Demand Forecasting Accuracy in Airline Revenue Management: Analysis of Practical Issues with Forecast Error Reduction

by

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Abstract

In any business, conventional wisdom dictates that lower forecast errors lead to better revenue performance. However, in airline revenue management, traditional methods of measuring forecast error show no clear relationship with revenues. This can be attributed to the deficiency in the basic forecasting assumption of independence of demand in a particular fare class on a flight from other fare classes/flights. In real life, passengers sell-up to higher fare classes and travel on other flights in the same or higher fare class. This leads to a situation where determination of actual realised demand becomes dependent on the state of the entire network at a particular time i.e., which fare classes are available for booking on the flight in question as well as other flights in the market, both of the airline and its competitors. Traditional measures of forecast accuracy that rely on estimated values of actual demand thus are of little use in evaluating forecasting performance in airline revenue management.

This thesis initially identifies and discusses these practical issues in determination of "actual" demand. It then attempts to gain an insight into the forecasting fundamentals by first analysing forecast accuracy in a simple monopoly network. A controlled case is set up to replicate the basic assumptions of reservations forecasting and set up a base comparison of forecasting methods. The analysis is then extended to include a competitive network. The results show that there is no clear relationship between forecast error and revenues as the realised demand under every network configuration is state-dependent.

Throughout the analysis, arbitrary forecast inflation leads to higher revenues under the leg-based forecasting and optimisation scheme. This arbitrary inflation of demand forecasts appears to compensate for the failure of leg-based optimisation to dynamically account for the arrival pattern of demand and the inability of leg-based forecasting to correctly estimate demand in multi-leg markets. Thus, this arbitrary forecast inflation leads to higher revenues despite being "less accurate" when analysed under the traditional forecast-error metrics. The analysis shows that use of path-based forecasting, to avoid partial detruncation problems on multi-leg paths and a future protect algorithm to dynamically adjust to the arrival pattern of demand results in much improved revenues that are comparable to those obtained through forecast inflation.

Thesis Advisor: Dr. Peter Paul Belobaba
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Chapter 1: Introduction

Demand Forecasting Accuracy in Airline Revenue Management:

Analysis of Practical Issues with Forecast Error Reduction

Motivation

Airline Revenue Management strives to sell seats to passenger at a fare that approximates their maximum willingness to pay. This is the same economic principle of price differentiation which refers to some theoretical maximum possible revenue derived by charging a different price to every consumer according to the value of product/service to them or alternatively charging passengers their maximum willingness to pay (WTP). In practice however such price differentiation is impossible to achieve and therefore airlines offer a stratified fare structure and group their seats under these different fares.

The revenue management problem thus essentially becomes one of correctly defining these different fares, the pricing part, and accurately allocating seats under these different fares, the seat inventory control part. In the real world, even though the fares are regularly updated however they are comparably static and it is the seat inventory control that gets the limelight in revenue management systems. The revenue management thus essentially becomes allocation of seats to a specific fare class with its associated restrictions – which for profit maximisation becomes protection of reasonable number of seats for higher-fare classes that traditionally book closer to departure at the same time ensuring the aircraft is adequately full by selling otherwise empty seats at lower-fares.

Since the higher-fare paying passengers traditionally book closer to departure date, airlines have to estimate the number of seats to reserve for them as well as figure out how
many seats to offer to lower-fare, early-booking passengers. If the airline reserves too few seats for higher-fare classes, it will tend to turn away that demand when it arrives closer to departure date and thus lose revenue. On the other hand if it reserves too many seats, it runs the risk of flying empty seats since it will already have turned away the early-booking, lower-fare paying passengers. It is in this context that reservation forecasting assumes a highly important role.

As is apparent from the nature of the problem, the revenue management problem is a juggling act between trying to minimise lost revenue through turning away late demand as well as minimising “spoilage” by flying substantially empty aircraft. This very nature dictates that in theory there is a maximum revenue figure that can be achieved with accurate forecasts, and inaccurate forecasts result in negative deviations from this maximum attainable revenue. Thus minimising and even eliminating forecasting errors has become a subject of prime import in revenue management.

In principle, measuring forecast accuracy is a simple enough concept. It requires comparing the forecasted figures against actual figures. There are various traditional measures of forecasting accuracy from the basic forecast bias that directly compares forecast with actual to mean squared percentage error that is based on proportional errors. Each of these measures is geared toward a particular type of relationship between forecast error and its corresponding opportunity cost.

Historically, the efforts at determining the airline reservation forecasting accuracy have focused on these traditional measures. Initially the booking data available through airline’s historical database was directly compared with the forecasts. This process however ignores detruncation, a fact arising out of the inherent inability of the databases to capture demand (as opposed to bookings). The airline databases only record the number of bookings made for the flight in question, ignoring the number of passengers who were denied bookings in the class of their first choice as it reached its determined booking limit. These passengers then either did not travel or travelled subsequently in a higher fare class, on another flight, or went to competitors. Thus airline booking data is under representative of the market demand and requires unconstraining or detruncation to
reflect the higher existent demand. This is required as the forecasting is done for unconstrained demand. Later studies have tried to redress this issue by either unconstraining the recorded bookings before comparison or using data that does not include any constrained bookings.

However, the issue in measurement of airline reservation forecasting error is much more fundamental than these problems with comparison of like quantities. It is the impossibility of determination of actual demand data to enable comparison. As highlighted above the booking data in any airline’s database is not the representative of true demand. The actual bookings that are recorded by the airline are a product of numerous factors that include the booking limits imposed by the revenue management system, the status of booking in not only requested class but other fare classes not only on the same flight but on other flights in airlines network and even competitor flights in the same market.

The added complication is due to the iterative nature of the airline reservation process. The booking data from one iteration for a particular time interval before the departure is used in forecasting demand for the remaining time intervals. Thus a forecast in airline revenue management is state dependent, which simply implies that it is dependent on the state of the system at the moment. Another issue arises from one of the basic assumption of demand forecasting i.e. leg-class/ path-class independence. This basically means that airlines forecast demand for a particular fare class on a particular flight leg or a path and assume this to be independent of other fare classes. In actuality the closure of lower fare classes results in some passengers paying higher fare on the same leg/ path (sell-up) and some passengers drifting to other airlines (spill). Thus this independence assumption does not hold true in reality. Airlines in practice can not even determine what the actual total market demands is, a far simpler proposition compared to the detailed fare class demand on a single leg or path required by the revenue management system.

It is this limiting factor that makes the traditional quest for zero forecast error a dubious undertaking. Employing traditional measures of forecast accuracy in consort with some arbitrary measure of actual does not respect the conventional belief of error-revenue
relationship. This makes the study of issues surrounding forecasting accuracy a subject of utmost importance and impact. It is entirely possible that “inaccurate” forecasting methods might give better revenue performance than so-called “accurate” methods. It is also possible to manipulate forecast generated by traditional methods to encourage better revenue performance which translates into erosion of accuracy performance.

Objectives

In light of the issues discussed, the primary approach of this thesis is to measure forecast accuracy using traditional measures under different forecasting/detruncation combination in a simulated airline network under PODS – the Passenger Origin-Destination Simulator. A combination of different seat allocation optimization algorithms will also be used to observe accuracy under different conditions. These measures will be compared vis-à-vis the corresponding revenue performance of these combinations. This comparison will be used to highlight the fact that the conventional belief of accuracy-revenue relationship does not hold true in practice. In fact it is entirely plausible for highly inaccurate forecasting/detruncation combinations to outperform their more accurate counterparts.

Thus the primary goal of this thesis is to establish the fact that the term “forecast accuracy” has an entirely different connotation in the context of airline revenue management; distinct from other forms of business. It is impossible to measure actual demand for use as basis for comparison. To further reinforce this result, some arbitrary forecast manipulation techniques will be used on top of the traditional forecasting methods. A similar comparison will highlight the fact that higher “inaccuracy” does not translate into worse revenue performance rather these inaccurate forecasts result in higher revenues.

To gain insight into the revenue-accuracy relationship, similar analysis will be conducted in a monopoly network to validate the above results. These results will show that even in absence of competitive effects the traditional accuracy-revenue relationship does not hold and inaccurate forecast methods continue to do better.
As a secondary objective, the superior revenue performance of these arbitrary forecast manipulation methods will be analysed in detail. These methods have shown revenue potential under leg-based revenue management method. The analysis will focus on the difference between leg-based and path-based revenue management methods to ascertain the underlying reasons for this revenue potential, thereby suggesting more methodical ways to improve revenue performance.
Chapter 2: Theory

Forecast Accuracy: Issue Definition and Literature Review

Defining Issues Related to Concept of Accuracy in Historical Context

Revenue Management Basics

Revenue management represents the concentrated effort by airlines to maximise their revenues and has been the subject of research for past few decades.

The revenue management problem is essentially addressed by two means, a) by offering several fare products on a single flight, each with different set of restrictions targeted to match its intended passenger base (pricing) and b) by using a seat optimization algorithm that appropriately limits the seats available to lower-fare classes (inventory control).

Today, revenue management mainly revolves around optimizing the seat allocation process, which has led to new forecasting techniques as well as a number of seat allocation algorithms.

The Seat Inventory Control Process

The revenue management process is an iterative process made up of various steps. It can be illustrated through a simplified flowchart, from Skwarek (1997) as shown in Figure 1.
Construction of a fare structure and changes to existing fare structure are performed less often than the seat inventory control process and therefore emphasis is on the latter process.

The historical database (HDB) contains the data from all previous departures of interest. This data is used to estimate the forecasting parameters as employed in the forecasting process.

Forecasting by fare class is the next important step. The pertinent data in historical database is first selected. The selection process might involve exclusion of previous flights that can constitute outliers in the data set and show unusual demand behaviour. This data prior to being used in forecasting is detruncated if it has been constrained by booking limits, since seat-optimisation algorithms require detruncated demand forecasts. This data then goes into forecaster. There are various forecasting methods that will be explained in later sections.

The resultant forecast is input to the seat optimization algorithms which also take as input fare values by class and come up with booking limits on each fare class that maximise expected revenues. The information about No Shows, Denied Bookings, output of seat optimizers and airline’s opportunity cost calculations are used by an overbooking model to set booking limits by fare class.
Once the adjusted booking limits for fare class are made available, the passenger booking process starts where reservations and cancellations from passengers are considered. These steps are repeated over booking intervals and continue till the flight departure day. On departure day the information about No Shows and Denied Boarding and final loads is recorded in the Historical Database and the whole process repeats itself.

**Need for Forecasts**

A forecast is defined as “a quantitative estimate (or a set of estimated) about the likelihood of future events which is developed on the basis of past and current information”.\(^1\) As highlighted above, forecasting forms the backbone of the entire seat inventory control process and is thus integral to revenue management. The revenue management problem boils down to being able to predict in advance how many seats to protect for late-booking, higher-fare paying passengers, while at the same time selling enough seats to early-booking, lower-fare paying passengers to ensure minimum spoilage and healthy load factors.

The seat optimisation algorithms require forecast by fare class for every iteration of inventory control process in order to determine how many seats to protect for each fare class. Thus the forecasting of passenger demands by flight, date and fare class, also known as micro-level forecasting, is a basic requirement. This demand can be forecasted at many different levels, for example on a flight leg level, on a fare class level, or on an Origin-Destination Fare level.

**Need for Accurate Forecasts**

Intuitively we would expect forecasts to have a very strong bearing on revenues, as “better” or “accurate” forecasts seem to suggest that they would lead to higher revenues. Curry\(^2\), in his technical brief showed the results of forecasting impact on revenue using Monte Carlo Simulations. Under each set of different conditions, the percent revenue achieved was computed; defined as the revenue achieved with forecast error divided by

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\(^1\) Pindyck, Robert S. & Rubenfeld, D. (1998)

\(^2\) Curry, R.E. (1994)
the revenue that could have been achieved with full knowledge of demand. This resulted in the following asymmetric curves, shown in Figure 2.

![Figure 2: Forecast Error-Revenue Relationship](image)

This appears to follow our intuition. At lower demands, forecast errors have less impact on revenue as there are fewer inventory restrictions, regardless of the forecast. However at higher demand levels, forecast errors can have significant impact on revenues achieved.

Too high a forecast leads to increased revenue loss with overprotection, as too many seats would be protected for higher-fare paying passengers that will ultimately go empty (spoilage). In contrast, under-protection will see the seats filled up by more lower-fare paying passengers at the cost of denying seats to higher-fare paying passengers.

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4 Lee, Anthony O. (1990)
Lee⁵ in his thesis also made a similar analysis of revenue impact of forecast errors and his results also followed the general parabolic shape depicted above.

It should be noted that the above results are strongly dependent on the assumption that fare class demands are independent from each other. In reality a passenger denied a seat in lower fare class might sell-up to a higher fare class. Also in a networked environment with other airlines present, there is always a possibility of passenger spill-ins from other carriers. These considerations will lead to an asymmetric shape of the revenue-error parabolic curve.

**Review of Forecast Accuracy Studies**

This section will focus on reviewing salient works among the literature available on airline forecasting techniques and the comparative studies undertaken between various forecasting models.

**Revenue Management Forecasting Models**

The forecasting models can be grouped according to the data employed.⁶

- **Historical Bookings Model:** These models are based on booking data available in historical data base (HDB). This data is used as input to predict the increase in bookings on the current flight in the period from forecast interval to flight departure. These models are based on the assumptions that booking patterns for future flight departures are similar to the historical flights in the database.

