Characterization of Structural Properties and Dynamic Behavior using Distributed Accelerometer Networks and Numerical Modeling

by

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ABSTRACT

Both vibration-based structural health monitoring methodologies and seismic performance analysis rely on estimates of the base-line dynamic behavior of a structure. A common method for making this estimate is through measuring structural motions using sensors deployed in the structure of interest. This procedure was applied to the Green Building, a 20 story structure on the campus of the Massachusetts Institute of Technology. Using a network of 36 accelerometers installed in the structure by the United States Geological Survey, the response accelerations from ambient vibrations, seismic loading, and firework excitations were collected. Spectral analysis methods were applied to the collected data to identify the frequencies and general mode shapes of eight normal modes of the structure. These frequencies were 0.68 Hz, 2.45 Hz, and 8.10 Hz in the east-west direction; 0.75 Hz, 2.85 Hz, and 8.25 Hz in the north-south direction; and 1.45 Hz and 5.05 Hz in torsion. The building was found to have strong torsional responses, an asymmetry in the dynamic behavior of the eastern and western sides, and substantial base rocking motion, even under ambient excitations. Using the original design documents, the Green Building was numerically modeled with a lump-mass stick model and a mixed-element beam-shell finite element model. These models were validated and refined using the collected acceleration data. Initial simulations of seismic excitations demonstrated both models to have good agreement with measured values. The numerical models and structural characterization of the Green Building will be used to further develop vibration-based damage detection methodologies and to predict structural performance during strong seismic events.

Thesis Supervisor: Eduardo Kausel
Title: Professor of Civil and Environmental Engineering
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1 Introduction

1.1 Motivation and Background

The work contained in this thesis is part of a larger project into structural damage detection using distributed sensor networks funded by the Shell Oil Company. It is a collaboration between Shell Oil, Draper Laboratory, the MIT Computer Science and Artificial Intelligence Laboratory (CSAIL), and the MIT Department of Civil and Environmental Engineering (CEE). The goal of this project is to develop a sensor-based damage detection system which specifically tackles the problems of damage detection methodologies, sparse sensor availability, and novel sensing methods. The tasks for accomplishing this goal include investigation of structural health assessment methods based on material and structural behavior, design of reliable and cost-effective sensor networks, development of algorithms for sensor placement optimization and data inferencing, and field testing of a sensor system on a campus building. The study presented in this thesis seeks to identify the vibrational characteristics of a unique structure located in Cambridge, Massachusetts and to create predictive numerical models of its dynamic behavior. This corresponds to the project goals by demonstrating and developing the feasibility of vibration-based damage detection in large civil structures using accelerometer sensors, by establishing a base-line understanding of a structure's dynamic properties in anticipation of future field-testing, and by creating numerical models which can be used for both testing of algorithm testing and as tools for damage detection. A secondary motivation for this study is to characterize the dynamic behavior of a structure in the New England region, in the context of seismic performance.

Damage detection is a central concern of structural health monitoring (SHM). The objective of SHM is to use emergent sensing technologies, coupled with knowledge of material and structural properties, to infer the physical condition of a structure. Damage detection specifically seeks to detect, localize, and estimate the degree of damage sustained by a structural system. The damage can be long-term, such as corrosion, or a short-term event, such as a sudden member failure. SHM damage detection technologies are especially pertinent given the large quantity and poor maintenance condition of critical infrastructure systems—such as bridges and dams—found in the United States and other countries. SHM is also highly useful in structures where human access is difficult or dangerous—such as nuclear facilities, lengthy pipelines, or off-shore oil platforms. The benefits of SHM and damage detection include reducing the need, cost, and risks incurred by human inspections of structural systems; increasing the lifetime of structural systems by easing the detection of degradation; and preventing the economic, human, and national security damage caused by failure of infrastructure systems and other structures.
Many approaches exist for SHM, but one early-developed and robust method is vibration based damage detection. This technique is founded on the axiom that changes to a structure’s stiffness or mass will result in detectable shifts in its vibrational properties. Specifically, the structure’s natural frequencies, mode shapes, and modal damping ratios can be used as parameters for damage detection (Alvandi & Cremona, 2005), (Salawu, 1997). The use of natural frequencies is especially attractive since they can be easily acquired using accelerometer networks and efficiently computed. These modal parameters can also be used for damage localization and evaluation, since different locations and degrees of damage have different effects on the locations and modes of the structure. It is possible to couple this behavior with numerical model updating or subset selection algorithms to determine specific scenarios of damage which may be responsible for the observed property shifts. The accuracy of these approaches is, to an extent, driven by the density and resolution of the sensor arrays. Data inferencing can be used to increase the amount of information provided by sparse sensor networks. A basic, though not necessarily elegant, method for accomplish this is through coupling sensor measurements with numerical models to predict structural responses at un-sampled locations.

The second motivation of this study is to characterize the vibrational properties of a structure in the New England region, particularly in response to seismic loading. Boston and most of New England lie in a “moderate” earthquake hazard zone, which means that the area can experience level VI events on the Modified Mercalli Intensity Scale. The New England region rests on top of old continental crust which was substantially stressed by continental rifting during the Mesozoic era. This created many ancient faults which are difficult to locate and whose seismic activity is nearly impossible to predict (Kafka, 2011). Furthermore, large areas of Boston and Cambridge are built on fill material which may amplify ground motion during seismic events (Britton, et al., 2002). It is unclear what effect these geological features would have on buildings in the greater Boston area, should a stronger earthquake strike the region. In the past two years, two earthquakes have been felt in the Boston area. In 2011, a 5.8 magnitude earthquake occurred near Richmond, Virginia, and it was felt at Level VII in Boston. In 2012, a 4.0 magnitude earthquake occurred near Hollis Center Maine.

By determining a structure’s dynamic characteristics, such as natural frequencies, mode shapes, damping, and deflections, it is possible to estimate its general response to seismic excitations and the damage it may experience (Celebi, 2007). The finite element method is valuable in this regard, since it can be used to perform dynamic simulations of seismic events on numerical representations of the structure. Information about the dynamic characteristics of local structures is particularly useful in New England, since very few buildings in the area have been evaluated in this manner for their seismic response behavior. Given an understanding of a structure’s seismic performance, the results can be extended to other similar buildings in the region.
1.2 Objectives and Approach

The general objectives of the work undertaken in this thesis are to characterize the dynamic behavior of the Green Building—a structurally unique building on the Cambridge, Massachusetts campus of the Massachusetts Institute of Technology—and to develop computational models by which this behavior can be predicted. The overarching purpose is to demonstrate and further develop vibration based methods for structural damage detection, with the secondary purpose of discovering vibrational properties which can be used for a seismic evaluation of a Boston area structure. The specific goals to accomplish these objectives are fourfold and include the following:

1. To determine several natural frequencies of the Green Building and to estimate their corresponding mode shapes.
2. To identify features of dynamic behavior and link these to structural and geotechnical design choices
3. To represent the Green Building using a finite element numerical model which accurately predicts the structure's dynamic characteristics
4. To demonstrate the use of distributed sensors networks to characterize and monitor a structure's behavior, and to motivate the use of such systems for structural damage detection

Goals 1 and 2 will be accomplished in part by using a sensor array of 36 accelerometers installed in the Green Building by the United States Geological Survey (USGS). The system will be used to record building motions in response to various excitations. The resulting data will be analyzed using classical spectral analysis methods—such as Fourier analysis, power spectrum, and coherency spectrum—as well as other methods to locate natural frequencies and estimate deflection shapes. A comparative analysis using these methods will also help determine novelties in building motions, which can then be linked to known structural features. Goal 3 will be accomplished by using the building's design documents and standard finite element techniques to create numerical models of the Green Building. The frequencies and motion characteristics from Goals 1 and 2 will then be used to validate and refine the models as necessary. Goal 4 will proceed from successful completion of Goals 1, 2, and 3.

1.3 Thesis Organization

This thesis can be divided into three sections, each of which addresses part of the first three goals—background material and theory (Chapters 2 to 4), sensor data collection and analysis (Chapters 5 and 6), and development of numerical models (Chapter 7). The contents of each chapter in this thesis are briefly summarized below.
- **Chapter 1: Introduction**
  Presents background information and motivations for the research work

- **Chapter 2: Physical Assets**
  Describes the campus building (Green Building) under study and the capabilities of the accelerometer network used for data collection

- **Chapter 3: Time Series Analysis Methods**
  Reviews the basic theory and methods for analyzing the time-series data collected using the accelerometer array

- **Chapter 4: Data Pre-Processing**
  Presents some of the signal processing methods—such as filtering and base-lining—applied to the collected data

- **Chapter 5: Collected Data Sets**
  Summarizes the recorded accelerometer data sets and their corresponding excitations

- **Chapter 6: Feature Identification**
  Describes the application of the analysis methods from Chapter 3 to the data sets listed in Chapter 5, and presents the Green Building's natural frequencies, structural behaviors, and other observed results

- **Chapter 7: Green Building Modeling**
  Outlines the theory and process for creating a lumped-mass stick model and a mixed element beam-shell finite element model of the Green Building; presents initial validation results

- **Chapter 8: Future Work and Applications**
  Proposes areas of future work and possible applications of the research carried out

- **Chapter 9: Conclusions**
  Summarizes the research and results documented in this thesis
2 Physical Assets

2.1 The Green Building

Characterization studies of structural properties and dynamic behavior focused on the Green Building, also known as Building 54. The Green Building is an academic building on the campus of the Massachusetts Institute of Technology in Cambridge, Massachusetts. It is home to the lab, office, and classroom space of the Department of Earth and Planetary sciences. The Green Building was designed by I.M. Pei and constructed between 1962 and 1964. It is currently the tallest building in Cambridge. The roof supports radio and meteorological equipment, including a weather radar system enclosed within a large radome.

The Green Building is 83.7 m (274'-9") tall with a footprint of 16.5 x 34 m (54’ x 111’). The short edges of the building are aligned at about 25° north-west. Henceforth, the short and long directions of the Green Building will be referred to as North-South and East-West (NS and EW), respectively. The building has 21 stories and a below-grade basement level which connects to the MIT tunnel network at the South-West and North-East corners. The first floor is 10 m above grade and houses a large lecture hall. The inter-story height of the first two floors is about 7.8 m, while that of the remaining floors is 3.5 m. Mechanical rooms are located on the top two floors, and the heavy meteorological and radio equipment mentioned above is asymmetrically mounted on the roof. The radome is on the south-west corner. Three elevator shafts are located on the eastern side of the building, while the building’s two stairwells are placed symmetrically at the North-East and North-West corners. The elevator shafts insert large voids, measuring about 9.75 x 2.21 m (32’ x 7.25’), into the otherwise continuous floorslabs. The elevator shafts (and associated void-space) run the height of the building. The footprint of each stairwell is about 4.27 x 2.29 m (14’ x 7.5’). The building is constructed of cast-in-place reinforced concrete with two compressive strengths of concrete used. Interior beams, slabs, and stairs are composed of 3750 psi concrete while all other components are constructed from 4000 psi concrete. The Eastern and Western facades are composed of 0.25 m (10") thick shear walls which run the height of the building. Floor slabs are typically 0.1016 m (4") thick. Figure 2-1 shows the elevation of the Green Building and a typical floor plan.

The foundation system consists of footings supported by 14” diameter circular piles with pile caps. The piles are part of a floating foundation system and each has a capacity of 50 tons. The basement floor elevation is 3.8 m (12 ft. 6 in) below grade, and grade level is about 6.1 m (20 ft.) above sea level and 36.6 – 40.1 m (120 – 130 ft.) above bedrock (Celebi, et al., 2013). The site is adjacent to the Charles
River Basin and it is primarily composed of fill material. Geotechnical investigations of the area have found the fundamental site frequency to be about 1.5 Hz. (Celebi, et al., 2013).

Figure 2-1: Elevation and plan views of the Green Building; (Top Left) photograph of the Green Building’s south façade; (Top Right) elevation view of the south façade; (Bottom) plan view of a typical floor in the Green Building.

NORTH

87'

111'-6"

WEST

STAIRWELL

WEST SHEAR WALL

STAIRWELL

ELEVATOR
SHAFTS (3x)

STAIRWELL

SOUTH

EAST

SHEAR WALL

3'-6" x 11" REINFORCED
CONCRETE BEAM

9' O.C.

9' O.C.

COLUMNS

GROUNDFLOOR-EL = 0'
BASEMENT-EL = -12'

ROOF-EL = 275'
20TH FLOOR-EL = 258'
19TH FLOOR-EL = 248'
18TH FLOOR-EL = 237'
17TH FLOOR-EL = 225'
16TH FLOOR-EL = 213'
15TH FLOOR-EL = 202'
14TH FLOOR-EL = 190'
13TH FLOOR-EL = 178'
12TH FLOOR-EL = 167'
11TH FLOOR-EL = 151'
10TH FLOOR-EL = 144'
9TH FLOOR-EL = 132'
8TH FLOOR-EL = 120'
7TH FLOOR-EL = 109'
6TH FLOOR-EL = 97'
5TH FLOOR-EL = 85'
4TH FLOOR-EL = 74'
3RD FLOOR-EL = 62'
2ND FLOOR-EL = 51'
1ST FLOOR-EL = 33'
GROUND FLOOR-EL = 0'
BASEMENT-EL = -12'
2.2 Green Building Accelerometer Array

The Green Building is instrumented with 36 uniaxial EpiSensor ES-US force balance accelerometers produced by Kinemetrics. These sensors are designed primarily for structural monitoring purposes. The accelerometer placement and orientations are shown in Figure 2-2. This array and the associated data recording system have been in operation since October 2010, and they are the property of the USGS. Each sensor collects 200 samples per second with a recording range of ±4g. The accelerometers are cable-connected to a central data recording station and time-synchronized using GPS—accurate to within 1 microsecond. The data recorder is the Granite model, also produced by Kinemetrics. The system is internet-connected and set to trigger if accelerations over 1 gal (1 cm/s²) are detected. Acceleration data can also be collected through on-demand recordings and real-time streaming. An EpiSensor accelerometer and a Granite data recorder are pictured in Figure 2-3. The array is designed to monitor translations in the NS and EW directions, torsion, and base rocking motion. Torsional behavior is measured using parallel, NS-oriented sensors located at opposite ends of instrumented floors, while base rocking is determined using the four vertically aligned accelerometers in the basement. It is also possible to calculate floor drift ratios using vertically parallel accelerometer pairs. The displacement and velocities of each sensor can be calculated by numerical or frequency-domain integration, coupled with appropriate filtering to remove artifacts. The sensor system continuously records building motion and stores acceleration data in an adjustable buffer for 5 days.

