Tropical Cyclone Size in Observations and in Radiative-Convective Equilibrium

by

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Submitted to the Department of Earth, Atmospheric, and Planetary Science
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Abstract

Tropical cyclone size remains an unsolved problem in tropical meteorology, yet size plays a significant role in the damage caused by tropical cyclones due to wind, storm surge, and inland freshwater flooding. This work explores size, defined as the radius of vanishing wind, in observations and at equilibrium in an idealized numerical model.

First, a climatology of size is created from the QuikSCAT database of near-surface wind vectors for the years 1999-2008. Globally, the distribution of the outer radius is found to be log-normal, with statistically significant variation across ocean basins, but with minimal correlation with various dynamic and thermodynamic parameters.

Second, the sensitivity of the structure of a numerically-simulated axisymmetric tropical cyclone at statistical equilibrium to the set of relevant model, initial, and environmental external parameters is explored. The analysis is performed in a highly-idealized state of radiative-convective equilibrium (RCE). The non-dimensional equilibrium radial wind profile is found to be modulated primarily by a single non-dimensional parameter given by the ratio of the storm radial length scale to the parameterized eddy radial length scale. The relevant storm length scale is shown to be the ratio of the potential intensity to the Coriolis parameter, matching the prediction for the “natural” storm length scale in prevailing axisymmetric tropical cyclone theory. The outer storm circulation is further modulated by a second non-dimensional parameter that represents the non-dimensional Ekman suction rate.

Third, size is explored in three-dimensional “tropical cyclone world” simulations, with preliminary results confirming the relevant length scale obtained in axisymmetry.

Ultimately, the results of the equilibrium storm analysis are insufficient to explain the observed distribution of tropical cyclone size, but they provide the first steps toward a more fundamental understanding of the dynamics of size.

Thesis Supervisor: Kerry A. Emanuel
Title: Cecil & Ida Green Professor of Atmospheric Science
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It’s been a wild ride here – sometimes a struggle, occasionally a big struggle, but more often than not a memorable adventure. I honestly cannot believe that my graduate career is already over, having gazed out from this large 17th floor window taking for granted the clouds, trees, and child’s toy of a building that have greeted me every morning for the past five years.

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Chapter 1

Introduction

1.1 Motivation

1.1.1 Scientific

What sets the size of a tropical cyclone? Though seemingly a basic question, it remains largely unanswered in the field of tropical meteorology, despite over three decades of remarkable progress elucidating the dynamics of tropical cyclones (TCs). Indeed, the fundamental air-sea interaction instability that underlies their existence has been identified and placed within the context of a more general theory of tropical cyclones as a Carnot heat engine (Emanuel, 1986). The interaction of the TC with its environment has been studied in great detail, particularly the role of vertical wind shear and the associated time-dependent dynamics of TC intensification, which has been successfully incorporated into this Carnot engine framework (Tang and Emanuel, 2010). Furthermore, both theory and relatively simple dynamical models (Ooyama, 1969; Emanuel, 1995a; Rotunno and Emanuel, 1987) can reproduce the characteristic features of mature tropical cyclones, including maximum wind speed, central sea level pressure, and thermodynamic structure. Yet despite such tremendous scientific progress, as well as widespread recognition of the strong sensitivity of both storm surge (Irish et al., 2008) and wind damage (Iman et al., 2005) to storm size, size remains largely unpredictable.
As a simple motivating example, Figure 1-1 displays a visible satellite image of the tropical Atlantic ocean basin taken on 16 September 2010 from the GOES-East satellite. Three TCs are identifiable: Karl in the Gulf of Mexico, Igor in the western Atlantic, and Julia in the central Atlantic. Though each exhibits similar qualitative cloud structure, characterized by a circular central dense overcast (including an eye in the cases of Julia and Igor) surrounded by wispy bands of clockwise-rotating cirrus within the outflow, this structure manifests itself at distinct horizontal length scales for each storm. Indeed, at 1200 UTC on the 16th, the mean radius of 34-kt winds \( (r_{34kt}) \), as recorded in the National Hurricane Center Extended Best Track database (Demuth et al., 2006), was 95 km for Karl, 194 km for Julia, and 389 km for Igor—i.e. a near-exact doubling in size moving from small to medium and again from medium to large.

Given that the storms are at nearly identical latitudes (Karl: 19.6N; Julia: 21.8N; Igor: 20.8N), their distinct sizes cannot be attributed to variations in the ambient rotation rate. Nor can their distinct sizes be attributed to variations in peak wind speed, as Julia (90 kt) and Igor (120 kt) are of comparable intensities and, 24 hours later, Karl intensifies to 110 kt while its mean \( r_{34kt} \) expands only slightly to 139 km. Finally, the differences likely cannot be attributed to variations in potential intensity, whose approximate September climatological value (1982-1995; Source: http://wind.mit.edu/~emanuel/pcmin/climo.html) is largest for Karl (175 kt) and slightly smaller for Igor (160 kt) and Julia (140 kt); sea surface temperature anomalies, which may be used as a proxy for local anomalies in the potential intensity, are only significant for Julia (+1 K; Source: NOAA NCDC http://www.ncdc.noaa.gov/oa/climate/research/sst/weekly-sst.php).

Clearly, storm size is an enigma, one that defies conventional intuition.

1.1.2 Societal

In addition to the fundamental scientific motivation for understanding TC size, there exists tremendous societal motivation as well. Landfalling U.S. hurricanes are responsible for seven of the top ten costliest insured property losses due to natural disaster
Figure 1-1: Visible satellite image of the tropical Atlantic ocean basin from the GOES-East satellite taken on 16 September 2010 at 1445 UTC. Three TCs of very different sizes are identifiable: Karl (small) in the Gulf of Mexico, Igor (large) approaching the Caribbean, and Julia (medium) in the central Atlantic. Source: http://upload.wikimedia.org/wikipedia/commons/d/d5/20100916-1845UTC-GOES-East_visible.jpg.

worldwide since 1980 (Munich Re, 2011). Despite a projected decrease in the overall number of hurricanes globally, the potential for increases in the frequency and intensity of the strongest hurricanes due to climate change (Knutson et al., 2010) has raised concerns about similar increases in total economic damage in the future (Mendelsohn et al., 2012; Peduzzi et al., 2012). Moreover, 50% of damages due to hurricanes in the United States during the period 1870-2005, normalized for changes in population, wealth, and inflation, was caused by only eight storms (Pielke Jr. et al., 2008), highlighting the fact that U.S. economic damage by TCs is a fat-tailed phenomenon (Katz, 2012), though recent work argues that this tail behavior is linked primarily to that of the distribution of coastal economic value itself (Chavas et al., 2013).

For the purposes of risk assessment and emergency management, it is desirable to explain the observed variability in economic damage in terms of the characteristics of the storms themselves and their associated wind, storm surge, and rainfall hazards. Historically, studies have sought relationships between damages and the maximum wind speed. These relationships are typically found to follow power laws whose scal-
ing exponents range from 3 to 9 (Pielke Jr., 2007), indicating that damages increase very rapidly with wind speed. However, a significant amount of the variance in the historical damage database cannot be explained by variations in peak wind speed alone. More recent research has begun to appreciate the importance of storm size in modulating damage as well, as larger storms have a larger area of wind and rainfall exposure and are capable of generating higher storm surge. Irish and Resio (2010) demonstrated that storm surge is a complex function of multiple variables associated with storm track and structure as well as landfall location, but of particular importance are storm size and the slope of the local continental shelf; the latter has been found to be useful in explaining variability in tail of the U.S. damage distribution (Chavas et al., 2013).

Indeed, the modulation of storm surge by storm size can have devastating consequences, as demonstrated by the contrast between Hurricanes Camille (1969) and Katrina (2005), both of which made landfall near New Orleans, LA. Figure 1-2 displays the 1-minute sustained surface wind fields of each storm just prior to landfall, as analyzed by the NOAA Hurricane Research Division H*Wind Project (Powell et al., 1998). Although Camille was a much more intense storm, with a peak 1-minute sustained wind speed of 165 kt (Category 5 on the Saffir-Simpson scale), as compared to Katrina (113 kt; Category 3), Katrina’s radius of maximum wind (48 km) was twice as large as that of Camille. Consequently, Katrina’s peak recorded storm surge (8.5 m) was 1.6 m higher than that generated by Camille (6.9 m). The resulting societal impacts were drastically different: Katrina’s storm surge breached the local levee system, submerging much of the city of New Orleans. Ultimately, Katrina killed at least 1833 people and caused an estimated $81 billion (USD 2005) as compared to 259 fatalities and an estimated $23 billion (USD 2005) caused by Camille (Pielke Jr. et al., 2008). Though the latter case is still undoubtedly terrifying, the large disparity in outcome is primarily attributable to the difference in storm size.

A second, straightforward example of the role of storm size in causing damage is Hurricane Sandy in 2012, which killed 72 people and caused an estimated $50 billion (2012 USD) in damage within the United States, making it the sixth-costliest U.S.
Figure 1-2: Estimated 1-minute sustained surface wind fields of Hurricanes Camille (1969; left) and Katrina (2005; right) just before landfall near New Orleans (NOAA Hurricane Research Division H*Wind Project, Powell et al. (1998)). At the times shown, their respective peak 1-minute sustained wind speeds were 165 kt and 113 kt and radii of maximum wind were 24 km and 48 km.

hurricane landfall in history after normalizing for inflation and changes in population and wealth (Blake et al., 2013). Such tremendous destruction occurred despite the fact that Sandy’s peak sustained wind speed at landfall was a mere 70 kt, barely surpassing the threshold for designation as a hurricane (64 kt). Instead, Sandy was one of the largest TCs\(^1\) ever recorded in the Atlantic basin, with an estimated radius of gale force wind of \(r_{34kt}=1610\) km at landfall. Sandy’s large size enabled the storm to generate tremendous storm surge along the New Jersey and New York coastlines, inundating a significant fraction of New York City (peak surge of 9.23 ft at the Battery), and its enormous wind field left a large swath of destruction and cut power to millions of people for up to two weeks across the Northeast. Clearly, in the case of Sandy, storm size rather than peak wind speed was the dominant factor modulating total economic damage.

\(^1\)Sandy was in fact no longer a pure tropical cyclone at landfall, as it was beginning to undergo extra-tropical transition.
1.2 Review of existing research

To date, relatively little research has been performed to investigate the factors underlying storm size variability. Here we review the existing literature from the standpoint of observations, modeling, and theory.

1.2.1 Observations

Despite wide recognition of the importance of storm size in determining storm surge and the spatial extent of wind damage, size in nature remains largely unpredictable. In the absence of land interaction, the horizontal extent of the outer circulation is observed in nature to vary only marginally during the lifetime of a given tropical cyclone prior to recurvature into the extra-tropics, but significant variation exists from storm to storm, spanning a wide range of values from ~100-2000 km, regardless of basin, location, and time of year (Merrill, 1984; Frank, 1977; Chavas and Emanuel, 2010; Cheng-Shang et al., 2010). For example, the radius of gale force wind ($r_{34kt}$) was only 19 km for Tropical Storm Marco in the North Atlantic basin in 2008 (Demuth et al., 2006), whereas this radius reached a maximum of 1110 km for Super Typhoon Tip in the West Pacific basin in 1974 (Dunnavan and Diercks, 1980). Size is found to correlate only weakly with latitude and intensity (Merrill, 1984; Weatherford and Gray, 1988; Chavas and Emanuel, 2010; Chan and Chan, 2012), as the outer and inner core regions appear to evolve nearly independently. Characteristic storm sizes are typically 30-50% larger in the Pacific than in the Atlantic (Merrill, 1984; Liu and Chan, 1999), perhaps a consequence of the existence of large gyre TCs originating from monsoon depressions (Cocks and Gray, 2002). Similarly, Kimball and Mulekar (2004) determined from Atlantic Extended Best Track data that as a storm intensifies the radius of outermost closed isobar ($r_{OCI}$) remains approximately constant despite changes in the radial structure of the intermediate wind field. Moreover, they found that the radius of maximum winds ($r_m$) and intermediate wind radii are smaller, but $r_{OCI}$ is larger, in Gulf of Mexico storms relative to North Atlantic storms at comparable latitude.
In terms of the relationship between storm size and the synoptic-scale environment, Quiring et al. (2011) combined the Extended Best Track and NCEP/NCAR Reanalysis datasets but find minimal useful predictors for $r_m$ or $r_{34kt}$, with the exception of a slight positive correlation between mid-level relative humidity and $r_m$. Liu and Chan (2002) analyzed synoptic-scale weather patterns associated with Western North Pacific TCs of different sizes and found that small TCs typically form within ridge and monsoon-gyre patterns. Indeed, the West Pacific monsoon can develop a large gyre circulation that generates midget tropical cyclones on its periphery and, on occasion, the entire circulation subsequently evolves into a very large TC (Lander, 1994). Finally, TCs are known to expand during the process of extra-tropical transition (Hart and Evans, 2001; Evans and Hart, 2003; Elsberry, 1995), though the underlying mechanisms of this dynamic process are still an active area of research.

From a broader perspective, Merrill (1984) found that frequency distributions of $r_{OCI}$ in the Atlantic and Western North Pacific are qualitatively log-normal, though no formal statistical test was performed. Dean et al. (2009) found that the distribution of storm size, defined as the radius of vanishing winds divided by the ratio of the potential intensity to the Coriolis parameter, is close to log-normal in the Atlantic basin, though this analysis was based on the radius of gale force wind ($r_{34kt}$) taken from two datasets that employ very different methodologies and whose $r_{34kt}$ values for overlapping cases disagree markedly. To improve upon this effort, Chavas and Emanuel (2010) analyzed QuikSCAT scatterometer data and found that the global distribution of $r_0$ is approximately log-normal, though distinct median sizes exist within each ocean basin, suggesting that the size of a given TC is not merely a global random variable but instead is likely modulated either by the structure of the initial disturbance, the environment in which it is embedded, or both.

1.2.2 Modeling

In addition to observational work, numerical modeling also provides insight into the underlying dynamics of TC size. Hill and Lackmann (2009) and Xu and Wang (2010) showed using the full-physics WRF and TCM-4 models, respectively, that TCs tend
to be larger when embedded in moister mid-tropospheric environments due to the increase in spiral band activity and subsequent generation of diabatic potential vorticity which acts to expand the wind field laterally, a result also corroborated by Braun et al. (2012) exploring the role of near-core dry air patches on TC development. Fudeyasu and Wang (2011) combined budget calculations based on output from TCM-4 with solutions to the Sawyer-Eliassen equation and concluded that diabatic heating associated with mid-upper-tropospheric stratiform anvil clouds outside the eyewall in active spiral rainbands generates a mid-tropospheric inflow that transports absolute angular momentum inward to spin up the outer-core circulation, while the azimuthal-mean diabatic heating rate in the eyewall (where it is largest) contributes minimally to this spin-up process due to the high inertial stability in the inner-core region. Using a simple three-layer axisymmetric model, DeMaria and Pickle (1988) found that storm size at peak intensity increased with increasing background rotation rate but was constant with increasing sea surface temperature, while Smith et al. (2011) found in a separate three-layer model an optimum in storm size as a function of rotation rate, which they attributed to the inhibitive effect of inertial stability on boundary-layer inflow as the rotation rate is increased. Finally, the seminal work of Rotunno and Emanuel (1987) found in an idealized axisymmetric framework a strong relationship between the horizontal length scales of the initial and mature vortex. Xu and Wang (2010) corroborate this result, noting an additional sensitivity to the time-evolution of storm size, as a small initial vortex leads to a much slower increase in the inner-core size with time due to the weak surface entropy fluxes beyond the eyewall and associated dearth of spiral rainband activity.

Beyond modeling of individual TCs, Held and Zhao (2008) explore the “tropical cyclone world” of rotating f-plane radiative-convective equilibrium and find that TC size scales inversely with $f$, in apparent qualitative agreement with a scaling with either the ratio of the potential intensity to the Coriolis parameter (Emanuel, 1986) or the Rossby deformation scale, though they could not distinguish between the two. No work has been done thus far to quantify storm size in global general circulation models.
1.2.3 Theory

Finally, extant theory offers insightful models for TC structure. First and foremost, the maximum wind speed is bounded by the potential intensity (Emanuel, 1986, 2010), $V_p$, given by

$$V_p^2 = \frac{C_k}{C_d} \frac{T_{sst} - T_{tpp}}{T_{tpp}} (k^*_{sst} - k)$$

where $C_k$ and $C_d$ are the surface exchange coefficients of enthalpy and momentum, respectively, $T_{sst}$ is the sea surface temperature, $T_{tpp}$ is the outflow temperature near the tropopause, $k$ is the enthalpy of the unperturbed boundary layer air, and $k^*_{sst}$ is the saturation enthalpy at the sea surface temperature and pressure at the radius of maximum winds. Given that a TC can be viewed as Carnot heat engine that extracts heat at the warm surface and expels heat at the cold tropopause (Emanuel, 1986), this quantity can be derived from the balance between net production and net dissipation of energy in the system (Emanuel, 2003). Energy input is associated with two processes: surface fluxes of enthalpy, whose magnitude depends on the local wind speed and air-sea thermodynamic disequilibrium and is given by

$$F_k = C_k \rho |u|(k^*_{sst} - k)$$

where $u$ is the near-surface wind speed, and sensible heating due to internal frictional dissipation within the boundary layer, given by

$$F_{diss} = C_d \rho |u|^3$$

The local net energy production is given by the sum of Eqs. 1.2 and 1.3 multiplied by the Carnot efficiency, $\frac{T_{sst} - T_{tpp}}{T_{sst}}$. The effect of including dissipative heating is simply to change the denominator in the Carnot efficiency from $T_{sst}$ to $T_{tpp}$ (Bister and Emanuel, 2002). Local net energy loss due to frictional dissipation is also given by Eq. 1.3, but absent any efficiency multiplier. Assuming that the radial integrals of each of these processes are dominated by their contributions at the radius of maximum winds, and approximating the full wind by its azimuthal component, $V$, one may equate
the integrands directly and arrive at Eq. 1.1. Overall, the potential intensity is a function of the local, undisturbed thermodynamic environment, and it has been shown to provide a credible bound on the peak intensity of real TCs in nature (Emanuel, 2000).

As for radial structure, Emanuel (2004) developed a complete radial profile as a patchwork of asymptotically-matched solutions for the eye, the convecting inner core, and the non-convecting outer circulation. They combine angular momentum balance in a simple slab boundary-layer model with the constraints imposed by the hypothesis of subcloud layer enthalpy quasi-equilibrium (Raymond, 1995) in the convecting inner region, and with the constraint that the Ekman suction rate at the top of the boundary layer must match the radiative subsidence rate in the lower free troposphere in the non-convecting outer region. The eye solution is assumed to be in near-solid body rotation with the given maximum wind speed and radius of maximum winds due to the fast time-scales of turbulent eddies in the eye, which rapidly transport angular momentum radially inwards (Emanuel, 1997; Smith, 1980).

More recently, Emanuel and Rotunno (2011) derived a full analytical solution for the radial structure of the axisymmetric azimuthal gradient wind at the top of the boundary layer, whose asymptotic solution is given by

\[ V(r) = 2r_m \frac{V_m + \frac{1}{2}fr_m^2}{r_m^2 + r^2} - \frac{1}{2}fr \]  

(1.4)

where \( V_m \) is the maximum gradient wind speed, \( r_m \) is the radius of the maximum gradient wind speed, and \( f \) is the Coriolis parameter. Importantly, neither \( V_m \) nor \( r_m \) are free parameters, as \( V_m \) is a function solely of the ratio of the surface exchange coefficients and is given by

\[ \frac{V_m}{V'_p} = \left( \frac{1}{2} \frac{C_k}{C_d} \right)^{\frac{1}{2}} \]  

(1.5)

where \( V'_p \) is a nominal version of the potential intensity, \( V_p \), that does not include dissipative heating and uses the environmental saturation entropy in lieu of its ambient boundary layer value. Meanwhile, \( r_m \) is defined relative to the outer radius.
of vanishing wind \( V = 0 \), \( r_0 \), according to the analytical solution for the ratio of angular momentum at \( r_m \) to its value at \( r_0 \), given by

\[
\frac{M_m}{M_0} = \left( \frac{1}{2} \frac{C_k}{C_d} \right)^{2-\frac{1}{2}}
\]

where \( M_m \approx V_m r_m \) and \( M_0 = \frac{1}{2} f r_0^2 \). This solution is based on the assumption that small-scale, mechanically-driven turbulence in the TC outflow fixes the Richardson number to its critical value, thereby defining the radial dependence of the outflow temperature along the tropopause. For a sub-critical, slantwise moist neutral vortex, the distribution of the outflow temperature leads directly to the radial distribution of entropy in the boundary layer and, through thermal wind balance, the radial distribution of the azimuthal winds at the top of the boundary layer.

However, this latest solution is defined relative to a single free parameter given by the outer radius, \( r_0 \) – an elegant representation of our collective ignorance on TC size. Indeed, though reasonable theoretical models for storm structure exist, they are necessarily imposed onto an overall radial length scale that itself lacks theoretical guidance. The lone exception to this statement lies within the original potential intensity theory of Emanuel (1986), which includes a scaling for the theoretical upper-bound on \( r_0 \) that is given by the ratio of the potential intensity to the Coriolis parameter, \( \frac{V_p}{f} \), the derivation and physical insight for which we review here.

From the Carnot heat engine perspective, potential intensity theory assumes a balance between the net heat input into the system, \( \Delta Q \), and the work done by the system, \( W \). The heat input is in the form of wind-speed-dependent surface fluxes of enthalpy into the boundary layer from the lower boundary, whose existence is owed to the ambient air-sea disequilibrium of a greenhouse climate (Emanuel, 1987), as well as dissipative heating in the boundary layer; heat is expelled radiatively at the cold tropopause. The work performed is primarily that required to maintain the vortex wind field against frictional dissipation, \( W_{bl} \). However, the system must also do work to restore angular momentum in the outflow to its ambient value, \( W_{out} \), in order to connect the outflow leg with the boundary-layer inflow leg and thus energetically
close the Carnot engine. This restoration is believed to occur via vertical transport of angular momentum by small-scale turbulence in the outflow (Emanuel and Rotunno, 2011). The full balance is given by

$$\Delta Q = W_{bl} + W_{out}$$

(1.7)

Thus, implicit in Eq. 1.7 is the fact that any outflow work necessarily detracts from the work available to power the boundary layer winds and thus weakens the equilibrium storm. The outflow work, $W_{out}$, is simply proportional to the change in kinetic energy required to return the angular momentum back to its original value, which, assuming the process occurs at large radii, gives

$$W_{out} \approx \frac{1}{2} f (M_0 - M)$$

(1.8)

If we approximate $M$ with $M_m$, then combining Eqs. 1.8 and 1.6 results in

$$W_{out} \approx \frac{1}{2} f^2 r_0^2 \left( 1 - F \left( \frac{C_k}{C_d} \right) \right)$$

(1.9)

where $F \left( \frac{C_k}{C_d} \right)$ is given by the RHS of Eq. 1.6, thereby demonstrating that the outflow work is proportional to $r_0^2$ (i.e. the area of the storm) and thus a larger storm requires more work be performed in the outflow.

The manifestation of this size effect on the steady-state intensity of the vortex arises in the original potential intensity theory of Emanuel (1986), though it is more easily seen in a subsequent iteration of this theory. Eq. 20 of Emanuel (1995b) gives a non-dimensional relation for the central pressure perturbation, $P_c$,

$$P_c \sim 1 - \frac{1}{4} \frac{r_0^2}{r_0^2}$$

(1.10)

where $r_0$ has been non-dimensionalized by $\frac{\sqrt{\chi_s}}{f}$ and $\sqrt{\chi_s}$ is a velocity scale equal to the potential intensity with $C_k = C_d$. This relation dictates that the central pressure perturbation vanishes for a sufficiently large storm relative to this theoretical
length scale. Moreover, this length scale is employed to non-dimensionalize radius in Emanuel (1989) and Emanuel (1995a) and thus is viewed as a “natural” length scale for a TC.

