Damping Optimization Using Transfer Function Criteria

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ABSTRACT

Seismic performance has become a key point in the design of every type of structures. Even the simplest buildings need protection in areas of high seismic activity. However, there is no method defined by codes or general knowledge to help engineers make choices about the design of seismic protection devices. Even though several theories to optimize the use of devices have been developed, there is little practical application in structural engineering. The purpose of this paper is first to settle on the elements that can be used to protect structures. By looking at their effects on structures, it was found that dampers are the easiest to use in an optimization process. After describing the need of progress in the field of earthquake protections, this paper focuses on the impact of additional damping in a 2-D frame. Finally, the method developed by Izuru Takewaki was studied and implemented. By looking at the limitation of the interstory drift, the algorithm produced the optimal distribution of the damping. In order to estimate the performance of the method, the results were compared to empirical damping distributions. A complete program was developed in order to apply the optimization method to a wide range of custom 2-D frames.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering
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1 General knowledge about earthquakes

1.1 Need to protect structures against earthquakes

The following section explains why we need to protect our structures and how since the first engineering discoveries, builders have tried to make earthquake-proof constructions.

1.1.1 Earthquakes

To understand how earthquakes can damage structures, we have to first review the nature of an earthquake:

“A sudden violent shaking of the ground, typically causing great destruction, as a result of movements within the earth’s crust or volcanic action.”

The Oxford dictionary

The waves that impact the structure are described as Rayleigh waves, their action provokes transversal and vertical acceleration of the ground.

1.1.2 Consequences of the earthquakes over the ages

The first step is to identify some indicators that estimate how powerful an earthquake is. The first information relayed after an earthquake is the number of victims. However this indicator is sometimes independent of the power of the disaster. Figure 1-1 shows that the most powerful earthquakes are not the deadliest, in fact the impact of an earthquake mainly depends on the level of construction in the hit area.
The deadly factor is mainly due to collapses and falls of debris, whereas the waves themselves just shake the ground. Even if we are able to define areas of high risks, predicting when and where earthquakes happen is still impossible. For this reason, we need to protect our population by building safe constructions. Figure 1-2 presents the ten most costly earthquakes in terms of destruction. An important point is that eight out of ten earthquakes happened between the earthquake of Loma Prieta (USA, 1989) and the earthquake of Tōhoku (Japan, 2011). When we combine the fact that earthquakes do not become stronger (Figure 1-3) with time, and the results of the damage (Figure 1-2), we understand that our structures, dealing with new constraints, need to become more resistant to earthquakes. Among the costliest earthquakes we can quote:

- Tōhoku earthquake, Japan (2011, magnitude 9)
- Great Hanshin earthquake, Japan (1995, magnitude 6.9)
- Sichuan earthquake, China (2008, magnitude 8)
- Northridge earthquake, United States (2010, magnitude 8.8)
Figure 1-2: Earthquakes with largest damage

Figure 1-3 presents a historical record of the earthquakes since 1900. This apparent increase in the number of earthquakes is only due to the multiplication of accelerometers around the world. The main idea presented in this document is that the earthquakes nowadays are not stronger than those in the past, but they generate more financial damage.

Finally, it is important to keep in mind that small structures (less than ten stories) are stiffer than high-rise buildings. This is a key criteria to establish which edifices are the most susceptible to collapse. We have to consider the stiffer structures.
Indeed the more flexible a construction is, the less amount of energy it needs to stay still. Figure 1-4 presents the transfer function of a system made of a mass, a spring and a damper. The transfer function presents the amplification factor that relates the ground displacement and the mass displacement. This coefficient varies with the frequency of the ground motion. For rigid structure (large ratio), the amplification converges to one, whereas soft structures (small ratio) can reach a ratio that can diminish the effects of the ground motion:

![Transfer function diagram](image)

*Figure 1-4: Transfer function*

The problem is that in our cities, these constructions with less than ten floors are the most common and they constitute the main tenements for the populations of large cities. Another interesting piece of information is a study carried out to estimate the consequences of medium earthquakes (magnitude 6 on Richter scale) hitting important cities presented in table 1-A below.

A major issue concerns the level of protection of the actual constructions. Because most of structures are ancient or designed without considerations of ground motions, they need to be upgraded to be able to resist and protect their inhabitants. Moreover, a significant number of large cities are located in dangerous areas where most of the constructions do not fit the requirements to resist remain safe during earthquakes.
<table>
<thead>
<tr>
<th>Cities</th>
<th>Inhabitants (M)</th>
<th>Possible causalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kathmandu, Nepal</td>
<td>1</td>
<td>69,000</td>
</tr>
<tr>
<td>Istanbul, Turkey</td>
<td>13.8</td>
<td>55,000</td>
</tr>
<tr>
<td>Delhi, India</td>
<td>21.7</td>
<td>38,000</td>
</tr>
<tr>
<td>Quito, Ecuador</td>
<td>2.2</td>
<td>15,000</td>
</tr>
<tr>
<td>Manila, Philippines</td>
<td>1.6</td>
<td>13,000</td>
</tr>
<tr>
<td>Islamabad, Pakistan</td>
<td>1.2</td>
<td>12,500</td>
</tr>
<tr>
<td>San Salvador, El Salvador</td>
<td>0.6</td>
<td>11,500</td>
</tr>
<tr>
<td>Mexico City, Mexico</td>
<td>8.8</td>
<td>11,500</td>
</tr>
<tr>
<td>Izmir, Turkey</td>
<td>2.8</td>
<td>11,500</td>
</tr>
<tr>
<td>Jakarta, Indonesia</td>
<td>10.2</td>
<td>11,000</td>
</tr>
<tr>
<td>Tokyo, Japan</td>
<td>13.2</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Table 1-A: Causalities predictions under a strength-6 earthquake [1]

1.1.3 History of earthquake-resistant protections

Unfortunately for the human being, the attractive areas of the globe (primary resources, geographical accesses) are located near high hazard seismic areas. Important cites all over the globe can be hit by powerful earthquakes (Los Angeles, West coast of the South America).

![Figure 1-5: Global seismic hazard map [2]](http://www.seismo.ethz.ch/static/GSHAP/)


Figure 1-5 shows that city like Istanbul and some others in China are located in the high danger areas. In order to continue to develop these cities, we need to understand how the first earthquake-proof structures of these areas were built.

An interesting point made by historians is the relationship between earthquakes and ancient great civilizations. They had to develop tools because of their geographical implantations. In fact most of the first large cities were located close to areas with dense seismic activity. The first discovery is that the power of the earthquakes appeared to create damage when structures have become more and more stiffer. When small villages were hit, only the stone chimneys and the stone temples collapsed; meanwhile, all the wooden houses were just deformed. Typically, historians agree to consider three types of structures as the first earthquake-proof tools.

1.1.3.1 Pyramidal shapes

It is a good starting point to understand first how earthquakes damaged ancient structures but also how ancient great builders civilizations developed tools to protect structure. The first point concerns the evolution of the stiffness of the constructions. The main factors are the available space on the ground and the technology to raise high structures. Because of the lack of strong, light and efficient material and mainly because of the available space (constructions were not restrained to fit in an available space located in a dense area), structures tend to be highly rigid.

*Figure 1-6: Mayan pyramid*
The first great constructions were the pyramids; their wide bases and their slenderness ratio make them highly rigid and therefore quasi earthquake-proof. The structure is so rigid that it moves as one rigid body following the ground motion. So that it prevents high drift values. The key factor of the pyramids is their vertical load repartition, which follows their triangular shape. Because of a large amount of mass located close to the ground, the center of gravity of the whole structure is closer to the ground than it would be for an equivalent rectangular building. The direct consequence is a reduction of the effect of ground motion.

*Figure 1-7: Egyptian pyramids*

1.1.3.2 Mortarless assemblies

One of the first civilizations to encounter the consequences of large ground motion is the Inca. The case of the city of Machu Picchu is one of the first constructions to use earthquake proof systems. We can observe some of the great walls of the city standing after hundreds years of violent earthquakes. Their longevity is due to their ability to deform and dissipate energy. Inca civilization discovered that to resist during an earthquake, the structure has to move to reduce the internal stresses. The key point in building these walls lies mainly in the knowledge of rock cutting. The Inca’s walls are mortarless, only the cut shapes enable the walls to stand. The flexibility created by this type of connections enables the structure to easily deform and thus reduces the internal effort (shear and bending). It means because the structure is a bit weaker and less rigid, its capacity to absorb waves energy is less
important and the flexibility of the connection enables the dissipation of the energy through friction.

![Figure 1-8: Example of mortarless wall](image)

Comparing this case and the first one, we can see that either super strong or super soft structures are able to resist during earthquakes.

1.1.3.3 First multi floor slender constructions

Nowadays, some of these first multi-floor structures still resist to powerful earthquakes. This is the case of the Japanese pagodas, these towers, built in wood or brick, still stand after thousands year. In fact, after the 1995 earthquake in Hyogo, only the pagodas remained undamaged even those located in fully devastated regions. The secret is the heart of these wooden towers. A central column carries all floors independently enabling differential displacement. The elements of a tower are made of a floor and the roof that supports its, it means the structure can experience high value of drift during ground motion but because of the lack of inter-story connections, the amount of drift does not create high values of stress. Moreover, the wood used allows larger deflections without any plastic deformations. These factors enable the pagodas to be the first earthquake-proof multistory structures. Even nowadays, projects are based on the pagoda structural scheme.
In conclusion, the evolution of the design constrains (height, lack of space) has created new issues about the dynamic behavior of structures. By combining the seismic hazard in the highly demographic areas and the lack of space, constructions have moved to gravity constraints to dynamic ones. Indeed, most of the new high-rise structures innovate in their damping systems to reduce the effect of lateral loading. From tuned mass damper to base isolation, structural engineers try to reduce the vulnerability of their project and constantly raise the bar for complexity and height. Figure 1-10 shows the evolution of the record for the tallest building.
1.2 Need of knowledge and improvement

One of the issues brought by high-rise slender buildings is their need for stiffer material; however, the typical construction materials like steel and concrete are not as good as was wood in resisting to drift during an earthquake. That is why some new tools need to be developed to increase the capacity of these slender buildings against ground motion.

One of the challenges of earthquake engineering is to gather data and to elaborate tools to predict and estimate the impact of earthquakes. At the point of development of our civilization, it is impossible to relocate people to avoid area of powerful earthquakes. It means we have to evolve in our methods to build so that our tall and slender structures can become resistant to earthquakes. In this case, to resist does not just mean to stand, but also means to avoid the maximum structural and non-structural damage. Even if an earthquake does not kill, it always has a period where the economy is paralyzed during the time needed for people to fix the damage. That is why, nowadays, new earthquake engineering principles tend to minimize the investment in an earthquake-proof structure by reducing the cost of repairs. This new aspect is tricky because it needs to merge several aspects of construction and engineering and costs estimation. Indeed, to work efficiently, this process requires data and information about how earthquakes impact structures, but also on tools to reduce the impact of waves. There are so many parameters that impact the damage of a structure that only global knowledge can be gathered to get some ideas about what type of design is efficient. In finality, each type of structure has to be tested and design following a independent process.

