AN ASPECT OF "NEO-CLASSICAL" AND "KEYNESIAN"
THEORIES OF GROWTH AND DISTRIBUTION

by

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I. Introduction

While Keynes was never quite interested in the problem of long-run growth, nor in the problem of income distribution, in any systematic manner, we now have "Keynesian" theories of both. The extension of the Keynesian system in the shape of a "generalization of the General Theory" is due to the works of a number of economists, of whom Mrs. Joan Robinson and Mr. Nicholas Kaldor have been particularly active. Whether these extensions have gone beyond what can be fairly called "Keynesian" is a question on which a paper can be written. However, this is not that paper.

The alternative aggregative approach that has been used quite a bit in the recent years is the approach variously named as "Classical," following Keynes' own terminology, or "Neo-classical," to differentiate it from the works of economists like Ricardo, Malthus, and Marx, or "marginalist," to emphasize the importance of concepts like marginal

*My greatest debt is to Robert Solow for his detailed comments and criticisms. I have also benefited from the comments of Messrs. Richard Eckaus, A. L. Olgaard, and Paul Samuelson.


productivity in this school of thinking. Professor James Meade, to whom we owe the most comprehensive attempt to present the growth element of this approach, seems to prefer the term "Neo-classical" in the title of his book, and the term "Classical" in the Preface of it. We shall use the former, particularly because a number of modern attempts have been made to use the older and more classical models of Smith, Ricardo, Malthus, and Marx, for studying problems of economic growth and development, so that what one means by a "Classical" model is not quite clear. The Neo-classical aggregative models have been developed in recent years by Solow, Haavelmo, Swan, Hicks.


and Meade,9 among others, and also by Tobin,10 in terms of a somewhat wider model. Neo-classical distribution models have had a much longer history, and in some sense the last major work on it is Hicks' The Theory of Wages.11 though of course it has been used in one form or another in most of the modern literature on distribution.

The object of this note is to study the difference between the "Neo-classical" systems on the one hand and the "Keynesian" systems on the other, and to see to what extent they can be combined, and what happens when they are combined. A fair number of controversies have already taken place on this, e.g., between Joan Robinson and Trevor Swan,12 between Luigi Pasinetti and Robert Solow,13 between Ronald Findlay and Nicholas Kaldor,14 between Hiroshi Atsumi and Nicholas

Kaldor,15 and between James Tobin and Nicholas Kaldor.16 It does appear, however, that the issues involved in the debate have not become quite as clear as one would like them to be. While this note does not, perhaps, achieve that objective, it is an attempt in that direction. We shall confine ourselves, in this paper, to the comparison of "Neo-classical" and "Keynesian" approaches to these problems, and not have the occasion to go into the classical systems, which in some respects differ more fundamentally from both of these, than they seem to from each other.

Further, we shall concentrate only on one particular aspect of the controversy, overlooking other equally serious ones. We shall assume (a) that there is a homogeneous capital good, which is perfectly malleable and which does not depreciate (or depreciates by "evaporation," as Professor Meade puts it, i.e., depreciates at a fixed proportional rate per year of the total stock), and (b) at any point of time there is a given production function relating quantities of labor and capital to output (land is free), and a movement along it does not shift it in that period, though of course it affects the shape of the production function in the future. The difficulties with the first assumption Mrs. Robinson has done much to emphasize,17


and those with the second have been pointed out by Nicholas Kaldor.\textsuperscript{18}
Both are genuine difficulties, but not merely do they hit at the root of a number of existing theories, they also make further theorization excessively difficult. Mrs. Joan Robinson can cope with heterogeneous, specific capital goods only in terms of her model of "Golden Age," which rules out all situations excepting those of a very special type of equilibrium. Mr. Nicholas Kaldor has to go ahead with a rigidly given "technological progress function" uniquely relating proportional rates of growth of output with proportional rates of growth of capital, which is difficult to rationalize except in terms of a neutrally shifting Cobb-Douglas function. Besides, Mr. Kaldor does not ask whether a movement along this new function may not shift it, just as a movement along the old-fashioned production function (he claims) moves that. Just as no unambiguous relationship might exist, at any point of time, between the stock of capital (and labor) and of output, the result being dependent on the rate of capital accumulation, similarly there might be no unambiguous relationship between the rate of capital accumulation and the rate of output growth, because the connection might depend upon the rate at which the rate of capital accumulation is speeding up or slowing down. It is easy to construct economic cases where such a dependence can be expected. All this is not to deny the importance of these considerations, for these questions do go at the very root of most economics.

but they are fundamentally very unwieldy problems and need a good deal more work to be fully incorporated in a model of production, distribution, or economic growth.

