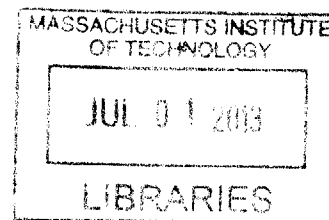


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# Essays in Financial Economics

by

Yifan Zhou

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Requirements for the Degree of

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## ABSTRACT

In Chapter 1, we seek to understand the relation between liquidity and market imperfections from two dimensions: 1) Across liquidity measures, we compare the influence of imperfections on two commonly used measures, Kyle's lambda and price reversal; 2) Across imperfections, we study the interaction between two sources of market imperfection, information asymmetry and participation cost. We show that the two liquidity measures may be affected in opposite directions by the same imperfection, or may not capture liquidity changes at all; imperfection interactions can cause the market to appear "less illiquid" than single-imperfection benchmarks. Our model also suggests that imperfections and liquidity shocks may influence expected returns in opposite directions, which complicates the liquidity-asset price cross-sectional relation.

In Chapter 2, joint with Andrew Lo, we perform an empirical comparison of systemic risk measures. In a recent survey paper, Bisias et al. (2012) provide a summary of 31 proposed measures for systemic risk in the financial system. In this paper we examine a subset of these measures to determine their time series properties before, during, and after the Financial Crisis of 2007–2009. By comparing their empirical properties over time, we hope to identify which measures were most informative for navigating through the 1998 and the 2007–2009 crises. By constructing rolling-window estimates of these measures using only prior data, we control for the most blatant forms of look-ahead bias to assess the value of these measures as "early-warning signals". Finally, we explore the possibility of combining these measures to produce even more informative indicators of systemic risk.

In Chapter 3, joint with Andrew Lo and Silvia Sgherri, we construct two global systemic risk indicators as well as a panel of regional indicators, using monthly hedge fund data. Results show that our geographic-focus global indicator provided contemporaneous characterization of financial distress; the hedge fund style-category global indicator generated early-warnings for the 2007 quant crisis and the 2011 European debt crisis, and typically led the geographic-focus indicator by 1~2 months. In addition, we use Granger causality network to visualize the interconnectedness of regional risks and track the transmission of crisis over time.

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# Chapter 1

## Liquidity, Asymmetric Information, and Endogenous Participation Costs

### 1.1 Introduction

#### 1.1.1 Background and Preview

The presence of financial imperfections often hinders market participants' willingness or ability to trade, which reduces the amount of liquidity available in the market. For example, when there's information asymmetry with respect to the fundamentals of a risky security, the buyer of the security may attribute the selling pressure to unfavorable private information that his counter-party may have, and therefore requires additional compensation for bearing the risk; on the other hand, the seller may also be reluctant to enter the market due to concerns that the orders may reveal his motive and private information to the market, and causes unfavorable movements in prices. Additional trade-offs for market participants may include: participation itself is costly; the difficulty to quickly locate a counter-party may force traders give price concessions; and so on. The impact of each imperfection on liquidity has been studied extensively in a large and growing literature. However, as pointed out by Vayanos and Wang (2009), existing models of market imperfection mostly focus on one specific type of imperfection, because the modeling assumptions of one imperfection often rules out the impact of another. For example, the prevalence of noise traders and risk-neutral market makers in asymmetric information models eliminates risk-sharing motives that are central in other trading-cost models. In reality, multiple imperfections are likely to be at work simultaneously, and empirical evidence also

suggests that modeling each imperfection separately may not be sufficient. For example, Spiegel (2008) argues that cross-market liquidity patterns cannot be explained by a single sources of market imperfection. These questions calls for a modeling approach that incorporates multiple imperfections under a fixed set of assumptions.

A closely related issue is how to define “liquidity”, which can not be observed directly but rather needs to be proxied by other market measures. This question is central in the empirical study of the cross-sectional relation between liquidity and asset prices. A positive return-illiquidity relationship was proposed by Amihud and Mendelson (1986) and since then has been examined in a number of empirical studies. Illiquidity measures employed in these studies at least include: (1)  $\lambda$  as proposed by Kyle (1985), which measures the price sensitivity to signed order flow, employed in for example Amihud and Mendelson (1986), and Brenna and Subrahmanyam (1996). (2) Price reversal  $\gamma$ , as proposed by Pastor and Stambaugh (2003), which is minus the auto-covariance of prices. (3) Bid-ask spread, for example in Roll (1994). These measures capture different aspects of illiquidity, and raises the question of whether the choice of illiquidity measure will affect the cross-sectional relation between illiquidity and expected returns.

To address these questions, we start by developing a unifying model that nests two types of imperfections, *asymmetric information* and *participation cost*, and investigate the behavior of two illiquidity measures,  $\lambda$  and  $\gamma$ . Following Vayanos and Wang (2009), we construct a three-period rational expectations equilibrium model. All agents are risk-averse and born identical in Period 0. They are endowed with a fixed share of of stocks which pays off at the end of their life-span. Between Period 0 and 1, nature split agents into two types who faces different endowment, information, and trading costs. Liquidity demanders will receive a non-traded endowment correlated with the stock’s payoff. They also have the opportunity to purchase private information on the stock’s payoff and trade their assets in Period 1 by paying a fixed cost. The fraction  $\mu$  of traders who decides to participate will be determined endogenously. Liquidity suppliers do not receive the non-traded endowment and will not be able to purchase private information either; they can, however, trade costlessly at all time. We study the relation between market illiquidity and market imperfections from two dimensions: across illiquidity, and across imperfections. In addition, since in this model endogenous trading needs are generated not only by imperfections but also by the non-traded risks, we also study the impact of liquidity shocks on illiquidity measures, and compare the two underlying causes of

liquidity trades.

Tables 1.1 and 1.2 summarize the impact of each imperfection and liquidity shock on trader’s participation rate, illiquidity measures, and expected return. Results in (cyan) color are new. In Table 1.1 we study the two degenerated models where one type of imperfection is set to zero. This allows us to compare our result to those from existing literature. In Table 1.2 we study the general model that incorporates both imperfections.

From the tabulated results we can at least make the following observations. Firstly, imperfections do not always increase illiquidity measure, and the relation between illiquidity and imperfection can even be non-monotonic. For example, illiquidity  $\lambda$  does not capture the increasing participation cost when it is the only imperfection in the market, price reversal  $\gamma$  does not change monotonically with information asymmetry  $\sigma_v^2$ . Secondly, the two illiquidity measures can be influenced in opposite directions by the same imperfection, or by liquidity shock. In particular, in the general model where the participation rate  $\mu$  is interior, both imperfections as well as liquidity shock induces opposite movements in  $\lambda$  and  $\gamma$ . Thirdly, imperfections do not always increase expected return, and the correlation between illiquidity measures and expected return is not always positive.

	$\mu$	$\lambda$	$\gamma$	Return		$\mu$	$\lambda$	$\gamma$	Return
$\sigma_v^2$	-	+, $\mu = 1$ +, $\mu < 1$	+, $\mu = 1$ -, $\mu < 1$	+, $\mu = 1$ +, $\mu = 1$	$\kappa$	-	0, $\mu = 1$ 0, $\mu < 1$	0, $\mu = 1$ -, $\mu < 1$	0, $\mu = 1$ -, $\mu < 1$
$\sigma_Z^2$	+	-	+	+	$\sigma_Z^2$	+	0	+	+

(a) Degenerated model with only **asymmetric information**.

(b) Degenerated model with only **participation cost**.

**Table 1.1: Impact of imperfections and liquidity shock in degenerated models.** The general dual-imperfection model is collapsed into two benchmark cases where one type of imperfection is set to zero.  $\lambda$  is price sensitivity to signed order flow,  $\gamma$  is minus auto-covariance of prices, “return” is the ex-ante expected return of the stock between Periods 0 and 2. Results in (cyan) color are new.

	$\mu$	$\lambda$	$\gamma$	Return
$\sigma_v^2$	-	$+, \mu = 1$	$+, \mu = 1$	$+, \mu = 1$
		$+, \mu < 1$	$-, \mu < 1$	$+, \mu < 1$
$\kappa$	-	$+, \mu = 1$	$0, \mu = 1$	$0, \mu = 1$
		$+, \mu < 1$	$-, \mu < 1$	$-, \mu < 1$
$\sigma_Z^2$	+	-	+	+

Table 1.2: **Impact of imperfections and liquidity shock in the general dual-imperfection model.** The general model incorporates both types of imperfections.  $\lambda$  is price sensitivity to signed order flow,  $\gamma$  is minus auto-covariance of prices, “return” is the ex-ante expected return of the stock between Periods 0 and 2. Results in (cyan) color are new.

### 1.1.2 Previous Literature

First of all, our model is an extension of Vayanos and Wang (2009), which establishes a unified framework for studying a wide range of market imperfections without making differed sets of assumptions that are specific to each imperfection. In the literature, models of asymmetric information usually assume noise traders or deep-pocketed risk-neutral market makers, whereas models on trading costs often assume risk aversion and generate trading needs through risk-sharing motives. The Vayanos and Wang framework allows us to compare the effects across imperfections, holding constant other assumptions such as trading motives, and risk attitudes.

In this exercise, we extend their set-up by nesting asymmetric information and participation cost into a dual-imperfection model, with the following differences: (1) the endogenous participation decisions are made only by liquidity demanders and not liquidity suppliers; (2) The correlation between stock payoff and the non-traded endowment is non-perfect; (3) Informed traders receive incomplete, but perfect information on the stock’s payoff. (4) We solve the model under more general parametric values which allows for corner solutions in the participation equilibrium. As will be described in Section 5, a degenerated version of our model coincides with the asymmetric information benchmark in Vayanos and Wang (2009) and is also consistent with their results, our result for participation cost is new and can be explained by the asymmetric liquidity responses from the buy side and sell side; our result for the dual imperfection model is new.

Our set-up is also closely related to the classic model in Grossman and Stiglitz



(1980) because we have the same information structure where agents endogenously choose to become informed or uninformed through the purchase of private signals, and the uninformed agents infer the private signal through observing the price and the participation decisions of other agents. The difference is that Grossman and Stiglitz (1980) models non-informational trading through exogenous shocks to the asset supply, while we model it through an endowment received by the informed. In addition, we also study the equilibrium asset price before the participation equilibrium in order to study the ex-ante effects of market imperfections.

A key intuition in our model is that costly participation may deter informed agents from participating and consequently revealing private information. In this regard, our model is related to Cao, Coval, and Hirshleifer (2002) where sidelined investors facing set-up cost chooses to delay trading, which triggers the blockage of information transfer and the market price does not fully reflect all the information available. Instead, trading endogenously generates the gradual incorporation of existing information as well as higher participation of sidelined investors. By comparison, we do not focus on the dynamic interaction between the participation cost and information asymmetry. Rather, we only analyze the impact of partial revelation on (static) illiquidity measures.

Finally, most liquidity models in the literature focus on the endogenous supply of liquidity and its price impact, whereas in this exercise we mainly study the endogenous demand of liquidity. Our perspective and solution concept is thus similar to Allen and Gale (1994), and Huang and Wang (2009). In Huang and Wang (2009), the aggregate order imbalances arise because costly participation generates the non-synchronization between liquidity demanders and suppliers; while in this exercise, we assume an aggregate liquidity shock and derive trading needs as a consequence of hedging demand.

## 1.2 The Model

We develop an inter-temporal generalization of Grossman and Stiglitz (1980) and incorporate two market imperfections, information asymmetry and participation cost. We describe the economy and information structure in Section 2. Section 3 solves the REE of the model by backward induction in the general model, as well as in two benchmark cases where one type of imperfection is removed from the general set-up. In Section 4 we construct two illiquidity measures. Section 5 analyzes the

comparative statistics and compare the intuition in the dual-imperfection model to individual single-imperfection benchmarks. Section 6 concludes.

### 1.2.1 Security Market

The model has three periods:  $t = 0, 1, 2$ . A stock is traded in a competitive asset market. It yields a risky dividend  $D = v + n$  at time  $t = 2$ , where  $v$  is the stock's fundamental value, and  $n$  is the idiosyncratic noise.  $v$  and  $n$  are normally distributed and mutually independent.  $v$  has a mean of  $\bar{v}$  and volatility of  $\sigma_v$ ;  $n$  has a mean of zero and volatility of  $\sigma_n$ . Let  $S_t$  denote the ex-dividend stock price at time  $t$ .

In addition, there is a short-term risk-free bond, which yields zero interest rate.

### 1.2.2 Agents

At  $t = 0$ , a set of agents are born who live for three periods until  $t = 2$ . Agents are born identical and endowed with  $\bar{\theta}$  of the stock which they can invest in the stock and the bond. They sell all their assets for consumption at time  $t = 2$ . Agents have CARA utility:

$$-\exp(-\alpha C_2) \tag{1.1}$$

where  $C_2$  is consumption in Period 2, and  $\alpha$  is the coefficient of absolute risk aversion.

Agents are split into two types who will face different endowment and trading costs. The first type of agents are "traders" who receives a non-traded payoff  $E$  at the end of his life-span, given by

$$E = Zn \tag{1.2}$$

where  $Z$  is a normal random variable with a mean of zero and a volatility of  $\sigma_Z$ .  $Z$  is independent with  $v$  and  $n$ . The non-traded payoff is therefore correlated with the stock's payoff through the idiosyncratic noise. The rest of the agents are "market makers" who do not receive or observe the non-trade payoff. The population weight of the traders and the market makers are  $\pi$  and  $1 - \pi$  respectively.

Given the correlation between the non-traded payoff and the stock payoff, traders want to adjust their stock positions in order to hedge against the non-traded payoff and share risk with others. Traders can decide whether or not to participate in the stock market at a cost. The population of traders who participate is  $\mu\pi$ , where  $\mu$  will be determined endogenously. Market makers are present at all times to provide liquidity for others. We will refer to participating traders as "liquidity demanders",

and market makers as “liquidity suppliers”.

### 1.2.3 Imperfections

#### Asymmetric Information

We assume some agents can observe a private signal  $s$  on the stock’s payoff  $D$  before trading in Period 1. Traders can purchase the private signal by paying a fixed cost, whereas market makers will not be able to do so. For simplicity, the signal is

$$s = v \tag{1.3}$$

which coincides with the stock’s fundamental payoff.  $v$  can be thought of as the trader’s best estimate of the stock’s payoff, after processing all publicly or privately available information. In a more generally setting, a trader would wish to optimally allocate his capital among research expense, processing cost, and trading capital; the signal quality improves when there is more input into research. Given the limited scope of this exercise, we will not delve into this direction but only model a fixed signal quality. A trader can be equally informed as anyone else who puts in the same effort.

#### Participation Cost

All agents can trade in the market at no cost at the beginning and the end of their life-span. In Period 1, market makers can trade costlessly, whereas traders need to pay a fixed cost  $\kappa$  to enter the market. For example,  $\kappa$  may include the “buy-in” expense to obtain exchange membership, as well as the opportunity cost of human capital allocated to trading. For reasons explained below, the participation cost is assume to be the bundled with the information acquisition cost. The participation decision is made ex-ante before private signals are revealed.

In this model we bundle the purchasing cost of acquiring private information, and the entry cost to participate in trading into a single fixed cost  $\kappa$  paid by all liquidity demanders who decide to trade. In other words, liquidity demanders are allowed to participate only if they pay an entry cost; and as long as they participate, they will make the effort to process all available information and obtain the private signal  $v$ . All costs incurred are included in  $\kappa$ .

### 1.2.4 Time-Line

We now describe the timing of events shown in Figure 1.1. In Period 0, agents are born with  $\bar{\theta}$  shares of stock as endowment. All agents are free to trade costlessly, and the market equilibrium determines the price  $S_0$ .

In Period  $\frac{1}{4}$ , nature splits agents into traders and market makers; agents learn their types.

In Period  $\frac{1}{2}$ , traders receive an offer of purchasing the private signal  $s = v$  on the stock's payoff and enter the market at cost  $\kappa$ . Those who choose to enter the market and observe the signal will then trade among themselves as well as with market makers; those who turn down this offer will leave the market. In equilibrium, a fraction  $\mu$  of traders chooses to enter.

In Period  $\frac{3}{4}$ , the private signal  $v$  is revealed to all participating traders; the liquidity shock  $Z$  is revealed to all traders, including those who stay out of the market.

In Period 1, participating traders and market makers submit orders, and complete the exchanges.

In Period 2, traders receive the non-traded payoff; agents sell all their assets for no additional cost, and consume their wealth.

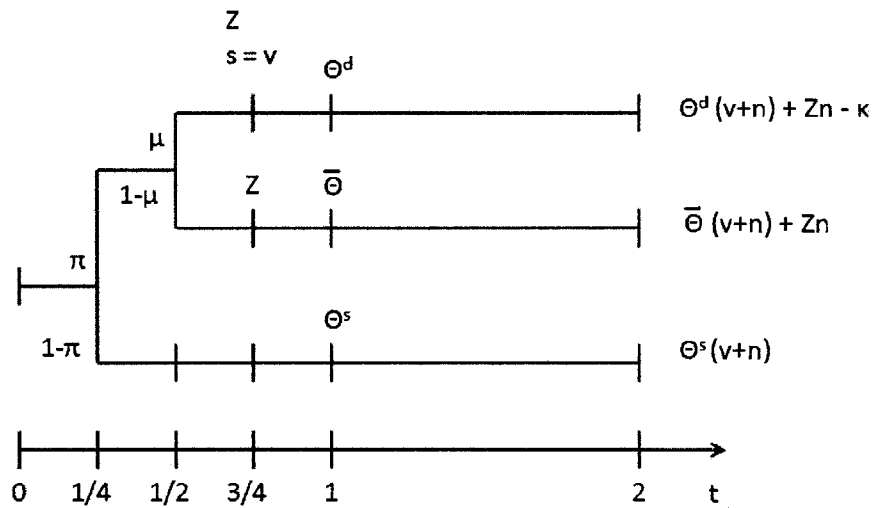


Figure 1.1: The Time Line of the Economy.

### 1.2.5 Discussions and Simplifications

Letting the set of participating liquidity demanders coincide with the set of informed agents is a modeling choice to avoid signal free-riding. As will be shown in Section 3, the equilibrium price in Period 1 is affine in the signal  $v$  and liquidity shock  $Z$ , which will be revealed to all liquidity demanders. Assume, instead, we allow a liquidity demander to participate without purchasing the private signal, then he would be able to perfectly infer the private signal  $v$  from the price.

Additionally, in a more general setting, we would separate the information acquisition decision from the entry decision, and replace  $\kappa$  with a combination of the acquisition cost of private information  $c_A$  and the entry cost of participation  $c_B$ . This set-up would incur at least the following complications: Firstly, after nature splits agents into traders and market makers, each of the entry and acquisition decisions will further split the agents into two sub-types. All together this set up creates too many subgroups for the problem to be analytically tractable. Secondly, the timing sequence of the two decisions may bring up the question of whether the participation decision should be modeled as ex-ante before observing the private information, or ex-post. We will not delve into this direction given the limited scope of the current exercise. In the rest of the paper we will continue to refer to  $\kappa$  as “participation cost”.

Before proceeding further, it should be pointed out that since all agents have CARA utility in which the agent’s risk attitude does not depend on wealth level, there will be **no** income effect in our model. This might seem puzzling at first glance because liquidity, by definition, should be a measure of the total amount of capital available (i.e. aggregate wealth) in the economy. Therefore, the channel for illiquidity to affect market behavior should naturally be through the income effect. In our model, since all agents are initially endowed with the per-capita share of the risky and risk-free assets, the total amount of capital is equivalently captured by the total number of agents participating in trading. Under information asymmetry and costly participation, the market is less liquid because a smaller number of agents are pulling out their capital for trading. In other words, limited participation is just income effect in disguise.

## 1.3 Equilibrium

In this section, we solve for the economy’s competitive equilibrium by backward induction. First, solve for the market equilibrium at  $t = 1$ , given agents’ participation

decisions, the realization of the private signal and the liquidity shock. Second, solve for the participation equilibrium at  $t = \frac{1}{2}$ , given the market equilibrium and agent's initial stock holdings. Third, solve for the market equilibrium at time  $t = 0$  and obtain the full equilibrium of the economy.

We will start with the equilibrium in the general model with both asymmetric information and participation cost, and then examine the two degenerated cases with one type of imperfection each.

### 1.3.1 Market Equilibrium at $t = 1$

At Period 1, there are  $\mu\pi$  traders and  $1 - \pi$  market makers present in the market. Trader's participation rate  $\mu$  is the equilibrium outcome of their participation decisions made ex-ante at  $t = \frac{1}{2}$ . The participating traders submit orders based on their knowledge of the private signal  $v$  and the liquidity shock  $Z$ , both learned at  $t = \frac{3}{4}$ . The price therefore reveals some of the trader's information to the uninformed market makers, who cannot observe either  $v$  or  $Z$  directly. Following Vayanos and Wang (2009), we conjecture a price function affine in the private signal  $v$  and the liquidity shock  $Z$ , i.e.

$$S_1(\mu, v, Z) = a + b(v - \bar{v} - cZ) \quad (1.4)$$

for three constants  $(a, b, c)$ . Using the knowledge of the price function, agents are now able to learn about the signals by observing the price  $S_1$ , and formulate demands that maximizes their expected utilities. We will then confirm this price function indeed clear the market.

A liquidity demander chooses to hold  $\theta_1^d$  shares of the stock, given the private signal  $v$  and the liquidity shock  $Z$ ; the price  $S_1$  do not convey no additional information. Consumption in Period 2 is

$$C_2^d(\theta_1^d; \mu, v, Z) = W_1 + \theta_1^d(v + n - S_1) + Zn \quad (1.5)$$

i.e. wealth in Period 1, plus capital gains from the stock and the non-traded payoff. His expected expected utility in Period 1 is

$$EU_{t=1}^d(\theta_1^d; \mu, v, Z) = E_n[-\exp(-\alpha C_2^d)] \quad (1.6)$$

which can be written as

$$-\exp \left\{ -\alpha \left[ W_1 + \theta_1^d (v - S_1) - \frac{1}{2} \alpha \sigma_n^2 (Z + \theta_1^d)^2 \right] \right\} \quad (1.7)$$

A liquidity supplier observes the price  $S_1$  and updates his belief on the distribution of the private signal  $v$ . Let  $\xi = v - \bar{v} - cZ$ , his posterior beliefs are

$$E[v|S_1] = \bar{v} + \beta_\xi \xi \quad (1.8)$$

$$\sigma^2[v|S_1] = \beta_\xi c^2 \sigma_Z^2 \quad (1.9)$$

where

$$\beta_\xi \equiv \frac{\sigma_v^2}{\sigma_v^2 + c^2 \sigma_Z^2} \quad (1.10)$$

He chooses holding  $\theta_1^s$  of the stock based on the posterior beliefs. Consumption in Period 2 and expected utility in Period 1 are

$$C_2^s = W_1 + \theta_1^s (v + n - S_1) \quad (1.11)$$

$$EU_{t=1}^s(\theta_1^s, \mu, v, Z) = E_n[-\exp(-\alpha C_2^d)] \quad (1.12)$$

The optimal demand functions follows directly from solving the maximization problems, summarized in the following proposition:

**Proposition 3.1** *Agents' demand functions for the stock in Period 1 are*

$$\theta_1^d = \frac{v - S_1}{\alpha \sigma_n^2} - Z \quad (1.13)$$

$$\theta_1^s = \frac{E[v|S_1] - S_1}{\alpha [\sigma^2[v|S_1] + \sigma_n^2]} \quad (1.14)$$

In equilibrium, all agents enter Period 1 with  $\bar{\theta}$  shares of stock holdings. The price (1.4) clears the market if the aggregate demand equals the supply for all realizations of  $(v, Z)$ :

$$\mu \pi \theta_1^d + (1 - \pi) \theta_1^s = (\mu \pi + 1 - \pi) \bar{\theta} \quad (1.15)$$

This determines the affine price coefficients in (1.4), solved in the Appendix and summarized as below:

**Proposition 3.2** *Given the participation equilibrium outcome  $\mu$  in Period  $\frac{1}{2}$ ,*

the equilibrium market price in Period 1 in is given by

$$S(\mu, v, Z) = a + b(v - \bar{v} - cZ) \quad (1.16)$$

where

$$a = \bar{v} - \alpha \bar{\theta} \cdot \frac{\mu\pi + 1 - \pi}{\frac{\mu\pi}{\sigma_n^2} + \frac{1-\pi}{\sigma_n^2 + \sigma^2[v|S_1]}} \quad (1.17)$$

$$b = \frac{\mu\pi \cdot (\sigma_n^2 + \sigma^2[v|S_1]) + (1 - \pi) \cdot \beta_\xi \sigma_n^2}{\mu\pi (\sigma_n^2 + \sigma^2[v|S_1]) + (1 - \pi) \sigma_n^2} \quad (1.18)$$

$$c = \alpha \sigma_n^2 \quad (1.19)$$

### 1.3.2 Participation Equilibrium In Period $\frac{1}{2}$

Given the market equilibrium in Period 1, we now determine the equilibrium participation rate  $\mu$  of liquidity demanders. In Period  $\frac{1}{2}$ , all agents hold  $\theta_0$  shares of the stock obtained from trading in Period 0, before discovering their types. Those assigned to be traders in Period  $\frac{1}{4}$  decides whether or not to participate in the market in Period 1, before any private information is revealed.

A marginal liquidity demander takes as given the participation decision of others, and anticipates the market price to be  $S_1(\mu, v, Z)$  in Period 1. If he chooses to pay the fixed cost  $\kappa$ , he will be able to observe both the private signal  $v$  and the liquidity shock  $Z$ , then hedge against the non-traded risk through trading in Period 1. If instead he chooses to forgo the cost  $\kappa$ , he will only observe the liquidity shock  $Z$  and will not be able to adjust his position  $\theta_0$ . The participation decision is made by comparing the expected utilities from the two outcomes, where the expectation is taken over all realizations of the private signal  $v$  and the liquidity shock  $Z$ . Let  $EU^d(\theta_0; \mu)$  and  $EU^{np}(\theta_0; \mu)$  denote his interim utility at Period  $\frac{1}{2}$  if he chooses to participate or not to participate, respectively. We have

$$EU^d(\theta_0; \mu) = E_{v,Z} \left\{ \max_{\theta_1^d} E_n [-\exp(-\alpha C_2^d) | v, Z] \right\} \quad (1.20)$$

$$EU^{np}(\theta_0; \mu) = E_{v,Z} \{ E_n [-\exp(-\alpha C_2^{np}) | v, Z] \} \quad (1.21)$$

Next we compute the two interim expected utilities. The liquidity demander who



chooses to participate pays the cost  $\kappa$  and his wealth in Period 1 is

$$W_1 = W_0 + \theta_0 (S_1 - S_0) - \kappa \quad (1.22)$$

He anticipates the market equilibrium to be as described in the previous subsection 3.1 and submits the demand function (1.13) in Period 1. His terminal payoff will be

$$C_2^d = W_1 + \theta_1^d (v + n - S_1) + Zn \quad (1.23)$$

The liquidity demander who chooses not to participate hold on to his position  $\theta_0$  until the end of his life span. Consumption in Period 2 will be

$$C_2^{np} = W_0 + \theta_0 (v + n - S_0) + Zn \quad (1.24)$$

In equilibrium, agents choose to hold  $\theta_0 = \bar{\theta}$  shares of the stock after trading in Period 0. The optimal decision for trader  $i$  is to participate if and only if

$$\gamma(\bar{\theta}; \mu) \equiv \frac{EU^d(\bar{\theta}; \mu)}{EU^{np}(\bar{\theta}; \mu)} < 1$$

The expectation in (1.20) and (1.21) can be obtained using (1.13) and (1.3.1), calculated in the Appendix. The results are summarized in the following proposition:

**Proposition 3.3** *In equilibrium, the interim expected utility for a marginal liquidity demander who chooses to participate is*

$$EU^d(\bar{\theta}; \mu) = - \frac{\exp(-\alpha F^d(\bar{\theta}; \mu)) \exp(\alpha \kappa)}{\sqrt{\Omega^d}} \quad (1.25)$$

where

$$F^d(\bar{\theta}; \mu) = \left\{ \bar{\theta}a + \frac{(\bar{v} - a)^2}{2c} \right\} - \frac{\alpha (\sigma_v^2 + c^2 \sigma_Z^2 - \alpha c \sigma_v^2 \sigma_Z^2)}{\Omega^d} \left[ \bar{\theta}b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]^2 \quad (1.26)$$

$$\Omega^d = 1 + \frac{\alpha}{c} (1-b)^2 [\sigma_v^2 + c^2 \sigma_Z^2] - \alpha c \sigma_Z^2 - \alpha^2 (1-b)^2 \sigma_v^2 \sigma_Z^2 \quad (1.27)$$

*The interim expected utility for a marginal liquidity demander who chooses not to participate is*

$$EU^{np}(\bar{\theta}; \mu) = -\frac{\exp\{-\alpha F^{np}(\bar{\theta}; \mu)\}}{\sqrt{\Omega^{np}}} \quad (1.28)$$

where

$$F^{np}(\bar{\theta}; \mu) = \bar{\theta}\bar{v} - \frac{1}{2}\alpha\sigma_v^2\bar{\theta}^2 - \frac{1}{2}\frac{\alpha\sigma_n^2\bar{\theta}^2}{\Omega^{np}} \quad (1.29)$$

$$\Omega^{np} = 1 - \alpha^2\sigma_n^2\sigma_z^2 \quad (1.30)$$

The overall equilibrium requires that liquidity demanders are indifferent at the margin,  $\gamma(\bar{\theta}; \mu) = 1$ . and only some participate ( $0 < \mu < 1$ ). In addition, the price  $S_1(\mu; v, Z)$  becomes more informative when the participation rate  $\mu$  is higher, which reduces the utility gain from participating and acquiring private information, i.e. increases  $\gamma(\bar{\theta}; \mu)$ . If the trading gains from participation is sufficiently high so that  $\gamma(\bar{\theta}; 1) < 1$ , then the marginal liquidity demanders chooses to participate even though the presence of a large number of traders has attenuated the profitability of doing so, and in equilibrium all traders chooses to participate. On the other hand, if the utility gain from trading is not enough to offset the loss from participation cost even when no other liquidity demander choose to participate  $\gamma(\bar{\theta}; 0) > 1$ , then the marginal liquidity will choose to forgo the entry cost and in equilibrium no liquidity demander participates.

