Microseismic mapping and source characterization for hydrofracture monitoring: a full-waveform approach

by

Fuxian Song

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B.S. Acoustics, Nanjing University, 2003

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Author ..........................................................................................................................

Department of Earth, Atmospheric, and Planetary Sciences

March 8, 2013

Certified by ......................................................

M. Nafi Toksöz
Robert R. Shrock Professor of Geophysics
Thesis Supervisor

Accepted by ......................................................

Robert van der Hilst
Schlumberger Professor of Earth and Planetary Sciences
Head, Department of Earth, Atmospheric and Planetary Sciences
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Abstract

The objective of this thesis is to improve the microseismic mapping capability for hydrofracture monitoring by using full-waveform information and understand fracturing mechanisms via microseismic source mechanism inversion.

First, we develop an array-based correlation approach to improve the detection of small magnitude events with mechanisms and locations similar to a nearby template event.

Second, we extend the correlation detector to the subspace detector by including waveforms from multiple template events. Empirical procedures are presented for building the signal subspace from clusters of events. The distribution of the detection statistics is analyzed to determine subspace detection parameters. The benefits of the subspace detector are demonstrated on a dual-array hydrofracture monitoring dataset.

Next, a full-waveform approach is developed for complete moment tensor inversion. By using synthetic data, we show that, for events in the near-field of a single monitoring well, a stable, complete moment tensor can be retrieved by matching the waveforms without additional constraints. At far-field range, we demonstrate that the off-plane moment tensor component is poorly constrained by waveforms recorded at one well. Therefore, additional constraints must be introduced. The complete moment tensor inversion approach is demonstrated with a single well dataset from the Bonner sands hydrofracturing. Moment tensor inversion results show that most events have a dominant double-couple component with the fracture plane orientation close to the average fracture trend derived from the multiple event locations. It suggests that in a reservoir with a high horizontal differential stress like the Bonner sands, the microseismicity occurs predominantly by shearing along natural fractures subparallel to the average fracture trend.

Finally, the full-waveform based complete moment tensor inversion method is applied to a dual-array hydrofracture monitoring dataset in Barnett shale at Fort Worth Basin. The determined microseismic source mechanisms reveal both tensile opening on hydraulic fracture strands trending subparallel to the unperturbed maximum horizontal principal stress direction and the reactivation of pre-existing natural fractures along the WNW and N-S directions.

Two main contributions are: 1) Improving hydrofracture mapping by developing advanced event detection and relocation algorithms using full waveforms; 2)
Understanding the fracturing mechanisms through complete moment tensor inversion and geomechanical analysis.

Thesis Supervisor: M. Nafi Toksöz
Title: Robert R. Shrock Professor of Geophysics
I would like to thank many people for their help towards the completion of this thesis and graduate education in Earth Resources Laboratory (ERL) at MIT.

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Chapter 1

Introduction

1.1 Objective

Unconventional gas resources including tight gas, coalbed methane, and shale gas are playing an increasingly important role in supplying low carbon fuel for a growing global energy demand. In US, unconventional gas production accounts for about half of the total gas output in 2010 and is projected to reach the 67% of the total US gas production by 2015 (IHS, 2012). Horizontal drilling and hydraulic fracturing are the two key technologies in developing these low permeability reservoirs. Statistics show that more than $3 Billion is spent annually on more than 20,000 hydraulic fracturing treatments in the continental US. However, it was reported that more than 2/3 of all hydrofracture stimulations do not perform up to expectations (Naik, 2007). This staggering number clearly points to a need to better understand the fracturing process.

Microearthquakes occur during the hydrofracture stimulation because of the stress perturbations and fracturing fluid leakage resulted from the hydraulic fracture. Understanding the fracture geometry is crucial to developing effective stimulation treatments and improving the economics of drilling and completing a well. Microseismic event mapping provides a way to image the overall geometry of the hydraulic fracture and assess the volume of rock enhanced by the hydrofracture stimulation.
The primary goal of this thesis is to improve the microseismic mapping capability for hydrofracture monitoring by using the full-waveform information and to understand the fracturing mechanisms in unconventional oil and gas reservoirs via microsesimic source mechanism inversion. Accurate microseismic maps and reliable source mechanism estimates not only reveal important information about the fracturing process, but also allow fracture characterization away from the wellbore, providing critical constraints for building fractured reservoir models.

Microseismic monitoring (MS) is typically conducted with downhole geophone arrays. In most cases, only one geophone array is available. This limited one-dimensional (1D) geophone coverage requires a use of the P-wave polarization information to derive three-dimensional (3D) locations. Unfortunately, for hydrofracture induced microearthquakes, normally P-waves are small compared to S-waves. On the other hand, the signal to noise ratio (SNR) of the recorded microseismic data varies enormously from one dataset to another, and it can often be very low. In the downhole monitoring case, the data are often contaminated by correlated noises such as borehole waves. These issues pose a significant challenge for microseismic event detection and location. In terms of event detection, the low SNR values of recorded microseismic waveforms set a detection limit. As such, the minimum detectable event magnitude increases with increased distance from monitoring geophones due to the increased signal attenuation with distance. This causes the viewing-distance bias, which can be a significant issue when interpreting the completeness of the fracture geometry (Maxwell et al., 2010b; Warpinski, 2009).

In this thesis, one of the main objectives is to improve microseismic event detection by exploring the full waveform information instead of only using incoherent energy information as in the conventional detectors. The array-based correlation detector is developed to detect small-magnitude events by matching the recorded data with the waveforms of a known template event, known as the master event. The additional processing gain from stacking the correlations across different components and geophones further improves the detector performance. In terms of location, we propose a transformed spectrogram method to improve the P- and S-phase arrival
picks. We further extend the correlation detector to the subspace detector to include waveforms from multiple template events. The signal subspace representation of a target source region derived from multiple template events honors the waveform variabilities that may exist due to variations in event locations and source mechanisms. The subspace detector is applied to a dual-array hydrofracture monitoring dataset. The comparison between the subspace detector, array correlation method, and array short-time average/long-time average (STA/LTA) detector is performed on the data from the far monitoring well to demonstrate the improved detection capability of the far well by using the subspace detector. Following event detection, a signal subspace projection method is developed to enhance the microseismic signals.

Another major objective is to better understand the fracturing mechanism. Although numerous efforts have been spent on understanding hydraulic fracture growth and the interaction between natural fractures and hydraulic fracture through laboratory tests (Warpinski and Teufel, 1987; Warpinski et al., 1993), and numerical modeling (Dahi-Taleghani and Olson, 2011; Busetti et al., 2012), very limited microseismic observations have been reported to shed light on the fracturing process by exploring the microseismic source information. Among those limited studies, Rutledge and Phillips (2003) is a classic one. They studied the microseismic source mechanisms in the Cotton Valley tight gas sands and concluded that the microearthquakes occur as shear failures on pre-existing natural fractures trending subparallel to the maximum horizontal stress direction. This is probably true for a simple tight gas sands reservoir with a high horizontal differential stress. However, this source assumption of shearing along a single plane is definitely not compatible with the complex location patterns as observed in the Barnett shale waterfrac case. Moreover, location analysis of microseismic events during an hydrofracture stimulation in the Barnett Shale, Fort Worth Basin, Texas, has indicated the possibility of complex interactions between natural fractures and hydraulic fractures (Roth and Thompson, 2009). Therefore, in this thesis, we develop a grid search based complete moment tensor inversion approach to study the complex source mechanisms that may
arise during the hydrofracture stimulation of complex fractured reservoirs. This approach matches the observed data with the full waveform synthetics generated by either the discrete wavenumber integration method or finite difference method. The grid search based inversion approach can not only determine the microseismic source mechanisms but also improve event locations. The complete moment tensor inversion makes no double-couple earthquake assumption about the underlying microseismic events. Therefore, it could retrieve microseismic source information for both shearing and tensile failures. The complete moment tensor inversion approach is studied in both single-well and multiple-well monitoring scenarios. This source inversion method is applied to two different microseismic datasets, a single-array dataset from hydraulic fracturing in the Bonner tight gas sands and a dual-array dataset from the waterfrac treatment in the Barnett shale. A comparison between the inverted source mechanisms from the two datasets reveals different fracturing behaviors and different mechanisms to enhance gas production in these two different reservoirs.

This thesis combines two basic scientific approaches: numerical modeling and field data analysis. Details of previous research and our research are discussed in the next section of this chapter.

1.2 Previous studies and our research

Over the last few years, Microseismic monitoring has evolved into a standard hydraulic fracture diagnostic technology, with numerous applications through all of the major tight gas and shale gas plays in North America. Originally, MS monitoring used seismic arrays deployed near the reservoir depth in offset, vertical observation wells (e.g. Warpinski et al., 1998). The main efforts in MS since its inception have been focused on developing better event detection and location algorithms. The locations of microseismic events, with sufficient resolution, provide information on fracture geometry and properties (Warpinski et al., 1998; Phillips et al., 2002; Maxwell, 2010a).
Various algorithms have been proposed to determine the microearthquake hypocenters given a known velocity model. They fall into two main categories: 1) travel time based methods and 2) migration based approaches. Travel time based methods use the difference between the P- and S-wave arrival times to calculate travel distances. The hypocenter is assigned to the intersecting region of these hemispheres (Lay and Wallace, 1995). In downhole monitoring with a single geophone array, due to its limited azimuthal coverage, the P-wave polarization information is used to determine the event azimuth in the three-dimensional (3D) space (Rutledge and Phillips, 2003). Alternatively, S-waves could be used to derive the event azimuth (Eisner et al., 2009). In either case, the arrival time picking is required, which could be a problem in the noisy environment especially for weak P waves. A review of advanced location algorithms such as proposed by Rabinowitz (1988); Pujol (1992); Joswig (1999) and Lomax et al. (2000) is given in Thurber and Rabinowitz (2000).

The migration based location methods, on the other hand, require less accurate arrival pickings. They select a window around either P- or S-wave arrivals and back-propagates the energies inside the signal window from all geophones to all possible mesh points in the formation according to their arrival times for different time steps. These steps span the time interval up to the maximum travel time observed from the target of interest to each geophone. An earthquake location is determined when the extrapolation of all geophone signals converges, which is supposed to occur at the origin time of the event (Rentsch et al., 2007; Lu, 2008; Zhao et al., 2010).

The first step towards microseismic mapping is the event detection. Several approaches have have been proposed for the automatic P-wave arrival detection (e.g., Allen, 1978; Baer and Kradolfer, 1987; Earle and Shearer, 1994; Anant and Dowla, 1997; Bai and Kennett, 2000; Saragiotis et al., 2002; Zhang et al., 2003) using energy analysis, short-term-average and long-term-average (STA/LTA) ratios, statistical analysis, frequency analysis, wavelet analysis, polarization analysis/particle motion or a combination of those. However, all of the above approaches have only used part of the information contained in the waveforms. None of them have tried to use full waveforms.
In earthquake seismology, waveform correlation of strong events, known as master events, is used to detect weaker events (Richards et al., 2004; Gibbons and Ringdal, 2006). These correlation based detectors are especially useful to lower the detection threshold and increase the detection sensitivity. In Chapter 2, we adapt the correlation detection method to hydrofracture monitoring by choosing a master event and using it as the cross-correlation template to detect small events, which share a similar location, fault mechanism and propagation path as the master event. We extend the conventional single-component single-geophone correction detector by stacking the correlations across multiple components and geophones to bring additional processing gains. To improve the arrival picking, a transformed spectrogram approach is developed by capturing the two features of a phase arrival in the time-frequency domain: high energy and high rate of energy increase. The effectiveness of this array-based correlation detector and the transformed spectrogram based arrival picking method is demonstrated using a field dataset from hydraulic fracturing stimulation of a carbonate reservoir.

Next, the correlation detector is further extended to the subspace detector to include waveforms from multiple template events. The signal subspace representation of a target source region derived from multiple template events honors waveform variabilities that may exist due to variations in event locations and source mechanisms (Harris, 2006). In Chapter 3, we present empirical procedures to build signal subspace from clusters of template events. We also develop a method to quantitatively determine the parameters of the subspace detector including the signal subspace dimension and detection threshold. The developed subspace detector is applied to a dual-array hydrofracture monitoring dataset. The comparison between the subspace detector, array correlation method, and array short-time average/long-time average (STA/ LTA) detector is performed on the data from the far monitoring well to demonstrate the improved detection capability of the far well by using the subspace detector. Following event detection, a signal subspace projection method is also proposed and tested to enhance weak microseismic signals.
Besides event locations, other source characteristics can also be determined, such as magnitude or moment as a measure of the source strength, fault-plane solutions (FPS, including fracture strike and dip) and slip direction. In general, slip across an internal surface can be modeled by a moment tensor matrix consisting of six independent elements, known as the complete moment tensor (Aki and Richards, 2002). Until recently, most microseismic source studies have been focused on determining double-couple (DC) mechanisms instead of the general source mechanisms represented by the complete moment tensor (Rutledge and Phillips, 2003; Sarkar, 2008; Li et al., 2011). One major reason for this DC assumption is based on the observation of high S/P-wave amplitude ratios which “could not be explained by tensile opening” (Phillips et al., 1998; Warpinski, 1997; Pearson, 1981). Therefore, it was speculated that hydrofracture induced events are predominantly shear failures along pre-existing natural fractures (Rutledge et al., 2004). However, there has been an ongoing debate on whether the microearthquakes are generated from shear failures or from tensile failures (Šilený et al., 2009, Bohnhoff et al., 2010). Moreover, non-double-couple (non-DC) mechanisms for the hydrofracture events were observed in an increasing number of studies (Šilený et al., 2009, Warpinski and Du, 2010). Knowledge of non-double-couple components, especially the volumetric component, is essential to understand the fracturing process. Vavryčuk (2007) showed that, for shear faulting on non-planar faults, or for tensile faulting, the DC source assumption is no longer valid and can severely distort the retrieved moment tensor and bias the fault-plane solution. Therefore, the complete moment tensor inversion is crucial not only to the retrieval of the volumetric component but also to the correct determination of the fault-plane solution.

Currently, most moment tensor inversion methods rely only on far-field direct P- and S-wave amplitudes (Nolen-Hoeksema and Ruff, 2001; Vavryčuk, 2007; Jechumtálová and Eisner, 2008; Warpinski and Du, 2010). Vavryčuk (2007) used the far-field approximation of the P- and S-wave Green’s function in homogeneous isotropic and anisotropic media to show that a single-azimuth dataset recorded in one vertical well cannot resolve the dipole perpendicular to the plane of geophones and
the hypocenter. Thus, the complete moment tensor of the general source mechanism is underdetermined with data from one well. To overcome this problem, previous studies proposed to use data recorded in multiple monitoring wells at different azimuths (Vavryčuk, 2007; Baig and Urbancic, 2010). Unfortunately, downhole microseismic monitoring datasets are frequently limited to a single array of geophones in one vertical well. Therefore, the issue of complete moment tensor inversion from one-well data remains to be solved.

In Chapter 4, we try to address this problem from the standpoint of full-waveform inversion. We propose a grid search based full-waveform approach for moment tensor inversion and event relocation. The source parameters including the FPS, seismic moment, and moment tensor component percentages are then derived from the inverted complete moment tensor. The influence of event-geophone distance and geophone azimuthal coverage on the condition number of the inversion sensitivity matrix is studied. Based on the results from the condition number study, two different inversion strategies, unconstrained inversion and constrained inversion, have been proposed to invert the complete moment tensor from one-well data for near-field and far-field events, separately. The influence of velocity model errors, source mislocations and data noise on the extracted source parameters is investigated using synthetic data. We further describe the application of the constrained inversion to a single-array MS dataset in Bonner sands from East Texas. By applying the constraint on the fracture strike and dip range, we show that a reliable, complete moment tensor solution and source parameters can be obtained for each event. The implications of inverted source mechanisms on the fracturing mechanism in Bonner sands reservoir are compared with the Barnett shale case and further illustrated in Chapter 5.

Finally, we turn our focus to fracturing mechanisms in a complex naturally-fractured reservoir with low horizontal differential stress. In Chapter 5, a dual-array waterfrac dataset from the Barnett shale at Fort Worth Basin is investigated for this purpose. In this study, we use the grid search based full-waveform approach and adopt a general dislocation model, i.e. the tensile earthquake model by Vavryčuk (2001) to study the source complexity in the Barnett shale. The source parameters
derived from the inverted complete moment tensor include the FPS, the slip direction, seismic moment, Vp/Vs ratio in the focal area, and moment tensor component percentages. We analyze the microseismicity in the Barnett shale using hydraulic fracture geomechanics. Based on the findings from geomechanical analysis, we propose a method to determine the fracture plane from the moment tensor. The significance of the occurrence of non-DC components is studied by the F-test. The influence of velocity model errors, event mislocations, and additive data noise on the extracted source parameters is quantified via a Monte-Carlo study using synthetic data. The determined microseismic source mechanisms reveal both tensile opening on hydraulic fractures in the unperturbed maximum horizontal principal stress direction and the reactivation of pre-existing natural fractures along the WNW and N-S directions. An increased fracture connectivity and enhanced gas production in the Barnett shale are achieved through the formation of a complex fracture network during hydraulic fracturing via rock failures on the weak zones of different orientations.

### 1.3 Thesis outline

This thesis contains six chapters, all related to microseismic event detection, location and hydrofracture source characterization. Each chapter, except for the first and last chapter, is written as an independent paper. Some of these papers are already published, and others are being prepared for publication.

In Chapter 1, the thesis objectives are stated and the background and previous studies pertained to this thesis are reviewed.

Chapter 2 describes the array-based correlation detector for microseismic event detection and the transformed spectrogram method for phase picking. The comparison with the array based STA/LTA detector is presented to demonstrate the effectiveness of the array-based correlation method. After event detection, the transformed spectrogram method is employed to pick the P- and S-arrivals. The picking results are
further compared with manual picks and STA/LTA picks. The bulk of this chapter has been published as:


*Geophysics, 75*(6), A47-A52.

Chapter 3 extends the correlation detector described in Chapter 2 to the subspace detector in order to include waveforms from multiple template events. The signal subspace representation of a target source region derived from multiple template events honors waveform variabilities that may exist due to variations in event locations and source mechanisms (Harris, 2006). In this chapter, we present empirical procedures to build the signal subspace from clusters of template events. The distribution of the detection statistics is analyzed to determine the parameters of the subspace detector including the signal subspace dimension and detection threshold. The effect of correlated noise is corrected in the statistical analysis. The proposed subspace design and detection approach is illustrated on a dual-array hydrofracture monitoring dataset. The comparison of event detections and false alarm triggers between the subspace detector, array correlation method, and array STA/LTA detector is performed to demonstrate the benefits of subspace detectors. Following event detection, a signal subspace projection method is also proposed and tested to enhance weak microseismic signals. The improvement in detection capability and weak signal enhancement offered by the subspace detector facilitates microseismic event location and interpretation. The bulk of this chapter has been submitted for publication as:


Chapter 4 moves on to the microseismic source characterization in a tight gas sands reservoir. In this chapter, we develop a grid search based approach to invert for complete moment tensor from full-waveform data recorded at a vertical geophone array. We use the discrete wavenumber integration method to calculate full wavefields in the layered medium. By using synthetic data, we show that, at the near-field range,
a stable, complete moment tensor can be retrieved from single-well data by matching the waveforms without posing additional constraints. At the far-field range, we demonstrate that the off-plane moment tensor component is poorly constrained by waveforms recorded at one well. Therefore, additional constraints must be introduced to retrieve the complete moment tensor. We study the inversion with three different types of constraints. For each constraint, we investigate the influence of velocity model errors, event mislocations and data noise on the extracted source parameters by a Monte-Carlo study. We test our method using a single well microseismic dataset obtained during hydraulic fracturing of the Bonner sands in East Texas. By imposing constraints on the fracture strike and dip range, we are able to retrieve the complete moment tensor for events in the far field. Field results show that most events have a dominant double-couple component. The results also indicate the existence of a volumetric component in some events. The derived fracture plane orientation generally agrees with that derived from multiple event location. It suggests that the microseismicity in Bonner sands occurs as predominantly shearing along a major fracture plane. In a reservoir with a high horizontal differential stress like the Bonner sands reservoir, an enhanced production from hydraulic fracturing is obtained through the improved fracture conductivity. The bulk of this chapter has been published as:


Chapter 5 presents a systematic microseismic source mechanism study in the Barnett shale, a complex naturally-fractured reservoir with a low horizontal differential stress. In this chapter, we perform the complete moment tensor inversion with a dual-array dataset from a hydraulic fracturing stimulation in the Barnett shale at Fort Worth Basin. The microseismicity in the Barnett shale is firstly analyzed using hydraulic fracture geomechanics. With the insights gained from geomechanical analysis, we propose a method to distinguish the fracture plane from the auxiliary plane. The tensile earthquake model is then used to extract complex source mechanisms from the inverted moment tensor. The source information derived
consists of the fault plane solution (FPS), the slip direction, the Vp/Vs ratio in the focal area, and the seismic moment. The significance of the occurrence of non-DC components is further investigated by F-test. The influence of velocity model errors, event mislocations, and additive data noise on the extracted source parameters is also studied via a Monte-Carlo test using synthetic data. In the end, the results of source mechanism analysis in the Barnett shale are presented for the best signal-to-noise ratio (SNR) events with low condition numbers. Finally, the information regarding the fracturing mechanism in the Barnett shale is discussed using the determined microseismic source mechanisms. The bulk of this chapter has been submitted for publication as:


Chapter 6 summarizes the major conclusions of this thesis and is followed by three appendices.

Appendix A describes the design set event selection and waveform alignment through the single-link algorithm, which is used in Chapter 3 for signal subspace construction.

Appendix B describes the derivation of equation (3-25) via the analysis of the detection statistics, which is used in Chapter 3 to determine parameters for the subspace detector.

Appendix C demonstrates why the off-plane moment tensor component m_{22} can be inverted from one-well data at near field using full waveform based moment tensor inversion approach proposed in Chapter 4.

### 1.4 References


Maxwell, S. C., B. Underhill, L. Bennett, C. Woerpel, and A. Martinez, 2010b, Key criteria for a successful microseismic project: SPE Paper 134695.


Chapter 2

Full waveform based microseismic event detection and phase picking: the array-based correlation approach¹

Abstract

The ability to detect small microearthquakes and identify their P and S phase arrivals is a key issue in hydrofracture downhole monitoring because of the low signal-to-noise ratios. We apply an array based waveform correlation approach (matched filter) to improve the detectability of small magnitude events with similar mechanisms and locations as a nearby master event. After detecting the event, we use a transformed spectrogram method to identify the weak P arrivals. We have tested the technique on a downhole monitoring dataset of the microseismic events induced by hydraulic fracturing. We show that, for this case, two events with a signal-to-noise ratio around 6dB, which are barely detectable using a short-time average/long-time average (STA/LTA) detector under a reasonable false alarm rate, are readily detected on the array-stacked correlation traces. The transformed spectrogram analysis of the detected events improves P and S phase picking.

¹ (the bulk of this Chapter has been) published as: Song, F., Kuleli H. S., Toksoz M. N., Ay E., and H. Zhang, 2010, An improved method for hydrofracture-induced microseismic event detection and phase picking: *Geophysics, 75*(6), A47-A52.
2.1 Introduction

Low-permeability oil reservoirs and gas shales are problematic to produce, often requiring multiple stages of hydraulic fracturing in order to create connected pathways through which hydrocarbons may flow. During hydrofracturing, many induced microearthquakes occur. These induced microearthquakes are extremely important for mapping the fractures and evaluating the effectiveness of hydraulic fracturing. Their locations are used to determine fracture orientation and dimensions, which is further used to optimize the late-stage treatment (Walker, 1997; Maxwell and Urbancic, 2002; Philips et al., 2002). Microearthquake locations also provide helpful information on reservoir transport properties and zones of mechanical instability, which can be used for reservoir monitoring and new well planning (Kristiansen et al., 2000; Willis et al., 2008; Willis et al., 2009). In this chapter, we propose a systematic approach to improve the low-magnitude hydrofracture event detection and phase identification.

Most microearthquakes are small and often are hard to detect. A noisy borehole environment further complicates the detection process. For downhole monitoring, as is the case for our study, additional difficulties for event location come from the limited receiver geometry, where usually only one monitoring well is available. In this case, additional information on wavefront propagation direction must be obtained to constrain the event azimuth (De Meersman et al., 2009; Eisner et al., 2009a). Although S-wave polarization has been proposed to compute the event azimuth (Eisner et al., 2009b), most methods still rely on P-wave polarization. However, most hydrofracture events typically radiate smaller P-waves than S-waves. Therefore, identification of the weak P-wave arrivals is crucial for downhole microearthquake location. The quality of P-wave arrival picking determines the precision of microearthquake locations (Pavlis, 1992), and the accuracy of event azimuth relies heavily on the P-wave vector (Eisner et al., 2009a).

In earthquake seismology, waveform correlation of strong events, known as master events, is used to detect weaker events (Richards et al., 2004; Gibbons and
Ringdal, 2006; Michelet and Toksöz, 2007). These correlation based detectors are especially useful to lower the detection threshold and increase the detection sensitivity. In this study, we adapt the method to hydrofracture monitoring by choosing a master event and using it as the cross-correlation template to detect small events, which share a similar location, fault mechanism and propagation path as the master event (Eisner et al., 2006). We compare the single component, single geophone correlation detector with an array stacked three-component (3-C) correlation detector. A significant improvement results from array stacking and matching the polarization structure. Moreover, the array stacking of correlation traces suffers no coherence loss and requires no knowledge of velocity model as is the case with a conventional beam of array waveforms dependent on a plane-wave model (Kao and Shan, 2004).

To locate detected events, we need to identify their P- and S-wave arrivals. Typically the STA/LTA type algorithm is used to pick P- and S-wave arrivals (Earle and Shearer, 1994). The problem with this algorithm is that it is very sensitive to background noise level, which can change significantly during hydraulic fracturing. We propose a transformed spectrogram based approach to identify P- and S-wave arrivals where the influence of high background noise is reduced. This method can act as an initial picking of P- and S-wave arrivals. The transformed spectrogram picking results can be further refined using an iterative cross-correlation procedure proposed by Ronen and Claerbout (1985), and Rowe et al. (2002).

2.2 Methodology

2.2.1 Correlation detector

The seismic waveforms observed at any receiver can be modeled as a convolution of the source, medium and receiver response (e.g. Stein and Wysession, 2002):

\[ D(t) = S(t) \ast G(t) \ast R(t), \]  

(2-1)

where \( D(t) \) is the recorded seismic data, \( S(t) \), \( G(t) \), and \( R(t) \) represent the source wavelet, medium Green's function and receiver response,
respectively. Thus, nearby events sharing a similar source mechanism will have similar waveforms observed at the same receiver (Arrowsmith and Eisner, 2006). This is the basis for the correlation detector. Once an event with a good signal-to-noise ratio is identified by the conventional STA/LTA type detector, it can be used as the master event to cross-correlate with nearby noisy record. If the 3-C waveforms of the master event are denoted as \( w_{N,At}^{lk}(t_M) \):

\[
w_{N,At}^{lk}(t_M) = [w_{N,At}^{lk}(t_M), w_{N,At}^{lk}(t_M + \Delta t), \cdots, w_{N,At}^{lk}(t_M + (N - 1)\Delta t)]^T,
\]

(2-2)

where component index is \( k = 1,2,3 \); geophone index is \( j = 1,2,\cdots J \); \( t_M \) is the starting time of the master event which is determined by the STA/LTA detector. The inner product between \( w_{N,At}^{lk}(t) \) and \( w_{N,At}^{lk}(t_M) \) is defined as

\[
(w_{N,At}^{lk}(t), w_{N,At}^{lk}(t_M)) = \sum_{i=0}^{N-1} w_{N,At}^{lk}(t_M + i\Delta t)w_{N,At}^{lk}(t + i\Delta t),
\]

(2-3)

and the single-component, single-geophone correlation detector is given by Gibbons and Ringdal (2006),

\[
C_w^{lk}(t_{N,At}) = C[w_{N,At}^{lk}(t), w_{N,At}^{lk}(t_M)] = \frac{(w_{N,At}^{lk}(t_M), w_{N,At}^{lk}(t_M))}{\sqrt{(w_{N,At}^{lk}(t_M), w_{N,At}^{lk}(t_M))}},
\]

(2-4)

Data redundancy contained in the array and three components can be utilized by introducing another two forms of correlation detector, that is,

\[
C_{w}^{k}(t_{N,At}) = \Sigma_{j=1}^{l} C_{w}^{lk}(t_{N,At}),
\]

(2-5)

\[
C_{w}(t_{N,At}) = \Sigma_{k=1}^{3} \Sigma_{j=1}^{l} C_{w}^{lk}(t_{N,At}).
\]

(2-6)

Equation (2-5) represents the single-component, array-stacked correlation detector (Gibbons and Ringdal, 2006). Equation (2-6) gives the three-component, array-stacked correlation detector. We will see later in this chapter that stacking of the correlation traces across the array and over all three components brings additional processing gain which will facilitate the detection of events with low signal-to-noise ratios. It is worth pointing out that for detection purposes, the stacking of correlation traces is performed without move-out correction. An implicit assumption is that we are dealing with events close to the master event. On the other hand, the move-out in the \( C_{w}(t_{N,At}) \) across the array can be used to locate events relative to the master event.
if sufficient receiver aperture is available, such as the surface monitoring case with a two-dimensional receiver coverage (see Eisner et al., 2008).