1.3 Defining storm size

There are a variety of metrics to which the term “size” refers, often to great confusion. Operationally, the common size radii are the radius of maximum wind ($r_m$), the radii of 64, 50, and 34 kt wind ($r_{64kt}$, $r_{50kt}$, $r_{34kt}$, respectively), and the radius of outermost closed isobar $r_{OCI}$. However, as noted earlier, the inner core, typically encompassing $r_m$ and $r_{64kt}$ and perhaps $r_{50kt}$, and the outer circulation, typically encompassing $r_{34kt}$ and $r_{OCI}$, tend to evolve independently of one another, with the latter more stable in time during the lifetime of a TC.

Here we use the term “size” to refer to a measure of the broad outer circulation of the storm, and we formally define size as the outer radius, $r_0$, where the radial wind profile vanishes, following theoretical convention given by the combination of Eqs. 1.4 and 1.6 as well as in earlier versions of potential intensity theory (e.g. Emanuel (1986)). Though less tangible operationally, $r_0$ is the relevant theoretical free parameter in need of constraint. Moreover, $r_0$ represents a universal metric of size that is independent of any specific choice of wind speed, whether dimensional (e.g. $r_{34kt}$) or non-dimensional (e.g. radius of 50% of the maximum wind speed), whose radius is used as a basis for comparison across storms.

1.4 Objectives

This work seeks to build upon the small base of existing research on TC size by characterizing the distribution of size in nature and exploring the determinants of equilibrium size in radiative-convective equilibrium, the simplest representation of a tropical atmosphere. Chapter 2 describes the creation of a climatology of tropical cyclone size based on QuikSCAT scatterometer data. Chapter 3 explores the
modulation of tropical cyclone size and structure by dimensional parameters and surface exchange coefficients in an idealized state of axisymmetric radiative-convective equilibrium. Chapter 4 explores tropical cyclone size in an identical thermodynamic environment but in three dimensional “tropical cyclone worlds”. Finally, Chapter 5 concludes with a synthesis of key findings and discussion across all chapters and explores the many opportunities for future work.
Chapter 2

A QuikSCAT Climatology of Tropical Cyclone Size

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2.1 Introduction

In the absence of land interaction, the horizontal extent of the outer circulation of a tropical cyclone (TC) is observed in nature to vary only marginally during the lifetime of a given TC prior to recurvature into the extra-tropics (Merrill, 1984; Frank, 1977), but significant variation exists from storm to storm, regardless of basin, location, intensity, and time of year. Kimball and Mulekar (2004) determined from Atlantic Extended Best Track data that as a storm intensifies the radius of outermost closed isobar (ROCI) remains approximately constant despite changes in the radial structure of the intermediate wind field. More recently, modeling work by Hill and Lackmann (2009) and Wang (2009) showed that TCs tend to be larger when embedded in moister mid-tropospheric environments due to the increase in spiral band activity and subse-

1Permission to use figures, tables, and brief excerpts from this chapter in scientific and educational works is hereby granted provided that the source is acknowledged: Chavas, D. R., and K. A. Emanuel (2010), A QuikSCAT climatology of tropical cyclone size, Geophys. Res. Lett., 37, L18816. Figures 2-3, 2-4, and 2-5 and additional discussion in Section 2.4.4 have been added to the original publication text for the purposes of the presentation herein. Additionally, a few minor text edits from the original publication were made for the purpose of clarification.
quent diabatic generation of potential vorticity which acts to expand the wind field laterally.

From a broader perspective, Merrill (1984) found frequency distributions of ROCI in the Atlantic and Western North Pacific that qualitatively resemble log-normal distributions. Dean et al. (2009) [hereafter D09] found that the distribution of normalized storm size, defined as the radius of vanishing winds divided by the ratio of the potential intensity to the Coriolis parameter, is close to log-normal in the Atlantic basin. However, the result of D09 is based on the radius of gale force winds (R34) taken from two datasets that employ very different methodologies and whose R34 values for overlapping cases disagree markedly.

Ideally, one would prefer to characterize the size distribution based upon direct surface wind measurements taken from a single, consistent source. Thus, this work examines the global distribution of TC size, defined here as the radius of vanishing winds, using an independent, high-resolution dataset generated by the QuikSCAT satellite microwave scatterometer. The following sections outline the data and methodology used to generate a climatology of TC size, discuss its characteristic values and distribution, and explore the intra-storm evolution of size.

2.2 Data

Ocean near-surface (10m) wind vector data are taken from the QuikSCAT Level 2B dataset on a 12.5 km x 12.5 km grid for the period beginning July 19, 1999 (the start of the satellite’s operational life) through December 31, 2008; this dataset is available at http://podaac.jpl.nasa.gov/DATA_CATALOG/quiKSCATinfo.html. Owing to rain contamination of the signal, QuikSCAT data quality is highest away from strong precipitation, and the instrument is considered very accurate in the range 3–20 m s$^{-1}$ (NASA Jet Propulsion Laboratory, 2010); Chou et al. (2010) found RMS differences between QuikSCAT wind speeds and dropwindsonde data of 2.6 m s$^{-1}$. For a complete discussion of potential errors, see Hoffman and Leidner (2005).

Tropical cyclone 6-hourly location and intensity data are taken from the National
Hurricane Center HURDAT Best Track database (Jarvinen et al., 1984). For calculation of the normalization factor, \( \frac{H}{f} \), which is the natural tropical cyclone length scale (Emanuel, 1986), potential intensity values are taken from monthly mean re-analysis data (Bister and Emanuel, 2002) bi-linearly interpolated to the storm location.

2.3 Methodology

2.3.1 Locating TCs

To create a climatology of tropical cyclones as seen by QuikSCAT, Best Track location and intensity \( (V_{BT}) \) data are spline interpolated iteratively forward until reaching the minimum distance, \( d \), to any valid (i.e. non-rain-flagged) QuikSCAT datapoint of a given pass. Cases for which \( d > 100 \) km or the interpolated intensity \( V_{BT} \leq 17.451 \text{ ms}^{-1} \) are skipped.

Next, to identify the TC center of circulation we take as a first guess the interpolated Best Track location, about which we extract all data (including rain-flagged) within a 4° x 4° box. All TC centers are then subjectively identified based on the full QuikSCAT wind vector field in this box. Only those cases for which there exists a single, clearly-defined center of cyclonic circulation are included, based upon the criteria that a) the center is consistent with the wind vectors in the immediate vicinity in all directions, and b) the broad “outer” circulation (i.e. 1-4 degrees from center) is easily discernible and is consistent with the location identified by criterion (a). The authors sought to be conservative in this procedure; when ambiguous, the case was omitted. All data within 2500 km of the center are then used for subsequent analysis.

Only cases over water and for which the potential intensity \( PI > 40 \text{ ms}^{-1} \) are included in order to avoid cases in which storms are rapidly transitioning to regions of cold sea surface temperatures where mature tropical cyclones cannot be sustained. The TC translation vector, calculated directly from the full spline interpolation of the Best Track dataset, is then subtracted from all wind vectors. All vectors are projected onto their pure-azimuthal component relative to the TC center and vector
magnitudes are signed: positive for cyclonic, negative for anti-cyclonic. Lastly, wind speeds are azimuthally-averaged within 10-km wide rings moving radially outward from center to obtain a radial wind profile for each TC fix.

Finally, we select a single azimuthal-average wind speed, $V_{QS}$, and for each TC fix determine its radius, $r_{QS}$, and extrapolate outward to $r_0$ using a theoretical model of outer wind structure that assumes minimal deep convection in the outer region. This model is described in detail in Emanuel (2004) and is reviewed below.

### 2.3.2 Selecting $V_{QS}$

Selection of an optimal QuikSCAT wind speed, $V_{QS}$, necessitates balancing three key constraints. First, the assumption of constant background flow, represented by the single translation vector subtracted from all points, loses validity far from center; this constraint renders any effort to extract $r_0$ directly from the QuikSCAT data invalid. Second, the number of data points increases dramatically as one moves outward from the TC center. Finally, Brennan et al. (2009) found that QuikSCAT observed winds have a near-zero bias due to rain in the range of $10-15\text{ ms}^{-1}$. The validity of a given azimuthal-average wind speed depends on the trade-offs between the above three factors. Based on these criteria we set $V_{QS} = 12\text{ ms}^{-1}$.

The final result is a dataset of 2154 TC fixes spread across five basins: Atlantic (482), East Pacific (367), West Pacific (640), Indian Ocean (78), and Southern Hemisphere (587).

### 2.3.3 Estimating Outer Radius $r_0$

To estimate the outer radius, $r_0$, we employ the outer wind structure model derived in Emanuel (2004) (for an abridged form, see D09) to extrapolate radially outwards from the QuikSCAT-defined azimuthal-average radius, $r_{QS}$, of the wind speed $V_{QS}$ described above. Here, we briefly review the model’s characteristics. The flow is assumed to be steady and axisymmetric. The model assumes that there is no deep convection beyond $r_{QS}$, resulting in a local balance between subsidence warming and
radiative cooling. Furthermore, given that both the lapse rate and the rate of clear-sky radiative cooling are nearly constant in the tropics, the equilibrium subsidence velocity, $w_{rad}$, can be taken to be approximately constant. In equilibrium, this subsidence rate must match the Ekman suction rate into the boundary layer in order to prevent the creation of large vertical temperature gradients across the top of the boundary layer. The radial profile of azimuthal velocity is therefore determined as that which provides the required Ekman suction, and is given

$$
\frac{\partial (rV)}{\partial r} = \frac{2r^2 C_D V^2}{w_{rad} (r_0^2 - r^2)} - fr
$$

where $r$ is the radius, $V$ is the azimuthal wind speed, $f$ is the Coriolis parameter, $C_D$ is the bulk aerodynamic drag coefficient. We set $C_D = 10^{-3}$ and $w_{rad} = 1.6$ cm$s^{-1}$.

To our knowledge, this nonlinear first order differential equation has no analytical solution. $D09$ neglected the partial derivative term to derive a simple analytical solution for $r_0$. However, (1) can also be solved numerically for $r_0$, and the solution to the full equation is 30-150 km larger than the approximated solution over the typical range of tropical latitudes and $r_{QS}$ values (not shown). Thus, for our purposes we elect to use the full numerical solution.

## 2.4 Results

### 2.4.1 Basic statistics

Figure 2-1a displays the median radius of 12 m$s^{-1}$, $r_{12}$, and $r_0$ values and the standard deviation of $r_0$ both globally and by basin.

The global median outer radius is 423 km and ranges from a minimum of 341 km in the East Pacific to a maximum of 488 km in the West Pacific. The standard deviation of $r_0$ is 168 km and scales across basins in a similar fashion to the median value. The median distance between $r_{12}$ and $r_0$ is 226 km. These values compare reasonably well with those of previous studies (e.g. Merrill (1984)). Moreover, $r_0$ is relatively insensitive to variations in $w_{rad}$ and $C_D$ (assumed constant), with changes
Figure 2-1: Top: Median values of $r_{12}$ (blue) and $r_0$ (green) of $r_0$ (red) globally and by basin. All units in [km]. Error bars denote range of two standard deviations from the mean. Bottom: Correlation coefficients between $r_0$ and various parameters globally and across basins; “day” represents day of the hurricane season. Basins listed are Atlantic (AL), East Pacific (EP), West Pacific (WP), Indian Ocean (IO), and Southern Hemisphere (SH).

of approximately 25 km for the rather extreme cases of a halving or doubling of the ratio $\frac{C_D}{u_{rad}}$ for $\phi = 20^\circ$ and $r_{12} = 200 \text{ km}$.

Figure 2-1b displays correlation coefficients between $r_0$ and various parameters of interest. The lone correlation of note exists between $r_0$ and intensity $V$ ($r = .36$) and is relatively consistent across basins; this matches the weak correlation ($r = 0.28$)
found by Merrill (1984). Meanwhile, \( r_0 \) is effectively independent of latitude, which contradicts the typical finding that TCs tend to expand as they recurve poleward (e.g. Merrill (1984)).

2.4.2 Size distribution

Table 1 lists the p-values for the statistical fit to various distributions of \( \log(r_{12}) \), \( \log(r_0) \), \( \log(r^*_{12}) \), and \( \log(r^*_0) \), where the asterisk denotes normalization by \( \frac{D}{f} \) following D09. All p-values are calculated using the Kolmogorov-Smirnoff test statistic. In the case of the normal and log-normal test distributions, the observed data were rescaled to have zero mean and unit variance for comparison to the standard normal parent distribution \( N(0,1) \). P-values approaching unity indicate that the observed distribution is close to the parent distribution.

Table 1. Kolmogorov-Smirnoff p-values for statistical fits to various parent distributions for \( r_{12} \), \( r_0 \), \( r^*_{12} \), and \( r^*_0 \). Log-normal refers to the normal fit of \( \log(r) \).

<table>
<thead>
<tr>
<th>Probability Distribution</th>
<th>( r_{12} )</th>
<th>( r_0 )</th>
<th>( r^*_{12} )</th>
<th>( r^*_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-normal</td>
<td>.028</td>
<td>.626</td>
<td>.248</td>
<td>.226</td>
</tr>
<tr>
<td>Normal</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Weibull</td>
<td>.001</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rayleigh</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gamma</td>
<td>.05</td>
<td>.11</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The goodness of fit between the distribution of \( r_0 \) and a log-normal parent distribution is the most significant from among the variables and distributions tested here. The null hypothesis that \( r_0 \) is gamma distributed (\( p = .11 \)) also cannot be rejected at the 95% confidence level, though based on a \( \chi^2 \) metric (\( p = .043 \)) it can be rejected.

For a direct comparison of \( r_{12} \) and \( r_0 \), their global frequency distributions, along with the Gaussian fit to the mean and variance of the logarithm of each dataset, are displayed in Figure 2-2. Globally, \( p = .028 \) for \( r_{12} \), which indicates that the null hypothesis of a log-normal distribution can be rejected at the 95% confidence.
interval. On the other hand, $p = .626$ for $r_0$, which indicates that the distribution is reasonably close to log-normal. Moreover, \textit{D09} determined that normalization of $r_0$ by $\frac{P_I}{I}$ results in a distribution that is much closer to log-normal. Our results indicate that the distribution of $r_0$ is significantly closer to log-normal than that of $r_{12}$, but that the subsequent normalization of $r_0$ in fact makes the log-normal fit worse. Though normalization does improve the fit for $r_{12}$, this may be understood in a crude mathematical sense given that $\log(\frac{r_I}{f}) = \log(r) + \log(\frac{f}{P_I})$. The distribution of $f$ itself has a p-value of $p = .165$, which is greater than that of $r_{12}$ but less than $r_0$, and thus normalization would be expected to improve the fit for $r_{12}$ but to reduce it for $r_0$. In either case, the important result here is that normalization is not necessary to observe a size distribution that is relatively close to log-normal. These results are
found to be largely insensitive to the choice of $V_{QS}$ over the range $8 - 15 \text{ m s}^{-1}$ (not shown). The findings are qualitatively similar within individual basins (Figure 2-3).

2.4.3 Control experiments

To what extent is this log-normal distribution an artifact of the outer wind structure model employed here? Given that our version of $r_0$ is only a function of $r_{12}$ and $f$, we perform three test experiments. First, we recalculate $r_0$ using the observed distribution of $f$ but set all values of $r_{12}$ to be constant and equal to the median value, $r_{12} = 197.15 \text{ km}$, which results in a p-value of $p=.002$. Second, we recalculate $r_0$ using the observed distribution of $r_{12}$ but set all values of $f$ to be constant and equal to the median value, $f = 5 \times 10^{-5} \text{ s}^{-1}$, which results in a p-value of $p=.222$.

Finally, we recalculate $r_0$ using the observed distribution of both $r_{12}$ and $f$ but randomly reshuffle their pairings, the purpose of which is to address the question of whether nature “matches” $r_{12}$ and $f$ in some optimal way as to generate a log-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2-3.png}
\caption{P-values for Gaussian fit to the distribution of $\log(r_{12})$ (blue) and $\log(r_0)$ (red) across basins.}
\end{figure}
normal distribution. For 100 runs, the p-value for the observed pairings of \( r_{12} \) and \( f \) is larger than approximately 80% of cases with randomized pairings, which suggests that, though not optimized, how \( r_{12} \) and \( f \) are paired in nature may play a role in bringing the distribution of \( r_0 \) closer to log-normal.

Taken together, these experiments indicate that, though a component of the observed distribution is simply due to the nature of the outer structure model chosen in this work, the actual distributions of \( r_{12} \) and \( f \) are also central to generating the log-normal distribution.

### 2.4.4 Intra-storm evolution

For the 241 distinct TCs with 4 or more QuikSCAT observations in the dataset used here, the mean intra-storm rate of change of \( r_{12} \) and \( r_0 \), taken as the slope of the linear least-squares fit to the data, is 18.1 and 10.9 km day\(^{-1}\), or approximately 9 and 2.5 % day\(^{-1}\) of the median value, respectively. The distribution for the case of \( r_0 \) is shown in Figure 2-4. The respective standard deviations are 43.1 and 53.2 km day\(^{-1}\), indicating significant variance across individual storms; the distribution of rates of change is approximately Gaussian about the mean. Though relatively small, these mean expansion rates are statistically significantly different from zero at the 95% confidence interval (\( p = 0 \) and .002, respectively). A slow broadening of the wind field with time has also been noted in previous studies (e.g. Cocks and Gray (2002), Merrill (1984)).

Closer inspection reveals that much of this expansion appears to occur early in the storm’s evolution. For the 215 distinct TCs whose first 4 observations occur within a 100 hour period, the expansion rate of \( r_{12} \) and \( r_0 \) over these first 100 hours is 24.0 and 18.7 km day\(^{-1}\), respectively. Meanwhile, for the 35 distinct TCs with 4 or more observations at least 100 hours after the initial observation, the expansion rate beyond 100 hours declines substantially to 8.3 and –0.8 km day\(^{-1}\), respectively, neither of which are statistically significantly different from zero (\( p = .28 \) and .92). Significant variance exists, though, as standard deviations are 43.1 and 53.2 km day\(^{-1}\), respectively. If the outer radius of a mature TC truly remains approximately constant.
Figure 2-4: Distribution of $\frac{\partial r_o}{\partial t}$ for all storms with at least four observations. Red lines denote mean (solid) and one standard deviation (dashed) growth rates, with a mean value of 10.9 km day$^{-1}$.

with time, then this result may be an indication that our threshold minimum intensity
of 17.5 ms$^{-1}$ is capturing TCs at the tail end of the genesis process during which the
outer radius has yet to reach its quasi-steady state, but further investigation is needed
to validate such a claim.

Finally, there are six TCs that are observed at least 14 times during their lifespans,
enabling a closer look at the evolution of storm size over the lifecycle of a few long-lived
storms. Their time evolutions are plotted in Figure 2-5. In most cases the size of these
storms remains quite steady in time. In particular, the red curve, which corresponds
to the longest lived and most observed storm in the database (22 observations),
stays at a remarkably constant size throughout its entire lifespan and at a value
tantalizingly close to our global median value of 400 km. This subset appears to
provide a convenient, representative sample of our collective knowledge of storm size
evolution: during its lifecycle, a tropical cyclone typically does not change significantly
in size (red, black, green, blue), occasionally it grows gradually (cyan, yellow), but it rarely if ever contracts.

2.4.5 Case study: Alberto (2000)

The most-observed case (red) in Figure 2-5 corresponds to Hurricane Alberto (2000) in the Atlantic basin, whose lifecycle spanned the period August 3-25, 2000. Alberto was one of the top ten longest-lived storms in the Atlantic basin in recorded history (Beven, 2000). Alberto was a classic Cape Verde-type TC that developed from a strong African Easterly Wave (AEW; Thorncroft and Hodges (2001)) off the coast of West Africa that spent its entire life at sea. A map showing the track and evolution of Alberto is displayed in Figure 2-6.

During the course of its life-cycle, Alberto traversed a large range of latitudes. It first develops into a Tropical Depression at 11° N, begins to recurve poleward near
Figure 2-6: Map displaying the NHC Best Track and evolution of Hurricane Alberto (2000). Colors correspond to Saffir-Simpson category; maximum wind speeds from NHC Best Track database. Circles denote QuikSCAT observation, with marker size scaled by QuikSCAT-based estimate of $r_0$ (see legend). Contours denote climatological distribution of $V_p$ for August (Bister and Emanuel, 2002).

45° W, undergoes a large, 5-day anti-cyclonic loop between 33° N and 39° N, and finally moves rapidly northward starting at 0600 UTC on 22 August. Importantly, Alberto was able to move a significant distance poleward before encountering extratropical disturbances, as extratropical transition was observed to begin only on the final leg of its track poleward of 45°N. Meanwhile, Alberto underwent three different periods of intensification to Hurricane status, the strongest of which allowed the storm to attain Category 3 status ($V_m = 110$ kt) on 12 August. Figure 2-7 displays a time-series of the evolution of $V_m$, $V_p$, $r_m$, and $r_0$. Data for $r_m$ are taken from the Extended Best Track dataset (Demuth et al., 2006).

The stability of the size of Alberto throughout its lifecycle is remarkable given the significant variations in $r_m$, $V_m$, $V_p$ and $f$ that the storm endures. Alberto provides a clear example of the apparent independence of storm size from the variables typically
Figure 2-7: Time series of $V_m$ and $V_p$ (top), $r_m$ and $r_0$ (bottom) for Alberto (2000). Colors for $V_m$ correspond to Saffir-Simpson category as in Figure 2-6.

considered to be operationally-relevant to TC evolution, including both environmental parameters and characteristics of the inner-core of the storm. In particular, this case study provides anecdotal evidence that the increase in storm size with latitude during the life-cycle of a storm, which has been noted in previous observational studies (Merrill, 1984; Weatherford and Gray, 1988), may simply be the signal associated with those storms undergoing extratropical transition rather than any fundamental process associated with changes in $f$ (indeed, equilibrium dynamics would predict a smaller storm at larger $f$, as discussed in Chapter 3). Though extratropical transition climatologically begins as a storm begins to recurve poleward at around $35^\circ N$ (Hart and Evans, 2001), Alberto demonstrates that storms are occasionally able to move substantially poleward while retaining their pure tropical structure, in which case storm expansion may not be expected to occur.
2.5 Discussion and Conclusions

Given the high resolution and high precision of QuikSCAT data, the results presented here provide credible evidence that the global distribution of tropical cyclone size, defined as the radius of vanishing winds calculated using an outer wind structure model that assumes vanishing deep convection beyond the azimuthally-averaged radius of 12 $ms^{-1}$ winds, is approximately log-normal. While the distribution of $r_{12}$ is qualitatively log-normal, the distribution of $r_0$ is quantitatively much closer to log-normal. Moreover, in contrast to the work of D09, we find here that the normalization by the natural length scale of tropical cyclones, defined as the ratio of the potential intensity to the Coriolis parameter, reduces rather than improves the goodness of fit of the observed distribution to log-normal, suggesting that this length scale is not fundamental to storm size as it is observed under the current Earth climate. Control experiments indicate that the choice of the outer wind model alone is insufficient to explain the observed p-values for the distribution of outer radius; the distributions observed in nature of $r_{12}$ and $f$, from which the distribution of $r_0$ is derived, appear to play an important role as well. Finally, analysis of the intra-storm evolution of size indicates that both $r_{12}$ and $r_0$ tend to expand very slowly with time early in the storm lifecycle, after which size appears to remain nearly constant, although significant variance exists across storms.

What is the implication of the log-normal distribution in the context of tropical cyclones? As noted earlier, in the absence of significant external environmental forcing, there is evidence that the spatial extent of a given tropical cyclone remains relatively constant throughout its lifetime, suggesting that the existence of this distribution may be a manifestation of the processes that generate tropical cyclones in the first place and/or of the distribution of their precursor disturbances. However, with respect to size, there is no obvious single multiplicative process during genesis that is amenable to isolation. This will be the subject of future work.
Chapter 3

Equilibrium Tropical Cyclone Size and Structure in Axisymmetry

3.1 Introduction

Considerable progress has been made over the past three decades in elucidating the dynamics of tropical cyclones (TCs). Theory has been developed suggesting that TCs may be viewed as a Carnot heat engine whose heat source arises from the ambient thermodynamic disequilibrium of the tropical oceans (Emanuel, 1986). Furthermore, both theory and relatively simple dynamical models (Ooyama, 1969; DeMaria and Pickle, 1988; Rotunno and Emanuel, 1987; Emanuel, 1995a) are able to reproduce many of the characteristic features of mature tropical cyclones, including maximum wind speed, central sea level pressure, and thermodynamic structure. Most recently, Emanuel and Rotunno (2011) derived a full analytical solution for the radial structure of the axisymmetric balanced tropical cyclone wind field at the top of the boundary layer.

