Essays on Institutions for Financial Stability

by

Roberto Benelli

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Author .......................................................... Department of Economics

May 10, 2002

Certified by .............................................. Stephen A. Ross

Franço Modigliani Professor of Finance and Economics
Thesis Supervisor

Certified by .............................................. Bengt Holmström

Paul A. Samuelson Professor of Economics
Thesis Supervisor

Accepted by .............................................. Peter Temin

Elisha Gray II Professor of Economics
Chairman, Department Committee on Graduate Studies

ARCHIVES


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Abstract

This thesis includes three essays on the interaction between financial market institutions and market liquidity, and its implications for financial stability. The first essay studies an overlapping generations model of a risky asset market in which some agents face a participation cost. Market participation, by affecting the size of the pool of potential holders of the risky asset, determines the liquidity of the asset market. This essay studies how the frictions that are associated with capital requirements on financial institutions affect their incentives to supply liquidity to the market. The participation decision generates a positive and a negative externality, and the interaction between the two externalities can give rise to multiple equilibria in participation, i.e. to “liquidity cycles”. The second essay studies the complementary problem of the optimal design of incentive systems for financial institutions in the context of limited market liquidity. In a contract between a borrower and a lender, financial incentives are provided by requiring the borrower to finance a sufficiently large share of her investment project. In the states of nature in which many projects are liquidated simultaneously, liquidation in private contracts is excessive relative to the efficient (second-best) contract chosen by a planner who internalizes the externality working through the liquidation price. This essay studies whether capital requirements on the borrowers can implement the second best allocation, and if not what kind of policy instruments can implement it. The last essay presents a model of international lending that is built on a basic form of contractual incompleteness: foreign investors cannot commit to provide state-contingent or long-term finance to domestic entrepreneurs. This form of contractual incompleteness implies that there is excessive liquidation of socially viable projects in the competitive equilibrium that emerges in decentralized markets. Institutions that manage to limit liquidation have the potential to improve welfare.

Thesis Supervisor: Stephen A. Ross
Title: Franco Modigliani Professor of Finance and Economics

Thesis Supervisor: Bengt Holmström
Title: Paul A. Samuelson Professor of Economics
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This thesis is dedicated to my parents Alba and Bruno and to my brother Alessandro.
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\(^1\)This chapter has been written jointly with Roberto Rigobon.
Chapter 1

Introduction

This thesis includes three essays on the interaction between financial market institutions and market liquidity, and its implications for financial stability. Liquidity admits several interpretations, and the three essays emphasize different dimensions of this concept; chapters 2 and 3 focus on the role and implications for financial stability of capital requirements, while chapter 4 discusses institutional arrangements that are common in international financial markets.

In Chapter 2, I study an overlapping generations model of a risky asset market in which some agents face a participation cost. Market participation, by affecting the size of the pool of potential holders of the risky asset, determines the liquidity of the asset market. In this chapter, I study how the frictions that are associated with capital requirements on financial institutions affect their incentives to supply liquidity to the market. The participation decision generates a positive and a negative externality. Participation, by improving the allocative efficiency of the price system, reduces the dividend risk. However, it also raises the liquidity risk borne by short-horizon agents by increasing the responsiveness of the asset price to the arrival of information. The interaction between the two externalities can give rise to multiple equilibria in participation. When there are multiple equilibria in participation, the stochastic process that selects participation has to be correlated over time. That is, the equilibrium dynamics of the economy exhibit “liquidity cycles” in which high and low participation levels tend to cluster in time. I study a few examples that highlight the main features of the model, and discuss some of its normative implications.

Chapter 3 studies the problem that is complementary to the analysis of Chapter 2, since
it studies the optimal design of incentive systems on financial institutions in the context of a (given) limited market liquidity. Financial incentives and termination threats coexist as incentive devices in many private contracts in financial markets, as well as in the regulation of financial institutions. This chapter addresses the optimal balance between financial incentives and termination threats in a contract between a borrower and a lender. Financial incentives are provided by requiring the borrower to finance a sufficiently large share of her investment project. Based on information about the borrower’s performance, the borrower-lender relationship can be terminated by liquidating a fraction of the project at a price that depends on the aggregate volume of liquidation. In the states of nature in which many projects are liquidated simultaneously, liquidation in private contracts is excessive relative to the efficient (second-best) contract chosen by a planner who internalizes the externality working through the liquidation price. I study whether capital requirements on the borrowers can implement the second best allocation, and if not what kind of policy instruments can implement it. This allows for an interpretation of the actual risk-based capital requirements, and highlights some of their limitations.

Chapter 4, written jointly with Roberto Rigobon, applies some of the insights of Chapter 3 to the context of international lending. The model of international lending that we study in this chapter is built on a basic form of contractual incompleteness: foreign investors cannot commit costlessly to provide state-contingent or long-term finance to domestic entrepreneurs. This form of contractual incompleteness implies that there is excessive liquidation of socially viable projects in the competitive equilibrium that emerges in decentralized markets. As a result, institutions that limit liquidation have the potential to improve welfare. We compare the bailout of the domestic entrepreneurs through ex-post government intervention with a workout mechanism based on a majority rule. We then extend the model to allow the domestic entrepreneurs to issue two types of external claims, short-term “hot money” and a more stable source of finance, foreign direct investment (FDI). We show that in the competitive equilibrium FDI is excessive, because the entrepreneurs and the FDI investors do not take into account the negative externality that works through the short-term investors’ decisions to demand the early repayment of their claims. This result sheds a new light on controls on short-term capital inflows that aim at reducing a country’s vulnerability to the volatility of short-term flows.
Chapter 2

Market Participation and Liquidity Risk

2.1 Introduction

In this chapter I study a model of a risky asset market in which market participation is costly for some agents. An agent’s decision to participate in the market is a decision to supply liquidity; the demand for liquidity originates from the agents who reach the end of their investment horizon, and from the agents who do not find it convenient to incur the participation cost and therefore sell their holdings of the risky asset. Thus, this chapter presents a model of demand and supply of liquidity.

Market participation affects the quality of the aggregate information, and therefore the properties of the equilibrium price of the risky asset. A key assumption is that the market aggregates the agents’ dispersed information, which is produced only if the agents incur the participation cost. By affecting the quality of the aggregate information, an agent’s participation decision generates externalities on other agents. Participation generates a positive externality because it improves the quality of the aggregate information about the risky asset’s dividend, thus reducing the dividend risk borne by the market participants. However, participation can also generate a negative externality. Intuitively, if an increase in participation in the current period implies that participation in the future increases as well, then current participation improves the quality of the information that will be available in the future. If the asset price’s respon-
siveness to information increases with the quality of information, then an increase in current participation raises the future asset price volatility from the perspective of the current period. This increase in volatility represents an additional source of risk for agents with a finite investment horizon. Because this risk arises from the liquidation of assets at the end of the agents' horizon, it admits the interpretation of liquidity risk. Thus, the negative externality generated by participation operates through an increase in the liquidity risk borne by finite horizon agents when trading in the asset market is ongoing over time.

The interaction between the two externalities can give rise to multiple equilibria in participation levels. That is, there can be different levels of market participation that can occur in any period and that are consistent with equilibrium in the asset market and individually optimal participation decisions. Which level occurs in any period is determined by an exogenous random selection process. Though exogenous, the selection process must be consistent with the equilibrium in the asset markets and with the individually optimal participation and portfolio decisions. I discuss a few examples that highlight under which conditions on the selection process multiple equilibria in participation can occur. As the previous discussion suggests, the serial correlation in the selection process (that is, the existence of "liquidity cycles" if one interprets market participation as market liquidity) is a key condition for the existence of multiple equilibria, and therefore for liquidity risk.

Participation in the risky asset market is a state variable for the equilibrium price of the risky asset. Because it affects the quality of information aggregated by the market, participation is also a state variable for the volatility of the equilibrium asset price. The obvious implication of the model is therefore that volatility comoves with liquidity. More interestingly, this model predict that when volatility and liquidity change over time, they have to be serially correlated.

With regard to the normative implications, the chapter points out that increasing the flow of information to the market can magnify the liquidity risk faced by some agents. This point is worth emphasizing because official documents on market "transparency" tend to hold the view that an improvement in the transmission of information is always beneficial for market participants; this chapter points out one cost associated with better information revelation.¹

This chapter has implications for the debate on the countercyclicality of capital require-

¹See Morris and Shin (1999) for a discussion of another cost of better information transmission.
ments and of mark-to-market practices more generally. The 1996 Amendment to the Basel Accord [BIS (1996a,b)] established a common international standard for the regulation of the market risk in the banks' portfolios, the risk of losses on the proprietary trading activities. The Amendment exemplifies a trend in capital markets toward risk-sensitive capital requirement for financial institutions.\(^2\) One implication of this type of risk-management systems is the phenomenon that some have blamed for the amplification and propagation of financial crises, namely that capital requirements against portfolios of risky assets are countercyclical, as they are tightened in periods of market "turbulence" characterized by market downturns and/or increases in volatility. According to this interpretation, financial institutions are induced to cut back on their positions precisely when the market volatility increases, thus exacerbating market volatility and reducing liquidity. For example, the events in the summer and fall of 1998 have been described as having "[mimicked] those of a margin call, albeit on a worldwide scale" as "the surge in Value-at-Risk (VaR) levels above predefined levels during the crisis compelled market participants to unwind positions. Because of the widespread use of similar models, similar behavior was adopted by numerous investors. The resulting simultaneous pressure to unwind positions dried up liquidity of markets and therefore exacerbated market volatility." [BIS (1999, pp. 14 and 41)].\(^3\) The opportunity cost of the capital that the financial institutions hold against their risky positions represents one of the costs of market-making and trading activities. Furthermore, since the required capital holdings depend positively on measures of market volatility (such as VaR), the opportunity cost of capital is increasing in market volatility. The model that I present in this chapter will permit me to analyze this argument in detail. The participation cost can be interpreted as the opportunity cost of supplying liquidity, a cost that has to be incurred ex-ante before the trading opportunities can be exploited. Financial instability takes the form of multiple equilibria in participation levels and the possibility that a failure in the coordination of the agents' participation decisions leads to low levels of participation. In the

\(^2\)The proposed revision of the Basel Capital Accord for the regulation of banks' credit risk has just confirmed the trend. The risk weights in the revised standardized approach will be more reflective of actual risk, and banks will be allowed to use their own credit assessment in the internal ratings based approach. Furthermore, capital incentives between the standardised and internal ratings based approach should encourage banks to the latter approach to credit risk. See Jorion (2000, Chapter 3) for an overview of the existing regulation.

\(^3\)The Value-at-Risk of a portfolio is the quantile at a prespecified confidence level of the distribution of the net profits over a given time horizon.
model, financial instability is a more serious problem than suggested by the interpretation of the financial crisis in 1998 just given. The existence of multiple equilibria and coordination failures does not rely necessarily on the sensitivity of the participation cost to market variables such as market volatility, but is a feature of the coordination problem in a dynamic context.

The chapter is organized as follows. In Section 2, I present the model, and I discuss some related literature. In Section 3, I derive the equilibrium in the asset market for fixed market participation. In Section 4, I study the individually optimal participation decisions, and characterize market participation consistent with equilibrium in the asset market. In Section 5, I discuss the trade-off between the positive and negative externalities generated by the participation decisions, and present a few examples that highlight the main features of the model. Section 6 briefly concludes. All the proofs are in the Appendix.

2.2 The model

There are two assets, a risky and a riskless asset. Time is discrete, indexed by \( t = 0, 1, 2, \ldots \). There is a single good in each period, used as numeraire. There are infinitely many overlapping generations of agents; each agent has a two-period horizon (that is, an agent is “alive” for two periods). In each period, participation in the market for the risky asset is costly for a subset of the agents. At the end of the first period of their life, agents have to decide whether to participate in the market for the risky asset in the following period; if they participate, they have to incur an exogenous cost (the “participation cost”). For simplicity, participation in the market for the riskless asset is costless for the agents who are just born. Participation in the market for the riskless asset is costless for all the agents.

Figure 1 shows the timing of the events within a given period \( t \). At the beginning of the period, the risky asset pays out the dividend \( X_t \) on any share of the risky asset. A new generation of \( I \) agents are born. I will refer to the agents born in period \( t \) as the “young” agents in period \( t \), and to the agents born in \( t - 1 \) and alive in \( t \) as the “old” agents in period \( t \). The \( N \) agents who in period \( t - 1 \) incurred the participation cost for period \( t \) receive a private signal about the dividend \( X_{t+1} \) that will be paid out at the beginning of period \( t + 1 \). The participation level \( N \) for period \( t \) is known to all market participants in period \( t \). The market
for the risky asset clears in period $t$ at the (ex-dividend) price $p_t$; the market clearing price aggregates the private signals of the market participants, and is observed by all the market participants. At the time of market clearing in period $t$, the participation level $N'$ for period $t + 1$ has not been determined yet, and thus is random from the perspective of the agents who participate in period $t$. After market clearing and before entering period $t + 1$, the young agents born at the beginning of period $t$ have to decide whether to participate in period $t + 1$. Market participation is costly: if an agent chooses to participate, she has to incur the participation cost $C/2$ (the normalization by $1/2$ is made entirely for notational convenience). If an agent does not participate, she will have to liquidate her holdings of the risky asset acquired in period $t$ at the beginning of period $t + 1$. On each share of the risky asset, an agent who does not participate receives $X_{t+1} + p_{t+1}$, the dividend in period $t + 1$ and the resale value of the asset (the market clearing price in $t + 1$). Since the liquidation decision does not depend on the actual realizations of the price and dividend, it is a "market order".
2.2.1 Assets

The riskless asset yields a gross rate of return of \(1+r\) per period, with \(r > 0\). The (net) interest rate \(r\) has to be strictly positive to ensure the existence of the equilibrium. The supply of the riskless asset is infinitely elastic; all the agents can freely borrow and lend at the interest rate \(r\) regardless of their participation decision.

The risky asset is infinitely lived. It pays a random dividend \(X_t\) in period \(t\). The dividend process \(\{X_t\}\) is a sequence of independent and identically distributed (iid) normal random variables,

\[ X_t \sim N(\bar{X}, \sigma_X^2). \]

\(p_t\) is the \textit{ex-dividend} price of the risky asset in period \(t\), that is, the price of the flow of dividends \(X_{t+1}, X_{t+2}, \ldots\). There is a constant supply of \(Z > 0\) shares of the risky asset.

2.2.2 Preferences

The new generation born in period \(t\) consists of a finite number \(I\) of types, indexed by \(i = 1, \ldots, I\). The types differ for their private information (described below), and may differ for their risk aversion. Each type consists of a continuum of unit mass of identical agents; thus, a mass of \(I\) agents is born every period. For simplicity, I refer to the mass of agents who participate as the participation “level”, or participation for short. The assumption that there is a continuum of agents of each type permits me to avoid the “schizophrenia” problem pointed out by Hellwig (1981).

Agents have mean-variance preferences over consumption at the end of their horizon. Let \(w_2\) be consumption at the end of the horizon of an agent born at the beginning of the current period. The expected utility of the agent is given by

\[
U(w_2) = \mathbb{E}[w_2] - \frac{1}{2} \rho(i) \mathbb{V}[w_2],
\]

where \(\mathbb{E}[w_2]\) and \(\mathbb{V}[w_2]\) are the expected value and variance conditional on any information that is available whenever the agent’s utility is evaluated; the parameter \(\rho(i)\) is the risk-aversion of type \(i\). The types are ordered so that \(\rho(i + 1) \geq \rho(i)\) for \(i = 1, \ldots, N - 1\).

If the final wealth were normally distributed conditional on the available information, the
preferences in (2.1) could be derived from the standard CARA utility function. However, as it will be shown below, the final wealth \( w_2 \) will not always be normally distributed. Consider for instance an agent at the beginning of her two-period horizon. The value of her wealth at the end of the first period of her horizon will depend on the state variable \( N' \) (the participation in the following period), which will be realized between the first and the second period of the agent’s life. Furthermore, her portfolio choice for the following period will depend on the future realizations of the private signals (aggregated by the equilibrium price). While final wealth \( w_2 \) will be normally distributed conditional on this information, \( w_2 \) is not normally distributed before this information becomes known. Thus, the preferences in (2.1) cannot be derived from CARA utility.

It turns out, however, that even by assuming the preferences in (2.1), the analysis of the equilibrium is difficult. Thus, I will take a further approximation to the preferences in (2.1) that can be justified as follows. First note that before the realization of next period’s participation \( N' \), the variance of the final wealth can be decomposed as

\[
\mathbb{V}[w_2] = \mathbb{E}_{N'} \left[ \mathbb{V}[w_2|N'] \right] + \mathbb{E}_{N'} \left[ \left( \mathbb{E}[w_2] - \mathbb{E}[w_2|N'] \right)^2 \right],
\]

implying that

\[
\mathbb{E}[w_2] - \frac{1}{2} \rho(i) \mathbb{V}[w_2] \approx \mathbb{E}[w_2] - \frac{1}{2} \rho(i) \mathbb{E}_{N'} \left[ \mathbb{V}[w_2|N'] \right]
\]

if

\[
\mathbb{E}_{N'} \left[ \left( \mathbb{E}[w_2] - \mathbb{E}[w_2|N'] \right)^2 \right] \approx 0,
\]

where the moments \( \mathbb{E}[w_2|N'] \) and \( \mathbb{V}[w_2|N'] \) are conditional on the participation level \( N' \) and the expectation \( \mathbb{E}_{N'}[\cdot] \) is taken with respect to the state variable \( N' \). That is, the approximation is accurate if the variance of the final wealth from the perspective of the current period is mostly accounted by the differences in the final wealth’s variance across the states \( N' \) rather than by the differences in the wealth’s expected value across the states \( N' \). I will work with this approximation to the mean variance preferences in (2.1), that is, I will ignore the term \( \mathbb{E}_{N'} \left[ \left( \mathbb{E}[w_2] - \mathbb{E}[w_2|N'] \right)^2 \right] \). In other words, I will assume that agents have mean-variance preferences conditional on the interim state \( N' \); the preferences of an agent of type \( i \) born at the
beginning of the current period are described by

\[ U(w_2) = \mathbb{E}_{N'} \left\{ \mathbb{E}[w_2|N'] - \frac{1}{2} \rho(i) \mathbb{V}[w_2|N'] \right\}, \]  

(2.2)

The main problem with this formulation is that the state \( N' \) is an endogenous variable. That is, the preferences depend on a variable that is being determined as a part of the equilibrium, and the equilibrium depends in turn on the agents' preferences. Nevertheless, I work with the preferences in (2.2), since they preserve the tractability of CARA preferences.

### 2.2.3 Costly asset market participation

A key assumption is that participation in the market for the risky asset is costly for at least some agents. If participation were costless, all agents would always participate, since they retain the option to choose a zero demand for the risky asset once in the market.

Participation in the market for the \textit{riskless} asset is costless every period for all the agents who are alive in the period. With regard to the \textit{risky} asset, the young agents in generation \( t \) face no cost to participate in period \( t \). On the other hand, an old agent born in period \( t \) who decides to participate in the risky asset market in \( t + 1 \) has to incur a fixed cost \( \frac{C}{2} > 0 \) in terms of the numeraire good. Once the participation cost has been incurred, an old agent can freely choose her demand for the risky asset when the market opens in period \( t + 1 \). If she does not participate, she will have to liquidate any holdings of the risky asset acquired in period \( t \) at the price \( p_{t+1} \) that will be determined in period \( t + 1 \) (after having received the dividend \( X_{t+1} \) at the beginning of period \( t + 1 \)), and hold the proceeds in the riskless asset until consumption occurs at the end of period \( t + 1 \).

As mentioned above, I often drop for notational simplicity the time index from the participation levels and denote by \( N \) the mass of agents born in period \( t - 1 \) who participate in period \( t \) (the current period) and by \( N' \) the mass of old agents born in period \( t \) who incur the participation cost and participate in period \( t + 1 \) (the next period); obviously, \( N \leq I \) and \( N' \leq I \). The total mass of agents who participate in the risky asset market in period \( t + 1 \) is thus given by \( I + N' \), since the participation of the young agents is costless. For simplicity, I also focus on the case in which the agents of the same type \( i \) make the same participation
2.2.4 Private information

The $N$ old agents who incur the participation cost in period $t - 1$ and participate in the risky asset market in period $t$ receive some private information about the asset's dividend. An old agent of type $i$ born in period $t - 1$ who incurred the participation cost for period $t$ receives the private signal $y^i_t$ about the dividend $X_{t+1}$ paid at the beginning of $t + 1$, given by

$$y^i_t = X_{t+1} + \epsilon^i_t,$$  \hspace{1cm} (2.3)

where $\epsilon^i_t$ is a zero mean normal random variable with variance $\sigma^2$. I assume that the dividends and the noise terms in the private signals are jointly normal, and that $\epsilon^i_t$ are iid across time and types; the shocks $\epsilon^i_t$ are independent of $X_s$ for all $t$ and $s$. Obviously, the signal $y^i_t$ is correlated with the payoff $X_{t+1}$. The private signal is observed only after the participation cost has been incurred.

2.2.5 The budget constraints

Agents are born with the same initial wealth $w_0$. I denote by $\theta^{i,t}_t$ and $\theta^{i,t}_{t+1}$ the holdings of the risky asset (number of shares) in period $t$ and $t + 1$ respectively of an agent of type $i$ born in period $t$. If she participates in the risky asset market in $t + 1$, the wealth at the beginning of period $t + 1$ and $t + 2$ is

$$w^{i}_{t+1} = [X_{t+1} + p_{t+1} - (1 + r)p_t]\theta^{i,t}_t + (1 + r)w_0$$

$$w^{i}_{t+2} = [X_{t+2} + p_{t+2} - (1 + r)p_{t+1}]\theta^{i,t}_{t+1} + (1 + r)w^{i}_{t+1} - C/2$$

$$= [X_{t+2} + p_{t+2} - (1 + r)p_{t+1}]\theta^{i,t}_{t+1} + (1 + r)w^{i}_{t+1} - C/2 \hspace{1cm} (2.4)$$

---

4Thus, since type contains a unit mass of agents, $N$ is both the number of types and the mass of agents who incur the participation cost (the participation level).

5Given the mean variance preferences and the assumption of an infinitely elastic supply of the riskless asset, this assumption about the initial wealth is not substantial.
If agent $i$ does not participate in the risky market in period $t+1$, then she invests $w_{t+1}^i$ in the riskless asset and her wealth is $w_{t+2}^i = (1 + r)w_{t+1}^i$ at the beginning of period $t+2$, that is, $\theta_{t+1}^{i,t} \equiv 0$ if the agent does not incur the participation cost $C/2$. The agent consumes her wealth $w_{t+2}^i$ at the beginning of period $t+2$. I will drop the type index $i$ when no confusion arises from its omission.

The total payoff from holding one share of the risky asset between periods $t$ and $t+1$ is the sum of the dividend $X_{t+1}$ paid by the asset in period $t+1$ and the price at which the asset can be resold, $p_{t+1}$. Thus, the excess return from holding one unit of the risky asset between periods $t$ and $t+1$ over the riskless asset is $X_{t+1} + p_{t+1} - (1 + r)p_t$.

I have assumed in (2.4) that the portfolio $\theta_{t}^{i,t}$ chosen by the young agents of type $i$ does not depend on the participation decision that they expect to make after the risky asset market clears in period $t$. I will show below that this conjectured behavior is optimal in equilibrium.

### 2.2.6 Discussion

Equilibrium models in which agents are subject to constraints on their portfolio choices such as borrowing, margin or VaR constraints, tend to be difficult to study. These constraints affect the agents’ economic incentives to provide liquidity, and thus determine the liquidity of the market. In this chapter, I make a simplifying assumption about the friction that limits the agents’ ability to supply liquidity, namely that agents have to incur an exogenous cost before participating in the risky asset market. This assumption allows me to separate the participation decision (the decision to supply liquidity) from the portfolio allocation decision, and implies that market liquidity will be the outcome of a coordination game between potential market participants. The interpretation of liquidity as the outcome of a coordination game emphasizes the public good nature of liquidity, and highlights the determinants of the agents’ incentives to supply liquidity.

Many papers concerned with liquidity risk (for example, Loewenstein and Willard (2001)) emphasize one side of “liquidity trading”, the forced sales of asset holdings for liquidity-related motives: agents initially active on some market face the risk of being forced to liquidate their

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6See, for example, Yuan (1999), Gromb and Vayanos (2001), Kyle and Xiong (2001), Basak and Shapiro (2001).
asset holdings at some point in time regardless of the existing market conditions, the source of the liquidity risk. The OLG extension introduces the other side of liquidity trading by assuming that some agents are forced into the market for the risky asset every period (they are “born”). In practice, such traders could be fund managers who purchase a security because their objective is to track an index.

The assumption about the timing of the participation decision seeks to capture one important aspect of the decision to supply liquidity, namely that it has to be made before the trading opportunities from supplying liquidity are realized. In the model, the agents have to compare the expected benefit from supplying liquidity (which depends on the endogenous asset price properties) with the participation cost. The aggregate supply of liquidity will depend on the mass of agents who participate in the risky asset market. In other formulations, as in Grossman (1988), the supply of liquidity depends on the amount of capital allocated to this purpose by market makers. Both formulations share the presumption that the supply of liquidity cannot adjust instantaneously to changes in the demand for liquidity. The uncertainty about future participation decisions (that is, the uncertainty about the future supply of liquidity) gives rise to the liquidity risk faced by the current market participants.

The participation cost has been introduced as an exogenous fixed cost; it is crucial that participation is costly for a nontrivial participation decision. The assumption that participation in the risky asset market is costless for the young agents is made entirely for simplicity, and the model could easily be extended to allow for costly participation of the young agents as well. There is empirical evidence on the limited financial market participation of households: households tend to hold very undiversified portfolios, often consisting of very few securities. This evidence has been invoked in support of various market imperfections, including costly market participation. While the traditional interpretation of the participation cost is that it

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7See also Allen and Gale (1994) for a model in which liquidity depends on the “cash in the market”.

8If participation were costless, then it would never be optimal for an agent to liquidate “early”, since liquidation takes place at the equilibrium price. An agent who participate retains the option to choose her portfolio after observing the equilibrium prices, and hence the option to choose a zero demand for the risky asset. Therefore, if participation were costless, it would always be worth observing the price before deciding to liquidate. This contrasts, for example, with the mechanism in Diamond and Dybvig (1983), where the sequential service constraint on deposit withdrawals implies that there is a benefit in being at the beginning of the line of withdrawals during a bank run.

9DeLong et al. (1990) quote this evidence as supportive of limited rationality; Allen and Gale (1994) as supportive of costly participation. The traditional reference on limited participation is Mankiw and Zeldes
represents an information cost, \(^{10}\) for this chapter the most natural interpretation is that it represents the cost that must be incurred to carry out trading and market-making activities. In particular, since financial institutions hold capital against the risk in their trading and market-making activities (for risk-management and regulatory purposes), the participation admits the interpretation of the opportunity cost of such capital. The agents considering to participate have to compare this cost with the value of the trading opportunities that will be available to the agents who supply liquidity. I will show below that the value of participation can be concisely summarized in equilibrium by the Sharpe ratio of the risky asset.

The rationale for the assumption that the agents receive private information only if they choose to participate in the risky asset market in the second period of their life is that it takes time to develop the necessary expertise to acquire and process information about an asset. Therefore, only the agents who have previously been in the market have access to privileged information. There are models that explicitly describe the process of information acquisition through active market participation. In a model of price experimentation, Leach and Madhavan (1993) study how market makers can manipulate their pricing strategy in order to extract valuable information from their market orders. Clearly, this argument requires the prior market participation of the agents who receive private information about an asset’s payoff. In Cao and Lyons (1999), dealers infer valuable information from their customer orders. Extracting information from customer orders presumes that the dealers have been willing to clear these orders in the past. The assumption made here that private information is received only conditional on prior market participation seeks to retain the flavor of these channel; its simplicity however will permit me to study the general equilibrium of the economy.

### 2.3 Equilibrium in the asset market with fixed participation

In this section I derive the rational expectations equilibrium in the asset market in a given period \(t\), taking the participation level in the current period as given. In the next section, I will use the equilibrium price function determined in this section to study the optimal participation

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\(^{10}\) For instance, this is the interpretation given given by Grossman (1976), Merton (1987), and Allen and Gale (1994).
decisions.

The main proposition in this section is a straightforward generalization of the classic result by Grossman (1976) on the rational expectations equilibrium (REE) in a static economy in which agents receive private information about an asset’s dividend. The derivation of the REE is slightly more elaborate in the OLG framework because I have to take into account that the relevant payoff from holding one share of the risky asset for one period includes the asset’s resale value in addition to the asset’s dividend, and the resale value is determined endogenously in equilibrium.

In this section I suppose that the market participation $N$ for period $t$ has already been determined. Participation consists of the agents who in period $t - 1$ incurred the participation cost for period $t$. With a slight abuse of notation, I also denote by $N$ the set of agents who participate; the set of agents who participate can always be chosen as $\{1,...,N\}$. Since the $I$ young agents born in period $t$ participate in the risky asset market, market participation in period $t$ consists of $N + I$ agents. I denote by $\mathcal{N}$ the set of possible participation levels that can occur in the following period. Market participation in the following period, $N'$, is selected from the set $\mathcal{N}$ according to the probability distribution $Q_N$. In the next section, I will describe how the set $\mathcal{N}$ is determined in equilibrium. In general, I allow the probability distribution $Q_N$ to depend on the current participation level $N$; the (potential) dependence of $Q_N$ on the current participation level justifies the subscript $N$.\footnote{The set $\mathcal{N}$ is the set that contains all the possible participation levels that can occur in any period. If some participation level cannot occur in the next period given the current participation level $N$, it receives probability zero by the probability distribution $Q_N$.} Each agent takes the distribution $Q_N$ as given.