**Proposition 3.4** *The participation equilibrium is give as follows:*

- 1) *If  $\gamma(\bar{\theta}; \mu^*) = 1$  for some  $0 < \mu^* < 1$ , and  $S_1$  is given by (1.3.1) as in the market equilibrium in Period 1, then  $(\mu^*, S_1(\mu^*))$  is the overall equilibrium;*
- 2) *If  $\gamma(\bar{\theta}; 1) < 1$ , then  $(1, S_1(1))$  is an overall equilibrium where all liquidity demanders participate;*
- 3) *If  $\gamma(\bar{\theta}; 0) > 1$ , then  $(0, S_1(0))$  is an overall equilibrium where no liquidity demander participate.*

### 1.3.3 Market Equilibrium at $t = 0$

In Period 0, all agents are identical and enter the market without the knowledge of whether he will become a trader or a market maker. He chooses to hold  $\theta_0$  shares of the stock so as to maximize the unconditional expected utility

$$EU_0(\theta_0) = \pi EU^L(\theta_0) + (1 - \pi) EU^s(\theta_0; \mu^*) \quad (1.31)$$

where  $EU^L(\theta_0)$  and  $EU^s(\theta_0)$  are the interim expected utilities in Period  $\frac{1}{2}$  of

being a trader and market maker respectively, after his type has been revealed in Period  $\frac{1}{4}$ .

If he discovers himself to be a trader in Period  $\frac{1}{4}$ , then he will make the optimal participation decision as described in the previous subsection 3.2,

$$EU^L(\theta_0) = \max \{EU^d(\theta_0; \mu^*), EU^{np}(\theta_0; \mu^*)\} \quad (1.32)$$

He has wealth

$$W_1 = W_0 + \theta_0(S_1 - S_0) \quad (1.33)$$

If instead he discovers himself to be a market maker, then he will observe the price  $S_1$  and formulate optimal demand as described in subsection 3.1 in Period 1. The expectation in Period  $\frac{1}{2}$  is taken over all realizations of the private signal, and liquidity shock:

$$EU^s(\theta_0; \mu^*) = -E_{v,Z}[\exp(-\alpha C_2^s) | S_1] \quad (1.34)$$

which can be obtained by substituting  $\theta_1^s$  from (1.14),  $S_1$  from Eq. (1.4), and  $W_1$  from (1.33). The expected utility depends on the liquidity shock  $Z$  since  $Z$  affects the price  $S_1$ .

The expected utilities are calculated in the Appendix. The results are summarized in the following proposition:

**Proposition 3.5** *The interim utility in Period  $\frac{1}{2}$  for an agent with initial holding  $\theta_0$  is:*

*If he becomes a trader and decides to participate:*

$$EU^d(\theta_0; \mu) = -\frac{\exp(-\alpha F^d(\theta_0; \mu)) \exp(\alpha \kappa)}{\sqrt{\Omega^d}} \quad (1.35)$$

where

$$F^d(\theta_0; \mu) = \left\{ (\bar{\theta} - \theta_0) S_0 + \theta_0 a + \frac{(\bar{v} - a)^2}{2c} \right\} - \frac{\alpha (\sigma_v^2 + c^2 \sigma_Z^2 - \alpha c \sigma_v^2 \sigma_Z^2)}{\Omega^d} \left[ \theta_0 b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]^2 \quad (1.36)$$

$$\Omega^d = 1 + \frac{\alpha}{c} (1-b)^2 [\sigma_v^2 + c^2 \sigma_Z^2] - \alpha c \sigma_Z^2 - \alpha^2 (1-b)^2 \sigma_v^2 \sigma_Z^2 \quad (1.37)$$

If he becomes a trader and decides not to participate:

$$EU^{np}(\theta_0; \mu) = -\frac{\exp(-\alpha F^{np}(\theta_0; \mu))}{\sqrt{\Omega^{np}}} \quad (1.38)$$

where

$$F^{np}(\theta_0; \mu) = \bar{\theta}S_0 + \theta_0(\bar{v} - S_0) - \frac{1}{2}\alpha\theta_0^2 \left( \sigma_v^2 + \frac{\sigma_n^2}{1 - \alpha^2\sigma_n^2\sigma_Z^2} \right) \quad (1.39)$$

$$\Omega^{np} = 1 - \alpha^2\sigma_n^2\sigma_z^2 \quad (1.40)$$

If he becomes a market maker:

$$EU^s(\theta_0; \mu) = \frac{\exp\left\{-\alpha \left[ A_1^s - \frac{\alpha (B_1^s)^2 \sigma_\xi^2}{2(1 + \alpha C_1^s \sigma_\xi^2)} \right]\right\}}{\sqrt{1 + \alpha C_1^s \sigma_\xi^2}} \quad (1.41)$$

where

$$F^s = A_1^s - \frac{\alpha (B_1^s)^2 \sigma_\xi^2}{2(1 + \alpha \sigma_\xi^2 C_1^s)} \quad (1.42)$$

$$\Omega_s = 1 + \alpha \sigma_\xi^2 C_1^s \quad (1.43)$$

$$A_1^s = \bar{\theta}S_0 + \theta_0(a - S_0) + \frac{(\bar{v} - a)^2}{2c(1 + \alpha\beta_\xi c\sigma_Z^2)} \quad (1.44)$$

$$B_1^s = \theta_0 b - \frac{(\bar{v} - a)(b - \beta_\xi)}{c(1 + \alpha\beta_\xi c\sigma_Z^2)} \quad (1.45)$$

$$C_1^s = \frac{(b - \beta_\xi)^2}{c(1 + \alpha\beta_\xi c\sigma_Z^2)} \quad (1.46)$$

The equilibrium price in Period 0 needs to clear the market, so that the aggregate demand  $\theta_0$  coincides with aggregate supply  $\bar{\theta}$ . The maximization problem is calculated in the Appendix.

**Proposition 3.6** *The equilibrium price in Period 0  $S_0$  is given by:*

$$0 = \frac{\partial EU_0(\theta_0; \mu)}{\partial \theta_0} \Big|_{\theta_0 = \bar{\theta}} \quad (1.47)$$

### 1.3.4 Degenerated Models

In this subsection we consider the two degenerated cases nested in the general model where only one type of imperfection is present. Firstly, when  $\kappa = 0$ , there is no participation cost in Period 1, and the degenerated model only incorporates information asymmetry. This simplification will not change the structure of the general set-up and the solutions. Secondly, when  $\sigma_v = 0$ , there is no information asymmetry concerning the stock's fundamental payoff, and the degenerated model only incorporates participation cost. Liquidity demanders only participate to hedge against the non-traded risk, but there's no incentive to trade on information advantage. The degenerated model can be solve by either setting  $\sigma_v = 0$  in the general solution, or solved directly as summarized follows.

**Proposition 3.7** *In the absence of information asymmetry, agents' demand functions for the stock in Period 1 are*

$$\theta_1^d = \frac{\bar{v} - S_1}{\alpha\sigma_n^2} - Z \quad (1.48)$$

$$\theta_1^s = \frac{\bar{v} - S_1}{\alpha\sigma_n^2} \quad (1.49)$$

Taking as given the participation rate  $\mu$  as the participation equilibrium outcome in Period  $\frac{1}{2}$ , market price in Period 1 is again determined by the market clearing condition.

$$\mu\pi\theta_1^d + (1 - \pi)\theta_1^s = [\mu\pi + 1 - \pi]\bar{\theta} \quad (1.50)$$

Substituting the optimal demands in (1.48) (1.49) into the market clearing condition, we obtain the following:

**Proposition 3.8** *In the absence of information asymmetry, and given the participation equilibrium outcome  $\mu$  in Period  $\frac{1}{2}$ , the equilibrium market price in Period 1 in is given by*

$$S_1 = \bar{v} - \alpha\sigma_n^2(\bar{\theta} + \Delta Z) \quad (1.51)$$

where

$$\Delta = \frac{\mu\pi}{\mu\pi + 1 - \pi} \quad (1.52)$$

The interim utilities are also simplified once information asymmetry is removed:

**Proposition 3.9** *In the absence of information asymmetry, the interim utility at  $t = \frac{1}{2}$  for an agent with initial holding  $\theta_0$  is:*

*If he becomes a trader and decides to participate:*

$$EU^d(\theta_0; \mu) = -\frac{\exp(-\alpha F^d(\theta_0; \mu)) \exp(\alpha \kappa)}{\sqrt{\Omega^d}} \quad (1.53)$$

where

$$F^d(\theta_0; \mu) = \left\{ (\bar{\theta} - \theta_0) S_0 + \theta_0 a + \frac{(\bar{v} - a)^2}{2c} \right\} - \frac{\alpha c^2 \sigma_Z^2}{2 \Omega^d} \left[ \theta_0 b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]^2 \quad (1.54)$$

$$\Omega^d = 1 + \alpha (1-b)^2 c \sigma_Z^2 - \alpha c \sigma_Z^2 \quad (1.55)$$

*If he becomes a trader and decides not to participate:*

$$EU^{np}(\theta_0; \mu) = -\frac{\exp(-\alpha F^{np}(\theta_0; \mu))}{\sqrt{\Omega^{np}}} \quad (1.56)$$

where

$$F^{np}(\theta_0; \mu) = \bar{\theta} S_0 + \theta_0 (\bar{v} - S_0) - \frac{1}{2} \alpha \theta_0^2 \frac{\sigma_n^2}{1 - \alpha^2 \sigma_n^2 \sigma_Z^2} \quad (1.57)$$

$$\Omega^{np} = 1 - \alpha^2 \sigma_n^2 \sigma_Z^2 \quad (1.58)$$

*If he becomes a market maker:*

$$EU^s(\theta_0; \mu) = \frac{\exp \left\{ -\alpha \left[ A_1^s - \frac{\alpha}{2} \frac{(B_1^s)^2 \sigma_\xi^2}{1 + \alpha C_1^s \sigma_\xi^2} \right] \right\}}{\sqrt{1 + \alpha C_1^s \sigma_\xi^2}} \quad (1.59)$$

where

$$F^s = A_1^s - \frac{\alpha}{2} \frac{(B_1^s)^2 (c^2 \sigma_Z^2)}{1 + \alpha (c^2 \sigma_Z^2) C_1^s} \quad (1.60)$$

$$\Omega_s = 1 + \alpha (c^2 \sigma_Z^2) C_1^s \quad (1.61)$$

$$A_1^s = \bar{\theta} S_0 + \theta_0 (a - S_0) + \frac{(\bar{v} - a)^2}{2c} \quad (1.62)$$

$$B_1^s = \left[ \theta_0 - \frac{(\bar{v} - a)}{c} \right] b \quad (1.63)$$

$$C_1^s = \frac{b^2}{c} \quad (1.64)$$

Finally, the equilibrium price in Period 0  $S_0$  is determined so that agent's optimal demand  $\theta_0$  coincides with per-capita supply  $\bar{\theta}$ .

**Proposition 3.10** *In the absence of information asymmetry, the price in Period 0 is*

$$S_0 = \bar{v} - \alpha\sigma_n^2\bar{\theta} - \frac{\pi M}{1 - \pi + \pi M}\Delta_d\Delta\bar{\theta} \quad (1.65)$$

where

$$\Delta = \frac{\mu\pi}{\mu\pi + 1 - \pi} \quad (1.66)$$

$$\Delta_d = \frac{\alpha\sigma_n^2 \cdot \alpha^2\sigma_n^2\sigma_z^2}{1 - \alpha^2\sigma_n^2\sigma_z^2(2\Delta - \Delta^2)} \quad (1.67)$$

$$M = \exp(\alpha\kappa) \exp\left(\frac{1}{2}\alpha\Delta_d\bar{\theta}^2\right) \frac{\sqrt{1 + \alpha^2\sigma_n^2\sigma_z^2\Delta^2}}{\sqrt{1 - \alpha^2\sigma_n^2\sigma_z^2(2\Delta - \Delta^2)}} \quad (1.68)$$

## 1.4 Illiquidity Measures

Following Vayanos and Wang (2009), in this section we construct the two common used measures: Kyle's Lambda and Price Reversal. The impact of liquidity trades on these two measures will be discussed in the next Section for the two degenerated models, as well as the general model with dual imperfections.

### 1.4.1 Kyle's Lambda

Kyle's Lambda is the regression coefficient of the price change between Periods 0 and 1 on the signed volume of participating liquidity demanders in Period 1. When the market is more illiquid, prices are more sensitive to trading volume, therefore  $\lambda$  is larger.

$$\lambda = \frac{Cov(S_1 - S_0, \mu\pi(\theta_1^d - \bar{\theta}))}{Var[\mu\pi(\theta_1^d - \bar{\theta})]} \quad (1.69)$$

Eq.(1.4) implies that the price change between Periods 0 and 1 is

$$S_1 - S_0 = a + b(v - \bar{v} - cZ) - S_0 \quad (1.70)$$

Eq. (1.13) implies that the signed volume of liquidity demanders is

$$\mu\pi (\theta_1^d - \bar{\theta}) = \mu\pi \left( \frac{v - S_1}{\alpha\sigma_n^2} - Z - \bar{\theta} \right) \quad (1.71)$$

Substitute the price change and volume into the definition of  $\lambda$ , we obtain the following:

**Proposition 4.1** *In the general model with both types of imperfections, illiquidity as measured by Kyle's Lambda is given by*

$$\lambda_{dual} = \frac{c}{\mu\pi} \frac{b}{1-b} \quad (1.72)$$

where the affine coefficients are given as in (1.18)(1.19), and the participation equilibrium outcome  $\mu$  is given as in Proposition 3.4 .

### 1.4.2 Price Reversal

Price reversal is defined as the negative auto-covariance of price changes. When markets are more illiquid, liquidity trades cause larger deviations of the stock price from its fundamental value, hence the prices dynamics exhibit higher auto-correlation,  $\gamma$  is higher:

$$\gamma = -cov(S_2 - S_1, S_1 - S_0) \quad (1.73)$$

Substituting the price  $S_1$  from Eq. (1.4), we obtain the following:

**Proposition 4.2** *In the general model, under dual imperfections, illiquidity as measured by price reversal is given by*

$$\gamma_{dual} = b(b-1)\sigma_v^2 + b^2c^2\sigma_z^2 \quad (1.74)$$

where the affine coefficients are as given in (1.18)(1.19).

### 1.4.3 Illiquidity in Degenerated Models

As seen in the previous section, setting  $\kappa = 0$  does not simplify the structure of the model and the illiquidity measures will take the same form as in the general model. When  $\sigma_v = 0$ , the illiquidity measures can be simplified by Proposition 3.8:

$$\mu\pi (\theta_1^d - \bar{\theta}) = \mu\pi [\bar{\theta} - (1 - \Delta) Z] \quad (1.75)$$

$$S_1 - S_0 = \bar{v} - \alpha\sigma_n^2 (\bar{\theta} + \Delta Z) - S_0 \quad (1.76)$$



$$S_2 - S_1 = n + \alpha\sigma_n^2(\bar{\theta} + \Delta Z) \quad (1.77)$$

Substituting the above into (1.69) (1.73), we obtain the following:

**Proposition 4.3** *When information asymmetry is absent in the general model and participation cost is the only market imperfection in Period 1, illiquidity and price reversal are given by*

$$\lambda_{pc} = \frac{\alpha\sigma_n^2}{1 - \pi} \quad (1.78)$$

$$\gamma_{pc} = (\alpha\sigma_n^2)^2 \Delta^2 \sigma_Z^2 \quad (1.79)$$

Note that  $\lambda_{pc}$  only depend risk-aversion, population of market makers, and the variance of idiosyncratic shock;  $\lambda_{pc}$  does not vary with participating cost or the variance of liquidity shock.

## 1.5 Illiquidity and Imperfections

In this section we investigate how illiquidity and asset prices are affected by each of the two imperfections incorporated in the general model, as well as by the variance of the liquidity shock. Our findings suggest the following: First, the same type of imperfection may influence the two commonly used illiquidity measures in opposite directions, or in certain cases may not be able to influence the illiquidity measure; Second, when both imperfections are present, the market may be measured “less illiquid” compared to two benchmark scenarios when only one of the imperfections are incorporated. Third, the two types of imperfections can influence expected return in opposite directions.

The comparative statistics are obtained by numerical simulations, due to analytical intractability. We simulate the model around the baseline parameter values:  $\sigma_v^2 = .3$ ;  $\sigma_z^2 = 1$ ;  $\sigma_n^2 = .1$ ;  $\alpha = 2.2$ ;  $\pi = 0.4$ ;  $\bar{v} = 1.5$ ;  $\bar{\theta} = 1$ ;  $\kappa = 0.015$ , and then vary the degree of each imperfection and liquidity shock ( $\sigma_z^2$ ) in the neighborhood of the baseline values; the results are included at the end of this report. Simulations with other baseline parameter values are qualitatively the same.

### 1.5.1 Illiquidity in Degenerated Models

In this subsection we briefly examine the two degenerated models where only one type of imperfection is present; in the next subsection we will use the results here to explain the interaction between the two imperfections. This part of the analysis is closely related to the single-imperfection benchmarks in Vayanos and Wang (2009); our analysis here considers more general parameter values which allows for both corner and interior solutions in the participation equilibrium. Comparative statistics are carried out by setting one imperfection to zero ( $\sigma_v^2$  or  $\kappa$ ), and varying the degree of the other imperfection and the liquidity shock ( $\sigma_Z^2$ ).

#### 1.5.1.1 Asymmetric Information

We start with the restricted case where  $\kappa = 0$  and all liquidity demanders choose to participate,  $\mu = 1$ . The results are shown in Figure 1.2a and Figure 1.2b, summarized the following Propositions:

**Proposition 5.1** *When asymmetric information is the only source of imperfection and all liquidity demanders chooses to participate, an increase in variance of private information  $\sigma_v^2$  raises illiquidity  $\lambda$ , price reversal  $\gamma$ , as well as the price discount in Period 0.*

This set-up coincides with the full-information benchmark in Vayanos and Wang (2009) and our results are also consistent. As  $\sigma_v^2$  increases, there's more uncertainty regarding the stock's fundamentals, and, the information asymmetry between liquidity demanders and suppliers also becomes more severe. Liquidity suppliers cannot distinguish whether the selling pressures are due to risk-sharing motives, or due to unfavorable realization of the stock's fundamentals which they have no private information about. Hence they are less willing to supply liquidity and demand a higher price drop for doing so. These two effects both reduces mark Illiquidity, which raises  $\lambda$  and  $\gamma$  to increase unambiguously.

On the other hand, the Hirshleifer effect implies that risk-sharing is best under no-information ( $\sigma_v^2 = \infty$ ) and worst under full-information ( $\sigma_v^2 = 0$ ). The increase in  $\sigma_v^2$  therefore improves risk-sharing and decreases the price discount as  $\sigma_v^2$  increases. However, this effect is dominated by the uncertainty and learning effects, and the price discount still increases with  $\sigma_v^2$ .

We also find that the liquidity demander's participation rate decreases with  $\sigma_v^2$ , for the following reasons. From the perspective of liquidity suppliers, fluctuations in

liquidity shock  $Z$  is regarded as “noise” for them to infer the private signal  $v$  from movements in the realization of  $S_1$ . When  $\sigma_v^2$  is sufficiently high, the fluctuations in  $Z$  becomes negligible and liquidity suppliers can infer  $v$  much more accurately. Therefore, the value of acquiring the private signal is weakened, and liquidity demanders facing diminished trading gain will choose not to participate, the participation rate  $\mu$  will continue to fall. The interior solutions of  $0 < \mu < 1$  are not displayed in Fig. 1.2a, because in the absence of participation cost, it would require a large deviation from the baseline parameter values to reach interior  $\mu$ ; this will be seen more clearly in the next subsection, where the introduction of participation cost will allow us to reach interior  $\mu$  with lower  $\sigma_v^2$ , and the intuitions are the same.

**Proposition 5.2** *When asymmetric information is the only source of imperfection and all liquidity demanders chooses to participate, an increase in the variance of liquidity shock  $\sigma_Z^2$  lowers illiquidity  $\lambda$ , raises price reversal  $\gamma$ , and raises the price discount in Period 0.*

A larger variance  $\sigma_Z^2$  of the liquidity shock provides higher incentive for traders to hedge against non-traded risks. Higher aggregate demand for liquidity increases both price reversal  $\gamma$  and the price discount in  $S_0$ . However, higher  $\sigma_Z^2$  makes it more difficult for liquidity suppliers to infer the private signal  $v$  from movements in  $S_1$ . Therefore, on a per-trade basis, prices are less sensitive to volume,  $\lambda$  decreases.

**Observation**  $\lambda$  and  $\gamma$  are influenced in opposite directions by  $\sigma_Z^2$ .

### 1.5.1.2 Participation Cost

In the other single-imperfection benchmark which incorporates only participation cost, the comparative statistics are shown in Figure 1.3a and Figure 1.3b, summarized in the Propositions below:

**Proposition 5.3** *When participation cost is the only source of imperfection, an increase in participation cost  $\kappa$  lowers the participation rate  $\mu$ , keeps illiquidity  $\lambda$  unchanged, lowers price reversal  $\gamma$ , and lowers the price discount in Period 0.*

**Observation** Price discount is influenced in opposite directions by asymmetric information and participation cost.

When  $\kappa$  is sufficiently low, all traders prefer to participate because private information is very cheap to obtain, and the utility gain from trading on private information as well as hedging non-traded risks overwhelms the participation cost. As  $\kappa$  continue to increase, some traders will decide to leave the market. Competition becomes less intense among those who choose to stay, seizing a larger share of the

trading gain. The participation rate  $\mu$  continues to decrease until the marginal trader is indifferent between participating or not. Finally, when  $\kappa$  is sufficiently high, no trader participates ( $\mu = 0$ ).

As the participation rate  $\mu$  decreases, the market becomes “less illiquid” in the sense that there’s less demand of liquidity. This effect is captured by price reversal  $\gamma$  and price discount in  $S_0$ , both of which decrease with  $\kappa$ . As aggregate demand for liquidity is smaller, suppliers require a lower discount, and overall there is less transitory movement in the stock price. This effect differs from the participation-cost-only benchmark in Vayanos and Wang (2009), where the liquidity suppliers are making the participation choice. In their set-up, higher participation cost discourages liquidity suppliers from participating, which makes the market more illiquid, and the price discount increases with  $\kappa$ .

Illiquidity  $\lambda$  remains constant because the decrease in volume offsets the decrease in price movement. Liquidity suppliers are aware that for each additional unit of trade, liquidity demanders are compensated for paying an additional unit of participation cost. Liquidity suppliers’ learning ability is not affected by variations in participation, hence the price sensitivity per trade is unchanged.

**Proposition 5.4** *When participation cost is the only source of imperfection, an increase in the variance of liquidity shock  $\sigma_Z^2$  raises the participation rate  $\mu$ , keeps illiquidity  $\lambda$  unchanged, raises price reversal  $\gamma$ , and raises the price discount in Period 0.*

For interior values of  $\mu$ , there are two channels for  $\sigma_Z^2$  to influence market illiquidity: Firstly keeping the participation rate  $\mu$  constant, an increase in  $\sigma_Z^2$  causes more transitory movements in the prices; also, a larger  $\sigma_Z^2$  provides more incentive for liquidity demanders to hedge against non-traded risks, which raises the participation rate  $\mu$  and increases the demand for liquidity. Both effects contribute to increase price reversal  $\gamma$  and the price discount in  $S_0$ . Once  $\mu$  reaches unity, the second channel is no longer in effect, and  $\gamma$  increases with  $\sigma_Z^2$  at a lower rate. However, due to the increase in trading volume, illiquidity  $\lambda$  remains constant, prices are equally sensitive per trade for all values of  $\mu$  and  $\sigma_Z^2$ .

Here the impact of liquidity shock again differs from the participation-cost-only benchmark in Vayanos and Wang (2009), because liquidity suppliers and demanders react asymmetrically to liquidity shocks. When liquidity suppliers are making participation decisions, the demand side is fixed, an increase in  $\sigma_Z^2$  implies that providing liquidity is more profitable. The rise in liquidity supply causes a drop in price dis-

count. In contrast, if liquidity demanders are making participation decisions, since the decisions are made ex-ante before the realization of  $Z$ , the increase in  $\sigma_Z^2$  implies liquidity demanders are more willing to pay the participation cost up front because there's higher need to hedge against large liquidity shocks. Faced with a fixed liquidity supply, the price discount raises to compensate

**Observation**  $\lambda$  does not capture the changes in illiquidity due to increased participation cost, or increased variance of liquidity shock;  $\gamma$  should be a better measure of illiquidity in this case.

## 1.5.2 Illiquidity in the General Dual-Imperfection Model

In this section we investigate how illiquidity and asset prices are affected by each of the two imperfections incorporated in the general model, as well as by the variance of the liquidity shock. Numerical simulations are shown in Figures 1.4a, 1.4b, and 1.5a; the results are summarized in the following Propositions:

**Proposition 5.5** *An increase in the variance  $\sigma_v^2$  of information asymmetry lowers the participation rate  $\mu$  among traders, raises illiquidity  $\lambda$ , and lowers the price  $S_0$  in Period 0.*

*When all traders are participating,  $\sigma_v^2$  raises price reversal  $\gamma$ ; when only a fraction of traders are participating,  $\sigma_v^2$  lowers price reversal  $\gamma$ .*

In the general model, liquidity demanders who pay the fixed cost and participate in trading have two folds of incentive to do so: Firstly, trading allows liquidity demanders to hedge against the non-traded payoff and share risk with liquidity suppliers; Secondly, liquidity demanders who can observe the private signal  $v$  and liquidity shock  $Z$  are better informed than suppliers who observe none of them, so liquidity demanders may well take advantage of their superior knowledge through trading.

$\sigma_v^2$  lowers the participation rate  $\mu$  for the same reason as in the degenerated model with only asymmetric information, shown in Figure 1.4a. Two major differences should be noted between 1.2a and 1.4a: Firstly, for the same range of  $\sigma_v^2$ , we end up with interior solutions of  $\mu$ ; Secondly, the relation between  $\gamma$  and  $\sigma_v^2$  is no longer monotonic.

For each value of  $\sigma_v^2$ , when  $\kappa \neq 0$  liquidity demander are more reluctant to participate because they require a higher trading gain to cover the loss from participating cost,  $\mu$  decreases. The thinner volume contributes to raise price sensitivity, illiquid  $\lambda$  grows even faster in Figure 1.4a than in Figure 1.2a, and  $\lambda$  eventually converges to infinity as  $\mu$  converges to zero. Also because volume is low, the overall price impact

measured is smaller and causes price reversal  $\gamma$  to decrease with  $\sigma_v^2$ .

**Observation** When participation rate  $\mu$  is interior,  $\lambda$  and  $\gamma$  move in opposite directions as information asymmetry increases between informed liquidity demanders and non-informed liquidity suppliers.

**Observation** Price reversal  $\gamma$  can be lower under dual imperfections ( $\sigma_v^2 \neq 0$ ,  $\kappa \neq 0$ ) than under only participation cost ( $\sigma_v^2 = 0$ ,  $\kappa \neq 0$ ); illiquidity  $\lambda$  is unambiguously higher under dual imperfections.  $\lambda$  should be a better measure of illiquidity in this case.

Now we examine the effect of participation cost  $\kappa$ , holding information asymmetry  $\sigma_v^2$  constant,  $\sigma_v^2 \neq 0$ . The comparative statistics are presented in Figure 1.4b, summarized in the following Proposition:

**Proposition 5.6** *An increase in the participation cost  $\kappa$  lowers the participation rate  $\mu$  among traders, raises illiquidity  $\lambda$ , lowers price reversal  $\gamma$ , and lowers the price discount in Period 0.*

Participation cost  $\kappa$  lowers  $\mu$  for the same reason as in the degenerated model with only participation cost. Compare the effect of  $\kappa$  in Figure 1.4b to Figure 1.3b, the comparative statics on  $\mu$ ,  $\gamma$  and  $S_0$  are qualitatively the same, while the presence of information asymmetry further lowers participation rate  $\mu$ , raises price reversal  $\gamma$ , and raises price discount for each value of  $\kappa$ . Again we observe that price reversal  $\gamma$  and price discounts are lower when liquidity demanders face a higher cost to participate.

The major difference is that, in Figure 1.3b where participation cost is the only imperfection in the market, the increase in  $\kappa$  will not change  $\lambda$ , i.e. each additional unit of demand will not make the market more or less illiquid because the change in demand only reflects the change in the cost of doing so. In the general model, however, liquidity suppliers are well aware that their counter-parties are trading not only to share risks, but also to benefit from private information. When  $\kappa$  increases, market makers are aware that those who remain in the market are confident that their trading gain are sufficient to compensate the ever-increasing cost of doing so. Therefore, prices becomes more sensitive to each additional unit of trade,  $\kappa$  increases  $\lambda$ .

**Observation** Price reversal  $\gamma$  can be lower under dual imperfections ( $\sigma_v^2 \neq 0$ ,  $\kappa \neq 0$ ) than under only asymmetric information information ( $\sigma_v^2 \neq 0$ ,  $\kappa = 0$ );

illiquidity  $\lambda$  is unambiguously higher under dual imperfections.  $\lambda$  should be a better measure of illiquidity in this case.

The second half of this observation can be shown analytically as follows:

By Proposition 3.2,

$$\frac{b}{1-b} = \frac{\mu\pi(\sigma_n^2 + \beta_\xi c^2 \sigma_z^2) + (1-\pi)\beta_\xi \sigma_n^2}{(1-\beta_\xi)(1-\pi)\sigma_n^2} \quad (1.80)$$

Hence, by Proposition 4.1, illiquidity  $\lambda$  in the dual imperfection model is

$$\lambda = \frac{c}{\mu\pi} \frac{b}{1-b} = c \cdot \frac{(\sigma_n^2 + \beta_\xi c^2 \sigma_z^2) + \frac{1-\pi}{\mu\pi} \beta_\xi \sigma_n^2}{(1-\beta_\xi)(1-\pi)\sigma_n^2}$$

where  $\beta_\xi = \frac{\sigma_v^2}{\sigma_v^2 + c^2 \sigma_z^2}$ . An increase in  $\sigma_v^2$  raises  $\beta_\xi$  and lowers  $\mu$  when it is interior, hence  $\sigma_v^2$  raises  $\lambda$  unambiguously. An increase in  $\kappa$  keeps  $\beta_\xi$  unchanged and lowers  $\mu$  when it is interior, hence  $\kappa$  raises  $\lambda$  when  $\mu$  is interior. Therefore it is impossible to achieve  $\lambda_{dual} < \min\{\lambda_{asym}, \lambda_{pc}\}$ ,  $\lambda$  is always higher under dual imperfections than under only one type of imperfection.