However, this latest solution remains defined relative to a single free parameter: the outer radius, $r_0$. Indeed, despite wide recognition of the sensitivity of both storm surge (Irish et al., 2008) and wind damage (Iman et al., 2005) to storm size, size remains largely unpredictable, and relatively little observational or modeling work has been performed to elucidate the factors underlying its variability. In the absence
of interaction with land or extratropical disturbances, size is observed in nature to vary significantly more from storm to storm than within the lifetime of a given storm, regardless of basin, location, and time of year (Merrill, 1984; Frank, 1977; Chavas and Emanuel, 2010; Cheng-Shang et al., 2010). Size is found to correlate only weakly with both latitude and intensity (Merrill, 1984; Weatherford and Gray, 1988; Chavas and Emanuel, 2010), as the outer and inner core regions appear to evolve nearly independently. Chavas and Emanuel (2010) found that the global distribution of $r_0$ is approximately log-normal, though distinct median sizes exist within each ocean basin, suggesting that the size of a given TC is not merely a global random variable but instead is likely modulated either by the structure of the initial disturbance, the environment in which it is embedded, or both.

Recent research has begun to explore the sensitivity of storm size to local thermodynamic variables. Observationally, Quiring et al. (2011) combine the Extended Best Track and NCEP/NCAR Reanalysis datasets to demonstrate that various local environmental variables have at best a secondary influence on the radius of maximum wind ($r_{max}$) and the radius of gale force wind in the Atlantic basin, with the exception of a positive correlation between mid-level relative humidity and $r_{max}$. Idealized modeling studies in Hill and Lackmann (2009) and Xu and Wang (2010) found that TCs tend to be larger when embedded in moister mid-tropospheric environments due to the increase in spiral band activity and subsequent generation of diabatic potential vorticity which acts to expand the wind field laterally. Using a simple three-layer axisymmetric model, DeMaria and Pickle (1988) found that storm size at peak intensity increased with increasing background rotation rate but was constant with increasing sea surface temperature, while Smith et al. (2011) found in a separate three-layer model an optimum in storm size as a function of rotation rate attributed to the inhibitive effect of inertial stability on boundary-layer inflow as the rotation rate is increased. Finally, Rotunno and Emanuel (1987) found in an idealized axisymmetric framework a strong relationship between the horizontal length scales of the initial and mature vortex.

A dynamical systems approach may provide a path forward in improving our
understanding of tropical cyclone size. Tang and Emanuel (2010) demonstrated analytically that tropical cyclone intensity may be viewed as a non-linear dynamical system that evolves towards a stable equilibrium whose value depends on the local environmental and initial conditions. This behavior has been verified in a modeling context on both short time-scales (e.g. Rotunno and Emanuel (1987)) and long time-scales over which the storm's maximum wind speed has achieved statistical equilibrium (Hakim, 2011). However, no such theory exists for the dynamical evolution of tropical cyclone structure, and the tropical cyclone at statistical structural equilibrium remains unexplored. This is of particular relevance given the large range of sizes observed in nature (Chavas and Emanuel, 2010).

This work seeks to build upon the small base of literature on tropical cyclone size by systematically exploring the sensitivity of the structure of an axisymmetric tropical cyclone at statistical equilibrium to the set of relevant model, initial, and environmental variables. Expanding on the work of Hakim (2011), we perform our analysis in the simplest possible model and physical environment: a highly-idealized state of radiative-convective equilibrium (RCE). The results of the sensitivity analysis are then synthesized via dimensional analysis to quantify the relationship between equilibrium storm structure and the set of relevant input parameters. Section 2 details the methodology, including model description and experimental design. Section 3 derives a useful alternative formulation of the maximum potential intensity in the context of our idealized RCE environment. Results and comparison with existing theory are presented in section 4, and discussion of some key findings are presented in section 5. Finally, section 6 provides a brief summary and conclusions.

3.2 Methodology

3.2.1 Model description

This work employs version 15 of the Bryan Cloud Model (CM1), a non-hydrostatic atmospheric cloud-system resolving model (CSRM; original version described in Bryan
and Fritsch (2002)) that has been applied to the study of a variety of convective systems including topographic flow (Miglietta and Rotunno, 2010), tropical cyclones (Bryan and Rotunno, 2009b; Bryan, 2011), and mid-latitude squall lines (Parker, 2008). CM1 was originally written with the goal of incorporating state of the art numerics and physics, in particular for moist processes, while satisfying near-exact conservation of both mass and energy in a reversible saturated environment. The model is set up in three-dimensions but can also be configured with identical parameters for two-dimensional axisymmetric (radius-height) geometry, a convenient property that will be exploited in this work.

CM1 solves the fully compressible set of equations of motion in height coordinates on an f-plane for flow velocities \((u, v, w)\), non-dimensional pressure \((\pi)\), potential temperature \((\theta)\), and the mixing ratios of water in vapor, liquid, and solid states \((q_x)\) on a fully staggered Arakawa C-type grid in height coordinates. The model has a rigid lid at the top with a 5-km thick damping layer beneath and a wall at the domain’s outer horizontal edge with an adjacent damping layer whose thickness is set to approximately \(\frac{1}{15}\) of the domain’s width. The damping time-scale is set to its default value of 6 minutes. Model horizontal (x-y) and vertical grid spacing are each constant in the domain. Model microphysics is represented using the Goddard-LFO scheme based on Lin et al. (1983), which is a mixed-phase bulk ice scheme with prognostic equations for water vapor, cloud water, rainwater, pristine ice crystals, snow, and large ice. For full details, see Bryan and Fritsch (2002). Lastly, in lieu of a comprehensive scheme for radiative transfer, an idealized scheme (discussed below) is imposed due to its simplicity.

Turbulence is parameterized using a Smagorinsky-type closure scheme (Smagorinsky, 1963), which assumes steady and homogeneous unresolved turbulence, modified such that different eddy viscosities are used for the horizontal/radial and vertical directions to represent the differing nature of turbulence between the radial and vertical directions in a highly anisotropic system such as in the inner core of a tropical cyclone. In the context of tropical cyclones, turbulence fulfills the critical role of counteracting eyewall frontogenesis by the secondary circulation that, in the inviscid limit, would
lead to frontal collapse (Emanuel, 1997). Meanwhile, in a three-dimensional RCE state, turbulence has a minimal impact on the mean state.

### 3.2.2 Idealized model/environmental RCE set-up

We construct a highly-idealized model and environmental configuration with the objective of reducing the model atmospheric system to the simplest possible state (i.e. minimal number of dimensional variables) that supports a tropical cyclone. Model horizontal and vertical grid spacings are set to $dx = dy = dr = 4 \text{ km}$ and $dz = .625 \text{ km}$, respectively, and no grid stretching is applied. This horizontal resolution was selected with the goal of minimizing both the sensitivity of storm structure to grid spacing and the overall computational load. Surface pressure is set to 1015 hPa. Radiation is represented simply by imposing a constant cooling rate (which is typical of the clear-sky mean tropical troposphere, see Hartmann et al. (2001)), $Q_{cool}$, to the potential temperature everywhere in the domain where the absolute temperature exceeds a threshold value, $T_{tpp}$; below this value, Newtonian relaxation back to this threshold is applied:

$$\frac{\partial \theta}{\partial t} = \begin{cases} -Q_{cool} & T > T_{tpp} \\ \frac{\theta(p,T_{tpp}) - \theta(p,T)}{\tau} & T \leq T_{tpp} \end{cases} \quad (3.1)$$

where $\theta$ is potential temperature, $T$ is absolute temperature, and $\tau$ is the relaxation timescale, set to 40 days (except in the damping layer as noted above). Thus, all water-radiation and temperature-radiation feedbacks are neglected. The lower-boundary sea surface temperature, $T_{sst}$, is set constant. Surface fluxes of enthalpy and momentum are calculated using standard bulk aerodynamic formulae

$$F_k = C_k \rho |u|(k_{\ast}^s - k) \quad (3.2)$$

$$\tau_s = -C_d \rho |u| u \quad (3.3)$$

where $F_k$ is the surface enthalpy flux, $\rho$ is the near-surface air density, $u$ is the near-surface (i.e. lowest model level) wind velocity, $k$ is the near-surface enthalpy, $k_{\ast}^s$
is the saturation enthalpy of the sea surface, \( \tau_s \) is the surface stress, and the exchange coefficients for momentum, \( C_d \), and enthalpy, \( C_k \), are set constant, despite their acknowledged real-world dependence on wind-speed (Powell et al., 2003). Finally, background surface enthalpy fluxes are required to balance column radiative cooling in order to achieve RCE in the absence of significant resolved wind perturbations (such as a tropical cyclone). Because axisymmetric geometry precludes the direct imposition of a background flow, we instead simply add a constant gustiness, \( u_{sfc} \), to \(|u|\) for the model calculation of (3.2) and (3.3). This set-up is conceptually similar to that of Hakim (2011) with the important exceptions that here we employ a non-interactive radiative scheme and we include background surface fluxes throughout the domain.

This configuration provides a simplified framework for the exploration of equilibrium tropical cyclone structure in RCE. Nolan et al. (2007) demonstrated that, in the presence of a full radiation scheme, the f-plane RCE state depends only on \( T_{sst} \), \( u_{sfc} \) and very weakly on the Coriolis parameter, \( f \). For this work, the idealized radiation scheme introduces two additional degrees of freedom, \( T_{tpp} \) and \( Q_{cool} \), to which the RCE state is sensitive. Thus, we initialize each axisymmetric simulation with the RCE solution from the corresponding three-dimensional simulation on a 196 x 196 km\(^2\) domain with identical \( T_{sst} \), \( T_{tpp} \), \( Q_{cool} \), and \( u_{sfc} \); the RCE state is indeed found to be nearly insensitive to \( f \) (not shown) and thus it is held constant at its Control value to reduce computational load. This domain size is specifically chosen to be large enough to permit a large number of updrafts but small enough to inhibit convective self-aggregation (Bretherton et al., 2005) over a period of at least 100 days. The RCE solution is defined as the 30-day time- and horizontal-mean vertical profiles of potential temperature and water vapor, with the threshold for equilibrium defined as \( \frac{\partial \theta}{\partial t} < \frac{1}{30} K \, day^{-1} \) over the equilibrium period at all model levels; in most cases, this period corresponds to simulation days 70-100, though in a few cases (primarily those with low radiative cooling rates for which equilibration is slow) the simulation is extended until the equilibrium criterion is met. Overall, this approach ensures that each axisymmetric simulation begins very close to its "natural" model-equilibrated
background state (first emphasized in Rotunno and Emanuel (1987)) and thus is absent any significant stores of available potential energy that may exist by imposing an alternate initial state, such as a mean tropical sounding.

The result of the above methodology is a model RCE atmosphere comprised of a troposphere capped by a nearly isothermal stratosphere at temperature $T_{pp}$. More generally, this model tropical atmosphere may be thought of as an extension of the classical fluid system in which a fluid is heated from below and cooled from above (albeit throughout the column), but with two key modifications: 1) the energy input into the system is dependent on wind-speed, thereby permitting a wind-induced surface heat exchange (WISHE; Emanuel (1986)) feedback; and 2) the energy lost from the system is dependent on an externally-defined temperature threshold, $T_{pp}$, which conveniently corresponds to the convective outflow temperature central to the maximum potential intensity theory of tropical cyclones. Both modifications facilitate a more straightforward methodology and analysis of the factors that modulate equilibrium storm size and structure.

### 3.2.3 Initial perturbation

Bister and Emanuel (1997) demonstrated that the fundamental process during tropical cyclogenesis is the near-saturation of the column at the mesoscale in the core of the nascent storm. Thus, we superpose an initial perturbation upon the background RCE state by saturating the air at constant virtual temperature in a region above the boundary layer bounded by $z = [1.5, 9.375] \text{ km}$ and $r = (0, r_0)$ within a quiescent environment. We also test an initial mid-level vortex of the form used in Rotunno and Emanuel (1987), characterized by a radius of vanishing wind $r_0$ and a peak wind of $V_{m_0} = 12.5 \text{ m s}^{-1}$ at $r_{m_0} = r_0/5$, centered at $z = 4.375 \text{ km}$ with azimuthal wind speeds above and below decaying linearly to zero over a distance of 2.875 km. However, as is shown in Fig. 3-7, the two approaches have similar results, and thus for the sake of simplicity we elect to initialize all other simulations with the mid-level moisture anomaly. In addition to this initial disturbance, random perturbations with magnitudes uniformly distributed on the range $[-1, 1] \text{ K}$ are added to the potential
temperature field at every point to break the initial horizontal symmetry of the model.

3.2.4 Control simulation parameter values

For the Control simulation, values of the key external parameters for the model, environment, and initial condition are provided in Table 3.1. The values of the horizontal and vertical mixing lengths, $l_h$, and $l_v$, respectively, used in the Smagorinsky-type parameterization of three-dimensional turbulence are typical values taken from the literature (Bryan and Rotunno, 2009a). The corresponding initial three-dimensional RCE vertical profile of potential temperature and water vapor is displayed in Figure 3-1.

The domain size for the Control run requires special attention. Prior research modeling tropical cyclones typically place the outer wall of the domain at a distance of 1000-1500 km (e.g. Rotunno and Emanuel (1987); Hakim (2011)). However, as shown

Figure 3-1: Initial three-dimensional radiative-convective equilibrium vertical profile of temperature (red dashed), potential temperature (red solid), and water vapor mixing ratio (blue) for the Control simulation.
Table 3.1: Parameter values for the Control simulation. The parameters $l_h$ and $l_v$ correspond to the horizontal and vertical mixing lengths, respectively, in the turbulence parameterization; $H_{domain}$ is the height of the model lid; $L_{domain}$ is the radius of the outer wall in the axisymmetric model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Value</th>
<th>Environment</th>
<th>Value</th>
<th>Initial Perturbation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_h$</td>
<td>1500 m</td>
<td>$T_{set}$</td>
<td>300 K</td>
<td>$r_{0_w}$</td>
<td>200 km</td>
</tr>
<tr>
<td>$l_v$</td>
<td>100 m</td>
<td>$T_{typ}$</td>
<td>200 K</td>
<td>$r_{0_w}$</td>
<td>400 km</td>
</tr>
<tr>
<td>$C_k, C_d$</td>
<td>.0015</td>
<td>$Q_{cool}$</td>
<td>1 K day$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{domain}$</td>
<td>25 km</td>
<td>$u_{sfc}$</td>
<td>3 m s$^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{domain}$</td>
<td>12288 km</td>
<td>$f$</td>
<td>$5 \times 10^{-5}$ s$^{-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 3-2, which depicts the day 100-150 mean radial profile of the azimuthal component of the gradient wind at $z = 1.56$ km, storm size is dramatically influenced by the radius of the outer wall up to an upper bound; the storm seems content to simply fit into the box into which it is placed. Beyond this upper bound, the equilibrium storm is largely insensitive to the location of the wall. The theoretical basis underlying the existence of this upper bound is discussed below.

Thus, because the outer wall is purely a model artifact, we set it conservatively at $L_{domain} = 12288$ km for all simulations run herein. This has the added benefit of ensuring that the storm itself is not significantly altering the background environment, which could modify the potential intensity from its RCE value.

3.2.5 Characterizing equilibrium storm structure

All simulations are run for 150 days in order to allow sufficient time for the full tropical cyclone structure to reach statistical equilibrium, and data is output at 6-hour intervals. We then calculate a 2-day running mean of the radial profile of the azimuthal gradient wind at $z = 1.56$ km to reduce noise in the pressure field. Results are not sensitive to the output frequency nor the averaging period length. We calculate the gradient wind, $V_g$, directly from model prognostic variables based on gradient wind balance:

$$V_g = -\frac{1}{2}fr + \left(\frac{1}{4}f^2r^2 + rC_p\theta_v\frac{\partial \pi}{\partial r}\right)^{\frac{1}{2}}$$

(3.4)
Figure 3-2: Time-mean radial gradient wind profiles at $z = 1.56\ km$ for days 100-150 as a function of domain width. Note the convergence in storm size beyond $L_{\text{domain}} \approx 3000\ \text{km}$.

where $r$ is radius, $C_p$ is the specific heat of air at constant pressure, $\theta_v$ is the virtual potential temperature, and $\pi$ is the Exner function. The pitfalls of using the full $\pi$ (i.e. including contributions from both the balanced and unbalanced flow) for calculating $V_g$ are discussed in Bryan and Rotunno (2009a). The equilibrium radial wind profile is defined as the time-mean of the 30-day period after day 60 with the minimum time-variance in the maximum gradient wind speed, $V_m$. For two cases ($f = 10^{-4}\ \text{s}^{-1}$, $l_h = 750\ \text{m}$), the equilibrium period was adjusted manually to account for ongoing structural variability. Though simplistic, this definition provides a clean signal in many of the details discussed below. A dynamic equilibrium period is preferable to a static one (e.g. day 70-100 mean) to account for simulations that exhibit significant long-period variability in storm structure.

Following the theory presented in Emanuel and Rotunno (2011), we would ideally characterize the structure of the tropical cyclone wind field near the top of the
boundary layer with three variables: the maximum gradient wind speed, $V_m$, the radius of maximum gradient wind, $r_m$, and the outer radius of vanishing wind, $r_0$. However, variability in the radial profile of the gradient wind in the eyewall, which typically (though not always) exhibits a double-humped structure due to the existence of super-gradient flow (see Bryan and Rotunno (2009a) for discussion), renders $r_m$ noisy. Thus, as a proxy we will track the radius of 75% of $V_m$ outside of the eyewall, hereafter denoted $r_{ew}$, which is more stable and typically scales closely with $r_m$.

Meanwhile, direct calculation of $r_0$ is problematic due to the large variability toward the outer edge of the model storm ($V \leq 0.1 V_m$) and correspondingly large sensitivity of the precise value of $r_0$ to this variability. Instead, we employ the outer wind structure model derived in Emanuel (2004) to represent the outer portion of the storm circulation and to estimate $r_0$. This outer wind model assumes that the flow is steady, axisymmetric, and absent deep convection, resulting in a local balance between subsidence warming and radiative cooling. Furthermore, the equilibrium radiative subsidence velocity, $w_{cool}$, can be taken to be approximately constant with radius. In equilibrium, this subsidence rate must match the rate of Ekman suction-induced entrainment of free tropospheric air into the boundary layer in order to prevent the creation of large vertical temperature gradients across the top of the boundary layer. The radial profile of azimuthal velocity is therefore determined as that which provides the required Ekman suction and is governed by the following differential equation

$$\frac{\partial (rV)}{\partial r} = \frac{2r^2 C_d V^2}{w_{cool}(r_0^2 - r^2)} - fr$$

where $r$ is the radius and $V$ is the azimuthal wind speed. Eq. (3.5) is a Riccati equation with no known analytical solution. The value of $w_{cool}$ is calculated from the assumed balance between subsidence-induced warming and radiative cooling

$$w_{cool} \frac{\partial \theta}{\partial z} = Q_{cool}$$

where $\frac{\partial \theta}{\partial z}$ is set to its pressure-weighted mean value in the layer $z = 1.5 - 5 \text{ km}$ (i.e. directly above the boundary layer) for the background state (see Section 3.2.8).
For the Control run, this gives \( w_{\text{cool}} = 0.25 \text{ cms}^{-1} \), which agrees well with the value of 0.24 obtained by calculating the mean (negative) vertical velocity in the region \( r = [400, 600] \text{ km} \) and \( z = [1.5, 5] \text{ km} \) from the equilibrium state of the Control simulation. Finally, given a single point in r-V space, (3.5) may be solved numerically for \( r_0 \) using a shooting method. Details of the application of this analytical model are reprised in a later section.

The equilibrium maximum gradient wind speed, \( V_m \), is defined as the time-mean of its 2-day running mean value over the equilibrium period in order to account for shifts in \( r_m \) that would act to smooth out \( V_m \), though the difference between this value and the simple time-mean value is typically small (\( \leq 1\% \)). Meanwhile, equilibrium values of the two size variables, \( r_{ew} \) and \( r_0 \), are calculated directly from the final equilibrium radial profile.

### 3.2.6 Experimental approach: parametric sensitivities and dimensional analysis

We begin by running a Control simulation whose parameter values are given above and the evolution of which is discussed below. We then perform a wide range of experiments in which we independently and systematically vary all eight external dimensional parameters that are potentially relevant to the dynamics of the system: \( T_{\text{sst}}, \ T_{\text{tp}}, \ Q_{\text{cool}}, \ u_{\text{sfc}}, \ l_h, \ l_v, \ f, \) and \( r_0 \). For each of \( l_h, \ l_v, \ f, \ r_0 \), we run simulations successively halving and doubling from the Control value, while for the four thermodynamic parameters we run simulations each varying one parameter from Control as follows: \( T_{\text{sst}} = 285, 287.5, 290, 292.5, 295, 300, 305, 310 \text{ K}; \ T_{\text{tp}} = 238, 225, 213, 200, 188, 175, 163, 150 \text{ K}; \ u_{\text{sfc}} = 10, 5, 4, 3, 2, 1, 0.5 \text{ ms}^{-1}; \) and \( Q_{\text{cool}} = 0.25, 0.375, 0.5, 0.75, 1, 1.5, 2, 3, 4 \text{ K day}^{-1} \). These ranges, listed in order of increasing \( V_p \), span a reasonable range of values of \( V_p \) from 50 – 150 \text{ ms}^{-1}.

Some important modifications are made to accommodate the wide range of simulations presented here. The domain height is increased by 5 km in cases where the troposphere is deeper than Control to ensure that the upper damping layer lies suffi-
ciently far above the tropopause. For the above sensitivity experiments in which the equilibrium radius of maximum wind is less than the Control value, the simulation is re-run at doubled horizontal resolution (i.e. $dx = 2\,\text{km}$, $L_{\text{domain}} = 6144\,\text{km domain}$) to ensure that the inner storm core is comparably resolved. Lastly, the time step is halved in cases where the CFL condition is violated.

The final scaling results indicate to which dimensional variables the equilibrium storm structure is systematically sensitive. Dimensional analysis is then applied to synthesize the results in a non-dimensional framework.

### 3.2.7 Potential Intensity in RCE

The architecture of this model RCE state enables the equation for the maximum potential intensity to be reformulated in a useful manner. The generalized potential intensity (Emanuel, 2010) is given by

$$V_p^2 = \frac{C_k T_{\text{sst}} - T_{\text{tpp}}}{C_d T_{\text{tpp}}} (k^*_0 - k) \quad (3.7)$$

Combining (3.7) with the surface enthalpy flux equation in (3.2) gives

$$V_p^2 = \frac{T_{\text{sst}} - T_{\text{tpp}}}{T_{\text{tpp}}} \frac{F_k}{\rho C_d |\mathbf{u}|} \quad (3.8)$$

In RCE, column energy balance requires that the surface enthalpy flux into the column be exactly balanced by the column-integrated radiative cooling, which in this idealized set-up is given by

$$F_k = -\int_{p_s}^{p_0} C_p \frac{\partial T}{\partial t} \frac{dp}{g} = -\int_{p_s}^{p_0} C_p \frac{\partial \theta}{\partial t} \left( \frac{p}{p_0} \right)^{R_d/C_p} \frac{dp}{g} = C_p Q_{\text{cool}} \frac{\overline{\Delta p}}{g} \quad (3.9)$$

where $C_p$ is the specific heat of air, $\overline{\Delta p}$ given by

$$\overline{\Delta p} = \frac{p_0}{1 + \frac{R_d}{C_p}} \left( \left( \frac{p_s}{p_0} \right)^{1+\frac{R_d}{C_p}} - \left( \frac{p_{\text{tpp}}}{p_0} \right)^{1+\frac{R_d}{C_p}} \right) \quad (3.10)$$
is the mean pressure depth of the troposphere, reduced slightly by the adiabatic expansion term in the integrand of (3.9), and we have ignored any small contribution from Newtonian relaxation in the stratosphere. Substituting (3.9) into (3.8) results in

\[ V^2_p = \frac{T_{sst} - T_{tpp} C_p Q_{cool} \overline{\Delta p}}{T_{tpp} g \rho C_d |u|} \] (3.11)

Thus, (3.11) makes it readily apparent that the potential intensity in RCE with constant tropospheric cooling is a function of four externally-defined parameters: \( T_{sst}, T_{tpp}, u_{sfc}, \) and \( Q_{cool} \), with the tropospheric thickness \( \Delta p \) primarily a function of \( T_{tpp} \). Note that \( |u| \) represents the mean background wind speed, including both the resolved mean wind speed and the gustiness, \( u_{sfc} \); because the TC occupies only a small areal fraction of the very large domain, its contribution to the mean wind is small.