A critical assumption is when the uncertainty about next period’s participation is resolved. I assume that when agents choose their portfolios in period $t$ and the asset market clears, they are uncertain about participation in period $t + 1$. The uncertainty associated with the next period’s participation level is the source of the liquidity risk, and is summarized by the probability distribution $Q_N$. This uncertainty is resolved after the market for the risky asset has cleared in period $t$, so that when the agents born in period $t$ make their participation decision for period $t + 1$, they correctly anticipate that participation in period $t + 1$ is $N'$, and at this conjecture $N'$ agents find it profitable to participate. One can think that the selection of $N'$ for period $t + 1$ is governed by a sunspot variable $\xi$ with support $\mathcal{N}$ and probability distribution...
$Q_N$. When the agents make their participation decisions, they rely on the sunspot variable $\xi$ to coordinate their beliefs, that is, they observe the realization $\xi \in \mathcal{N}$ after the market has cleared in period $t$ and before participation decisions for $t+1$ are made. In equilibrium, $N' = \xi$.

The following lemma gives the young and old agents’ expected utilities, which in turn yield their asset demands used for the derivation of the equilibrium in the asset market. I denote by $\overline{y}_t$ the average of the signals received by the old agents who participate in the risky asset market in period $t$,

$$\overline{y}_t = \frac{\sum_{i \in \mathcal{N}} y^i_t}{N} = X_{t+1} + \frac{\sum_{i \in \mathcal{N}} e^i_t}{N}. \tag{2.5}$$

The average signal $\overline{y}_t$ describes the aggregate information in the economy. I assume in the Lemma that the equilibrium price $p_t$ reveals $\overline{y}_t$, a property that holds in equilibrium.

**Lemma 1 (Expected utilities).** Suppose that $N$ old agents participate in period $t$. The expected utility, at the time of market clearing in period $t$, of a young agent of type $i$ born at the beginning of period $t$ is given by

$$U_{i,t}^t = (1 + r)^2 w_0 + (1 + r) \left[ \mathbb{E}[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t \right] \theta_i^{i,t}$$

$$- \frac{1}{2} \rho(i)(1 + r)^2 \mathbb{E}_{Q_N} [\mathbb{V}[X_{t+1} + p_{t+1}|p_t]] (\theta_i^{i,t})^2 + \kappa_i^i, \tag{2.6}$$

where $\kappa_i^i$ is the value (in utility terms) of the option to participate in the risky asset market in $t+1$, which depends on the current participation level $N$ but not on the portfolio choice $\theta_i^{i,t}$.

The expected utility of an old agent of type $i$ born at the beginning of period $t - 1$ is given by

$$U_{i,t-1}^{t-1} = (1 + r)w_t + \left[ \mathbb{E}[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t \right] \theta_i^{i,t-1}$$

$$- \frac{1}{2} \rho(i) \mathbb{E}_{Q_N} [\mathbb{V}[X_{t+1} + p_{t+1}|p_t, N']] (\theta_i^{i,t-1})^2. \tag{2.7}$$

The demand for the risky asset of a young agent and an old agent of type $i$ is given respectively by

$$\theta_i^{i,t} = \frac{1}{(1 + r) \rho(i) \mathbb{E}_{Q_N} [\mathbb{V}[X_{t+1} + p_{t+1}|p_t, N']]} \mathbb{E}[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t \tag{2.8}$$

$$\theta_i^{i,t-1} = \frac{\mathbb{E}[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t}{\rho(i) \mathbb{E}_{Q_N} [\mathbb{V}[X_{t+1} + p_{t+1}|p_t, N']]} \tag{2.9}$$
The expected utilities $\mathcal{U}_t^i$ and $\mathcal{U}_t^{i-1}$ are evaluated in period $t$ conditional on current participation $N$ and the average signal $\bar{y}_t$ (revealed by the equilibrium price). The demands (2.8) and (2.9) and the assumption that $r > 0$ imply that the young agents are effectively more risk averse than the old agents. This property is an implication of mean-variance preferences, which assume away any wealth effect on portfolio demand. Because consumption occurs at the end of an agent’s two-period horizon, the final consumption of an agent is equal to the wealth at the end of her first period reinvested for one period in the riskless asset, $(1 + r)w_{t+1}$, augmented by the excess return (over the riskless asset) from the investment in the risky asset in the second period. Mean-variance preferences imply that $\kappa^i_t$, the ex-ante value (in utility terms) of the option to invest in the risky asset market in the second period, enters additively in (2.6) and is independent of $w_{t+1}$. Therefore, the portfolio choice in the first period affects final consumption only through $(1 + r)w_{t+1}$. Since the variance of this term is proportional to $(1 + r)^2$ while its expected value is proportional to $(1 + r)$, the agents with the two-period horizon are effectively more risk averse than the agents with the one-period horizon (by the factor $\frac{1}{1+r}$).

I denote by $\bar{\varrho}$ and $\bar{\varrho}_p$ the inverse of the risk tolerance of the young agents and of the old agents who participate in the risky asset market respectively,

\[
\frac{1}{\bar{\varrho}} = \frac{1}{I} \sum_{i \in I} \frac{1}{\varrho(i)}
\]

\[
\frac{1}{\bar{\varrho}_p} = \frac{1}{N} \sum_{i \in N} \frac{1}{\varrho(i)}
\]

Since all the generations are identical, $1/\bar{\varrho}$ is the average risk tolerance within any generation, a parameter of the model. Unless $\varrho(i)$ is constant, the average risk tolerance $1/\bar{\varrho}_p$ depends on the participation decisions of the old agents. The following proposition describes the rational expectations equilibrium (REE) for period $t$ as a function of the current participation level $N$.

**Proposition 1.** Let $\mathcal{N} \subseteq \{1, ..., I\}$ be the set of potential participation levels. Suppose that $N \in \mathcal{N}$ old agents born in period $t - 1$ participate in the risky asset market in period $t$, together with the $I$ young agents born in period $t$. The participation level $N'$ of old agents in period $t + 1$ is drawn randomly from the set $\mathcal{N}$ according to the probability distribution.
$Q_N$ after the market for the risky asset has cleared. The unique linear rational expectations equilibrium price function is given by

$$p_t = a + b(\bar{y}_t - \bar{X}),$$  \hspace{1cm} (2.10)

where $a = a(N)$ and $b = b(N)$ depend on $N$ and on the pair $(N, Q_N)$ The constant $a(N)$ is the solution to the following linear functional equation,

$$a(N) = \frac{1}{1 + r} \left[ \bar{X} - \Lambda(N) \frac{Z}{I + N} \frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N + \sigma_e^2} \right] +$$

$$\frac{1}{1 + r} \sum_{N' \in N} \left[ a(N') - \Lambda(N) \frac{Z}{I + N'} \frac{1}{1 + r} \frac{\sigma_X^2}{\sigma_X^2 N' + \sigma_e^2 / N'} \right] Q_N(N'),$$  \hspace{1cm} (2.11)

where $1/\Lambda(N)$ is a function of the average risk tolerances of the young and old agents who participate, $1/\bar{p}$ and $1/\bar{p}_p$

$$\frac{1}{\Lambda(N)} = \frac{N}{I + N} \left( \frac{1}{\bar{p}_p} \right) + \frac{1}{1 + r} \frac{I}{I + N} \left( \frac{1}{\bar{p}} \right).$$  \hspace{1cm} (2.12)

The constant $b(N)$ is given by

$$b(N) = \frac{1}{1 + r} \frac{\sigma_X^2}{\sigma_X^2 + \sigma_e^2 / N'}.\hspace{1cm} (2.13)$$

To understand the form of the equilibrium price function (2.10), note that mean-variance preferences imply that the equilibrium price of the risky asset is equal to the price that would be paid by risk neutral agents reduced by a risk premium that depends on the amount of risk borne on average by the population of agents participating in the market for the risky asset,

$$p_t = \frac{1}{1 + r} \left\{ E[X_{t+1} + p_{t+1} | p_t] - \Lambda(N) \frac{Z}{I + N} E_Q[X_{t+1} + p_{t+1} | p_t, N'] \right\}.$$

One share of the asset purchased in period $t$ at price $p_t$ pays out the dividend $X_{t+1}$ and can be resold at the equilibrium ex-dividend price $p_{t+1}$. Therefore, the relevant payoff for pricing is $X_{t+1} + p_{t+1}$, whose first two moments have to be computed conditional on the available
information. The relevant measure of average risk tolerance is $1/\Lambda(N)$, the weighted average of the risk tolerances within the young and old agents who participate in the risky asset market in period $t$. In particular, the definition of the average or “market” risk aversion $\Lambda(N)$ in (2.12) takes into account that the young agents are effectively more risk averse than the old agents. If the individual risk aversion parameter $\rho(i)$ is constant across agents, this feature implies that an increase in the participation of old agents raises the average risk-tolerance $1/\Lambda(N)$, and *ceteris paribus* decreases the risk premium. If the risk aversion $\rho(i)$ is not constant across agents and the more risk-tolerant agents participate first, an increase in the participation of old agents has an ambiguous effect on the average risk tolerance $1/\Lambda(N)$, since participation reduces the average risk tolerance $1/\bar{\rho}_p$ of the old agents. It seems a plausible characteristic of the model that market participation can reduce the average risk aversion.

The main part of the proof consists in computing the expected value and variance of $X_{t+1} + p_{t+1}$ conditional on the information available to the agents at the time of market clearing. As in Grossman (1976), the equilibrium asset price is a sufficient statistic for the set of private signals of the old agents who participate in period $t$. The constants $a(N)$ and $b(N)$ in (2.11) and (2.13) ensure that the equilibrium price function is consistent over time. To understand the form of these coefficients, note that the coefficient $a$ is the solution to the following functional equation,

$$
a(N) = \frac{1}{1 + r} \left[ \bar{X} - \frac{\Lambda(N)}{I + N} \frac{Z}{\sigma_x^2 N} \frac{\sigma_e^2}{\sigma_x^2 + \sigma_e^2} \right] \left\{ \text{risk premium due to dividend risk} \right\} + \frac{1}{1 + r} \mathbb{E}_{Q_N} \left[ a(N') - \Lambda(N) \frac{Z}{I + N} \frac{1}{(1 + r)^2} \frac{\sigma_x^4}{\sigma_x^2 + \sigma_e^2} \right] \left\{ \text{risk premium due to price risk} \right\}.
$$

The expected dividend, discounted at the interest rate $r$, is $\frac{\bar{X}}{1 + r}$. The risk premium due to the uncertainty about the dividend (the dividend risk) depends on the market risk aversion $\Lambda(N)$, the per capita average supply of the asset $\frac{Z}{I + N}$, and the variance of the dividend conditional on the available information, $\frac{\sigma_x^2 \sigma_e^2}{\sigma_x^2 N + \sigma_e^2}$. The expected equilibrium resale value of the asset under the probability distribution $Q_N$, discounted at the interest rate $r$, is $\mathbb{E}_{Q_N}[a(N')]/(1 + r)$. The
risk premium due to the uncertainty about this value (the price risk) depends on the market risk aversion, the per capita supply of asset, and the expected variance of the resale price, $\mathbb{E}_{Q_N} \left[ \frac{1}{(1+r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_t^2/N_t} \right]$. Since the properties of the asset price in period $t+1$ depends on participation in period $t+1$ (uncertain as of the time of market clearing in period $t$), this risk dimension represents the liquidity risk. Furthermore, I show in the proof of the proposition that the dividend and the resale value are uncorrelated, and hence no covariance term appears in the equilibrium price function.

The difference equation for $a(N)$ can be iterated forward, yielding

$$a(N_t) = \frac{X}{r} - \sum_{s=0}^{\infty} \left( \frac{1}{1+r} \right)^{s+1} \mathbb{E}_t \left[ \Lambda(N_{t+s}) Z \left( \frac{\sigma_X^2 \sigma_t^2}{\sigma_X^2 N_{t+s} + \sigma_t^2} + \frac{1}{(1+r)^2 \sigma_X^2 + \sigma_t^2/N_{t+s+1}} \right) \right],$$

that is, the risk premium depends the entire future (stochastic) path of market participation. Changes in the expected future behavior of liquidity can affect the current risk premium even before they are actually realized. This is not a surprising result given the forward looking nature of asset pricing.

**Remark.** The approximation in (2.2) of the mean-variance preferences (2.1) is equivalent to the following approximation to the equilibrium price,

$$\mathbb{V}[p_{t+1}|p_t] = \mathbb{E}_{Q_N}[\mathbb{V}[p_{t+1}|p_t, N']]$$

$$+ \mathbb{E}_{Q_N}[\mathbb{E}[p_{t+1}|p_t, N'] - \mathbb{E}[p_{t+1}|p_t]^2]$$

$$\approx \mathbb{E}_{Q_N}[\mathbb{V}[p_{t+1}|p_t, N']].$$

The term $\mathbb{E}[p_{t+1}|p_t, N'] - \mathbb{E}[p_{t+1}|p_t]$ is the innovation in the expected equilibrium price due to the realization of participation $N'$ for the following period. The approximation is accurate if the expected value of the square of this innovation is sufficiently small relatively to the average of the variance of the equilibrium price across the states $N'$, that is, if changes in the variance of the price across states is relatively more important than changes in its expected value across states for the total price risk.
2.4 Optimal participation decisions and equilibrium participation

Having studied the equilibrium in the asset market for a given set of participation levels $\mathcal{N}$, I turn next to the determination of this set. For convenience, I define the risk premium $Y_t$ on the risky asset in period $t$ as

$$Y_t \equiv \mathbb{E}[X_{t+1} + pt+1|pt] - (1 + r)pt.$$  \hspace{1cm} (2.14)

The following lemma contains a simple result on $Y_t$ that will be useful below.

**Lemma 2.** The equilibrium risk premium on the risky asset in period $t$ when $N$ old agents participate in the risky asset market is given by

$$Y_t = \Lambda(N) \frac{Z}{I + N} \left\{ \frac{\sigma_X^2 \sigma_r^2}{\sigma_X^2 N + \sigma_r^2} + \mathbb{E}_{Q_N} \left[ \frac{1}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 N + \sigma_r^2 / N'} \right] \right\}.$$

\[ \square \]

The important property of the risk premium $Y_t$ is that it depends only on the current participation level $N$. This property would not hold in more general versions of the model, for example if the supply of the risky asset were random and unobservable by the agents.

The following lemma provides a second useful result.

**Lemma 3.** Suppose that an agent of type $i$ believes at the time of market clearing in period $t$ that participation in period $t + 1$ is $N'$. Then, conditional on the information in period $t$ and her belief, the risky asset’s excess return over the riskless asset between period $t + 1$ and $t + 2$, that is, $X_{t+2} + pt+2 - (1 + r)pt+1$, is uncorrelated with the value of the agent’s wealth $u_{t+1}^i$ at the beginning of period $t + 1$. \[ \square \]

These two results can be used to derive the following proposition.

**Proposition 2.** Suppose that an agent of type $i$ born in period $t$ believes that participation in period $t + 1$ is $N'$. The increase in the agent’s expected utility from participation in the risky asset market, $\Delta U^p_{i,t+1}$, is given by
\[ \Delta u_{t+1}^P = \frac{1}{2} \left[ \frac{1}{(\rho(t))^2} \mathbb{E}_{Q_{t}} \left[ \mathbb{V}[X_{t+2} + p_{t+2} | p_{t+1}, N''] \right] - C \right]. \]

\( N' \) is an equilibrium participation level, that is, \( N' \in \mathcal{N} \), if it satisfies the following inequality,

\[ \Lambda(N')^2 \left( \frac{Z}{I + N'} \right)^2 \left[ \frac{\sigma^2 \sigma_i^2}{\sigma^2 N' + \sigma_i^2} + \sum_{N'' \in \mathcal{N}} \frac{1}{(1 + r)^2} \frac{\sigma^2_X}{\sigma^2_X + \sigma_i^2/N''} Q_N(N'') \right] \geq \rho(N') C \] (2.15)

and this inequality fails when evaluated at \( N' + 1 \). If the types are ranked in order of increasing risk aversion, then agents of type \( i \) participates if \( i \leq N' \), while they do not participate if \( i > N' \).

\[ \square \]

For simplicity, the equilibrium condition (2.15) will be used somewhat loosely in the following, since I will ignore the possibility that there does not exist a type for which this condition holds exactly. Furthermore, I assume that the agents of a given type always make the same participation decision. Therefore, I will work with (2.15) holding as an equality rather than with the pair of inequalities required by Proposition 2. The equilibrium participation condition then states that if \( N \in \mathcal{N} \) is an equilibrium participation level, then it satisfies the following condition

\[ \Lambda(N)^2 \left( \frac{Z}{I + N} \right)^2 \left[ \frac{\sigma^2 \sigma_i^2}{\sigma^2 N + \sigma_i^2} + \sum_{N'' \in \mathcal{N}} \frac{1}{(1 + r)^2} \frac{\sigma^2_X}{\sigma^2_X + \sigma_i^2/N''} Q_N(N'') \right] = \rho(N) C. \] (2.16)

The left-hand side is the square of the Sharpe ratio, evaluated in equilibrium. The condition then states that the equilibrium in participation occurs when the benefit from participation, summarized by the Sharpe ratio, compensates the agents for the cost that they incur to participate, adjusted for the risk aversion \( \rho(N) \) of the marginal type participating in the market. If the types are ranked in order of increasing risk aversion, then all the agents of type \( i \leq N \) find it convenient to participate, while agents of type \( i > N \) do not.

The equilibrium participation condition (2.16) depends explicitly on the family of probability distributions \( \{Q_N\}_{N \in \mathcal{N}} \) that govern the evolution of equilibrium participation. Each element \( N \in \mathcal{N} \) is a solution to the equilibrium condition (2.16), given all the other elements in \( \mathcal{N} \) and \( \{Q_N\}_{N \in \mathcal{N}} \). The dependence of current participation on future participation derives
from the fact that the uncertainty about future participation is a source of risk for the agents who choose to participate, because it generates uncertainty about the resale value of the asset. The magnitude of this risk clearly depends on the probability distributions \( \{Q_N\}_{N \in \mathcal{N}} \), whose support \( \mathcal{N} \) is being determined. It is a weakness of the model that while the family of distributions \( \{Q_N\}_{N \in \mathcal{N}} \) is important for determining the total risk borne by market participants, it is not uniquely pinned down by an equilibrium condition; the only requirement is that for any \( N \in \mathcal{N} \), the probability distribution \( Q_N \) satisfies (2.16).

It is clear from the form of the equilibrium price function in (2.10) that agents participating in the risky asset market face two types of risk. The first one is the uncertainty about the dividend \( X_t \) and the noise terms in the private signals \( \epsilon_t \). I interpret this type of uncertainty as fundamental risk, since it is related to the uncertainty about the dividend and the "technology" available to the agents to learn information about it (the competitive market for the risky asset). The second one is the uncertainty about the equilibrium participation levels in \( \mathcal{N} \) that will occur in the future, summarized by the family of probability distributions \( \{Q_N\}_{N \in \mathcal{N}} \). I interpret this type of uncertainty as liquidity risk, since it is related to the individual decisions to participate (that is, to supply liquidity to the market). Current participation is a state variable for the equilibrium price of the risky asset if the selection process \( Q_N \) depends on current participation \( N \). As it is apparent from the price function, the two risks are priced in equilibrium, but it is not straightforward to distinguish their separate impact on the risk premium.

To summarize the results of the last two sections, the equilibrium dynamics of the economy is governed by the pair \( (\mathcal{N}, \{Q_N\}_{N \in \mathcal{N}}) \), where \( \mathcal{N} \) is the set of equilibrium participation levels and \( \{Q_N\}_{N \in \mathcal{N}} \) is the family of probability distributions for the selection of next-period participation: when the current participation is \( N \), participation in the next period is \( N' \) with probability \( Q_N(N') \). For each equilibrium participation level \( N \) in \( \mathcal{N} \), the equilibrium condition in (2.16) holds given \( Q_N \). If \( N \) is the participation in the current period, the equilibrium price for the risky asset is

\[
p = a(N) + b(N) \left( X_{t+1} + \frac{\sum_{\ell \in N} \epsilon_{t}^{\ell}}{N} - \bar{X} \right),
\]

where \( a(N) \) and \( b(N) \) are given in Proposition 1 by (2.11) and (2.13). In the current period, the \( I \) young agents participate, together with \( N \) old agents. If the risk-aversion is not constant.
across agents, these agents are the $N$ agents with the lowest risk aversion.

### 2.5 Dividend risk, price risk, and multiple equilibria in participation

The analysis in the previous section implies that all the equilibrium participation levels have to be determined simultaneously; moreover, the set of equilibrium participation levels depends on the family of distributions $\{Q_N\}_{N \in \mathcal{N}}$ that govern the selection of future participation. Accordingly, the problem of determining the set of equilibrium participation levels is difficult, since their number, which is equal to the number of equations that have to be satisfied simultaneously, is not known apriori. In this section, I try to gain some insight on the properties of the equilibrium through a series of examples. In the examples, I make a conjecture about the number of participation levels, and then verify that the conjecture can be supported in equilibrium by a proper choice of the process $\{Q_N\}_{N \in \mathcal{N}}$ for equilibrium selection.

To illustrate the mechanism underlying the existence of multiple equilibria in participation, suppose that the risk aversion coefficient is equal to $\rho$ across agents, and rewrite the equilibrium participation condition in (2.16) as follows,

$$V[X_{t+1}|p_t, N] + \mathbb{E}_{Q_N}[V[p_{t+1}|p_t, N']] = \frac{\rho}{\Lambda(N)^2} \left( \frac{C}{I+N^2} \right)^2. \tag{2.17}$$

This rearrangement will be useful later in the section. The right-hand side is the participation cost in utility terms ($\rho C$) divided by the average risk-aversion of the participating agents ($\Lambda(N)$) and the per-capita asset holding ($Z/(I+N)$). These last two terms are standard in asset-pricing models in the CARA-normal framework, and are the direct channels through which market participation affects the equilibrium Sharpe ratio in a mean-variance world, the market risk aversion $\Lambda(N)$ and the per capita holdings of the risky asset $\frac{Z}{I+N}$. It is easy to check that the right-hand side of (2.17) is increasing in current market participation.\(^{12}\) The average risk aversion $\Lambda(N)$ falls with market participation as more old agents participate, because the old

\(^{12}\)Note that

$$\frac{I+N}{\Lambda(N)} = \frac{(1+r)N + I}{(1+r)\rho}$$
agents are effectively less risk averse than the young agents; obviously, the per-capita asset holdings falls with market participation as well.\footnote{The effect of market participation on the average risk aversion stems from the difference in the horizon of young and old agents, and the fact that participation is costly for the old agents. In models of costly participation where agents differ only for their individual risk-aversion, participation increases the average risk-aversion, as more and more risk-averse agents are drawn into the market (see for instance Pagano (1989)). The property that participation improves the market’s ability to bear risk does not seem unrealistic, though, and does not does not drive the results below. As it will be clear, if participation increases the average risk aversion, the likelihood of multiple equilibria is higher.}

The interesting term in (2.17) is the left-hand side, the risk per share borne by an agent who in period $t$ holds one share of the risky asset. This term is the variance of the total payoff $X_{t+1}+p_{t+1}$ from holding one share of the risky asset between $t$ and $t+1$,\footnote{For simplicity, I refer to $E_{Q_N}[V[p_{t+1}|p_t,N']]$, the expected variance of the price conditional on next period’s participation, as the variance of the price, even though this term is different from the variance conditional on the current information, $V[p_{t+1}|p_t]$.} and captures the effect of participation on risk through the quality of information aggregated by the market. Suppose that $N$ agents decided in period $t-1$ to participate in period $t$. For each share purchased in period $t$, a holder receives the dividend $X_{t+1}$ at the beginning of period $t+1$; the variance of the dividend depends on the participation level $N$, and is the first term on the left-hand side. As more agents participate in period $t$, that is, the higher $N$, the lower the variance of the dividend, $V[X_{t+1}|p_t,N]$, because more private signals are aggregated by the market: higher participation in period $t$ improves the quality of the information available to predict the dividend in $t+1$. Thus, the individual decision to participate in the risky asset market generates a positive externality on other agents by increasing the quality of the aggregate information and reducing the dividend risk. Each share purchased in period $t$ will be sold at the equilibrium price $p_{t+1}$ that will be determined in period $t+1$. Proposition 1 shows that $p_{t+1}$ depends on the participation level $N'$ in period $t+1$; the expected variance of $p_{t+1}$ is the second term on the left-hand side of (2.17). Suppose that high participation in the current period tends to be followed by high participation in the following period, and similarly for low participation to be followed by low participation, because the selection process $Q_N$ is serially correlated. Then, an

$$
\Lambda(N)^2 \left( \frac{Z}{I+N} \right)^2 = \left[ \frac{(1+r)pZ}{(1+r)N+I} \right]^2.
$$

so that the product of the two terms in the denominator of the right-hand side is
increase in current participation $N$ tends to *increase* the chances of high participation $N'$ in the next period, and thus the variance of the price, $\mathbb{E}_{Q_N}[\nu[pt+1|pt', N']]$, since from the equilibrium price function (2.10) the variance of the price conditional on $N'$ is increasing in $N'$,

$$\nu[pt+1|pt', N'] = \frac{1}{(1 + r)^2} \frac{\sigma^2_X}{\sigma^2_X + \sigma^2_t/N'},$$

and high participation levels become more likely when current participation increases. Intuitively, the increase in the variance of the price is due to the fact that when more agents participate, the equilibrium price becomes more responsive to the aggregate information (the average signal $\bar{y}_{t+1}$), because the quality of the aggregate information improves with participation. To the agents born in period $t - 1$ who participate in period $t$ and liquidate their asset holdings at price $pt+1$, the increased sensitivity of price to information represents an additional source of risk. Depending on the properties of the selection process $\{Q_N\}$, the individual decision to participate can thus generate a *negative externality* on other agents through the variance of the resale price. As a result of the two externalities, the variance of the total payoff $xt+1 + pt+1$ can be nonmonotonic in market participation and multiple equilibria in participation are possible.

This heuristic discussion highlights the trade-off between the improvement in the allocative efficiency of the price system and liquidity risk. At one extreme, if no information were available, the resale price would be constant, and no price risk would be borne; however, the dividend risk would be the highest (equal to the unconditional variance $\sigma^2_X$). At the other extreme, if participation were high, the dividend could be predicted very precisely, but the resale price would be very responsive to the arrival of future information. It is worth emphasizing the point that (future) information revelation contributes to the magnitude of the liquidity risk of the agents currently in the market, because policy discussions on market “transparency” tend to suggest that that better information transmission is always beneficial to market participants. This chapter points out one cost associated with better information revelation.\(^{15}\)

The role of the short horizon of market participants has been emphasized in papers by DeLong *et al.* (1990) and Shleifer and Vishny (1997), where the additional source of price risk originates from the erratic but systematic behavior of noise traders. In this chapter, the origin

\(^{15}\)See Morris and Shin (1999) for a discussion of another cost of better information transmission.
of the additional risk lies ultimately in the coordination problem between market participants whose investment activity involves a friction (the participation cost). The externalities generated by the participation decisions imply that the value of market participation depends on the agents’ beliefs about what other agents simultaneously choose to do. If the specific friction is an institutional feature that serve some other purpose, the problem described by this chapter is more fundamental than noise trading, and would survive the elimination of "irrational" behavior.

In the following of this section I present a few examples that illustrate the logic of the heuristic argument just discussed.

2.5.1 The static model

Clearly, the trade-off between the allocative efficiency of the price system and liquidity risk is absent in static versions of the model where the resale value of the risky asset is identically zero at the terminal date. Consider a three period version of the model. In period 0, agents are borne and make the participation decision. In period 1, the agents who participate receive a private signal about the dividend paid in period 2; the risky asset market clears. In period 2, the dividend is paid out. It is easy to see that the analogue of the equilibrium participation in (2.16) is

\[ \Lambda(N)^2 \left( \frac{Z}{I + N} \right)^2 \frac{\sigma^2_{e} \sigma^2_{x}}{\sigma^2_{x} N + \sigma^2_{e}} = \rho C, \]  

(2.18)

where the left-hand side is the equilibrium Sharpe ratio. The Sharpe ratio depends on the variance of the dividend \( \mathbb{V}[X_{t+1}|p_t, N] \) conditional on the market participation \( N \). The uncertainty about the dividend is the only source of risk, since the resale value of the asset is zero. Thus, only the positive externality from market participation is at work in the static model. Suppose that all agents have the same risk aversion \( \rho \), implying that the average risk aversion \( \Lambda(N) \) is constant and equal to \( \rho \).\(^{16}\) The equilibrium condition (2.18) then implies that the equilibrium participation is unique. However, there can be multiple equilibria if risk aversion is not constant across the agents. In this case, the average risk aversion \( \Lambda(N) \) increases with market participation, as more risk averse agents are brought into the market. If \( \Lambda(N) \) increases sufficiently

---

\(^{16}\)In the static model, it is no longer the case that \( \Lambda(N) \) depends on participation even though \( \rho \) is constant, since there is no difference in the time horizon of market participants.
fast with market participation, that is, if the ability to bear additional risk in the economy falls sufficiently fast, there can be multiple solutions for \( N \) in the equilibrium participation condition (2.18).