To prevent aliasing, the incoming data is digitally filtered at the recorder by several Finite Impulse Response (FIR) filters (Kinemetrics, 2008). Accelerometer data is collected in 1 second packets and output in MATLAB .m files as raw ADC counts across a 24 bit range. Equation (2.1) is used to convert the raw ADC counts to engineering units. The installed accelerometers operate on ±2.5 volt range and a sensitivity of 0.625 V/g. If converting to units of m/s², Equation (2.1) reduces to Equation (2.2).

\[ \ddot{u} = \frac{(\text{Raw ADC})(\text{Voltage Range})(\text{Gravitational Acceleration})}{(\text{ADC Scale})(\text{Sensitivity})} \]  
\[ \ddot{u}(m/s^2) = \frac{(2.5)(9.81)}{(8388607)(0.625)} = (\text{Raw ADC})(4.678 \times 10^{-6}) \]
Figure 2-2: Distribution and orientation of USGS accelerometers within the Green Building

Figure 2-3: Accelerometer and data recorder; (left) Episensor ES-US Accelerometer; (right) Granite data recorder (not to scale)
3 Time Series Analysis Methods

Due to the complexity of the system under study, the presence of random noise, and numerous measurement uncertainties, the collected Green Building accelerometer time histories were analyzed using statistical and spectral methods. These time histories belong to a class of random, or stochastic, processes, to which a brief introduction is found in Section 3.1. The remainder of this chapter will present an overview of the analysis techniques applied to the Green Building data. However, detailed estimator procedures and algorithms will be omitted for the sake of brevity. These analysis techniques include Fourier analysis (Section 3.2), covariance (Section 3.3), correlation (Section 3.4), power spectral density (Section 3.5), coherence spectra (Section 3.6), and a method of visual colormap analysis (Section 3.7). The application of these methods to the Green Building data and the associated results are reported in Chapter 6. The analysis techniques describe here are found in most signal analysis texts. The following works were consulted for the data analysis methods reported in this thesis and for writing this Chapter: (Bendat & Piersol, 1993), (Brandt, 2011), (Jenkins & Watts, 1968), (Smith, 1997)

3.1 Random Processes

Time series, such as those recorded by the Green Building accelerometer array, track the evolution of a physical quantity with time, and they can be classified as either deterministic or random processes. A deterministic process is one whose behavior as a function of time can be predicted exactly. With a random process, future behavior cannot be predicted with acceptable accuracy. A superposition of the two types of processes also occurs—where a deterministic component is summed with a random noise component. While a random process is indeterminate, it can still be analyzed and compared using its statistical and spectral properties. Ambient vibrations, such as those measured in the Green Building are assumed to belong to a random process. This is due to the complexity of the system and the many random input loadings. In cases were the Green Building is subjected to strong, transient-type input loadings (such as earthquakes), the data belongs to the superimposed category; however, spectral analysis methods still apply.

It is assumed that a time series resulting from a random process is represented by a random variable \( x(t) \), sampled from some underlying probability density function. A measured time series is only one realization of an infinitely many possible outcomes. This full set of possible outcomes is called an ensemble. In the case of multiple measurements of the same process, the observed time series are all drawn from this ensemble. Statistical properties—including mean, variance, covariance, and power spectrum—can be calculated for each sampled time series. If the statistical properties of a time series are constant with time, then the process is considered to be stationary. If the process is stationary and the
statistical properties are also invariant across the individual time-series in the ensemble, then the process is also considered to be ergodic. The criteria for stationarity and ergodicity are summarized in Table 3-1. Conversely, if the properties of the time history change with time, the system is non-stationary. It follows that for a stationary-ergodic process, the statistical behavior of the system can be estimated using only one time series.

Table 3-1: Criteria for stationary and ergodic processes. $E$ is some statistical moment, $f$ and $g$ are two time histories sampled from the same ensemble, and $t_1$ and $t_2$ are two arbitrary times

<table>
<thead>
<tr>
<th>Stationary Process</th>
<th>$E[f(t_1)] = E[f(t_2)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary-Ergodic Process</td>
<td>$E[f(t)] = E[g(t)]$</td>
</tr>
</tbody>
</table>

Few physical systems are completely stationary, especially on large time-scales. However, for civil structures with data sampling over durations of minutes, stationarity and ergodicity are generally assumed. Formal tests for stationarity can be carried out, the simplest of which is the Frame Test. In this test, a sampled time history is partitioned into several intervals (or frames), and lower-order statistical moments are computed for each frame. If the statistical moments are invariant with time across the frames, then the process is considered stationary. An example of the Frame Test using Green Building ambient vibrations is presented in Figure 3-1. Here, a 320 second acceleration time-series sampled on June 22, 2012 is partitioned into 4 frames of 80 seconds each. Two lower order moments—the mean and standard deviation—are calculated for each frame. These moments are largely the same across the four frames, indicating that the ambient vibrations can be approximated as being stationary.
Acceleration time series collected using the Green Building accelerometer array are considered to be either stochastic or superimposed stochastic-deterministic processes—corresponding to ambient vibrations and strong-source events, respectively. As mentioned above, and demonstrated in Figure 3-1, Green Building motions are assumed to be stationary and ergodic. It follows that spectral and statistical methods, namely Fourier analysis, covariance, correlation, power spectral densities, and coherence should be used in the analysis of Green Building vibrations. These methods are explored in greater detail below.

### 3.2 Fourier Analysis

Fourier analysis is a principal method for depicting a signal in the frequency domain, and it is a key component in the estimators for some of the statistical analysis methods described later in the chapter. A brief overview of the Fourier analysis, series, and transform is presented here. The cornerstone of Fourier analysis is the assumption (largely true) that a signal can be accurately represented by a superposition of appropriate sines and cosines. The relative contribution of these sinusoidal terms is representative of the energy distribution as a function of frequency, found in the original time series. Fourier analysis can be applied to either continuous or discrete signals, which in turn can be either

---

**Figure 3-1:** The frame test applied to an ambient acceleration time history recorded at Accelerometer 26.
periodic or aperiodic. For continuous periodic signals, the analysis is performed by expanding the time history into the Fourier series, where the signal is represented by sinusoids of discrete frequencies, corresponding to the fundamental frequency, \( \omega_1 \) and its harmonics (integer multiples of \( \omega_1 \)). The Fourier series decomposition of a signal \( x(t) \) with a period \( T \) is shown in Equation (3.1), with the formulas for the coefficients \( A_n \) and \( B_n \) provided in Equations (3.2) and (3.3). It is often convenient to express the Fourier decomposition in complex notation with a single coefficient, \( C_n \) (Equations (3.4) and (3.5)). The magnitude of the coefficient \( C_n \) indicates how a signal's power is distributed across different frequencies. Another quantity emerging from the Fourier coefficients is the phase angle (Equation (3.6)).

\[
x(t) = 0.5A_0 + \sum_{n=1}^{\infty} (A_n \cos(\omega_1 t) + B_n \sin(\omega_1 t))
\]  

(3.1)

\[
A_n = \frac{2}{T} \int_0^T x(t) \cos(\omega_n t) \, dt \quad n = 0, 1, 2, ...
\]

(3.2)

\[
B_n = \frac{2}{T} \int_0^T x(t) \sin(\omega_n t) \, dt \quad n = 0, 1, 2, ...
\]

(3.3)

\[
x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_n t}
\]

(3.4)

\[
C_n = 0.5(A_n - jB_n) = \frac{1}{T} \int_0^T x(t)e^{-j\omega_n t} \, dt \quad n = \pm(1, 2, 3, ...)
\]

(3.5)

\[
C_0 = 0.5A_0
\]

\[
\theta_n = \tan^{-1}\left(\frac{B_n}{A_n}\right)
\]

(3.6)

The principle of Fourier series can be extended to the case of discrete, aperiodic signals—such as those encountered in the Green Building acceleration data—by introducing the Discrete Fourier Transform (DFT). The basic formula for the DFT is found in Equation (3.7). In this equation, \( x(t) \) is a signal in the time domain sampled at a time interval of \( \Delta t \) with a total of \( N \) samples, \( x = x(n\Delta t) \), and \( X \) is the resulting Fourier spectra. The time-domain signal can be recovered by applying the same formula to its frequency-domain representation (Equation (3.8)). The specific variety of DFT used was the Fast Fourier Transform (FFT) algorithm, as implemented in MATLAB. Fourier analysis will be applied to the
Green Building accelerometer data to identify the structural periodicities, calculate the power spectra (Section 3.5), and to perform frequency domain averaging (Section 4.1).

\[ X_k = \sum_{n=0}^{N-1} x_n e^{-2j\pi kn/N} \quad k = 0,1,2,\ldots N - 1 \]  \hspace{1cm} (3.7)

\[ x_n = \sum_{k=0}^{N-1} X_k e^{2j\pi kn/N} \quad n = 0,1,2,\ldots N - 1 \]  \hspace{1cm} (3.8)

### 3.3 Covariance

The covariance is a statistical measure which quantifies how two signals change with one another. Specifically, covariance tracks the linear dependence of two random variables. The covariance function compares the signals as a function of a time lag \( \tau \) between them. The lag variable can also represent other quantities besides time, such as spatial separation. For random variables \( x(t) \) and \( y(t) \) with sample size \( N \), the covariance and the cross-covariance functions are computed using Equations (3.9) and (3.10). A positive covariance indicates the signals are changing together in the same direction, a negative covariance indicates that they are changing together but in opposite directions, while a covariance of zero means that the signals have no common trends. In the special case where the two signals are the same \( (x(t) = y(t)) \), the covariance reduces to the variance of the signal, given by Equation (3.11). The magnitude of the covariance does not have a meaningful physical interpretation. Thus, it is difficult to establish what the strength of the relationship between two signals is, and it makes comparison of different signal pairings impossible. The covariance will primarily be used to compute the correlation function, which is described in the next section.

\[ Cov(x, y) = \sigma_{xy} = E[x - E(x)] \cdot E[y - E(y)] = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}) \]  \hspace{1cm} (3.9)

\[ C_{xy}(\tau) = E[x(t) - E(x)] \cdot E[y(t + \tau) - E(y)] = \frac{1}{N - \tau} \sum_{t=1}^{N-\tau} (x_t - \bar{x})(y_{t+\tau} - \bar{y}) \]  \hspace{1cm} (3.10)

\[ Cov(x) = \sigma_x^2 = E[x^2] - E(x)^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i^2) - \bar{x}^2 \]  \hspace{1cm} (3.11)
3.4 Correlation

Correlation, like covariance, is a statistical measure which estimates the linear dependence between two signals as a function of the time shift ($\tau$) between them. As with covariance, $\tau$ can also represent shifts in other quantities, such as spatial position. Two correlation values are of interest—the correlation function and the correlation coefficient function (sometimes known as the Pearson product-moment correlation coefficient), which will henceforth be referred to as the correlation coefficient. The correlation function is defined in Equation (3.12). The correlation coefficient function is found by dividing the cross-covariance function of two signals by the product of their standard deviations. For random variables $x(t)$ and $y(t)$ with sample size $N$, the correlation coefficient is given by Equation (3.13).

A special case of the cross-correlation is the auto-correlation, in which the signal is compared to itself. In this case, Equation (3.13) reduces to Equation (3.14). Dividing by the standard deviation normalizes the covariance, and gives a correlation coefficient on the range of $-1$ to $1$. This gives information on the strength of the linear relationship between the signals and makes it possible to compare different signal pairs. Signals with correlation values of $\pm 1$ have a strong linear relationship (changing together in the same or opposite direction), while those with correlations of 0 are completely random or have a perfectly non-linear relationship. A common rule of thumb holds that correlations of 0.7 to 1.0 are strongly correlation, values between 0.3 and 0.7 are moderately correlated, and values between 0 and 0.3 are weakly correlated and can be considered random. These conventions are summarized in Table 3-2.

In the context of the Green Building accelerometer data, correlations will be used to estimate the degree to which energy propagates between successive floors and to compute the power spectral density function. Applied to spatial lags, the correlations will be used to find the area of effect of certain sources and to compare the behavior of the east and west sides of the building.

$$R_{xy}(\tau) = E[x(t)y(t+\tau)] = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (x_t)(y_{t+\tau})$$  \hspace{1cm} (3.12)

$$\rho_{xy}(\tau) = \frac{C_{xy}(\tau)}{\sigma_x\sigma_y} = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} (x_t - \bar{x})(y_{t+\tau} - \bar{y})$$  \hspace{1cm} (3.13)

where

$$\sigma_x = \sqrt{E[x^2] - E(x)^2} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i^2) - \bar{x}^2}$$
\[ \rho_{xx}(\tau) = \frac{C_{xx}(\tau)}{\sigma_x^2} = \frac{1}{N-\tau} \sum_{i=1}^{N-\tau} (x_i - \bar{x})(x_{i+\tau} - \bar{x}) \]  

(3.14)

Table 3-2: Rules of thumb for interpreting correlation values

<table>
<thead>
<tr>
<th>Correlation Value</th>
<th>Nature of Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 to 1.0 (or -0.7 to -1.0)</td>
<td>Strong</td>
</tr>
<tr>
<td>0.3 to 0.7 (or -0.3 to -0.7)</td>
<td>Moderate</td>
</tr>
<tr>
<td>0 to 0.3</td>
<td>Weak or Random</td>
</tr>
</tbody>
</table>

3.5 Power Spectral Density

The power spectral density (PSD) is a frequency decomposition of a random process. It is estimated by taking the FFT of the correlation function. Like correlation, the PSD can be computed for two different time series (cross-PSD) and for the same signal (Auto-PSD), as shown in Equations (3.15) and (3.16). Conversely, the correlation function can be recovered, by taking the inverse FFT of the PSD. The PSD may also be computed by taking the FFT of the input time series directly. This approach is often computationally and conceptually simpler, and Welch’s Method (a type of FFT-based PSD estimator) is the most commonly used method to compute the PSD. Welch’s method is omitted here, but the standard procedure for FFT-based PSD estimation is summarized below. Given two random processes, \( x(t) \) and \( y(t) \), a set of \( N \) time histories may be collected. For a time history \( n \), the FFTs of the variables are given by \( X_n(t) \) and \( Y_n(t) \). The PSDs may be computed by multiplying \( Y_k(t) \) with the complex conjugate of \( X_k(t) \) and averaging over the \( N \) collected sample histories, as shown in Equations (3.17) and (3.18). Due to the double-sided integral in the Fourier transform, the PSD has mirrored positive and negative frequencies. To ease physical interpretation, the one-sided cross-PSDs and auto-PSDs—denoted as \( G \) in Equations (3.19) and (3.20)—are often used in signal analysis.

\[ S_{xy}(\omega) = FFT \left( R_{xy}(\tau) \right) = \int_{-\infty}^{+\infty} R_{xy}(\tau) e^{-i\omega \tau} d\tau \]  

(3.15)

\[ S_{xx}(\omega) = FFT \left( R_{xx}(\tau) \right) = \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-i\omega \tau} d\tau \]  

(3.16)
\[ S_{XY}(\omega) = E(X_n^*(\omega)Y_n(\omega)) = \frac{1}{N} \sum_{i=1}^{N} X_i^*(\omega) Y_i(\omega) \] (3.17)

\[ S_{XX}(\omega) = E(|X_n(\omega)|^2) = \frac{1}{N} \sum_{i=1}^{N} |X_i(\omega)|^2 \] (3.18)

\[
G_{XY}(\omega) = \begin{cases} 
2S_{XY}(\omega) & \omega > 0 \\
S_{XY}(\omega) & \omega = 0 
\end{cases}
\] (3.19)

\[
G_{XX}(\omega) = \begin{cases} 
2S_{XX}(\omega) & \omega > 0 \\
S_{XX}(\omega) & \omega = 0 
\end{cases}
\] (3.20)

The PSD amplitude for some time-history, \( y(t) \), has units of \([y(t)]^2/Hz\). In the case of acceleration time histories, this comes out to be \((m/s^2)^2/Hz\). The auto-PSD can be thought of as representing the average power carried by the time series as a function of frequency, and the area beneath the PSD curve in a selected frequency band is the squared RMS value for that band. To ease interpretation, the PSD can be integrated as in Equation (3.21), to yield the cumulated spectrum power (CSP). This gives a better idea of the relative contributions of particular frequencies. Unlike the auto-PSD, the cross-PSD is usually complex valued. The real and imaginary components of the PSD are called the coincident spectral density function (or cospectrum) and quadrature spectral density function (or quadspectrum). The magnitude and phase angle spectrum of the PSD can be computed by decomposing the PSD into its real and imaginary components (\( C_{xy} \) and \( Q_{xy} \)) and applying the standard equations. The amplitude spectrum measures how relative signal power varies between the two signals as a function of frequency, while the phase spectrum indicates frequency phase shift.

\[
CSP(\omega) = \int_{0}^{\omega} G_{XY}(\omega) \, d\omega
\] (3.21)