A high cross-correlation coefficient on $C_{w}(t)_{N,At}$, $C_{w}(t)_{N,At}$ or $C_{w}(t)_{N,At}$ indicates the arrival of a microseismic event. A simple threshold for the cross-correlation coefficient serves as an efficient event detector. A further advantage of this detection method is that the master event can be updated with time to capture the hydrofracture propagation.

2.2.2 Transformed spectrogram phase picking

The correlation detector determines the occurrence of microseismic events. To locate the events, P and S arrivals must be picked at each 3-C geophone. Weak P arrivals pose a special challenge for time picking. To alleviate this problem, we use a transformed spectrogram approach to enhance weak P arrivals and to facilitate the P and S phase picking. We apply the multi-taper method, proposed by Thomson (1982), to calculate the spectrogram. The basic idea of the multi-taper spectrogram is that the conventional spectral analysis method suppresses the spectral leakage by tapering the data before Fourier transforming, which is equivalent to discarding data far from the center of the time series (setting it to small values or zero). Any statistical estimation procedure which throws away data has severe disadvantages, because real information is being discarded. The multi-taper method begins by constructing a series of N orthogonal tapers, and then applies the tapers to the original data to obtain N sets of tapered data. Because of the orthogonality of the tapers, there is a tendency for the N sets of tapered data to be nearly uncorrelated. If the underlying process is near-Gaussian, those N sets of tapered data are therefore nearly independent. Thus, the sum of Fourier transforms of these N sets of tapered data will give us an unbiased, stable and high-resolution spectral estimate. The multi-taper spectrogram is then differentiated with respect to time to enhance the phase-arrival. Next, a transformed spectrogram is formed by multiplying the differentiated spectrogram with the original spectrogram to highlight two features of a phase arrival: high energy increase and
high energy (Gibbons et al., 2008). Mathematically, let the spectrogram estimate within time window \([t, t + L]\) be \(A(f, t, L)\), the transformed spectrogram \(S(f, t)\) can be expressed as:

\[
S(f, t) = (\log[B(f, t, L)] - \log[B(f, t - L, L)]) \log[B(f, t, L)].
\] (2-7)

\[
B(f, t, L) = A(f, t, L)/ \min_{f_1} A(f, t, L).
\] (2-8)

The characteristic function of this transformed spectrogram is defined over the signal frequency range \([f_1, f_2]\) as:

\[
\tilde{S}([f_1, f_2], t) = \max \left\{ \frac{1}{N_f} \sum_{f=f_1}^{f_2} S(f, t), 0 \right\}.
\] (2-9)

where \(N_f\) is the number of frequency points over the microseismic signal frequency range \([f_1, f_2]\). The expression for \(S(f, t)\) is a multiplication of two terms: the first differential term represents the energy change from the previous time window \([t - L, t]\) to the current time window \([t, t + L]\), while the second term gives the energy within the current time window. The normalized spectrogram \(B(f, t, L)\) ensures a positive value of the second term in equation (2-7) so that \(S(f, t)\) is a monotonically increasing function with respect to the first energy change term. For any time \(t\), equation (2-9) looks for a positive energy change, i.e. energy increase. The two positive peaks on \(\tilde{S}([f_1, f_2], t)\) give the P- and S-wave arrivals. Furthermore, considering P- and S-waves may have different signal-to-noise ratios (SNR) on different components, this transformed spectrogram phase picking approach is applied to all 3-C data. The P- and S-wave arrivals are identified on the transformed spectrogram of the component that has the maximum SNR.

### 2.3 Field data example

A microseismic survey was performed during the hydraulic fracturing stimulation of a carbonate reservoir in Oklahoma. An 8-level geophone array was deployed in the monitoring well at a true vertical depth from 4545 ft to 4895 ft (Level-1 was the shallowest 3-C geophone). The treatment well is approximately 1450 ft away from the monitoring well. The perforation was conducted at a true vertical depth of 5030 ft.
Figure 2-la shows a segment of the continuous microseismic record. Unfortunately, level-8 failed to work, so only waveforms from 7-levels are available. Figure 2-1b shows that the most energetic part of low-frequency noise is concentrated mainly below 75 Hz. Additional signal spectral analysis demonstrates that most signal energy is below 300 Hz. Therefore, a band-pass filter of [75 300] Hz was applied to the raw data to get an enhanced signal as shown in Figure 2-1c. Figure 2-2 shows the three components (z, x, y) of the band-pass filtered data. The band-pass filtered data in Figure 2-2 show several microseismic events. The three largest events, noted as event 1, 2, and 3 with S-wave arrivals on level-1 at approximately 19.3 s, 8.3 s, and 28.0 s, are detected by the standard STA/LTA event detection algorithm. Another two smaller events (event 4, 5) around 13.5 s and 2.3 s are noticeable, but are hard to detect by the STA/LTA detector with a reasonable false alarm rate. To calculate the SNR of these 5 events, we define:

$$\text{SNR}(dB) = 10 \log_{10} \left( \frac{\sum_{k=1}^{3} \sum_{j=1}^{N_1} s_{at}^{jk}(i)^2}{\sum_{k=1}^{3} \sum_{j=1}^{N_2} n_{at}^{jk}(i)^2} \right) / N_1$$

(2-10)

where \(s_{at}^{jk}(i)\) and \(n_{at}^{jk}(i)\) denote the k-th component data of the event and noise recorded at the j-th receiver, with \(N_1\) and \(N_2\) being microseismic signal and noise window length. The calculated SNRs for event 1-5 on the band-pass filtered data are 15.3 dB, 12.4 dB, 11.7 dB, 6.5 dB and 6.1 dB, respectively. The largest event around 19.3 s is selected as the master event. Figure 2-3 shows the vertical component (z component) cross-correlation template, where both P- and S-wave arrivals are included. We apply three forms of correlation detector to the data in Figure 2-2. Figure 2-4b gives the one-geophone one-component correlation result (Level 1, vertical component), while Figure 2-4c and Figure 2-4d give the array-stacked correlation traces using only the vertical component and all three components respectively. Compared to the band-pass filtered data on Figure 2-4a, the one-geophone one-component correlation detector does not increase the SNR, which indicates the existence of some correlated noise. Figure 2-4c, however, gives better SNRs for two weak events 4 and 5 by stacking the vertical component correlation traces across all 7 geophones. The noise correlation level has decreased from 0.2 in
Figure 2-4b to 0.05 in Figure 2-4c after cross-geophone stacking. The correlation level for the weakest event 5 in Figure 2-4c is 0.45. This means that, by stacking the one-component correlation traces, the SNR for the weakest event 5 has increased from 6.1 dB in Figure 2-4a to 19.0 dB in Figure 2-4c. Figure 2-4d represents the array-stacked correlation traces across all three components. The noise correlation level further decreases to 0.03. The SNR for the weakest event 5 increases to 22.5 dB in Figure 2-4d. This additional 3.5 dB SNR gain over Figure 2-4c comes from matching in polarization structure by using all three components. Even for the master event (i.e. the strongest event), the SNR on the 3-C array-stacked correlation detector has been boosted from the original 15.3 dB in Figure 2-4a to 30.4 dB in Figure 2-4d. Two weak events 4 and 5 are easy to identify in Figure 2-4d. This shows that the three-component array-based correlation detector can effectively enhance the SNR of small microseismic events, and therefore is suitable to detect small-magnitude events with similar waveforms to a master event. In practice, we can use the STA/LTA detector to identify several large events, which can then be used as master events to detect their nearby weak events.

For each detected event, we use the transformed spectrogram approach as described in equations (2-7) ~ (2-9) to identify its P- and S-wave arrivals and compare it to standard STA/LTA picks (Earle and Shearer, 1994). We calculate the characteristic function \( \tilde{S}(\{f_1, f_2\}, t) \) out of all 7 geophones for all 5 detected events to pick the P- and S-wave arrivals on each 3-C geophone. Here \([f_1, f_2]\) is set as the microseismic signal frequency range, \([75, 300]\). The method is applied to all three components to get the optimal P- and S-picks. Take level-1 geophone for example, Figure 2-5 compares the manual picks (solid line), transformed spectrogram picks (dash line) and STA/LTA picks (dash-dot line) for the master event. P- and S-waves have the highest SNR on the horizontal and vertical component separately. Thus, P-wave arrival is picked from the horizontal component, while S-wave arrival is obtained from the vertical component. For this large event, the arrivals given by both methods are close to the manual picks, which means that we can use STA/LTA detector to identify master events and select \(t_m\). The arrivals identified by the peaks

55
on $\tilde{S}([75,300],t)$ are close to the onset of phase arrivals while the STA/LTA picks tend to give the peak arrival times. For the weakest event, as shown in Figure 2-6, the STA/LTA picks can hardly agree with manual picks due to the high noise level while the transformed spectrogram picks are consistent with manual picks. This illustrates that the transformed spectrogram facilitates picking of weak arrivals. The noise level has less influence on the characteristic function due to the differentiation term in equation (2-7). The shape of the characteristic function depends on the signal energy distribution over the time and frequency, and the window length $L$. The choice of $L$ depends on the balance between the sharpness of the P and S peaks (i.e. the resolution of arrival picks) and the occurrence of spurious peaks. From our experience, three to four times the dominant period is a good value.

2.4 Summary

In this chapter, we have proposed a systematic approach for hydrofracture event detection and phase picking. By field test, we have demonstrated that once a large event is detected by the standard STA/LTA detector, it can be used as the master event. A three-component array-stacked correlation detector using this master event template can effectively increase the detectability of nearby small-magnitude events. The three-component, array-stacked processing is superior to a single-component, single-geophone correlation detector. This processing gain increases with the increased number of geophones. The limitation of the correlation detector is that it is only capable to detect the events nearby a master event. However, as fracture propagates, we can also update the master event accordingly from newly detected large events either by our approach or by the STA/LTA detector. For phase picking, we applied the transformed spectrogram approach to 3-C data to identify the weak arrivals. The P- and S-wave arrivals are picked from the component which has the highest single-to-noise ratio for P- and S-wave vector separately. The transformed spectrogram captures the two features of a phase arrival in the time-frequency domain: high energy and high rate of energy increase, and therefore improves phase
picking. Detection and phase identification of small-magnitude microseismic events have potential for not only hydrofracture monitoring but also reservoir surveillance.

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**2.5 References**


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Figure 2-1: (a) A 32s raw vertical velocity data record from a three-component downhole geophone array. (b) Amplitude spectrum of the panel in (a) after summing over all traces. (c) The panel in (a) after [75, 300] Hz band-pass filtering.
Figure 2-2: [75, 300] Hz band-pass filtered velocity data: (a) z component (same as Figure 2-1(c)), (b) x component, (c) y component (Events 1, 2, 3 are detected by the STA/LTA detector, with event 1 selected as the master event for the correlation detector. Events 4 and 5, although visible, are hard to detect by the STA/LTA detector.).
Figure 2-3: Master event waveform as the cross-correlation template (vertical component of event 1 as shown in Figure 2-2(a)).
Figure 2-4: (a) [75, 300] Hz band-pass filtered vertical velocity data from geophone 1. (b) One-component one-geophone correlation detector output. (c) One-component, array-stacked correlation detector output. (d) Three-component, array-stacked correlation detector output.
Figure 2-5: Comparison of manual picks (solid line), transformed spectrogram picks (dash line), and STA/LTA picks (dash-dot line). (a) P-wave arrival picks on band-pass filtered x component data from geophone 1 for event 1 (the master event). (b) S-wave arrival picks on band-pass filtered z component data from geophone 1 for event 1. (c) Characteristic function $S([75, 300], t)$, as specified in equation (2-9), for the x component data, where P-wave arrival is identified as the first major peak. (d) $\tilde{S}([75, 300], t)$ for the z component data, where S-wave arrival is identified as the second major peak. (e) STA/LTA function for x component data. (f) STA/LTA function for z component data.
Figure 2-6: Comparison of manual picks (solid line), transformed spectrogram picks (dash line), and STA/LTA picks (dash-dot line). (a) P-wave arrival picks on band-pass filtered x component data from geophone 1 for event 5 (the weakest event). (b) S-wave arrival picks on band-pass filtered z component data from geophone 1 for event 5. (c) Characteristic function $S([75, 300], t)$, as specified in equation (2-9), for the x component data, where P-wave arrival is identified as the first major peak. (d) $S([75, 300], t)$ for the z component data, where S-wave arrival is identified as the second major peak. (e) STA/LTA function for x component data. (f) STA/LTA function for z component data.
Chapter 3

Full Waveform Based Microseismic Event Detection and Signal Enhancement: The Subspace Approach

Abstract

Microseismic monitoring has proven to be an invaluable tool for optimizing hydraulic fracturing stimulations and monitoring reservoir changes. The signal to noise ratio (SNR) of the recorded microseismic data varies enormously from one dataset to another, and it can often be very low especially for surface monitoring scenarios. Moreover, the data are often contaminated by correlated noises such as borehole waves in the downhole monitoring case. These issues pose a significant challenge for microseismic event detection. On the other hand, in the downhole monitoring scenario, the location of microseismic events relies on the accurate polarization analysis of the often weak P-wave to determine the event azimuth. Therefore, enhancing the microseismic signal, especially the low SNR P-wave data, has become an important task. In this study, a statistical approach based on the binary hypothesis

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test is developed to detect the weak events embedded in high noise. The method constructs a vector space, known as the signal subspace, from previously detected events to represent similar, yet significantly variable microseismic signals from specific source regions. Empirical procedures are presented for building the signal subspace from clusters of events. The distribution of the detection statistics is analyzed to determine the parameters of the subspace detector including the signal subspace dimension and detection threshold. The effect of correlated noise is corrected in the statistical analysis. The subspace design and detection approach is illustrated on a dual-array hydrofracture monitoring dataset. The comparison between the subspace approach, array correlation method, and array short-time average/long-time average (STA/LTA) detector is performed on the data from the far monitoring well. It is shown that, at the same expected false alarm rate, the subspace detector gives fewer false alarms than the array STA/LTA detector and more event detections than the array correlation detector. The additionally detected events from the subspace detector are further validated using the data from the nearby monitoring well. The comparison demonstrates the potential benefit of using the subspace approach to improve the microseismic viewing distance. Following event detection, a signal enhancement method is proposed by projecting the total energy into the signal subspace. Examples on field data are presented indicating the effectiveness of the subspace-projection-based signal enhancement procedure.

3.1 Introduction

Microseismic monitoring has become a valuable tool for understanding physical processes in the subsurface. Besides its most common use in hydrofracture monitoring, it is also widely used for reservoir surveillance, geothermal studies, and monitoring of CO₂ sequestration (Phillips et al., 2002; Maxwell et al., 2004; Warpinski, 2009).

The occurrence of microearthquakes follows a frequency-magnitude power law relation similar to tectonic earthquakes (Maxwell et al., 2006). The majority of
microseismic events occur in the low magnitude range with a typical Richter magnitude $M_L < -1$. Moreover, the recorded microseismic waveforms are usually contaminated by the high amplitude noise. In downhole monitoring of hydraulic fracturing, as is the case for this study, the high amplitude noise may come from various sources, most notably from the borehole waves excited by pumps located at the surface. Therefore, the recorded microseismic data normally have a very low signal-to-noise ratio (SNR). This low SNR poses a great challenge in processing microseismic data and leads to two major consequences. Firstly, the accurate time picking of the P- and S-wave arrivals for individual events becomes a difficult task, which impacts the accuracy of the microearthquake location and, indeed, the success of the microseismic monitoring. Secondly, the low SNR values set a detection limit. As such, the minimum detectable event magnitude increases with increased distance from monitoring geophones due to the increased signal attenuation with distance. This causes the viewing-distance bias, which can be a significant issue when interpreting the completeness of the fracture geometry (Maxwell et al., 2010; Warpinski, 2009).

Known methods for automated microseismic event detection include short-time-average/long-time-average (STA/LTA) detectors and correlation-type detectors. The STA/LTA detector calculates the energy ratio of short-time window to long-time window and declares the appearance of seismic events when the ratio exceeds a threshold (Earle and Shearer, 1994). The correlation detector screens seismic events by calculating a correlation coefficient between the received signal and a template event known as the master event, assuming events that are to be detected have similar waveforms as the master event (Gibbons and Ringdal, 2006; Song et al., 2010).

Simple STA/LTA detectors are broadly applicable, but suffer from high false alarm rates when an aggressive threshold is set to detect smaller signals. Correlation detectors are highly sensitive, having high detection probability at low false alarm rates. However, they are applicable only to repetitive sources confined to very compact source regions.

Unlike the above two approaches, a detection method based on statistical hypothesis testing has been proposed to take into account the statistics of both signal
and noise (Bose et al., 2009). In their detection algorithm, the microseismic event signal recorded at the downhole geophone array is assumed to be a scaled and delayed version of a common trace. The common trace is modeled as a deterministic Ricker wavelet signal convolved with a finite impulsive response (FIR) filter. The FIR filter is determined by maximizing the detection likelihood. Although the common trace can be adjusted from one detection window to another, there is only one microseismic signal template in each detection window; it therefore faces similar difficulties as correlation detectors. Moreover, this method cannot take advantage of previously detected events.

In order to overcome these limitations, we adapt the subspace detection method of Harris (2006) to replace the single matching template in a correlation detector with a suite of basis vectors (known as the signal subspace) that are combined linearly to match occurrences of variable signals from a specific source region. We extend the surface monitoring setup to the downhole monitoring configuration and consider correlated noises with different variances on different geophone channels. We introduce a systematic procedure to determine the parameters for the subspace detector.

The subspace design and detection approach is demonstrated on a dual-array hydrofracture monitoring dataset. We compare the subspace approach, array correlation method, and array short-time average/long-time average (STA/LTA) detector using the data from the far monitoring well. The additionally detected events from the subspace detector are further validated using the data from the nearby monitoring well. The comparison illustrates the effectiveness of using the subspace approach to improve the detection capability. Furthermore, we develop a subspace projection approach to enhance the SNR of detected microseismic signals. Signal enhancement results on the field dataset are presented.

Exposition in this chapter is necessarily mathematical. The number of symbols is sufficiently large that a table of symbols has been included (Table 3-1). To keep the number of symbols to a minimum, a few conventions have been adopted. First, the underlined lower-case symbol indicates a column vector, while a matrix is shown as
the underlined upper-case symbol. Second, a symbol with a “hat” denotes the estimated value. When it refers to the embedding space dimension, the effect of correlated noise has been corrected.

3.2 Methodology

3.2.1 Subspace detector theory: statistical binary hypothesis testing

Current practice in seismic event detection is concentrated at the extremes of a spectrum of possibilities determined by the amount of information available about the temporal structure of signals to be detected. On one end, incoherent energy detectors, such as STA/LTA detectors, assume little knowledge of the underlying signals. On the other end, correlation detectors assume a completely known signal and coherently use the fine temporal and spatial structure of detected seismic signals to enhance the sensitivity (Harris, 1991). STA/LTA detectors are broadly applicable, but are insensitive to waveform information and thus have a high false alarm rate, while correlation detectors are sensitive to waveforms and have fewer false alarms, but are less flexible and are applicable only to repetitive sources. Therefore, the subspace detector was proposed to manage the tradeoff between sensitivity and flexibility (Scharf and Friedlander, 1994; Harris, 2006).

The basic idea of subspace detection is to replace the single matching template in the correlation detector with a suite of basis vectors, known as the signal subspace, that are combined linearly to match the occurrence of variable signals from a specific source region. The subspace detector is implemented as a binary hypothesis test on a window that slides along a continuous data stream.

The microseismic data recorded at multiple channels are multiplexed into a continuous stream according to the following equation:

\[ x((n-1) \cdot N_c + i) = x_i(n), \]  

(3-1)
wherein $x_i(n)$ is the $n$-th time sample from the $i$-th channel, where $n = 1, 2, ..., i = 1, 2, ..., N_c$. In the downhole setup, the three component (3C) data from all levels of geophones are multiplexed. The data within each window may be presented as

$$
\mathbf{x}(n) = \begin{bmatrix} x_1(n) & x_2(n) & \cdots & x_{N_c}(n) & x_1(n+1) & x_2(n+1) & \cdots & x_{N_c}(n+N_T-1) \end{bmatrix}^T.
$$

(3-2)

$\mathbf{x}(n)$ is an $N \times 1$ vector with $N = N_c \times N_T$, where $N_T$ is the temporal length of the window.

The subspace detection is posed as a binary hypothesis testing problem. Under the null hypothesis ($H_0$), the windowed data $\mathbf{x}(n)$ are assumed to consist of noise. In the alternative hypothesis ($H_1$), the data consist of both signal $\mathbf{s}$ and noise $\mathbf{n}$.

$$
\begin{align*}
\{ & \mathbf{x}(n) = \mathbf{n} \quad \text{under hypothesis } H_0 \\
& \mathbf{x}(n) = \mathbf{s} + \mathbf{n} \quad \text{under hypothesis } H_1
\end{align*}
$$

(3-3)

The noise $\mathbf{n}$ is assumed to be zero-mean Gaussian noise with an unknown variance $\sigma^2$. It is also assumed to be temporally and spatially uncorrelated. The signal $\mathbf{s}$ is assumed to be deterministic but dependent upon a vector of unknown parameters $\mathbf{a}$, and is expressed as an unknown linear combination of basis waveforms:

$$
\mathbf{s} = \mathbf{U} \mathbf{a},
$$

(3-4)

where the $N \times d$ matrix $\mathbf{U}$ represents $d$ unknown signal subspace bases. The signal subspace dimension $d$ may take any value from 1 to $N$. Without losing generality, $\mathbf{U}$ can be made orthonormal:

$$
\mathbf{U}^T \mathbf{U} = \mathbf{I}_d.
$$

(3-5)

Therefore, energy captured in the signal subspace may be simplified to

$$
E_c = \mathbf{a}^T \mathbf{a}.
$$

(3-6)

Under the above assumptions, the probability function for the recorded data is

$$
p(\mathbf{x}(n)|H_0) = \frac{1}{(2\pi \sigma^2)^{N/2}} \exp \left( -\frac{1}{2\sigma^2} \mathbf{x}^T(n)\mathbf{x}(n) \right)
$$

(3-7)

under the null hypothesis $H_0$ (no event present), and

$$
p(\mathbf{x}(n)|H_1) = \frac{1}{(2\pi \sigma^2)^{N/2}} \exp \left( -\frac{1}{2\sigma^2} \left( \mathbf{x}(n) - \mathbf{Ua} \right)^T \left( \mathbf{x}(n) - \mathbf{Ua} \right) \right)
$$

(3-8)

under the alternative hypothesis $H_1$ (event present). The generalized log likelihood ratio can be derived as (Van Trees, 1968),
\[ l(\tilde{x}(n)) = \ln \left( \Lambda(\tilde{x}(n)) \right) = \ln \left[ \frac{\max_{\sigma} \sigma p(\tilde{x}(n)|H_1)}{\max_{\sigma} \sigma p(\tilde{x}(n)|H_0)} \right] = -\frac{N}{2} \ln[1 - c(n)]. \] (3-9)

The ratio of energy in the signal projected into the signal subspace \( \mathbf{U} \) to the energy in the original data is represented by the quantity \( c(n) \), known as the subspace detection statistics,

\[ c(n) = \frac{x_p^T(n)x_p(n)}{x^T(n)x(n)}. \] (3-10)

The projected signal \( x_p(n) \) is the least-squares estimate of the signal \( \tilde{x}(n) \) in the detection window,

\[ x_p(n) = \mathbf{U}^T \tilde{x}(n). \] (3-11)

The subspace detection statistics \( c(n) \) is a positive quantity with values ranging between 0 and 1. The generalized likelihood ratio test detects an event of interest if the generalized log likelihood ratio exceeds a certain threshold \( \alpha \), that is

\[ l(\tilde{x}(n)) = -\frac{N}{2} \ln(1 - c(n)) > \alpha. \] (3-12)

It means that an event is declared if the subspace detection statistics \( c(n) \) is larger than a detection threshold \( \gamma \),

\[ c(n) = \frac{x_p^2(n)x_p(n)}{x^2(n)x(n)} > \gamma. \] (3-13)

### 3.2.2 Subspace detector implementation: subspace design

To perform subspace detection based on equations (3-11) and (3-13), the first step is to construct the signal subspace bases \( \mathbf{U} \). Harris (2006) suggested a way to build the signal subspace from previously detected seismic events. The objective in constructing the signal subspace representation \( \mathbf{U} \) is to obtain acceptably accurate orthogonal bases for seismic signals characteristic of events of interest in the target source region. A representation with a larger dimension provides a higher possibility of detecting weak events by capturing more of the energy of an incompletely known signal. However, a higher order dimension representation may also be expected to increase the false alarm rate by allowing the detector to match noise with great probability. Consequently, a parsimonious representation with an adequate signal...
energy capture is desired. Assuming that $D$ previously detected events are selected to construct the signal subspace, for a given representation order of $d$, the signal subspace bases $U$ should capture as much energy in the $D$ design set events as possible.

For each event in the design set, the aligned channel-multiplexed data vector may be written as,

$$s(n) = [s_1(n) s_2(n) \ldots s_{N_c}(n) s_1(n+1) s_2(n+1) \ldots s_{N_c}(n+N_T-1)]^T.$$  \hspace{1cm} (3-14)

The design data matrix $S$ is assembled with $D$ channel-multiplexed column vectors, with each column representing one design set event,

$$S = [s_1(n) s_2(n) \ldots s_D(n)].$$  \hspace{1cm} (3-15)

To prevent large events in the design set from dominating the design data matrix, data from each event are normalized to have unit energy, that is,

$$s_i^T(n)s_i(n) = 1, \quad i = 1, 2, \ldots, D.$$  \hspace{1cm} (3-16)

The singular value decomposition (SVD) of the design data matrix $S$ is

$$S = \Sigma \Sigma^T \Sigma \Sigma^T = WA,$$  \hspace{1cm} (3-17)

where $A = \Sigma \Sigma^T$ is the representation coefficient matrix. According to Eckart and Young (1936), the best approximation to $S$ in the least-squares sense for a given order $d$ is the truncated SVD of the matrix to the rank $d$. Consequently,

$$S = WA = [W_d \quad W_{D-d}] [\Sigma_d \ 0] \begin{bmatrix} V_d^T \ 0 \end{bmatrix} \Sigma_{d}^{-1} V_d^T \Sigma_{d}^{-1} = U_A d$$  \hspace{1cm} (3-18)

$$U = W_d$$  \hspace{1cm} (3-19)

$$A_d = \Sigma_{d} V_d^T$$  \hspace{1cm} (3-20)

The matrix of coefficients $A_d$ provides an expression of the energy captured in the signal subspace $U$ corresponding to the first $d$ largest singular values for the $D$ events in the design set. Consider, for example, the $i$-th design set event,

$$s_i \sim U a_i,$$  \hspace{1cm} (3-21)

where $a_i$ is the $i$-th column of $A_d$, the fractional energy capture for this event is

$$f_i = \frac{a_i^T a_i}{s_i^T(n)s_i(n)} = a_i^T a_i.$$  \hspace{1cm} (3-22)
The average fraction of energy captured for all D events in the design set may accordingly be calculated as:

$$\overline{f_c} = \frac{1}{D} \sum_{i=1}^{D} f_c^i = \text{trace}(\Sigma_d \Sigma_d^T)/\text{trace}(\Sigma^T \Sigma).$$

(3-23)

The fractional energy capture for each of the D events and the average fraction of energy captured for all D events may be plotted as a function of the dimension of representation d, also referred to as the signal subspace dimension. Each fractional energy capture curve extends from 0 to 1, and increases with increased dimension of representation. When d reaches a certain value, the average fraction of energy captured for all design events exceeds a predetermined threshold (e.g., 80%). This may be used as an aid to determining d. In the field study section, an example will be shown to determine the signal subspace dimension d by generating and processing plots, as described above.

The question remaining is to select the D design set events to represent the source region of interest. The single-link clustering algorithm is used to serve this purpose (Israelsson, 1990).

A template event library is built upon the previously identified events, for example based on the output of a STA/LTA detector with a conservative threshold. The multi-channel time series data are converted to a single channel multiplexed data vector according to equation (3-1). Pair-wise correlation coefficients for the template events in the library are then calculated from the single channel multiplexed data vector. Assuming that there are M events in the library, the event dissimilarity matrix $K$ with a size of $M \times M$ is constructed using the following equation:

$$K_{p,q} = 1.001 - \lambda_{p,q}.$$  

(3-24)

$K_{p,q}$ may be viewed as a measure of inter-event distance in waveform similarity space for events p and q, where $\lambda_{p,q}$ is the maximum waveform correlation between events p and q.

Next, the events are automatically clustered based on the dissimilarity matrix $K$. Various clustering algorithms may be used in different embodiments. The choice of clustering algorithm depends on objectives and expectations in characterizing a
source. In the context of hydraulic fracturing, the assumption is often that the source region to be characterized through representative waveforms may have significant variation in source mechanisms, some variation in source time function and location, or some combination of all these attributes. To this end, in this chapter, an aggressive algorithm for linking events into relatively extensive chains spanning the space of waveform variations, in the form of a single-link method is employed.