**2.5.2 Unique equilibrium**

Suppose that there is a unique equilibrium participation level: there is a unique solution to the equilibrium participation condition (2.16), that is \( \mathcal{N} = \{ \bar{N} \} \). Then, the equilibrium price (2.10) is an iid sequence; the expected value of the equilibrium price of the risky asset can be computed from (2.11),

\[
a(\bar{N}) = \frac{1}{r} \left\{ \bar{X} - \Lambda(\bar{N}) \right\} \frac{Z}{I + \bar{N}} \left[ \frac{\sigma_{X}^{2} \sigma_{\varepsilon}^{2}}{\sigma_{X}^{2} \bar{N} + \sigma_{\varepsilon}^{2}} + \frac{1}{(1 + r)^{2}} \frac{\sigma_{X}^{4}}{\sigma_{X}^{2} + \sigma_{\varepsilon}^{2}/\bar{N}} \right],
\]

and the constant \( b(\bar{N}) \) is given by

\[
b(\bar{N}) = \frac{1}{1 + r} \frac{\sigma_{X}^{2}}{\sigma_{X}^{2} + \sigma_{\varepsilon}^{2}/\bar{N}}.
\]

The variance of the equilibrium price of the risky asset is given by

\[
\mathbb{V}[p_{t}] = \frac{1}{(1 + r)^{2}} \frac{\sigma_{X}^{2}}{\sigma_{X}^{2} + \sigma_{\varepsilon}^{2}/\bar{N}}
\]

This is essentially the equilibrium that obtains in the static model studied by Grossman (1976), except for the the additional term due to the price risk borne by the agents.

Suppose that the risk aversion is constant across agents. The equilibrium participation condition (2.16) implies that an increase in the participation cost \( C \) reduces market participation \( \bar{N} \). The risk-premium on the risky asset increases (because \( a(\bar{N}) \) is increasing in \( \bar{N} \), and \( \bar{N} \) falls), and the variance of the equilibrium price increases.

**2.5.3 State-independent equilibrium selection**

Suppose that there exists a finite number of distinct equilibrium participation levels \( N = \{ N_{1}, ..., N_{M} \} \) that jointly satisfy the equilibrium participation condition (2.16) for a given family of probability distributions \( \{ Q_{N_{1}}, ..., Q_{N_{M}} \} \). Suppose that equilibrium selection is an inde-
dependent process across periods, that is, the participation level \( N_j \) is selected with probability \( q_j \) independent of the participation level in the previous period. The participation level \( N_j \) must satisfy the equilibrium participation condition (2.16). Given the assumed equilibrium selection process, \( N_j \) must satisfy

\[
\left[ \frac{(1 + r)\rho Z}{(1 + r)N_j + I} \right]^2 \left[ \frac{\sigma_X^2 \sigma_t^2}{\sigma_X^2 N_j + \sigma_t^2} + \frac{1}{(1 + r)^2} \sum_{k=1}^{M} \frac{g_k - \frac{\sigma_X^4}{\sigma_X^2 + \sigma_t^2/N_k}}{\sigma_X^2 + \sigma_t^2/N_k} \right] = \rho C.
\]

Clearly, the second term in squared brackets does not depend on \( N_j \). Therefore, this equation cannot hold simultaneously for any other \( N_j' \neq N_j \).

This proves the following proposition.

**Proposition 3.** A necessary condition for the existence of multiple equilibria in participation levels \( \{N_1, \ldots, N_M\} \) is that the equilibria selection be state-dependent for at least some participation level, that is, there must exist \( j' \neq j \) such that \( Q_{N_{j'}} \neq Q_{N_j} \).

\( \square \)

### 2.5.4 State-dependent equilibrium selection: an interpretation of risk management systems

Suppose that there exist two distinct equilibrium participation levels, \( N_H > N_L \), the high-participation state \( N_H \) and the low-participation state, with the interpretation that \( N_H \) represents a liquid market and \( N_L \) an illiquid market. Suppose that the selection of the participation level is state-dependent and symmetric, in the following sense. When the current participation level is \( N_H \), it also selected in the following period with probability \( q \), and \( N_L \) is selected with probability \( 1 - q \). Similarly, if the current participation level is \( N_L \), it is also selected with probability \( q \) in the following period, and \( N_H \) is selected with probability \( 1 - q \).

First, define the adjusted participation cost in utility terms \( G(N) \) (the right-hand side of (2.17)) as a function of market participation,

\[
G(N) \equiv \left[ \frac{(1 + r)N + I}{(1 + r)\rho Z} \right]^2 \rho C. \tag{2.19}
\]

Define also the variance \( V(N, N', q) \) of the total payoff from holding one share of the risky asset

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for one period when current participation is \( N \) in the current period, and is \( N \) in the following period with probability \( q \) and \( N' \) with probability \( 1 - q \),

\[
V(N, N', q) \equiv \frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N + \sigma_e^2} + \frac{q}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N} + \frac{(1 - q)}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N'}.
\] (2.20)

Note that \( V(N, N', q) \) is decreasing in \( N \),

\[
\frac{\partial V(N, N', q)}{\partial N} = -\frac{\sigma_X^4 \sigma_e^2}{(\sigma_X^2 N + \sigma_e^2)^2} \left( 1 - \frac{q}{(1 + r)^2} \right) < 0,
\]

and its intercept at \( N = 0 \) is given by

\[
V(0, N', q) = \sigma_X^2 + \frac{(1 - q)}{(1 + r)^2} \frac{\sigma_X^4 N'}{\sigma_X^2 N' + \sigma_e^2},
\]

which is decreasing in \( q \). That is, if there is no participation in the current period \( (N = 0) \) and no participation is expected for the following period \( (q \approx 1) \), the variance of the total payoff is equal to the dividend’s unconditional variance \( \sigma_X^2 \). The reason is that no information is available to reduce the dividend’s variance below its unconditional variance when current participation is zero; moreover, no price risk is borne because the price is constant when market participation is zero (and no information reaches the market in the following period as well). On the other hand, if \( q < 1 \), some price risk is borne in addition to the dividend risk, since there is a positive probability of switching to the state with positive participation \( N' \), where the price becomes responsive to the arrival of some information.

With these definitions, the equilibrium participation conditions (2.17) for \( N_H \) and \( N_L \) can be written as

\[
G(N_H) = V(N_H, N_L, q)
\]

\[
G(N_L) = V(N_L, N_H, q).
\]

Figure 2 plots \( G(N) \), \( V(N, N_L, q) \) (the variance of the total payoff when the current participation is \( N \), and participation in the next period is \( N \) with probability \( q \) and \( N_L \) with probability \( 1 - q \)), and \( V(N, N_H, q) \) (the variance of the total payoff when the current participation is \( N \), \( 36 \)
and participation in the next period is $N$ with probability $q$ and $N_H$ with probability $1-q$). The loci $V(N, N_L, q)$ and $V(N, N_H, q)$ are drawn under the assumption that $N_H > N_L$, and thus $V(N, N_L, q) > V(N, N_H, q)$ at any $N$, since the price risk is high when the state can switch from $N$ to $N_H$ than from $N$ to $N_L$. The intersection between $V(N, N_L, q)$ and $G(N)$ defines $N_H$, and the intersection between $V(N, N_H, q)$ and $G(N)$ defines $N_L$; in equilibrium such intersections must coincide with the values $N_H$ and $N_L$ used to draw the loci $V(N, N_L, q)$ and $V(N, N_H, q)$. However, it is clear from Figure 2 that any solution implies necessarily $N_H < N_L$, contradicting the assumption that $N_H > N_L$. Therefore, the only solution to the pair of equilibrium conditions is $N_H = N_L$, that is, equilibrium participation is unique.

Equilibrium uniqueness is not due to the assumed symmetry of the equilibrium selection process. Consider the following example in which the market is liquid “almost surely”, $q_H \approx 1$. That is, when the current participation level is high ($N_H$), it is almost certainly high in the following period as well; however, there is a small probability that market participation falls in the following period to some value $N_L < N_H$. When participation falls to the low level, it
remains low with probability $q_L$, and switches back to the high level with probability $1-q_L$. The probability $q_L$ can thus be thought of as the severity of the liquidity crisis, since it determines the persistence of the low participation state. High participation $N_H$ is defined by

$$V(N_{H,L}, 1) = G_H(N_H)$$  \hspace{1cm} (2.21)

where the dot indicates that $N_H$ does not depend on $N_L$ when $q_H \approx 1$. Low participation $N_L < N_H$ is defined as before by

$$V(N_L, N_H, q_L) = G_L(N_L)$$  \hspace{1cm} (2.22)

In (2.21) and (2.22), I have introduced the subscripts $H$ and $L$ in the function $G$, but for the moment I suppose that $G_L(.) \equiv G_H(.)$. The equilibrium conditions (2.21) and (2.22) define a recursive system in $(N_L, N_H)$; to be admissible, a solution to this system has to satisfy $N_L < N_H$. Figure 3 plots two loci $G_L(.)$ and $G_H(.)$ corresponding to two different participation costs, and
the loci $V(N, *, 1)$ and $V(N, N_H, q_L)$. Ignoring for the moment $G_L(\cdot)$, the figure shows that it is impossible to have a distinct solution for $N_L$ and $N_H$: equilibrium participation is unique, corresponding to $N_H$ at which $G_H(N)$, $V(N, *, 1)$, and $V(N, N_H, 1)$ intersect. However, it is also clear that it would be possible to have two participation levels in equilibrium if the participation cost depended on the participation level. Consider for example the following participation cost:

$$C(N) = \begin{cases} 
C & \text{if } N \geq \hat{N} \\
C + \Delta_c & \text{if } N < \hat{N}
\end{cases}$$  \hspace{1cm} (2.23)

where $C$, $\Delta_c$ and $\hat{N}$ are positive constants. With this simple modification, there can exist two participation levels, $N_L < N_H$. The condition (2.21), with $G_H(\cdot)$ defined from (2.19) using $C$ as participation cost, yields the high participation level $N_H$. Given $N_H$, the condition (2.22), with $G_L(\cdot)$ defined from (2.19) using $C + \Delta_c$ in place of $C$ as participation cost, defines $N_L$. The function $G_L(\cdot)$ lies strictly above the function $G_H(\cdot)$ if and only if $\Delta_c > 0$, in which case $N_L < N_H$. To be consistent with the participation cost used in the derivation of $N_H$ and $N_L$, the solutions for $N_H$ and $N_L$ have to satisfy $N_H \geq \hat{N} > N_L$.

The following proposition summarizes the discussion so far.

**Proposition 4.** Suppose that there exists two equilibrium participation levels, $N_H \geq N_L$.

Suppose that: (i) when the current equilibrium is $N_H$, it is selected in the following period with probability $q_H \approx 1$; (ii) when the current equilibrium is $N_L$, it is selected in the following period with probability $q_L < 1$. Then, if $\Delta_c = 0$ in the participation cost (2.23), $N_H = N_L$.

Define $N_H$ the solution to (2.21) when the participation cost is $C$ and $N_L$ the solution to (2.22) when the participation cost is $C + \Delta_c$, and set $\hat{N} = N_H$. Then, $\Delta_c > 0$ if and only if $N_H > N_L$. □

Figure 3 shows that the total risk per share of the risky asset (the vertical intercept) is larger in the low participation equilibrium. This property supports the interpretation of the participation cost as the opportunity cost of the capital that an agent has to hold against her holdings of the risky asset in compliance with risk-based capital requirement and risk-management systems. To see why, note that in the low participation equilibrium, each agent participating in the market holds a larger portfolio of the risky asset; furthermore, the total
risk per share is higher in the low participation equilibrium. Thus, the portfolio's Value at Risk (VaR)\textsuperscript{17} of each agent considering whether to participate in the market is higher when $N_L$ agents participate than when $N_H$ agents participate. If the amount of capital that has to be allocated against a position is increasing in the portfolio's VaR, then the total capital requirement is higher in the low participation equilibrium, justifying $\Delta_c > 0$; furthermore, the higher $\Delta_c$ the higher the variance of the total payoff (dividend and resale value) on a unit position. According to this interpretation, then, risk-based capital requirements can contribute to the existence of "bad" equilibria in participation and coordination failures, and thus can be destabilizing.

The equilibrium showed in Figure 3 has two interesting implications:

- for fixed $q_L$, that is, severity of the liquidity crisis, the higher $\Delta_c$ the lower the equilibrium participation $N_L$ in the low-participation equilibrium;

- for fixed $\Delta_c$, the higher $q_L$ the lower the equilibrium participation $N_L$ in the low-participation equilibrium.

The two properties state that the serial correlation in equilibrium selection and the sensitivity of the participation cost to market participation tend to reinforce each other in reducing market participation and increasing the volatility of total return. That is, the dynamic structure of pricing amplifies the potential instability caused by state-dependent participation costs.

2.5.5 Liquidity crashes with slow recoveries

Rather than an exogenous feature, I show next that the state dependence of the participation cost can result from the dynamic interaction between market participation and asset price volatility. Consider the following example, in which I conjecture that there exist three participation levels $N_H > N_M > N_L$ (high, intermediate, and low states respectively) under the following restrictions to the transition probabilities:

\textsuperscript{17}Recall that the Value at Risk of a portfolio is the quantile at a prespecified confidence level of the distribution of the net profits and losses over a given time horizon.
• if the current state is high, \( N_H \), it is chosen "almost surely" in the following period; with a small probability, however, liquidity "collapses", that is, the state switches to the low state \( N_L \);

• if the current state is low, \( N_L \), it remains low in the following period with probability \( 0 < q_L < 1 \), and switches to the intermediate state with probability \( 1 - q_L \);

• if the current state is intermediate, it switches back to the low state with probability \( 0 < q_M < 1 \), and switches to the high state with probability \( 1 - q_M \).

The first restriction captures the notion that liquidity crises are rare but possible events. A liquidity crisis takes the form of a crash, as participation (liquidity) switches from the highest to the lowest level without a transition through the intermediate state. The second and third restrictions state that the recovery from the liquidity crash is slow, since it does not start for sure \( (q_L > 0) \), and when it does (the state switches from low to intermediate), there is a positive probability that liquidity crashes again to the low state \( (q_M > 0) \). The transition probabilities \([q_{hk}]\) are summarized in the following table, where \( q_{hk} \) is the probability of the state \( k \) in the next period given that the current state is \( h \), for \( h, k \in \{N_H, N_M, N_L\} \):

<table>
<thead>
<tr>
<th>1 - ε</th>
<th>0</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - q_M</td>
<td>0</td>
<td>q_M</td>
</tr>
<tr>
<td>0</td>
<td>1 - q_L</td>
<td>q_L</td>
</tr>
</tbody>
</table>

For these transition probabilities, the equilibrium condition (2.17) implies that \( N_H, N_M \) and \( N_L \) have to satisfy the following system,

\[
\frac{\sigma^2_X \sigma^2_t}{\sigma^2_X N_H + \sigma^2_t} + \frac{1}{(1 + r)^2} \frac{\sigma^4_X}{\sigma^2_X + \sigma^2_t/N_H} = G(N_H) \tag{2.24}
\]

\[
\frac{\sigma^2_X \sigma^2_t}{\sigma^2_X N_M + \sigma^2_t} + \frac{(1 - q_M)}{(1 + r)^2} \frac{\sigma^4_X}{\sigma^2_X + \sigma^2_t/N_H} + \frac{q_M}{(1 + r)^2} \frac{\sigma^4_X}{\sigma^2_X + \sigma^2_t/N_L} = G(N_M) \tag{2.25}
\]

\[
\frac{\sigma^2_X \sigma^2_t}{\sigma^2_X N_L + \sigma^2_t} + \frac{(1 - q_L)}{(1 + r)^2} \frac{\sigma^4_X}{\sigma^2_X + \sigma^2_t/N_M} + \frac{q_L}{(1 + r)^2} \frac{\sigma^4_X}{\sigma^2_X + \sigma^2_t/N_L} = G(N_L) \tag{2.26}
\]

where the function \( G(.) \) has been defined in (2.19). The assumption that the probability is close to one implies that (2.24) defines the high state \( N_H \). I verify the conjecture by proving
the three following claims (proved in the Appendix).

Claim 1. Suppose that \( N_H > N_L \). Then \( N_H > N_M \). □

Claim 2. Suppose that \( N_H > N_M \). Then \( N_H > N_L \). □

The first two claims show that \( N_H > N_L \) and \( N_H > N_M \). To verify the conjecture, it remains to check that \( N_M > N_L \).

Claim 3. If \( q_M = q_L \), then \( N_M > N_L \). If \( q_M \neq q_L \), it is still the case that \( N_M > N_L \) if \( q_M \) and \( q_L \) are sufficiently close to each other. □

Consider for instance Claim 2, that is, why there exists a low participation \( N_L \) strictly below the high participation level \( N_H \). Suppose that the current participation is low, so that the dividend risk is given by \( \sigma_X^2 \sigma_z^2 / (\sigma_X^2 N_L + \sigma_z^2) \), the first term in the left-hand side of (2.26). Under the assumed transition probabilities, the recovery to the high state is slow and goes through the interim state \( N_M \); it follows that the price risk borne by the market participants is lower than the price risk they would bear if the transition occurred directly to the high state, that is, if with probability \( 1 - q_L \) the state switched to the high state rather than to the intermediate state. In other words, given that the state is currently low, the slow transition keeps the price risk lower. The same participation levels \( N_H \) and \( N_L \), with \( N_H > N_L \), would be consistent with equilibrium if the participation cost were higher in the low state and recovery occurred directly from the low to the high state (with probability \( 1 - q_L \)), implying a larger price risk. Indeed, the proof of Claim 2 shows that (2.26) can be written as

\[
\frac{\sigma_X^2 \sigma_z^2}{\sigma_X^2 N_L + \sigma_z^2} + \frac{(1 - q_L)}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_z^2 / N_H} + \frac{q_L}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_z^2 / N_L} = \tilde{G}(N_L)
\]

where \( \tilde{G}(.) > G(.) \). The pair of equilibrium conditions for \( N_H \) and \( N_L \) is thus formally equivalent to the pair of equilibrium conditions discussed in the previous section with state-dependent participation cost. A similar argument can be made with regard to Claims 2 and 3.

From the equilibrium conditions (2.24)-(2.26) and the fact that the function \( G(.) \) defined in (2.19) is increasing in market participation, the variance of the total payoff \( X_{t+1} + p_{t+1} \) can be ranked in terms of the participation level from high to low (that is, the variance is high when participation is high, and so on). However, the same ranking does not hold necessarily for the variance of the rate of return \( \frac{X_{t+1} + p_{t+1}}{p_t} \). The reason is that in spite of the variance of the total
return being higher when participation is high, the market ability to bear risk is also higher.\textsuperscript{18} As a result, the equilibrium risk premium can be lower, and the variance of the rate of return lower, in the high participation state than in the intermediate and low participation states.

2.6 Conclusions

In this chapter, I have studied a model of a risky asset market in which market participation is costly for some agents. Participation generates a positive externality, since it improves the allocative efficiency of the price system by reducing the uncertainty about the risky asset’s dividend. However, it can also generate a negative externality through the increased responsiveness of the price to the arrival of new information, because the agents with a finite horizon who liquidate their asset holdings at the equilibrium price face a more volatile price (that is, the increased responsiveness of price to information raises the liquidity risk of current market participants). The interaction between the two externalities can give rise to multiple equilibria in market participation. Thus, as market participation admits the interpretation of market liquidity, market liquidity can change over time in equilibrium.

In equilibrium, participation (liquidity) is a state variable for the equilibrium price of the risky asset. By affecting the quality of the aggregate information, participation is also a state variable for the volatility of the equilibrium asset price. The straightforward implication of the model is therefore that volatility comoves negatively with liquidity. More interestingly, when volatility and liquidity change over time, they have to be serially correlated. The association between the liquidity of a market and the serial correlation in asset price volatility is an interesting prediction of the model, and suggests a direction for future empirical research.

With regard to the policy implications, this chapter emphasizes one cost of better information revelation when some agents are exposed to liquidity risk due to their short investment horizon. The chapter also confirms the potential destabilizing effect of capital requirements indexed to market volatility. In the model, financial instability takes the form of the existence of low participation equilibria and coordination failures. The problem of financial instabil-

\textsuperscript{18}Recall that the average risk aversion $\lambda(N)$ is decreasing in market participation. Furthermore, given the fixed supply of the risky asset, the per capita holding of the risky asset is the lowest in the high participation equilibrium.
ity, though, can be more severe than argued in policy discussions, as the existence of “bad” equilibria does not rely necessarily on the sensitivity to market variables (such as volatility) of capital requirements and risk management systems, but is a feature of the coordination problem between agents in a dynamic context.

This chapter has emphasized the public nature of liquidity, and the role of coordination between private agents in supporting the liquidity of a market. Some aspects of the behavior of liquidity in certain markets can be ascribed to the coordination between private agents. An example is the difference in liquidity between on-the-run and off-the-run issues of the US Government securities: securities that have essentially the same fundamentals have markedly distinct liquidity properties, as the on-the-run issues are regarded as liquidity vehicles by many market participants.\(^{19}\) One weakness of this approach is that it is to some extent vacuous in pinning down the determinants of the liquidity of a financial security, since the equilibrium selection is driven by exogenous sunspot variables. An interesting issue for future research is the relation between the characteristics of a security and its issuer on one hand and the coordination problem of market participants on the other.

### 2.7 Appendix

#### 2.7.1 Proof of Lemma 1

I will show that the equilibrium price \( p_{t+1} \) will depend on the participation level \( N' \) in period \( t + 1 \), which is distributed according to the probability distribution \( Q_N \).

Conditional on the current market participation \( N \) and the asset price \( p_t \) (which reveals the average of the private signals \( \bar{y}_t \)), the expected utility of an old agent of type \( i \) born in period \( t - 1 \) who participates in the risky asset market in period \( t \) is given by

\[
U_{t,i,t-1} = (1 + r)w_t + \{E[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t\} \theta_{t,i,t-1}^{t,t-1}
\]

\[
-\frac{1}{2} \rho(t)E_{Q_N} [\{X_{t+1} + p_{t+1}|p_t, N'\}](\theta_{t,i,t-1}^{t,t-1})^2
\]

\(^{19}\) See, for instance, Krishnamurthy (2001).
which is maximized by the portfolio demand

\[ \theta_{t}^{i,t-1} = \frac{\mathbb{E}[X_{t+1} + p_{t+1} | p_{t}] - (1 + r)p_{t}}{\rho(i)\mathbb{E}_{Q_{N}}[\mathbb{V}[X_{t+1} + p_{t+1} | p_{t}, N']]} \].

I will show in Lemma 2 that the risk premium \( \mathbb{E}[X_{t+1} + p_{t+1} | p_{t}] - (1 + r)p_{t} \) is nonstochastic conditional on the current participation level \( N \). This property simplifies the derivation of the equilibrium considerably.

Consider now a young agent of type \( i \) born in period \( t \). The agent’s final wealth is \( w_{t+2} = (1 + r)w_{t+1} + \theta_{t+1}^{i,t} [X_{t+2} + p_{t+2} - (1 + r)p_{t+1}] \), where \( \theta_{t+1}^{i,t} \) is the demand for the risky asset that will be chosen in period \( t+1 \) by the agent (it is identically zero if the agent does not participate). The demand \( \theta_{t+1}^{i,t} \) will depend on the participation level \( N' \) that will be occur in period \( t+1 \) (which is still unknown when market clears in period \( t \)). Furthermore, Lemma 3 proved below implies that the excess return \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) in period \( t+1 \) is uncorrelated with the agent’s wealth \( w_{t+1} \) at the end of period \( t \). Therefore, what matters for a young agent’s demand for the risky asset in period \( t+1 \) is \( (1 + r)w_{t+1} \). This argument implies that

\[ \mathcal{U}_{t}^{i,t} = (1 + r)\mathbb{E}[w_{t+1} | p_{t}] - \frac{1}{2} \rho(i)(1 + r)^2 \mathbb{E}_{Q_{N}}[\mathbb{V}[w_{t+1} | p_{t}, N']] + \kappa_{i}^{t}, \]

where \( \kappa_{i}^{t} \) depends on \( N \) but not on the portfolio demand \( \theta_{t+1}^{i,t} \) for period \( t \). This expression gives in turn the optimal portfolio demand of a young agent in period \( t \),

\[ \theta_{t}^{i,t} = \frac{1}{(1 + r)} \frac{\mathbb{E}[X_{t+1} + p_{t+1} | p_{t}] - (1 + r)p_{t}}{\rho(i)\mathbb{E}_{Q_{N}}[\mathbb{V}[X_{t+1} + p_{t+1} | p_{t}, N']]} \).

### 2.7.2 Proof of Proposition 1

Suppose that \( N \) old agents participate in the risky asset market in period \( t \). Denote by \( N' \in \mathcal{N} \) the participation level that can occur at \( t+1 \), and by \( Q_{N}(.) \) the probability distribution of \( N' \) over \( \mathcal{N} \). All the young \( I \) agents participate in the risky asset market. The equilibrium price function is as conjectured,

\[
p_{t} = a + b(\bar{y}_{t} - \bar{X}) = a + b\left(\bar{X}_{t+1} - \bar{X}\right) + b\frac{\sum_{i \in \mathcal{N}} \mu_{i}^{t}}{\mathcal{N}}
\]
where \( a = a(N) \) and \( b = b(N) \).

As in Grossman (1976), the equilibrium price provides a sufficient statistic for the set of private signals. Using the conjectured equilibrium, the projection of \( X_{t+1} \) on \( p_t \) is given by

\[
\mathbb{E}[X_{t+1}|p_t] = \bar{X} + \frac{1}{b} \frac{\sigma_X^2}{\sigma_X^2 + \sigma_t^2/N} (p_t - a)
\]

\[
= \bar{X} + (1 + r)(p_t - a)
\]

\[
= \bar{X} + \frac{\sigma_X^2}{\sigma_X^2 + \sigma_t^2/N} (\bar{y}_t - \bar{X})
\]

where the second equality follows from the definition of \( b \) in (2.13), and

\[
\mathbb{V}[X_{t+1}|p_t, N] = \frac{\sigma_X^2 \sigma_t^2}{\sigma_X^2 N + \sigma_t^2}.
\]

By using the asset demands (2.8) and (2.9) in the market clearing condition for the risky asset in period \( t \),

\[
\sum_{i \in I} \theta_i^{t,t} + \sum_{i \in N} \theta_i^{t-1} = Z
\]

the equilibrium price in period \( t \) is given by

\[
p_t = \frac{1}{1 + r} \left[ \mathbb{E}[X_{t+1} + p_{t+1}|p_t] - \Lambda(N) \frac{Z}{I + N} \mathbb{E}_{Q_N}[\mathbb{V}[X_{t+1} + p_{t+1}|p_t, N']] \right]
\]

where \( 1/\Lambda(N) \) is defined in the statement of the proposition.

By using the conjectured equilibrium price function into \( \mathbb{E}[X_{t+1}|p_t] \) computed at the beginning of the proof, the equilibrium price at \( t \) is

\[
p_t = \frac{1}{1 + r} \left[ \bar{X} + \frac{\sigma_X^2}{\sigma_X^2 + \sigma_t^2/N} (\bar{y}_t - \bar{X}) + \mathbb{E}[p_{t+1}|p_t] \right]
\]

\[
- \frac{1}{1 + r} \Lambda(N) \frac{Z}{I + N} \left[ \frac{\sigma_X^2 \sigma_t^2}{\sigma_X^2 N + \sigma_t^2} + \mathbb{E}_{Q_N}[\mathbb{V}[p_{t+1}|p_t, N'] + 2\text{Cov}(X_{t+1}, p_{t+1}|p_t, N')] \right]
\]

By equating this price function to the initial conjecture, I find the expressions for the coefficients \( a(N) \) and \( b(N) \),
\[ a(N) = \frac{1}{1+r} \left[ \bar{X} + \mathbb{E}[p_{t+1} | p_t] - \frac{\Lambda(N)}{I + N} \frac{Z}{\sigma^2 + \sigma^2 \varepsilon} + \mathbb{E}_{N'} \left[ \mathbb{V}[p_{t+1} | p_t, N'] + 2\text{Cov}(X_{t+1}, p_{t+1} | p_t, N') \right] \right] \]

\[ b(N) = \frac{1}{1+r} \frac{\sigma^2}{\bar{X} + \sigma^2 \varepsilon / N}. \]

Finally, I have to compute the conditional moments \( \mathbb{E}_{N'}[\mathbb{V}[p_{t+1} | p_t, N']] \) and \( \mathbb{E}_{N'}[\text{Cov}(X_{t+1}, p_{t+1} | p_t, N')] \).

First note that the covariance between the next period’s dividend \( X_{t+1} \) and next period’s asset price \( p_{t+1} \) conditional on \( N' \) is zero:

\[ \text{Cov}(X_{t+1}, p_{t+1} | p_t, N') = \text{Cov}(X_{t+1}, a(N') + b(N')(X_{t+2} + \sum_{i \in N'} \frac{\epsilon_{i+1}^t}{N'} - \bar{X}) | p_t, N') \]

\[ = \text{Cov}(X_{t+1}, b(N') X_{t+2} + b(N') \sum_{i \in N'} \frac{\epsilon_{i+1}^t}{N'} | p_t, N') Q_N(N') \]

\[ = 0 \]

because \( X_{t+1} \) is independent of \( X_{t+2} \) and the noise terms \( \epsilon_{i+1}^t \).