Another useful application of the PSD is in mode-shape estimation. Assuming that natural frequencies have been determined and that there are a sufficient number of sampling locations on a structure, the approximate mode shape of the structure can be found using Equation (3.22), where \( \phi_n \) is the mode shape of the \( n^{th} \) natural frequency \( (f_n) \) at location \( m \), and \( x_m(t) \) is a time history sampled at location \( m \). The sampled motions must all be in the same direction, and for best results, the sampling
locations should be distributed evenly throughout the structure. To estimate the shape of a mode \( p \), time histories from about \( p \) locations will be required. This relation is best used only for small modal damping ratios, generally about \( \xi < 0.05 \). For the Green Building dynamic characterization, the PSD will be used to compute signal coherence (covered in the next section), to identify periodicities in structural motions, and to determine approximate mode shapes.

\[
\phi_n(x_m(t)) = \sqrt{G_{x_m x_m}(f_n)}
\]  

(3.22)

### 3.6 Coherence

The coherence function, sometimes called the magnitude squared coherence, is a normalized measure of the correlation of two signals in the frequency domain. It conveys similar information as phase angles, but it is visually easier to interpret. The coherence spectrum is computed using the signal PSD’s and Equation (3.23), below. It is often useful to compute the square-root of the coherence function, Equation (3.24), to determine the sign of the linear relationship of the signals. The coherence values follow the same rule-of-thumb as correlation (Table 3-2), with values of \( \pm 1 \) signifying the signals to be changing together (either positively or negatively) and 0 indicating a completely random relationship between the two. The coherence function is useful for estimating the effect of noise on a system, and it will be applied to the Green Building for natural frequency identification and mode shape estimation by assessing how different parts of the structure move with one another as a function of frequency.

\[
\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}
\]

(3.23)

\[
\gamma_{xy}(f) = \frac{|G_{xy}(f)|}{G_{xx}(f)G_{yy}(f)}
\]

(3.24)

### 3.7 Visual Colormap Analysis

To better recognize structural behavior and identify modal characteristics, it is often helpful to examine the signal set holistically, as opposed to by individual time-history. A convenient and conventional tool for this purpose is the colormap—a visual representation of a system where a color-scale is used to represent the value of some quantity of interest. In this case, the system is the Green Building as represented by 36 accelerometers, and the quantities of interest are accelerations, displacements, and how these change with time.
The colormaps were created as two-dimensional images using MATLAB. First, a blank synthetic image was created with the same aspect ratio as a desired façade of the Green Building. Next, the appropriate sensor time-histories were associated with points on this image, corresponding to the actual accelerometer locations in the building. The motions at the remaining points were found by assuming a linear interpolation between the defined accelerometer locations—first in the vertical and then in the horizontal direction. A color-scale was then applied to the image, based on the extreme motion values. For extra flare, the colormap was made slightly transparent and superimposed on a photograph of the Green Building. An example of a single colormap from the 10/16 Hollis Center earthquake is pictured in Figure 3-2. This process was repeated for each time point, and the resulting images were then collected into a movie.

These colormap animations are useful since they allow motions to be seen and interpreted throughout the structure and as an evolving function of time. Using them, it is possible to view the characteristics of energy propagation through a structure and to estimate the locations of excitation sources. More importantly, they can be used to estimate mode shapes and verify the identity of modal frequencies. This is performed by applying a band-pass filter to the full data set around some frequency band of interest, per the filtering methods described in Section 4.2 below. The visualization is then run on the filtered data set, and the resulting motions describe the structural response due to the selected frequency. If the selected frequency is a normal mode of vibration, the visualized motion is the corresponding mode shape. This method was used to check the motions associated with the identified natural frequencies and to estimate the location of nodal points in mode shapes.

Figure 3-2: Colormap plot of Green Building accelerations in the NS direction due to the 10/16 Hollis Center Earthquake
4 Data Pre-Processing

For analysis of the Green Building data and similar data sets, the methods outlined in the preceding section can be applied to the collected data. It is also often desirable to integrate or differentiate the data to get a full set of displacement, velocity, and acceleration responses. However, real systems and instrumentation often admit features into recorded data which make direct integration and application of these analysis methods error-prone. One problem is that of signal noise—either electronic or stemming from ambient vibrations—which can mask features of interest in the time-domain and implant spurious frequencies in the frequency domain. Significant problems also arise from DC errors occurring in electronic sensing systems and low-frequency variations in either the sensors or the structural system under study. Both features complicate time-domain integration and analysis done in the frequency domain. Due to these errors, collected time histories must usually undergo some pre-processing to eliminate the corrupting features. Three pre-processing methods used for the Green Building data are summarized in this Chapter. Section 4.1 covers frequency domain averaging, filtering is reviewed in Section 4.2, and a summary of base-lining is presented in Section 4.3. To demonstrate the necessity of these methods, Figure 4-1 shows a comparison between minimally processed displacements and those which have been filtered and base-lined. Both time histories from the same ambient signal, and they were integrated from accelerations using the trapezoidal method.

![Figure 4-1: Comparison of displacement time histories from a minimally processed signal (left) and from a filtered and base-lined signal (right)](image-url)
4.1 Frequency Domain Averaging

Frequency domain averaging—sometimes referred to as ensemble averaging—was used to reduce frequency noise and amplify periodicities in data. As the name implies, the method consists of taking several related frequency spectra and averaging their values at each frequency point. Assuming noise to have a semi-random frequency distribution, this has the effect of driving frequencies associated with noise down to zero, while amplifying the periodicities of the system (which are assumed to occur consistently across the averaged spectra).

For the Green Building data, this method was applied in several ways to achieve different ends. For time histories containing primarily ambient vibrations of a random nature, the averaging process was applied to individual recordings as a de-noising method. The Fourier spectrum of each history was divided into a reasonable number of segments (usually about 8), with each segment overlapping by 50%. These were then multiplied by a Hanning window function and averaged. Frequency averaging was also used in both transient and ambient signals to distinguish structural motions and building-wide excitation frequencies from local vibrations. This was done by averaging the Fourier Spectra of time histories collected at several different spatial locations or recording dates. This reduced periodicities occurring only on certain floors or times, while amplifying consistent structural frequencies.

4.2 Filtering

Filtering—the process of eliminating certain frequencies from a signal—was used to remove the effects of DC errors and low-frequency oscillations, suppress high-frequency noise which had no effect on building motions, and to isolate frequencies of interest for the visual analysis procedure described in Section 3.7. The collected signals were all high-pass filtered with a pass-frequency of 0.4 Hz. This was found to be the minimum frequency required for accurate double integration of the acceleration signals. Low-pass filtering was used on a case-by-case basis, depending on the high-frequency content of each signal. For band-pass filtering, a combination of a high-pass and low-pass filters were applied around the desired frequency. A 2nd order Butterworth digital filter designed in MATLAB was used for all signal filtering. The Butterworth filter was chosen for its flat frequency response in the pass-band. The corresponding disadvantage of filter is that the stop-band has a relatively slow roll-off.

4.3 Base-lining and De-trending

Base-lining of the recorded data was used to remove artificial linear trends and offsets which arose from electronic drift and integration artifacts. The Green Building data was base-lined by subtracting either the mean of the signal (a “constant” base-line) or a straight line fitted to the data. Linear trends in the signals were determined by applying least-squares linear regression. Since the raw
accelerations were not zero-centered, mean base-lining was always applied to these signals to bring their equilibrium state down to zero. Linear de-trending was rarely necessary for the accelerations. However, the integrated velocities and displacements both required linear de-trending to remove artifacts and make possible further integration.
5 Collected Data Sets

5.1 Overview

Five sets of acceleration data were collected using the Green Building Accelerometer array. Of these, 2 sets represented ambient structural behavior, while 3 recordings were of strong external excitations. In addition to these, 79 sets of ambient vibration data were provided by Dr. Mehmet Celebi of the USGS. These data sets were analyzed to characterize the Green Building's dynamic behavior and structural characteristics and to validate numerical models of the Green Building, the results of which are reported in Chapters 6 and 7, respectively. In this chapter, details of the recording procedures, environmental conditions, and data set quality will be presented for the analyzed events. Weather and seismic activity conditions were found by consulting records from Weather Underground and the Weston Observatory. All times are given according to the 24 hour clock. Ambient vibration recordings will be discussed in Section 5.2, the unidentified May 14 event in Section 5.3, the July 4th Fireworks show in Section 5.4, and the October 16, 2012 Hollis Center earthquake in Section 5.5. The time histories for each accelerometer channel and recorded event are found in Appendix A through Appendix D.

The peak recorded accelerations and displacements generally increased with excitation in the following order: ambient vibrations, 4th of July fireworks show, May 14 event, and Hollis Center earthquake. In terms of recorded frequencies, the 4th of July fireworks induced the most high-frequency content in the Green Building, followed by the May 14 event. While the lower structural translational and torsional modes were all found to be within the 0-10 Hz band (see Chapter 6), the high-frequency (> 20 Hz) vibrations are thought to result from machinery or dynamic live-load sources such as pedestrian traffic.

5.2 Ambient Vibrations

Recordings of the Green Building's ambient motions come from two sources. The first of these comprises a set of on-demand recordings made by the USGS and provided to MIT by Dr. Mehmet Celebi of the USGS. These consist of seventy-nine 330 second recordings made every hour between Monday April 4, 2011 at 7:30:28 and Thursday April 7, 2011 at 13:30:28. During this three day period, no seismic activity was detected, and weather conditions ranged from calm to rainy and windy. The second class of ambient recordings consists of 2 on-demand measurements made by the MIT team on Friday June 22, 2012 at 00:11:16 and Monday April 15, 2013 at 02:35:02. The recordings are 320 seconds and 470 seconds long, respectively. Two recordings were made to check that ambient measurements were both repeatable and representative of regular building behavior. Since the signal sets show very similar properties, only one—that from June 22—will be presented in this thesis. During the June 22 recording,
there was no seismic activity and weather conditions were clear with 1 mph winds out of the West, and

gusts of up to 4 mph. A representative acceleration time history at the 2nd floor, NS direction (channel 14)
is presented in Figure 5-1. A summary of the peak accelerations due to the 6/22 ambient vibrations is

found in Table 5-1. The full set of acceleration time histories collected from the 6/22 ambient vibrations
can be found in Appendix A. The full Fourier spectra from floor 7 (Channel 20, east side, NS direction) is

presented in Figure 5-2. Most of the response energy is contained in the 0-10 Hz range, but the signal has

substantial high-frequency content up to the Nyquist frequency, probably resulting from local
disturbances such as machinery and pedestrian traffic.

![Acceleration at Channel 14](image)

Figure 5-1: Acceleration time-history at the 2nd floor of the Green Building (NS direction) resulting from
ambient excitation on June 22, 2012

<p>| Table 5-1: Peak accelerations in the NS, EW, and vertical directions due to ambient excitation |
|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|</p>
<table>
<thead>
<tr>
<th>Peak Acceleration (m/s²)</th>
<th>North-South Direction</th>
<th>East-West Direction</th>
<th>Vertical Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (Channel)</td>
<td>35</td>
<td>34</td>
<td>6</td>
</tr>
<tr>
<td>Time from start of recording (s)</td>
<td>283.755</td>
<td>109.09</td>
<td>214.805</td>
</tr>
</tbody>
</table>
The May 14 data set was collected via an automatically triggered recording in response to an unidentified excitation which occurred on Monday, May 14, 2012 at 2:04:15. The recording began at 2:03:46 and lasted 150 seconds, and the duration of the event—from the beginning of the disturbance to attenuation of major motion—was 1.4 seconds. Weather conditions were clear and calm with wind speeds of 5 mph out of the south. No seismic activity was recorded in the area, and a survey of the local news found no reports of anomalous explosions or other activities. A representative time history, with a detail of the event is shown in Figure 5-3. A comparative examination of the waveform seems to suggest a blast or explosive source. Furthermore, the excitation was first detected at the lower accelerometers, and the eastern NS accelerometers reported higher peak accelerations that their counterparts along the western shear wall. The EW oriented accelerometers consistently measured higher accelerations than those along the NS axis. This suggests a ground-based shockwave very near to the building’s eastern side. The peak values of acceleration in the NS, EW, and vertical directions are summarized in Table 5-2. A representative Fourier Spectra from Floor 7 (NS direction, Channel 20) is shown in Figure 5-4. As with the ambient vibrations, the high frequency noise persists. Certain peaks match-up with those of ambient vibrations, reinforcing the view that these are either caused by machinery within the building or represent higher structural modes. The presence of higher structural frequencies under ambient and low-energy conditions would be particularly beneficial for damage detection purposes, since these higher modes are
more sensitive to changes in structural properties. The full set of acceleration time histories collected during the May 15 event can be found in Appendix B.

Figure 5-3: Acceleration time histories from the ground floor of the Green Building (EW Direction), resulting from an unidentified excitation on May 14, 2012; (right) detail of the event

Table 5-2: Peak accelerations in the NS, EW, and vertical directions due to the 5/14 Event excitation

<table>
<thead>
<tr>
<th></th>
<th>North-South Direction</th>
<th>East-West Direction</th>
<th>Vertical Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Acceleration (m/s²)</td>
<td>0.1321 (0.0135g)</td>
<td>0.6651 (0.0678g)</td>
<td>0.0133 (0.0014g)</td>
</tr>
<tr>
<td>Location (Channel)</td>
<td>14</td>
<td>34</td>
<td>5</td>
</tr>
<tr>
<td>Time from start of recording (s)</td>
<td>29.25</td>
<td>29.18</td>
<td>29.37</td>
</tr>
</tbody>
</table>
5.4 July 4th Fireworks Show

An on-demand recording was made of structural accelerations resulting from the beginning of the hour long “Boston Pops Fireworks Spectacular” Independence Day fireworks show, occurring on Wednesday July 4th, 2012. The recording began at 22:32:05 and lasted 510 seconds. Of this, the firework detonations start at 22:38:40 and occupy the last 115 seconds. Weather conditions were rainy with sustained wind speeds of 3 mph from the West and wind gusts of 9 mph. There was negligible vehicle traffic; however, the area experienced extensive pedestrian activity throughout the evening. Fireworks were launched off a series of barges placed in the Charles River, about midway between the Harvard and Longfellow bridges (Figure 5-5). The approximate horizontal distance from the barges to the Green Building was 600 m. For analysis purposes, the fireworks were assumed to detonate at elevations between 100 m to 250 m—the conventional altitude of professional fireworks. A representative time-history of accelerations with a detail of the fireworks, recorded at the 2nd floor in the NS direction (accelerometer channel 15) is presented in Figure 5-6. The sharp, high-frequency clustered peaks correspond to the structural response due to the positive pressure waves from individual firework detonations. The peak acceleration of 0.0184g (0.1808 m/s²) occurred at the second floor, in the EW direction (accelerometer channel 13) at 0.525 seconds into the show. A summary of the peak accelerations in the NS, EW and vertical directions is presented in Table 5-3. Once again, the Fourier spectra at floor 7 (NS direction, Channel 20) is shown in Figure 5-7. There is an overall increase in high-frequency content over that found in the ambient vibration recordings. This is expected, given the acoustic noise, high pedestrian
traffic, and inclement weather affecting the area. A full set of acceleration recordings collected during the 4th of July Fireworks show can be found in Appendix C.