1.2.1 Different type of damage

With the evolution of construction materials, structures become taller and more slender. These new physical aspects involve new construction methods. Indeed, the effect of an earthquake is worse on this type of tall flexible structures. Moreover, with the financial investment that a high-rise structure represents, owners want
their structure to stand but also to match serviceability in order to ensure the comfort of its users. This new criteria impacts the new design rules, in fact old structures still stand because they encounter large deformations during earthquake. However, these deflections are not acceptable to fully ensure the comfort during ground motion. Equilibrium has to be found in order to satisfy each part.

1.2.1.1 Structural damage

From a structural point of view, the drift of a story creates a shear force in the frames. During earthquakes, because of the loads (internal masses) involved in a structure and the displacements created by the ground motion, the shear force usually causes the first floor to drift and collapse (Figure 1-10). The process of shear collapse starts when the first floor under the impact of the ground motion begins to move. The inertia of the structure above the first floor creates a delay in the response. It takes time for the rest of the floors to follow the motion. This differential drift creates high shear stress, which causes the collapse.

![Figure 1-10: Shear collapse after the 1989 San Francisco earthquake](image)

Figure 1-11: Shear collapse after the 1989 San Francisco earthquake

Figure 1-11 shows typical drift-induced damage. The first floor is totally shifted and the top floors are still straight and do not seems to encounter damage. This effect is due to the response of the structure, which can be interpreted as the sum of the responses of each mode. Because the drift is larger in the first floor in the first mode.
of a moment frame structure, the highest internal forces are located in this first frame, which carries the whole structure. The combination of high drift and P-delta effect generates high shear which causes the failure of the structure by shear drift. Even if, after an earthquake, a structure seems to be safe, some expertise usually has to be used in order to look for cracks or relevant signs of structural weakness, which costs money and time.

The main difference between drift induced and acceleration-induced damages is that acceleration creates mostly non-structural damage whereas drift directly impacts on the structure (Figure 1-11).

1.2.1.2 Non-structural damage

(Elements presented below come from the FEMA website: http://www.fema.gov)

FEMA defined three risks generated by non-structural damage:
- Life safety (Could anyone be hurt?)
- Property loss (Could a large propriety loss result?)
- Functional loss (Could the loss of a component cause an outage or interruption?)

Even if these risks do not deal with the collapse or loss of the structure, they must be taken into account in the design process of earthquake-proof systems. Indeed, functional loss damage can cause tremendous financial loss for companies and may take time to be repaired.

1.2.1.2.1 Life safety

Life safeties take into account every risk of injuries. The main cause of injuries caused by non-structural damage is the fall of components (glass, pipes...). One of the life risks is caused by the fall and is often forgotten in all the potential components. This fall prevents people to reach safety exits and therefore become exposed to other type of risks. Basically, the way nonstructural components are attached can be the cause of severe injuries (lights and fake ceiling).
There are a lot of examples where people were stuck in buildings because the fall of components prevented them to exit the structure. Even if this is not a deadly event, this type of situation delays the rescue and can prevent injured people from being treated on time. Another consequence concerns elevators. The typical nonstructural damage happens when the counterweight comes out from its rails and sometimes hits the cab.

1.2.1.2 Property loss

The risk of property loss concerns everything that is damaged or destroyed inside a structure during an earthquake. It takes into account material, commodities, and equipment. It is this type of element that causes the first risk (Life safety). It can be caused not just by fall but also by fire or water and so is considered an indirect loss. Because all of the components inside a building typically represent the price of the construction cost, the economical consequences can be really important. For instance during the Loma Prieta earthquake of 1989, a library of San Francisco spent more than 10% of the repair to rebind a few rare books.
1.2.1.2.3 Functional loss

A functional loss is a result of nonstructural damage that prevents the structure to be used for its main functions. It can be caused by life safety risk due to post earthquake or simply power or water outage. Nowadays, a simple power outage can paralyze the activities of every type of company from factories to banks. This risk is often the most expensive loss for companies after an earthquake. For instance after the earthquake of Tōhoku (Japan, 2011), SONY announced $3.2billion loss because of material shortage due to the inactivity of its suppliers. In areas with powerful earthquakes, police and fire stations have special procedures to remained operational in case of strong earthquakes, even companies that need to function 24 hours a day, like data servers, have back up plans to transfer their buy and sell orders to safety servers. However, the best solution is that structures become safe place to stay and work instantly after the shock.

1.2.2 Conclusion

Because the two types of damage are coupled during an earthquake, the disaster creates large loss of money and time. That is why a part of the financial investment dedicated to the construction of a project needs to be allocated to get a better understanding of the impacts of an earthquake in order to create efficient earthquake-proof tools.
2 Protection devices

2.1 Different control devices

The different tools currently used in structure to reduce the effect of lateral loadings are grouped into two different families. First there are the passive tools, they do not need an extra source of energy to reduce the impact of lateral loadings. Secondly, there are active control devices, which operate during a specific time. They are supplied by external source of energy and sensors set in the structures control their action.

Among the long list of tools developed, the most common ones are:

- Tuned mass damper
- Base isolation
- Viscous damper

Most of them can be equipped with active actuators in order to control the protection.

2.2 Different models of protection

2.2.1 Tuned mass dampers

The idea of the tuned mass damper (TMD) is to add a device in the structure in order reduce the displacements of the main structure. A TMD is made of a mass, a stiff connection to the structure and a damping component. One of the most typical localization of TMD is the top of high-rise structure. The mass of the TMD and its stiffness are calibrated to match one of the main frequencies of the structure. Doing so creates large oscillations of the TMD but significantly decreases the response of the structure. It is due to the relative motion of the TMD compare to the structure. Its motion opposed to the structure, and so it returns the structure to its straight position. Even if TMD are powerful tools, their effect is limited by their calibration based on a typical excitation frequency. That is why TMD are widely used to reduce the effect of the wind but they are not a good tool against earthquakes, first because
earthquakes are composed of a large range of frequencies and secondly because it takes some time for the TMD to reach their optimal efficiency.

Figure 2-1: TMD in Taipei 101

Figure 2-1 presents the TMD located in the top part of the Taipei 101 tower. The yellow sphere is the mass suspended with cables. This pendulum has an equivalent stiffness that depends of the length of the cable. The elements presents on the floor are dampers; they are attached to the frame of the building.

A specific point of the calibration of the tuned mass damper is that the parameters of the TMD need to be set at a certain value depending on the characteristics of the structure. Contrary to additional damping where an increase of the damping generally decreases the response, there is an optimal value of the TMD damping, which if exceeded will increase the response.

2.2.2 Base isolation

The principle of the base isolation is to separate the structure and the ground by setting a soft layer of rubber able to deform a lot during an earthquake. By installing a soft layer under the building, the potential deformation under ground motion is significantly reduced. In fact, the structure will stay still (Figure 2-2). This key principle will prevent high shear in the first level of the building and so decrease the damage due to drift.
The theory of the base isolation design is based on the global stiffness of the structure. Figure 2-3 presents the transfer function of a single degree of freedom. The variable $\rho$ is defined as the ration of the period of the ground motion compare to the period of the structure. If the structure is too stiff (Figure 2-3: $\rho < 1$), the minimum displacement experienced will be the ground motion. It means there is no attenuation of the excitation in the response. However, if $\rho$ increases up to 3, the response is only half of the excitation. In conclusion, to reduce the effects of the ground motion, the global stiffness has to be reduced. However, in a typical structure, the frames are designed using the codes. Something else must be used to decrease the global stiffness. That is where the base isolation comes into play.
The materials used to create this soft pillow under the structure are usually laminated rubber. The soft base will absorb the energy and deform, leaving the structure with a new ability to deform. Without the base isolation, a building behaves like a cantilever. However, with the new type of soft connections, the entire structure becomes able to shift at its base. This idea is based on the study of the transfer function of the entire structure behaving as one rigid body.

The active technology can be applied to base isolation. This coupling is used when the relative displacement of the structure compared to the ground is too large. In order to reduce it and prevent remaining oscillations after an earthquake, some active damping can be added to the device. This active damping is activated when excessively large displacements happen. Contrary to the TMD, the base isolation is not designed for a specific excitation frequency, which makes it really efficient to reduce the impact of ground motion. In fact because of the progress of earthquake engineering a lot of owners decide to upgrade their properties. However, the range of efficiency of the base isolation depends largely on the type of ground and on the fundamental frequencies of the structure itself. Also, under wind load the structure will experience a large deflection. Thus, some fuses need to be set in order to activate the system only during earthquakes. Moreover, in order to avoid unequal repartition of the softness under the structure, the aging of the rubber has to be followed and all the pieces installed must have the same age.

Figure 2-4: Typical laminated rubber
Figure 2-4 presents a typical configuration of a base isolation. The straight steel member on the right side is the fuse.

2.2.3 Dampers

2.2.3.1 General effects

The final tool consists of adding damping in the structure using different types of dampers:
- Viscous dampers
- Hysteretic dampers
- Friction dampers

2.2.3.2 Friction damper

The Coulomb law defines the friction damping. The device develops an action of constant magnitude $F$ and in the same direction as the speed vector.

$$ F = F \text{sign}(\dot{u}) $$  \hspace{1cm} (2.01)

If a friction damper is linked to a body under periodic displacement, the general behavior of the device can be described as follow:

$$ u(t) = \dot{u} \sin(\Omega t) $$  \hspace{1cm} (2.02)

The work developed in one cycle consists of the shaded area in Figure 2-5:

$$ W_{\text{cycle}} = 4F\dot{u} $$  \hspace{1cm} (2.03)
Even if this type of damper presents a good behavior to limit the displacement of a structure, the fact that the developed force does not depend on the magnitude of the velocity limits its utilization.

2.2.3.3 Hysteretic damper

This second type of damper is based on the hysteretic effect. The main example of this non-linear effect can be described by the yielding of a steel piece. Like the elastic behavior, the inelastic depends on the properties of the material. In order to protect the device against damage due to high-deformation, the force developed is limited to $F_y$. Again if the device is slaved using a harmonic displacement

$$u(t) = \dot{u}\sin(\Omega t)$$  \hspace{1cm} (2.04)

The force will be defined depending on the value of $u$:

$$F = k_h u$$

where:

$k_h$ is the elastic stiffness (N/m) 

Then, the material reaches its limit and so from B to C:

$$F = F_y$$  \hspace{1cm} (2.06)$$

Then from C to D, it is a simple elastic behavior. However the difference is the residual strain (D). Under perfect elastic deformation, the material should reach its initial shape after deformation. Finally, we can define the worked done during a single cycle:

$$W_{cycle} = 4F_y \left( \frac{u}{u_y} - 1 \right)$$

$$W_{cycle} = 4k_h u_y^2 \left( \frac{u}{u_y} - 1 \right)$$  \hspace{1cm} (2.07)$$

This device is widely used, however, its behavior is too complex to be used in an optimization process. Because of this non-continuous behavior, the mechanism of this device needs to be discretized between the different working phases.