The other two conditional moments \( \mathbb{E}[p_{t+1} | p_t] \) and \( \mathbb{E}_{N'}[\mathbb{V}[p_{t+1} | p_t, N']] \) depend on the set of possible participation levels \( N' \) that can occur in the following period. Given the conjectured price function, they are given by

\[ \mathbb{E}[p_{t+1} | p_t] = \sum_{N' \in N} \mathbb{E}[p_{t+1} | p_t, N'] Q_N(N') = \sum_{N' \in N} a(N') Q_N(N') \]

\[ \mathbb{E}_{N'}[\mathbb{V}[p_{t+1} | p_t, N']] = \sum_{N' \in N} \frac{1}{(1+r)^2} \frac{\sigma^4}{\bar{X} + \sigma^2 \varepsilon / N'} Q_N(N'). \]

Thus, the constant term \( a(N) \) satisfies the following equation:

\[ a(N) = \frac{1}{1+r} \left[ \bar{X} + \sum_{N' \in N} a(N') Q_N(N') \right] - \]

\[ \frac{1}{1+r} \frac{\Lambda(N)}{I + N_t} \frac{Z}{\sigma^2 + \sigma^2 \varepsilon} + \sum_{N' \in N} \frac{1}{(1+r)^2} \frac{\sigma^4}{\sigma^2 N_t + \sigma^2 \varepsilon / N'} Q_N(N'). \]
2.7.3 Proof of Lemma 2

First note that from Proposition 1,

\[ \mathbb{E}[X_{t+1}|p_t] = \bar{X} + (1 + r)(p_t - a(N)) \]
\[ \mathbb{E}[p_{t+1}|p_t] = \mathbb{E}_{Q_N}[a(N')|p_t] \]

Substituting into the definition of the risk premium,

\[ Y_t = \mathbb{E}[X_{t+1} + p_{t+1}|p_t] - (1 + r)p_t = \]
\[ = \bar{X} + (1 + r)(p_t - a(N)) + \mathbb{E}_{Q_N}[a(N')|p_t] - (1 + r)p_t \]
\[ = \bar{X} + \mathbb{E}_{Q_N}[a(N')|p_t] - (1 + r)a(N) \]
\[ = \Lambda(N) \frac{Z}{I + N} \left\{ \frac{\sigma_x^2 \sigma_y^2}{\sigma_x^2 N + \sigma_y^2} + \mathbb{E}_{Q_N} \left[ \frac{1}{(1 + r)^2} \frac{\sigma_x^4}{\sigma_x^2 + \sigma_y^2/N'} \right] \right\} \]

where the last equality follows from the functional equation (2.11) for \( a(N) \) in Proposition 1.

2.7.4 Proof of Lemma 3

I will show that conditional on the information available to the agent in period \( t \) and the belief \( N' \), \( w_{t+1}^t \) and \( [X_{t+2} + p_{t+2} - (1 + r)p_{t+1}] \) are orthogonal, since \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) is equal to the residual in the projection of \( X_{t+2} + p_{t+2} \) on \( p_{t+1} \) plus a term that depends on \( N' \) only. Since conditional on the information available to the agent, the only stochastic component in \( w_{t+1}^t \) is \( X_{t+1} + p_{t+1} \) (see (2.4)), then \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) is orthogonal to \( w_{t+1}^t \). To see this point, note that from the projection of \( X_{t+2} \) on \( p_{t+1} \) (see proof of Proposition 1),

\[ \mathbb{E}[X_{t+2}|p_{t+1}, N'] = \bar{X} + (1 + r)(p_{t+1} - a(N')) \]
\[ \mathbb{E}[p_{t+2}|p_{t+1}, N'] = \mathbb{E}[a(N'')|p_{t+1}, N'] + \mathbb{E}[b(N'')(X_{t+3} + \sum_{i \in N''} \bar{c}_{t+2}^{i} - \bar{X}|p_{t+1}, N') \\
= \mathbb{E}[a(N'')|p_{t+1}, N'] + \mathbb{E}[b(N'')\mathbb{E}[X_{t+3} + \sum_{i \in N''} \bar{c}_{t+2}^{i} - \bar{X}|N'', \bar{p}_{t+1}|p_{t+1}, N'] \\
= \mathbb{E}[a(N'')|p_{t+1}, N'] \\
= \mathbb{E}_{Q_N}[a(N'')] \]

where \( N'' \) is the participation in \( t + 2 \). Therefore,

\[ X_{t+2} + p_{t+2} - \mathbb{E}[X_{t+2} + p_{t+2}|p_{t+1}, N'] \\
= X_{t+2} + p_{t+2} - \bar{X} - (1 + r)(p_{t+1} - a(N')) - \mathbb{E}_{Q_N}[a(N'')] \\
= X_{t+2} + p_{t+2} - (1 + r)p_{t+1} - (\bar{X} - (1 + r)a + \mathbb{E}_{Q_N}[a(N'')]) \]

i.e. \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) is equal to the residual in the projection of \( X_{t+2} + p_{t+2} \) on \( p_{t+1} \) plus the term \( \bar{X} - (1 + r)a + \mathbb{E}_{Q_N}[a(N'')] \) that depends on \( N' \) only,

\[ X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \]

\[ = [\bar{X} - (1 + r)a + \mathbb{E}_{Q_N}[a(N'')]] + [X_{t+2} + p_{t+2} - \mathbb{E}[X_{t+2} + p_{t+2}|p_{t+1}, N']] \]

This result implies that \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) is orthogonal to \( p_{t+1} \). It is also orthogonal to \( X_{t+1} \), since \( \{X_t\} \) is an iid sequence. Therefore, \( X_{t+2} + p_{t+2} - (1 + r)p_{t+1} \) is orthogonal to \( w_{t+1}^{i} \).

### 2.7.5 Proof of Proposition 2

Consider the participation decision problem before entering period \( t + 1 \) of an agent of type \( i \) born in period \( t \). Before observing her own signal and the equilibrium price at \( t + 1 \), she has to decide whether to incur the participation cost \( C \) and participate in the risky asset market, or to liquidate her risky asset holdings and invest in the riskless asset until \( t + 2 \), when consumption occurs. When agent \( i \) makes her participation decision, she believes that
the market participation will be $N'$.  

Consider first the case in which agent $i$ decides to participate. If she participates, she will demand

$$\theta_{i,t+1}^t = \frac{E[X_{t+2} + p_{t+2}[pt_{t+1}, N'] - (1+r)pt_{t+1}}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]} = \frac{Y_{t+1}}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]}$$

Both the numerator and denominator depend only on $N'$, and hence are nonstochastic from the perspective of an agent who anticipates the participation level $N'$ for period $t + 1$. Hence, if she participates, her consumption at the beginning of period $t + 2$ will be

$$w_{i,t+2}^t = \left[ X_{t+2} + pt_{t+2} - (1+r)pt_{t+1}\right] \frac{Y_{t+1}}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]} + (1+r)w_{i,t+1}^t - \frac{1}{2} C$$

This expression can be used to compute the expected value and variance of the agent’s consumption if she participates. The expected value of wealth is given by:

$$E[w_{i,t+2}^t|pt,N'] = \frac{Y_{t+1}^2}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]} + (1+r)E[w_{i,t+1}^t|N'] - \frac{1}{2} C$$

With regard to the variance, Lemma 3 implies that

$$E_N[V[w_{i,t+2}^t|N']] = \left[\frac{Y_{t+1}}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]}\right]^2 E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']] + (1+r)^2 E[w_{i,t+1}^t|N']$$

Then, it follows that the expected utility from participating, $U_{i,t+1}^P$, is given by

$$U_{i,t+1}^P = U_{i,t+1}^L + \frac{1}{2} \left[ \frac{Y_{t+1}^2}{\rho(i) E_N[V[X_{t+2} + p_{t+2}[pt_{t+1}, N'']]} - C \right]$$

where I have denoted by $U_{i,t+1}^L$ agent $i$’s expected utility if she does not participate,
\[ \mathcal{U}_{i,t+1}^L = (1 + r)E[w_{i,t+1} | N'] - \frac{1}{2} \rho(i)(1 + r)^2 V[w_{i,t+1} | N']. \]

Therefore, an agent of type \( i \) participates if

\[ \frac{1}{\rho(i)} \frac{Y_{t+1}^2}{E_{Q_N}[V[X_{t+2} + p_{t+2} | p_{t+1}, N']]} \geq C \]

and not participate if this inequality does not hold. An equilibrium participation \( N' \) satisfies

\[ \frac{Y_{t+1}^2}{E_{Q_N}[V[X_{t+2} + p_{t+2} | p_{t+1}, N']]} = \rho(N') C. \]

Lemma 1 then implies that the equilibrium condition can be written as

\[ \Lambda(N)^2 \left( \frac{Z}{I + N} \right)^2 \left[ \frac{\sigma^2 \sigma^2}{\sigma^2 N + \sigma^2} + \sum_{N' \in N} \frac{1}{(1 + r)^2 \sigma_X^2 + \sigma^2 / N'} Q_N(N') \right] \geq \rho(N) C. \]

and the reverse inequality for type \( N + 1 \).

### 2.7.6 Proof of Claim 1

The equilibrium condition (2.17) for \( N_M \) can be written as

\[ \frac{\sigma^2 \sigma^2}{\sigma^2 N_M + \sigma^2} + \frac{1}{(1 + r)^2 \sigma_X^2 + \sigma^2 / N_M} = \hat{G}(N_M) \tag{2.27} \]

where I have defined a new adjusted cost for participation \( \hat{G}(.) \),

\[ \hat{G}(N) \equiv G(N) - \frac{(1 - q_M)}{(1 + r)^2} \left[ \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_H} - \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N} \right] - \frac{q_M}{(1 + r)^2} \left[ \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_L} - \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N} \right] \]

\( \hat{G}(.) \) is an increasing function in \( N \). Note first that \( \hat{G}(N_H) > G(N_H) \) if and only if

\[ \frac{q_M}{(1 + r)^2} \left[ \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_L} - \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_H} \right] < 0 \]
which is satisfied if and only if $N_H > N_L$. Also, $\hat{G}(0) < G(0)$. Define $0 < N^* < N_H$ such that $\hat{G}(N^*) = G(N^*)$, that is,

$$(1 - q_M) \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_H} + q_M \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N_L} = \frac{\sigma_X^4}{\sigma_X^2 + \sigma^2 / N^*}.$$ 

A sufficient condition for $N_H > N_M$ is that $N_H > N^*$, in which case $N_M > N^*$. Suppose that $N_H \leq N^*$. Then, (2.24) implies that

$$\frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_H + \sigma_e^2} + \frac{1}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N^*} \geq G(N_H)$$

and using the definition of $N^*$,

$$\frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_H + \sigma_e^2} + \frac{1}{(1 + r)^2} \left[ (1 - q_M) \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_H} + q_M \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_L} \right] \geq G(N_H)$$

but if $N_H > N_L$, from (2.24),

$$\frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_H + \sigma_e^2} + \left[ (1 - q_M) \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_H} + q_M \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_L} \right] < G(N_H).$$

This shows that $N_H > N^*$, and therefore $N_H > N_M$.

### 2.7.7 Proof of Claim 2

Rewrite (2.26) as

$$\frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_L + \sigma_e^2} + \frac{(1 - q_L)}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_H} + \frac{q_L}{(1 + r)^2} \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_L} = \tilde{G}(N_L) \quad (2.28)$$

where

$$\tilde{G}(.) \equiv G(.) + \frac{(1 - q_L)}{(1 + r)^2} \left[ \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_H} - \frac{\sigma_X^4}{\sigma_X^2 + \sigma_e^2 / N_M} \right] > G(.)$$

The previous analysis on state-dependent participation cost can be applied directly to the present case. Since $\tilde{G}(.) > G(.)$, it must be the case that $N_H > N_L$. 

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2.7.8 Proof of Claim 3

Suppose that \( N_M \leq N_L \). Then, (2.25) and (2.26) imply that

\[
\frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_M + \sigma_e^2} + \frac{(1-q)}{(1+r)^2} \frac{\sigma_X^4}{\sigma_X^2 N_M + \sigma_e^2/N_H} \leq \frac{\sigma_X^2 \sigma_e^2}{\sigma_X^2 N_L + \sigma_e^2} + \frac{(1-q)}{(1+r)^2} \frac{\sigma_X^4}{\sigma_X^2 N_L + \sigma_e^2/N_M}
\]

This inequality is impossible: since \( N_H > N_M \) (Claim 2) and \( N_M \leq N_L \), the left-hand side is strictly larger than the right-hand side.
Chapter 3

The Design of Financial Contracts and Capital Requirements

3.1 Introduction

Many financial contracts include covenants that can trigger actions with termination-type features. Examples include credit lines, securities lending, and margin accounts at futures exchanges; these contracts are often secured by the initial provision of collateral and allow for "margin calls", the demand of additional collateral during the life of the contract. The failure to meet a margin call typically leads to the termination of the contract. Financial incentives and termination threats do not coexist only in private contracts. *Ex-ante* measures (such as capital and reserve requirements) are usually associated with more or less explicit performance-contingent penalties in the regulation of financial institutions. A notable example of such coexistence is the Amendment to the Basle Accord regarding the proprietary trading activities of banks (BIS (1996a,b)). The Amendment sets up an incentive system based on a combination of capital requirements (increasing in the market risk of the bank's trading portfolio) and penalties dependent on the realized trading outcomes.\(^1\) The Amendment recognizes that the problem of distinguishing the outcomes under the control of the banks from exogenous shocks

\(^1\) The penalties take the form of an increase in future capital requirements. I will describe the current regulation in some more detail in Section 5. The quotes that follow are from BIS (1996b).
is more difficult in times of market volatility, when “noise” is more important. Accordingly, the Amendment recommends the national regulators to relax the penalties if poor performance is due to “an unexpected bout of high market volatility, which nearly all [risk] models may fail to predict”, that is, when poor performance is due to “bad luck” rather than to a flaw in the bank’s risk model or to imprudent behavior.

This chapter addresses the optimal balance between financial incentives and termination threats in a contract between a borrower and a lender. The analysis of this problem presumes an incentive problem, a role for financial incentives, and a technology for termination. A simple way to introduce an incentive problem is to consider a moral hazard model of investment. Moral hazard also permits me to introduce an endogenous capital structure decision, and thus a role for financial incentives, parsimoniously. In the model presented in Section 2, an agent with an investment opportunity (the “borrower”) turns to an outside investor (the “lender”) to finance a fraction of the initial cost of an investment project. The borrower’s investment is subject to moral hazard. That is, the borrower, who I interpret as a financial institution that borrows from the capital markets, makes an investment decision that is unobservable by the lender and that affects the distribution of her investment project. The moral hazard problem implies that the contract has to provide the borrower with incentives to choose the efficient action. I consider two types of incentive devices, financial incentives and termination threats. Financial (or pecuniary) incentives require the borrower to finance a sufficiently large share of the project with own funds. Because the borrower’s cost of capital is higher than the lender’s, though, financial incentives are costly to provide. Incentives are also provided by the threat of termination. Based on information about the borrower’s performance, the borrower-lender relationship can be (partially) terminated.

I am interested in a particular form of termination, the liquidation of a fraction of the project on an asset market. The liquidity of the asset market, that is, how the liquidation price for the project responds to the aggregate volume of liquidation, affects the optimal balance between the two incentive devices. Furthermore, the quality of the information available for performance evaluation determines the value of termination as an incentive device. By construction, performance evaluation is difficult in any state of nature in which poor performance is diffuse across

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2In Section 2, I discuss empirical evidence that justifies this implication of market volatility.
different borrowers. The likelihood of these states of nature determines the quality of information for performance evaluation. It is costly to terminate the project when poor performance is due to an aggregate state beyond the borrower’s control, but the performance information may not permit the lender to discriminate between the reasons of poor performance. The optimal contract trades off the borrower’s benefit of accessing a source of finance that is cheaper than her own capital (that is, of raising leverage) against the cost of termination.

Two contracts interact in the asset market. The liquidation decision privately agreed in a contract generates a negative pecuniary externality on the other contract through the liquidation price in any state of nature the projects are liquidated simultaneously. In equilibrium, simultaneous liquidation occurs in the (only) state of nature in which the uncertainty about the reasons of poor performance is not resolved. In Section 3, I study the normative implications of this externality by defining the efficient (second-best) contract as the contract that would be chosen by a central planner who internalizes the externality in the problem of maximizing the value of a representative borrower but who is subject to the same contracting constraints as the private agents, the incentive compatibility constraint and the lenders’ participation constraint.

As intuition suggests, the negative pecuniary externality that works through the liquidation price implies that the decentralized equilibrium is inefficient relative to the second best because there is excessive liquidation in the state of nature in which the projects are liquidated simultaneously. As a result of excessive liquidation, the liquidation price for the projects is lower in the decentralized equilibrium. Depending on the liquidity of the asset market, the liquidation revenues can be higher or lower in the decentralized equilibrium relative to the second best allocation. Because the collateral value of the projects includes the liquidation revenues, the liquidity of the asset market affects the borrowers’ leverage in the decentralized equilibrium relative to their leverage in the second best. This point in turn has implications for the implementation of the second best contract when liquidation in private contracts cannot be limited directly.

This is because the performance information is garbled by an aggregate state, a “noise” shock, that causes the borrower’s performance to be poor regardless of her action. This structure will permit me to formalize the concern that “bouts of high market volatility” may worsen the performance evaluation problem.

I refer to the allocation corresponding to this contract as the second-best allocation. In Section 3, I also discuss how the main results are sensitive to incorporating the surplus of the liquidity suppliers into the planner’s welfare criterion.
If the asset market is sufficiently liquid, excessive liquidation translates into *excessive leverage* in the decentralized equilibrium. The second-best allocation can be implemented by limiting leverage in private contracts, that is, by imposing a *capital requirement* on the borrowers. The capital requirement is set *ex-ante* to deal with a problem (excessive liquidation) that manifests itself at a later stage in some states of nature. By forcing agents to rely more in their contracts on financial incentives, a capital requirement induces them to rely less on liquidation. However, if the market is sufficiently illiquid, leverage in the decentralized equilibrium falls below leverage in the second best in spite of the higher volume of liquidation, because the price effect of excessive liquidation is sufficiently large to depress the liquidation revenues and therefore the collateral value of the projects. Being a limit to leverage from above, capital requirements are ineffective in this situation and a more direct intervention in support of asset prices in the asset market is needed to implement the second best. The intervention can be interpreted as the introduction of a contingent claim (a “liquidity” option) combined with a *liquidity requirement* on the borrowers. This result deserves some attention because in the last few years the emphasis in the regulation of financial institutions has largely been on capital requirements. This chapter shows that in a simple environment this type of regulation can be ineffective, and other policies are called for; liquidity requirements are an example of such policies.

In Section 4, I study how the quality of the information available for performance evaluation affects the second-best contract, and in particular the capital requirement that implements the second best allocation. To carry out the thought experiment of varying the quality of information in a clean way, I need to separate the effects of changes in the quality of information from changes in the value of a project. This is done by varying the (exogenous) correlation between the two projects. In addition to providing a degree of freedom for comparative statics, the implications of changes in correlation have independent interest because empirically the correlation among many assets tends to increase with market volatility. By raising the chances that the projects’ performance will be poor at the same time, an increase in correlation can exacerbate the moral hazard problem and thus raise the capital requirement that implements the second best allocation. In the final part of the section, I take the model’s predictions as a benchmark for the evaluation of the existing regulation of the banks’ trading activities. The chapter provides an interpretation of the actual risk-based capital regulation, but also
Figure 3-1: The timeline

emphasizes the importance of considering policies different from capital requirements to deal with the problem of excessive liquidation in response to large shocks, when capital requirements may become ineffective.

Section 5 briefly concludes. An Appendix contains the derivation of some results used in the chapter. Some related literature is discussed at the end of Section 2 and Section 3.

3.2 The model

The central element of the model is the relationship between a risk-neutral borrower who has access to an investment opportunity (the project) and a risk-neutral lender who finances a fraction of the initial cost of the investment. There is a moral hazard problem since the borrower makes an investment decision, unobservable by the lender, that affects the outcome of the project. Figure 1 shows the timeline of the events in the model. There are three periods, 0, 1 and 2, and one good.

In period 0, a financial contract is signed between the borrower and the lender; a borrower is matched to a competitive lender. The size of the initial cost of the investment is normalized to one. After the contract has been signed, the borrower chooses an action; the action is not observed by the lender. The borrower can choose between two actions, “high” \(e^H\) and “low” \(e^L\) effort. The borrower can improve the distribution of the project’s payoffs by exerting
high effort (being "diligent"), but enjoys a private benefit $B > 0$ if she chooses low effort (that is, if she "shirks"). After the action has been chosen, an exogenous state is determined (the "aggregate state"). The aggregate state $\theta$ can be "high" ($\theta^H$) or "low" ($\theta^L$); neither the borrower nor the lender observe $\theta$ before period 2.

In period 1, the borrower and the lender receive some imperfect information about the final payoff (the "interim" information); based on this information, termination takes place as specified in the contract. In period 1, after the borrower has chosen effort, the project generates a signal. The signal can take two values, high ($s^H$) and low ($s^L$), "good" and and "poor" performance respectively. Below, I will consider two projects, 1 and 2; the agents in the first contract (contract 1) are referred to as borrower 1 and lender 1, the agents in the second contract (contract 2) as borrower 2 and lender 2. The two projects generate the signals $s_1$ and $s_2$ in period 1.

In period 2, the payoff is observed and the contractual payments are made.

The contract between the borrower and the lender specifies the share of the project financed by the borrower, liquidation contingent on the interim information, and the contingent payments to the borrower and the lender from the liquidation revenues and the project’s payoffs.

### 3.2.1 Projects’ payoffs and information structure

The purpose of this subsection is to construct a simple environment in which the outcome of the project is affected by the borrower’s effort and by an exogenous state beyond her control, and in which information about the borrower’s performance is released gradually. Consider the following example of an environment with these properties. Suppose that there are three exogenous states, the "good luck" (G) state, the "bad luck" (B) state, and the "neutral" (N) state. The exogenous state is observable only when the project’s final payoff is realized; however, an interim signal is observed before the payoff is realized. Effort matters for the outcome of the project only in N. In this state, if the project succeeds, the interim signal is high, while if the project fails the signal is low. In G, the project’s payoff is high, and the interim signal is high. In B, the project’s payoff is low, and the interim signal is low. The interim information is imperfect because the high signal can be observed either in G or N, and because the low signal can be observed either in B or N. Below, the uncertainty at the interim stage will imply
that termination is a costly incentive device. Since the focus of this chapter is on termination, which is relevant as an incentive device only in case of poor performance, I will abstract from the fact that the high signal can be due to good luck.

Thus, I suppose that the project’s payoff depends on the aggregate state $\theta$ and on the borrowers’ effort (see Figure 2). The aggregate state $\theta$ can take two values (corresponding to N and B in the previous example), high ($\theta^H$) or low ($\theta^L$), with probability $\gamma$ and $1 - \gamma$ respectively:

- if the state is high, the payoff is $R^H$ (the project “succeeds”) with probability $e$ and zero (the project “fails”) with probability $1 - e$, where $e$ is effort;

- if the state is low, the payoff is $R^L$ regardless of the borrower’s effort.

I assume that $e^HR^H \geq R^L > 0$, that is, the project’s expected value conditional on $\theta^H$ is higher than conditional on $\theta^L$. The payoffs $R^H$ and $R^L$ are verifiable and fully pledgeable to the lenders. As indicated above, effort $e$ can be high ($e^H$) or low ($e^L$), with $\Delta_e \equiv e^H - e^L > 0$.

In addition to the monetary payments from the project’s payoffs specified in the contract, the
borrower derives a private benefit \( V > 0 \) from the project. The benefit is reduced proportionally by termination: if in period 1 a fraction \( \tau \) of the project is terminated, the private benefit is reduced to \((1 - \tau)V\). The private benefit represents the borrowers' future income that cannot be pledged to the lenders and that is lost in case of termination.\(^5\)

The borrower and the lender receive interim information in period 1 in the form of signals. Project 1 generates a signal \( s_1 \) about its final payoff. The signal \( s_1 \) can take two values, high \((s^H)\) and low \((s^L)\): if the aggregate state is high, they observe \( s^H \) if the project succeeds and \( s^L \) if the project fails; if the aggregate state is low, they observe \( s^L \). Thus, the low signal \( s^L \), which represents poor performance, can be observed in either aggregate state. Since the aggregate state is not observable, the low signal is garbled as performance measure by the uncertainty about the aggregate state.\(^6\)

The two projects are subject to the same aggregate state, and have the same payoff and information structure described in Figure 2. Conditional on the high aggregate state, the projects' payoffs are independent regardless of the borrowers' effort.

Remark. The payoff structure in Figure 2 can be generalized to allow for a payoff \( r^H \geq 0 \) in case of failure. As long as \( R^H > r^H \), the analysis is essentially unchanged.\(^7\) Thus, the assumption that the payoff \( R^L \) in the low aggregate state is higher than the (zero) payoff in case of failure is not restrictive; however, setting \( r^H \) to zero simplifies somewhat the analysis. Moreover, the assumption that \( R^L > 0 \) is not inconsistent with \( \theta^L \) being the "low" state from the \textit{ex-ante} point of view (which matters for contracting reasons), since the expected value of the project conditional on \( \theta^H \) is higher than the expected value of the project conditional on \( \theta^L \).

Together with the assumptions on the liquidation price made below in (3.3), the assumption that \( R^L > 0 \) will imply that termination is a costly incentive device. Finally, the assumption that the volatility of the project's payoff in the low aggregate state is lower than in the high aggregate state is not substantial. Because all the agents are risk-neutral, \( R^L \) can be thought of as the expected value of a random cash flow.

\(^5\)See Holmström and Tirole (2001) for a similar assumption.

\(^6\)The signal admits the interpretation of earning announcement; as it will become clear below, it is somewhat problematic to interpret it as a forward looking stock price.

\(^7\)In the optimal contract the payoff \( r^H > 0 \) would be pledged entirely to the lender. The assumption that \( R^H > r^H \) simply states that effort improves the distribution of the project's payoffs.
3.2.2 The uncertainty at the interim stage

The second feature that I want to capture with the information structure is that the uncertainty about the aggregate state is highest when poor performance is common across projects in the economy. Thus, I assume that in addition to the information about their project, the borrower and the lender observe some information about the other project’s payoff. For simplicity, I assume that they observe the signal $s_2$ perfectly. The observation of $s_1$ and $s_2$ resolves the uncertainty about the aggregate state in period 1 unless both signals are low, $s_1 = s_2 = s^L$. It is clear in fact that whenever $s_1 = s^H$, the signal $s_2$ about project 2 is irrelevant for project 1, since $s_1 = s^H$ implies that project 1 will succeed. On the other hand, if $s_1 = s^L$ and $s_2 = s^H$, agents in contract 1 learn from $s_2 = s^H$ that the aggregate state is high, so that $s_1 = s^L$ implies that project 1 will fail: relative performance evaluation resolves the uncertainty about the aggregate state that would persist if only $s_1 = s^L$ were observed. However, when $s_1 = s^L$ and $s_2 = s^L$, it remains unclear whether the aggregate state is low, $\theta^L$, or whether it is high and both projects will fail: the uncertainty about the reason of poor interim performance is unresolved even by resorting to relative performance evaluation.

Therefore, from the period 0’s perspective of the agents in contract 1, there are three possible information states (states of nature) in period 1, which I denote by $\omega = (\omega^H, \omega^{LH}, \omega^{LL})$

- $\omega^H \equiv (s_1 = s^H)$, agents in contract 1 learn that project 1 will succeed;
- $\omega^{LH} \equiv (s_1 = s^L, s_2 = s^H)$, agents in contract 1 learn from signal 2 that $\theta = \theta^H$, and thus that project 1 will fail;
- $\omega^{LL} \equiv (s_1 = s^L, s_2 = s^L)$, the available information does not reveal the underlying aggregate state.

Since the simultaneous occurrence of poor performance does not resolve the uncertainty about the cause of poor performance, I refer to the state $\omega^{LL}$ as the “confusion” state. The uncertainty is highest when poor performance is common across borrowers: performance evaluation is more difficult in $\omega^{LL}$ than in the other states, as all the uncertainty is unresolved in $\omega^H$ and $\omega^{LH}$.

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*I discuss at the end of this section the empirical justification for this feature of the model. In Section 4, it will permit me to vary the quality of the interim information.*

62
Suppose that borrower 1 chooses effort $e_1$ (and borrower 2 chooses high effort $e^H$). Some simple algebra yields the probability of the information states,

\[
\begin{align*}
\Pr(\omega^H; e_1) &= \gamma e_1 \\
\Pr(\omega^{LL}; e_1) &= (1-e_1)(1-e^H)\gamma + 1 - \gamma
\end{align*}
\] (3.1)

and the posterior probability of the low aggregate state in the confusion state $\omega^{LL}$,

\[
\Pr(\theta^L|\omega^{LL}; e_1) = \frac{1-\gamma}{(1-e_1)(1-e^H)\gamma + 1 - \gamma} \geq \Pr(\theta^L|s_1 = s^L) = \frac{1-\gamma}{(1-e_1)\gamma + 1 - \gamma} \geq 1 - \gamma.
\] (3.2)

The first-inequality in (3.2) implies that $\omega^{LL}$ is a stronger signal for $\theta^L$ than a single realization of $s^L$. The probability of the confusion state $\omega^{LL}$ is a measure of the quality of the information about performance.

### 3.2.3 The opportunity cost of capital

The contract specifies the fraction $k_b$ of the initial cost of the investment financed by the borrower and the fraction $k_l$ financed by the lender; thus, $k_l$ is a measure of the borrower’s leverage.

The lender and the borrower have enough initial endowment to finance the project. The lender requires a gross rate of return of at least one. The borrower receives an endowment in period 0, large enough to finance the project entirely. However, the borrower’ opportunity cost of capital $\rho$ is higher than the lenders’, $\rho > 1$. The opportunity cost of capital $\rho$ represents the gross rate of return on an alternative (divisible) investment opportunity, and the wedge $\rho - 1$ a measure of the borrower’s scarcity of funds. This last assumption implies that if the borrower’s effort were contractible, it would be optimal for the borrower to finance the project entirely with external funds (that is, $k_b = 0$). However, with moral hazard it will not be in

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9A variety of channels could explain the relatively high cost of equity capital; for instance, see Myers and Majluf (1984), Gorton and Pennacchi (1991), and Diamond and Rajan (2000). An alternative approach to model the borrower’s scarcity of funds would be to suppose that the borrower has a fixed endowment of capital and chooses the size of the investment; the borrower’s equity investment is constant in absolute terms, and leverage varies with the investment size.
general optimal to finance the project entirely with external funds, since the borrower has to be provided with incentives to be diligent.