- **Advance Bookings Model:** In these models, Bookings-in-hand (BIH) data from the future flight is used as input to predict the bookings to come from forecast interval to flight departure.

---

⁵ Lee, Anthony O. (1990)
⁶ Skwarek, Daniel K. (1997)
• **Combined Bookings Model**: These models employ both data from HDB and BIH data of current flight to predict the bookings to come on the current flight for the period from forecast interval to flight departure.

Note that only the data used as input to predict the bookings increase is the basis for this grouping of models.

### Categorisation of Revenue Management Forecasting Models

**Historical Bookings Models**

Scandinavian Airlines in 1978, proposed a basic model that employed arithmetic mean of historical bookings at the end of booking process, calculated over selected departures in the HDB. The same paper also highlights the amount of data necessary for “accurate” forecasting and how to remove outlier data points.

In a closely related work, Ducanson suggested exponential smoothing to weight the most recent departures more in calculating the average, since it realistically represented the current trend.

Wickham, in his 1995 thesis, offered a formal version of the above mentioned models. His versions were based on fare classes. They are represented by the following equations:

- **Simple**: $\hat{BIH}(0)_f = \frac{1}{M-t} \sum_{i=f-M}^{f-t} BIH(0)_i$

- **Exponential**: $\hat{BIH}(0)_f = \sum_{i=f-M}^{f-t} \frac{\alpha_i}{M-t} BIH(0)_i \text{ s.t. } \sum_{i=f-M}^{f-t} \alpha_i = 1, \alpha_{f-M} < ... < \alpha_{f-t}$

where $BIH(0)_f$ is final bookings on day 0 of the flight $f$ being forecast

$M$ is the number of flights considered in the forecast plus the number of flights leaving before $f$ but not yet departed

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7 SAS (1978)
8 Ducanson (1974)
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$t$ is the number of booking interval from which the prediction is being made.

Sa in 1997\textsuperscript{10} proposed another historical bookings model using Box-Jenkins ARIMA (Auto-Regressive, Integrated Moving Average) model. This he calibrated for a single fare-class on a particular flight. However since his results showed high standard errors due to variability of data, he subsequently changed his approach.

Another variation of Historical Booking Models is the “pickup” or “historical moving average” method models. These are based on estimation of average increase in bookings from subject interval to flight departure using selected flights from HDB. Another variation, the “advanced pickup” model, developed by L’Heureux\textsuperscript{11} of Canadian Pacific Airlines, uses more recent information from soon to depart flights that have as yet incomplete booking histories. This was done to increase response to variation in demand much more quickly.

The classical pickup models have the following equations\textsuperscript{12}:

Equal Weighting:

\[
\hat{BIH}(0)_f = \frac{1}{M-t} \sum_{i=f-M}^{f-t} (BIH(0)_i - BIH(t)_i) + BIH(t)_f
\]

Exponential Weighting:

\[
\hat{BIH}(0)_f = \sum_{i=f-M}^{f-t} \frac{\alpha_i}{M-t} (BIH(0)_i - BIH(t)_i) + BIH(t)_f \text{ s.t. } \sum_{i=f-M}^{f-t} \alpha_i = 1, \alpha_{f-M} < \ldots < \alpha_{f-t}
\]

The advanced pickup model has the following equation\textsuperscript{13}:

\[
\hat{BIH}(0)_f = \frac{1}{M-t} \left[ \sum_{i=f-M}^{f-t} (BIH(0)_i - BIH(1)_i) + \sum_{i=f-M+1}^{f-t+1} (BIH(1)_i - BIH(2)_i) + \ldots + \sum_{i=f-M+t}^{f-1} (BIH(t-1)_i - BIH(t)_i) + BIH(t)_f \right]
\]

\textsuperscript{10} Sa, Joao (1987)
\textsuperscript{11} L’Heureux, Ed (1986)
\textsuperscript{12} Skwarek, Daniel K. (1997)
\textsuperscript{13} Ed L’Heureux (1986)
Advance Bookings Models

Early work on advanced booking models was taken up by Harris and Marucci of Alitalia. They developed a simple model that provided aggregate booking forecasts for groups of selected flights. The method employed using two data sets, one a snapshot at different intervals of time for the subject flights and the second included total booking on all flights for a 45 day period. This aggregation level limited the usefulness of this model to the forecasting problem under study since specificity and sensitivity to variation in individual flights is lost.

Lee\textsuperscript{14} provided a modified regression model that was based on three groups of terms. First group included terms for bookings-in-hand for the subject flight, second group included terms to cater for external causal factors and the third group accounted for random error. However this model was neither calibrated nor external factors specified.

Wickham\textsuperscript{15} proposed a reduced version of Lee's model, using only Bookings-in-hand data at departure and at time interval of interest from HDB flights and then using the equation with current booking history of subject flight to predict final loads. This non-causal regression model is given by the following equation:

\[
\hat{BIH}(0)_f = \sum_{i=N}^{I} \theta_i \times BIH(i)_f + g \times W(f,i) + v(f,i)
\]

where $\theta_i$ are the coefficients on BIH in previous time periods

$g$ is a vector of coefficients on exogenous factors

$W$ is a vector of exogenous factors

$v$ is a random error term

Lee\textsuperscript{16} also proposed another advance bookings model that regarded booking process as a Poisson process, distributed with certain probability of booking request/cancellation in a specified period. He thus developed a censored Poisson model incorporating detruncation as well. This model assumes a constant probability for booking request, cancellation as well as constant booking limits within the forecasting interval. This was a very complex

\textsuperscript{14} Lee, Anthony O. (1990)
\textsuperscript{15} Wickham, Robert R. (1995)
\textsuperscript{16} Lee, Anthony O. (1990)
model limited by its practical application. The first issue was that the model was very computationally intensive requiring Maximum Likelihood estimation of two variables for every flight, between every interval. It is not possible to decrease the number of intervals as the assumption of constant request and cancellation probabilities and constant booking limit will not hold. The second issue was that the assumption about arrival, cancellation rates and booking limit was unrealistic. Since Lee adopts three intervals, he assumes booking curve and cumulative cancellation probability to be linear with kinks at the intervals. Lee has defined booking limit as ‘maximum bookings in a fare class” which does not remain constant within an interval due to nested nature of fare classes. In order to fulfil the Poisson requirement of this number being constant, intervals will need to be reduced further increasing computing requirements.

**Combined Historical and Advance Bookings**

Sa\(^{17}\), in his same paper, used calibrated causal regression models that employed both HDB data as well BIH data from the subject flight. This effort was more successful than his ARIMA approach.

Ben Akiva\(^{18}\) also suggested a forecasting model by flight and fare class. This model combined a non-causative regression model with a time-series model. The regression model employed advance bookings data and time-series model employed HDB information. The analysis was done on a monthly basis and no validation tests were carried out to check forecasting ability on individual future flights.

Lee, also suggest a non-causal, “full-information” model that combined appropriately weighted final bookings information from HDB flights, BIH for current time interval from flights yet to depart and BIH information from the subject flight. This gives weight to recent flight data that reflects recent changes in demand. This model also combined detruncation and forecasting in a recursive substitution method. However his treatment makes this approach computationally intensive.

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\(^{17}\) Sa, Joao (1987)

\(^{18}\) Ben-Akiva, Moshe (1987)
An alternative computationally “efficient” forecaster has been proposed by Hopperstad\textsuperscript{19} which uses all available booking information but does not utilise the computationally intensive maximum likelihood estimation. This is similar to Lee’s approach as it combines the forecaster and detruncator in a single model and L’Heureux’s work in using all available booking information.

**Review of Comparative Assessment Studies**

This section deals with the review of various studies undertaken to compare forecasting performance of various methods against each other. The various models have usually been compared on basis of forecasting “accuracy” employing traditional measures of forecast error.

Sa\textsuperscript{20}, in his thesis, compared ARIMA time-series models versus regression models. These models were employed for short-term forecasting and the comparison was based on goodness-of-fit tests. The bookings data was taken from ten markets. He dismissed ARIMA models on basis of their poor performance in one fare class on one of these ten markets. Regression models were also estimated for all of these markets and they differed in overall model fit and statistical significance of coefficients. No tests were done by forecasting using a different data set. Also no information was provided about time-series fit on the remaining nine markets. This comparative testing was very data specific and thus has limited applicability in terms of deciding the relative performance of models that were tested.

Ben-Akiva\textsuperscript{21} used a combined model with an ARIMA time-series component and a regression component, as mentioned earlier. The correlation coefficients between the predicted and actual observations declined when the two components were run separately. In relative terms, regression model fit the data better than time-series. There were no tests done using a different data set. The data used was monthly. Also no consideration was given to effect of booking limits on demand. The fare classes were aggregated as well.

\textsuperscript{19} Hopperstad, Craig (1991)  
\textsuperscript{20} Sa, Joao (1987)  
\textsuperscript{21} Ben-Akiva, Moshe (1987)
These limitations translate into limited applicability of this analysis where forecasting accuracy for RM systems is concerned.

Wickham's\textsuperscript{22} was the first study to comprehensively employ the traditional measures of forecast error. He used a historical database with booking history by fare class and day of week. He employed various models including classical and advanced pickup, time-series (both with and without weights) and regression models. He performed these measurements over various HDB sizes and forecasting periods. He also utilised a simple detruncation method. Based on these tests, he concluded that the Pickup forecasting model outperformed both regression and time-series models. He also showed that increasing the forecasting period improved the performance of advanced pickup model and that advanced pickup was more susceptible to sudden demand shifts. All models invariably over forecasted demand compared to actual booking data detruncated by Wickham's own unconstraining algorithm. This algorithm employed multiplication of constrained data at every time frame by a corresponding fixed unconstraining percentage developed from unconstrained data. This resulted in an under-estimate of "actual demand", causing forecasts to appear too high. Increasing HDB size was not a significant factor affecting forecasting performance.

Even though Wickham's study is the first complete evaluation based on traditional forecasting measures, there are some considerations. He selected two fare classes from the HDB without considering how this choice will affect the performance of forecasting models. He also aggregated data over 24 markets without consideration of difference in characteristics in terms of stage-length or passenger type.

Lee\textsuperscript{23} compared his own censored-Poisson and full-information models with regression and pickup models. He employed a single-market data set with forecasting horizon of two months and the database consisted of nine months of data. The models used three forecasting intervals generating class-specific forecasts. The models were compared using three measures of accuracy. His results ordered the models in terms of decreasing performance as full-information, censored-Poisson, regression and pickup. Even in

\textsuperscript{22} Wickham, Robert R. (1995)
\textsuperscript{23} Lee, Anthony O. (1990)
expanded tests taking into account several markets and fare-classes showed that these results hold though the differences in performance were small. However his study does not offer answer to the question that use of a computationally intensive, more accurate forecaster for fewer times during the booking process is preferable to more frequent use of less accurate yet less computationally intensive method. The latter method will rapidly take into account the changes during the booking process. No revenue performance comparison was carried out in this regard.

Skwarek was the very first study that discussed the issues surrounding measurement of forecasting performance through traditional measures of accuracy. He correctly identified various factors that make the accuracy measurement an impossibility vis-à-vis airline reservation forecasting. He was the first one to advocate abandoning the use of traditional measures of forecast accuracy in airline forecasting. Instead he argued that revenue performance should be used as the primary platform for comparison among various forecasting methods as zero forecast error is impossible to achieve. He was also the first one to utilise initial Passenger Origin Destination Simulator (PODS) versions to simulate single market conditions.

His principal findings indicated that pickup forecasting usually performs at least as well as regression forecasting and significantly better under certain conditions like high demand variability). However under high system-wide demand variability, regression came out superior. His qualified ranking was pickup forecasting first, regression second and efficient forecaster third. He also tested various detruncation methods and concluded that under high variability of demand booking curve detruncation with moderate or extreme scaling and projection detruncation perform better.

Zickus’s thesis deals with interaction between forecasting and detruncation methods and seat-optimization algorithms. The thesis expands previous PODS-related research to a network scenario and analyses the effects of different forecasting and detruncation algorithms in the yield management context. These results are compared with earlier

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24 Skwarek, Daniel K. (1997)
26 Zickus, Jeffrey S. (1998)
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revenue management simulations in order to determine the sources of gains from yield management system improvement. It also simulates a realistic competitive scenario with each airline able to vary its choice of seat optimization, forecasting and detruncation method. His thesis highlights the differences among various combinations of forecaster, detruncator and seat optimiser while giving insights into the reasons for performance differences. It also tests the robustness of different RM algorithms.

Zickus’s thesis deals only with the relative revenue merits of different forecasting methods namely pickup and regression forecasting. It does not deal with the relative accuracy of these methods. This work, though, provides a good base case as it deals with the effects on system revenue under different forecasting methods in a variety of competitive scenarios. Furthermore, it explores the compatibility of these forecasting methods with each of the different seat optimizers. It does however raise some very relevant questions especially regarding actual demands.

Forecast Accuracy as a Concept

Forecast Accuracy in general, insofar as achievement of lower errors is considered, is a very valid goal for any revenue management application. However, in the specific field of airline revenue management, accuracy as a concept and reduction of forecast errors as a management goal do not hold similar weight. In order to understand this we need to delve deeper into the basic assumptions of airline reservation forecasting and contrast that with real world passenger behaviour. Also we need to look at the traditional measures of forecasting accuracy and how they are incompatible with the airline revenue management since the choice of base dataset, inherent to all traditional measures, plays a significant role by introducing bias.