The single-link algorithm aggregates event clusters based solely on the single pair of events (one event in each of two clusters under consideration for merging) with a large waveform correlation (Israelsson, 1990). The hierarchical agglomerative clustering procedure is presented as a dendrogram, as will also be illustrated in the field study section. At each clustering step, the cophenetic correlation coefficient is calculated to measure how well the clustering models the actual event dissimilarity behavior, which is described in matrix \( K \) (Rowe et al., 2002). The cophenetic correlation coefficient may serve as an aid to select the design set events. Sudden decreases in the cophenetic correlation indicate that the cluster just formed has made the dendrogram less faithful to the data and thus may suggest that the clustering process should be terminated between this cluster and the previous one. The events that have been clustered up to that point are then automatically selected as the design set events. The design set is a set of events that are to be used to construct the signal subspace bases. Therefore, it is desirable for the design set to not only represent the actual inter-event correlation behavior described by the original dissimilarity matrix \( K \), but also to comprise most of the larger events in the event library. To this end, the waveform root-mean-square amplitudes of the automatically selected events are checked to ensure that most of the larger events in the library are included. The details of the single-link clustering algorithm and design set event selection are further illustrated in Appendix A.

### 3.2.3 Subspace detector implementation: parameter determination and performance evaluation
Once the signal subspace \( \mathcal{U} \) is constructed, in order to conduct subspace detection based on equations (3-11) and (3-13), it is necessary to determine the detection threshold \( \gamma \). Harris (2006) studied the distribution of the subspace detection statistics \( c(n) \) and derived the threshold using the Neyman-Pearson criterion (Van Trees, 1968). Under this criterion, the subspace dimension \( d \) is firstly determined by maximizing the probability of detection \( P_D \) for a fixed false alarm rate \( P_F \). The threshold \( \gamma \) is then derived from the false alarm rate using the following equation:

\[
\left\{ \begin{array}{l}
1 - F_{d,N-d} \left( \frac{\gamma}{1 - \gamma} \right) = P_F \\
1 - F_{d,N-d} \left( \frac{\gamma}{1 - \gamma} \cdot \bar{t}_c \cdot N \cdot \text{SNR}, (1 - \bar{t}_c) \cdot N \cdot \text{SNR} \right) = P_D
\end{array} \right.
\]  

(3-25)

where \( P_F \) is evaluated from the cumulative central \( F \) distribution \( F_{d,N-d}(\cdot) \) with \( d \) and \((N-d)\) degrees of freedom under null hypothesis \( H_0 \), while \( P_D \) is expressed in terms of the cumulative doubly non-central \( F \) distribution with the same degrees of freedom and a non-centrality parameters of \( (\bar{t}_c \cdot N \cdot \text{SNR}) \) and \([(1 - \bar{t}_c) \cdot N \cdot \text{SNR}] \) for the numerator and denominator term, respectively. \( \bar{t}_c \) is the average fraction of energy captured for all \( D \) design set events, defined in equation (3-23). \( N \) denotes the embedding space dimension, out of which \( d \) is the signal subspace dimension, and \((N-d)\) is the dimension of the orthogonal complement of the signal subspace. \( \text{SNR} \) is the signal-to-noise ratio in the detection window, defined as

\[
\text{SNR} = E / \sigma^2 / N,
\]  

(3-26)

where \( E \) is the signal energy over the detection window of length \( N \), and \( \sigma^2 \) denotes the unknown Gaussian noise variance. The details of the derivation of equation (3-25) are further presented in Appendix B.

When deriving equation (3-25), two implicit assumptions are made. Firstly, the Gaussian noise variance \( \sigma^2 \) is identical across all \( N_c \) channels. This is not generally true. As we will see in the following field study section, the noise variances on different channels are estimated from the pre-event noise and the microseismic data from different channels are normalized by their relative variances before multiplexing (equation (3-2)). Secondly, the noise in the detection windows is assumed to be statistically uncorrelated. If the detection window is \( N \) samples long, the dimension of
the embedding space, where the signal subspace resides, is also $N$. As Wiechecki-Vergara et al. (2001) point out, the effective dimension of the embedding space can be significantly lower than $N$ if the data are filtered prior to detection. Even without filtering, noise is typically correlated and helps reduce the effective dimension of the embedding space. To correct for the influence of the correlated noise, according to Wiechecki-Vergara et al. (2001), the effective dimension of the embedding space $\bar{N}$ is related to the variance of the sample correlation coefficient $\hat{c}_{ij}$ between noise data $\hat{n}$ and event signal $s$ by:

$$\bar{N} = 1 + \frac{1}{\sigma_{\hat{c}}^2} N.$$  \hspace{2cm} \text{(3-28)}$$

Considering a decrease in the effective embedding space dimension resulted from the correlated noise and/or data pre-processing, equation (3-25) is rewritten as

$$\left\{ \begin{array}{ll} 1 - F_{d, N-d} \left( \frac{\gamma \cdot \bar{N} - \gamma}{\gamma \cdot \bar{N} - \gamma} \right) = P_F \\
1 - F_{d, N-d} \left( \frac{\gamma \cdot \bar{N} - \gamma}{\gamma \cdot \bar{N} - \gamma} \cdot \tilde{r}_c \cdot N \cdot \text{SNR}, \left(1 - \tilde{r}_c \right) \cdot N \cdot \text{SNR} \right) = P_D \end{array} \right. \hspace{2cm} \text{(3-29)}$$

Equation (3-29) gives the average probability of detection for the events in the design set assuming the design events are all equally likely. If the signals in the design set span the range of signals produced by the source of interest, the calculated average probability of detection $P_D$ in equation (3-29) also indicates the detection probability for all possible events from this source region.

According to equation (3-29), assuming a given false alarm rate $P_F$, the detection threshold $\gamma$ can be related to the signal subspace dimension $d$. Therefore, for the given false alarm rate $P_F$, the average probability of detection $P_D$ over a SNR range of interest is a sole function of the signal subspace dimension $d$. A value for the signal subspace dimension $d$ and the detection threshold $\gamma$ are thus determined.
As illustrated in the previous section, the subspace dimension $d$ can also be derived from the average fractional energy capture plots. We will discuss the $d$ values determined by equation (3-29) and by the fractional energy capture plots in the field study section.

Note that when using equation (3-29) to determine $d$ and $r$, a known $P_F$ is assumed. In this study, we will compare the subspace detection results with those from array correlation and STA/LTA detection. Therefore, we propose a method to determine $P_F$ from the correlation detector and compare the three types of detectors under the same $P_F$.

The array correlation detector on the channel-multiplexed data can be written as:

$$
\hat{c} = \frac{s_m^T \mathbf{x}}{\sqrt{(s_m^T s_m)(\mathbf{x}^T \mathbf{x})}},
$$

(3-30)

where $s_m$ is the correlation template, i.e. master event data, and $\mathbf{x}$ is the windowed data to be detected. Both of them have been band-pass filtered and, therefore, have a zero mean. A template event with a good SNR in both P- and S-waves is selected as the correlation template. The correlation detection threshold $\rho_c$ can be estimated from the histogram plot of template event-noise correlation and correlation between template events, which will be illustrated in the field study section. A comparison between equation (3-10) and equation (3-30) shows that the correlation coefficient $\hat{c}$ is equivalent to the square root of the subspace detection statistics $c(n)$ with a signal subspace dimension of $d = 1$. Therefore, the correlation detector (equation (3-30)) can be implemented as a subspace detector with $d = 1$,

$$
c(n) = \frac{(s_m^T \mathbf{x})^2}{(s_m^T s_m)(\mathbf{x}^T \mathbf{x})},
$$

(3-31)

In this chapter, we define $c(n)$ in equation (3-31) as the correlation detection statistics.

The detection threshold associated with equation (3-31) is

$$
\gamma_c = \rho_c^2.
$$

(3-32)

According to equation (3-29), both detectors have a false alarm rate of

$$
P_F = 1 - F_{1,N-1} \left( \frac{\gamma_c}{1 - \gamma_c} \right).
$$

(3-33)

The array correlation algorithm claims an event if

$$
c(n) > \gamma_c.
$$

(3-34)
The array STA/LTA detector is employed on the channel multiplexed data as,

\[
    r(n) = \frac{\mathbf{x}_{\text{STA}}(n)^T \mathbf{x}_{\text{STA}}(n)}{\mathbf{x}_{\text{LTA}}(n)^T \mathbf{x}_{\text{LTA}}(n)}/N_{\text{STA}}
\]

and

\[
    \begin{align*}
        \mathbf{x}_{\text{STA}}(n) &= [x_1(n)x_2(n) ... x_{N_c}(n)x_1(n + 1)x_2(n + 1) ... x_{N_c}(n + N_{\text{STA}} - 1)]^T \\
        \mathbf{x}_{\text{LTA}}(n) &= [x_1(n - N_{\text{LTA}}) ... x_{N_c}(n - N_{\text{LTA}}) ... x_1(n - 1) ... x_{N_c}(n - 1)]^T
    \end{align*}
\]

\[N_{\text{STA}} \text{ and } N_{\text{LTA}} \text{ are the STA, LTA window lengths in samples, respectively. In the field study, typical values of 3 and 15 times the dominant period are selected as the STA and LTA window lengths (Song et al., 2010). Following the analysis in Appendix B, under the null hypothesis H_0, the STA/LTA detection statistics } r(n) \text{ has a central F distribution with } \nu_1 \text{ and } \nu_2 \text{ degrees of freedom. Therefore, at the same false alarm rate } P_F \text{ as specified in equation (3-33), the array STA/LTA detection threshold } \gamma_r \text{ can be calculated by}
\]

\[
P_F = 1 - \mathcal{F}_{\nu_{\text{STA}}, \nu_{\text{LTA}}}(\gamma_r).
\]

\[\nu_{\text{STA}} \text{ and } \nu_{\text{LTA}} \text{ are the effective embedding space dimension of the STA and LTA window, respectively, which can be calculated using the method proposed in Wiechecki-Vergara et al. (2001). An event is declared from the array STA/LTA algorithm if}
\]

\[
r(n) > \gamma_r.
\]

The processing steps of subspace detection and signal enhancement can be summarized as follows:

1. analyze the spectrum of the recorded data and determine the filter parameters;
2. apply the band-pass filter; multiplex the filtered continuous data using only channels with good SNRs to form the channel-multiplexed data stream shown in equation (3-1);
3. perform the initial detection on the channel-multiplexed data stream with STA/LTA algorithm according to equation (3-35);
4. form the template event library and noise data library out of the initial detection; estimate the noise mean and variance on each geophone channel; normalize the filtered data in step 2 by noise variances and form the new channel-multiplexed data stream;
(5) calculate the waveform correlation pairwise for all M events in the template event library; carry out the single-link clustering algorithm on the template event correlation data and select D out of M template events to construct the design set; align the waveforms of D design set events and extract the temporal window including both P- and S-waves that will be used to form the design data matrix S on equation (3-15) and define the signal subspace template;

(6) estimate the effective embedding space dimension \( \tilde{N} \) using equations (3-27) and (3-28); carry out the array correlation detection (equations (3-31), (3-34)) on the new channel-multiplexed data stream and determine the correlation detection threshold \( \gamma_c \) and the corresponding false alarm rate \( P_F \) from equations (3-32) and (3-33);

(7) for the calculated false alarm rate \( P_F \) in step 6, determine the signal subspace dimension \( d \) by equation (3-29) and the fractional energy capture plots; calculate the subspace detection threshold \( \gamma \) by equation (3-29); construct the signal subspace \( U \) from the design data matrix \( S \) according to equations (3-18) and (3-19); calculate the array STA/LTA detection threshold \( \gamma_r \) by equation (3-37);

(8) conduct the subspace detection (equations (3-10), (3-13)) and array STA/LTA (equations (3-35), (3-38)) on the new channel-multiplexed data stream; compare the subspace detection, array STA/LTA and array correlation detection (step 6) results;

(9) for detected events, employ the subspace projection approach (equation (3-11)) to enhance the weak microseismic signals.

3.3 Field study

3.3.1 Field setup

A microseismic survey was conducted during the fracture stimulation of the Mesa Verde and Cameo Formations in the Mamm Creek Field of the Piceance Basin, Colorado at a depth approximately from 1310 m (4300 ft) to 1981 m (6500 ft) (Weijers et al., 2009). A total of 40 stages of hydraulic fracturing treatments in five wells were mapped. Figure 3-1 shows the located microseismic events from one stage.
stimulation in fracturing well 24D. The microseismic data were collected using two
twelve-level, three-component (3C) geophone arrays deployed in two offset
monitoring wells at a depth from 1905 m (6250 ft) to 2050 m (6726 ft). The
monitoring well 13B is approximately 457 m (1500 ft) away from the fracturing well
24D, while the monitoring well 24C is closer to the fracturing well 24D, at a distance
of about 117 m (386 ft). Therefore, on Figure 3-1, fewer events are observed from the
far well 13B than the nearby well 24C. Two event clusters appear on Figure 3-1, with
a cluster comprising the majority of the events located around well 24C and another
minor cluster close to well 13B. Clearly, the microseismicity map is discontinuous
and has a gap between the two clusters. This illustrates the footprint of a limited
viewing distance from far well 13B, which hinders the interpretation of microseismic
maps.

In this study, we apply the subspace approach to improve the detection capability
for far well 13B. We also perform STA/LTA and array correlation algorithms on the
data from well 13B and compare the detection results with those from the subspace
detector. In the following section, we will follow the processing flow proposed in the
methodology section and begin with data pre-processing and signal subspace
construction. Next, we will determine the detection parameters and conduct the
subspace, array STA/LTA and array correlation detections. After that, we will discuss
the results from all 3 detectors. Finally, we will present the signal enhancement results
using the subspace projection method.

3.3.2 Data pre-processing and signal subspace design

Figure 3-2 shows the 3C raw data for a typical microseismic event recorded by the
12 geophones in well 13B. It is clear that data from geophones 7-12 have higher
SNRs. On Figure 3-2c, even high-amplitude S-waves are hard to see on geophones 1-
6. Therefore, in this chapter, only data from geophones 7-12 in well 13B are used in
the following analysis.

Noise suppression is an essential step prior to detection. Figure 3-3 gives the
spectrum analysis of the raw data for a typical event and noise file. A comparison of
the average amplitude spectrum between noise and microseismic signals indicates a
dominant signal frequency of [100, 400] Hz, which is depicted as the black square in
Figure 3-3c. Thus, prior to detection, a band-pass filter of [100, 400] Hz was applied
to the raw data to get an enhanced signal as shown in Figure 3-4. The filtered data in
Figure 3-4b demonstrates an enhanced SNR over the raw data in Figure 3-4a. The
band-pass filtered 3C continuous data from geophones 7-12 were next multiplexed to
form a channel-multiplexed data stream according to equation (3-1).

In order to build the template event library for subspace construction, an initial
detection was conducted on the channel-multiplexed data using the STA/LTA
algorithm (equation (3-35)). The detection results on a 30-minute continuous record
are plotted in Figure 3-5b. At a conservative threshold of 30, 20 events with good
waveforms are identified and shown as red stars in Figure 3-5b. Therefore, an initial
template event library comprising $M = 20$ events was built. It is worth noting that
several false alarms (fake events) appear on Figure 3-5b. This is due to the lack of
sensitivity to waveforms, which is also the motivation to develop correlation and
subspace algorithms.

As discussed in the methodology section, when developing the subspace detection
theory, we assumed the noises on different channels are Gaussian distributed with a
zero mean and an identical variance. However, in practice, this assumption may not
be true. For geophones at different depths and distances from the microseismic event,
the noise variance may vary. Figure 3-6a-c shows the noise standard deviation of the
identified 454 noise files from the initial detection on different geophones and
components. It is clear that the noise standard deviation varies over different
geophones and components. It is interesting to point out that the vertical component
data, shown on Figure 3-6c, generally have larger standard deviations compared to
horizontal components. This may be due to the poorer coupling of the vertical
component geophone (Song and Toksöz, 2011). Figure 3-6d-f gives the noise mean
across different geophones and components. It is presented as a multiple of the
waveform absolute maximum value on the corresponding channel. It is observed on
Figure 3-6d-f that the band-pass filtered noise data have negligible mean values. To
comply with the noise model assumed by the subspace detector, the continuous data from different channels were normalized by their noise standard deviation values and multiplexed to form a new channel-multiplexed data stream. The new channel-multiplexed data have zero mean and identical variance. In the following section, we will apply array STA/LTA, correlation and subspace detection algorithms on the new channel-multiplexed data and compare their detection results.

Figure 3-7 plots the identified 20 template events from Figure 3-5 after the noise standard deviation normalization. On the common geophone plot of Figure 3-7a, significant waveform variations across events are observed. The motivation of subspace detection is to preserve these variations in the subspace representation and detect more events under the same false alarm rate. The template event data were windowed to include both P- and S-waves and the pair-wise correlation between template events was calculated. On the single event plot of Figure 3-7b, coherent P- and S-wave arrivals across the geophone array are seen.

The pair-wise dissimilarity distances between template events were calculated from event correlation values according to equation (3-24). The template events were then clustered based on the pair-wise event dissimilarity distance $K$ via the single-link algorithm. In the single-link algorithm, events are aggregated and aligned in a sequential manner (Israelsson, 1990). The dendrogram in Figure 3-8 shows the hierarchical clustering process. Events 11 and 19 have the smallest dissimilarity distance $K$ (i.e. the largest correlation) and are clustered first. Next, the dissimilarity distances between the remaining 18 events and the newly formed event cluster (11, 19) are updated using the single-link algorithm (see Appendix A for details) and the clustering continues with the second smallest dissimilarity distance and forms a bigger cluster (11, 19, 5). The clustering goes on until all 20 events in the template event library have been clustered. The result is shown in Figure 3-8. At each clustering step, the cophenetic correlation coefficient (see Appendix A for details) is calculated to measure how well the clustering preserves the actual event dissimilarity behavior. A sudden decrease in the cophenetic correlation indicates the termination of the clustering and the formation of the design set. In this study, we also consider the event
dissimilarity distance threshold when forming the design set. The dissimilarity distance threshold cannot be too small or too large, since a small threshold cannot preserve the waveform variations and a large threshold will lose the sensitivity to waveforms. Another consideration for the choice of design set events is to include as many large amplitude events as possible. Therefore, we select a dissimilarity distance threshold of 0.6. It generates a design set of $D = 12$ events as shown in Figure 3-8.

As the clusters are aggregated, the waveforms from the design set events are also aligned to form the design data matrix $S$ (see Appendix A for details on the waveform alignment). Proper alignment is crucial, as poor alignment will result in a subspace operator with a larger than necessary number of dimensions (Harris, 2006). Figure 3-9 shows the alignment result. Figure 3-9a gives the design set event waveforms before alignment, while Figure 3-9b presents the aligned waveforms. Several sub-clusters show up on Figure 3-9b, as also seen on Figure 3-8. Overall, the dominant phases such as the P- and S-waves are clearly aligned.

In order to construct the signal subspace operator $U$ from the design data matrix $S$ by equations (3-17) and (3-19), the signal subspace dimension $d$ has to be determined. One way to determine $d$ is to look at the fractional energy capture $f_c^D$ for the $D$ design set events as a function of $d$. Figure 3-10a shows the fractional energy captured for each design set event in blue. The fractional energy capture curves all begin at 0 and end at 1 and increase with increased dimension of representation (i.e. signal subspace dimension). The average fraction of energy captured for all design events $\bar{f}_c$ is plotted in red. When $d$ reaches 4, the average fraction of energy captured for all design events exceeds 80%. This acts as an aid to determining $d$. Another way is to look at the increase in the average fractional energy capture $\Delta \bar{f}_c$ as a function of the increased subspace dimension $d$,

$$\Delta \bar{f}_c = \bar{f}_c(d + 1) - \bar{f}_c(d).$$

(3-39)

It is observed from Figure 3-10b that $\Delta \bar{f}_c$ decreases rapidly as $d$ is increased, which indicates a marginal benefit in the signal energy capture at large $d$ values. However, with increased $d$, the noise energy capture also increases, which will lead to an increased
number of false alarms. Therefore, an intermediate value of $d = 4$ is chosen as the signal subspace dimension. We will revisit this in the next section when we calculate the detection probability curve.

So far, the design data matrix $S$ and the subspace dimension $d$ have been derived. Finally, the signal subspace is built based upon equations (3-17) and (3-19).

### 3.3.3 Detection parameter estimation and performance evaluation

As discussed in the methodology section, in order to derive the detection threshold $\gamma$, it is necessary to determine the false alarm rate $P_F$ first.

Figure 3-11a gives the histogram of the correlation between template event and noise, while Figure 3-11b presents the histogram of correlation values between template events. The effective dimension of the embedding space $\tilde{N}$ is related to the variance of the sample correlation coefficient $\tilde{c}_{ij}$ under null hypothesis by equation (3-28). From Figure 3-11a, it is estimated that

$$\tilde{N} = 1 + \frac{1}{\sigma_{\tilde{c}}^2} = 402.$$  

According to Wiechecki-Vergara et al. (2001), the green line on Figure 3-11a shows the theoretical null probability density function of $\tilde{c}_{ij}$,

$$f_R(r) = \frac{1}{\Gamma(\frac{N - 1}{2})} \left(1 - r^2\frac{R}{2}\right)^{\frac{N - 1}{2} - 2},$$

which fits well with the observed histogram.

To produce only 1 false alarm out of all 6240 correlation samples shown as the red line on Figure 3-11a, a correlation detection threshold of $\rho_c = 0.385$ is chosen. At this threshold plotted as the red line on Figure 3-11b, around 16% template events are detected from the correlation detector. According to equation (3-33), the false alarm rate for the chosen correlation detection threshold is calculated as

$$P_F = 1 - F_{1,\tilde{N}-1}\left(\frac{\rho_c^2}{1 - \rho_c^2}\tilde{N} - 1\right) = 10^{-15}.$$  

At this false alarm rate $P_F$, the threshold for correlation detection statistics in equation (3-31) and array STA/LTA detection statistics in equation (3-35) are given by,
\[ \gamma_c = \rho_c^2 = 0.149, \]
\[ \gamma_s = 3.989. \]

The threshold \( \gamma \) for the subspace detection statistics in equation (3-10) is determined by maximizing the detection probability \( P_D \) for the given false alarm rate \( P_F = 10^{-15} \). Figure 3-12 illustrates the probability of detection \( P_D \) as a function of SNR. Twelve individual \( P_D \) curves appear in the figure, one for each possible signal subspace dimension \( d \). It is clear that the \( P_D \) curve for \( d = 1 \), shown in yellow, does not reach 1 even at high SNRs such as 10 dB. This feature occurs because of the low signal energy capture for most events when the signal subspace consists of a single vector (see Figure 3-10a). This also justifies the disadvantage that comes with the correlation detector and the detection method proposed by Bose et al. (2009), both of which use only one basis vector to represent the signal subspace. On Figure 3-12, the \( P_D \) curve climbs quickly as \( d \) increases from 1 to 2, because the added dimension increases the signal energy capture significantly, as seen in Figure 3-10. The detection probability continues to improve until \( d \) increases to 4. By the time the subspace dimension grows to 4, the average fractional energy capture is above 0.8 for all design set events and good detection performance \((P_D \sim 1)\) is achieved for a SNR as low as -13.5 dB. Performance continues to improve, but marginally, until \( d \) reaches 8, beyond which the detection probability actually begins to decline. This is because the marginal increase in signal energy capture afforded by additional increments to the signal subspace representation does not offset the increase in noise energy capture. The analysis suggests that a dimension of 4 would be a good choice for this example in terms of maximizing the probability of detection for a given false alarm rate. This is consistent with the fractional energy capture analysis in Figure 3-10. Given that \( d = 4 \), and \( \bar{N} = 402 \), the subspace detection threshold \( \gamma \) is calculated from \( P_F \) using equation (3-29),

\[
P_F = 1 - F_{d,N-d} \left( \frac{\gamma}{1-\gamma} \frac{N-d}{d} \right) = 10^{-15} \Rightarrow \gamma = 0.174.
\]

3.3.4 Comparison of array STA/LTA, correlation and subspace detectors
The new channel-multiplexed data are formed by multiplexing the x, y, z component data from geophones 7-12 in well 13B, after noise standard deviation normalization. The array STA/LTA, correlation and subspace detectors are then applied to the new channel-multiplexed data. Figure 3-13 compares the detection results of the three detectors on the 30-minute continuous record. The false alarm rate has been set at \( P_F = 10^{-15} \) for all three detectors. The detection thresholds are calculated from the previous section and indicated in Figure 3-13 with the black lines. The correlation detector has a lower threshold (0.149) than the subspace detector (0.174), since it has a single dimension and, thus, should match background noise less well. It is seen from the figure that the background level of the correlation statistics is lower than that of the subspace statistics commensurate with the calculated threshold.

At \( P_F = 10^{-15} \), there are 604, 1571, and 2730 triggers on the 30-minute continuous record by array correlation, subspace, and STA/LTA detectors, respectively. It is difficult to analyze such a large number of triggers. We proceed with two alternative approaches.

First, we analyze a 1-minute portion (in this case, from 1100 seconds to 1160 seconds on Figure 3-13) of the data to compare the performance of three detectors under the same expected probability of false alarms. Table 3-2 presents the results. It is clear that at a constant false alarm rate \( P_F = 10^{-15} \), the correlation detector gives the least amount of false alarms while STA/LTA detector generates the most false alarms. This is due to the fact that an incoherent detector, such as STA/LTA detector, has no sensitivity to waveforms and thus is prone to false alarms. The correlation detector, on the contrary, matches signals both temporally and spatially to enhance sensitivity and, therefore, has less chance to generate false alarms. It is interesting to point out that several design set events, shown as yellow crosses on Figure 3-13c, are missed by the correlation detector even at a low threshold of 0.149. However, they are detected as red crosses by both STA/LTA and subspace algorithms in Figure 3-13b and d. Figure 3-14 plots the four design set events missed by the correlation detector at a threshold of 0.149 (see Figure 3-13c). Substantial variations between the waveform of the four design set events on Figure 3-14a-d and the template event waveform on
Figure 3-14e are observed. This demonstrates the inability of correlation detectors to capture waveform variations, which result from variations in the source mechanisms, locations, and source time functions.

Secondly, due to the double-precision limit of the machine used, it is hard to set a false alarm rate less than $10^{-15}$. Instead, we look at a reduced number of triggers. Figure 3-15 presents the first 35 largest triggers from three detectors on the 30-min continuous record. On Figure 3-15b, the crosses represent the 21 events detected by the array STA/LTA algorithm with the minimum detected event plotted in red, while the false alarm with the largest STA/LTA detection statistics is shown as the green square. One event, detected by the STA/LTA algorithm but missed by the subspace detector, is seen as the magenta cross on Figure 3-15b. Figure 3-16a plots the 3C waveforms of the minimum detected event, where coherent P- and S-waves are seen across the array. Figure 3-16b shows the STA/LTA event missed by the subspace detector. Figure 3-16c gives the false alarm with the largest STA/LTA statistics, where no coherent arrivals are observed.

Similarly, out of the largest 35 triggers by the correlation detector, 10 events are detected and plotted as crosses in Figure 3-15c. The minimum detected event and the correlation template event are shown as the red and magenta cross, respectively. Figure 3-17a and Figure 3-17b shows the 3C waveforms of the minimum detected event and the template event. Coherent P- and S-waves appear on both figures. A good degree of waveform similarity is seen between the detected and template event especially for the dominant S-waves. Figure 3-17c shows the waveforms of the false alarm with the largest correlation statistics, plotted as the green square on Figure 3-15c. No coherent P-waves are seen. It is worth noting that dominant coherent energy at the moveout of S-waves is observed on Figure 3-17c, which justifies the appearance of the high correlation. Several factors could contribute to the missing P waves. Firstly, P-waves usually have small amplitudes and are embedded in high noise. Alternatively, the geophone array may be close to the nodal points of the P-wave radiation pattern. Either way, P-waves cannot be identified. However, the polarization information of P-waves is essential for locating microseismic events from
downhole geophones (Warpinski et al., 2005; Rentsch et al., 2007). Therefore, for the purpose of location, we consider event triggers without P-waves as false alarms.

Among the 35 largest triggers by the subspace detector, 21 events are detected and plotted as crosses on Figure 3-15d. All 12 events, comprising the design set, are detected and shown as black crosses. The 9 additional events, denoted as magenta crosses, are also detected at the threshold of 0.619. Out of the 9 events, 2 events, labeled as ‘1’ and ‘2’ on Figure 3-15d, are missed by both the STA/LTA and correlation detectors. Figure 3-18a and Figure 3-19a show the 3C waveform for events ‘1’ and ‘2’, respectively. Comparing Figure 3-18a and Figure 3-19a with Figure 3-16a and Figure 3-16b, it is clear that the STA/LTA algorithm allows more waveform variations than the subspace detector, but at the expense of increased false alarms. Moreover, the 10 events detected by the correlator are a subset of the subspace detections. Looking at the correlation and subspace statistics for the 10 correlation events, it is found that the subspace detector has an increased processing gain (larger statistics when events are present).

