlike ASCA, XMM mirrors are not foils but still monolithic shells. It is the novel process used to make the mirrors that allows them to be more than 25 times thinner than the one used on Chandra and thus to be more densely nested (see Table 1.3). Instead of the conventional figuring and polishing technology, XMM used a replica technique by electroforming nickel mirrors from superpolished mandrels. Based on mass constraints and on the performance of the Nickel electroforming technology, a 0.47 mm wall thickness for the smallest mirror diameter was baselined. The thickness variation then increases linearly with shell diameter in order to guarantee sufficient stiffness. Another advantage of the thin-walled shells is the reduction of the weight of the optics. As can been seen in Table 1.3, the mass of the eight mirrors aboard Chandra is 950 kg against only 750 kg for the three mirror modules on XMM. However because of the errors incurred during mirror forming and coalignment, the resolution of XMM is only 20 arcseconds against 0.5 arcseconds for Chandra.

1.5 A look ahead

In the race to unravel the mysteries of the universe, new discoveries bring new questions. Improvements of several orders of magnitude in sensitivity or angular resolution, such as those provided by Chandra and XMM, have led in the past to the discovery of unsuspected phenomena in known sources or of entirely new classes of sources. These discoveries tend to open entirely new fields of research and determine the direction of future activities. For this reason, the need for better angular resolution and higher sensitivity will always be fed. The future of X-ray astronomy will require the deployment of high throughput satellites with good angular resolution for deep surveys, and for spectroscopy and variability studies of faint sources and of extended objects having low surface brightness [18, 20].

The traditional full shell approach is incapable of meeting the goals set for the future telescopes, such as the array of four Spectroscopy X-ray Telescopes (SXT) on the Constellation X-ray Mission (Constellation-X) or the X-ray Evolving Universe
Table 1.4: Mirror parameters for future X-ray telescopes.

<table>
<thead>
<tr>
<th></th>
<th>Constellation-X</th>
<th>XEUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>operational energy range (keV)</td>
<td>0.25-10</td>
<td>0.05-30</td>
</tr>
<tr>
<td>effective area at 1.0 keV (cm²)</td>
<td>&gt; 15,000 (4 modules)</td>
<td>60,000</td>
</tr>
<tr>
<td>effective area at second point (cm²)</td>
<td>6,000 (6.4 KeV)</td>
<td>30,000 (8keV)</td>
</tr>
<tr>
<td>mirrors (Wolter Type I)</td>
<td>70 nested</td>
<td>??</td>
</tr>
<tr>
<td>outer diameter (m)</td>
<td>1.6</td>
<td>4.5</td>
</tr>
<tr>
<td>mirror mass (kg)</td>
<td>415</td>
<td>8900</td>
</tr>
<tr>
<td>mass with support structure (kg)</td>
<td>750</td>
<td>??</td>
</tr>
<tr>
<td>focal length (m)</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>angular resolution (arcsec)</td>
<td>&lt; 15 up to 6.4 keV</td>
<td>2 at 1 keV</td>
</tr>
</tbody>
</table>

Spectroscopy Mission (XEUS) [4]. As can be seen in Table 1.4, both missions will have very large effective area (XEUS would have a sensibility 100 times higher than XMM) for an overall mass still limited by the capacity of the actual launch vehicles.

The technology of figuring and polishing mirror substrates to the precise Wolter design seems to have reached its practical and economic limit with Chandra. Even though it produces very smooth and accurate mirror surfaces, the mirror substrates remain relatively thick. The number of shells that can be nested is thus limited and so is the collecting area. These mirrors are also quite heavy and take a lot of resources to fabricate. The combination of these factors drives the production cost of such telescope high. Therefore, a telescope with scaled up versions of Chandra’s mirrors, to get the desired collecting area, would be extremely costly to produce and impossible to launch into space with current launch vehicles. The replica technique used to produce XMM’s thin shell mirrors is a major advance to gain collecting area while keeping the weight low. However this technique is, at this point, incapable of efficiently producing mirrors even thinner than the one used on XMM.

An alternative to the full shell technology is to use segmented foil optics. In this approach, thin foils coated with a smooth layer of high density metal (Figure 1-17) are densely packed in modular units, generally a quadrant (Figure 1-18). The modular units are then combined to form a complete nested optic as shown on Figure 1-19. Examples of X-ray telescopes employing foil mirrors are BBXRT, ASCA and Astro-E,
Figure 1-17: Thin foil coated with gold, similar to those used on Astro-E, standing on a table.

Figure 1-18: An Astro-E mirror quadrant. About 170 thin foils are stacked up in this modular unit.
Figure 1-19: An Astro-E assembled telescope. For a better illustration the four quadrants have been outlined in white.

<table>
<thead>
<tr>
<th></th>
<th>ASCA</th>
<th>Astro-E XRT-S</th>
<th>Astro-E XRT-I</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of telescopes</td>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>diameter (cm)</td>
<td>34.5</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>focal length (m)</td>
<td>3.5</td>
<td>4.5</td>
<td>4.75</td>
</tr>
<tr>
<td>foil substrate</td>
<td>Al</td>
<td>Al</td>
<td>Al</td>
</tr>
<tr>
<td>foil thickness (μm)</td>
<td>127</td>
<td>155</td>
<td>155</td>
</tr>
<tr>
<td>surface coating</td>
<td>Au</td>
<td>Pt</td>
<td>Au</td>
</tr>
<tr>
<td>number of nestings</td>
<td>120</td>
<td>168</td>
<td>175</td>
</tr>
<tr>
<td>foils per telescope</td>
<td>960</td>
<td>1344</td>
<td>1400</td>
</tr>
<tr>
<td>effective area at 1.5 keV/telescope (cm²)</td>
<td>300</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>effective area at 7 keV/telescope (cm²)</td>
<td>150</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Angular resolution (arcmin)</td>
<td>3</td>
<td>&lt; 1.5</td>
<td>&lt; 1.5</td>
</tr>
<tr>
<td>Weight/telescope (kg)</td>
<td>9.84</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 1.5: Mirror parameters for X-ray telescopes using segmented foil optics.
Such thin foil mirror system can be extensively nested to greatly enhance the effective collecting area. As seen in Table 1.5, each of ASCA’s four telescopes consists of 120 layers and Astro-E has about 180 layers in each of its five telescopes. The increase in sensitivity is particularly significant at higher energies. Many foils can be positioned in the inner part of the system. These foils have a smaller angle to the optical axis, and allow smaller angles of incidence of the incoming X-rays. Therefore, a thin-foil telescope can have a higher throughput at higher energy at which the critical angle for reflection is small.

In contrast to full shell mirrors, the segmented foil mirror system is very light. Even if each of the four ASCA telescopes is composed of 960 foils, it weighs less than 10 kg (Table 1.5). And each Astro-E telescope weighs only approximately 16 kg. The weight factor is further multiplied by the possibility of using a lighter support system for the optics.

Finally, the combination of a low weight and a relatively straightforward process of fabrication of the thin foils reduces the cost of production of this kind of system, as compared with a thick-shell system.

The high throughput, low mass and low cost of segmented foil optics could be essential to the next generation of X-ray observatories. The current design of XMM is based on this approach and it is considered as a serious option to build the four SXTs on Constellation X. A scaled up version of the Asto-E foils could meet the mass and sensitivity requirement detailed in Table 1.5. However the angular resolution of this type of system must be improved in order to meet the SXT’s specification of less than 15 arcseconds. Because in practice thin foils cannot yet be made to the precise Wolter geometry, a cylindrical section of a cone is usually taken as an approximation. This conical approximation and the difficulties with the overall alignment of the foils limit the angular resolution of the telescope to about a minute of arc in the case of Astro-E.

At the MIT Space Nanotechnology Laboratory (SNL), we have been working with a team of researchers from NASA Goddard Space Flight Center (GSFC) and the
Harvard Smithsonian Astrophysical Observatory (SAO) to improve both the figure and the alignment of the foils. We also propose our own telescope design which uses more wisely the advantages of segmented foil optics.

1.6 Description of our concept

In our effort to achieve a simpler, lower cost and more manufacturable concept, we propose, instead of the traditional Wolter I optical design, to explore novel wide-field off-axis Kirkpatrick-Baez optics designs. Traditional KB designs have already been described in Section 1.3.3. KB optics have a number of advantages over Wolter I optics [22]. The main one being the rectangular form-factor, which leads to highly compact modular designs and also simplifies the assembly process.

The segmented off-axis KB design allows an even more compact system as depicted in Figure 1-20. This novel optical design achieves an 80 % increase in packing density over traditional Wolter I, for an equivalent field of view. Indeed, filling in the interior portions of the optics in Figures 1-20a and b would restrict the field of view [8]. In our concept the telescope aperture is segmented into $140 \times 100 \times 400$ mm$^3$ modules, each holding 120 mirrors, with each module producing a distinct image on a dedicated imaging detector. Each module is composed of one $x$ and one $y$ module frame and each module frame focuses X-rays along a single direction. Figure 1-21 represents such a module frame.
Figure 1-21: Scale model of a wide-field Kirkpatrick-Baez optic module frame. An additional frame would focus the cross axis. Each frame holds 30 parabolic and 30 hyperbolic foils; only six are shown in the figure.

Another crucial aspect of KB designs is that the maximum deviation from a perfect KB optic surface to a plane, for reasonable design parameters, is typically <15 μm. The use of planar surfaces tremendously simplifies the shaping, metrology and assembly problems.

Nevertheless, whatever optical design you might consider using, producing high-resolution foil optics still requires progress in several key technology areas, including precision assembly engineering, shaping technology and foil metrology. Table 1.6 summarizes the angular resolution error budgets for Astro-E and Constellation-X/SXT [38]. To meet Constellation-X/SXT requirements, both the alignment and the shaping of the foil substrates must be improved by a factor of 6.

On Astro-E the foils are held in place by radial bars, with grooves cut via Electrical-Discharge Machining (EDM). Figure 1-22 shows one of those bars and a close-up view on some grooves. EDM can only machine parts to a tolerance of ±10 μm, and a surface roughness of about 1 μm [29]. To compensate for this inaccuracy and to facilitate the loading of the foils, the nominal dimension of the grooves is 25 μm wider than the foil thickness. In the final assembly, the foils are not fixed, they can move freely inside
Table 1.6: Angular resolution error budgets for Astro-E and Constellation-X/SXT. The overall errors are computed as root-sum-squared. The first two terms are included twice, once for each reflection stage. The intrinsic errors are due to the conical shapes of the foils.

<table>
<thead>
<tr>
<th>Component</th>
<th>Astro-E (arcsec)</th>
<th>Current Status (arcsec)</th>
<th>Constellation-X/SXT (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mandrels</td>
<td>40</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Foil substrate</td>
<td>40</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Alignment</td>
<td>40</td>
<td>40</td>
<td>7</td>
</tr>
<tr>
<td>Intrinsic</td>
<td>16</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Overall</td>
<td>90</td>
<td>45</td>
<td>14</td>
</tr>
</tbody>
</table>

Figure 1-22: A mounting bar from Astro-E and a close-up view on some of its EDMed grooves.
their respective grooves. This crude mounting and assembly scheme cannot achieve the 15 arcseconds resolution imposed by Constellation-X/SXT, which translates into a foil positioning accuracy of better than 2 μm [38]. To provide an alternative to this traditional assembly technique, we have developed a method for mounting foils with sub-μm accuracy which is based on the separation of alignment and mounting fixtures and uses extremely precise micro-electro-mechanical systems (MEMS) devices as alignment bars. In Chapter 2, I describe this novel assembly technique and present placement accuracy results obtained with a prototype assembly truss.

In Chapter 3, I report on progress in developing a low-cost method for shaping thin-foil glass optics. Such optics might help reduce the errors associated with the shape of the foil substrate and serve as components for X-ray mirrors in missions such as Constellation-X. This method is based on novel thermal shaping techniques that achieve the desired shape with high accuracy, avoiding the need for replication.

And finally in Chapter 4, I address one crucial question concerning the use of glass optics as substrates for X-ray mirrors aboard satellites. Can they survive to the acoustic loads occurring during launch?
Chapter 2

High-accuracy assembly

The alignment and mounting fixtures used on previous segmented foil systems to support the foil optics must be improved in order to meet the needed refinement in angular resolution for future X-ray missions. This section presents a method of assembling the optical elements into a complete mirror assembly. The fundamental feature of the concept provides tooling and assembly such that the non-precision flight hardware is permanently assembled around the unattached, precision positioned mirrors. Mirrors are precision positioned using silicon alignment micro-structures fabricated to sub-μm tolerances. Herein we discuss the alignment of foil optics having a rectilinear geometry such as Kirkpatrick-Baez systems. This will facilitate the development of the concept and the fabrication of a breadboard test system. The application of this technique to axially symmetric Wolter I systems used on Constellation-X/SXT is discussed in other papers [5].

2.1 Precision alignment concept

There are many requirements of the structures and tooling used to mount foil x-ray optics. Mounting structures must allow for simple assembly of a large number of foils. The foils must be aligned to their proper position with accuracy sufficient to reach the desired angular resolution of the optic. The foils must be held in their assembled position by a structure that can survive launch stresses, vibrations, and
acoustic loading, and can maintain foil positions in a space environment.

We seek to greatly improve mounting of segmented optics by achieving these requirements with separate structures for alignment and flight. Alignment is accomplished with an assembly truss utilizing a metrology frame (Figure 2-2), which uses etched silicon microstructures, referred to herein as microcombs (see Section 2.3) of \(\mu\text{m}\) or sub-\(\mu\text{m}\) dimensional accuracy that are mechanically mated to a precision reference surface. After alignment, foils are bonded to a separate flight frame that meets the mechanical and thermal requirements of the launch and space environments, but does not have stringent requirements for accuracy. The concept of separating the metrology and mechanical structures of a system is a common technique, particularly in cases where sub-\(\mu\text{m}\) accuracy is required [44], but has not been used in previous segmented foil x-ray optic missions.

The flight frame structure holds the foils during launch and in orbit. Once the foils have been aligned to their proper position by the assembly truss and metrology frame, they are bonded to alignment bars (referred to as "coarse combs" in contrast to the highly accurate microcombs) similar to those used in Astro-E (Figure 1-22). Because the foils are bonded into oversized grooves of the coarse combs, the tolerance requirements on the flight frame are minimal. The flight frame need only to provide high stiffness and good thermal expansion match with the foils. For example, Kovar or thermally matched composite frames made with machine shop tolerance are acceptable.

A simplified description of the assembly process follows (Figure 2-1). Rectangular foil optics are installed in the flight frame and temporarily held in place by the oversized slots in the coarse combs. The flight frame is then installed in the assembly truss, which supports both the spring microcombs and the metrology frame. Once the foils are properly trussed into their proper shape and held there by the microcombs, the coarse combs are briefly removed, precise beads of epoxy are applied into their oversized grooves, and they are reinstalled in the flight frame. Once the adhesive cures, the flight frame may be removed from the assembly truss. The tooling and all precision surfaces are then reusable for building multiple sets of flight hardware.
Figure 2-1: Assembly process. Foil mirrors are held loosely in the flight module by coarse combs. The flight module is then inserted inside the assembly truss that uses silicon microcombs to precisely align the foil mirrors. The foil mirrors are then bonded to the flight frame, and the assembly truss is removed and can be reused to align another flight module.
Figure 2-2: Assembly system and flight frame. The outer structure, including the base plate, reference flat, and top plate comprises the assembly truss. The reference flat and the reference microcombs constitute the metrology frame. Spring and reference microcombs slide in grooves in the baseplate of the assembly truss (see inset).
Figure 2-3: Scanning electron microscope (SEM) images of our silicon spring (left) and reference (right) microcombs’ teeth.

### 2.2 Assembly truss and metrology frame

#### 2.2.1 Foil alignment

Alignment of foils is achieved with an assembly truss utilizing a metrology frame (Figure 2-2). Foils are clipped by silicon microcombs with a point-like contact at their top and bottom edges which provide accurate positioning of the foils (Figure 2-4 and 2-5). The microcombs in turn are referenced with point-like contact against an ultra-flat reference surface.

The assembly truss uses high-accuracy silicon microcombs of two types: reference microcombs and spring microcombs (see Figure 2-3 and Section 2.3). The circular extremities of the reference microcombs come into a precision point contact with the reference flat in order to provide a precise reference between the foils and the reference flat. The teeth of the reference microcombs then form accurate reference surfaces for the microsheets to register against (Figure 2-4). The reference flat used on our prototype assembly truss is a diamond turned plate that serves as the reference plane for the entire module during assembly of the metrology frame. It has a global flatness
Figure 2-4: Foils are referenced by the spring microcombs against the reference microcombs which in turn are referenced against a flat reference surface. All the references are made by point contacts as illustrated in the top view of a foil being clipped in between a reference tooth and a spring tooth.

Figure 2-5: Side view of a spring and reference microcomb installed in the assembly truss, shown alone (left) and mounting a thin glass mirror (right).
of ±0.5 μm over an area of 130 × 150 mm². Spring microcombs can be actuated and provide sufficient force to push the foils against the reference microcombs. As the spring microcomb slides in its groove in the assembly truss, each foil is pushed against its corresponding tooth on the reference microcomb (Figure 2-4). Furthermore their special shape allows us to deal with the thickness variation of the foils. Since the microcombs are part of the metrology frame they need to be durable enough for laboratory handling and repeated use in the alignment procedure.