Forecasting Assumptions in Airline Revenue Management

In airline seat inventory management, forecasts are generated for demand by fare class level for a given flight. The forecasting methods assume certain properties for this level
of aggregated demand which is also applicable to the historical dataset from which the forecast are made.

The forecasting assumptions for the fare-class level are as follows:

1. **Demands are segregated by fare class and are independent from each other**

   This is the fundamental assumption of forecasting at this level however it does not reflect the real world passenger behaviour. The unrealistic nature of assuming demand segregation by fare class is discussed in further detail in the following section. The assumption of independence among these demands is also fairly naïve given the opportunities for sell-up and recapture in an airline’s network. Sell-up refers to the opportunity whereby a passenger denied reservation in a particular fare class trades up to higher fare-class on the same flight which is still open to booking. Recapture opportunities arise due to multiple path opportunities between the same origin-destination cities in a network. Horizontal recapture occurs when the same passenger decides to book a seat in the same fare-class on another flight in the airline’s network. Vertical recapture represents the situation where the same passenger books in a higher-fare class on another flight in the network.

   With a competitive network with multiple carriers, opportunities for spill-in and spill-out arise whereby the same passenger decides to travel on another carrier, either in the same or higher-fare class and vice versa situation where passengers from other carriers who are denied their first choice, book with the subject airline. All these situations effectively invalidate to the mutual independence assumption of fare-class demand.

2. **Demands by fare class are not constrained by the booking limits**

   Forecasting is distinct from detruncation and this assumption is pivotal to unbiasedness of any forecaster. The forecaster, in general, assumes that the input data is unconstrained in order to output an unbiased forecast for use by optimizer.

   The forecaster employs dataset containing observations from both previous flights (courtesy of Historical DataBase) and current flight's booking. If the data from HDB is constrained, it will invariably lead to lower forecasts. The lower forecasts will
cause lower booking limits on higher fare classes, making more seats available to lower-value, early-booking passengers, ultimately resulting in lower revenues.

The assumption that demand by fare class is not constrained by booking limit is also valid for the current flight's booking data. This assumption does not introduce any distortion as the seat optimizer determines the number of seats to offer to a particular fare class. In situations where a fare class is being constrained, it will lower the booking limit for low-value fare classes.

3. **Demands are normally distributed**
   In order to detruncate (unconstrain) and forecast, it is necessary to assume a distribution for passenger demand. The detruncators use this distribution to extrapolate demand from constrained data set and forecasters utilise parameters estimated from the same assumed distribution. However, this distribution is censored at the booking limit, and at the same time is truncated at zero (for there are no negative bookings). Empirical reservation pattern analysis\(^{27}\) shows that normal demand distribution pattern holds for moderate demand level. Positive skewness is associated with low-demand levels and a 'spike' at capacity level associated with high-demand levels. Thus this assumption seems to be conveniently reasonable for medium demand and is followed in all seat inventory control processes. A subsequent, yet small, empirical study\(^{28}\) hints at presence of natural skewness in the underlying demand, suggesting a lognormal distribution instead of normal. Lee in his thesis\(^{29}\) argues for Poisson distribution. American Airlines' analysis suggests a gamma distribution for underlying demand. There are trade-offs involved, however, between these distribution assumptions. Normal distribution is computationally simple in situations with moderate censoring and truncation and is most commonly used.

4. **Cancellation rates are similar between Historical Database and forecast flights**
   A forecaster employs gross measures of booking from HDB (which include reservations that are later cancelled) to predict final bookings-in-hand which is a net

\(^{27}\) Belobaba, Peter P. (1985)
\(^{28}\) Brummer, Mark et. al. (1988)
\(^{29}\) Lee, Anthony O. (1990)
measure (since cancellations cannot occur after close of booking period). The distortion in forecast is avoided by assuming same gross/net relationship, in terms of cancellations, occurs on the predicted flight $f$ as on the flights in HDB. The forecaster thus predicts a certain proportion of cancellations for the forecast flight on the basis of prior demand. Prediction of higher cancellation than actual causes less final bookings and vice versa.

**Contrast with Real World Passenger Behaviour**

An airline's pricing structure is composed of fare products employing elements of both differential and discriminatory pricing. The aim is to distribute demand into several fare classes. However the fare products result in imperfect segmentation with the result that each fare class is not composed of homogenous type of passengers. Rather these passengers opting for a particular fare product will switch to other fare classes if their first choice fare is not available.

Thus demand for a fare class is not independent of demand in other fare classes. It is defined only with respect to and situated within the particular suite of other fare products offered in the market, by the same and competing airlines. In reality, demand occurs as demand by passenger type which is fairly independent rather than demand by fare-class as the characteristics differ considerably. However even by type, interdependencies exist. If we consider Time and Price Sensitivity to be the two prime determinant of passenger type, we can define following four types of passengers (Refer to Figure 3):

- **Type I:** Time-sensitive and price-insensitive
- **Type II:** Time-sensitive and price-sensitive
- **Type III:** Time-insensitive and price-sensitive
- **Type IV:** Time and price insensitive
Whereas Type I passengers are most influenced by service variables, like frequency and in-flight amenities etc.; Type II passengers consider price to be of prime importance in their decision. However both Type I and Type II passengers are sensitive to frequency and related service variables while differing in their price sensitivity.

In practice, this disparity between the assumption of independence and real-world behaviour needs to be recognised and adjusted for in the forecasting and seat optimisation steps. Another important consideration is the context of fare products in the historical database and forecast period. If the fare product changes between the two periods then it requires further adjustment to the historical data base.

Adjustment for independence of demand is further exacerbated by passenger behaviour under constrained situations – where reservations reach booking capacity. Under the independence assumption, the denied lower-fare passengers do not travel upon denial, whereas in reality they either sell-up, are recaptured on other flights or spilled onto other carriers.

\[30\] Belobaba, Peter P. (1987)
Issues of Inherent Bias due to Choice of Base

In previous sections, the focus has been on the arbitrary nature of the concept of forecast accuracy due to the fallibility of the basic assumptions of forecasting that lead to incorrect forecasts. The current section deals with the equally important issue of the difficulty in choice of a “base” or “actual” demand in employing traditional measures of forecast error. This exposes the inapplicability of these traditional measures of error to airline reservation forecasting.

Definition of Inherent Bias

The main issue arising in comparative assessment of alternative forecasting methods based on error metrics is what base to use in measurement of error. Depending on the base used in error definition, the existence of constrained observations in the dataset and the forecaster used, inherent forecast biases will be present (Refer to Figure 4).

![Figure 4: Inherent Biases in Measurement of Forecast Error](image)

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Bias exists when the summed difference between predicted and actual bookings over all flights being forecasts is not equal to zero. Failure to eliminate inherent biases mixes the ranking of forecasters as the observed differences can be due to construction of the experiment as to inherent performance differences.

Skwarek (1997) has listed various types of analysis and the resulting biases. In a Type I analysis that utilises a constrained data set and a constrained error measurement, the bias is uncertain as it varies from forecaster to forecaster on their treatment of constrained observations. Most comparative studies of forecasting have been of this type.

For Type II analysis, an unconstrained data set is used in combination with a constrained base. This will invariably result in a positive inherent bias. Wickham's study involves this type of forecast error where all forecasters displayed positive bias.

In Type III, the dataset includes flights with constrained data points, but final booking data is unconstrained by some method. This will result in negative bias in contrast to Type II error.

Finally, Type IV error analysis involves a dataset which is unconstrained and a base for the measure of forecast error which employs similar unconstraining procedure. This type of analysis has been rarely performed. Forecast errors calculated under these conditions can be theoretically considered free of induced bias. However, there are still two major sources of bias unaccounted for.

One is the systematic bias in the detruncation procedure itself. This will affect the forecast error calculations, as the base unconstrained bookings on which forecast error is calculated must be estimated via detruncation for every closed flight. Thus a Type IV analysis will only be truly unbiased if detruncation method is unbiased or the there are no constrained observations in the dataset. In the latter case, however all analyses types will have no bias.

The other important source of bias is when the detruncation scheme between calculating base for forecast error and unconstraining historical dataset for forecasting are different. In this case the forecaster/detruncation combination using a different detruncation
methodology than the base, will suffer from an inherent bias differential from the combination that employs same detruncation scheme as the base.

**Is Zero Forecast Error Possible or Desirable?**

From the above discussion of limitations of forecasting assumptions and inherent biases arising out of analyses, it appears that under some ideal circumstances where these issues are appropriately accounted for, zero forecast error is achievable. Also since intuitively forecast error is related to revenue performance, zero forecast error will result in maximum revenue performance. However even if definitional issues about forecast error as mentioned above are somehow resolved, the achievement of zero forecast in actuality is nearly impossible. 32 The problem remains with the assumption that an input with zero forecast error will result in flight loads that exactly stick with “zero error” predictions and thus maximise revenue.

Skwarek (1997) has presented the following example that helps explain this. Consider that a constrained flight $f$ departs and the airline is certain that passengers on the next $f+1$ departure of the same flight will have exactly similar demand characteristics. After detruncation, the airline inputs the expected unconstrained bookings as its forecast for the $f+1$ departure, based on constrained bookings from departure $f$. Even then this “zero error” forecast will not result in a “zero error” result, with passenger reservations materialising as predicted.

As the seat optimiser will adjust optimal seat booking limits to these inputs causing different class closures, sell-up and/or lost passengers and thus a different resultant constrained booking pattern than departure $f$ will be observed. The associated unconstrained booking level for this $f+1$ departure will also be different and as a result there will be a non-zero forecast error.

Only in conditions where either there is low-demand resulting in no fare-class being constrained or each passenger being placed according to his/her WTP, is the above not applicable. This again points out to the arbitrary nature of passenger demand by fare class.

as only in these two limited conditions described above, a perfect mapping between passenger type and fare class occurs. Otherwise, observed demands in each fare class are state dependent: a result of the time in the booking class that lower-fare classes close and the segmentation ability of each fare class.

Thus the conventional managerial emphasis on lowering forecast errors and comparing forecasters’ performance through traditional measures is somewhat misguided.

**Detruncation**

Detruncation, unconstraining or uncensoring a distribution refers to the process of estimating parameters of a distribution based on a sample from which some values have been removed or censored. In the airline case this refers to the process of estimating the unconstrained demand, in the event that bookings-in-hand reach the booking limit for a fare class. This particular fare-class is marked closed and all further requests will be refused till it becomes open again due to cancellations etc. As soon as requests are refused, it becomes almost impossible to infer the actual demand directly from the bookings data and thus the need for detruncation.

**The Need for Detruncation**

Theoretically speaking, not detruncating such constrained data can have severe revenue consequences for the airline. If a forecaster under predicts demand for higher-value fare classes as a result of not unconstraining the demand, then more low-value passengers will be accepted and later-booking higher-value passengers will have to be turned away. This result then becomes part of historical database for future flights, depressing further the high-value fare class forecasts. This yield dilution becomes extreme as bookings of high-value fare class passengers spirals iteratively downwards, replaced by the low-fare paying passengers.

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33 Schneider (1986)
**Entwined Nature of Detruncation and Forecasting**

Detruncation and Forecasting are closely linked as both contribute to generate demand forecast. However, detruncation is applied earlier in the process of generating a demand. Detruncation unconstrained demand whereas forecasting consists of getting an estimated of future demand, given previous demand, including the previously estimated unconstrained demand. Both these are essential to reliable forecasting.

The seat optimiser routines generate booking limits on unconstrained demand and in turn require unconstrained demand forecasts as input. As discusses in previous sections, unconstrained historical dataset as input to forecaster results in reliable forecasts and thus detruncation is essential to forecasting.

**Review of Previous Forecast Accuracy Studies in context of Detruncation**

As briefly mentioned in previous sections, the majority of comparative studies on forecasters have ignored detruncation both for historical database as well as actual dataset for error measurement. In this most of the studies assumed that their datasets are unconstrained.

Wickham\(^{34}\) in his thesis has discussed extensively the comparison of several forecasting models with and without a detruncated dataset. He developed his own detruncation algorithm in this regard that has been described in a previous section. His salient findings were:

- Detruncation had no significant effect on higher booking classes.

- Detruncation decreased the spread of performance metrics among the forecasting models that were tested.

- Some performance metrics even improved after detruncation was used

\(^{34}\) Wickham, Robert R. (1995)
• Inherent positive forecast bias significantly increased.

These results are intuitive apart from the decrease in spread of performance metrics. High value fare classes will be least affected by detruncation as they only close on flights with extremely high demand, which occurs rarely. The inherent positive bias increase is attributed to the Type II analysis conducted by Wickham as mentioned earlier.

There have been few comparative studies of different detruncation methods being used in context of forecast accuracy. Skwarek\textsuperscript{35} used PODS to undertake a comparative analysis of different detruncators; however he disregarded accuracy as performance basis and instead focused solely on revenue performance. His results showed that even at low demand factors, the impact of detruncation models on revenues can be as high as 3.5%, provided only one of the two airlines in the network is employing detruncation.

**Summary**

This chapter serves as the introduction to the main issue at hand, namely forecast accuracy. The need for forecasting was highlighted followed by the requirement for forecasts to be as accurate as possible. In the same context of forecasting accuracy, past academic works are then reviewed to impart a context to this current study. The basic assumptions underlying the forecasting phenomenon in airline revenue management were then critically discussed in detail. The concept of forecast bias was introduced and different types of biases were discussed. It was also highlighted that the target of zero forecast error is impossible to achieve practically. Detruncation was defined followed by the need for detruncation. The close nature of forecasting and detruncation was also discussed. A critical review of previous forecasting studies with respect to detruncation was also undertaken.