To study the 2 events from well 13B that are missed by both the STA/LTA and correlation detectors, we look at detections from nearby well 24C. By considering the possible arrival time difference on well 24C and 13B (for a well separation of 1100 ft, the maximum two-way travel time difference is around 0.3 seconds), and possible difference between STA/LTA picks and subspace picks (depending on the size of the detection window, in this study, around 0.5 seconds), we search the data from well 24C over an interval of +/- 1 second around the subspace picks. We look for the maximum STA/LTA peak within the searched time interval. Figure 3-18b and Figure 3-18b give the 3C waveform plots associated with the maximum STA/LTA peak on nearby well 24C. Coherent S-waves are seen on Figure 3-18b, while P-waves are missing. As discussed before, for location purposes, we consider this event not detectable on well 24C. On the contrary, on Figure 3-19b, both coherent S- and P-waves are observed. We treat this event as a valid detection from well 24C. Although only the largest 35 triggers from this 30-min record are analyzed, we are able to detect one event (Figure 3-18a), which is not seen on nearby well 24C. This demonstrates the
capability of the subspace algorithm in improving the detection capability of far well 13B.

As previously mentioned, limited by the double-precision machine, we could not perform a constant false alarm detection at $P_F < 10^{-15}$. However, the false alarm rate corresponding to a given detection threshold can still be calculated to an arbitrary precision by analytical approaches (Kendall and Stuart, 1979). The last column of Table 3-2 summarizes the detection results on the 30-min record for the largest 35 triggers. At a much lower expected false alarm rate, the STA/LTA detector ($P_F = 8 \times 10^{-102}$) gives the same amount of false alarms as the subspace detector ($P_F = 4 \times 10^{-82}$). This is consistent with the constant false alarm rate case shown in the middle column of Table 3-2, where, due to its limited sensitivity to waveforms, the STA/LTA detector generates many more false alarms than the subspace detector at the same false alarm rate. On the other hand, even at a larger allowed false alarm rate, the correlation detector ($P_F = 7 \times 10^{-56}$) cannot detect as many events as the subspace detector ($P_F = 4 \times 10^{-82}$). This indicates the inability of the correlation detector to capture the waveform variations.

3.3.5 Signal enhancement based on subspace projection

Once weak events are detected, the corresponding 3C data are projected into the signal subspace $U$ according to equation (3-11) to obtain the enhanced signal $x_p(n)$. As an example, Figure 3-20 shows the enhanced signal for two weak subspace events ‘1’ and ‘2’. Compared to the original data on Figure 3-20a and Figure 3-20c, the enhanced signals on Figure 3-20b and Figure 3-20d have larger SNRs especially for the weak P-waves. For both events, the median value in the SNR gains from subspace projection, across six geophones and three components, is around 16 dB and 19 dB for P- and S-waves, respectively. The major contribution in the SNR increase comes from the pre-event noise suppression. Since noise mostly exists in the orthogonal complement of the signal subspace, the noise energy projected into the signal subspace is minimal.
3.4 Summary

In this chapter, we introduced a full-waveform based event detection and signal enhancement approach for microseismic monitoring. The method constructs a vector space, known as the signal subspace, to represent variable microseismic signals from specific source regions. It models the signals to be detected as a linear combination of the orthogonal bases of the subspace. Unlike correlation detectors, the subspace approach is more broadly applicable. Furthermore, the subspace detector is sensitive to waveforms and, therefore, offers a lower probability of false alarms, compared to STA/LTA detectors.

A systematic procedure based on the statistical hypothesis testing theory was presented to build the signal subspace from previously detected events and determine the detection parameters. The subspace detector provides a way to manage the tradeoff between sensitivity and flexibility by adjusting the detection parameters such as the detection threshold and the subspace dimension, i.e. the number of bases used to present the signal subspace. The subspace design and detection approach was demonstrated on a dual-array hydrofracture monitoring dataset. The application of the subspace, STA/LTA, and correlation detectors to the data from the far monitoring well was presented. It is found that, at the same false alarm rate, the subspace detector gives fewer false alarms than the array STA/LTA detector and more event detections than the array correlation detector. The additionally detected events from the far monitoring well by the subspace detector were compared with the detections from the nearby well. It was demonstrated that, with the subspace detector, we are able to detect additional events that are not seen on the nearby well. The limitation of the subspace detector is the complexity and relatively large computation cost in building the signal subspace. Fortunately, the signal subspace construction could be done offline, which makes real-time subspace detection possible. Moreover, the template event library used to form the signal subspace could be dynamically updated as detection goes on. When the subspace detector is used as a post-processing tool, it
would be more efficient to build the signal subspace from the spatial clusters of events.

The data of detected events were projected into the signal subspace to form the enhanced microseismic signals. It was shown that the SNR of detected weak microseismic events is improved after applying the subspace-projection-based signal enhancement procedure. By using both full waveforms from multiple events and the signal/noise statistics, the proposed subspace detection and signal enhancement approach is capable of handling strong noise and offers the potential for future application of hydrofracture monitoring with the treatment well, as the noise in the treatment well is much higher than the offset monitoring well.

**Acknowledgements**

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**3.5 References**


Table 3-1: Symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>The null hypothesis: event not present in the detection window</td>
</tr>
<tr>
<td>$H_1$</td>
<td>The alternative hypothesis: event present in the detection window</td>
</tr>
<tr>
<td>$x(n)$</td>
<td>The n-th sample in the channel multiplexed continuous data stream</td>
</tr>
<tr>
<td>$x_i(n)$</td>
<td>The n-th sample in the continuous data recorded by the i-th channel</td>
</tr>
<tr>
<td>$\mathbf{x}(n)$</td>
<td>The $N*1$ data vector in the subspace/correlation detection window starting at n-th time sample</td>
</tr>
<tr>
<td>$\mathbf{x}_{\text{STA}}(n)$</td>
<td>The channel multiplexed data vector in the STA window starting at n-th time sample</td>
</tr>
<tr>
<td>$\mathbf{x}_{LTA}(n)$</td>
<td>The channel multiplexed data vector in the LTA window ending at n-th time sample</td>
</tr>
<tr>
<td>$N, N_T$</td>
<td>The number of data samples, time samples in each subspace/correlation detection window</td>
</tr>
<tr>
<td>$N_c$</td>
<td>The number of recorded channels</td>
</tr>
<tr>
<td>$\tilde{N}$</td>
<td>The effective embedding space dimension of the subspace/correlation detection window</td>
</tr>
<tr>
<td>$N_{\text{STA}}, N_{\text{LTA}}$</td>
<td>The number of time samples in each STA, LTA window</td>
</tr>
<tr>
<td>$\tilde{N}<em>{\text{STA}}, \tilde{N}</em>{\text{LTA}}$</td>
<td>The effective embedding space dimension of the STA, LTA window</td>
</tr>
<tr>
<td>$\mathbf{s}, \mathbf{n}$</td>
<td>The signal and noise vector in the detection window</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>The unknown noise variance</td>
</tr>
<tr>
<td>$\mathbf{U}$</td>
<td>The $N*d$ matrix, comprising $d$ signal subspace bases</td>
</tr>
<tr>
<td>$\mathbf{a}$</td>
<td>The $d*1$ coefficients, used to project the signal vector $\mathbf{s}$ into the signal subspace $\mathbf{U}$</td>
</tr>
<tr>
<td>$\mathbf{a}_d^i$</td>
<td>The $d*1$ coefficients, used to project the data vector $\mathbf{S}'(n)$ from the i-th design set event into the signal subspace $\mathbf{U}$</td>
</tr>
<tr>
<td>$E_c$</td>
<td>The energy captured in the signal subspace after projection</td>
</tr>
<tr>
<td>$p(\cdot</td>
<td>H_0)$</td>
</tr>
<tr>
<td>$p(\cdot</td>
<td>H_1)$</td>
</tr>
<tr>
<td>$l(\mathbf{x}(n))$</td>
<td>The generalized log likelihood ratio function of the detection data vector</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The subspace detection threshold, associated with the subspace detection statistics $c(n)$ defined in equation (3-10)</td>
</tr>
<tr>
<td>$\mathbf{x_p}(n)$</td>
<td>The projection of the detection data vector $\mathbf{x}(n)$ into the subspace $\mathbf{U}$</td>
</tr>
<tr>
<td>$\mathbf{S}'(n)$</td>
<td>The $N*1$ normalized data vector from the i-th design set event</td>
</tr>
<tr>
<td>$\mathbf{S}$</td>
<td>The design data matrix, comprising $D$ data vectors of design set events</td>
</tr>
<tr>
<td>$\mathbf{w}(n)$</td>
<td>The projection of the detection data vector $\mathbf{x}(n)$ into the orthogonal complement to the subspace $\mathbf{U}$</td>
</tr>
<tr>
<td>$\mathbf{W}, \Sigma, \mathbf{V}$</td>
<td>The SVD of the $N*D$ design data matrix $\mathbf{S}$</td>
</tr>
<tr>
<td>$\mathbf{A}$</td>
<td>The exact representation coefficient matrix of size $N*D$</td>
</tr>
<tr>
<td>$\mathbf{A}_d$</td>
<td>The approximate representation coefficient matrix of size $d*D$</td>
</tr>
<tr>
<td>$D, d$</td>
<td>The number of design set events, the signal subspace dimension</td>
</tr>
<tr>
<td>$f^i$</td>
<td>The fractional energy captured in $\mathbf{U}$ for the i-th design set event</td>
</tr>
<tr>
<td>$f_c$</td>
<td>The average fractional energy captured in $\mathbf{U}$ for all $D$ design set events</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\Delta f_c$</td>
<td>The increase in average fractional energy capture</td>
</tr>
<tr>
<td>$M$</td>
<td>The number of template events</td>
</tr>
<tr>
<td>$K$</td>
<td>The original template event dissimilarity distance matrix of size $M \times M$</td>
</tr>
<tr>
<td>$K^g$</td>
<td>The template event dissimilarity distance matrix of size $M \times M$ at clustering step $g (g=1,2,\ldots,M-1)$</td>
</tr>
<tr>
<td>$K_{p,q}$</td>
<td>The original dissimilarity distance between template event $p$ and $q$</td>
</tr>
<tr>
<td>$K_{p,q}^g$</td>
<td>The dissimilarity distance between template event $p$ and $q$ at clustering step $g (g=1,2,\ldots,M-1)$</td>
</tr>
<tr>
<td>$\lambda_{p,q}$</td>
<td>The maximum waveform correlation between template event $p$ and $q$</td>
</tr>
<tr>
<td>$P_F$</td>
<td>The probability of false alarms, i.e. the false alarm rate</td>
</tr>
<tr>
<td>$P_D$</td>
<td>The probability of detection</td>
</tr>
<tr>
<td>$c_{ij}$</td>
<td>The sample correlation coefficient between noise data $\eta_j$ and event signal $s_i$</td>
</tr>
<tr>
<td>$c_g$</td>
<td>The cophenetic correlation coefficient between $K$ and $K^g$</td>
</tr>
<tr>
<td>$\sigma_c^2$</td>
<td>The variance of the sample correlation between noise and event</td>
</tr>
<tr>
<td>$s_m$</td>
<td>The $N \times 1$ correlation template vector, i.e. master event data vector</td>
</tr>
<tr>
<td>$Y_c$</td>
<td>The correlation detection threshold, associated with correlation detection statistics $c(n)$ defined in equation (3-31)</td>
</tr>
<tr>
<td>$Y_r$</td>
<td>The STA/LTA detection threshold, associated with STA/LTA detection statistics $r(n)$ defined in equation (3-35)</td>
</tr>
<tr>
<td>$f_R(\cdot)$</td>
<td>The probability density function of sample correlation coefficient under $H_0$</td>
</tr>
<tr>
<td>$F_{\cdot,\cdot}(\cdot)$</td>
<td>The cumulative distribution function of subspace/correlation detection statistics, could be central F distribution or doubly non-central F distribution</td>
</tr>
</tbody>
</table>
Table 3-2: Summary of detections results on a 30-minute continuous record in far well 13B by the STA/LTA, correlation, and subspace detectors.

<table>
<thead>
<tr>
<th>Type of detectors</th>
<th>Performance type</th>
<th>Constant false alarm rate(^1)</th>
<th>Constant # of triggers(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array STA/LTA detector</td>
<td>(# of detected events / # of false alarms)</td>
<td>10 / 139</td>
<td>21 / 14</td>
</tr>
<tr>
<td></td>
<td>(expected false alarm rate (P_F))</td>
<td>(P_F = 10^{-15})</td>
<td>(P_F = 8 \times 10^{-102})</td>
</tr>
<tr>
<td>Array correlation detector</td>
<td>(# of detected events / # of false alarms)</td>
<td>5 / 9</td>
<td>10 / 25</td>
</tr>
<tr>
<td></td>
<td>(expected false alarm rate (P_F))</td>
<td>(P_F = 10^{-15})</td>
<td>(P_F = 7 \times 10^{-56})</td>
</tr>
<tr>
<td>Subspace detector</td>
<td>(# of detected events / # of false alarms)</td>
<td>6 / 27</td>
<td>21 / 14</td>
</tr>
<tr>
<td></td>
<td>(expected false alarm rate (P_F))</td>
<td>(P_F = 10^{-15})</td>
<td>(P_F = 4 \times 10^{-82})</td>
</tr>
</tbody>
</table>

Note 1: only the detection results from a 1-minute segment of the total 30-minute record are listed here under a constant false alarm rate \(P_F = 10^{-15}\).

Note 2: the largest 35 triggers of the detection results from the total 30-minute record are analyzed and listed here.
Figure 3-1: (a) Horizontal plane view of the microseismic event locations from one stage treatment plotted as black stars. The blue and black squares denote the monitoring wells 13B and 24C, respectively, while the fracturing well is shown as the red triangle. The origin (0, 0) corresponds to the wellhead location of well 13B. (b) The side view of the microseismic events. The blue squares and black squares represent the two twelve-level geophone arrays deployed in well 13B and 24C separately (from deep to shallow depths: geophone 1 to 12). The perforation locations are depicted as the red triangles in fracturing well 24D. Fewer events are detected on the far well 13B. Data from the far well 13B will be used in this study for subspace detection and signal enhancement.
Figure 3-2: The three-component raw data plot for a typical event recorded in the far well 13B: (a) x component, (b) y component, (c) z component.
Figure 3-3: (a) The raw x component data of a 0.5s event record from geophones 7-12 in well 13B. (b) The raw x component data of a 0.5s noise segment recorded by geophones 7-12 in well 13B. (c) Amplitude spectrum of the raw event and noise data in the panels (a) and (b), averaged over all 6 geophones. The black square demonstrates the dominant signal frequency range of [100, 400] Hz.
Figure 3-4: (a) The raw x component data of a 0.5s continuous record from geophones 7-12 in well 13B. (b) The [100, 400] Hz band-pass filtered result of the panel (a).
Figure 3-5: Array STA/LTA detection on a 30-min continuous record from far well 13B. a) The x component [100, 400] Hz band-pass filtered continuous data from one geophone in well 13B. b) The STA/LTA detection results on the channel-multiplexed data. The x, y, z component data from geophones 7-12 are used in the STA/LTA detection. The template event library for the subspace detector, comprising the $M = 20$ identified events using a conservative STA/LTA threshold of 30, is plotted in red stars.
Figure 3-6: The standard deviation and mean of identified 454 noise data files across the six geophones (geophones 7-12 from well 13B). Left columns: noise standard deviation. a) x component. b) y component. c) z component. Right columns: noise mean as a multiple of its corresponding absolute maximum value. d) x component. e) y component. f) z component.
Figure 3-7: Waveform plot of the detected 20 template events (as described in Figure 3-5) after noise standard deviation normalization. a): Band-pass filtered unaligned waveforms of all 20 events from one geophone in well 13B. b): Band-pass filtered unaligned waveforms of one template event from all six geophones in well 13B (geophones 7-12).
Figure 3-8: Template event clustering and design set event selection through the dendrogram using the single-link algorithm. The red line shows the termination of clustering with a maximum event dissimilarity distance of 0.6, which gives a design set comprising $D = 12$ events (events 11 to 16).
Figure 3-9: The waveform alignment of design set events using the single-link algorithm. a) The unaligned z component waveform plot from one geophone. b) The waveform plot of panel a) after alignment.
Figure 3-10: a) Fractional energy capture $f_c$ as a function of dimension of representation $d$ (also known as the signal subspace dimension) for each design set event is plotted in blue, while the average fractional energy capture $\bar{f}_c$ for all $D=12$ design set events as a function of $d$ is shown in the red curve. A threshold of at least 80% average fractional energy capture plotted as the vertical red line gives an optimal subspace dimension $d = 4$. The horizontal red line shows the theoretical detection threshold for the subspace detector with $d = 4$, and false alarm rate of $P_F = 10^{-15}$. b) The increase in the average fractional energy capture $\Delta f_c$ as a function of an increased subspace dimension $d$. 
Figure 3-11: a) The histogram of correlation values between template event and noise. b) The histogram of correlation values between template events.
Figure 3-12: The probability of detection as a function of the SNR at a fixed false alarm rate $P_F = 10^{-15}$. In this case, the detection probabilities are calculated as a function of SNR for subspace dimensions ranging from 1 to 12. The detection probability curve for the selected subspace detector with $d = 4$ is plotted in red, while the yellow and black curves demonstrate the detection probability curves for the subspace detector with $d = 1$ and $d = 12$, respectively.
Figure 3.13: The comparison of detection results on a 30-min continuous record in far well 13B at a fixed false alarm rate $P_F = 10^{-15}$. The new channel-multiplexed data, formed by the x, y, z component data from geophones 7-12 after noise standard deviation normalization, are used in the detection. a) The [100, 400] Hz band-pass filtered x component data from one geophone in well 13B. b) The STA/LTA detection, c) the correlation detection, and d) the subspace detection $(d=4)$ results on the new channel-multiplexed data. The threshold values at $P_F = 10^{-15}$, plotted as the black horizontal lines, are 3.989, 0.149, and 0.174 for the STA/LTA, correlation, and subspace detector, respectively. The four design set events missed by the correlation detector, but captured by STA/LTA and subspace detectors, are plotted as yellow and red crosses.
Figure 3-14: The band-pass filtered $x$ component waveform plot. The dashed and solid black lines represent the P and S arrival picks on geophones 7-12 (geophone index: 1-6) in well 13B. a-d) The four design set events missed by the correlation detector, but captured by STA/LTA and subspace detectors at $P_F = 10^{-15}$. e) The correlation template event.
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Chapter 4

Microseismic Moment Tensor Inversion Using Full Waveforms: Theoretical Analysis and a Field Example From Single Well Monitoring

Abstract

Downhole microseismic monitoring is a valuable tool in understanding the efficacy of hydraulic fracturing. Inverting for the moment tensor has gained increasing popularity in recent years as a way to understand the fracturing process. Previous studies only utilize part of the information in the waveforms such as direct P- and S-wave amplitudes and make far field assumptions to determine the source mechanisms. The method gets hindered in downhole monitoring where only limited azimuthal coverage is available. In this study, we developed an approach to invert for complete moment tensor using full-waveform data recorded at a vertical borehole. We use the discrete wavenumber integration method to calculate full wavefields in the layered medium. By using synthetic data, we show that, at the near-field range, a stable, complete

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moment tensor can be retrieved by matching the waveforms without additional constraints. At the far-field range, we demonstrate that the off-plane moment tensor component is poorly constrained by waveforms recorded at one well. Therefore, additional constraints must be introduced to retrieve the complete moment tensor. We study the inversion with three different types of constraints. For each constraint, we investigate the influence of velocity model errors, event mislocations and data noise on the extracted source parameters by a Monte-Carlo study. We test our method using a single well microseismic dataset obtained during hydraulic fracturing of the Bonner sands in East Texas. By imposing constraints on the fracture strike and dip range, we are able to retrieve the complete moment tensor for events in the far field. Field results show that most events have a dominant double-couple component. The results also indicate the existence of a volumetric component in the moment tensor. The derived fracture plane orientation generally agrees with that derived from multiple event location.

4.1 Introduction

Downhole microseismic monitoring is a valuable tool for fracture mapping. The locations of microseismic events, with sufficient resolution, provide information on fracture geometry and properties (Warpinski et al., 1998; Phillips et al., 2002). Besides location, seismic moment tensor is also derived to understand the microseismic source mechanisms and stress state (Nolen-Hoeksema and Ruff, 2001; Baig and Urbancic, 2010). The complete moment tensor of the general source mechanism consists of six independent elements (Aki and Richards, 2002). Some researchers (Phillips et al., 1998; Warpinski, 1997) observed high S/P-wave amplitude ratios which "could not be explained by tensile opening" (Pearson, 1981) and concluded that the induced events are shear failure along pre-existing joints in rocks surrounding hydraulic fracture due to elevated pore pressure. Thus, most studies have been focused on double-couple mechanisms (Rutledge and Phillips, 2003). However, recent studies have shown the existence of non-double-couple mechanisms for some
hydrofracture events (Šílený et al., 2009; Warpinski and Du, 2010). Knowledge of non-double-couple components, especially the volumetric component, is essential to understand the fracturing process. Moreover, Vavryčuk (2007) showed that, for shear faulting on non-planar faults, or for tensile faulting, the deviatoric source assumption is no longer valid and can severely distort the retrieved moment tensor and bias the fault-plane solution. Therefore, the complete moment tensor inversion is crucial not only to the retrieval of the volumetric component but also to the correct estimation of the fault-plane solution.

Currently, most moment tensor inversion methods rely only on far-field direct P- and S-wave amplitudes (Nolen-Hoeksema and Ruff, 2001; Vavryčuk, 2007; Jechumtálová and Eisner, 2008; Warpinski and Du, 2010). Vavryčuk (2007) used the far-field approximation of the P- and S-wave Green’s function in homogeneous isotropic and anisotropic media to show that a single-azimuth dataset recorded in one vertical well cannot resolve the dipole perpendicular to the plane of geophones and the hypocenter. Thus, the complete moment tensor of the general source mechanism is underdetermined with data from one well. To overcome this problem, previous studies proposed to use data recorded in multiple monitoring wells at different azimuths (Vavryčuk, 2007; Baig and Urbancic, 2010). Unfortunately, downhole microseismic monitoring datasets are frequently limited to a single array of geophones in one vertical well. Therefore, the issue of complete moment tensor inversion from one-well data remains to be solved.

In this chapter, we try to address this problem from the standpoint of full-waveform inversion. We propose a full-waveform approach for moment tensor inversion using data from one monitoring well. It uses the discrete wavenumber integration method to calculate elastic wavefields in the layered medium. By matching the waveforms across the geophone array, we show that, when the events are close to the monitoring well, the inversion can be stabilized so that the complete moment tensor can be retrieved from data recorded in a single borehole without making additional source assumptions. We quantify the closeness of events by studying the condition number of the sensitivity matrix. For events far from the
monitoring well, as is the typical case of hydraulic fracturing, we demonstrate that additional constraints must be introduced to retrieve the off-plane dipole component (also pointed by Vavryčuk, 2007; Jechumtálová and Eisner, 2008). Three types of constraints have been studied in this chapter to invert the complete moment tensor for events at far field. Furthermore, we investigate the influence of velocity model errors, source mislocations and data noise on the extracted source parameters using synthetic data. Finally, we describe the application of the constrained inversion to a field dataset from East Texas. By applying the constraint on the fracture strike and dip range, we show that a reliable, complete moment tensor solution and source parameters can be obtained for each event.

4.2 Methodology

4.2.1 Full-waveform based complete moment tensor inversion

The complete moment tensor of a microseismic event is characterized by the 6 independent elements of the 3 by 3 symmetric moment tensor matrix $m_{jk}$. To improve the complete moment tensor inversion with a single borehole, we use all phases that are embedded in the full waveform data. Our approach starts from fast full elastic waveform modeling in a layered medium with the discrete wavenumber integration method (DWN; Bouchon, 2003). The $i$-th component (North, East, Down) of the observed waveform at geophone $n$ is modeled as:

$$v_i(x^n_i, x_s, t) = \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} G_{ij,k}(x^n_i, x_s, t) \ast s(t), \quad (4-1)$$

where $\ast$ denotes the convolution operation (same hereinafter); $G_{ij,k}(x^n_i, x_s, t)$, the spatial derivative of the Green's function, is the $i$-th component of the elementary seismograms at the $n$-th geophone $x^n_i$ due to a point moment tensor source $m_{jk}$ at $x_s$; $s(t)$ is the source time function. In this study, a smooth ramp function with a center frequency of 550 Hz is used as $s(t)$ according to the spectral analysis of field data. The sampling frequency is 4 kHz in both synthetic and field study. Considering that
the moment tensor matrix \( m_{jk} \) has only six independent elements, equation (4-1) can be written as:

\[
\sum_{i=1}^{6} A_{il}(x^n_r, x_s, t) M_i(x_s) = v_i(x^n_r, x_s, t).
\]  (4-2)

Here \( M_i \) is the \( l \)-th moment tensor element: \( M_1 = m_{11}, M_2 = m_{22}, M_3 = m_{33}, M_4 = m_{12}, M_5 = m_{13}, M_6 = m_{23} \), while \( A_{il} \) denotes the \( i \)-th component of the elementary seismograms at geophone \( x^n_r \) due to a point moment tensor source \( M_i \) at \( x_s \). In matrix form, equation (4-2) becomes:

\[
A M = D.
\]  (4-3)

Here the sensitivity matrix \( A \) (i.e. data kernel) is composed of six columns, with each column consisting of the elementary seismograms from a point moment tensor source \( M_i \). The six element vector \( M \) represents the complete moment tensor:

\[
M = [M_1, M_2, M_3, M_4, M_5, M_6]^T.
\]  (4-4)

Data column vector \( D \) is comprised of all available components recorded at all geophones ranging from time \( t_{on} \) to \( (t_{on} + T_n) \), where \( t_{on} \) and \( T_n \) are the starting time, and the duration of recorded data used in the inversion from geophone \( n \), respectively.

In this study, we choose \( T_n \) to include both P- and S-wave trains and keep it fixed for all \( N \) geophones. \( t_{on} \) is determined from the event origin time \( t_0 \) and the P-wave travel time from the event to geophone \( n \). Event origin time is obtained by a grid search around its initial estimate within the dominant signal period. The initial estimate of the origin time can be found by cross-correlating the synthetic and observed waveforms.

To reduce the influence from errors in source locations, during the inversion, we also perform a grid search around the initial location. The spatial search range and grid size are selected based on the location uncertainty. The uncertainty in locations from a vertical array is estimated from the standard deviations of P- and S-wave arrival times and P-wave polarization angles (Eisner et al., 2010). For the field data, we calculate standard deviations and obtain 3.0 m (10 ft) in the radial direction, 7.6 m (25 ft) in the vertical direction and 5° in P-wave derived back-azimuths. We further determine the location uncertainty in the horizontal directions (North, East) from the
standard deviations of the radial distances and P-wave derived back-azimuths for a monitoring array at a typical distance of 100.6 m (330 ft). The standard deviation is estimated to be 9.1 m (30 ft). Therefore, in this study, we use a spatial grid size of 5 ft and a spatial search cube with the size of 15*15*11 grids (North, East, Down). The best solution of the event location \( x_s \), origin time \( t_0 \) and moment tensor \( M_t \) is determined by minimizing the squared L-2 norm of the waveform fitting error:

\[
J(x_s, t_0, M_t) = \sum_{n=1}^{N} \sum_{i=1}^{N_c} \sum_{k=1}^{N_t} (d_i(x^n_s, k\Delta t) - v_i(x^n, x_s, k\Delta t))^2.
\]  (4-5)

where \( N \) is the number of geophones, \( N_t \) is the number of time points, and \( N_c \) is the number of components used in the inversion. \( \Delta t \) is the sampling interval of the recorded data.

To further stabilize the inversion, both synthetic data and observed data are band-pass filtered. Based on the spectral analysis of the signal and pre-event noise from the field data example, a band-pass filter of [200, 900] Hz is used in this study. For \( N \) geophones, the sensitivity matrix \( A \) has a size of \( NN_cN_t \) by 6. In this study, as we will explain in the field study, only two horizontal components are used in the inversion due to poor signal-to-noise ratios (SNRs) in the vertical component. Therefore in this study, \( N_c = 2 \). However, the method itself is not limited to two components. If matrix \( A \) is good conditioned, a least-squares solution to the over-determined system can be obtained using the generalized inverse,

\[
M = (A^T A)^{-1} A^T D.
\]  (4-6)

The condition number of matrix \( A \) will be discussed in the synthetic study.

The processing steps can be summarized as follows:
1) generate a Green’s function library, calculate the elementary seismograms and apply the band-pass filter to the elementary seismograms for each possible event location;
2) apply the same band-pass filter to the recorded waveforms;
3) estimate the initial event origin time at every possible event location;
4) carry out a cascaded grid search around the initial estimated event origin time and location. For each said event location, conduct a grid search on event origin time. For each origin time and location, find the least-square solution \( M_t(x_s, t_0) \)
according to equation (4-6), and evaluate the L-2 waveform fitting error according to equation (4-5);
5) determine the best solution of moment tensor, event location and origin time with the least waveform fitting error.