The effect of liquidity shock in the general model is a synthesis of its effects in the two single-imperfection benchmarks.

**Proposition 5.7** *An increase in the variance of liquidity shock  $\sigma_z^2$  raises the participation rate  $\mu$  among traders, lowers illiquidity  $\lambda$ , raises price reversal  $\gamma$ , and lowers the price  $S_0$  in Period 0.*

$\sigma_z^2$  raises  $\mu$  for the same reason as in the two degenerated models, and the higher demand for liquidity makes the market more illiquid. Comparing Figures 1.2b, 1.3b to 1.5a, the presence of asymmetric information causes illiquidity  $\lambda$  to decrease with  $\sigma_z^2$  in the general model; asymmetric information and participation cost both cause  $\gamma$  to increase with  $\sigma_z^2$ , and price discount to increase with  $\sigma_z^2$  in the general model.

Our findings suggest that while  $\lambda$ ,  $\gamma$  and expected return are all valid proxies of illiquidity, they do not always yield consistent conclusions. The discrepancy amongst the three proxies arises because they capture different aspects of market movements and information. The price movements captured by  $\lambda$  has two parts: the permanent component, which arises from uncertainty in the fundamentals; and the transitory component, which arises because agents are risk-averse and requires a price-drop in Period 1 for bearing the liquidity risk.  $\lambda$  also uses the volume information which

is available only after the realization of the liquidity shock and the private signal.  $\gamma$  only captures the the transitory component, and also uses the price  $S_1$  which is realized after the liquidity shock and the private signal. Price discount in Period 0 encompasses the ex-ante effect of all imperfections.

We have shown that market under two imperfections can appear “less illiquid” than single-imperfection benchmarks, when illiquidity is measured by price reversal  $\gamma$ . The transitory price movement is attenuated by the interaction between two imperfections for the following reasons: (1) The adverse-selection effect. Liquidity suppliers are well aware that their counter-parties have superior information, hence demand a higher discount for bearing liquidity risk; this in turn undermines liquidity demander’s willingness to participate, because they face a higher cost for merely sharing risks even if the realization of  $v$  is favorable. (2) The volume effect. Participation cost reduces the population of liquidity demanders in the market, which weakens liquidity supplier’s ability of inferring the private signal from price, lowers the profit of supplying liquidity and causes less transitory movements in price.

On the other hand, the above observation does not hold for  $\lambda$  because the permanent component in price movements is not affected by the adverse-selection and volume effects. This, however, does not suggest that  $\lambda$  dominates  $\gamma$  as the “better” illiquidity measure. In particular, when participation cost is the only imperfection,  $\lambda$  does not capture the liquidity change due to higher participation cost  $\kappa$ , or higher liquidity shock  $\sigma_Z^2$ .

One can separate the permanent component into  $\lambda_{perm}$  which is the regression coefficient of the price change between Periods 0 and *Period 2* on signed volume, and separate the residual into  $\lambda_{trans} = \lambda - \lambda_{perm}$ . However, calculation shows  $\lambda_{trans} = -\frac{\alpha\sigma_n^2}{\mu\pi}$  is a constant, therefore the transitory component in  $\lambda$  do not have the same behavior as  $\gamma$ . As for the price discount, since both the adverse-selection and volume effect depend on the heterogeneity among agents after the realization of imperfections. Therefore, they do not show up ex-ante in Period 0 and not captured by the expected return between Period 0 and Period 2.

Finally, our results suggest that the correlation between illiquidity measures and expected return may not be unambiguously positive. As shown in Table 1.2, as the underlying cause of illiquidity changes from asymmetric information to participation cost to liquidity shock, neither measure co-moves with expected return consistently. Moreover, when the participation rate  $\mu$  is interior, liquidity shock  $\sigma_Z^2$  influences  $\mu$ ,  $\lambda$ , and  $\gamma$  all in opposite directions, which weakens the identification power of cross-



sectional tests. One possible solution is to sort securities on the basis of information asymmetry, participation cost, and idiosyncratic risks respectively, then restrict the illiquidity-return test to the three subsets of assets where only one source of cross-sectional variation dominates.

## 1.6 Conclusion

We developed a rational expectation equilibrium model that incorporates two types of imperfections: (1) Asymmetric Information, (2) Participation Cost, and studied the impact of imperfection on two illiquidity measures: (1) Kyle's  $\lambda$ , (2) Price reversal  $\gamma$ . We find that while both measures are valid proxies for illiquidity, they often gives opposite conclusions on the direction of change in illiquidity. In the general model where both imperfections are present, the market may be measured "less illiquid" by  $\gamma$  compared to two benchmark scenarios when only one of the imperfections are incorporated. However, this does not imply  $\lambda$  dominates  $\gamma$  as a better illiquidity measure. Under several scenarios, we have shown that  $\lambda$  is not capable of capturing the liquidity change when participation cost or liquidity shock rises. Moreover, imperfection do not always raise expected return, and the correlation between illiquidity and expected return is not unambiguously positive.

One possible extension of this exercise to separate the information acquisition cost from the participation cost, and examine their individual impacts. This would require imposing more structures on the receiving and revelation of private signals, in order to avoid generating too much dispersion in agents. Another direction is to extend this model into multiple periods or infinite horizon, and allow asymmetrically informed agents who faces fixed participation cost to choose trading horizon optimally. These dynamic models may allow us to highlight the interaction between the two imperfections more clearly.

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# Appendix

## Proof of Proposition 3.1

Liquidity demanders' optimal demand in Eq. (1.13) follows by maximizing the term inside the exponential in Eq. (1.7).

Liquidity suppliers choose  $\theta_1^s$  to maximize the expectation in (1.12), which can be written as

$$\begin{aligned}
 EU_{t=1}^s(\theta_1^s; \mu, S_1) &= E_{v,n}[-\exp\{-\alpha[W_1 + \theta_1^s(v + n - S_1)]\} | S_1] \\
 &= E_{v,n}[-\exp\{-\alpha[W_1 + \theta_1^s(E[v|S_1] - S_1 + v - E[v|S_1] + n)]\} | S_1] \\
 &= -\exp\left\{-\alpha\left[W_1 + \theta_1^s(E[v|S_1] - S_1) - \frac{1}{2}\alpha(\theta_1^s)^2(\sigma_n^2 + \sigma^2[v|S_1])\right]\right\}
 \end{aligned}$$

Eq. (1.14) follows from maximizing the exponential term over  $\theta_1^s$ .

## Proposition 3.2

Substituting  $E[v|S_1]$  and  $\sigma^2[v|S_1]$  from Eqs. (1.8) and (1.9), we can write the market clearing condition Eq. (1.4) as

$$\mu\pi \cdot \left[ \frac{(\bar{v} - a) + (1 - b)\xi}{\alpha\sigma_n^2} + \left( \frac{c}{\alpha\sigma_n^2} - 1 \right) Z \right] + (1 - \pi) \frac{(\bar{v} - a) + (\beta_\xi - b)\xi}{\alpha[\sigma^2[v|S_1] + \sigma_n^2]} = [\mu\pi + 1 - \pi] \bar{\theta} \quad (1.81)$$

Eq. (1.81) can be viewed as an affine equation in the variables  $(\xi, Z)$ . Set coefficient of  $Z$  to zero:

$$\begin{aligned}
 \frac{c}{\alpha\sigma_n^2} - 1 &= 0 \\
 \boxed{c = \alpha\sigma_n^2} & \quad (1.82)
 \end{aligned}$$

Set coefficient of  $\xi$  to zero and substitute  $c = \alpha\sigma_n^2$

$$\begin{aligned}
 \mu\pi \left( \frac{1 - b}{\alpha\sigma_n^2} \right) + (1 - \pi) \left( \frac{\beta_\xi - b}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} \right) &= 0 \\
 \boxed{b = \frac{\mu\pi \cdot (\sigma_n^2 + \sigma^2[v|S_1]) + (1 - \pi) \cdot \beta_\xi \sigma_n^2}{\mu\pi(\sigma_n^2 + \sigma^2[v|S_1]) + (1 - \pi)\sigma_n^2}} & \quad (1.83)
 \end{aligned}$$

Set constant term to equal RHS of Eq. (1.81):

$$\begin{aligned} \mu\pi \cdot \frac{\bar{v} - a}{\alpha\sigma_n^2} + (1 - \pi) \frac{\bar{v} - a}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} &= [\mu\pi + 1 - \pi] \bar{\theta} \\ \bar{v} - a &= \alpha\bar{\theta} \frac{\mu\pi + 1 - \pi}{\frac{\mu\pi}{\sigma_n^2} + \frac{1 - \pi}{\sigma_n^2 + \sigma^2[v|S_1]}} \\ \boxed{a = \bar{v} - \alpha\bar{\theta} \cdot \frac{\mu\pi + 1 - \pi}{\frac{\mu\pi}{\sigma_n^2} + \frac{1 - \pi}{\sigma_n^2 + \sigma^2[v|S_1]}} & \quad (1.84) \end{aligned}$$

### Proof of Proposition 3.3

Proposition 3.3 follows from setting  $\theta_0 = \bar{\theta}$  in Proposition 3.5, shown below.

### Proof of Proposition 3.5

- Liquidity Demanders who participates

Assume the participating liquidity demander chooses  $\theta_0$  at Period 0 and choose to participate. He will rebalance his portfolio to hold  $\theta_1^d$  shares of the stock at Period 1, where the optimal  $\theta_1^d$  is given in Eq. (1.13). Liquidity demander's interim utility in Period 1 has been calculated in Proposition 3.1 as

$$EU_{t=1}^d(\theta_0; \bar{\theta}, v, Z) = E_n(-\alpha C_2^d | v, Z) = -\exp(-\alpha X_1^d)$$

where

$$X_1^d(\theta_0; \bar{\theta}, v, Z) = W_1 + \theta_1^d(v - S_1) - \frac{\alpha\sigma_n^2}{2}(\theta_1^d + Z)^2$$

$$W_1 = \bar{\theta}S_0 + \theta_0(S_1 - S_0)$$

To calculate liquidity demander's interim utility at Period  $\frac{1}{2}$ , first note that  $\theta_1^d = \frac{v - S_1}{\alpha\sigma_n^2} - Z$ , and the affine price coefficient  $c = \alpha\sigma_n^2$ ,  $X_1^d(\theta_0; \bar{\theta}, v, Z)$  can be re-written as

$$X_1^d(\theta_0; \bar{\theta}, v, Z) = (\bar{\theta} - \theta_0)S_0 + \theta_0S_1 + \frac{c}{2}[(\theta_1^d)^2 - Z^2]$$

Next we use Lemma A.1 in Vayanos and Wang (2009) to calculate the expectation

$$EU^d = E_{v,Z} [-\exp(-\alpha X_1^d)]$$

Change of variables: Let

$$x \equiv \begin{bmatrix} \xi \\ Z \end{bmatrix}$$

$$\Sigma^d \equiv \begin{bmatrix} \sigma_v^2 + c^2 \sigma_z^2 & -c \sigma_z^2 \\ -c \sigma_z^2 & \sigma_z^2 \end{bmatrix}$$

Then the participant's optimal demand can be re-written as  $\theta_1^d = \frac{(1-b)\xi + (\bar{v}-a)}{c}$ , and  $S_1 = a + b\xi$ . Hence

$$X_1^d(\theta_0; \bar{\theta}, v, Z) = A_1^d + \{B_1^d\}' x + \frac{1}{2} x' (C_1^d) x$$

where

$$A_1^d(\theta_0; \bar{\theta}, v, Z) = \left\{ (\bar{\theta} - \theta_0) S_0 + \theta_0 a + \frac{(\bar{v} - a)^2}{2c} \right\}$$

$$B_1^d(\theta_0; \bar{\theta}, v, Z) = \begin{bmatrix} \theta_0 b + \frac{(1-b)}{c} (\bar{v} - a) \\ 0 \end{bmatrix} \equiv \begin{bmatrix} b_{11} \\ 0 \end{bmatrix}$$

$$C_1^d(\theta_0; \bar{\theta}, v, Z) = \begin{bmatrix} \frac{(1-b)^2}{c} & \\ & -c \end{bmatrix}$$

Therefore

$$I + \alpha C_1^d \Sigma^d = \begin{bmatrix} 1 + \alpha \frac{(1-b)^2}{c} (\sigma_v^2 + c^2 \sigma_z^2) & -\alpha (1-b)^2 \sigma_z^2 \\ \alpha c^2 \sigma_z^2 & 1 - \alpha c \sigma_z^2 \end{bmatrix}$$

$$\left( \Sigma^d (I + \alpha C_1^d \Sigma^d)^{-1} \right)_{1,1} = \frac{1}{\det(I + \alpha C_1^d \Sigma^d)} [\sigma_v^2 + c^2 \sigma_z^2 - \alpha c \sigma_v^2 \sigma_z^2]$$

$$\begin{aligned} \alpha (B_1^d)' \left[ \Sigma^d (I + \alpha C_1^d \Sigma^d)^{-1} \right] (B_1^d) &= \alpha (b_{11})^2 \left( \Sigma^d (I + \alpha C_1^d \Sigma^d)^{-1} \right)_{1,1} \\ &= \frac{\alpha}{\det(I + \alpha C \Sigma)} (b_{11})^2 (\sigma_v^2 + c^2 \sigma_z^2 - \alpha c \sigma_v^2 \sigma_z^2) \end{aligned}$$

Therefore

$$EU^d(\theta_0; \bar{\theta}, v, Z) = -\frac{\exp(\alpha\kappa) \exp(-\alpha F^d(\theta_0; \bar{\theta}, v, Z))}{\sqrt{\det(I + \alpha C_1^d \Sigma^d)}}$$

where

$$F^d(\theta_0; \bar{\theta}, v, Z) = A_1^d - \frac{1}{2} \alpha (B_1^d)' \Sigma^d (I + \alpha C_1^d \Sigma^d)^{-1} B_1^d$$

$$F^d(\theta_0; \bar{\theta}, v, Z) = \left\{ (\bar{\theta} - \theta_0) S_0 + \theta_0 a + \frac{(\bar{v} - a)^2}{2c} \right\} - \frac{\alpha (\sigma_v^2 + c^2 \sigma_Z^2 - \alpha c \sigma_v^2 \sigma_Z^2)}{2 \det(I + \alpha C \Sigma)} \left[ \theta_0 b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]^2$$

In equilibrium,

$$F^d(\bar{\theta}; \bar{\theta}, v, Z) = \left\{ \bar{\theta} a + \frac{(\bar{v} - a)^2}{2c} \right\} - \frac{\alpha (\sigma_v^2 + c^2 \sigma_Z^2 - \alpha c \sigma_v^2 \sigma_Z^2)}{2 \det(I + \alpha C_1^d \Sigma^d)} \left[ \bar{\theta} b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]^2$$

$$\frac{\partial F^d(\bar{\theta}; \bar{\theta}, v, Z)}{\partial \theta_0} = (a - S_0) - \alpha b \frac{(\sigma_v^2 + c^2 \sigma_Z^2 - \alpha c \sigma_v^2 \sigma_Z^2)}{\det(I + \alpha C_1^d \Sigma^d)} \left[ \bar{\theta} b + \left( \frac{1-b}{c} \right) (\bar{v} - a) \right]$$

- Non-participants

If the liquidity demander does not participate, he will hold on to  $\theta_0$  shares of the stock until the end of his life-span.

$$C_2^{np} = (\bar{\theta} - \theta_0) S_0 + \theta_0 (v + n) + Zn \quad (1.85)$$

Using the Law of Iterated Expectations, the non-participant's interim utility at  $t = \frac{1}{2}$  is

$$EU^{np}(\theta_0; \bar{\theta}) = E_Z [E_{v,n} [-\exp(-\alpha C_2^{np}) | Z]]$$

The conditional expectation over  $(v, n)$  can be calculated as

$$E_{v,n} [-\exp(-\alpha C_2^{np}) | Z] = -\exp(-\alpha X_1^{np})$$

where

$$X_1^{np} = [\bar{\theta}S_0 + \theta_0 (\bar{v} - S_0)] - \frac{1}{2}\alpha\sigma_v^2\theta_0^2 - \frac{1}{2}\alpha\sigma_n^2 (\theta_0 + Z)^2$$

The unconditional expectation becomes

$$EU^{np}(\theta_0; \bar{\theta}) = -\exp\left(-\alpha\left\{[\bar{\theta}S_0 + \theta_0(\bar{v} - S_0)] - \frac{1}{2}\alpha\sigma_v^2\theta_0^2\right\}\right) E_Z\left[\exp\left(\frac{1}{2}\alpha^2\sigma_n^2(\theta_0 + Z)^2\right)\right]$$

Hence

$$\boxed{EU^{np}(\theta_0; \bar{\theta}) = -\frac{\exp(-\alpha F^{np}(\theta_0; \bar{\theta}))}{\sqrt{1 - \alpha^2\sigma_n^2\sigma_Z^2}}} \quad (1.86)$$

where

$$F^{np}(\theta_0; \bar{\theta}) = [\bar{\theta}S_0 + \theta_0(\bar{v} - S_0)] - \frac{1}{2}\alpha\sigma_v^2\theta_0^2 - \frac{1}{2}\frac{\alpha\sigma_n^2}{1 - \alpha^2\sigma_n^2\sigma_Z^2}\theta_0^2 \quad (1.87)$$

In equilibrium,

$$F^{np}(\bar{\theta}; \bar{\theta}) = \bar{\theta}\bar{v} - \frac{1}{2}\alpha\sigma_v^2\bar{\theta}^2 - \frac{1}{2}\frac{\alpha\sigma_n^2}{1 - \alpha^2\sigma_n^2\sigma_Z^2}\bar{\theta}^2 \quad (1.88)$$

$$\frac{\partial F^{np}(\bar{\theta}; \bar{\theta})}{\partial \theta_0} = \bar{v} - S_0 - \alpha\sigma_v^2\bar{\theta} - \frac{\alpha\sigma_n^2}{1 - \alpha^2\sigma_n^2\sigma_Z^2}\bar{\theta} \quad (1.89)$$

- Liquidity Suppliers

Agent holds  $\theta_0$  shares of stock after trading in Period 0. If he becomes to a market maker in Period  $\frac{1}{4}$ , he will rebalance the portfolio to hold  $\theta_1^s$  shares of the stock at Period 1 as described in Eq. (1.14). Interim Utility at Period 1 has been calculated in Proposition 3.2:

$$EU_{t=1}^s(\theta_0; \bar{\theta}, S_1) = E_{v,n}[-\exp(-\alpha C_2^s) | S_1] = -\exp(-\alpha X_1^s)$$

where

$$\begin{aligned} X_1^s(\theta_0; \bar{\theta}, S_1) &= \bar{\theta}S_0 + \theta_0(S_1 - S_0) + \theta_1^s(E[v|S_1] - S_1) \\ &\quad - \frac{1}{2}(\theta_1^s)^2\alpha(\sigma_n^2 + \sigma^2[v|S_1]) \end{aligned}$$

To calculate the interim utility in Period  $\frac{1}{2}$ , note that  $S_1 = a + b\xi$ ,  $E[v|S_1] = \bar{v} + \beta_\xi \xi$ ,  $\theta_1^s = \frac{E[v|S_1] - S_1}{\alpha(\sigma_n^2 + \sigma^2)}$ , and the affine price coefficient  $c = \alpha\sigma_n^2$ . Hence  $X_1^s(\theta_0; \bar{\theta}, S_1)$  can be re-written as

$$\begin{aligned} X_1^s(\theta_0; \bar{\theta}, S_1) &= \bar{\theta}S_0 + \theta_0(S_1 - S_0) + \frac{1}{2} \frac{(E[v|S_1] - S_1)^2}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} \\ &= \bar{\theta}S_0 + \theta_0(a + b\xi - S_0) + \frac{1}{2} \frac{[(\bar{v} - a) + (\beta_\xi - b)\xi]^2}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} \\ &= A_1^s + B_1^s \xi + \frac{1}{2} C_1^s \xi^2 \end{aligned}$$

where

$$A_1^s(\theta_0; \bar{\theta}, S_1) = \bar{\theta}S_0 + \theta_0(a - S_0) + \frac{(\bar{v} - a)^2}{2\alpha[\sigma^2[v|S_1] + \sigma_n^2]} \quad (1.90)$$

$$B_1^s(\theta_0; \bar{\theta}, S_1) = \theta_0 b + \frac{(\bar{v} - a)(\beta_\xi - b)}{\alpha[\sigma^2[v|S_1] + \sigma_n^2]} \quad (1.91)$$

$$C_1^s(\theta_0; \bar{\theta}, S_1) = \frac{(\beta_\xi - b)^2}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} \quad (1.92)$$

where

$$\sigma^2[v|S_1] = \beta_\xi c^2 \sigma_z^2 = (1 - \beta_\xi) \sigma_v^2$$

Again using Lemma A.1 in Vayanos and Wang (2009), the expected utility is

$$EU^s(\theta_0; \bar{\theta}, S_1) = -\frac{\exp(-\alpha F^s)}{\sqrt{1 + \alpha \sigma_\xi^2 C_1^s}} \quad (1.93)$$

where

$$F^s(\theta_0; \bar{\theta}, S_1) = A_1^s - \frac{\alpha (B_1^s)^2 \sigma_\xi^2}{2(1 + \alpha \sigma_\xi^2 C_1^s)} \quad (1.94)$$

In equilibrium,

$$A_1^s(\bar{\theta}; \bar{\theta}, S_1) = \bar{\theta}a + \frac{(\bar{v} - a)^2}{2\alpha[\sigma^2[v|S_1] + \sigma_n^2]} \quad (1.95)$$

$$B_1^s(\bar{\theta}; \bar{\theta}, S_1) = \bar{\theta}b + \frac{(\bar{v} - a)(\beta_\xi - b)}{\alpha[\sigma^2[v|S_1] + \sigma_n^2]} \quad (1.96)$$

$$C_1^s(\bar{\theta}; \bar{\theta}, S_1) = \frac{(\beta_\xi - b)^2}{\alpha(\sigma_n^2 + \sigma^2[v|S_1])} \quad (1.97)$$



$$\boxed{F^s(\theta_0; \bar{\theta}, S_1) = A_1^s(\bar{\theta}; \bar{\theta}, S_1) - \frac{\alpha\sigma_\xi^2 \{B_1^s(\bar{\theta}; \bar{\theta}, S_1)\}^2}{2(1 + \alpha\sigma_\xi^2 C_1^s)}} \quad (1.98)$$

$$\boxed{\frac{\partial F^s(\bar{\theta}; \bar{\theta}, S_1)}{\partial \theta_0} = (a - S_0) - \alpha\sigma_\xi^2 \cdot b \cdot \frac{B_1^s(\bar{\theta}; \bar{\theta}, S_1)}{1 + \alpha\sigma_\xi^2 C_1^s}} \quad (1.99)$$

### Proof of Proposition 3.7

Eqs. (1.48) (1.49) follows similarly from Proposition 3.1, and setting  $\sigma_v^2 = \sigma_v^2[v|S_1] = 0$ .

### Proof of Proposition 3.8

Substitute participating agents' optimal demand functions Eqs. (1.48) (1.49) into the market clearing condition Eq. (1.50), and divide both sides by  $(\mu\pi + 1 - \pi)$ , we have

$$\Delta \left( \frac{\bar{v} - S_1}{\alpha\sigma_n^2} - Z \right) + (1 - \Delta) \frac{\bar{v} - S_1}{\alpha\sigma_n^2} = \bar{\theta}$$

$$S_1 = \bar{v} - \alpha\sigma_n^2 (\bar{\theta} + \Delta Z)$$

### Proof of Proposition 3.9

Proposition 3.9 follows directly by setting  $\sigma_v^2 = 0$  in Proposition 3.6.

### Proof of Proposition 4.1

Substitute participating demander's optimal demand from Eq. (1.13) into the definition of  $\lambda$  in Eq. (1.69), and rewrite  $v = \bar{v} + cZ + \xi$ ,  $S_1 = a + b\xi$ ,  $c = \alpha\sigma_n^2$ , we find

$$\theta_1^d = \frac{\bar{v} - a}{c} + \frac{1 - b}{c} \xi$$

$$\begin{aligned}
\lambda &= \frac{\text{cov} \left( a + b\xi - S_0, \mu\pi \left( \frac{\bar{v}-a}{c} + \frac{1-b}{c}\xi \right) \right)}{(\mu\pi)^2 \text{var} \left[ \frac{\bar{v}-a}{c} + \frac{1-b}{c}\xi \right]} \\
&= \frac{\mu\pi \cdot \frac{b(1-b)}{c} \sigma_\xi^2}{(\mu\pi)^2 \left( \frac{1-b}{c} \right)^2 \sigma_\xi^2} \\
&= \frac{c}{\mu\pi} \frac{b}{1-b}
\end{aligned}$$

### Proof of Proposition 4.2

Substituting the prices  $S_2 = v + n$  and  $S_1$  from Eq. (1.4), we find

$$\begin{aligned}
\gamma &= -\text{cov} [S_2 - S_1, S_1 - S_0] \\
&= -\text{cov} [v + n - a - b(v - \bar{v} - cZ), a + b(v - \bar{v} - cZ) - S_0] \\
&= b(b-1)\sigma_v^2 + b^2c^2\sigma_Z^2
\end{aligned}$$

### Proof of Proposition 4.3

In the degenerated model with  $\sigma_v = 0$ , substitute agents' optimal demands and equilibrium market price  $S_1$  from Proposition 3.7 and 3.8, we find

Illiquidity:

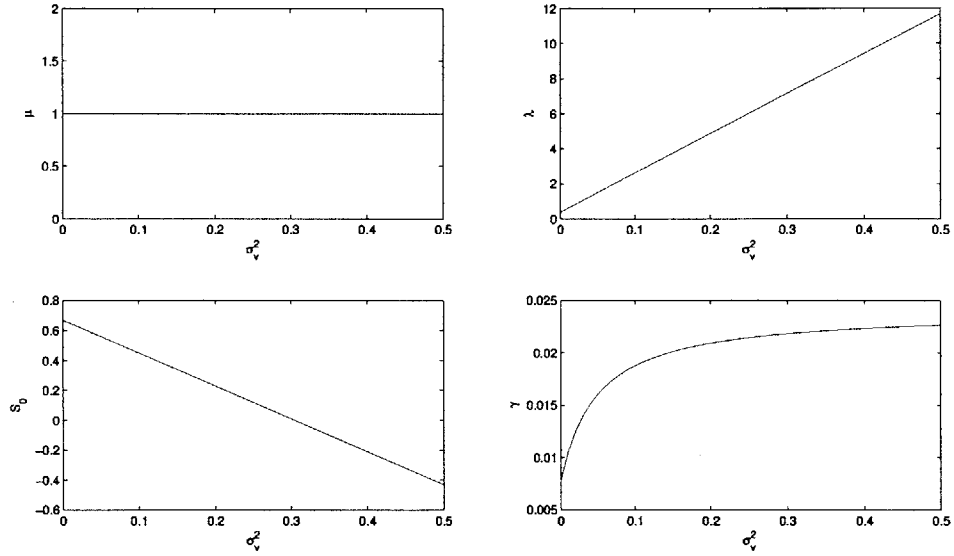
$$\begin{aligned}
\lambda &= \frac{\text{Cov} (S_1 - S_0, \mu\pi (\theta_1^d - \bar{\theta}))}{\text{Var} [\mu\pi (\theta_1^d - \bar{\theta})]} \\
&= \frac{\text{Cov} (\bar{v} - \alpha\sigma_n^2 (\bar{\theta} + \Delta Z), \mu\pi (-1) (1 - \Delta) Z)}{\text{Var} [-\mu\pi (1 - \Delta) Z]} \\
&= \frac{\alpha\sigma_n^2}{\mu\pi} \frac{\Delta}{1 - \Delta} \\
&= \boxed{\frac{\alpha\sigma_n^2}{1 - \pi}}
\end{aligned}$$

Price reversal:

$$\begin{aligned}\gamma &= -cov(S_2 - S_1, S_1 - S_0) \\ &= -cov(\bar{v} + n - \bar{v} + \alpha\sigma_n^2(\bar{\theta} + \Delta Z), \bar{v} - \alpha\sigma_n^2(\bar{\theta} + \Delta Z) - S_0) \\ &= \boxed{(\alpha\sigma_n^2)^2 \Delta^2 \sigma_Z^2}\end{aligned}$$

Figure 1.2: Illiquidity in Degenerated Model I:  $\sigma_v \neq 0, \kappa = 0$

(a) Effect of Asym. Info.