Even when, however, these problems are ignored there seems to remain a lot of differences between the "Keynesian" and the "Neoclassical" schools, and we shall be concerned with these aspects of the problem for the rest of the paper. We shall first introduce the main differences between the two schools in terms of an extremely simple model, a model like the one put forward by Frank Ramsey,¹⁹ where the homogeneous capital good happens to be indistinguishable from the homogeneous consumer good.

We shall first discuss the problem in terms of the interrelationship between the variables in a period of time. The process of growth can be viewed as a series of such pictures. Many of the differences between the two schools come out clearly in the "static" model.

We first construct a "model" that includes all the behavior postulates that the "Keynesians" and the "Neoclassicists" make, along with taking the definitional identities. The resulting model, as one may expect, is overdetermined. It is, however, useful to pose the problem this way for a preliminary sorting out of the issues involved in this controversy.

II. An Overdetermined System and Ways Out

We assume a given production function that is homogeneous of the first degree. We assume that both the product market and the factor market are perfect. Wages are brought to equality with marginal product, in a Neoclassical fashion, at full employment. We also assume given saving habits of the workers and the entrepreneurs, and also a specific accumulation function of the latter. We have the following set of equations:

\[ Q = T(K, L), \]  \hspace{1cm} (1)

which is the production function (homogeneous of the first degree) with the technological knowledge given at the point of time, relating quantities of capital and labor to the amounts of output.

\[ Q = P + L \cdot w, \]  \hspace{1cm} (2)

which states the identity that profits plus the wages bill equal total product, and that the wages bill equals the level of employment \( L \) multiplied by the wage rate \( w \).

\[ w = \frac{\partial Q}{\partial L}, \]  \hspace{1cm} (3)

which equates the wage rate to the marginal product of labor. Since the production function is homogeneous of the first degree, equations (2) and (3) ensure that the profit rate will equal the marginal product of capital.

\[ I = A (e, \ldots), \]
which is the investment function relating investment to profit expectations (e), and also among other things, to what Keynes called "the animal spirits" of the entrepreneurs. There are various ways of formulating the investment function, and different Keynesians do it differently, but as long as it is an independent function, not equated automatically to the level of expected savings, the problem remains the same. We put it below in its simplest form, viz., I is equal to an independently determined amount A

\[ I = A \]  

(4)

We shall discuss this function in greater detail later, and see how the withdrawal of the rigid independence assumption affects our results.

20. "In short," concludes John R. Meyer and Edwin Muh, the authors of perhaps the most important recent empirical study on the investment function, "there is a varying amount of empirical truth in each theory but nothing to justify any claim to unique superiority for any one theory above all other alternatives." J. R. Meyer and E. Muh, The Investment Decision, (Cambridge, Mass., 1957), p. 204.

21. Traditionally the equality of ex ante saving and investment, when assumed to come about through the mechanism of the interest rate. Another possible interpretation is to assume that business savings predominate total savings, and business savings are motivated by the desire to invest. See, for example, Lori Tarshis, "Business Funds, Consumption and Investment," Post-Keynesian Economics (New Brunswick, New Jersey, 1954). But Professor Tarshis points out:

...even though we accept the view that a change in business savings will lead to a change in the same direction in investment, this does not imply a return to Say's law and its consequences. There is no reason to suppose that every attempt to save more succeeds, or that if it does succeed, it leads to an equal increase in investment. (p. 386)
which equates investment to savings, the latter being equal to savings out of profits (with a propensity to save of $s$) and savings out of wages (with a propensity to save of $s'$).