This analytical result will be leveraged below, though all values of potential intensity presented herein are calculated from the background state sounding (defined in the subsequent section) using the detailed Emanuel sub-routine (Bister and Emanuel, 2002) with zero boundary layer wind speed reduction under pseudo-adiabatic thermodynamics and including dissipative heating.

### 3.2.8 Defining the background state

Though we initialize each axisymmetric simulation with the three-dimensional RCE state, ultimately the more relevant background state for the equilibrium tropical cyclone is that of the ambient environment beyond the storm circulation in the axisymmetric model itself. Thus, we define the background state as the area-weighted mean vertical profile of potential temperature and water vapor averaged over the radial grid points 2000-2500, which corresponds to the region \( r = [8000, 10000] \ km \) for our Control domain size. This quantity is largely insensitive to radius or averaging time period so long as it is calculated beyond the primary storm circulation. From this background state, we may calculate relevant quantities for our analysis, including the potential intensity, radiative-subsidence rate, and deformation radius.
Figure 3-3: Comparison of the potential intensity, $V_p$, calculated from the initial three-dimensional RCE state and the final axisymmetric RCE state outside of the storm across simulation sets varying each of the four governing thermodynamic parameters.

The potential intensity for the axisymmetric RCE state is typically 80-90% of the value of the corresponding three-dimensional RCE state, though they do not differ precisely by a constant factor across simulations. Figure 3-3 displays the fractional reduction of $V_p$ in axisymmetry relative to its three-dimensional counterpart across the simulation sets varying each of the four governing thermodynamic parameters. As in the three-dimensional case, the axisymmetric $V_p$ is predominantly a function of these thermodynamic parameters. In the cases of varying $T_{tp}$ and $u_{sfc}$, there is systematic variation in this fractional reduction, such that this reduction increases with increasing $V_p$. Meanwhile, this quantity does not vary significantly with $l_h$, $f$, or domain size, suggesting that the difference in $V_p$ between axisymmetry and three-dimensions is not attributable to the existence of the storm itself (i.e. the relative contribution of the storm circulation to the domain-mean resolved near-surface wind) but rather is related to the differing nature of convection in the two geometries. This
is curious and warrants further investigation.

3.3 Results

3.3.1 Control run

Figure 3-4 displays the time evolution of the 2-day running mean of $V_m$, $r_m$, and an estimate of the outer radius, $r_0$ (calculated as in Section 3.3.4), for the Control simulation as well as estimated time-scales to equilibrium for each individual variable. The time-scale to equilibrium, $\tau_x$, where $x$ is the variable of interest, is defined as the starting time of the first 30-day interval whose mean value is within 10% of the equilibrium value and whose average daily rate of change over this period does not exceed 1% of the mean. All three variables exhibit similar qualitative evolutions: rapid increase during genesis to a super-equilibrium value followed by a more gradual decay to equilibrium. However, the maximum excess over equilibrium is very large for $r_0$ and $r_m$ (\(~100\%) and relatively small (\(~30\%\)) for $V_m$, the latter of which matches the overshoot value found in Hakim (2011) for the same radial turbulent mixing length. Moreover, the time-scales to equilibrium for storm size are significantly longer for size ($\tau_{r_m} = 70 \text{ days}$ and $\tau_{r_0} = 61 \text{ days}$) than for intensity ($\tau_V = 29 \text{ days}$). The details of the transient phase of the structural evolution will be explored in future work. Ultimately, the Control simulation's equilibrium storm structure is characterized by $V_m^{eq} = 70 \text{ ms}^{-1}$, $r_m^{eq} = 46 \text{ km}$, $r_0^{eq} = 694 \text{ km}$. Importantly, the Control case exhibits non-negligible long-period variability of \(~20\%\) about the estimated equilibrium value, leaving some ambiguity regarding the precise values for each structural variable at equilibrium.

These results suggest that modeling tropical cyclones over a period sufficient to achieve quasi-equilibrium in intensity (typically 10-20 days), as is commonly done in the literature, may result in a storm that has not reached structural equilibrium or else has done so artificially due to the domain-limitation imposed by the model's outer wall.
Figure 3-4: For the Control simulation, time evolution of the 2-day running mean $V_m$, $r_m$, and $r_0$ normalized by their respective equilibrium values (upper-right corner). For this simulation, $V_p = 79 \text{ m s}^{-1}$ and $f = 5 \times 10^{-5} \text{ s}^{-1}$. Pink line denotes 30-day period used for equilibrium calculation, and black dashed lines denote $\pm 10\%$ of the equilibrium value. Markers along the abscissa denote estimated time-scales to equilibration.

### 3.3.2 Radial profile sensitivity tests

The principal objective is to collapse the radial wind profiles across all simulations to a single curve based on external parameters alone. Thus, we begin simply with the dimensional radial gradient wind profiles for eight simulation sets, each of which correspond to one of the eight external dimensional parameters, displayed in Figure 3-5, in order to highlight a few basic but important features. First, both storm intensity and inner-core size (e.g. $r_m$) increase with increasing potential intensity across all four thermodynamic parameters (panels 1-4). Second, storm size decreases with increasing $f$ and increases with increasing $l_h$, the latter primarily only within the inner core, while storm intensity decreases with increasing $f$ and $l_h$. Detailed analysis of the
effects of the horizontal mixing length is found in Bryan and Rotunno (2009b) and Rotunno and Bryan (2012). Third, the equilibrium storm forgets the initial condition, $r_{0_2}$ (panel 8), with an identical result for an initial mid-level vortex (not shown; see Figure 3-7 for scalings). Finally, storm intensity and overall size are not systematically sensitive to the vertical mixing length, $l_v$ (panel 7), which corroborates the results of Bryan and Rotunno (2009b) and Rotunno and Bryan (2012); larger vertical mixing length magnitudes do correspond to a slow expansion of the eye at the apparent expense of the eyewall, though its overall effect remains small relative to that of $l_h$, so long as $l_v$ is much smaller than the depth of the troposphere as is easily the case for the range of plausible values. A much deeper discussion of the role of $l_v$ in the boundary layer in the broader context of classical vortex flow solutions with frictional boundary layers is discussed in Rotunno and Bryan (2012). Thus, based upon these results, we hereafter elect to neglect the effects of both the initial condition and the vertical mixing length, leaving only six external dimensional parameters.

Given the structural similarity apparent in the dimensional curves in Figure 3-5, we propose to normalize $V$ by $V_m$ and $r$ by $r_{ew}$ (the radius of 75% of the maximum wind); the result is shown in Figure 3-6. Remarkably, this single normalization removes a large majority of the variability in each case and, conveniently, separates any residual variability between the inner core region and the outer circulation. In effect, Figure 3-6 provides a road map for analysis, beginning first and foremost with the relationship between the internal variables $V_m$ and $r_{ew}$ and our external dimensional parameters, followed by an exploration of the residual variability in the eye, eyewall, and outer region of the storm.

Based on Eq. (3.11) and the common scaling of both intensity and size with $V_p$, we hypothesize that the primary role of the dimensional parameters $T_{sat}$, $T_{tpp}$, $Q_{cool}$, and $u_{sfc}$ is to modulate the potential intensity, $V_p$. From among the four thermodynamic external parameters, the tropopause temperature is the simplest theoretically, such that its variability should affect only the potential intensity and the depth of the troposphere, $H$. It will also slightly modulate the column-integrated radiative cooling, but due to the exponential decay in density with height, the mass of the troposphere

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Figure 3-5: Equilibrium radial profiles of the gradient wind for simulation sets in which each of the eight dimensional external parameters is varied. The top four panels correspond to the four thermodynamic parameters, for which shading reflects potential intensity from low (light grey) to high (black); the bottom four panels correspond to relevant dynamic parameters, for which shading reflects parameter magnitude from low (light grey) to high (black).

varies by $\lesssim 15\%$ over the range of tropopause temperatures explored here. Given that $H$ is not expected to be relevant to the dynamics of the system so long as $\frac{L}{H} \ll 1$ as noted earlier, we argue that $T_{tpp}$ represents the “base” case that isolates the variability in storm structure due strictly to variations in $V_p$. We focus first on this base case,
Figure 3-6: As in Figure 3-5, but with radial profiles normalized as follows: \( V \) by \( V_m \) and \( r \) by \( r_{ew} \). Only those six parameters exhibiting strong structural sensitivity are shown.

\( V_p(T_{pp}) \), before proceeding to analysis of the other three parameters, which may have additional effects on the system superimposed upon that associated with \( V_p \).

### 3.3.3 Base case: Inner core

Figure 3-7 displays the scaling of \( V_m \) and \( r_{ew} \) with the set of relevant input physical parameters. Both structural variables exhibit systematic sensitivity to three param-
eters: $V_p(T_{tpo})$, $f$, and $l_h$, with minimal sensitivity to the other parameters as noted above.

Figure 3-7: Scaling of the equilibrium value of $V_m$ (top) and $r_{ew}$ (bottom) with relevant dimensional parameters, $X$, normalized by their respective Control values (abscissa). Parameters to which a structural variable exhibits systematic sensitivity are plotted in solid black.

Rather than analyzing the role of each parameter independently, though, we may synthesize the results quantitatively via dimensional analysis. The Buckingham-Pi theorem states that the number of independent non-dimensional parameters in a
dimensional system is equal to the difference between the number of independent di-
mensional parameters and the number of fundamental measures. For our purposes, we
have three relevant dimensional parameters and two fundamental measures, distance
and time, thereby giving only one independent non-dimensional parameter, hereafter
$C_1$. Any output non-dimensionalized quantity, $Y$, can be expressed as a function of
the set of non-dimensional parameters. For our system, the result is

$$Y = f(C_1)$$

(3.12)

The form of this functional relationship can only be determined by experimentation.

Thus, we define the dominant non-dimensional number in this system as

$$C_1 = \frac{V_p}{fl_h}$$

(3.13)

We choose to non-dimensionalize $V_m$ by $V_p$ and $r_{ew}$ by $V_p/f$.

The scalings between the two non-dimensional structural variables and $C_1$ for a
suite of experiments varying one or more of $V_p$, $f$, or $l_h$ are displayed in Figure 3-8;
parameters for the set of experiments are given in Table 3.2. A linear relation in log-
log space corresponds to a power-law scaling whose exponent is given by the linear
slope, i.e.

$$Y = C_1^\alpha$$

(3.14)

The linearly-regressed slopes are also given in Figure 3-8. In the case of $r_{ew}$, the power
law indeed provides the best statistical fit. In the case of $V_m$, though, the log-log plot
exhibits slight negative curvature, particularly towards low values of $C_1$, indicating
that a logarithmic relationship, $Y \sim \beta \times log_{10}(C_1)$, provides a slightly better fit; this
regression with $\beta = .37$ is plotted as well (dash-dot line). Though statistically slightly
less precise, the power law relationship is much more amenable to theoretical physical
insight. The resulting non-dimensional power-law relationships are given by

$$\frac{V_m}{V_p} \sim \left(\frac{V_p}{fl_h}\right)^{27}$$

(3.15a)
Figure 3-8: Scaling of the equilibrium values of the non-dimensionalized structural variable $V_m$ (top), $r_{ew}$ (bottom) with the non-dimensional number $C = \frac{V_p}{\bar{f}t_h}$. Best-fit linear regressions plotted (dash), whose linearly-regressed slopes, corresponding to the estimated power-law scaling exponent in (3.14), and associated 95% confidence intervals listed (parentheses) and r-square values adjusted to account for the number of estimators (top-left corner). For $V_m$, a logarithmic regression is also shown (dash-dot). Grey fill highlights those simulations for which $V_p(T_{pp})$ alone is modulated. Grey bars indicate the full range of variability of the 30-day running mean after day 60.
\[
\frac{r_{ew}}{V_p} \sim \left( \frac{V_p}{fl_h} \right)^{-0.55}
\]

We may then solve (3.15) for the corresponding dimensional scalings:

\[
V_m \sim V_p^{1.27} (fl_h)^{-0.27}
\]

\[
r_m \sim \left( \frac{V_p}{f} \right)^{0.45} (l_h)^{0.55}
\]

Thus, equilibrium storm intensity is found to scale super-linearly with the potential intensity and, more weakly, inversely with both the background rotation rate and the radial turbulent mixing length. The equilibrium \( r_{ew} \), which scales closely with the radius of maximum gradient wind, is found to scale approximately as the geometric mean of the ratio of the potential intensity to the Coriolis parameter and the radial turbulent mixing length, weighted slightly towards the latter. Note that the direct non-dimensional scaling for \( r_m \) has an exponent of \( \alpha = -0.52 \) and \( r_{adj}^2 = 0.84 \), both statistically indistinguishable from \( r_{ew} \) at the 95% confidence level.

Curiously, the power dissipation (Emanuel, 2005) follows the scaling

\[
PDI \sim V_{m}^{3.42} \sim V_p^{4.7} f^{-1.7} l_h^3
\]

which exhibits only a very weak dependence on \( l_h \).

### 3.3.4 Base case: Outer wind field

We may now quantify the scaling of the overall storm size. We reiterate that \( r_0 \) is difficult to extract directly from numerical model output, and thus elect to use the analytical outer wind model of Emanuel (2004) to represent the outer circulation. Following the above non-dimensional scaling results, we first non-dimensionalize \( V \) by \( V_p \) and \( r \) by \( \frac{V_p}{f} \) in Eq. (3.5), giving

\[
\frac{\partial (\tilde{r} \tilde{V})}{\partial \tilde{r}} = \frac{C_d V_p}{w_{cool}} \frac{2 \tilde{r}^2 \tilde{V}^2}{(r_0^2 - \tilde{r}^2)} - \tilde{r}
\]
Table 3.2: Parameter values for each simulation used to test the scaling relationships associated with Eq. (3.12), where $C_1 = \frac{V_p}{f l_h}$. Control values are listed in Table 3.1.

<table>
<thead>
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<th>$f \times 10^{-5} \text{s}^{-1}$</th>
<th>$l_h \text{[m]}$</th>
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where tildes denote non-dimensional quantities. Chavas and Emanuel (2010) employed this model to estimate $r_0$ in observations by fitting the model to the radius of 12 $\text{ms}^{-1}$. Here we find that Eq. (3.18) can credibly reproduce the entire equilibrium radial wind profile outside of the eyewall for many simulations with a simple empirical
modification of the first term on the RHS of (3.18) by a constant factor, taken here to be $c = .3$. As an example, Figure 3-9 depicts the Control simulation equilibrium profile compared against Eq. (3.18) fit without and with this modification (i.e. $c = 1$ and $c = .3$, respectively). To fit this analytical model, we begin at $r_{ew} = r(.75V_m)$ from the equilibrium radial wind profile and integrate Eq. (3.18), with the first term on the RHS multiplied by $c = .3$, outwards to $r_0$. Remarkably, the empirically-modified Eq. (3.18) captures nearly the entire equilibrium radial wind profile beyond $r_m$. This empirical fit across our simulation sets is explored in the next section.

**Outer wind field model fit**

The fit of Eq. (3.18) to the equilibrium radial wind profile in the outer region of the storm can be improved significantly by multiplying the first term on the RHS of Eq. (3.18) by a constant. Figure 3-10 shows a histogram of the optimal constant, $c$, for
Table 3.3: Optimized $c$ for each simulation (see text for details); corresponding histogram is plotted in Figure 3-10. Asterisk denotes a likely outlier.

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<th>$u_{sfc}$</th>
<th>$Q_{cool}$</th>
<th>$T_{pp}$</th>
<th>$l_h$</th>
<th>$f (\times 10^{-5})$</th>
<th>$c$</th>
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<td>.26</td>
<td>.75</td>
<td>.2</td>
<td>3000</td>
</tr>
<tr>
<td>295</td>
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<td>2</td>
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<td>1</td>
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<td>300</td>
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<td>305</td>
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<td>310</td>
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<td>3</td>
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<td>4</td>
<td>10.0*</td>
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</table>

Each simulation, and the optimal values are provided in Table 3.3. Optimal values are obtained by minimizing the mean square error within the region $r(0.1V_m < V < 0.75V_m)$ over the range $c = [0.1, 10]$. The median is $c = 0.26$, and most simulations lie in the range $c = 0.2 - 0.4$, skewed slightly towards higher values. For varying $T_{pp}$, all cases are tightly clustered at $c = 0.25 - 0.27$, with the exception of $T_{pp} = 238 \, K$, which is likely an outlier. Thus, for this work we choose $c = 0.3$.

Taking $c = 0.3$, Figure 3-11 displays the radial profile of the error, defined as $V_{E04} - V_{CM1}$, for all simulations varying each of the six relevant dimensional parameters. Mean absolute errors are less than 2 $ms^{-1}$ across most simulations. The most significant deviation occurs for $Q_{cool}$, which exhibits a systematic trend in mean error that reflects an overestimation of the wind field at low cooling rates and an underestimation at high cooling rates, indicating that the sensitivity to $w_{cool}$ is not as strong in the numerical model as would be predicted by Eq. (3.18). This behavior is also reflected in the steady increase in the optimal value of $c$ in Table 3.3 for radiative cooling rates of $0.375 - 1.5 \, K \, day^{-1}$. More precisely, increasing $Q_{cool}$ (and thus $w_{cool}$) by a factor of 4 over this range corresponds approximately to a doubling in $c$, suggesting that the sensitivity of the true radial wind profile to the radiative-subsidence rate is overestimated by a factor of two. At very high radiative cooling rates, convection progressively increases beyond the eyewall region such that the entire wind field ex-
Figure 3-10: Histogram of optimized constant, $c$, applied to first term on the RHS of Eq. (3.18) across simulation sets (color). Control simulation is in black. The values are calculated by fitting Eq. (3.18) to $r(.75V_m)$ and then minimizing the mean square error in the region $r(.1V_m < V < .75V_m)$. Values are tested over the range $c = [.1, 10]$. The median is $c = .26$.

...pands significantly and the analytical model provides a poor fit due to the significant mismatch in the vicinity of $r_{ew}$.

Nonetheless, the broad success of this simple empirical modification indicates that this analytical model, despite its simplicity and many documented deficiencies in the inner core of a TC (Smith and Montgomery, 2008; Persing and Montgomery, 2003;
Figure 3-11: Radial profiles of error in the fit of the analytical outer wind model (Eq. (3.18), empirically modified with \( c = .3 \)) to the equilibrium radial wind profile, defined as \( V_{EO4} - V_{CM1} \), for all simulations varying each of the six relevant dimensional parameters. Error profiles are smoothed with a 10-pt smoother. In the top four panels, shading reflects potential intensity from low (light grey) to high (black); in the bottom two panels, shading reflects parameter magnitude from low (light grey) to high (black). Analytical model is fit to \( r(0.75V_m) \), with the range \( r(0.75V_m < V < .1V_m) \) solid and \( r(V < .1V_m) \) dashed; radii are normalized by \( r_0 \) as calculated from the outer wind model given by Eq. (3.18). Red line depicts mean error over the inner range, and the corresponding mean absolute error (MAE) for the simulation set is listed in the top left corner.
Smith and Vogl, 2008), likely captures the essential physics of the equilibrated, non-convecting outer circulation at least within the idealized approach employed here. Ultimately, this simple slab boundary layer model, which is derived from a balance between the net convergence of angular momentum by the radial wind in the boundary layer and its frictional sink at the surface, assumes that all quantities, such as $V$ and $u\frac{\partial M}{\partial r}$, are constant with height below $z = 1.56 \ km$. The cumulative effect of violations of these assumptions will manifest itself as a misfit between Eq. (3.18) and the “true” equilibrium wind profile. Because Eq. (3.18) is derived from a simple two-term balance between radial advection of angular momentum and frictional loss of angular momentum at the surface, this misfit may be represented simply by a single multiplicative factor. Why this factor should remain roughly constant both with radius and across many simulations is not obvious, and a preliminary analysis of the model assumptions (not shown) reveals no single, dominant assumption that is consistently violated and from which an improved theoretical model might be developed.

Thus, for our purposes, we elect simply to use $c = .3$, noting that the scaling results are not sensitive to the precise value chosen. Though one may be tempted to optimize the value of $c$ for each individual simulation, such an approach introduces an additional degree of freedom that, absent an underlying theoretical justification, will add additional complexity to the problem with minimal new physical insight. A deeper analysis of the physics behind this empirical modification, and of the validity of this model more generally in the outer non-convecting region of the storm circulation, is an important endeavor for future work.

**Outer radius**

We apply Eq. (3.18) with the aforementioned empirical modification to estimate the outer radius and to explore variability in the outer region of the equilibrium radial wind profiles.

The top panel of Figure 3-12 displays outer radial wind profiles for varying $T_{tpp}$, normalized as in Figure 3-6. Overlaid on top of these radial profiles are the solutions of Eq. (3.18), each of which provide an estimate of the outer radius, $r_0$ (blue dots).
The scaling of $r_0$ with $V_p$ is shown as an inset. As noted earlier, the normalized equilibrium radial wind profiles exhibit a systematic expansion of the far outer wind field with increasing $V_p$, a qualitative behavior that is correctly predicted by Eq. (3.18).

Indeed, Eq. (3.18) is itself modulated by a second non-dimensional parameter, $C_2$, given by

$$ C_2 = \frac{C_1 V_p}{w_{cool}} $$  \hspace{1cm} (3.19)

We may quantify the impact of $C_2$ by simply holding it fixed at its Control value (53) when solving Eq. (3.18); the result is shown in the bottom panel of Figure 3-12. Comparison of the red curves in the top and bottom panels of Figure 3-12 reveals that the effect of $C_2$ manifests itself primarily only at large radii in the far outer region of the storm circulation. Additionally, these new curves provide an estimate of an adjusted outer radius, $r_0^*$ (blue dots, bottom panel), which is analogous to $r_0$ but with $C_2$ fixed at its Control value. The scaling of $r_0^*$ with $V_p$ is shown as an inset. Physically, fixing $C_2$ acts to partially collapse the curves in the far outer region, reducing 80% of the variance in the normalized outer radius.

Though the influence of $C_2$ is minimal at smaller radii where wind speeds are an appreciable fraction of the maximum value, it exerts a significant influence on the precise value of $r_0$. This is of particular importance given that $C_2$ includes a factor $V_p$. As a result, the true $r_0$ is a function of $C_1$ and $C_2$, both of which include variability with $V_p$, one of our critical dimensional parameters.

Thus, Figure 3-13 displays the joint scaling of $r_0$ with $C_1$ and $C_2$ over a wide range of values of each. The values of $r_0$ are calculated beginning with the empirically-derived relationships for $\frac{V_m}{V_p}$ (exponential) and $\frac{r_{ew}}{V_p}$ (power-law) as a function of $C_1$ displayed in Figure 3-8 and given by

$$ \frac{V_m}{V_p} = -.3 + .37 \times \log_{10}(C_1) \hspace{1cm} (3.20a) $$

$$ \frac{r_{ew}}{V_p} = 0.73 \frac{C_1}{C_1^{0.55}} \hspace{1cm} (3.20b) $$
Figure 3-12: As in Figure 3-6 for varying $T_{tip}$, focused on the outer region of the storm. Black curves are the equilibrium radial wind profiles; red curves are solutions of Eq. (3.18) fit directly (top), and fit with $C_dV_p$ held fixed at its Control value (bottom). Blue dots indicate corresponding $r_0$ (top) and $r_0^*$ (bottom), and the corresponding scalings with $V_p$ are shown as insets.