Figure 5-5: Location of fireworks barges for Boston’s 4th of July fireworks show

Figure 5-6: Acceleration time histories from the second floor of the Green Building (NS Direction) during Boston’s 4th of July fireworks show; (left) full time history; (right) detail of the show beginning

Table 5-3: Peak accelerations in the NS, EW, and vertical directions from the July 4th fireworks show

<table>
<thead>
<tr>
<th></th>
<th>North-South Direction</th>
<th>East-West Direction</th>
<th>Vertical Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Acceleration (m/s^2)</td>
<td>0.0663 (0.0068g)</td>
<td>0.1808 (0.0184g)</td>
<td>0.0162 (0.0017g)</td>
</tr>
<tr>
<td>Location (Channel)</td>
<td>36</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>Time from Start of Show (s)</td>
<td>78.72</td>
<td>0.525</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Figure 5-7: Fourier spectrum of 4th of July fireworks induced vibrations at Channel 20; larger than usual amount of high frequency content is visible

5.5 October 16, 2012 Hollis Center Earthquake

Acceleration response data was collected via an automatically triggered recording due to an earthquake near Hollis Center, Maine, which occurred on Tuesday October 16, 2012. The recording began at 19:12:35 EST, and lasted 155 seconds. Weather conditions were clear, with negligible wind. The earthquake was of 4.0 magnitude, and the epicenter was located about 4 km outside of Hollis Center, Maine—about 142 km from Cambridge, Massachusetts (Figure 5-8). Thus, the seismic waves arrived at the Green Building along an axis oriented about 15 degrees north-east. The event started at 19:12:23 and was detected in Cambridge at 19:12:47. A time-history of the event—as recorded at the Green Building’s ground level in the NS direction (accelerometer channel 9)—is shown in Figure 5-9. The event duration—from arrival of the p-waves to the attenuation of the s-waves—was about 32.5 s. The p-wave arrival time was 12 seconds into the recording, while the s-wave reached the Green Building at 28.5 seconds, giving a time delay of 16.5 seconds. A peak acceleration response of 0.0124g (0.1218 m/s²) occurred 19 seconds into the event, and it was recorded at the roof-top level in the NS direction (accelerometer Channel 35). A summary of the peak accelerations in the NS, EW and vertical direction is presented in Table 5-4. The majority of the earthquake’s frequency content was found between 1.2 and 11 Hz. The characteristic
high-frequency content observed in lower-energy excitations is present, but insignificant compared to the seismic excitation frequencies. However, it does become more prominent with height. This can be seen in Figure 5-10 and Figure 5-11, which shows the Fourier Frequency spectrum of accelerations recorded at ground level in the NS direction (channel 8) and at the 7th floor (NS direction, channel 20). The full set of acceleration recordings collected during the Hollis Center earthquake can be found in Appendix D.

Figure 5-8: Location of the 10/16 Hollis Center Earthquake

Figure 5-9: Ground response at ground-level NS direction (Station 9) with a detail (right).
Table 5-4: Peak accelerations in the NS, EW, and vertical directions due to the 10/16 earthquake

<table>
<thead>
<tr>
<th></th>
<th>North-South Direction</th>
<th>East-West Direction</th>
<th>Vertical Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Acceleration (m/s²)</td>
<td>0.1218 (0.0124g)</td>
<td>0.1214 (0.0124g)</td>
<td>0.0389 (0.0040g)</td>
</tr>
<tr>
<td>Location (Channel)</td>
<td>35</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Time from Arrival (s)</td>
<td>9.005</td>
<td>8.075</td>
<td>10.105</td>
</tr>
</tbody>
</table>

Figure 5-10: Fourier spectrum of the Hollis Center earthquake; from a recording taken at ground level in the NS direction
Figure 5-11: Fourier spectrum of the Hollis Center earthquake; from a recording taken at the 7th floor in the NS direction
6 Feature Identification

The acceleration data sets collected from the Green Building and summarized in Chapter 5 were analyzed using the methods described in Chapter 3 to determine the vibrational and structural characteristics of the structure. A total of 8 natural frequencies were identified and estimates were made of their mode shapes. These findings and the analysis underpinning them are detailed in Section 6.1. Significant base-rocking behavior—in which the foundation undergoes a rigid-body rotation—was observed in both the NS and EW directions and foundation flexure was found to occur alongside torsional motion. These features are detailed in Section 6.2. The Green Building was also observed to exhibit strong torsional behavior in response to most excitations (Section 6.3). A comparative analysis of motions on the Eastern and Western sides also revealed response asymmetries which were rooted in differences between the structural layout of the two ends of the Green Building. These differences in motion and the proposed causes underlying them are presented in Section 6.4. A general summary of all identified dynamic characteristics is provided in Section 6.5. For the sake of brevity, each section contains only a small selection of Fourier spectra, power spectra, and coherences to motivate the identified features. Likewise, correlation functions are not reported here as they do not directly motivate the results. The time histories for all the recorded excitations and accelerometer channels can be found in their respective appendices (Appendix A through Appendix D).

6.1 Natural Frequencies

In this section, the natural frequencies of the Green Building corresponding to motions in the East-West, North-South, and torsional directions will be presented. The East-West and North-South motions arise from rigid-body structural translations, bending deformations, and shear deformations in these directions. Torsion is a whole-building twisting deformation along the structure’s height. The natural frequencies were identified by visual inspection of the Fourier spectra, power spectra, coherence spectra, phase angles, and band-passed visual animations (described in Section 3.7) of the collected accelerometer and displacement data. For frequency analysis purposes, the 6/22 ambient vibration, 4th of July firework, and 5/14 Event recordings were used. As an additional check, the identified modes were also found using some of the remaining ambient recordings, since in practice all the ambient recordings should be interchangeable. For the sake of brevity, only ambient results from the 6/22 ambient vibration measurements will be presented in this section. The October 16 earthquake was not used for mode identification since the high-energy content of the excitation masked the underlying structural frequencies.

Reporting of the mode identification will be partitioned into subsections containing analysis of fundamental modes (6.1.1), second modes (6.1.2), and third modes (6.1.3). This division was used since
the energies required to excite these modes often required the use of different signal sets with different excitation sources. The fundamental modes were easily distinguishable in the ambient recordings, while higher modes occurred more clearly in the firework and May 14 event data sets.

The general procedure for mode frequency identification was to identify periodicities in the signals using the Fourier and power spectra. These periodicities were then compared with signals from other accelerometers oriented with both the same and different orientation to determine the directionality of motion. Next, the coherence spectra and phase angles between spatially separate time histories were checked to confirm building motions and to help estimate mode shapes. The visual animations were used to corroborate these estimates. Using these approaches it was possible to determine some of the Green Building’s natural frequencies. In total, eight natural frequencies were positively identified, and a summary of these is found in Table 6-1. To reduce redundancy, only a selection of the computed results will be presented in the sections below.

Table 6-1: Natural frequencies of the Green Building identified using data from the accelerometer array

<table>
<thead>
<tr>
<th>Modal Frequencies (Hz)</th>
<th>East-West</th>
<th>North-South</th>
<th>Torsional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Mode</td>
<td>0.68</td>
<td>0.75</td>
<td>1.45</td>
</tr>
<tr>
<td>2nd Mode</td>
<td>2.45</td>
<td>2.85</td>
<td>5.05</td>
</tr>
<tr>
<td>3rd Mode</td>
<td>8.10</td>
<td>8.25</td>
<td>NA</td>
</tr>
</tbody>
</table>

6.1.1 Fundamental Modes

The first EW, NS, and torsional modes were identified using ambient vibration measurements and confirmed using the 4th of July and May 14 event recordings. As summarized in Table 6-1, the frequencies of these modes were found to be 0.68, 0.75, and 1.45 Hz, respectively. Figure 6-1 shows the Fourier spectra of accelerations from the 6/22 ambient vibration recording. The spectra are taken at two floors in both the EW and NS directions. Focusing on the interval below 1.5 Hz, three consistent peaks can be distinguished. A peak at 1.45 Hz is present in all four spectra, while peaks at 0.68 and 0.75 occur in only in the EW and NS directions, respectively. This peak distribution is present in all recordings and at all locations in the Green Building. Figure 6-5 and Figure 6-6 show these peaks in both the July 4th firework and May 14 event recordings. This suggests that these frequencies correspond to structural modes and that the peaks at 0.68 and 0.75 are directed along the EW and NS axis.
Figure 6-1: Fourier spectra of accelerations occurring during the 6/22 ambient vibration recording. The top row shows recordings in the EW (left) and NS (right) directions on the eastern side of floor 19, while the bottom row shows the same plots at floor 12.

To establish the nature of motion associated with these frequencies, the signal coherences were taken between pairings of appropriate accelerometers. Figure 6-2 shows the pairing of NS oriented accelerometers against the sensor at floor 19, and Figure 6-3 shows the same for accelerometers in the EW direction. This is done to establish whether the sensors are moving together—as in a fundamental mode—or with some accelerometers out of sync—as associated with higher modes. To check for torsional motions, the coherences were also computed between parallel NS oriented accelerometers located on the same floors (Figure 6-4). These checks were also performed using phase angles, with 0 and 180 degree angles corresponding to in-sync and out of sync motions, respectively. For unwrapped phase angles, this is indicated by odd and even multiples of pi at the frequency of interest.
Referencing Figure 6-2, all the NS sensor pairings at 0.75 Hz show a very strong positive coherence of +1. Likewise in Figure 6-3, the EW sensor pairings at 0.68 Hz also show a strong positive coherence. This shows the motions to be synchronous and indicates that these two frequencies are the fundamental modes in the NS and EW directions. In Figure 6-4, all pairings of parallel sensors show strong negative coherence at 1.45 Hz, indicating that the sensors are accelerating in opposite directions. This corresponds to the twisting motion of the first torsional mode. In the plot, all the pairs also have strong positive coherence at 0.75 Hz over the height of the building. This again indicates that the building is swaying symmetrically in the NS direction—as expected of a NS mode.

![Coherence of NS accelerometers](image)

Figure 6-2: Coherences of NS oriented accelerometers paired against the floor 19 sensor; from the 6/22 ambient vibration. Note the positive coherence at 0.75 Hz and 1.45 Hz.
6.1.2 Second Modes

The second EW, NS, and torsional modes were found to be at 2.45 Hz, 2.85 Hz, and 5.05 Hz. These modes appear in all the recorded signals, but they are most apparent in the July 4th fireworks and the May 14 event recordings. This is due to the greater excitation energy needed to mobilize higher modes. The analysis process for the second (and higher) modes was similar to that used for the
fundamental modes. Periodicities were identified using the Fourier and power spectra and details of building motions were then examined with coherence spectra, phase angles, and visual analysis. As can be seen in Figure 6-1, above, the ambient vibrations contain the 2nd mode frequencies in question. However, these are diffused and mixed in with other peaks of unknown origin. The acceleration Fourier spectra of two floors in the NS and EW directions from the July 4th fireworks show and the May 14 event are presented in Figure 6-5 and Figure 6-6, respectively. In both figures, the peak at 5.05 Hz is present, suggesting a whole-building motion, possibly corresponding to a torsional mode. Conversely, the 2.45 and 2.85 Hz peaks only appear in EW and NS spectra.

Figure 6-5: Fourier spectra of accelerations occurring during the Fourth of July fireworks show. The top row shows recordings in the EW (left) and NS (right) directions on the eastern side of floor 19, while the bottom row shows the same plots at floor 12.
Signal coherences with the same pairings as used in Section 6.1.1 are presented below. Pairs of NS accelerometers are shown in Figure 6-7, Figure 6-8 contains EW pairings, and Figure 6-9 compares parallel NS oriented accelerometers. All these coherences are from the May 14 event. In Figure 6-7, the coherences at 2.85 Hz are all either strongly positive or negative. Specifically, the channel 32-29 pairing is nearly +1 coherent, while the rest of the channels paired with 31 are nearly -1 coherent. This indicates that at this frequency, channels 31 and 29 are moving in the same direction while the other NS channels are moving in the opposite direction. This suggests the presence of a mode shape nodal point located between floors 13 and 18—a configuration which intuitively makes sense, given the 2nd mode shape of a cantilevered beam. Furthermore, in Figure 6-9 all NS accelerometers on the same floor have a strong...
positive coherence. Both these features indicate the 2nd NS mode to be at 2.85 Hz. Analogous behaviors are observed in the EW coherence spectra. In Figure 6-8, a similar split occurs in the coherences at 2.45 Hz, with channels 31-29 strongly positive while the rest are strongly negative. Once again, this points to a second EW mode. It is worth noting that the cross-over point of the EW second mode shape is in the same region as that of the second NS mode—between floors 13 and 18.

Regarding the second torsional mode, the 5.05 Hz frequency is present in all directions and all recorded excitations, thus suggesting to the presence of a structural frequency. A torsional mode is a probable cause for such pervasive whole-building motion. Examining the parallel NS coherences in Figure 6-9, there is a trend towards negative coherence, but it is not as strong or universal as that seen with the 1.45 Hz fundamental mode. However, Figure 6-8 shows clear separation of coherences between certain EW sensors. Some separation is also visible with the NS sensors in Figure 6-7. Due to the motions associated with twisting, it is expected for some sensors to be moving together along their respective axis while others move in the opposite direction. For the EW sensors, the pairs of 31-25 and 31-22 are negatively coherent, while the remaining sensors have positive coherence. In both EW and NS cases, this corresponds to the region between floors 8 and 17—the central region of the building—twisting in a different direction than the others. Once more, this matches the expected motion for the 2nd torsional mode, and the cross-over region is the same as that for the two 2nd translation modes. Regarding the ambiguous coherences of the parallel NS accelerometers, it is possible that due to various structural asymmetries, the 2nd torsional mode couples with some NS and EW translation motion, in addition to classical twisting. This coupling may account for the significant, weakly coherent noise around 5.05 Hz in Figure 6-9.
Figure 6-7: Coherences of NS oriented accelerometers paired against the floor 19 sensor; from the May 14 event. Note coherences at 2.85 Hz suggesting a cross-over point.

Figure 6-8: Coherences of EW oriented accelerometers paired against the floor 19 sensor; from the May 14 event. Note coherences at 2.45 Hz suggesting a cross-over point, and the coherence separation at 5.05 Hz.
In identifying the second modes—2.45, 2.85, and 5.05 Hz in the EW, NS, and torsional directions, respectively—it is apparent that they stack in the same order as the fundamental modes. Furthermore, the ratios between the second and fundamental modes in each direction, summarized in Table 6-2, are all similar and within 6 percent of each other. Such uniform frequency scaling between modes is expected given that classical estimates of beam frequencies vary only with the mode number.

<table>
<thead>
<tr>
<th>EW Modes</th>
<th>NS Modes</th>
<th>Torsional Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.6</td>
<td>3.8</td>
<td>3.4</td>
</tr>
</tbody>
</table>

6.1.3 Third Modes

Due to high frequency noise sources, contaminating motions, and the higher energies required to excite them, higher structural modes are more difficult to positively identify. However, candidates for the third EW and NS translation modes were located using the May 14 event data and partially corroborated using the 4th of July fireworks measurements. These modes are estimated to be at 8.1 and 8.25 Hz, for the EW and NS directions. Figure 6-5 and Figure 6-6, above, both display a peak at 8.1 Hz in the EW accelerometers and some feature located around 8 Hz in the NS directions. To further identify these frequencies as structural periodicities, the power spectra of NS and EW accelerations from two floors
during the May 14 event are presented in Figure 6-10. The peaks at 8.1 and 8.25 Hz can be easily distinguished, along with their energy content relative to the other frequencies.