If we assume full employment of capital and labor, this model has four "unknowns," viz., $Q$, $P$, $w$, and $I$, and has five equations, so that it is an overdetermined system. Alternative ways of bridging this gap give us alternative theories of production, distribution, and growth.

(a) The Pure "Keynesian" System: If equation (5) is dropped, we have a perfectly determinate system, which corresponds to Mr. Kaldor's "Keynesian" system completely, and which can also be found in Mrs. Robinson's works. From equations (4) and (5), we have:

$$A = P + s + L + w + s'$$

Taking (6) together with equation (2), we get:

$$\frac{P}{Q} = \frac{A}{Q (s - s')} - \frac{s'}{s - s'}$$  \hspace{1cm} (7)

This determines the relative distribution of income. The absolute size of income is given by equation (1) above, and when combined with equation (7), we get the absolute size of profits and of wages.


23. See, for example, *Exercises in Economic Analysis*, pp. 70-80.
So much for the "Keynesian" distribution theory. The growth process will consist of a succession of steps like this over time. Each year the capital stock is enhanced by investment (equation 4), and labor force grows exogenously. Technological knowledge might be made to grow exogenously, or made to depend on the pattern of capital accumulation, but, for any given year, we have a system of determination of output, wages and profits as described above. The process can continue.

(B) The "Neo-classical" System: If equation (4) is dropped, we get the basic Neo-classical system. Equation (1) as before gives the total output. Given that, equations (2) and (3) give the distribution of income. Equation (5) gives the rate of capital accumulation, and along with the growth of population and of technological knowledge, determine the process of growth over time. Various aspects of this distributional and growth process have been studied by Solow, Swan, Haavelmo, and Meade, but the essential aspect of the models of all of them come to this. The underlying assumption could vary, but the simplest is to assume investment to be infinitely elastic to interest rate, which brings it automatically to equality with planned savings.

(C) Leif Johansen's System: Professor Johansen puts forward a set of models24 that combine (a) an independent investment function (equation 4), thus differing from the above "Neo-classical" model, and (b) the marginal equality of productivity and wages (equation 3),

thus differing from the pure "Keynesian" model. How does he do this, in view of the overdeterminacy discussed above? The comparison is not easy, since even his simplest model (the "One-Sector Model") includes international trade, government expenditure, taxes, and so on; but the essential difference lies in its lack of an independent savings function (our equation 5). In Johansen's model, the consumption of "domestically produced goods and of competitive imports" as well as of "noncompetitive imports of consumer goods" depend on the relative prices and on the average consumption expenditure, but the latter which is treated as an "unknown" of the system, is not behavioristically related to the size of the income per head or to any other variable. To make the model comparable with our system outlined above, we can either (a) relate the average consumption of all goods taken together to the average size of income (and its distribution), or (b) relate the consumption of individual goods to relative prices and directly to income. In the first case we add an equation, and in the second we lose an unknown. In either case the model becomes overdetermined by one degree.

It is to be noted, however, that this formulation does not mean that Professor Johansen in fact denies the existence of the empirical relationship between consumption and income. What he does assume is that this is neutralized by the government policy of taxes and subsidies. He states:

In most macro-economic models total consumption outlays are related to total income, which is further related to net

25. Johansen, p. 29, equations 2.1; 6-7.
production. We have not introduced any relation of this kind. In fact we have assumed that the chain between Y [income] and the economic result of production is broken by net direct taxes (taxes minus subsidies to consumers), which are not explicit in the model.25

So what the Neo-classicists do by dropping the independent investment equation, what the generalizers of the General Theory do by dropping the marginal equation, Professor Johansen does by dropping the aggregate consumption function, i.e., in our model, the savings function.26 It is to be noted, however, that unlike the "Neo-classical" or the "Keynesian" models, the Johansen system strictly implies specific government policies.