2.2.3.4 Viscous damper

This paper focuses on the use of viscous dampers, first because they are widely used and they are also easy to set in existing structures without many modifications. A typical viscous damper looks like a piston. The main difference is that there is no external energy applied to move the arm. The two chambers are full of a fluid of high viscosity. Some holes in the piston head enable the fluid to travel between the chambers. When a displacement with a specific speed is applied on the piston, the arm moves in the fluid, which creates the viscous reaction of the damper.

\[ F_{developed} = c \times \dot{u} \]

where:
\( c \) is the damping coefficient (\( N \cdot s/m \))
\( \dot{u} \) is the speed of the arm (\( m/s \))  

(2.08)

Using once again the forcing displacement constrain:

\[ F = c \Omega \dot{u} \]

\[ u = \dot{u} \sin \Omega t \]

\[ \dot{u} \]

\[ t = \frac{\pi}{\Omega} \]

Figure 2-7: Behavior of a viscous damper [1]

This time it is interesting to notice the dependence on the period of excitation. It means, if this device is used to limit the effect of earthquake, the effect will be different based on the spectrum of the ground motion.

The principle to change the damping coefficient is to play on the size of the holes. If the size increases, the size of the obstacle decreases and the fluid can migrate easily between the chambers.

![Figure 2-8: Typical composition of a viscous damper [1]](image)

The reason why viscous dampers are common tools in building engineering is because their effects decrease the value of the displacement of the structure but generally (when damping is proportional to the stiffness), it does not change the parameters of the structure like the fundamental period. Finally, because one device can be set in each moment frame of the structure, the combinations to mesh the framing with dampers are large. Moreover, because dampers are used several at a time, the effect on the earthquake protection is sprayed in the entire structure and in consequence the bracing does not have to transfer large loads but several small ones instead.

2.2.3.5 Different configuration

Because the force developed by a damper is principally a function of its velocity, it is crucial that when installing the damper, the bracing configuration maximizes the elongation of the device.

\[ \varepsilon_{\text{damper}} = k \cdot \varepsilon_{\text{structure}} \text{ where } k > 1 \]

where:

- \( \varepsilon_{\text{damper}} \) is the elongation of the damper at floor \( n \)
- \( \varepsilon_{\text{structure}} \) is the drift of the floor \( n \)

Several configurations have been tried in order to maximize the effect of the damper. The most commonly used are presented in Figure 2-9. The main parameters are the angle between the members and the frame and also the angle of the damper.

![Figure 2-9: Typical damper configurations](image)

(a) Diagonal-brace damper
(b) K-brace-damper
(c) Upper toggle-brace-damper
(d) Lower toggle-brace-damper
The choice of the type of configuration is mainly based on architecture and availability for the project. For instance, (a), (c) and (d) hide openings.

2.3 Proposal

Even if all the tools presented previously have advantages to be set in a structure, the versatility of the viscous damper and their adaptability for every type of construction make them one of the most popular tools to reduce the impact of earthquakes. Following this trend, this report will focus on the impact of the repartition of dampers in a multi-story model with the idea of maximizing their effect.
3 Effect of viscous dampers

3.1 Single degree of freedom experiment

To understand the most important parameters for the optimization process, it is important to know how they impact the structure. If we consider a single degree of freedom system made of a mass, a spring and a viscous damper:

![Figure 3-1: Single-degree of freedom system](image)

The main parameters are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>Mass of the system [kg]</td>
</tr>
<tr>
<td>$c$</td>
<td>Damping parameter [Ns/m]</td>
</tr>
<tr>
<td>$k$</td>
<td>Stiffness [N/m]</td>
</tr>
</tbody>
</table>

*Table 3-A: Main parameters of a single-degree of freedom system*

Without any damping, when the excitation frequency equals to the fundamental period of the system, the mass enters into resonance. However because of the properties of the materials used to build structures, the damping effect exists naturally in the structure. For a steel frame the damping will be around 2% and 5% for a concrete structure. To improve the performances of the structure, we can make the stiffness and the mass vary.
The equilibrium equation, is:

\[ M \ddot{u} + c \dot{u} + ku = -M \ddot{u}_g \]  \hspace{1cm} (3.01)

Using complex value to introduce the frequency of the excitation:

\[ \frac{U}{\bar{U}_g} = \frac{-M}{-M \omega^2 + ic \omega + k} = H(\omega) \]  \hspace{1cm} (3.02)

where:

- \( i \) is the complex constant
- \( \omega \) is the pulsation of the excitation

The first idea is to adjust the mass of the system. However, because the fundamental period and the damping of the structure depend on the mass, the variation of the response is not linear with respect to \( M \).

The following content presents the variation of the response when the main parameters vary one by one. The idea is to determine the most efficient variation, which reduces the maximum value of the transfer function. However, in order to contain the effect of the variation, the chosen parameter will have to impact only the maximum of the transfer function. The modulus of the complex transfer function is obtained using the definition of \( H \) in equation 3.02.
First case: Variation of the mass

![Figure 3-2: Effect of the mass variation](image)

**Table 3-B: Transfer function parameters**

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass</th>
<th>Damping</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>$M = 10^2$ kg</td>
<td>$\xi = 0.1$</td>
<td>$k = 10^4$ N/m</td>
</tr>
<tr>
<td>H2</td>
<td>$M = 5 \times 10^2$ kg</td>
<td>$\xi = 0.1$</td>
<td>$k = 10^4$ N/m</td>
</tr>
<tr>
<td>H3</td>
<td>$M = 10^3$ kg</td>
<td>$\xi = 0.1$</td>
<td>$k = 10^4$ N/m</td>
</tr>
</tbody>
</table>

Second case: Variation of the stiffness

![Figure 3-3: Effect of the stiffness variation](image)
<table>
<thead>
<tr>
<th></th>
<th>$M = 5 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H1$</td>
<td>$\xi = 0.1$</td>
</tr>
<tr>
<td></td>
<td>$k = 10^5 \text{N/m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$M = 5 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H2$</td>
<td>$\xi = 0.1$</td>
</tr>
<tr>
<td></td>
<td>$k = 10^4 \text{N/m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$M = 5 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H3$</td>
<td>$\xi = 0.1$</td>
</tr>
<tr>
<td></td>
<td>$k = 5 \times 10^4 \text{N/m}$</td>
</tr>
</tbody>
</table>

Table 3-C: Transfer function parameters

Third case: Variation of the damping

![Figure 3-4: Effect of the damping ratio variation]

<table>
<thead>
<tr>
<th></th>
<th>$M = 5.1 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H1$</td>
<td>$\xi = 0.1$</td>
</tr>
<tr>
<td></td>
<td>$k = 10^4 \text{N/m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$M = 5.1 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H2$</td>
<td>$\xi = 0.2$</td>
</tr>
<tr>
<td></td>
<td>$k = 10^4 \text{N/m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$M = 5.1 \times 10^2 \text{kg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H3$</td>
<td>$\xi = 0.05$</td>
</tr>
<tr>
<td></td>
<td>$k = 10^4 \text{N/m}$</td>
</tr>
</tbody>
</table>

Table 3-D: Transfer function parameters
3.2 Need for optimization

Although acting on mass and stiffness seems to be a good starting point to develop an optimization theory, it may not be feasible for a real structure. First of all, stiffness, mass and fundamental period are strongly coupled. It means that in order to reduce the amplitude of the response, the stiffness needs to be increased. And this change creates a modification in the value of the fundamental period. One of the potential consequences could be that the new period matches the frequency content of the earthquake.

The table below describes the typical acceleration spectrum of an earthquake in Cambridge, MA defined by ASCE 7. In this example, the stiffness is increased in order to reduce the drift-induced damage. However, the increase in the stiffness reduces the fundamental period that varies from 0.6 s to 0.4 s. The consequence of this variation is that the fundamental period now stands for the highest ground acceleration (0.17g to 0.28g). Based on this simple experiment, it becomes clear that one of the most powerful earthquake protections is one based on the increase of damping value in the structure.

![Response spectrum ASCE 7](image)

*Figure 3-5: Acceleration spectrum*
In conclusion, because the effect of stiffness or mass variation are too complex to be easily coupled with an optimization process, it appears to be more efficient to deal only with damping and let the stiffness be defined by the codes. The idea is to proceed in two steps:
- Design of the stiffness for wind load (codes)
- Damping repartition defined for earthquakes

The consequence of this division is that not only the structure will match the requirements of the codes but it will also prevent the damage of earthquakes.

### 3.3 Damping proportional to stiffness

In order to define a percentage of damping, the notion of Rayleigh damping is used. The general formulation sets that we can find two coefficients in order to verify the following equation:

\[ C = \mu M + \lambda K \]

where:
- \( C \) is the damping matrix
- \( M \) is the mass matrix
- \( K \) is the stiffness matrix
- \( \mu \) is the mass proportional Rayleigh damping coefficient
- \( \lambda \) is the mass proportional Rayleigh damping coefficient

However, the objective of this report is to apply damping optimization to structural engineering. Because in this field the dampers are linked to the moment frames that create the lateral rigidity of the structure, it is more accurate to apply the equation 3.01 using the following simplified form:

\[ C = \lambda K \]  \hspace{1cm} (3.04)

The next step is to relate the rate of damping \( \xi_{eq} \) to the property of the system.

\[ \xi_{eq} = \frac{1}{2} \lambda \omega_1 \]  \hspace{1cm} (3.05)
Where $\omega_1$ is the first fundamental period of the structure. The optimization process uses an equal repartition of the damping to initialize the process. The choice is made to use the following equation (3.06) to relate the damping value and the damping ratio:

$$c = 2 \frac{\xi_{eq}}{\omega_1}$$

(3.06)

Where $c$ is the initial amount of damping set at each level of the structure, $k_1$ is the lateral stiffness of the first floor.

3.4 Extension to multiple degrees of freedom system

This section of the report presents the extension of the previous part, because there is going to be more that one drift value, we first need to define criteria. Then by changing the damping value at each floor, the optimal configuration will be found. In the previous part of this paper, the damping ratio parameter appeared to be the best starting point to reduce the impact of ground motion. In this section, the number of degree of freedom is increased in order to see if general conclusions can be generated or if the optimal solution behavior appears to be changing too much. In this last case, a step-by-step solution will need to be constructed in order to reach the optimal criteria by convergence.

Before running some analyses, a criteria needs to be defined. As this study focuses on the reduction of the drift value, a combination of the values of the drift at each floor.