The microcomb etching process causes a small amount of undercutting (see Section 2.3.5), resulting in a slight taper in the surface of the microcomb that contacts the foil. This ensures that a point contact is effectively made between the microcombs and foil, and that the contact points of the spring and reference microcombs are on directly opposite sides of the foil (Figure 2-4). This balance of opposing forces is necessary to prevent the microcombs from applying a local torque at the mounting point, causing distortion in the glass.

2.2.2 Alignment tolerance and implications on design

The tolerance requirements on the alignment system are driven by the dependence of the optical performance on the various dimensions. Specifically, small displacement errors in the mounting point in the plane of the foil do not have a strong effect on the optical performance, but displacement errors perpendicular to the foils (and the optical axis) have a stronger effect on optical performance. As an ensemble, the assembly system is required to provide < 2μm alignment of the foils, but to achieve this it is only necessary that the reference flat and the reference microcombs, which comprise the metrology frame, have μm accurate tolerances. In fact, an error in the angle of a microcomb as it sits in its groove in the assembly truss is relatively unimportant because it will not have a strong effect on the positioning of the contact point in the direction that is important, perpendicular to the foils.

As a result, the only parts of the system where μm accuracy is required are the dimensions of the microcombs and the points of reference, where the reference microcombs meet the reference flat and where the foils meet the reference microcombs.
Errors such as non-perpendicularity between the microcombs and reference flat only contribute as higher order errors. In the current system, the reference and spring microcombs, which are first attached to steel support bars that lend additional strength and rigidity, are assembled to both the top and base plates by springs. Those springs facilitate the precise actuation of the microcombs inside the plates’ grooves and reduce the number of precision surfaces required. The first set of springs, hook springs (Figure 2-6), pushes the microcombs against the precision surface 2 (Figure 2-7), while the second, serpentine springs, maintains them against the precision surface 3. Because of the relax alignment tolerance of the microcombs inside their grooves the precision surfaces are not fancy diamond turned surfaces, but are made using traditional machine shop tooling. The reference flat, however, is a diamond turned aluminum plate which also acts as a structural member supporting the base plate and top plate. In this design, the forces involved in bolting the system together cause distortions at the level of 0.5 \( \mu \text{m} \) in the reference flat. In future designs, the reference flat will be an optical flat that is kinematically mounted to the assembly truss, and will not be a structural member of the system.

2.3 Silicon microcombs

Our high accuracy alignment technique is enabled by silicon microcombs that are etched from a silicon wafer using microelectromechanical systems (MEMS) technology. They offer two distinct advantages over other techniques for mounting segmented foil optics. First, our fabrication technique provides the sub-\( \mu \text{m} \) accuracy which is a general characteristic of the lithographic process used in the MEMS industry, allowing for highly accurate foil placement. Second, it is possible to make intricate structures such as the leaf springs on the spring microcombs and the precisely round reference surface on the teeth and at the end of the reference microcomb.
Figure 2-6: Close-up on a spring microcomb pushing a foil, loosely held by coarse combs, against a reference microcomb (not shown). The hook springs, holding the microcombs and their support bars against the baseplate, are also shown.

Figure 2-7: CAD drawing of the base plate and its precision surfaces.
2.3.1 Design requirements: generalities

Many of the requirements of the microcombs design relate directly to the properties of the foil optics being mounted, particularly edge roughness, thickness variation, and figure errors. The baseline for the segmented foil Constellation-X/SXT design is to use epoxy replication with a glass substrate. Because the edges of the foil are generally rough compared to interior surfaces, the mounting point should ideally be a small distance away from the edge so that contact can be made with the smooth mirror surface (Figure 2-8). The alignment bars of previous generation had no provision to mount the foils with a contact point other than the edge [40](see Figure 1-22). The reference microcomb teeth have a rounded reference surface so the point of contact is 1.5 mm away from the edge of the foil and the rough edge is thus avoided (Figure 2-9).

To ensure a point-like contact between the reference flat and the reference microcombs, the latters have round extremities, which are preferred to sharp tip edges to limit stress levels. The top of the spring and reference microcomb teeth is also curved so that the abrasion between parts is minimized when the foils are loaded in the
assembly truss. Assembly of the microcombs inside the assembly truss is facilitated by the holes designed in their bases. They allow the hook springs to perform their task of maintaining the support bars inside the assembly truss grooves.

Replicated foils have typical thickness variations of 20 μm, which is due mostly to the glass substrates. To accommodate them, we use micromachined leaf springs that force the foil against the reference surface of the reference microcomb. The replicated foil optics will generally have figure errors where the shape of the foil differs from the ideal, so that the foil will have to be distorted to make contact with the reference microcomb. The leaf springs are designed so that for the expected range of thickness variation, enough force is applied to the foil to move it into place, overcoming frictional forces and the effects of figure errors in the foils.

2.3.2 Leaf spring design requirements

We have developed a simple description of the mechanical requirements of the spring microcombs leaf spring that is used to optimize its dimensions. First, it is necessary to force the foil into position against the reference microcombs, overcoming frictional
forces and bending the foil against any intrinsic figure errors. Second, since a single spring microcomb is used to force into position multiple foils, we must take into account the expected range of foil thicknesses to ensure that the thickest and thinnest foils can be accommodated. The general problem is that while a very thick, rigid leaf spring could be designed to supply sufficient force to move the mirrors, it might not have the range of motion needed to mount a particularly thick foil without fracturing the leaf spring (exceeding the 566 MPa breaking strength of silicon). Conversely, a leaf spring with a large range of motion might not be able to apply a large enough spring force to move the foils. Our analysis, based on analytic calculations and finite element modeling, allows us to search all possible leaf spring geometries (parameterized by spring length and thickness) to minimize internal stresses given the substrate mounting requirements.

The minimum load that the leaf spring should be able to apply, $F_{\text{min}}$, is defined as the sum of three terms,

$$F_{\text{min}} = F_{\text{figure}} + F_{\text{bending}} + F_{\text{friction}}.$$  \hspace{1cm} (2.1)

$F_{\text{min}}$ is the force required to force the foils into the desired shape. The term $F_{\text{figure}}$ represents a force imparted to the leaf spring due to a foil’s intrinsic figuring error. It is related to the figuring technique used to shape the foils. The worst-case figuring error for thermally formed glass substrates (see Chapter 3) is typically smaller than 5 \(\mu\text{m}\). $F_{\text{bending}}$ is due to any slight bending of the foil that we desire to impart using the microcombs. The nominal shape of the thermally formed substrates may indeed be slightly different from their optimal optical shape. Currently we are forming flat substrates for which the largest deviation to the final optical shape is no more than 15 \(\mu\text{m}\) in the case of KB optics. Latter on, we plan to form substrates with a cylindrical profile which will be closer to the optical shape.

The sum $F_{\text{shape}} = F_{\text{figure}} + F_{\text{bending}}$ represents the load required to locally displace the foil into its desired shape. According to the previous discussion, the maximum foil local displacement required is less than 20 \(\mu\text{m}\). Table 2.1 gives the force required
to locally displace the glass substrate by 20 μm in two different cases. In the first one, six microcombs are used to hold the glass (configuration shown in Figure 2-10) whereas in the second one only four are used. The results have been computed for a 140 mm × 100 mm D-263\(^1\) glass sheet of two different thickness \((e = 200 \mu m\) and \(e = 400 \mu m\)).

\(F_{friction}\) is due to the fact that the bottom of the foil, resting on the microcomb, gives frictional resistance when the foil is pushed. As we had no idea, at the time of the first design, of the exact value of the friction coefficient between the D-263 glass sheet and the silicon microcombs, we overestimated it to be equal to unity. As the foil is always resting at least on two microcombs, this means that \(F_{friction}\) should at the minimum be equal to one half the weight of the glass substrate considered. For this conservative design we will choose \(F_{friction}\) to be equal to the weight of the substrate. Appendix B provides a better estimate of the friction coefficient obtained experimentally. Table 2.2 summarizes the various contributions to the minimum load \(F_{min}\). For a 400 μm thick glass substrate, the minimum load that the leaf spring should be able to apply is approximately 0.18 N.

But the leaf spring displacement must accommodate not only an "equilibrium" displacement, \(\delta^* = \delta^* (F_{min})\), but also the maximum foil-to-foil thickness variation \(\delta_{max}\) as can be seen on Figure 2-11. \(\delta_{max}\) has already been estimated to 20 μm.

With these two numbers in mind, \(F_{min}\) and \(\delta_{max}\), we can start designing the leaf spring such that the maximum stress encountered, when applying to it a displacement \(\delta = \delta^* + \delta_{max}\), remains below an allowable stress level, defined as \(\sigma_A = 300\) MPa, inferior to the nominal breaking strength of silicon, \(\sigma_{max} = 566.4\) MPa.

2.3.3 Leaf spring analytical model

Since the overall length of the leaf spring is much greater than its width \(h\), it can be analytically modeled as a beam with one end rigidly fixed and the other end receiving a load \(F\) (Figure 2-12). This particular type of beam is also known as cantilever. The

\(^1\)D-263 glass are provided by Schott.
Figure 2-10: Constraints applied in the finite element model used to determine the force required to locally displace the glass substrate by an amount $d$.

<table>
<thead>
<tr>
<th>$d = 20\mu m$</th>
<th>Six microcombs</th>
<th>Four microcombs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Force at center point</td>
<td>Force at corner point</td>
</tr>
<tr>
<td>$e = 200\mu m$</td>
<td>$4.5 \times 10^{-3} \text{N}$</td>
<td>$1.2 \times 10^{-3} \text{N}$</td>
</tr>
<tr>
<td>$e = 400\mu m$</td>
<td>$35.8 \times 10^{-3} \text{N}$</td>
<td>$8.9 \times 10^{-3} \text{N}$</td>
</tr>
</tbody>
</table>

Table 2.1: Force required to locally displace a $140 \times 100 \text{mm}^2$ glass sheet of thickness $e$ by an amount $d = 20 \mu m$. Two different configurations are considered. In the first one, six microcombs are used (Figure 2-10), whereas in the second one only four microcombs, one at each corner, are used.

<table>
<thead>
<tr>
<th>$e$</th>
<th>$F_{\text{shape}}$ (N)</th>
<th>$F_{\text{friction}}$ (N)</th>
<th>$F_{\text{min}}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$200\mu m$</td>
<td>$4.5 \times 10^{-3}$</td>
<td>$69 \times 10^{-3}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$400\mu m$</td>
<td>$35.8 \times 10^{-3}$</td>
<td>$138 \times 10^{-3}$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Table 2.2: Various contributions to $F_{\text{min}} = F_{\text{shape}} + F_{\text{friction}}$, the force required to force foils into the desired shape. $F_{\text{friction}}$ is obtained by multiplying the glass weight by the friction coefficient.
Figure 2-11: Illustration of the relationship between the two requirements on the leaf spring design, $F_{\text{min}}$ and $\delta_{\text{max}}$.

Figure 2-12: Cantilever with an end load.
modelization of the leaf spring as a cantilever is quite crude mainly because it is curved and not straight. However, it provides the opportunity to perform a preliminary structural analysis to quickly get an estimate of the various parameters and thus to check if the design is feasible. To complete the analysis a more sophisticated Finite Element Method will be used.

The maximum deflection, $\delta$, of a cantilever is given by the following equation:

$$\delta = \frac{FL^3}{3EI}$$

(2.2)

where $E$ is the elastic or Young’s modulus and $I$ the flexural moment of inertia. In our case, the beam cross-section is a rectangle and the moment of inertia $I$ is therefore:

$$I = \frac{1}{12} h^3 e$$

(2.3)

where $e$ is the thickness of the leaf in the plane perpendicular to the view of Figure 2-12, i.e. the thickness of the silicon wafer (see Section 2.3.5). The bending stress inside the beam is evaluated thanks to the expression:

$$\sigma = \frac{M_y}{I} y$$

(2.4)

The maximum stress is thus found at the surface of the beam base where both $M_y$, the bending moment, and $y$ are maximum.

$$\sigma_{max} = \frac{Pl \ h}{I \ 2}$$

(2.5)

Dropping the subscript “max” for the maximum stress, equations (2.2), (2.3) and (2.5) lead to:

$$\frac{F}{\delta} = \frac{Eh^3 e}{4l^3}$$

(2.6)

$$\frac{\sigma}{F} = \frac{6l}{eh^2}$$

(2.7)

Therefore when a displacement $\delta = \delta^* + \delta_{max}$ is applied at the end of the cantilever,
the maximum stress is given by:

\[ \sigma = \frac{3}{2} E \frac{h}{l^2} (\delta^* + \delta_{\text{max}}) \]  

(2.8)

where \( \delta^* \) is an “equilibrium” displacement function of \( F_{\text{min}} \), \( \delta^* = \delta^*(F_{\text{min}}) \).

\[ \delta^* = \frac{4l^3}{Eh^3c} F_{\text{min}} \]  

(2.9)

From equations (2.8) and (2.9), the maximum stress in the leaf spring can be expressed as a function of the requirements, \( F_{\text{min}} \) and \( \delta_{\text{max}} \), and of the design parameters, \( h \) and \( l \).

\[ \sigma = \frac{3}{2} E \frac{h}{l^2} \left( \frac{4l^3}{Eh^3c} F_{\text{min}} + \delta_{\text{max}} \right) \]  

(2.10)

In order to solve the initial design problem of finding values of \( l \) and \( h \) such that the stress is below a given level, the last equation is rewritten as a third order polynomial equation in \( l \):

\[ l^3 - 2\sigma \frac{wl^2}{3} + hE\delta_{\text{max}}w = 0 \]  

(2.11)

with

\[ w = \frac{eh^2}{4F_{\text{min}}} \]  

(2.12)

This algebraic equation can be explicitly solved for \( l \). It has three different roots. For the positive numerical values considered here, one of those roots is always negative and thus not a solution of the problem. The other two are real and positive if and only if:

\[ h \geq \frac{9}{\sigma} \sqrt[3]{\frac{E\delta_{\text{max}}F_{\text{min}}^2}{2e^2}} \]  

(2.13)

In this case, there are two meaningful solutions to equation (2.11). In the \((h,l)\) plane, the aggregation of those two solutions represents the set of design parameters satisfying the requirements at a given level of stress \( \sigma \). Figure 2-13 shows several of those stress level contour lines obtained for \( F_{\text{min}} = 0.18 \text{ N} \) and \( \delta_{\text{max}} = 20 \mu \text{m} \).

The best dimensional design for the leaf spring is then chosen on the stress contour line for which \( \sigma = \sigma_A = 300 \text{ N} \), such that the beam hypothesis \( l \gg h \) holds.
this point, a large number of design possibilities remains even though all the initial requirements have been satisfied. To reduce this design freedom, another constraint is added. The length of the leaf spring should be made as small as possible. Indeed as the silicon wafer, from which the microcombs are made, are only 4 inches in diameter, keeping the height of the microcombs small will help packing as many of them as possible on a single wafer and thus reduce the processing time. In order to satisfy this new requirement we look for minimum of $\sigma$ with respect to $h$. Taking the derivative with respect to $h$ of equation (2.10) and equating the result with zero lead to the following relation between $h$ and $l$:

$$l = \frac{\sqrt{3E\delta_{\text{max}}}}{8F_{\text{min}}} h$$  \hspace{1cm} (2.14)$$

The intersection in the $(h, l)$ plane of this line with the stress contour line already selected corresponds to our optimal design. Figure 2-13 shows that for chosen $\sigma = 300$ MPA, we have $l = 2.5$ mm and $h = 0.26$ mm. Note that $l$ is not exactly the
overall length of the leaf spring. In fact, $l$ is the distance between the base of the leaf spring and point B (Figure 2-9), where the contact with the foil is made and thus where forces are applied. Since B is located about 1 mm from the leaf spring free extremity, the overall length of the leaf spring is $l_0 = l + 1$ (mm).

2.3.4 Leaf spring finite element model

ANSYS finite element modeling of the exact leaf spring geometry is performed to refine the previous analysis.

In order to create the ANSYS model, the first step is to design the spring microcomb under a CAD software such as ProEngineer using the parameters derived from the analytical model. Then the model is exported in a finite element solver, such as ANSYS. Because of the symmetry of the problem with respect to a plane located halfway of the microcomb thickness, the size of the problem can be reduced by considering only one half of the spring microcomb. In this case, only half the force we would apply to the complete model should be used (because the force is applied in the plane of symmetry). The mesh of the model was composed of solid tetrahedral elements with ten nodes each. It had to be iteratively refined until the level of stress obtained converged to a stable value. The final mesh had a global element size of 0.08 and extra-constraints applied along the edges where the stress concentration was higher. We imposed a number of 20 nodes on the curved thick lines shown in Figure 2-14 and 30 nodes on the thick straight line. As seen in Table 2.3, the mesh was usually composed of 40,000 to 50,000 nodes. This high number of nodes led to a long computation time. In the future, this time could be reduced by removing the lower-left portion of the model where the stress is negligible (Figure 2-15). Table 2.3 shows that for parameters derived from the analytical model, we got a stress under the loads imposed by the requirements ($\sigma_u$) of 506 MPa, greater than the intended 300 MPa. The stress encountered when the gap between the leaf spring and its base is closed ($\sigma_m$) was also higher than the maximum stress that silicon can sustain. Therefore some modification of the model characteristic dimensions was needed. Adjusting the leaf spring dimensions to $l = 3.5$ mm and $h = 0.35$ mm yielded a final $\sigma_u$ of
Figure 2-14: Close-up on the region of the mesh where the stress is concentrated. On the thick lines, extra-meshing constraints were imposed such that the mesh density reflects the stress concentration.