(D) An Unemployment Equilibrium? A Possible method of combining all the equations might appear to be in making the size of the labor force a variable. With employment as a further unknown, of course within an upper limit, we may be able to strike a point of equilibrium that will satisfy everybody. Diagram I illustrates this for the simple case when capitalists save all they earn, and wage earners spend

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27. To summarize: equations (1), (2), (3) and (5) give the Neo-classical system; equations (1), (2), (4) and (5) give the "Keynesian" system; and equations (1), (2), and (3) and (4) give Johansen's system.
all they get, i.e., $s=1$, and $s's=0$, so that equation (5) reduces to $I=P$. The curve $OQ$ relates the total output to the level of employment. The independently given investment is given by the amount $OI$ represented by the straight line $IA$. This is also equal to the profits because of equation (5). When $OI$ is subtracted from each level of output, we get the residual, which in this model corresponds to the wage bill. The positive values of this are shown by the curve $BW$. There is one point on the curve $BW$, if it is continuous and of the usual shape, where the marginal and the average is equal, where all the equations of the system are satisfied. At this point, $M$ in diagram I, the wage rate $(OE/OE)$ is equal to the rate of change of the wage bill with changes in employment (the slope of $BW$ at point $M$). The latter corresponds to the rate of change of total output (since profits are fixed) with changes in employment, i.e., the marginal product of labor (the slope of $OQ$ at $R$). So all the equations are happily satisfied, provided, of course, the size of the potential labor force is at least as large as $OE$.

This way out is, however, not as interesting as it might appear at the first sight. If we do believe in the Neo-classical world, why should the wage not fall in this situation driving up the level of employment and driving down the level of marginal product? The Neo-classical theory of factor price determined by equating it to the corresponding marginal product does not work too happily in the absence of full employment. It would need something very special to do the trick. Of course, one might assume wage rigidity to prevent full employment from being established by production substitution,
and in fact rigidity of real wages can be assumed even when money wages are variable; but even then there is no particular reason why the wages would turn out to be rigid at just the level (ME/OE) required for the equilibrium depicted in diagram I. There does not seem to be any mechanism in the system that will ensure just the right degree of unemployment.

To avoid another possible false line of approach, and a very false one at that, we should make it quite clear that the problem has got nothing to do with different propensities to consume of the capitalists and of the wage earners. Though this is an important assumption in the "Keynesian" system (really due more to Michael Kalecki than to Lord Keynes), the conflict is not resolved by putting $s = s'$. Equation (5) still remains an independent equation: $I = s \cdot Q$. What it does destroy, however, is the possibility of achieving the desired amount of investment through a redistribution of income, maintaining full employment. In this case the "Keynesian" distribution mechanism stops giving us any results, and far from reducing the overdeterminacy of the model, it even makes the sub-system of equations (1), (4), and (5) itself overdetermined having only two unknowns ($Q$ and $I$) to play with. Thus, this is no road to the solution of the problem. However, it incidentally shows how heavily dependent the "Keynesian" distribution mechanism is on the difference between the two propensities. It is also to be noted that if $s$ is close to $s'$, any change in the "animal spirits" will lead to a violent shift of income distribution in the "Keynesian" system.
III. **Expected and Actual Prices**

The obvious way of taking all the relationships into account is to allow the expected price of the commodity to differ from the actual one. Let the expected price be \( q \) and the actual price \( p \). Let \( m \) represent the level of money wages. We now have the following modified system:

\[
\begin{align*}
Q &= T(K, L) & (1) \\
Q - p &= \frac{P + L \cdot m}{P} & (2A) \\
\frac{m}{\frac{\partial Q}{\partial L}} &= q & (3A) \\
I &= A & (4) \\
\frac{I - p \cdot P \cdot s + L \cdot m}{s} &= (5A)
\end{align*}
\]

Equation (1), the production function, is unchanged. Equation (2A) states that money income is equal to the sum of total money profits and total money wage bill. Equation (3A) states that the money wage rate is equal to the expected value of the marginal product. Equation (4) repeats the independent investment decision, given in real terms. Equation (5A) states the equality of the money value of real investment with savings in monetary terms. With the "widow's curse" assumption of no savings out of wages, and all savings from profits, the last becomes:

\[
I - p \cdot P = \frac{P}{s} & (5B)
\]