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean</td>
<td>$\frac{1}{n} \sum_{i=1}^{n} Drift_i$</td>
</tr>
<tr>
<td>SRSS</td>
<td>$\sqrt{\sum_{i=1}^{n} (Drift_i)^2}$</td>
</tr>
<tr>
<td>SAV</td>
<td>$\sum_{i=1}^{n} \sqrt{(Drift_i)^2}$</td>
</tr>
</tbody>
</table>

Table 3-E: Drift criteria
### 3.4.1 Two-degree of freedom

The model used is described in Figure 3-6.

![Figure 3-6: Dynamic model two-degree of freedom](image)

Table 3-F presents the different parameters used to obtain the values of the different drifts.

<table>
<thead>
<tr>
<th>( M = \begin{bmatrix} m &amp; 0 \ 0 &amp; m \end{bmatrix} )</th>
<th>( K = \begin{bmatrix} 2k &amp; -k \ -k &amp; k \end{bmatrix} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = \begin{bmatrix} 2c &amp; -c \ -c &amp; c \end{bmatrix} )</td>
<td>( T = \begin{bmatrix} 1 &amp; 0 \ -1 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>( A = K + i\omega_1 C - \omega_1^2 M )</td>
<td>( \bar{U} = T \cdot A^{-1} \cdot \begin{bmatrix} -m \ -m \end{bmatrix} )</td>
</tr>
</tbody>
</table>

**Table 3-F: Parameters of the study**

The first step is to look at how the minimum SRSS drift varies in function of the amount of damping but also which damping repartition creates the lowest value of drift. The following figures are obtained for a damping ratio of 10%.
Figures 3-7 and 3-8 show Min1 and Min2, which are not the minimum for the case plotted but for the corresponding SRSS. It enables us to see how different the solution between the minimums of the drift are compared to the drift that composed the optimal value of the SRSS.

With 10% damping, the minimum drift for each floor is obtained by deploying all the damping available in the first floor. Thus, the minimum SRSS and SAV
correspond to this configuration (See Figure 3-8: Drift first floor). This is due to the fact that the criteria of the drift are based on the first mode response.

![Drift criteria with a damping ratio of 0.1](image)

*Figure 3-9: SRSS and AM criteria*

With this quick experiment, we can easily show that for a specific amount of damping, at least an optimal configuration of damping repartition can be found in order to minimize global criteria based on the drift.

If we repeat the experiment by increasing the amount of damping, we can create optimal solutions that consist in a repartition between the two floors and not just in the first one. The results are presented on Figures 3-10, 3-11 and 3-12. In this case, the optimal configuration to minimize the drift in the second floor is no longer the same that minimizes the first floor drift. Moreover, the corresponding solution minimizes the drift at one floor but creates a large drift in the other one. That is why the SRSS and the SAV are an intermediate solution.
In Figure 3-12 the indications Min1 and Min2 are not the minimum for the case plotted but for the corresponding SRSS. It enables to see how different the solutions between the minimums of the drift are compared to the drift that composed the optimal value of the SRSS.
Finally, in order to see how important the amount of damping is in causing the SRSS to vary, Figure 3-12 presents the minimum SRSS for a two degree of freedom system function of the damping ratio.

Figure 3-12: SRSS and AM criteria

Figure 3-13: 2-DOF SRSS function of the damping ratio
It is interesting (3-13 and 3-14) to notice that the increase of the damping ratio for the first percentages (1% to 10%) has a great impact on the reduction of the damping (whereas it is the SRSS criteria or the minimum drifts). However, the values of damping ratio above this value of 10% do not have important impact on the reduction. From an economical point of view, it means that the financial investment for dampers will be much more efficient that the money spent to increase the damping value from 10% to 15%. This point is critical for two reasons; First, it shows how important the impact of a good optimization can be. Secondly, it informs us about the idea of filling the structure with dampers can be inefficient and economically disastrous.
Figure 3-15: Evolution of the optimal repartition function of the damping ratio

All the results for the 2-DOF are detailed in appendix 3-1.

3.4.2 Three-degree of freedom

Before looking at a potential optimization process, a last study will focus on the optimization of a three-degree of freedom system.

The model used in part 3.3 is reused and updated as follow.

\[
M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \quad K = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \\
C = \begin{bmatrix} 2c & -c & 0 \\ -c & 2c & -c \\ 0 & -c & c \end{bmatrix} \quad T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \\
A = K + i\omega_1 C - \omega_1^2 M \quad \ddot{U} = T \cdot A^{-1} \cdot \begin{bmatrix} m \\ -m \end{bmatrix}
\]

Table 3-G: Parameters of the study
Previously, we used two different criteria (SRSS and SAV). However, they showed small differences in the optimal repartition of the damping over the whole structure. That is why the study of the three-degree of freedom system is done using only the SRSS. As previously done, the first step consists in finding the optimal repartition using an allowable damping ratio. The following plots represent the variation of a drift criteria depending with C1 and C2, the values of the dampers installed in the first 2 floors. Because the system has three degrees of freedom, the condition on the constant sum of the dampers is used to express the value of C3. In these figures, the value of each damper varies from 0% to 100% of the allowable total damping (based on a damping ratio of 15%).

**Drift of the floor number 1**

![Drift 1 with a damping ration of 0.15](image)

![Drift 2 with a damping ration of 0.15](image)

![Drift 3 with a damping ration of 0.15](image)

*Figure 3-16: 3-DOF Drift 1*
The damping repartition that creates the minimum drift at the first floor consists of installing all the damping in the first floor. This configuration is the one expected from the study done for a two-degree of freedom system.

**Drift of the floor number 2**

In this case, the damping combination that minimizes the drift of the second floor is an even distribution of the damping between C1 and C2. The extreme dark blue line stands for $C3 = 0$. This seems normal as the drift of floor number 2 is defined as the relative motion of the second floor compared to the first floor. In order to minimize this value, both floors need to be damped.
Drift of the floor number 3

In this last case, the available amount of damping is too small to be able to use some in the third floor. That is why all the damping is sprayed between the first two floors.
SRSS criteria

In this case, all the optimal solutions are reached by splitting the damping between C1 and C2. This phenomenon was observed in the previous part concerning the two-degree of freedom system. We can easily generalize this solution. By making the assumption that in case of a lack of global damping (\(\xi\) too small), the damping will be set in the first floor. This is due to the deformed shape of the first mode (second degree curve). The highest floor drift is reached for the bottom floor and so the dampers are located in this section in order to damped the vibrations and so prevent the drift induced-damage.

The previous experiment is now done with a higher damping value (30%), the results are presented on Figures 3-20, 3-21 and 3-22.
In the second situation, the damping ratio is sufficient to make the distribution complex. First, the configuration that minimizes the drift of the third floor is a combination of C1, C2 and C3 (all non-zero). Then the SRSS is composed of a damper in the first two floors, but the SRSS is not obtained when only the first floor drift is reached.

The final step of the study is to figure out if the location of the SRSS moves continuously when the damping increases or if it jumps between optimal configurations.
As previously stated, the amount of damping installed in the structure becomes less efficient after 15%. On the other hand, the money used to increase the amount of damping varies linearly with $\xi$. That is one of the reasons why when dampers are chosen the amount of damping is usually close to 15%.

Figure 3-22 presents the evolution of the repartition of the dampers as a function of the damping ratio. As shown previously, when the amount of damping is too low, all the damping is allocated to the first floor. What is interesting is the behavior when $\xi$ becomes sufficient to install damping in the second floor. We could expect the damping to be mainly located on the first floor and the damping of the second floor to increase slowly. In fact, around 20% of the damping is transferred on the second floor. (Figure 3-22)
Figure 3-22: 3-DOF Optimal distribution function of the damping ratio

The plot on Figure 3-22 presents a general shape of the curves but composed of smaller oscillations. These are due to the approximation of the code. It is defined by cutting the amount of damping into 400 slots, which are superimposed to create the final distribution. However, this approximation does not prevent the results to be interpreted and the general shape can be extracted from Figure 3-26.

In conclusion, the study of the multiple cases developed above shows the existence of an optimal configuration of the damping. However, the computation to reach this optimum is time consuming. In the next section of this paper, an optimization process will be presented and the experiments of the current section will be used as controls tests. Another important point to construct a program that converges to the minimum is that the global variation of the minimum creates a bowl where the optimal configuration is located. It is crucial because it means the optimization process will not have to make the difference between a local and a global minimum.

All the results for the 3-DOF are detailed in appendix 3-2.
4 Optimization Process

4.1 Method

The current section presents a method to obtain the optimal damping distribution. Then a case study will be used to evaluate the efficiency of the method.

4.1.1 Global idea

Because the method has to be easily applicable to structural engineering, a good start consists in the following hypotheses:

- The stiffness is defined using the codes (ASCE)
- An initial global amount of damping should be allocated
- All the dimensions of the structure are known
- The analysis will be run in a 2D plan
- The dampers used are viscous dampers

Secondly, we need to defined criteria on which the optimization will focus. Based on the damage created by an earthquake, two possibilities can be explored. Acceleration, can be considered, because it provokes damage on the equipment inside the structure, but the effect of drift impacts directly the safety of the structure. It means that drift will be the only effect damaging the structural members.

The idea behind the process developed is to set up the dampers in the most efficient way in order to reach optimal settings. Nowadays, when structural firms have to put viscous dampers to counteract earthquake effects, usually dampers are set in the first floors with the idea to reach a certain damping ratio (usually 15-20%).

With the method developed in this paper, the idea is the return to the basics. By looking at how the transfer functions behave when we modify the damping coefficient, we are able to create a step-by-step process updating all the values of the dampers.
4.1.2 Theory

The method used in this paper is based on the optimization process developed by Izuru Takewaki (2009). The particular point of this method is that it is strongly based on the variation of the transfer functions.

4.1.2.1 Notations

<table>
<thead>
<tr>
<th>$N$</th>
<th>Number of floors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_i$</td>
<td>Displacement of floor $i \in [1, N]$</td>
</tr>
<tr>
<td>$\ddot{u}_g$</td>
<td>Ground acceleration</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Mass allocated to floor $i \in [1, N]$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Stiffness allocated to floor $i \in [1, N]$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Damping allocated to floor $i \in [1, N]$</td>
</tr>
<tr>
<td>$T$</td>
<td>Drift matrix $[N \times N]$</td>
</tr>
<tr>
<td>$U_i(\omega)$</td>
<td>Fourier transform of $u_i$, $i \in [1, N]$</td>
</tr>
</tbody>
</table>

*Table 4-A: Notations*

4.1.2.2 Model

As defined previously, the initial amount of damping used to start the computation is based on the proportionality with the stiffness.

$$
C = \lambda \ast K \\
\xi_{eq} = \frac{1}{2} \lambda \omega_1
$$

(4.01)

We start with the equilibrium equation defined at each time $t$ using matrix notation for the system defined in figure 4-1.

$$
K u(t) + C \dot{u}(t) + M \ddot{u}(t) = -M \ddot{u}_g(t)
$$

(4.02)
Using the complex notation for a system under harmonic excitation, equation 4.01 becomes:

\[(K + i\omega C - \omega^2 M)U(\omega) = -m\ddot{U}_g(\omega)\]  \hspace{1cm} (4.03)

Because the optimization focuses on the main cause of structural damage which is drift, we need to transform the vector \(U(\omega)\) into the drift vector \(\delta(\omega)\).