Table 2.3: Finite Element Model results for various steps in the iterative design process. \( h \) and \( l \) are the parameters of the design problem. \( \sigma_u \) is the maximum stress in the leaf spring when both \( F_{\min} \) and \( \delta_{\max} \) are applied while \( \sigma_m \) is the maximum stress when the gap between the leaf spring and its base is closed. This gap is 70 \( \mu \)m wide in the first three designs and 60 \( \mu \)m wide in the final design.
Figure 2-15: Finite element analysis of stress profiles in the leaf spring.
330 MPa. Because an error was made in the early extrapolation of ANSYS data to evaluate $\sigma_u$, even though $\sigma_u$ was still greater than $\sigma_A = 300$ MPa, those dimensions were selected for the final design of the leaf spring. However since both $\sigma_u$ and $\sigma_m$ are still well below the nominal breaking strength of silicon, the leaf spring can safely perform its tasks.

### 2.3.5 Fabrication

The microcombs are fabricated from 100 mm diameter, 380 $\mu$m thick double-side-polished silicon wafers using contact lithography followed by time multiplexed deep reactive ion etch (TMDRIE) [6]. The microcomb pattern, designed with CAD software, is written to a lithographic mask. The process allows the pattern of the microcombs to be transferred from the lithographic mask to a wafer with a contact masking step. The pattern is then etched through the entire thickness of the wafer with TMDRIE. The processing is done at the MIT Microsystems Technology Laboratories. For a detailed description of the procedure, see Chen et al. [13] and Chen [12]. The inexpensive process we presently use limits accuracy to about 0.5 $\mu$m. However a state-of-the-art process would give us a 15 nm accuracy.

One important aspect of the process is that the wafer is etched with a very small amount of undercutting, about 0.8°, resulting in a slightly angled face on all the reference surfaces (see top view in Figure 2-4). This ensures that a point-like contact is made between the microcomb teeth and the foil. Thorough cleaning of the microcombs after processing is required to remove photoresist and oxide, and careful handling in a cleanroom environment is necessary for repeatable sub-$\mu$m assembly. We plan to develop a process to grow a thermal oxide layer on the surface of the microcombs, which will provide a hard layer of protection against damage.

### 2.4 Tests of the metrology frame

Direct measurements of the dimensions of the microcombs using a microscope with a reticle eyepiece and precision translation stage have shown that the microcombs are
A) Design microcomb with CAD software.

C) Develop resist, etch chrome, and strip resist.

E) Pattern transfer by inductively coupled plasma reactive ion etch ($\text{SF}_6 + \text{C}_4\text{F}_8$).

B) Prepare quartz mask and perform electron beam lithography.

D) Replicate mask by UV contact print onto resist coated silicon wafer.

F) Finished microcomb.

Figure 2-16: Silicon microcomb fabrication process.
fabricated to a higher tolerance than the 2 \( \mu m \) resolution of the measurement. To more accurately characterize microcombs, we have designed a series of experiments that uses our breadboard test assembly system to specifically measure the alignment capabilities of the microcombs. The system has a rectilinear geometry and is designed to mount a nest of parallel foil optics. Since we are currently not able to manufacture foils with accuracy comparable to the alignment accuracy of the microcombs, we instead used rigid flat plates to demonstrate the alignment capacity. The plates are 102 mm \( \times \) 102 mm \( \times \) 2.3 mm fused silica with a flatness specified to be less than 2 \( \mu m \). This level of flatness is large compared to the microcomb alignment errors we are trying to measure, but we have taken care to make differential measurements where the error can be subtracted out. Utilizing rigid flat plates also removes as a source of error the flexibility of thin foils, which could potentially be deformed during the assembly process making arcsecond accurate measurements difficult. Though the issue of deformation of foils when mounted in the assembly truss is not addressed by the rigid flat plate tests, it is an important issue to be treated in future tests. The degradation of microcomb reference precision due to wear is addressed by examining
Figure 2-18: Schematic of test operation. The repeatability is the standard deviation of the measured angles $\theta_A$ or $\theta_B$. The slot-to-slot accuracy is the difference between the angles $\theta_A$ or $\theta_B$ and the mean deviation of all slots. The reference flat measurement is the averaged value of $\theta_A$ or $\theta_B$.

relevant parts microscopically before and after measurements.

The tests are designed to measure the parallelism of rigid flat plates mounted in different “slots” of the assembly truss, defined by the microcomb teeth (a microcomb slot is the space between one reference tooth and its corresponding spring tooth). As there are currently 10 teeth on one microcomb, there are 10 microcomb slots in a microcomb pair. They are referenced by ascending numbers from the reference flat. The goal is to achieve $\sim 1$ $\mu$m alignment of the plates, measured by their degree of parallelism. The tests address whether the plate is held consistently in the same position when placed in the same microcomb slot (repeatability), whether the plate remains parallel when installed in different slots (slot-to-slot accuracy), and whether the plate is parallel to the reference surface.

The assembly system is designed to mount a thin, and therefore flexible, foil at six points (three on the top and bottom), but in these tests, since a rigid plate is used, three mount points are used (two on the bottom and one on the top) to ensure a clear interpretation of the test results.
Figure 2-19: Left: Autocollimator pointing at a rigid flat plate in the assembly truss. Right: Closeup of microcombs mounting a rigid plate used for testing purposes.

2.4.1 Experimental setup

Using three spring-reference microcomb pairs (Figure 2-19), we installed a rigid flat plate in the assembly truss in various configurations. The plate is always held in the same way, with two microcomb pairs at the bottom corners and one microcomb pair in the top middle. Angles are measured with a Newport quadrant detector autocollimator (model LAE500-C), which reads out µrad in pitch and yaw with a resolution of 0.1 µrad. Both the assembly truss and the autocollimator are fixed on a rigid optical bench. The three-point mounting of the plate simplifies conversion of angles into linear displacement along the length of the microcombs. The two pairs of microcombs on the bottom are separated by 81 mm, giving a yaw conversion of 12 µrad per µm. The top pair is 100 mm above them, giving a pitch conversion of 10 µrad per µm. For more details about those conversions please refer to Appendix C. It is useful to note that in these tests, 0.5 µm corresponds to about 1 arcsecond.

2.4.2 Autocollimator drift

During early tests we noticed a significant variability in the angle of the reference flat as given by the autocollimator (more than 1 µrad per minute). This drift might be due
to air turbulence, heating of the electronics inside the autocollimator or instabilities in the mounting of the assembly truss. To minimize this effect, we installed the autocollimator very close to the reference flat and waited for about one hour after turning on the autocollimator, so that it could equilibrate before beginning the tests. These precautions reduced the drift below 0.3 μrad per minute. Accuracy was further improved by interpolating between calibration measurements taken with the reference flat before and after each measurement of a mounted plate. The error associated with instabilities in the autocollimator measurements is estimated to be ∼0.03 μm.

2.4.3 Early results and associated problems

In the first round of tests, five different reference microcombs have been used. Those microcombs were issued from the first generation of microcombs. As we shall see next, they did not perform as expected. This leads us to bring modifications to their fabrication process and handling. Figure 2-20 shows the different configurations tested.

The repeatability of mounting of a plate in a given slot was measured by a repeated process of sliding a rigid flat plate against the reference microcomb teeth in a given slot, measuring its angle, then removing it. Results of repeatability tests (Table 2.4) show a typical 1 σ mounting repeatability error of about 0.11 μm in both axes.

To measure mounting variations between slots, the plate was installed in several different slots and for each slot, five measurements were made. In Table 2.4, the mounting deviation in each slot is compared to the mean deviation from all slots for each different setup. The variation between slots is on average about 0.56 μm for the pitch and 0.59 μm for the yaw. However many of the data collected show a 1 σ slot-to-slot mounting error of more than 1 μm.

To measure alignment between the mounted plate and the reference flat, the angle of the plate and reference flat were compared with the plate mounted in several different slots. The measurements exposed in Table 2.4 shows a misalignment of more than 15 μm for the first three setups, far from the sub-μm alignment capacity that we intend to demonstrate.

87
Figure 2-20: Microcombs configurations for early round of tests. Numbers identify microcomb pairs from each other.

Table 2.4: Early results of measurements of the assembly system.

<table>
<thead>
<tr>
<th>setup</th>
<th>slot</th>
<th>displacement error (μm)</th>
<th>slot to slot</th>
<th>reference flat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>repeatability</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>pitch yaw</td>
<td>pitch yaw</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>0.20 0.17 0.1 0.4</td>
<td>-19.9 56.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.15 0.04 -0.8 -2.0</td>
<td>-11.3 -23.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.05 0.11 1.6 -0.2</td>
<td>20.0 6.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.04 0.17 0.5 -0.5</td>
<td>-4.2 -1.1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>x</td>
<td>x 0.17 0.1 0.4</td>
<td>-19.9 56.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>x 0.17 0.1 0.4</td>
<td>-19.9 56.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td>x 0.17 0.1 0.4</td>
<td>-19.9 56.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>x 0.17 0.1 0.4</td>
<td>-19.9 56.1</td>
<td></td>
</tr>
</tbody>
</table>
The errors detected in the slot-to-slot measurements and in the reference flat measurements led us to think that some of the microcombs might be defective. A close examination with a high-power microscope of the microcombs’ reference surfaces quickly confirmed our thinking. On all the microcombs that looked the most suspicious from the measurements collected (2, 3 and 5), we found defects. Figure 2-21 suggests that the biggest slot-to-slot variations observed are introduced by the presence of remaining oxide sticking out from the reference surfaces. Traces of oxide are also present in the vicinity of a defect found on microcomb 3 (Figure 2-22). The presence of oxide around the defects oriented our investigation toward the fabrication process. In order to prevent premature feature erosion (mostly erosion of the leaf spring) due to backside etching, an oxide layer is grown both on the top and back sides of the silicon wafer (see [12] for more details). Therefore, at the end of the
etching run, the microcombs and their respective dummy features (see Figure 2-17) are still bonded together through the 1.6 μm thick oxide layer. This bond is not strong enough to prevent the microcombs from being extracted but could generate local defects such as those previously mentioned.

To demonstrate the sub-μm alignment capacity, a new generation of microcombs had to be made. For this new generation, the resist and oxide stripping process has been improved [12] and extra care has been taken when handling the microcombs. The tests presented in this section have then been repeated with the new microcombs.

2.4.4 Results

As for the earlier round of tests, results of repeatability tests (Table 2.5) show a typical 1 σ mounting repeatability error of ~0.1 μm in both axes. This is not really surprising as the type of defects encountered in the previous generation should not have any impact on repeatability.

The new slot-to-slot measurements are much more encouraging than the previous were. Among all the measurements, variation between slots corresponds to a 1 σ slot-to-slot mounting error of less than 0.5 μm in both pitch and yaw (Table 2.5).

Even more encouraging are the reference flat results. The measurements showed a typical misalignment of about 2 μm in pitch and about 1 μm in yaw. Results of reference flat measurements, corrected to only show errors due to the microcombs (see below), are shown in Table 2.5.

The errors detected in the repeatability measurements (~0.1 μm) and in the slot-to-slot measurements (~0.5 μm) are typical of what we expect based on the sub-μm accuracy of the fabrication techniques used in making the microcombs. It is not necessarily surprising that the results of comparing the plate to the reference flat are slightly worse (2 μm in pitch and about 1 μm in yaw) because there are several additional sources of error. Among these, non-flatness of the reference flat and rigid plate are each estimated to contribute a ~0.5 μm error to the reference flat tests. Scratches in the relatively soft surface of the reference flat (which is a diamond turned aluminum plate) could cause additional errors.
Table 2.5: Results of measurements of the assembly system.

To isolate the error due to the reference flat, we repeated the measurements of Table 2.5 with the three pairs of microcombs permuted into different positions. This tests the hypothesis that in addition to the ~0.5 μm variations seen in the slot-to-slot tests, there is an offset (due to non-flatness or scratches in the reference flat) that will be present regardless of which microcombs are used. In comparing the old and new data sets, we found that there was an offset of -2.0 μm in pitch and -1.1 μm in yaw. These offsets were subtracted from reference flat measurements to remove the error contribution from the reference flat leaving only errors in the microcomb dimensions and their ability to make a consistent reference with the reference flat. The offset-subtracted data has a mounting accuracy standard deviation of 0.3 μm in pitch and 0.4 μm in yaw. This result is encouraging for two reasons. First, it suggests that the reference between the microcombs and reference flat is as good as the reference between the microcombs and the rigid plate, both being accurate to about 0.5 μm. Second, we expect that the error due to the reference flat will be removed by replacing the aluminum reference flat with a kinematically mounted quartz reference flat that will have λ/4 or better flatness and will be more resistant to scratching.

### 2.4.5 Degradation of reference precision

After performing the tests the microcombs were examined with a high-power microscope to search for evidence of damage from wear. Damage was noticeable on some
of the reference microcomb teeth (Figure 2-23). The reduction in alignment accuracy caused by this damage has not yet been quantified, but it appears not to be very significant. Another type of damage has been seen on the base of the microcombs (Figure 2-24). The friction between the silicon and the plate being slid in position inside the assembly station seems to create fretting. We expect that those damages will be reduced when much less massive thin microsheets are used instead of rigid plates. We are also developing a technique to harden the microcombs by growing a hard surface layer of thermal oxide.

The microscope also revealed that some of the reference surfaces were contaminated by dust particles, several \( \mu \text{m} \) in size. We expect that this will not be a significant problem in future tests, which unlike these, will be performed in a cleanroom environment.

### 2.5 Impact of the spring microcomb actuation on the metrology frame accuracy

The assembly truss utilizes etched silicon microcombs to provide sub-micron placement accuracy for the reflecting surfaces of the X-ray optics. Tests exposed in the
previous section have demonstrated the alignment capacity of the reference microcombs. This section addresses the impact of the spring microcomb, used to push the foils against the reference microcombs, on the placement accuracy. The two main issues to consider here are:

- Does the displacement of the spring microcomb with respect to the metrology frame affect the placement accuracy?

- Does the force applied on the back of the glass by the spring microcomb affect the placement accuracy?

The experimental setup used for those tests is almost identical to the one already described in Section 2.4.1. The top spring microcomb is now attached to a micrometer (Figure 2-25) which enables accurate translation with respect to the metrology frame. The micrometer used is a Mitutoyo differential screw translator which features a non-rotating spindle and a resolution of 0.1 μm.

As the top spring microcomb moves, we observe the induced displacement of the rigid flat plate through the variation in angles of its normal vector. The same conversion of angles into linear displacement along the length of the microcombs is applied.
2.5.1 Impact of spring microcomb translation

Concerns

The issue addressed in this section is the deterioration of placement accuracy due to the translation of the spring microcomb with respect to the metrology frame. In order to prevent the microcombs from applying a local torque at the mounting point, there is no spacing between the reference and spring microcombs (see top view of Figure 2-4). Therefore the contact point of the spring and reference microcombs are on directly opposite sides of the foil. But friction might transmit displacement from the spring microcomb to the reference microcomb, which is only held to the assembly truss by stiff hook springs.

Furthermore if the axis of translation of the spring microcomb and the groove in which it slides are not parallel, forces could be applied to the assembly truss and distort the metrology frame.
Tests and result

To quantify the previous concerns, the following tests have been performed. While the reflecting plate is resting against three reference teeth\(^2\), the top spring microcomb is moved. At no point does the spring tooth touch the fused silica plate. The pitch angle of the plate is measured in \(\mu\text{rad}\) with the autocollimator and then translated into corresponding linear displacement in \(\mu\text{m}\). For each round of test about fifteen different positions, spread out over approximately 1 mm, are measured. The standard deviations of those measurements are reported in Table 2.6.

<table>
<thead>
<tr>
<th>Test Pitch ((\mu\text{m}))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10</td>
<td>0.22</td>
<td>0.09</td>
<td>0.06</td>
<td>0.09</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
<td>0.07</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 2.6: Standard deviation of Pitch errors in \(\mu\text{m}\).

The impact of the displacement of the spring microcomb on the placement accuracy is therefore smaller than 0.1 \(\mu\text{m}\) for this setup. This number is relatively small compared to the global placement accuracy goal of 2 \(\mu\text{m}\). Moreover this error should be reduced by replacing the aluminum diamond-turned reference flat with a kinematically mounted quartz reference flat and by making the assembly structure stiffer.

2.5.2 Force applied by the spring microcomb

Spring microcomb role and characteristics

The spring microcombs are used in the assembly frame to push the foils against the reference microcombs. They can locally apply a force on the foils (Figure 2-26) in order to give them their desired shape. Moreover they can also handle the thickness variation of the different foils\(^3\).

\(^2\)A slight tilt of the assembly station ensures that gravity holds the plate against the reference teeth.

\(^3\)The thickness variation is about 20 \(\mu\text{m}\) for the D-263 Schott glass.
The stiffness of the current spring microcomb design, according to detailed ANSYS modeling, has been found to be equal to $k = 6 \text{ N/mm}$. As the gap between the leaf and the base of the spring is 60 $\mu$m wide, the maximum force that can be applied, in “normal” operations, is therefore about 0.36 Newtons.

**Experimental results**

To determine whether the force applied on the back of the plate will have an effect on the placement accuracy, almost the same procedure described earlier (Section 2.5.1) has been used. However, the focus will now be made on the domain where the spring tooth is in contact with the plate.

A series of tests has consistently shown that the force applied indeed affects the placement accuracy. In fact, as shown on Figure 2-27, the error introduced is quasi-linear with respect to the leaf spring displacement and therefore with respect to the force. The maximum error is about 0.7 $\mu$m and is obtained when the leaf bottoms out, i.e. when the force is 0.36 N.