4.2.2 Source parameter estimation

The complete moment tensor can be decomposed into the isotropic (ISO), compensated linear vector dipole (CLVD) and double-couple (DC) components. In this thesis, we use the decomposition of a moment tensor proposed by Vavryčuk (2001). The symmetric moment tensor matrix $m_{jk}$ can be diagonalized and represented as the sum of the deviatoric moment $M^{dev}$ (i.e., the moment tensor with zero volumetric component), and the isotropic moment $M^{iso}$. Parameter $\varepsilon$ is introduced to measure the size of CLVD relative to DC:

$$\varepsilon = -\frac{\lambda_{dev}^{min}}{\lambda_{dev}^{max}},$$

where $\lambda_{dev}^{min}$ and $\lambda_{dev}^{max}$ are the minimum and maximum absolute eigenvalues of the deviatoric moment, respectively. For a pure DC, $\varepsilon=0$, and for a pure CLVD, $\varepsilon=\pm 0.5$. Parameter $\varepsilon$ is positive for tensile sources and negative for compressive sources. The percentages of each component (ISO, CLVD, DC) can be calculated as

$$c^{ISO} = \frac{\text{trace}(m_{jk})}{3M_0},$$

$$c^{CLVD} = 2\varepsilon(1 - |c^{ISO}|),$$

$$c^{DC} = 1 - |c^{ISO}| - |c^{CLVD}|,$$

where $M_0$ is the seismic moment in N*m, defined as the largest absolute eigenvalue of the moment tensor matrix $m_{jk}$:

$$M_0 = \max_{|i|} |\lambda_i|.$$ (4-11)

The moment magnitude is calculated as:

$$M_w = \frac{2}{3} log_{10}(M_0) - 6.607.$$ (4-12)
According to Jost and Hermann (1989), the eigenvector $b$ of the moment tensor matrix $m_{jk}$ corresponding to the intermediate eigenvalue gives the null axis, while the eigenvectors $t$ and $p$ corresponding to the maximum and minimum eigenvalues give the tension and compression axis, respectively. The fracture plane normal $n$ and the slip vector $v$ can be derived from the $t$ and $p$ axes after compensating for the non-zero slope angle $\alpha$ (Vavryčuk, 2001) as follows:

$$\sin(\alpha) = 3\frac{\lambda_{\text{dev}}^{\text{max}} + \lambda_{\text{dev}}^{\text{min}}}{\lambda_{\text{dev}}^{\text{max}} - \lambda_{\text{dev}}^{\text{min}}}$$

$$v = \frac{1}{\sqrt{2}} (\sqrt{1 + \sin(\alpha)t} + \sqrt{1 - \sin(\alpha)p}),$$

$$n = \frac{1}{\sqrt{2}} (\sqrt{1 + \sin(\alpha)t} - \sqrt{1 - \sin(\alpha)p}).$$

The fracture plane solutions including strike $\phi$, dip $\delta$, and rake $\lambda$ can be further derived from the fracture plane normal $n$ and the slip vector $v$ (Jost and Hermann, 1989).

4.3 Synthetic study

4.3.1 Condition number of the sensitivity matrix in full waveform inversion

In this section, we study the influence of borehole azimuthal coverage and the source-receiver distance on the condition number of the sensitivity matrix and discuss its implications in complete moment tensor inversion using synthetic data from a single well.

Figure 4-1 gives the source receiver configuration. In this experiment, we fix the microseismic event at (0, 0, 3946 m). An array of six-level three-component (3C) geophones is deployed in each vertical well at the same depth range as the field setup from 3912 m (12835 ft) to 3944 m (12940 ft). The horizontal location of the well is adjusted so that the mean source-receiver distance falls into the range between $4\lambda_s$ and $36\lambda_s$, where $\lambda_s$ is the dominant S-wave wavelength. For each mean source-receiver distance, we calculate the elementary seismograms and apply the [200, 900] Hz band-pass filter to obtain the filtered elementary seismograms and form the
sensitivity matrix $A$. Figure 4-2 shows the one dimensional (1D) P- and S-wave velocity models derived from the field study. We use this velocity model to generate elementary seismograms for the condition number study.

Figure 4-3(a) shows the condition number of the sensitivity matrix $A$ as a function of both borehole azimuthal coverage and the mean source-receiver distance when all 3C data are used in the inversion. Three observations are clearly seen on Figure 4-3(a). Firstly, the condition number increases dramatically with the increased mean source-receiver distance for the one-well case. This signifies that the resolvability of complete moment tensors deteriorates at far field when only one-well data are used in moment tensor inversion. In addition, the eigenvector corresponding to the minimum eigenvalue gives the least resolvable moment tensor element. In the case of well B1 at the azimuth of $0^\circ$, the off-plane element $m_{22}$ ($m_{22}$) is the least resolvable moment tensor element. This is consistent with the far field study in the homogeneous media.

Secondly, the condition number for the multiple-well cases is significantly lower than that of the one-well case at large source-receiver distances, while the condition number is low for all cases at small source-receiver distances. This indicates that complete moment tensor inversion is possible even with one-well data when the receivers are at the near field range. There is no clear distinction between near field and far field. At a noise level of 10%, as is the case in the following synthetic study, a rule of thumb is that at a mean source-receiver distance that is less than five times the S-wave wavelength, a stable complete moment tensor solution can be determined from the one-well data. Finally, the condition number of the two-well case is similar to that of the eight-well case. This seems to imply that, with two wells separated at $45^\circ$, the resolvability of complete moment tensor is comparable to that of eight wells, although, for more complex scenarios such as a laterally heterogeneous medium, eight wells can bring additional benefits in enhancing the source azimuthal coverage and improving SNRs of recorded events. The condition number of the two-well case barely increases with increased source-receiver distances. This indicates that the complete moment tensor inversion is feasible for both near field and far field with two-well data. Figure 4-3(b) compares the condition number of the sensitivity matrix
of the one-well case using all 3Cs and only two horizontal components. The result suggests that two horizontal components have a similar capability of constraining the moment tensor as three components.

4.3.2 Complete moment tensor inversion of events in the near field

As we see in the previous section, for events that are close to the monitoring well, it is possible to invert the complete moment tensor from one-well data. Figure 4-4(a) shows the total wave-fields of the two horizontal components recorded in the well B1 at an azimuth of 0°. The synthetic data are generated with the reference velocity model plotted in Figure 4-2. Without losing generality, a non-double-couple microseismic source with 74% of DC, 15% of CLVD, and 11% of ISO component is used in the simulation. The microseismic source has a strike of 108°, dip of 80°, and rake of 43°. The distance from the source to six receivers ranges from one to six dominant S-wave wavelengths. At a distance of one to two dominant S-wave wavelengths, complex waveforms are seen on geophones 5 and 6 due to the near-field effects. At a distance larger than three S-wave wavelengths, distinct P and S phases are observed on geophones 1 to 4. Figure 4-4(b) gives the near-field terms of the two horizontal components. It is seen on Figure 4-4(b) that the near-field terms decrease fast with the increased source-geophone distance. To quantify the contribution of near-field information, we calculate the peak amplitude ratio of the near-field term to the total wave-fields for each component on each geophone. The average peak amplitude ratios of the two horizontal components are 9%, 11%, 14%, 18%, 22% and 60% for geophones 1 to 6, respectively. Therefore, the major contribution of near-field information to the inversion comes from geophones 5 and 6, which are close to the microseismic source.

Figure 4-5(a) shows the noisy seismograms by adding zero-mean Gaussian noise with a standard deviation reaching 10% of the average absolute maximum amplitude of the two components across all six geophones. Figure 4-5(b) gives the band-pass filtered data used to invert for the complete moment tensor.
The P- and S-wave velocity models are randomly perturbed up to a half of the velocity difference between adjacent layers so that the sign of the velocity difference between adjacent layers does not change. The perturbation is independent between different layers and P- and S-wave velocities are independently perturbed. The perturbed velocity model is used as the approximate velocity model for moment tensor inversion throughout the chapter. As mentioned in the methodology section, to mimic the field example, the event location is randomly perturbed up to 9.1 m (30 ft) in North and East directions and 7.6 m (25 ft) in the vertical direction. In the inversion, a grid search is carried out around the randomly perturbed event location. The moment tensor solution corresponding to the minimum L-2 waveform fitting error is selected as the inversion result. Figure 4-6 gives the best waveform fitting for one Gaussian noise realization. A good agreement between modeled data in black and band-pass filtered synthetic data in red is seen on both components.

The source parameters are then estimated from the inverted complete moment tensor. In order to obtain statistically relevant results, we perform 100 moment tensor inversions and source parameter estimations, each with a different noise realization. Figure 4-7 shows the histograms of the ISO, CLVD, DC, seismic moment, strike, dip, rake errors for the non-double-couple event. The average absolute errors in the percentages of the ISO, CLVD, and DC components are about 4%, 4%, and 6%, respectively, while the average absolute relative error in seismic moment is around 6%. The average absolute error in the strike, dip and rake is smaller than 2 degrees. Moreover, the complete moment tensor inversion using the horizontal component data from geophones 5 and 6 gives comparable results in the inverted source parameters. This indicates that the near-field information contributed to the retrieval of $m_{22} (m_{ee})$ mainly comes from geophones 5 and 6. Considering the inaccuracies in the source location and velocity model together with 10% Gaussian noise, the inverted source parameters agree well with the true values. This demonstrates that for events in the near field (i.e., at a mean source-receiver distance smaller than 5 times S-wave wavelength), the complete moment tensor inversion is feasible with one-well data
using only two horizontal components. The retrieval of $m_{22}$ with one-well data at near field is further illustrated in Appendix C.

### 4.3.3 Complete moment tensor inversion of events in the far field

As we see in the condition number study, for events that are far from the monitoring well (i.e., at a mean source-receiver distance larger than five times S-wave wavelength), the condition number of the sensitivity matrix using one-well data is high compared to those near-field events. In the case of well B1 at the azimuth of 0°, the off-plane element $m_{22}$ is the least resolvable moment tensor element from full-waveform inversion.

Figure 4-8 shows the condition number of the sensitivity matrix when inverting for all six moment tensor elements and five moment tensor elements, except $m_{22}$, with only two horizontal components. It is observed that at far field in the layered medium, when $m_{22}$ is excluded from the inversion, the condition number of the sensitivity matrix is reduced to the level of complete moment tensor inversion at near field. This shows that the full-waveforms are mainly sensitive to the five moment tensor elements, except $m_{22}$. Therefore, for events in the far field, additional constraints must be introduced to retrieve $m_{22}$.

The basic idea of the constrained inversion is to invert for the rest five moment tensor elements using waveforms assuming a known value of $m_{22}$. The source parameters are then estimated from the complete moment tensor as a function of $m_{22}$. As suggested by Jechumtálová and Eisner (2008), we test the $m_{22}$ value between $-10M_5$ and $10M_5$, where $M_5$ is the maximum absolute value of the five inverted elements. By using a priori source information (for example, fracture orientations) as constraints, $m_{22}$ can be determined. Finally, the complete moment tensor and the source parameters are derived.

It is also seen from Figure 4-8 that in the layered medium, the condition number is not a monotonous function of mean source-receiver distance for the case of constrained inversion, while the condition number in the homogeneous medium is a
monotonous function of mean source-receiver distance. This can be explained by the difference in the take-off angle coverage at the source between the homogeneous medium and layered medium.

Eaton (2009) pointed out that in the homogeneous medium, the condition number is inversely proportional to the solid angle at the source subtended by the geophone array. In the homogeneous medium, only direct rays are available, and therefore the take-off angle coverage at the source is fully characterized by the solid angle. However, in the layered medium, as is the case in this study, not only direct but also reflected and refracted rays exist, even if the source and geophone array are situated in the same layer. Therefore, the take-off angle coverage at the source has been increased in the layered medium compared to the homogeneous medium scenario, considering the additional reflected and refracted rays.

The increase in the take-off angle coverage at the source produces a decreased condition number. Hence, in the layered medium, the condition number is controlled by the geometry of the receiver array relative to not only the source, but also the velocity model. An increase in the mean source-receiver distance will reduce the take-off angle coverage of the direct rays. It may, however, increase the take-off angle coverage from reflected and refracted rays. There is also a critical distance for the refracted rays to occur. Thus, the non-monotonous behavior for the constrained inversion case in the layered medium is probably due to the complex interaction of the increased take-off angle coverage from the reflected and refracted rays and the decreased take-off angle coverage of direct rays.

Several types of constraints may be applied in the constrained inversion. In this chapter, we study three types of constraints. In type I constraint, the range of the strike and dip is assumed to be known. This will give a permissible range of $m_{22}$ values. We further assume that the source mechanism is mostly double-couple, and therefore we determine the $m_{22}$ value by maximizing the DC percentage within that permissible range. Figure 4-9 gives an example of applying type I constraint. In this example, we use the same non-double-couple source and source receiver configuration as the previous near-field case, shown in Figure 4-4. The mean source-
receiver distance increases to 91.4 m (17.5λs). In Figure 4-9, we invert for the five moment tensor elements, except $m_{22}$, from the band-pass filtered noise-free horizontal component data recorded in well B1. Assuming that the strike, and dip range is known to be +/- 15° around the true values, the cyan strip gives the permissible range of $m_{22}$ values. The vertical line in green denotes the determined $m_{22}$ value by maximizing the DC percentage within that permissible range.

In type II constraint, we assume that the exact strike value is known so that the $m_{22}$ value is determined directly. In type III constraint, the fracture plane solution is unknown; instead, we assume the event is predominantly double-couple. This suggests that the $m_{22}$ value is obtained by maximizing the DC percentage among all possible values.

Table 4-1 compares the non-double-couple source inversion results under three different constraints using noise-free horizontal component data from well B1. For each constraint, it shows the deviation of the inverted source parameters from the original input source parameters. Two observations are seen on Table 4-1. Firstly, in this case, type I constraint gives the same result as type III constraint; this indicates the strike, dip range from type I constraint may be too large to bring additional information in constraining $m_{22}$ for this noise-free dataset. Secondly, among all three constraints, type II constraint gives the least error in the inverted source parameters. This is because maximizing the DC percentage, as in type I & III constraint, is not a good assumption about the actual source (the true moment tensor is non-double-couple, with 74% of DC, 15% of CLVD, and 11% of ISO component). Moreover, knowing strike value not only helps constrain the fracture plane geometry such as the strike, dip, and rake values, but also enables the recovery of $m_{22}$ and, eventually, moment component percentages.

Next, we add 10% Gaussian noise into the synthetic horizontal component data and perform 100 moment tensor inversions on the band-pass filtered noisy data, each with a different noise realization. The histograms of the inverted source parameters are plotted in Figure 4-10 for this non-double-couple source.
Table 4-2 summarizes the statistics of the histograms in Figure 4-10. It gives the mean absolute errors in the inverted source parameters under three different inversion constraints. With data noise, we observe that mean absolute errors in the strike, dip, rake of the type I constraint are smaller than those of the type III constraint; this implies that even a rough knowledge of the strike, and dip range helps reduce the uncertainty of $m_{22}$ and, eventually, the fracture plane solution (strike, dip, rake). The errors in strike, and dip estimates are also bounded, as explicitly specified in type I constraint ($\pm 15^\circ$ for Table 4-2).

Knowing the exact strike value, as in the type II constraint, greatly reduces the errors in the estimated fracture plane solution and seismic moment. However, the mean absolute errors in the CLVD, DC percentages seem to be slightly higher than those of type I constraint. This may indicate a tradeoff in errors between the fracture plane solution and moment component percentages for the noisy data scenario. Furthermore, a comparison between the noise free case (Table 4-1) and 10% Gaussian noise case (Table 4-2) shows that random noise does not cause a serious distortion in the inverted source parameters. Compared to the random noise, the closeness of the applied constraints to the true source model probably plays a bigger role in the constrained moment tensor inversion for events at far field.

Similar to Figure 4-10, we conduct a Monte-Carlo study of the constrained moment tensor inversion for a double-couple source with the same strike, dip, and rake values as the previous non-double-couple case. The histograms of the inverted source parameters are given in Figure 4-11.

Table 4-3 summarizes the double-couple source inversion results under three different constraints. We see that maximizing DC percentage, as in type III constraint, gives the smallest mean absolute errors in component percentage estimates while knowing strike value, as in type II constraint, helps reduce the errors in the fracture plane solution. In general, from Table 4-2 and Table 4-3, we see that, with a reasonable amount of data noise, and errors in velocity model and source location, the complete moment tensor can be inverted from one-well data at far field by imposing additional constraints such as the fracture plane orientation.
It is worth noting that the synthetic study conducted here is not a complete test on the influence of velocity model errors, since only one random perturbation of the velocity model is used in the inversion. Furthermore, one should be cautious that the influence of velocity model errors can be more serious when the source and the geophone array are situated in two different velocity layers.

4.4 Field study

4.4.1 Field setup

A microseismic survey was conducted during the hydraulic fracturing treatment of the Bonner sands in the Bossier play at a depth approximately from 3956 m (12980 ft) to 3981 m (13060 ft). The microseismic data were collected using a twelve-level, three-component geophone array deployed in the vertical monitoring well at a depth from 3874 m (12710 ft) to 3944 m (12940 ft). The treatment well is approximately 151 m (495 ft) away from the monitoring well. The recorded data were analyzed and located for hydraulic fracturing mapping as outlined by Griffin et al. (2003), and Sharma et al. (2004). The velocity model for location, shown in Figure 4-2, was derived from the well logging data and calibrated using perforation shots (Warpinski et al., 2003). The information on local geology was also considered when building the velocity model.

In this study, we test our method on several located microseismic events to invert for the complete moment tensor and estimate source parameters. The microseismic data from the bottom six geophones at a depth from 3912 m (12835 ft) to 3944 m (12940 ft) are selected due to their higher signal-to-noise ratios (SNRs). The P-waves on the upper 6 geophones are barely identifiable due to the larger distance from the events. The average S-wave SNR on the upper 6 geophones is also 10 dB lower than that on the bottom 6 geophones. Moreover, due to the poor clamping of vertical component geophones, the average SNR of the band-pass filtered vertical component data is at least 10 dB lower than that of the band-pass filtered horizontal component data. On the other hand, from Figure 4-3(b), it is observed that two horizontal
components have a similar capability in resolving the moment tensor as three components. Therefore, only the two horizontal components from the bottom 6 geophones are used in the following moment tensor inversion.

Figure 4-12 illustrates the horizontal plane view of the located events, with monitoring well at the origin. The average fracture trend is seen along the N87°E or N-93°E direction (Sharma et al., 2004). Seven events at a depth from 3975 m to 3993 m are selected and plotted as red circles. The mean source-receiver distance for the selected events is around 15λs (106.7 m). The average noise level as a percentage of maximum absolute signal amplitude is about 7% for the selected events, which is lower than the 10% noise level used in the synthetic study.

In the following section, we will begin with one event, named test event 1, to demonstrate the procedure of the constrained moment tensor inversion and source parameter estimation using full waveforms. After that, we will present and discuss the results from all seven chosen events.

4.4.2 Moment tensor inversion and source parameter estimation

As discussed in the synthetic study, for events that have a mean source-receiver distance larger than 5λs, the complete moment tensor can be inverted from full waveforms by imposing additional constraints. Warpinski and Du (2010) used direct P- and S-wave amplitudes from this one-well dataset and applied a zero-trace (deviatoric source) constraint to invert for the source mechanisms and reported a large amount of scatter in the inverted strike and dip values.

In this study, instead of the deviatoric source constraint, a more realistic constraint on the fracture geometry is applied in the inversion. A conservative strike range of +/-60° around the average fracture trend and a dip range of 60°-90° is used as the type I constraint in this field example. The source parameters including the fracture plane solution, seismic moment, and component percentages are estimated from the inverted complete moment tensor.
Figure 4-13 shows the constrained inversion for test event 1 with type I constraint. The cyan strip gives the permissible range of $m_{22}$ values. The $m_{22}$ value is determined by the green vertical line representing the maximum DC percentage within the allowed strike, dip range. Thus, the complete moment tensor is obtained.

Figure 4-14a) and Figure 4-14b) give the waveform fitting for test event 1 between modeled and observed data. A good agreement of dominant P- and S-wave trains is seen in both Figure 4-14a) and Figure 4-14b). This gives confidence in the event location and 1D velocity models. The un-modeled wave packages are probably due to random noise and the un-modeled lateral heterogeneities.

The source parameters of test event 1 estimated from the complete moment tensor are listed in Table 4-4. The seismic moment for event 1 is around $1.8\times10^4$ N·m, suggesting a moment magnitude around -3.22. The two strike values estimated from the double-couple component correspond to the orientation of the fracture plane and the auxiliary plane, respectively. It is hard to distinguish the two planes with only one event. The estimated strike, dip, and rake values for all test events are listed in Table 4-4. The first set of values agrees well with the average fracture trend of N87°E or N-93°E observed by Sharma et al. (2004), and is chosen as the fracture strike. Although the constraint used in the inversion assumes a strike range of +/- 60° around the average fracture trend, the actual inverted strike values for the six out of seven events have a maximum deviation from the average fracture trend of less than +/- 35°. In other words, additional information brought by the constrained inversion improves our a priori knowledge on source parameters, more specifically the fracture strike. The difference between the inverted strike values and the average fracture trend comes from the fact that the orientation of small local fractures described by individual event differs from the average fracture orientation given by multiple event location (Rutledge and Phillips, 2003). Furthermore, noise contamination may also contribute to the difference.

Table 4-4 also summarizes the estimated component percentages. The results indicate a dominant double-couple component for most events. However, even considering the errors in the component percentage estimates as discussed in the
synthetic study, a non-negligible volumetric component is observed for some events such as test events 3 and 6.

For each event, the corner frequency is estimated from the far-field S-wave displacement spectrum (Walter and Brune, 1993). The approximate source radius is then determined from the corner frequency estimate according to Madariaga’s model (Madariaga, 1976; Talebi and Boone, 1998). The corner frequencies of all seven test events range from 450 Hz to 750 Hz. The derived source radii indicate a small rupture area on the order of 1 m². The moment magnitude of the test events ranges from -4 to -2, which is consistent with previous studies of hydrofracture events from downhole observations (Warpinski, 2009).

4.5 Summary

In this chapter, we developed a full-waveform based complete moment tensor inversion approach for hydraulic fracture monitoring using microseismic data recorded at a vertical borehole. The study involved both synthetic data and field data. Condition number study showed that two monitoring wells at an azimuthal separation of 45° have a similar resolving power of the moment tensor as eight wells with full azimuthal coverage. By exploring full wavefields in a layered medium instead of using only far-field direct P- and S-wave amplitudes, we demonstrated that the complete moment tensor can be retrieved for events that are close to the monitoring well. The near-field and non-direct wave (i.e., reflected/refracted waves) information in a layered medium contribute to the decrease in the condition number. On the other hand, when the events are in the far-field range, two monitoring wells are desirable for complete moment tensor inversion.

By synthetic tests, we demonstrated that, complete moment tensor from one-well data at far field is possible if one imposes some appropriate constraints. Far-field tests with different constraints indicate that a priori information on fracture orientation helps recover the complete moment tensor and reduce the uncertainty of not only the fracture plane solution but also seismic moment and moment component percentages.
Synthetic study also shows that a reasonable amount of error in source location and the velocity model, together with random noise, do not cause a serious distortion in the inverted moment tensors and source parameters.

Proper constraints on the source play a big role in complete moment tensor retrieval using one-well data at far field. The strike, and dip range constraints were applied in a field study to invert for complete moment tensor from one-well data at far field. The results indicate the existence of both double-couple and non-double-couple components in the source. The fracture strike values, derived by the inversion, generally agree with the average fracture trend determined from multiple event location.

Potential errors in source parameter estimates from one-well data at far field primarily come from the inaccuracies in the a priori information that has been used in the inversion. Future work will include testing the method against the results from two-well inversion. An extended study on the influence of velocity model errors will also be carried out in the future. The full-waveform approach has the potential to improve the source properties study of microseismic events monitored using borehole sensors even in a single well.

Acknowledgements

The authors would like to thank Pinnacle - A Halliburton Service for providing the data and for funding this research. We are grateful to Dr. Norm Warpinski, Dr. Jing Du, Dr. Erkan Ay and Dr. Qinggang Ma from Halliburton Energy Services Company; Dr. Bill Rodi, Dr. H. Sadi Kuleli and Dr. Michael Fehler from MIT for their helpful discussions. We thank Halliburton Energy Services Company and Anadarko Petroleum Corporation for permission to publish this work. We would like to thank the reviewers and the associate editors for the incisive and helpful comments. Their suggestions contribute significantly to the improvement of the paper.
4.6 References


Table 4-1: Summary of microseismic source inversion with one-well data under different constraints. The inversion is performed with noise-free data and using the approximate velocity model and the mislocated source. The average source-receiver distance is 91.4 m (300 ft). The true moment tensor of this non-double-couple source is described in Figure 4-4.

<table>
<thead>
<tr>
<th>Type of inversion constraints</th>
<th>I</th>
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<tr>
<td>Isotropic component percentage (%)</td>
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<td>-28</td>
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<td>CLVD component percentage (%)</td>
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<td>-15</td>
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<td>Dip (Degrees)</td>
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<td>1</td>
<td>-9</td>
</tr>
<tr>
<td>Rake (Degrees)</td>
<td>-8</td>
<td>-4</td>
<td>-8</td>
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</table>
Table 4-2: Statistics of non-double-couple microseismic source inversion with one-well data under different constraints (Refer to Figure 4-13). The inversion is performed with 10% Gaussian noise contaminated data and using the approximate velocity model and the mislocated source. The average source-receiver distance is 91.4 m (300 ft). The true moment tensor is described in Figure 4-4.

<table>
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<th>Type of Inversion constraints</th>
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<tr>
<td>Dip (Degrees)</td>
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<td>Rake (Degrees)</td>
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Table 4-3: Statistics of double-couple microseismic source inversion with one-well data under different constraints (Refer to Figure 4-11). Table caption is analogous to Table 4-2.

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<th>II</th>
<th>III</th>
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<td>Dip (Degrees)</td>
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Table 4-4: Results of source parameter determinations for the seven selected test events using constrained inversion with Type I constraint.