3.2.4 The market for liquidated projects

The project can be (partially) liquidated on an asset market that opens in period 1. I assume that the price of the project is zero when the project is known to fail (in state $\omega^{LH}$), and is $R^H$ when it is known to succeed (in state $\omega^H$). In the confusion state $\omega^{LL}$, let $P(x)$ denote the inverse demand function for the liquidated projects, where $x$ is the aggregate volume of liquidation, with $P'(x) < 0$ for all $x \geq 0$.

10 If a fraction $\tau_1$ of project 1 is liquidated in the interim period, it yields the revenues $\tau_1 P(\tau_1 + \tau_2)$, while the remaining fraction $1 - \tau_1$ yields the project’s payoff in period 2, zero if the aggregate state is high and the project fails, and $(1 - \tau_1)R^L$ if the state is low. Moreover, I assume that

$$\Pr(\theta^L|\omega^{LL})R^L > P(x) > 0 \text{ for all } x.$$  \hspace{1cm} (3.3)

This inequality implies that liquidation is valuable if the aggregate state is high but the project fails yielding a zero payoff ("early" intervention prevents a further loss, because $P > 0$), while it is "disruptive" if the low signal is due to "bad luck" (the low aggregate state) beyond the borrower’s control (since $\Pr(\theta^L|\omega^{LL})R^L > P(x)$). The difference $\Pr(\theta^L|\omega^{LL})R^L - P(x)$ is the liquidity premium that is incurred in state $\omega^{LL}$ by liquidating the project. The magnitude of the negative response of the liquidation price to the aggregate volume of liquidation is a measure of liquidity or depth of the asset market.

3.2.5 The feasible contracts

A contract specifies the shares of the project financed by the borrower and the lender, and how the project’s payoffs and the liquidation revenues are split between them. Precisely, the contract $\Omega_1 \equiv (k_b, k_l, R^H_b, R^L_b, \tau_1, y^{LL})$ determines:

- the shares of the project financed by the borrower ($k_b$) and lender ($k_l$);
the contingent payments $R_b^H$ and $R_b^L$ to the borrower from the project’s payoffs $R^H$ and $R^L$ respectively;\(^{11}\)

- the fractions $\tau_1 \equiv (\tau_1^{LH}, \tau_1^{LL})$ of the projects liquidated in the information states $\omega^{LH}$ and $\omega^{LL}$;

- the payment $y^{LL}$ to the borrower from the liquidation revenues in state $\omega^{LL}$.

Since the initial cost of the investment is fixed and normalized to one, the implementation of the project requires that $k_b + k_l \geq 1$. I assume that the borrower is protected by limited liability. Together with the assumption that the private benefit is nonpledgeable, limited liability implies that the borrower cannot receive negative payments, $R_b^H \geq 0$ and $R_b^L \geq 0$. The contingent payments $R_b^H$ and $R_b^L$ are limited by the project’s payoffs, $R_b^H \leq R^H$ and $R_b^L \leq (1 - \tau_1^{LL})R^L$, where the first inequality takes into account that in the contracts I will consider it will never be optimal to liquidate when the project is known to succeed (in state $\omega^H$); the second inequality takes into account that the liquidation of the fraction $\tau_1^{LL}$ of the project reduces the final payoff proportionally. The payment $y^{LL}$ cannot be negative, and is limited by the liquidation revenues, $0 \leq y^{LL} \leq \tau_1^{LL}P(\tau_1^{LL} + \tau_2^{LL})$. In the contracts that I will consider below, the project will always be liquidated entirely when it is known to fail, $\tau_1^{LH} = 1$. However, since in state $\omega^{LH}$ the liquidation price is zero, there are no liquidation revenues to split between the borrower and the lender. As already mentioned, no liquidation takes place in state $\omega^H$, so there are no liquidation revenues in this state either.

3.2.6 The borrower’s and lender’s values

The expected value of the contract to the borrower is

$$U_b(e_1) = \Pr(\omega^H)(R_b^H + V) + \Pr(\omega^{LL})[y^{LL} + \Pr(\theta^L|\omega^{LL})R_b^L + (1 - \tau_1^{LL})V] + \Pr(\omega^{LH})(1 - \tau_1^{LH})V - \rho k_b$$  

(3.4)

---

\(^{11}\)There is no loss in generality in letting the payment to a borrower in case of success, $R_b^H$, be independent of the interim performance of the other borrower, since the likelihood ratio for the project’s high payoff $R^H$,

$$\frac{\Pr(R^H, s_1 = s^H, s_2 = e^H)}{\Pr(R^H, s_1 = s^H, s_2 = e^L)} = \frac{e^H}{e^L}$$

does not depend on the realization of the other borrower’s signal, $s_2$. This is an application of the Sufficient Statistic Theorem in Holmström (1979).
The first three terms are the expected payments and the expected value of the private benefit of continuation (net of the loss due to liquidation). The last term is the opportunity cost at the gross rate of return \( \rho \) of financing a share \( k_b \) of the initial investment. The expected value of the contract to the lender is

\[
U_l(e_1) = \Pr(\omega^H)(R^H - R^H_b) + \\
\Pr(\omega^{LL})\left\{\tau_1^{LL} P(\tau_1^{LL} + \tau_2^{LL}) - y^{LL} + \Pr(\theta^L|\omega^{LL})[(1 - \tau_1^{LL})R^L - R^L_b]\right\} - k_l
\]

(3.5)

The first term is the expected payment from the project in case of success net of the payment to the borrower. The second term is the expected payment received in state \( \omega^{LL} \): the sum of the expected liquidation revenues net of the payment \( y^{LL} \) to the borrower and the expected residual value of the project net of the payment \( R^L_b \) to the borrower. The last term is the opportunity cost of financing a share \( k_l \) of the initial investment. The individual rationality (IR) constraint for the lender is

\[
U_l(e_1) \geq 0.
\]

(3.6)

That is, the expected income pledged to the lender must be at least as large as the lender’s opportunity cost of capital.

Finally, note that effort \( e_1 \) affects the distribution of the information states (see (3.1) and (3.2)), and thus the expected values of the contract.

### 3.2.7 The incentive constraint

Because effort is nonobservable and costly, the contract has to provide the borrower with incentives to be diligent. In this section I describe the incentive compatibility (IC) constraint that induces the borrower to choose high effort. There are two channels through which effort affects incentives: the traditional one, that is, the probability of the project’s success, and a new one, namely that effort affects the probability of confusion and hence the power of termination as an incentive device.

Holding the other borrower’s effort fixed at \( e^H \), effort affects the probability distribution of
the information states \( \omega \). Equation (3.1) implies that

\[
\begin{align*}
\Pr(\omega^{H}; e^{H}) - \Pr(\omega^{H}; e^{L}) &= \gamma \Delta_e \\
\Pr(\omega^{LL}; e^{H}) - \Pr(\omega^{LL}; e^{L}) &= -\gamma \Delta_e (1 - e^{H}).
\end{align*}
(3.7)
\]

That is, effort raises the probability of success and reduces the probability of confusion \( \omega^{LL} \).

Using this result and (3.4), the borrower finds it optimal to be diligent if \( U_b(e^{H}) \geq U_b(e^{L}) + B \), that is,

\[
\gamma \Delta_e (R^H_b + V) \geq \gamma \Delta_e (1 - e^{H}) [y^{LL} + (1 - \tau^{LL}_1) V] + \gamma \Delta_e e^{H} (1 - \tau^{LH}_1) V + B
(3.8)
\]

As I will argue in the next subsection, in an optimal contract the borrower will receive no payment from the liquidation revenues \( y^{LL} = 0 \), because any positive payment raises the minimum incentive-compatible contingent payment \( R^H_b \), reducing the borrowing capacity. Moreover, in state \( \omega^{LH} \) it will be optimal to liquidate the project entirely \( (\tau^{LH}_1 = 1) \). Thus, the IC constraint simplifies to

\[
R^H_b + V \geq \frac{B}{\gamma \Delta_e} + (1 - e^{H})(1 - \tau^{LH}_1)V.
\]

That is, the contingent payment for the project’s success (augmented by the private benefit \( V \)) must be sufficiently large. How large depends on two terms. The first term \( B/(\gamma \Delta_e) \) is the standard effect in moral hazard models: the agent must retain a sufficiently high stake in the final payoff for the improvement in the project’s payoff distribution to offset the private cost of effort \( B \). The second term is due to confusion in the interim period. If the borrower shirks, she increases the probability of confusion (see (3.7)). In state \( \omega^{LL} \) termination is less than complete \( (\tau^{LL}_1 < 1) \) because of the uncertainty about the project’s final payoff. Thus, by shirking the borrower can reduce the expected loss of the private benefit \( V \) (relative to state \( \omega^{LH} \)). The contingent payment \( R^H_b \) must be sufficiently large to offset the borrower’s benefit of increasing the likelihood of confusion. Intuitively, the effect of shirking on the probability of the confusion state in (3.7) represents the “temptation” to shirk due to the uncertainty about the reason of poor performance. Finally, note that for termination to have an incentive effect the private benefit must be strictly positive, \( V > 0 \).
3.2.8 The optimal contract in the decentralized equilibrium

The optimal contract maximizes the borrower’s expected value \( U_b(e^H) \) subject to the IC constraint (3.8) and the lender’s IR constraint (3.6), taking as given the other contract.\(^{12}\) That is, an optimal contract maximizes the borrower’s value while inducing high effort and promising the competitive (zero) interest rate on the lender’s investment. Note that the liquidation \( \tau_2^{LL} \) in state \( \omega^{LL} \) agreed in the other contract fixes the position of the price schedule \( P(\tau_1^{LL} + \tau_2^{LL}) \) for the borrower’s project in case of liquidation, and thus the borrower’s pledgeable income. I study the symmetric decentralized equilibrium between the contracts, simply defined as a symmetric Nash equilibrium:

**Definition.** A symmetric decentralized equilibrium (SDE) is a pair of contracts \((\Omega_1, \Omega_2)\) such that:

1) the contract \( \Omega_1 \) is optimal given contract \( \Omega_2 \), and the contract \( \Omega_2 \) is optimal given \( \Omega_1 \);

2) the \( \Omega_1 \) and \( \Omega_2 \) are symmetric: \( \Omega_1 = \Omega_2 \). \( \square \)

Here I discuss informally the characteristics of the optimal contract; its formal derivation is in the Appendix.

The IC and IR constraints are binding in the optimal contract. Suppose that the IC were not binding. Then, the contingent payment to the borrower could be reduced by \( \delta R_b^H \), reducing the borrower’s expected payoff by \( \Pr(\omega^{LL})\delta R_b^H \). Holding liquidation unchanged, the IC would still be satisfied. By pledging \( \delta R_b^H \) to the lender, the lender’s investment could increase by \( \Pr(\omega^{LL})\delta R_b^H \), saving \( \rho \Pr(\omega^{LL})\delta R_b^H > \Pr(\omega^{LL})\delta R_b^H \) to the borrower in terms of cost of capital. Similarly, if the IR constraint were not binding, the borrower could increase the share of the project financed by the lender. Moreover, the investment from the lender is maximized by pledging to her any income received after \( \omega^{LL} \) is realized, that is, \( R_b^L = y^{LL} = 0 \).

Consider next the optimal liquidation when the project is known to fail in state \( \omega^{LH} \). Since the liquidation price of the project is zero, the contract specifies full liquidation if the improvement in incentives due to termination in \( \omega^{LH} \) more than offsets the expected loss of the borrower’s private benefit \( V \). As intuition suggests, this is the case if the borrower’s cost of

\(^{12}\) Obviously, the feasibility constraints \( 0 \leq \tau_1^{IH} \leq 1 \) and \( 0 \leq \tau_1^{IL} \leq 1 \) for the fractions of the project that are liquidated in states \( \omega^{IH} \) and \( \omega^{IL} \) have to be satisfied.
capital $\rho$ is sufficiently large ($\rho e^H > 1$); I assume that this condition is satisfied in the following, so that $\tau^{LH} = 1$. Furthermore, it is never optimal to liquidate in state $\omega^H$ when the project succeeds.\textsuperscript{13}

Given the liquidation of the fraction $\tau^{LL}_1$ in state $\omega^{LL}$, the IC constraint yields the contingent payment to the borrower in case of success,

$$R^H_b = (1 - e^H)(1 - \tau^{LL}_1)V + \frac{B}{\gamma \Delta_e} - V. \quad (3.9)$$

By substituting $R^H_b$ into the binding IR constraint (3.6), I find the share $k_l$ of the initial investment financed by the lender (that is, the borrower’s leverage). The share $k_l$ is the maximum income that the borrower can pledge to the lender without harming her incentive to be diligent; it depends on the fractions $(\tau^{LL}_1, \tau^{LL}_2)$ of the two projects liquidated in state $\omega^{LL}$ because they determine the price of the projects in the asset market. Simple algebra allows me to write this incentive compatible share as $k_l = \bar{k}_l + k_l^*(\tau^{LL}_1, \tau^{LL}_2)$, where

$$\bar{k}_l \equiv \gamma e^H R^H + (1 - \gamma)R^L - \gamma e^H \left( \frac{B}{\gamma \Delta_e} - e^H V \right) \quad (3.10)$$

$$k_l^*(\tau^{LL}_1, \tau^{LL}_2) \equiv \Pr(\omega^{LL})Y(\tau^{LL}_1, \tau^{LL}_2) + \gamma e^H (1 - e^H)\tau^{LL}_1 V. \quad (3.11)$$

In (3.11) I have defined $Y(\tau^{LL}_1, \tau^{LL}_2)$ as the revenues from liquidating the fraction $\tau^{LL}_1$ net of the expected loss of the project’s payoff,

$$Y(\tau^{LL}_1, \tau^{LL}_2) \equiv -[\Pr(\theta^L|\omega^{LL})R^L - P(\tau^{LL}_1 + \tau^{LL}_2)]\tau^{LL}_1. \quad (3.12)$$

Under the assumption in (3.3), $Y$ is negative, since a liquidity premium in incurred in state $\omega^{LL}$.\textsuperscript{14} The share $k_l$ can be decomposed into a minimum share $\bar{k}_l$ independent of liquidation and an additional share $k_l^*(\tau^{LL}_1, \tau^{LL}_2)$ that depends on liquidation in the two contracts. The minimum share $\bar{k}_l$ is the pledgeable income when liquidation is not available in state $\omega^{LL}$ as an

\textsuperscript{13}Recall that in state $\omega^H$ the price of the project is $R^H$, so that there is no benefit in terms of liquidation revenues. Any liquidation in state $\omega^H$ would only have negative effects, since it would destroy the borrower’s private benefit and worsen the borrower’s incentives by punishing her when the project succeeds.

\textsuperscript{14}Under the previous assumption on the price function, $Y$ is decreasing in its two arguments, and concave in its first argument. Moreover, the function $Y(x, x)$ is decreasing and concave in $x$. 

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incentive device. It is equal to the project’s expected payoff minus the expected value of the share of $R^H$ that must be retained by the borrower for incentive purposes.\footnote{This point deserves a comment. First, recall that in state $\omega^{LH}$ liquidation is complete. By exerting effort, the borrower raises the probability of state $\omega^H$ and reduces the probability of state $\omega^{LL}$. Since there is no liquidation in these states (under the maintained assumption that no liquidation is possible in state $\omega^{LL}$), the change in the expected private benefit from continuation induced by effort is $[\gamma\Delta_e - \gamma\Delta_e(1 - e^H)]V = \gamma\Delta_e e^HV$. The contingent payment $R^H$ can thus be reduced by $e^HV$. The expected value of this reduction is $\gamma(e^H)^2V$, the term that appears in $\bar{k}_t$.} Leverage can be raised beyond $\bar{k}_t$ by the amount $k^*_t(\tau^L_1, \tau^L_2)$ that depends on liquidation (in state $\omega^{LL}$) in the two contracts. First, the liquidation of the fraction $\tau^L_1$ generates the net revenues $Y(\tau^L_1, \tau^L_2)$ (the first term in (3.11)). Second, an increase in $\tau^L_1$ (the threat of termination) improves the borrower’s incentives; the improvement in incentives in turn raises the share of the project’s payoff that can be pledged to the lender (the second term in (3.11)).

The optimal liquidation $\tau^L_1$ in state $\omega^{LL}$ maximizes the following objective function,

$$L(\tau^L_1, \tau^L_2) = \rho k^*_t(\tau^L_1, \tau^L_2) - [\Pr(\omega^{LH})V + \gamma e^H(1 - e^H)V] \tau^L_1. \quad (3.13)$$

From the Lagrangian $L(\tau^L_1, \tau^L_2)$, it is clear that the contract trades off the borrower’s benefit of leverage against the cost of liquidation required to sustain leverage. The cost includes the expected loss of private benefit $V$ (the first term in square brackets) and the dilution of the borrower’s stake in the project as a larger fraction of $R^H$ is pledged to the lender (the second term in squared brackets). Note that $\partial k^*_t/\partial \tau^L_2 < 0$, that is, an increase in liquidation in the other contract reduces the borrower’s expected value because it reduces the project’s liquidation value.

The discussion so far is summarized in the following proposition.

**Proposition 1.** The optimal liquidation $\tau^L_1$ in state $\omega^{LL}$ satisfies the following first-order condition (also sufficient under the maintained assumptions on the liquidation price),

$$\frac{\partial k^*_t(\tau^L_1, \tau^L_2)}{\partial \tau^L_1} = \Pr(\omega^{LH})V \rho + \gamma e^H(1 - e^H)V \rho, \quad (3.14)$$

where $\partial k^*_t/\partial \tau^L_1$ is the partial derivative of the leverage function in (3.11) with respect to its first argument.
A necessary condition for an interior liquidation $\tau_1^{LL}$ is that the borrower’s cost of capital $\rho$ is sufficiently large, $\rho e^H > 1 + \frac{1-\gamma}{\gamma(1-e^H)}$. If this condition holds, there exists a range of values $[V_{\min}, V_{\max}]$ for the private benefit $V$ such that there exists a unique solution for $\tau_1^{LL}$. □

If there is no private benefit from continuation, $V = 0$, the first-order condition (3.14) for liquidation reduces to $\partial k_t^*/\partial \tau_1^{LL} = 0$: because liquidation involves no private cost to the borrower, it is optimal to choose liquidation in order to maximize leverage and thus economize on the use of the borrower’s capital. However, since liquidation reduces pledgeable income (because a liquidity premium is paid, i.e. $Y < 0$) without improving incentives, leverage is maximized by setting $\tau_1^{LL} = 0$. If $V > 0$, liquidation can be used as an incentive device, that is, $\tau_1^{LL}$ appears in the IC constraint. If $\rho$ is below the threshold given in the proposition, the value of termination as an incentive device (which permits the borrower to economize on the use of her own capital) is not large enough to offset the cost of liquidation in terms of the loss of private benefit $V$ and the liquidity premium paid on the asset market, and thus $\tau_1^{LL} = 0$. If $\rho$ is above that threshold, though, termination becomes valuable to the borrower as an incentive device. Under this condition, the value of termination as an incentive device is increasing in the private benefit $V$: an increase in $V$ raises the marginal benefit of termination (due to the improvement in incentives and the resulting increase in leverage) more than it raises the marginal cost of termination (due to the loss of private benefit and the liquidity premium paid in case of termination). Thus, there is a range of values $[V_{\min}, V_{\max}]$ for $V$ such that there is a unique solution for $\tau_1^{LL}$ in the unit interval.

The following proposition summarizes the symmetric decentralized equilibrium.

**Proposition 2.** At a SDE, the optimal liquidation $\tau^{LL,D}$ in state $\omega^{LL}$ satisfies the first-order condition (3.14) for $\tau_1^{LL} = \tau_2^{LL} \equiv \tau^{LL,D}$. The project is liquidated completely in state $\omega^{LH}$, and continues in state $\omega^H$. The borrower receives the contingent payment $R_t^H$ in case of success defined by (3.9), and borrows $k_t = \bar{k}_t + k^*_t(\tau^{LL,D}, \tau^{LL,D})$, where $\bar{k}_t$ and $k^*_t$ are given by (3.10) and (3.11) respectively. The SDE is unique. □

**Remark 1.** It is worth comparing the optimal liquidation in state $\omega^{LL}$ with the liquidation that would be optimal if the projects’ payoffs were revealed in period 1 (the perfect information
case). Suppose that the price of a project known to fail is zero, and is $P$ (depending on the aggregate volume of liquidation) if the project's payoff is $R^L$, with $P < R^L$. Under the same condition that ensures full liquidation in state $\omega^{LH}$ in the imperfect information case, full liquidation occurs whenever the project fails. No liquidation occurs when the project's payoff is $R^L$, because $P < R^L$ and termination has no incentive value as the outcome $R^L$ is beyond the borrower's control. Thus, relative to the full information case, confusion at the interim stage affects liquidation in state $\omega^{LL}$ in two ways. First, liquidation is too low when $\theta = \theta^H$ and the project fails; this reduces the incentive benefit of termination. Second, liquidation is too high when $\theta = \theta^L$; this causes an excessive loss of the monetary payoff $R^L$.

**Remark 2.** In equilibrium, liquidation takes place in state $\omega^{LL}$ even if a liquidity premium is incurred. Following the argument in Diamond and Rajan (2000), renegotiation may be impossible because of the collective action problem among a large number of competitive lenders.\(^{16}\)

### 3.2.9 Discussion

In the information structure in Figure 2, the uncertainty about the fundamentals of the assets in the economy (the projects) is highest when poor performance is diffuse across borrowers. In practice, asset price volatility changes over time: it is inversely related to the level of asset prices, and tends to increase following large price movements, especially downwards.\(^{17}\) The aspect of asset price volatility that is more relevant for the assumed information structure is that empirically it has proved difficult to relate the changes in volatility to changes in underlying

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\(^{16}\)Consider the following example. Suppose that there is a mass one of identical lenders, who in period 0 agree on the contract derived in this section. In state $\omega^{LL}$, each lender has a claim to a payment $\tau^{LL,D}P$, and a claim to a payment $(1 - \tau^{LL,D})R^L$ in period 2 if the state is $\theta^L$. The borrower may try to reduce liquidation and to finance a larger payment to the lenders by reducing the overall liquidity premium paid in period 1. Suppose that in state $\omega^{LL}$ the borrower offers a lender to swap her claim to $\tau^{LL,D}P$ with a claim promising an expected payment of $S > \tau^{LL,D}P$ in period 2. However, suppose that this new claim is required to be junior to the pre-existing claims on the project output $R^L$. The collective action problem implies that this swap may not be feasible. To see why, suppose that all the lenders refuse the offer to swap their claims, and consider the problem of a lender who is considering whether to swap her claim. If she accepts the claim, she receives a claim worth $S$. However, since the claim is junior to the existing claims and the other lenders do not accept the offer, the project's payoff $(1 - \tau^{LL,D})R^L$ is just sufficient to cover the senior claims. Thus, $S = 0$, and it is not optimal for the lender to swap her claim.

fundamentals. For example, Schwert (1989) finds the stock return volatility to be significantly larger during macroeconomic recessions but no strong association with other measures of volatility in the economy's fundamentals. This "volatility puzzle" suggests that changes in "noise" rather than in fundamentals mostly drive the changes in asset price volatility. In a recent paper, David and Veronesi (2001) present evidence that is closely related to the assumed information structure. They find that periods of high uncertainty about the fundamentals of the economy (inflation and earning growth rates) are associated with high volatilities in stock and bond returns. The model implication that performance evaluation is more difficult when poor performance is common across borrowers captures in a stylized fashion the fact that volatility increases in market downturns, reducing in turn the quality of the information about fundamentals. It is plausible that the worsening in the quality of information also affects the performance evaluation of financial institutions that hold and trade portfolios of assets whose prices are more likely to be driven by noise when market volatility increases. Some evidence in this sense is provided by Berkowitz and O'Brien (2001) in their study of actual profits and losses of the trading activities of six large commercial banks. They report that the violations of the VaR limits tend to be large when they occur, to cluster in time and across institutions, and coincide with periods of market volatility. This pattern is consistent with the assumed information structure if one interprets a violation of the VaR limit as poor performance.

The threat of termination as a disciplinary device has been examined in the literature, see Stiglitz and Weiss (1983) for an early treatment. Papers in the banking literature, e.g. Calomiris and Kahn (1991), have argued that a bank run triggered by the release of information indicating a bank's poor performance can be beneficial as an incentive device (but may nevertheless lead to costly contagious bank runs). Hellman, Murdock and Stiglitz (2000) show that competition between banks can erode their continuation value, and thus reduce the value of termination as

18 See, for instance, Cutler, Poterba, and Summers (1989), and Roll (1988).
19 Uncertainty over the fundamentals is significant in statistical and economic terms to account for volatilities in asset returns after controlling for the past return volatility, the fundamentals' volatility and measures of the business cycle. Uncertainty over the fundamentals is computed from the updated distribution over the (unobservable) states of the fundamentals in a Markow switching model.
20 A similar argument is made by Diamond and Rajan (2000). The fragility of bank's demand liabilities, that is, the threat of liquidation, permits the bank to commit not to renegotiate on any precommitted payments. The collective action problem among the bank's depositors provides the bank with a commitment device.
an incentive device. \footnote{See also Bolton and Scharfstein (1990) for an example of how the threat of termination can induce the truthful revelation of private information.} Shleifer and Vishny (1997) interpret their model of performance-based arbitrage as an incentive mechanism according to which the amount of funds available to an arbitrageur is a function of past performance. In practice, the role of termination in financial markets is likely to have increased in the last decade with the strong surge in the use of collateral (BIS (2001)).

The assumptions on the liquidity of the asset market admit several interpretations; \footnote{Kyle and Xiong (2001) make an assumption about the behavior of long-term investors that is similar to the assumption on liquidity supply made here. Long-term investors provide liquidity only if the spread between the fundamental price and the actual price of an asset is sufficiently large, that is, if there is a "safety margin".} I discuss briefly two of them. Following Shleifer and Vishny (1992), the first interpretation is that the liquidity problem in state $\omega^{LL}$ arises because the borrower's assets have to be transferred to inferior users; thus, the price $P$ in state $\omega^{LL}$ is what can be paid by the marginal highest valuation user of the assets. The second interpretation is inspired by Kyle (1985). In the interim period, liquidity is supplied by competitive risk-neutral market makers. The interim signals generated by the two projects represent the public information available to market makers and uninformed "traders" (the agents in the two contracts). There are other agents, the "speculators", who observe the true aggregate state and can sell assets that from the perspective of the market makers are indistinguishable from the projects. I study a model along these lines in a separate Appendix, where I show that it is possible to construct pooling equilibria in which the uninformed and informed agents sell the same quantity $\tau^{LL}$ in the confusion state $\omega^{LL}$. Because of the adverse selection problem due to informed trading, the price paid by the market makers in state $\omega^{LL}$ that is consistent with zero expected profits satisfies the inequality in (3.3). For the agents in the contracts, the difference between $\Pr(g^{L} | \omega^{LL}) R^{L}$ and the price paid by the market makers is a liquidity premium. \footnote{Furthermore, if the likelihood of informed trading depends positively on the speculators' opportunity to trade, that is, $\tau^{LL}$, the equilibrium in this model generates an inverse relationship between the liquidation price and the aggregate volume of liquidation. This is the case if information acquisition involves a fixed cost whose size depends negatively on the opportunity to trade, for instance because of an information externality among the informed agents, and competition between speculators drives the expected profits from informed trading to zero. The price function then describes a set of equilibria along which the market makers and the speculators are indifferent.}
does not depend on effort is made to simplify the analysis. This assumption states that the low aggregate state $\theta^L$ can account for low performance beyond the borrower's control, that is, it represents "bad luck". The assumptions that $R^L > 0$ and that a liquidity premium is incurred in case of termination are not essential for incentive purposes, but provide a justification to the supply of liquidity on the asset market, as just discussed. It is the assumption that the private benefit $V$ is positive and reduced by termination that makes termination "bite" as an incentive device.

### 3.3 The efficient (second best) contract

In this section I study the problem of a central planner who is subject to the same contracting constraints as the individual agents but who internalizes the external effect ignored by them in the decentralized equilibrium, the effect of aggregate liquidation on the liquidation price.

#### 3.3.1 The second best liquidation

The planner maximizes the \textit{ex-ante} value of a representative borrower, subject to the incentive constraint and the lender' participation constraint. The program solved by the central planner is the same as the one studied in the previous section except for the fact that the planner takes into account that in the confusion state $\omega^{LL}$ the aggregate volume of liquidation is $2\tau^{LL}$, and thus that the net revenue from liquidation is $Y(\tau^{LL}, \tau^{LL}) = [P(2\tau^{LL}) - Pr(\theta^L | \omega^{LL})R^L] \tau^{LL}$. I refer to the solution of the planner's problem as the efficient (second-best) contract, and to the implied allocation as the second-best allocation. In the second-best contract, the IC and IR constraints are binding and the borrower receives no payment when the aggregate state is low, $R_b^L = 0$, and from the liquidation revenues, $y^{LL} = 0$. Furthermore, the project is liquidated when it fails in state $\omega^{LH}$, $\tau^{LH} = 1$.