It is hypothesized that this error is due to bending of the reference tooth. However, a simple beam bending calculation (Equation 2.15), where $d=0.38 \text{ mm}$, $h=1.02 \text{ mm}$,
Figure 2-27: Error in placement accuracy due to the force applied by the spring microcomb.

l=1.5 mm and E=1.61×10^5 N/mm^2, would give only a displacement of the contact point of 0.068 μm for a force of 0.36 N. Therefore we have to look somewhere else to explain the displacement observed.

\[
\frac{F}{\delta} = \frac{Eh^3d}{4l^3}
\]  

(2.15)

Contact problems between curved surfaces: theory

We are interested in the elastic deformation at the contact between the flat plate and the curved reference microcomb. The Hertz theory [44] provides a useful set of equations for the contact stresses between surfaces with only one radius of curvature each. Contact stresses occur when curved surfaces of two bodies are pressed together by external loads. For this calculation, the surface of the reference microcomb in contact with the plate will be approximated as a cylinder of length L and diameter \(d_1\) loaded by a force \(F/L\) along its length. The contact area between this cylinder and the flat plate is a rectangle of length \(L\) and width \(2b\):
\[ b = \left( \frac{2Fd_1}{\pi LE_e} \right)^{1/2} \] (2.16)

where \( E_e \) is the equivalent modulus of elasticity of the system based on the elastic modulii and Poisson ratios of the two materials in contact

\[ E_e = \frac{1}{\frac{1 - \eta_1^2}{E_1} + \frac{1 - \eta_2^2}{E_2}} \] (2.17)

For an elastic cylinder of diameter \( d_1 \) compressed against a flat rigid surface \( (E_2 \) is infinite in equation 2.17), the displacement of the contact surface relative to the center of the cylinder is

\[ \delta_{cylinder} = \frac{2F}{\pi LE_e} \left[ \ln \left( \frac{2d_1}{b} \right) - \frac{1}{2} \right] \]
\[ = \frac{2F(1 - \eta_1^2)}{\pi LE_1} \left[ \ln \left( 2d_1 \left( \frac{\pi LE_1}{2Fd_1(1 - \eta_1^2)} \right)^{1/2} \right) - \frac{1}{2} \right] \] (2.18)

For an elastic cylinder on an elastic flat plate, we cannot use Equation 2.18. Indeed, to determine the displacement of the center of a cylinder with respect to a point at distance \( d_0 \) below the surface, a two-part solution is required. The first part is the displacement due to the deformation of the cylinder as given by Equation 2.18. The second part is for the deformation of the elastic flat plate as a rigid cylinder is pressed into it:

\[ \delta_{flat} = \frac{2F}{\pi LE_e} \left[ \ln \left( \frac{2d_0}{b} \right) - \frac{\eta_2}{2(1 - \eta_2)} \right] \]
\[ = \frac{2F(1 - \eta_2^2)}{\pi LE_2} \left[ \ln \left( 2d_0 \left( \frac{\pi LE_2}{2Fd_1(1 - \eta_2^2)} \right)^{1/2} \right) - \frac{\eta_2}{2(1 - \eta_2)} \right] \] (2.19)

Typically, \( d_0 \) is set equal to the cylinder diameter \( d_1 \). Note that in this case \( E_1 \) is infinite in Equation 2.17. The total deflection of the system is thus

\[ \delta_s = \delta_{cylinder} + \delta_{flat} \] (2.20)
Contact problems between curved surfaces: application

For the problem at hand, we have the following numerical values:

\[
\delta_s = \frac{F(1 - \eta_2^2)}{\pi L E_1} \left[ \ln \left( \frac{2d_1 \pi L E_1}{F(1 - \eta_1^2)} \right) - 1 \right] + \frac{F(1 - \eta_2^2)}{\pi L E_2} \left[ \ln \left( \frac{2d_1 \pi L E_2}{F(1 - \eta_2^2)} \right) - \frac{\eta_2}{(1 - \eta_2)} \right]
\] (2.21)

Silicon

\[
\begin{align*}
d_1 &= 15.3 \text{ mm} \\
E_1 &= 1.61 \times 10^{11} \text{ Pa} \\
\eta_1 &= 0.24
\end{align*}
\]

Fused silica

\[
\begin{align*}
E_2 &= 7.32 \times 10^{10} \text{ Pa} \\
\eta_2 &= 0.167
\end{align*}
\]

In order to resolve the deformation as a function of the force applied in Equation 2.21, the only remaining parameter to be determined is the length L of the cylinder. Because the silicon wafer, from which the spring microcombs are made, are 380 \(\mu\)m thick, we know that L must be smaller than this number. Moreover, as the fabrication process of the microcombs introduces undercut (Section 2.3.5), L should only be a small fraction of this number. Table 2.7 suggests that L=40 \(\mu\)m would give a deflection similar to the one already experimentally measured, about 0.7 \(\mu\)m for an applied force of 0.36 N.

<table>
<thead>
<tr>
<th>L ((\mu)m)</th>
<th>380</th>
<th>300</th>
<th>150</th>
<th>50</th>
<th>40</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_s) ((\mu)m)</td>
<td>0.09</td>
<td>0.11</td>
<td>0.22</td>
<td>0.60</td>
<td>0.73</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Table 2.7: Total deflection of the system as a function of L for \(F=0.36\) N, calculated from Equation 2.21.
Figure 2-28: Comparison between the error in placement accuracy as predicted by Hertz theory for L=40 μm and experiments.

L being fixed, we can now plot the deformation of the contact area predicted by the Hertz theory as a function of the force applied, or equivalently as a function of the leaf spring displacement, and compare it to different experimental results. Figure 2-28 shows that most of the experimental data indeed follow the curve predicted by Hertz theory. In general, the 1σ spread of the experimental data with respect to the theoretical curve is 0.18 μm.

**Conclusion**

The distortion of the contact interface between the reference microcomb and the substrate surface can lead up to 0.8 μm of error in placement accuracy (for a force of 0.36 N). The fact that the actual substrate of the optic will be made of D-263 Schott glass, and not of fused silica, does not modify this result as both materials have comparable Young’s modulus.

On the one hand, since the main component of the forces applied to the foils during the assembly process can be predicted, we can take their effect into account while designing the reference microcombs. On the other hand, some effects such as friction
Figure 2-29: Spring microcomb applying a force on the back of a plate covered by a thin epoxy layer.

or thickness variation cannot be predicted and therefore could introduce errors. For example, the effect of the thickness variation of the substrate (20 μm) would lead to an error between the placement of the thicker substrate and the thinner one of about 0.26 μm.

2.5.3 Impact of a layer of epoxy between the substrate surface and the front of the reference microcomb

Tests and result

Unless foils can be made without epoxy replication, a thin epoxy layer\textsuperscript{4} must be used between the substrate surface (glass) and the reflection surface (gold). The presence of this extra layer could amplify the deformation discovered earlier.

The tests reported in Section 2.5.2 have been repeated with a substrate on which gold has been replicated. The thickness of the epoxy\textsuperscript{5} layer on the plate, used for those tests, has been measured to be equal to 69 μm. The thickness of the gold layer

\textsuperscript{4}The epoxy layer can be 25 to 102 μm thick.
\textsuperscript{5}EPO-TEK 301
Figure 2-30: Comparison between the error in placement accuracy as predicted by Hertz theory (no epoxy) and experiments with an epoxy layer between the substrate and reflection surfaces.

is typically about 20 nm. Figure 2-30 suggests that the epoxy layer does affect the placement accuracy, as the pitch error measured is now somewhat bigger than the error predicted by Hertz theory without the layer. For an applied force of 0.3 N, the error is about 0.25 μm larger than what is predicted by the theory for glass alone.

Finally it should be noted that the piece used for the test was only partially covered with replicated gold and that the contact point on the gold was made right on the edge of one of the replicated areas (because none of those areas extend up to the edge of the substrate, where the contact with the micromcombs is made).

**Conclusion**

Substrates that are thicker than average will require more leaf spring displacement, which results in more force on the substrate and thus more distortion. Table 2.8 summarizes the average distortion introduced in the system per unit of force applied in the two different cases, with and without epoxy layer. Table 2.9 gives the percentage of the thickness variation of the substrate that will contribute to placement errors for the current leaf spring design.
Table 2.8: Surface distortion per unit force, which results in placement errors.

<table>
<thead>
<tr>
<th>Epoxy</th>
<th>No epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 \mu m/N</td>
<td>1.7 %</td>
</tr>
<tr>
<td>2.8 \mu m/N</td>
<td>1.24 %</td>
</tr>
</tbody>
</table>

Table 2.9: Surface distortion per spring displacement, which relates directly to thickness variation of substrate.

<table>
<thead>
<tr>
<th>Epoxy</th>
<th>No epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 %</td>
<td>1.24 %</td>
</tr>
</tbody>
</table>
Chapter 3

Thin-foils optics shaping

The huge number of foils to be nested for the next generation of X-ray telescopes demands a substrate which can be easily fabricated in large numbers. Simple engineering constraints on the weight and available area on any realistic satellite platform require substrates of reasonably high strength to weight ratio and thickness \( \sim 0.2-0.5 \) mm. The substrate material must have sufficient strength to accommodate both the mounting stresses, and the stresses that the mirrors will see during a rocket launch. It must also maintain its figure over long period of time, more than ten years for modern satellite platforms. The advantages of thermally-formed glass substrates over epoxy replicated aluminum substrates used on some foil optics telescopes of the previous generation will be discussed in this chapter.

Contrary to Wolter type I optics, Kirkpatrick-Baez optics deviates from flat surface by only a few microns. For several realistic optical designs that we have considered, the maximum deviation of the KB optical shape from a tangential plane ranges from 3 \( \mu \)m to 15 \( \mu \)m. As we will see latter, planar surfaces tremendously simplify the shaping and metrology problems. However thin flat glass does not exist. Uniformly thick glass substrates are available, but glass with a flat natural relaxed shape cannot be found. And polishing techniques, used on more traditional optics, cannot be used to conform such thin substrates to their proper shapes. Indeed, mirrors are traditionally made of solid thick plates of glass with a thickness about 1/6th of their diameter whereas our glass substrates have a thickness only about 1/1000th of their
size. For this reason, we had to investigate novel shaping methods. The initial idea is to produce flat glass substrates using thermal shaping methods and then to approximate the anastigmatic parabolic/hyperbolic KB shapes by applying very small bends to those flat foils during assembly.

3.1 ERAF vs Thermally Formed Glass

For moderate resolution (~ few arcminute) and moderate cost X-ray optics aluminum is an ideal substrate. It can either be rolled to the appropriate figure as for SODART [47] or stress-relieved on a mandrel to obtain the figure as for ASCA [43]. For both SODART and ASCA the remaining small ripple (smaller than a few hundred microns) was removed using the lacquer polishing technique first used on X-ray optics by Catura et al. [10]. The thin gold reflecting surface is then deposited onto the lacquer-polished mirror. To obtain better performance on Astro-E the lacquer polishing technique was replaced by epoxy replication [43]. This technique allows the suppression of longer wave surface deformation. The rolled aluminum foils are pressed onto a mandrel coated with gold, with a thin layer of epoxy in between to smooth out the surface ripple in the foils and mandrel. Epoxy replicated aluminum foils (ERAF) produce very good X-ray reflecting surfaces. Similar techniques were also used to make X-ray mirrors on EXOSAT and some XMM prototype mirrors.

While this approach has already been successful in producing good quality X-ray optics, there are a number of drawbacks which motivated us to seek alternatives. A description of the tedious ERAF production process, and the fact that thousands of substrates must be produced, illustrates one problem with ERAF. The aluminum substrate must be rolled, sprayed with epoxy, the mandrel coated with gold, then sprayed with epoxy and then the substrate must be carefully separated from the mandrel. To minimize epoxy outgassing, which can affect surface quality, all those steps have to be done in an elaborate vacuum facility inaccessible to most university groups. In addition, ERAFs are intrinsically less stable substrates than alternatives, especially when very thin. They are subject to diffusion of water into or out of the
epoxy and the adiabatic nature of the epoxy curing process means that volatiles are continually devolving out of the epoxy, and the epoxy is continually shrinking over its lifetime. This can lead to figure distortions with time and/or temperature. Bimaterial bending, due not to temperature effects but to epoxy shrinkage, may also limit ERAFs if subarcminute performance is desired.

In an effort to manufacture inexpensive substrates with appropriate surface quality and figure to achieve sub-arcminute performance, we decided to focus on recently developed, thermally formed glass substrates. Thermally-formed glass has many desirable properties. Glass optic, can be fabricated in appropriate flight sizes, and high production throughput can be obtained relatively inexpensively. All the reasons that glass is a preferred material for optical wavelengths apply to X-rays as well. Glass has favorable mechanical properties. It is stiff, stable, has a low thermal expansion coefficient, does not suffer from plastic deformation like metal foils and has a low density (2.5 g/cm³), making lightweight, high-efficiency optics realizable. Glass can be produced with a very smooth surface, it can be made very flat over large areas and it can be produced in very thin uniform sheets.

In the past, it was difficult to find a glass which possessed all of these favorable characteristics. The emergence of glass as the substrate of choice in flat panel displays has led to the development of several glass products which are ideal for producing X-ray optics. The glasses, known generically as “microsheets”, are used as the substrate on which active elements of flat panel displays are deposited. They must be extremely smooth on all length scales up to the size of the displays (~ 30 cm). The glass microsheet of choice is Schott Desag D-263 or AF-45. These glasses are produced by a method, known as “overflow”, in which the glass sheet is never in mechanical contact with any surface. Deviations from uniformity are small, with the most pronounced error being a variation in thickness of about 20 μm. Chuck Hailey’s group at Columbia University pioneered the use of these materials for X-ray optics. They have developed methods for thermal shaping, or slumping, microsheets into precision conical mandrels for hard X-ray Wolter I optics [23]. At MIT we have performed a large number of experiments slumping these sheets onto flat mandrels of various
types, with promising results [24].

3.2 Microsheet slumping

Perfectly flat sheets are not initially required. The requirement is that the sheets have a total flatness of less than a couple of microns with long spatial frequency error.

The glass microsheet material, 200 μm thick Schott D-263 borosilicate glass, has been selected on its availability in high quality and volume. The sheets are easy to work, incredibly strong and have a sub-nanometer roughness. However as supplied, those microsheets are not particularly flat. Using an in-house surface metrology tool described next, we have found that the typical warpage of 200 μm thick, 100×100 mm² sheet is about 200 μm. Figure 3-4a is a topograph of a 50×50 mm² microsheet warped by about 100 μm. However, due to the amorphous structure of glass the low-spatial frequency figure can be vastly improved through thermal shaping.

3.2.1 Foil metrology

High-accuracy metrology forms part of all research in high-resolution optics. A good rule of thumb is that a metrological system must have ~5× better precision and accuracy than the figure goal of an optic. State-of-the art figure and assembly metrology for monolithic optical systems is currently at the ~0.1 nm level. Foil optics pose special problems due to their susceptibility to fixturing strain, gravity sag, and environmental effects such as vibration, thermal gradients, and air turbulence. However Kirkpatrick-Baez optics deviate from flat surfaces by only a few microns, which simplifies metrology system design and reduces referencing errors.

To study the shape of the thermally formed microsheet we have built a foil optic surface metrology station based on the Hartmann test [33]. The technique essentially measures the deviation of an array of narrow beams reflected from the test optic and imaged onto a CCD, as compared to that reflected from a flat reference surface (see Figure 3-1). The extreme simplicity of this metrology system is to be compared with the very complicated and error prone systems used to measure Wolter type I optics.
Figure 3-1: Hartmann foil surface metrology measurement tool. A grid of light sources is reflected from the test optic and imaged on a CCD. The deviation of the grid image from that of a flat reference directly measures the test surface topography.

[28]. Currently, the measurement accuracy of glass sheet topography is limited since at optical wavelengths a non-negligible percentage of the light is reflected from the back surface of the microsheets. Repeated surface height measurements of the same sheet agree to a level of $\sim 0.25 \, \mu m$, and the lateral resolution is about 2 mm. Large improvements in accuracy, repeatability and resolution are expected from a future Shack-Hartmann wavefront sensor that operates at UV wavelengths [34].

### 3.2.2 Slumping procedure

In order to flatten the microsheet, we have performed a large number of experiments that involve thermal shaping or slumping. The basic idea is to slump the sheet against a flat fused silica mandrel. In this process one heats the sheet placed on top of the fused silica mandrel using a standard oven, and gravity causes the glass microsheet to slump against the flat fused silica plate. To achieve any good results, excellent temperature uniformity and a carefully prescribed thermal profile are required. Enclosing the mandrel and the sheet in a Ni-coated copper box ensures a suitable temperature uniformity. The experiments are performed inside a class 100 cleanroom.
Figure 3-2: Slumping settings. The glass microsheet and the fused silica mandrel are enclosed in a Ni-coated copper box inside the oven. Six thermometers are placed inside the copper box above the glass to ensure that a good uniformity is attained.

Six thermometers placed on the top of the copper box, as shown on Figure 3-2, enable us to measure the uniformity and the temperature actually reached by the copper box, which is slightly different from the oven temperature as can been seen on Figure 3-3. The thermal profile adopted for our slumping experiments is composed of four different phases (Figure 3-3). First, the oven is programmed to reach its highest temperature, or slumping temperature, with a 2.3°C/min rate. Then it remains at this temperature during a “holding” period before starting a low rate decrease, 0.5°C/min, towards 400°C. Finally the temperature is brought back to the room temperature at 5°C/min. The data collected by the thermometers show that the temperature is uniform within a degree inside the Ni-coated copper box during all of the process steps.