(b) Effect of Liquidity Shock

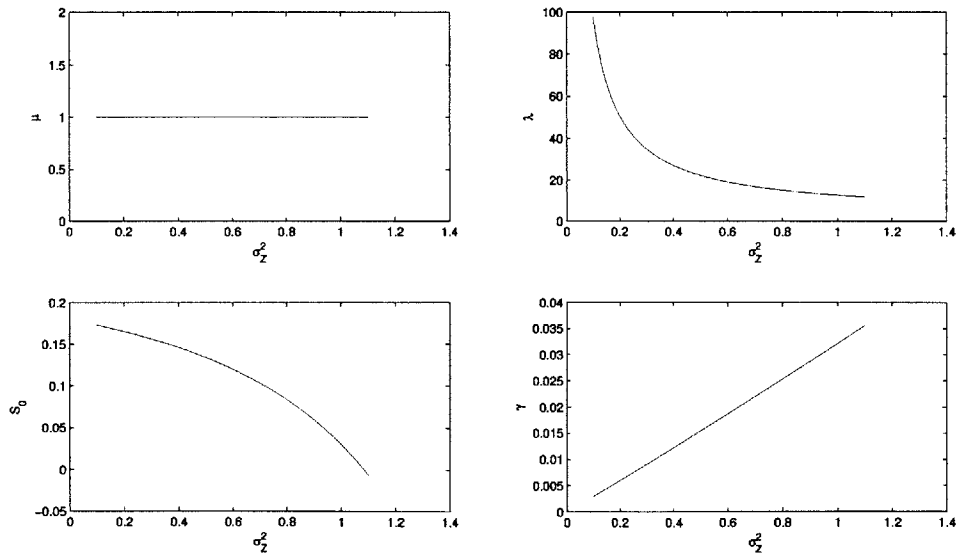
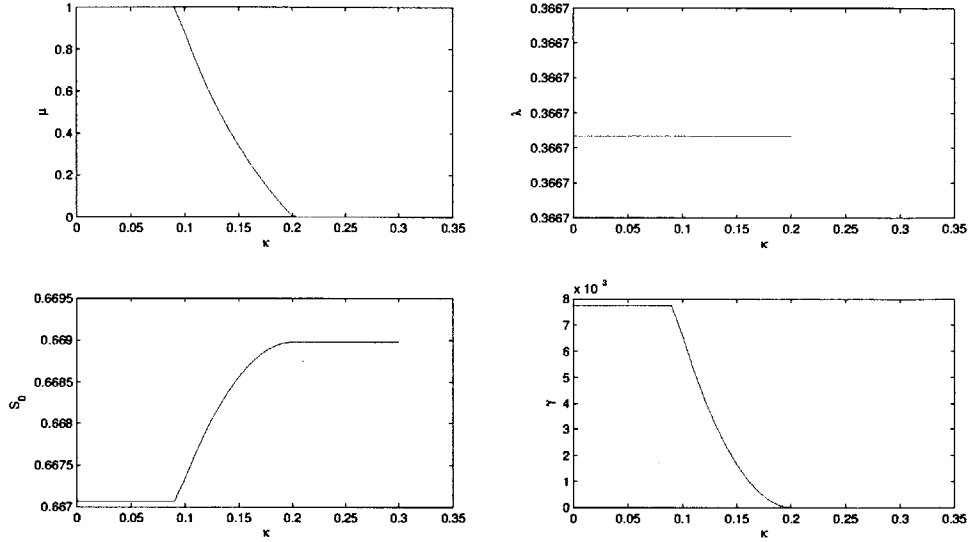


Figure 1.3: Illiquidity in Degenerated Model II:  $\sigma_v = 0, \kappa \neq 0$

(a) Effect of Participation Cost



(b) Effect of Liquidity Shock

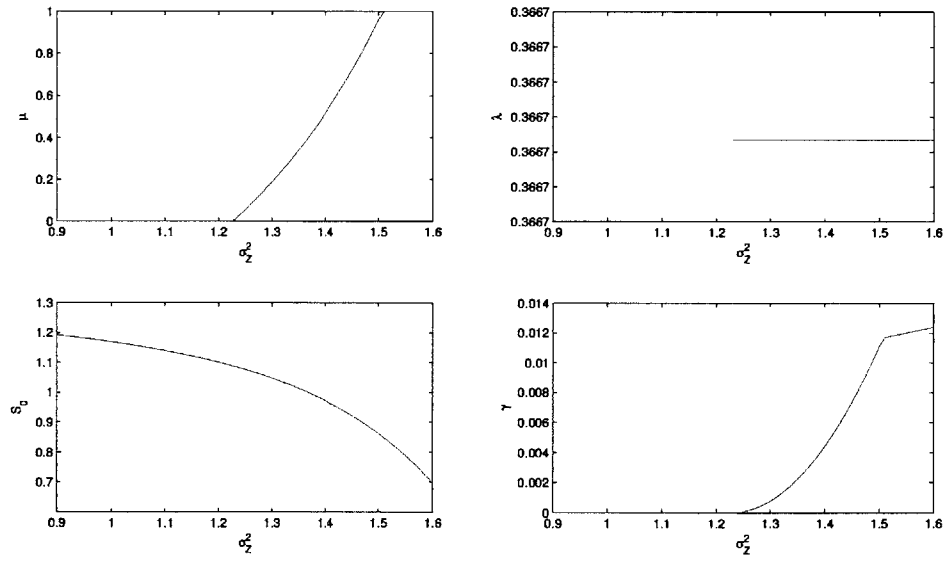
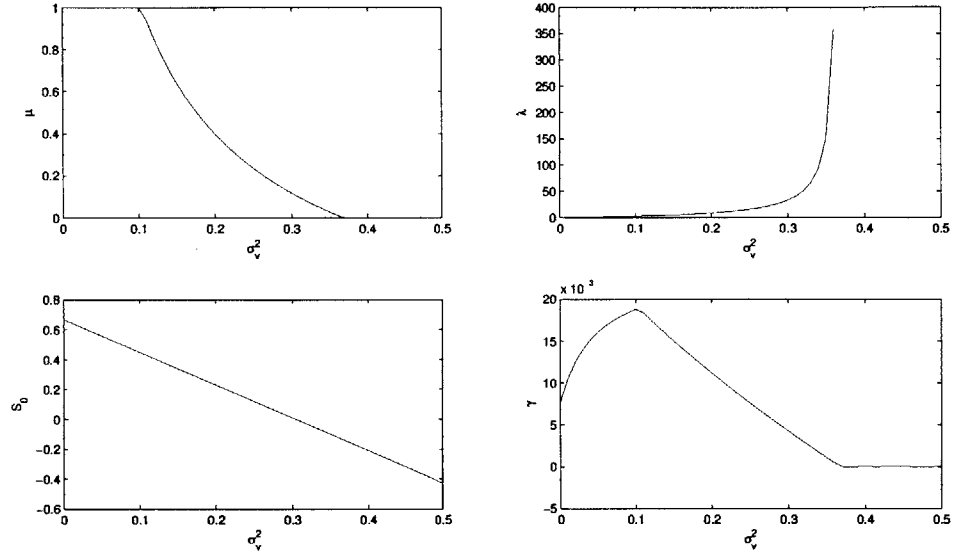


Figure 1.4: Effect of Imperfections in the General Model

(a) Effect of Information Asymmetry



(b) Effect of Participation Cost

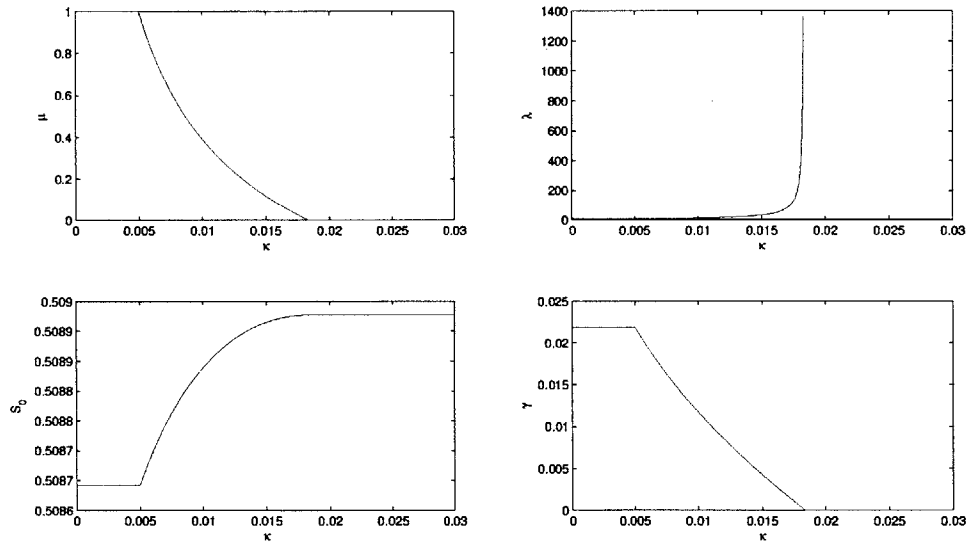
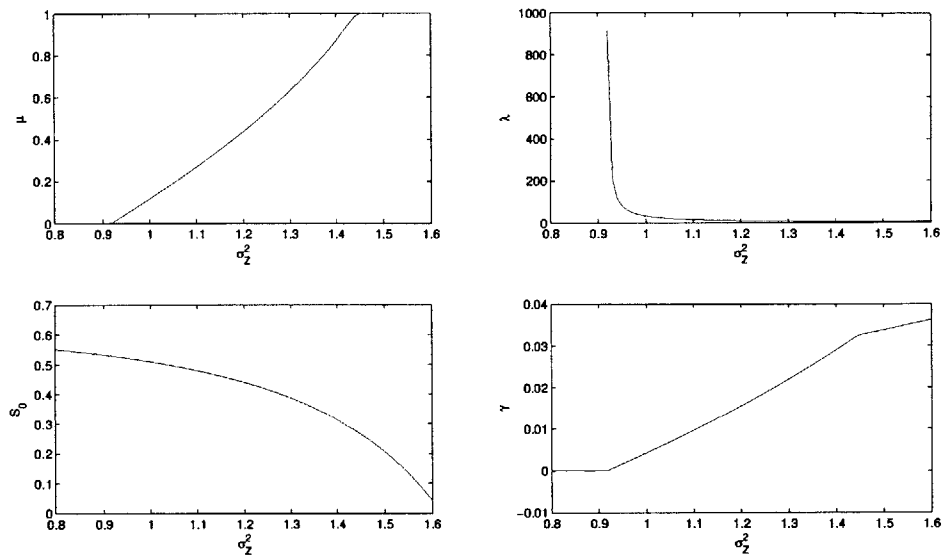


Figure 1.5: Effect of Liquidity Shock in the General Model

(a) Effect of Liquidity Shock







## Chapter 2

# An Empirical Comparison of Systemic Risk Measures

### 2.1 Introduction

The 2007~2009 financial crisis has highlighted the critical importance of measuring and monitoring global systemic risk. In recent years, this topic has received overwhelming interest from both academia and regulators. In the United States, the *Dodd Frank Wall Street Reform and Consumer Protection Act* has established a new role for the Federal Reserve Board as a systemic risk regulator. In the international community, institutions such as the Group of 20 (G-20) and the International Monetary Fund have also called for multilateral collaborative efforts in financial reform for systemic risk regulation. In order to restore global economic stability after the worst recession since World War II and take stock of the lessons learned during the crisis, it is important to develop analytic tools that help policy makers to understand what exactly went wrong from various angles. In the academic community, numerous research has been conducted to build new models and econometric techniques for measuring systemic risk, as described by the Office of Financial Research's recent survey (Bisias, Flood, Lo, and Valavanis (2012)). In this paper, we follow up and conduct an empirical comparison among these measures, in an effort to explore which of these new academic research could add the most value for providing policy guidance.

From an operational perspective, the current literature on systemic risk can be broadly divided into two categories: one is to develop system-level measures in the time-series, which asks the question whether we can find early-warning indicators

that anticipate systemic events, or capture the building-up of systemic risk contemporaneously; the other is to examine in the cross-section whether an individual firm should be designated as a Systemically Important Financial Institution (SIFI) that poses a threat to financial stability. Our focus in this paper is the former. If we could travel backward in time and equip ourselves with the kind of systemic risk measurement technologies available now, could it ever be possible to be alerted against the impending crisis, and get out of harm's way? Moreover, when some of these measures are warning against impending danger while the rest remains quiet and peaceful, which one(s) should one listen to and what decision should be made? These are the type of questions that we seek to answer here.

Currently there's no consensus on how to define systemic risk, much less how to quantify it. For our purposes, we adopt the working definition that a "systemic event" is any set of circumstances that threatens the confidence in or the stability of the financial system, hence systemic risk is the risk of such an event. This definition may seem too vague and generic to be of practical value, but yields some surprisingly novel distinctions when applied to specific contexts. For example, under this definition, the 2006 collapse of the \$9 billion hedge fund Amaranth Advisers was not systemic, but the 1998 collapse of the \$5 billion hedge fund Long Term Capital Management (LTCM) was, because the latter event affected a much broader swath of financial markets and threatened the viability of several important financial institutions, unlike the former. And the failure of a few regional banks is not systemic, but the failure of a single highly interconnected money market fund can be. Of course, this is just one of several possible definitions of systemic risk.

Using this working definition, we conduct formal empirical comparisons of various measures through which we can determine which ones are most effective for detecting threats to financial stability. We start by reconstructing several systemic-risk measures surveyed in Bisias, Flood, Lo, and Valavanis (2012) and extending them to the most recent period. Currently we have included the following measures: Mahalanobis Distance, which measures the statistical unusualness of a set of asset returns given their historical distribution pattern; Absorption ratio, which estimates the number and importance of common factors driving the returns of financial institutions, and associated periodic spikes in their correlations; GDP Stress Test, which measures the maximum drop in national GDP growth during a crisis period; Granger Causality, which is a statistical measure of interconnectedness among hedge funds, banks, broker/dealers, and insurance companies based on network analysis applied

to the monthly returns of these financial institutions; CoVaR, which estimates the Value-at-Risk of the entire financial system conditional on the stress of a particular financial institution; Marginal Expected Shortfall, which estimates extreme losses and “tail risk” during broad market declines; Measures of illiquidity risk, concentration, and the probability of market dislocation in the hedge fund industry, such as return auto-correlation, return-smoothing, and regime-switching.

In order to move forward, we need to provide an objective function for ranking the effectiveness of various measures. Again, currently there is no consensus of what objective function should be used. This issue seems particularly difficult in the cross-section: What policy conclusion should be drawn, when the SIFI ranking generated by one measure does not agree with another? And how can we interpret the SIFI ranking variation over time? Along the time-series dimension which this paper is focused on, we propose to evaluate systemic risk measures in two respects: as useful contemporaneous indicators of financial distress, and as early warning indicators of impending shocks. Using a list of systemic events proposed by the International Monetary Fund, we estimate the ability of each measure to successfully detect such events and compare the success rate with the potential for “false positives” during non-event periods, which yields an estimate of the indicator’s “signal-to-noise” ratio. Along the same lines, we also compare the performance of each measure in contemporaneous and forecasting Logit regressions to rank them in terms of statistical significance, goodness of fit, and persistence. Rankings measures by past performance helps us identify a subset of candidate measures that could have been the most informative for navigating through the 1998 and the 2007~2009 crises.

While each of these existing measures offers its unique insight to capture potential threats to financial stability, the sheer size and multiplicity of the current framework has also prohibited it from being an intuitive ready-to-use policy tool. Another important goal is to explore whether we could actually benefit from the capacity to assess systemic risk from a multitude of angles, in an effort to develop a new composite measure that improves the performance over the set of individual measures. We contribute to the literature by establishing the framework for conducting a systematic comparison among existing models, as well as constructing a composite measure that extracts information from individual measures. Furthermore, no paper that we are aware of has connected the systemic risk models to a closely-related strand of research which focuses on developing quantitative measures of financial conditions.

Similar to the mission of this paper, the financial condition index literature is

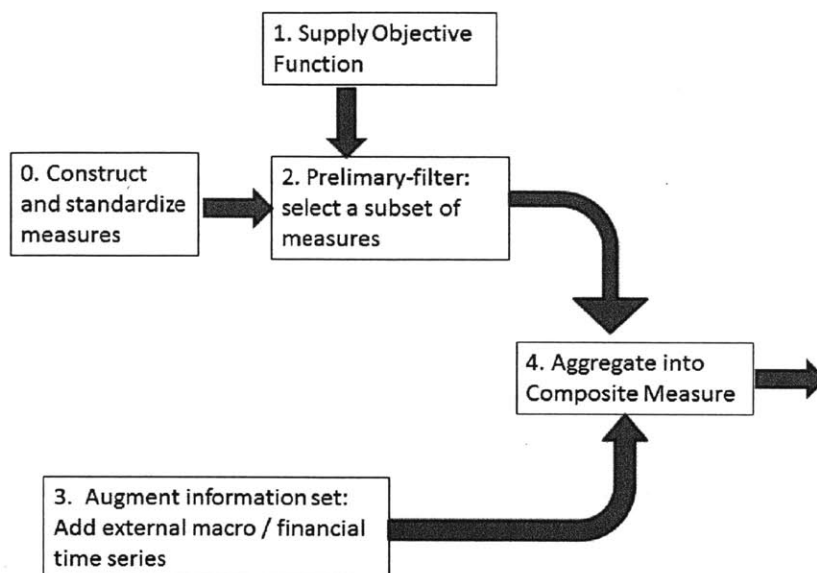
also aimed at finding macro and financial indicators that could potentially warn of an impending episode of financial distress. We compare the asset class coverage between the two strands of literature, and identify additional factors that are empirically good measures of financial distress, but have not been fully studied or modeled by the systemic risk literature. The candidate set of well-performing systemic risk measures are then combined with the selected external factors in order to improve the composite measure performance. Moreover, there are existing econometric techniques in the FCI literature that can be well imported into systemic risk research, which allows for the comparison and aggregation of individual indicators available at different frequencies and start at different points in time. See, for example, Stock and Watson (2000), Rosenberg (2009), Hakkio and Keeton (2009), Rose and Spiegel (2011), and so on.

The gist of our nested statistical model can be summarized in Figure 2.1. We start by constructing and standardizing individual measures as surveyed in Bisias, Flood, Valavanis, and Lo (2012), then select a subset of candidate measures based on an objective function that evaluates the measures' past performances in picking up or forecasting a given set of systemic events. The candidate measures are then combined with a panel of external factors to construct a composite measure, which will again be evaluated by its contemporaneous and forecasting performance for the same set of systemic events. The details of our approach will be elaborated in the methodology part of later sections.

We find that CoVaR, MES, and Granger causality networks measures are good contemporaneous indicators of systemic risk, whereas hedge-fund regime-switching measures are the only category of leading indicators available so far. In fact, we are able to develop a composite hedge-fund-only measure that generated early warnings in both 1998 and 2007, before the onset of market dislocations. Traditionally, financial stability analysis mainly focuses on macroeconomic and banking data; our results demonstrate that policy makers can greatly benefit from examining alternative asset classes such as hedge funds.

The rest of the paper proceeds as follows: Section 2 briefly reviews the set of measures that we have included in this study, data requirements, and other related literature; Section 3 describes the empirical comparison methodology on the set of existing measures; Section 4 constructs a composite measure from the existing measures and external factors, and discusses what the composite tells us about the current state of systemic risk; Section 5 concludes.

Figure 2.1: Modular Graph for the Nested Statistical Approach.



## 2.2 Existing Measures and Literature Review

In this section we review the literature on the set of existing measures that have been implemented in this paper, as well as related strands of literature on quantifying financial conditions, and econometric techniques for constructing aggregate indices. To begin with, we are faced with two major challenges: Firstly, the 31 different measures in Bisias et al (2012) use 31 different kinds of data, many of which are proprietary and not available to us; Secondly, several papers only describe their data without providing sufficient details on the actual inputs needed for replication. In those cases, we do our best to get similar data.

We focus on nine categories of measures that we were able to obtain the data most easily: the Mahalanobis distance, the absorption ratio, GDP stress test, Granger-causality networks, CoVaR, marginal expected shortfall, hedge fund illiquidity proxied by return auto-correlation, hedge-fund return-smoothing models, and hedge-fund regime-switching models. The methodologies for constructing these measures are briefly summarized as below. This selection of measures does not reflect any opinion, we picked those categories only because they were the easiest for us to implement.

These nine categories span into a total of twenty seven time series after taking into account similar measures constructed from different hedge-fund investment styles. We construct these measures and extend to the most current period possible (up to December 2011); for the rest, we compiled a list of data inputs and sources. We believe this effort could contribute to the systemic risk research community in general.

## 2.2.1 Review of Existing Measures

### 1. Granger Causality Network

This measure was developed in Billio, Getmansky, Lo, and Pelizzon (2012). The authors study the return inter-connectedness across hedge funds, banks, brokers, and insurance companies, which provides indirect information about the build-up of systemic risk among the four sectors. For a given 36-month window, they select the 25 largest firms from each sector as determined by average market capitalization or AUM. Pair-wise Granger causality test is conducted between institutions: X is said to “Granger-cause” Y if past values of X contain information that helps predict Y above and beyond the information contained in past values of Y alone:

$$X_t = \sum_{j=1}^m a_j X_{t-j} + \sum_{j=1}^m b_j Y_{t-j} + \epsilon_t \quad (2.1)$$

A directional network of these 100 institutions is hence constructed for the period of interest. To investigate the dynamic propagation of systemic risk, they calculate the Dynamic Causality Index ( $DCI_t$ ):

$$DCI_t = \frac{\text{number of causal relationships}}{\text{total possible number of causal relationships}} \quad (2.2)$$

An increase in the Dynamic Causality Index (DCI) indicates a higher level of system interconnection. Data are obtained from CRSP and the Lipper/TASS databases.

### 2. Mahalanobis Distance

This measure was developed in Kritzman and Li (2010). The authors define “financial turbulence” by the Mahalanobis distance, which measures the statistical unusualness of a set of asset returns given their historical patterns of

behavior:

$$d_t = (y_t - m)' \Sigma^{-1} (y_t - m) \quad (2.3)$$

where

$$\begin{aligned} d_t &= \text{turbulence at time } t \\ y_t &= (n \times 1) \text{ vector of asset returns} \\ m &= (n \times 1) \text{ sample average vector of asset returns} \\ \Sigma &= (n \times n) \text{ sample covariance matrix of asset returns} \end{aligned}$$

In principle, this methodology can be applied to any cross-section of asset returns at any frequency, as long as balanced-panel data is available. Following Kritzman and Li (2010), we implement this measure on a monthly frequency on the five series asset returns: MSCI US stock index, MSCI non-US stock index, US Bonds, real estate, and commodities. The original paper did not specify the data source for the last three, therefore based on data availability we choose to use the Bloomberg/EFFAS US Government Bond Index, the Dow Jones-UBS Commodity Index, and the Case-Shiller Seasonally-Adjusted Home Price Index. The path of financial turbulence is generated by running the metric in (2.3) over time.

### 3. Absorption Ratio

This measure was developed in Kritzman, Li, Page, and Rigobon (2010), which measure the extent to which various markets are tightly coupled. The intuition for this measure is that when sources of risks are unified, any shock to the market is more likely to propagate quickly and broadly across sectors, which implies a higher level of systemic risk. The authors use principal analysis and define the “absorption ratio” as the fraction of total variance of asset returns explained by a fixed number of eigen vectors:

$$AR = \frac{\sum_{i=1}^n \sigma_{E_i}^2}{\sum_{j=1}^n \sigma_{\alpha_j}^2} \quad (2.4)$$

where

$$\begin{aligned}n &= \text{number of eigen vectors used in calculating } AR \\ \sigma_{E_i}^2 &= \text{variance of eigen vector } i \\ \sigma_{\alpha_j}^2 &= \text{variance of asset } j\end{aligned}$$

Again, in principle, this measure can be implemented on any set of balanced-panel return data, at any frequency. We follow the descriptions in Kritzman, Li, Page, and Rigobon (2010) to apply (2.4) on daily returns for the 51 countries of the MSCI US index, and use 500-day rolling window to estimate the sample covariance matrix. In order for this measure to be comparable with the rest, we re-sample at the monthly frequency by taking the time-series maximum over that period.

#### 4. GDP Stress Test

There are numerous way to conduct stress tests; here we implement the model developed in Alfaro and Drehmann (2009). Domestic macroeconomic conditions typically weaken ahead of crises, and once the stress emerges output drops substantially. The authors propose a simple AR model of GDP growth

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \dots + \epsilon_t \tag{2.5}$$

where  $y_t$  denotes the real GDP growth rate at time  $t$ ; the Bayesian Information Criterion is used to determine the appropriate number of lags for each country. Countries are shocked by the worst negative forecast error in (2.5) during its most recent crisis. We use quarterly real GDP data from Bloomberg to conduct the stress test in (2.5); the countries in our sample include: Argentina, Australia, Belgium, Brazil, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Indonesia, Ireland, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, the Philippines, Singapore, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States. The aggregate systemic risk measure is constructed by taking the cross-sectional sum over all countries.



## 5. Co-VaR

This measure was developed in Adrian and Brunnermeier (2010). The authors propose to measure systemic risk by the Value-at-Risk of the entire financial system conditional on the distress of a particular financial institution  $i$ :

$$Pr(X^{system} \leq CoVaR_q^i | X^i = VaR_q^i) = q \quad (2.6)$$

where  $X^i$  and  $X^{system}$  are the growth rates of market-valued total financial assets for institution  $i$  and the financial system, respectively. To capture the time variation in the joint distribution of  $X^i$  and  $X^{system}$ , they run the following quantile regression on a vector of lagged state variables  $M_{t-1}$ :

$$X_t^i = \alpha^i + \gamma^i M_{t-1} + \epsilon_t^i \quad (2.7)$$

$$X_t^{system} = \alpha^{system|i} + \beta^{system|i} X_t^i + \gamma^{system|i} M_{t-1} + \epsilon_t^{system|i} \quad (2.8)$$

The set of state variables  $M_t$  includes the VIX, the liquidity spread between three-month repo and three-month treasury, weekly change of three-month treasury, weekly change of three-month ten-year yield spread, weekly change of the BAA/ten-year treasury credit spread, weekly equity market return and weekly real estate sector excess return. The time-varying  $CoVaR_t$  and  $VaR_t$  are then generated from the regression predicted values:

$$VaR_t^i(q) = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1} \quad (2.9)$$

$$CoVaR_t^i(q) = \hat{\alpha}^{system|i} + \hat{\beta}^{system|i} VaR_t^i(q) + \hat{\gamma}^{system|i} M_{t-1} \quad (2.10)$$

## 6. Marginal Expected Shortfall

This measure was developed in Acharya, Pedersen, Philippon, and Richardson (2010). A firm's marginal expected shortfall (MES) is the loss of the equity market value of financial firms during days in the prior year when the stock market losses were in its 5% worst-case periods. They focus on 102 financial firms with at least 5 billion USD in market capitalization which includes depositories, broker-dealers, insurance agents, non-depository institutions, real estate, and so on. The original measure was developed to evaluate the marginal impact of a single stock on the market, which yields a cross-sectional ranking. In order for this measure to be comparable with the rest, we calculate the cross-sectional

sum and standard deviation of the individual marginal expected shortfalls over the entire 102 firms; intuitively, higher dispersion among losses at different firms implies a higher level of systemic risk.

#### 7. Hedge fund return smoothing

This measure was developed in Getmansky, Lo, and Makarov (2004), and examined hedge fund return profile at the individual fund level. In their model, hedge funds report smooth returns  $R_t^0$  instead of their true returns:

$$\begin{aligned} R_t^0 &= \theta_0 R_t + \theta_1 R_{t-1} + \dots + \theta_k R_{t-k}, \theta_j \in [0, 1], j = 0, \dots, k \\ 1 &= \theta_0 + \theta_1 + \dots + \theta_k \end{aligned}$$

Smoothed returns have the same observed mean as the true returns but lower variance and higher serial correlation. This effect is quantified by

$$\xi \equiv \sum_{k=0}^k \theta_j^2 \in [0, 1] \quad (2.11)$$

Funds that engage in more return smoothing have more spread-out  $\theta$ 's, which implies lower  $\xi$ . For any individual fund returns, the  $\theta$ 's can be either estimated by maximum-likelihood or a linear factor model. Data is taken from Lipper TASS.

#### 8. Hedge fund illiquidity proxied by return auto-correlation.

This measure was developed in Getmansky, Lo, and Makarov (2004) and Chan, Getmansky, Haas, and Lo (2006), which examined hedge fund risk-return profiles at the aggregate-industry level. The authors propose to use rolling first-order auto-correlation to proxy for hedge-fund illiquidity exposure, and define an overall measure of systemic risk in the hedge fund sector as the cross-sectional weighted average. Let  $\rho_{t,i}$  denote hedge fund  $i$ 's first-order auto correlation in month  $t$  using a window of past returns (the authors use 36 months), the aggregate measure of illiquidity  $\rho_t^*$  is given by

$$\rho_t^* \equiv \sum_{i=1}^{N_t} \omega_{it} \rho_{t,i} \quad (2.12)$$

where  $N_t$  denotes the number of hedge funds in the sample at time  $t$ . The

weight  $w_{it}$  of hedge fund  $i$  is given by

$$w_{it} \equiv \frac{AUM_{it}}{\sum_{j=1}^{N_t} AUM_{jt}} \quad (2.13)$$

where  $AUM_{jt}$  are the assets under management for fund  $j$  at time  $t$ . Data is from Lipper TASS.

#### 9. Hedge fund regime-switching model.

This measure was also developed by Chan, Getmansky, Haas, and Lo (2006). The authors hypothesize that hedge fund returns can be modeled as a switching between two states of the world: a normal regime and a distressed regime, each with its own mean and variance. Denote by  $R_t$  the return of a hedge fund index in period  $t$  and assume the following specification:

$$R_t = I_{it} \cdot R_{1t} + (1 - I_{it}) \cdot R_{2t}, \quad R_{it} \sim \mathcal{N}(u_i, \sigma_i^2) \quad (2.14)$$

where

$$I_t = \begin{cases} 1 & \text{with probability } p_{11} \text{ if } I_{t-1} = 1 \\ 1 & \text{with probability } p_{21} \text{ if } I_{t-1} = 0 \\ 0 & \text{with probability } p_{12} \text{ if } I_{t-1} = 1 \\ 0 & \text{with probability } p_{22} \text{ if } I_{t-1} = 0 \end{cases}$$

The model in (2.14) is applied to the CSFB/Tremont hedge fund return indexes. Maximum-likelihood estimation allows us to determine the parameters in each state, as well as the probability of transition between the two states. Data is available on a monthly basis from January 1994 and includes the following investment styles: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event-driven distressed, event-driven multi-strategy, event-driven risk arbitrage, fixed-income arbitrage, global macro, long/short equity, managed futures, and multi-strategy. For each hedge fund investment style, a systemic risk measure is derived as the probability of being in the low-mean-return state.

### 2.2.2 Review of Other Related Literature

This paper is related to several strands of literature that seek to quantify financial conditions, and we contribute to the literature by establishing a linkage between the systemic risk literature and the financial conditions literature. Many papers have used a weighted average of a panel of macro and financial series to construct financial distress measures. Typically the input series is standardized by numbers of sample standard deviations from the sample mean, and high levels of the standardized index serves as warning signs of financial distress. For example, Rosenberg (2009) developed the Bloomberg US Financial Conditions Index (BFCIUS) which uses yield spreads and indices from US money markets, equity markets, bond markets. It assigns equal weight on each sector and within each sector equal weights on individual components. Compared to existing systemic risk measures, the BFCIUS covers asset classes not yet examined by the systemic risk literature, in particular credit spreads (corporate bond spread, muni spread, agency spread) and asset bubbles (NASDAQ/S&P 500 ratio, S&P Home builders / S&P 500 ratio). While Rosenberg (2009) is a useful first stab at creating an early warning system to predict economic fall outs, we also seek to make further improvements. For example, the weights on individual series should reflect their relative importance in constructing an aggregate measure, therefore equal weights may not be sufficiently justified. Moreover, the BFCIUS standardize their input series by sample mean and sample variance, which introduces an look-ahead bias into the process.