\footnote{It can be relaxed as follows. Suppose that effort affects the outcome of the project also in $\theta^L$, but that in this state the realization of the project outcome is delayed to the second period. When the agents observe $s^L$, they are uncertain whether it is due to the project's failure (in $\theta^H$) or to the delay of the projects (in $\theta^L$). By defining $R^L$ to be the expected value (at high effort) of the project in $\theta^L$, that is, $R^L \equiv e^H R^H$, the previous analysis remains valid. For any contract that specifies a payment to the borrower in the low state, one can find an equivalent contract (in terms of incentives and agents' payoffs) that pledges $R^L$ entirely to the lender. Intuitively, since the agents are risk neutral, their utilities are linear in the monetary payments. Thus, the monetary incentives can be provided by a single large payment to the borrower in case of success in the high aggregate state.}
The key difference between the SDE and the second-best contract is the liquidation in state \( \omega^{LL} \), which in turn affects pledgeable income and leverage. The concavity of \( Y(\tau^{LL}, \tau^{LL}) \) guarantees that the second-best liquidation is unique and strictly below liquidation in the SDE. This discussion leads to the following proposition.

**Proposition 3.** The second-best liquidation \( \tau^{LL,C} \) in state \( \omega^{LL} \) satisfies the following first-order condition (also sufficient under the maintained assumptions on the liquidation price),

\[
\frac{dk^*_t(\tau^{LL,C}, \tau^{LL,C})}{d\tau^{LL}} = \text{Pr}(\omega^{LL}) \frac{V}{\rho} + \gamma e^H (1 - e^H) \frac{V}{\rho}
\]  

where, from the definition of \( k^*_t \) in (3.11),

\[
\frac{dk^*_t(\tau^{LL,C}, \tau^{LL,C})}{d\tau^{LL}} = \text{Pr}(\omega^{LL}) \left[ Y_1(\tau^{LL,C}, \tau^{LL,C}) + Y_2(\tau^{LL,C}, \tau^{LL,C}) \right] + \gamma e^H (1 - e^H) V
\]

(3.16)

In the SDE, there is excessive liquidation in the confusion state \( \omega^{LL} \), that is, \( \tau^{LL,C} < \tau^{LL,D} \). \( \square \)

Excessive liquidation stems from the negative externality that works through the liquidation price. When choosing liquidation, the agents do not internalize the negative effect of their liquidation decisions on the liquidation price available for the other project. Thus, the marginal net liquidation revenue in state \( \omega^{LL} \) from the perspective of individual agents, \( Y_1 \), is higher than the marginal net liquidation revenue from the perspective of the planner, \( Y_1 + Y_2 \), because \( Y_2 = \tau^{LL} P' < 0 \). As a result, the second best liquidation in the state of nature in which the projects are simultaneously liquidated is lower than liquidation in the SDE, \( \tau^{LL,C} < \tau^{LL,D} \). In turn, excessive liquidation pushes down the liquidation price in the SDE relative to the price in the second best allocation.

**Remark 1.** The analysis leading to Proposition 3 had a partial equilibrium flavor. By assuming that the planner maximizes the representative borrower’s expected value, I have not taken into account the welfare effects that changes in the price of the projects on the asset market may have on the agents who supply liquidity (and on any other agents who trade in the market). Suppose that \( P \) represents the price paid by the marginal highest valuation user of the projects, and that the planner attaches a weight \( \psi \) to the net surplus of the buyers (the
liquidity suppliers) from purchasing an amount $2\tau^{LL}$ at price $P(2\tau^{LL})$.\footnote{That is, the planner maximizes $2U_0(e^H) + \psi S(2\tau^{LL})$ subject to the IC and IR constraints, where the net surplus $S(2\tau^{LL})$ is given by}

\[ S(2\tau^{LL}) = Pr(\omega^{LL}) \int_0^{2\tau^{LL}} \left[ P(x) - P(2\tau^{LL}) \right] dx. \]

Then, the planner has to trade off the borrowers' benefit of reducing liquidation in the SDE against the loss of the liquidity suppliers' surplus. At a SDE, the planner's marginal benefit of reducing liquidation still exceeds the marginal cost, that is, liquidation is excessive in the SDE, if the borrower's cost of capital $\rho$ is sufficiently large relative to the weight $\psi$. Intuitively, excessive liquidation reduces the collateral value of the project, and thus the borrower's borrowing capacity; as a result, the higher the cost of capital $\rho$, the stronger the welfare effect on borrowers of excessive liquidation. For ease of comparison of the second-best allocation with the SDE, I have chosen to work with the case $\psi = 0$.\footnote{For example, this would be the case of a small open economy whose assets are sold abroad in case of liquidation, implying that the planner attaches no weight to the welfare of the foreign liquidity suppliers. More generally, $\psi < 1$ if the planner assigns a social value to the borrowers' strength, e.g. because the better capitalized the financial institutions (here, the borrowers) the lower the systemic risk.}

Remark 2. The excessive liquidation result in Proposition 3 follows from the noncooperative interaction between two "large" units on the asset market: agents do not internalize the liquidity externality on the other agents. Yet, they take into account the effect of their contractual decisions on their own expected value, and thus restrain their liquidation. Excessive liquidation is exacerbated if the agents are "small" and behave competitively in the asset market for the liquidated projects.\footnote{The previous model can be adapted to the case of small agents as follows. Suppose that there is a mass one of borrowers of type 1 and 2, each with one project of type 1 and 2. The projects of the same type are perfectly correlated. Each borrower is matched to a lender. To keep the incentive constraint unchanged, assume that the agents in a contract observe the interim signal of their own project and the interim signal of the projects of the other type, but not the performance of the other projects of the same type.} There is a competitive equilibrium in which liquidation $\tau^{LL,CE}$ in state $\omega^{LL}$ is interior and such that $\tau^{LL,C} < \tau^{LL,D} < \tau^{LL,CE}$. Because price-taking agents do not internalize any effect of liquidation on the liquidation price, liquidation is higher and the the price is lower in the competitive equilibrium relative to the SDE.
3.3.2 Excessive liquidation and leverage

Next I study how the inefficiency in the SDE, excessive liquidation, affects leverage in the SDE relative to leverage in the second best contract (the second best leverage for short). The relationship between leverage in the SDE and the second best leverage will have important implications for the issue of implementation discussed below.

Define \( \tau_{\max} \) as liquidation in state \( \omega^{LL} \) that maximizes leverage, \( \frac{dk^*_t(\tau_{\max}, \tau_{\max})}{d\tau^{LL}} = 0 \), and assume that \( \tau_{\max} \) exists and is interior. Because \( k^*_t(\tau^{LL}, \tau^{LL}) \) is concave in \( \tau^{LL} \), the first-order condition (3.15) for \( \tau^{LL,C} \) implies \( \tau^{LL,C} \leq \tau_{\max} \), and \( \tau^{LL,C} = \tau_{\max} \) if and only if the borrower’s private benefit from continuation \( V \) is zero. That is, the second best liquidation is never above \( \tau_{\max} \), and whenever liquidation is privately costly to the borrower the second best liquidation is strictly below \( \tau_{\max} \).

The following proposition, proved in the Appendix, describes the relationship between the second best leverage and leverage in the SDE. For notational simplicity, define \( k^*_t(D) \equiv k^*_t(\tau^{LL,D}, \tau^{LL,D}) \) and \( k^*_t(C) \equiv k^*_t(\tau^{LL,C}, \tau^{LL,C}) \):

**Proposition 4.** Suppose that \( V > 0 \). There exists a threshold \( t < \tau_{\max} \) such that the relationship between the second best leverage \( k^*_t(C) \) and leverage \( k^*_t(D) \) in the SDE is as follows:

i) if \( 0 \leq \tau^{LL}_t \leq t \), leverage in the SDE exceeds the second best leverage, \( k^*_t(D) > k^*_t(C) \);

ii) if \( t < \tau^{CL}_t \leq \tau_{\max} \), the second best leverage exceeds leverage in the SDE, \( k^*_t(D) < k^*_t(C) \).

This proposition states that the relationship between second best leverage and leverage in the SDE can be described entirely as a function of the second best liquidation \( \tau^{LL,C} \) in state \( \omega^{LL} \). This result admits an intuitive interpretation in terms of the liquidity of the asset market. The borrower’s collateral includes the liquidation revenues. If the asset market is sufficiently liquid, the price effect of excessive liquidation is not strong enough to offset the quantity effect (that is, the fact that a larger fraction of the project is sold) and the fact that a larger fraction of the project can be pledged to lenders when there is more liquidation (see (3.11)). As a result, the borrowers can borrow more in the SDE relative to the second best contract. However, if the asset market is sufficiently illiquid, the price effect can be so strong to reduce the collateral value of the projects and thus leverage (relative to the second best).

Figure 3 illustrates this relationship. The top panel shows the leverage function \( k^*_t(\tau^{LL}, \tau^{LL}) \),
that is, the leverage that the borrower can achieve when liquidation in both contracts is $\tau^{LL}$ (the dashed curve). The panel also plots $k_t^*(\tau^{LL}, \tau^{LL,D})$ and $k_t^*(\tau^{LL}, \tau^{LL,D'})$, the leverage that the borrower can achieve by liquidating the fraction $\tau^{LL}$ of her project, given that the liquidation in the other contract is at two different levels, $\tau^{LL,D} < \tau^{LL,D'}$ (the two solid curves). Because higher aggregate liquidation depresses the liquidation price, $k_t^*(\tau^{LL}, \tau^{LL,D})$ is strictly below $k_t^*(\tau^{LL}, \tau^{LL,D'})$. By definition, $k_t^*(\tau^{LL}, \tau^{LL,D})$ and $k_t^*(\tau^{LL}, \tau^{LL,D'})$ intersect $k_t^*(\tau^{LL}, \tau^{LL})$ at $\tau^{LL} = \tau^{LL,D}$ and $\tau^{LL} = \tau^{LL,D'}$ respectively. The bottom panel plots the total derivative of $k_t^*(\tau^{LL}, \tau^{LL})$ (the dashed curve) and the partial derivatives of $k_t^*(\tau^{LL}, \tau^{LL,D})$ and $k_t^*(\tau^{LL}, \tau^{LL,D'})$ with respect to their first argument. At any liquidation $\tau^{LL}$, the slope of $k_t^*(\tau^{LL}, \tau^{LL,D'})$ is smaller than the slope of $k_t^*(\tau^{LL}, \tau^{LL,D})$. The bottom panel also shows (as an horizontal line) the marginal cost of leverage for the borrower, the right-hand side of the first-order conditions (3.14) and (3.15). The two lines correspond to two different levels of the marginal cost. For example, if the cost of capital is low (high), the adjusted marginal cost of leverage is high (low), since termination is less (more) attractive as an incentive device.

In the bottom panel, the second best liquidation $\tau^{LL,C}$ and $\tau^{LL,C'}$ for the two levels of the marginal cost of leverage are at the intersections of the marginal cost lines with the total derivative of the leverage function. The second best leverage implied by $\tau^{LL,C}$ and $\tau^{LL,C'}$ can be read off $k_t^*(\tau^{LL}, \tau^{LL})$ in the top panel. The optimal liquidation for the borrower, when liquidation in the other contract is $\tau^{LL,D}$, is at the intersection of the partial derivative $\partial k_t^*(\tau^{LL}, \tau^{LL,D})/\partial \tau^{LL}$ with the marginal cost line. At a SDE, this intersection occurs at $\tau^{LL,D}$. Liquidation in the two SDE’s corresponding to the two levels of marginal cost are $\tau^{LL,D}$ and $\tau^{LL,D'}$. The excessive liquidation result is apparent from the fact that if liquidation in contract 2 is larger than the second best liquidation ($\tau^{LL,D} > \tau^{LL,C}$), the slope of $k_t^*(\tau^{LL}, \tau^{LL,D})$ at $\tau^{LL,C}$ is larger than the slope of $k_t^*(\tau^{LL}, \tau^{LL})$ at $\tau^{LL,C}$. Thus, the borrower would choose liquidation above $\tau^{C}$ as well.

If the second best liquidation is sufficiently small, $\tau^{LL,D} < \tau_{\text{max}}$ at the SDE. A lower

\[k_t^*(\tau^{LL}, \tau^{LL}) > \partial k_t^*(\tau^{LL}, \tau^{LL,D})/\partial \tau^{LL}< \partial k_t^*(\tau^{LL}, \tau^{LL,D'})/\partial \tau^{LL} \]

\[\text{for } \tau^{LL} < \tau^{LL,D} \text{ and } k_t^*(\tau^{LL}, \tau^{LL}) < k_t^*(\tau^{LL}, \tau^{LL,D}) \text{ for } \tau^{LL} > \tau^{LL,D} \].

\[\text{This follows from } \partial k_t^*(\tau^{LL}, \tau^{LL,D})/\partial \tau^{LL} = \text{Pr}(\omega^{LL})Y_{12} < 0.\]

\[\text{The concavity of } k_t^*(\tau^{LL}, \tau^{LL,D}) \text{ in its first argument and the first-order conditions (3.14) and (3.15) imply that } \partial k_t^*(\tau^{LL, C}, \tau^{LL,D})/\partial \tau^{LL} > \partial k_t^*(\tau^{LL,C}, \tau^{LL,D})/\partial \tau^{LL}.\]

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Figure 3-3: Leverage in the SDE and second best leverage
marginal cost of liquidation makes liquidation more attractive, and raises the second best liquidation, \( \tau^*_{LL,CL} > \tau^*_{LL,CL} \). In the SDE, liquidation is higher as well, \( \tau^*_{LL,DL} > \tau^*_{LL,DL} \). If the downward shift in the horizontal marginal cost line is sufficiently pronounced, the leverage \( k^*_i(\tau^*_{LL,DL}, \tau^*_{LL,CL}) \) in the SDE falls below the second best leverage \( k^*_i(\tau^*_{LL,CL}, \tau^*_{LL,CL}) \), as showed in the figure. In terms of Proposition 4, the second best leverage \( \tau^*_{LL,CL} \) has crossed the threshold \( t \).

3.3.3 Implementation of the second-best contract

In this subsection I discuss how the planner (such as a financial regulator) can implement the second best allocation. In practice, it is plausible that there is an asymmetry with regard to the instruments that can be used to address the inefficiency in the decentralized equilibrium. While limiting leverage in private contracts seems feasible, raising it might not be so. I take this asymmetry into account in discussing the implementation of the second best.

Implementation with capital requirements

Proposition 4 states that leverage in the SDE is excessive relative to the second best leverage, \( k^*_i > k^*_C \), if the second best liquidation is sufficiently small, \( \tau^*_{LL,CL} < t \): excessive liquidation implies excessive leverage. The second-best allocation can be implemented by setting a minimum capital requirement on the borrowers equal to \( 1 - k^*_C \), that is, by limiting the permitted leverage to \( k^*_C \). The individually optimal contract has to be derived under the additional constraint that \( k_i \leq k^*_C \); this constraint is binding, since \( k^*_D \) in the unregulated equilibrium is larger than \( k^*_C \).

Given \( k^*_C \), the IR constraint yields the smallest liquidation that guarantees the zero rate of return required by the lender, that is, \( \tau^*_{LL,CL} \). Finally, the IC constraint yields the contingent payment \( R^*_H \) to the borrower.

The logic for capital requirements is straightforward. The inefficiency derives from the excessive reliance on liquidation in the SDE; by limiting leverage, the planner ensures that

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31 In performing comparative statics exercise on the marginal cost of liquidation, one has to be careful not to affect the leverage function as well. This is the case for changes in the cost of capital \( p \).

32 The choice of \( \tau^*_{LL,CL} \) in the two contracts is indeed a Nash equilibrium. Given that the other borrower chooses \( \tau^*_{LL,CL} \), a choice of \( \tau' > \tau^*_{LL,CL} \) permits the borrower to raise more external finance than permitted by the capital requirement. On the other hand, a choice of \( \tau' < \tau^*_{LL,CL} \) would not violate the capital requirement. However, raising \( \tau' \) up to \( \tau^*_{LL,CL} \) permits the borrower to economize on her costly capital.
incentives are provided more through pecuniary measures, so that agents need to rely less on liquidation. In welfare economics, the standard textbook policy intervention to deal with a negative externality is a tax on the activity that generates it: by raising marginal costs, the tax induces private agents to internalize the externality. Taxing liquidation, the activity that generates the negative externality, does not seem feasible in this context. In a broader perspective, it would for example imply that the decision to suspend a credit line, roll over a debt or issue a margin call should be taxed. Capital regulation has the nature of a quantity constraint on the activity that generates the externality, and implements indirectly the second best allocation.

This implementation relies critically on the planner’s ability to observe and control the borrower’s balance-sheet: by setting an upper limit to leverage at \( k^C \), the planner sets liquidation at the desired level (through the binding \( IR \) constraint). The case for capital requirements may be strengthened, however, if the planner controls the borrower’s balance-sheet only partially.

To illustrate the point, consider the extension of the model in which the scales \( I_1 \) and \( I_2 \) of the projects can be varied freely, and in which the liquidation price is given by \( P(\tau^L_1 I_1 + \tau^L_2 I_2) \).\(^{33}\) The negative externality through the liquidation price operates now at the level of the fraction of the project liquidated in the confusion state and at the level of the investment size. Suppose that the planner can limit leverage, but may not be able to limit the investment size, for instance because the borrower can engage in off-balance sheet derivatives transactions that are difficult to monitor closely. That is, there may be a second moral hazard problem associated with the choice of the investment size. Through the \( IR \) constraint (which now applies to each dollar invested by the lender), the planner can still control liquidation by setting a limit on leverage. However, the planner has to take into account the incentive constraint associated with the choice of the investment size, that is, the effect of liquidation on the investment size when the latter cannot be regulated directly. Since investment is larger when it is not regulated directly (because the agents do not take into account the externality through the liquidation price), the marginal cost of liquidation from the perspective of the planner is larger at any liquidation fraction (because

\(^{33}\)The model discussed so far is a particular case with \( I_1 = I_2 = 1 \). The borrower’s activity is scaled up proportionally to the size \( I \); in particular, the cost of effort is \( BI \) and the private benefit for continuation is \( V/I \). The borrower’s opportunity cost of capital and the rate of return required by the lenders are \( \rho \) and one respectively. Note that the \( IR \) constraint in (3.6) applies to each dollar invested by the lender.
the price for the project depends negatively on the aggregate volume of liquidation). Thus, the
planner restrained liquidation even more, and hence tightens the capital requirement, relative to
the case in which the investment size can be regulated directly. Intuitively, the planner seeks to
offset the inefficiency by relying more aggressively on the instrument (the capital requirement)
over which she has control.\textsuperscript{34}

\textbf{Intervention on the asset market, liquidity options and liquidity requirements}

Proposition 4 also states that leverage in the SDE is too low relative to the second best leverage,
$k^P_l < k^C_l$, if the second best liquidation is sufficiently large, $\tau^{LL,C} > t$. This is the case when
the asset market is sufficiently illiquid and the price effect of excessive liquidation so large that
the collateral value of the projects in the SDE falls below its value in the second best allocation.
This situation can be thought of as a liquidity crisis, in which the liquidation price has collapsed
in the asset market because of the large volume of liquidation, and this in turn forces the agents
to resort to a large volume of liquidation. Capital requirements are ineffective, because they
set a limit to leverage from above while the inefficiency in the SDE takes the form of a \textit{capital
squeeze} on the borrowers, in the sense that the agents with the higher opportunity cost of
capital are required to invest more than in the second best allocation. A tax on liquidation
when liquidity has collapsed in key asset markets seems even less feasible than in the previous
case.\textsuperscript{35}

If leverage cannot be raised directly, the planner can seek to support market liquidity. For
example, suppose that the planner announces the plan $(\bar{\tau}, \bar{P})$, according to which the liquidation
price in the interim stage is not allowed to fall below the threshold $\bar{P} \equiv P(2\tau^{LL,C})$ in state $\omega^{LL}$,
at which the planner stands ready to purchase up to the fraction $\bar{\tau} \equiv \tau^{LL,C}$ (per borrower). If the
market is expected to be more liquid, borrowers can borrow more at lower expected liquidation;

\textsuperscript{34}However, there are two more effects to consider in addition to the one discussed in the text. By allowing
more liquidation (that is, by relaxing the capital requirement) the planner would discourage investment in the
unregulated investment case (because each dollar of investment is liquidated more frequently). The reduction in
investment raises the liquidation price; relaxing the IR constraint is beneficial to the borrower. The reduction in
investment, though, is costly to the borrower. Unless the first effect is so strong to offset the second effect and
the one in the text (because $P'$ is very large), liquidation is \textit{less} aggressive and capital requirement \textit{tighter} if the
planner cannot control the borrower’s entire balance sheet.

\textsuperscript{35}There is however anecdotal evidence that bank regulators sometimes exercise “moral suasion” on the banks
with the aim of preventing fire sales of collateral, e.g. commercial real estate.
in turn, if liquidation is lower, the recovery in the liquidation price generates sufficiently large revenues that no net disbursement from the planner at the interim stage is necessary.\textsuperscript{36} The plan \((\tau, \overline{P})\) can be reinterpreted as a portfolio consisting of \(\tau\) units of a contingent claim that pays \(\overline{P}\) in state \(\omega^{LL}\). This contingent claim completes the asset markets with respect to the risk of excessive liquidation: by holding this contingent claim, a holder gets at least \(\overline{P}\) in \(\omega^{LL}\). Since this claim guarantees a minimum price in case of liquidation, it admits the interpretation of a liquidity option (see Scholes (2000)). The second best can be implemented with the liquidity requirement on the borrowers to hold \(\tau\) units of this claim. Alternatively, the second best can be implemented by \(\tau\) units of a put option on the project at the strike price \(\overline{P}\).\textsuperscript{37} Since the exercise of the put option depends on the market price of the borrower's assets rather than on the liquidity event \(\omega^{LL}\), the practical implementation of liquidity requirements with the put is less demanding in terms of the verifiability of the relevant state of nature. On the other hand, in more general contexts a put option on the borrower’s asset may create additional incentive problems, for example if the borrower chooses the risk of her assets; the asset substitution problem may be less severe if the exercise of the liquidity option is contingent on liquidity events that are (at least in part) beyond the borrower’s control.

The effectiveness of this intervention relies critically on the ability of the planner to prevent any further liquidation beyond what prescribed by the plan \((\tau, \overline{P})\). The planner may not have this ability if she monitors the actions of the agents in the economy imperfectly and trading in the asset market is sufficiently anonymous.\textsuperscript{38} The planner may then be unable to implement the second best, but may nevertheless improve on the decentralized equilibrium. To illustrate this point, suppose that the planner announces the plan \((\tau, \overline{P})\), with the same interpretation

\textsuperscript{36}The intervention has some features of the classic lending-of-last-resort function of central banks, say a discount window for the borrowers' assets. In particular, the collateral (the project) is valued at the “pre-crisis” price \(\overline{P}\).

\textsuperscript{37}The liquidity contingent claim pays out \(\overline{P}\) only in state \(\omega^{LL}\); in this state, the seller of the claim receives assets worth \(2\tau\overline{P}(2\tau) = 2\tau\overline{P}\). The no-arbitrage price of this claim is thus zero. The seller of the put option pays out \(2\tau\overline{P}\) in state \(\omega^{LL}\); furthermore, since the price of a project that fails is zero, the put is exercised also in state \(\omega^{LH}\). The seller of the put receives assets worth \(2\tau\overline{P}(2\tau) = 2\tau\overline{P}\) in state \(\omega^{LL}\), and zero in state \(\omega^{LH}\). The zero expected profit price of the put is thus \(\Pr(\omega^{LH} | \overline{P})\) (note that contrarily to the previous case, this is not the no-arbitrage price of the put, since some risk is borne by the seller of the put).

\textsuperscript{38}In other words, while the plan \((\tau, \overline{P})\) meets the IR and IC constraints, one has also to ensure that it is individually rational for the agents to behave as prescribed by it. This point is similar to the problem, described by Jacklin (1987), of ensuring the coexistence of optimal liquidity insurance through deposit contracts and trading in financial markets.
as before that in state $\omega^{LL}$ the agents in a contract can sell to the planner up to the fraction $\tilde{\tau}$ of the project at price $\tilde{P}$. Moreover, the agents in a contract can freely agree (at the time of contracting) to liquidate any additional fraction $\tau^o_1$ on the asset market, with $0 \leq \tau^o_1 \leq \tilde{\tau}$.

The upper bound $\tilde{\tau}$ is a technological constraint that describes the planner’s ability to restrict the agents’ trades in the asset market (that is, $\tilde{\tau}$ represents the planner’s monitoring ability). Taking into account the plan $(\tau, P)$, the agents face the residual inverse demand function $P^o(\tau^o_1 + \tau^o_2) \equiv P(\tau^o_1 + \tau^o_2 + 2\tilde{\tau})$ for the additional fraction $\tau^o_1$. If $\tilde{\tau} > 0$, the residual price function is steeper than absent any intervention, suggesting that the agents might be cautious in their liquidation. I also assume that the planner is required to break even on the plan $(\tau, P)$ in equilibrium or, equivalently, that the seller of the liquidity option makes zero expected profits.

Given the plan $(\tau, P)$, the decentralized equilibrium in the additional fractions $\tau^o_1$ and $\tau^o_2$ is similar to the one that obtains absent any intervention. At a symmetric equilibrium and under the requirement that the planner breaks even, these fractions depend on $\tilde{\tau}$ only, that is, $\tau^o_1 = \tau^o_2 \equiv \tau^o(\tilde{\tau})$. The function $\tau^o(\tilde{\tau})$ is a constraint in the planner’s problem of choosing the intervention plan $(\tau, P)$ in order to maximize the expected value of a representative borrower; the planner breaks even by setting $\bar{P} = P(2\tau^o(\tilde{\tau}) + 2\tilde{\tau})$. It is possible to show that unless the agents’ ability to trade in the asset market is sufficiently limited (that is, $\tilde{\tau}$ is sufficiently small), the planner cannot improve on the SDE by offering a plan of the form $(\tau, P)$. Intuitively, there is a free rider problem caused by the agents’ ability to trade in the asset market in state $\omega^{LL}$. The free rider problem, if sufficiently severe, undermines the planner’s ability to improve coordination and simultaneously break even on her intervention. However, the planner can improve on the decentralized equilibrium, by offering a plan $(\bar{\tau}, \bar{P})$ with $\bar{\tau} > 0$, and simultaneously break even if she can restrain the agents’ additional trades in the asset market sufficiently well. In the reinterpretation of the plan $(\tau, P)$ as a liquidity option, the break-even condition is a zero profit condition on the sellers of the option. This discussion then suggests that the allocation in the decentralized equilibrium can be improved upon by the

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39Given $(\tau, \tilde{P})$, borrower 1 and lender 1 agree on the reaction function $\tau^o_1(\tau^o_2)$ to maximize $L^o(\tau^o_1, \tau^o_2)$, where the Lagrangian $L^o$ is defined from $L$ in (3.13) by replacing the price function $P$ with $P^o$. In a symmetric equilibrium, $\tau^o_1 = \tau^o_2 \equiv \tau^o(\tilde{\tau})$. Define $V^o(\tau, \tilde{P})$ to be the borrower’s indirect utility at a symmetric decentralized equilibrium in the liquidation fractions $\tau^o_1$ and $\tau^o_2$ given the planner’s policy $(\tau, \tilde{P})$. The problem of the planner is to maximize $V^o(\tau, \tilde{P})$ subject to the break-even condition $\bar{P} = P(2\tau^o(\tilde{\tau}) + 2\tilde{\tau})$. 

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introduction of the liquidity option by competitive sellers combined with a liquidity requirement on the borrowers only in the presence of sufficiently active monitoring by the planner. A fully decentralized solution to the excessive liquidation problem is not possible in this example.

3.3.4 Discussion

The inefficiency in the SDE is the manifestation of a more general phenomenon: market incompleteness with regard to liquidity needs leads to excessive demand for liquidity when liquidity is scarce. In this chapter, the scarcity of liquidity is modeled as a downward sloping demand for the projects in the asset market. The inefficiency originates from the negative spillover that an agent’s demand for liquidity has on other agents’ collateral through the liquidation price. Externalities working through liquidity have been described in the literature. Shleifer and Vishny (1992) highlight the positive externality that works through the supply of liquidity: firms in the same sector can provide each other partial insurance against negative idiosyncratic shocks by offering a resale market for liquidated assets. The same logic applies with regard to the demand for liquidity (that is, when an agent demands liquidity, she makes it unavailable to other agents), and is relevant when agents need to simultaneously liquidate assets following a common shock.\textsuperscript{40} The externality resembles the inefficiency discussed by Holmström and Tirole (1998). In their model, financial markets fail in general to allocate liquidity among private agents efficiently even in the absence of aggregate uncertainty. The reason is that individual firms hoard liquidity (that is, demand excessive liquidity by holding more liquid assets than they need \textit{ex-post}) making it unavailable to other firms. The (second-best) efficient allocation can be implemented through a mechanism that pools the individual risks, say within financial intermediaries that provide liquidity as needed by the firms. The principle underlying the inefficiency in this chapter is similar (agents demand too much liquidity from a market in the state of nature in which they have a simultaneous demand for liquidity); this chapter investigates alternative instruments to implement the second-best allocation.

\textsuperscript{40}In Caballero and Krishnamurthy (2000), agents decide how to allocate their capital between two sectors, and indirectly determine the demand and supply of liquidity; the capital allocation decisions generate externalities on other agents because the agents do not take into account the effect of their choices on the likelihood that the economy’s collateral constraint will be binding. A decentralized equilibrium can be constrained Pareto inefficient if the return from supplying liquidity (that is, from accumulating assets that can be used as collateral with foreign lenders) is lower than the social value of liquidity because of contracting constraints in the economy.
The main positive implication of the analysis of the second-best contract is the prediction that markets where tighter ex-ante margin and collateral requirements are in place should be more resilient to the realization of negative shocks than markets with looser requirements. Anecdotal evidence from the behavior of over-the-counter derivatives markets relative to exchange-based markets during times of financial turbulence seems to be in line with this prediction. Barth, Caprio and Levine (2001) present some supportive evidence in the banking context: in a broad cross section of countries, they find that tighter banking regulation tends to reduce banks' fragility.