Then, for each $C_1$, Eq. (3.18) is applied to the corresponding $(r_{ew}/r_f, V_{m}/V_p)$ using a range of values of $C_2$. In this way, we exploit the fact that the direct impacts of $C_1$ and $C_2$ are effectively independent in radius, with the former modulating the inner core
of the storm and the latter modulating the outer circulation. The non-dimensional outer radius decreases slowly with increasing $C_1$ and increases more quickly with increasing $C_2$, particularly at high values. Additionally, there is non-linearity in the joint scaling for small $C_1$ and large $C_2$, which is associated with the more rapid decline of the exponential relationship for $\frac{V_p}{V_p}$ (Eq. (3.20a)) at very small $C_1$.

To quantify the variation of non-dimensional $r_0$ with $C_1$ and $C_2$, a simple estimate of the separable power-law scaling can be obtained using multiple linear regression. The result is given by

$$\frac{r_0}{V_p} \sim C_1^{-2} C_2^{33}$$

and is plotted (contours) in Figure 3-13. This statistical fit performs reasonably well except in regions of significant curvature, i.e. for small $C_1$ and large $C_2$ and vice versa. Notably, there is minimal curvature in the neighborhood of the Control simulation.

Additionally, we may probe the scaling with $C_1$ and $C_2$ independently. First, the
scaling of the non-dimensional outer radius with $C_1$ while holding $C_2$ fixed at its Control value, which corresponds to $r_0^*$ in Figure 3-12, is equivalent to the application of dimensional analysis to $r_0^*$ as was done above for $V_m$ and $r_{ew}$. Indeed, the direct estimate of the non-dimensional scaling for $r_0^*$ is shown in Fig. 3-14. The empirically-derived power law scaling exponent is $-0.15$, which closely matches the result from multiple linear regression over the combined $(C_1, C_2)$ parameter space given in Eq. (3.21). The corresponding dimensional scaling is

$$r_0^* \sim \left( \frac{V_p}{f} \right)^{0.85} \left( l_h \right)^{1.15} \quad (3.22)$$

Thus, Eq. (3.22) dictates that, at fixed $C_2$, overall storm size is found to scale nearly linearly with the ratio of the potential intensity to the Coriolis parameter, with a slight expansion for increasing radial turbulent mixing length. This scaling matches the existing axisymmetric theoretical prediction for the scaling of the upper bound
on the size of a tropical cyclone (Emanuel, 1986, 1989, 1995a). This “natural” length scale is \( \sqrt{X_s} \), where \( X_s \) is a velocity scale that is equivalent to the potential intensity with \( C_s = C_d \) and neglecting dissipative heating and the pressure dependence on the saturation vapor pressure of water. As first described in Emanuel (1986), the existence of this theoretical upper bound is most easily understood from the perspective of a Carnot heat engine, in which the work required to restore lost angular momentum in the anticyclone aloft increases with increasing storm size, and by conservation of energy there remains less energy available to overcome frictional dissipation at the surface, i.e. a weaker storm. To the extent that the inclusion of the pressure dependence of saturation vapor pressure and dissipative heating do not alter this fundamental principle, our modeling results appear to confirm this prediction.

Meanwhile, the scaling of the non-dimensional outer radius with \( C_2 \) while holding \( C_1 \) fixed at its Control value represents an expansion of the far outer circulation, whose scaling is \( C_2^{33} \), above and beyond this primary storm scaling associated with \( C_1 \). Importantly, in order to isolate the theoretical scaling of (Emanuel, 1986) in a dimensionally-consistent manner, one must first hold \( C_2 \) constant, as we have done in calculating \( r^* \); this seems reasonable given that the theory is applicable only to the ascending region of the storm and so should not be expected to represent variability in the non-convecting outer circulation. Moreover, the scaling result for \( r^*_0 \) is very similar when applying Eq. (3.18) beginning at \( r(2V_m) \) (scaling exponent of \(-.11\)), as shown in Figure 3-15, indicating that this result is not an artifact of the analytical outer wind model.

### 3.3.5 Physical interpretation

More generally, the non-dimensional parameter, \( C_1 \), represents the ratio of the storm radial length scale, \( \frac{V_m}{f} \), to the parameterized eddy radial length scale, \( l_h \), and thus it is the values of each of these parameters relative to one another, rather than their absolute values, that is fundamental to the structure of the storm. For example, though one would expect \( V_m \) to scale linearly with \( V_p \) all else equal, the super-linearity is a manifestation of the fact that a larger value of \( V_p \) results in a storm that is
more intense and larger. Because radial turbulence acts to reduce radial gradients in scalars such as temperature (and thus gradient azimuthal wind speed, through gradient thermal wind balance) over a distance proportional to the prescribed mixing length, a larger storm at constant $l_h$ implies a reduction in $C_1$, and thus the storm will feel a weaker effective turbulence. Indeed, from (3.14) for constant $C_1$ we do indeed recover the linear scaling $V_m \sim V_p$.

In addition, these findings corroborate prior work demonstrating the importance of radial turbulence in determining inner-core storm structure (Bryan and Rotunno, 2009a; Bryan, 2011; Rotunno and Bryan, 2012). In particular, the strong scaling relationship between $r_m$ and $l_h$ reflects the critical role of radial turbulence in countering eyewall frontogenesis by the secondary circulation that, in the inviscid limit, would lead to frontal collapse (Emanuel, 1997). Meanwhile, the influence of radial turbulence as parameterized here only weakly modifies storm structure near the outer edge of the storm.
Notably, the combination of $f$ and $l_h$ in the denominator suggests that, in the inner core of the non-dimensional system, both variables are dynamically equivalent. This notion appears reasonable given that both variables modulate the eyewall structure (Figure 3-6) in an identical manner such that the two-hump structure is replaced by a single hump as either parameter progressively increases in magnitude.

Meanwhile, $C_2$ represents the reciprocal of the non-dimensional Ekman suction rate in the outer wind region, where the requirement that $w_{Ek} = w_{cool}$ has been imposed. This can be seen more clearly by deriving Eq. (3.18) starting explicitly from the definition of the Ekman suction velocity, given by the divergence of the frictionally-induced inflow, $u$, integrated over the boundary layer depth, $h$,

$$w_{Ek} = \int_0^h \frac{1}{r} \frac{\partial (ru)}{\partial r} \partial z$$

and non-dimensionalizing as above. Combining Eq. (3.23) with $u$ derived from angular momentum balance in the boundary layer leads to an expression for $\frac{w_{Ek}}{C_dV_p}$ that can be rearranged to give Eq. (3.18) (this scaling for $w_{Ek}$ also appears in the traditional Ekman solution for a vertically-uniform boundary layer, which corresponds to this same derivation for Eq. (3.18) but in the limit $rV << \frac{1}{2}fr^2$, i.e. near $r_0$). Physically, decreasing the Ekman suction rate implies through Ekman dynamics a weaker (negative) vorticity and thus a more gradual decay of the radial wind profile. In non-dimensional space, the non-dimensional suction rate can be decreased either by decreasing $w_{cool}$, which by assumption implies a smaller dimensional $w_{Ek}$, or by increasing the scaling factor $C_dV_p$. In this way, $C_2$ governs the rate of decay of the wind profile with radius at large radii in the non-dimensional system.

### 3.3.6 Estimating $l_h$

Given the sensitivity of the equilibrium structure, particularly $r_m$, to the turbulent radial mixing length, an accurate estimation of $l_h$ in the inner core of a real tropical cyclone is important but lacks any theoretical or observational foundation, as it is not a physical parameter that can be determined as a function of physically calcu-
lable natural variables. Bryan and Rotunno (2009b) and Bryan (2011) attempt to estimate its value by tuning it to match the steady-state model intensity to either the theoretical potential intensity or the theoretical maximum gradient wind speed of Emanuel and Rotunno (2011). We note here that the theory of Emanuel and Rotunno (2011) does not include the effect of radial turbulence, so it is not clear whether it is the appropriate quantity against which to tune. Nonetheless, the above results suggest that the more relevant objective is to tune the ratio $\frac{V_{\alpha}}{V_p}$ to the horizontal mixing length non-dimensionalized by the storm scale $\frac{V}{f}$ (i.e. the reciprocal of $C_1$). We thus estimate this parameter value as that which gives the theoretical result from Emanuel and Rotunno (2011) of $\frac{V_{\alpha}}{V_p} = \frac{1}{\sqrt{2}}$ in the case of $\frac{C_b}{C_d} = 1$, as shown shown in Figure 3-16. Given that this exercise favors statistical precision over theoretical insight, we perform this estimation using the logarithmic rather than the power law fit to the data. The resulting best estimate is $\frac{V_{\alpha}}{V_p} = .0017$, or approximately $\frac{1}{690}$ of the storm radial length scale. For our Control values for $V_p$ and $f$, this result translates to $l_h \approx 2700$ m. This value seems reasonable in the context of previous work that finds optimal values in the range 1000 – 1500 m given that those simulations were performed in domains approximately half the size required to avoid influencing storm size (Figure 3-2).

3.3.7 Sensitivity to potential intensity

We now return to the hypothesis that the sensitivity of storm structure to each of the four thermodynamic parameters collapses to a sensitivity to potential intensity. Figure 3-17 displays the respective scalings of $V_m$, $r_{ew}$ and $r_0^*$ with $V_p$. Indeed, across all four parameters there is a systematic, direct scaling with $V_p$ in both intensity and size, with several interesting deviations. Implicitly, any variability above and beyond the scaling with $V_p(T_{pp})$ is necessarily a result of modulation of some other aspect of the system that is correlated with $V_p$. For $V_m$, the slightly super-linear scaling with $V_p$ matches that found for $T_{pp}$ in all cases with the exception of $u_{sfc}$, for which the scaling is more gradual, and at high values of $Q_{cool}$. For $r_{ew}$, the scaling with $V_p$ for both $u_{sfc}$ and $Q_{cool}$ is faster than for $T_{pp}$, and the scaling diverges non-linearly at
Figure 3-16: Estimation of the optimal value of the radial mixing length normalized by $\frac{\nu_e}{f}$ by matching the logarithmic fit to the data (black dash) to the theoretical relationship of $\frac{V_m}{V_p} = \frac{1}{\sqrt{2}}$ (black dash-dot) for $\frac{C_s}{C_d} = 1$ in Emanuel and Rotunno (2011). The best estimate (grey dot) is approximately $\frac{1}{600}$ of the storm scale.

very high radiative cooling rates. Finally, for $r_0^*$ (the outer radius at fixed $C_2$), the scalings largely collapse with the exception of high radiative cooling rates as was seen for $r_{ew}$. The two cases with coldest $T_{sst}$ (285, 287.5 K) do not conform well to the overall scaling but instead are weaker and smaller than expected given their potential intensities, though these simulations exhibit significant ongoing variability during the post-equilibration period.

For $r_0^*$, we may also apply our analytical model beginning at much larger radii to test the sensitivity of the scaling result to the use of (3.18). Figure 3-18 displays the scaling for $r_0^*$ where we apply (3.18) beginning at $r(.2V_m)$. The result is very similar to the original result, reflecting the fact that (3.18) does a reasonable job representing the radial wind profile radially-inwards of $r(.2V_m)$ (Figure 3-11)

Additionally, we may calculate $r_0^*$ using the optimized values of $c$ specific to each
Figure 3-17: Scaling of the respective equilibrium values of $V_m$, $r_{ew}$, and $r_0^*$ (ordinate) with $V_p$ (abscissa) for the four thermodynamic parameters. Dashed line indicates best fit to all data. For each individual parameter, best fit linear slope (95% CI) and corresponding adjusted r-square statistic listed in lower right corner. Grey bars indicate the full range of variability of the 30-day running mean after day 60.
Figure 3-18: As in Fig. 3-17 but with $r_0^*$ calculated by applying (3.18) to the numerical equilibrium solution beginning at $r(.2V_m)$.

simulation, as shown in Figure 3-19. The overall scaling with $V_p$ is preserved, though the scaling appears to be more rapid for $u_{sfc}$ and $Q_{cool}$ than in the original case. However, it is important to note that variations in $c$ are mathematically equivalent to variations in $C_2$, and therefore $r_0^*$ is no longer a function solely of $C_1$. It is likely that variations in $c$ actually reflect variations in the true dependence on the factors that comprise $C_2$ (or some other, more complex set of factors) that may not be properly captured by such a simple model as (3.18) and thus for which we cannot control, except by setting the product $c \times C_2$ constant at the Control value as we have done in Figures 3-17 and 3-18. Moreover, the data overall may be noisier (e.g. the excluded outlier at the warmest value of $T_{cpp}$), as the estimate of $c$ will be sensitive to long-period variability in storm structure at large radii approaching $r(.1V_m)$; subsequently, $r_0$ is quite sensitive to $c$. This is again an indication that a more complete understanding of the validity of (3.18) in the outer region would be highly beneficial for determining the precise nature of the variability far outer wind field. Nonetheless, the consistent signal of the scaling of overall storm size with $V_p$ is insensitive to these details.

In combination, these results indicate that storm structure scales predominantly with $V_p$, though the inner core of the storm has a tendency to expand more rapidly for
increasing $Q_{\text{cool}}$ and $u_{\text{sfc}}$ than what is expected from increasing $V_p$ alone, particularly for very high radiative cooling rates at which the size of the entire storm (both $r_{\text{ew}}$ and $r_0$) increases. The scaling for $r_{\text{ew}}$ necessarily integrates the variability of the eye and eyewall together. To isolate effects associated solely with the eye, we also extract from the equilibrium storm structure the counterpart to $r_{\text{ew}}$ (i.e. $r(.75V_m)$) that lies within the eye, which we denote $r_e$. Figure 3-20 displays the scaling of the ratio of $r_e$ to $r_{\text{ew}}$ with $V_p$. Positive slope indicates that the eye is expanding relative to $r_{\text{ew}}$ for increasing $V_p$, whereas zero slope indicates that the entire inner core structure of the storm scales uniformly. There are two prominent deviations from zero.

First, $u_{\text{sfc}}$ exhibits a statistically-significant positive scaling with an estimated slope of .5. Indeed, one can observe in Figures 3-5 and 3-6 a distinct difference in the behavior of the eye as compared to $T_{\text{tpp}}$, such that the radial wind profile in the eye expands outward as the gustiness is decreased. This expansion of the eye, which is positively correlated with the expansion associated with increasing $V_p$, may explain the difference in the scaling of $r_{\text{ew}}$ between $u_{\text{sfc}}$ and $T_{\text{tpp}}$ noted earlier.

Second, $Q_{\text{cool}}$ exhibits a substantial expansion of the eye, but only for large radiative cooling rates. The strange behavior in the eye is evident in Figure 3-5, as the
Figure 3-20: As in Figure 3-17, but for the scaling of the ratio of $r_e$ to $r_{ew}$. Positive slope indicates that the eye is expanding relative to the width of the eyewall.

inner edge of the eyewall is rapidly pushed to larger radii. In the normalized plots in Figure 3-6, this behavior manifests itself as the progressive erosion of the inner hump in the two-humped structure of the gradient wind profile in the eyewall region. At equilibrium, the eye above the boundary layer is characterized by radiative-subsidence balance, which, coupled with Ekman upwelling within the boundary layer, necessarily implies from mass continuity an outward mass flux above the boundary layer that should act to push the eyewall radially outwards. One expects that this mass flux would scale with the radiative subsidence rate and thus with the radiative cooling rate, but why this effect appears prominent only for high cooling rates is unclear.

As noted above, both $r_{ew}$ and $r_0^*$ expand at high radiative cooling rates. This may be due to the development of significant convection beyond the eyewall, which may cause an expansion of the wind field (Hill and Lackmann, 2009) and also likely explains the poor fit of the analytical outer wind model to the equilibrium radial wind profile near $r_{ew}$ in these cases. It should be noted, though, that the variability in storm
structure increases significantly at these more extreme conditions. For the largest radiative cooling rate ($4 \ K \ day^{-1}$), the storm develops wind maxima at radii around 1000 km before eventually developing a more traditional radial wind structure only around day 120. More generally, this behavior may reflect the fact that in radiative-convective equilibrium, precipitation must balance free tropospheric radiative cooling. Thus, total precipitation necessarily must increase with $Q_{cool}$, but given constraints on the terminal velocity of raindrops (to which $V_m$ was found to be sensitive in Bryan and Rotunno (2009a)) as well as the decrease in boundary layer water vapor mixing ratio with increasing $Q_{cool}$, this implies that the areal coverage of precipitation may necessarily increase, which may result in a larger storm. At sufficiently high values of $Q_{cool}$, a precipitation distribution that is confined to the eyewall region, as it appears to be in typical axisymmetric simulations, may no longer be sufficient to balance radiative cooling, and thus convection will necessarily develop at larger radii. A more detailed analysis of this hypothesis is beyond the scope of this work.

3.3.8 Rossby deformation radius

Given that the structure of a tropical cyclone is characterized by a warm anomaly embedded within a rotating fluid, one potentially-relevant length scale from conventional geostrophic adjustment theory that has not been discussed to this point is the Rossby deformation radius, defined as

$$L_R = \frac{N_v H}{f}$$

(3.24)

where $N_v^2 = \frac{g}{\theta_v} \frac{\partial \theta_v}{\partial z}$ is the buoyancy frequency, $\theta_v$ is the virtual potential temperature, and $H$ is the fluid depth (Emanuel (1994), p. 166). One plausible explanation for the finding that the relevant storm length scale is $L_R$ is that this quantity simply covaries with the deformation radius. Indeed, in a three-dimensional rotating radiative-convective equilibrium simulation, Held and Zhao (2008) noted a scaling for the size of their tropical cyclone that was consistent with either $L_R$ or $\frac{V_p}{f}$ but could not distinguish between the two based on the given parameter space. Here we test this
hypothesis in Figure 3-21, which is analogous to the bottom panel of Figure 3-17 but for the scaling of \( r_0^* \) with \( L_R \) rather than \( V_p \) (which is equivalent to \( \frac{V}{f} \) since \( f \) is fixed). The deformation radius is calculated from the background state vertical profiles of potential temperature and water vapor, where \( H \) is the depth of the troposphere, taken to be the linearly-interpolated altitude where the temperature first drops below \( T_{\text{pp}} \), and \( N_v \) is taken as the tropospheric pressure-weighted mean. In the case of varying \( T_{\text{sat}} \) and \( T_{\text{pp}} \), \( r_0^* \) indeed scales in the same direction for both \( L_R \) and \( V_p \). In contrast, the scaling of \( r_0^* \) with \( u_{\text{sfc}} \) and \( Q_{\text{cool}} \) is of the opposite sign. Taken together, Figure 3-21 suggests that \( L_R \) is not fundamental to the scaling of the equilibrium storm, noting that this conclusion applies equivalently to \( V_m \) and \( r_m \) given that both exhibit similar qualitative scaling behavior (i.e. positive and monotonic with \( V_p \)).

Physically, the distinct scaling relationship of these two parameters is the manifestation of their convenient effect on our idealized RCE state: an increase in \( Q_{\text{cool}} \) and a decrease in \( u_{\text{sfc}} \) both act to increase the air-sea thermodynamic disequilibrium,
which increases $V_p$ (Eq. (3.11)) while simultaneously decreasing $N_v$ and $H$. This latter effect is explained as follows: in our idealized set-up, the requirement of column energy balance between surface enthalpy fluxes (Eq. (3.2)) and net radiative cooling (Eq. (3.9)) reduces to a mutual constraint on $k^* - k$, $u_{sfc}$, and $Q_{cool}$; the effects of the associated changes in $\rho$, $\Delta p$, and the mean resolved wind speed are relatively small. Thus, decreasing $u_{sfc}$ at constant $Q_{cool}$ necessitates an increase in $k^* - k$ in order to maintain constant surface enthalpy fluxes, as does increasing $Q_{cool}$ at constant $u_{sfc}$ in order to increase surface enthalpy fluxes to match the enhanced radiative cooling. In either case, an increase in the air-sea thermodynamic disequilibrium at constant $T_{sst}$ implies a decrease in the specific humidity at the lowest model level and of the boundary layer overall. Given that the RCE state is constrained to approximately follow a moist adiabat associated with some measure of the sub-cloud layer entropy (Betts, 1986) and, moreover, that the air-sea disequilibrium is predominantly in the form of latent heat, $N_v$ is directly proportional to the sub-cloud layer specific humidity. Furthermore, for fixed $T_{tip}$, $H$ scales with the mean lapse rate, which is proportional to $N_v$. Thus, decreased sub-cloud layer water vapor translates to a decrease in $N_v$ and $H$ and therefore, given fixed $f$, a decrease in $L_R$.

Additionally, $L_R = 0$ for the case of a dry tropical cyclone, which would preclude its existence if $L_R$ were indeed the fundamental length scale. Yet Mrowiec et al. (2011) demonstrated a quasi-steady dry tropical cyclone in an axisymmetric model, and we have successfully generated a quasi-steady dry storm in this modeling environment as well. Figure 3-22 (left) displays the time-evolution of the radial profile of the full azimuthal wind speed at $z = 1.5 \text{ km}$ for a dry version of the Control simulation. This dry simulation is initialized using the RCE sounding calculated from the analogous small-domain three-dimensional simulation absent any water, and no initial disturbance is input. The dry case is more variable than its moist counterpart, though the wind field remains reasonably steady for days 80-140. The time-mean radial wind profile averaged over days 100-130 is also shown in Figure 3-22 (right).

The higher degree of variability in the dry case is likely due to the fact that in dry RCE, the static stability is zero, and thus radiative cooling cannot be balanced
by subsidence warming. Instead, it can only be balanced by convection, including in regions that are typically convection-free in the moist case, such as inside the eye and perhaps in the subsiding region within the outer region of the storm (though here the storm itself may generate positive static stability). Furthermore, in contrast to the moist case, the dry case must be characterized by updraft-downdraft symmetry, yet the distribution of convection in a moist TC is typically concentrated into a small region within the eyewall. Both differences may explain the existence of a secondary wind maximum just beyond the edge of the primary TC ($r \approx 800 \text{ km}$) in Figure 3-22, a feature that is not typically observed in the moist simulations. A deeper analysis of the differences between the dry and moist simulations may prove to be quite insightful and is left to future work; here we simply note that such a dry storm can indeed be generated in this modeling environment.

3.4 Varying $\frac{C_k}{C_d}$

Beyond the non-dimensional parameters identified above, an additional non-dimensional parameter of interest is the ratio of the surface exchange coefficients of enthalpy and momentum, $\frac{C_k}{C_d}$. Though current theory suggests that it is only their ratio that is relevant to storm structure, here we perform tests varying both $C_k$ and $C_d$ indepen-
Table 3.4: Simulations varying either $C_k$ or $C_d$ and the corresponding values for the potential intensity and for the value of the horizontal mixing length used in order to fix $\frac{V_p}{flh}$ to its Control value.

<table>
<thead>
<tr>
<th>$C_k \times 10^{-3}$</th>
<th>$C_d \times 10^{-3}$</th>
<th>$\frac{C_k}{C_d}$</th>
<th>$V_p \ [ms^{-1}]$</th>
<th>$l_h \ [m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1.5</td>
<td>.5</td>
<td>80</td>
<td>1316</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
<td>92</td>
<td>1500</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>2</td>
<td>105</td>
<td>1720</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>4</td>
<td>124</td>
<td>2026</td>
</tr>
<tr>
<td>12</td>
<td>1.5</td>
<td>8</td>
<td>155</td>
<td>2532</td>
</tr>
<tr>
<td>1.5</td>
<td>2.12</td>
<td>.71</td>
<td>76</td>
<td>1247</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
<td>92</td>
<td>1500</td>
</tr>
<tr>
<td>1.5</td>
<td>1.06</td>
<td>1.41</td>
<td>110</td>
<td>1800</td>
</tr>
<tr>
<td>1.5</td>
<td>0.75</td>
<td>2</td>
<td>133</td>
<td>2180</td>
</tr>
<tr>
<td>1.5</td>
<td>0.53</td>
<td>2.82</td>
<td>161</td>
<td>2642</td>
</tr>
<tr>
<td>1.5</td>
<td>0.375</td>
<td>4</td>
<td>201</td>
<td>3284</td>
</tr>
<tr>
<td>1.5</td>
<td>0.1875</td>
<td>8</td>
<td>261</td>
<td>4276</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0938</td>
<td>16</td>
<td>346</td>
<td>5672</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0625</td>
<td>24</td>
<td>449</td>
<td>7359</td>
</tr>
</tbody>
</table>

dently, spanning a large range of values of $\frac{C_k}{C_d}$: the range of $C_k$ is $.75-12 \times 10^{-3}$ (i.e. $\frac{C_k}{C_d}$ spans $\frac{1}{2}$-8x the Control value) and the range of $C_d$ is $.0625-2.1 \times 10^{-3}$ (i.e. $\frac{C_k}{C_d}$ spans $\frac{1}{\sqrt{2}}$-24x the Control value). Because $V_p$ is itself a function of the exchange coefficients, varying the latter will also modulate the dominant non-dimensional parameter, $\frac{V_p}{flh}$, based on our earlier result. Thus, in order to isolate the effect of varying $C_k$ or $C_d$ alone, we simultaneously modulate $l_h$ so as to fix $\frac{V_p}{flh}$ to its Control value. The values for $V_p$ and $l_h$ are listed in Table 3.4.