Many authors construct an aggregate financial conditions index by extracting the first principal component their input series. By construction, this extracts the common driver of the panel of financial series, and the factor loadings also reflect the systemic importance of each indicator. A prominent example is the Chicago Fed National Activity Index (CFNAI) which capture a single latent factor extracted from 85 variables describing US economic activity. For instance, Hakkio and Keeton (2009) estimate their Kansas City Fed Financial Stress Index (KCFSI) from a sample of US financial indicators on the health of the banking system, debt, equity, and money markets.

Further alterations of the standard PCA is called for when data varies in frequency and availability. In the FCI literature, many macro series are available with a long history but low quarterly frequency, whereas financial data are often available at a higher frequency but only becomes available at much later times, partly due to the emergence of new instruments in equity and credit derivatives, as well

as new markets such as hedge funds. Stock and Watson (2000, 2002) shows that this issue can be resolved by generalizing the standard PCA to an iterative estimation strategy. With a balanced panel, PCA reduces to ordinary least squares (OLS) estimation; for unbalanced panel, in each iteration missing values are replaced by their expectation conditional on the observed data, and estimates of factor and loadings are updated until the sum of squared errors converges. Bai and Ng (2004) show that this estimation is consistent for dynamic factor models as the size of the cross-section grows. This strategy has been used in recent research such as Hatzius, Hooper, Mishkin, Schoenholtz, and Watson (2010), where the authors are able to construct an aggregate index over a broad range of interest rates, asset prices, quantitative and survey-based series of data dating back to the 1970s. Brave and Butters (2010) further extend this iterative strategy by incorporating the temporal aggregation and accumulation algorithms in Harvey (1989) and Aruoba (2009). The authors constructed a weekly index of financial conditions by building upon an unbalanced panel of more than 100 individual series which spans over money markets, debt/equity markets, and the banking system, and are drawn with mixed frequencies ranging from weekly to quarterly.

This paper is also related to Lo, Sgherri, and Zhou (2012), which uses monthly hedge-fund regime-switching probabilities to construct contemporaneous and early-warning systemic risk indicators. The differences can be outlined as follows: Firstly, as described in the introduction section of this paper, one major obstacle in comparing systemic risk models is that there's no consensus on what objective function people should use to evaluate and compare different measures. In this current paper, we take an event-based approach and quantify systemic risk as the probability of discrete systemic events; in contrast, Lo, Sgherri, and Zhou (2012) models high/low systemic risk regime as continuous blocks of time. Secondly, the majority of systemic risk literature has predominantly focused on applying US data, including this paper; Lo, Sgherri, and Zhou (2012) takes on an international perspective and aims at comparing systemic risk stress level among different geographic regions.

## 2.3 Comparison of Individual Measures

### 2.3.1 Methodology

Our main approach in this paper has been illustrated in the introduction section Figure 2.1, and here we describe with more detail our motivation for taking this approach. Currently there is no consensus on what framework should be used when comparing different systemic risk measures. We approach this problem by dividing into the following modules:

In Module 0, we construct and standardize each systemic risk measure as a monthly time-series up to December 2011. Most standardization in the literature uses  $z$ -scores calculated from sample mean and sample standard deviation; here we standardize the individual series into  $z$ -scores using *rolling* mean and *rolling* standard deviation:

$$X_{it} = \frac{\xi_{it} - \hat{\mu}_{it}}{\hat{\sigma}_{it}} \quad (2.15)$$

where

$$\begin{aligned} \xi_{it} &= \text{systemic risk measure } i \text{ on month } t \\ \hat{\mu}_{it} &= \frac{1}{t} \sum_{s=1}^t \xi_i(s) \\ \hat{\sigma}_{it} &= \sqrt{\frac{1}{t} \sum_{s=1}^t [\xi_i(s) - \hat{\mu}_{it}]^2} \end{aligned}$$

This is to avoid the look-ahead bias as much as possible, so the resulting measures are actually implementable in the sense that if you implement them in the 2008 you actually see those results in 2008.

After constructing and standardize the measures, in Module I we first need to choose one particular objective function and bring the collection of measures under one unifying framework. The objective function chosen in this paper is event-based: we evaluate how well each measure can identify a given set of “systemic events”.

As shown in Table 2.1, we start by identifying – just by judgment – the set of systemic events in the 2007~2009 crisis and 1997~1998 crisis respectively, in order to examine which existing measures are able to pick up these events contemporaneously, and whether any of them are able to serve as an early warning signals. For the most recent crisis, we follow the IMF GFSR and include the following events: the Quant

meltdown in August 2007, Bear Stearns failure in March 2008, Lehman failure in September 2008, Global Central Bank Intervention in October 2008, and the Greek debt crisis April 2010. For the earlier 1997~1998 crisis, we chose to include the Thai Baht devaluation in July 1997, Russian Debt crisis in August 1998, LTCM debacle in September 1998, as well as the Japanese Yen appreciation in October 1998.

We also construct a control group of bad market events that caused tension in certain market sectors but weren't systemic, just to see whether or not we are getting false positives. The following events are included in our control group: the Tech bubble burst in March 2000; terrorist attack in September 2001; equity retreat in July 2002; oil price spikes in April 2004 and June 2008; US credit downgrade in Aug 2011.

One issue that comes up is whether there is some subjectivity in the choice of events, and people may have different views on which events should be included. Ultimately, this subjectivity is unavoidable, since there are so few events compared to the diversity of significant facets within each event. Moreover, by taking a modular view on this approach, the inclusion / exclusion of events can be seen as parameters of one module of the system; the other modules are selection of measures, and aggregation of measures. The overall framework is set up as a combination of all parts, and we can certainly fine-tune each module. Alternatively we could use distribution parameters of the output composite measure to define a quantitative threshold, and define systemic events as those who caused the measure to exceed this threshold. However, this also raises circularity in which systemic events are first used as benchmarks for selecting measures, then the composite measure is used to define systemic events. After all, our motivation is just is to set up a framework for people to start thinking about how to compare and contrast them. In order for the framework to be self-consistent, we only require that the composite measure able to capture what it's designed to capture in the first place, i.e. the set of systemic events that we started with.

Furthermore, in Table 2.2 we compare the objective functions that have been adopted in related literature. Authors either choose to focus on a subjectively-selected set of events, or choose a macro variable (e.g. GDP growth rate) as a proxy for financial conditions. In particular, our approach is most similar to Carlson, Lewis, and Nelson (2012), which also evaluate systemic risk measures by calibrating their performances for identifying daily events of policy interventions out of systemic risk concerns, and the events they select also spans the two major crisis that we

Table 2.1: List of Systemic Events and Control Group of Non-Systemic Events

Events	
Thai Baht devaluation	Jul 1997
Russian debt crisis	Aug 1998
LTCM debacle	Sep 1998
Yen appreciation	Oct 1998
Quant meltdown	Aug 2007
Bear Stearns failure	Mar 2008
Lehman and TARP failure	Sep 2008
Global central bank intervention	Oct 2008
Greek debt crisis	Apr 2010
<i>(event selection follows IMF GFSR)</i>	

Non-Events	
Tech bubble burning Up	Mar 2000
Terrorist attack	Sep 2001
DJIA sank to lowest level in nearly 4 years; NASDAQ and SP500 at lowest levels since 97	Jul 2002
Oil price hit a 3.5 year high	Apr 2004
Crude oil price tops \$100 a barrel	Jun 2008
S&P downgrades US credit rating	Aug 2011



aim to cover. For the 1998 crisis, Carlson, Lewis, and Nelson (2012) select the 9/23/1998 LTCM bailout as the benchmark event; here we choose to also include several market events that preceded, and probably precipitated, the last straw that broke the camel's back. Oet, Eiben, et al (2011) uses volatility regime benchmark to identify 50 systemic risk event weeks from 1991 to 2010; here we choose not to subjectively identify systemic events with another subjectively-selected measure, especially out of concerns for the volatility paradox (that leverage builds up and systemic risk rises during low-volatility periods, not high-volatility periods).

Besides, one may want to adopt an event-free approach, such as to test for pair-wise Granger causality across existing measures, and rank individual measures by the number of other measures that can be Granger-caused by itself; or to estimate pair-wise correlations and predictive relations, and rank individual measures by the number of other measures that are significantly correlated with itself, or can be predicted by itself. Such analysis might also allow us to track the building up and propagation of systemic risk across different sectors. Regardless of which criterion is applied, the objective is to select a subset of systemic risk measures that have better performance than the rest of the group. In fact, such event-free approaches does not directly address the central issue of systemic risk research – even if we found one measure capable of predicting some other measures, or have the highest cross-correlation with other measures, it doesn't necessarily imply policy makers will find it more useful to monitor systemic events; this may well be capturing “sector-rotation” between asset classes unrelated to the building up of systemic fragility.

Module II is to apply the supplied objective function as a preliminary filter to select a subset of candidate measures. The questions we want to ask are: Do measures pick up those systemic events? Secondly, do those measures generate false positives during the control groups that are bad market events that are not deemed to be systemic? And whether or not these measures can actually forecast these events; in other words, do these measures serve as early warning signs? More formally, we use the following criteria as an initial filtering of systemic risk measures:

One is the “signal-to-noise ratio”: we compute the average of this measure during these systemic events that we are targeting, and then compute the average during all the other periods (not just during the control group, but during all other periods that were not systemic), and then calculate the ratio of the event average and the

Table 2.2: Comparison of Methodology: Objective Function (Module I)

Index	Objective Function
Board Carlson, Lewis, and Nelson (2012)	policy interventions (Fed, FDIC, Treasury) out of systemic risk concerns -> 36 daily events (9/23/1998; 2007~2010 intervention events)
Chicago Fed Brave and Butters (2011)	list of 50+ weeks of financial crisis or market disruptions (incl. Enron, Y2K)
Cleveland Fed Oet, Eiben, et al (2011)	Use volatility regime benchmark 50 systemic risk event weeks (1991Q1: 2010Q4)
Kansas Fed Hakkio and Keeton (2009)	GDP growth rate
St Louis Fed Kliesen and Smith (2010)	N/A (calculate principal component)
Bank of Canada Illing and Liu (2006)	historical crisis periods
Hatzius, Hooper, Mishkin, Schoenholtz, and Watson (2010)	GDP growth rate

non-event average. In other words, SNR is defined as

$$\bar{X}_{ie} \equiv \frac{1}{\#events} \sum_{t \in events} X_{it} \quad (2.16)$$

$$\bar{X}_{in} \equiv \frac{1}{T - \#events} \sum_{t \notin events} X_{it} \quad (2.17)$$

$$SNR_i = \bar{X}_{ie} / \bar{X}_{in} \quad (2.18)$$

where  $X_{it}$  is the original measure normalized by its rolling mean and rolling standard deviation. Another possible definition of SNR is to compute the ratio of event-months versus the non-event control group. However, this definition is sensitive to the selection of control-group, and therefore more subjective compare to the approach in (2.18).

The other approach we undertake is to use Logit regression and examine whether these measures are significant during the systemic event months. We use the two following specifications:

1. Contemporaneous regression

$$Z_t = G(\alpha + \beta_{i,0}X_{i,t} + \beta_{i,1}X_{i,t-1} + \beta_{i,2}X_{i,t-2} + \dots + \epsilon_{it}) \quad (2.19)$$

2. One-step ahead forecast regression

$$Z_t = G(\alpha + \beta_{i,1}X_{i,t-1} + \beta_{i,2}X_{i,t-2} + \dots + \epsilon_{it}) \quad (2.20)$$

The independent variable for the logit regression is an indicator variable for systemic events: 1 for months during which there has been in systemic event, 0 otherwise. The dependent variables are the the individual systemic risk measures with its lags. In the baseline case, we use a uni-variate regression with no lags; similar analysis are conducted with higher numbers of lags. We rank measures by whether or not they can get an explanatory power in the logit regression, i.e. the significance of  $\beta_{i,0}$  in the contemporaneous regression (2.19) or  $\beta_{i,1}$  in the one-step ahead forecast (2.20).

Along the same lines, we also compare the goodness-of-fit and examine how much of the variation in the systemic event indicator can be explained by variations in the individual measures. Measures are ranked by  $R^2$  from the Logit regressions.

Our current approach in Module II is to filter out measures with negative signal-to-noise ratios, or non-significant coefficients in the Logit regressions. An alternative approach could be assigning relative weights to the individuals measures based on their past performance: the worse-performing measures are assigned lower weights than the better-performing ones.

Another characteristic we examine is persistence, in which we compute the first-order auto correlation of these measures. Although it is interesting to compare auto-correlation among different categories of measures, a more persistent measure doesn't necessarily imply it is a better measure since the cause for high auto-correlation may not be related to the building up of financial distress. Therefore we will not use auto correlation as a benchmark to filter out measures, but nonetheless report the rankings.

Module III is to augment the information set by incorporating additional data series that are currently not yet examined in the systemic risk literature, but are empirically useful measures of financial distress. Details on external factors will be provided in Section 2.4.1. Going forward as new theory models are established, these external factors would eventually become new systemic risk measures and migrate

into Module 0.

Finally, Module IV combines the candidate systemic risk measures with the external factors, and aggregate them into a composite measure. In this paper, we choose to adopt unbalanced-panel principal component analysis, which allows for the fact that the individual measures and external factors start at different points in time, and that the factor loadings on the individual measures are derived from their commonality instead of assigning subjective weights. In principle, the aggregation algorithm can be as simple as a “majority rule”, or much more sophisticated approaches such as machine learning and other various filtering techniques. Regardless, all these alternatives can be refinements of Module IV, but the overall framework remains unchanged.

### 2.3.2 Results

The summary statistics for all measures are reported in 2.7. In Table 2.8 we rank the measures by their signal-to-noise ratio. Our results suggest that the following five measures have the highest signal-to-noise ratio as defined in (2.18), in other words they give the highest stress level readings during the event-months relative to all other months: the marginal expected shortfall standard deviation comes first, followed by Granger causality network for financial firms, the CoVaR, and two regime-switching measures based on event-driven-distressed and multi-strategy hedge funds, respectively.

Among different hedge-fund regime-switching measures, managed futures and equity-market-neutral have the lowest signal-to-noise ratio. The Absorption ratio and GDP stress test both fall into the lower end of rankings. We also observe that some measures have registered negative signal-to-noise ratios, all of which are based on return-smoothing. As a first-step filter, we would exclude those low SNR measures in constructing the composite index.

In Table 2.9 we report the baseline contemporaneous Logit regression results, in other words this is the univariate regression of systemic event indicator on the current period systemic risk measure with no further lags. We also performed the same regression with additional lags, and the rankings turned out to be similar. As shown in Table 2.9, using the Logit regression we get a different ordering but consistent results as the SNR analysis. By ranking measures by the significance (not magnitude) of Logit regression coefficient in (2.19), we see that CoVaR and marginal expected shortfall cross-sectional standard deviation again have very good

performance, followed by marginal expected shortfall cross-sectional sum; hedge-fund regime-switching measures comes next, with the categories being multi-strategy and event-driven multi-strategy. Three of the top five performers in contemporaneous logit regression are the same as SNR.

Moving towards the lower end, the absorption ratio and GDP stress test have a close-to-zero coefficient and also insignificant; we observe again that some measures provide the wrong direction in the logit regression (2.19), with return-smoothing equity-market-neutral at the bottom, similar to the SNR results. These measures will be excluded from the composite measure.

In Table 2.10 we report the one-step-ahead Logit regression rankings in the baseline univariate case. Again measures are ranked by the significance of the regression coefficient  $\beta_1$  in (2.20), not necessarily by its magnitude. This time, multi-strategy-based hedge fund regime-switching measure comes at the top, followed by marginal-expected-shortfall cross-sectional standard deviation, event-driven hedge-fund regime-switching measure; global-macro has newly emerged in the top five. Compared with Table 2.9, our results suggest that hedge-fund regime-switching measures are better predictors than contemporaneous indicators.

In terms of explanatory power, in Table 2.11 we report the R-squares for the contemporaneous (Panel a) and one-step-ahead logit regressions (Panel b), both in the baseline univariate case. Using standard methods of constructing these R squares, in the contemporaneous regression we are looking at 32% at the top of the panel. In other words, we can actually get reasonable explanatory power using CoVaR, marginal expected shortfall, regime switching for hedge funds, followed by the Granger causality measures for market sector interconnectedness.

With the one-step ahead, we are again getting slightly different rank orderings but the same five measures at the top. Now the highest R-square for the one-step ahead is 15%, which is approximately half of the contemporaneous case but still reasonable starting point for predictive analysis.

In Table 2.12 we rank the measures by their first-order auto-correlation. As can be seen from the results, auto-correlations are in general quite high in these integrated series; not surprisingly, hedge funds have the highest serial correlation, and the top five are all hedge-fund regime-switching measures with the investment styles being multi-strategy, long-short equity, all-styles, emerging-markets, and global macro. Intuitively, hedge funds have monthly mark-to-market requirements, therefore they are the first in line to sense any changes in financial and credit conditions. Signs

of distress usually show up in hedge fund returns – particularly the illiquid ones – before other markets are hit. GDP stress test comes next, which implies that this measure does not provide a salient contrast between event-months versus the rest of the period, but past levels of stress test provides quite a good estimate for the period ahead. The same can be said for the absorption ratio. Finally, the marginal expected return cross-sectional sum and standard deviation are among the lowest in terms of serial-correlation, although both are top performers in the SNR and Logit regressions. This is consistent with our previous discussion that serial correlation rankings may not be the appropriate criterion for selecting measures.

Overall, our results indicate that marginal expected shortfall, CoVaR, Granger causality networks, and hedge fund regime-switching measures are among the most informative for policy makers to navigate through previous crises. Furthermore, hedge-fund regime-switching measure is a better leading measure than contemporaneous, whereas Granger Causality network and CoVaR are better contemporaneous measures than leading.

The predictive power of hedge fund regime-switching measures can be understood by the examining the dynamics of how financial distress is propagated across asset classes. Hedge funds borrow from the banks and are highly leveraged. Therefore, they are usually the first in line to sense changes in credit and financial conditions. Many recent literature has emphasized the role of serial deleveraging as a crucial mechanism in causing financial crisis (see, for example, Brunnermeier, Nagel, and Pedersen (2008)). Essentially, Once there is a shock to the system, prices continue to drop until someone is forced to liquidate in order to meet margin calls. Hedge fund liquidation starts from the most liquid markets and this act further weakens the market by drying up liquidity. When too many people are trying to flee too quickly, the rest also become caught up. Eventually, when nobody can liquidate anymore, people move on to liquidate in the next market, and the downward spiral starts. In our view, signs of distress shows up when we start to see liquidation across the board from hedge funds in all asset classes.

Granger causality network examines the interconnections between hedge funds, banks, brokers, and insurance companies. Risk spill-over from hedge funds to other sectors is slower than within hedge funds, therefore counting the number of inter-sector causality links is much more likely to a contemporaneous measure than a leading measure.

CoVaR identifies the tail-risk of the entire financial system by individual institu-

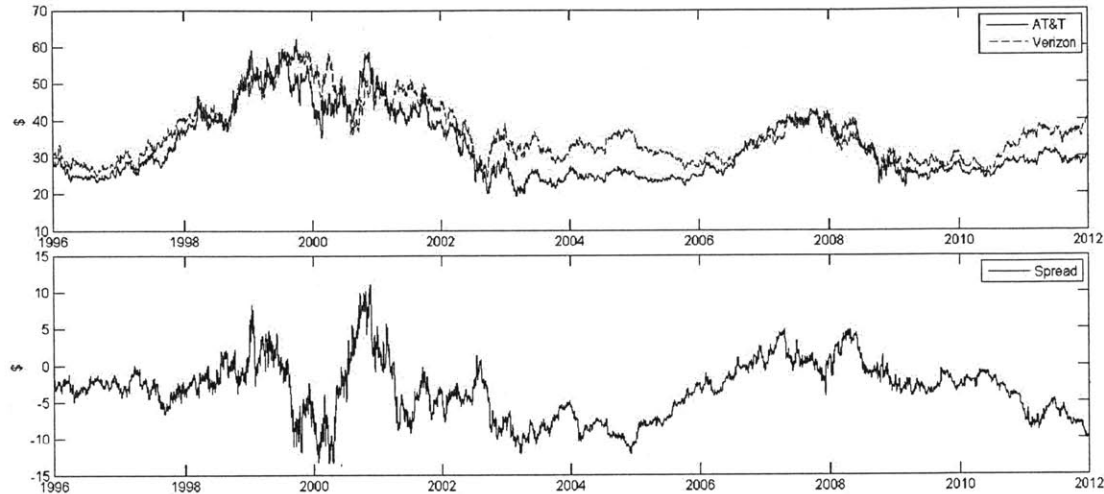
tions. Conceptually, this measure can be used to anticipate systemic risk because it does not rely on contemporaneous price movements. However, the key variable in the CoVaR definition (2.6) is the growth rate of the firm's asset value, which is computed through quarter data of firm's leverage, book equity value, and shares-outstanding. This information lag in data inputs could reduce the output measure's predictive performance. Marginal expected shortfall is conceptually similar to CoVaR and also focuses on the left-tail of the return distribution; by taking the cross-sectional sum, we are measuring the "size" of the left-tail, whereas by taking the cross-sectional standard deviation, we are measuring the "fatness" of the left tail. Yet by construction MES only uses information from equity returns and not quarterly balance-sheet information, and it turns out to have better forecasting power as reported in Table 2.10.

Our entire analysis include three categories measures that are constructed exclusively from hedge fund returns and AUM's: regime-switching, return-smoothing, and illiquidity proxied by return-autocorrelation. With the same information set, regime-switching models outperformed the other two categories as indicators of systemic risk. At this point it should also be of interest to examine across different hedge-fund investment styles, and we will follow the investment style definitions from Credit Suisse / Tremont Hedge Fund Index.

To start with, event driven funds "seek to profit from potential mispricing of securities related to a specific corporate or market event. Such events can include: mergers, bankruptcies, financial or operational stress, restructurings, asset sales, ..., as well as other types of corporate events." In particular, as a sub-category, event-driven distressed funds "typically invest across the capital structure of companies subject to financial or operational distress or bankruptcy proceedings. Such securities often trade at discounts to intrinsic value due to difficulties in assessing their proper value, lack of research coverage, or an inability of traditional investors to continue holding them." Considering the five event months for the 2007-2009 crisis, two are directly linked to major corporate bankruptcies (Lehman and Bear Stearns) where as the others also lead to significant equity downturns. Consistent with intuitions, event-driven funds are quite useful as systemic risk indicators.

Another set of regime-switching indicator comes from global macro funds, which "typically focus on identifying extreme price valuations and leverage is often applied on the *anticipated* price movements in equity, currency, interest rate and commodity markets. Managers typically employ a top-down global approach to concentrate on

Figure 2.2: Stylized example of Equity Market Neutral: Equity pairs trading AT&T vs Verizon, the spread is not indicative of macro or systemic distress.



*forecasting* how political trends and global macroeconomic events affect the valuation of financial instruments.” As can be observed from Table 2.8, 2.9 and Table 2.10, the regimes of global macro fund returns turn out to be a better leading indicator than a contemporaneous indicator, and are in general quite consistent with global economic trends.

In contrast, equity market neutral funds ended up as the least indicative of systemic risk. Here we should note that our objective is not to rank the profitability or risk-return characteristics across various investment categories, but rather whether those return regimes are consistent with systematic risk evolution. Equity market neutral funds “typically take both long and short positions in stocks while seeking to reduce exposure to the systematic risk of the market ... exploit investment opportunities unique to a specific group of stocks, while maintaining a neutral exposure to broad groups of stocks defined for example by sector, industry, market capitalization, country, or region.” As a stylized example, AT&T (T) and Verizon (VZ) are a pair of technology stocks with similar products and clientele. As shown in Figure 2.2, their price movements are usually in line with one another. When there is a short-term divergence between the two, an equity-market-neutral investor would take on relative-value positions with the expectation that the spread would eventually converge, and the timing would be driven mostly by company-specific events rather than systemic events. Therefore, it is unsurprising that the regime probabilities of equity market neutral funds are not particularly informative for identifying systemic risk.



In the current set of results, hedge fund illiquidity proxied by auto-correlation didn't appear as particularly informative, which is somewhat surprising considering that fire-sale liquidation and deleveraging were the central drivers of the 2007 crisis (Khandani and Lo (2011)). However, the liquidity measure implemented in this comparison study is calculated using 36-month rolling window, in other words by 2007 this measure is capturing the average first-order auto-correlation from 2005~2007. To capture the sudden liquidity changes during those highly tumultuous episodes, we should be conducting a microscopic study with higher frequency data and over a narrower window; this, for now, is beyond the original scope of the empirical comparison project. Furthermore, Zhou (2010) also demonstrate under the joint influence of multiple frictions and liquidity shocks, autocovariance (which can be measured with daily returns) and Kyle's lambda (which can be measured with transaction-level data) may indicate opposite changes in liquidity levels. Therefore, the choice of liquidity measure could also have affected the comparison outcome.

Finally, in the above rankings, GDP stress test does not seem to have very good performance either in the contemporaneous or leading measure. This may be due to the fact that aggregating over static, one-country-at-a-time stress-tests is only a starting point for the time being; however, a dynamic view of stress-testing would be more appropriate. Across sectors, when there's a shock to banks, the ripple effect immediately reaches the hedge funds, causing the repo market to seize up and banks are forced to take the next action; across countries, many authors have documented that (see, for example, Pepinski, T. (2012) ) the US subprime crisis has led to a global repatriation of portfolio capital in which international investors rebalanced their portfolio away from the US and back to home countries in which their fund were domiciled. Our reported results should not be interpreted as GDP stress tests are not useful channels for systemic risk management.

## 2.4 Construction of A Composite Measure

### 2.4.1 Methodology

Our next goal is to construct an aggregate systemic risk measure from the individual measures. To begin with, we compare our approach to other papers in related literature as shown in Table 2.3 and examine what they have included as the input data series. The common approach is to select 10~20 financial series from different asset

Table 2.3: Comparison of Methodology: Input Data Series (Module II)

Index	Objective Function
Board Carlson, Lewis, and Nelson (2012)	12 series of liquidity, credit, and uncertainties standardized by long-run mean and stdev
Chicago Fed Brave and Butters (2011)	100 financial indicators: money markets (28) debt/equity (27), banking system (45)
Cleveland Fed Oet, Eiben, et al (2011)	11 series of bank loans, FX credit, equity, debt transformed into CDF
Kansas Fed Hakkio and Keeton (2009)	18 series of interest rates, yield spreads, bond volatility index, equity volatility index
St Louis Fed Kliesen and Smith (2010)	11 series of credit spreads, equity/bond correlation, VIX, price/return dispersion
Bank of Canada Illing and Liu (2006)	rolling beta for banking industry, liquidity and credit spread, equity vol, exchg rate vol
Hatzius, Hooper, Mishkin, Schoenholtz, and Watson (2010)	45 series of interest rates, asset prices; quantity variables (e.g. CP, ABS issuance); surveys.

classes (risk-free rates, credit, equity, volatility, and so on), while some others choose to be as broad as possible (for example, Brave and Butters (2011)), which raises the questions of whether more is necessarily better. Given the above considerations, we also seek to incorporate information from various asset classes but will limit our exposure to the better-performing candidate measures as described in the previous section.

Next comes the question of which asset classes we can cover in our aggregate index. What we look for is to enrich the information set in our composite index by incorporating other financial and macroeconomic time series that have been frequently used as measures of financial distress, but not yet examined in the systemic risk literature. In Table 2.4 we compare the asset class coverage and methodology in the systemic risk literature versus the financial conditions literature.

The systemic risk literature has the unique advantage of using hedge-fund data and network analysis to develop measures that have not been studied in the financial conditions literature. Additionally, several systemic risk models are based on nonlinear correlation measures that focuses on tail risk (e.g., the CoVaR and the Co-Risk), whereas financial condition indexes typically use simple linear correlations among different asset classes. On the other hand, the financial condition literature covers a

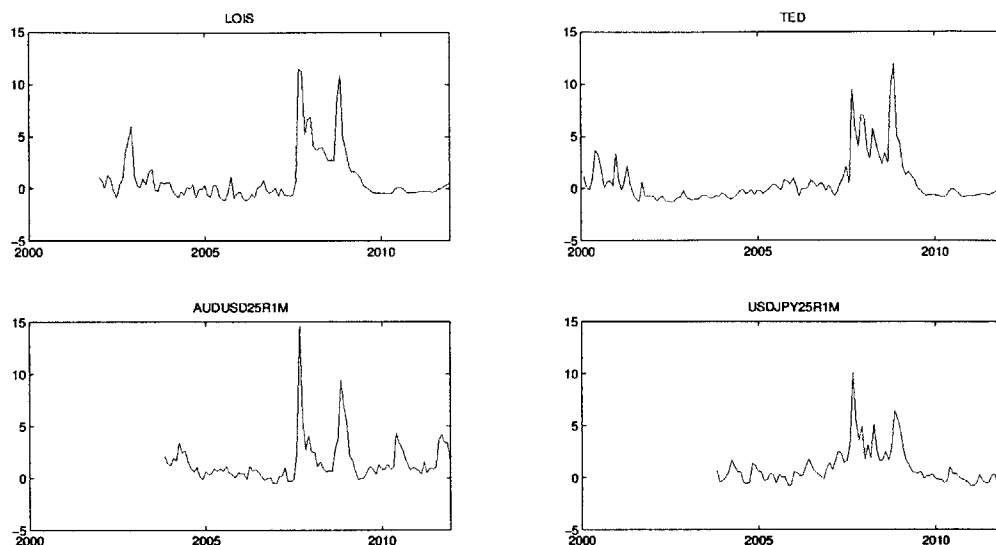
Table 2.4: Comparison of Asset Class Coverage in the Literature: Systemic Risk and Financial Condition Index (FCI).