Public regulation of financial institutions can be justified at least on two grounds. The first one is that regulation is useful if financial institutions do not anticipate or observe all the relevant dimensions of the price process. For example, Kyle and Xiong (2001) point out that risk managers should take into account (among other things) the wealth effect on convergence traders and the importance of market liquidity. Public regulation can be welfare improving if the regulator enjoys a better access to information about these dimensions of the price process, for instance the size of aggregate market positions. The rationale for regulation is essentially a measurement problem (Ross (2001)). The second approach is that individual financial institutions are rational but do not internalize some important externality. This is the rationale for regulation in this chapter, where the externality works through the liquidation price. Furthermore, there is a rationale for regulation even if the financial contracts are endogenous, contrarily to what is often assumed in the literature on banking regulation. In this literature, the contract is typically a deposit contract that pays the riskless interest rate (because the lenders are protected by deposit insurance). The interaction among the deposit contract, the borrower’s limited liability, and the asset substitution problem gives rise to excessive risk taking, and provides a rationale for regulation: since the depositors have no incentive to monitor

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41 "The credit protection provided by collateral and frequent margining may somewhat moderate the tendency of credit and liquidity flows in wholesale financial markets to seize up under stress (...). For example, repo markets and exchange-traded futures markets are often relatively resilient and subject to limited credit rationing in periods of market turbulence." (BIS (2001, p. 24))

42 See Table 8. They find that a Capital Regulatory Index and an Official Supervisory Index tend to reduce the probability of a banking crisis. However, the evidence on the separate role of these indexes is not statistically strong.

43 However, the regulator would be subject to the same information constraints if the main problem is to deal with extreme scenarios (rare events), a problem that has been emphasized with regard to the financial crisis in 1998.
the bank’s investment decisions, the regulator acts as the depositors’ representative.\footnote{See Freixas and Rochet (1997, Chapter 9) for a survey of this literature. The representation hypothesis is discussed in more detail by Dewatripont and Tirole (1994).}

3.4 Correlation and confusion

Empirically, correlation among many assets tends to increase in periods of market volatility and downturns; the increase in correlation seems to have occurred during the 1998 financial crisis.\footnote{The traditional approach of computing the simple correlation coefficient between asset returns over periods characterized by different volatility shows that correlation tend to increase with market volatility. It has been subsequently pointed out that this finding may be a statistical artefact; more formal analysis, though, tends to confirm it. See Forbes and Rigobon (1998) and Ang and Chen (2000); see BIS (1999, Table 5) for evidence on the 1998 crisis.} If one interprets the projects of the borrowers as the trading strategies of financial institutions operating in similar markets and adopting similar investment “styles”, then the projects are likely to be correlated, with the size of the correlation depending on the correlation between the underlying assets. The model can be easily extended to study the implications of changes in correlation between projects on incentives, and in turn on the second best contract.

To illustrate why this extension is interesting, consider the comparative statics exercise of increasing the probability $\gamma$ of the high aggregate state $\theta^H$. Holding the second best liquidation $\tau^{LL,C}$ constant, the straightforward implication of an increase in $\gamma$ is to raise pledgeable income.\footnote{By increases because $e^H R^H \geq R^L$; $k_i^*$ increases because under (3.3), $Y < 0$ and $Pr(\omega_{LL})$ falls with an increase in $\gamma$.} The probability $\gamma$ does not affect only the pledgeable income, though, but also the probability distribution of the information states. Equation (3.1) implies that an increase in $\gamma$ reduces the probability of the confusion state $\omega^{LL}$, and thus affects the borrower’s incentives. For convenience, rewrite the first-order condition (3.15) for $\tau^{LL,C}$ as

$$P + 2\tau^{LL,C} P' + \frac{(\rho - 1) \gamma e^H (1 - e^H) V}{\Pr(\omega^{LL})/\rho} = \Pr(\theta^L | \omega^{LL}) \left( R^L + \frac{V}{\rho} \right)$$

The left-hand side is the marginal benefit of liquidation (in terms of revenues on the asset market and incentive value of termination); the right-hand side is the marginal cost of liquidation (the loss of monetary payoff and private benefit from continuation). From (3.7), an increase in $\gamma$ raises the effect of shirking on the probability of confusion, and thus the borrower’s temptation.
to shirk (because the aggregate state in which effort matters for performance becomes more likely); this effect appears as an increase in the numerator of the last term on the right-hand side. At the same time, an increase in $\gamma$ lowers the probability of confusion, and thus the borrower's expected loss of the private benefit $V$; this effect appears as a fall in the denominator of the last term on the right-hand side. Finally, an increase in $\gamma$ reduces the posterior probability of the low state, $\Pr(\theta^L|\omega^L)$, and thus the opportunity cost of liquidation. These three effects raise the second-best liquidation in the optimal contract. Because the leverage function $k^*_l$ is increasing in liquidation in the range relevant for the planner, the increase in liquidation reinforces the effect on pledgeable income. Thus, an increase in $\gamma$ lowers the capital requirement that implements the second-best allocation.\footnote{Changes in $e^H$ (or $e^L$), the probability of success under the borrower's control, could be analyzed in a similar way by separating its effect on pledgeable income and on the probability distribution of the information states. By inspection of the first-order condition (3.15) and the leverage function defined by (3.10)-(3.11), though, the multiple appearance of $e^H$ makes the analysis much more difficult.}

The adjustment in the second best contract is due to the change in the value of the projects, which affects the income pledgeable to the lenders, and to the change in the borrower's incentives linked to the quality of the interim information, that is, the likelihood of confusion. It seems worth highlighting the implications of changes in the likelihood of confusion separately. Correlation between the projects will permit me to carry out this thought experiment.

\subsection{Projects' correlation}

I introduce correlation by supposing that a borrower's choice determines the correlation of her project with an exogenous investment opportunity (a "factor"), and that the factors affecting the two projects are correlated. Figure 4 shows the payoff structure of a project in the high aggregate state $\theta^H$; as before, the project's payoff in state $\theta^L$ is $R^L$.\footnote{All the statements in this subsection about the joint distribution of the projects' payoffs will be conditional on $\theta^H$.} Borrower 1's investment opportunity is described by an exogenous random variable, the factor $\varepsilon_1$. The factor can take two values, high ($\varepsilon^H$) or low ($\varepsilon^L$) with equal probability; the realization of the factor is not observed in period 1.

Effort $e_1$ determines $(p_1, q_1)$, a vector of conditional probabilities; high effort $e^H$ gives $(p^H, q^H)$ and low effort $e^L$ gives $(p^L, q^L)$. The pair $(p_1, q_1)$ defines the distribution of the project
conditional on factor $\varepsilon_1$:

\begin{align*}
    p_1 & \equiv \Pr(R_1 = R^H | \varepsilon_1 = \varepsilon^H) \quad (3.17) \\
    q_1 & \equiv \Pr(R_1 = R^H | \varepsilon_1 = \varepsilon^L). \quad (3.18)
\end{align*}

By definition, the marginal probability of success is $e^H \equiv \frac{1}{2}p^H + \frac{1}{2}q^H$ and $e^L \equiv \frac{1}{2}p^L + \frac{1}{2}q^L$ for high and low effort respectively. I assume that effort raises the probability of success for each realization of the factor: $\Delta_p \equiv p_H - p_L > 0$ and $\Delta_q \equiv q_H - q_L > 0$. For concreteness, I assume that the project is positively correlated with its factor, that is, $p^H \geq q^H$ and $p^L \geq q^L$.\footnote{The project is perfectly correlated with its factor if $p^H = 1$ and $q^H = 0$ (assuming that the borrower is diligent).} Below, I will use the assumption that

\begin{equation}
    \Delta_q > \Delta_p \Leftrightarrow p^H - q^H < p^L - q^L \quad (3.19)
\end{equation}
The inequality states that effort is relatively more important for the project's success following the low realization of the factor (when the conditional probability of success is \( q^H \) or \( q^L \) for high or low effort respectively) than the high realization of the factor (when the conditional probability of success is \( p^H \) and \( p^L \) for high or low effort respectively).\(^5\)

The projects are correlated through the correlation between the factors. Denote by \( \pi \) the conditional probability that the factors take the same value,

\[
\pi \equiv \Pr(\varepsilon_1 = \varepsilon^H | \varepsilon_2 = \varepsilon^H) = \Pr(\varepsilon_1 = \varepsilon^L | \varepsilon_2 = \varepsilon^L) \geq \frac{1}{2}. \tag{3.20}
\]

For expository brevity but slightly imprecisely, I will refer to \( \pi \) as the "correlation" between the factors. If \( \pi > 1/2 \), the two factors are positively correlated.

The probability of the failure of one project conditional on the failure of the other project will determine the probability of confusion at the interim stage. Thus, define \( \nu_1 \equiv \Pr(R_1 = R^H | R_2 = 0; \varepsilon_1) \), the probability that the project succeeds (when effort is \( \varepsilon_1 \)) conditional on the failure of the other project (and given that the other borrower chooses high effort). Simple algebra shows that

\[
1 - \nu_1 = 1 - e_1 + \frac{2\pi - 1}{4} \frac{(p_1 - q_1) (p^H - q^H)}{1 - e^H}. \tag{3.21}
\]

\( \nu_1 \) can take two values, \( \nu^H \) and \( \nu^L \) corresponding to high and low effort. If the projects are not independent,\(^5\) then \( 1 - \nu_1 > 1 - e_1 \): project 2’s failure raises the likelihood of project 1’s failure. The increase in the likelihood of failure is purely informational: because \( p^H > q^H \), project 2’s failure raises the likelihood that the second factor is low \( (\varepsilon_2 = \varepsilon^L) \), and hence the likelihood that the first factor is low \( (\varepsilon_1 = \varepsilon^L) \), whenever the factors are positively correlated, making project 1’s failure more likely (relative to the unconditional probability).

\(^5\)This assumption can be motivated as follows (this motivation is loosely inspired by the discussion in Holmström and Tirole (2000)). The borrower’s investment is subject to “market” uncertainty (represented by the factor); the borrower can implement risk-management measures to hedge her project. The hedging activity is unobservable by outsiders and privately costly to the borrower. For example, the borrower could use off-balance sheet derivatives transactions to hedge, and thus limit the expected loss due to the negative realization of the factor, rather than to “gamble” with derivatives. The private benefit \( B \) would then represent the gain to the borrower from gambling.

\(^5\)Equation (3.21) shows that the two projects are independent, that is, \( \nu_1 = e_1 \), for either choice of effort if either \( p^H = q^H \) and \( p^L = q^L \) (a project is independent of its factor, and a fortiori of the other project) or if \( \pi = 1/2 \) (the two factors are independent).

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3.4.2 The distribution of the interim information states

Correlation does not affect the type of uncertainty at the interim stage. The three information states $\omega_H, \omega^{LH}$ and $\omega^{LL}$ still identify the possible states of nature in period 1. However, their relative likelihood depends on correlation. Simple algebra shows that

\[
\begin{align*}
\Pr(\omega_H; e_1) &= \gamma e_1 \\
\Pr(\omega^{LL}; e_1) &= (1 - \nu_1)(1 - e^H)\gamma + 1 - \gamma
\end{align*}
\]

(3.22)

where $1 - \nu_1$ is the probability of failure conditional on the failure of the other project, defined in (3.21). Since $\nu^H > \nu^L$, the probability of confusion is higher if the borrower shirks. Holding effort constant, the probability of confusion is increasing in $\pi$, since an increase in correlation shifts the probability mass towards those states of nature in which the two projects have the same outcome, high or low.\textsuperscript{52}

The incentive constraint is modified by the presence of correlation in a straightforward way. As before, in the optimal contract the liquidation revenues and the project’s payoff in the low aggregate state are pledged to the lender, and the project is liquidated in state $\omega^{LH}$. The IC constraint can thus be written as

\[
R^H_b + V \geq B \frac{\Delta_v}{\Delta_e} + \frac{\Delta_v}{\Delta_e} (1 - e^H)(1 - \pi^L) V
\]

(3.23)

The key difference with (3.8) is that the temptation to shirk due to confusion (the second term on the right-hand side) depends on the term $\Delta_v/\Delta_e$, where I have defined $\Delta_v \equiv \nu^H - \nu^L$, the effect of effort on the probability of success conditional on the failure of the other project. When the projects are independent, $\Delta_v = \Delta_e$, and the IC constraint in (3.8) obtains. With

\textsuperscript{52}It is still true that $Pr(\theta|\omega^{LL}) \geq Pr(\theta|s_i = s^L)$, that is, $\omega^{LL}$ is a stronger signal for the low aggregate state $\theta^L$ than a single realization of the low signal. The value of the additional interim information provided by the other project’s signal, though, is decreasing in the project’s correlation, that is, the higher $\pi$ the closer the posterior probability $Pr(\theta|\omega^{LL})$ to the posterior probability $Pr(\theta|s_i = s^L)$. Intuitively, the more correlated the projects, the more likely the occurrence of $\omega^{LL}$ when the aggregate state is high, and thus the lower the value of observing an additional signal that is more likely to replicate the existing one.

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correlation, it is easy to show that

$$\Delta_{\nu} = \left(\frac{1}{2} - \Upsilon\right) \Delta_p + \left(\frac{1}{2} + \Upsilon\right) \Delta_q > 0$$  \hspace{1cm} (3.24)$$

where

$$\Upsilon \equiv \left(\pi - \frac{1}{2}\right) \frac{p^H - q^H}{2 - (p^H + q^H)} \leq \frac{1}{2}$$

Under the assumption in (3.19), \(\Delta_{\nu} > \Delta_{\varepsilon}\), and \(\Delta_{\nu}\) is increasing in \(\pi\). The intuition for this result is as follows. Equation (3.24) shows that \(\Delta_{\nu}\) is a weighted average of \(\Delta_p\) and \(\Delta_q\), with a higher weight on \(\Delta_q\) because the failure of the other project (more likely when the factor is low) raises the chances that the own factor is low (in which case, \(q^H\) or \(q^L\) is the relevant probability of success depending on effort). Thus, if \(\Delta_q > \Delta_p\), then \(\Delta_{\nu} > \Delta_{\varepsilon}\). Furthermore, correlation raises the weight on \(\Delta_q\) in (3.24), and thus \(\Delta_{\nu}\), because it raises the strength of the other project’s failure as a signal that the own factor is low.

To summarize, under the assumption in (3.19) the minimum incentive compatible contingent payment \(R^H_b\) is larger when the projects are (positively) correlated than when they are independent; furthermore, an increase in correlation \(\pi\) raises \(\Delta_{\nu}\), and hence \(R^H_b\). In short, correlation magnifies the temptation to shirk due to confusion.

Remark 1. From (3.24), low effort raises the probability of confusion, a fact that may suggest that low effort raises the correlation between the projects. In general, though, this is not true. Intuitively, when the borrower shirks and switches to \((p^L, q^L)\), there are two opposing effects on the projects’ correlation. On the one hand, \(p^L < p^H\) implies that the project tends to succeed less frequently when the other project succeeds; this effect reduces correlation. On the other hand, \(1 - q^L > 1 - q^H\) implies that the project tends to fail more frequently when the other project fails; this effect raises correlation. If \(q^L\) is sufficiently small, the second effect prevails and low effort raises the correlation between the projects. This observation illustrates the point that what matters for the likelihood of confusion, and hence for incentives, is not the unconditional correlation between the projects, but the probability of failure conditional on the failure of the other project, that is, the conditional correlation.

Remark 2. The implication that an increase in the factors’ correlation reduces the value of the interim information by raising the likelihood of confusion contrasts with standard results
in the literature on relative performance evaluation. In this literature, an increase in the correlation between the activities of different agents improves the quality of information (it increases the signal to ratio noise of the realized performance) and thus reduces the agency costs.\footnote{See Holmström (1982). For example, in Acemoglu and Zilibotti (1998), capital accumulation increases the number of firms in the economy and hence reduces the noise in the signal represented by the firms' average performance.} Here, an increase in correlation shifts the probability mass from the state in which the interim information is perfectly informative (the state $\omega^{LH}$) to the state in which the signals are less than perfectly informative (the confusion state $\omega^{LL}$). This discreteness explains the negative association between correlation and the quality of information.

### 3.4.3 The comparative statics of correlation

In this subsection I discuss (somewhat informally) the comparative statics of the factors' correlation for the second best contract; the Appendix contains the details of the analysis. These results will be used in the next subsection to interpret the existing regulation. A change in $\pi$ affects the probability of confusion $\Pr(\omega^{LL})$ and the temptation to shirk due to confusion $\Delta_\nu$ (see (3.22) and (3.24)). It is useful to highlight the two channels separately.

Consider first the effect of correlation on the second-best liquidation in state $\omega^{LL}$. Under assumption (3.19), an increase in $\pi$ raises $\Delta_\nu$ and, for the sake of argument, suppose that $\Pr(\omega^{LL})$ is held constant. From the IC constraint in (3.23), the increase in $\Delta_\nu$ raises the temptation to shirk due to confusion, which has to be countered by raising liquidation in state $\omega^{LL}$. An increase in $\Pr(\omega^{LL})$ (holding $\Delta_\nu$ constant) raises the expected loss of the private benefit from continuation $V$, but also the expected cash flow from the asset market (since the state in which liquidation occurs becomes more likely). If the private benefit is sufficiently large, an increase in $\Pr(\omega^{LL})$ lowers liquidation.

Next consider the effect of correlation on leverage, holding liquidation constant. There are two effects to consider. First, an increase in $\Delta_\nu$ reduces pledgeable income, because the worsening in the borrower's incentives raises the minimum incentive compatible contingent payment to the borrower (see the IC constraint). Second, an increase in $\pi$ shifts the probability mass from state $\omega^{LH}$ to the confusion state $\omega^{LL}$, where a liquidity premium is paid (because $Y < 0$), reducing pledgeable income. These two effects reduce leverage, holding liquidation
constant. Changes in liquidation cannot offset the reduction in pledgeable income if the cash flow from the project's payoffs is more important for the repayment of the lender than the liquidation revenues from the asset market; this is the case if the asset market is sufficiently illiquid. Thus, an increase in correlation reduces leverage in the second-best contract, that is, it tightens the capital requirement on the borrowers that implements the second best allocation.

For convenience, I summarize this discussion in the following informal proposition.

**Proposition 5.** An increase in the factors' correlation $\pi$ raises the probability of confusion $\Pr(\omega^{LL})$ and, under assumption (3.19), the temptation to shirk $\Delta_\nu$. The net impact on the second best liquidation $\tau^{LL}_{C}$ is in general ambiguous, and is positive if the effect through $\Delta_\nu$ prevails, that is, if the moral hazard problem is sufficiently severe. If the asset market is sufficiently illiquid, an increase in correlation raises the capital requirement that implements the second best allocation. □

### 3.4.4 An interpretation of the regulation of trading books

In this subsection, I take the model's predictions as the benchmark for assessing the existing regulation of the proprietary trading activities of banks. Thus, I interpret the borrower as a bank and the project as the bank's portfolio of traded financial securities; the bank makes an unobserved portfolio decision, and raises external funds from the capital markets to (partially) finance its activity.  

The 1996 Amendment to the Basle Accord introduced a capital requirement for the *market risk*, "the risk of losses in on and off-balance-sheet positions arising from movements in market

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54The approach of interpreting a borrower as a financial institution whose investment decision is subject to moral hazard is common in the literature on banking regulation. In this literature, the moral hazard problem is often modeled as risk-shifting, and the financial contract as a given debt (deposit) contract. See Freixas and Rochet (1997), Chapter 9, for a review of this literature. Other examples of moral hazard include absconding by the bank's managers (Calomiris and Kahn (1991)) and the choice between risky and safe assets (Hellman, Murdock and Stiglitz (2000) and John, Saunders and Senbet (2000)). The previous analysis has implications for the entire activity of the bank rather than for the trading activities only. That is, it follows a "whole-bank" approach, and does not address the reasons why the banks' trading book is in practice regulated separately from the lending book. From the theoretical point of view, though, it is not clear why they should be the object of separate regulation. See Holmström and Tirole (2000) for a discussion of this point.

prices” of securities that a bank holds in its portfolio for speculative purposes or to carry out its brokering and market making activities (the “trading book”). In many countries, the trading book consists of traded securities that the banks are required to mark to market. The Amendment allows the banks that meet certain qualitative requirements to use their own internal risk management models to compute the capital requirement for market risk. The capital requirement is increasing in market risk, measured as the Value at Risk (VaR) of the bank’s portfolio (the upper quantile at a prespecified confidence level of the distribution of portfolio losses over a given time horizon).\footnote{The (daily) capital requirement is proportional to the moving average of the last sixty (daily) realized VaR measures; however, if the last VaR measure exceeds the number that results from this computation, the capital requirement is equal to the last VaR measure.} Moreover, the banks are subject to an ex-post verification process known as “backtesting”. Backtesting is a statistical test of the accuracy of a bank’s risk model, and “typically consists of a periodic comparison of the bank’s daily Value-at-Risk measures with the subsequent daily profit or loss—the trading outcome.” Under the null hypothesis that the bank’s risk model is accurate, over a sufficiently long period the fraction of times a daily trading outcome exceeds its predetermined VaR measure (an “exception”) should be sufficiently close to the confidence level used to compute the VaR measure. The penalties for exceeding the VaR limits are increasing in the number of exceptions, and take the form of tighter future capital requirements. However, the Amendment emphasizes explicitly that considerations over the power of the test “should be prominent (...) in interpreting the results of a bank’s backtesting program”, and that the intensity of the regulatory response should depend on the “strength of the signal generated from the backtest”. As a result, the penalties should not be automatic, for “the Committee has no interest in penalizing banks solely for bad luck”. Thus, the regulator should not intervene if the exceptions are due to “an unexpected bout of high market volatility, which nearly all models may fail to predict” rather than to a “correctable problem in the bank’s model”, that is, when the exceptions are due to “bad luck”.

Because asset price volatilities and correlation tend to move together, capital requirements increasing in VaR tend to increase with recent market volatility and to be correlated across financial institutions. Changes in volatilities and correlation are also likely to affect incentives through the mechanism described in this chapter.\footnote{Some evidence on the correlation of VaR measures at a quarterly horizon across institutions and with market} In the simple model presented in this chap-
ter, confusion at the interim stage is a discrete phenomenon, whose likelihood increases with the correlation between the factors. However, I argued in Section 2 that the empirical evidence suggests that the quality of the information available for performance evaluation worsens continuously with market volatility. Thus, I conduct the following thought experiment. Suppose that there is a change in the quality of information that will be available in the future for performance evaluation, say because of a change in market volatility and correlation expected in the future (in the following, market volatility for short). How does the second best contract respond to such a change? In particular, what is the adjustment in the ex-ante capital requirement that implements the second best? The answers to these questions (summarized in Proposition 5) provide the model's benchmark for the stylized features of the current regulation.\footnote{volatility is provided by Jorion (2001); see especially Figure 2. However, Berkowitz and O'Brien (2001) fail to find a strong correlation across VaR measures at a daily horizon. One explanation for this divergence may be that the supply of capital to trading activities is likely to be very inelastic over a very short horizon, implying that institutions have to adjust their asset positions to meet their capital requirements with a constant endowment of capital. The implication could then be that actual VaR measures display little variation and covariation at a very high frequency. The relation between a "bouf of high market volatility" and the performance evaluation problem is clearly on the regulators' minds, as it is apparent from the above quotes.} According to Proposition 4, provided that the second-best liquidation is sufficiently small (that is, the asset market is sufficiently liquid), a capital requirement implements the second-best allocation; furthermore, under the conditions in Proposition 5, it is optimal to raise the capital requirement with (expected) market volatility. In short, when capital requirements are effective, they exhibit a risk-based pattern. Proposition 5 also formalizes the concern about the automatic application of the ex-post penalties: if the continuation value is sufficiently large, performance-based measures should be relaxed when the quality of performance information deteriorates (modeled as an increase in $\Pr(\omega^{LL})$). However, this argument overlooks the negative incentive implications of confusion ($\Delta_v$ in the model), and thus can be incomplete.\footnote{This approach is consistent with the spirit of VaR-based regulation, which aims at being forward looking in its measurement of risk. The current regulation requires banks to update the information used to estimate the asset price distribution at least once a year. However, the common industry practices of weighting recent observations more heavily (exponential weighting) imply that VaR measures tend to be sensitive to the most recent market behavior.}
The capital requirement prescribed by the Accord is increasing in market volatility, possibly in a very steep fashion. By contrast, the model points out that there are situations where capital requirements become ineffective and liquidity considerations predominant (in the sense of Proposition 4), e.g. when large shocks to volatility (or directly to market liquidity) reduce liquidity and in turn the agents’ access to external finance. In these circumstances, other policies aimed more directly at the asset market liquidity are needed. However, these are precisely the circumstances when actual risk-based capital requirements are likely to be tightened.

3.5 Conclusions

This chapter has studied the optimal balance between financial incentives and termination threats in a financial contract between a borrower and a lender. Financial incentives are provided by requiring the borrower to finance a sufficiently large share of her project. Termination takes the form of liquidating the project at a price that depends on aggregate liquidation. Because agents in private contracts do not internalize the negative externality that works through the liquidation price, there is a role for public policy to implement the efficient second-best allocation. If the asset market is sufficiently liquid, the inefficiency in the decentralized equilibrium takes the form of excessive leverage, and a capital requirement on the borrowers implements the second best allocation. However, capital requirements are ineffective if the asset market is sufficiently illiquid, since the inefficiency takes the form of a capital squeeze on the borrowers. The introduction of a liquidity option combined with a liquidity requirement on the borrowers can implement the second best allocation. The implementation, though, cannot be fully decentralized, since the planner has to deal with the free rider problem caused by the agents’ ability to trade in the asset market.

Starting from the Basle Accord in 1988, risk-based capital requirements have increasingly assumed a central role in the regulation of financial institutions. This chapter has provided an interpretation of risk-based capital requirements. I have shown that the worsening in the quality of information associated with diffuse poor performance can have negative incentive effects that can be countered by an increase in capital requirements, and I have argued that this worsening in information is an implication of market volatility. However, this chapter has
also emphasized the importance of considering institutions or policies different from capital
requirements to deal with the excessive liquidation problem in response to large shocks, when
capital requirements become ineffective; liquidity requirements are an example of such policies.
A comprehensive study of the conditions under which liquidity requirements should complement
capital requirements is a promising direction for future research. The relationship between
liquidity requirements and capital requirements raises interesting questions about their modus
operandi, since the former affect the composition of the asset side of the balance sheet while the
latter the composition of the liability side. In turn, the practical implementation of liquidity
requirements raises additional issues, for example with regard to their incentive implications
and the verifiability of the liquidity events; these issues were briefly touched in the construction
of the liquidity option in Section 3.

In this chapter I have studied two links between the financial contracts. The contracts
interact because the performance of a borrower reveals information useful for the performance
evaluation of the other borrower, and because the liquidation price available for a project
results from the liquidation decisions made simultaneously in the two contracts. However, the
model has some partial equilibrium features, which limit somewhat the interpretation of the
results. I discuss briefly three of these limitations that can be important. In the analysis
of the implications of the quality of information, I have separated the effect of a change in
the correlation between the projects (which determines the quality of information) from the
effect of a change in the value of the projects (against which the borrowers can raise external
finance and capital requirements are set). In practice, though, the value of the projects and their
second moments are likely to move together, and to be determined endogenously in equilibrium.
The second caveat concerns the assumption, made to focus on the determinants of the cost of
liquidation, that the opportunity cost of capital to borrowers and lenders is constant. However,
in more general contexts changes in the demand for capital are likely to affect the required
rate of return. Finally, while I have assumed an exogenous supply of liquidity in the form of a
downward sloping demand for the projects, the economic incentives to supply liquidity should
be considered explicitly. These incentives are likely to depend on the contracts in place, and
thus be affected by the existing institutional arrangements. A more complete investigation of
the efficiency properties of risk-based capital requirements needs to acknowledge the feedback
effects among these channels.