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<tr>
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<th>$M_w$</th>
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<th>ISO%</th>
<th>CLVD%</th>
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<tr>
<td></td>
<td>$10^4$N·m</td>
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<tr>
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<td>-3.22</td>
<td>96</td>
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<td>-3</td>
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<td>68</td>
<td>3</td>
<td>-29</td>
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<tr>
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<td>-3.06</td>
<td>52</td>
<td>48</td>
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<td>5.8</td>
<td>-2.89</td>
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<td>31</td>
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</tr>
<tr>
<td>5</td>
<td>1.4</td>
<td>-3.30</td>
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</tr>
<tr>
<td>6</td>
<td>3.2</td>
<td>-3.05</td>
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<tr>
<td>7</td>
<td>3.3</td>
<td>-3.05</td>
<td>82</td>
<td>18</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Event</th>
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<th>Dip</th>
<th>Rake</th>
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<tbody>
<tr>
<td></td>
<td>Degrees</td>
<td>Degrees</td>
<td>Degrees</td>
</tr>
<tr>
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<td>108</td>
<td>81</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>107</td>
<td>62</td>
<td>8</td>
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<td>73</td>
<td>-43</td>
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Note: The strike, dip, rake, and slope values are defined according to the conventions set forth by Aki & Richards [2002].
Figure 4-1: (a) Horizontal plane view of the source and receiver array distribution in the condition number study. The microseismic event, labeled as the plus sign, lies in the center, with 8 monitoring wells, B1 to B8, evenly spreading from the North direction to the North-West direction. The azimuthal separation between two adjacent wells is 45°. (b) 3D view of the single well configuration used in the inversion study (B1 well, at the azimuth of N0°E). The grey star denotes the hypocenter location of the microseismic event, while the six receivers, deployed in the well, are shown as black triangles. (North: x, East: y, Down: z)
Figure 4-2: One-dimensional P- and S-wave velocity model derived from field study.
Figure 4-3: The condition number of the waveform sensitivity matrix $A$, plotted as a function of the mean source-receiver distance, shown in multiples of the dominant S-wave wavelength. The matrix $A$ is formed using: a) three-component full waveforms under different well configurations; b) full waveforms of three components or two horizontal components from the six-receiver array in B1 well at the azimuth of $0^\circ$. Well azimuth is defined as East of North.
Figure 4-4: Synthetic seismograms recorded by the six receivers in well B1 from a non-double-couple microseismic source (horizontal components only, with North component in red, East component in blue). a) total wave-fields. b) near-field terms only. Each source-receiver distance is shown as multiples of the dominant S-wave wavelength ($\lambda_s = 5.2$ m). The average source-receiver distance is 18.3 m (60 ft). The scaling factor for each trace is also listed. The source has a strike of 108°, dip of 80°, and rake of 43°. The source is composed of: 74% DC component, 15% CLVD component, and 11% isotropic component.
Figure 4-5: Synthetic data from the non-double-couple microseismic source. a) After adding 10% Gaussian noise to the horizontal component data shown in Figure 4-4. b) After applying the [200, 900] Hz band-pass filter to the noise contaminated data in a). The North component is plotted in red, while the East component is shown in blue. The scaling factor is 30.
Figure 4-6: Comparison between the modeled data in black and band-pass filtered synthetic data in red for the non-double-couple source in Figure 4-4. The modeled data are generated from the inverted microseismic moment tensor matrix (6 independent elements). The unconstrained inversion is performed with the band-pass filtered horizontal components in Figure 4-5b). a) North component plot. b) East component plot. The scaling factor is 30. All the inversions in this study are performed with only horizontal components from well B1, and using the approximate velocity model and the mislocated source (see text).
Figure 4-7: The histograms of errors in the inverted source parameters. The microseismic source is non-double-couple. The true moment tensor and source-receiver locations are described in Figure 4-4. The unconstrained inversion is performed with the band-pass filtered horizontal components from well B1.
Figure 4-8: The condition number of the waveform sensitivity matrix $A$, plotted as a function of the mean source-receiver distance, shown in multiples of the dominant S-wave wavelength. The matrix $A$ is formed using full waveforms of two horizontal components recorded by the six-receiver array in the monitoring well B1. The condition number of the unconstrained inversion in the layered medium for all six independent moment tensor elements is plotted in red, while the condition numbers of the constrained inversion in the layered and homogeneous medium for five independent moment tensor elements except $m_{22}$ are shown in black and blue, respectively.
Figure 4-9: Synthetic test on non-double-couple source mechanism: Top plot: strike (red line), dip (black line), and rake (blue line) of DC component of the full moment tensor as a function of the unconstrained component \( m_{22} \). Middle plot: components of the full moment tensor as a function of the unconstrained component \( m_{22} \). Red line, double-couple (DC); black line, isotropic (ISO); blue line, compensated linear vector dipole (CLVD). Bottom plot: inverted seismic moment as a function of the unconstrained component \( m_{22} \), with \( M_0 \) as the true seismic moment. The inversion is performed with type I constraint, where the range of inverted strike, dip is specified a priori. The cyan strip represents the allowed strike, dip range. The constrained inversion recovers \( m_{22} \) by seeking to maximize the DC percentage within the cyan strip. The correct solution is represented by the vertical green line. The inversion is performed with noise-free data from well B1. The average source-receiver distance is 91.4 m (300 ft). The true moment tensor is described in Figure 4-4.
Figure 4-10: The histograms of errors in the inverted source parameters (non-double-couple source). The true moment tensor and the source-receiver configuration are described in Figure 4-9. The constrained inversion is performed with 10% Gaussian noise contaminated data. Left column: inversion with Type I constraint. Middle column: inversion with Type II constraint. Right column: inversion with Type III constraint. See main text for details on different constraint types.
Figure 4-11: The histograms of errors in the inverted source parameters (double couple source). The source has a strike of 108°, dip of 80°, and rake of 43°. The source-receiver configuration is described in Figure 4-9. The rest of the figure description is analogous to Figure 4-10.
Figure 4-12: Horizontal plane view of microseismic event locations for the Bonner dataset. Seven selected test events for moment tensor inversion are shown as red circles.
Figure 4-13: Constrained inversion for test event 1 with Type I constraint. The figure description is analogous to Figure 4-9.
Figure 4-14: Waveform fitting for test event 1. Modeled seismograms derived from constrained inversion are shown in black, while the observed seismograms are plotted in red. a) North component. b) East component.
Chapter 5

Microseismic Source Characterization in the Barnett Shale Using Dual Array Data: Linking Microseismicity to Reservoir Geomechanics

Abstract

Microseismic source mechanisms contain important information for understanding the reservoir, natural fractures, stress state, and fracturing mechanisms. In its complete form, the microseismic source is represented by a symmetric moment tensor having six independent components. Difficulties arise when attempting to invert for the complete moment tensor with the conventional amplitude inversion method if only a single monitoring well is available. With the full waveform approach, as previous studies have shown, the near-field information and non-direct waves (i.e. refracted/reflected waves) help stabilize the inversion and retrieve the complete moment tensor from the single-well dataset. However, for events which are in the far field from the monitoring well, a multiple-well dataset is required to invert for

4(The bulk of this Chapter has been) submitted as: Song, F., Warpinski N. R., and M. N. Toksöz, Full-waveform Based Microseismic Source Mechanism Studies in the Barnett Shale: Linking Microseismicity to Reservoir Geomechanics, for Geophysics.
complete moment tensor. In this study, we perform the complete moment tensor inversion with a dual-array dataset from a hydraulic fracturing stimulation in the Barnett shale at Fort Worth Basin. Determining the source mechanism from the moment tensor requires the use of a source model, which in this study is the general dislocation model or, equivalently, the model of tensile earthquakes. The tensile earthquake model could describe the microearthquake source more adequately and predict the non-DC components. The source information derived consists of the fault plane solution (FPS), the slip direction, the Vp/Vs ratio in the focal area, and the seismic moment. The primary challenge of extracting the source parameters from the moment tensor is to distinguish the fracture plane from the auxiliary plane. In this study, we analyze the microseismicity in the Barnett shale using hydraulic fracture geomechanics. With the insights gained from geomechanical analysis, we are able to determine the fracture plane from the moment tensor. Furthermore, we investigate the significance of the occurrence of non-DC components by F-test. We also study the influence of velocity model errors, event mislocations, and additive data noise on the extracted source parameters using synthetic data. The results of source mechanism analysis are presented for the best signal-to-noise ratio (SNR) events triggered by waterfrac treatment. Some microseismic events are shown to have fracture planes with similar orientations to natural fractures delineated by core analysis, suggesting the reactivation of natural fractures during the hydrofracture treatment. Other events occur as predominantly tensile events striking along the unperturbed maximum horizontal principal stress (SHmax) direction, indicating an opening mode failure on the hydraulic fracture strands trending sub-parallel to the unperturbed SHmax direction. The microseismic event source mechanisms not only reveal important information about the fracturing mechanism, but also allow fracture characterization away from the wellbore, providing critical constraints for understanding fractured reservoirs.

5.1 Introduction
Microseismic mapping has proven valuable for monitoring stimulations in unconventional reservoirs such as gas shales (Fisher et al., 2004; Shemeta et al., 2007; Maxwell et al., 2010; Birkelo et al., 2012). Besides location, microseismic waveforms contain important information about the source mechanisms and stress state (Baig and Urbancic, 2010). The complete moment tensor of the general source mechanism consists of six independent components (Aki and Richards, 2002). Previous studies have demonstrated that conventional methods using only far-field P- and S-amplitudes from one vertical well cannot retrieve the off-plane moment tensor component and therefore have to make additional assumptions such as assuming a deviatoric source (Vavryčuk, 2007).

However, recent studies have shown the existence of non-double-couple (non-DC) mechanisms for some hydrofracture events (Šilený et al., 2009; Warpinski and Du, 2010). Knowledge of the complete moment tensor, especially the non-DC components, is essential to understand the fracturing process especially the failure mechanisms (Šilený et al., 2009). Moreover, Vavryčuk (2007) showed that, for shear faulting on non-planar faults, or for tensile faulting, the deviatoric source assumption is no longer valid and can severely distort the retrieved moment tensor and bias the fault plane solution (FPS: strike, dip, and rake angles). Therefore, the complete moment tensor inversion is crucial not only to the retrieval of the non-DC components but also to the correct estimation of the fracture plane orientation.

To overcome the difficulty associated with single-well complete moment tensor (MT) inversion, Song and Toksöz (2011) proposed a full waveform approach to invert for the complete moment tensor. They demonstrated that the complete moment tensor can be retrieved from a single-well dataset by inverting the full waveforms, if the events are close to the monitoring well. It has been shown that the near-field information and nondirect waves (i.e., reflected/refracted waves) propagated through a layered medium contribute to the decrease in the condition number of the sensitivity matrix. However, when the events are in the far-field range, at least two monitoring wells are needed for complete moment tensor inversion. Therefore, in this chapter, we
invert for the complete moment tensor to determine the microseismic source mechanisms in the Barnett shale by using dual array data.

Determining the source mechanism from the moment tensor requires the use of a source model. As pointed out by Vavryčuk (2011), one of the models describing the earthquake source more adequately and predicting significant non-DC components is the general dislocation model or, equivalently, the model of tensile earthquakes (Vavryčuk, 2001). This model allows the slip vector defining the displacement discontinuity on the fracture to deviate from the fracture plane. Faulting can thus accommodate both shear and tensile failures. Consequently, the fracture can possibly be opened or closed during the rupture process. Tensile earthquakes have been reported in hydraulic fracturing and fluid injection experiments (Zoback, 2007; Šílený et al., 2009; Baig and Urbancic, 2010; Warpinski and Du, 2010; Song and Toksöz, 2011; Fischer and Guest, 2011). Moreover, field and experimental observations reveal that simple, planar hydraulic fractures, as commonly interpreted in many reservoir applications, are relatively rare (Busetti et al., 2012). The location analysis of microseismic events during the hydrofracture stimulation in the Barnett Shale, Fort Worth Basin, Texas, reveals complex location patterns that depend on the local stress state and proximity to folds, faults, and karst structures (Roth and Thompson, 2009; Warpinski et al., 2005). Therefore, in this study, we adopt the tensile earthquake model to determine the microseismic source mechanisms from the inverted moment tensor. The extracted source parameters include the FPS, the slip direction, the Vp/Vs ratio in the focal area, and the seismic moment. The determined source mechanisms are aimed to help better understand the formation of the observed complex location patterns and eventually the fracturing process in the Barnett shale.

We select several events with good signal-to-noise ratios (SNR) and low condition numbers out of a dual-array microseismic dataset from a hydraulic fracture stimulation of the Barnett shale at Fort Worth Basin, USA. We use the discrete wavenumber integration method to calculate elastic wavefields in the layered medium (Bouchon, 2003). By matching the waveforms across the two geophone arrays, we invert for the moment tensor of each selected event. To derive the source parameters
from the moment tensor, the fracture plane has to be separated from the auxiliary plane. To address this problem and better understand how the microseismicity is related to the fracturing process, we study the hydraulic fracture geomechanics in the Barnett shale. Based on the observations from geomechanical analysis, we describe an approach to determine the source parameters from the inverted moment tensor. To quantify the uncertainty of extracted source parameters, we conduct a Monte-Carlo test on synthetic data to study the influence of velocity model errors, source mislocations and additive data noise. Furthermore, we also investigate the significance of the occurrence of non-DC components by F-test. We show that apart from the DC component, the majority of the events have significant non-DC components, in the appearance of an off-fracture-plane slip vector. Finally, we discuss the estimated microseismic source mechanisms and their implications in understanding the fracturing process and the reservoir.

5.2 Methodology

5.2.1 Tensile earthquake model

To describe the complexity in the earthquake source that gives rise to the occurrence of significant non-DC components, a general tensile earthquake model was proposed by Vavryčuk (2001) and further illustrated by Vavryčuk (2011). In this study, we follow the convention of Vavryčuk (2011). As shown in Figure 5-1, the fracture plane normal \( n \) and the slip vector \( v \), defined in the (north, east, downward) coordinate system, are expressed for the tensile source in terms of strike \( \phi \), dip \( \delta \), rake \( \lambda \), and slope angle \( \alpha \) as follows:

\[
\begin{align*}
    n_1 &= -\sin\delta\sin\phi \\
    n_2 &= -\sin\delta\cos\phi \\
    n_3 &= -\cos\delta \\
    v_1 &= (\cos\delta\sin\lambda\sin\phi + \cos\lambda\cos\phi)\cos\alpha - \sin\delta\sin\phi\sin\alpha \\
    v_2 &= (-\cos\delta\sin\lambda\cos\phi + \cos\lambda\sin\phi)\cos\alpha + \sin\delta\cos\phi\sin\alpha \\
    v_3 &= -\sin\delta\sin\lambda\cos\alpha - \cos\delta\sin\alpha .
\end{align*}
\]
Here, strike $\phi$ is measured clockwise round from North. The dip $\delta$ is defined as the angle between the fracture plane and the horizontal. The rake $\lambda$ is measured in the fracture plane as the angle between the strike vector and the projected slip vector. The slope angle $\alpha$ is defined as the inclination of the slip vector from the fracture plane. A positive $\alpha$ indicates a tensile earthquake, while a negative $\alpha$ represents a compressive event.

The seismic moment tensor $M$ for this source in an isotropic medium is,

$$M_{kl} = \lambda_p v_l n_k \delta_{kl} + \mu (v_k n_l + v_l n_k)$$  \hspace{1cm} (5-3)

where $\lambda_p$ and $\mu$ are the Lamé coefficients at the focal area (to avoid confusion with fault rake angle $\lambda$, the Lamé first parameter is denoted as $\lambda_p$ in this chapter), $\delta_{kl}$ is the Kronecker delta, $n_l$ and $v_l$ are the slip vector and fracture plane normal shown in Equations (5-1) and (5-2), respectively. The symmetric moment tensor $M$ can be diagonalized and decomposed into double-couple (DC), isotropic (ISO), and compensated linear vector dipole (CLVD) components,

$$M = M_{\text{DEV}} + M_{\text{ISO}} = M_{\text{CLVD}} + M_{\text{DC}} + M_{\text{ISO}}$$  \hspace{1cm} (5-4)

According to Vavryčuk (2011), the eigenvector $b$ of the moment tensor matrix $M$ associated with the intermediate eigenvalue gives the null axis, while the eigenvectors $t$ and $p$ corresponding to the maximum and minimum eigenvalues give the tension and compression axis, respectively. The fracture plane normal $v$ and the slip vector $u$ can be derived from the $t$ and $p$ axes after compensating for the non-zero slope angle $\alpha$ (Vavryčuk, 2001) as follows:

$$\sin \alpha = \frac{3 (\lambda_{\text{max}}^\text{dev} + \lambda_{\text{min}}^\text{dev})/(\lambda_{\text{max}} - \lambda_{\text{min}}^\text{dev})}{\sqrt{1 + \sin \alpha t + \sqrt{1 - \sin \alpha p}}}, \hspace{1cm} (5-5)$$

$$v = \frac{1}{\sqrt{2}} (\sqrt{1 + \sin \alpha t + \sqrt{1 - \sin \alpha p}}), \hspace{1cm} (5-6)$$

$$n = \frac{1}{\sqrt{2}} (\sqrt{1 + \sin \alpha t - \sqrt{1 - \sin \alpha p}}). \hspace{1cm} (5-7)$$

$\lambda_{\text{max}}^\text{dev}$, $\lambda_{\text{min}}^\text{dev}$ denote the maximum and minimum eigenvalues of the deviatoric moment tensor $M_{\text{DEV}}$. Based on equations (5-1), (5-2), (5-5) and (5-6), the source parameters, slope angle $\alpha$, strike $\phi$, dip $\delta$, and rake $\lambda$, could be determined from the moment tensor $M$. The ratio between the Lamé coefficients $\lambda_p$ and $\mu$ at the focal area is another
source parameter, defined as \( k \) and can be derived from the moment tensor \( M \) as follows:

\[
k = \frac{\lambda_p}{\mu} = \frac{2}{3} \left( \frac{\text{tr}(M)}{\lambda_{\text{max}} + \lambda_{\text{min}}} - 1 \right).
\] (5-8)

According to Vavryčuk (2001), the stability conditions imposed on an isotropic medium requires

\[
k = \frac{\lambda_p}{\mu} > -\frac{2}{3}, \quad \mu > 0.
\] (5-9)

This also poses a lower limit for the \( V_p/V_s \) ratio at the focal area of the earthquakes that follow the tensile earthquake model,

\[
V_p/V_s = \sqrt{k + 2} > 1.15
\] (5-10)

According to this limit, all measurable physical properties in the focal area including \( V_p, V_s, \) the bulk modulus and the shear modulus are positive, in spite of the fact that for some cases, the Lamé first parameter \( \lambda_p \) may be negative.

Other source parameters including seismic moment \( M_\text{b} \), moment tensor magnitude \( M_w \), and DC, ISO, and CLVD component percentages could also be determined from the moment tensor (Vavryčuk, 2001, Song and Toksöz, 2011).

### 5.2.2 Full-waveform based source mechanism determination using dual-array data

According to our earlier study, the near-field information and nondirect waves (i.e., reflected/refracted waves) propagated through a layered medium contribute to the decrease in the condition number of the sensitivity matrix, and therefore stabilize the moment tensor inversion (Song and Toksöz, 2011). In this chapter, we adopt the full waveform inversion approach of in Song and Toksöz (2011) to determine the complete moment tensor of microseismic events in the Barnett shale.

To reduce the influence from errors in source locations, during the moment tensor inversion, we perform a grid search around the initial source location (Song and Toksöz, 2011). The spatial search range and grid size are selected based on the location uncertainty. The location uncertainty in the downhole monitoring scenario is estimated from the standard deviations of P- and S-wave arrival times and P-wave polarization.
angles (Eisner et al., 2010). For the dual-array dataset used in this study, we calculate standard deviations and obtain 4.6 m (15 ft) in the radial direction, 7.6 m (25 ft) in the vertical direction and 2° in P-wave derived event back-azimuths constrained by two geophone arrays. We further determine the location uncertainty in the horizontal directions (North, East) from the standard deviations of the radial distances and P-wave derived event back-azimuths at a typical distance of 305 m (1000 ft) for the selected 42 events. The standard deviation is estimated to be 10.6 m (35 ft). Therefore, a spatial grid size of 3 m (10 ft) and a spatial search cube with the size of 7*7*5 grids (North, East, Down) are used throughout this paper.

In this study, we match full waveforms from two vertical wells. In principal, complete moment tensor can be extracted from two observation wells for any event not situated on the observation well plane. As pointed out by Eaton (2009), in the homogeneous medium, the condition number of the sensitivity matrix for moment tensor inversion is inversely proportional to the solid angle at the source subtended by the geophone array. The nondirect waves propagated through a layered medium increase the source take-off angle coverage and, therefore, reduce the condition number (Song and Toksöz, 2011). In either case, an azimuthal angle at the source subtended by two vertical geophone arrays close to 90° is desirable to reduce the condition number of the sensitivity matrix. Therefore, in this paper, we select several events that have both good SNRs and azimuthal angles to the two geophone arrays close to 90°. In this way, low condition numbers are assured.

In this study, there was a significant difference in noise standard deviations from geophones at different wells. Thus, a weighted least-squares inversion is performed inside the grid search loop of event location and origin time. The weights are determined from the pre-event noise standard deviation at each geophone, for each component. The weight for the n-th geophone, l-th component, $w_{nl}$, is calculated as the inverse of the pre-event noise standard deviation at the corresponding channel:

$$w_{nl} = 1/\text{std}(n_l(x^p, t)), \quad (5-11)$$

where $n_l(x^p, t)$ is the l-th component data of the pre-event noise at n-th geophone.
The best solution of the event location $x_s$, origin time $t_0$, and moment tensor $M_l$ (1 = 1,2,...,6) is determined by minimizing the squared L-2 norm of the weighted waveform fitting error:

$$J(x_s, t_0, M_l) = \sum_{k=1}^{N_k} \sum_{n=1}^{N_n} \sum_{l=1}^{N_L} w_{nl}(d_l(x^n_l, k\Delta t) - v_l(x^n_l, x_s, k\Delta t))^2.$$  (5-12)

Equivalently, the grid search based complete moment tensor inversion is meant to maximize the variance reduction $\text{VAR}$, defined as,

$$\text{VAR}(x_s, t_0, M_l) = 1 - J(x_s, t_0, M_l) .$$  (5-13)

In this study, we noticed a poor SNR in the vertical component data, as also seen in our earlier study (Song and Toksoz, 2011). Therefore, only horizontal components are used in the inversion. The reasons for the poor SNRs associated with the vertical component may come from two sources. Firstly, vertical component geophones are normally harder to couple into the formation compared to horizontal component geophones in a vertical borehole. Secondly, surface noise such as pumping and culture noise coupled into the borehole propagates as guided wave modes like Stoneley-waves, which have predominant motion in the vertical component.

5.3 Field study

5.3.1 An overview of the Barnett gas shale reservoir

The Fort Worth Basin was bordered on its outboard side by an island-arc system which supplied very little coarse-grained sediment to the Barnett Shale. Limestone interbeds in the Barnett (including the middle Forestburg Member) formed as mass-gravity or turbidity flows of skeletal material derived from surrounding carbonate platforms. Immediately after black-shale deposition, a temporary expansion of the western carbonate produced the overlying Marble Falls Formation. The Mississippian stratigraphic section in the Fort Worth Basin consists of limestone and organic-rich shale. The Barnett Shale formation, in particular, consists of dense, organic-rich, soft, thin-bedded, petrolierous, fossiliferous shale and hard, black, finely crystalline, petrolierous, fossiliferous limestone (Lancaster et al., 1993).
The Barnett Shale, as determined by core and outcrop studies, is dominated by clay- and silt-size sediment with occasional beds of skeletal debris. In lithologic descriptions, the Barnett shale is a mudstone rather than shale. It is highly indurated, with silica making up approximately 35–50% of the formation by volume and clay minerals less than 35% (Bruner and Smosna, 2011). This silica-rich nonfissile shale behaves in a more brittle fashion and fractures more easily than clay-rich shales, responding well to stimulation.

The Barnett shale reservoir has characteristic features of very low matrix permeability in the range of microdarcies to nanodarcies (Johnston, 2004), and some degree of natural-fracture development (Bruner and Smosna, 2011). From core studies, two major sets of natural fractures were identified. One fracture system had an azimuth of north-south (N-S) and another, west-northwest-east-southeast (WNW) (Gale et al., 2007; Gale & Holder, 2010). Surprisingly the natural fractures in the Barnett shale were completely healed and filled with calcites.

5.3.2 Field setup

A microseismic survey using two vertical wells at a separation of about 487 m (1600 ft) was conducted during the waterfrac treatment of the Barnett shale in the Fort Worth Basin at depths of about 2290 m (7500 ft). Each observation well had twelve-level, three-component geophones spaced approximately 12 m (40 ft) apart, with the tool situated just above the shale interval that was being stimulated. The recorded data were analyzed and located for hydraulic fracturing mapping as outlined by Warpinski et al. (2005). The velocity model for location, shown in Figure 5-2a, was derived from the well logging data and calibrated using perforation shots. The information on local geology was also considered when building the velocity model.

A typical anisotropy parameter for the Barnett shale is reported as $\varepsilon = 0.1, \Delta = 0.2, \gamma = 0.1$ (note that the Thomsen parameter which controls the near-vertical anisotropic response is denoted as $\Delta$ in this chapter to avoid the confusion with fracture dip angle $\delta$) (Warpinski et al., 2009). From the examination of the ray paths from all microseismic events to two geophone arrays, it is found that the ray paths are mostly...
horizontal, with a maximum deviation from the horizontal less than 22° (Warpinski et al., 2009). According to the weak anisotropy theory of Thomsen (1986), the P-wave velocity variation within this range would be less than 0.5%, while the SH velocity variation would be less than 2%. Therefore, we may conclude that, for this dataset, the effect of anisotropy on the waveform modeling is small relative to the general uncertainty in velocity. In the study, the perforation-calibrated horizontal velocity model described in Figure 5-2a is used and the anisotropy effect is neglected. Table 5-1 lists the seismic properties of the layer sequence in the Barnett shale reservoir, which are used to generate synthetic seismograms for moment tensor inversion. The density information is extracted from the density log. The P- and S-wave Q factor values are determined by considering both the lithology and amplitude decay measured across the geophones (Toksoz and Johnson, 1981; Rutledge et al., 2004).

Figure 5-3 gives the horizontal plane view of the microseismic event locations from waterfrac treatment in the Barnett shale using the isotropic velocity model shown in Figure 5-2a. The majority of the microseismic events occur in the lower Barnett shale interval. The two vertical observation wells 1 and 2 are presented as the yellow and green squares on Figure 5-3, respectively, while the treatment well trajectory is plotted as the cyan line with treatment wellhead shown as the blue square. The origin (0, 0) corresponds to the location of observation well 1. The green dashed line represents the observation well plane. As stated previously in the methodology section, we select several events that have both good SNRs and azimuthal angles to the two geophone arrays close to 90° for complete moment tensor inversion. A total of 42 events are selected. Among the chosen events, 4 event groups appear and are denoted as G1, G2, G3, and G4, respectively.

In the following section, we will follow the processing flow proposed in the methodology section, and conduct a systematic study to evaluate the uncertainty of the inverted source parameters for each event group using synthetic data. After that, we will proceed to the geomechanical analysis section to gain some insights on how the microearthquakes are generated. We will also propose an approach to distinguish the fracture plane from the auxiliary plane. Finally, we will discuss the field study results.
5.3.3 Uncertainty of the inverted source parameters from synthetic study

In this section, we study the influence of velocity model errors, source mislocations and additive data noise on the inverted source parameters by performing a Monte-Carlo test using synthetic data.

Firstly, we study the influence of data noise and source mislocations. In this test, we generate noise-free synthetic seismograms for each example event within the four event groups using the reference velocity model shown in Figure 5-2a to mimic the field case. Without losing generality, four tensile earthquakes with \((\phi, \delta, \lambda, \alpha, k)\) of \((60^\circ, 80^\circ, 60^\circ, 20^\circ, -0.3), (30^\circ, 75^\circ, -160^\circ, 15^\circ, 0.8), (55^\circ, 85^\circ, 80^\circ, 25^\circ, -0.5),\) and \((10^\circ, 50^\circ, 75^\circ, -20^\circ, 0.1)\) were simulated to represent events for group G1, G2, G3, and G4, respectively. The double-couple component percentages for each of these four tensile earthquakes are 53\%, 51\%, 48\% and 48\%. The same source model is used throughout the synthetic study section. It is worth noting that a larger slope angle \(\alpha\) is chosen with a higher dip \(\delta\) in this model. The motivation for this choice will be further illustrated in the geomechanical analysis section.

For each well, the noisy synthetic data were formed by adding zero-mean Gaussian noise with a standard deviation reaching 10\% of the absolute maximum amplitude of the two horizontal components averaged across the twelve geophones. The noise was added independently for each geophone array at the same noise level of 10\%. The noise level of 10\% was set to represent the estimated noise level in the field dataset.

To investigate the influence of source mislocations, the true event location is randomly perturbed up to 10.6 m (35 ft) in each horizontal direction and 7.6 m (25 ft) in the vertical direction to represent the location uncertainty in the field example. In the inversion, a grid search is carried out around the perturbed event location. The moment tensor inversion is performed on the [100, 300] Hz band-pass filtered noisy synthetic data using the correct velocity model. The moment tensor solution corresponding to the minimum L-2 waveform fitting error is selected as the inversion result. The source parameters are then estimated from the inverted complete moment tensor. In all synthetic tests, we distinguish the fracture plane from the auxiliary plane by selecting the one with
a smaller error in source parameter estimates. However, in the field study section, where no knowledge about the true source parameters is available, we will propose a method to distinguish the fracture plane from the auxiliary plane according to the insights from the geomechanical analysis.

In order to obtain statistically relevant results, we perform 100 moment tensor inversions and source parameter estimations, each with a different noise realization. Table 5-2 summarizes the average absolute errors of the inverted source parameters for four example events. The condition number of the sensitivity matrix for each example event from the weighted least squares inversion is also listed. The example event G4 has the largest condition number due to the smallest azimuthal angle at G4 subtended by the two geophone arrays, which is seen on Figure 5-3. Overall, the inverted source parameters agree well with the true values, with average absolute errors in both FPS and slope angle $\alpha$ less than 2 degrees. The average absolute errors in component percentages, $k$, and $M_0$ are also negligible. This indicates that with a correct velocity model, microseismic source mechanisms can be reliably determined from the dual-array dataset by the grid search based full waveform inversion approach, as long as the event mislocation is within the location uncertainty and the condition number is reasonably low. Additive data noise has a minimal effect on the inversion, which is also reported in Song and Toksöz (2011). It is interesting to point out that, at the same noise level, errors in the inverted source parameters tend to be higher at a larger condition number. This is reasonable, since the errors propagated into the moment tensor solution from data noise are controlled by the condition number.

Next, we perform the DC inversion instead of complete MT on the same band-pass filtered noisy synthetic data. In this inversion, the event source mechanism is forced to be double-couple. Therefore, it provides no information on $\alpha$, $k$, and component percentages. Table 5-3 lists the average absolute errors of the inverted seismic moment and FPS for four example events. Compared to Table 5-2, it is clear that DC inversion severely biased the estimates of fracture plane orientation even with a correct velocity model. This is understandable, since the DC source clearly is not a good assumption about the
underlying tensile earthquakes, which have a DC component percentage of only about 50%.

Finally, we investigate the influence of velocity model errors on the inversion. In this test, the P- and S-wave velocity models are randomly perturbed up to 10% and 20% of the velocity difference between adjacent layers so that the sign of the velocity difference between adjacent layers does not change. A larger perturbation for S-wave velocity is to take into account the fact that the S-wave velocity is generally less reliably determined than the P-wave velocity. The perturbation is independent between different layers and P- and S-wave velocities are independently perturbed. The density model is kept unchanged, as the velocity perturbation is dominant in determining the characteristics of the waveforms. The Qp and Qs model is also kept constant to study the influence of the velocity perturbation. The velocity models are perturbed 100 times, as shown in Figure 5-2b. We then conduct 100 moment tensor inversions and source parameter estimations, each with a different velocity model and noise realization. In each inversion, the 10% Gaussian noise and the same amount of source mislocations as the case for Table 5-2 are also included.

Figure 5-4 demonstrates the process of the grid search based moment tensor inversion of the synthetic tensile event G1 for one velocity model and noise realization. It plots the normalized variance reduction as a function of searched event location and origin time. The black star denotes the initial source location and origin time estimate, while the white star gives the source location and origin time after full waveform matching. It is clear that the variance reduction function VAR is maximized at the inverted source location and origin time, suggesting a better waveform fit than the initial event location and origin time. The moment tensor solution, event location, and origin time are then determined. Figure 5-5 shows the best waveform fitting for the synthetic event G1. A good agreement between modeled data in black and band-pass filtered synthetic data in red is seen on both components.