To further identify these modes, details of the EW and NS paired coherences are presented in Figure 6-11 and Figure 6-12. Unlike for the lower modes, the pairs were made against accelerometers located on the second floor, since these pairings demonstrated the building's behavior more clearly. In a third translational mode, the expected motion of a prismatic building is for roughly the bottom and top third of the structure to move in unison, while the middle third moves in the opposite direction. This forms two nodal points in the resulting mode shape at the intersections of the three regions. This mode shape can be glimpsed at 8.25 Hz, as shown in Figure 6-11. At this frequency, accelerometer 20 has positive coherence with accelerometers 32, 29, 17, and 14 and negative coherence with 23 and 26. In terms of floor levels, this means that the ground floor to floor 7 and floors 18 up to the roof are moving together, while the two cross-overs occur somewhere between floors 7 and 18. A similar pattern is repeated in the EW coherences (Figure 6-12), where the separation is even clearer. At 8.1 Hz, accelerometer 19 is strongly coherent with channels 31, 28, and 16, and it has strong negative coherence with the remaining channels. This indicates synchronous motion between floors 6, 7, 18, and the roof and opposite motion with the other floors. In both EW and NS cases, these motions correspond to the mode shape expected for third translational modes. While the coherence spectra seems to suggest other frequencies have motions similar to those of normal modes, only the 8.1 and 8.25 Hz frequencies are strongly and consistently represented in the Fourier and power spectra.

In comparing the frequency spectra of various excitations, the 8.1 Hz EW frequency generally appears more often than its 8.25 Hz NS counterpart. The majority of the NS motion of the Green Building seems to be carried by lower translational and torsional modes, as can be seen in preceding figures. Furthermore, the 8.1 Hz EW mode is most detectable towards the top of the building. Finally, motions at the 8.1 Hz mode seem to be strongly mobilized only by certain excitations. As seen in the full set of Fourier and Power spectra, while the 8.1 Hz mode is present in all recordings, only the May 14 event consistently excites this EW mode. Since this frequency represents a higher mode, it is expected that higher energies and specific source characteristics will be conducive to exciting it. Its strong occurrence in the May 14 event may be due to the excitation source's frequency content, energy content, and directionality. As noted in Section 5.3, the May 14 source is speculated to have been located close to the building's eastern façade. Like the second modes, the third EW and NS modes also stack in the same order as the fundamental modes. Their ratios with the fundamental modes are 11.9 and 11, respectively.
Figure 6-10: Power spectra of accelerations occurring during the May 14 event. The top row shows recordings in the EW (left) and NS (right) directions on the eastern side of the roof level, while the bottom row shows the same plots at floor 12.
Figure 6-11: Detail of coherences of NS oriented accelerometers paired against the floor 7 sensor; from the May 14 event. Note coherence distribution around 8.25 Hz

Figure 6-12: Detail of coherences of EW oriented accelerometers paired against the floor 7 sensor; from the May 14 event. Note separation of coherences around 8.2 Hz
6.2 Foundation Motion

Base rocking behavior was determined by examining the frequency spectra, coherences, and phase angles of the vertically aligned accelerometers located in the basement level of the Green Building (Figure 6-13). Base rocking is a form of soil-structure interaction, where the foundation of a building undergoes a rigid-body rotation. In the case of the Green Building, this is accompanied by pile uplift and sinking. A depiction of this motion is found in Figure 6-14. A variation (often coupled) of this phenomenon is base flexure, where differential motions in the building cause the foundation to bend. These behaviors are functions of the site's geotechnical layering as well as the building's foundation and structural design.

![Diagram of foundation motion](image)

Figure 6-13: Distribution of vertical accelerometers in the Green Building's basement level

![Diagram of Green Building](image)

Figure 6-14: Base Rocking in the Green Building; along the NS and EW axis (left and center). Rocking consists of a rigid-body rotation (exaggerated) and pile uplift and sinking (right)
Since differential base motions are expected to be coupled with the building's bending and translational modes, the behavior of the vertical accelerometers was scrutinized around the lower natural frequencies identified in Section 6.1, above. The Fourier spectra of the four vertical base accelerometers are found in Figure 6-15. Peaks at 0.68, 0.75, and 1.45 Hz, corresponding to the fundamental EW, NS, and torsional modes, respectively are clearly identifiable. This confirms some coupling of base motions with the classic modes. Figure 6-16 presents the coherence between the vertical accelerometers in the 0-10 Hz range. Examining these values at the 0.75 Hz NS mode, it can be seen that accelerometers located on the same North/South façade of the building (3-5 and 4-6) are positively coherent, while those on opposite or diagonal sides (3-4, 5-6, 3-6, and 4-5) have negative coherence. This implies that sensor pairs on the same North/South side are moving vertically together, while those on opposite sides are moving in opposite directions. Such motions correspond to a rotation of the base in the NS direction (about the building’s EW axis) which is coupled with the fundamental NS mode.

Using this same approach, the base behavior at 0.68 and 1.45 Hz can be examined. A detail of Figure 6-16 is shown in Figure 6-17. At 0.68 Hz, a similar pattern to that of 0.75 Hz emerges. Accelerometers on the same East or West façade of the building (3-4 and 5-6) have positive coherence, while those on opposite or diagonal sides (3-5, 4-6, 3-6, and 4-5) are negatively coherent. However, the coherence values are not as high as those for NS motions. While there is some base rocking about the NS axis associated with the EW fundamental mode, the coupling and motion is not very strong. This corresponds to expectations, since the EW axis is in the building’s long direction, and significant base rocking would need to displace the foundation piles larger distances. Examining coherences at the 1.45 Hz torsional mode, accelerometers on opposite diagonals of the base (3-6 and 4-5) are highly coherent while all others have strong negative coherence. This motion—synchronous movement of opposite corners—is expected of the torsional response in prismatic buildings. Its presence in the vertical sensors indicates some foundation flexure coupled with the fundamental torsional mode. The strength of the coherences also suggests a very powerful torsional response of the building, a feature which will be covered in detail in Section 6.3.
Figure 6-15: Fourier frequency spectra of the vertical base accelerometers subjected to the 4th of July fireworks show excitation
Figure 6-16: Coherence between vertically-aligned accelerometers in the basement of the Green Building subjected to the 4th of July fireworks show

Examinaing the coherence spectrum in Figure 6-16 at the higher modes, base-rocking behavior also seems to occur close to the 2nd and 3rd EW modes, at 2.45 and 8.1 Hz, respectively. These coherence
peaks are very narrow, and the amplitude of this rocking is unknown. Both of these will be studied in future work. Base rocking is noticeably absent in the higher NS and torsional modes. Why the foundation should prefer to rock in the EW direction over other directions—particularly at higher frequencies—is unclear, but it could be related to the as-built conditions of the foundation piles, which is detailed in the paragraph below. Another peculiarity of base motion observed from the coherence and frequency data is the lack of clear vertical motions. The coherence spectrum of any of the collected data sets does not show a region of strong positive coherence which would be indicative of a vertical translational mode. Regions of positively-trending coherence do exist in the 3-5 Hz and 8.5-9.5 Hz intervals, but these are not sufficiently coherent to conclude synchronous vertical motion. Also, no single peak frequency corresponding to a fundamental vertical mode was observed.

The peak vertical movements of the accelerometers also indicate an asymmetry between the instrumented corners. The peak displacements of the vertical stations in response to ambient, firework, and 5/14 event excitations are presented in Table 6-3. Channel 5, at the foundation’s south-west corner, consistently experiences the highest displacements. Since a tunnel connection is at that location, this can be the result of poor sensor mounting coupled with foot and maintenance traffic causing more vibration noise than at the other locations, thus leading to higher reported motion. It is also possible that the movement is indeed structural. This could be due to an issue with the tunnel connection, or it could stem from the large number of piles of the south-west group being different than their counterparts. The as-built foundation plans label several piles in this corner as either “lost” or having a “torn shell buried”. It is assumed that these notes indicate deviations in quality which may ultimately increase overall movement at that location.

<table>
<thead>
<tr>
<th>6/22 Ambient Vibration</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fireworks</td>
<td>1.13E-06</td>
<td>1.15E-06</td>
<td>1.56E-06</td>
<td>1.20E-06</td>
</tr>
<tr>
<td>5/14 Event</td>
<td>9.37E-07</td>
<td>5.88E-07</td>
<td>1.57E-06</td>
<td>5.78E-07</td>
</tr>
<tr>
<td>10/16 Earthquake</td>
<td>4.18E-05</td>
<td>3.08E-05</td>
<td>3.77E-05</td>
<td>3.41E-05</td>
</tr>
</tbody>
</table>

Table 6-3: Peak displacements at the vertical accelerometers

6.3 Torsional Behavior

Based on the frequency spectra of recorded events, the Green Building was found to consistently exhibit strong torsional behavior. As can be observed in the Fourier and power spectra presented earlier in
this chapter, the 1.45 Hz peak associated with the first torsional mode, and to a lesser extent the 5.05 Hz peak corresponding to the second torsional mode, makes a large and consistent contribution to the building’s motion and its frequency content—particularly in the spectra of NS oriented accelerometers. These peaks often rival those of the translational modes. As mentioned in Section 6.2 above, torsional motion is also coupled with base flexure at the foundation level. Torsional behavior of the building is ubiquitous across the various excitations, times, and locations within the Green Building. An example of this is found in Figure 6-18, which shows the cumulative spectrum power of accelerations at the floor 12 during ambient vibrations and the May 14 event. The torsional frequencies—1.45 and 5.05 Hz—are associated with the largest jumps in power content of the recorded motions. A more dramatic example of the contribution of torsional motion is presented in Figure 6-19, which depicts the absolute majority of spectral power at floor 19 to be in the fundamental torsional mode.

Figure 6-18: Cumulative spectral power at floor 12 during June 22 ambient vibrations (left) and the May 14 event (right). Note the steep rises around 1.45 Hz and 5.05 Hz.
The torsional behavior of the Green Building can be accounted for by a number of features. The most prominent of these is an asymmetry between the eastern and western sides of the building. All three elevator shafts and their associated void space are located on the eastern side, while the two stairwells are placed in the northwest and northeast corners. Furthermore, heavy meteorological research equipment is unevenly distributed on the roof. These asymmetries encourage differential twisting movement between the eastern and western sides. A significant structural discontinuity also occurs between the first and ground floors. Here, the densely placed column lines terminate and transition to four large columns and a void area which runs through the building along the north-south axis. This discontinuity causes a local decrease in structural stiffness at the ground floor. A visual analysis of the building motions has shown that this region is especially prone to twisting, and it may be what drives the torsional behavior of the entire structure. As noted in Section 6.2, there are also foundation asymmetries in pile quality and distribution, which may encourage twisting motions. Moreover, the fundamental frequency of the surrounding ground is about 1.5 Hz—very nearly the fundamental torsional frequency of the Green Building. This may help amplify the Green Building’s twisting motion and make it more prominent in the frequency spectra.

6.4 East-West Asymmetries

Another related feature observed in the collected data was an asymmetry in the dynamic behavior of the eastern and western sides of the Green Building. This feature was found by comparing the motions of parallel pairs of NS oriented accelerometers located on the same floors. The eastern side was found to
generally undergo higher displacements and accelerations. The eastern side signals also had a higher noise floor. These patterns were observed over the height of the building, beginning at the second floor, and in response to all recorded events. Figure 6-20 shows the time histories at the eastern and western sides of floor 12, subjected to ambient vibrations and the Hollis Center earthquake. The eastern side has appreciably higher accelerations due to both excitations. Figure 6-21 superimposes the noise envelope from ambient vibrations at floor 18. Again, the higher noise levels on the eastern side are apparent. Higher ambient noise levels are also found in EW oriented accelerometers compared to their NS counterparts. The noise levels recorded for individual accelerometers have a consistent magnitude across different times and excitation events. That is, for the same accelerometer, the noise level was observed to be the same in each recording. Finally, Table 6-4 compares the peak accelerations between the eastern and western sides of the Green Building at floor 18, subjected to ambient vibrations, the May 14 event, and the Hollis Center earthquake. During the May 14 event, the eastern side experienced accelerations 1.42 times higher than the western side. With these differences in motion, the east-west asymmetry could potentially be the source of structural damage modes, given a sufficiently energetic excitation. These inconsistencies in the dynamic behavior of the eastern and western sides are most likely the result of the same features which cause strong torsional behavior and which are detailed in Section 6.3, above.
Figure 6-20: Accelerations from ambient vibrations (top) and the Hollis Center earthquake (bottom) at floor 12. Recordings from NS oriented accelerometers on the East side (left) and West side (right).
Figure 6-21: Noise levels at floor 18 due to ambient excitation. Comparison between NS oriented accelerometers located at opposite sides of the floor (left) and between NS and EW oriented accelerometers at the eastern side.

Table 6-4: Comparison of peak accelerations between parallel NS oriented accelerometers at floor 18 due to various excitations

<table>
<thead>
<tr>
<th></th>
<th>East Side (m/s²)</th>
<th>West Side (m/s²)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/22 Ambient</td>
<td>0.0077</td>
<td>0.0055</td>
<td>1.39</td>
</tr>
<tr>
<td>May 14 Event</td>
<td>0.0457</td>
<td>0.0322</td>
<td>1.42</td>
</tr>
<tr>
<td>Hollis Center Earthquake</td>
<td>0.0582</td>
<td>0.0420</td>
<td>1.39</td>
</tr>
</tbody>
</table>

6.5 Summary

The acceleration data collected using the accelerometer array was analyzed to determine the vibrational characteristics of the Green Building. The frequencies of eight normal vibrational modes were identified, including EW modes at 0.68 Hz, 2.45 Hz, and 8.1 Hz; NS modes at 0.75 Hz, 2.85 Hz, and 8.25 Hz; and two torsional modes at 1.45 Hz and 5.05 Hz. For each mode, these frequencies stack in the order of EW, NS, and torsion. The second and third modes are at ratios of about 3.6 and 11.5 relative to the fundamental modes. No vertical translational modes were observed. However, the building was found to significantly experience soil-structure interaction effects called base rocking and base flexure. Base rocking was coupled strongly with all three fundamental modes, and it was also associated with higher EW frequencies. Base flexure was observed to occur with the torsional modes. The base motion behavior is thought to be due to the geotechnical characteristics of the site and asymmetries in the Green Building’s pile foundation.
In general, the Green Building was observed to undergo strong torsional movement and to exhibit slightly different motions at its eastern and western ends. Torsional frequencies were represented in all the collected data sets, and their power often rivaled that of the translational modes. The eastern side of the building experienced higher accelerations and higher ambient noise levels. Both the strong torsional response and the differences in east-west behavior are thought to result from structural asymmetries, uneven dead load distribution (in the form of roof-mounted meteorological equipment and other machinery), and irregularities in the pile foundation. The ground’s fundamental frequency of 1.5 Hz—which is close to the building’s fundamental torsional frequency of 1.45 Hz—also probably amplifies twisting motion.

These frequencies and vibrational characteristics will be used in the development of predictive numerical modes of the Green Building, in further analysis and identification of the Green Building’s structural system, and as a base-line for future damage-detection and seismic analysis of the structure.
7 Green Building Modeling

Two numerical models of the Green Building were developed to perform predictive simulations of dynamic behavior. The models are capable of computing natural frequencies, mode shapes, floor displacements, floor accelerations, and inter-story drift. They will be used to predict Green Building responses to various excitations and damage scenarios, to extrapolate motion data from the accelerometer network, and to perform an assessment of seismic performance. The models will also generate synthetic data sets for the testing of sensor placement optimization algorithms, data inferencing algorithms, and damage detection methodologies created by the CSAIL and the CEE research groups. These models consist of a lumped-mass stick model, in which each floor is condensed to a 6 degree of freedom mass, and a physically realistic mixed element beam-shell finite element model, which was developed using commercial software. Both models were tuned and validated against frequency and vibrational characteristics identified using the collected accelerometer data. The lumped mass and beam-shell models are covered in greater detail in Sections 7.2 and 7.3, respectively. In order to create these models, structural and geotechnical plans was first interpreted and converted into a physically realistic, 3-dimensional CAD solid model—a process which is summarized in Section 7.1, below.