This system is perfectly determinate with \( Q, P, m, I, p, \) and \( q \).
as the unknowns, and either $p$ or $q$ can be put equal to unity as the unit of account. In this system, the capitalists equate the expected value of the marginal product to the wage rate, but the actual value of the marginal product may turn out to be different. The relationship between expected and actual prices can be found out. For example if we combine equation (58) with equations (2A) and (4), we get:

$$p = \frac{L}{Q - A}$$  

which combined with (3A) gives:

$$p = \frac{L}{Q - A} \frac{\partial Q}{\partial L} q$$  

Only when it so happens that

$$\frac{Q - A}{L} = \frac{\partial Q}{\partial L}$$  

will the price expectations be fulfilled, $p = q$. But this is in the nature of a special case.

If the picture we have tried to paint above is correct, then two reasonably interesting statements can be made about the system. First of all, the assumption of "perfect foresight" of the "Neo-classical" model would appear to be, in this context, internally contradictory.

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28. An asymmetry between investment and consumption behaviors should be noted in this model. Investment is a real demand, but consumption decisions are taken to be fixed relations of money income. In fact the price level adjusts itself in such a way as to make the value of money expenditure on consumption to absorb the difference between full employment output and real investment.
This additional equation, viz., $p = q$, may not at all be consistent with the rest of the system. Except through an accident, when (10) is satisfied, $p$ will be different from $q$, no matter what $q$ is. This implies that in some sense, a disequilibrium in the factor market may be more natural than equilibrium. Secondly, given the propensities to consume, if the capitalists behave the way the Neoclassicists assume them to (viz., equating the expected value of the marginal product to the wage rate), and if they also behave the way "Keynesians" assume them to (viz., carrying out an independently determined real amount of investment), it is the "Keynesian" result that will hold good, not the Neoclassical. The actual value of the marginal product will differ from the expected value as well as from the wage rate, and the income distribution will be given by the "Keynesian" dictates.

The absence of symmetry between the "Neoclassical" result and the "Keynesian" result can be best understood in the following way. The "Neoclassical" model assumes behavior patterns such that the wage rate is equated to the expected value of the marginal product of labor, and then with the further assumption of correct expectations, the equality of the wage rate and the actual marginal product is obtained. But this overlooks the possibility that the effect of the prediction on the thing predicted—what some philosophers call the "Oedipus Effect"—might make correct expectations impossible, because there might not be any expectation, which—if held—will be realized.
It is to be noted that the existence of the Oedipus Effect does not necessarily imply/correct expectations will be impossible, as Professor Karl Popper seems to suggest in his well-known Poverty of Historicism. There could be cases where the system of simultaneous equations allow fulfilled expectations, involving some cases where all expectations must necessarily be fulfilled. But if the above model is correct, it would appear that the expectation of any price level and of any value of the marginal product of labor will contradict itself if that expectation is given shape—in the "Neo-classical" manner—in the determination of factor prices.

It might seem that there is a similar problem of expectation and fulfillment in the "Keynesian" model. Unless the entrepreneurs anticipate the price level correctly, how are they going to decide on the amount of money investment that will lead to the desired amount of real investment? But all that is needed to assume here is that the entrepreneurs go on bidding up prices until they have got hold of enough goods for their predetermined real investment. If the entrepreneurs' propensity to save is bigger than that of the wage earners, the entrepreneurs will succeed in securing the real resources for their investment plan by altering the distribution of income. It is to be noted, however, that if the propensities to save depend on the level of prices, the "Keynesian" system will be in difficulty. Of course the equality

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29. "The idea, in short, of an exact and detailed calendar of social events is self-contradictory; and exact and detailed scientific social predictions are therefore impossible." Karl Popper, Poverty of Historicism, (Boston, 1957), pp. 13-14.
represented by equation (7) will still hold with the rates of saving that operate at the point of equilibrium, but without a function relating the propensities to save to the level of prices, the system will not be able to fulfill any predictive functions. Similarly, if investment is decided in money terms rather than in real terms, but continues to be independent of the ex ante level of savings, both the "Keynesian" and the "Neo-classical" results will be in jeopardy. The "Neo-classical" system avoids the jeopardy by equating the level of investment to the level of ex ante savings, while the "Keynesians" make the level of real investment fixed. Neither is a very good assumption, but of the two the latter seems to make a somewhat smaller demand on one's imagination. But on this, opinions could conceivably differ.