100 moment tensor inversions, each with one inaccurate velocity model and noise realization, are performed to study the influence of velocity model errors on the inverted source parameters. Figure 5-6 plots the errors of the inverted event location along (N, E,
D) directions in stars for the synthetic tensile source G1 as a function of different velocity model realizations. The event location error is shown as multiples of search grid size. The black line represents the search limit in the vertical direction for the grid search based moment tensor inversion, while the green line demonstrates the identical search limit in the north and east directions. It is observed that all the location errors are bounded in the search limit. This indicates that our search range is sufficient for the assumed velocity model errors. Figure 5-7 gives the histograms of errors in the inverted source parameters for the synthetic event G1.

Likewise, Figure 5-8 gives the best waveform fitting for the synthetic event G4, which is located close to well 2 and far from well 1. A good agreement between modeled data in black and band-pass filtered synthetic data in red is also observed on both components. This indicates the effectiveness of weighted least squares inversion in dealing with the significant difference in noise standard deviation at different geophone arrays. Figure 5-9 plots the histograms of errors in the inverted source parameters for the synthetic event G4.

A similar Monte-Carlo test was also conducted for synthetic events G2 and G3. Table 5-4 summarizes the average absolute errors of the inverted source parameters for all 4 synthetic events. The median value of the condition number of the inversion matrix across the 100 inversions is also listed for each example event. Three observations are seen in Table 5-4. Firstly, compared to Table 5-2, the errors in the inverted source parameters are clearly increased for all events. This signifies that the velocity model errors have a more profound influence in the moment tensor inversion than data noise and source mislocations. Secondly, at the same noise level and with the same amount of velocity model perturbations, the example event with the smallest median condition number (event G3) tends to have the least error in source parameter estimates. For the assumed velocity model errors, the event G1, with the largest condition number, has an average absolute error of 0.9, 14°, 22° and 21% for k, α, φ and CLVD component percentage, respectively. Finally, among all 4 inverted source parameters (φ, δ, λ, α) related to the fracture plane orientation and slip direction, the dip angle δ is the most reliably determined, with a maximum error up to 5°, while the strike angle φ is the least
accurate estimate. The errors in the inverted slope angle $\alpha$ are also small, indicating that $\alpha$ can be accurately estimated.

5.3.4 Hydraulic fracture geomechanics in the Barnett shale

To understand how microearthquakes are generated in the Barnett shale, it is essential to look at the hydraulic fracture mechanics. Microseismicity associated with hydraulic fracturing has considerably different geomechanical aspects than tectonic earthquakes, rockbursts, or geothermal shear dilation. The inflation of a hydraulic fracture with internal pressure induces very large stresses in the surrounding formation. The stress perturbations are often greater than the stress difference that existed in the formation prior to fracturing. In addition, the leakoff of the high pressure fluid, at pressures well above the minimum in situ stress, reduces the normal stress and destabilizes any natural fractures or other permeable weakness planes. These combined factors create the unstable zones around the hydraulic fracture where the microseismicity would occur (Warpinski et al., 2012). In this section, we calculate the hydraulic fracture induced stress perturbations in the Barnett shale and consider the pore pressure increase resulting from fracturing fluid leakage to study possible failure types that could occur in the Barnett shale.

Looking at a single hydraulic fracture for simplicity, there are several models available to calculate the stress field induced by the fracture, including both finite element and analytical models. For scoping calculations, analytical models are sufficient. Among the various analytical models, the most versatile one is a pressurized three-dimensional (3D) elliptic crack (Green and Sneddon, 1950). This model requires a homogeneous, isotropic, linear-elastic formation and a uniform fluid pressure inside the hydraulic fracture, but these simplifications still allow for adequate evaluation of the characteristics of the stress field around the hydraulic fracture and the influence of the stress field on rock failure behavior. As described in Figure 5-10, the stress perturbations have two characteristic zones, a tip-influenced region along the hydrofracture tip direction and a broadside region along the hydrofracture normal direction, and these are considered separately. Prior to fracturing, the Barnett shale reservoir is in the normal faulting regime (Bruner and Smosna, 2011; Agarwal et al., 2012). Therefore, the
broadside region is along the unperturbed minimum horizontal principal stress (Shmin) direction and the tip region is along the unperturbed SHmax direction. Only a vertical fracture is considered here.

Table 5-5 lists the hydrofracture and formation parameters typical of the Barnett shale waterfrac treatment (Agarwal et al., 2012). The broadside region, the area alongside the hydrofracture after the tip has passed, can be assessed using the analytic model of Green and Sneddon (1950) for typical elongated fractures (length > height). Figure 5-11a gives the stress decay moving away from the hydrofracture face along the centerline of the hydrofracture, with respect to both length and height. The largest stress perturbation is the compressive stress along the Shmin direction. While the stress perturbation in the SHmax direction is also compressive, it is considerably less. This behavior suggests the stress perturbations imposed by the hydrofracture are highly stabilizing in the broadside region. The reason is twofold. First, the shear stress in the formation is significantly reduced since the horizontal differential stress is decreased after the hydrofracture perturbation. Second, the total normal stress is increased, since compressive stress is added to both SHmax and Shmin stresses. The combined effect is to increase frictional strength and reduce the available shear stress, making it very difficult for microearthquakes to occur. One possibility to generate microseismicity in the broadside region is to have the high pressure fracturing fluid leak off into permeable weak zones such as natural fractures, since the increase in the pore pressure from fluid leakage will destabilize the weak zones and cause microearthquakes to happen (Warpinski et al., 2012). For an over-pressured gas reservoir such as the Barnett shale reservoir, the pore pressure increase resulting from fracturing fluid leakage is actually much greater than the stress perturbation due to the opening of the hydrofracture, since the pore pressure change is on the order of the fracturing pressure minus the ambient pore pressure, while the stress change, the net pressure, is on the order of the fracturing pressure minus the unperturbed Shmin stress.

The tip region of the hydrofracture has a different stress perturbation pattern. Figure 5-11b plots the stress perturbations due to the presence of the hydrofracture ahead of the length tip along the centerline of the hydrofracture with respect to height and width. Here,
all the stress changes are tensile. The largest tensile stress is along the SHmax direction, 
and a slightly smaller tensile stress occurs along the Shmin direction. This has the effect 
of slightly decreasing the horizontal differential stress and significantly decreasing the 
total stress. The net effect could be destabilizing the tip region and inducing 
microearthquakes if any favorably oriented weakness planes are encountered. This zone 
is relatively small, at most a few meters, and provides a mechanism for microearthquakes 
to occur slightly ahead of the hydrofracture tip. In contrast to the broadside region, there 
is no fluid leakage in this zone, and therefore the pore pressure stays as the ambient pore 
pressure. The above calculations are related to a single hydraulic fracture. Although the 
geomechanics become considerably more complex in the case of multiple hydraulic 
fractures during the multiple-stage, multiple-perforation treatment, the general features of 
stress perturbations from the single hydraulic fracture analysis still hold (Warpinski et al., 
2012; Agarwal et al., 2012).

Fischer and Guest (2011) proposed a way to identify four different types of 
earthquakes as shown in Figure 5-12: tensile ($\sigma_n < 0, \tau = 0, \alpha > 0$), hybrid tensile 
($\sigma_n < 0, |\tau| > 0, \alpha > 0$), pure shear ($\sigma_n = 0, |\tau| > 0, \alpha = 0$) and compressive shear 
($\sigma_n > 0, |\tau| > 0, \alpha < 0$) events. The Mohr circle was used to represent in-situ stress state, 
and the Griffith failure criterion was adopted to describe both shear and tensile failures 
(Ramsey and Chester, 2004). The Griffith failure criterion is written as,

$$\tau^2 = 4T_0(\sigma_n + T_0), \quad (5-14)$$

$$S_0 = 2T_0, \quad (5-15)$$

where $S_0$ and $T_0$ are the inherent cohesion strength and the tensile strength of the rock. 
According to the Griffith failure criterion, rock will fail along a fracture plane where the 
shear stress $\tau$ reaches the level specified by Equation (5-14).

Only the fluid leakage effect was considered by Fischer and Guest (2011). However, 
the stress perturbations from the hydrofracture are important for the analysis of 
microseismicity associated with hydraulic fracturing (Warpinski et al., 2012). In this 
study, we take into account both the fluid leakage effect and stress perturbations due to 
the presence of the hydrofracture. We consider two possibilities, microseismicity
occurring in the intact rock and on the weak zones such as natural fractures and induced hydraulic fractures.

Different cohesion strength values were proposed to describe the intact rock and the weak zones inside the Barnett shale. The cohesion strength is normally derived from the tensile strength according to Equation (5-15). It is generally accepted that the tensile strength value is highly variable. In Gale and Holder (2008), a tensile strength value ranging from 12 to 44 MPa was reported for the Barnett shale samples tested, while in Tran et al. (2010), a tensile strength value of the Barnett shale ranging from 1.38 MPa (200 psi) to 20.7 MPa (3000 psi) was proposed. In this study, we found that a tensile strength of 10 MPa for the intact rock and 1 MPa for the weak zones inside the Barnett shale seems to adequately explain the observed microseismicity. The core analysis indicates that the natural fractures inside the Barnett shale are calcite filled while the rock matrix is mostly siliceous, suggesting a weak bond between the calcite filling and the surrounding rock matrix (Gale et al., 2007). Therefore, a one-tenth of the tensile strength of the intact rock is assigned as the tensile strength of the natural fractures in this study. The difference between the tensile strength of the intact rock used in this study and that reported by Gale and Holder (2008) may be attributed to the scale effect and possible data selection bias in the laboratory study. The observed microseismicity typically occurs at a much larger scale than the size of core samples used in the laboratory test. Moreover, stronger rock samples with higher tensile strengths are easier for laboratory testing, and thus may incur the data selection bias. Overall, the parameters used for the geomechanical analysis of the Barnett shale are listed in Table 5-5.

In Figure 5-13a, the 3D Mohr-circle shows the locus of the shear stress $\tau$ and the effective normal stress $\sigma_n$ on an arbitrarily oriented fracture in the Barnett shale. The blue circle on the right corresponds to the ambient pore pressure $p_0$, while the left circle is associated with the maximum possible pore pressure case, that is, when the pore pressure is elevated to the fracturing pressure $p_f$. The Griffith failure envelope for the intact rock with the inherent cohesion strength $S_0$ of 20 Mpa is plotted in Figure 5-13a as the red curve. It is discovered that even at the maximum possible pore pressure, rock failure is very unlikely to occur in the intact rock because of its large cohesion strength. It is worth
mentioning that only pore pressure increase is considered here, since the pore pressure increase resulting from fracturing fluid leakage is actually much greater than the stress perturbation due to the opening of the hydrofracture under the treatment parameters listed in Table 5-5.

Figure 5-13b gives the failure analysis in the tip region. In this region, no fracturing fluid leakage occurs. According to Figure 5-11, the stress perturbations due to the hydraulic fracture are assumed to be \(-0.77p_{\text{net}}, -p_{\text{net}}\) and \(-0.1p_{\text{net}}\) along the Shmin, SHmax and vertical directions, respectively. The black, green and cyan crosses denote the principal stresses in the original unperturbed Shmin (NW-SE), SHmax (NE-SW) and vertical directions, respectively. It is interesting to see that the relative magnitude of the Shmin and SHmax principal stresses has changed due to the stress perturbation from the hydraulic fracture. The original Shmin (NW-SE) direction is now becoming the maximum in-situ horizontal stress direction. The Griffith failure envelope for the weak zones inside the Barnett shale with the inherent cohesion strength \(S_{\text{ow}}\) of 2 Mpa is plotted as the red curve. It is found from Figure 5-13b that compressive shear events could happen on some preferred weak zones in the tip region. As described in Figure 5-13b, the angle between the failure point and the maximum principal stress \(\sigma_v\) is equal to \(2\delta\), that is, twice the dip angle of the fracture plane (Zoback, 2007). This suggests that compressive shear events \(\alpha < 0\) with a dip around 50° could occur on weak zones such as natural fractures in the tip region.

Figure 5-14a presents the failure analysis in the broadside region. The stress perturbations from the hydraulic fracture are assumed to be \(+0.5p_{\text{net}}, +0.1p_{\text{net}}\) and 0 in the Shmin, SHmax and vertical directions, respectively. The decrease of horizontal differential stress, together with the increase in the total stress, stabilizes the broadside region. Therefore, the fracturing fluid leakoff into the weakness zones is essential for microearthquakes to occur in this region. The pore pressure increase is assumed to be equal to the net fracturing pressure \(p_{\text{net}}\) minus a pressure drop term. The pressure drop is inversely proportional to the square root of the permeability of the natural fractures, which is unknown. In Figure 5-14, a pressure drop of 200 psi is assumed, as suggested by Agarwal et al. (2012). The selection of this value is not intended to estimate the pressure
drop but to serve as a scoping parameter. The black, green and cyan crosses denote the principal stresses along the original unperturbed Shmin (NW-SE), SHmax (NE-SW) and vertical directions, respectively. The interchange of Shmin and SHmax directions resulting from the hydrofracture induced stress changes is also seen. The red, green and blue pluses demonstrate the shear and effective normal stresses on the fracture planes with strike angles of (80°, 140°), (10°, 70°), and (-15°, 45°), respectively (corresponding to a +/- 30° range around the WNW, N-S, NW-SE directions). The corresponding dip angles are also listed in the figure. The Griffith failure envelope for the weak zones with the inherent cohesion strength $S_{ow}$ of 2 Mpa is plotted as the red curve. It is observed in Figure 5-14a that both compressive shear and tensile events could happen on some preferred fractures in the broadside region with the existence of fluid leakage. Similar to Figure 5-13, because of the decreased horizontal differential stress after hydrofracture stress perturbation, the 3D Mohr circle behaves like a 2D Mohr circle with almost identical principal stresses in Shmin and SHmax directions. Therefore, for reservoirs with a low horizontal differential stress and in normal faulting regimes, such as the Barnett shale reservoir, rock failure could occur along almost any strike direction. However, the fracture plane dip angle does play an important role in determining the failure type. Figure 5-14b gives the zoomed version of Figure 5-14a. It is clear that in spite of different strike angles, tensile events could only occur at high dip angles such as $\delta = 80^\circ$ in this figure, while compressive shear events are observed at a low dip angle like $\delta = 45^\circ$.

It is worth pointing out that the stress perturbation values chosen for the tip and broadside region in the analysis above are not meant to be an accurate representation of the in-situ stress changes but to serve as the typical scoping parameters. Nevertheless, some general conclusions regarding microseismicity in the Barnett shale can still be drawn. Firstly, microseismicity is very unlikely to occur in the intact rock because of its large cohesion strength. Therefore, weak zones like natural fractures are critical for hydraulic fracturing in the Barnett shale (Gale et al., 2007; Gale & Holder, 2010). Secondly, rock failure could happen on the preferred weak zones in both the tip region and the broadside region. The pore pressure increase due to fracturing fluid leakage is essential for microseismicity in the broadside region, while tensile stress perturbations
incurred by the hydraulic fracture facilitate the generation of microearthquakes in the tip region. Possible weak zones in the Barnett shale include natural fractures and the newly created hydraulic fractures. Two sets of dominant natural fractures were reported to be in the WNW and N-S directions, respectively (Gale et al., 2007; Gale & Holder, 2010). Finally, for reservoirs with a low horizontal differential stress and in normal faulting regimes, such as the Barnett shale reservoir, rock failure could occur along almost any strike direction. The tensile events tend to occur at high dip angles, while compressive shear events are normally associated with low dip angles. This observation suggests that we could assign the high dipping plane as the fracture plane for tensile events and treat the low dipping plane as the fracture plane for compressive shear events. This justifies the synthetic sources we assumed in the previous synthetic study section. In the following field study section, we will use this approach to distinguish the fracture plane from the auxiliary plane.

5.3.5 Moment tensor inversion and source mechanism determination: results and discussions

In this section, we apply the grid search based full waveform inversion approach to the 42 selected events to invert for the complete moment tensor. The tensile earthquake source parameters including FPS (strike $\phi$, dip $\delta$, rake $\lambda$), the slope angle $\alpha$, $k$, the Vp/Vs ratio at the focal area, seismic moment $M_0$, moment tensor magnitude $M_\sigma$, and DC, ISO, and CLVD component percentages are also estimated from the inverted moment tensors. We will begin with one field event, named ‘GI-1’, to demonstrate the procedure of the complete moment tensor inversion and source parameter estimation using full waveforms. After that we will present the source mechanism results for all 42 chosen events and discuss their implications in understanding the fracturing process and the reservoir.

Figure 5-15 demonstrates the process of the grid search based moment tensor inversion of the field event GI-1 using the layered model illustrated in Table 5-1 and Figure 5-2a. On Figure 5-15a, the normalized variance reduction is plotted as a function of searched event location and origin time. The black star denotes the initial source location and origin time estimate, while the white star gives the inverted source location
and origin time. It is clear that the variance reduction function VAR is maximized at the inverted source location and origin time, suggesting a better waveform fit than the initial event location and origin time. Figure 5-15b presents the VAR at the inverted source location as a function of origin time. It is observed that the VAR is periodical with respect to the time shift. A comparison between Figure 5-15a and Figure 5-15b seems to indicate that the periodicity of VAR with respect to the time shift is more pronounced than that to the source location. This is caused by inverting seismograms of a limited frequency band between 100 and 300 Hz. A wider frequency band gives a better resolution but a less stable inversion result. This is because a larger frequency bandwidth requires a more accurate velocity model and an energetic signal across a wide frequency band, which is difficult to achieve in the field. Therefore, the selection of the filtering bandwidth of [100, 300] Hz is to balance the tradeoff between the inversion stability and the solution resolution.

The moment tensor solution, event location, and origin time are then determined. Figure 5-16 shows the best waveform fitting for the field event GI-1. A good agreement in dominant P- and S-wave trains between modeled data in black and observed data in red is seen on both components. It is worth pointing out that the noisy feature on the modeled data of well 2 in Figure 5-16a is not due to numeric noise but as a result of the large scaling factor of 11.65 used in the plot. The actual waveform amplitude of the North component from well 2 is much smaller than that from well 1. In this example event, we did not notice significant unmodeled wave packages. In some other events, we did see some degree of unmodeled wave packages between P- and S-arrivals, which probably points to the presence of a complex laterally inhomogeneous structure in this area. Overall, a good agreement in dominant P- and S-wave packages between modeled data and observed data is observed for all 42 events.

Next, we estimate the source parameters from the inverted moment tensor for this field event GI-1 using the method proposed in the methodology section. Two planes with strike, dip and rake of (16°, 79°, 70°), (343°, 32°, 229°) are derived. The slope α is estimated to be 37°. Even considering the possible error of 14° in the slope angle due to data noise, source mislocations and velocity model errors as discussed in the synthetic
study, the field event G1-1 is considered to be tensile. Moreover, as illustrated in the synthetic study, the dip angle is the most reliably determined parameter (see the analysis in Table 5-4). Therefore, the plane with the larger dip angle of 79° is selected as the fracture plane following the conclusion drawn from geomechanical analysis. The fracture strike is estimated to be 16°. As illustrated in the synthetic study, the strike angle $\phi$ is the least accurate source parameter estimate with an error up to 22° for event group G1 (see Table 5-4). The fracture strike associated with field event G1-1 is considered to be consistent with the N-S direction. Therefore, event G1-1 is attributed to the tensile opening of the N-S natural fracture.

To further confirm the non-DC components presented in event G1-1, the F test has been performed to test the significance of non-DC components by taking into account the variance reductions in the MT and pure DC inversions, and the corresponding numbers of degrees of freedom in the observed data (Šílený et al., 2009). It turns out for event G1-1, at a confidence level of 99.9%, the MT model is better than the DC source model in satisfying the observed data. Actually, for all the 42 events under investigation, at a confidence level higher than 95%, the MT model is preferred to describe the observed data. In other words, the probability of the existence of the non-DC source is significant.

The same procedure is then applied to all the selected events. Table 5-6 summarizes the determined source parameters for all 42 events. It is observed that all the events except the 6 underlined events follow the tensile earthquake model. The 6 underlined events have $k$ values beyond the physical limit described in Equation (5-9) and, therefore, cannot be modeled by the tensile earthquake model of Vavryčuk (2001). The reason for this behavior is not clear. It may be due to the higher complexity in these 6 events that cannot be modeled by the simple tensile earthquake model. Nevertheless, we will focus our attention on the remaining 36 events in the following discussion.

Considering the possible error in the strike estimate as described in the synthetic study (see Table 5-4), we group the 36 events in Table 5-6 into 3 groups: 11 events striking in the NE-SW direction are shown in black ("black events" hereinafter), 3 events striking along the WNW direction are depicted in blue ("blue events" hereinafter), and the remaining 22 events striking approximately along the N-S direction are listed in red.
("red events" hereinafter). As mentioned previously, Gale et al. (2007) identified two sets of dominant natural fractures along the WNW and N-S directions, respectively. Pre- and post-injection borehole image logs and cored intervals suggest that, in structurally complex areas, multiple hydraulic fracture strands are likely to propagate along the SHmax direction (Warpinski et al., 1993, Fast et al., 1994). Geologic discontinuities, such as joints, faults, and bedding planes, were found to contribute to the creation of multiple hydraulic fracture strands mapped during mineback experiments and generated in laboratory tests (Warpinski and Teufel, 1987). Recently, numerical studies also indicate that the interaction between pre-existing natural fractures and the advancing hydraulic fracture is a key condition leading to complex hydraulic fracture patterns (Dahi-Taleghani and Olson, 2011). Therefore, it is likely that multiple hydraulic fractures oriented sub-parallel to the SHmax direction, i.e. the NE-SW direction, would form because of the interaction of the main advancing hydraulic fracture and pre-existing natural fractures in the Barnett shale. Hence, we may attribute the identified 3 groups of events in black, blue and red to rock failures on the hydraulic fractures in the NE-SW direction, the WNW and N-S oriented natural fractures, respectively.

It is observed in Table 5-6 that all 11 black events striking along the NE-SW direction have positive slope angles. Even if the possible errors in the slope estimate are considered, at least 9 black events have non-negligible positive slope angles, despite that the other 2 black events have slope angles close to 0°. It is believed that these events striking along the NE-SW direction may indicate the tensile opening of multiple hydraulic fractures trending sub-parallel to the SHmax direction.

The fracture plane orientation of the blue and red events is close to the natural fracture orientation. It is speculated that these events correspond to the reactivation of WNW and N-S oriented natural fractures. The majority of these events have positive slope angles, in spite of the possible errors in the slope estimate as described in Table 5-4. This seems to indicate the existence of tensile opening associated with the reactivation of natural fractures. Nevertheless, non-negligible negative slope angles are also seen for some blue and red events, such as events G1-3, G1-11, G1-14, G1-18, G3-1 and G3-3. One question arises, that is, how could these compressive shear events on natural
fractures improve the permeability and enhance gas production? One possible explanation would be the fracture asperity. The shearing process causes the calcite filling inside the natural fractures to break, which creates open spaces. The compressive stress may decrease the volume of the newly created void space, but the asperities in these natural fractures help preserve some of the newly created flow paths and, therefore, support an increase in permeability.

The moment magnitude for all the events is found to range from 0 to -3, with the majority falling into the range of -1 to -3, even after taking into account a possible error in seismic moment estimate up to 30%.

It is observed in Table 5-6 that the Vp/Vs ratio in the focal area is generally lower than that of the surrounding medium where seismic waves propagate. This behavior was also reported in the seismological study of tensile faulting by Fojtíková et al. (2010). It is also interesting to see that some of the largest derived Vp/Vs ratios (Vp/Vs >1.7 for events G4-8, G1-17, G2-2) appear in the events occurring on the hydraulic fractures trending sub-parallel to the SHmax direction. Even considering the possible uncertainty in the k estimate resulting from data noise and velocity model inaccuracies, this observation still holds. These large Vp/Vs ratios, close to that of the surrounding medium, might be a sign of newly formed hydraulic fractures instead of aged natural fractures.

Furthermore, in terms of component percentages, many events from the group G1, G4 seem to have CLVD as the dominant component. Two possible reasons for this behavior are (1) errors in CLVD component and (2) the mechanism associated with hydraulic fracturing in these complex fractured gas shales.

The possibility of a large error in CLVD component percentage for event groups G1 and G4 is very real because of their larger condition numbers, as seen from Table 5-4. There may also be a possibility of data selection bias. Good quality events generally have good P-waves, but P-waves are quite small for pure DC events.

Alternatively, for some events in the groups G1 and G4, the analysis might be correct and a large CLVD component may be physical, reflecting the properties of the earthquake source or of the medium in the focal area. On one hand, this could be an indicator of the presence of tensile faulting, manifested by a positive correlation between
the ISO and CLVD components (Vavryčuk, 2001). On the other hand, the large CLVD component can arise from near-simultaneous faulting on fractures of different orientations or on a curved fracture surface (Nettles and Ekström, 1998).

Finally, it is worth drawing a comparison of the microseismic source mechanisms between the Barnett shale case and the Bonner tight gas sands case (Song and Toksöz, 2011). The microseismic map in the Bonner tight gas sands delineates a simple planar geometry. Although only one-well dataset is available for the Bonner tight gas sands case, Song and Toksöz (2011) were able to use the constrained inversion to invert the source mechanisms for some events by matching full waveforms. The determined microseismic FPS in the Bonner sands also suggested a dominant fracture plane orientation close to the average fracture trend derived from multiple event locations. The retrieved source mechanisms indicated a predominant DC component. This seems to suggest that in a simple reservoir with a high horizontal differential stress (around 3MPa), such as the Bonner sands, the microseismicity occurs as predominantly shearing along natural fractures subparallel to the average fracture trend. Increased production is obtained in reservoirs like Bonner gas sands through the improved fracture conductivity. On the contrary, in a fractured reservoir with a low horizontal differential stress (around 0.7 MPa), such as the Barnett shale, the microseismic source mechanism study indicates that both tensile and compressive shear events could occur on preferred weak zones such as pre-existing natural fractures and newly created hydraulic fracture strands. In the normal faulting regime, tensile events tend to have higher dips. A complex fracture network is formed together with complex non-DC events. An enhanced production is achieved in reservoirs like the Barnett shale through the increased fracture connectivity.

To summarize, weak zones such as newly created hydraulic fracture strands and calcite filled natural fractures inside the Barnett shale play a critical role, not only in the production enhancement but also in the generation of microearthquakes during the hydrofracture treatment. The determined microseismic source mechanisms provide a wealth of information about the fracturing process and the reservoir. Results from geomechanical analysis indicate that all the microearthquakes occur on the weak zones surrounding the hydraulic fracture. Microearthquakes happen as the response of the
reservoir to the hydrofracture perturbation. Therefore, in addition to hydraulic fracture mapping, microseismic monitoring could serve as a reservoir characterization tool.

5.4 Summary

In this chapter, we presented a comprehensive microseismic source mechanism study in the Barnett shale at Fort Worth Basin. We used a grid search based full waveform inversion approach to determine the complete moment tensor from a dual-array dataset. We estimated the source parameters for each event according to the tensile earthquake model. Both shear and tensile failures were accommodated in this model. The derived source parameters include the fault plane orientation, the slope angle, the Vp/Vs ratio in the focal area, and the seismic moment.

We analyzed the microseismicity in the Barnett shale using hydraulic fracture geomechanics. We considered both the pore pressure increase due to fracturing fluid leakage and the stress perturbations resulting from the hydraulic fracture in our analysis. We used the Griffith criterion and the 3D Mohr circle to determine the failure types. Results indicate that weak zones are critical to the generation of microseismicity in the Barnett shale. It is found that both tensile and compressive shear events could occur on preferred weak zones including natural fractures and hydraulic fractures. In the normal faulting regime, such as that encountered in the Barnett shale, tensile events tend to have higher dips. We proposed a method to distinguish the fracture plane from the auxiliary plane. The fracture plane is selected as the high dipping plane for events with positive slope angles, and the low dipping plane for events with negative slope angles.

In the synthetic study, we investigated the influence of velocity model errors, event mislocations, and additive data noise on the extracted source parameters via a Monte-Carlo test. We demonstrated that with a correct velocity model, the errors in the inverted source parameters are minimal. We also showed that a reasonable amount of error in source location and the velocity model, together with data noise, do not cause a serious distortion in the inverted moment tensors and source parameters. In our synthetic test, the fracture dip is proven to be the most reliable source parameter estimate with respect to
velocity model errors, while the fracture strike has the largest inversion error resulting from velocity model inaccuracies. The synthetic test also indicates that with the same amount of velocity model errors and data noise, large source parameter errors occur when the condition number of the sensitivity matrix is high.

We determined the source mechanisms for 42 good signal-to-noise ratio and low condition number microseismic events induced by waterfrac treatment in the Barnett shale. Results show that most events follow the tensile earthquake model and possess significant non-DC components. We demonstrated the significance of the occurrence of non-DC components in these events by F-test. The inverted source mechanisms reveal both tensile opening on the hydraulic fracture strands trending sub-parallel to the unperturbed SHmax direction and the reactivation of pre-existing natural fractures along WNW and N-S directions. An increased fracture connectivity and enhanced gas production in the Barnett shale are achieved through the formation of a complex fracture network during hydraulic fracturing via rock failures on the weak zones of various orientations.

Potential errors in source parameter estimates from dual-array data primarily come from the unmodeled velocity and attenuation model errors. An extended study of the influence of attenuation and anisotropy will be carried out in the future. Full waveform based microseismic source mechanism study not only reveals important information about the fracturing mechanism, but also allows fracture characterization away from the wellbore, providing critical constraints for understanding fractured reservoirs.