7.1 Geometric Solid Model

To create accurate numerical models, information about the structure design of the Green Building was required. This structural and layout information was gathered by consulting the original, hand-drawn design documents, dating from 1962. These are kept by the Archives of MIT’s Department of Facilities, but they can be viewed by members of the MIT community upon request. The particular documents consulted for modeling purposes were electronic scanned copies of the original design packages, and they were provided by Professor Oral Buyukozturk of the MIT Civil and Environmental Engineering Department. The structural and geotechnical design documents were also used in the interpretation of vibrational features found in the accelerometer data, as mentioned in Sections 6.3 and 6.4.

Due to the poor quality of the scanned documents, select floorplans were first transcribed into CAD drawings using AutoCAD. Information from these and the original drawings was then used to produce a physically realistic, part-based 3D solid model, again using AutoCAD. The model contained most of the documented structural features, including exterior walls, floor slabs, beams, columns, structural posts, foundation walls, and pile caps. Excluded features included stairs, roof-mounted equipment, foundation piles, and the MEP infrastructure. All interior walls were assumed to be non-structural and hence were omitted from the model. Also ignored were material inhomogeneities, such as
reinforcing steel in the concrete. Several renderings of the full building model are pictured in Figure 7-1. Renderings of the individual floors models can be found in Appendix E.

![Figure 7-1: 3D solid model of the Green Building; (left) view from above; (center) view from below; (right) with shear walls and floor slabs removed](image)

For use in the beam-shell finite element model, the entire solid model was converted into a system of wireframes (representing beams and columns) and 2D regions (representing shells), as outlined in Section 7.3. In the case of the lumped-mass model, individual solid floor models were converted into beam-shell models as described in Section 7.2.5, and these were then used to find the equivalent beam stiffness matrices as detailed in Section 7.2.4. The solid floor models were also used to compute total floor masses (see Section 7.2.6 for more details). In general, the advantage of the solid models lay in allowing for the rapid creation of various finite element models and for rapid propagation of changes in the models themselves.

### 7.2 Lumped Mass Stick Model

#### 7.2.1 Overview

The lumped-mass stick model is an idealized finite element model of a structure, where a building is represented as a series of point masses connected by a network of equivalent beam and column elements which capture the stiffness properties of the structure. Lumped-mass stick models have the
advantage of providing fairly accurate information about floor accelerations, displacements, and inter-
story drifts while being computationally inexpensive and conceptually simple to implement. This makes
them attractive choices for estimating dynamic structural responses due to seismic events and other
excitations. The corresponding disadvantage is that the high degree of idealization limits the accuracy of
these models when representing complex structures. These models also require an accurate (and
sometimes non-trivial) condensation of a real structure’s stiffness properties. Furthermore, the behavior
predicted by stick-models is true in aggregate, so information about what happens at particular areas of
the structure is not determined.

In the case of the Green Building, the mass of each story was lumped to a point at the center of
the floor slab. Each floor mass was connected by a single vertical equivalent column element, which
approximated the stiffness properties of the actual connecting structural components. The foundation was
modeled as a clamped boundary condition—no translations and no rotations—at the ground level. Thus,
soil-structure interaction effects and base rocking were not measurable with the model. However, the
boundary can also be modeled with a spring system to better represent the effects of foundation and soil
properties. Each floor mass admitted six degrees of freedom—three translations and three rotations in the
Cartesian coordinate system. Thus, the connecting beam elements admit 12 degrees of freedom (6 at each
end). A diagram of the model idealization is shown in Figure 7-2.

![Green Building Diagram](image-url)

Figure 7-2: Lumpung of Green Building properties for the lumped-mass stick model. The mass of each floor is
modeled as a point mass (represented on the right as a circle with an ‘m#' annotation), while the story
stiffness are modeled as equivalent beams (represented on the right by the ‘k#’ annotated lines connecting the masses).

The lumped-mass stick model (henceforth referred to as the lumped-mass model) predicts structural deformations by solving the classical dynamic equation of motion (Equation (7.1)). Here, M, C, and K represent the mass, damping, and stiffness matrices for the assembled model; P(t) is a vector of time-dependent loads applied to the appropriate degrees of freedom; and u, \( \dot{u} \), and \( \ddot{u} \) are vectors of displacement, velocity, and acceleration at some given time. To arrive at the modal frequencies and mode shapes of the lumped mass model, the eigenvalue problem (Equation (7.2)) is solved, where \( \omega_i \) is the modal frequency (in rad/s) and \( \phi_i \) is a vector representing the mode shape for mode i. As with most structural models, loadings can be applied either through defining the load vector P(t) or through prescribing displacements.

\[
M\ddot{u} + C\dot{u} + Ku = P(t) \tag{7.1}
\]

\[
(K - \omega_i^2 M)\phi_i = 0 \tag{7.2}
\]

For seismic excitations, a support motion, such as a ground acceleration \( \ddot{u}_g \), is typically defined. Since relative displacements are the quantities of interest for estimating structural damage, it is helpful to solve Equation (7.1) by defining the equations of motion in terms of relative displacements (Equation (7.3)) and applying the ground acceleration as an effective inertial force \( P_{eff} \) at the appropriate degrees of freedom. Here, \( \zeta \) is the influence vector, which applies the effective inertial force only to degrees of freedom which are in the direction of excitation. The resulting equation of motion is found in Equation (7.4) below. The seismic excitation problem may also be solved in terms of absolute displacements. In this case, a correction term of \( \zeta u_g \) must be applied to the displacement and velocity terms, which results in Equation (7.5).

\[
u_r = u_{total} - \zeta u_g \tag{7.3}
\]

\[
M\ddot{u}_r + C\dot{u}_r + Ku_r = P_{eff} = M\zeta\ddot{u}_g \tag{7.4}
\]

\[
M\ddot{u}_{total} + C(\ddot{u}_{total} - \zeta u_g) + K(u_{total} - \zeta u_g) = 0 \tag{7.5}
\]

The Green Building lumped-mass model was coded in MATLAB and used the Rayleigh damping model and the Newmark-\( \beta \) numerical integration method, both of which are summarized in Sections 7.2.2 and 7.2.3, respectively. The global stiffnesses of individual floors were computed with Autodesk
Simulation Multiphysics (Section 7.2.5) and condensed to equivalent columns by the procedure detailed in Section 7.2.4. Seismic support excitations were applied to the model using Equation (7.4) above.

### 7.2.2 System Damping

In reality, system damping is a function of many variables—including motion magnitude, frequency, material properties, and behavior of individual building components—all of which make structural damping difficult to precisely determine. To estimate the system damping matrix for the lumped mass model, Rayleigh damping was assumed. Rayleigh damping is a commonly used classical (and proportional) damping model which has been shown to have acceptable agreement with experimental damping measurements in the elastic material regime. It is a superposition of mass and stiffness proportional damping and a function of frequency and some known (or estimated) damping ratios. The equation for the Rayleigh damping matrix is shown in Equation (7.6). The coefficients $a_0$ and $a_1$ (Equation (7.7)) are functions of user-specified frequencies and damping ratios. The frequencies ($\omega_1$ and $\omega_2$) are upper and lower bounds of the frequency range representative of the system being computed. The corresponding damping ratios ($\zeta_1$ and $\zeta_2$) can either be estimated or acquired through more rigorous methods (see below). For simplicity, it can be assumed that the damping ratios are equal for both frequencies ($\zeta_1 = \zeta_2 = \zeta$), in which case the expressions for $a_0$ and $a_1$ reduce to those in Equation (7.8).

Given some $a_0$ and $a_1$, the damping ratio as a function of frequency can be computed using Equation (7.9). A plot of this damping ratio for the lumped mass model is found below in Figure 7-3.

$$C = a_0 M + a_1 K$$

$$a_0 = \frac{2\omega_1 \omega_2 (\zeta_2 \omega_1 - \zeta_1 \omega_2)}{(\omega_1 + \omega_2)(\omega_1 - \omega_2)} \quad a_1 = \frac{2(\zeta_1 \omega_1 - \zeta_2 \omega_2)}{(\omega_1 + \omega_2)(\omega_1 - \omega_2)}$$

$$a_0 = \frac{2\zeta \omega_1 \omega_2}{\omega_1 + \omega_2} \quad a_1 = \frac{2\zeta}{\omega_1 + \omega_2}$$

$$\zeta(\omega) = \frac{a_0}{2\omega} + \frac{a_1 \omega}{2}$$

The damping ratios can be estimated through experimental vibration tests, from measured accelerations, by system identification, through comparison with existing data from similar structures, by referencing common design values, or through a mix of these approaches combined with engineering judgment. Rigorous experimental vibration tests are often impractical, so in practice the last four options are commonly used. For the Green Building lumped mass model, a constant damping ratio of 0.03 for $f_1 =$...
0.45 Hz and \( f_2 = 5.00 \) Hz was used. These frequencies correspond roughly to the model’s 1st East-West translational and 2nd torsional modes. Given the frequency spectra from the accelerometer array and the applied excitations, higher modes where not expected to be strongly represented. The damping ratio was estimated based on tabulated values in (Chopra, 2007) and the results from an auto-regressive extra input (ARX) system identification model of the Green Building created by (Celebi, et al., 2013). A plot of the modal damping ratio as a function of the frequency is found in Figure 7-3.

![Damping Ratio vs. Frequency for the Green Building Lumped Mass Model](image)

**Figure 7-3**: The modal damping ratio as a function of frequency for the Green Building lumped mass model; *(left)* the damping ratio at low frequencies; *(right)* damping ratios at high frequencies

A serious limitation of Rayleigh damping is that modes inside the frequency interval of \( f_1 \) to \( f_2 \) (or \( \omega_1 \) to \( \omega_2 \)) tend to be slightly under-damped, while the damping outside this range increases proportionately with frequency, with the contribution of higher modes being practically eliminated. This behavior can be seen in Figure 7-3 above, and it underscores the importance of an appropriately chosen frequency interval and corresponding damping ratio. As mentioned above, based on the natural frequencies of the lumped mass model and the frequency and energy content observed in the Green Building, strong higher-mode contributions are not expected, and the applied frequency interval is assumed to be acceptable.

### 7.2.3 Numerical Integration

The lumped-mass model solves the equations of motion in the time domain using the Newmark-\( \beta \) method—a commonly used implicit numerical integration algorithm. This method relies on two integration parameters, \( \alpha \) and \( \delta \), to ensure stability and accuracy. For values of \( \delta \geq 0.5 \) and \( \alpha \geq 0.25(0.5+\delta)^2 \) the Newmark-\( \beta \) method becomes unconditionally stable. For the lumped-mass model, values of \( \delta = 0.5 \) and \( \alpha = 0.25 \) were chosen. These values reduced the Newmark-\( \beta \) method to the constant average acceleration, or trapezoidal, method. The full Newmark-\( \beta \) algorithm can be found in many texts,
including (Bathe, 2006). Since the method is unconditionally stable, the choice of timestep \((\Delta t)\) depends only on accuracy considerations. Since generally only a select number of normal modes contribute to a structure’s dynamic response, the minimum timestep can be estimated using Equation (7.10), where \(f_u\) is the frequency of the highest expected contributing mode. In the case of the Green building, the contributing modes are not expected to be higher than 10 Hz, which gives a minimum timestep of 0.0025 seconds. In both the lumped-mass and beam-shell models, timesteps of 0.001 seconds were used for additional accuracy. This temporal sampling interval is five times higher than that used in the deployed accelerometer array.

\[
\Delta t_{\text{min}} = \frac{1}{40f_u}
\]  

(7.10)

### 7.2.4 Stiffness Computation

As mentioned in Section 7.2.1, the global lumped-mass model is assembled from a collection of equivalent, 12 degree of freedom (6 degrees at each end) beam elements—shown in Figure 7-4—whose properties are representative of individual floors in the Green Building. The full 12x12 stiffness matrix of an element can be determined from any of the 6x6 sub-matrices by applying a rigid body translation and making the appropriate geometric transformations (Kausel, 2012a). Given the prismatic element and stiffness matrix in Figure 7-4, a vertical rigid body translation can be defined in which the displacement at point A is written in terms of the displacement at point B. The two are related by a transformation matrix \(T\), given in Equation (7.11), where \(L\) is the element length. This relation can then be substituted into the stiffness equation (Equation (7.12)). Here, the internal force vector is zero since the displacements are rigid body motions. Using Equation (7.12), the relations between the stiffness sub-matrices can be derived in terms of a single sub-matrix—here taken to be \(K_{AA}\). These relations are summarized in Equation (7.13). By determining only a floor’s \(K_{AA}\), the full beam element matrix can be found. With the element matrices, the global structural stiffness matrix can be assembled using the standard method.

\[
A \rightarrow M/2, k(6x6)
\]

\[
B \rightarrow M/2, k(6x6)
\]

**Figure 7-4: Green Building floor with equivalent beam element**
\[ u_A = Tu \]
\[ T = \begin{bmatrix}
1 & 0 & 0 & 0 & -L & 0 \\
0 & 1 & 0 & L & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \]

\[ \{F\} = [K]\{u\} \rightarrow \{F_A\} = \begin{bmatrix} K_{AA} & K_{AB} \end{bmatrix} \{u_A\} \rightarrow \{0\} = \begin{bmatrix} K_{AA} & K_{AB} \end{bmatrix} \{T\} \]

\[ K_{BA} = -K_{AA}T \quad K_{BA} = -T^T K_{AA} \quad K_{BB} = T^T K_{AA} T \]

To determine \( K_{AA} \), a floor and its corresponding lumped-mass representation are assumed to be fixed at point B, as shown in Figure 7-5. The challenge of modeling a structure as a 1D lumped-mass stick model lies in accurately condensing the distributed stiffness properties of the actual floor down to a 6 degree-of-freedom cantilevered beam. There are three general ways in which this can be accomplished. For all three methods, the motion of the floor is defined relative to a point at the center of each floor slab. The motions of the floor are assumed to be driven by the slab. This assumption results from the floor mass and the degrees of freedom being idealized as acting at center-slab in the lumped-mass stick model. If the floor structure has a relatively simple frame construction, the effective beam stiffness can be found by summing the stiffnesses of individual structural members—with geometric transformations applied as needed to determine rotational stiffnesses (Ghali, et al., 2009). This method was not used due to the relatively complex structure of the Green Building and the greater accuracy afforded by methods 2 and 3. The second method applies the stiffness (or flexibility) method to physically-realistic finite element floor models. Using static analysis, unit displacements (or forces) are applied to the slab degrees of freedom, and a spatial averaging of the resulting reactions (or displacements) yields the effective stiffness matrix. This method was attempted for the Green Building stiffness estimate, but it was ultimately abandoned in favor of the simpler, and less computationally expensive, direct condensation method.
Figure 7-5: Set-up for lumping of mass and stiffness properties of a Green Building floor to an equivalent lumped-mass element

The direct condensation method arrives at the equivalent stiffness by manipulating the global stiffness matrix of the full floor finite element model (Kausel, 2012b). The creation of these models is detailed in Section 7.2.5 below. After extracting the global matrix, all degrees of freedom which are not located on the surface of the floor slab are condensed out. This is done by applying the static condensation method (Ghali, et al., 2009). First, the global stiffness matrix is rearranged such that the slab degrees of freedom are contained within one sub-matrix $K_{SS}$, the remaining degrees of freedom are in another sub-matrix $K_{RR}$, and the cross-terms are in sub-matrices $K_{SR}$ and $K_{RS}$. Using these matrices, the effective slab stiffness is computed with Equation (7.14). The slab matrix must next be condensed to a point on its surface, which is collinear with the center of mass of the entire structure. In the case of the Green Building, this point roughly coincides with the geometric center of each floor slab. To reduce the slab to 6 degrees of freedom at a single point, rigid body motions and their associated geometric transformations are applied. The specific geometric transformations depend on the degrees of freedom carried by the finite elements defining the floor slab—for instance brick elements admitting 3 translational degrees of freedom.

$$K_{Global} = \begin{bmatrix} [K_{SS}] & [K_{SR}] \\ [K_{RS}] & [K_{RR}] \end{bmatrix} \quad K_{Slab} = [K_{SS}] - [K_{SR}][K_{RR}]^{-1}[K_{RS}]$$  \hspace{1cm} (7.14)

As detailed in Section 7.2.5, the individual floor models of the Green Building are composed of beam, shell, and rigid link elements, with the slabs represented by shell elements. While shell elements admit only 5 degrees of freedom per node, they are defined with 6 to insure stability. The 6th degree—corresponding to rotation about an out-of-plane axis—is assigned a fictitious value equal to a twisting coefficient (typically 0.001) multiplied by the lowest bending stiffness of the element. As mentioned below, the addition of rigid links also promotes coupling of the slab with out-of-plane rotational degrees.
of freedom. Due to both these coupling features, the condensed slab matrix is assumed to behave as a system with 6 degrees of freedom per node.