\textsuperscript{35} Skwarek, Daniel K. (1997)
Chapter 3: Simulation Results

PODS Simulation Results Analysis

Investigating Forecast Accuracy in a simulated network environment

In this chapter, we will present findings of our analysis into revenue-accuracy relationship using various forecasting/detruncation combination employing Passenger Origin-Destination Simulator (PODS). However, before we delve into the analysis details, it is important to briefly introduce the simulation environment that comprises of the PODS simulation model, the simulated network, the optimizers, forecasting and detruncation schemes.

PODS: A Brief Introduction

The Passenger Origin Destination Simulator or PODS (as it is widely known) was developed at Boeing by Hopperstad et al.\(^36\). This simulator is an evolution of the Decision Window Model (DWM)\(^37\) also developed at Boeing. However, PODS is a much more capable development that not only incorporates the DWM characteristics of schedule, airline image, aircraft type, but additional characteristics of fare products in determining passengers preferences in an environment that allows two or more airlines to compete; simulating the effects of various optimisation, forecasting and detruncation schemes on their network revenues.

PODS Model

The PODS simulator is composed of four interacting components, namely:

\(^36\) Hopperstad, Berge and Filipowski, (1995)
\(^37\) The Boeing Company (1994)
• Historical Database,

• Forecaster,

• Optimiser and

• Passenger Choice Model.

These four components are interlinked as shown below in Figure 5:

![Figure 5: PODS Architecture, Source: Hopperstad, The Boeing Company](image)

The Historical Database (HDB) stores booking information from previous flights and provides this historical data to forecaster. The forecaster’s historical database is manually initialised at the beginning of each simulation run. The forecaster utilises this historical bookings data and the bookings currently on hand furnished by the optimiser to forecast future demand for the current flight. The forecaster’s output, the expected future bookings are then fed into the Optimiser. The Optimiser determines seat protections and availability for various fare classes, based on the forecaster’s output and the actual path/class bookings and cancellations. This seats availability is fed to the Passenger Choice Model which generates new passenger bookings/cancellations based on the passengers’ decision criteria.
Working of a Simulation

Each Simulation has specific input parameters which determine the type of forecasting/detruncation combination and Revenue Management Optimiser that will be employed by each of the airline as well as defining the network configuration. Each simulation is a set of trials (5 in this case) with each trial composed of 600 samples. Each sample represents one set of flight departures for the network, representing a single day of operations. The first 200 of these samples are discarded to avoid initial condition effects. These trials/samples combination guarantees statistically significant and stable results.

At the trial level, the flow of events is detailed in figure 6. Each trial is made up of 600 repetitions of the same day (sample) with no trends. Bookings are spread over 16 time frames for each day. For a more exhaustive explanation of working of PODS and internal structure, the reader is encouraged to read Wilson\textsuperscript{38} and Lee\textsuperscript{39}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{PODS_Flow_Chart.png}
\caption{PODS Flow Chart, Source: Zickus, Jeffrey S. (1998)}
\end{figure}

\textsuperscript{38} Wilson, John L. (1995)
\textsuperscript{39} Lee, Alex Y. (1998)
Network D

The simulation models a network of operations for the competitor airlines. The network employed is titled ‘Network D’, the current stable configuration. It is made up of 40 spoke cities and two hubs, one for each airline. The geographical configuration roughly simulates a domestic network spanning the continental United States, with the hubs located roughly in the middle and cities distributed equally to both east and west sides of the hubs. This helps to simulate a variety of routes and O-D markets via hubs.

The network is unidirectional; with flights originating from West Coast and terminating at East Coast (refer to Figure 8). This set of 40 spoke cities associated with two hubs forms 482 origin-destination markets, connected by 252 flight legs. There are three banks per day per airline, i.e. each market is served by three flights per day, resulting in 2892 paths. The network flow diagram (Figure 7): shows this network configuration.

![Flow Diagram for Network D](image1)

![Geographical Arrangement of Network D](image2)
Revenue Management Methods

PODS can utilise six different Optimisers or Revenue Management (RM) Algorithms, in addition to the primitive First Come First Serve (FCFS) (until leg capacity is reached). The commonly employed optimisers are, namely:

- Fare Class Yield Management/Expected Marginal Seat Revenue (FCYM/EMSR)
- Displacement Adjusted Virtual Nesting (DAVN)
- Heuristic Bid-Pricing (HBP)
- Prorated Bid-Pricing (ProBP)

This section will only briefly discuss those optimisers that were employed in simulations. For a detailed explanation of each of these revenue management algorithms, a reading of Williamson40, Bratu41 and Lee42 is recommended.

The fare structure employed in simulation/analysis comprises of four classes, Y, B, M and Q in descending order of value. The Revenue Management (RM) algorithms have been classified into two broad categories, namely, Fare Class RM algorithms and Origin-Destination (OD) algorithms, based on whether they optimise on flight-leg level or OD path level.

Fare Class Revenue Management Algorithms

This set of algorithms set booking limits for a fare class on a flight leg level, based on demand forecasts on leg basis. This is the shortcoming of these algorithms, as it is entirely possible that short-haul passengers will be allowed bookings on flight segments making up a path against a long-haul higher-fare passenger wishing to travel on that path.

40 Williamson, Elisabeth L. (1998)
41 Bratu, Stéphane J-C (1998)
42 Lee, Alex Y. (1998)
These situations lead to optimisation issues and lower network revenues. This group of algorithms, detruncate, forecast and protect seat inventory on leg basis.

**Fare Class Yield Management (FCYM)**

Fare Class Yield Management method employs the ‘Expected Marginal Seat Revenue’ (EMSR) algorithm which was developed by Belobaba\(^{43}\), and is one of the earliest algorithms in the airline industry, for nested booking classes on a flight leg. This algorithm employs leg-based forecasts by fare-class and leg-based seat protection levels.

The main idea underlying the algorithm is that the uncertainty of passenger making a booking on a given flight in a fare class should be considered for setting booking limits. EMSR sets booking limits according to expected marginal revenue of every incremental seat sold, hence its name. Expected marginal seat revenue is the product of probability of selling additional seat in a fare class and its associated fare. This probability decreases as the number of seats already protected for the fare class increases. Hence the booking limit for a class is set when its marginal revenue equals the fare of next lower class.

The EMSR\(_b\)\(^{44}\) version goes a step ahead in protecting joint upper classes from the class immediately below. Thus given our fare structure, the EMSR\(_b\) algorithm will calculate a protection level for joint Y and B classes from M class. The class demands are assumed to be independent and normally distributed, making it easier to calculate demand for these joint classes.

**Origin-Destination Algorithms**

This set of algorithms allows for different availability for different Origin-Destination pair. The term O-D control method encompasses a Revenue Management algorithm employing any type of O-D seat inventory control scheme. Thus in practice as well as PODS there are many possible combinations constituting O-D control method e.g. leg forecast and protection with O-D virtual mapping, leg-based protection or vice versa and O-D forecast with O-D protection.

\(^{43}\) Belobaba, Peter P. (1987)
\(^{44}\) Belobaba, Peter P. (1992)
**Displacement Adjusted Virtual Nesting (DAVN)**

Displacement Adjusted Virtual Nesting (DAVN) employs O-D forecasting and network optimization with leg-based inventory control. DAVN employs solving a deterministic, revenue maximisation, Linear Programming (LP) problem\(^{45}\) to determine the shadow price for each leg in the network. The shadow price is the minimum the airline is willing to accept for an additional seat on a given leg or alternatively speaking, it is the expected revenue increase observed from relaxing the capacity constraint. The bid price is then employed to calculate pseudo-fares that accounts for these displacement costs. For local passengers, the pseudo fare is the actual fare however, for the connecting passenger on a two-leg path is the difference between the fare on the leg the passenger is booking on and the shadow price on the other leg (where a local passenger is being displaced by him). This algorithm has higher performance in terms of revenue than the FCYM, in PODS simulations.

**Heuristic Bid Price (HBP)**

Heuristic Bid Price method was developed by Belobaba\(^{46}\). This method accounts for displacement of local passengers by connecting passengers using the notion of bid prices. The bid price is a minimum threshold price. Instead of seat protection level, the algorithm dynamically determines the bid price; the minimum price that should be paid by a passenger in order to make a booking. This bid price, for local passengers, is based on the EMSR value of the last seat available on a flight leg and for a connecting passenger, on a two-leg path, it is the sum of a) EMSR value of the last available seat on first leg and b) product of percentage of local passengers on both legs and EMSR value of the last available seat on connecting flight leg. The booking decision is essentially based on comparison of fare with the bid price on each leg. For connecting passenger on a two-leg path, this translates to acceptance only when the fare is higher than the maximum of bid prices of both legs. This algorithm employs forecasting, detruncation and optimisation on flight-leg/booking class basis.

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\(^{45}\) Williamson, Elisabeth L. (1992)

\(^{46}\) Belobaba, Peter P. (1998)
**Prorated Bid Price (ProBP)**

Prorated Bid Price is the most recent RM algorithm to be added to PODS repertoire. This method was developed by Bratu (1998). Prorated Bid Price algorithm again uses a bid price method to determine acceptance of a booking request. However it differs from HBP since it calculates bid price for each leg depending on the O-D and splits the actual total fare, for connecting passengers, among the legs traversed thus taking into account the network structure and demand on each leg. This method uses EMSR_c, the critical EMSR value of a leg (i.e. the Expected Marginal Revenue of the last seat available on the leg) for calculating prorated fares. One issue with the calculation of the prorated fares is that the EMSR_b algorithm uses the total itinerary fare of an ODF traversing leg to determine EMSR_c value and thus overestimates it. An iterative convergence model has been developed to address this issue. This algorithm, during PODS simulation runs, has shown significant improvement over other RM schemes.

**Forecasting Methods**

Forecasting aims to provide a quantitative estimate of the future demand for a flight, based on similar past flights and current bookings on the subject flight. PODS allows the user to simulate two forecasting techniques, which are briefly discussed here, namely:

- Pickup Forecasting

- Regression Forecasting

For a detailed description of these forecasting methods, one is referred to Zickus.  

**Pickup Forecasting**

Pickup Forecasting technique is more detailed than a regular time series as it employs the number of passengers picked up from one time period to the next, besides using the average of previous observed bookings or unconstrained demand for previous flights. The

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47 Zickus, Jeffrey S. (1998)
forecasting is done on path-class or leg-class basis, dependent on the requirements of the optimiser.

Pickup forecaster averages the pick-up (number of passengers booked between two consecutive time periods) from one time period to the next for a fixed number of previous flights. This average is then added to the current or forecast number of bookings for the current time period.

L’Heureux (1986) developed a modified pickup forecasting scheme involving weighted averages, with more weight given to recent observations. This scheme reduced the impact of an outlier flight in the dataset, however, at the same time it introduced susceptibility to prolonged periods of unusual booking activity. PODS, however, utilises the basic Pickup forecasting scheme without any relative weighting of observations.

**Regression Forecasting**

Regression Forecasting in PODS employs least-square regression technique. This method relates the number of bookings accumulated at a given point in time for a given flight to the final bookings. The basic formulation is as follows:

\[ X_{F,i} = \alpha_n + \beta_n X_{F,i-n} + \varepsilon_n \]

where
- \( X_{F,i} \) is the total number of bookings after time frame \( i \)
- \( X_{F,i-n} \) is the total number of bookings at \( n \) time frames prior to departure
- \( n \) is the number of time frames over which the model is calculated
- \( \alpha_n \) and \( \beta_n \) are the intercept and slope respectively of the linear regression model for time period \( i \)
- \( \varepsilon_n \) is the error in the model.

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\(^{48}\) Gorin, Thomas O. (2000)
Detruncation Methods

Detruncation methods estimate the unconstrained demand, which is necessary for good revenue management forecasting. Detruncation analyses historical booking patterns and applies these patterns to closed observations to evaluate demand for a given flight. PODS uses various detruncation techniques of which the two used in simulation are described; Booking Curve Detruncation and Projection Detruncation.

Booking Curve Detruncation

Booking Curve detruncation is the simplest method used in PODS to estimate unconstrained demand. This detruncation scheme assumes that bookings follow a similar pattern for all flights. Thus if a certain percentage of total bookings for a fare class or OD class have occurred by a particular time frame in the past, it is logical to infer the same for the current flight. This detruncation scheme simply uses the unclosed flights to estimate what percentage of passengers books from a certain time period to the last time period and utilises this to evaluate unconstrained demand on closed flights.

The Booking Curve detruncation algorithm calculates the expected increase in relative bookings from one booking period to the next and then computes the relative overall variation in bookings from one period to the last period, giving a booking curve. This booking curve is used to determine unconstrained demand. A detailed explanation of the algorithm is given by Zickus49.

Projection Detruncation

This algorithm assumes that the conditional probability of underestimating the unconstrained demand for a flight and fare class, given that this particular fare class closed, is a constant value. Again, for detailed description of algorithm, one is referred to Zickus50.