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5.5 References


Table 5-1: Seismic properties of the layer sequence in the Barnett shale gas reservoir. The listed P- and S-wave velocities are the values calibrated by perforation timing. $Q_p$ and $Q_s$ values are determined by considering both the lithology and amplitude decay measured across the geophones (Toksoz and Johnson, 1981; Rutledge et al., 2004).

<table>
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<th>Layer number (Rock type)</th>
<th>Property</th>
<th>$V_p$ (Km/s)</th>
<th>$V_s$ (Km/s)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$Q_p$</th>
<th>$Q_s$</th>
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<td>2.44</td>
<td>2.4</td>
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<td>60</td>
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<td>100</td>
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<td>2.6</td>
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<td>100</td>
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<tr>
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<td>4.11</td>
<td>2.29</td>
<td>2.4</td>
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<td>2.29</td>
<td>2.55</td>
<td>100</td>
<td>60</td>
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<td>2.44</td>
<td>2.5</td>
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Table 5-2: Statistics of complete moment tensor (MT) inversion with two-well synthetic data. The inversion is performed with 10% Gaussian noise contaminated data and uses the correct velocity model and the mislocated source. The values listed in this table summarize the statistics of the inverted source parameters for 100 different additive noise realizations. The true moment tensor for the example event in each event group is described in the main text. The condition number of the inversion matrix for each example event at the inverted source origin time and location is listed below the event ID.

<table>
<thead>
<tr>
<th>Example event (condition number)</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
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<tbody>
<tr>
<td>Mean absolute errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the inverted source parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seismic moment (%)</td>
<td>2.8</td>
<td>0.5</td>
<td>0.7</td>
<td>1.5</td>
</tr>
<tr>
<td>$k = \frac{\lambda_p}{\mu}$</td>
<td>0.05</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Slope (°)</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Strike (°)</td>
<td>1.5</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Rake (°)</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>DC component percentage (%)</td>
<td>1.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Isotropic component percentage (%)</td>
<td>1.4</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>CLVD component percentage (%)</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Table 5-3: Statistics of double-couple (DC) inversion with two-well synthetic data. The inversion is performed on the same noisy data as Table 5-2 and uses the correct velocity model and the mislocated source. The values listed in this table summarize the statistics of the inverted source parameters for 100 different additive noise realizations. The true moment tensor for the example event in each event group is also identical to that of Table 5-2. DC inversion provides no information on $k$ and moment tensor component percentages.

<table>
<thead>
<tr>
<th>Example event</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seismic moment (%)</td>
<td>12</td>
<td>6</td>
<td>27</td>
<td>40</td>
</tr>
<tr>
<td>Strike (°)</td>
<td>61</td>
<td>37</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>Dip (°)</td>
<td>38</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Rake (°)</td>
<td>49</td>
<td>160</td>
<td>29</td>
<td>56</td>
</tr>
</tbody>
</table>
Table 5-4: Statistics of complete moment tensor (MT) inversion with two-well synthetic data. The inversion is performed on the same noisy data as Table 5-2 and uses an approximate velocity model and mislocated source. The values listed in this table summarize the statistics of the inverted source parameters for 100 different perturbed velocity model realizations. Different additive noise realizations are used for different velocity model realizations. The true moment tensor for the example event in each event group is also identical to that of Table 5-2. The median condition number of the inversion matrix among 100 different velocity model realizations for each example event at the inverted event origin time and location is listed below the event ID.

<table>
<thead>
<tr>
<th>Mean absolute errors in the inverted source parameters</th>
<th>Example event (condition number)</th>
<th>G1 (23)</th>
<th>G2 (6)</th>
<th>G3 (4)</th>
<th>G4 (17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seismic moment (%)</td>
<td>17</td>
<td>15</td>
<td>13</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>( k = \frac{\lambda_p}{\mu} )</td>
<td>0.9</td>
<td>0.4</td>
<td>0.1</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Slope (°)</td>
<td>14</td>
<td>3</td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Strike (°)</td>
<td>22</td>
<td>7</td>
<td>2</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Dip (°)</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Rake (°)</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>DC component percentage (%)</td>
<td>14</td>
<td>4</td>
<td>5</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Isotropic component percentage (%)</td>
<td>14</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>CLVD component percentage (%)</td>
<td>21</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Table 5-5: Parameters for a typical waterfrac treatment in the Barnett shale taken from (Agarwal et al., 2012).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic fracture half length $x_f$</td>
<td>150 m (492 ft)</td>
</tr>
<tr>
<td>Hydraulic fracture height $h_f$</td>
<td>60 m (197 ft)</td>
</tr>
<tr>
<td>Young’s modulus, $E$</td>
<td>45 GPa ($6.53 \times 10^6$ psi)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>Minimum horizontal stress $S_{h_{\text{min}}}$</td>
<td>33.78 MPa (4900 psi)</td>
</tr>
<tr>
<td>Maximum horizontal stress $S_{h_{\text{max}}}$</td>
<td>34.47 MPa (5000 psi)</td>
</tr>
<tr>
<td>Vertical stress $S_v$</td>
<td>48.26 MPa (7000 psi)</td>
</tr>
<tr>
<td>Ambient pore pressure $p_0$</td>
<td>26.89 MPa (3900 psi)</td>
</tr>
<tr>
<td>Net fracturing pressure $P_{\text{net}}$</td>
<td>3.45 MPa (500 psi)</td>
</tr>
<tr>
<td>Inherent cohesion strength of the intact rock $S_0$</td>
<td>20 MPa (2900 psi)</td>
</tr>
<tr>
<td>Inherent cohesion strength of weak zones $S_{0w}$</td>
<td>2 MPa (290 psi)</td>
</tr>
<tr>
<td>Treatment depth</td>
<td>2.29 km (7500 ft)</td>
</tr>
</tbody>
</table>
Table 5-6: Results of source mechanism determinations for the 42 selected microseismic events during the waterfrac treatment in the Barnett shale. The full-waveform based complete MT inversion is employed on this two-well dataset to determine the source parameters.

<table>
<thead>
<tr>
<th>Event ID</th>
<th>$M_0$ ($10^7$ N m)</th>
<th>$M_w$</th>
<th>$k$</th>
<th>$V_p/V_s$</th>
<th>$\alpha$ (°)</th>
<th>$\phi$ (°)</th>
<th>$\delta$ (°)</th>
<th>$\lambda$ (°)</th>
<th>DC (%)</th>
<th>ISO (%)</th>
<th>CLVD (%)</th>
<th>Cond. Num.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1-1</td>
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<td>-1.4</td>
<td>0.10</td>
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<td>37</td>
<td>16</td>
<td>79</td>
<td>70</td>
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<td>28</td>
<td>48</td>
<td>6</td>
</tr>
<tr>
<td>G1-2</td>
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<td>-1.1</td>
<td>0.02</td>
<td>1.42</td>
<td>40</td>
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<td>86</td>
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<td>52</td>
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</tr>
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<td>-1.4</td>
<td>0.17</td>
<td>1.47</td>
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<td>4</td>
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<td>-43</td>
<td>14</td>
</tr>
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<td>29</td>
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198
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<th>Dip</th>
<th>Rake</th>
<th>Slope</th>
<th>Magnitude</th>
<th>Depth</th>
<th>Distance</th>
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<td>156</td>
</tr>
</tbody>
</table>

Note 1: The strike, dip, rake, and slope angles follow the convention of Aki & Richards [2002], and are defined in the Figure 5-1.

Note 2: The underlined events are classified as events that can not be modeled by the tensile earthquake model of Vavryčuk [2001]. The highlighted events in red and blue are classified as events associated with reactivation of natural fractures striking along N-S and WNW directions, respectively. The rest of the events in black, except the underlined events, correspond to the events striking along SHmax (NE-SW) directions. Please see the main text for details.
Figure 5-1: A model for the tensile earthquake (after Vavryčuk, 2011; Aki & Richards, 2002). See the main text for the definition of strike $\phi$, dip $\delta$, rake $\lambda$, and slope angle $\alpha$. 
Figure 5-2: (a) One-dimensional P- and S-wave velocity model derived from the field study shown in the black. The blue lines on the left and right sides denote the observation wells 1 and 2, respectively. The red triangles represent the depth of the 12 geophones in each observation well. The rock type for each layer is also listed in the figure. The waterrefrac treatment is performed in the lower Barnett interval, with the majority of microseismic events occurring in the lower Barnett interval also. (b) The red and blue lines depict the perturbed P- and S-wave velocity models to study the influence of velocity model errors on the inverted source parameters. Please see the main text for details.
Figure 5-3: Horizontal plane view of the microseismic event locations from waterfrac treatment in the Barnett shale plotted as red circles. The yellow and green squares denote the two vertical observation wells 1 and 2, respectively, while the treatment well trajectory is plotted as the cyan line with treatment wellhead shown as the blue square. The origin (0, 0) corresponds to the location of observation well 1. The green dotted line represents the observation well plane. A total of 42 events located off the observation well plane with good signal-to-noise ratios are selected for source mechanism study in this chapter. Among the selected events, 4 event groups are seen and denoted as G1, G2, G3, and G4, respectively.
Figure 5-4: Moment tensor inversion of a synthetic tensile source located within the event group G1 (see Figure 5-2): the normalized variance reduction as a function of searched event origin time and event location. 10% Gaussian noise is added to the noise-free data of the synthetic tensile event G1 to form the noisy synthetic data for inversion. The complete moment tensor inversion is applied to the band-pass filtered horizontal components from two wells. The inversion is performed with an inaccurate velocity model and a mislocated source. The variance reduction described in this figure corresponds to one noise and velocity model realization. The initial event location and origin time is shown as the black star, while the grid search inverted event location and origin time is plotted as the white star. Detailed information regarding this synthetic test is explained in the main text.
Figure 5-5: Comparison between the modeled data in black and band-pass filtered noisy synthetic data in red for the synthetic tensile source G1. a) North component plot. b) East component plot. The relative scaling factors between well 1 (geophones 1-12) and well 2 (geophones 13-24) are listed. The modeled data are generated from the inverted microseismic moment tensor matrix (6 independent elements). The waveform comparison presented in this figure corresponds to the same inaccurate velocity model and noise realization as shown in Figure 5-4. Detailed information regarding this synthetic test is described in Figure 5-4 and explained in the main text.
Figure 5-6: The errors of the inverted event location in (N, E, D) directions for the synthetic tensile source G1 are shown as stars and plotted as a function of velocity model realizations. 100 moment tensor inversions, each with one inaccurate velocity model and noise realization, are performed to study the influence of velocity model errors on the inverted source parameters. The event location error is shown as multiples of search grid size. The black line represents the search limit in the vertical direction for the grid search based moment tensor inversion, while the search limit in the north and east directions is identical and plotted as the green line. Detailed information regarding this synthetic test is described in Figure 5-4 and explained in the main text.
Figure 5-7: The histograms of errors in the inverted source parameters for the synthetic tensile source G1. 100 moment tensor inversions, each with one inaccurate velocity model realization, are performed to study the influence of velocity model errors on the inverted source parameters. Detailed information regarding this synthetic test is described in Figure 5-4 and explained in the main text.
Figure 5-8: Comparison between the modeled data in black and band-pass filtered noisy synthetic data in red for a compressive source located within the event group G4 (see Figure 5-2). The rest of the figure description is analogous to Figure 5-5.
Figure 5-9: The histograms of errors in the inverted source parameters for the synthetic compressive source G4. The rest of the figure description is analogous to Figure 5-7.
Figure 5-10: The horizontal plane view of the three-dimensional (3D) elliptic hydraulic fracture model and its characteristic neighbourhood regions. The out of the paper direction is the vertical (fracture height) direction. Two characteristic neighbourhood regions: tip region and broadside region, are classified according to the different features of stress perturbations induced by the 3D elliptic hydraulic fracture. Please see the text for details.
Figure 5-11: The calculated stress perturbations due to the 3D elliptic hydraulic fracture described in Figure 5-10. a) Stress decay normal to fracture face along centerline of fracture in the broadside region. b) Stress decay ahead of the length tip along centerline of fracture in the tip region.
Figure 5-12: Schematic illustration of the generation of four different failure types using the Mohr Circle and Griffith failure envelope. According to the relations between shear stress $\tau$ and normal stress $\sigma_n$, the tensile, hybrid tensile, pure shear and compressive shear failure modes are defined (Modified after Fischer and Guest, 2011).
Figure 5-13: a) Representation of the shear and effective normal stress on an arbitrarily oriented fracture with the 3D Mohr circle for a typical Barnett shale waterfrac treatment (treatment parameters are listed in Table 5-5). The blue circle on the right corresponds to the ambient pore pressure $p_0$, while the left circle is associated with the maximum possible pore pressure case, that is, the pore pressure is increased to the fracturing pressure $p_f$. The Griffith failure envelope for the intact rock with the inherent cohesion strength $S_0$ of 20 Mpa is shown as the red curve. b) The 3D Mohr-circle representation of the tip region. The black, green and cyan crosses denote the principal stresses along the original unperturbed $S_{\text{min}}$ (NW-SE), $S_{\text{max}}$ (NE-SW) and vertical directions, respectively. In this figure, the hydrofracture induced stress perturbations are considered and no fracturing fluid leakage occurs in the tip region. The Griffith failure envelope for weak zones with the inherent cohesion strength $S_{0w}$ of 2 Mpa is plotted as the red curve. See the main text for detailed discussions.
Figure 5-14: a) The 3D Mohr-circle representation of the broadside region. In this figure, the hydrofracture induced stress perturbations are considered. Fracturing fluid leakage is assumed in the broadside region. See the main text for detailed discussions. The red, green and blue pluses demonstrate the normal and shear stresses on the fracture planes with strike angles of $(80^\circ, 140^\circ)$, $(10^\circ, 70^\circ)$, and $(-15^\circ, 45^\circ)$, respectively (corresponding to WNW, N-S, NW-SE directions). The corresponding dip angles of these fracture planes are also listed in this Figure. The rest of the figure description is analogous to Figure 5-13b. b) Zoomed version of Figure 5-14a.
Figure 5-15: Moment tensor inversion for the field event G1-1. a) The normalized variance reduction as a function of searched event origin time and event location. The initial event location and origin time is shown as the black star, while the grid search inverted event location and origin time is plotted as the white star. b) The normalized variance reduction as a function of searched event origin time at the optimum event location. The initial and inverted event origin times are plotted as the black and red stars, respectively.
Figure 5-16: Waveform fitting for field event G1-1. Modeled seismograms derived from grid search based complete moment tensor inversion are shown in black, while the observed seismograms are plotted in red. a) North component. b) East component. The relative scaling factors between well 1 (geophones 1-12) and well 2 (geophones 13-24) are listed. The inversion is performed on the band-pass filtered horizontal components and uses the layered model shown in Figure 5-2a) and Table 5-1.
Chapter 6

Conclusions

In this thesis, we improved the microseismic mapping capability for hydrofracture monitoring by using full waveform information and developed a full waveform based microseismic source mechanism inversion approach to better understand the fracturing mechanisms in unconventional oil and gas reservoirs.

In terms of improving microseismic mapping, both the array-based correlation and the subspace detector have been developed to increase event detections while keeping low false alarm triggers. A transformed spectrogram method that captures two basic features of a phase arrival, i.e. high energy and high energy increase in the time-frequency domain, was proposed to improve the phase arrival pickings for better location. The subspace projection approach was developed to enhance the weak microseismic signals. The effectiveness of these proposed methods has been demonstrated using field data.

To better understand fracturing mechanisms in unconventional oil and gas reservoirs, a grid search based full waveform inversion approach was developed to invert for complete moment tensor and determine microseismic source mechanisms using data from downhole arrays. This approach matches the observed data with the full waveform synthetics generated by either the discrete wavenumber integration method or finite difference method. The grid search based inversion approach can not only determine the microseismic source mechanisms but also refine event locations.
The complete moment tensor inversion makes no double-couple source assumption for the microseismic events. Therefore, the method could retrieve microseismic source information for both shearing and tensile failures. The complete moment tensor inversion approach is studied in both single-well and multiple-well monitoring scenarios. Two different microseismic datasets, a single-array dataset from hydraulic fracturing in the Bonner tight gas sands and a dual-array dataset from the waterfrac treatment in the Barnett shale, are used in the study. The inverted source mechanisms are compared and they reveal different fracturing mechanisms in these two reservoirs. Detailed conclusions have been given at the end of each chapter. Some general conclusions resulted from this dissertation work are:

1) Compared to an inherent energy detector such as the STA/LTA detector that is routinely used in today's microseismic processing, field studies show that the correlation detector can enhance the detection capability of small magnitude events with mechanisms and locations similar to a nearby template event, known as the master event. The gain in the detection sensitivity of correlation detectors comes from waveform matching. Additional processing gain is achieved by stacking the correlations over multiple components and geophones. The transformed spectrogram method is demonstrated to improve the automatic P- and S-phase arrival picking.

2) The subspace detector that constructs a vector space, known as the signal subspace, is a powerful tool for detecting microseismic signals from a specific source region. Yet, it has not been used in hydrofracture mapping. The method models the signals as a linear combination of the orthogonal bases of the subspace. Field results demonstrate that, unlike correlation detectors, the subspace approach is more broadly applicable. The subspace detector is also sensitive to waveforms and, therefore, offers a lower probability of false alarms, compared to STA/LTA detectors. The main limitation of the subspace detector is the complexity and relatively large computation cost in building the signal subspace from multiple template events. Fortunately, the signal subspace construction could be done off-line, which makes real-time subspace detection possible. The analysis of
the detection statistics provides a rigorous way to quantitatively determine the subspace detection parameters. The subspace detector offers a way to manage the tradeoff between detection sensitivity and flexibility. The improved detection results will help to better interpret the microseismicity in the reservoir, especially in the regions far from the monitoring well. Field test results demonstrate that the SNR of detected weak microseismic events is improved after applying the subspace-projection-based signal enhancement procedure.

3) Synthetic and field studies indicate that full waveform inversion could recover the complete moment tensor using data recorded at a single geophone array, when the event is in the near-field range of the array. The near-field and non-direct wave (i.e., reflected/refracted waves) information in a layered medium contribute to the decrease in the condition number of the sensitivity matrix. On the other hand, when the events are in the far-field range, appropriate source constraints need to be imposed to recover complete moment tensor. Additional constraints, such as the average fracture orientation derived from the event location trend, help recover the complete moment tensor and reduce the uncertainty of not only the fracture plane solution but also seismic moment and moment component percentages.

4) Field and synthetic studies demonstrate that a weighted least squares based waveform inversion of data from multiple wells could retrieve the complete moment tensor without posing additional source constraints. Field and synthetic tests also show that the grid search based inversion approach is capable of refining microseismic event locations when a good velocity model is available. The derived source parameters reveal important information regarding fracturing mechanisms in unconventional oil and gas reservoirs.

5) A Monte-Carlo test based approach is proposed and applied in this thesis to evaluate the errors in the inverted source parameters due to additive data noise, velocity inaccuracies and event location errors. The errors in the inverted moment tensor and source parameters are more sensitive to velocity model errors and less sensitive to additive data noise and source mislocations.
6) In a reservoir such as Bonner tight gas sands with a high horizontal differential stress (for the Bonner sands reservoir, the horizontal differential stress is around 3 MPa), the microseismic event locations show a simple, planar geometry. Field studies show that most microearthquakes have a dominant double-couple component, a reasonable amount of the isotropic component, and a negligible CLVD component. This suggests that the microseismicity in Bonner sands occurs predominantly by shearing along natural fractures sub-parallel to the average fracture trend. An enhanced production in the Bonner tight gas sands reservoir from hydraulic fracturing is obtained mainly through the improved fracture conductivity.

7) In a fractured reservoir with a low differential stress such as the Barnett shale (for the Barnett shale reservoir, the horizontal differential stress is around 0.7 MPa), microearthquake locations delineate a complex network. Weak zones inside the Barnett shale such as pre-existing natural fractures play a critical role in generating the microseismicity during hydrofracture treatment. Geomechanical analysis shows that, in the normal faulting regime, tensile events are associated with higher dip angles, while compressive events occur at lower dip angles. The determined microseismic source mechanisms reveal both tensile opening on hydraulic fracture strands trending subparallel to the unperturbed maximum horizontal principal stress direction and the reactivation of pre-existing natural fractures along the WNW and N-S directions. An increased fracture connectivity and enhanced gas production in the Barnett shale are achieved through the formation of a complex fracture network during hydraulic fracturing via rock failures on the weak zones of various orientations.

8) Microseismicity occurring during hydrofracture treatment contains a wealth of information about the fracturing process and the reservoir. Therefore, in addition to hydraulic fracture mapping, microseismic monitoring could serve as a reservoir characterization tool.
Appendix A

Design set event selection and waveform alignment through the single-link algorithm

The single-link algorithm has been proposed for seismic event clustering and been used in the subspace algorithm (Israelsson, 1990, Harris, 2006). In this appendix, we review the steps of design set event selection and waveform alignment via the single-link algorithm.

The single-link clustering method begins by treating all events as individual clusters containing one event each. In each step of the clustering method, the minimum distance pair (i.e., largest correlation measurement) is selected and the two clusters (events), to which it corresponds, are merged. As two clusters are combined, the dissimilarity distances between the two clusters and any third remaining cluster are combined by selecting the smaller of the dissimilarity distance measurements to represent the inter-event distance of the new cluster with the third cluster. An updated dissimilarity matrix $K^m$ is formed to reflect the inter-event distance changes caused by the clustering. This process of aggregation continues until a single cluster remains. The clustering results are summarized by a dendrogram, which shows the successive fusions of events. At each clustering step, a cophenetic correlation coefficient ($C_p$) is calculated to measure how well the clustering models the actual similarity behavior, which is described in matrix $K$. Assuming that there are $M$ events in the template
event library, the original dissimilarity matrix $K$ has a size of $M \times M$. The cophenetic correlation is computed as the correlation coefficient between $K$ and $K^g$, for successive steps $g = 1, 2, \ldots, M-1$,

$$C_g = \frac{\sum_{q=1}^{M} \sum_{p=1}^{M} K_{p,q}K_{p,q}^g}{\left[\sum_{q=1}^{M} \sum_{p=1}^{M} K_{p,q}K_{p,q} \sum_{q=1}^{M} \sum_{p=1}^{M} K_{p,q}^gK_{p,q}^g\right]^{1/2}}. \quad (A-1)$$

As clustering progresses, the correlation between the $K^g$ matrix and the original $K$ matrix will continue to decrease as the original entries are replaced with the dissimilarity distances calculated for the growing clusters. Overall, values of $C_g$ will thus decline.

The design set is a set of events in the template event library that are to be used to construct the signal subspace bases. Therefore, it is desirable for the design set to comprise not only most of the larger template events, but also to represent the actual inter-event correlation behavior described by the original dissimilarity matrix $K$. Therefore, a sudden decrease in $C_g$ is used as an indicator to terminate clustering.

Besides the cophenetic correlation criteria, in this appendix the event dissimilarity distance threshold is also considered to ensure reasonable waveform variability when forming the design set.

The waveform alignment is done simultaneously with the design set event selection. The delays used for waveform alignment are calculated relative to the reference event, i.e., event 11 as shown in Figure 3-8. For each event that belongs to the left nodes of the dendrogram and is directly connected to the reference event (event 19, 5, 18, 2, 20, and 6 in Figure 3-8), the delay is the point in the cross-correlation function where the correlation between that event and event 11 is maximized. The rest of the design set events are connected to the reference event through intermediate left node events. The delay of each of these events is calculated as the sum of all the delays on the connection path to the reference event. For example, the delay for event 16 is the sum of the delays from event pairs (16, 6) and (6, 11). Likewise, the delay for event 13 is the sum of the delays from event pairs (13, 12), (12, 2) and (2, 11). The waveform alignment results for all $D=12$ design set events after applying the delays are displayed in Figure 3-9.
References


Appendix B

Derivation of equation (3-25) via the analysis of the detection statistics

In this appendix, we derive the subspace detection probability and false alarm rate from the analysis of the subspace detection statistics. According to Harris (2006), the subspace detection statistics \( c(n) \) defined in equation (3-10) can be transformed into a F-distributed variable,

\[
c'(n) = \frac{[x_p^T(n)x_p(n)]/\sigma^2 / d}{[w^T(n)w(n)]/\sigma^2 /(N-d)}, \tag{B-1}
\]

where \( w(n) \) is the projection of the detection data vector \( x(n) \) into the orthogonal complement to the subspace \( U \),

\[
w(n) = [I_N - uu^T]x(n). \tag{B-2}
\]

Under null hypothesis \( H_0 \), \( x_p(n) \) and \( w(n) \) are two independent zero-mean Gaussian distributed variables with an identical variance of \( \sigma^2 \). Therefore, \([x_p^T(n)x_p(n)]/\sigma^2 \) and \([w^T(n)w(n)]/\sigma^2 \) are independent and chi-square distributed, with \( d \) and \((N-d)\) degrees of freedom, respectively. Hence, \( c'(n) \) in equation (B-1) follows the central \( F \) distribution under null hypothesis. From equation (3-13), the false alarm occurs when

\[
c'(n) > \frac{\gamma N-d}{1-\gamma}d, \tag{B-3}
\]
where $\gamma$ is the threshold for $c(n)$. Thus, the false alarm rate is calculated as

$$P_F = 1 - F_{d,N-d} \left( \frac{\gamma}{1 - \gamma} \frac{N-d}{d} \right),$$

(B-4)

where $F_{d,N-d}(\cdot)$ denotes the cumulative central F distribution with $d$ and $(N-d)$ degrees of freedom.

Similarly, under alternative hypothesis $H_1$, $x_p(n)$ and $w(n)$ are two independent Gaussian distributed variables with an identical variance of $\sigma^2$, but with non-zero mean values. Therefore, $\frac{[x_p^T(n)x_p(n)]}{\sigma^2}$ and $\frac{[w^T(n)w(n)]}{\sigma^2}$ are independent and noncentral chi-square distributed, with $d$ and $(N-d)$ degrees of freedom, respectively. Considering the fractional energy captured in the signal subspace $U$, the noncentrality parameters of $\frac{[x_p^T(n)x_p(n)]}{\sigma^2}$ and $\frac{[w^T(n)w(n)]}{\sigma^2}$ are $\frac{\alpha^T\alpha}{\sigma^2}$ and $\frac{(1 - \alpha^T\alpha)}{\sigma^2}$, respectively. Thus, $c'(n)$ in equation (B-1) follows the doubly noncentral F distribution under alternative hypothesis. An event is then detected according to equation (B-3). The detection probability is then derived as

$$P_D = 1 - F_{d,N-d} \left( \frac{\gamma}{1 - \gamma} \frac{N-d}{d}, \frac{\alpha^T\alpha}{\sigma^2}, (1 - \frac{\alpha^T\alpha}{\sigma^2}) \right).$$

(B-5)

As discussed in the main text, if we assume 1) the signals in the design set span the range of signals produced by the source of interest, and 2) the design events are all equally likely, the noncentrality parameters $\frac{\alpha^T\alpha}{\sigma^2}$ and $\frac{(1 - \alpha^T\alpha)}{\sigma^2}$ for any event can be replaced by the ratio of the average energy captured in the subspace and its orthogonal complement to the noise variance. This gives

$$P_D = 1 - F_{d,N-d} \left( \frac{\gamma}{1 - \gamma} \frac{N-d}{d}, \frac{\hat{E}^T\hat{E}}{\sigma^2}, (1 - \frac{\hat{E}^T\hat{E}}{\sigma^2}) \right).$$

(B-6)

Substituting SNR from equation (3-26) into equation (B-6) yields equation (3-25).
References

Appendix C

Retrieval of $m_{22}$ from one-well data at near field

In this appendix, we study the ability to retrieve $m_{22}$ using two horizontal component data from one vertical well at near field. Previous studies have shown that, with far field P- and S-wave amplitudes, it is impossible to invert for $m_{22}$ using data from one vertical well (Nolen-Hoeksema and Ruff, 2001; Vavryčuk, 2007). In this study, we use a pure $m_{22}$ source to generate synthetic seismograms. The true moment tensor, in this case, has only one non-zero element, $m_{22} = 1$. The source is comprised of 66.7% of CLVD and 33.3% of isotropic component. The source receiver configuration is the same as the near-field study. We invert for the complete moment tensor with band-pass filtered horizontal component data after adding 10% Gaussian noise. During the inversion, we use the approximate velocity model and a spatial grid search around the mislocated source (Please see the main text for details). Figure C-1 shows the source parameters derived from the inverted complete moment tensor. In this case, there is no double-couple component. Therefore, there is no definition for the strike, dip, and rake (Jechumtálová and Eisner 2008). The mean absolute errors in DC, ISO, CLVD percentages are 3%, 1% and 3%, respectively, while the mean absolute error in seismic moment is 2.4%. Considering the noise we add and the errors in velocity model and source location we assume in the inversion, the errors in
the inverted source parameters are negligible. This shows that complete moment
tensor inversion can be inverted from near-field waveforms.

Figure C-1: The histograms of errors in the inverted source parameters. The true
moment tensor has only one non-zero element, $m_{22} = 1$. The source receiver
configuration is described in Figure 4-4, with an average source-receiver distance of
18.3 m (60 ft). The unconstrained inversion is performed with 10% Gaussian noise
contaminated horizontal components from well B1.
References