The slumping temperature and the holding time result from a tradeoff between the final flatness of the sheet and the sticking issue. Indeed the higher the temperature or the longer the holding time is, the flatter the foil will be; but above a certain temperature or holding time, some areas of the glass microsheet eventually stick on the fused silica mandrel, leaving the latter unusable for further processing. For this
Figure 3-3: Thermal profile used during slumping. The temperature of the oven is first raised over the annealing point of the glass and then slowly decreased until it reaches 400°C. From this point the temperature is brought back to the room temperature much faster. A “holding period” can also be used when the temperature of the oven is at its maximum. The temperature seen by the glass inside the copper box (TC1) is always slightly lower than the preset oven temperature.

reason we have selected a temperature about 50°C above the annealing point of the glass (~560°C for D-263 glass, see Appendix A) and a holding time ranging from 0 to 25 minutes.

3.2.3 Slumping to flat plate

For the first slumping experiments, the mandrel used was an optically flat fused silica plate (Figure 3-5a). No particular cleanliness precautions were taken. Figure 3-4b shows a typical microsheet topograph after thermal shaping demonstrating tremendously improved flatness. The largest deviation from a flat surface, initially bigger than 100 μm, has indeed been reduced to about 5 μm.

Several observations are noted. First edge effects are generally seen, especially at corners, which manifest as sharp bends up to 10 μm in height. Suppressing edge defects such as cracks or fissures before slumping or cutting oversized sheets to the final dimensions after slumping could eliminate this problem. Second, the topographs
Figure 3-4: a) Hartmann topograph of a 50×50 mm\(^2\) glass microsheet before thermal slumping, showing a \(\sim 100\) \(\mu\)m warp. b) Topograph of sheet after slumping, showing \(\sim 5\) \(\mu\)m warp. Note the 15x change of vertical scale between the two images.

indicate the sheets are deformed by many fine bumps one to five microns high and around 5 mm in diameter. After much experimentation we have determined that these deformations are due to \(\mu\)m size dust particles on the mandrel that print through the slumped glass sheet. Essentially, even in the clean room, the mandrel and microsheet never actually touch, but are held off by dust particles.

A good deal of effort has been expended trying to understand and mitigate the dust issue. Our findings are somewhat counter-intuitive. For example, super-cleaning the mandrel and microsheet with RCA\(^1\) [30] before slumping is a disaster. Due to optical contact bonding of the surfaces and mismatch of the coefficient of thermal expansion, the glass shattered during the slumping process. We therefore concluded that the presence of dust particles is actually essential to the success of slumping to a smooth mandrel, since it prevents direct contact between parts.

In order to minimize deformation caused by dust particles trapped between mandrel and glass microsheet we followed a novel approach. We have tried to slump

\(^1\)RCA is a cleaning solution based on ammonia and hydrogen peroxide used in the silicon semiconductor industry.
Figure 3-5: Glass microsheet shaping process. a),b): Sheet is slumped against a flat fused silica mandrel. c),d): sheet is slumped against a pin chuck fused silica mandrel.

against a silicon pin chuck that has been micromachined into very flat array of thousands of $25 \times 25 \mu m^2$ pins. Figure 3-5 illustrates the advantage of slumping against a pin chuck instead of a standard flat plate. The underlying idea is that dust particles fall into the space between the pins which are deep enough to take up to $\mu m$ sized dust particles and prevent them from deforming the glass. While this does give improved results, it introduces a new problem. Indeed, many silicon pins broke and stuck to the microsheet during the thermal process.

3.2.4 Slumping to pin chuck

Much thought has led us to use the very same idea but with pin chuck made of fused silica, or more exactly of fused silica, and not silicon. On the one hand, fused silica being much stronger than silicon, the pins should not break as they did before. Fused silica is also easier to work with and to figure for use as a shaping mandrel. On the other hand, the fabrication of fused silica pin chuck is not as straightforward as for silicon [24]. Figure 3-6 displays the pattern that is transferred to a $100 \times 100 \times 2.3 \ mm^3$ fused silica photomask during fabrication. In fact, the size of the pins on the finished mandrels is closer to $15 \ \mu m$ and their height is about $2.5 \ \mu m$.

The first slumping results on fused silica pin chuck were not very encouraging
because several areas of the glass microsheet had been damaged during the slumping. Indeed small pieces of glass had been pulled out from the foil. A careful examination of the fused silica mandrel revealed that these defects were linked to defects in the mandrel itself. In fact, between the pins there were, at some places, islands or plateaus where the glass got stuck during slumping (Figure 3-7).

Once we had fixed this problem, we obtained excellent results. Much fewer deformations that can be traced to dust particles were seen. The central area of the foil was then flat within $\sim 1.5 \mu m$ (Figure 3-8). However edge effects were still perceptible especially at corners and variations in final surface quality were observed between different slumping runs.

To get a better handle on the final result, we have developed a very simple technique for determining problematic areas beforehand. The RCA cleaned glass microsheet is first positioned on the fused silica pin chuck. Then using a transparent plastic bag filled with air, we push the microsheet against the fused silica mandrel. Because it is filled with air, the bag applies an almost uniform pressure against the entire surface of the microsheet (Figure 3-9). Therefore the shape of the microsheet at this point is close to the one it should take after slumping. Another important feature of the bag is its transparency. Indeed it allows us to observe the interference fringes that arises when a monochromatic source is used to illuminate the microsheet and the mandrel. If the gap between the microsheet and the base of the fused silica
Figure 3-7: Microscope image of a defect in the fused silica mandrel that has produced a crack in the glass during the thermal shaping process.

Figure 3-8: (Left) Hartmann topograph of a glass sheet after slumping on a fused silica pin chuck mandrel. (Right) Histogram of surface angular deviation showing a global resolution of $\sim 18$ arcsecond.
Figure 3-9: Schematic of the “bag test” used to anticipate the results of a slumping run. A plastic bag filled with air is used to apply a uniform pressure against the glass microsheet. If a dust particle prevents the sheet from flattening to the top of the pins the resulting wedge between the bottom of the glass and the mandrel surface will produce concentric interference fringes.

Pin chuck is not constant, the path length difference, $\Delta d$, between the ray reflected from the back surface of the glass, labelled 1 on Figure 3-9, and the one reflected from the surface of the mandrel, labelled 2, varies. This modulation of path length difference can be perceived as interference fringes. Let suppose that

$$\Delta d = n \frac{\lambda}{2}$$

(3.1)

where $\lambda$ is the wavelength of the monochromatic source used and $n$ is an arbitrary number. Then an even integer value of $n$ corresponds to a light fringe and an odd integer value to a dark fringe. The light source used emits the mercury “green” line, $\lambda = 546$ nm. Therefore the interval between a light and a dark fringe corresponds to a path length difference of $\lambda/2 = 273$ nm, or a microsheet height variation of $\lambda/4 = 136$ nm.

Usually most of the area observed is composed of a single fringe, which means that the gap is constant and thus that the glass is flat. But there is always a few
spots where there are concentric fringes. Typically five to twenty light concentric fringes can be seen. Those spots are associated either with dust particles bigger than the current size of the pins or rarely with defects in the mandrel. Hopefully the next generation of pin chuck will have taller pins (about 10 μm) and thus the probability that a dust particle is sticking out from the mandrel will be much smaller.

Currently we try to eliminate as much of the trapped dust particles as possible by using a CO₂ snow cleaning gun before every slumping run. CO₂ snow cleaning is a surface cleaning method in which a combination of high velocity small dry ice particles and carbon dioxide gas streams are directed towards a sample surface. The interactions between the solid and gaseous CO₂ and the surface particles and hydrocarbon contamination lead to surface cleaning. Cleaning with CO₂ snow leads to removal of even micron and submicron sized particles [2]. However, even inside the cleanroom and with the help of the CO₂ snow gun, it is difficult to trap fewer than ~ one particle per 10 cm². Thus at the end of the CO₂ snow cleaning process, we can usually see one set of concentric fringes in the central portion of the microsheet and other fringes along the edges.

Because glass and mandrel are both super-cleaned at this stage, an optical bond is generally observed between the two pieces. This bond is so strong that it is impossible to move the glass on the mandrel anymore. To break the bond without shattering the glass, we have to use an air gun and blow air in between the pins under the glass microsheet. As already mentioned, the strong bond that exists between two super-cleaned surfaces can damaged the glass during the thermal shaping process. Indeed contact bonding to the pins and/or friction prevents the glass from expanding and contracting freely relative to the mandrel. And once the glass sticks to the top of a pin for some time at high temperatures the probability for fusing increases dramatically.

For this reason, a 3 μm thick non-sticking TiO₂ coating is evaporated on the top surface of the mandrels. The roughness of this “anti-stick” layer is found to be the key to a useful shaping mandrel. Figure 3-10 shows the atomic force microscopy (AFM) data of two different TiO₂-coated mandrels. The rougher coating was evaporated on a four inch square pin chuck which has been successfully used to slump several
Figure 3-10: Atomic Force Microscopy data of two TiO$_2$-coated fused silica mandrels. For each mandrel, a 3D plot of the measured surface along with a height profile and the overall roughness are given.
Figure 3-11: Microscope view of sticking evidence. Looking through the glass, both the fused silica pins and the damages they left on the glass, because of sticking, can be seen.

microsheet to a remarkable level of flatness. Figure 3-12 displays the Hartmann topograph and angular resolution of the central portion of one of those microsheets. The foil is flat within 1 μm and the central portion of it shows rms surface height variations of only 0.2 μm. Compared to the initial warp, the thermal forming process has led to an ~100× improvement. An rms surface angular deviation of 10 arcseconds is achieved. In Figure 3-13 the histogram of angular deviation is shown for the mandrel and the slumped sheet from Figure 3-12. As can be seen, the flatness of the slumped microsheet is very close to the flatness of the mandrel itself. The rms angular deviation for an optical flat is comparable to the value obtained for the mandrel and indicates the measurement accuracy of our Hartmann system.

Those very promising results encouraged us to pursue our efforts in this direction. But as we tried to slump bigger foils by evaporating the same coating on larger fused silica mandrels, we found that the process that led to this very rough coating was hardly reproducible. And slumping very clean glass microsheets on non-rough mandrels inevitably induced sticking. Figure 3-11 shows the damages left by sticking on a glass microsheet slumped against a non-rough TiO₂-coated pin chuck.

Right now, we are seeking alternatives to roughen the top of the pins and thus avoid sticking. We are also trying to replace our surface metrology measurement tool, which has now reached its limits at the sub-micron level.
Figure 3-12: (Left) Hartmann topograph of a glass sheet after slumping on a TiO$_2$-coated fused silica pin chuck mandrel. (Right) Histogram of surface angular deviation showing a global resolution of $\sim$10 arcsecond.

Figure 3-13: Histogram of angular deviation for the pin chuck (solid line, rms = 4 arcseconds) and for the slumped microsheet from Figure 3-12 (dashed line, rms = 10 arcseconds).
3.3 Microsheet bending

The thermal process described in the previous section will provide flat glass substrates. But in order to use those substrates to focus X-rays, we still need to bend them to their parabolic/hyperbolic Kirkpatrick-Baez shapes. Because the shape of KB optics deviates from a plane by only a few microns, the initial idea is to apply very small bends to the microsheets during assembly using the silicon microcombs. For a KB telescope with a 2 m focal length, the shape of the reflective surfaces deviates from a plane by only $\sim 15 \mu m$ and for a 10 m focal length this number reduces to $\sim 4 \mu m$.

Since we plan to modify the shape of the microsheet by applying constraints only at a finite number of points along its edges, the final shape will not perfectly match the ideal KB shape. In order to evaluate the error introduced by using this technique, we have made several finite element model calculations of glass microsheet shapes. The initially flat microsheet surface is deformed by applying displacement at several points along its longer edges and then this is compared to the perfect KB surface. The glass sheet material is Schott D-263 and the sample considered is 200 $\mu m$ thick and measures 160 $\times$ 80 mm$^2$.

Figures 3-14 and 3-15 show that, as one might have expected, increasing the number of constrained points along the edges clearly reduce the error introduced. Nevertheless even with three points that can be constrained along both edges, the error introduced here is too big for the expected overall resolution of the optics. The numbers presented in both Figure 3-14 and 3-15 have been computed in the case of a KB optics with a 2 m focal length for which the maximum deviation of the optical surface from a plane is 15 $\mu m$. In the better case, although most of the surface is accurate within 0.25 $\mu m$, some areas located on the free edges are really much worse (1$\mu m$ error). The averaged angular deviation, which is the most important number for our purpose, is about 7 arcsecs but 25% of the area is worse than 5 arcsecs. To suppress those bad areas, we have tried to change the microsheet shape by cutting circular parts out of the rectangle along the unconstrained edges. The averaged angular deviation is reduced to about 4 arcsecs but 18.5% of the area is still worse.
Figure 3-14: (Left) Surface error topograph and (Right) histogram of surface angular deviation of an initially flat foil bent to its KB optical shape using two points along its longer edges.

Figure 3-15: (Left) Surface error topograph and (Right) histogram of surface angular deviation of an initially flat foil bent to its KB optical shape using three points along its longer edges.
than 5 arcsecs and 13.6% of the total surface has been lost in the process.

On the plus side, if we consider the 10 m focal length design for which the maximum deviation of the optical surface from a plane is only 4 \( \mu \text{m} \), the error introduced by using the silicon microcombs to bend the foil in its final shape is about 1 arcsec. The deviation of the actual to the desired shape is only \( \sim 0.1 \mu \text{m} \).

With those numbers in mind, we chose a different approach than the one originally envisaged to shape the glass microsheets into their final Kirkpatrick-Baez figure. Instead of flat pin chucks, mandrels with cylindrical shapes approximating the optimal Kirkpatrick-Baez figure will be used to slumped the glass microsheets. Then the silicon microcombs will be used to bend the microsheets by the tiny amount (less than 4 \( \mu \text{m} \)) that separates the cylindrical approximation from the Kirkpatrick-Baez figure.
Chapter 4

Flight integration

Scientific observations from space require instruments which can operate in the orbital environment and which can, first of all, reach their orbit safely. Indeed despite all the technological progress that have been made since the dawn of the space age in 1957, access to space is still not straightforward. Severe loads are applied on the spacecraft during its launch.

In this chapter, the practicability of wide-field off-axis Kirkpatrick-Baez designs is considered. Preliminary design calculations and tests are presented. The resistance of glass microsheets to acoustic loads is addressed and the tradeoff between strength and weight illustrated through the study of a strawman mission concept.

4.1 Launch environment overview

4.1.1 Loads on the spacecraft in the launch environment

The spacecraft and its payload are accelerated by (1) thrust and vibration transmitted across the launcher interface and (2) acoustic excitation via the atmosphere within the launcher fairing or payload bay doors.

The direct interface loads comprise steady thrust, low-frequency (1-40 Hz) transient accelerations, and random vibration. The accelerations vary as fuels burn and stage succeeds stage. Lateral interface loads result from wind shear and trajectory
corrections during ascent. The acoustic noise excites random vibration, particularly of spacecraft panels, directly. This noise originates from air turbulence due to efflux from the rocket engines at lift-off (or the airplane engines in the case of Pegasus) and from the turbulent flow around the whole vehicle. The first stage burn begins with the vehicle held firmly on the pad. From a few seconds before lift-off, rocket motor efflux generates an intense noise field (possibly 140 dB above the standard reference amplitude of 20 mPa). When hold-down arms are released, there is a transfer of stress which excites transiently the axial modes, particularly the lowest “pogo” frequency which may be near 15 Hz. The pogo mode is one of vertical motion in which masses above and below the center of gravity oscillate in opposite directions.

As the vehicle climbs, the sound field is intensified by ground reflection, leading to a maximum random-acoustic excitation. The acoustic peak decays as the vehicle accelerates past unity Mach in thinner air, but mechanical transmission through the structure will continue. Until the dynamic pressure \( \frac{1}{2} \rho v^2 \) peaks a little further into the flight, the spectral character of the vibration may change as boundary layer turbulence grows. Supersonic shockwave movement with flow separation will add its effects.

So peaks may occur early in flight and later at maximum dynamic pressure. As propellant is burned the flight-path acceleration increases and the vehicle compression strain increases; the release of strain energy at main engine cut-off transiently excites the vehicle axial modes, but at higher frequency (perhaps 30 Hz, due to reduced propellant mass). And there may be some ‘chugging’ (combustion instability, due to interaction with propellant feed).

The upper stage motors, burning beyond the atmosphere, transmit their random vibrations through vehicle and spacecraft structure. Shock and high-frequency transients are caused at separation by the release of energy, both elastic and chemical, at the firing of pyrotechnic fasteners.

Launch environment data will be found in the launch vehicle manual. Figures 4-1 and 4-2 are extracted from Pegasus users guide. Pegasus should be our baseline launch vehicle.
Figure 4-1: Payload Interface Random Vibration Specification. Extracted from Pegasus users guide.

Figure 4-2: Payload Acoustic Environment. Extracted from Pegasus users guide.
4.1.2 Spacecraft mechanical design

For spacecraft and their instrumentation, the engineering disciplines of mechanical and structural design work together, and both are founded on the study of the mechanics of materials. We create designs, then prove (by calculation and test) that they will work in the environments of rocket launch and flight.