Thus, given a slab with $N$ nodes having 6 degrees of freedom each, a reference point may be chosen which governs the motion of the entire slab. For simplicity, the origin of the local coordinate system of the slab will be located at this reference point. The displacements of that point, given by the vector $u_c$, define 3 translations and 3 rotations about that point. This displacement vector, $u_c$, can be related to the displacements of each node on the slab, $u$, by an appropriate transformation matrix $T$ (Equation (7.15)). The external forces applied at the reference point, $P_c$, can also be related to the nodal forces, $P$, using a similar relation, given in Equation (7.16). For 6 degree of freedom elements, the form of the transformation matrix $T$ is shown in Equation (7.17). Each sub-matrix $T_n$ relates the displacements of the controlling point to those of a particular node $n$, where $x_n, y_n, z_n$ represent the coordinates of node $n$. Using Equations (7.15) and (7.16), the stiffness equation of the system, Equation (7.18), can now be written in terms of the displacements and forces acting at the controlling point. From the stiffness equation, the effective 6x6 beam stiffness matrix of the controlling point can be found with Equation (7.19).

\[ u_c = \{u_{cx}, u_{cy}, u_{cz}, \theta_{cx}, \theta_{cy}, \theta_{cz}\}^T \quad \{u\} = [T] \{u_c\} \]  
\[ P_c = \{F_{cx}, F_{cy}, M_{cz}, M_{cy}, M_{cz}\}^T \quad \{P\} = [T] \{P_c\} \]  
\[ T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} \quad T_n = \begin{bmatrix} 1 & 0 & 0 & 0 & z_n & -y_n \\ 0 & 1 & 0 & -z_n & 0 & x_n \\ 0 & 0 & 1 & y_n & -x_n & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]  
\[ \{P\} = [K] \{u\} \quad \Rightarrow \quad \{P_c\} = [T]^T [K] [T] \{u_c\} \]  
\[ K_{eff} = [T]^T [K] [T] \quad \Rightarrow \quad K_{eff} = [T]^T [K_{slab}] [T] \]  

In summary, the process to determine the element stiffness matrix of a floor in the Green Building is as follows:

1. Extract the global stiffness matrix of a 3D finite element model of a given floor.
2. Condense the global stiffness matrix to the slab degrees of freedom using static condensation (Equation (7.14)).

3. Condense the effective slab stiffness matrix down to a 6x6 stiffness matrix corresponding to a point on the slab surface using geometric transformations (Equations (7.17) and (7.19)).

4. Determine the 12x12 floor stiffness element matrices using the transformations listed in Equation (7.13).

The 12x12 floor element stiffness matrices will ultimately be assembled into the global stiffness matrix of the lumped-mass stick model. The process in depicted graphically in Figure 7-6, below. The 6x6 stiffness matrices for three floors—ground floor, 1st floor, and the typical floor—were calculated, and their values can be found in Appendix G.

\[
K_{global} = \begin{bmatrix} K_{MM} & K_{MS} \\ K_{SM} & K_{SS} \end{bmatrix} \quad K_{slab} = K_{MM} - K_{MS}K_{SS}^{-1}K_{SM} \quad K_{eff} = T^TK_{slab}T
\]

Figure 7-6: The condensation of a floor stiffness matrix of a full finite element model to the 6x6 equivalent stiffness matrix used in the lumped-mass model

7.2.5 Finite Element Floor Models

In order to apply the direct condensation method, finite element models of individual floors were created. This was done using Autodesk Simulation Multiphysics, a commercial finite element software package. The geometric models of the floors were created using the 3D solid models described in Section 7.1 as a basis. These models were collapsed down to systems of 2D regions and lines to represent shell and beam elements, respectively. The slabs and shear walls of each floor were modeled using shell elements, while beams and columns were represented by beam elements. The out-of-plane rotations of the shells were coupled to other degrees of freedom at appropriate locations using rigid link elements. For simplicity, the floor beams were modeled with their axis lying in the plane of the floor slab. Fixed boundary conditions were applied to the bottom vertices of the floor models. A picture of a typical floor finite element model is shown in Figure 7-7. The material applied to the models corresponds to high
strength structural concrete, and its properties are summarized in Table 7-1. The global stiffness matrix of the floor model used in the direct condensation method was acquired by running a model check of the floor in Autodesk Simulation and requesting an output of the resulting stiffness matrix. Three individual floors were used for the Green Building lumped-mass model—the ground floor, 1st floor, and the typical floor. Images of these can be found in Appendix F.

![Finite element floor model of a typical floor in the Green Building. Lines represent beam elements.](image)

Table 7-1: Material properties applied to the solid and FE models of the Green Building. These values correspond to high strength structural concrete

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2400</td>
</tr>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>31</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### 7.2.6 Mass Computation

The global mass matrix of the lumped-mass model is assembled by overlapping the 12x12 mass matrices of each equivalent beam element. As with the stiffness matrix, only the 6x6 mass matrix of an element node is needed to define the full element mass matrix. This 6x6 matrix is found by computing the mass properties of the appropriate floor, and then assuming that each of the two nodes defining the equivalent beam receives one half of these mass properties, as depicted in Figure 7-4, above.
The form of an individual floor mass matrix is found in Equation (7.20), where $m$ is the full floor mass; $J_{xx}$, $J_{yy}$, and $J_{zz}$ are the mass moments of inertia with respect to a point aligned with the center of mass of the building and located on the slab; and $J_{xy}$, $J_{xz}$, and $J_{yz}$ are the mass products of inertia, with respect to the same point. Conceptually, the mass moments of inertia can be thought of as measures of an object’s tendency to rotate about a particular axis, while the mass products are rotational coupling terms which are related to asymmetries in the mass distribution of the object. They represent the tendency of an object to rotate about some axis, given a torque applied along some other axis. Given a set of orthogonal axis, $u$, $v$, and $w$ and a material density $\rho$, the equations for the mass moments and mass products of inertia are given in Equations (7.21) and (7.22), respectively. In practice, the mass products are very small and can be neglected. Since the center of mass of each floor is collinear with that of the entire building, the residual moment terms in the mass matrix go to zero, thus leaving an uncoupled diagonal matrix. The elements of the floor mass matrices were found by loading the solid floor models detailed in Section 7.1 into Autodesk Simulation Multiphysics, applying the material properties in Table 7-1, and executing a mass property computation. The mass properties of the typical floor are summarized in Table 7-2, below, and the properties of all the modeled floors can be found in Appendix H, along with the 6x6 mass matrices used in the lumped-mass model.

$$M_{Floor} = \begin{bmatrix}
m & 0 & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 & 0 \\
0 & 0 & m & 0 & 0 & 0 \\
0 & 0 & 0 & J_{xx} & J_{xy} & J_{xz} \\
0 & 0 & 0 & J_{yx} & J_{yy} & J_{yz} \\
0 & 0 & 0 & J_{zx} & J_{zy} & J_{zz} \\
\end{bmatrix}$$

(7.20)

$$I_{uu} = \int \left(u^2 + w^2\right) \rho dV$$

(7.21)

$$I_{uv} = -\int uv \rho dV$$

(7.22)

Table 7-2: Mass properties of a typical floor

<table>
<thead>
<tr>
<th>Floor Mass (kg)</th>
<th>$J_{xx}$ (kg·m⁴)</th>
<th>$J_{yy}$ (kg·m⁴)</th>
<th>$J_{zz}$ (kg·m⁴)</th>
<th>$J_{xy}$ (kg·m⁴)</th>
<th>$J_{xz}$ (kg·m⁴)</th>
<th>$J_{yz}$ (kg·m⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.22 \times 10^3$</td>
<td>$1.46 \times 10^7$</td>
<td>$5.05 \times 10^7$</td>
<td>$6.33 \times 10^7$</td>
<td>$-1.85 \times 10^6$</td>
<td>$-2.20 \times 10^6$</td>
<td>$-7.62 \times 10^6$</td>
</tr>
</tbody>
</table>
7.2.7 Validation and Refinement

A first-order validation of the model was performed by comparing the predicted natural frequencies and mode shapes with those observed in the collected Green Building data. A comparison between the first six natural frequencies computed by the refined lumped mass model and those observed in the Green Building is found in Table 7-3. The translational frequencies of the model closely matched the real data; however, matching the torsion proved difficult. In the process of model refinement, it was observed that tuning the translational modal frequencies often caused the torsional results to become inaccurate. A reason for this inversely proportional relationship has not been determined. Once acceptable agreement with the measured natural frequencies was achieved, a second-order validation of the model was carried out by subjecting the model to prescribed ground motions from the 10/16 Hollis Center earthquake. The predicted accelerations were then compared to those recorded in the Green Building during the event. Agreement between peak predicted and measured accelerations was found to be acceptable. For comparison, the accelerations at the roof level (NS axis) predicted by the model are found in Figure 7-8, while those measured in the Green Building are presented in Figure 7-9.

<table>
<thead>
<tr>
<th>Table 7-3: Comparison between lumped-mass model modal frequency predictions with those determined using the Green Building accelerometer data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modal Frequencies (Hz)</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1st EW</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Acclerometer Data</td>
</tr>
<tr>
<td>Lumped-Mass Model</td>
</tr>
</tbody>
</table>
Based on comparisons with measured natural frequencies, several iterations of refinement were carried out. This process consisted of creating successively more detailed and physically realistic finite
element floor models and then re-computing the floor stiffness matrices as detailed in Section 7.2.4. In the cases where this did not produce sufficiently satisfactory results, the dimensions of members where modified by small amounts to produce greater frequency agreement. More rigid links were also applied at crucial locations as needed to promote coupling with the out-of-plane degrees of freedom of the shell elements.

7.3 Beam-Shell Model

7.3.1 Overview

The beam-shell model is a fully three-dimensional and physically realistic finite element representation of the Green Building. The extensive tuning required for the lumped-mass model created uncertainty as to whether computed results mimicked the Green Building’s behavior or the model’s artificially tuned parameters. It was hoped that a more detailed and physically representative model would organically predict Green Building motions with higher fidelity. Another advantage of the beam-shell model was that it allowed for motions and stresses to be computed at practically any location within the building. This gave a better understanding of the behavior of monolithic sub-structures such as floor slabs and shear walls. The primary disadvantage of such beam-shell models is that they require a great deal of time and computational resources to perform simulations of complex structures.

Development of the beam-shell model was similar to that of the individual floor finite element models described in Section 7.2.5. The AutoCAD solid model of the Green Building was decomposed and collapsed into a series of lines, representing the beams and columns, and 2D planar regions representing the slabs and shear walls. These geometries were then loaded into Autodesk Simulation, where the model was meshed using beam and shell elements, and the materials properties in Table 7-1 were applied. Quadrilateral shell elements (with triangles used as needed) were used. The element edge lengths were about 0.8 meters. Rigid link elements were placed at appropriate locates to couple the out-of-plane rotations of the shells elements with other degrees of freedom. A clamped boundary condition at the building base was used. Like with the lumped-mass model, it was also possible to specify a boundary system of equivalent springs in order to better simulate the impact of the foundation and soil conditions. The transient solution was computed using modal superposition, for which a critical damping ratio of 0.3 was used. Seismic loadings were applied as prescribed base motions via the software’s default base-excitation feature. Other loading types can be defined as nodal forces. A picture of the full beam-shell model is found in Figure 7-10. Lines represent beam elements and the red triangles at the bottom nodes are fixed boundary conditions.
7.3.2 Validation and Refinement

As with the lumped-mass model, a first-order validation check of the beam-shell model was made by comparing the predicted and measured natural frequencies of the Green Building (Table 7-4). The model shows close agreement with both the first four translational and torsional modes, and it was decided that further refinement was not necessary. For second-order validation, the model was subjected to an acceleration base excitation (through the feature mentioned in Section 7.3.1, above) from the 10/16 Hollis Center earthquake. The agreement between peak predicted and measured displacements were found to be reasonable. A comparison of displacements at the roof level in the NS direction is shown in Figure 7-11. The peak displacement predicted by the model is $1.99 \times 10^{-4}$ meters, as compared to the measured peak value of $2.1 \times 10^{-4}$ meters. This is a difference of about 5.2 percent.

![Figure 7-10: Meshed beam-shell finite element model of the Green Building](image)

Table 7-4: Comparison of beam-shell model modal frequency predictions with those determined using the Green Building accelerometer data

<table>
<thead>
<tr>
<th></th>
<th>Modal Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1^{st}$ EW</td>
</tr>
<tr>
<td>Accelerometer Data</td>
<td>0.68</td>
</tr>
<tr>
<td>Beam-Shell Model</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Irgted Displacements at Roof Level due to Seismic Excitation

Figure 7-11: Comparison between predicted and measured Green Building displacements at roof level (NS axis) due to the Hollis Center Earthquake; (left) as predicted by the beam-shell model and (right) as measured by accelerometers.

While ultimately no refinement was carried out, the planned tuning procedure for the beam-shell model was similar to that used for the lumped-mass model. In order of its application, this process would have included modeling the entire structure with additional details, perturbing member dimensions by small amounts, and applying rigid links at appropriate locations to increase shell element coupling with the other degrees of freedom.
8 Future Work and Applications

Several research tasks are identified for future work. These build upon the findings reported in this thesis, and make use of the installed accelerometer array, the outlined analysis techniques, and the finite element models of the Green Building. These include performing rigorous predicative simulations of seismic and other excitations using the finite element models, performing a sensitivity study of structural changes using these models, continuing the monitoring and analysis of data from the accelerometer array, applying system identification algorithms to determine structural properties, and establishing source identification criteria from the collected data.

The finite element models described in Chapter 7 will be used to simulate the Green Building’s response to various excitations. The goal is to predict the Green Building’s dynamic behavior and, when possible, to compare this against data collected from the accelerometer array. Such comparisons will assist in establishing the accuracy and effectiveness of structural modeling for precise motion predictions. To increase result accuracy, the numerical models will be tuned using the collected data, as necessary. Several types of excitations will be simulated, among them seismic, blast and wind loading. The varieties of blast loading to be applied include airbursts (analogous to fireworks), ground-based blasts, and blasts originating from inside the structure. Wind loading simulations will be carried out for both ambient and extreme events, such as the hurricane-force gusts which occasionally visit Boston. The seismic excitations will be primarily of classic strong-motion events such as Northridge and El Centro. The Green Building’s response to earthquake loading will be particularly enlightening since little is known of the seismic performance of structures in the New England region. Results from these simulations will be used to compute shear, bending, and torsional deformations and inter-story drift ratios. In a separate thrust, a sensitivity analysis will be carried out using the finite element modes. This will consist of varying local material properties, member locations, and member sizing to gain an understanding of how these properties affect the vibrational behavior of the structure.