The drift at level \(i\) is defined as:

\[
\delta_i(\omega) = \begin{cases} 
(U_i - U_{i-1})(\omega) & \text{if } i \neq 1 \\
U_1(\omega) & \text{if } i = 1
\end{cases}
\]  \hspace{1cm} (4.04)

To extend this relationship to the entire model, we use the matrix \(T\).

\[
T = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & \cdots & 0 & 0 \\
0 & -1 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \cdots & -1 & 1 \\
0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
\]  \hspace{1cm} (4.05)
\[ \delta(\omega) = T U(\omega) \]  

(4.06)

We have seen in part 3 that the response is the most extreme around the fundamental frequencies and especially around the first one \( \omega_1 \). Following this idea, we define:

\[ \bar{U}(\omega_1) = U(\omega_1)/\bar{U}_g(\omega_1) \]  

(4.07)

So equation 4.02 applied for the fundamental first period becomes:

\[ (K + i\omega_1 C - \omega_1^2 M) \bar{U}(\omega) = - \left( \begin{array}{c} m_1 \\ \vdots \\ m_n \end{array} \right) \]  

(4.08)

To simplify the notation, we define:

\[ A(\omega). \bar{U}(\omega) = - \left( \begin{array}{c} m_1 \\ \vdots \\ m_n \end{array} \right) \]  

(4.09)

4.1.2.3 Optimization problem

From a mathematical approach, an optimization is defined with one quantity to minimize. Usually, it is a sum of positive terms. Then depending on the problem, a second equation can be used, in our case the damping amount has to stay constant. Following this principle, our problem can be express using the following equations:

\[ S_{amp} = \sum_{i=1}^{N} |\delta_i| \]  

(4.10)

\[ \sum_{i=1}^{N} c_i = c_{tot} \]  

(4.11)

Equation 4.10 expresses that the criteria to minimize is the sum of the inter-story drift amplitudes. Equation 4.11 restraints the sum of the damping values to a certain amount \( c_{tot} \). Summing the absolute value is a good indicator because it is fairly precise but more important; it is linear, which enables computation to run faster.
A good approach to this problem is to look at it from an energy point of view. Indeed, the system consists of damping actions and displacements. The Lagrangian relates these two parameters:

\[ L(C, \lambda) = \sum_{i=1}^{N} |\delta_i(C)| + \lambda \left( \sum_{i=1}^{N} c_i - C_{tot} \right) \]  

(4.12)

where:

\( \lambda \) is a Lagrangian factor

Using the partial differentiate with respect to \( c_j \), equation (4.12) becomes:

\[ \left( \sum_{i=1}^{N} |\delta_i(C)| \right)_j + \lambda = 0 \ (\forall j \in [1, N]) \]  

(4.13)

Because equation 4.13 depends on \( \lambda \), it would be useful to use the fact that 4.13 is true for each value of \( j \). By combining the different equations 4.13, we are able to create a new value that enables us to define the final parameter to optimize. To do so, each equation for \( j > 1 \) is subtracted to equation corresponding to \( j = 1 \). Then we can create the parameter \( \gamma_j \ (\forall j \in [1, N - 1]) \).

\[ \gamma_j = \frac{\left( \sum_{i=1}^{N} |\delta_i(C)| \right)_{j+1}}{\sum_{i=1}^{N} |\delta_i(C)|}_{j+1} \ (\forall j \in [1, N - 1]) \]  

(4.14)

In the equation 4.14, the subscript 1 is used to define \( \gamma_j \). This can be explained using experience and knowledge about damping. If the goal is to reduce the overall motion of a structure, a damper has to be installed in the first frame. That is why the criteria are defined based on this position, which is always equipped with a viscous damper if the stiffness distribution is roughly constant. Based on the definition of 4.14, the optimal situation for \( C_{j+1} \) is reached when \( \gamma_j = 1 \).
4.1.2.4 Solution to the problem

Before going into the process itself some parameters need to be defined especially those depending on partial differentiation with respect to $c_j$. First, the differentiation of equation 4.06 gives:

$$A \ddot{U}_j + A_j \dot{U} = 0$$

$$\begin{bmatrix}
1 & 0 & \cdots & \cdots & 0 \\
0 & 0 & & & \vdots \\
\vdots & \ddots & \ddots & & \vdots \\
\vdots & & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & 0 & 0
\end{bmatrix}$$

$$A_1 = 
\begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & & \vdots \\
\vdots & & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & 0
\end{bmatrix}$$

$$A_j = 
\begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & & \vdots \\
\vdots & & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & 0
\end{bmatrix}$$

$$\forall j \in [2, N]$$

Because of the way $A$ is defined, we can assume it is regular, so it can be defined using equation 4.13:

$$\ddot{U}_j = -A^{-1}A_j \dot{U}$$

Then using equation 4.04 for $\omega = \omega_1$:

$$\delta_j = -TA^{-1}A_j T^{-1} \delta$$

The next step consists in switching back to real numbers. To do so, the length of the vector $\delta$ must be defined.

First, we have for $\delta_i$ component of the vector $\delta$:

$$|\delta_i| = \sqrt{(Re[\delta_i])^2 + (Im[\delta_i])^2}$$

$$|\delta_i| = \frac{1}{|\delta_i|} \left(Re[\delta_i]Re[\delta_i] + Im[\delta_i]Im[\delta_i] \right)$$
Equation 4.21 is obtained by differentiating a function composed of a square root and a square. Concerning the real and imaginary function their differentiate are defined as follows:

\[
(Re(f))_j = Re((f)_j) \\
(Im(f))_j = Im((f)_j)
\]  

(4.22)

Then the differentiate of \( \delta_j \) with respect to \( c_k \).

\[
\delta_{jk} = TA^{-1}A_kA^{-1}A_jT^{-1}\delta - TA^{-1}A_jT^{-1}\delta_k
\]  

(4,22)

In order to proceed with a step-by-step optimization, we need to define the linear increment of \( y_j \) (equation 4.14) called \( \Delta y_j \):

\[
\Delta y_j = \left( \frac{1}{B_1} \frac{\partial B_{j+1}}{\partial c} - \frac{B_{j+1}}{B_1^2} \frac{\partial B_1}{\partial c} \right) \Delta c
\]

\[
= \frac{1}{B_1} \left( \frac{\partial B_{j+1}}{\partial c} - \frac{\partial B_1}{\partial c} y_j \right) \Delta c
\]

(4,23)

where

\[
B_j = \left( \sum_{i=1}^{N} |\delta_i| \right)_j
\]

(4,24)

And

\[
\frac{\partial B_i}{\partial c} = \{ |\delta_i|_{i1} + \ldots + |\delta_n|_{i1} , \ldots , |\delta_i|_{in} + \ldots + |\delta_n|_{in} \}
\]

(4,25)

\[
|\delta_{jk}| = \frac{1}{|\delta_i|^T} \{ Re[\delta_{ik}] Re[\delta_{ij}] + Re[\delta_i] Re[\delta_{jk}] + Im[\delta_{ik}] Im[\delta_{ij}] \\
+ Im[\delta_i] Im[\delta_{jk}] \} - |\delta_{ik}| \{ Re[\delta_i] Re[\delta_{jk}] + Im[\delta_i] Im[\delta_{jk}] \}
\]

(4,26)

Where:

\[
\begin{bmatrix}
\delta_{1jk} \\
\vdots \\
\delta_{njk}
\end{bmatrix} = TA^{-1}A_kA^{-1}A_jT^{-1} \begin{bmatrix}
\delta_{1} \\
\vdots \\
\delta_{n}
\end{bmatrix} - TA^{-1}A_jT^{-1} \begin{bmatrix}
\delta_{1k} \\
\vdots \\
\delta_{nk}
\end{bmatrix}
\]

(4,27)

In order to complete the system to optimize, we need one more equation. The last relationship is defined using 4.11:
\[ \sum_{i=1}^{N} \Delta c_i = 0 \quad (4.28) \]

The next step of the problem is to define an optimization step. This parameter is crucial because a step too large will bring bad results, and too small will take too much time to run.

\( N_{\text{step}} \) is defined as the number of step by cycle in the optimization process. So we now have:

\[ \Delta y = \frac{1}{N_{\text{step}}} (y_f - y_0) \quad (4.29) \]

### 4.2 Adaptation of the algorithm

#### 4.2.1 Particular case

When the algorithm turns negative one of the coefficients \( c_j \). The first step is to deal with this case without updating the c’s coefficients. Then the value of the \( c_j \) is equally divided between the other coefficients. At the next step, the matrix \( R \) is computed. The column \( j-1 \) and the line \( j \) are removed to solve Equation 4.23.

The next particular case occurs when the first coefficient \( c_1 \) becomes zero. In that case \( B_1 \) becomes 0, so equation 4.14 is irrelevant. That is why the main coefficient, which used to be \( c_1 \) has to be turned into another one. In our case, it seems logical that the next subscript selected is the next non-zero damping coefficient. In this particular case, all the parameters needed to solve the optimization problem are recalculate from the beginning to match the new value of the main parameter.
4.2.2 Points to define

In the method defined by Izuru Takewaki, some arrangements need to be defined. This is mainly due to the fact that the method was defined using perfect tools (springs, dampers, mass). For instance, the method to obtain the stiffness matrix needs to be developed; secondly some points of the iteration process need to be defined.

4.2.2.1 Iteration process

A choice needs to be made for a particular step of the process: when one of the $c_j$ becomes 0. The optimization algorithm stands that a new vector $\Delta y$ has to be used based on the new value of $\gamma_0$. Because a new parameter has been created to find the variation of each dampers value, it seems logical to start a “new loop.” To simplify the consequences of this choice, the main loop will see its boundary conditions increased by another $N_{step}$ steps. It means the longest loop can reach $(N - 1) * N_{step}$ where $N$ is the number of floor.

4.2.2.2 Definition of the stiffness matrix

As explained previously, the method defined by Takewaki is based on perfect spring and mass dynamic model. However, the first step to transform a 2D frame into a spring and mass model is to define a process to extract the stiffness matrix. A process needs to be created to obtain the stiffness matrix from every type of frame. Because the method is based on the model of the shear beam stiffness (approximately same stiffness at each level), we have to investigate using different loadings and constrain to see which method reaches the closest shear beam behavior.