The twin objectives are to create a structure which is (i) strong enough not to collapse or break, and (ii) stiffly resistant against excessive deformation. Forces may be static, or dynamic, but the dynamic forces are dominant in rocket flight, and vibrations are a harsh aspect both of the launch environment and of environmental testing, generating often large dynamics forces.

The need for strength to survive the launch environment is clear enough, but stiffness is a less obvious requirement. Some space instruments, such as optical devices for X-ray imaging, must maintain their dimensions to high accuracy in order to perform well. The property of stiffness is the capability to resist deformation under the action of force. Even if a piece does not need to be accurately positioned, it must in any case be stiff, to resist vibration. Stiffness dictates that all naturally resonant frequencies of the structure will be high, and the vibration oscillations, induced by the noisy launch, acceptably small. We shall see that both the choice of material and also the design decisions for the disposition of structure parts are important for stiffness. It is convenient to specify stiffness in terms of an arbitrary minimum natural frequency, perhaps 80-120 Hz for a small instrument, 60 Hz for an average subsystem, 24 Hz for a large instrument such as the Hubble Space Telescope and 20 Hz axially for a whole spacecraft.

The aims of structural-mechanical design are therefore summarized as adequate strength, to withstand tests and launch, adequate stiffness, to minimize deflections, and least weight, at allowable cost.
4.1.3 Vibration tests

We have already noted that dynamic loads are critical for the spacecraft designer. Also we have drawn attention to the role of stiffness, in relation to given mass, for promoting acceptably high natural frequencies. The vibrations found in rocket vehicle during the spacecraft’s launch are random, as with other transport vehicles. They may be very intense, particularly in the vicinity of rocket efflux, which radiates a strong acoustic field. At release and staging there are shocks, which excites transient modes of higher frequency vibration. The proving of designs against vibrations is a necessity which spawn tests, usually more severe than the actual vibration they are intended to simulate.

If we consider vibrations at those higher frequencies that excite resonances of an instrument’s structure many complexities immediately enter the picture. The instrument assembly is a piece of secondary structure. At this stage there is neither a detailed mathematical model of the instrument structure, nor knowledge of how the spacecraft will modify the launcher excitations.

In any case, the most exacting conditions imposed are usually the random vibration tests to qualification levels. Because of the uncertainty of flight vibrations, their complexity, and the statistical element in specifying them, these tests are likely to be more severe than flight conditions and so may become the dominant design requirements.

In the early stages of development of new designs arbitrary overtests are less risky than no tests. Such tests are of much value in exposing weakness in design, but they fall short of being simulations of the environment. Indeed, for proof-of-design qualification and flight acceptance, tests of complete structures should be demanded.

The random excitation vibration test has been widely adopted as the principal test for proof of mechanical design but there is, however, an alternative which is a test in an acoustic chamber.
4.1.4 Conclusion

A space instrument must survive the launch environment and therefore it must survive the greater rigors of the vibration tests. The following practice is recommended:

- Every detail of design should be examined to eliminate stress-raising sharp corners and sudden change of section.

- Stocky, sturdy constructions are to be sought, well braced and buttressed. Flimsy and flexible parts are suspect. While there will always be some low-frequency modes, the aim should be to get as many as possible above 200 Hz.

- Every screw fastening should have a locking feature.

- Materials should be specified with precision so quality can be assured.

In order to know if our initial design would survive the harsh launch environment, we have made some initials calculations to determine the natural resonant frequencies of both the flight frame and the glass microsheets.

4.2 Flight frame design

The flight frame, that will hold the Au-coated glass microsheets together, does not need to be really accurate. Standard machining shop tolerances should be sufficient. However, it needs to be really stiff. Indeed, distortion in the flight frame could introduce warpage in the mirror assembly. That is why the flight frame should be designed to have the greatest first natural resonant frequency possible. To limit displacement amplitude to a couple microns, frequencies should be at least above 1000 Hz.

In order to determine the stiffness of the flight frame, we have exported its model into a Finite Element Solver and have calculated the natural resonant frequencies. Calculations have been made assuming that the flight frame is made out of aluminum but in fact thermally matched composites will be used. The use of composites should bring an additional 20% stiffness and reduce weight.
We have considered several different designs. Three will be discussed here. The first one is represented on Figure 4-3, where all the “walls” have a thickness of 5 mm. The second is similar but with a wall thickness of 7 mm. The third one (see Figure 4-4), compared to the first one, has a 1 mm wide cross reinforcing each open side.

The results of the analysis are summarized in the following table (values are given in Hertz):

<table>
<thead>
<tr>
<th>First Design</th>
<th>Second Design</th>
<th>Third Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>464</td>
<td>621</td>
<td>726*</td>
</tr>
<tr>
<td>565</td>
<td>732</td>
<td>729*</td>
</tr>
<tr>
<td>828</td>
<td>1068</td>
<td>805</td>
</tr>
<tr>
<td>904</td>
<td>1088</td>
<td>951*</td>
</tr>
<tr>
<td>991</td>
<td>1292</td>
<td>954*</td>
</tr>
<tr>
<td>1169</td>
<td>1385</td>
<td>969</td>
</tr>
<tr>
<td>1525</td>
<td>1889</td>
<td>974</td>
</tr>
<tr>
<td>1636</td>
<td>2054</td>
<td>1024</td>
</tr>
</tbody>
</table>

Table 4.1: Natural resonant frequencies (in Hertz) of the three different flight frame designs studied. The frequencies marked with an asterisk are only affecting the reinforcing crosses and do not involve displacement in the other parts of the structure.

So as one might have expected, thicker walls make the flight frame stiffer but would also make it heavier. However we could then reduce the weight of our structure by removing some material out of the low stress regions. The other alternative seems much more interesting. Indeed, the additional crosses reinforce the box corners and thus eliminate the first two natural resonant frequencies that are associated with maximum local displacements on the corners, as shown on Figure 4-5. If we keep in mind the fact that aluminum will be replaced by stiffer composite materials, the first resonant frequency is now quite close to the 1000 Hz objective. The third design seems therefore the best one to fulfill the goals of the flight frame. The next step would be to study where the stress is concentrated in order to improve and lighten our structure.
Figure 4-3: Full-sized structural model of a flight frame. The coarse combs with their oversized grooves can be seen.

Figure 4-4: Third flight frame design considered. The aperture loss due to the reinforcement cross is about 3%.
Figure 4-5: Resonant modes of the flight frame first design. For each mode, both the corresponding resonant frequency and deformed shape are given.
4.3 Microsheets natural resonant frequencies

4.3.1 Cylindrical inflection of rectangular plates

In this section, we will consider the glass microsheets as rectangular plates constrained along two edges as shown on Figure 4-6. To simplify the calculations, only cylindrical inflections about the $y$ axis will be considered, which means that $y$-modes are ignored. The deflection $v(x, t)$ is thus independent from the $y$ coordinate.

For this particular case, the equation of motion, with internal viscous damping, should be written

$$D \frac{\partial^4 v}{\partial x^4} + D' \frac{\partial}{\partial t} \frac{\partial^4 v}{\partial x^4} + \rho h \frac{\partial^2 v}{\partial t^2} = 0$$

(4.1)

where $D = \frac{E h^3}{12(1 - \nu^2)}$, $D' = \frac{\eta h^3}{12(1 - \nu^2)}$, $E$ is the Young's modulus of glass, $\eta$ is the coefficient of viscosity, $\nu$ is the Poisson's ratio, $\rho$ is the density, and $h$ is the thickness of the plate.
Let say that \( v(x, t) = f(x)g(t) \), then
\[
\frac{D}{\rho h} f^{(4)}(x) = \frac{\ddot{g}(t)}{g(t)} + \frac{D'}{D} \dot{g}(t) = \text{constant} = -w^2
\] (4.2)

We obtain two uncoupled differential equations. The one to consider first is
\[
f^{(4)}(x) = \frac{\rho hw^2}{D} f(x)
\] (4.3)

The general solution of this ODE is
\[
f(x) = \alpha \cos \left( \sqrt{\frac{\rho hw^2}{D}} x \right) + \beta \sin \left( \sqrt{\frac{\rho hw^2}{D}} x \right) + \gamma \cosh \left( \sqrt{\frac{\rho hw^2}{D}} x \right) + \delta \sinh \left( \sqrt{\frac{\rho hw^2}{D}} x \right)
\] (4.4)

The boundary conditions for this problem are given by
\[
\begin{align*}
v(0, t) &= v(L, t) = 0 \Rightarrow f(0) = f(L) = 0 \\
\frac{\partial v}{\partial x}(0) &= \frac{\partial v}{\partial x}(L) = 0 \Rightarrow f'(0) = f'(L) = 0
\end{align*}
\] (4.5)

Therefore a non-trivial solution of Equation 4.3 exists if and only if
\[
\cos \left( \sqrt{\frac{\rho hw^2}{D}} L \right) \cdot \cosh \left( \sqrt{\frac{\rho hw^2}{D}} L \right) = 1
\] (4.6)

This condition for the existence of resonance contains the term \( w \). This term can be interpreted as a pulsation through the second differential equation,
\[
\ddot{g} + w^2 \frac{D'}{D} \dot{g} + w^2 g = 0
\] (4.7)

Indeed, if the damping is neglected, i.e. \( D' = 0 \), the solution for Equation 4.7 is of the type
\[
g(t) = a \cos (w t) + b \sin (w t)
\] (4.8)
Table 4.2: First four solutions to the equation \( \cos(X) \cdot \cosh(X) = 1 \)

Thus if \( X_n \) is a solution of \( \cos(X) \cdot \cosh(X) = 1 \) (see Table 4.2), the natural resonant frequencies of the rectangular plate are given by

\[
f_n = \frac{w_n}{2\pi} = \frac{X_n^2}{2\pi} \sqrt{\frac{E}{12\rho(1 - \nu^2)}} \frac{h}{L^2}
\]  

(4.9)

If we consider D-263 glass, the first two resonant frequencies are numerically given by

\[
f_1 = 5.67 \times 10^3 \left( \frac{h}{L^2} \right)
\]  

(4.10)

\[
f_2 = 15.61 \times 10^3 \left( \frac{h}{L^2} \right)
\]  

(4.11)

where \( h \) and \( l \) are expressed in meter and \( f \) in Hertz.

Thanks to this analytical result, we are now able to predict the influence of the microsheet dimensions on its natural resonant frequencies. The following table gives the first natural resonant frequency in Hertz of some microsheet shapes that we might want to use.

<table>
<thead>
<tr>
<th>L=100 mm</th>
<th>h=200 ( \mu m )</th>
<th>h=400 ( \mu m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>226</td>
<td></td>
</tr>
<tr>
<td>L=140 mm</td>
<td>58</td>
<td>116</td>
</tr>
</tbody>
</table>

Table 4.3: First natural resonant frequencies (in Hertz) of a glass microsheet as given by Equation 4.10 for various values of the design parameters.

The microsheet with a first resonant frequency of 58 Hz, which optimizes the telescope resolution and weight, might cause problems. Indeed this frequency is not high enough for such a small instrument. For those particular parameters, the glass microsheet might not survive the launch environment. To resolve this issue, we will
Table 4.4: Natural resonant frequencies (in Hertz), obtained using Finite Element Methods, of a glass microsheet of various thickness and held in different configurations. The values in bold are problematic values.

<table>
<thead>
<tr>
<th>6 fixed points</th>
<th>8 fixed points</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=200 μm</td>
<td>h=400 μm</td>
</tr>
<tr>
<td>41</td>
<td>86</td>
</tr>
<tr>
<td>67</td>
<td>136</td>
</tr>
<tr>
<td>124</td>
<td>255</td>
</tr>
<tr>
<td>157</td>
<td>316</td>
</tr>
<tr>
<td>162</td>
<td>328</td>
</tr>
<tr>
<td>253</td>
<td>519</td>
</tr>
<tr>
<td>260</td>
<td>527</td>
</tr>
<tr>
<td>294</td>
<td>596</td>
</tr>
<tr>
<td>335</td>
<td>672</td>
</tr>
<tr>
<td>389</td>
<td>791</td>
</tr>
</tbody>
</table>

need either to use a thicker microsheet or to modify the way the microsheet is attached to the flight frame.

Although this analytical result has been very helpful, it is based on several approximations and therefore is not as accurate as a finite element model would be.

### 4.3.2 Finite element model

Using ANSYS, we have been modeling the microsheets as rectangular plates that are not constrained along their whole edges but only on a finite number of points. Thus this model is much closer to reality than the previous analytical model.

It turns out that natural resonant frequencies calculated by ANSYS are smaller than the predictions of the analytic method. The numbers presented in Table 4.4 have been computed for a 140×100 mm² D-263 glass microsheet. Two different configurations are studied. In the first configuration, the microsheet is fixed to the flight frame by three points along each of its shorter edges. And in the second one, another mounting point is added in the middle of the longer edges. For each configuration, two different microsheet thickness are considered. Table 4.4 displays the first ten natural resonant frequencies, in Hertz, of the four different cases and Figure 4-7 depicts
the deformed shapes of the modes associated with the case where the microsheet is 400 μm thick and held at six points.

Numbers shown in bold in Table 4.4 are values which may cause problems because they are located in or near a high power spectral density area of the payload interface random vibration specification for Pegasus (see Figure 4-1). So if we cannot change the mirror length, we might want to, as anticipated during the analytical analysis, use the thicker glass or use the eight mounting points configuration. Yet in the first case the mass of the optic would increase tremendously and/or the collecting area achievable would be reduced.

4.4 Microsheets deformation and stress

To get a better appreciation of the level of stress and deformation that the microsheets might encounter during a rocket launch, the OverAll Sound Pressure Level, given in Pegasus Users guide (see Figure 4-2), has been applied to a finite element model of a glass microsheet. The model used retains the eight mounting point configuration and the thickness of the microsheet considered is 0.2 mm.

Since the OverAll Sound Pressure Level is given in decibels, we should first convert it in a more conventional pressure unit using the following equation [1]

\[ K = 20 \log \frac{P}{P_0} \quad (4.12) \]

where \( K \) is the sound pressure level in decibels and \( P_0 = 2 \times 10^{-5} \) N/mm\(^2\) is a reference pressure level. Thus a 130.8 dB OASPL is equivalent to a pressure of 69.35 N/mm\(^2\). It is also interesting to note that this pressure level is also equivalent to a transient acceleration \( \gamma = 14 \) g given by

\[ \gamma = \frac{P}{h \rho g} \quad (4.13) \]

where \( h \) is the thickness of the microsheet, \( \rho \) the density of D-263 glass and \( g = 9.81 \) m/s\(^2\).

The results from the finite element analysis are displayed in Figure 4-8. The
Figure 4-7: Resonant modes of a 140×100 mm$^2$, 400 μm thick glass microsheet. For each mode, both the corresponding resonant frequency and deformed shape are given.
Figure 4-8: Stress and displacement of a 0.2 mm thick glass microsheet loaded with a static pressure of 69.35 N/m² (Pegasus OverAll Pressure Sound Level).
Figure 4-9: Stress repartition around microsheet mounting fixture. The arrows are vectors representing the stress principal directions and intensities. The empty elements are the elements bonded to the flight frame.

maximum deflection experienced by the microsheet is about 0.5 mm and the maximum stress is less than 25 MPa. Those numbers are very encouraging because they indicate that the gap between two microsheets inside the flight frame, 3 mm, should be large enough to accommodate their deflections and that the stress of the microsheet should remain below the ultimate tensile stress of D-263 glass (110 MPa). However because modes of the glass microsheet could be excited by the acoustic environment and thus resonance could occur, we cannot conclude based on this single analysis that the microsheet would survive a rocket launch.

On Figure 4-8, we can see that the maximum stress levels are concentrated around the mounting points. To have a better knowledge of the stress repartition around those points, a more realistic model of the way the microsheet is actually held to the flight frame has been realized. Figure 4-9 displays the result of this analysis.

Finite Element Methods have also been used to quantify the amount of deformation expected on a glass microsheet due to gravity sag. It turns out that a 0.2 mm thick microsheet fixed at eight points and held horizontally, will experience a maximum deformation under its own weight in the Earth's gravitational field of about
37 μm (see Figure 4-10). For a 0.4 mm thick microsheet the maximum deformation under gravity is 10 μm. Those numbers are enormous if compared to the surface accuracy required to achieve the targeted optical resolution of one arcsecond or less. Fortunately, the microsheets will not experience this gravitational field once launched in space. But on Earth, this means that special care has to be taken when measuring and inferring from the measurements the figure of a microsheet.