The demand is assumed to be normally distributed. The algorithm first computes the mean demand over the unclosed observations. Then it employs an arbitrary value, \( \tau \), to

49 Zickus, Jeffrey S. (1998)
evaluate the detruncated value of the closed observation. The $\tau$ represents the conditional probability by which the demand for a flight and fare class is underestimated. It is conditional on the fact that closure occurred for that particular flight and fare class. This conditional probability is assumed to be a constant value. The closed observation is projected to a new value such that the ratio of probability higher than this new value to the probability higher than original value is equal to $\tau$, as shown in Figure 9.

$$\tau = \frac{A}{A + B}$$

![Figure 9: Projection Detruncation]

**PODS Forecast Accuracy Measures**

PODS allows the user to analyse forecasts in detail through a number of forecast-related outputs. These various forecast measures include:

- Number of Observations contributing to the measure

- Mean Forecast of remaining demand from a certain timeframe to departure

- Mean Forecast Error
  This is also referred to as Mean Bias. This is calculated as the difference between “actual” and forecast values where “actual” values for open observations are actual bookings and for closed observations are bookings unconstrained with booking curve detruncation algorithm.
Demand Forecasting Accuracy in Airline Revenue Management

- Mean Absolute Error
  
  This is also referred to as Mean Absolute Deviation (MAD). This is calculated as
  
  \[ \frac{\sum_{n=1}^{N} |actual_{n} - forecast_{n}|}{N} \]
  
  where N is the number of forecasts generated over a certain period of time

- Standard Deviation of Absolute Forecast Error

PODS also outputs these measures separately for open and closed observations as well as total observations. We employ some of these traditional forecasting accuracy measures to show that these measures do not provide any meaningful relationship vis-à-vis revenue performance of various forecasting methods. It is again reminded that “actual” values calculated for these measures are booking curve detruncated for closed observations.

**Research Methodology**

Primary research uses PODS to simulate various network configurations. In line with the goals defined, the simulation and subsequent analysis focuses on, first, establishing that there exists no meaningful relationship between the traditional measures of forecast accuracy and their corresponding revenue performance, and second, understanding the factors underlying the better revenue performance of some arbitrary forecast manipulation techniques.

Thus the simulation results can be divided into two broad parts. The first part compares and discusses the revenue performance of various forecasters and their relative accuracy under traditional measures. The second part focuses on investigation into the better revenue performance of the arbitrary forecast inflation.

The goals dictated a detailed analysis of forecast accuracy that is free from distortions due to competition and network effects under conditions that satisfy basic assumptions of forecasting. This called for a monopoly network with single path per market with the

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50 Lee, Anthony O. (1990)
added restriction of passenger’s first choice being his only choice as the first step in analysis. Monopoly network ensures that there is no spill-out of potential demand to competitors as well as spill-in from competitors, thus no competitive impacts. A single path per market setup avoids recaptures, both vertical and horizontal – passengers who choose to travel on other paths in the same market if their first choice path is not available. First choice being a passenger’s only choice ensures that fare class independence is maintained as a passenger will not be able to sell up to a higher fare class if his first choice fare class is closed. Such a network configuration would ensure that the basic assumptions of fare class/path demand independence are satisfied. However, limiting the analysis to these idealised conditions would be of limited practical value, thus there was a need for gradual addition of complexity in the simulations and analysis to terminate at a configuration that for most parts resembles the real world.

Thus the following logical course of action for simulation and analysis has been adopted in this thesis:

- **Part 1: Analysis of Revenue versus Forecast Accuracy relationship**
  - Single Path per Market, First Choice Only Choice, Monopoly Network
  - Single Path per Market, Monopoly Network
  - Multiple Paths per Market, First Choice Only Choice, Monopoly Network
  - Multiple Paths per Market, Monopoly Network

- **Part 2: Use of Path-based Forecasts and Future Protect**
  - Monopoly Network Configurations
  - Competitive Network Configurations

Each of these steps is explained in more detail below.
Part 1: Forecast Accuracy vs. Revenue Relationship

Simulation Setup

The simulations done for the analysis of revenue-forecast accuracy share common setup and it is useful to describe its detail before launching into their results and analysis. The individual difference will be described wherever they are applicable.

Monopoly Network Description

To remove competitive effects from the duopoly network $D$, primarily Spill-in revenue, the network was reduced to a monopoly configuration. This configuration thus has only one hub and 120 markets and 440 flight legs. There are still three banks per day resulting in each market being served by multiple paths.

Revenue Optimiser Methods

Unless otherwise specified, the analysis is mostly focused on Expected Marginal Seat Revenue algorithm (EMSR$_b$) and Displacement Adjusted Virtual Nesting (DAVN). This contrasts a traditional fare class, leg-based optimisation scheme with a more sophisticated Origin-Destination based scheme. In most cases, EMSR$_b$ with leg-based forecasting forms the base case against which revenue performance and accuracy is measured.

Detruncation Schemes

As described above, PODS supports two detruncation schemes, namely, Booking Curve Detruncation and Projection Detruncation. We have utilised both schemes in combination with different forecasters.

Booking Curve detruncation in combination with Pickup forecasting usually forms the performance base case, as this is the basic forecasting/detruncation scheme employed in the industry. This combination is termed “PUBC” throughout the analysis. Booking curve detruncation scheme is also paired with Regression forecaster. This combination is termed as “RGBC”
It is important to mention here that PODS simulation utilises Booking Curve detruncation to calculate “actual” demand data which is then subsequently employed to determine the various forecasting accuracy related outputs. Thus all PODS forecasting measures are biased in favour of forecasting/detruncation combination that include Booking Curve detruncation scheme.

Projection Detruncation has always been used with Regression Forecasting, the combination serving as the more sophisticated scheme than Pickup-Booking Curve combination. This combination is abbreviated as “PDRG”. Three primary ‘τ’ values have been used in the analysis; 0.15, 0.30 and 0.40, and the combinations termed PDRG 0.15, PDRG 0.30 and PDRG 0.40 respectively. The lower the τ value, the more aggressive the detruncation performed, resulting in higher forecasts. In most cases only two of these PDRG combinations have used to show the trend when moving from aggressive to conservative detruncation.

**Forecasters**

In the subsequent analysis, two forecasting schemes have been used, Pickup Forecasting and Regression Forecasting. As mentioned above, Pickup Forecasting is the simplest scheme and usually serves as the base combination, in conjunction with Booking Curve detruncation. Regression Forecasting is considered the more sophisticated of the two schemes. It is used with both Booking Curve and Projection Detruncation algorithms.

**Forecast Multiplication**

As introduced earlier, it tends to be the case that the current forecasting schemes under-predict existent demand thus losing out on revenue potential. Many airlines in the real world use some technique to manually manipulate or inflate forecasts in some of their markets.

Forecast Multiplication is an option in PODS that models this manual, arbitrary inflation of forecasts given by the current forecasters, before these forecasts are fed to the Optimisers. In PODS, this is modelled by a user-input factor, called Forecast Multiplier which is used to increase initial forecasts generated by the Forecaster. For example, a
Forecast Multiplier value of 1.1 implies the forecasts will be inflated by 10% before input to the optimiser. This is termed as FM 1.1 in the subsequent description of analysis. Also Pickup Forecaster/Booking Curve Detruncation combination with Forecast Multiplication is termed simply as FM, whereas other forecaster/detruncation combinations are clearly marked, e.g. RGBC+FM1.1 implies regression forecaster/projection detruncation with forecasts inflated by 10%.

In the course of this analysis, Forecast Multiplication will serve both as a diagnostic tool and yardstick against which the revenue potential of various forecasting/detruncation combinations will be judged. In a separate but related PODS study, the value of Forecast Multiplier that gives maximum revenue gain over base case has been determined, under various network configurations. The results show that under FCYM, Forecast Multiplication performs much better in relative percentage terms than under DAVN. Also, under FCYM, the revenue benefits of Forecast Multiplication extend over a broader range of multiplier values than under DAVN. Similarly the maximum Multiplier values for the duopoly cases are higher than monopoly cases.

In our case, only that value of Forecast Multiplier which yields the maximum revenue for that particular network configuration is used. This helps put the performance of sophisticated forecasters and detruncation algorithms in perspective.

Simulation and Results

As discussed above, the assumptions on which airline forecasting is based do not fully hold in a competitive, multiple paths per market network with multiple fare products and therefore as a first step in analysis, a network configuration was chosen where all the basic forecasting assumptions hold true.

Single Path per Market, First Choice Only Choice, Monopoly Network Analysis

Single Path per Market, First Choice only Choice, Monopoly Network serves as the logical launching point for forecasting accuracy analysis as it adheres to the forecasting assumption of fare class demand independence.
This configuration differs from the Monopoly Network D described above as it limits each market to be served by a single path. This was essential to remove network effects of recapture.

Recapture results when passengers who could not obtain their first choice in terms of fare class or path, subsequently book on other paths offered by the same carrier. In the case where the passenger just changes paths and books within the same fare class, it is termed as ‘Horizontal Recapture’ and in case where the passenger not only books on a different path but a different fare class than his first choice, it is known as ‘Vertical Recapture’. When Recapture occurs, in either horizontal or vertical case, the forecasting assumption of independence of demand is violated, thus to prevent recapture all paths except one are removed from the markets.

Similarly, First Choice only Choice ensures that passengers who do not get their first choice in terms of fare class then subsequently do not sell up to a higher fare class on the same path. If this situation, called Sellup is allowed, it again violates the basic assumption of mutual independence of fare class demands. Thus this initial and highly simplified configuration is aimed at ensuring that all forecasting assumptions hold true. This would then provide the most honest environment to ascertain if a revenue-forecast error relationship exists.

**FCYM**

A glance at the revenue performance results under FCYM in Figure 10 show that two forecasting/detruncation combinations, RGBC and PDRG 0.30 when used with Forecast Multiplication (FM) result in highest revenue gain of 0.61% over the base case. The base in this case uses Pickup/Booking Curve Detruncation combination without Forecast Multiplication.
Among the forecasting/detruncation combinations without Forecast Multiplication, PDRG at a $\tau$ value of 0.15 does the best, with a revenue gain of 0.47%. PDRG with $\tau$ value 0.15 is considered very aggressive detruncation that results in much higher forecasts than the base. However, the same combination, when used in conjunction with FM, results in revenue gains that are not the highest. This can be attributed to even higher forecasts that result in overprotection and subsequent spoilage loss, i.e. seats that were protected for higher-fare classes remain unbooked due to lack of demand.

One important point needs to be highlighted here that in PODS, as in the real world under Advance Purchase restrictions, a restricted lower-fare class can not be sold after the restriction period kicks in. Thus if seats are available in terminal time frames, they can not be offered to lower fare-classes that have closed, even if incremental low-fare class demand exists in the market. The apparent losses in Q-class revenues, despite the higher forecast, ascertained from a fare class profile analysis of revenue gains over the base case confirm this.
As can be seen in Figure 11, among the non-FM combinations (combinations without Forecast Inflation), PDRG 0.3 shows the highest increase in Y-class revenue, as well as gains in B and M classes. Thus slightly aggressive detruncation leads to higher Y-class forecasts. PDRG 0.3 with Forecast Multiplication shows the highest Y-class revenue increase among the FM combinations as well but it also suffers the highest revenue loss in Q-class. This clearly indicates a case of over protection.

A look at the mean forecast for Y-class in Figure 12 confirms this. There is a distinct difference between the mean forecasts of non-FM and FM combinations, with the revenue-maximising forecast level closer to the FM combinations.
PDRG0.15 without FM results in highest forecast among the non-FM combinations which is not surprising since it is the most aggressively detruncated combination. Similarly, PDRG0.15 with FM is the highest of all the combinations simulated, with the best performing (revenue-wise) combinations of RGBC+FM and PDRG0.3+FM returning slightly lower forecasts.

As is apparent from this distribution of mean forecasts, high forecasts tend to return higher revenue but too high a forecast results in revenue losses. Even though PUBC+FM combination results in much higher forecasts than PDRG0.15, which are closer to the maximum revenue achieving combinations of PDRG0.3+FM and RGBC+FM, yet its revenue gain is lower than the PDRG0.15 combination.

The Mean Absolute Deviation (MAD) or Mean Forecast Error chart (Figure 13) also affirms this conclusion. Although both RGBC+FM and PDRG0.3+FM combinations show the lowest MAD values among the FM combinations, the same does not hold true for PDRG0.15, the best revenue performer among the non-FM combinations. Almost all non-FM combinations have lower MAD values than the base case of PUBC. The only inference that can be drawn is that the best revenue-wise performing combinations have
MAD values that are closest on either side to the base case MAD values. Thus under this configuration, forecasts that are too far removed from base case do not yield better revenues.

The most aggressive combination of PDRG0.15+FM has the highest positive mean bias which is expected as can be seen from Figure 14. Both combinations employing booking curve detruncation algorithm show the lowest mean bias, with PUBC achieving zero bias over all time frames. This is hardly surprising, given that the actual bookings used to determine the bias were calculated using booking curve detruncation scheme. This biases the results in favour of the PUBC and RGBC combinations.

The salient results from analysis under FCYM can be summarised as:

- Higher mean forecasts than base correspond to higher revenue performance than base, though too high forecasts may result in revenue loss.

- In terms of Mean Absolute Deviation, FM Combinations tend to be on the higher side of the base PUBC case and all non-FM combinations fall on the lower side of the base, with higher revenue performance combinations closer to the base on either side.
- As with the mean forecasts, all combinations have higher mean bias than base except RGBC which has zero bias.

![Figure 14: Y-Class Mean Bias - FCYM](image)

In the context of the primary goal of this analysis, it is a significant observation that even under a network configuration that abides by the basic assumption of forecasting for FCYM, there is no apparent relationship between revenue and forecast error as proposed in Figure 2.