The accelerometer system will remain deployed for the conceivable future, and it will continue to be used for detection of special events and for on-demand recordings of specific excitations. Excitations which will be recorded in the future include passing subway trains, elevators moving within the building, vehicle traffic on the nearby Memorial Drive, and construction activity in the surrounding area. Recorded events will continue to be analyzed using the methods outlined here, with the intent of identifying more normal modes and structural characteristics. More advanced analysis methods will also be applied, including Operational Deflection Shape (ODS) analysis for the estimation of mode shapes and Experimental Modal Analysis (EMA) for determining modal frequencies, shapes, and damping ratios. Classical system identification algorithms, such as autoregressive extra input (ARX), autoregressive...
moving average (ARMA), and their variants will be used to determine system damping parameters. Through continued monitoring, it is hoped that changes in the Green Building’s dynamic behavior—and therefore structural properties—can be tracked over time. The recorded motions will be used to compute shear, bending, and torsional deformations and to estimate inter-story drifts. The signals will also be analyzed to better understand wave propagation within the structure and to identify paths of mechanical energy flow. Finally, further signal analysis will be carried out to establish defining properties of the recorded excitations. The goal is use data from instrumented structures like the Green Building to detect, locate, and correctly identify sources of vibration in the surrounding environment.

The applications of the work summarized in this thesis are threefold. Firstly, it serves as a demonstration of methods for characterizing and predicting the dynamics of existing structures. The observed parameters can be applied to assess the seismic response of other similar structures in the New England region. Perhaps most importantly, it serves as a jumping-off point for characterization of structural damage and for the development of new damage detection methods. For instance, damage to structures manifests itself as changes in the frequency and time domain behavior of a building. By characterizing these behaviors, the healthy base-line case of the Green Building has been identified. Through continued monitoring, it can be shown that changes to the structure will result in detectable shifts in the building’s frequency response and possible alterations to normal dynamic motions. Furthermore, the excitations recorded by the sensor array can be coupled with the structural models of the Green Building to predict possible structural damage locations resulting from the recorded, real-world loadings. Comparison of numerically predicted and recorded motions can also suggest the presence and location of damage. Such model and sensor based techniques for damage detection can be applied to a variety of structures—from high-rise buildings to oil platforms. Numerous other damage detection approaches are possible using sensor systems and computational models as a starting point. Among them are wave propagation and path-based methods. These two properties will be the immediate next focus of future work.
9 Conclusions

A study of the dynamic behavior of the Green Building—a 20 story academic building on the campus of the Massachusetts Institute of Technology—was carried out. An array of structural health monitoring accelerometers installed in the building by the USGS was used to collect structural accelerations resulting from various events, including ambient vibrations, Boston’s 4th of July fireworks show, the October 16, 2012 earthquake near Hollis Center, Maine, and an unidentified blast-type event from May 14, 2012.

The data was analyzed using spectral analysis methods. From this, the building’s vibrational and structural characteristics were determined. The frequencies and general mode shapes of eight normal modes were identified. The frequencies were at 0.68 Hz, 2.45 Hz, and 8.10 Hz in the east-west direction; 0.75 Hz, 2.85 Hz, and 8.25 Hz in the north-south direction; and 1.45 Hz and 5.05 Hz in torsion. No clear vertical translational modes were found. Strong base-rocking motion—a form of soil-structure interaction—was observed, and it was coupled with all the fundamental modes as well as the higher east-west modes. This motion is rarely observed under ambient, low-amplitude excitations. The Green Building was also found to undergo strong torsional motions and to have asymmetric responses between its eastern and western sides. The eastern end of the building had consistently greater accelerations and ambient noise levels. The base rocking behavior is likely due to the geotechnical characteristics of the site and the irregular placement and quality of foundation piles. Both torsion and the east-west motion asymmetry are thought to result from structural asymmetries, uneven dead-load distribution, and irregularities in the pile foundation. Torsion is probably amplified by similarities in the fundamental frequencies of the structure and of the surrounding ground—1.45 Hz and 1.5 Hz, respectively.

Two finite element models of the Green Building—a lumped-mass stick model and a mixed element beam-shell model—were developed using the original design documents. The models were validated and tuned using the measured natural frequencies and dynamic characteristics. The models reported fundamental frequencies of 0.71 Hz, 0.79 Hz, and 3.01 Hz for the lumped-mass model and 0.65 Hz, 0.81 Hz, and 1.26 Hz for the beam-shell model, all of which (except for the lumped-mass torsion) are within 20 percent of the measured values. Initial simulations using the Hollis Center earthquake as an excitation show good agreement for both models, as compared to the recorded accelerometer data.

This study achieved a basic characterization of a unique structure’s vibrational properties using a distributed accelerometer network, and successfully demonstrated the use of numerical models to predict structural motions. It also showed that the bending, shear, and torsional deformations of the Green Building—and by extension other large civil structures—can be accurately detected using similarly designed accelerometer arrays. Using these findings, it will be possible to assess the seismic performance
of the Green Building. Accurate numerical models for the prediction of structural motions and for the generation of synthetic data were also successfully developed. Furthermore, the installed sensor system and established analysis methods will be used as a jumping-off point for future investigations of structural damage detection methodologies. Damage detection capabilities are highly desirable, both for the maintenance of existing critical infrastructure and for providing structural health assessments of special structural systems—such as oil rigs or nuclear facilities—for which conventional inspection methods are costly or difficult.
Appendix A: Ambient Vibration Data

A.1 Excitation Details

Table A-1: Summary of 6/22 ambient vibration recording

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Friday 6/22/2012 at 00:11:16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recording Duration</td>
<td>320 seconds</td>
</tr>
<tr>
<td>Event Duration</td>
<td>NA</td>
</tr>
<tr>
<td>Excitation Type</td>
<td>Ambient</td>
</tr>
<tr>
<td>Weather Conditions</td>
<td>Clear with winds of 1 mph out of the west and gusts of 4 mph</td>
</tr>
<tr>
<td>Seismic Activity</td>
<td>None</td>
</tr>
</tbody>
</table>

More details about this recording can be found in Section 5.2.

Figure A-1: Distribution of accelerometers in the Green Building, labeled by channel number
A.2 Acceleration Time Histories

![Acceleration at Channel 1]

![Acceleration at Channel 2]

![Acceleration at Channel 3]

![Acceleration at Channel 4]

![Acceleration at Channel 5]

![Acceleration at Channel 6]
Appendix B: May 14 Event Data

B.1 Excitation Details

Table B-1: Summary of May 14th event recording

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Monday 5/14/2012 at 02:04:15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recording Duration</td>
<td>150 seconds</td>
</tr>
<tr>
<td>Event Duration</td>
<td>1.4 seconds</td>
</tr>
<tr>
<td>Excitation Type</td>
<td>Unknown. Speculated to be a blast/pressure load</td>
</tr>
<tr>
<td>Weather Conditions</td>
<td>Clear with no sustained winds and gusts of 5 mph out of the south</td>
</tr>
<tr>
<td>Seismic Activity</td>
<td>None</td>
</tr>
</tbody>
</table>

More details about this recording can be found in Section 5.3.

Figure B-1: Distribution of accelerometers in the Green Building, labeled by channel number
B.2 Acceleration Time Histories
Acceleration at Channel 7

Acceleration at Channel 8

Acceleration at Channel 9

Acceleration at Channel 10

Acceleration at Channel 11

Acceleration at Channel 12
Appendix C: July 4th Fireworks Data

C.1 Excitation Details

Table C-1: Summary July 4th fireworks show recording

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Wednesday 7/4/2012 at 22:38:40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recording Duration</td>
<td>510 seconds</td>
</tr>
<tr>
<td>Event Duration</td>
<td>115 seconds</td>
</tr>
<tr>
<td>Excitation Type</td>
<td>Fireworks (blast loading)</td>
</tr>
<tr>
<td>Weather Conditions</td>
<td>Raining and storming with 3 mph winds out of the west and gusts at 9 mph</td>
</tr>
<tr>
<td>Seismic Activity</td>
<td>None</td>
</tr>
</tbody>
</table>

More details about this recording can be found Section 5.4.

Figure C-1: Distribution of accelerometers in the Green Building, labeled by channel number
C.2 Acceleration Time History

Acceleration at Channel 1

Acceleration at Channel 2

Acceleration at Channel 3

Acceleration at Channel 4

Acceleration at Channel 5

Acceleration at Channel 6
Acceleration at Channel 19

Acceleration at Channel 20

Acceleration at Channel 21

Acceleration at Channel 22

Acceleration at Channel 23

Acceleration at Channel 24
Appendix D: Hollis Center Earthquake Data

D.1 Excitation Details

Table D-1: Summary of Hollis Center earthquake recording

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Date and Time</td>
<td>Tuesday 10/16/2012 at 19:12:35</td>
</tr>
<tr>
<td>Recording Duration</td>
<td>155 seconds</td>
</tr>
<tr>
<td>Event Duration</td>
<td>32.5 seconds</td>
</tr>
<tr>
<td>Excitation Type</td>
<td>Earthquake</td>
</tr>
<tr>
<td>Weather Conditions</td>
<td>Clear with negligible wind</td>
</tr>
<tr>
<td>Seismic Activity</td>
<td>4.0 magnitude earthquake</td>
</tr>
<tr>
<td></td>
<td>near Hollis Center, Maine</td>
</tr>
</tbody>
</table>
|                        | about 142 km from Cambridge,
|                        | Massachusetts. The angle of |
|                        | approach was from about 15  |
|                        | degrees north-east.         |

More details about this recording can be found in Section 5.5.

Figure D-1: Distribution of accelerometers in the Green Building, labeled by channel number
D.2 Acceleration Time Histories

Acceleration at Channel 1

Acceleration at Channel 2

Acceleration at Channel 3

Acceleration at Channel 4

Acceleration at Channel 5

Acceleration at Channel 6
Appendix E: Solid Floor Models

Figure E-1: Solid model of the ground floor of the Green Building (top view)

Figure E-2: Solid model of the ground floor of the Green Building (bottom view)
Figure E-3: Solid model of the first floor of the Green Building (top view)

Figure E-4: Solid model of the first floor of the Green Building (bottom view)
Figure E-5: Solid model of a typical floor of the Green Building (top view)

Figure E-6: Solid model of a typical floor of the Green Building (bottom view)
Appendix F: Beam-Shell FE Floor Models

Figure F-1: Finite element model of the Green Building ground floor. Wireframe (left) and 3D visual (right)

Figure F-2: Finite element model of the Green Building first floor. Wireframe (left) and 3D visual (right)

Figure F-3: Finite element model of the Green Building typical floor. Wireframe (left) and 3D visual (right)


Appendix G: Floor Stiffness Matrices

Table G-1: 6x6 equivalent stiffness matrix of the Green Building ground floor for the lumped mass model

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>$\theta_x$</th>
<th>$\theta_y$</th>
<th>$\theta_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2.28E+09</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-8.82E+09</td>
<td>0</td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>9.09E+08</td>
<td>0</td>
<td>3.52E+09</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>0</td>
<td>5.48E+10</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>0</td>
<td>3.52E+09</td>
<td>0</td>
<td>1.41E+12</td>
<td>1.27E+10</td>
<td>-3.89E+06</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>-8.82E+09</td>
<td>0</td>
<td>0</td>
<td>1.27E+10</td>
<td>1.03E+12</td>
<td>-9.70E+05</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-3.89E+06</td>
<td>-9.70E+05</td>
<td>1.91E+12</td>
</tr>
</tbody>
</table>

Table G-2: 6x6 equivalent stiffness matrix of the Green Building first floor for the lumped mass model

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>$\theta_x$</th>
<th>$\theta_y$</th>
<th>$\theta_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>6.01E+09</td>
<td>-2.74E+03</td>
<td>-4.30E+02</td>
<td>-9.38E+02</td>
<td>-4.24E+09</td>
<td>-1.44E+04</td>
</tr>
<tr>
<td>y</td>
<td>-2.74E+03</td>
<td>1.62E+10</td>
<td>-3.05E+02</td>
<td>4.88E+09</td>
<td>1.16E+02</td>
<td>-3.57E+03</td>
</tr>
<tr>
<td>z</td>
<td>-4.30E+02</td>
<td>-3.05E+02</td>
<td>1.40E+11</td>
<td>1.46E+02</td>
<td>2.12E+02</td>
<td>-1.36E+03</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>-9.38E+02</td>
<td>4.88E+09</td>
<td>1.46E+02</td>
<td>2.90E+12</td>
<td>-2.49E+10</td>
<td>-3.25E+04</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>-4.24E+09</td>
<td>1.16E+02</td>
<td>2.12E+02</td>
<td>-2.49E+10</td>
<td>2.27E+12</td>
<td>2.30E+05</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>-1.44E+04</td>
<td>-3.57E+03</td>
<td>-1.36E+03</td>
<td>-3.25E+04</td>
<td>2.30E+05</td>
<td>4.66E+12</td>
</tr>
</tbody>
</table>

Table G-3: 6x6 equivalent stiffness matrix of the Green Building typical floor for the lumped mass model

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>$\theta_x$</th>
<th>$\theta_y$</th>
<th>$\theta_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1.79E+10</td>
<td>-1.01E+02</td>
<td>1.34E+03</td>
<td>2.13E+02</td>
<td>-1.23E+10</td>
<td>6.98E+02</td>
</tr>
<tr>
<td>y</td>
<td>-1.01E+02</td>
<td>3.94E+10</td>
<td>1.08E+03</td>
<td>1.71E+10</td>
<td>2.47E+02</td>
<td>3.98E+03</td>
</tr>
<tr>
<td>z</td>
<td>1.34E+03</td>
<td>1.08E+03</td>
<td>2.60E+11</td>
<td>-9.62E+02</td>
<td>-1.67E+03</td>
<td>-1.65E+02</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>2.13E+02</td>
<td>1.71E+10</td>
<td>-9.62E+02</td>
<td>3.90E+12</td>
<td>4.36E+10</td>
<td>1.61E+04</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>-1.23E+10</td>
<td>2.47E+02</td>
<td>-1.67E+03</td>
<td>4.36E+10</td>
<td>3.21E+12</td>
<td>1.59E+04</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>6.98E+02</td>
<td>3.98E+03</td>
<td>-1.65E+02</td>
<td>1.61E+04</td>
<td>1.59E+04</td>
<td>4.65E+12</td>
</tr>
</tbody>
</table>
Appendix H: Floor Mass Properties

Table H-1: 6x6 mass matrix of the Green Building ground floor for the lumped mass model

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>θx</th>
<th>θy</th>
<th>θz</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>4.22E+05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>4.22E+05</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>0</td>
<td>4.22E+05</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>θx</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.46E+07</td>
<td>0</td>
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</tr>
<tr>
<td>θy</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5.05E+07</td>
<td>0</td>
</tr>
<tr>
<td>θz</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.33E+07</td>
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</tbody>
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Table H-2: 6x6 mass matrix of the Green Building 1st floor for the lumped mass model

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<th>y</th>
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<th>θx</th>
<th>θy</th>
<th>θz</th>
</tr>
</thead>
<tbody>
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<td>x</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y</td>
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<td>7.61E+05</td>
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<td>0</td>
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<tr>
<td>z</td>
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<td>0</td>
<td>7.61E+05</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>θx</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4.26E+07</td>
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<td>0</td>
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<tr>
<td>θy</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1.04E+08</td>
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<td>0</td>
<td>0</td>
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<td>1.22E+08</td>
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</tbody>
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Table H-3: 6x6 mass matrix of the Green Building typical floor for the lumped mass model

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<th>y</th>
<th>z</th>
<th>θx</th>
<th>θy</th>
<th>θz</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y</td>
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<td>7.50E+05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>z</td>
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<td>0</td>
<td>7.50E+05</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1.27E+08</td>
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</tbody>
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References


Kausel, E., 2011. 1.581/2.060 *Advanced Structural Dynamics [Class Notes]*. Cambridge: Massachusetts Institute of Technology.


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