The first load combination consists in applying a point load $F$ at the top of the frame and computes the coefficient $k$ at floor $I$ using:
\[ k_i = \frac{F}{(u_i - u_{i-1})} \] 

(4.30)

The second method is based on what could be the loading corresponding to the first mode shape. A load \( F \) is applied at each floor (N loads) at the same time and the stiffness coefficients are computed as follows:

\[ k_i = \frac{F \times (N + 1 - i)}{(u_i - u_{i-1})} \] 

(4.31)

The final process is based on a more theoretical approach; a load \( F \) is applied step-by-step at each floor and for each load case, a coefficient \( k_i \) is computed.

\[ k_i = \frac{F_i}{(u_i - u_{i-1})} \] 

(4.32)

We notice that the most consistent results are obtained for rigid body constraint and fixed supports which is logical as you constrain the structure from bottom to top in order to make it react as a rigid homogeneous mass and spring structure. Secondly, the type of loading (Equations 4-30, 4-31, 4-32) does not have a large impact on the results. That is why the custom loading (Equation 4-32) seems to be a good approach to get the stiffness matrix.

**4.2.3 Results using theoretical models**

Before applying the method to real frames, it is to understand how it behaves when parameters like stiffness or damping vary. Using the results obtained in Appendix 4-1, a model is built. The different cases are:

<table>
<thead>
<tr>
<th>Number of floors</th>
<th>Mass per floor [kg]</th>
<th>Stiffness per floor [N/m]</th>
<th>Damping ratio [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( 10^5 )</td>
<td>( 4.10^7 )</td>
<td>10 (1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 20 )</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 8.10^7 )</td>
<td>10 (3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>20 (4)</td>
</tr>
</tbody>
</table>

*Table 4-B: Cases to study*
To obtain the results, a MATLAB code is used. The choice of MATLAB is based on the easy way to manipulate non-constant size matrices. In order to extend the study, another method is compared to the Takewaki method. This other way to set the damping is based on the number of the floor $i$ compared to the rest:

$$c_i = \frac{N - i + 1}{\sum_{i=1}^{N} i} \cdot c_{total}$$

where:
- $N$ is the number of floors
- $c_{total}$ is the total amount of damping

The results will focus on the way the optimization converges to the final damping distribution but also on a comparison between the transfer functions of each story.

<table>
<thead>
<tr>
<th>Dampers</th>
<th>Case (1)</th>
<th>Case (2)</th>
<th>Case (3)</th>
<th>Case (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 [%]</td>
<td>56</td>
<td>42</td>
<td>56</td>
<td>42</td>
</tr>
<tr>
<td>C2 [%]</td>
<td>44</td>
<td>36</td>
<td>44</td>
<td>36</td>
</tr>
<tr>
<td>C3 [%]</td>
<td>0</td>
<td>22</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>C4 [%]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C5 [%]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 4-C: Final damping distribution*

The first comment about the four tests is that we have equivalent optimal configurations for situation (1) and (3) and for (2) and (4). The results seem to show different configuration for different damping ratio. However, if the mass is constant and the stiffness varies, it does not impact the results. The detailed results are presented in Appendix 4-1.
4.2.4 Irregular stiffness distribution

4.2.4.1 General results

Because this method can be applied to various structures and geometries, some cases can be defined with non-uniform stiffness. In these particular situations, the first floor is typically stiffer than the others, which from a dynamic point of view impacts its response by reducing the displacement. Thus the damper in the first floor can be transferred to the second floor. The following example presents the optimal configuration in a three-degree of freedom structure where the first floor is twice as stiff as the second and third ones.

In this example, all optimum distributions (for a single drift or the SRSS criteria) present a set of dampers made of at least C2 and C3. It means if the Takewaki method needs to be extended to this kind of irregular problem, the problem of changing the main damper coefficient, which by default, is the first one needed to be allowed during the computation.
4.2.4.2 Behavior of the algorithm

We need to understand the main changes when the algorithm sets the first (and main) damper to zero. The example of a three-degree of freedom is used to show what changes are made concerning the elimination of rows and columns in the matrix defined by Equations 4.23 and 4.28:

4.2.4.2.1 First case \( c = c_1, \ c_2 = 0 \) or \( c_3 = 0 \)

This case is the more general one. Its specificity is that:

\[
\forall j \in [1, N] \quad j > j_{\text{main}}
\]  
(4.34)

That is why the general formulation of the optimization coefficient has the same form for all \( j \):

\[
\gamma_j = \frac{\left(\sum_{i=1}^{N}\delta_i^j\right)_{j+1}}{\left(\sum_{i=1}^{N}|\delta_i^j|\right)_{j+1}}
\]  
(4.35)

The following two equations present the deleting process if \( c_2 = 0 \) (4.36) and if \( c_3 = 0 \) (4.37).

\[
\begin{pmatrix}
\frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_1} - \frac{\partial B_1}{\partial c_1} \gamma_1\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_2} - \frac{\partial B_1}{\partial c_2} \gamma_1\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_3} - \frac{\partial B_1}{\partial c_3} \gamma_1\right) \\
1 & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_1} - \frac{\partial B_1}{\partial c_1} \gamma_2\right) & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_2} - \frac{\partial B_1}{\partial c_2} \gamma_2\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_3} - \frac{\partial B_1}{\partial c_3} \gamma_2\right) \\
1 & 1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
\Delta c_1 \\
\Delta c_2 \\
\Delta c_3
\end{pmatrix} =
\begin{pmatrix}
\Delta \gamma_1 \\
\Delta \gamma_2 \\
0
\end{pmatrix}
\]  
(4.36)

\[
\begin{pmatrix}
\frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_1} - \frac{\partial B_1}{\partial c_1} \gamma_1\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_2} - \frac{\partial B_1}{\partial c_2} \gamma_1\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial c_3} - \frac{\partial B_1}{\partial c_3} \gamma_1\right) \\
1 & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_1} - \frac{\partial B_1}{\partial c_1} \gamma_2\right) & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_2} - \frac{\partial B_1}{\partial c_2} \gamma_2\right) & 1 & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial c_3} - \frac{\partial B_1}{\partial c_3} \gamma_2\right) \\
1 & 1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
\Delta c_1 \\
\Delta c_2 \\
\Delta c_3
\end{pmatrix} =
\begin{pmatrix}
\Delta \gamma_1 \\
\Delta \gamma_2 \\
0
\end{pmatrix}
\]  
(4.37)
4.2.4.2.2 Second case \( (\text{Main } c=c_2, c_1=0 \text{ or } c_3=0) \)

This time equation 4.34 is no longer verified. Some indeces are smaller than \( j_{main} \). That is why the definition of the criteria \( \gamma_j \) is more complex:

\[
\gamma_j = \begin{cases} 
  j & \text{if } j < j_{main} \\
  j + 1 & \text{if } j > j_{main}
\end{cases}
\]

(4.38)

And so is defined

\[
\gamma_j = \frac{(\sum_{i=1}^{N} |\delta_i|)_j}{(\sum_{i=1}^{N} |\delta_i|)_{j_{main}}}
\]

(4.39)

And finally the final matrix equation is written:

The following two equations present the deleting process if \( c_1 = 0 \) (4.40) and if \( c_3 = 0 \) (4.41).

\[
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_1} - \frac{\partial B_2}{\partial c_1} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_1} - \frac{\partial B_2}{\partial c_1} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_2} - \frac{\partial B_2}{\partial c_2} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_2} - \frac{\partial B_2}{\partial c_2} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_3} - \frac{\partial B_2}{\partial c_3} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_3} - \frac{\partial B_2}{\partial c_3} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\Delta c_1 \\
\Delta c_2 \\
\Delta c_3 \\
\end{pmatrix} =
\begin{pmatrix}
\Delta \gamma_1 \\
\Delta \gamma_2 \\
0 \\
\end{pmatrix}
\]

(4.40)

\[
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_1} - \frac{\partial B_2}{\partial c_1} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_1} - \frac{\partial B_2}{\partial c_1} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_2} - \frac{\partial B_2}{\partial c_2} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_2} - \frac{\partial B_2}{\partial c_2} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{B_2} \left( \frac{\partial B_1}{\partial c_3} - \frac{\partial B_2}{\partial c_3} \right) \gamma_1 \\
\frac{1}{B_2} \left( \frac{\partial B_3}{\partial c_3} - \frac{\partial B_2}{\partial c_3} \right) \gamma_2 \\
\end{pmatrix}
\begin{pmatrix}
\Delta c_1 \\
\Delta c_2 \\
\Delta c_3 \\
\end{pmatrix} =
\begin{pmatrix}
\Delta \gamma_1 \\
\Delta \gamma_2 \\
0 \\
\end{pmatrix}
\]

(4.41)

4.2.5 Conclusion

The evolution of damping values through optimization seems to follow the same convergence process: first the dampers of the higher floors are set to zero whereas the values of the lower vary without stable behavior. Then the values converge slowly to make the optimization criteria reach the value of 1 indicating that the best
damping repartition has been found. In all cases the algorithm stops because one of
the \( \gamma \)'s (Equation 4-39), usually it is the first one that reaches the optimal criteria.
For a same stiffness, increasing the damping ratio adds more dampers in the
structure but also brings not only one optimal criterion close to one, but an
additional one.

The study of the transfer function is an important point. Indeed the optimization is
based only on the behavior of the structure for the first mode. Getting an extended
overview of the optimization is a good way to understand what happens for the
higher modes. For the first case, we can see that the optimization reduces
significantly the drift transfer function for floors 1, 2 and 3. However it creates a
larger response for the pulsations around 10 to 30 rad/s for floor 3, 4 and 5. It can be
easily explained by the fact that the Takewaki method deletes the dampers at these
higher floors to concentrate the damping at the lower floors. For the transfer
functions of the third and fifth floor, we can see a peak appearing near 26.2 rad/s,
which is the third mode of the structure. Even if it means that for these particular
floors in these particular frequencies domain the Takewaki method is not as good as
the other methods, it is important to keep in mind the difference between the value
of the peak corresponding to the first mode. For instance, in the third floor, 0.03 SI
compared to the one obtained for the third mode at the same floor 0.007, there is
still a difference of around 80\% between the drift values. (Appendix 4-4 presents a
flowchart of the algorithm)

4.3 Comparison between theory and algorithm

4.3.1 Definition of the loops

In order to estimate the power of the optimization process, we need to define
relevant criteria based on the results. First of all, for the cases with less than four-
degree of freedom, we can find the optimal criteria by making the values of the
dampers vary. For the algorithm, after several tests, a good estimation of the
number of steps needed has been determined. The comparison between the algorithm and the theoretical results are presented in the table below.

For a system with N-degree of freedom:

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of steps (Maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>$(N - 1) \times 1000$</td>
</tr>
<tr>
<td>Genetic algorithm</td>
<td>$(100)^{N-1}$</td>
</tr>
</tbody>
</table>

*Table 4-D: Number of steps for the worst case*

An average computer is able to do two million operations per minute. By assuming that the looping processes are made of approximately 200 operations for the optimization process and around 20 for the genetic algorithm, a five-degree of freedom system would take:

<table>
<thead>
<tr>
<th>Method</th>
<th>Operations</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization</td>
<td>$200 \times 4 \times 1000$</td>
<td>$\sim 25\text{s}$</td>
</tr>
<tr>
<td>Genetic</td>
<td>$20 \times 100^4$</td>
<td>$\sim 33\text{h}$</td>
</tr>
</tbody>
</table>

*Table 4-E: Time consumption*

Table 4-E shows how powerful the algorithm is. By creating a step-by-step method that can converge to the optimal solution instead of searching randomly, an efficient process has been created.