IV. The Process of Adjustment

The preceding analysis is, in some ways, very incomplete. Price expectations are not fulfilled, and the value of the marginal product turns out to be different from the one expected. But this is a situation of disequilibrium. Will the system now adjust itself to establish an equilibrium? This does not seem to follow at all. In fact a perfectly natural expectation in a case like this seems to be an unstable inflation or deflation.50 We discuss various possible patterns

50. Professor James Meade supplements his analysis with a "central monetary authority" which sees that the rate of interest is "set at such a level as to preserve a constant cost-of-living index." "If the price of consumption good tends to fall, the rate of interest is lowered, ..." (Op. cit., p. 3). This will certainly be a corrective in the process, but whether this will succeed or not will depend on (a) the response
adjustment.

We start with two strong behavioral assumptions, which we shall release later. First of all we assume that the entrepreneurs expect the price in the future to be what it is today. Secondly, they decide to invest a fixed proportion of their total output, i.e., they keep \((A/Q)\) constant. Both these assumptions will be dispensed with later.

We also assume that the production function is homogeneous of the first degree and is of the Cobb-Douglas variety. Technological progress is neutral in the "Hicksian" sense.

\[
Q = G \cdot e^{at} \cdot L^a \cdot K^b \cdot \lambda
\]  

(18)

where \(t\) represents time, and \(a, \lambda,\) and \(G\) are all constants.

In this case, at any given point of time,

\[
\frac{3Q}{3L} = \frac{Q \cdot \lambda}{L}
\]

(11)

Now, for \(p = q\), we need, from equation (10), the following:

\[
\frac{3Q}{3L} = \frac{Q \cdot \lambda}{L}
\]

i.e., from (11),

\[
\frac{A}{Q} = 1 - \lambda
\]

(12)

Now we have the following process (subscripts refer to the time dimension). From (9),

\[
\frac{B_1}{Q} = \left(\frac{L}{2 - \frac{1}{Q}}\right) \left(\frac{\partial Q}{\partial L}\right) Q
\]

of investment and expenditure to the interest rate, and (b) the extent of endogenous instability in the model taking the economy away from such a price constancy. But, it must be said, that if it does succeed in bringing the level of ex ante investment to the level of ex ante savings, the Neo-classical results will follow in this government-interfered economy.
From this and equation (11), we get:

\[
P_1 = \left( \frac{q_1}{q_2 - A_q} \right) q_2
\]

\[
= \left( \frac{q_2}{1 - A_q/q_1} \right) q_2
\]

\[
= r \cdot q_1 ,
\]  \hspace{1cm} (13)

where \( r \) is a constant over time.

The rest is very simple:

\[
q_2 = P_1
\]

\[
p_2 = r \cdot q_2 = r^2 \cdot q_1
\]

\[
q_3 = P_2
\]

\[
p_n = r^{n-1} \cdot p_1 = r^n \cdot q_1
\]  \hspace{1cm} (14)

There is a constant exponential inflation in the system.

Now, let us alter the assumptions about expectations and about the constancy of \((A/Q)\). The expectation assumption above is too gentle. The entrepreneurs notice a price rise from the previous period but go on assuming that the price of the present period will continue. They are much more likely to assume a price rise, perhaps in the same proportions as in the previous period. This, however, introduces the possibility of the proportional rate of inflation getting faster and faster over time. If, for example, \( r = 1.02 \), the actual price turns out to be 2 per cent more than what they expect. So there is a tendency for them to revise their price expectations continually upwards, because however much inflation they expect, the actual inflation turns out to be 2 per cent more. So here is a tendency to run into a hyperinflation.
The other question concerns the reactions of \((A/Q)\). If the ratio of investment to the scale of output tends to rise (fall) whenever actual price and profits turn out to be more (less) than the expected price and profits, the value of \(r\) will itself be rising (falling), as is clear from equations (10) and (10A), so that there will be a tendency for the proportional growth rate of prices to speed itself up.