A good way to double-check those numbers is to use the classical formula for the maximum deflection $w_{\text{max}}$ of a rectangular plate subjected to an uniformly distributed load $[7]$. The deflection at the center of the plate is given by the equation

$$w_{\text{max}} = C(1 - \nu^2)(pb^4/Eh^3)$$  \hspace{1cm} (4.14)

where $p$ is the uniformly distributed load per unit of area, $b$ is the short span length, $E$ is the modulus of elasticity of the material in the plate, $h$ is the plate thickness and, $\nu$ is the Poisson’s ratio, and $C$ is a dimensionless constant whose value depends upon the ratio $b/a = \alpha$ of the sides of the plate and upon the type of support at the edge of the plate. This solution applies only for rectangular plates undergoing small elastic displacements, typically smaller than one-half the plate thickness. If we
suppose that all the edges are fixed (in fact there is only eight fixed points along the edges), then $C$ is given by

$$C = \frac{0.032}{1 + \alpha^4} \quad (4.15)$$

In the case of interest, the load is the gravity felt by the microsheet. We can convert it into a uniformly distributed surface load in the following way

$$F = m\gamma = Shp\gamma = pS \quad (4.16)$$

$$\Rightarrow p = h\rho\gamma \quad (4.17)$$

where $F$ is the overall force applied to the plate, $S = ba$ is the surface of the plate, $\rho$ the density of the material in the plate, and $\gamma = 1g = 9.81 \text{ m/s}^2$ is the acceleration felt by the plate. Gathering Equations 4.14, 4.15 and 4.17 leads to the following expression for the maximum deflection

$$w_{max} = (1 - \nu^2) \frac{0.032}{1 + 0.4 \left(\frac{b}{a}\right)^4} \frac{\rho\gamma b^4}{Eh^2} \quad (4.18)$$

Thus for a $140\times100\times0.2 \text{ mm}^3$ D-263 glass microsheet, the maximum deflection should be $w_{max} = 23.4 \mu\text{m}$. This number is consistent with the FEM result exposed earlier. The fact that it is smaller was expected since the actual microsheet is fixed at eight points only and not along all edges.

The previous analysis resolved the deformation of an horizontal glass microsheet under gravity. It has shown that gravity distorts the flat microsheet by an enormous amount compared to the desired accuracy. Fortunately during assembly and measurement the microsheet is not held horizontal but vertical. However, if the microsheet is not held perfectly vertical, the effects of gravity might affect its shape. In the worst case, the microsheet is only held by the oversized grooves of the coarse combs. As the grooves are 1 mm wide and the microsheet is 140 mm long, the angle between the gravity field and the plane defined by the microsheet should not exceed $0.82^\circ$. Figure 4-11 shows that, in this case, the deformation induced by gravity is
Figure 4-11: Deformation of a plate held almost vertical under the action of gravity. The angle between the gravity field and the plate is 0.82°. Two different thickness of the plate and two different mounting configurations are considered.
not negligible. For the 200 μm thick plate fixed at six points along its smaller edges, the distortion is even more than not negligible, it is about five time worse than the placement accuracy achieved with the current assembly station and than the averaged figure error of a thermally figured glass microsheet! Those results illustrate the need to accurately control the angle between the gravity field and the plate during figure measurement. In order to get a contribution from gravity to the overall figure error of less than 0.1 μm, this angle must be controlled or known within an ~0.04°, or 2.4 arcminutes, accuracy. The future Shack-Hartmann metrological system [34] will be mounted on an air bearing table equipped with levels that can be buy with a resolution up to 5 arcseconds. Microsheets should therefore be aligned with respect to the gravity field within less than 1 arcminute.

4.5 Weight/Strength tradeoff

In this section, I want to illustrate the tradeoff between the overall weight of the optical system and the strength of the individual glass microsheet. To do so I will study a strawman mission concept for which I will compare the various design configurations.

The strawman mission concept considered is a “Super Chandra” which has a focal length of 10 m, an aperture of 2 m for an effective area at 1 keV of 1.3 m². The optical design is based on the wide-field off-axis Kirkpatrick-Baez concept described in Chapter 1. For this reason, it uses a segmented focal plane. This design has roughly 13 times Chandra’s effective area and a resolution of 0.1 arcsecond due to diffraction from the 3 mm space between each individual glass microsheet.

As the front surface of a flight module measures 150 x 110 mm², an array of 13 x 18 flight modules is used to fill the 2 m telescope aperture. Therefore we need 234 flight modules. Each flight module being composed of one x and one y module frame, a total number of 468 flight frames needs to be assembled. Table 4.5 gives the contributions of various components to the global weight of a flight frame.

To realize this “Super Chandra”, several designs are considered. Those designs differ either by the presence of a reinforcement cross structure on the flight frame or
<table>
<thead>
<tr>
<th>Component</th>
<th>Dimensions ((\text{mm}))</th>
<th>Volume ((\text{cm}^3))</th>
<th>Weight ((\text{g}))</th>
<th>Number</th>
<th>Total Weight ((\text{g}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microsheet</td>
<td>100×140×0.2</td>
<td>2.8</td>
<td>7</td>
<td>60</td>
<td>422</td>
</tr>
<tr>
<td></td>
<td>100×140×0.4</td>
<td>5.6</td>
<td>14</td>
<td>60</td>
<td>843</td>
</tr>
<tr>
<td>Flight frame structure ((\text{Al}))</td>
<td>204×150×110 5 mm thick wall</td>
<td>405.5</td>
<td>1100</td>
<td>1</td>
<td>1100</td>
</tr>
<tr>
<td>Cross reinforcement</td>
<td>1×5×172</td>
<td>1.8</td>
<td>5</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>Lightweight</td>
<td>15 mm wide X shape</td>
<td>-39.6</td>
<td>-107</td>
<td>2</td>
<td>-214</td>
</tr>
</tbody>
</table>

Table 4.5: Evaluation of the mass of the various components of a flight frame.

by the number of mounting points used to bond the glass microsheets to the flight frame or by the thickness of the microsheets. The properties of the different designs studied are summarized in Table 4.6.

As can been seen, reinforcing the flight frame structure and adding mounting points on the glass microsheets (Designs 5 and 6) really improve the stiffness of both the flight frame and the microsheets for a limited overall weight cost of ≈15 kg and an aperture loss of ≈5 %. However increasing the thickness of the microsheets is much more costly. Indeed the transition from Design 5 to Design 6 is paid by an 32 % increase of the overall weight. Therefore if the 0.2 mm thick microsheets can survive the launch environment, there is no need to consider Design 6 and a large amount of weight can thus be saved. But if it is not strong enough, thicker microsheets must be used. In this case, the optics might end up being to heavy and then the number of flight modules, and therefore the targeted effective area, would have to be reduced.

Finally, it is interesting to illustrate the advantage of foil optics over traditional full shell approaches by computing the collective area to mass ratio for Designs 5 and 6 and compare it to previous X-ray telescopes. XMM mirrors, which offer the largest effective area to date, collect an impressive 6000 cm\(^2\) at 1 keV for an overall mass of 750 kg. The area-to-mass ratio for XMM is thus 8 cm\(^2\)/kg at 1 keV. For Chandra this ratio is only 1.2 cm\(^2\)/kg. Using foil optics, we achieve, according to Table 4.6, an area-to-mass ratio of 17.4 cm\(^2\)/kg at 1 keV with Design 5 and 13.2 cm\(^2\)/kg with Design 6. This is roughly twice greater than for XMM. Yet ASCA, which offers 1300 cm\(^2\) at
<table>
<thead>
<tr>
<th>Properties</th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
<th>Design 5</th>
<th>Design 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross reinforcement</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Microsheet thickness</td>
<td>0.2 mm</td>
<td>0.4 mm</td>
<td>0.2 mm</td>
<td>0.4 mm</td>
<td>0.2 mm</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Mounting points</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Frame frequency</td>
<td>464 Hz</td>
<td>464 Hz</td>
<td>805 Hz</td>
<td>805 Hz</td>
<td>805 Hz</td>
<td>805 Hz</td>
</tr>
<tr>
<td>Microsheet frequency</td>
<td>41 Hz</td>
<td>86 Hz</td>
<td>41 Hz</td>
<td>86 Hz</td>
<td>97 Hz</td>
<td>193 Hz</td>
</tr>
<tr>
<td>Aperture loss</td>
<td>0 %</td>
<td>0 %</td>
<td>2.5 %</td>
<td>2.5 %</td>
<td>5.3 %</td>
<td>5.3 %</td>
</tr>
<tr>
<td>Frame weight (Al)</td>
<td>1310 g</td>
<td>1730 g</td>
<td>1320 g</td>
<td>1740 g</td>
<td>1340 g</td>
<td>1760 g</td>
</tr>
<tr>
<td>Module weight (Al)</td>
<td>2620 g</td>
<td>3460 g</td>
<td>2640 g</td>
<td>3480 g</td>
<td>2680 g</td>
<td>3520 g</td>
</tr>
<tr>
<td>Total glass mass</td>
<td>198 kg</td>
<td>395 kg</td>
<td>198 kg</td>
<td>395 kg</td>
<td>198 kg</td>
<td>395 kg</td>
</tr>
<tr>
<td>Total mass (no support)</td>
<td>613 kg</td>
<td>810 kg</td>
<td>618 kg</td>
<td>814 kg</td>
<td>627 kg</td>
<td>824 kg</td>
</tr>
<tr>
<td>Support mass (10 %)</td>
<td>62 kg</td>
<td>81 kg</td>
<td>62 kg</td>
<td>82 kg</td>
<td>63 kg</td>
<td>83 kg</td>
</tr>
<tr>
<td>Overall mass</td>
<td>675 kg</td>
<td>891 kg</td>
<td>680 kg</td>
<td>896 kg</td>
<td>690 kg</td>
<td>907 kg</td>
</tr>
</tbody>
</table>

Table 4.6: Properties of various design configurations for a “Super Chandra” mission concept. The mass of the flight frame module is based on an aluminum structure. An actual flight model would use composite materials and thus save about 33% in mass.
1 keV for only 40 kg, carries a remarkable area-to-mass ratio of 32.5 cm²/kg. At the end, we should get closer to ASCA's ratio because the structure of the flight frames could be further lightweight and because we have used in our preliminary calculations as a baseline for the structure of the flight frames Aluminum while they will actually be made of thermally-matched composite materials. Since the mass saved by using composite materials instead of aluminum can be estimated on the order of 33%, the actual area-to-mass ratio for design 5 should be closer to 22.7 cm²/kg at 1 keV.

4.6 Acoustic tests on glass microsheets

In order to prove the resistance of the glass microsheets to realistic rocket launch conditions, to check their natural resonant frequencies and to validate the design decisions, an in-house acoustic test chamber has been set up. Inside this chamber, a 200 μm thick glass microsheet is exposed to acoustic loads similar to those experienced in a rocket launch (see Figure 4-2). The way the glass is attached to the test structure accurately reproduces the way it would be bonded to a flight frame.

4.6.1 Acoustic test chamber setup

The equipment used for the acoustic test is configured as shown in Figure 4-12. The function generator creates a sine wave at a desired frequency. This wave is subsequently amplified and converted to an acoustic pressure wave by a loud speaker. The multimeter connected at the output of the audio amplifier allows us to control the power transmitted to the speaker and thus avoid breaking it. Inside the chamber, two microphones measure the acoustic pressure. One is adjacent to the microsheet under test and the other is located behind the microsheet. The acoustic test chamber is enclosed in a metal container with a hinged lid to allow the operator to open the access port to install the flight frame in the test chamber. The space between the test chamber and the outer metal container is filled with plastic bags containing lead shot to minimize sound coupling and to stiffen the inner chamber.
Figure 4-12: System setup of the acoustic test chamber.
A detailed description of the pieces of equipment used for those tests follows.

**Test equipment**

**function generator** The function generator used for this test is a Wavetek model FG3B. The output from the function generator is a sine wave at a desired frequency. The output of the generator is at the BNC connector marked “output”.

**signal amplifier** The amplifier used for this test is a RadioShack MPA45, model 32-2035. The output from the amplifier is an amplified version of the output from the function generator. The amplification will be set by reference to the calibrated microphones. The 16 ohm output from the signal amplifier is used to drive the speakers.

**microphone** The microphones are Bruel & Kjaer type 4136, both of which have a calibrated response of about 1.34 mV/Pa.
**microphone amplifier**  The microphone amplifier is a unity-gain Brüel & Kjaer type 2804 with two channels. In operation, microphone 1, adjacent to the object under test, is connected to Input 1. Microphone 2, behind the object under test, is connected to Input 2. The rotary switch on the amplifier will either connect or cross the outputs, so that with Output 1 connected to the voltmeter the position of the rotary switch determines which microphone is being read.

**acoustic chamber**  The acoustic chamber contains the speaker, microphones, and holder for the object under test.

**speaker**  The speaker used for this test is a RadioShack 8 inches (203 mm) woofer speaker. The RMS capacity of the speaker is 50 W and its effective range is 35 to 3,000 Hz.

**voltmeter**  The voltmeter used for this test is a Hewlett-Packard model 34401A multimeter, which is a true rms voltmeter. The true rms response is necessary to accommodate the varying frequencies of the signal of interest.

### 4.6.2 Microsheet glueing

For the acoustic tests to have any meaning, it is crucial that the mounting scheme of the microsheet to the test structure reproduces as closely as possible the one that will eventually be used for the flight model. An important aspect to consider is that thousands of glass microsheets will need to be assembled to flight frames. Therefore the process used to bond the microsheets to the flight frames must allow automatization.

The current approach is to use an automatized glue dispenser that would insert a precision tip into holes drilled above the oversized grooves of the coarse combs (see Figure 4-14 and 4-15) and then dispense a precise amount of glue to fill the grooves. This technique has been tested while bonding glass microsheets to the acoustic test structure. A glue dispenser, model 1000 XL provided by EFD, connected to a plant air supply, has been used to provide consistent deposits of glue. Since the glue must remain inside the groove in which it is applied even under the action of gravity, a
Figure 4-14: Cross-section view of a glass microsheet being bonded to a coarse comb. Cylindrical holes in the coarse combs allow a thin precision tip to dispense glue inside the oversized grooves.

Figure 4-15: Glass microsheet glued to a coarse comb. The holes through which the glue is dispensed can be seen.
high-viscosity epoxy has been selected. TRA-BOND 2116 is a thixotropic\textsuperscript{1}, low vapor pressure epoxy system that passes NASA Outgassing Specification. It is typically used for critical aerospace applications where a high-fill, non-sag reliable adhesive is required for bonding and enhancing the mechanical and structural rigidity of assemblies. This epoxy is readily mixed, handled, used and cured at room temperature. For this viscous epoxy to be pushed out from the 0.58 mm-wide dispensing tip used, the operating pressure of the dispenser has to be set up to 36 psi and the timer to 6. As the pot life of the glue is only about 30 minutes, the glueing operations must be completed quickly.

The use of a robotic arm (or several) instead of the operator arm should, latter on, reduce the time required to execute the task. The epoxy currently used will also eventually be replaced by an epoxy thermally matched to the glass microsheet. HYSOL EA-9313 is an epoxy that passes NASA Outgassing Specification for optical components and that could be thermally matched to D-263 glass. In fact, this kind of epoxy has already been successfully used and thermally matched on Chandra. The technique used to matched the epoxy on Chandra could easily be applied to our project. A powder of D-263 glass, or of an other glass with a slightly smaller CTE\textsuperscript{2}, is mixed to the regular epoxy so that on average the CTE of the mix is equal to the CTE of D-263 glass.

4.6.3 Acoustic chamber calibration

The response of the speaker to a constant voltage depends upon the frequency. Figure 4-16 shows that as we sweep the frequency on the function generator and maintain constant the amplification level, the acoustic pressure inside the chamber varies significantly.

Since we wish to study the behavior of a glass microsheet under a given sound pressure level, the pressure inside the chamber has to remain constant. Therefore, we need to adjust both the frequency and the amplification level at the same time.

\textsuperscript{1}Thixotropy is the property of various gels of becoming fluid when disturbed (as by shaking).
\textsuperscript{2}CTE is an abbreviation for Coefficient of Thermal Expansion.
Figure 4-16: Pressure in the acoustic chamber as a function of the frequency when a constant voltage is used to drive the speaker.

Figure 4-17: Voltage to be applied to the speaker to get a constant sound pressure level inside the chamber as a function of the frequency.
Figure 4-17 gives the voltage to be applied to the speaker to get a constant sound pressure level as a function of the frequency considered.

4.6.4 Acoustic tests procedures

Natural resonant frequencies determination

To discuss the physical behavior of the glass microsheet when exposed to the acoustic field radiated by the speaker, we will assimilate the microsheet to a mechanical oscillator of mass $m$, stiffness $s$ and viscous damping constant $r$ being driven by an alternating force $F_0 \cos(\omega t)$, where $F_0$ is the amplitude of the force.

The mechanical equation of motion, i.e., the dynamic balances of forces, is given by

$$m\ddot{x} + r\dot{x} + sx = F_0 \cos(\omega t)$$

(4.19)

The complete solution for $x$ in the equation of motion consists of two terms. A “transient” term which dies away with time and is, in fact, the solution to the equation $m\ddot{x} + r\dot{x} + sx = 0$. The second term is called the “steady state” term, and describes the behavior of the oscillator after the transient term has died away. The steady state behavior of $x$ is given by [37]

$$x = \frac{F_0}{\omega Z_m} \sin(\omega t - \phi)$$

(4.20)

where

$$Z_m = \left( r^2 + (\omega m - s/\omega)^2 \right)^{1/2}$$

(4.21)

and

$$\tan \phi = \frac{\omega m - s/\omega}{r}$$

(4.22)

The total phase difference between the displacement and the force is very small at very low frequencies. But at high frequencies the displacement lags the force by $\pi$ radians and is exactly out of phase. At a frequency $\omega_0$ where $\omega_0 m = s/\omega_0$, $\phi = 0$ and therefore the phase of the displacement is $90^\circ$ behind that of the force. $\omega_0$ is said to
be the frequency of velocity resonance since the velocity of the oscillator reaches its maximum at this particular frequency. The displacement resonance will occur when the denominator $\omega Z_m$ is a minimum. This takes place when

$$\omega = 0$$

or

$$\omega^2 = \omega_0^2 - \frac{r^2}{2m^2} \quad (4.23)$$

Thus the displacement resonance occurs at a frequency slightly less than $\omega_0$, the frequency of velocity resonance. For a small damping constant $r$ or a large mass $m$ these two resonances, for all practical purposes, occur at the frequency $\omega_0$.