Note that in our idealized thermodynamic environment the potential intensity is much more sensitive to $C_d$ than $C_k$. Indeed, Eq. (3.11) predicts that $V_p$ should be independent of $C_k$, though the detailed calculation still indicates an increase in $V_p$ with $C_k$, which is amplified by the pressure dependence on the saturation vapor pressure.

The dimensional equilibrium radial profiles for the simulation sets varying $C_k$ and $C_d$ are displayed in Figure 3-23. For increasing $C_k$ (i.e. slowly increasing $V_p$), the equilibrium storm structure remains nearly fixed in space but exhibits a gradual re-
duction in the peak wind speeds within the eyewall. Meanwhile, for decreasing $C_d$ (i.e. rapidly increasing $V_p$), the storm intensifies and expands significantly. Notably, for a given value of $\frac{C_k}{C_d}$, the dimensional storm structure is markedly different depending on whether one varies this ratio via $C_k$ or $C_d$.

### 3.4.1 Inner Core

Following our prior procedure, we normalize $V$ by $V_m$ and $r$ by $r_{ew}$, with the result displayed in Figure 3-24. Once again, this normalization removes much of the variability in both cases and divides residual variability between the inner and outer regions. For $C_k$, there is minimal additional variability. For $C_d$, there are two additional degrees of variability: a contraction of the far outer wind field and an expansion of the eye with increasing $V_p$ (i.e. decreasing $C_d$). Explanations for these additional modes of
variability are explored further below.

In the case of $V_m$, a theoretical relationship exists that we may test using our simulation results. For a reasonably intense vortex, i.e. $V_m \gg f r_m$, Emanuel and Rotunno (2011) derive an equation (Eq. (41)) for the non-dimensional maximum gradient wind speed that is a function solely of the ratio of the exchange coefficients, given by

$$\frac{V_m}{V_p} \sim \left( \frac{1}{2 C_d} \right)^{\frac{1}{2} \frac{C_k}{C_d}}$$

(3.25)

The simulated equilibrium values of $V_m$ are compared to the scaling of Eq. (3.25) in Figure 3-25. The data closely match the theory over the entire range of values when varying both $C_k$ and $C_d$, and the respective best-fit scaling constants are identical and near unity (1.02). Indeed, the non-dimensional equilibrium intensity appears to closely follow the theoretical relation of Emanuel and Rotunno (2011) and depends

Figure 3-24: As in Figure 3-23, but with radial profiles normalized as follows: $V$ by $V_m$ and $r$ by $r_{ew}$. 

\[\text{Figure 3-24: As in Figure 3-23, but with radial profiles normalized as follows: } V \text{ by } V_m \text{ and } r \text{ by } r_{ew}.\]
Figure 3-25: Scaling between $\frac{V_m}{V_p}$ and $\frac{C_k}{C_d}$ in simulations (marker) and the theoretical scaling relation given by Eq. (3.25) (line) from Emanuel and Rotunno (2011) for varying $C_k$ (black, x) and $C_d$ (grey, diamond). The horizontal mixing length, $l_h$, is varied such that the non-dimensional parameter $\frac{V}{l_h}$ remains fixed at its Control value. For each exchange coefficient, best-fit scaling constants and 95% confidence intervals are listed at the top and adjusted r-square values are listed in the lower-left corner.

only on the ratio of the exchange coefficients, which is remarkable given that the dimensional storms differ so greatly depending on whether this ratio is varied by $C_k$ or $C_d$. Additionally, note that the experiments with very small $C_d$ correspond to extremely large values of $V_p$ (450 ms$^{-1}$; Table 3.4), which may lead one to question whether such a storm can be properly resolved in the given modeling environment. However, the combination of the rapid decline of $\frac{V_m}{V_p}$ at large values of $\frac{C_k}{C_d}$ ($V_m$ actually decreases as $C_d$ is reduced from $\frac{1}{15}$ to $\frac{1}{24}$ of its Control value despite the corresponding large increase in $V_p$) and the rapid expansion of the storm both serve to maintain a well-resolved dimensional storm.

As for $r_{ew}$, Figure 3-26 displays the scaling relationships between the ratio of exchange coefficients and $r_{ew}$ when varying $C_k$ and $C_d$. These scalings do not collapse
Figure 3-26: Scaling of $r_{ew}$ with the ratio of exchange coefficients, $\frac{C_k}{C_d}$, for varying $C_k$ (black, x) and $C_d$ (grey, diamond). Best fit scaling exponents with 95% confidence intervals are listed in the lower-right corner, and adjusted r-square values are given in the top-left corner.

to a single scaling with $\frac{C_k}{C_d}$, though in each case the scaling is relatively weak and is given by

$$\frac{r_{ew}}{V_p/f} \sim C_k^{-0.15} C_d^{-1}$$  \hspace{1cm} (3.26)

Thus, the inner core of the non-dimensional storm contracts slowly with both increasing $C_k$ and $C_d$.

Given that for the case of varying $C_k$, the inner core appears to scale uniformly, whereas for $C_d$ this is not the case, an alternative possible interpretation is the following approximate scaling

$$\bar{r}_{ew} \sim C_d^{-0.25} \left(\frac{C_k}{C_d}\right)^{-0.15}$$  \hspace{1cm} (3.27)

This perspective would suggest that the inner core storm size does in fact scale with
Figure 3-27: Scaling of the ratio of $r_e$ to $r_{ew}$ with $C_d$. Best fit scaling exponents with 95% confidence intervals are listed in the lower-right corner, and adjusted r-square values are given in the top-left corner.

But superimposed upon this scaling is an additional modification associated with $C_d$ alone. This seems plausible given the variability observed within the eye (Figure 3-24), characterized by an expansion of the eye relative to the eyewall for decreasing $C_d$. Such behavior is consistent with a negative scaling exponent. To quantify this relationship, Figure 3-27 displays the scaling of the ratio $\frac{r_e}{r_{ew}}$ as a function of $C_d$. The resulting scaling is $\frac{r_e}{r_{ew}} \sim C_d^{-0.21}$, though because this is a scaling relative to $r_{ew}$, it cannot explain the additional scaling with $C_d$ in Eq. (3.27). A theoretical argument for this relative scaling is explored in Section 3.5.

### 3.4.2 Outer radius

Finally, we explore sensitivity of the outer wind field to variations in the exchange coefficients following our earlier procedure. Figure 3-24 indicates that for varying $C_k$, there is little residual variability in the outer wind field, with the possible exception
Table 3.5: As in Table 3.3, but for $C_k$ and $C_d$, with $C_1 = \frac{V_p}{f L_n}$ held fixed at its Control value.

<table>
<thead>
<tr>
<th>$C_k \times 10^{-3}$</th>
<th>$c$</th>
<th>$C_d \times 10^{-3}$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.75</td>
<td>.28</td>
<td>2.12</td>
<td>.27</td>
</tr>
<tr>
<td>1.5</td>
<td>.26</td>
<td>1.5</td>
<td>.26</td>
</tr>
<tr>
<td>3</td>
<td>.31</td>
<td>1.06</td>
<td>.44</td>
</tr>
<tr>
<td>6</td>
<td>.26</td>
<td>.75</td>
<td>.45</td>
</tr>
<tr>
<td>12</td>
<td>.24</td>
<td>.53</td>
<td>.92</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>.375</td>
<td>.73</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>.188</td>
<td>2.52</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>.094</td>
<td>2.28</td>
</tr>
</tbody>
</table>

of very near $r_0$ where wind speeds are close to zero. Meanwhile, for varying $C_d$, there is significant additional variability in the outer wind field that, though quite noisy, suggests a systematic contraction as $C_d$ is decreased. Given that $V_p \sim C_d^{-\frac{1}{2}}$, the quantity $C_d V_p$ varies directly with $C_d$. Thus, the role of $C_2$ in Eq. (3.18) predicts that the far outer wind field will indeed contract as $C_d$ is decreased, as is observed.

The fit of Eq. (3.18), empirically modified with $c = .3$, to the equilibrium outer wind profiles is shown in Figure 3-28, which is analogous to Figure 3-9. Eq. (3.18) provides a good fit for all values of $C_k$, corroborated by the constancy of the optimized $c$ as $C_k$ is varied (Figure 3-10 and Table 3.5). However, Eq. (3.18) is too sensitive to changes in $C_d$ relative to the numerical model output, resulting in large errors at all radii as $C_d$ is progressively decreased toward an extreme value of $\frac{1}{24}$ of Control. Unfortunately, the optimized value of $c$ does not vary smoothly with $C_d$, rendering a quantitative estimate of the degree of oversensitivity difficult. We emphasize, though, that in this case we are exploring values of $C_d$ that are far removed from those typically associated with an Earth-like atmospheric boundary layer beneath a tropical cyclone.

Though estimation of the true $r_0$ may thus be difficult for $C_d$, the outer radius with the influence of $C_2$ removed, $r_0^*$, still provides a sensible metric for the overall size of the storm that is independent of the outer wind field variability. Thus, Figure 3-29 displays the scaling of $r_0^*$ with $\frac{C_k}{C_d}$ for varying $C_k$ and $C_d$, where $r_0^*$ is calculated
Figure 3-28: As in Figure 3-11, but for simulations varying $C_k$ and $C_d$ while holding $\frac{V_p}{fL_h}$ fixed to its Control value. Radial profiles of error in the fit of the analytical outer wind model (Eq. (3.18), empirically modified with $c = .3$) to the equilibrium radial wind profile, defined as $V_{eq} - V_{CM1}$, for all simulations varying each of the six relevant dimensional parameters. Error profiles are smoothed with a 10-pt smoother. In the top four panels, shading reflects potential intensity from low (light grey) to high (black); in the bottom two panels, shading reflects parameter magnitude from low (light grey) to high (black). Analytical model is fit to $r(.75V_m)$, with the range $r(.75V_m < V < .1V_m)$ solid and $r(V < .1V_m)$ dashed; radii are normalized by $r_0$ calculated using the outer wind model. Red line depicts mean error over the inner range, and the corresponding mean absolute error (MAE) for the simulation set is listed in the top left corner.
with Eq. (3.18), fit at \( r_{ew} \) as it was above. The resulting scaling is

\[
\frac{r_0^*}{\sqrt{\frac{V_p}{f}}} \sim C_k^{-3} C_d^{-1.5}
\]

(3.28)

Thus, overall non-dimensional storm size contracts slowly with increasing \( C_d \) and more rapidly with increasing \( C_k \).

### 3.4.3 Comparison with structural theory

Emanuel and Rotunno (2011) also derive a structural relationship between \( r_m \) and \( r_0 \) (Eq. (42)), which can be rewritten in non-dimensional form as

\[
\frac{\tilde{r}_m}{\tilde{r}_0^2} = \left( \frac{1}{2} \right)^{\frac{3}{2}} \left( \frac{C_k}{C_d} \right)^{\frac{1}{2}}
\]

(3.29)

where tildes denote non-dimensionalization by \( \frac{V_p}{f} \) and we use our \( r_0^* \) to represent their
Figure 3-30: Scaling of $\frac{e_{\text{avg}}}{r_0^2}$ with $\frac{C_k}{C_d}$ in simulations (marker) with linear fit (dashed) and the theoretical scaling relation given by Eq. (3.29) (solid) from Emanuel and Rotunno (2011) for varying $C_k$ (black, x) and $C_d$ (grey, diamond). The horizontal mixing length, $l_h$, is varied such that the non-dimensional parameter $\frac{V_p}{f l_h}$ remains fixed at its Control value. For each exchange coefficient, best-fit scaling constants and 95% confidence intervals are listed at the top.

Throughout this theoretical $r_0$. Taking the scaling results from Eqs. (3.27) and (3.28) together, Figure 3-30 shows that the scaling of this non-dimensional ratio with $\frac{C_k}{C_d}$ is indeed nearly a square-root dependence on $\frac{C_k}{C_d}$ when varying either $C_k$ or $C_d$, despite the fact that the scalings for the respective radii differ between the two cases. The best-fit constant is in each case is in the range .65 – .75.

This result is somewhat surprising given that the theory of Emanuel and Rotunno (2011) is valid only in the ascending region of the storm, and thus it’s not clear that such a structural relationship for the outer region of the storm would apply. The scalings are nearly identical when fitting Eq. (3.18) to $r(0.3V_m)$, suggesting that they are relatively robust even though the fit of Eq. (3.18) to the true outer wind field is far from perfect. However, in the context of the earlier result that the theoretical length
scale of $\frac{r}{f}$ also emerges from $r_0^*$ this result provides additional evidence that extant tropical cyclone theory accurately captures the radial structure of the tropical cyclone, and any additional variability in the outer wind field associated with variations in $w_{cool}, C_d$, etc., is simply passively superimposed onto this underlying framework.

3.5 Theoretical scaling for eye

Our results indicate that the radial wind profile in the eye is found to be predominantly sensitive to $u_{sfc}, C_d$ and $l_h$ (the last modulates the entire eyewall). Specifically, the eye expands for decreasing $u_{sfc}$ and $C_d$ and for increasing $l_h$. These sensitivities can be interpreted via analysis of the integrated angular momentum balance of the eye. The eye at equilibrium within the boundary layer should be characterized by a balance between inward radial turbulent transport of angular momentum from the eyewall and the integrated angular momentum loss at surface (Emanuel, 1997; Smith, 1980).

The local tendency of scalar quantities due to turbulence is parameterized as

$$D_M = \frac{1}{r} \frac{\partial}{\partial r} \left( rK_h \frac{\partial M}{\partial r} \right)$$

where the radial turbulent diffusivity, $K_h$, is

$$K_h = l_h^2 S_h = l_h^2 \sqrt{2 \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial V}{\partial r} - \frac{V}{r} \right) \left( \frac{\partial V}{\partial r} \right)}$$

and $S_h$ is the horizontal component of the local deformation. Integrating from the center out to some radius $R$ gives the net inward turbulent flux of angular momentum across $R$. This flux must equal the integrated angular momentum loss due to friction at the surface,

$$\frac{C_d}{h_{BL}} \int_0^R (V + u_{sfc}) (rV) r dr \approx \left[ l_h^2 r V \sqrt{2 \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial V}{\partial r} - \frac{V}{r} \right) \left( \frac{\partial V}{\partial r} \right) \right]_R$$
given zero flux across \( r = 0 \) and assuming that the net vertical flux of angular momentum out the top of the boundary layer is negligible; \( h_{BL} \) is the depth of the well-mixed boundary layer and we have taken \( M \approx rV \). Neglecting the term on the right hand side of Eq. (3.32) involving the radial shear of the radial wind, the resulting balance is given by

\[
\frac{C_d}{h_{BL}} \int_0^R (V + u_{sfc})(rV)rdr \approx \left[ l_h^2 r V \sqrt{\frac{\partial V}{\partial r}} \left( \frac{\partial V}{\partial r} - \frac{V}{r} \right) \right]_R
\]  

(3.33)

The RHS is evaluated at some \( r = R \) just inside \( r_m \), whereas the LHS must be integrated.

Though \( V(r) \) in the eye is typically assumed to be close to a state of solid body rotation (Emanuel, 1997), it cannot be exactly so or else the turbulent angular momentum flux (RHS) vanishes. As a simple ansatz, we apply a power-law solution for \( V(r) \) of the form

\[
\frac{V}{V_m} = \left( \frac{r}{r_m} \right)^\alpha
\]

(3.34)

in Eq. (3.33), which leads to the scaling

\[
r_m^3 \sim \frac{h_{BL} l_h^2}{C_d \left( 1 + \frac{2\alpha + 3}{\alpha + 3} \frac{u_{sfc}}{V_m} \right)}
\]

(3.35)

where we have taken \( R = r_m \), ignoring the details of the eyewall structure.

Thus, this scaling predicts \( r_m \sim l_h^{\frac{2}{3}} \) and \( r_m \sim C_d^{-\frac{1}{3}} \). For \( l_h \), the scaling is a reasonable estimate of the empirically-derived exponent of \(-.55\) though slightly exceeds it. For \( C_d \), the scaling also gives a decent estimate, whose exponent was found above to be \(-.2\). The scaling with \( u_{sfc} \), however, is qualitatively consistent with the observed relationship but is quantitatively incorrect. The scaling predicts significant sensitivity only for \( \frac{u_{sfc}}{V_m} \sim 1 \), which corresponds only to the cases with high values of \( u_{sfc} \) (for \( u_{sfc} = 10 \), this quantity is approximately \( .2 \)), yet we see systematic sensitivity at all values of \( u_{sfc} \), suggesting that the power-law solution is not appropriate to capture the effect of \( u_{sfc} \) on eye structure. This is true even for large \( \alpha \), a consequence of the
fact that the surface sink of angular momentum is area-weighted and the region where \( \frac{u_{\text{stc}}}{V_m} \sim 1 \) lies near \( r = 0 \) and thus occupies a relatively small area within the eye. Note that the scalings for \( l_h \) and \( C_d \) are independent of the specific profile of \( V(r) \) and can be extracted via scale analysis of Eq. (3.33). This model also does not represent the effects of \( Q_{\text{cool}} \), which was observed to expand the eye at high values. Overall, though, this model based on angular momentum balance appears to conceptually capture a few of the key processes that contribute to modulation of the eye diameter.

### 3.6 Discussion

Though our scaling results are physically intuitive, the representation of the full spectrum of turbulent eddies via a single radial mixing length scale, \( l_h \), as is required in axisymmetric geometry, is less than ideal. We have demonstrated that the more relevant external parameter is this radial mixing length normalized by the radial storm scale, yet there exists no accepted theory for the "correct" value of this parameter nor is it understood that this parameter is necessarily a constant in both time and space. In principle, given that no eddies are resolved in axisymmetry, \( l_h \) represents the length scale of the largest eddy, which plausibly corresponds to the circumference of the eyewall and therefore should scale with the radius of maximum wind. Notably, application of such an ansatz to our scalings results in \( V_m \sim V_p \) and \( r_m \sim r_0 \sim \frac{V_p}{f} \) as would be expected from dimensional considerations alone. However, lacking additional information, the combination of this structural uncertainty and the vagaries of modeling storm size render a transition from quantifying scaling estimates to more precise predictions of \( V_m \) and \( r_m \) potentially dubious using axisymmetric models in their current form.

Nonetheless, to the extent that the qualitative dynamics of the effect of eddies on storm structure are reasonably captured in this framework, as appears to be the case based on theoretical considerations as well as recent work that finds favorable comparisons between axisymmetric and three-dimensional simulation output (Bryan, 2011), much may yet be gleaned from the analysis of computationally-cheaper axisymmetric...
simulations and comparisons with theory. In particular, it is perhaps unsurprising in retrospect that the relative rather than absolute eddy length scale is the relevant parameter in the context of extant tropical cyclone theory that is itself phrased entirely in terms of relative rather than absolute radial length scales, a topic discussed in Emanuel (1995b). Indeed, it seems plausible that a similar argument would hold for the parameterized vertical turbulence, i.e. the relevant parameter is the vertical turbulent mixing length relative to the depth of the troposphere, though sensitivity of storm structure to this parameter is in any case small for commonly-accepted values relevant for the modern Earth atmosphere.

Furthermore, details of the dynamics of the eye may play a role in modulating $r_m$, at least in the context of the simplified axisymmetric set-up explored here. While radial turbulence is clearly the dominant factor, whose effect is to shift the entire eyewall outwards, variations in the drag coefficient, radiative cooling rate, and gustiness all appear to add secondary variability to eye size. The extent to which such processes are important in a three-dimensional tropical cyclone containing the full spectrum of eddies is unclear. Emanuel (1997), following from Smith (1980), argues that the role of turbulent eddies is simply a passive response to an otherwise barotropically-unstable radial wind profile inside of the radius of maximum winds. The eddies rapidly transport angular momentum inwards from the radius of maximum winds (which is replenished by the secondary circulation), thereby driving the eye towards a state of solid-body rotation. Given this perspective, it seems likely that real three-dimensional eddies, which act on fast time scales and are capable of responding to changes in the local forcing (analogous to a time-varying $l_h$), may counteract these secondary effects in real-world tropical cyclones. Results from Khairoutdinov and Emanuel (2012) exploring RCE on an $f$-plane in three dimensions found preliminary evidence that, in addition to overall size, $r_m$ increased with increasing $T_{ast}$.

The extent to which these equilibrium results can be applied to real storms in nature is not clear for two key reasons. First, the time-scales to equilibrium identified here for the Control simulation are significantly longer than the lifespan of tropical cyclones on Earth. Second, storms in nature rarely exist in a truly quasi-steady back-
ground for more than a couple of days, if at all. Instead, storms live within an evolving thermodynamic environment due to along-track changes in potential intensity, vertical wind shear, interactions with land or extratropical disturbances, etc. Even given an idealized atmospheric state in a truly quiescent large-scale environment, the thermal stratification of a real ocean allows for time-dependent cooling of the sea surface driven by wind-driven turbulent mixing of the upper ocean, thus precluding equilibration of the background environment when a storm is present. Indeed, the large range in observed size distribution cannot be explained by the equilibrium results; Chavas and Emanuel (2010) noted that non-dimensionalization by $\frac{V_p}{f}$ had little impact on their results, and correlations between storm size and this length scale or with $V_p$ or $f$ alone were relatively small. However, equilibrium dynamics may potentially manifest itself more clearly at an aggregate level, such that shifts in the global distribution of $\frac{V_p}{f}$ within the main tropical cyclone basins may translate into shifts in the size distribution of tropical cyclones, even though variability within the global distribution is the result of more complex non-equilibrium processes. For example, global warming due to a doubling of atmospheric carbon dioxide concentrations is expected to lead to a global increase in potential intensity of $\sim 10\%$ (Emanuel, 1987; Knutson et al., 2010). Much more work is needed to assess the extent to which such a relationship is borne out in models while accounting for shifts in the spatial distribution of potential intensity as well as tropical cyclone genesis locations and tracks.

More broadly, the emergence of many details of the equilibrium storm structure in these idealized simulations is interesting given the simplistic set-up and the ongoing variability that characterize the time-evolution of many of the simulations even during the post-equilibration period. Though there are additional details of storm structure that are surely lost under these idealized conditions, and a more properly resolved boundary layer may be necessary to better quantify the effects of varying those parameters that significantly modulate the lower troposphere, particularly the sea surface temperature, this work furthers the notion that many of the fundamental dynamical processes of the tropical cyclone are in fact quite coarse-grained and can be reasonably captured by simple models that enable inflow near a lower boundary, ex-
change of enthalpy and momentum with that lower boundary, and outflow aloft where enthalpy can be expelled (i.e. a Carnot engine). This harps back to the analyses using very simple three-layer tropical cyclone models by Ooyama (1969) and DeMaria and Pickle (1988), the latter found interesting variations in storm size, which compare favorably to both observations and more recent modeling work, even at a very low horizontal resolution of 25 km.