Finally, appendices 4-2 and 4-3 present a three-degree of freedom system solved with both genetic and optimization algorithm. For the genetic algorithm, the looping is based on the following process:
The problem of this system concerns the level of coverage. Some combinations are useless, for instance when \( C_1 = C_{\text{total}} \) and \( C_2 = C_{\text{total}} \). In that case \( C_3 < 0 \) so the results are not computed. Using a cutting of thousands of the final value helps to reach accurate results.

\[
\begin{align*}
C_1 & \text{ Varies from 0 to } C_{\text{total}} \text{ (step 1/1000)} \\
C_2 & \text{ Varies from 0 to } C_{\text{total}} \text{ (step 1/1000)} \\
C_3 & \text{ Computed using the conservation of the amount of damping}
\end{align*}
\]

Regarding the optimization algorithm, the basic cut is a thousand steps for a typical looping, but when a damper reaches the zero value, another thousand steps are added to restart a new system.

4.3.2 Results

The results (Appendix 4-3) obtained using Takewaki’s method are really close to the optimal theoretical solution obtained using the genetic algorithm. This simple control enables us to extend the method to larger models.
5  Full program

5.1  Structure of the program

5.1.1  Need for structural optimization software

The final part of this thesis focuses on the development of a tool to easily apply the optimization with every type of 2-D frame. In fact because this program is flexible concerning the inputs, it turns out that increasing its range of action is easy if the user can turn its problem into $M$, $K$, $C$ matrices. However, such a process can be time-consuming to be done for each model. That is where the possibilities of computing can help the user to decrease the time allocated to the construction of models.

![Flowchart of the full algorithm](image)

*Figure 5-1: Flowchart of the full algorithm*

The idea is to move from a pure theoretical optimization process to a more user-friendly and open program. The main step to enable this interaction between the designer and its program is to keep the numbers in the systems and only use visual effect and real object to create the model of the problem. The theory developed by Takewaki is a perfect illustration of this principle. Where the algorithm needs stiffness, the user would prefer to use cross-section properties and other
characteristics that will impact the stiffness. The user does not want to access the full stiffness of the problem. Moreover, after the optimization, the program reaches its limits. However, the user wants to use and work with his/her model.

5.1.2 SAP2000 featuring MATLAB

The choice to use MATLAB and SAP2000 is based on the Application Programming Interface (API) developed for SAP200, which enables a lot of software, like MATLAB, to control SAP200 and use its output data. The advantage of this cooperation is that MATLAB is really efficient with iteration and operations with matrices whereas SAP2000 can analyze every possible type of 2-D frame defined by this problem. MATLAB will be used as the main control platform for the entire algorithm and SAP200 will create a real full model of the structure and save different type of dampers repartition (chosen by the users), in order to run analyses (time history, response spectrum). In the end, the user becomes able to try different custom shapes of 2-D frame without considering geometrical matrices (members and nodes), that is why, using SAP2000, the real model can be built and then MATLAB will work with the transformation of the real object to parameters needed for the optimization.
5.2 Program

Figure 5-3: Main steps of the program

Figure 5-3 presents the main steps of the process and the components used to execute the functions. Even if MATLAB controls SAP, the principle of the code is more to use MATLAB as a powerful calculator to optimize and then update the model in SAP2000.

5.2.1 Definition of the geometry using MATLAB

The first step of the process consists in turning geometric parameters into a full frame with custom geometry. The parameters of the user described below:

- Topology of the frame
- Members to delete
- Lateral system
- Dampers

5.2.2 Topology of the frame

This first geometrical definition had two ways to be done. The first one was to ask the user to input the matrix of the nodes and then based on the relationship between the nodes in a rectangular frame MATLAB would have meshed the framing. The second option starts with a global constant frame of a rectangular envelope. The program creates the envelop of a rectangular frame based on these parameters.
More importantly, they are numbered using the same numbering system as SAP200 (useful to control SAP2000 with its default command to create a 2-D frame and then work with the same numbering). Figure 5-4 presents a typical pre-frame. The green labels are the members, the yellow ones are the nodes and the red ones are the supports.

![Figure 5-4: Typical 2-D frame envelope](image)

### 5.2.3 Members to delete

The next step is to obtain the final geometry, which can be irregular, wishes by the user. Even if the user has large opportunities to define his/her frame, it is important to keep in mind that the frame must have only one “top.”

![Figure 5-5: Unallowable geometry (two tops 24 and 36)](image)
Figure 5-5 presents a geometry, which cannot be treated by this optimization, indeed suppressing members 28, 32 and 12 creating two different “tops.” It is due to the fact that the equivalent stiffness geometry cannot be expressed as tri-diagonal.

5.2.4 Lateral system

The next step, to be able to cover the largest number of cases, consists of giving the possibility of installing bracing in the structures. It is an option, which is set by indicating the 2 supports surrounding the braced column. Figure 5-6 presents the results of this option.

![Figure 5-6: Bracing installed]

5.2.5 Dampers

Finally, the location of the frames that are going to receive the dampers is input as the number of the node receiving the dampers number one. All the steps described enable the program to save data to send to SAP2000. However, in order to keep the user aware of the process, each modification of the envelope is displayed in real time to the user. That gives him/her the possibility to stop the process if something is wrongly defined. Moreover, in order to limit these input mistakes, each choice is incorporated to an infinite loop. The only way to exit the loop is to get an expected answer or to get the approval of the user.
Appendix 5-1 presents the different models that can be obtained for a single frame envelope.

5.2.6 SAP200 (1)

After this first interaction between the user and MATLAB, all the information needed to build a SAP2000 model of the structure is transferred using the API coding. The real advantage of this connection between software is the time spent. To build the model defined by Figure 5-8 requires a few minutes, whereas the program takes a few seconds. However, there are some elements that are defined only for this part of the algorithm:

- Type of supports
- Cross-sections used
- Type of constrains (Body or Diaphragm)
- Nodal masses

One solution to deal with the importation of external data, like pre-defined cross-section is to create a blank document with all the elements pre-imported. When MATLAB opens SAP2000, it also calls this pre-set file and fills it with the necessary information to build the model.
SAP2000 is now used to extract the stiffness coefficient associated at each floor. The process to obtain this information is based on two things. First, a load of constant magnitude is applied individually on each node of a straight frame going from bottom to top of the structure. Figure 5-9 shows the load definition for the leading example. It consists of four horizontal unitary loads applied on the left size of the frame.

The final process is based on a more theoretical approach: a load $F$ is applied step-by-step at each floor and for each load case, a coefficient $k_i$ is computed.
\[ k_i = \frac{F_i}{(u_i - u_{i-1})} \]

where \( i \) is the indicator of the floor
\( u_i \) is the displacement of the \( i \)th floor created by \( F_i \)
\( F_i \) is the force applied on the \( i \)th floor

The stiffness coefficients are collected by MATLAB and gathered in a vector.

### 5.2.7 Optimization

Now that the \( \mathbf{M}, \mathbf{K} \) matrices are defined, the optimization process can be used to generate the optimal stiffness distribution. The results obtained by this step are described in Appendix 5-2.

### 5.2.8 SAP2000 (2)

Finally, the choice is given to the user to use different models:
- Optimal damping distribution
- Solution with 1 to \( N \) identical dampers
- Proportional to the stiffness
- Constant distribution

All the different possible models are saved in a folder using date and hour as a name. Then, the user can run analyses with the saved files. Because there are too many options and coding this step would be useless, choice is allowed to define custom time history of acceleration spectrums.

Appendix 5-2 presents a full study. All the elements presented are obtained using the algorithm and the final analyses are run for each outputs files and then compiled in the final report.

In order to compare different states of damping, an additional internal damping (based on Rayleigh damping) will be added. In addition to the drift results, we can say that the optimal repartition based on the Takewaki’s method is more convenient.
because only three dampers need to be installed, and they are located at the bottom of the structure.
6 Conclusion

With the constant increasing of the population, the need to build in high seismic activity region is going to become common. In order to protect actual and futures constructions, engineers need to develop tools and technics to find efficient systems to reduce the effect of the ground motion. The method developed by Takewaki shows that it is possible to compute powerful and fast analytical tools to make the constructions costs economically profitable.

We saw that viscous dampers are a simple and efficient tool that can fit easily with architectural constrains. Even if this paper deals only with linear structure, we found out that it is possible by looking at a perfect model to not only converge to the optimal damping distribution but faster than a genetic algorithm.

With the increase in computing possibilities, we are now able to run non-linear analysis to estimate the damage after a series of earthquakes.

In order to cover more material, this study could be extended to 3-D models. In fact because the optimization works with matrices, only a method to convert 3-D frame into a typical stiffness matrix needs to be developed.
7 References


Kanno Y, “Damper Placement Optimization in a Shear Building Model with Discrete Design Variables: A Mixed-Integer Second-Order Cone Programming Approach”, Department of Mathematical Informatics, University of Tokyo, Tokyo 113-8656, Japan


8 List of documents

Appendix 3-1 Theoretical optimal distribution 2-DOF
Appendix 3-2 Theoretical optimal distribution 3-DOF
Appendix 4-1 Cases studied with the algorithm
Appendix 4-2 Results theory
Appendix 4-3 Comparison theory vs. algorithm
Appendix 4-4 Flowchart of the algorithm
Appendix 5-1 Possible bracing configurations
Appendix 5-2 Typical complete-case study
Appendix 3-1
Theoretical optimal distribution 2-DOF
RESULTS

Parameters

Number of DOF = 2
Mass = 80000 kg
Stiffness = 200000000 N/m
Damping ratio = 0.1
First period = 0.2 s
Total damping = 2588854 N.s/m

SRSS

Drift minimum = 0.003 m
C1 = 2588854 N.s/m (100 %)
C2 = 0 N.s/m (0 %)

SAV

Drift minimum = 0.004 m
C1 = 2588854 N.s/m (100 %)
C2 = 0 N.s/m (0 %)

Drift 1

Drift minimum = 0.003 m
C1 = 2588854 N.s/m (100 %)
C2 = 0 N.s/m (0 %)

Drift 2

Drift minimum = 0.002 m
C1 = 2588854 N.s/m (100 %)
C2 = 0 N.s/m (0 %)
--- Parameters ---

Number of DOF = 3
Mass = 80000 kg
Stiffness = 400000000 N/m
Damping ratio = 0.5
First period = 0.14 s
Total damping = 18305965 N.s/m