Consequently, the frequency at which resonance occurs is characterized by a phase difference between the displacement of the oscillator and the force applied of $-\frac{\pi}{2}$. Experimental determination of the resonance frequency can be achieved if a method for measuring phase difference is available.

If two sine waves are simultaneously fed to an oscilloscope (one to the vertical input an the other to the horizontal input) and the scope is set to operate in the $X$-$Y$ mode, the resulting display on the scope screen is referred to as a Lissajous pattern. The characteristics of the shape of the Lissajous pattern can be used to measure the phase difference between two sine waves of the same frequency. If the equations of the two waves are

$$X = C \sin (\omega t) \quad (4.24)$$

and

$$Y = B \sin (\omega t + \theta) \quad (4.25)$$

the phase difference is found from the Lissajous pattern by equation

$$\frac{A}{B} = \sin \theta \quad (4.26)$$

where $A$ is the point where the ellipse crosses the $y$ axis (Figure 4-18). If two sine
waves are of the same frequency and phase, the Lissajous pattern will be a diagonal line. If the sine waves are of the same frequency but out of phase by 90°, the pattern will be an ellipse; if the amplitudes are also equal, the ellipse will instead be a circle.

The Lissajous pattern obtained by simultaneously connecting the two microphones inside the chamber to the oscilloscope can thus be used to determine the resonant frequencies of the glass microsheet. If the phase difference between the microphone measuring the acoustic pressure radiated by the speaker and the microphone measuring the acoustic pressure as perturbed by the glass microsheet is equal to 90° (Figure 4-19), then the microsheet is at resonance and the Lissajous pattern observed on the oscilloscope is a circle.

This observation constitutes the core of the procedure used to measure the resonant frequencies of the glass microsheets. A microsheet is held to the test structure by coarse combs in a way similar that it would be attached to the flight frame. The two microphones are positioned as shown on Figure 4-19. One of them is placed on the side and records the acoustic pressure radiated by the speaker. The other one is placed behind the microsheet. The pressure inside the chamber is set to a value that does not threaten the glass but still allows an easy read-out on the screen of the

Figure 4-18: Determining phase angle from Lissajous patterns.
Figure 4-19: Acoustic chamber configuration. The top edge of the glass microsheet under test and the two microphones recording the acoustic pressure inside the chamber can be seen.
Mechanical resistance to payload acoustic environment

The resistance tests address the survival of glass microsheets to the acoustic environment encountered on a typical Pegasus rocket launch. Only one microphone is used for those tests. It measures the pressure generated by the speaker.

The frequency is slowly increased from 50 Hz to 1,000 Hz while the voltage applied to the speaker is carefully set to its prescribed value, so that the sound pressure level seen in the chamber corresponds to that of a rocket launch. Table 4.7 gives the various subdivisions of the frequency spectrum considered and their associated sound pressure level extrapolated from Figure 4-2.

After exposure to the sound pressure level of any subdivisions of the frequency spectrum, the microsheet is inspected for damages or cracks. If the glass successfully passes these tests then micro strain gauges [48, 19] will be, latter on, used to measure the level of stress experienced by the microsheet next to the mounting points.

<table>
<thead>
<tr>
<th>Frequency range (Hz)</th>
<th>50 - 150</th>
<th>150 - 600</th>
<th>600 - 700</th>
<th>700 - 900</th>
<th>900 - 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound pressure level (dB)</td>
<td>110</td>
<td>116</td>
<td>121</td>
<td>125</td>
<td>120</td>
</tr>
<tr>
<td>Microphone reading (mV)</td>
<td>8.47</td>
<td>16.91</td>
<td>30.07</td>
<td>47.66</td>
<td>26.80</td>
</tr>
</tbody>
</table>

Table 4.7: Subdivisions of the frequency spectrum and their associated sound pressure level.

oscilloscope (between 90 and 100 dB). The oscilloscope is operated in the X-Y mode.

At the starting frequency of 50 Hz, the display is a line with a slope of approximately 45°. As the frequency is increased and the pressure maintains constant, the display becomes eventually elliptic. The frequencies where the display is close to a circle, indicating that the microsheet is at resonance, are noted.
Table 4.8: Natural resonant frequencies, obtained using the experimental setup described in the previous section, of a 200 μm thick glass microsheet fixed at eight points.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>92 Hz</th>
<th>213 Hz</th>
<th>227 Hz</th>
<th>251 Hz</th>
<th>680 Hz</th>
</tr>
</thead>
</table>

4.6.5 Acoustic tests results

The natural resonant frequencies of a 200 μm thick glass microsheet fixed at eight points, measured by the experimental setup previously described, are displayed in Table 4.8. The smaller resonant frequency measured, about 92 Hz, is in agreement with the Finite Element analysis performed in Section 4.3.2. The other resonant frequencies turned out to be harder to determine because of the sensibility of the frequency generator. A number of them is located, as predicted by the FEM analysis, in the 140-260 Hz range where the signals of the two microphones are always out of phase. Finally, it should be noted that between 300 Hz and 500 Hz, no resonance frequencies can be found using this experimental procedure because the intensity of the acoustic field inside the chamber is not uniform enough.

The glass microsheet successfully passed the mechanical resistance tests. It has been exposed to the acoustic sound pressure levels given in Table 4.7 for a total duration time of 750 seconds. For frequencies larger than 700 Hz, the amplifier was not able to deliver enough power into the speaker to produce the required sound pressure level. For the 700-900 Hz band, the sound pressure obtained was only about 119 dB and for the 900-1,000 Hz band, it was about 118 dB.
Chapter 5

Conclusion

We have developed a system of alignment tooling that is designed to provide sub-\(\mu\)m alignment of segmented thin foil X-ray optics. Alignment is achieved with an assembly truss utilizing a metrology frame, featuring silicon microstructures that are fabricated to sub-\(\mu\)m tolerance. After alignment, foils are bonded to a separate flight structure. The assembly truss is then removed, and is not part of flight hardware (and is therefore reusable). Our approach of using a separate system for alignment and flight simplifies the alignment system by removing requirements such as mass limits and launch stress survival. It also simplifies the flight structure which does not have to provide high-accuracy alignment.

We have built and tested an experimental assembly truss to demonstrate this alignment and mounting technique. Tests performed thus far have demonstrated that the alignment system can mount rigid flat plates with a positioning tolerance typically better than 0.5 \(\mu\)m, resulting in a high degree of parallelism that for the segmented Constellation-X SXT design would correspond to a \(\sim\)1 arcseconds resolution. These tests have also underlined the effects of Hertzian contact stresses on the placement accuracy. Contact stresses occur when curved surfaces of two bodies are pressed together by external loads. Foil optics that are thicker than average will require more leaf spring displacement to be pushed against the reference microcombs. The distortion of the contact interface between the reference microcomb and the foil, due to the resulting extra force applied, can lead up to \(\sim\)0.3 \(\mu\)m error in placement.
accuracy for the current leaf spring design.

In the current design of the assembly truss, the forces involved in bolting the system together cause small distortions in the reference flat. In future designs, the reference flat will be an optical flat that is kinematically mounted to the assembly truss, and will not be a structural member of the system. Future tests will characterize the system's ability to accurately position thin foil optics. In addition to the same positioning accuracy demonstrated with rigid flat plates, these tests will require that we minimize the distorting forces applied to the flexible foil.

Novel methods for the shaping of flat thin-foil X-ray optics have also been investigated. We have obtained encouraging results from slumping to TiO$_2$ coated pin chucks. Away from the edges of 50x50 mm$^2$ microsheets we achieved rms angular deviations as low as 10 arcseconds with a lithographically defined pin chuck that serves as the slumping mandrel. This approach eliminates the need for epoxy replication from superpolished mandrels and offers a low-cost alternative for the production of flat light-weight mirrors.

A number of improvements can be made to the current pin chucks. The pin shape and size can be better controlled through the use of directional plasma etching instead of the isotropic wet HF etch employed so far. This will allow for taller pins of smaller cross sectional area, which should prevent deformation from larger dust particles and reduce the contact area between glass and mandrel even further. Unfortunately, the rough, non-sticking TiO$_2$ coating has proven to be difficult to reproduce reliably. We are therefore also investigating alternative methods to roughen the top of the pins. The sizes of our slumped glass sheets to date are relatively small. Experiments on larger mandrels and microsheets will be performed soon. The suppression of edge effects also needs to be improved. Nevertheless, we believe that the shaping of microsheets to flat mandrels can be improved to the arcsecond level through careful design of the mandrel surface, and scaled up in size in a well temperature-controlled environment.

Finally, the resistance of glass microsheets to the harsh acoustic environment typ-
ically encountered during a rocket launch has been addressed. The natural resonant
frequencies of various glass microsheet configurations have been determined. Acoustic
chamber tests have shown that the microsheet configuration that currently serves as
a baseline (140×100×0.2 mm³ glass sheets fixed at eight points) can withstand the
sound pressure levels of a Pegasus launch.

The level of stress occurring around the mounting points of the microsheet when
exposed to acoustic loads should be measured using micro strain gauges. Such a
measurement would provide us an estimate of the safety margin.
# Appendix A

## Material properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>D-263 glass</th>
<th>AF-45 glass</th>
<th>Silicon</th>
<th>Al</th>
<th>Fused silica</th>
<th>Copper</th>
<th>Sapphire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm³)</td>
<td>2.51</td>
<td>2.72</td>
<td>2.33</td>
<td>2.72</td>
<td>2.19</td>
<td>8.9</td>
<td>3.99</td>
</tr>
<tr>
<td>Young’s modulus (N/mm²)</td>
<td>72,900</td>
<td>66,000</td>
<td>160,000</td>
<td>69,000</td>
<td>73,200</td>
<td>117,000</td>
<td>345,000</td>
</tr>
<tr>
<td>Torsion modulus (N/mm²)</td>
<td>30,100</td>
<td>26,700</td>
<td>98,000</td>
<td>25,900</td>
<td>31,000</td>
<td>40,000</td>
<td>148,000</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.208</td>
<td>0.235</td>
<td>0.12</td>
<td>0.330</td>
<td>0.167</td>
<td>0.355</td>
<td>0.29</td>
</tr>
<tr>
<td>Tensile Strength (MPa)</td>
<td>110 (heated)</td>
<td>386</td>
<td>566</td>
<td>76-540</td>
<td>49.7</td>
<td>221</td>
<td>449</td>
</tr>
<tr>
<td>CTE (10⁻⁶/°C)</td>
<td>7.2</td>
<td>4.5</td>
<td>3.68</td>
<td>23</td>
<td>0.55</td>
<td>16.6</td>
<td>8.4</td>
</tr>
<tr>
<td>Strain point (°C)</td>
<td>529</td>
<td>627</td>
<td></td>
<td></td>
<td>1000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annealing point (°C)</td>
<td>557</td>
<td>663</td>
<td></td>
<td></td>
<td>1100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Softening point (°C)</td>
<td>736</td>
<td>883</td>
<td></td>
<td></td>
<td>1600</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.1: Mechanical properties of various materials used for this project.
Appendix B

Friction Tests

The goal of those tests was to evaluate the static friction coefficient between the D-263 glass sheet and the silicon microcombs.

B.1 Methodology

We have used a traditional method of measuring the macroscale static friction which is to place a block of one material on a plane composed of a chosen counterface material and slowly tilt the plane until relative motion just begins.

The tilt angle $\theta_s$ for which motion occurs is related to the static friction coefficient by the following equation:

$$\mu_s = \tan \theta_s , \quad (B.1)$$

where $\theta_s$ is commonly called the friction angle or the angle of repose.

B.2 Setup

Two microcombs were fixed on an angle bracket, which can be rotated thanks to a rotary table. Two rectangular pieces of glass sheet were glued to a cubic piece of aluminum such that the whole assembly can stand freely on the glass edges. This “biped” was place on top of the combs. Thus we have reproduced the exact way the
glass edges are going to slide along the combs during the assembly process (Figure B-1).

We used two different test “bipeds” and made them slide in both directions (clockwise and anti-clockwise). Tests 1 and 2 correspond to different sliding directions of the first biped while tests 3 and 4 are associated with the second biped. For each different setup, ten measurements are made.

In both cases, the glass pieces have a thickness $e = 200\mu$m and the edges used are the ones as cut by the manufacturer.

### B.3 Results

Table B.1 gathers the friction angles and their associated friction coefficients obtained for the different test setups. We can see that except for Test 2, all the angles measured are under $\theta_s = 32^\circ$. The higher-than-average values obtained during Test 2 might be due to fretting of glass, which seems to make the friction bigger after many tests. These phenomena should not occur on the glass substrates being used in the assembly truss because we will be sliding them along the microcombs only once. Therefore,
Table B.1: Results from friction tests.

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_S$ Max (degrees)</td>
<td>27.3</td>
<td>32.3</td>
<td>24.3</td>
<td>22.7</td>
<td>32.3</td>
</tr>
<tr>
<td>$\theta_S$ Average (degrees)</td>
<td>21.7</td>
<td>26.6</td>
<td>20.4</td>
<td>19.6</td>
<td>21.3</td>
</tr>
<tr>
<td>$\theta_S$ Stdev (degrees)</td>
<td>2.6</td>
<td>3.1</td>
<td>2.7</td>
<td>1.9</td>
<td>1.2</td>
</tr>
<tr>
<td>$\mu_S$ Max</td>
<td>0.52</td>
<td>0.63</td>
<td>0.45</td>
<td>0.41</td>
<td>0.63</td>
</tr>
<tr>
<td>$\mu_S$ Average</td>
<td>0.40</td>
<td>0.50</td>
<td>0.37</td>
<td>0.36</td>
<td>0.39</td>
</tr>
<tr>
<td>$\mu_S$ Stdev</td>
<td>0.05</td>
<td>0.07</td>
<td>0.05</td>
<td>0.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>

the maximum static friction coefficient would be:

$$\mu_S = \tan 32^\circ \approx 0.63$$ \hspace{1cm} (B.2)

This value is very conservative and if one considers only the averaged value of all the measurements made, it would give a friction angle of 21.3°, i.e. a static friction equal to \(\sim0.4\).
Appendix C

Conversion of angles into linear displacement for the assembly truss setup

To demonstrate the alignment capacity of the microcombs, we use a rigid flat plate which is always held in the same way, with two microcomb pairs at the bottom corners and one microcomb pair in the top middle. Angles are measured with a Newport quadrant detector autocollimator, which reads out μrad in pitch and yaw with a resolution of 0.1 μrad. The purpose of this section is to clarify the derivation of the formulas used to infer the relative position of the teeth of the reference microcombs from the angles read.

C.1 Definitions

The axis of the orthonormal referential used to describe the problem are chosen such that the y axis is along the direction of motion of the microcombs and pointing toward the autocollimator, the z axis is vertical and pointing up.

The three points of contact \((P_1, P_2, P_3)\), shown on figure C-1, are defined by their
coordinates in \((x, y, z)\):

\[
P_1: \begin{cases} 
A/2 & \delta y_1 \\
\delta y_2 & \delta y_2 \\
B & 0 
\end{cases}
\quad P_2: \begin{cases} 
0 & \delta y_1 \\
\delta y_2 & \delta y_3 \\
0 & 0 
\end{cases}
\quad P_3: \begin{cases} 
A & \delta y_3 \\
0 & 0 
\end{cases}
\]

Finally the linear displacement error between the two microcombs at the bottom and the error between the top and bottom combs are respectively defined as:

\[
\delta y = \delta y_3 - \delta y_2 \quad \text{(C.1)}
\]
\[
\delta p = \delta y_1 - \frac{\delta y_2 + \delta y_3}{2} \quad \text{(C.2)}
\]
C.2 Calculation

The three points $P_1, P_2, P_3$ define a plane. We can obtain a vector $\vec{n}$ normal to this plane by computing the cross product of $P_1P_2$ and $P_1P_3$:

\[
\vec{n} = \begin{bmatrix} -A/2 \\ \delta y_2 - \delta y_1 \\ -B \end{bmatrix} \wedge \begin{bmatrix} A/2 \\ \delta y_3 - \delta y_1 \\ -B \end{bmatrix} = \begin{bmatrix} B(\delta y_3 - \delta y_2) \\ -A.B \\ -A/2(\delta y_3 + \delta y_2 - 2\delta y_1) \end{bmatrix} = \begin{bmatrix} B.\delta y \\ -A.B \\ A.\delta p \end{bmatrix}
\]

The angles read on the autocollimator, Yaw and Pitch, are respectively the angle between the $y$ axis and the projection of $\vec{n}$ on the plane $(x, y)$ and the angle between the $y$ axis and the projection of $\vec{n}$ on the plane $(y, z)$. Therefore we derive the following relations:

\[
\tan(Yaw) = \frac{n_x}{n_y} = -\delta y/A \\
\tan(Pitch) = \frac{n_z}{n_y} = -\delta p/B
\]

C.3 Result

As the angles measured are very small (typically smaller than 150 $\mu$rad), we can approximate the tangent function by the identity. Moreover in the current setup $A$ is equal to 81 mm and $B$ to 100 mm. Therefore the conversions from angles (in $\mu$rad) to linear displacement (in $\mu$m) are given by:

\[
\delta y = \frac{1}{10}.Yaw \quad \text{(C.3)} \\
\delta p = \frac{81}{1000}.Pitch \quad \text{(C.4)}
\]
Bibliography