Forecast multiplication, an arbitrary method of inflation has outperformed the more sophisticated forecaster/detruncation combinations like regression and projection detruncation. However, it does not happen to result in most accurate forecasting as measured through conventional metrics. The revenue performance of forecast multiplication has also highlighted that there is demand in the market that is not being forecasted by these forecaster/detruncation combinations.
DAVN

Revenue results under DAVN are markedly different from EMSR_b (Figure 15). Under EMSR_b, all combinations showed revenue improvements over base case, whereas under DAVN only RGBC shows revenue gains over base among non-FM combinations and PUBC and RGBC show revenue gains over base among FM combinations. The magnitude of percentage revenue gains is also much smaller than EMSR_b. Another result of note is that the maximum revenue gain occurs for RGBC without FM, a gain of 0.19%, compared to the maximum for FM-combinations of 0.13% with PUBC+FM combination.

![Diagram showing revenue gains](image)

**Figure 15: Revenue Gains under DAVN-First Choice Only Choice, Single Path/Market, Monopoly**

For PDRG combinations, with FM or without FM, more aggressive detruncation, courtesy of a lower \( \tau \) value, results in greater revenue loss. The magnitude of these losses is much higher (-0.55% for PDRG 0.15 without FM, -0.95% for PDRG 0.15 with FM) than the maximum revenue gain (0.19% for RGBC without FM), as shown in Figure 15. This shows that under OD control (employing OD optimiser and forecasting); there is much less room for revenue performance improvement compared to leg-based optimisation. The revenue maximal forecast level is much closer to the base under DAVN and path-based forecast that it employs than EMSR_b and associated leg-based forecast. A
look at revenue breakdown by fare class (Figure 16) shows that the overall revenue performance is dictated by the revenue gains in Y-class and associated losses in Q-class, with very little gains/losses in M and B classes.

For non-FM combinations, both PDRG combinations show higher Y-class revenue gains than RGBC, with more aggressive PDRG 0.15 showing highest gain among non-FM combinations, as can be seen in Figure 16.

Figure 16: Revenue Gains by Fare Class under DAVN
Both non-FM PDRG combinations result in overprotection in Y-class, as is apparent from the comparatively big losses in Q-class that result in aggregate loss over base. This again shows that although there is First Choice Y-class demand in the market, the difference between revenue maximal forecast and base forecast is not that much.

Similar results are apparent for FM combinations though here the First Choice Y-class revenue gains are higher e.g. for PDRG combinations much higher than 1%. The corresponding losses, due to overprotection, are also much severe, reaching 2.13% for PDRG 0.15, the most aggressive detruncation.

A look at the Y-class mean forecasts for the more important combinations in Figure 17 shows that all these combinations result in higher mean forecast than the base. RGBC without FM, the best revenue performer, has mean forecasts that are only slightly higher than those of the base case PUBC, again emphasising the small performance improvement envelope available to ODF control, since switch from leg-based to ODF control takes up the major portion of potential revenue. The results show that overly aggressive detruncation is bad for revenues and use of aggressive detruncation in tandem with forecast multiplication results in even worse revenue performance.
The highlight in the Mean Absolute Deviation graph, Figure 18, is that RGBC without FM has the lowest MAD of all combinations. Similarly PUBC with FM, the best revenue performer among those combinations that employ forecast multiplication, has the highest MAD.

This is exactly opposite of the EMSRb case, where the combinations with better revenue performance were closer to base case in terms of MAD values. In this case, the two improved revenue performing combinations are farthest from the base PUBC without FM case and form the MAD envelope around it. Mean Bias analysis in Figure 19 shows that all combinations have much higher mean bias than the base PUBC without FM combination, barring RGBC without FM which has near zero bias. However, there is no correspondence of bias with the revenue performance of the combination.

Even though RGBC without FM, the best revenue performer, has near zero bias; it is more due to the fact that the detruncation scheme employed by PODS to estimate ‘actual’, is booking curve detruncation algorithm. This result is repeated again in coming analysis, where both PUBC and RGBC combinations without FM continue to show near zero/zero biases.
The results summary of forecast accuracy analysis under DAVN shows

- Higher forecasts than the base case of PUBC don’t systematically translate into revenue gains. In the instances with a positive revenue change, there is no discernable relationship between revenue gains and mean forecasts.

- Best revenue performing combinations usually are the farthest from base case of PUBC forming sort of an envelope. The non-FM combination, with MAD values lower than the base forming the lower edge and FM combination, with MAD values higher than the base, forming the upper edge of this envelope.

- All combinations have higher bias than base case of PUBC, which has zero bias. There is no clear revenue performance-bias relationship.

These results again highlight the fact that using the traditional error metrics; it is not necessarily true that the best revenue performing forecasting/detruncation combination is also the one with least forecast error. The other significant findings are that forecast multiplication fails to repeat the revenue performance it achieved under FCYM and an
enhanced forecaster like Regression does best in terms of revenue. This hints at some fundamental difference between leg-based FCYM and path-based DAVN that is responsible for the revenue performance differential, principally the use of path-based forecasts by DAVN.

**Single Path per Market, Monopoly Network Analysis with Sellup**

The next step in analysis was to remove the restriction of first choice being the only passenger choice. This only allows passengers to sell-up to higher classes, as there is still only one path offered in any market. By gradually increasing the complexity of the network, the aim is to carry forward the findings from simpler network configurations to real world scenarios.

The Network is again the same Monopoly Network D, carried from the above analysis but with the restriction of First Choice being the only passenger choice removed. There is still no possibility of recapture, both horizontal and vertical, and, by virtue of being a monopoly network, no possibility of spill-in/out, from and to competitors. The only two revenue sources available in this configuration are First Choice revenue and Sell-up revenue.

It is highlighted here that the base case under this configuration also has the restriction of First Choice only Choice removed. In other respects it is essentially the same base case, employing Pickup Forecasting and Booking Curve Detruncation with no Forecast Multiplication. Also the revenue maximal Forecast Multiplier under this configuration is 1.2, referred to as FM1.2 subsequently.

**FCYM**

A quick glance at the revenue performance of various combinations shows that all combinations perform better than the base case. Also, the revenue gains in most cases are slightly higher than the First Choice Only Choice case. This is expected due to increase revenue earning potential through Sell-up as passengers whose first choice is a low-value fare class, and who were turned down due to non-availability of their first choice in
previous network configuration are now offered the opportunity to book an available seat in a higher-fare class.

![Revenue Gains under FCYM - Single Path/Market, Monopoly with Sellup allowed](image)

**Figure 20:** Revenue Gains under FCYM - Single Path/Market, Monopoly with Sellup allowed

Figure 20 above, shows that PDRG 0.15 continues to outperform other non-FM combinations, however RGBC also performs comparably. Among the FM combinations, both RGBC and PDRG0.3 stand out with revenue gains of 0.68% and 0.69% respectively. The interesting observation is that non-FM combinations of RGBC and PDRG0.15 perform comparably to the FM combination of PDRG0.15. FM however continues to do best with gains of 0.68% and 0.69% in combination with RGBC and PDRG 0.3 respectively.

The revenue breakdown by category highlights one important fact that the introduction of sell-up opportunity does not significantly distort/change the results. FM still continues to outperform non-FM combinations. The revenue changes (gains) in First Choice category are still of primal importance compared to the revenue changes due to Sell-up.

This is a very interesting result as it seems to suggest that the possibility of sell-up in a market has an insignificant effect on the overall revenue outcome and thus does not void
the forecasting assumption of independence of class demands. Thus, the problem with measuring forecasting accuracy is two fold, one due to the presence of multiple paths in the market and the other due to state-dependent nature of forecasts, which makes estimation of ‘actual demand’ impossible.

The fare class profile is different even though the revenue changes in Y and Q classes remain important. The positive revenue changes in M and B classes for many forecasting/detruncation combinations now form a significant part of their overall revenue performance.

![Figure 21: Revenue Gains by Category under FCYM](image)

The introduction of sell-up behaviour is reflected in the fare class profile of PDRG0.15 without FM as seen in Figure 21, which now displays revenue gains in M and B classes. These gains for B class result from the selling up activity from Q class and for M class from the sell up activity in both B and Q classes.
The sell-up effect is quite apparent, in Figure 22, under the FM combinations. These combinations show significant gains in higher-value fare classes, with Y-class gains in excess of 2% (except 1.53% for PUBC). Higher forecasts yield increased protection.
levels for higher fare classes, and sell-up opportunity results in greater bookings. This leads to significant losses in Q-class revenues, thus lowering the final revenue outcomes.

The fare class profile of First Choice revenue, in Figure 23 above, clearly shows the increased gains in higher fare classes, for FM combinations, compared to base. All
combinations result in higher mean forecasts per leg, as can be seen in Figure 24, than the base PUBC combination. FM combinations have the highest mean forecasts that correspond with their highest revenue gain values. Among the non-FM combinations, PDRG 0.15 has the highest mean forecast and the highest revenue gain. Thus higher mean forecast loosely translate in higher revenue performance under this configuration.

![Figure 25: Mean Absolute Deviation under FYCM](image)

FM combinations also show much higher mean absolute deviation values than the base PUBC, as in Figure 25, though the relative positions of these combinations do not correspond directly with their revenue performance.

![Figure 26: Mean Bias under FCYM](image)
FM combinations also display the highest mean biases of all combinations (refer to Figure 26). Regression and pickup forecasters, without forecast multiplication, show near zero bias which is a distortion caused by the fact that determination of “actuals” use the same booking curve detruncation.

These results again highlight that Forecast Multiplication continues to return superior revenue performance while not giving the most ‘accurate’ forecasts under FCYM. This revenue performance is due to increased higher-fare first choice revenue suggesting that there is unforecasted first choice higher-fare class demand in the market.

**DAVN**

The revenue performance under DAVN (refer to Figure 27) in this configuration is similar to one under First-Choice-Only-Choice configuration. The regression forecaster continues to be the best revenue performer under DAVN. Regression without forecast multiplication results in a revenue gain of 0.30% over the DAVN base case of pickup forecaster. Even among the combinations using forecast multiplication, Regression shows a 0.24% revenue improvement over the base case.

Projection Detruncation, with $\tau$ value 0.15 proves to be too aggressive a detruncation, resulting in revenue losses over base, both with and without forecast multiplication. These revenue performance figures again highlight that under DAVN arbitrary forecast multiplication does not perform as well as it does under FCYM.

A look at the revenue breakdown by category in Figure 28 reveals that, unlike FCYM, revenue gain from sell-up plays a significant role in determining the revenue performance of a combination. The revenue gain from sell-up for RGBC combination is 1/5th of the total revenue gain for the combination. The sell-up gain, for RGBC with forecast multiplication accounts for about 1/3rd of the total revenue gain. This is a significant deviation from FCYM results as it shows that under DAVN, increased forecast and resultant greater seat protection for higher fare classes results in greater number of passengers selling up from lower classes whereas under FCYM it resulted in greater number of first-choice, higher fare-class passengers.
Analysis of the fare class profile in Figure 29 shows that, in comparison with first-choice-only-choice configuration, the combinations that show overall revenue gains have greater revenue gains in Y-class with small gains in M and B classes as well.
Similarly, the associated revenue losses in Q-class are smaller compared with the first choice only choice configuration. This again highlights how introduction of sell-up opportunity has resulted in greater revenue.
Y-class mean forecasts show (Refer to Figure 30) that regression without forecast multiplication has slightly higher Y-class mean forecasts than the base PUBC over all time frames. This slightly higher forecast allows this combination to protect for increased first choice Y-class demand while incurring lowest revenue loss in Q-class. It is however interesting to note that all other combinations result in much higher Y-class mean forecast than RGBC. Both PDRG combinations with Forecast Multiplication have the highest mean forecasts, clearly resulting in over protection for the Y-class and contributing to their lower than base revenue performance. RGBC with multiplication has the highest mean forecast of all combinations that turn in higher revenue performance than base.
The Mean Absolute Deviation comparison in Figure 31 highlights more interesting results. RGBC combination, both with and without multiplication, show lowest MAD values of all combinations, especially in later time frames. The worst performing PDRG combinations with forecast multiplication end up with highest MAD values over most of the time frames.
The Mean Bias analysis in Figure 32 also shows similar results with all positive revenue gain combinations displaying lowest bias values. RGBC without Forecast multiplication has near zero bias, next to base case of PUBC with zero bias over all time frames. This, however, is attributable to the choice of booking curve detruncation scheme for calculating ‘actuals’, as mentioned previously.

The salient result is that forecast multiplication fails to increase revenues under OD control. Also in the simple single path per market network there is little evidence of a consistent relationship between forecast error and revenue.

**Multiple Paths per Market, First Choice Only Choice, Monopoly Network Analysis**

The next major network configuration change involves introduction of multiple paths in every market. This makes the network more realistic as now the service frequency in every market has been increased to offer a potential passenger more choices, as is the case in the real world.

In our analysis so far, we had reduced the service frequency in every market to one, thus effectively making one path available in every market. Now we have tripled the service frequency in very market. This change results in three different paths being made available with associated three connecting banks at the hub. The introduction of multiple paths in the market gives rise to additional revenue potential through recapture of passengers previously turned away due to lack of availability of their first preference, either in terms of time or fare class, and their decision not to sell-up to higher fare classes. The airline is now able to satisfy the travel needs of a greater share of potential passenger demand. These very reasons are responsible for frequent service offered by airlines in select markets in the real world.