It is to be noted, however, that expected rate of profit per unit of capital for the current period is independent of the expected price, since the expected wage rate and the expected value of the capital stock all get adjusted to that price:

\[
e = \frac{2 \cdot q - L \cdot \frac{\partial q}{\partial L} \cdot q}{k \cdot q} = \left( \frac{q - L \cdot \frac{\partial q}{\partial L}}{k} \right)
\]

So, if the entrepreneurs decide on their rate of investment solely on the basis of the expected rate of profit for the current period, the fact that they now expect a higher price \((q)\) will not make the slightest difference to the real rate of investment. But there are three further factors to be considered here. First of all, an overfulfillment of profit expectations in the past might put them in a more adventurous mood, and if they do incur a bigger rate of investment their adventure will pay off in terms of a higher rate of profit. Secondly, they would notice that while in their expectational system the profit rate seems to be independent of the expected price level, the actual profit rate in fact does depend on the actual price level (since money wages are determined with respect to the expected price level, not the actual). This may make them more sceptical of their own decision models and more responsive to a rising price level. Finally, and most important,
a rise in the price level would mean a capital gain of real assets, so that investment may be stimulated not by a higher profit rate expectation but by a desire to make use of capital gains on real assets.\textsuperscript{31}

We have been ignoring so far any problem arising from excessive capital accumulation in depressing the productivity of capital. With a Cobb-Douglas production function of the first-degree homogeneous type, as in equation (18), we have the following expression for the proportional rate of growth of outputs:

\[
g = \frac{dq}{dt} = \frac{1}{Q} \left( a + \alpha \cdot n + (1 - \alpha) \cdot k \right)
\]

where \(n\) is the proportional growth rate of labor, and \(k\) that of capital.

For output to grow at the same rate as capital, we put \(g = k\), which makes:

\[
g = k = \frac{a + \alpha \cdot n}{L}
\]

(15)

This keeps the ratio of capital to output constant, say at level 0.

The corresponding ratio of investment to income, \((I/Q)\) or \((A/Q)\), is:

\[
I = \frac{K \cdot k}{Q} = \left( \frac{A + \alpha \cdot n}{L} \right) \cdot g
\]

(16)

The profit rate per unit of capital in this model will equal \(k\), since investments are equal to profits. There will be a constant capital-output ratio, and a constant rate of growth, except that prices will be continually rising or falling, and will be constant only if \((I/Q)\) given by this turns out accidentally to be the same required to keep prices

\textsuperscript{31} Cf. James Tobin, "A Dynamic Aggregative Model", op. cit.

Incidentally, the saving rate might also rise thanks to the so-called "Pigou effect," and this will act in the direction of pacifying the inflation.
We may next examine what happens if \( (I/Q) \) rises and is pushed beyond this equilibrium level. Of course the value of \( \frac{20}{21} \) or the marginal product of capital will be falling, but if this model is correct, the profit rate per unit of capital will be kept up as long as \( (I/K) \) keeps up. With \( (Q/K) \) falling, this will require \( (I/Q) \) to be rising at a constant profit rate. What will be happening is that the falling profit rate from diminishing productivity of capital will be kept at bay by continually raising the share of profits in the national income through a buoyant investment behavior. Whether everything else in the "Keynesian" model will hold in the face of such a continuous fall of the share of wages, is of course, excessively doubtful.

To conclude, it is interesting that the price mechanism described above has three distinct processes. First of all, given the ratio of investment to income and given that the capitalists expect prices to be the same this year as in the last, there will be a constant exponential rate of price rise (fall). Secondly, the price expectations themselves would move in the direction of the rise (fall), and discovering that whatever they expect, the actual price turns out to be consistently more (less) than what they expected, the capitalists are likely to add fuel to the fire by continually raising (reducing) their price expectations. And finally, the proportion of income invested is likely to move in the direction opposite to the one of equilibration, thereby increasing the proportional excess (shortfall) of the actual price \( \text{vis-a-vis} \) the expected price. All these lead to
a fairly strong tendency toward continual inflation (or deflation) with the process speeding itself up over time, for a period at least. Ultimately, however, this speeding will be checked by diminishing returns to capital, unless the profit rate is kept up by continually depressing the share of wages by an ever-increasing manifestation of "animal spirits."