3.7 Conclusions

This work combines highly idealized modeling, motivated by existing axisymmetric tropical cyclone theory, with dimensional analysis to systematically quantify the scaling relationship between the structure of a model tropical cyclone at statistical equilibrium and relevant model, initial, and environmental input parameters. We perform this analysis in a model world whose complexity is reduced so as to retain only the essential physics of the tropical atmosphere necessary to produce a tropical cyclone: radiative-convective equilibrium in axisymmetric geometry on an f-plane with constant tropospheric cooling, constant background gustiness (to provide a background source of water vapor), constant surface exchange coefficients for momentum and enthalpy, and constant sea surface and tropopause temperatures. Importantly, this model tropical atmosphere could in principle exist for all time in column-wise radiative-convective equilibrium, in which column-integrated radiative cooling is exactly balanced by surface fluxes of enthalpy, in the absence of a tropical cyclone. Following the theoretical work of Emanuel and Rotunno (2011), we characterize the full structural evolution of the storm by the time-series of three dynamical variables calculated near the top of the boundary layer: the maximum gradient wind speed, a proxy for the radius of maximum gradient wind, taken as the radius of 75% of the maximum wind speed, and the outer radius of vanishing wind.

We find that, under these simplified conditions, the inner core storm structure at statistical equilibrium is primarily a function of only three external parameters: the potential intensity, the Coriolis parameter, and the radial turbulent mixing length.
These three parameters comprise the dominant non-dimensional parameter, $\frac{V_p}{f l_h}$, for the equilibrium system. This parameter can be interpreted as the ratio of the storm radial length scale, $\frac{V_p}{f}$, to the radial eddy mixing scale, $l_h$, and it dictates that the critical role of parameterized radial turbulence in determining inner-core storm structure in axisymmetric geometry is manifest not in the absolute value of the radial turbulent mixing length but rather in its value relative to the natural length scale of the storm. A second non-dimensional parameter, $\frac{C_d V_p}{w_{	ext{wind}}}$, whose reciprocal represents the non-dimensional Ekman suction rate, exists within a pre-existing slab boundary layer outer wind model that, given a simple empirical modification, can reproduce the outer wind field of a tropical cyclone across a range of simulations. Controlling for this secondary mode of variability, we find that the overall size of the storm scales nearly linearly with $\frac{V_p}{f}$, which is the theoretical scaling for the upper bound on tropical cyclone size derived in Emanuel (1986) that is a consequence of the energetic contribution of outflow work in the Carnot framework. Contrary to conventional wisdom based on geostrophic adjustment, the Rossby deformation radius is shown not to be fundamental to equilibrium size. Finally, the ratio of the surface exchange coefficients, $\frac{C_k}{C_d}$, represents a third relevant non-dimensional parameter whose effect on non-dimensional storm structure appears to match the theoretical relationships for intensity and structure given in Emanuel and Rotunno (2011).

Opportunities for future work abound. First, further analysis of tropical cyclones within our idealized environment is warranted, including a better understanding of the deviations from the uniform scaling with potential intensity across our thermodynamic parameters, particularly within the eye. An exploration of storm size at the extremes, such as the existence of a theoretical lower bound, would be fruitful. New simulations run at higher resolution would be useful to test the sensitivity of the details of the results found herein to more realistic representations of real world storms. In particular, exploration of the analytical outer wind field boundary layer model and its application to the non-convecting outer region of a tropical cyclone is needed to understand both the physics of our empirical modification as well as the validity of the analytical model when applied to a simulation with a more properly resolved bound-
ary layer. Second, this work may be extended to environments of greater complexity. For example, application of an explicit temperature-dependent radiative scheme or full-physics radiation scheme could be useful in assessing the impact of radiative feedbacks on our results, which may be non-negligible given the apparent sensitivity of eye dynamics, and thus the radius of maximum wind, to the radiative cooling rate. The impact of factors that limit storm intensity, such as mid-level ventilation (Tang and Emanuel, 2010) and ocean mixing, on storm size and structure remains unexplored. Additionally, testing the validity of the results in more computationally-expensive three-dimensional simulations where three-dimensional turbulence is more properly resolved would provide insight into the role “real” turbulence plays in setting storm structure, as well as the extent to which axisymmetric parameterizations of turbulence accurately reproduce the effects of three-dimensional turbulence on storm size. Finally, application and extension of this work to real world tropical cyclones remains an open question, including the more complicated time-dependent dynamics associated with the transient phase of storm evolution in our idealized modeling environment. Ultimately, this may help provide a physical interpretation of the observed size distribution of tropical cyclones (Chavas and Emanuel, 2010). Such a fundamental physical understanding would ideally translate into a capacity for credible prediction of storm size, structure, and evolution at the level of individual storms, as well as insight into how the distribution of storm size may differ in other climate states. Both would be beneficial for the purposes of emergency preparedness and risk management alike.
Chapter 4

Exploration of Equilibrium Tropical Cyclone Size in Three Dimensional Simulations

4.1 Introduction

Tropical cyclone size remains an unsolved problem in tropical meteorology. Though significant advances have been made in understanding storm intensity (Emanuel, 1986; Bister and Emanuel, 1998; Emanuel and Rotunno, 2011) and its modification by interaction with the environment in which it is embedded, such as vertical wind shear (Tang and Emanuel, 2010; Zeng et al., 2007), the radial scale and structure of the storm is less well-understood. Recent theoretical work (Emanuel and Rotunno, 2011) derived a solution for the steady-state radial structure of the convecting inner core of the storm in an axisymmetric framework that assumes gradient thermal wind balance and moist slantwise neutrality. Additionally, Emanuel (2004) developed a complete radial profile as a patchwork of asymptotically-matched solutions for the eye, the convecting inner core, and the non-convecting outer circulation. In both cases, though, the solutions are defined relative to a free parameter given by the outer radius of vanishing wind, \( r_0 \), thus providing no constraint on the radial length.
scale of the storm as a whole.

Currently, no theoretical framework exists for determining \( r_0 \), though potential intensity theory (Emanuel, 1986) provides a theoretical upper bound for this quantity that scales with a length scale given by the ratio of the potential intensity, \( V_p \), to the Coriolis parameter, \( f \). In Chapter 3, tropical cyclone size at a statistical steady-state across a wide range of idealized climate states was explored in an axisymmetric modeling framework. Storm size, measured by an estimate of \( r_0 \) adjusted to remove a secondary mode of variability in the non-convecting outer region of the wind field, was found to scale predominantly with this theoretical length scale, \( \frac{V_p}{f} \).

Additionally, the inner core structure of the storm was found to be modulated primarily by a non-dimensional parameter given by the ratio of this storm length scale to the parameterized eddy radial mixing length, \( l_h \). This result provides a conceptually simple and qualitatively reasonable characterization of the fundamental role of radial turbulence in modulating eye and eyewall structure. Theoretically, Emanuel (1997) demonstrated the requirement of radial turbulence for preventing eyewall collapse due to the frontogenetic nature of the storm’s overturning circulation. Observationally, the inner core wind field exhibits significantly greater variability than the broader outer circulation during the lifetime of a storm (Frank, 1977; Weatherford and Gray, 1988; Merrill, 1984). Indeed, overall size, often defined as \( r_0 \) (Chavas and Emanuel, 2010), the radius of 34-kt winds, the radius of outermost closed isobar, etc., is only weakly correlated with maximum wind speed or the radius of maximum winds. In contrast to the relative quiescence of the minimally-convecting outer region of the storm, the convective inner core is subject to the asymmetric, chaotic, multi-scale variability associated with a multitude of processes, such as vortex Rossby waves excited by both internal (Schubert et al., 1999) and external factors (Reasor et al., 2000; Corbosiero et al., 2006; Reasor et al., 2004; Wang, 2002) and entropy ventilation (Tang and Emanuel, 2012; Molinari et al., 2012).

Despite the conceptual appeal of this simple axisymmetric framework, though, the crude axisymmetric representation of a critical and highly azimuthally-asymmetric process muddies direct application of the axisymmetric results to real world storms.
that live in three-dimensions. Thus, this work seeks to climb the next rung on the hierarchy of models (Held, 2005) from analytical theory to real world tropical cyclones by evaluating the primary results obtained in the axisymmetric framework within a fully three-dimensional model environment in which eddies with scales larger than the grid scale can be explicitly resolved. Specifically, we will focus on the sensitivity of TC size to variations in potential intensity as modulated by the tropopause temperature, which was identified in Chapter 3 as a useful "base" case from among the four governing thermodynamic parameters given that its dominant effect is simply to modulate the convective outflow temperature.

Section 4.2 describes the methodology, including a brief description of the model set-up and the key similarities and differences with respect to the axisymmetric simulations. Section 4.3 motivates the role of the Coriolis parameter with a simple example of self-aggregation. Section 4.4 presents the key results. Section 4.5 discusses implications of the results and the limitations of this study. Finally, Section 4.6 synthesizes key conclusions and explores avenues for future work.

4.2 Methodology

4.2.1 CM1 model: 3D

This work employs version 15 of the Bryan Cloud Model (CM1), a non-hydrostatic atmospheric cloud-system resolving model (CSRM; original version described in Bryan and Fritsch (2002)). CM1 solves the fully compressible equations of motion in height coordinates on an f-plane for flow velocities \((u, v, w)\), non-dimensional pressure \((\pi)\), potential temperature \((\theta)\), and the mixing ratios of water in vapor, liquid, and solid states \((q_x)\) on a fully staggered Arakawa C-type grid in height coordinates. Additional details can be found in Chapter 3.

Conveniently, the model can be configured using identical physics in both two-dimensional axisymmetric (radius-height) and fully three-dimensional geometry, with the important exception of the representation of turbulence. Because turbulence is
inherently a three-dimensional phenomenon, two-dimensional (axisymmetric) geometry cannot resolve turbulent eddies of any scale, and thus the effect of turbulent eddies is necessarily parameterized using a modified Smagorinsky-type scheme, in which the grid length scale is replaced with distinct horizontal and vertical mixing lengths, $l_h$ and $l_v$, respectively. The distinct mixing lengths are employed in order to represent the differing nature of turbulence between the radial and vertical directions in a highly anisotropic system such as in the inner core of a tropical cyclone. As demonstrated in Chapter 3, the horizontal mixing length has a pronounced effect on the inner-core structure of the equilibrium storm. Meanwhile, in three-dimensions, a proper Smagorinsky turbulence scheme is employed to represent the subgrid-scale effects of turbulence, while eddies whose scales exceed the grid-scale are explicitly resolved. There remain parameterized mixing length scales, but they are much smaller and are intended to represent only unresolved subgrid-scale turbulence rather than storm-scale eddies.

In this work, a set of three-dimensional simulations is performed under the identical idealized model and environmental set-up as was employed for the axisymmetric simulations analyzed in Chapter 3. Here we briefly review this set-up. Surface pressure is set to 1015 hPa. Radiation is represented by a constant cooling rate, $Q_{cool}$, applied to the potential temperature everywhere in the domain where the absolute temperature exceeds a threshold value, $T_{tpp}$; below this value, Newtonian relaxation back to this threshold is applied with a timescale of 40 days. At the lower boundary, the sea surface temperature is taken as a constant, $T_{sst}$, and surface fluxes of enthalpy and momentum are calculated using standard bulk aerodynamic formulae in which the exchange coefficients for momentum, $C_d$, and enthalpy, $C_k$, are held constant. Finally, background surface enthalpy fluxes are required to balance column radiative cooling in order to achieve radiative-convective equilibrium in the absence of significant resolved wind perturbations, such as a tropical cyclone. Though in three-dimensions this effect can be included via imposition of a mean background flow, for the sake of consistency and comparison with the axisymmetric simulation results we simply add a constant gustiness, $u_{sfc}$, to the magnitude of the wind speed, $|u|$, for
the model calculation of the surface fluxes.

For this environmental set-up, the radiative-convective equilibrium (RCE) vertical profile of potential temperature and water vapor is a function of the four governing thermodynamic parameters: \( Q_{cool}, \ T_{tpp}, \ T_{sst}, \) and \( u_{sfc}. \) Additionally, the generalized potential intensity (Emanuel, 2010), \( V_p, \) in RCE can be reformulated as a function of these four governing thermodynamic parameters, \( Q_{cool}, \ T_{tpp}, \ T_{sst}, \) and \( u_{sfc}, \) given by

\[
V^2_p = \frac{T_{sst} - T_{tpp} C_p Q_{cool} \Delta \rho}{g \rho C_d |u|} \tag{4.1}
\]

where \( C_p \) is the specific heat of air, \( g \) is the acceleration due to gravity, \( \rho \) is the near-surface air density, and \( \Delta \rho \) is a measure of the mean pressure depth of the troposphere.

For the purposes of the subsequent analysis, all values of potential intensity presented herein are calculated using the detailed Emanuel sub-routine (Bister and Emanuel, 2002) with zero boundary layer wind speed reduction under pseudo-adiabatic thermodynamics and including dissipative heating.

### 4.2.2 Description of simulations

Our objective is to explore the sensitivity of TC size to potential intensity, \( V_p, \) via modulation of the tropopause temperature, \( T_{tpp}. \) We define a Control simulation using the same values for the thermodynamic parameters and exchange coefficients as were used in the axisymmetric case; the values are provided in Table 4.1. We set the rotation rate, \( f, \) to \( 40 \times 10^{-5} \) s\(^{-1}\), or 8 times the value used in the axisymmetric Control simulation, with the expectation that this will generate smaller storms that require correspondingly smaller domains and therefore reduce the overall computational burden (see Section 4.3). This approach is similar to the work of Khairoutdinov and Emanuel (2012), which explored variations in TC size in \( f \)-plane RCE associated with changes in \( V_p \) via modulation of \( T_{sst}. \) The Control simulation is run on a \( 1536 \times 1536 \times 25 \) km\(^3\) doubly-periodic square domain with horizontal and vertical resolutions of \( dx = 4 \) km and \( dz = 625 \) m, respectively. The model has a rigid lid at the top with a 5-km thick damping layer beneath, and it employs doubly-periodic lateral
Table 4.1: Parameter values for the 3D Control simulation. See text for details.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{sst}$</td>
<td>300 K</td>
</tr>
<tr>
<td>$T_{tpp}$</td>
<td>200 K</td>
</tr>
<tr>
<td>$Q_{cool}$</td>
<td>1 K day$^{-1}$</td>
</tr>
<tr>
<td>$u_{sfc}$</td>
<td>3 m s$^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>$40 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>$C_k,C_d$</td>
<td>.0015</td>
</tr>
</tbody>
</table>

boundaries in contrast to an outer wall employed in the axisymmetric set-up. The model is initialized in the same manner as the axisymmetric simulations by using the RCE sounding, defined as the time- and horizontal-mean vertical profiles of potential temperature and water vapor for days 70-100, calculated from a three-dimensional simulation on a small 196 \times 196 km$^2$ domain that inhibits self-aggregation (for sufficiently small $f$, here set at $f = 5 \times 10^{-5}$ s$^{-1}$; see Section 4.3 for discussion of large rotation rates). Though ideally these RCE soundings should be recalculated from simulations using the full Smagorinsky turbulence scheme, turbulence does not have a strong impact on the initial RCE state.

We run four simulations: one Control simulation with $T_{tpp} = 200$ K (CTRL), two simulations with tropopause temperatures of 150 K (PI108), and 250 K (PI43), as well as one simulation with $f$ increased by a factor of 2 (FX2); parameter values for the experiments, including domain size, resolution, and average number of TCs extracted by the tracking algorithm at each output timestep are provided in Table 4.2. Note that PI108 only has one TC in the domain, the potential implications of which are discussed below.

4.2.3 Characterizing statistical equilibrium storm structure

One additional important distinction between the axisymmetric and three-dimensional approaches lies in the characterization of the equilibrium state. Axisymmetric simulation of a tropical cyclone seeks to exploit the near-circular symmetry of TC structure to model a TC directly, and so by definition only produces a single TC in its domain,
Table 4.2: Parameter values for each experiment. $N$ denotes the average number of storms extracted by the algorithm at each time step, and $T$ denotes the length of the statistical equilibrium period over which statistics are accumulated.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$T_{tp}$ [K]</th>
<th>$V_p$ [m s$^{-1}$]</th>
<th>$f$ [$\times 10^{-6}$]</th>
<th>$dx$ [km]</th>
<th>$L_{domain}$ [km]</th>
<th>$N$</th>
<th>$T$ [day]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI108</td>
<td>150</td>
<td>108.1</td>
<td>40</td>
<td>4</td>
<td>1536</td>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>CTRL</td>
<td>200</td>
<td>69.7</td>
<td>40</td>
<td>4</td>
<td>1536</td>
<td>1.8</td>
<td>34</td>
</tr>
<tr>
<td>PI43</td>
<td>250</td>
<td>42.6</td>
<td>40</td>
<td>2</td>
<td>1152</td>
<td>3.7</td>
<td>27</td>
</tr>
<tr>
<td>FX2</td>
<td>200</td>
<td>69.5</td>
<td>80</td>
<td>2</td>
<td>1152</td>
<td>3.7</td>
<td>24</td>
</tr>
</tbody>
</table>

regardless of domain size. In contrast, three-dimensional simulation of a tropical cyclone in RCE seeks to create a model atmosphere that is capable of supporting a TC, absent any constraints on the structure or behavior of a TC or even the number of TCs that develop in the system. Thus, in the former case, equilibration is defined based simply on the direct evaluation of the time-evolution of the simulated tropical cyclone wind field. In the latter case, the statistical equilibrium state may be characterized by the perpetual decay and regeneration of multiple TCs and therefore must be defined instead based upon the steadiness of some domain-wide quantity. Here we define our statistical equilibrium period based upon the domain-integrated water vapor, which reaches a quasi-steady state after 25-35 days in all simulations presented herein. The statistical equilibrium state is then characterized based on the statistics of the radial wind profiles associated with the array of TCs identified in the domain. Notably, one additional consequence is that the initial disturbance is rendered irrelevant in the three-dimensional case.

As in the axisymmetric case, the background state, from which we calculate relevant environmental quantities including the potential intensity and the radiative-subsidence rate, is ideally characterized as the mean state in the environment beyond the storm circulations. Thus, we define the background state as the mean vertical profile of potential temperature and water vapor at all gridpoints where the near-surface wind speed is below 1 m s$^{-1}$ and the pressure is greater than the mean (to exclude points in the eye of a TC). These profiles are then time-averaged over three
one-day periods corresponding to the first, middle, and last day of each simulation's equilibrium period.

At each three-hourly output time-step during the statistical equilibrium period, we use an objective tracking algorithm to locate TC centers in the domain. This algorithm takes the perturbation pressure field at the surface and zeroes out all data except those with magnitudes more than three standard deviations below the mean in order to isolate regions of data surrounding TC centers. Because of the effects of compressibility and non-hydrostatic accelerations, the minimum pressure value is occasionally offset from the true storm center. Thus, we subsequently apply a nine-point smoother 30 times to smooth the data in order to estimate the center of the TC's broader pressure distribution rather than taking the minimum of the raw pressure data. The algorithm then searches for local minima in the smoothed pressure field with magnitudes greater than 100 Pa within a square neighborhood whose side-length is set equal to the length scale $\frac{V}{f}$, which is the theoretical scaling length for TCs at equilibrium (Emanuel, 1986) identified in the axisymmetric simulations of Chapter 3.

We define the minimum pressure threshold relative to the mean, rather than as an absolute quantity, to account for the dependence of minimum pressure on storm size (small storms will have a higher minimum pressure than large storms, all else equal), as well as to attempt to consistently sample from the upper end of the intensity distribution specific to each simulation. As an example, Figure 4-1 displays snapshots of the wind field at $z = 1.5 \ km$ for each simulation, and objectively-identified TC centers are marked. In the cases of PI43 and CTRL, there is at least one TC that can be identified by eye that has not been identified by the algorithm, which is generally true of those simulations with multiple TCs. Clearly this is a conservative algorithm, but one that focuses on mature storms while avoiding more ambiguous cases of storms in the process of genesis or decay.

For each identified TC snapshot (i.e. no compositing), we extract the storm-centered wind field and project it onto the local azimuth in order to isolate the purely rotational component of the wind field. Finally, we calculate the azimuthally-averaged
Figure 4-1: Snapshots of the distribution of the full wind speed [$m s^{-1}$] at $z = 1.5 \ km$ for PI43 (top left, day 47.75), CTRL (top right, day 38.5), PI108 (bottom left, day 69.62), and FX2 (bottom right, day 47.88). White dots mark objectively-identified TC centers. Image size scales with domain size such that length scales are preserved.

radial profile of the azimuthal wind at $z = 1.5 \ km$, binning data in radial increments equal to the grid spacing of the simulation of interest. From this radial wind profile, storm structure is characterized by accumulating the statistics of the maximum wind speed, $V_m$, a proxy for the radius of maximum winds, $r_{ew} = r(0.75V_m)$, and two metrics for the outer radius, $r_0$, discussed below. The proxy for $r_m$ is used to capture the
outer edge of the eyewall while avoiding the noise in the wind profile around \( r_m \) itself.

Importantly, we focus here primarily on metrics of overall storm size. The maximum wind speed is very likely limited by the horizontal resolution, which is coarse relative to the small storm size that is an intentional consequence of the use of an artificially high rotation rate. Furthermore, though one would ideally analyze the gradient wind, this quantity is noisy and cannot be easily time-averaged without a more complex tracking algorithm that follows each individual storm in time. Given that the full wind is only expected to exceed the gradient wind within the eyewall (Bryan and Rotunno, 2009a) where wind speeds are already reduced due to the relatively coarse resolution in our simulations, we use the azimuthal component of the full wind for simplicity.

Following the results of Chapter 3, we estimate the outer radius using the simple slab boundary layer model of Emanuel (2004), which we briefly review here. The model assumes no deep convection, and combines mass continuity with the balance between radial advection of angular momentum and surface drag. The non-dimensional form of the resulting differential equation is given by

\[
\frac{\partial (\tilde{r} \tilde{V})}{\partial \tilde{r}} = \frac{C_d V_p}{w_{\text{cool}}} \frac{2 \tilde{r} \tilde{V}^2}{(\tilde{r}_0^2 - \tilde{r}^2)} - \tilde{r}
\]  

(4.2)

where \( r \) is the radius and \( V \) is the azimuthal wind speed, and tildes denote non-dimensional quantities, where we have non-dimensionalized \( V \) by \( V_p \) and \( r \) by \( \frac{V_p}{f} \). The primary assumption of Eq. (4.2) is a match between the Ekman suction rate at the top of the boundary layer and the radiative-subsidence rate, \( w_{\text{cool}} \), just above it. Eq. (4.2) is a Riccati equation with no known analytical solution. The value of \( w_{\text{cool}} \) is calculated from the assumed balance between subsidence-induced warming and radiative cooling

\[
w_{\text{cool}} \frac{\partial \theta}{\partial z} = Q_{\text{cool}}
\]  

(4.3)

where \( \frac{\partial \theta}{\partial z} \) is set to its pressure-weighted mean value in the layer \( z = 1.5 - 5 \) km (i.e. directly above the boundary layer) in the background state. Eq. (4.2) is solved numerically using a shooting method. As noted in Chapter 3, Eq. (4.2) carries with
it an additional mode of variability associated with the non-dimensional parameter \( C_2 = \frac{C_c V_p}{u_{out}} \), which represents the reciprocal of the non-dimensional Ekman suction rate and therefore controls the radial decay rate of the non-dimensional wind field in the far outer region of the storm.

Results from the axisymmetric simulations indicated that this outer wind model is capable of reproducing the entire axisymmetric equilibrium radial wind profile beyond the radius of maximum winds when the first term on the RHS of Eq. (4.2) is multiplied by a constant, taken as \( c = .3 \); this empirical adjustment accounts for deficiencies in the model assumptions, though the underlying physics are not currently understood and are the worthy subject of future work. Here we find that the best fit constant for each simulation is: CTRL \(-1.34\); PI43 \(-1.18\); PI108 \(-1.03\); FX2 \(-1.14\). This value is relatively constant across the simulations, with a mean of \( c = 1.17 \).

For the purposes of direct comparison with the axisymmetric results, we apply an identical methodology for estimating the outer radius by fitting the model to \( r_{ew} \) and integrating Eq. (4.2) radially outward, but taking the empirical best-fit constant \( c = 1.17 \). Additionally, as described in Chapter 3, we calculate an adjusted outer radius, \( r_0^* \), in which \( C_2 \) is fixed at its Control value, thereby providing a universal metric for storm size that is independent of the secondary variability that exists only in the non-convecting outer region at large radii.