The network, however, is still a monopoly network. This allows us to analyse the effect on revenue of introduction of multiple paths in the market without interference of competitive effect due to presence of other airline(s) in the market.
In order to further isolate the multiple paths effect, i.e. to observe what revenue changes occur simply by introduction of a heavier service schedule, the first choice only choice condition is inserted in this configuration. This condition ensures that there is no sell-up to higher fare classes on the same path as well as no recapture to other paths, both horizontally and vertically. The passengers, who choose to travel on the new paths introduced in the market, appear as increased first choice demand. This configuration, thus in some ways is similar to the single path per market, first choice only choice configuration. This abides by the demand independence by fare class assumption in forecasting.

**FCYM**

The network revenue for base case of PUBC under this configuration is exactly three times the base case revenue of single path per market, first-choice-only-choice monopoly network configuration. Revenue gains are shown in Figure 33.

![Figure 33: Revenue Gain over PUBC under FCYM](image)

All forecasting combinations show revenue gain over base case of PUBC. Combinations with forecast multiplication exhibit greater revenue gains than non-FM combinations. Thus under FCYM with multiple paths per market, advanced forecasting and
detrunckation combinations show improved results, though, forecast multiplication continues to do better. The revenue gain profile of various combinations is similar to the single path per market case, though the individual gain percentages are slightly lower.

The revenue gains for all combinations come primarily from Y-class, (refer to Figure 34) with small gains in M and B classes. For nearly all combinations, except for Regression without forecast multiplication, the gains in higher fare classes are associated with losses in Q-class. All FM combinations register a gain of more than 1% in Y-class, which is only
achieved by PDRG0.15 among non-FM combinations. This shows that there exists first-choice Y-class demand in the market that is not addressed by PUBC alone or by other forecasting/detruncation combinations without forecast multiplication.

Figure 35 shows that all combinations result in higher mean forecasts in Y-class than the base case of PUBC. Projection Detruncation with $\tau$ value 0.15, coupled with forecast multiplication results in the highest mean forecast, however, as the revenue gains comparison shows that it does not result in corresponding highest revenue gain. The probable cause is that the aggressive detruncation with forecast multiplication leads to overprotection for higher-fare classes leading to losses in Q-class and spoilage. The remaining forecasting/detruncation combinations show a relationship whereby higher mean forecast results in higher revenue gain figures.

A review of mean absolute deviations in Figure 36 show that all FM combinations exhibit higher MAD values than the base PUBC. Regression and PDRG 0.3 show lowest MAD values, and correspondingly the lowest revenue gain figures. Apparently, combinations whose MAD values are higher than but relatively closer to base PUBC enjoy the greatest revenue gains. These combinations result in forecasts that are higher than base to address
the first choice Y-class demand in the market but not so high as to suffer from losses due to over-protection.
Mean Bias comparison in Figure 37 also shows that all FM combinations have a much higher positive bias than the base. Both Regression and Pickup combinations with booking curve detruncation show near zero bias. This is due to choice of detruncation scheme employed in calculation of actuals. These results are roughly similar to single path per market case.

Even with introduction of multiple paths in a market, forecast multiplication continues to be the top revenue performer under FCYM even though its performance under traditional forecast metrics is not good.

**DAVN**

Under DAVN as well, the base case of PUBC enjoys three times the revenue for single path per market case. The associated revenue gains profile for various combinations is also similar to the single-path case. Forecast multiplication fails to repeat its revenue improving performance under FCYM since those revenue gains arise from the revenue potential that exists due to FCYM employing leg-based forecasts.

Regression continues to be the best forecaster for DAVN as can be seen in Figure 38. Analysis under the forecasting metrics employed shows (refer to Figure 39) that
regression results in slightly higher forecast than base case, yet it enjoys the lowest MAD values and lowest mean bias of all combinations. These forecast metrics and revenue relationships are almost unique for every network configuration tested.

Under DAVN, relatively sophisticated forecasters like Regression continue to perform best in terms of revenue. Forecast multiplication, with highest forecasts continues to do poorly. The consistency of these results further strengthens the view that the performance of forecast multiplication under FCYM has more to do with the fundamental differences between DAVN and FCYM methods, essentially dependence of FCYM on leg-based forecasts that do not cater for partial detruncation effects, resulting in overall lower forecasts.

**Multiple Paths per Market, Monopoly Network Analysis**

As the next step in analysis, the restriction of first choice only choice was removed since the impact of introducing multiple paths has been analysed in isolation. The removal of this restriction allows us to study forecasting/detruncation behaviour in a network that closely resembles the real world minus the competitive impact.
The removal of first choice only choice condition now allows the airline to take full benefits of offering multiple paths in any market. It is now possible to explore the revenue potential offered by both horizontal and vertical recapture and their effect on forecasting accuracy. The easing of restriction also allows sell-up to higher classes on the same path. Both sellup and recapture possibilities combine to offer a much greater revenue earning potential in this network configuration.

**FCYM**

The introduction of sell up and recapture results in all forecaster/detruncation combinations exhibiting much higher revenues than the base case of PUBC. Regression forecaster is the best revenue performer among all non-FM combinations and even outperforms PUBC with forecast multiplication. However, FM combinations still take the top honours in revenue performance with RGBC+FM resulting in highest revenue gain of 0.74%.

![Figure 40: Revenue gain over PUBC under FCYM](image)

These results as shown in Figure 40 clearly show that advanced forecasting and detruncation schemes like Regression and Projection Detruncation result in improved revenue performance compared to the pickup forecaster/booking curve detruncation.
combination. However there is still unforecasted higher-fare class demand in the market as indicated by the much higher revenue performance achieved through use of forecast multiplication.

The revenue breakdown by category analysis in Figure 41 makes for an interesting observation. For all non-FM combinations, gain through recapture revenue, especially horizontal recapture, holds the major share of overall revenue performance whereas for FM combinations, first choice revenue gain makes for the major share. Regression shows the largest increase in recapture revenue. This again shows that non-FM combinations do not tap into the first-choice higher-value demand. FM combinations show the greatest increases in Y-class first choice revenue category.

All forecaster/detruncation combinations return higher mean forecasts than the base case as seen in Figure 42. PDRG 0.15 with forecast multiplication returns the highest forecast but then it is clearly overforecasting as this does not translate into highest revenue performance. Regression with forecast multiplication has the next highest mean forecast and it corresponds with its best revenue gain figures of all combinations.
Figure 42: Y-class Mean Forecast per Leg under FCYM

Figure 43: Y-class Mean Absolute Deviation under FCYM
In terms of Mean Absolute Deviation values shown in Figure 43, all combinations cluster around the base case MAD values with forecast multiplication combinations returning values on the higher side, corresponding with their greater revenue gain figures. Among the non-FM combinations, both regression and PDRG0.3 have lower MAD values than base case and it corresponds with their returning the lowest gain figures of all combinations. Thus it appears that higher absolute deviations in Y-class from the base case resulting in higher forecasts lead to higher revenues. All these results point to the fact that higher forecasts in Y-class address the unforecasted Y-class demand in the market and may lead to higher revenues if over-protection losses are not overwhelming.

![Figure 44: Y-class Mean Bias under FCYM](image)

The Mean Bias metric in Figure 44 shows that all combinations, especially FM combinations have high positive bias compared with the base case of PUBC, which due to common detruncation scheme employed for calculating “actual” demand, enjoys zero bias with regression. Thus higher bias roughly corresponds with higher revenue performance.

In this final iteration of analysis in a near real world monopoly network configurations, the results are consistent with previous simulations. Forecast multiplier continues to offer the best revenue figures while there is little discernable relationship between revenue performance of forecaster/detruncation combinations and accuracy.
The results clearly show that EMSR₆ suffers from its inherent static nature combined with the dependence of FCYM on leg-based forecasts. This results in lower forecasts in higher-fare classes for FCYM. These deficiencies are catered to some extent by Forecast Multiplication through its inflation of forecasts and thus result in its higher revenue performance.

**DAVN**

Regression turns out to be the best forecaster under DAVN, both with and without forecast multiplication as seen in Figure 45.

![Figure 45: Revenue Gain over PUBC under DAVN](image)

Revenue breakdown analysis in Figure 46 shows results that are strikingly different than those under FCYM. Sell-up and recapture now assume paramount importance in the overall revenue gain figures. Regression forecasting returns the highest horizontal recapture gain of 0.38%.
A breakdown analysis of first choice revenue in Figure 47 reveals that there is much first choice Y-class demand in the market, though the associated losses in Q-class make the first choice revenue a negligible part of overall gain.
Figure 47: First Choice Revenue Fare Profile under DAVN

Mean absolute deviation values in Figure 48 show that the best revenue performing combinations (regression with and without forecast multiplication) return the lowest MAD values of all combinations. Similarly, regression combinations figure among the lowest mean bias values of all combinations.

Figure 48: Y-class Mean Absolute Deviation under DAVN
These DAVN results represent a slight departure from the now well established norm. It is the first time that a combination with Forecast Multiplication has returned the best revenue gain values under DAVN, though by a very small and insignificant margin of 0.03%.

**Part 2: Use of Path-based Forecast and Future Protect**

One of the salient results of analysis done in Part 1 is that Forecast Multiplication, an arbitrary forecast inflation scheme, continues to exhibit better revenue performance under FCYM. The Forecast Multiplication is primarily employed in the analysis as a diagnostic tool to ascertain the margin available for performance improvement to forecaster/detruncation combinations. It was observed that even under Single Path per Market, First Choice Only Choice Monopoly Network configuration that abides by the basic path-class independence of demand assumption, Forecast Multiplication continued to do better under FCYM. However, under DAVN algorithm that employed path-based forecasts and optimization, Forecast Multiplication was not beneficial. This phenomenon therefore warranted further study.

Thus, in line with the secondary goal of the thesis, the performance differential achieved by forecast multiplication under FCYM was investigated through use of Path-based forecast and Future Protect algorithm under different network configurations. The rationale behind the switch to path-based forecast was that use of leg-based forecasts under FCYM leads to partial detruncation being undertaken on closed paths comprising of more than one leg. Thus if one leg closes on a two leg path and the other does not, then in the next iteration, due to only detruncation on the leg that closed, the forecasts for the path as a whole will be lower as the detruncation and subsequent leg-based forecasting scheme will not take into account that these two legs constitute one path.

Similarly the EMSRb algorithm, employed in FCYM, is static in nature and does not dynamically protect seats for higher fare classes. Future Protect is seen as a solution to this shortcoming of EMSRb as it allows the EMSRb algorithm to dynamically adjust to demand in the market.
Future Protect

Future Protect represents a hedging technique in revenue management where additional seats are protected for higher-fare classes in anticipation of additional future sales. This hedging works to airline’s advantage in two kinds of scenarios. In markets where airline is experiencing high demand in higher-fare classes, it can protect more seats and make them available to these higher-fare classes. On the other hand, in markets where there exists a substantial low-fare/discount fare demand, airline can take seats away from lower-fare classes and make them available to higher-fare classes. In case these additional seats remain unsold, they can be offered to discount-fare passengers in terminal stages of reservation process. The only risk involved with this hedging technique is that the lower-fare demand that is denied availability early in the reservation process, does not return in the terminal stages when these additional seats are released at lower fares.

Future Protect represents a much more methodical and systematic way of ensuring greater seat availability to higher fare classes than the arbitrary forecast inflation through forecast multiplication. In PODS the Future algorithm is implemented as explained by the following example:

Booking Limits under Future Protect (EMSRₙ)

For Y class
Mean Y demand + FPM x Mean Future B Demand

For Y+B classes
Mean Y demand + Mean B demand + FPM x Mean Future M Demand

For Y+B+M classes
Mean Y demand + Mean B demand + Mean M demand + FPM x Mean Future Q Demand

Where
FPM = Future Protect Multiplier and,

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51 Hopperstad (2002)
Mean Future Demand = Expected demand from timeframe n+1 through 16 (departure) for optimization at timeframe n.

Thus Future Protect Multiplier is an input parameter that is under control of an individual airline. An airline can choose to be aggressive in protecting additional seats for higher fare classes through choice of a high value for Future Protect Multiplier.

Simulation Setup

The simulations done for the analysis of effect of path-based forecast and future protect utilised the same monopoly network configurations as employed in the part 1 analysis. However this analysis was further carried out on competitive network configurations to ascertain how use of path-based forecasts and future protect fares in a realistic scenario.

As mentioned above, the forecast multiplication only exhibited improved revenue performance under FCYM, thus this entire analysis was carried under FCYM. As a first step in this analysis, leg-based forecasts were replaced with path-based forecast to analyse the effect of this change in isolation. In the next stage, Future Protect was introduced with leg-based forecast to isolate the impact of employing future protect. In the last stage of analysis, both path-based forecast and future protect were employed in combination.

Single Path per Market, First Choice Only Choice, Monopoly Network Configuration

The first logical step for this analysis was again our simplest network configuration that abides by the basic forecasting assumption of path-class independence.

As a first step in this analysis, leg-based forecasts were replaced with path-based forecast to analyse the effect of this change in isolation. In the case of Regression Forecasting without forecast multiplication, this switch to Path-based forecasting resulted in an increase in revenue of 0.16% over leg-based forecasting as shown in Figure 49. This represents a significant improvement but is still short of 0.3% revenue gain by 0.14%, achieved by our diagnostic tool of forecast multiplication in combination with regression.
The use of Future Protect with leg-based forecasting as shown in Figure 50 resulted in much improved revenue performance. At Future Protect multiplier value of 0.7, regression exhibited a revenue gain of 0.31% over leg-based forecast alone.
The combined use of path-based forecasts and future protect showed highly impressive results, as seen in Figure 51, with their revenue performance outdoing that of forecast multiplier. This result is very significant as it highlights the shortcomings of FCYM scheme and explains the miraculous revenue performance achieved through the use of our diagnostic tool of forecast multiplication.