In practice, brakes in the inflationary process might arise partly from the squeezing of the real income of rentiers (if any), partly from a rise in the interest rate due to a growing demand for money, partly from a restraint in the consumption expenditure arising due to falling real value of money assets, as well as from a rise in the savings coefficients of entrepreneurs with a rise in their real profits. And of course the government is likely to intervene with monetary and fiscal measures. But the fact that in spite of strong government policy, in advanced free enterprise economies, inflation has gone on more or less continually, might support the view that there are inherently unstable elements in the system, such as the ones encountered in the above study.

V. Conclusions

The upshot of all this can be put in the form of a set of conclusions:

1. When we combine the "Neo-classical" and the "Keynesian" models of production, distribution and growth, we get an overdetermined system. The "Neo-classicists" make it determined by dropping the independent investment function; "Keynesians" do it by ignoring the equality of wages to marginal products; and Professor Leif Johansen does it by keeping both the above equations, and by dropping the
savings function, i.e., the function of aggregate consumption, through an assumption about government tax policy.

(2) If all the contesting parties are granted their behavior postulates, the model can be made determinate by allowing expected prices to differ from the actual ones. In this system the Neoclassical result of wages equaling marginal product will hold good only insofar as we consider the expected value of the marginal product. The actual income distribution will seem to be determined by the "Keynesian" rules, as long as investment decisions are taken in real terms and the propensities to consume are constant. When investment decisions are taken in fixed money terms, both the systems will be in jeopardy.

(3) The assumption of perfect foresight, in this context, will be internally inconsistent, except in a very special case. This indicates a methodological problem involved in the "Neo-classical" system that does not seem to arise in the "Keynesian" model.

(4) Insofar as the investment function is not entirely independent and will respond to prices, the adjustment may well be in the wrong direction from the one required to bring the actual and expected prices to equality. This suggests that if the behavior postulates of both the "Neo-classicists" and the "Keynesians" are right, the result may be an unstable inflation, or deflation, which might be speeding itself up. The speeding up will result from the capitalists trying to raise (reduce) their investment plans when the profits they expected turn out to be less (more) than what they get, in real terms; and also result from changing asset preferences. However,
the speeding up will occur only if the proportion of income invested rises (falls), so that it is not quite inevitable even when investment rises. In any case, the process of speeding up might not continue indefinitely since the rise in the rate of accumulation might bring about a fall in the profit rate per unit of capital unless compensated by continually squeezing the share of wages. The speeding up will, however, also tend to result from a continual upward revision of price expectations; for no matter how wild the expectations, they are consistently over- (or under-) fulfilled.

(5) The lack of stability in the system does not arise, it should be emphasized, from the lack of factor substitutability, as in Harrod's model. Just the contrary. Perfect competition and factor substitution ensures continuous full employment. The money wages are competitively determined by the expected value of the marginal product of labor at full employment. The actual prices—corresponding to the money wages—turn out to be, except in a very special case, different from the expected prices. Then the process feeds on itself in the manner described, and might even feast on itself.

(6) Insofar as the postwar years have been characterized by continuous inflations, this model, combining "Neo-classical" and "Keynesian" elements may have some relevance. However, it is important to remember that both the "Neo-classical" and the "Keynesian" systems share 32. When investment falls, the relative must, since full employment output will be rising over time. However, the model described is much more appropriate for an inflation than for a price deflation. The continual upward revision of money wages is more realistic than downward revisions in an economy with trade unions.
are based on high degrees of abstraction, so that the relationship may be spurious.

Incidentally, it is of some interest to note that like "the black girl" in Shaw's story who searched for God and found an Irish farmer, we have searched for a comprehensive model of production, distribution and growth, and have found a model of inflation. But that is what is obtained if the two systems are put together. To what extent this is a commentary on the real world, and to what extent merely on the inadequacy of the systems studied, is a question worth investigating with statistical data.