--- SRSS ---

Drift minimum = 0.0004 m
C1 = 12631116 N.s/m (69%)  
C2 = 5674849 N.s/m (31%)

--- SAV ---

Drift minimum = 0.001 m
C1 = 12210079 N.s/m (67%)  
C2 = 6095886 N.s/m (33%)

--- Drift 1 ---

Drift minimum = 0 m
C1 = 18305965 N.s/m (100%)  
C2 = 0 N.s/m (0%)

--- Drift 2 ---

Drift minimum = 0 m
C1 = 7084408 N.s/m (39%)  
C2 = 11221556 N.s/m (61%)
Appendix 3-2
Theoretical optimal distribution 3-DOF
RESULTS

---------Parameters---------

Number of DOF = 3
Mass = 100000 kg
Stiffness = 30000000 N/m
Damping ratio = 0.15
First period = 0.82 s
Total damping = 3502695 N.s/m

---------SRSS---------

Drift minimum = 0.03 m

C1 = 2641032 N.s/m (75 %)
C2 = 861663 N.s/m (25 %)
C3 = 0 N.s/m (0 %)

---------SAV---------

Drift minimum = 0.05 m

C1 = 2563972 N.s/m (73 %)
C2 = 938722 N.s/m (27 %)
C3 = 0 N.s/m (0 %)

---------Drift 1---------

Drift minimum = 0.019 m

C1 = 3502695 N.s/m (100 %)
C2 = 0 N.s/m (0 %)
C3 = 0 N.s/m (0 %)

---------Drift 2---------

Drift minimum = 0.018 m

C1 = 1772363 N.s/m (51 %)
C2 = 1730331 N.s/m (49 %)
C3 = 0 N.s/m (0 %)
-------Drift 3-------

Drift minimum = 0.011 m

C1 = 2297768 N.s/m (66 %)
C2 = 1204927 N.s/m (34 %)
C3 = 0 N.s/m (0 %)

---------------Parameters---------------

Number of DOF = 3
Mass = 100000 kg
Stiffness = 30000000 N/m
Damping ratio = 0.3
First period = 0.82 s
Total damping = 7005389 N.s/m

--------SRSS---------

Drift minimum = 0.016 m

C1 = 4168207 N.s/m (60 %)
C2 = 2837183 N.s/m (41 %)
C3 = 0 N.s/m (0 %)

--------SAV---------

Drift minimum = 0.027 m

C1 = 4084142 N.s/m (58 %)
C2 = 2921247 N.s/m (42 %)
C3 = 0 N.s/m (0 %)

--------Drift 1---------

Drift minimum = 0.009 m

C1 = 7005389 N.s/m (100 %)
C2 = 0 N.s/m (0 %)
C3 = 0 N.s/m (0 %)
--------Drift 2--------

Drift minimum = 0.008 m

C1 = 2528945 N.s/m (36 %)
C2 = 4476444 N.s/m (64 %)
C3 = 0 N.s/m (0 %)

--------Drift 3--------

Drift minimum = 0.006 m

C1 = 2591994 N.s/m (37 %)
C2 = 1541186 N.s/m (22 %)
C3 = 2872210 N.s/m (41 %)
Appendix 4-1
Cases studied with the algorithm
Evolution of damping values

Final damping values

Optimal criteria
CASE 4

Evolution of damping values

Final damping values

Optimal criteria
Appendix 4-2
Results theory
RESULTS

------------Parameters------------

Number of DOF = 3
Mass = 80000 kg
Stiffness = 40000000 N/m
Damping ratio = 0.15
First period = 0.63 s
Total damping = 3617567 N.s/m

------------SRSS------------

Drift minimum = 0.018 m
C1 = 2727646 N.s/m ( 75 %)
C2 = 889922 N.s/m ( 25 %)
C3 = 0 N.s/m ( 0 %)

------------SAV------------

Drift minimum = 0.03 m
C1 = 2648059 N.s/m ( 73 %)
C2 = 969508 N.s/m ( 27 %)
C3 = 0 N.s/m ( 0 %)

------------Drift 1------------

Drift minimum = 0.011 m
C1 = 3617567 N.s/m ( 100 %)
C2 = 0 N.s/m ( 0 %)
C3 = 0 N.s/m ( 0 %)

------------Drift 2------------

Drift minimum = 0.011 m
C1 = 1830489 N.s/m ( 51 %)
C2 = 1787078 N.s/m ( 49 %)
C3 = 0 N.s/m ( 0 %)
----------Drift 3----------

Drift minimum = 0.006 m

C1 = 2376742 N.s/m (66 %)
C2 = 1240826 N.s/m (34 %)
C3 = 0 N.s/m (0 %)

----------------------------
%RESULTS%----------------------------

----------Parameters----------

Number of DOF = 3
Mass = 80000 kg
Stiffness = 40000000 N/m
Damping ratio = 0.3
First period = 0.63 s
Total damping = 7235135 N.s/m

----------SRSS----------

Drift minimum = 0.009 m

C1 = 4304905 N.s/m (60 %)
C2 = 2930230 N.s/m (41 %)
C3 = 0 N.s/m (0 %)

----------SAV----------

Drift minimum = 0.016 m

C1 = 4218084 N.s/m (58 %)
C2 = 3017051 N.s/m (42 %)
C3 = 0 N.s/m (0 %)

----------Drift 1----------

Drift minimum = 0.006 m

C1 = 7235135 N.s/m (100 %)
C2 = 0 N.s/m ( 0 % )
C3 = 0 N.s/m ( 0 % )

--------Drift 2--------

Drift minimum = 0.005 m

C1 = 2611884 N.s/m ( 36 % )
C2 = 4623251 N.s/m ( 64 % )
C3 = 0 N.s/m ( 0 % )

--------Drift 3--------

Drift minimum = 0.004 m

C1 = 2677000 N.s/m ( 37 % )
C2 = 1591730 N.s/m ( 22 % )
C3 = 2966405 N.s/m ( 41 % )
Appendix 4-3
Comparison theory vs. algorithm
Evolution of damping values

Steps

Damping (N s/m)

C1
C2
C3

Final damping values

Floor

C (N s/m)

Optimal criteria

Index

Criteria
Appendix 5-1
Possible bracing configurations
Appendix 5-1: Possibilities offered by the algorithm

Frame:
- 4 by 4
- type of supports
=> Envelope

Dampers

Custom geometry

No bracing

Bracing

Dampers
Appendix 5-2
Typical complete-case study
User: T. DODY
Date: 04/15/13

PARAMETERS

Frame:
- Number of floors = 6
- Number of bays = 4
- Height of the floors = 4.5 m
- Width of the bays = 6 m
- No diagonal bracing

Steel sections:
- Beams = W 24 x 131
- Columns = W 16 x 89

Type of support:
- Fixed

Type of constrains:
- Body
SAP 2000 model:

![Diagram]

**Final frame**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 25 | 6 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 26 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 27 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 28 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 29 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 30 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

![Diagram]
## STIFFNESS

<table>
<thead>
<tr>
<th>NODES</th>
<th>LOAD CASE</th>
<th>DISP. [m]</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>k1 62111801.24 KN/m</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.000161</td>
<td>k2 61728395.06 KN/m</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.000324</td>
<td>k3 60606060.61 KN/m</td>
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<tr>
<td></td>
<td>3</td>
<td>0.000325</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.000049</td>
<td>k4 59880239.52 KN/m</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.000493</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.00066</td>
<td>k5 58823529.41 KN/m</td>
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<td>6</td>
<td>5</td>
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<td>k6 57142857.14 KN/m</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.000845</td>
<td></td>
</tr>
</tbody>
</table>

F = 10000 N

## OPTIMIZATION

Additional equivalent damping ration:
- 0.20

Nodal mass:
- 0.2*10^5 kg (per span)
- 0.8*10^5 kg (per floor)
Cases chosen:

- Constant repartition ✓
- Triangular repartition ✓
- Takewaki repartition ✓
- Stiffness proportional ✓

Mode:

- First mode $\omega_1 = 6.66$ rad/s ($T = 0.94$ s)

Damping repartition:

<table>
<thead>
<tr>
<th>Damper</th>
<th>Constant</th>
<th>Takewaki</th>
<th>Stiff. Prop.</th>
<th>Triangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3,730,292</td>
<td>8,473,261</td>
<td>3,858,447</td>
<td>6,394,786</td>
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<td>2</td>
<td>3,730,292</td>
<td>7,602,485</td>
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<td>3,719,820</td>
<td>3,197,393</td>
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<td>3,730,292</td>
<td>-</td>
<td>3,654,176</td>
<td>2,131,595</td>
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<tr>
<td>6</td>
<td>3,730,292</td>
<td>-</td>
<td>3,549,771</td>
<td>1,065,798</td>
</tr>
</tbody>
</table>

Evolution of the damping values:
Final damping distribution and optimal criteria:

Final damping values

Optimal criteria
Drift transfer functions:

For a simplicity purpose, the cyan and blue plots superimposed, same happens for the red and green.
Cases chosen:

- Constant repartition ✓
- Triangular repartition ✓
- Takewaki repartition ✓
- Stiffness proportional ✓

Because of the similar transfer function between several distributions, only the constant one and the Takewaki optimization will be compared.

Earthquakes chosen:

- Northridge ✓
- Loma Prieta ✓
- Imperial valley ✓

The earthquakes records are scaled in order to get a PGA of 1g.

Only the highest drift values will be extracted from the analysis.

Initial damping ratio:

The initial damping ratio is based on Rayleigh damping and defined as follow:

- First mode : 3%
- Second mode : 3%

Solving method:

- Linear analysis
- Direct integration
RESULT

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{imperial_valley.png}
\caption{Imperial valley}
\end{figure}

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{loma_prieta.png}
\caption{Loma Prieta}
\end{figure}
<table>
<thead>
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<th></th>
<th>NO DAMPING</th>
<th></th>
<th>CONSTANT</th>
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<tbody>
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<td></td>
<td>IMPERIAL</td>
<td>IMPERIAL</td>
<td></td>
</tr>
<tr>
<td>MAXIMUM [mm]</td>
<td>16.77</td>
<td>16.11</td>
<td>5.34</td>
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<tr>
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<td>14.83</td>
<td>12.69</td>
<td>4.74</td>
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<td>DRIFT 1-2</td>
<td>12.69</td>
<td>9.72</td>
<td>4.01</td>
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<tr>
<td>DRIFT 2-3</td>
<td></td>
<td>5.63</td>
<td>3.18</td>
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<td>2.26</td>
</tr>
<tr>
<td>DRIFT 4-5</td>
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<td></td>
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<td>LOMA MAXIMUM</td>
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<td></td>
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<td>DRIFT 0-1</td>
<td>13.27</td>
<td>12.57</td>
<td>8.96</td>
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<tr>
<td>DRIFT 1-2</td>
<td>11.19</td>
<td>9.19</td>
<td>7.84</td>
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