Figure 51: Path-based Future Protect Multiplier results compared with PUBC+Leg-based Forecast

The results demonstrate that the higher revenue performance of Forecast Multiplication under FCYM can be accounted for and even bettered by a combination of path-based forecast and Future Protect algorithm.

The analysis in Part 1 has clearly shown that Forecast Multiplication does well only under FCYM and fails to perform well under DAVN. The essential difference being the use of path-based forecast by DAVN resulting in higher forecasts. The use of path-based forecasts along with Future Protect; that dynamically protects seats for higher fare classes, bridges the revenue performance gap between FCYM with FM and DAVN.

Single Path per Market Monopoly Network Configuration with Sellup

The next step was to repeat the analysis in the same network with the restriction of first choice only choice relaxed to allow sell-up. Under this network configuration Forecast
Multiplication when used in combination with PDRG at $\tau$ value 0.4 results in a revenue gain of 0.75% over PUBC base case as can be seen in Figure 52.

![Figure 52: Path-based Forecast results compared with PUBC+Leg-based Forecast](image)

The switch to path-based forecast, shown in Figure 52 resulted in Regression showing a revenue improvement of 0.66% over the base case. This was a very encouraging outcome as it validates part of the results determined previously in single path per market, first choice only choice monopoly network.

![Figure 53: Leg-based Future Protect Multiplier results compared with PUBC+Leg-based Forecast](image)
Use of future protect alone, shown in Figure 53, resulted in a 0.67% revenue gain over base case for regression; revenue performance that is very comparable to forecast multiplication. This further lends weight that combination of path-based forecast and future protect would result in much improved revenue performance.

Figure 54 shows that the combined use of future-protect and path-based forecast resulted in 0.76% revenue gain over base case, thus again outperforming forecast multiplication. This result strengthened the earlier drawn conclusion that increased revenue performance of arbitrary forecast multiplication can be overcome by the use of path-based forecast and future protection.

**Multiple Path per Market, First Choice Only Choice, Monopoly Network Configuration**

The next step towards adding realism in this analysis involved application of path-based forecast and future protect in a multiple path per market scenario. However in the first stage, first choice only choice restriction was introduced to isolate the effects of introduction of multiple paths in every market.
Path-based forecast showed a healthy revenue performance with a gain of 0.4% over base case for regression, compared with 0.54% for forecast multiplication. This can be seen in Figure 55. Introduction of Future Protect under this configuration, as shown in Figure 56, resulted in a revenue gain of 0.45% over base case.

When future-protect and path-based forecasting were used in combination, refer Figure 57, they again outdid the forecast multiplication by returning a revenue gain of 0.55% for regression.
The above results again validated the conclusion regarding combined use of path-based forecast and future protect.

**Multiple Path per Market, Monopoly Network Configuration with Sellup and Recapture**

The relaxation of first choice only choice restriction results in a network configuration that resembles a real world airline’s hub and spoke network with multiple services in markets, though without the effect of competition distorting the results.
As shown in Figure 58 for regression forecasting, Path-based forecast return an impressive gain of 0.20% over leg-based forecast for a total of 0.58% gain, compared to 0.74% revenue gain over base for forecast multiplication. Future Protect returns even better revenue performance with a gain of 0.61% over base, an improvement of 0.23% for regression forecaster as shown in Figure 59.

![Figure 59: Leg-based Future Protect Multiplier results compared with PUBC+Leg-based Forecast](image)

![Figure 60: Path-based Future Protect Multiplier results compared with PUBC+Leg-based Forecast](image)
The combination of Future-Protect and Path-based forecasts (shown in Figure 60) results in revenue performance that is comparable to that to forecast multiplication; 0.67% for the combination compared with 0.74% for forecast multiplication.

![Bar chart showing revenue source comparison](image)

**Figure 61: Revenue Source Comparison of Path-based Future Protect with Forecast Multiplication**

A comparison of revenue sources in Figure 61 highlights significant difference between forecast multiplication and combination of future-protect and path-based forecasts. Forecast Multiplication revenue gain mainly comes through increased first choice revenues, while for the combination of future-protect and path-based forecast, recapture gains form the major part.

This shows that use of Forecast Multiplication results in predicting additional First choice segment of the demand in the market. This segment is being under-forecasted by traditional forecasters under FCYM. However, the use of Path-based forecasts and Future Protect is making use of presence of multiple paths in the market.

**Multiple Path per Market, Competitive Network Configuration**

The results of preceding analysis were highly encouraging as they supported the hypothesis that path-based forecast and future protect when used with a reasonable advanced forecaster/detruncation combination can account for the shortcomings of
traditional forecasters/optimizers. These impressive results led to extending the analysis to full competitive network scenario in network D.

The competitive network D analysis was carried out for two different scenarios. In the first case the other airline was a rudimentary competitor employing Pickup forecaster/Booking curve detruncation with leg-based forecast while the airline under analysis uses path-based forecast and future protect. The summarised results of the analysis in Figure 62 show that path-based FPM used with PDRG 0.4 returns revenue gains that are comparable to forecast multiplication.

A second scenario used for competitive network analysis employed a smart competitor that matched the airline under analysis at every step. In this analysis, both Airlines match each other in terms of Forecasting/Detruncation Method, Forecast Multiplier or Future Protect Multiplier values.
In case of airline A, path-based Future Protect performs comparably with forecast multiplication when used with regression; returning a revenue gain of 0.70% to 0.73% for forecast multiplication, as seen in Figure 63. In case of airline B, the closely matching competitor, the use of path-based Future Protect also results in revenue improvement over base that is comparable to forecast multiplication, shown in Figure 64.
Summary

This chapter details the core analysis undertaken as part of this thesis. Forecast accuracy and revenue performance of various forecaster/detruncation combinations was analysed under various network configurations that gradually grew complex and close to real world conditions. The results were repeated for both FCYM and DAVN. It was learnt that the relative revenue performance of various forecaster/detruncation combinations has little relationship on their accuracy when accuracy is defined in comparison with actuals that are state-dependent and whose estimation/calculation requires detruncation schemes that influence the accuracy measurement. In fact, under almost all network configurations and FCYM, forecast multiplication proved to be the best revenue performer yet it did not fare well on various error metrics employed in the analysis. However, forecast multiplication failed to perform under DAVN in almost all cases; whereas a more sophisticated forecaster like regression did better under DAVN. This led to closer analysis of performance differential of Forecast multiplication between FCYM and DAVN. The glaring difference between FCYM and DAVN was the use of path-based forecasts and optimization by the latter. This accounted for the shortcomings of leg-based forecasting and optimization in FCYM, namely static nature of EMSR_b and partial detruncation leading to lower forecasts.

It was determined through analysis and validated over various network configurations that the use of path-based forecast with future-protect, under FCYM, results in revenue performance for regression (and in one case projection detruncation) that is comparable to forecast multiplication. Thus path-based forecast in conjunction with Future Protect addressed the shortcomings of leg-based forecasting and optimization.
Chapter 4: Conclusion

Principal Findings of Analysis:

Results of Analysis and Future Research Directions

Synopsis of Thesis Objectives

The main focus of the thesis was to investigate the concept of forecasting accuracy in the context of airline revenue management with the objective to test the traditionally held assumption of a clear relationship between revenue performance of different forecasters and their forecast error as measured through conventional error metrics. The entire analysis was done through simulations using the Passenger Origin Destination Simulator.

In keeping with this focus, the primary analysis was the measurement of forecast accuracy using traditional measures under different forecasting/detruncation combinations. The accuracy was defined in terms of Mean Absolute Deviation and Mean Bias of the forecasts. These traditional error metrics require calculation of actual demand for comparison. The actual demand is estimated using historical booking data that is detruncated using a booking curve detruncation scheme. However, this scheme neglects the fact that the historical booking data is state dependent, that is depends on the state of the network at that particular instant in terms of what classes are closed/open both for the subject airline and competitors in the market. These error measurements were used in conjunction with the revenue performance of these forecasting/detruncation combinations, with the primary goal to establish that the conventional accuracy-revenue relationship does not hold in practice in airline revenue management.

In addition to the forecasting/detruncation combinations, arbitrary forecast multiplication was employed both as a diagnostic tool to highlight the margin in revenue performance improvement as well as an aid to reinforce the primary goal that mean lower forecast
error (based on traditional forecast accuracy metrics) does not necessarily translate into higher revenue performance.

As a secondary goal, the apparent revenue gain from forecast multiplication was investigated closely to ascertain the shortcomings in employment of traditional forecasting/detruncation schemes. The analysis was initiated on the most basic network configuration that abides by the basic forecasting assumption of path-class demand independence, free from distortion due to competition. The analysis was then shifted to network configurations that gradually increased in complexity to near real world competitive scenarios.

**Summary of Principal Findings**

The principal findings of the analysis are as follows:

- In airline revenue management, there is no clean relationship between revenue performance of a forecaster and accuracy of its forecasts, as traditionally defined using error metrics that employ estimation of actual unconstrained demand.

This is a very significant finding as a major thrust of airline revenue management research is aimed at coming up with ‘better’ forecasters. The term ‘better’ is a vague term, though it usually implies more ‘accurate’ forecasters, with improved revenue performance implicitly assumed. This comes from the traditionally held belief of a clear revenue-forecast error relationship. In the course of this analysis, an attempt has been made to dispel this notion of ‘better’ and more ‘accurate’ forecaster resulting in improved revenue performance. This is due to many factors, notable among them being the unrealistic nature of basic assumptions of forecasting and the inability to compute ‘actual’ demand to calculate forecasting accuracy through conventional metrics.
• Forecast Multiplication, an arbitrary forecast inflation scheme, returns higher revenues when used with various forecaster/detruncation combinations under Fare Class Yield Management

This result highlights the utility of forecast multiplication as an important diagnostic tool. Its superior revenue performance raised many issues that led to the secondary analysis being conducted in this thesis to ascertain the contributory factors of this revenue performance. Another significant finding was that the superior revenue performance of forecast multiplication did not correspond with accurate forecasts, instead on most occasions forecast multiplication returned the most inaccurate figures thus voiding any clear or direct relationship between revenue performance and forecast accuracy.

• Advanced or more sophisticated forecaster/detruncation combinations like Regression and Projection Detruncation exhibit better revenue performance under OD control (namely DAVN) compared to forecast multiplication

This result when analysed in tandem with the previous findings lead to the important conclusion that a significant part of better revenue performance of forecast multiplication stems from the unrealised revenue potential under leg-based forecasting and optimisation. Forecast Multiplication essentially gives higher forecasts in higher fare classes leading to greater revenue. Leg-based forecasting and optimisation suffer from the partial detruncation problem on multi-leg paths thereby resulting in lower forecasts. The partial detruncation problem arises when a path comprises of two or more than two legs. If one of the constituent legs closes, in a leg-based scheme only that particular leg demand will be detruncated. This shortcoming is not present under DAVN since it already uses path-based forecasts and optimisation. This difference is responsible for a significant part of the revenue differential between leg-based forecasts and Forecast Multiplication under FCYM.

• The use of path-based forecasts and Future Protect results in much improved revenue performance compared to leg-based forecasts, under Fare Class Yield Management.
This revenue performance is comparable (even higher) in many cases to that of forecast multiplication.

This is the most significant result from the second part of the analysis conducted in course of this thesis. This shows that the improved revenue performance of forecast multiplication is attributable to the static nature of EMSR\textsubscript{b} algorithm and issue of partial detruncation arising due to leg-based forecasts. A switch to path-based forecast and employing future protect counters both these shortcomings, resulting in revenue performance that is superior to forecast multiplication under most monopoly network configurations and comparable under a full-up competitive network configuration.

**Future Research Directions**

The discussion over the course of this thesis results in a number of interesting questions that would make for meaningful future research directions. The discussion about the unrealistic nature of basic forecasting assumption of path/class demand independence assumption naturally leads to the thought that why it should not be abandoned in favour of a more realistic assumption. There is a glaring need for extensive research into basic assumptions of airline revenue management forecasting. In industry practice, as well as PODS simulations, it has been observed that horizontal spill is more dominant than Sellup and Recapture. This leads to the hypothesis that demand independence by market/class is a more realistic assumption. Research under Market-based Revenue Management or MAC\textsuperscript{52} (acronym for Market/Airline/Class) has been initiated under the PODS consortium. Further research into Market-based forecasting, or ODF grouping of forecasts will lead to hopefully better forecasts. This also avoids the small numbers problem in ODF forecasts.

It has been emphasised throughout the thesis that the calculation of actual demand is difficult given the state-dependent nature of forecast in revenue management. Thus another interesting future research avenue is state-dependent forecasting or application of

\footnote{Hopperstad, Craig (2002)}
Bayesian forecasting techniques in the context of airline revenue management. Industry leaders in revenue management are considering the development of fourth-generation revenue management systems that incorporate state-based forecasting and optimisation. This seems to be the logical way forward.

Another related issue concerns the usefulness of current/traditional error metrics in airline revenue management. There is a large amount of literature available on statistical techniques for error or residual analysis in linear models in general and classical time series models in particular. However the focus in this classical error analysis is on residuals or “fitted” errors that measure a retrospective departure of data from model. This is not very helpful as the entire point of forecasting exercise is to predict future data accurately and thus the emphasis should be on predictive fit of forecasting models. This again relates to employment of Bayesian statistics in future forecasting research.
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