	Systemic Risk	FCI
Hedge Fund Measures	available	
Network Analysis Measures	available	
Housing Market Measures	available	available
Quantity Variables, Sentiment Surveys		available
Macroeconomic Series	some coverage	well-developed
Correlation Measures	non-linear	linear
Exchange Rate	examined in this paper	
Comparison and Aggregation of Measures	examined in this paper	available

broader range of macroeconomic series such money supply, quantity variables (e.g. MBS, CMBS, ABCP issuance), forward looking sentiment measures (e.g. option and swaption volatility expectations), and surveys (e.g. credit availability, the assets and liabilities of commercial and “shadow” banks). The most eminent examples are perhaps the LIBOR-OIS spread and the TED spread, both of which have a proven record of picking up episodes of high systemic risk, but as far as we know the systemic risk literature has not yet established theories to explain them. To start with, we include both the LIBOR-OIS spread and the TED spread into the set of external factors that will be combined with the subset of well-performing candidate measures.

Moreover, practitioners have commented that the increasing proportion of bank’s profits generated by carry trades may be an indicator of impending systemic risk. While it is difficult to directly conduct empirical tests on that front, we do find that exchange rates can contain important information about financial distress. As shown in Fig. 2.3, we compare the risk reversals for two currency pairs, AUD/USD and USD/JPY with the LIBOR/OIS spread and TED spread, all after rolling standardization. For any currency pair, the risk reversal is the difference between the implied volatility of an OTM European call and an OTM European put with equal moneyness. When the exchange rate distribution is negatively skewed under the risk-neutral measure, investors are willing to pay more to insure against currency depreciation, and the risk reversal becomes negative. Using data from Bloomberg, we find that the two currency pairs plotted in Fig. 2.3 are picking up very similar periods of financial distress as the LOIS and TED; in fact, both risk reversals shows the most acute warning signs for the August 2007 quant crisis, which is the onset for

Figure 2.3: Time Series Plot of External Factors: LIBOR/OIS Spread, TED Spread, and Risk Reversals



the turbulence in the following two years. So far the only drawback for using risk reversals is that they become available only at later times – the two series in Figure 2.3 are available on Bloomberg since 2003. Given their performance in capturing financial distress, we also include the risk reversal series as external factors.

In terms of aggregation framework, the financial conditions literature commonly uses two approaches for constructing aggregate indicators from a collection of individual series (Table 2.5): One approach is a simple weighted average, which begs the question of how to decide the weights on individual series. We choose not to do adopt this approach, because firstly, by optimizing performance of the aggregate indicator we risk over-fitting the model; secondly, the weights should reflect the relative importance of the individual measures, which we deem as a model output rather than a model input. The second approach is to calculate the first principal component from the individual measures. The easiest way to construct a composite index is to extract the first few principal components from the panel of systemic risk measures and external factors. One obvious drawback of this approach is that the individual series start at different points in time. While measures such as the GDP stress test can span a sample period of several decades, several other measures uses corporate and sovereign credit default swap data that become available only at much later

Table 2.5: Comparison of Methodology: Aggregation Framework (Module IV)

Index	Aggregation Framework
Board Carlson, Lewis, and Nelson (2012)	Logistic regression on level, vol, and corr -> probability of distress
Chicago Fed Brave and Butters (2011)	missing value Kalman filter, the Harvey accumulator and the EM algorithm
Cleveland Fed Oet, Eiben, et al(2011)	weighted by total dollar flows into each sector of bank loans, FX credit, equity, and debt
Kansas Fed Hakkio and Keeton (2009)	first principal component
St Louis Fed Kliesen and Smith (2010)	first principal component
Bank of Canada Illing and Liu (2006)	first principal component
Hatzius, Hooper, Mishkin, Schoenholtz, and Watson (2010)	unbalanced-panel principal component (Stock and Watson (2000))

times.

We follow the recent FCI literature and use unbalanced-panel principal component analysis (see, for example Bai and Ng (2008), Stock and Watson (2006), Stock and Watson (2000), and Hatzius et al (2010)), which generalizes the standard PCA through an iterative OLS strategy that improves upon initial guesses of factors and loadings over many rounds. Below we outline this iterative approach:

Suppose  $\{X_{it}\}$  is an unbalanced panel of individual systemic risk measures (de-meaned),  $i = 1, \dots, N$  where  $N$  is the total number of indicators included in the model. The goal is to decompose  $X_{it}$  into factor loadings and estimate the common factor  $F_t$  and the loadings  $\lambda_i$ :

$$X_{it} = \lambda_i' F_t + u_{it} \quad (2.21)$$

where  $F_t$  is a  $k \times 1$  vector,  $\lambda_i$  is the  $i$ -th row of  $\Lambda$ . The factors and loadings are chosen by minimizing the objective function

$$V(F, \Lambda) = \sum_{i=1}^N \sum_{t=1}^T (X_{it} - \lambda_i' F_t)^2 \quad (2.22)$$

When the panel is balanced, the solution to the least squares problem in (2.22) reduces to simply calculating the principal components of  $X_{it}$ , i.e. the eigen-vectors of the sample covariance matrix. With unbalanced panel where some observations on  $X_{it}$  are missing, the quadratic minimization needs to be modified. The objective function (2.22) is accommodated by summing over non-missing observations. During iteration  $j$ , the elements of the estimated balanced panel are constructed as

$$\hat{X}_{it} = \begin{cases} X_{it} & \text{if } X_{it} \text{ is observed} \\ \hat{\lambda}'_i \hat{F}_t & \text{otherwise} \end{cases} \quad (2.23)$$

The estimate of  $F_t$  is then updated by computing the eigen vectors corresponding to the largest  $k$  eigenvalues of

$$\frac{1}{N} \sum_i \hat{X}_i \hat{X}_i' \quad (2.24)$$

where  $\hat{X}_i = (\hat{X}_{i1}, \hat{X}_{i2}, \dots, \hat{X}_{iT})'$ . The estimate of  $\Lambda$  is updated by the OLS regression of  $\hat{X}$  onto this updated estimate of  $F$ .

## 2.4.2 Results

In Figure 2.6 we provide the time-series plot of the composite measure with the complete set of systemic events and control group highlighted. Table 2.6 compares the type I vs type II error for the original and the composite measure. As described in the previous section, we form the composite index by choosing the best-performing individual measures and combining them with a set of external factors; each measure is said to detect a systemic event if its 95% quantile is exceeded.

As reported in the top panel of Table 2.6, the original measures are quite good at detecting systemic events. Out of twenty seven measures in total, four of them can detect Bear Stearns failure, seven can detect Lehman failure, and sixteen are able to capture Global Central Bank Intervention. The composite has improved power especially for the Quant crisis, and also gave correct warning signals for the Bear Stearns failure, the Lehman and TARP failure, as well as the Global Central Bank Intervention, but missed the European debt crisis. Another benefit of using the composite measures is that we don't have any false positives, whereas there are a number of false positives using the individual measures. The tradeoff seems to be that we get less power in certain circumstances, but the size tends to be more accurate.

Table 2.6: Compare Type I and Type II Errors of the Original and Composite Measures

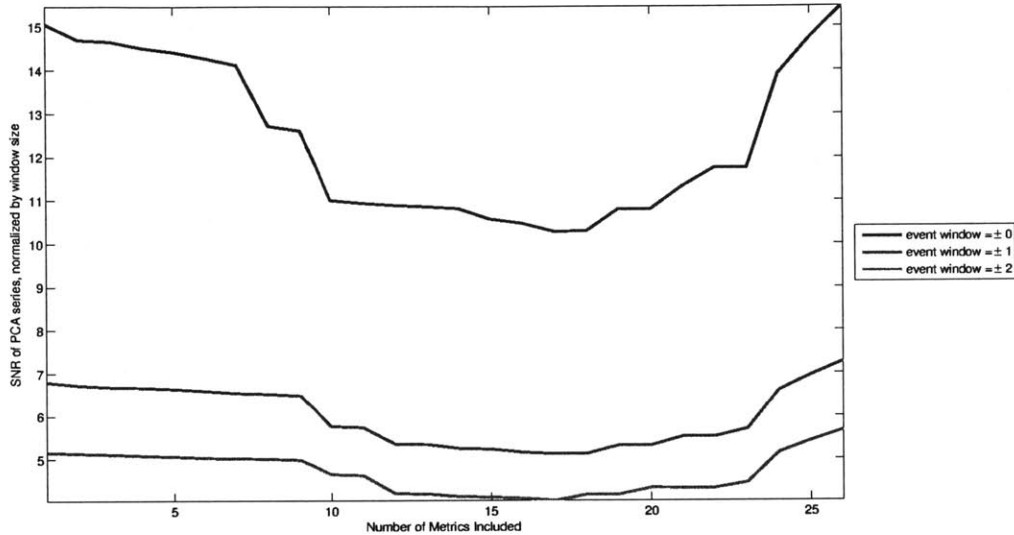
(a) Number of measures generating correct warning

Events		Original	Composite
Quant Meltdown	Aug 2007	1	TRUE
Bear Stearns failure	Mar 2008	4	TRUE
Lehman and TARP failure	Sep 2008	7	TRUE
Global central bank intervention	Oct 2008	16	TRUE
Greek debt crisis	Apr 2010	1	FALSE

(b) Number of measures generating false positives

Non-Events		Original	Composite
Tech Bubble Burning Up	Mar 2000	4	FALSE
Terrorist Attack	Sep 2001	0	FALSE
DJIA sank to lowest level in nearly four years; NASDAQ and SP500 at lowest levels since 97	Jul 2002	4	FALSE
International oil price hit a 3.5 year high	Apr 2004	1	FALSE
Crude oil price tops \$100 a barrel	Jun 2008	5	FALSE
S&P downgrades US credit rating	Aug 2011	2	FALSE

Figure 2.4: Composite Measure Signal-To-Noise Ratio with Varying Number of Original Measures Included.



Next we vary the number of original measures that are included in constructing the composite measure, and examine whether including more original measures can generate higher signal-to-noise ratio. In fact, it doesn't. As shown in Figure 2.4, the composite signal-to-noise ratio is the highest when we include only the most predictive or the most explanatory individual measures; as we add more measures in, the signal-to-noise ratio actually decreases with the number of measures included, unless the number of measures are close enough to including everything, where the resulting signal-to-noise ratio become comparable to the very best. The same observation holds true for different event windows (i.e. calculating SNR with only the current month, including  $\pm 1$  month, and including  $\pm 2$  months).

The composite measure's performance in Logit regression is reported in Table 2.13. Compared to Table 2.9 and 2.10, the composite measure outperforms the individual measure both in the contemporaneous Logit regression and the one-step-ahead Logit regression. In all contemporaneous regression with different lags, the probability of systemic event indeed increases with the composite measure, and the composite measure has positive and quite significant identification power. In the baseline case of uni-variate regression, the composite measure has  $\chi^2 = 19$  which improves upon the top individual measure performance  $\chi^2 = 9$  as shown in Table



2.9. For the one-step-ahead forecast, again the composite measure has positive and significant predictive power for all logit regressions with different lags. In the baseline case of uni-variate regression, the composite measure has  $\chi^2 = 11$  which improves upon the best individual measure performance  $\chi^2 = 6$  as shown in Table 2.10.

Finally, following the discussion in Section 2.3.2, hedge fund regime-switching measures are the only category whose forecasting performance is better than contemporaneous. To reinforce this idea, in Lo, Sgherri, and Zhou (2013) we construct another composite measure by aggregating over hedge funds only. There we show that the hedge-fund-only composite measure transitioned into high-risk regime during June~July 2007, immediately before the actual meltdown on 8/9/2007; on the recovery side, during January~February 2009, it transitioned back into the low-risk regime, immediately before the Dow Jones Industrial Average reached its bottom on 3/6/2009. This indicator also gave correct warnings for the European debt crisis and the mid-90's Latin American and Asian crises without generating any false positives.

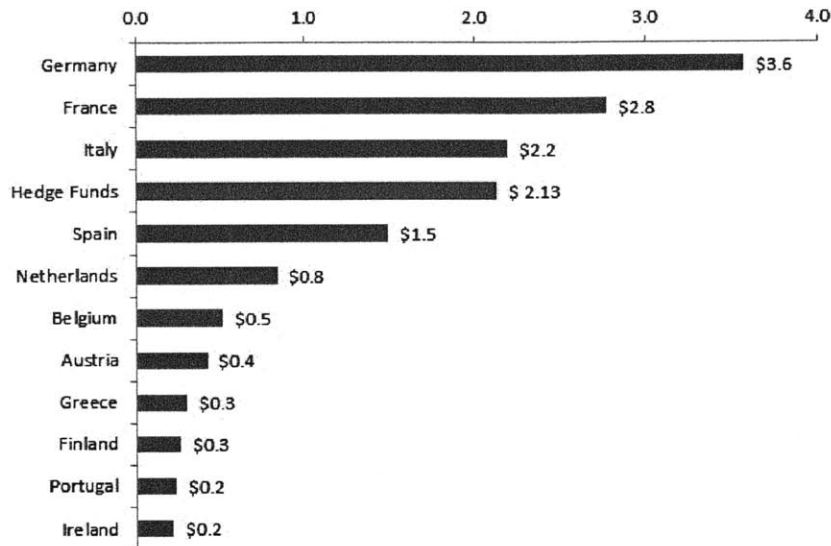
This set of results further supports our discussion in Section 2.3.2 that hedge fund measures are crucial leading indicators of systemic risk. By 2012, the hedge fund industry has more than \$2 trillion assets under management, which is comparable to the entire Italian GDP, or two thirds of Germany, seven times Greece, and ten times Ireland (see Fig. 2.5). Given their size and leading performance, it becomes evident that hedge fund measures can provide important insights unavailable from conventional asset classes. Furthermore, from a supervisory perspective, the best time to implement policy tools would be when the leading indicator is picking up early signs of financial distress but the contemporaneous has not yet. Ultimately, the classification of a collection of measures can provide additional timing information that cannot be made available from a single leading or contemporaneous measure.

### 2.4.3 Where Do We Stand Today?

Regarding the current state of systemic risk level, we turn to Figure 2.6 and examine what the composite measure tells us at the end of the sample period, namely December 2011. As shown in the graph, the composite measure reached its highest stress level in September 2008 and has quickly retreated to low stress level since the second half of 2009. By the end of 2011, the composite measure shows a low level of stress (zscore below 0).

Now that we have already arrived at the second half of 2012, with the benefit of hindsight we can compare the measure's indication versus the world's actual events.

Figure 2.5: Compare the Size of The Hedge Fund Industry with Country GDP's (\$ trillion)



Within the US, the Federal Reserve has made considerable use of forward communication tools for providing policy stimulus (Bernanke (2012)), and the series of FOMC statements published in 2011 have frequently stated that economic conditions would warrant the federal funds rates to remain exceptionally low for an extended period, at least through mid-2013. Furthermore, in August 2011 the Fed has also introduced the maturity extension program (MEP) to purchase \$400 billion of long-term Treasuries and sell an equivalent amount of shorter-term Treasuries over the period ending in June 2012. Forward guidance from the Federal Reserve reduces long-term interest rate by reducing future short-end expectations, therefore leading to more accommodative financial conditions.

Globally, however, the world economy was still suffering from clouds of uncertainties from the European countries. International investors were constantly concerned about the possibility of a disorderly Greece exit, the risks of individual bank failures, sovereign defaults, soaring yields in the core and peripheral euro-zone countries alike, as well as evaporating liquidity across the board. However, at this stage, most systemic risk models in the literature have been predominantly focused on the US, and therefore, so do our composite measure which is constructed from an ensemble of these individual measures and other US-benchmarked external factors. While this US-focused composite measure is not showing up much signs of distress by the end

of 2011, it also reminds us that it is important to develop region-specific indicators for systemically-important regions of the world, such as the Euro-zone.

## 2.5 Conclusion

To conclude, in this paper we develop a framework which allows people to start comparing individual systemic risk measures, and aggregating them into one composite measure. Among the measures currently constructed, we find that CoVaR, marginal expected shortfall, Granger causality networks, and hedge-fund regime-switching measures are the most informative for identifying systemic events. In particular, hedge-fund-based measures are the best leading indicators that we have found so far, which consistently generated early warnings for the 1998 and the 2007~2009 financial crises. We also construct a composite measure that outperforms the individual measures and has increased power of detecting systemic events. Credit spreads and currency risk reversals turn out to be empirically useful indicators of systemic risk, although the current literature has not yet studied the economic mechanisms of their impact.

In future research, we would like to incorporate more macroeconomic time series collected at different frequencies, and ultimately create a centralized platform where these measures can be computed on a regular basis. That would enable us to compare and get feedback from the public as to which ones of these measures are more useful, and provide policy guidance on a real-time basis.

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# Appendix

Table 2.7: Summary Statistics of Individual Systemic Risk Measures

	Name	Mean	Std	Min	Max
1	Granger Causality	0.33	1.37	-1.82	4.72
2	Mahalanobis Distance	0.22	1.24	-1.26	6.86
3	Absorption Ratio	0.44	1.42	-2.14	3.56
4	GDP Stress Test	0.66	1.55	-0.85	5.85
5	CoVaR	0.44	1.55	-1.46	8.80
6	MES sum	0.98	1.00	-1.05	4.66
7	MES std	0.19	1.18	-1.40	6.97
8	Smoothing: Convertible Arbitrage	0.90	1.83	-1.02	7.70
9	Smoothing: Equity Short Bias	0.48	1.76	-2.92	7.92
10	Smoothing: Emerging Markets	0.10	1.15	-1.63	2.79
11	Smoothing: Event Driven	-0.60	1.08	-4.24	2.56
12	Smoothing: Long/Short Equity	-0.15	1.44	-2.28	3.28
13	Smoothing: All Styles	0.36	1.21	-1.61	4.51
14	Hedge Fund Auto-Correlation	1.27	0.67	-0.03	2.34
15	Regime: All Styles	0.18	0.31	0.00	1.00
16	Regime: Convertible Arbitrage	0.45	0.42	0.01	1.00
17	Regime: Emerging Markets	0.17	0.31	0.00	1.00
18	Regime: Equity Market Neutral	0.02	0.13	0.00	1.00
19	Regime: Event Driven	0.18	0.33	0.00	1.00
20	Regime: Event Driven Distressed	0.09	0.24	0.00	1.00
21	Regime: Event Driven Multi-strategy	0.16	0.29	0.00	1.00
22	Regime: Event Driven Risk Arbitrage	0.32	0.30	0.03	1.00
23	Regime: Fixed Income Arbitrage	0.12	0.28	0.00	1.00
24	Regime: Global Macro	0.20	0.37	0.00	1.00
25	Regime: Long/Short Equity	0.19	0.32	0.00	1.00
26	Regime: Managed Futures	0.90	0.13	0.50	1.00
27	Regime: Multi-strategy	0.16	0.31	0.00	1.00

Table 2.8: Measures Ranked by Signal-To-Noise Ratio

	Name	Signal-To-Noise Ratio
1	MES std	30.35
2	Granger Causality	8.86
3	CoVaR	8.20
4	Regime: Event Driven Distressed	4.60
5	Regime: Multi-strategy	4.24
6	Regime: Event Driven Multi-strategy	3.96
7	Regime: Event Driven	3.60
8	Regime: Global Macro	3.32
9	Smoothing: Dedicated Short Bias	2.98
10	Regime: All Styles	2.77
11	MES sum	2.66
12	Regime: Long/Short Equity	2.62
13	Regime: Emerging Markets	2.34
14	Smoothing: Event Driven	1.85
15	Regime: Fixed Income Arbitrage	1.78
16	Absorption Ratio	1.76
17	Regime: Event Driven Risk Arbitrage	1.59
18	Regime: Convertible Arbitrage	1.57
19	Smoothing: Long/Short Equity	1.42
20	GDP Stress Test	1.08
21	Regime: Managed Futures	1.06
22	Hedge Fund Auto-Correlation	0.76
23	Mahalanobis Distance	0.28
24	Regime: Equity Market Neutral	0.09
25	Smoothing: Convertible Arbitrage	-0.29
26	Smoothing: All Styles	-1.31
27	Smoothing: Emerging Markets	-8.01



Table 2.9: Measures Ranked by Contemporaneous Logit-Regression

Name	$\beta_0$	Wald $\chi^2$	$p$ -value
1 CoVaR	0.7341	9.18	0.00
2 MES std	0.7900	9.06	0.00
3 MES sum	1.1554	8.15	0.00
4 Regime: Multi-strategy	3.1069	7.43	0.01
5 Regime: Event Driven Multi-strategy	2.8793	6.71	0.01
6 Granger Causality	0.7196	6.29	0.01
7 Regime: Event Driven	2.6823	6.07	0.01
8 Regime: Event Driven Distressed	2.4613	5.27	0.02
9 Regime: Global Macro	2.1667	4.64	0.03
10 Regime: All Styles	2.2058	3.96	0.05
11 Regime: Long/Short Equity	1.9526	3.31	0.07
12 Regime: Emerging Markets	1.5317	1.99	0.16
13 Regime: Event Driven Risk Arbitrage	1.6590	1.63	0.20
14 Regime: Convertible Arbitrage	1.4375	1.53	0.22
15 Smoothing: Dedicated Short Bias	0.2286	1.22	0.27
16 Regime: Managed Futures	4.9353	0.85	0.36
17 Regime: Fixed Income Arbitrage	0.9416	0.54	0.46
18 Absorption Ratio	0.1583	0.25	0.62
19 GDP Stress Test	0.0209	0.01	0.94
20 Smoothing: Emerging Markets	-2.0475	4.62	0.03
21 Smoothing: All Styles	-0.8540	2.55	0.11
22 Smoothing: Convertible Arbitrage	-2.0058	1.80	0.18
23 Smoothing: Event Driven	-0.4485	1.04	0.31
24 Hedge Fund Auto-Correlation	-0.7540	0.97	0.32
25 Mahalanobis Distance	-0.1232	0.09	0.77
26 Regime: Equity Market Neutral	-6.2035	0.03	0.87
27 Smoothing: Long/Short Equity	-0.0300	0.01	0.93

Table 2.10: Measures Ranked by One-Step-Ahead Logit-Regression

	Name	$\beta_1$	Wald $\chi^2$	$p$ -value
1	Regime: Multi-strategy	2.6913	5.99	0.01
2	MES std	0.4840	5.22	0.02
3	Regime: Event Driven	2.1550	4.15	0.04
4	MES sum	0.7430	3.90	0.05
5	Regime: Global Macro	1.9330	3.83	0.05
6	Regime: Event Driven Multi-strategy	1.9594	3.14	0.08
7	Regime: All	1.7709	2.49	0.11
8	Regime: Event Driven Distressed	1.7118	2.12	0.15
9	Regime: Long/Short Equity	1.4992	1.87	0.17
10	Granger Causality	0.3640	1.42	0.23
11	CoVaR	0.2393	1.30	0.25
12	Regime: Managed Futures	6.8044	1.15	0.28
13	Regime: Event Driven Risk Arbitrage	1.2135	0.84	0.36
14	Regime: Convertible Arbitrage	0.8891	0.66	0.42
15	Regime: Emerging Markets	0.9166	0.60	0.44
16	Smoothing: Emerging Markets	-2.6761	5.14	0.02
17	Smoothing: All Styles	-1.7367	4.74	0.03
18	Smoothing: Event Driven	-0.7105	2.57	0.11
19	Smoothing: Convertible Arbitrage	-2.0180	1.81	0.18
20	Hedge Fund Auto-Correlation	-1.0026	1.49	0.22
21	Absorption Ratio	-0.3184	0.80	0.37
22	Smoothing: Long/Short Equity	-0.2747	0.60	0.44
23	Regime: Equity Market Neutral	-270.4621	0.11	0.74
24	Regime: Fixed Income Arbitrage	-0.3092	0.03	0.86
25	Mahalanobis Distance	-0.0586	0.02	0.88
26	GDP Stress Test	-0.0112	0.00	0.97
27	Smoothing: Dedicated Short Bias	-0.0089	0.00	0.97

Table 2.11: Measures Ranked by  $R^2$ 

## (a) Contemporaneous Regression

Measure	$R^2$	Measure	$R^2$
1 CoVaR	31.98	15 Smoothing: Ded Sh BS	2.84
2 MES std	30.03	16 Regime: Managed Futures	3.12
3 MES sum	23.10	17 Regime: Fixed Income Arbitrage	1.25
4 Regime: Multi-strategy	20.00	18 Absorption Ratio	0.67
5 Regime: Event Driven Multi-strategy	16.82	19 GDP Stress Test	0.01
6 Granger Causality	15.35	20 Smoothing: Emg Mkts	2.99
7 Regime: Event Driven	15.92	21 Smoothing: All Styles	8.72
8 Regime: Event Driven Distressed	11.13	22 Smoothing: Cnvt Arb	4.51
9 Regime: Global Macro	12.37	23 Smoothing: Evnt Drvn	2.83
10 Regime: All Styles	9.53	24 Hedge Fund Auto-Corr	2.80
11 Regime: Long/Short Equity	7.98	25 Mahalanobis Distance	0.25
12 Regime: Emerging Markets	4.59	26 Regime: Equity Market Neutral	0.62
13 Regime: Event Drvn Risk Arb	4.10	27 Smoothing: Ln/Sh Eq	0.02
14 Regime: Conv Arb	4.51		

## (b) One-Step Ahead Regression

Measure	$R^2$	Measure	$R^2$
1 Regime: Multi-strategy	15.06	15 Regime: Emerging Markets	1.41
2 MES std	11.01	16 Smoothing: Emg Mkts	2.96
3 Regime: Event Driven	10.14	17 Smoothing: All Styles	2.23
4 MES sum	9.63	18 Smoothing: Event Drvn	6.96
5 Regime: Global Macro	9.85	19 Smoothing: Cnvt Arb	1.82
6 Regime: Event Driven Multi-strategy	7.23	20 Hedge-Fund Auto-Corr	4.74
7 Regime: All Styles	5.85	21 Absorption Ratio	2.34
8 Regime: Event Driven Distressed	4.47	22 Smoothing: Ln/Sh Eq	1.77
9 Regime: Long/Short Equity	1.77	23 Regime: Equity Market Neutral	2.32
10 Granger Causality	3.35	24 Regime: Fixed Income Arbitrage	0.08
11 Regime: Managed Futures	4.83	25 Mahalanobis Distance	0.06
12 CoVaR	2.85	26 GDP Stress Test	0.00
13 Regime: Event Driven Risk Arbitrage	2.11	27 Smoothing: Ded Sh BS	0.00
14 Regime: Convertible Arbitrage	1.82		

Table 2.12: Measures Ranked by Persistence

Measure	$\rho_1$	std
1 Regime: Multi-strategy	0.98	0.03
2 Regime: Long/Short Equity	0.97	0.02
3 Regime: All Styles	0.97	0.02
4 Regime: Emerging Markets	0.96	0.02
5 Regime: Global Macro	0.96	0.02
6 GDP Stress Test	0.96	0.02
7 Absorption Ratio	0.96	0.02
8 Smoothing: Emerging Markets	0.94	0.03
9 Hedge Fund Auto-Correlation	0.94	0.03
10 Regime: Event Driven	0.91	0.04
11 Smoothing: Dedicated Short Bias	0.89	0.04
12 Smoothing: Long/Short Equity	0.89	0.04
13 Regime: Event Driven Distressed	0.88	0.05
14 Smoothing: Event Driven	0.87	0.04
15 Regime: Convertible Arbitrage	0.85	0.05
16 Regime: Event Driven Multi-strategy	0.85	0.05
17 Granger Causality	0.84	0.05
18 Smoothing: All Styles	0.81	0.05
19 Regime: Event Driven Risk Arbitrage	0.80	0.05
20 Regime: Fixed Income Arbitrage	0.79	0.05
21 Smoothing: Convertible Arbitrage	0.74	0.06
22 CoVaR	0.73	0.06
23 Regime: Managed Futures	0.59	0.07
24 Mahalanobis Distance	0.59	0.07
25 MES sum	0.50	0.07
26 MES std	0.36	0.08
27 Regime: Equity Market Neutral	-0.03	0.09

Table 2.13: Composite Measure Performance

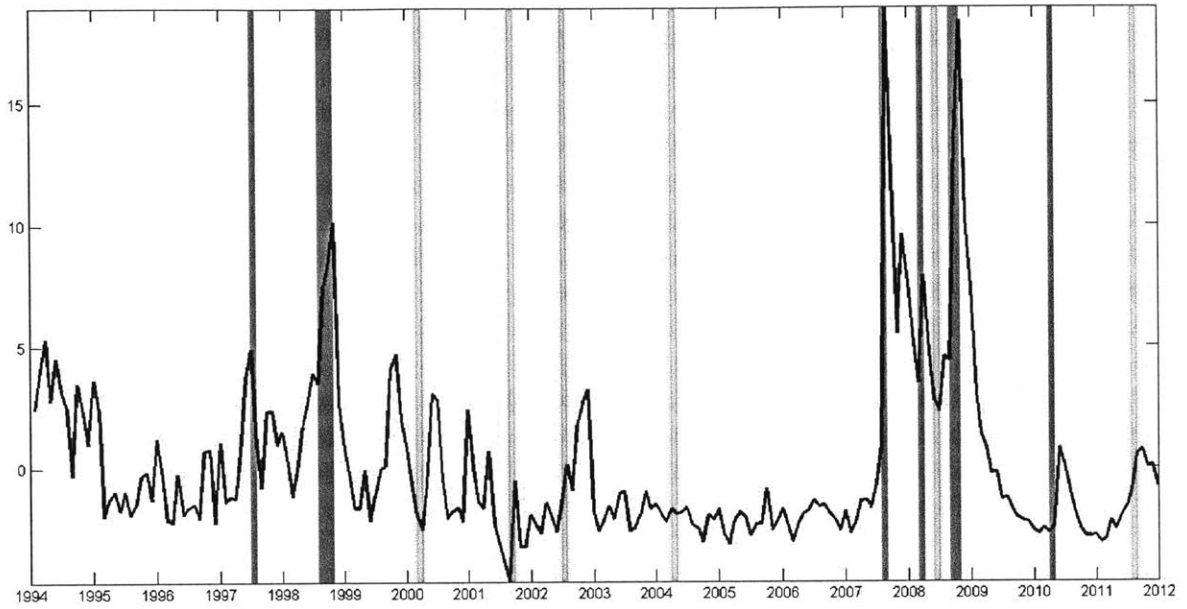
(a) Contemporaneous Logit Regression

Number of Lags	$\beta_0$		$\chi^2$	<i>p</i> - value
0	0.42	(0.10)	19.07	0.000
1	0.67	(0.19)	11.90	0.001
2	0.69	(0.21)	10.46	0.001
3	0.81	(0.25)	10.69	0.001
4	0.81	(0.25)	10.88	0.001
5	0.82	(0.25)	10.87	0.001
6	0.81	(0.25)	10.18	0.001

(b) One-Step-Ahead Logit Regression

Number of Lags	$\beta_1$		$\chi^2$	<i>p</i> - value
1	0.21	(0.06)	11.18	0.001
2	0.20	(0.09)	5.67	0.017
3	0.21	(0.09)	5.76	0.016
4	0.20	(0.09)	5.69	0.017
5	0.20	(0.09)	5.57	0.018
6	0.20	(0.09)	5.60	0.018

Figure 2.6: Composite Measure Time-Series Plot



## Chapter 3

# Monitoring Systemic Risk in Financial Markets

### 3.1 Introduction

The International Monetary Fund's three fundamental missions are surveillance, lending, and technical assistance. The key process known as "surveillance" involves regular monitoring of member countries' financial and economic policies, as well as identifying economic and financial weaknesses at the country (annual Article IV consultations with individual member countries), regional (policy examinations under currency unions), and global (executive board reviews of global economic trends and developments) levels. In the 2011 Triennial Surveillance Review (TSR) statement, Managing Director Madame Christine Lagarde emphasized that "Given the potential and speed with which developments in the financial sector can ignite and propagate crises, ensuring effective financial sector surveillance is in the interest of the entire membership. We all agree that financial stability analysis should be better integrated into surveillance; the issue is how to go about this systematically?"