### 4.3 TCs in Rotating RCE

#### 4.3.1 Self-aggregation

We first motivate the analysis of TCs in rotating radiative-convective equilibrium through a simple example of convective self-aggregation (Bretherton et al., 2005; Khairoutdinov and Emanuel, 2010) that was initially observed purely by accident. As noted above, the initial RCE state is defined based upon small-domain 3D simulations in a domain with side length \( L = 196 \, km \) and in which \( f \) was held fixed at a value of \( 5 \times 10^{-5} \, s^{-1} \). However, as \( f \) is successively increased, a threshold is crossed in
which the characteristics of convection transition from a homogenous, disaggregated state to an aggregated state in a finite period of time. This behavior is displayed in Figure 4-2, which depicts snapshots of the wind field at $z = 1.5 \, km$ at day 100 and the accumulated precipitation for days 98-100. In the disaggregated state, low-level wind speeds are uniformly small in magnitude and precipitation is randomly distributed within the domain. In the aggregated state, convection and precipitation are concentrated into a single region in the domain, and the horizontal wind field in this region resembles that of a very weak tropical cyclone. Aggregation does not occur for $f < 10 \times 10^{-5}$, but does occur for $f > 20 \times 10^{-5}$. In these latter cases in which aggregation occurs, aggregation is suppressed (at least on the time scale of 100 days) when the domain size is halved to $L = 92 \, km$ at fixed horizontal resolution.

As demonstrated by Held and Zhao (2008) and Khairoutdinov and Emanuel (2012), the rotation rate plays an important role in the length scale of organized convection in rotating RCE. This organization takes the form a TC-like vortex, whose theoretical length scale is proportional to $\frac{V_f}{f}$. The statistical equilibrium state may thus tend toward a “tropical cyclone world” (Khairoutdinov and Emanuel, 2012) characterized by one or more TCs, so long as this length scale is of the same order as, or smaller than, the domain size. Here, $\frac{V_f}{f} = 174 \, km$, corresponding to a diameter of $\sim 350 \, km$, which is approximately double the domain size, though the vortex that develops is clearly far weaker than would be expected given the thermodynamic environment, likely due to the limitations imposed by the domain size in combination with the coarse horizontal resolution. Though an interesting subject in its own right, a deeper analysis of the self-aggregation process is beyond the scope of this study. Here we simply use this observation as a launching point for analyzing TC size in the rotating RCE state.

4.3.2 Radial wind profiles in the presence of multiple TCs

Although rotating RCE can conveniently support multiple TCs simultaneously, it's not obvious that their respective wind fields will be sufficiently independent of one another to plausibly extract the radial wind profile of an individual storm. However,
the interaction of TCs appears to be primarily that of a simple mutual advection/co-rotation (Ritchie and Holland, 2006), such that storms that approach one another temporarily co-rotate before being repelled back to a distance where the interaction ceases. Direct mergers between two mature TCs are rare in these simulations; typically mergers occur only when one storm is already in a process of decay before being "absorbed" into another, mature storm.

As an example, we return to the PI43 snapshot in the top-left panel of Figure 4-2: Snapshot of windspeed (color) at day 100 and accumulated precipitation over days 98-100 (white contour, 2 cm interval) for four small-domain RCE simulations in which the rotation rate and domain size are varied (respective values listed above each plot). Horizontal resolution is 4 km in all cases.
Figure 4-3: Cross-section of the magnitude of the wind speed between the centers of the two dominant TCs in the lower half of the domain of the P143 snapshot (Figure 4-1, top-left panel).

4-1 in order to highlight the spacing between mature TCs and the honeycomb-like “moats” of very low wind speeds that separate individual storms. The wind profile cross-section connecting the center points of the two dominant TCs in the lower half of the domain is plotted in Figure 4-3. Remarkably, the wind speeds do in fact approach zero very near the mid-point of the two TC centers. Though this is a particularly clean example, such behavior appears to be commonplace at the interface between two or more adjacent storms, which is encouraging for the credible extraction of the azimuthally-averaged radial profile of each storm in the domain.

4.4 Size statistics

We begin with the statistics of the dimensional structural parameters. Figure 4-4 compares the PDFs and respective median values of $V_m$, $r_{cw}$, $r_0$, and $r_0^*$ across all objectively-identified TC snapshots within each simulation. Within the inner core, the distribution of $V_m$ increases slowly with increasing $V_p$, from a median of $22\,ms^{-1}$ in P143 to $30\,ms^{-1}$ in P108. The distribution decreases slightly going from CTRL to FX2 despite the doubled horizontal resolution in the latter. Similar qualitative
behavior is observed for \( r_{ew} \), whose median increases rapidly from 35 km (P143) to 207 km (P1108) with increasing \( V_p \), and whose respective distributions simultaneously broaden while retaining a largely symmetric shape. As for the broader circulation, our estimate of \( r_0 \) also increases rapidly with \( V_p \), while \( r_0^* \) increases slightly more gradually because \( C_2 \) has been held fixed.

A more insightful perspective lies in the analysis of the non-dimensional structural
parameters, where $V$ is non-dimensionalized by $V_p$ and $r$ by $\frac{V_p}{f}$, as shown in Figure 4-5. The distribution of non-dimensional $V_m$ is nearly constant across the simulations, with the exception of P143, whose median value is nearly twice as large. This suggests that each simulation is similarly resolved in non-dimensional space and thus lends credibility to a comparison across simulations. In the case of the low-$V_p$ simulation, the fact that it appears to be better resolved than the others is encouraging and is certainly preferable to being under-resolved relative to the other simulations.

The non-dimensional $r_{ew}$ still exhibits a significant increase with increasing $V_p$, with median values of .33, .62, and .77 for P143, CTRL, and P1108, respectively, indicating that this quantity in fact scales faster than the length scale $\frac{V_p}{f}$. Curiously, for FX2 the median non-dimensional $r_{ew}$ also increases significantly compared with the CTRL simulation, from .62 to .78. This behavior was not observed in the axisymmetric simulations. Moreover, the values for $r_{ew}$ are quite large relative to the axisymmetric results, in which the non-dimensional $r_{ew}$ values were $\sim$ .05 for the Control simulation.

Finally, the non-dimensional $r_0$ increases with increasing $V_p$, from 1.61 to 2.42 (Figure 4-5, lower-left panel), though the majority of this increase occurs between P143 and CTRL. Meanwhile, the non-dimensional $r_0^*$ brings the median values for the set of simulations into closer alignment (Figure 4-5, lower-right panel). All median values fall within the range 1.93 – 2.45. This result provides some evidence in support of the results of the axisymmetric simulations, in which it was found that $r_0^*$ scaled nearly linearly with $\frac{V_p}{f}$. Additionally, the much larger value for $c$ in three dimensions suggests that the outer wind field of 3D storms decay significantly more gradually than their axisymmetric counterparts.

The shape of the distributions of the outer radius provides additional information regarding TCs in this rotating RCE state. First, these distributions are relatively narrow, reflecting the fact that the outer circulation does indeed remain quite stable with time despite large variations in inner-core structure, corroborating both the axisymmetric results and observations of real TCs (Weatherford and Gray, 1988). Second, the distributions of $r_0$ have a prominent lower tail but only a minimal upper
tail above the mode of the distribution, indicating that TCs are not exceeding their characteristic size. This is in direct contrast to the axisymmetric results, in which TCs exhibit a transient super-equilibrium phase in both intensity and size, a state that is not likely to be missed by the tracking algorithm. Instead, the existence of the lower tail suggests that TCs either first intensify and then expand early in their life-cycle, or else first contract and then decay late in their life-cycle. Cursory observations of the output suggest that the former appears to be the common case. An algorithm
that tracks the life-cycle of individual TCs would allow for analysis of the trajectory of each storm through the joint \( (V_m, r_0^*) \) phase space, which will be the subject of future work.

Overall, TCs in this framework appear to be larger than their axisymmetric counterparts. In the latter, the typical magnitude of non-dimensional \( r_0^* \) was \( \sim 0.5 \). In three-dimensions, this magnitude is \( \sim 2 - 2.5 \), which qualitatively matches the results of Khairoutdinov and Emanuel (2012). Meanwhile, \( r_{ew} \) is significantly larger than in axisymmetry, though this result may be subject to several important caveats discussed below. This contrast in absolute sizes may reflect the tendency for convection in axisymmetry to be confined solely to the eyewall, whereas three-dimensional TCs must contend with the asymmetric nature of both radial turbulence and convection itself, as well as other asymmetric effects associated with translation and interaction with other TCs in the domain.

4.5 Discussion

Though these results are interesting, there are many caveats that must be acknowledged. First, the number of TCs in the domain may affect the structure of any single TC. In particular, the high-\( V_p \) simulation can only fit one TC in the domain. If this storm does not fit neatly within the domain, though, then the remaining space will be left open when it would otherwise be at least partially occupied by another TC in a larger domain. Because the entire domain is cooling radiatively, and radiative cooling must be energetically balanced by precipitation, which occurs almost entirely within TCs in the aggregated state, the existence of large regions of open space absent a TC implies that the lone TC in the domain must necessarily generate more precipitation than it might otherwise in a multi-TC state. Given constraints on the terminal velocity of rain and the constancy of boundary layer water vapor content as \( T_{ipp} \) or \( f \) is varied, this may lead to an expansion of the area of the precipitating region, which may cause an expansion of the TC. Indeed, the lone TC in PI108 weakens by about 50% during simulation days 55-65 before reintensifying back to its original
state, perhaps a consequence of the fact that the equilibrium state to which the TC tends cannot simultaneously exist at domain-wide equilibrium in the domain in which it is embedded. This effect may also be playing a role in the CTRL simulation as well, in which 2-3 TCs are typically present, yet it appears that a 4th storm could nearly fit into the remaining open space. More generally, one might expect this effect to decrease in influence as the domain is increasingly well-packed; how rapidly the magnitude of this effect would drop off is not known, and may be different depending on the specific quantity of interest.

Nonetheless, there seems to be a signal in overall storm size that emerges despite the above caveats. The distributions of $r_0$ are relatively narrow, a feature that is perhaps most evident in the case of P1108 despite the fact that its corresponding distribution of $r_{ew}$ is the broadest of all simulations. This provides evidence that scalings for overall storm size may be reasonably captured in the simulations presented here. Indeed, the radiative effect discussed above may require an expansion of the TC’s precipitation field, which may lead to an increase in the inner core size of the TC wind field, but if the broad outer circulation is not strongly affected by variability in the inner core, then it is plausible that the scaling for the outer circulation may remain largely unchanged.

4.6 Conclusions

Here we have explored the statistics of the size and structure of tropical cyclones that emerge within a set of idealized three-dimensional, rotating RCE simulations at statistical equilibrium. These RCE simulations are set-up under the identical thermodynamic environment as the axisymmetric simulations of Chapter 3. Importantly, here a full three-dimensional turbulence scheme is included that liberates us from the axisymmetric horizontal turbulent mixing length, $l_h$, which exerts a strong influence on the inner-core structure in axisymmetry.

The primary result of this analysis is the finding that a measure of the overall size of the storm, given by the outer radius calculated using a simple slab boundary layer
outer wind model and adjusted to account for dependence on the potential intensity in the far outer region of the storm, scales approximately linearly with \( \frac{V_p}{f} \) as was found in the axisymmetric simulations of Chapter 3. This provides further evidence that the theoretical length scale of TCs, \( \frac{V_p}{f} \), given by Emanuel (1986) does indeed govern the overall size of an equilibrium TC, corroborating the results of Khairoutdinov and Emanuel (2012) for varying \( T_{sat} \). Our proxy for the radius of maximum wind is surprisingly found to scale super-linearly with \( \frac{V_p}{f} \), though we have low confidence in the fidelity of this result given the coarse resolution of the model relative to the size of the TCs, which was made intentionally small in order to accommodate more TCs at lower computational expense.

Additionally, the distributions of size across our simulations appear to be firmly bounded from above, indicating that TCs do not undergo a transient super-equilibrium phase in size as was found in the axisymmetric case, at least in the context of storms that develop within the statistically-equilibrated TC world. This result suggests that the super-equilibrium phase may be a product of the parameterized turbulence scheme employed in axisymmetry or else to the unique dynamics associated with axisymmetric geometry. Moreover, this result appears to better match observations of TCs in nature, whose broad outer circulations are not typically observed to contract during its life-cycle (Chavas and Emanuel, 2010).

There remain several caveats to the analysis performed here that will be addressed in future work. In particular, for those simulations in which there are only one or a couple of TCs in the domain, the combination of the geometry of the domain and the need for precipitation to balance domain-wide radiative cooling may force the precipitating inner core of the TC(s) to expand relative to its true equilibrium value, which would be obtained in a domain that is neatly-packed with TCs. Additionally, dependencies on horizontal resolution must be further addressed. However, given the axisymmetric simulation results and the steadiness of the broad outer circulation relative to the turbulent inner core of real TCs (Weatherford and Gray, 1988), the behavior of the overall size of a TC may not be strongly affected by these issues. Finally, a better understanding of the physics of the outer wind region would help
improve our methodology for estimating $r_0$. 
Chapter 5

Conclusions and Future Work

5.1 Summary

Tropical cyclones in nature are known to span a large range of sizes, yet the factors that govern storm size remain enigmatic, whether in the context of an individual TC at genesis and during its life-cycle or in the context of its global distribution. Despite major advances over the past few decades in our understanding of the genesis, motion, intensity, and structure of TCs, as well as the two-way interaction between TCs and the environment (both atmosphere and upper ocean) in which they are embedded, size appears to behave largely independently of all of these factors and thus is effectively unconstrained. Theoretically, storm size, defined as the outer radius, $r_0$, where the winds vanish, is literally a free parameter, with the exception of a scaling for its theoretical upper bound given by the ratio of the potential intensity to the Coriolis parameter.

This work takes the first steps toward a more fundamental understanding of TC size. First, a climatology of size, as measured by an estimate of $r_0$, is created from the QuikSCAT satellite database. This climatology reveals that storm size is approximately log-normally distributed globally and spans a wide range of values ranging from 100 km to 1600 km, perhaps an indication that size is indeed an unconstrained random variable as intimated by current theory. However, the median values of size are distinct across basins, suggesting that size is modulated by variations in the ther-
modynamic environment, in the distribution of initial disturbances, or both. The
time-evolution of storm size exhibits a high degree of variability, but on average
storms are found to expand gradually early in their life-cycle before stabilizing.

TC size is then investigated in a highly-idealized thermodynamic environment
using the Bryan Cloud Model (CM1), with a focus on the determinants of the equi-
librium state. Overall, the dominant length scale for the equilibrium storm is found to
be $\frac{V_p}{f}$, matching the scale for the theoretical upper bound on TC size given by exist-
ing potential intensity theory. Contrary to conventional wisdom based on geostrophic
adjustment, the Rossby deformation radius is shown not to be fundamental to equi-
librium size.

In axisymmetry, the inner storm structure (i.e. intensity and size) of the equilib-
rium storm is modulated primarily by a single non-dimensional parameter, $\frac{V_p}{l_h}$. This
non-dimensional parameter represents the ratio of the storm length scale, $\frac{V_p}{l_h}$, to the
parameterized eddy mixing length, $l_h$. This parameter modulates the effective turbu-
lence felt by a storm of a given size; a higher value of $V_p$ corresponds to a storm that
is both more intense and larger, and therefore it feels a weaker effective turbulence,
resulting in a further increase in $V_m$ above and beyond a linear scaling with $V_p$. Mean-
while, the far outer region of the storm is modulated by a second non-dimensional
parameter, $\frac{C_d V_p}{U_{COOL}}$, that emerges from a simple slab boundary layer model of the outer
wind field that assumes no deep convection.

This idealized framework is then explored in a set of three-dimensional simula-
tions, liberated from the outsized influence of the axisymmetric parameterized turbu-
lent length scales that lack a strong theoretical basis. These simulations reveal that
overall storm size scales with $\frac{V_p}{f}$, as was found in axisymmetry, though this result
is the subject of ongoing work. When varying $V_p$, the radius of maximum wind was
surprisingly found to scale more rapidly than this length scale, though domain and
resolution dependencies limit the confidence that can be placed in this result.

In axisymmetry, the time-scales to structural equilibrium are found to be quite
long, as the model TC exhibits a transient phase during which storm size significantly
overshoots its equilibrium value before relaxing to equilibrium after more than 50 days
for the Control simulation. In three-dimensions, as well as arguably in observations, there is no evidence of such a super-equilibrium phase, suggesting that this behavior is specific to axisymmetric geometry.

Overall, this work has taken the first steps towards a more comprehensive, fundamental understanding of TC size. There remains a significant gap to be filled between the characterization of storm size in observations and in the idealized modeling world explored here. Nonetheless, these results in combination suggest that storm structure may potentially be represented simply as a combination of radial wind profile solutions in the convecting inner region and non-convecting outer region, each of which are subject to distinct, though reasonably well-understood, dynamics. Moreover, though the equilibrium results cannot explain the observed dynamics and distribution of storm size, they do indicate the end point towards which TC size evolves, even if such an evolution is actually too slow to be observed in nature. More research is needed to continue to build our understanding from all perspectives: observations, idealized models, and theory.

5.2 Future Work

There is a multitude of avenues for future work, as the work presented herein presents only the first steps towards understanding TC size in nature. Much more work remains to be done.

5.2.1 Ventilation and equilibrium size

The work performed here explored TCs in a veritable environmental paradise: axisymmetry in the absence of any vertical wind shear or other impediments to storm intensity. Tang and Emanuel (2010) derived a solution for a ventilation-modified steady-state potential intensity that accounts for the import of low-entropy air into the core of the TC by vertical wind shear. Thus, one sensible follow-on question is whether equilibrium TC size scales with the unmodified potential intensity in its original form or that modified by ventilation. Tang and Emanuel (2012) demonstrated
a framework for incorporating the effects of vertical wind shear, which is a non-axisymmetric process, into an axisymmetric model by imposing a locally-enhanced parameterized diffusion, which mixes external dry air into the core of the storm. Such a framework could be combined with the approach applied in the work presented here to explore how equilibrium size responds to a thermodynamic environment whose potential intensity is reduced due to the presence of steady ventilation. Additionally, one could start from a storm equilibrated to the unventilated potential intensity and quantify the time-scales for structural re-equilibration following the sudden onset of ventilation.

5.2.2 Transient size evolution

Given an equilibrium size and structure toward which a storm tends, what governs the time-dependent evolution towards this equilibrium solution? This work identified the existence of long time-scales to structural equilibrium in axisymmetry, particularly in the context of storm size. Further work is needed to quantify the processes and non-dimensional parameters that govern these time-scales, though some recent theoretical work (Emanuel, 2012), which derived an analytical solution for the time-dependent evolution of storm intensity, may provide some guidance. Though it is not clear whether this super-equilibrium period is applicable to the three-dimensional world, an understanding of the time evolution of size in both contexts would be illuminating. Storm size at genesis, as well as its subsequent evolution, remains largely unpredictable in nature, and analysis of the simplified axisymmetric framework may offer insights applicable to the real world, even in the context of an unphysical representation of turbulence. However, although the physics of the mature TC appears readily represented within a purely axisymmetric framework, it is not clear whether the genesis process can be properly captured in this way or whether it is fundamentally a three-dimensional process (e.g. Montgomery et al. (2006); Simpson et al. (1997)), particularly given the highly axially-asymmetric distribution of convection that typically pervades the incipient disturbance during its development.
5.2.3 Validation of outer wind field model

This work found that the simple slab outer wind model of Emanuel (2004), with a single empirical modification, could represent the entire outer wind field of the axisymmetric TC over a surprisingly large array of simulations. A deeper analysis of this outer wind model would be highly beneficial to understand the physics behind this modification and when it breaks down, as well as the extent to which the success of the model is a function of the simplicity of the numerical model set-up employed here. Perhaps if given a boundary layer that is resolved in much greater detail, the simple empirical modification will no longer be appropriate. Ideally, one seeks an updated outer wind model that properly captures the underlying physics of the outer wind region and therefore can be used to model the full outer wind field absent any empirical modifications. Though this slab model has been widely critiqued in the context of the inner-core wind field (Smith and Montgomery, 2008; Persing and Montgomery, 2003; Smith and Vogl, 2008), it has not been thoroughly tested in the context of the quiescent, non-convecting outer region, where simplicity and validity may more readily go hand-in-hand.

5.2.4 TC size in more complex environments

This work has identified the dominant scaling for equilibrium TC size within a highly idealized environment. Based on these results, more detailed simulations would be useful in exploring with much greater precision, particularly in three dimensions, how these results are modulated by the many real-world processes not resolved or included in this environment. Such processes include a detailed representation of the boundary layer, higher horizontal resolution within the eye, realistic radiative transfer, and inclusion of external environmental interactions such as vertical wind shear (as discussed above) and an interactive ocean. Moreover, though radiative-convective equilibrium is a simple and useful representation of a tropical atmosphere, the real tropics contains much greater horizontal heterogeneity, including overturning circulations such as the Hadley and Walker cells and intra-seasonal variability such
as Equatorial waves and the MJO (Kiladis et al., 2009). Thus, exploring TC size in idealized environments other than RCE, such as under the weak temperature gradient assumption (Sobel and Bretherton, 2000), may provide insight into TC size in different regional tropical climate regimes.

5.2.5 TC size in the current climate: explaining the log-normal

What determines the observed log-normal distribution of size in nature? Ultimately, a deeper understanding of the dynamics of TC size should entail an explanation of the nature and parameters (median, variance, etc.) of this distribution. Such an explanation may arise from the combination of a fundamental understanding of initial size and size evolution at the level of individual storms and the climatological statistics of those parameters deemed relevant to size. For example, evidence from observations and modeling suggests that the scale of the initial condition plays an important role in setting the size of the storm early in its life-cycle. If this relationship can be more precisely quantified (e.g. the specific variable/quantity whose length scale is relevant, the time-scales over which it is relevant), then an understanding of the statistics of initiating disturbances, as well as the underlying physics that generate such statistics, may be useful in explaining the statistics of storm size overall. Additionally, the statistics of the environments in which they are embedded may also be relevant, particularly in the context of a non-stationary climate, given evidence that the climate state can have a strong influence on the statistics of genesis itself (Rappin et al., 2010).

However, it should be noted that there is also evidence of more fundamental two-way interactions between TCs and climate which could potentially also manifest itself in terms of TC size. TC activity may have significant impacts on meridional ocean heat transport (Jansen and Ferrari, 2009) and the atmospheric general circulation (Hart, 2011) on seasonal or annual timescales. Moreover, Khairoutdinov and Emanuel (2010) put forth the hypothesis that the tropical climate is an example of a self-organized critical system, such that the tropics tend toward a critical phase transition
between disaggregated and aggregated convection, the latter of which includes TCs. Whether there is a role to play for a metric of TC activity that includes size, such as the Power Dissipation Index (Emanuel, 2005), in modulating climate is an open topic of research, but it is not out of the question that the log-normal distribution is one manifestation of such a complex interdependence within the climate system.

5.2.6 TC size under climate change

The effects of a changing climate on TC size is unknown. Theory (Emanuel, 1987) and GCMs (Knutson et al., 2010) indicate a global increase in potential intensity of ~10% due to a doubling of atmospheric carbon dioxide concentrations. Based on the \( \frac{V_f}{f} \) scaling for the equilibrium storm, one reasonable hypothesis entails a concurrent upward shift in the median of the log-normal size distribution. Such a hypothesis may be tested within GCM simulations. However, size in GCMs remains a largely unexplored topic under any climate. Quantifying variations in storm size in a changing climate and attributing these variations to variations in the statistics of potential intensity, initiating disturbances, genesis locations, tracks, and their underlying physical mechanisms, would be of great value in its own right, and also would provide useful datasets for application and validation of hypotheses for the theoretical relationships between TC size, the local environment, and climate. Undoubtedly, this is a particularly complex problem that integrates together the time-dependent physics of TC size at the individual storm level, the modulation of TC activity by the climate system, including the spatial and temporal distribution of genesis and intensification, and the large-scale changes in the climate system anticipated due to changes in radiative forcing, such as that associated with anthropogenic greenhouse gas emissions.
Bibliography