In this project we apply recent analytical tools developed in the systemic risk literature in an effort to assist with the Fund's ongoing policy work for monitoring systemic risk in financial markets, especially on the multilateral surveillance front. So far the academic literature has focused on developing systemic risk measures for various market sectors, and empirical studies have been conducted predominantly on US data. Our project takes a different approach by exploring the geographic dimension and constructing region-specific risk indicators. This perspective naturally aligns with the International Monetary Fund's role of providing policy recommendations to

member countries.

In the past, policy regulators including the International Monetary Fund have mostly focused on “core data”, that is, macroeconomic, international trade, and banking statistics. Traditionally, bank runs and the shortage of credit have been viewed as the classical channels of systemic risk transmission. Yet given the growing size of the hedge fund industry as well as their potential impact on financial institutions and markets in general, it seems that we need to shift focus outside the realm of traditional core data, and explore alternative asset classes to see if they can provide additional insights for multilateral surveillance.

The analytical framework here is based upon regimes of hedge fund returns and two characteristics of hedge fund investment: geographic focus and investment style. We use the entire Lipper/TASS hedge fund database including both live funds and graveyard funds, up to December of 2011. Along the regional dimension, we first create a regional hedge fund return index, which is the individual returns weighted by assets-under-management for all funds that have geographic focus in this region; the return index is then fit into a two-state Markov regime-switching model. The region’s systemic risk is measured as the probability of being in the “high-risk” state, and then we estimate the common factor behind all regional risk indicators to construct the global geographic-focus systemic risk indicator. Similarly, along the hedge-fund investment style dimension, we first construct style-specific return indexes and estimate the regime probability of the high-risk state per style; then construct the global investment-style systemic risk indicator. Furthermore, we examine the performance of the two global indicators over the course of major financial crises from the mid-90’s up to the 2011 European debt crisis. We find that the investment-style global indicator consistently leads the geographic-focus global indicator for about one to two months. Finally, we track regional risk spillovers by constructing the Granger causality network of individual indicators.

The rest of the paper proceeds as follows. Section 2 briefly reviews two strands of closely related literature, one on systemic risk measurement and regulation in the hedge fund sector, the other on the application of network analysis in systemic risk models; Section 3 describes the data and econometric models in this project; Section 4 reports the empirical results, describes the two panels of individual indicators along the geographic-focus dimension and investment-style dimension, compares the lead-lag performance of the two global indicators, as well as demonstrates the interconnectedness between regional indicators; Section 5 further discusses policy



implications and data requirements from this project; Section 6 concludes.

## 3.2 Literature Review

In this section we review two strands of literature that are closely related to the context of this paper, one on systemic risk in the hedge fund sector, the other on applying network analysis models to systemic risk analytics<sup>1</sup>.

### 3.2.1 Hedge Fund and Systemic Risk

Firstly, this current paper is related to Lo and Zhou (2012) where the authors conduct an empirical comparison on a collection of systemic risk measures. Their paper include 27 measures spanned over 9 categories including Granger causality network, Mahalanobis distance, absorption ratio, GDP stress test, Co-VaR, marginal expected shortfall, hedge-fund return smoothing, hedge fund illiquidity proxied by return autocorrelation, and hedge fund regime-switching model. Empirical results indicate that Granger causality networks and hedge fund regime-switching measures are among the top performers. A natural follow-up question is how to aggregate information from the well-performing measures. While Lo and Zhou (2012) uses a statistical approach and construct the composite measure by estimating the common factor for the subset of well-performing measures, in this paper we take a different perspective by nesting multiple models.

The hedge-fund regime-switching measure was developed in Chan, Getmansky, Haas, and Lo (2006), and originally applied to the CSFB/Tremont hedge fund return indexes on different investment style categories. Here we extend Chan, Getmansky, Haas, and Lo (2006) by clustering across the second dimension of geographic focus; also, we construct style-level AUM-weighted indexes by manually aggregating over all funds in the Lipper/TASS database, which allows for a much broader coverage of fund-level information. With the CSFB/Tremont indexes, the component funds first need to meet certain eligibility criterion such as “no investment lock-up period”, and “accepting new investments and redemptions” which potentially excludes many major players; secondly, for each representative strategy, the index composition is subject to a minimum (where available) of 10 funds and a maximum of 25 funds.

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<sup>1</sup>This section is partially based on the literature summary of networks and systemic risk that the author has written for the Macro Financial Modeling (MFM) group.

The Granger-causality measure was proposed in Billio, Getmansky, Lo, and Pelizzon (2012) where the authors develop multiple measures of systemic risk based on interconnections among the market returns of four types of institutions including hedge funds, banks, brokers, and insurance companies. The authors conduct pairwise Granger causality tests both among the four sector indexes and among the 100 largest institutions within the four sectors. Empirical results shows that just prior to crisis periods, all four sectors have become highly interrelated and less liquid, increasing the level of systemic risk in the finance and insurance industries; banks seem to have more significant impact on the other sectors than vice versa, and this asymmetry became highly significant just before the 2007~2009 financial crisis. In this current project we generalize the Granger causality approach from inter-institutional network to inter-regional regional network, which allows us to identify systemically important regions over time and also track the spillover of systemic risks.

The policy discussion of examining hedge-fund systemic regulation has already started before the recent financial crisis. For example, Kambhu, Schuermann, and Stiroh (2007) have stressed the importance for policy makers to extend their attention beyond the classic channels of systemic distress such as bank runs and credit shortages, and shift their focus more towards hedge funds. The authors point out that the unique features of hedge funds such as high leverage, opacity to outsiders, and convex compensation structure may generate intrinsic difficulties that exacerbate various market failures including agency problems, externalities, and moral hazard. The current practice of counterparty credit risk management (CCRM) has gone through many improvements, such as enhanced risk management techniques by counterparties, improved supervision, more effective disclosure and transparency, strengthened financial infrastructures, and more efficient hedging and risk distribution techniques. By examining the recent enhancements, the authors conclude that CCRM is still the appropriate starting point for limiting the potential for hedge funds to generate systemic disruptions.

### **3.2.2 Network Analysis and Systemic Risk**

The modern process of financial innovation has resulted in financial products and financial institutions that are increasingly complex and interconnected. Since the recent financial crisis, academic and policy researchers have established high priority for understanding the relationship between network structure and systemic risk. As a result, a new and growing literature have been developed on applying network

analysis models to the study of systemic risk, including theoretical, quantitative and simulation studies.

While in this paper our focus is on statistical network of regional risks, going forward each of the literature outlined below points to an interesting new direction for extending the current work. For example, physical linkages and statistical linkages may provide complementary insights: during episodes of financial distress, entities may become more interconnected in the *return* space, but less interconnected in the *position* space as institutions collectively suspend trading activities with a commonly regarded high-risk counterparty. After all, the work of safeguarding our financial system will depend on the efforts to understand systemic risk, and these research works are critical for empowering policy makers to make the right decisions.

Gai, Haldane, and Kapadia (2010) focus on the collapse of the interbank market and explore how complexity and concentration of financial linkages can give rise to systemic liquidity crises. Firstly, the authors set up a stylized network model of interbank claims to study the effect of funding liquidity shocks, and demonstrate analytically how the tipping points depend on the level of liquid asset holdings, the amount of interbank activity, as well as the size of haircuts on banks' assets. Secondly, they conduct numerical simulations to illustrate how greater complexity and concentration in the financial network may amplify the banking system's fragility, under six experimental settings: 1) A stylized systemic liquidity crisis where a random adverse haircut shock is applied to a single bank in an un-concentrated network; 2) Adding aggregate haircut shocks; 3) Systemic liquidity crises in a concentrated network; 4) the impact of targeted shocks in concentrated and less concentrated networks; 5) The impact of greater complexity; 6) Cyclicity in haircuts and the likelihood of systemic liquidity crises. Lastly, they demonstrate that the financial system could be made more resilient by policy measures, including tougher liquidity requirements, systemic liquidity requirements, haircut-dependent liquidity requirements, and greater network transparency.

Cont, Moussa, and Santos (2012) compare the role of balance sheet size versus network structure in the systemic importance of institutions by introducing and implementing the Contagion Index. This indicator is defined as the expected loss to a network triggered by the default of an institution under a macroeconomic stress scenario. Using the Contagion Index, the authors analyze a dataset of 2400 financial institutions chartered by the Brazilian Central Bank, which includes mutual exposures and capital levels reported at six quarters between 2007 and 2008. They find

that the Brazilian interbank network is highly heterogeneous in terms of counterparty distribution and exposure sizes; systemic risk is concentrated only on a small subset of financial institutions; network structure does matter in assessing systemic importance, using balance sheet size alone is insufficient; the compound effect of correlated market shocks and contagion can increase the proportion of contagious exposure in the network. Their policy recommendations are: 1) targeting the most contagious institutions is more effective in reducing systemic risk than increasing capital ratios uniformly across all institutions; and 2) capital requirements should not simply focus on the aggregate size of the balance sheet but depend on their concentration / distribution across counterparties: a minimal capital-to-exposure ratio can be more effective way of controlling contagious exposures.

Cohen-Cole, Kirilenko, and Patacchini (2012) propose a novel measure of systemic risk which is able to capture the precise cascade of behavior in agents. Following the social interactions literature, the authors measure systemic risk as the average impact of a shock that causes strategic reactions among interconnected market participants. This new measure captures two important features: 1) It is derived using all the direct and indirect connections in the entire network; 2) It suggests that large impacts can occur in the absence of defaults, with the flash crash of May 6, 2010 being a key example. The authors provide an application of this approach to a dataset from the Chicago Mercantile Exchange Dow futures market, which consists of 1,163,274 transactions between approximately 7335 trading accounts, and estimate the proposed systemic risk measure from transaction networks. They illustrate how correlated trading strategies can lead to correlated returns and how systemic risk is propagated through the network. Lastly, the authors point out that in order to assess financial stability policies, bankers needs to establish a clearly-defined objective function and a structural view of the economy. In parallel to the monetary policy literature, the current systemic risk literature only offers some descriptive insights without a systematic approach to evaluate policy. The authors call for welfare analysis that are specific to the actual network structure and reflect the incentives present in the market.

Yellen (2013) discusses the difficult task before policy makers and regulators with systemic risk responsibilities (such as the Federal Reserve) to balance the benefits of interconnectedness (growth and stability, risk sharing) while managing the potentially harmful side effects (amplify market frictions, information asymmetries, or other externalities). In response to the financial crisis, governments around the

global are adopting a multifaceted and coordinated reform agenda, such as the Basel Committee initiatives. Standardized OTC derivatives are now required to be cleared through central counterparties which can yield important advantages over a fully bilateral market structure. Some of the most significant policy tradeoffs arise in regulating the less standard OTC contracts which will continue on a bilateral basis. The proposed framework now requires collecting not only the variation margin but also the initial margin. However, higher initial margin requirements will make it more costly for market participants to use derivatives to hedge risk, which results in liquidity costs.

### 3.3 Methodology

#### 3.3.1 Econometric Model

First of all, for each region of geographic focus, we construct a regional return index  $R_t$  which is the AUM-weighted returns of all hedge funds with geographic focus in this particular region. The index return time series is then fed into a two-state Markov Switching model:

$$R_t = I_{it} \cdot R_{1t} + (1 - I_t) \cdot R_{2t}, R_{it} \sim \mathcal{N}(\mu_i, \sigma_i^2) \quad (3.1)$$

where

$$I_t = \begin{cases} 1 & \text{with probability } p_{11} \text{ if } I_{t-1} = 1 \\ 1 & \text{with probability } p_{21} \text{ if } I_{t-1} = 0 \\ 0 & \text{with probability } p_{12} \text{ if } I_{t-1} = 1 \\ 0 & \text{with probability } p_{22} \text{ if } I_{t-1} = 0 \end{cases}$$

Regional systemic risk is measured as the probability of the distressed regime. To serve the purpose of multilateral surveillance, it would be useful to extract the common driver of regional risks and construct a global indicator. Here the standard principal component analysis is insufficient, because data for each geographic region start at different times. To address this issue, we use unbalanced-panel principal component analysis that estimates the common factor through an iterative process. Along the same lines, we also cluster hedge funds by their investment style and extract the common driver across all styles.

The first principal components are again fit into a two-state Markov regime-

switching model, and global systemic risk is measured by the probability of the high-risk state. Subsequently, the two global indicators are referred to as the “geographic-focus indicator” and the “hedge-fund style-category indicator”.

To study the inter-regional risk spill-overs, we construct the Granger causality network among the set of regional indicators:

$$X_t = \sum_{j=1}^m a_j X_{t-j} + \sum_{j=1}^m b_j Y_{t-j} + \epsilon_t \quad (3.2)$$

where  $Y_t$  is the systemic risk time series of the “cause” region, and  $X_t$  is the systemic risk time series of the “effect” region. Region  $Y_t$  is said to Granger-cause the systemic risk of region  $X_t$  if the inclusion of  $Y_t$  improves the predictability of  $X_t$  in (3.2) (see, for example, Hamilton (1994) ). This methodology follows from Billio, Getmansky, Lo, and Pelizzon (2012); the difference is that the network linkages constructed here are uni-directional in order to demonstrate whether a particular region is the cause or the effect node in the Granger-causality relation.

### 3.3.2 Data Description

Our hedge fund data is taken from the Lipper/TASS hedge fund database, which includes monthly returns, asset-under-management, investment style, geographic focus, and other fund-level characteristics. The sample periods starts from early 1990s and ends on Dec 2011; in order to avoid the survivor-ship bias, we include both the live funds and the graveyard funds.

In Lipper TASS, AUM’s are reported in the following currencies: Australian Dollar (AUD), Brazilian Real (BRL) Canadian Dollar (CAD), Swiss Franc (CHF), Chinese Yuan (CNY), Czech Koruna (CZK), Deutsche Mark (DEM), Denmark Krone (DKK), Euro(EUR), French Franc (FRF), United Kingdom Pound(GBP), Hong Kong Dollar (HKD), Israel Shekel (ILS), Japanese Yen (JPY), Malaysia Ringgit (MYR), Dutch Guilders (NLG), Norway Krone (NOK), New Zealand Dollar (NZD), Poland Zloty (PLN), Swedish Krona (SEK), Singapore Dollar (SGD), US Dollar (USD), South African Rand (ZAR). The foreign-currency-denominated AUM is then converted into dollar AUM, using the average monthly exchange rate from CRSP. We exclude the observations where asset-under-management is missing since our following analysis will be constructed upon AUM-weighted index returns.

The summary statistics for monthly exchange rates are reported in Table Ta-

ble 3.1. Among the sample observations, asset-under-management is most commonly denominated in the US Dollar, Euro, Brazilian Real, British Pound, and the Swiss Franc. The sample includes three Euro legacy currencies, including the Dutch Guilder (NLG), the French Franc (FRF), and the Deutsche Mark (DEM). After normalizing with the mean level, the most volatile currency is the Brazilian Real (BRL) and the least volatile currency is the Israel Shekel (ILS).

Fund-level returns will be clustered across two dimensions: geographic focus, and investment styles. Our sample include the following regions of geographic focus: United States, United Kingdom, Japan, India, Russia, Asia Pacific, Asia Pacific Excluding Japan, Eastern Europe, Western Europe, Western Europe Excluding UK, North America, North America Excluding US, and Latin America. And the investment-style categories in our sample are: Fund of Funds, Managed Futures, Event Driven, Long Short Equity Hedge, Emerging Markets, Global Macro, Multi Strategy, Convertible Arbitrage, Equity Market Neutral, Dedicated Short Bias, Fixed Income Arbitrage, and Options Strategy.

## 3.4 Empirical Results

In this section we report the performance of the two global risk indicators that have been created, which provide contemporaneous and early-warning characterizations of the 2007 US quant crisis, the 2011 European debt crisis, as well as the mid-90's Latin American and Asian financial crises.

### 3.4.1 Two Global Indicators

In Table 3.2 and Table 3.3 we report the summary statistics for the two sets of index returns that were constructed from clustering fund-level returns along the geographic focus dimension and the investment style dimension, respectively. Indeed, there is quite considerable heterogeneity: across geographic regions, the average monthly return ranges from 0.71% for Japan to 1.86% for UK, the volatility ranges from 1.49% for Western Europe to 9.83% for UK, and the regions with the largest skewness (US, UK, Russia) also report the largest kurtosis; across investment styles, the average monthly return ranges from 0.27% for dedicated short bias to 1.88% for long/short equity, and the styles with the largest skewness (Fund of Funds, Fixed Income Arbitrage, and Convertible Arbitrage) also report the largest kurtosis. Besides, all regions

and all investment styles report positive first-order auto-correlation except dedicated short bias.

After feeding into the Markov regime switching model (3.1), the individual systemic risk indicators are summarized in Table 3.4 and Table 3.5. Heterogeneity across geographic regions is significant. For example, in Figure 3.1 we compare the time-series plots as a quick snap-shot of regional risk variations, the 1998 sovereign default caused more stress for Russia than the 2007 subprime crisis (Fig.3.1). Across different investment styles, Emerging Markets has lowest mean of 9.48% and options strategy has the highest mean of 45.06%, volatility has much less variation with global macro reporting the lowest volatility of 15.38% and convertible arbitrage reporting the highest volatility of 33.53%.

To construct the global geographic focus (investment style) indicator, we compute the unbalanced-panel first principal component of the cross-section of all regional (investment styles) indicators, as shown in the top panel of Figure 3.2. The two principal components are again fed in to the two-state Markov regime switching model (3.1) and the resulting time series of stress state probabilities are shown in the bottom panel of Figure 3.2. As illustrated in the graph, the global hedge fund investment style indicator (blue dashed line) leads the global geographic focus indicator (black solid line) during all episodes of high systemic stress.

To better compare the contemporaneous and early-warning performance of the global indicators, we show the heat map for four episodes of major financial crisis (Figure 3.3).

In August 2007, a number of quantitative long/short equity hedge funds suffered unprecedented losses, and the LIBOR-OIS spread crept up three-folds in one day from 13.4 bps to 39.95 bps on August 9th, 2007. This episode, commonly known as the “quant crisis”, marked the start of serial global deteriorations in the two years that followed. Our geographic-focus global indicator indeed entered into high-risk state by in August 2007; more importantly, the hedge fund style-category indicator is able to generate warning signals as early as June 2007 with 2-month lead. In the latter half of 2008, market participants started to ponder whether the sub-prime crisis was nearing the end. Both our global indicators showed that the financial markets were exiting from the high-risk state during January ~ February 2009, with the hedge-fund style-category indicator showing a steeper decrease in risk levels. Both indicators switched to low-risk state from March 2009 to early 2011. Indeed, the Dow reached its low of 6626.94 on March 6th, 2009 and has been rebounding since



then (Figure 3.5), while the real economy recovery started in mid-2009, as dated by the NBER and market commentaries.

For the first half of 2011, the hedge-fund style-category indicator again led the geographic-focus indicator in picking up rising levels of systemic risk, and the probability of high-risk state increased in Apr and May of 2011. Within the next two months, Greek, Portugal, and Ireland were downgraded, and the Berlusconi government started to adopt an austerity package for Italy. The geographic-focus global indicator showed that the risk level continued to intensify from May 2011 to August 2011 when the US was downgraded, and we have remained in the high-risk state until December 2011, which is the end of our sample period.

Regarding the mid-1990's crises: During 1993~1994, Turkey and Latin America were hit by economic downturns and currency speculative attacks. Again the hedge-fund style-category indicator led the geographic-focus global indicator by about one month both at the entry-side and the exit-side. Figure 3.6 shows that the Mexican peso / USD exchange rate plunged by 40% from 0.2886 to 0.1754 in the week of December 19th ~27th, 1994, and then remained relatively stable for the months after; our hedge-fund style-category indicator recovered in January~February 1995, whereas the geographic-focus indicator returned to low-risk state in March~April 1995.

During 1997~1999, the geographic indicator reported a wider window of high-risk state, compared to the hedge-fund indicator. In retrospect, the 1997~1999 period spanned over multiple crises. The geographic-focus indicator entered into high-risk-state as early as Jul 1997, when the Thai Baht and Malaysian Ringgit went under speculative attacks; in the following months, financial stress intensified (for example, it spilled over and triggered the Korean Kwon crisis in November 1997) but the Thai-crisis still remained as a localized event within a group of southeastern Asian countries. In contrast, the Russian sovereign default in Aug 1998 accentuated losses at major US hedge funds such as LTCM and eventually led to Fed bailout. Our hedge-fund style-category indicator started to show signs of distress in Apr~May 1998, in anticipation of the impending downward spirals that hit LTCM as well as the rest of the market. On the exit-side, the hedge-fund style-category indicator retreated to low-risk state in November 1998, two months after the September 1998 Fed bailout, leading the geographic-focus global indicator by 1~2 months.

### 3.4.2 Interconnectedness of Regional Risks

In this subsection we use network analysis to visualize regional interconnections and track the spill-over of systemic risks over time. To start with, we test for pairwise Granger causality relations between regional systemic risk indicators. On the network graph, two regions are linked if there is a significant Granger causality relation between their systemic risk indicators, whereas the direction of causality is shown by placing the cause region on the left, and the effect region on the right. Moreover, generalizing from the concept of SIFI (systemically important financial institution), we can also identify *systemically important regions* that are major hubs of cause or effect links on the Granger causality network.

The two panels in Figure 3.7 compares the Granger causality networks over the 1996~1998 period versus the 1997~1999 period. In the earlier period, Asia Pacific was causing most of the stress for the rest of the world. In particular, Japan's economy suffered from immediate spillovers because 40% of its export went to Asia – the network graph shows that Japan's systemic risk was indeed Granger-caused by Asia Pacific (data hadn't yet become available for Asia Pacific Excluding Japan during this period). A year later, the linkages looked much different: after its sovereign default, Russia had emerged as an important cause of financial distress for many other regions; Japan's role in causing other regional risks also intensified; as the crisis deepened, the total number of linkages had increased, indicating that regional risks had become much more intertwined and calling for closer monitoring from the international community.

Ten years later, over the two year period of 2006~2008 that includes the US subprime meltdown, we report the updated Granger causality network in Figure 3.8. One would have expected the US to emerge as a "systemically important region" and stand out as the main *cause* country of systemic risks for other parts of the world. However, what we observe from the data is that the regions have become so highly interconnected to the extent that it appears almost impossible to identify any single region as the sole source of global distress. Given the speed of risk spillovers in today's market, ideally we would conduct the same set of analysis on higher frequency data and over a narrower window (for example, daily returns or daily transactions over the course of July~August 2007).

Finally, in the post-Lehman period, systemic risk in the US began to gradually alleviate while the European debt crisis were just starting to unfold. As can be observed from Figure 3.9, Western Europe emerged as a main *cause* region of systemic

risk. Moreover, while the 2007~2009 crisis started in the advanced economies, its impact on emerging economies was also deep and profound. The impact on Russia has been accentuated by its structural vulnerabilities: dependence on the oil and gas sector, a narrow industrial gas and limited small and medium-size enterprise sector (World Bank (2008), World Bank (2009)). In late 2008, driven by the sharp decline of demand from advanced economies, the price of crude oil plummeted by 77% from peak \$145 / barrel to trough \$33 / barrel, the Ruble lost more than 36% in value from \$0.0432 to \$0.0275, and the MICEX (Moscow Interbank Exchange) equity index shed more than 73% from peak 1956 points to trough 513 points (Figure 3.10). Indeed, the systemic importance of Russia was evident in Figure 3.9 both on the *cause* side and on the *effect* side.

### 3.5 Policy Implication

Eight years after the LTCM debacle and one year prior to the US subprime crisis, hedge fund systemic risk has already drawn attention from financial stability regulators. On May 16, 2006, Chairman Bernanke gave the “Hedge Funds and Systemic Risk” speech at the Federal Reserve Bank of Atlanta:

“The collapse of Long-Term Capital Management in 1998 precipitated the first in-depth assessment by policy makers of the potential systemic risks posed by the burgeoning hedge fund industry.

“The debate about hedge funds and the broader effects of their activities on financial markets ... has now resumed with vigor – spurred, no doubt, by the creation of many new funds, large reported inflows to funds, and a broadening investor base.

“Concerns about hedge fund opacity and possible liquidity risk have motivated a range of proposals for regulatory authorities to create and maintain a database of hedge fund positions ... a system in which hedge funds submit position information to an authority that aggregates that information and reveals it to the market.”

From the implementation perspective, collecting hedge fund position information on a regular basis can be quite difficult. Moreover, it also raises other concerns such as data confidentiality and how the authority plans to use this information. In comparison, the data requirement for the two global indicators in this paper are

fairly high-level: we have only used monthly returns and descriptive characteristics such as geographic focus and investment style; yet we still observe a fairly consistent lead-lag relation between the contemporaneous and leading indicators, and do not generate any false positive or false negatives.

Along the same lines, after the 2007~2009 crisis the Office of Financial Research has been established with key responsibilities including conducting research on systemic risk measures, as well as identifying actionable data items that serves this mission. Again, the empirical results in this paper shows that we can develop quite useful systemic risk measures from a non-intrusive data collection process.

In the same speech, the Chairman also pointed out that:

“[The President’s Working Group on Financial Markets] focused on the potential for leverage to create systemic risk in financial markets.

“The concern arises because, all else being equal, highly leveraged investors are more vulnerable to market shocks. If leveraged investors default while holding positions that are large relative to the markets in which they have invested, the forced liquidation of those positions, possibly at fire-sale prices, could cause heavy losses to counterparties.

“These direct losses are of concern, of course, particularly if they lead to further defaults or threaten systemically important institutions; but in addition, market participants that were not creditors or counterparties of the defaulting firm might be affected indirectly through asset price adjustments, liquidity strains, and increased market uncertainty.”

Although this view was addressed toward the LTCM event, the critical role of leverage was also evident in the 1929 Great depression (Yellen (2013)) as well as the 2007~2009 crisis. Many policy analysts have called for strengthened capital requirements, extension of those requirements to investment banks, using short-term debt funding to replace long-term illiquid funding, as well as pro-cyclical capital requirements (Bullard, Neely, and Wheelock (2009)).

### 3.6 Conclusion

In this project we look into empirical methodologies to measure regional system risk and integrate into multilateral surveillance toolkit. Results show that data from

alternative asset classes such as hedge funds can be quite helpful for IMF's surveillance mission. More specifically, we develop three categories of products: Firstly, we provide a cross-section of regional risk indicators, which provides a snapshot of risk distribution. Secondly, we develop two global risk indicators, one is the based on geographic region clusters, which turns out to be a contemporaneous indicator, the other is based on investment style clusters, which turns out to be a leading indicator. Thirdly, we visualize the spill-over of risks by network analysis, and demonstrate the time-variation of interconnectedness.

Going forward, network analysis is likely to be a promising channel for new systemic risk models. This type of research would be very helpful for financial stability analysts to identify "systemically important regions". By construction, the type of network used in this project can be generalized to accommodate other types of analysis at higher resolutions. One possible extension is to study the physical link network, which has decreasing degrees of interconnectedness during financial crises. To identify the source of structural fragilities, it would be interesting to compare the networks constructed among the same set of entities but over different types of linkages.

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## Appendix

Table 3.1: Summary Statistics of Monthly Exchange Rates

Currency	No. Obs.	Start Date	End Date	min	max	mean	stdev	$\frac{stdev}{mean}$	skewness
AUD	4972	Nov-1992	Dec-2011	0.486	1.364	0.834	0.200	0.240	0.665
BRL	26750	Sep-2001	Dec-2011	0.267	2000	48.67	237.97	4.890	6.370
CAD	5295	Jan-1992	Dec-2011	0.624	1.061	0.814	0.105	0.129	0.290
CHF	11821	Mar-1992	Dec-2011	0.350	1.273	0.672	0.174	0.259	0.401
CNY	85	Jan-2008	Dec-2011	0.115	0.629	0.210	0.132	0.627	1.621
CZK	41	Sep-2009	Dec-2011	0.024	0.066	0.040	0.010	0.263	0.590
DEM	348	Sep-1992	Jan-1999	0.299	0.807	0.539	0.106	0.196	-0.090
DKK	416	Mar-1998	Dec-2011	0.113	0.212	0.161	0.026	0.160	-0.191
EUR	84214	Jan-1999	Dec-2011	0.845	1.579	1.206	0.198	0.164	-0.253
FRF	539	Jan-1991	Jun-1999	0.098	0.245	0.165	0.029	0.173	0.161
GBP	19807	Jan-1990	Dec-2011	1.079	2.437	1.709	0.242	0.141	0.753
HKD	196	Feb-2004	Dec-2011	0.123	0.204	0.134	0.016	0.121	3.146
ILS	12	Jun-2008	Jun-2009	0.204	0.309	0.245	0.024	0.097	0.453
JPY	6119	May-1994	Dec-2011	0.003	0.013	0.007	0.002	0.336	-0.069
MYR	58	Jun-2008	Dec-2011	0.230	0.471	0.356	0.067	0.188	-0.246
NLG	33	Jun-1995	Sep-1998	0.265	0.652	0.466	0.084	0.180	-0.105
NOK	882	Nov-2003	Dec-2011	0.104	0.211	0.155	0.025	0.159	0.191
NZD	101	Sep-2004	Dec-2011	0.394	1.349	0.686	0.186	0.271	1.064
PLN	183	Jun-2008	Oct-2011	0.215	0.569	0.324	0.073	0.224	0.751
SEK	4577	Apr-1995	Dec-2011	0.092	0.257	0.155	0.040	0.257	0.935
SGD	170	May-2007	Dec-2011	0.530	0.831	0.636	0.068	0.106	0.700
USD	480968	Dec-1975	Dec-2011	1.000	1.000	1.000	0.000	0.000	-
ZAR	230	Apr-2003	Dec-2011	0.084	1.341	0.359	0.306	0.851	1.692



Table 3.2: Summary Statistics of Hedge Fund Returns Clustered by Regions of Geographic Focus

Geographic focus	min	max	mean	median	stdev	$\frac{stdev}{mean}$
AsiaPacific	-6.435	7.735	0.967	1.073	1.718	1.776
AsiaPacificExcludingJapan	-12.984	11.832	0.760	0.911	2.655	3.493
Africa	-10.888	12.927	0.984	1.152	2.125	2.160
EasternEurope	-12.684	14.406	1.019	1.118	2.324	2.281
Global	-13.493	19.033	1.409	0.863	3.672	2.605
India	-14.247	16.660	0.800	0.844	3.639	4.548
Japan	-6.153	8.149	0.711	0.865	1.707	2.400
LatinAmerica	-13.170	13.629	1.113	1.136	2.374	2.133
NorthAmerica	-8.285	9.090	1.072	1.158	1.549	1.445
NorthAmericaExcludingUSA	-11.261	15.524	0.881	0.633	2.968	3.369
Other	-25.185	11.306	1.050	1.017	2.955	2.816
Russia	-27.417	50.521	1.686	1.242	7.863	4.664
UK	-37.070	72.050	1.863	0.825	9.828	5.275
US	-9.024	29.840	1.712	1.180	3.524	2.058
WesternEurope	-5.674	9.082	1.039	1.037	1.490	1.434
WesternEuropeExcludingUK	-9.492	14.000	0.937	0.824	2.545	2.717

Geographic focus	skewness	kurtosis	$\rho_1$	$\rho_2$	$\rho_3$
AsiaPacific	-0.521	2.861	0.206	0.184	0.056
AsiaPacificExcludingJapan	-0.328	5.276	0.137	0.063	0.064
Africa	-0.481	5.584	0.262	0.187	0.067
EasternEurope	-0.426	7.012	0.286	0.165	0.039
Global	0.610	2.999	0.321	0.150	0.061
India	0.224	3.763	0.280	0.134	0.000
Japan	-0.155	4.018	0.270	0.185	0.073
LatinAmerica	-0.210	7.363	0.341	0.208	0.051
NorthAmerica	-0.791	7.725	0.339	0.191	0.203
NorthAmericaExcludingUSA	0.705	5.393	0.309	0.110	0.038
Other	-2.157	20.864	0.275	0.151	0.124
Russia	1.714	10.677	0.095	0.210	0.065
UK	1.528	10.426	0.335	0.276	0.243
US	2.954	15.589	0.411	0.060	-0.058
WesternEurope	-0.063	5.453	0.176	0.193	-0.037
WesternEuropeExcludingUK	0.771	7.299	0.231	0.227	0.082

Table 3.3: Summary Statistics of Hedge Fund Returns Clustered by Investment Style

PrimaryCategory	min	max	mean	median	stdev	$\frac{stdev}{mean}$
Convertible Arbitrage	-12.086	10.900	0.773	0.919	1.919	2.481
Dedicated Short Bias	-13.173	23.374	0.268	-0.198	4.893	18.286
Emerging Markets	-18.795	19.412	1.444	1.691	4.446	3.079
Equity Market Neutral	-7.163	5.947	0.707	0.674	1.419	2.006
Event Driven	-6.880	5.108	1.097	1.189	1.318	1.201
Fixed Income Arbitrage	-7.779	4.871	0.625	0.725	1.377	2.205
Fund of Funds	-17.680	51.708	1.341	0.698	5.364	4.000
Global Macro	-21.250	21.500	1.393	0.999	4.175	2.996
Long/Short Equity Hedge	-12.813	20.178	1.883	1.454	4.271	2.268
Managed Futures	-15.529	19.432	1.432	0.817	4.426	3.090
Multi-Strategy	-10.860	15.892	1.179	1.011	3.176	2.694
Options Strategy	-5.189	8.095	0.497	0.311	1.660	3.337
Other	-4.909	4.157	0.617	0.652	1.049	1.700

PrimaryCategory	skewness	kurtosis	$\rho_1$	$\rho_2$	$\rho_3$
Convertible Arbitrage	-1.658	15.354	0.280	0.110	0.096
Dedicated Short Bias	0.550	1.098	-0.027	-0.104	0.027
Emerging Markets	-0.025	2.500	0.427	0.223	0.188
Equity Market Neutral	-0.317	6.186	0.223	0.142	0.048
Event Driven	-1.415	6.160	0.319	0.090	0.007
Fixed Income Arbitrage	-2.105	10.135	0.077	0.005	0.032
Fund of Funds	4.370	31.666	0.261	0.133	0.021
Global Macro	0.456	6.693	0.505	0.246	0.121
Long/Short Equity Hedge	1.141	4.467	0.145	0.047	0.190
Managed Futures	0.728	2.226	0.139	-0.093	-0.010
Multi-Strategy	0.829	4.712	0.491	0.320	0.251
Options Strategy	0.490	3.845	0.331	0.175	0.081
Other	-1.382	7.477	0.221	0.172	0.014

Table 3.4: Summary Statistics of Regime Switching Probabilities Clustered by Regions of Geographic Focus

Geographic Focus	mean	stdev	min	max
Africa	0.3836	0.2873	0.0426	0.9995
AsiaPacific	0.4929	0.4495	0.0008	1.0000
AsiaPacificExcludingJapan	0.3208	0.3064	0.0102	0.9996
EasternEurope	0.3687	0.4202	0.0019	1.0000
Global	0.3267	0.2945	0.0319	1.0000
India	0.2914	0.2775	0.0217	1.0000
Japan	0.2239	0.2734	0.0052	1.0000
LatinAmerica	0.0868	0.1866	0.0186	1.0000
NorthAmerica	0.3513	0.3758	0.0195	1.0000
NorthAmericaExcludingUSA	0.2511	0.3096	0.0162	0.9999
Other	0.2911	0.2898	0.0391	1.0000
Russia	0.2714	0.2425	0.0644	0.9990
WesternEurope	0.2579	0.3823	0.0023	1.0000
WesternEUropeExcludingUK	0.3207	0.2560	0.1054	1.0000
UK	0.2663	0.2627	0.0237	1.0000
US	0.3113	0.2655	0.0575	0.9991

Geographic Focus	skewness	25% quantile	50% quantile	75% quantile
Africa	0.5843	0.1309	0.2852	0.6274
AsiaPacific	-0.0330	0.0064	0.6062	0.9752
AsiaPacificExcludingJapan	0.9347	0.0672	0.1991	0.4761
EasternEurope	0.6176	0.0121	0.1068	0.9574
Global	0.9370	0.0919	0.1935	0.5398
India	1.1569	0.0736	0.1822	0.4557
Japan	1.4960	0.0274	0.1005	0.3142
LatinAmerica	3.8514	0.0204	0.0267	0.0446
NorthAmerica	0.7528	0.0369	0.1377	0.6761
NorthAmericaExcludingUSA	1.3418	0.0432	0.0863	0.3339
Other	1.2607	0.0799	0.1544	0.3688
Russia	1.6730	0.1065	0.1723	0.3353
WesternEurope	1.1831	0.0055	0.0198	0.3800
WesternEUropeExcludingUK	1.2655	0.1314	0.1998	0.4435
UK	1.3616	0.0760	0.1591	0.3587
US	1.1764	0.1105	0.2010	0.4624

Table 3.5: Summary Statistics of Regime Switching Probabilities Clustered by Investment Style

Primary Category	mean	stdev	min	max
Fund of Funds	0.2309	0.1910	0.0682	0.9991
Managed Futures	0.2153	0.3321	0.0012	1.0000
Event Driven	0.2022	0.3287	0.0034	1.0000
Long Short Equity Hedge	0.1931	0.2917	0.0086	1.0000
Emerging Markets	0.0948	0.2111	0.0025	1.0000
Global Macro	0.1608	0.1538	0.0631	0.9945
Multi Strategy	0.2575	0.2668	0.0467	0.9999
Convertible Arbitrage	0.2816	0.3353	0.0028	1.0000
Equity Market Neutral	0.1971	0.2925	0.0020	1.0000
Dedicated Short Bias	0.2250	0.2346	0.0303	1.0000
Other	0.1371	0.2435	0.0014	1.0000
Fixed Income Arbitrage	0.1913	0.2825	0.0034	1.0000
Options Strategy	0.4506	0.2810	0.0071	0.9987

Primary Category	skewness	25% quantile	50% quantile	75% quantile
Fund of Funds	1.9870	0.1072	0.1471	0.2891
Managed Futures	1.3512	0.0032	0.0298	0.2235
Event Driven	1.5454	0.0074	0.0182	0.2901
Long Short Equity Hedge	1.6700	0.0145	0.0359	0.2370
Emerging Markets	3.0645	0.0056	0.0127	0.0579
Global Macro	3.0487	0.0812	0.0993	0.1717
Multi Strategy	1.7374	0.0866	0.1469	0.2939
Convertible Arbitrage	1.1092	0.0360	0.1025	0.4246
Equity Market Neutral	1.6496	0.0113	0.0451	0.2724
Dedicated Short Bias	1.7140	0.0640	0.1221	0.2731
Other	2.3812	0.0124	0.0298	0.1147
Fixed Income Arbitrage	1.9303	0.0233	0.0776	0.1725
Options Strategy	0.2525	0.1998	0.4142	0.6812

Figure 3.1: Regional Systemic Risk Indicators

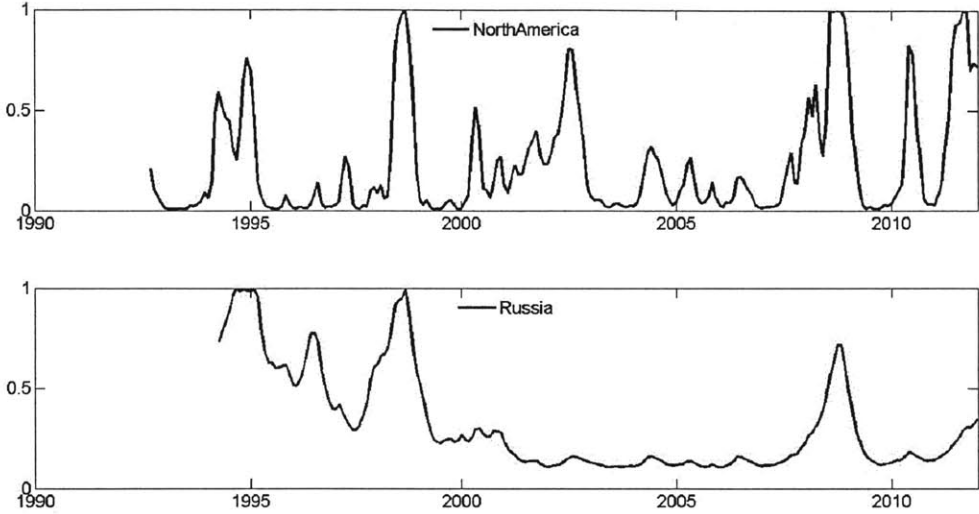


Figure 3.2: Construction of the Global Systemic Risk Indicators

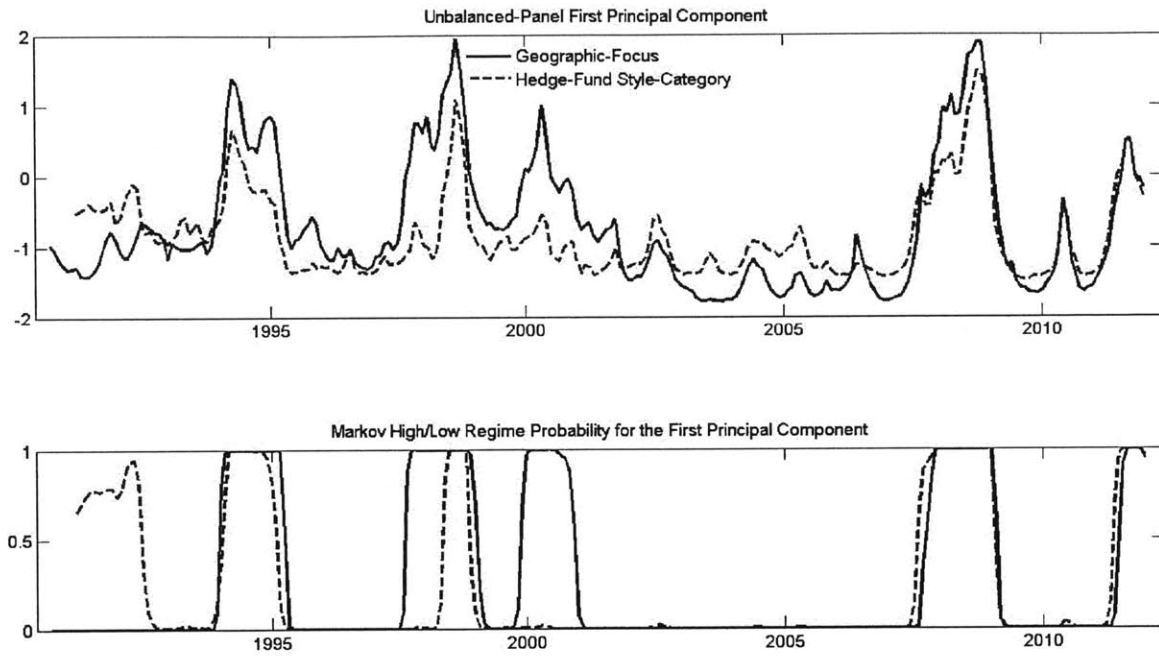


Figure 3.3: Heat-map of the Two Global Systemic Risk Indicators. “Geo” refers to the geographic-focus indicator; “HF” refers to the hedge-fund style-category indicator.

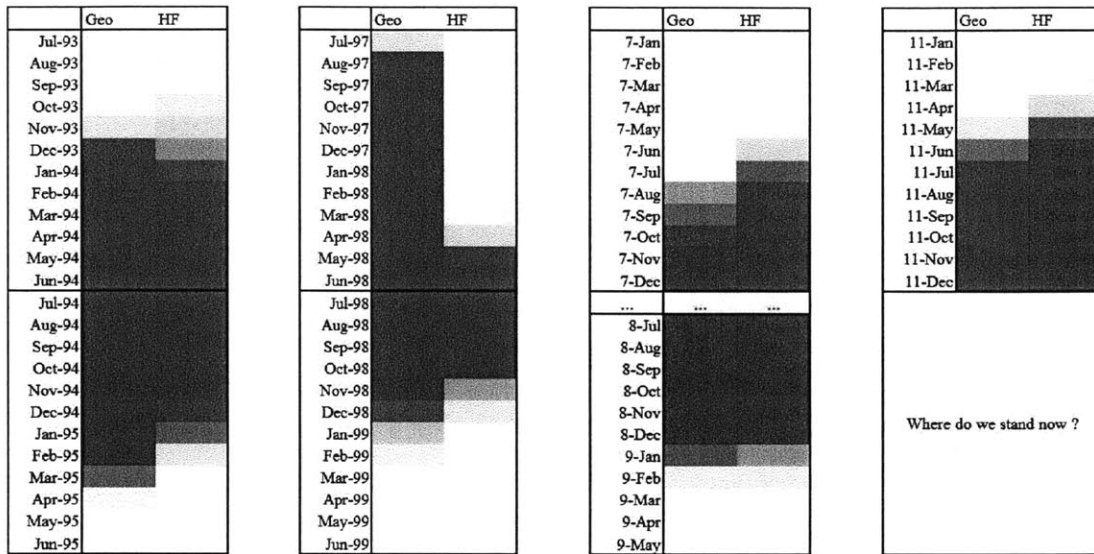


Figure 3.4: Time Series Plot of the Libor-OIS Spread at the Beginning of the 2007 US Quant Crisis.

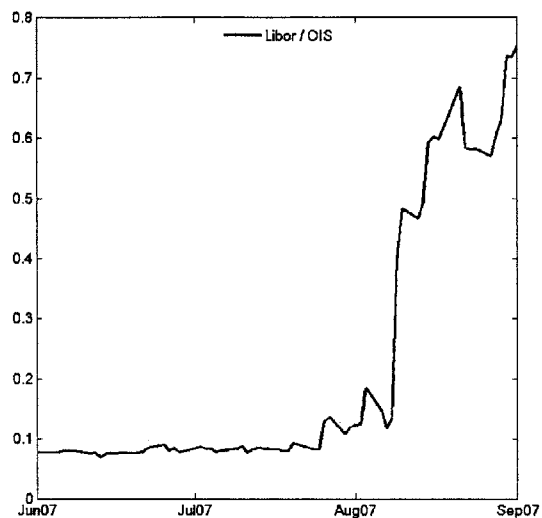


Figure 3.5: Time Series Plot of the Dow Jones Industrial Average from 2007 to 2012. The Dow reached its bottom on March 6th, 2009.

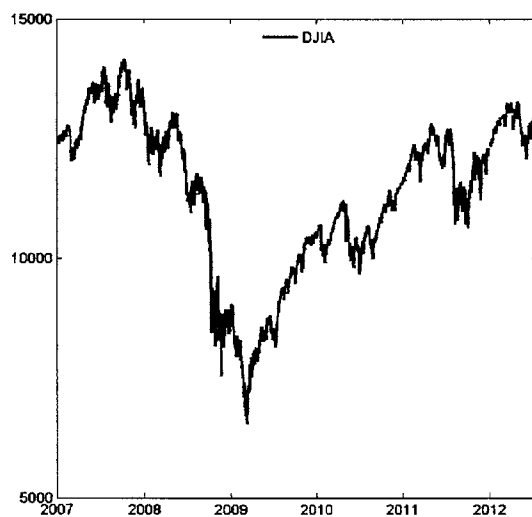




Figure 3.6: Time Series Plot of the Mexican Peso / US Dollar Exchange Rate From September 1994 to September 1995. The peso plunged by over 40% over one week in December 1994.

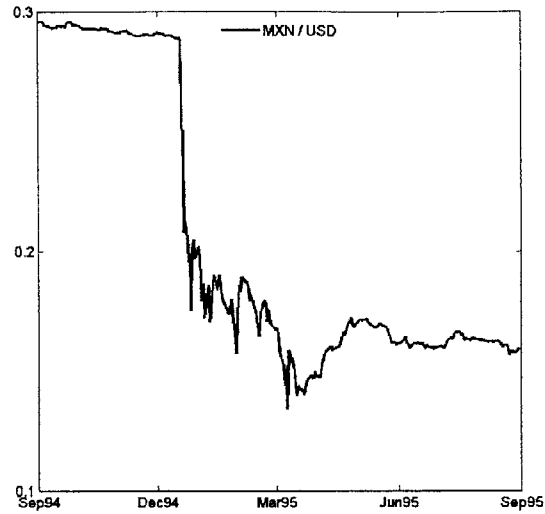
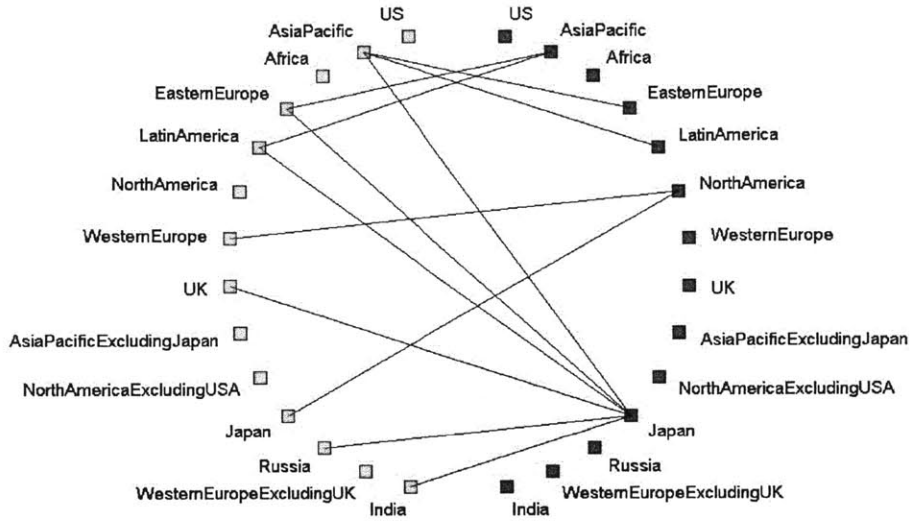


Figure 3.7: Granger Causality Network of Regional Risks

(a) Interconnections across regional risk indicators: 07/1996~06/1998



(b) Interconnections across regional risk indicators: 07/1997~06/1999

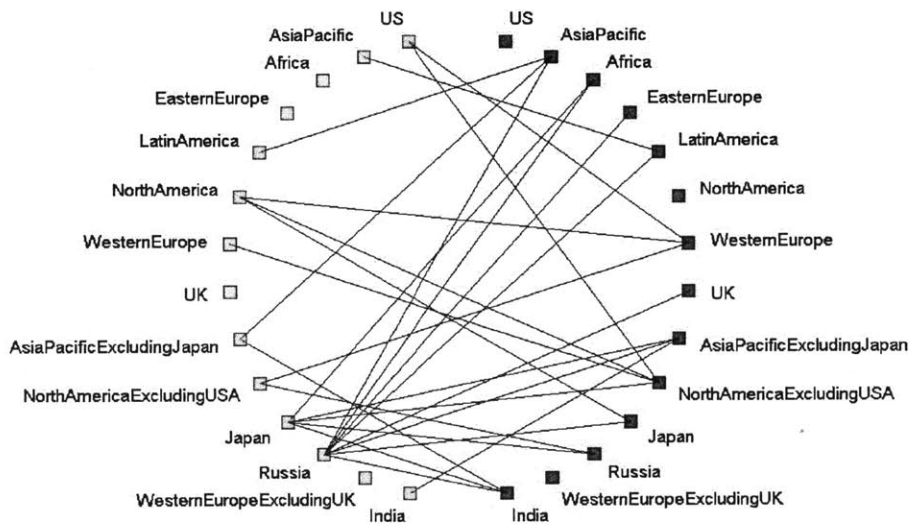


Figure 3.8: Regional Risk Interconnectedness: 07/2006~06/2008

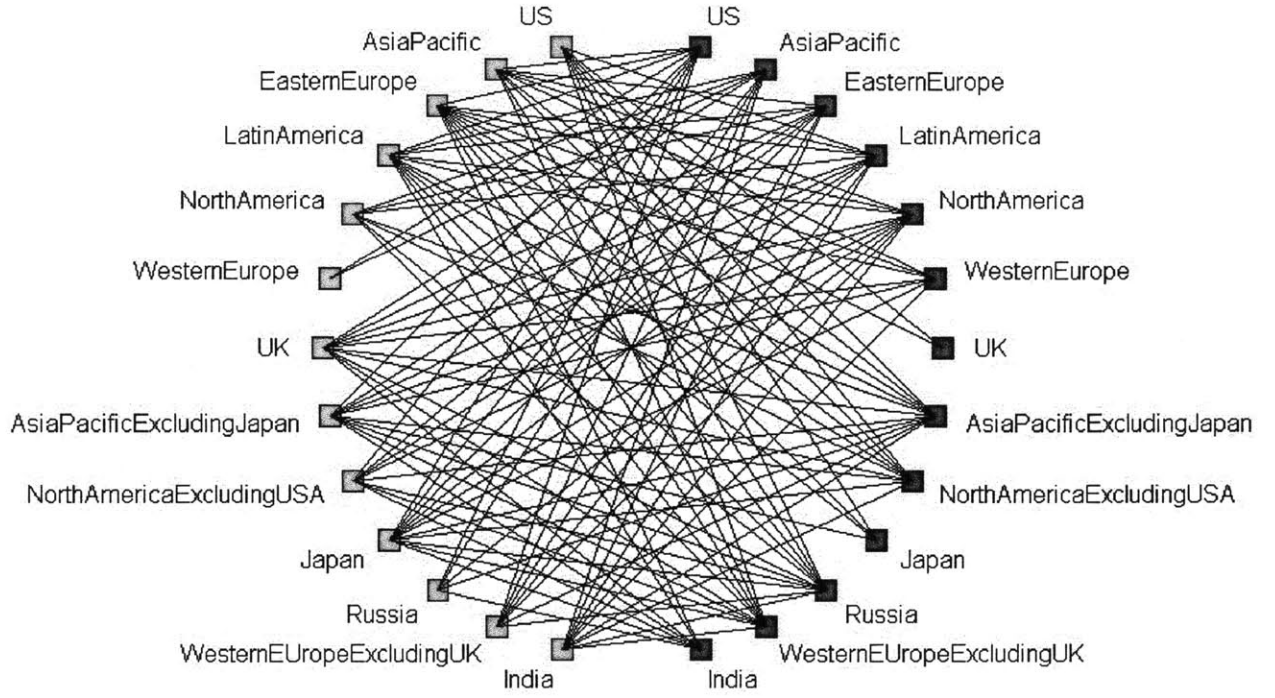


Figure 3.9: Regional Risk Interconnectedness: 12/2008~11/2010

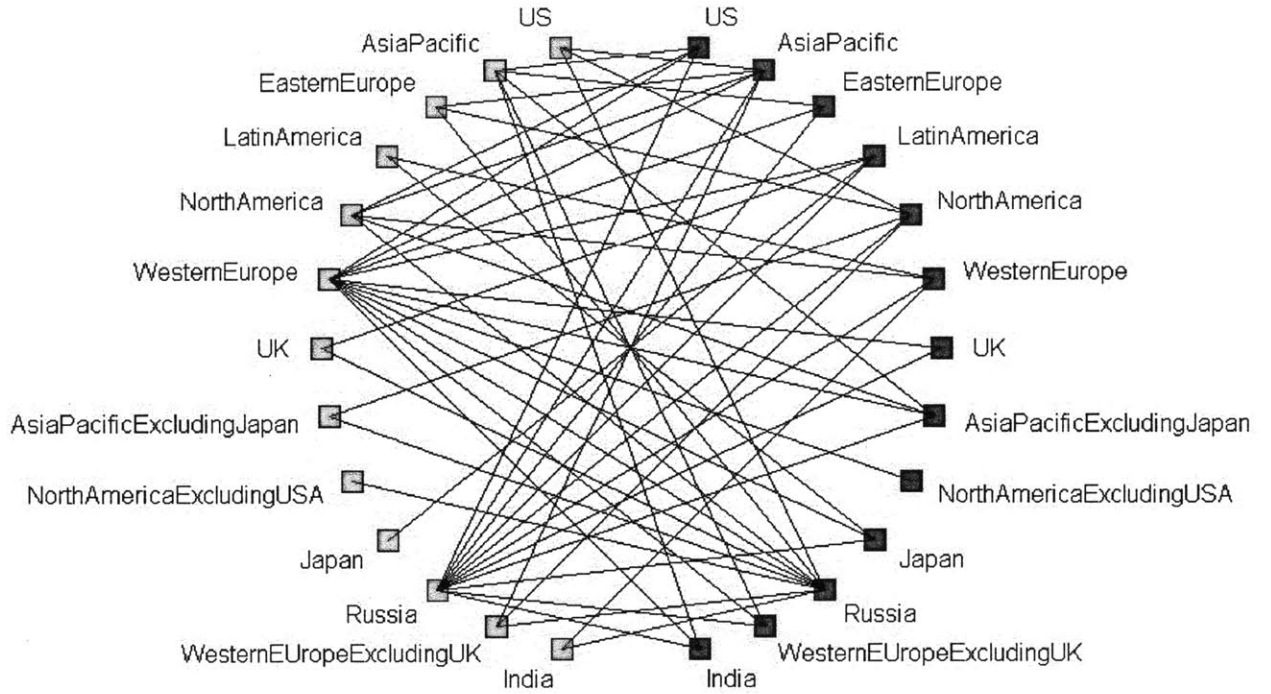
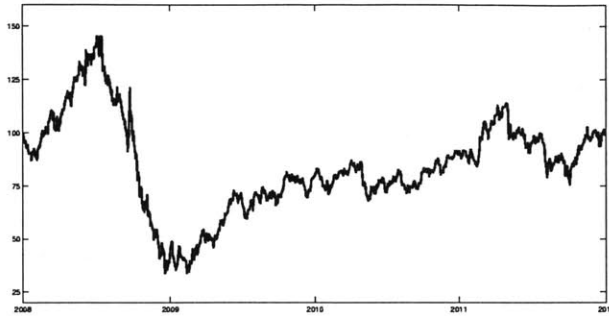
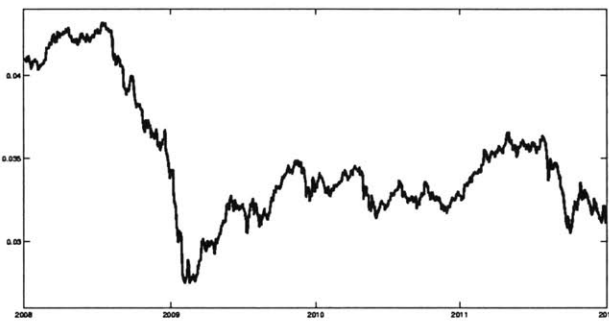


Figure 3.10: Key Financial Indicators during the 2008~2009 Russian Recession

(a) In late 2008, crude oil tumbled by 77% from peak \$145/barrel to trough \$33/barrel (Bloomberg WTI generic first crude oil price).



(b) In late 2008, the Ruble lost more than 36% in value from \$0.0432 to \$0.0275 (Bloomberg RUBUSD cross rate).



(c) In late 2008, the MICEX (Moscow Interbank Exchange) equity index shed more than 63% from peak 1956 points to trough 513 points.