3.6 Appendix

3.6.1 The optimal contract

Let $\alpha$ be the multiplier on the IC constraint; $\lambda$ the multiplier on the IR constraint; $\varphi_{LL}$ the multipliers on $y_{LL} \geq 0$; $\zeta_{LH}$ the multiplier on $\tau_{1L}^H \leq 1$; $\varrho$ the multiplier on $R^L_L \geq 0$. Furthermore, set $k_1 + k_2 = 1$. I do not need to specify all the other multipliers. The first-order condition (FOC) for $k_1$ yields $\lambda = \rho > 1$. The FOC for $R^H_L$ is given by $\gamma e^H + \alpha \gamma e^H = 0$, that is, $\alpha = e^H(\rho - 1)/\Delta e > 0$. Thus, the IC and IR constraints are binding. The FOC for $R^L_L$ is $(1 - \gamma) - \lambda(1 - \gamma) + \rho = 0$, which implies $R^L_L = 0$. The FOC for $y^{LL}$ yields $\varphi_{LH} = (\rho - 1)\Pr(\omega^{LL}) + \alpha \gamma \Delta e e^H > 0$, that is, $y^{LL} = 0$. The FOC for $\tau_{1L}^H$ is $\alpha \gamma \Delta e e^H V - (1 - e^H) e^H \gamma V = \zeta_{LH}$, implying that $\tau_{1L}^H = 1$ if $e^H \rho > 1$. Finally, the FOC for $\tau_{1L}^{LL}$ is

$$\alpha \gamma \Delta e (1 - e^H) V + \rho \Pr(\omega^{LL})[\tau_{LL} L^P + P - \Pr(\theta^L | \omega^{LL}) R^L_L] - \Pr(\omega^{LL}) V = 0$$

which can be rearranged as

$$\tau_{1L}^{LL} P^L + P + \Pr(\theta^{L} | \omega^{LL}) \frac{\rho e^H - 1}{(1 - e^H)} \frac{V}{\rho} = \Pr(\theta^L | \omega^{LL}) \left( R^L_L + \frac{V}{\rho} \right).$$

Since $P < \Pr(\theta^L | \omega^{LL}) R^L_L$ and $P' < 0$, a necessary condition for an internal solution for is $\tau_{1L}^{LL}$ is

$$\Pr(\theta^H | \omega^{LL}) \frac{\rho e^H - 1}{(1 - e^H)} > \Pr(\theta^L | \omega^{LL})$$

which can be rearranged as $\rho e^H > 1 + \frac{1 - \gamma}{\gamma(1 - e^H)}$, the condition given in the Proposition. Under this condition, there exists an interval $[V_{min}, V_{max}]$ for the values of the private benefit $V$ such that the solution for $\tau_{1L}^{LL}$ is interior. $V_{max}$ is the largest value of the private benefit such that the constraint $\tau_{1L}^{LL} \leq 1$ is not binding:

$$P(2) + P'(2) + \Pr(\theta^H | \omega^{LL}) \frac{\rho e^H - 1}{(1 - e^H)} \frac{V_{max}}{\rho} = \Pr(\theta^L | \omega^{LL}) \left( R^L_L + \frac{V_{max}}{\rho} \right).$$
\( V_{\text{min}} \) is the smallest value of the private benefit such that the constraint \( \tau_1^{LL} \leq 1 \) is not binding:

\[
P(0) + \Pr(\theta^H | \omega^{LL}) \frac{\rho e^H - 1}{(1 - e^H)} V_{\text{min}} \rho = \Pr(\theta^L | \omega^{LL}) \left( R^L + \frac{V_{\text{min}}}{\rho} \right).
\]

### 3.6.2 Proof of Proposition 4

Let \( \frac{dk_1^*}{d\tau^{LL}} \) and \( \frac{\partial k_1^*}{\partial \tau^{LL}} \) be the total and partial derivatives (with respect to the first argument) of the leverage function \( k_1^* \) in (3.11) with respect to liquidation, evaluated at \( \tau_1^{LL} = \tau_2^{LL} = \tau^{LL} \). It is easy to see that \( \frac{dk_1^*}{d\tau^{LL}} < \frac{\partial k_1^*}{\partial \tau^{LL}} \). Define the threshold \( t_0 \) as

\[
\frac{dk_1^*}{d\tau^{LL}} \bigg|_{t_0} = \frac{\partial k_1^*}{\partial \tau^{LL}} \bigg|_{\tau_{\text{max}}} > 0
\]

From its definition, \( t_0 < \tau_{\text{max}} \), since \( \frac{dk_1^*}{d\tau^{LL}} < \frac{\partial k_1^*}{\partial \tau^{LL}} \). According to this definition, if the second best liquidation \( \tau^{LL,C} \) coincides with the threshold \( t_0 \), \( \tau^{LL,C} = t_0 \), liquidation in the SDE maximizes leverage, \( \tau^{LL,D} = \tau_{\text{max}} \). By continuity and concavity of the function \( k_1^* \), there exists a threshold \( t \) such that \( t_0 < t < \tau_{\text{max}} \) such that if \( \tau^{LL,C} = t \), then \( k_1^*(t, t) = k_1^*(\tau^{LL,D}, \tau^{LL,D}) \).

This is the case because when the second best liquidation \( \tau^{LL,C} \) is equal to \( \tau_{\text{max}} \), the definition of \( \tau_{\text{max}} \) and \( \tau^{LL,C} < \tau^{LL,D} \) imply that \( k_1^*(\tau_{\text{max}}, \tau_{\text{max}}) > k_1^*(\tau^{LL,D}, \tau^{LL,D}) \), and when \( \tau^{LL,C} \) is sufficiently close to \( t_0 \), \( k_1^*(t_0, t_0) < k_1^*(\tau^{LL,D}, \tau^{LL,D}) \). From the concavity of \( k_1^*(x, x) \), it finally follows that when \( \tau^{LL,C} \in [t, \tau_{\text{max}}] \), the second best leverage exceeds leverage in the SDE, since \( k_1^*(\tau^{LL,C}, \tau^{LL,C}) > k_1^*(t, t) > k_1^*(\tau^{LL,D}, \tau^{LL,D}) \).

### 3.6.3 The comparative statics of correlation

I have discussed in the text how the IC constraint is modified by correlation. The borrower's leverage \( k_l = k_l^* + k_1^*(\tau_{LL,1}, \tau_{LL,2}) \) is now given by

\[
\bar{k}_l \equiv \gamma e^H \left( R^H - \frac{B}{\gamma \Delta e} \right) + (1 - \gamma) R^L + \gamma e^H \left[ 1 - \frac{\Delta u}{\Delta e} (1 - e^H) \right] V \tag{3.25}
\]

\[
k_1^*(\tau_{LL,1}, \tau_{LL,2}) \equiv \Pr(\omega^{LL}) Y(\tau_{LL,1}, \tau_{LL,2}) + \gamma e^H \frac{\Delta u}{\Delta e} (1 - e^H) \tau_{LL,1} V. \tag{3.26}
\]
The second-best liquidation $\tau^{LL,C}$ satisfies

$$Pr(\omega^{LL})[Y_1 + Y_2] + \gamma e^H \frac{\Delta_v}{\Delta_e} (1 - e^H)V = Pr(\omega^{LL})\frac{V}{\rho} + \gamma e^H \frac{\Delta_v(1 - e^H)}{\Delta_e} \frac{V}{\rho}.$$  (3.27)

It is immediate to see from (3.27) that an increase in $\Delta_v$ raises $\tau^{LL,C}$. Clearly, holding $\tau^{LL,C}$ constant, an increase in $\Delta_v$ reduces leverage. However, it is in principle possible that the increase in $\tau^{LL,C}$ has an offsetting effect on leverage. The total impact of $\Delta_v$ on leverage is

$$\frac{dk_1}{d\Delta_v} = \frac{\partial k_1}{\partial \Delta_v} + \frac{\partial k_1^*}{\partial \Delta_v} + \frac{dk_1^*}{\partial \tau^{LL,C}} \frac{\partial \tau^{LL,C}}{\partial \Delta_v}$$

$$= \frac{(1 - e^H)(1 - \tau^{LL,C})V}{\Delta_e} + \frac{dk_1^*}{\partial \tau^{LL,C}} \frac{d\tau^{LL,C}}{d\Delta_v}$$

$$= \frac{(1 - e^H)(1 - \tau^{C}_{LL})V}{\Delta_e}$$

$$+ \left[ Pr(\omega^{LL})(Y_1 + Y_2) + \gamma e^H \frac{\Delta_v}{\Delta_e} (1 - e^H)V \right] \frac{\gamma e^H (1 - e^H)(\rho - 1)}{-\Delta_e Pr(\omega^{LL})(4P' + 4\tau^{LL,C}P'') \rho}$$

where I have used

$$\frac{\partial (Y_1 + Y_2)}{\partial \tau^{LL}} = 4P' + 4\tau^{C}_{LL}P''$$

$$\frac{d\tau^{C}_{LL}}{d\Delta_v} = \frac{-\gamma e^H (1 - e^H)(\rho - 1)}{\Delta_e Pr(\omega^{LL})(4P' + 4\tau^{LL,C}P'') \rho} > 0,$$

where $P$, $P'$ and $P''$ are evaluated at the planner’s optimum. The sign for $\frac{dk_1}{d\Delta_v}$ is ambiguous. It is negative is the effect through $k_1$ prevails, that is, if

$$\left[ P + 2P' - Pr(\theta^L|\omega^{LL})R^L + \gamma e^H \frac{\Delta_v}{Pr(\omega^{LL})} (1 - e^H)V \right] \frac{1}{(4P' + 4\tau^{LL,C}P'')} < \frac{\rho(1 - \tau^{C}_{LL})}{\gamma e^H (\rho - 1)}$$

This condition is more likely to be satisfied the lower $P$ and the higher the absolute values of $P'$ and $P''$.

Finally, from (3.27), it is easy to see that

$$\frac{d\tau^{LL,C}}{dPr(\omega^{LL})} < 0 \iff \frac{V}{\rho} > P + 2\tau^{C}_{LL}P'.$$

Combining these results, if $V$ is sufficiently large (so that an increase in $Pr(\omega^{LL})$ does not raise
\( \tau^{LL, C} \) too much) and the asset market is sufficiently illiquid, an increase in \( \pi \) reduces leverage.
Chapter 4

Short-Term Capital Flows and Institutions for International Lending\textsuperscript{1}

The last decade has witnessed important changes in the level and composition of capital inflows into developing countries.\textsuperscript{2} Emerging countries depend on the external flows to finance their stabilization programs, their process of reform, and more generally their path toward development. Unfortunately, the increased reliance on capital inflows has also brought about a larger vulnerability to their reversal, and opened the possibility for a new type of currency crises. Indeed, the so-called third generation models of balance of payment crises have highlighted the interplay between the financial sector and capital outflows as the main engine for the collapse of exchange rate regimes (see, for example, Calvo (1998)). Moreover, capital flows have played a major role in the cause, propagation and deepening of international crises; Mexico 1994, Asia 1997, Russia 1998, and now Argentina, are examples of how the crises were exacerbated by the

\textsuperscript{1}This chapter has been written jointly with Roberto Rigobon.

\textsuperscript{2}On the pattern of capital flows, see Lane and Milesi-Ferretti (2000). On the composition of debt flows, see Rodrik and Velasco (1999). The main stylized facts about capital inflows can be summarized as follows: (i) syndicated bank loans and official flows were the most common form on international financing to developing countries in the late 1970's and early 1980's. (ii) portfolio (especially equity) and foreign direct investment (FDI) flows have increased substantially during the second part of the 1980's and especially in the 1990's (the "age of equity finance"). (iii) portfolio debt flows (bonds) have played an increasingly central role within debt finance, substituting for the decline in syndicated bank loans. (iv) short-term debt grew particularly rapidly during the 1990's.
interaction between capital flows and unsustainable exchange rate regimes.

The transformed nature of financial and currency crises has spurred a large and growing literature on the reform of the international financial system ("international financial architecture"). What role should be given to the international agencies such as the International Monetary Fund and the World Bank? What are the optimal institutions for international lending? To what degree should the private sector be involved in crisis resolution? This literature has largely focused on the implicit transfer to foreign investors that is supposed to take place through the international financial safety nets, and on the reforms aimed at limiting its size. The evidence on the actual magnitude and cost of this transfer is ambiguous, though.\textsuperscript{3}

This chapter builds on the main insight that capital flows to developing countries are characterized by a basic form of contractual incompleteness: the contracting possibilities are sufficiently limited that the international investors cannot commit to provide long-term or state-contingent financing to the domestic borrowers. We believe that this form of contract incompleteness is an important aspect of the poorer contractual environment and lower development of financial market institutions in developing countries. The large volatility and "on-off" nature of capital inflows (and asset prices) and the fact that long-term debt markets are missing in most developing countries provide indirect evidence of the inability of borrowers in developing countries to secure a stable source of finance.\textsuperscript{4}

In our model, domestic agents (the "entrepreneurs") have to raise external finance to implement their projects; a moral hazard problem limits the amount of the external finance that they can raise. In the basic version of model, the entrepreneurs can issue only short-term claims with the following properties. First, the foreign investors who purchase these claims can demand the repayment of their claims before the maturity of projects; second, they have priority over the proceeds generated by the liquidation of the domestic projects. Our key assumption is that the foreign investors cannot commit not to demand the early repayment of their claims. This lack of commitment leads to the excessive liquidation of the domestic projects. Large banks--especially

\textsuperscript{3}See Fernández-Arias and Hausmann (2000a) and Jeanne and Zettelmeyer (2001).

\textsuperscript{4}This assumption is similar to the main assumption in Holmström and Tirole (1998), i.e. that moral hazard limits the income that firms can pledge to outside investors, and thus limits the ability to raise funds to withstand liquidity shocks, and in Caballero and Krishnamurthy (2000), where domestic assets cannot be used internationally as collateral. Eichengreen and Hausmann (1999) refer to the situation in which the domestic currency cannot be used to borrow abroad or long-term as "original sin". Essentially, all the non-OECD countries are afflicted by the original sin; Eichengreen and Hausmann discuss some possible causes for it.
when participating in a syndicated loan—and official lenders are more likely to coordinate and
roll over or restructure their claims at times of crises than the dispersed holders of financial
claims. As a result, the relative decline in syndicated bank loans and official flows and the
rise of market-mediated finance are likely to have exacerbated the importance of this form of
contractual incompleteness. By interpreting liquidation as a measure of the volatility of capital
flows, this contractual incompleteness therefore implies that we should observe the increase in
the volatility of capital flows that has followed the capital account liberalization in developing
countries.

Institutions that limit the extent of liquidation raise welfare, since they help circumvent the
contractual incompleteness that gives rise to the inefficiency. We first compare two institutional
arrangements, a mechanism based on the bailout of the domestic entrepreneurs through ex-post
government intervention and a workout mechanism based on a majority rule, and then study
the role of policies that stimulate long-term finance.

As regards bailouts, the government can prevent the liquidation of the domestic projects by
implementing a (costly) corrective policy that offsets the effects of negative aggregate shocks.
We show that there are welfare gains from the government’s ability to commit ex-ante to a
bailout policy. However, a credible pre-commitment has to be backed by the willingness to
implement a larger intervention. We study then a workout mechanism based on the simple
majority rule that a foreign investor is allowed to demand the early repayment of her claims
only if a sufficiently large majority of investors choose to do the same. This mechanism is
consistent with recent proposed reforms of the international financial system that insist on
the coordination problem among numerous and dispersed holders of claims held by foreign
investors and traded in financial markets, most notably the Krueger (2001)’s proposal for a
workout mechanism in the context of a sovereign default. Under the plausible assumption that
financing government intervention is costly, the workout mechanism is a better institution than
the ex-post bailouts to deal with the excessive liquidation problem. The workout mechanism
operates directly as a coordination device for the agents’ expectations and does not require the
costly transfer of resources across periods and states of nature that underpins the government
intervention.

Finally, we extend the model to allow the domestic entrepreneurs to issue two types of claims,
a short term claim and a more stable source of finance that is not subject to the contractual incompleteness. This extension is motivated by the marked increase in the importance of foreign direct investment (FDI) flows to developing countries that has taken place in the last decade. By definition, FDI has a long maturity (coinciding with the life of the firm/project). Thus, this form of external finance should be less prone to the contractual incompleteness that plagues short-term finance and gives rise to excessive liquidation. Indeed, there is substantial evidence that FDI flows are less volatile than other forms of financial flows to developing countries.\footnote{See Albuquerque (2001) and the references therein for empirical evidence on the volatility of different forms of capital flows.} In addition to the long maturity, we also assume that FDI mitigates the agency problem between the domestic entrepreneurs and external investors but is costly to provide relative to the short-term source of finance ("hot money"). We show that in the competitive equilibrium the level of FDI is excessive. The reason is that FDI generates a negative externality through the contractual incompleteness because it raises the short-term investors’ incentives to demand the early repayment of their investment. This result deserves attention because it contrasts with the objectives of controls on short-term capital inflows that have been proposed as a way to reduce a country’s vulnerability to short-term flows.

The chapter is organized as follows. In Section 2 we present the basic model of international lending, and we show how the contractual incompleteness leads to excessive liquidation. In Section 3 we study how ex-post bailouts and a workout mechanism can mitigate the excessive liquidation problem. In Section 4, we extend the model to allow domestic borrowers to issue two types of external claims, short-term claims ("hot money") and long-term claims (FDI), and we study whether the optimal composition of claims is achieved in decentralized markets. Section 5 briefly concludes, and is followed by an Appendix.

4.1 The basic model

We consider a model of international lending with domestic firms (owned by domestic entrepreneurs), foreign investors, and the domestic government. The central element of the model is the relationship between a risk-neutral entrepreneur who has access to an investment opportunity (the project) and risk-neutral foreign investors who finance a fraction of the initial cost of
the investment. There is a moral hazard problem since the entrepreneur makes an investment decision, unobservable by the investors, that affects the outcome of the project. Figure 1 shows the timeline of the events in the model. There are three periods, 0, 1 and 2, and one good.

In period 0, a financial contract is signed between an entrepreneur and foreign investors. The size of the initial cost of the investment is normalized to one. After the contract has been signed, the entrepreneur chooses an action; the action is not observed by the lender, and determines the probability of success of the entrepreneur’s project. The borrower can choose between the high \( p_H \) and low \( p_L \) probability of success. The entrepreneur can improve the distribution if she is “diligent”, but enjoys a private benefit \( B > 0 \) if she “shirks”.

In period 1, the aggregate shock \( a \) is realized and observed by all the agents in the economy; the shock \( a \) affects the payoff in case of success of all the projects in the economy. In the next section, we will allow the government to take an action \( g(a) \) conditional on the shock \( a \). After the shock \( a \) has been realized (and, in the next section, after the government has taken its action), the foreign investors make their liquidation or continuation decisions.

In period 2, the payoff is observed and the contractual payments are made.
4.1.1 Entrepreneurs

Projects' Financing

There is a continuum of risk-neutral entrepreneurs, indexed by $z \in [z, \bar{z}]$, with distribution function $H$ and density function $h$. The entrepreneurs differ with regard to their initial wealth: entrepreneur $z$ has initial wealth $w(z)$. We assume that $w(z)$ is a continuous in $z$, and that the entrepreneurs are ranked in terms of their wealth, i.e. $w(z)$ is a nondecreasing function of $z$. Below it will be convenient to assume that there are many entrepreneurs of each type $z$, but for convenience we refer to entrepreneur $z$ rather than to an “entrepreneur of type $z$”.

The entrepreneurs have an identical investment project that requires an initial fixed investment cost normalized to one. All the entrepreneurs have to access the capital markets in order to finance their projects, i.e. $w(z) < 1$ for all $z \in [z, \bar{z}]$. External finance $d(z)$ is provided to entrepreneur $z$ by competitive risk-neutral investors who require an interest rate normalized to zero on their initial investment in the project.

Moral hazard in production

After the project has been financed, the entrepreneurs make an unobservable investment decision that affects the distribution of her project’s payoffs. The project has two possible outcomes, “success” and “failure”. If the project succeeds, the payoff is $a$, where $a$ is an aggregate shock realized in period 1; the payoff of the project in case of failure is normalized to zero for simplicity. The aggregate shock $a$ is distributed according to the continuous distribution function $F$ (with density function $f$).

The entrepreneur’s investment decision between two actions determines the probability of success of the project, $p \in \{p_H, p_L\}$, with $\Delta_p = p_H - p_L > 0$. The choice of the high probability of success $p_H$ entails the private cost $B > 0$ to the entrepreneurs. Together with the assumption that the entrepreneur’s action is unobservable, this last assumption implies that there is a moral hazard problem in production. The high probability of success $p_H$ is the socially efficient investment decision, i.e. $p_H E[a] > p_L E[a] + B$.
4.1.2 Foreign investors

In this section and in the next, we assume that foreign investors cannot commit at all to long-term finance; we relax this assumption in Section 4.

Short-term claims

After the aggregate shock $a$ is realized and observed, the investors can demand the repayment of their claims. Given that the projects are long-term (the payoff is realized in period 2), and assuming that the entrepreneurs do not have access to alternative sources of funding in period 1, the only way the entrepreneurs can meet the demand to repay is to liquidate their projects. Following Shleifer and Vishny (1992), the liquidation of a project takes place by transferring it to users with inferior abilities, in the sense that the project’s continuation value falls if the project is reallocated: if the project’s NPV is $V$ with the original entrepreneur, it is reduced by liquidation to $\lambda V$, where $0 < \lambda < 1$. The fraction $(1 - \lambda)V$ that is lost in case of liquidation is a real cost from the social point of view.

In practice, we can think of the investors’ demand of repayment as the issue of margin calls on the entrepreneurs, the suspension of a line of credit, or the refusal to roll over short-term debt. An entrepreneur who is asked to repay the claims that she issued to foreign investors is forced to liquidate her project and use the proceeds from liquidation to meet the repayment.

The second key assumption that we maintain is that the claims owned by the foreign investors are senior to the claims of the entrepreneurs. Thus, when the investors demand the repayment of their claims, they have priority over the proceeds from liquidation. We formalize this assumption next.

Optimal liquidation policy

Suppose that project $z$ has been implemented, and that the contract between entrepreneur $z$ and the foreign investors specifies that the entrepreneur will receive a payment $R_E(z)$ if the project succeeds. If in period 1 the aggregate shock $a$ is realized, the investors learn that the project’ payoff in case of success will be $a$, and that the expected value of their claims is $p_H(a - R_E(z))$. By assumption, the investors have priority over the entrepreneur on the proceeds from the liquidation of the project. Thus, the investors can demand the repayment of
their claims in period 1 even if liquidation reduces the project’s continuation value. Specifically, liquidation occurs if the value of the claims of external investors is below the liquidation value of the project,

\[ p_H(a - R_E(z)) \leq \lambda p_H a. \]  \hspace{1cm} (4.1)

The left-hand side is the expected value of the investors’ claim if the project is not liquidated; the right-hand side is the liquidation value of the project. Equivalently, liquidation occurs if the value of the entrepreneur’s claim exceeds the cost of liquidation,

\[ R_E(z) \geq (1 - \lambda)a. \]  \hspace{1cm} (4.2)

Since the cost of liquidation increases with \( a \), high realizations of \( a \) make continuation more profitable to the investors. Furthermore, the higher the stake of domestic entrepreneurs, the more likely the project is liquidated. Note that liquidation does not occur necessarily regardless of the realization of the shock \( a \) only if liquidation is costly, i.e. \( \lambda < 1 \). If liquidation were not costly, then liquidation would occur for any \( a \). However, in this case the entrepreneur would not find it profitable to implement the project.

The liquidation rule (4.2) defines the threshold \( \tilde{a}(z) \) below which project \( z \) is forced into liquidation by the investors’ demand to repay their claims,

\[ \tilde{a}(z) = \frac{R_E(z)}{1 - \lambda}. \]  \hspace{1cm} (4.3)

Remark 1. From the social point of view, liquidation is always costly, because \( (1 - \lambda)a \) is lost in case of liquidation. The criterion for liquidation in (4.1) is effectively a mark-to-market criterion, since liquidation occurs when the liquidation value of the project falls below the value of the investors’ claims. The investors hold a put option on the value of the project at a strike price that depends on \( R_E(z) \) and the liquidation value \( \lambda \). Therefore, liquidation admits the interpretation of default triggered by the fall in the market value of the entrepreneur’s asset below the value of her liabilities.

Remark 2. We have implicitly assumed that the contingent payment \( R_E \) cannot be contingent on the aggregate shock, i.e. that the aggregate shock is noncontractible. If the contingent
payment could be indexed to \( a \), contracts that prevent liquidation (i.e. for which (4.2) is never satisfied) could be written. We rule out these contracts, since they effectively circumvent the contractual incompleteness we want to emphasize.

**Remark 3.** The liquidation rule (4.2) assumes that no renegotiation takes place in period 1, even if liquidation is ex-post inefficient. The borrower may try to prevent liquidation by offering a payment \( P(a) \) to the investors, and to finance the payment by avoiding the liquidation cost. If the entrepreneur promises a payment equal to the liquidation cost, \( P(a) = (1 - \lambda)p_H a \), then liquidation occurs if \( p_H (a - R_E(z)) \leq (2\lambda - 1)p_H a \). Then, if \( \lambda > \frac{1}{2} \), renegotiation cannot eliminate some liquidation, and the analysis is essentially unchanged. If \( \lambda < \frac{1}{2} \), however, renegotiation implies that liquidation never occurs, because the return to the investors from liquidation is too low.\(^6\)

### 4.1.3 The financial contract

A financial contract between entrepreneur \( z \) and the foreign investors determines first whether the entrepreneur can implement her project. If so, the contract specifies \( d(z) \), the initial payment from the investors to the entrepreneur; \( \tilde{w}(z) \), the entrepreneur’s own contribution to the initial investment, with \( \tilde{w}(z) \leq w(z) \), the entrepreneur’s own wealth; \( R_E(z) \), the contingent payment to the entrepreneur in case of success, provided the project is not forced into liquidation in period 1. By assumption, the contract does not specify when the investors can demand the repayment of their claims. However, the investors have priority over the assets of the entrepreneur whenever the latter is unable to meet the demand of repayment. The project can be implemented provided that \( \tilde{w}(z) + d(z) \geq 1 \).

**Remark.** There is potentially a problem if \( \tilde{w}(z) < w(z) \) and thus \( w(z) + d(z) > 1 \), that is, if the entrepreneur does not invest all her initial wealth in the project. In this case, the entrepreneur could save \( w(z) + d(z) - 1 \) in order to try to prevent liquidation in the interim.

\(^6\)Otherwise, following the argument in Diamond and Rajan (2000) renegotiation may be impossible because of the collective action problem among a large number of competitive lenders. For example, suppose that there is a large number of investors in each project, and the aggregate shock \( a \) is below \( \tilde{a}(z) \), implying that the project will be liquidated if no renegotiation takes place. Let \( \nu > 0 \) be the fraction of investors who decide to liquidate, and \( 1 - \nu \) the fraction of investors who decide not to liquidate. Since \( \nu > 0 \), the liquidation cost is incurred, and \( P(a) = 0 \). Therefore, any investor who decides not to liquidate is exactly indifferent between liquidation and continuation, and liquidation occurs as a Nash equilibrium.
period. For simplicity, we rule out this possibility by assuming that the entrepreneurs do not have access to the capital markets for their savings and thus consume $\bar{w}(z) + d(z) - 1$ in period 0. This problem is an unpalatable feature of the fixed-size investment; if the entrepreneur could vary the scale of the project, she would use the available funds to expand the scale of her project, and this problem would not arise.\footnote{Holmström and Tirole (1998) study how firms can secure ex-ante the supply of liquidity in order to prevent future liquidation.}

4.1.4 The agents’ surplus

If entrepreneur $z$ invests $\bar{w}(z)$ of her wealth in the project and receives $R_E(z)$ in case of success, the entrepreneur’s net surplus from entering the financial contract with the investors is the expected value of the payment in case of success net of the opportunity cost of the investment $\bar{w}(z)$,

$$V_E(z) = \left[1 - F(\bar{a}(z))\right]p_z R_E(z) - \bar{w}(z),$$

where $p_z \in \{p_H, p_L\}$ depending on the probability of success chosen by the entrepreneur. If the entrepreneur cannot implement the project, her net surplus is zero.

The investors in project $z$ receive the payoff of the project net of the payment to the entrepreneur if the project succeeds when not liquidated in period 1, $a - R_E(z)$, and the liquidation value of the project $(1 - \lambda)$ if the latter is liquidated. Since the project is forced into liquidation when the aggregate shock $a$ is below the threshold $\bar{a}(z)$, the net surplus of the investors in project $z$ can be written as

$$V_I(z) = p_z E[a] - (1 - \lambda)p_z E[a | a < \bar{a}(z)] F(\bar{a}(z)) - p_z R_E(1 - F(\bar{a}(z))) - d(z)$$

that is, the investors receive the expected value of the project minus the expected liquidation cost, the payment to the entrepreneur, and the cost of capital.

Define the (expected) pledgeable income $y(R_E(z), \bar{a}(z))$ as the expected value of the income that can be pledged by entrepreneur $z$ to foreign investors given the contingent payment $R_E(z)$.
and the threshold $\hat{a}(z)$, that is,

$$
y(R_E(z), \hat{a}(z)) \equiv p_z E[a] - (1 - \lambda)p_z E[a|a < \hat{a}(z)]F(\hat{a}(z)) - p_z [1 - F(\hat{a}(z))]R_E(z). \quad (4.6)
$$

The break-even condition that the investors’ surplus is zero, $V_I(z) = 0$, is satisfied when the investors contribute exactly the pledgeable income, $d(z) = y(R_E(z), \hat{a}(z))$. Under the break even condition, the value of the contract to entrepreneur $z$ in (4.4) can be rewritten as

$$
V_E(z) = p_z E[a] - 1 - (1 - \lambda)p_z E[a|a < \hat{a}(z)]F(\hat{a}(z)), \quad (4.7)
$$

i.e. because the investors receive no surplus from financing the project, the entrepreneur receives all the surplus that is generated from implementing the project. The latter consists of the project’s net present value, reduced by the expected liquidation cost that is incurred whenever $\hat{a}(z) > a$.

### 4.1.5 The incentive compatibility constraint

The incentive compatibility (IC) constraint that induces entrepreneur $z$ to choose the high probability of success can be derived from the entrepreneur’s value in (4.4) by requiring that the increase in the expected payoff to entrepreneur $z$ from being diligent, given that the project is continued until maturity with probability $1 - F(\hat{a}(z))$, exceeds the private benefit from shirking $B$,

$$
\Delta_p[1 - F(\hat{a}(z))]R_E(z) \geq B \quad (4.8)
$$

The more likely liquidation (i.e. the higher $\hat{a}(z)$), the higher the minimum payment $R_E(z)$ that must be pledged to the entrepreneur for incentive reasons. Since in this model liquidation depends on the aggregate shock $a$ and is unrelated to the entrepreneur’s action, an increase in the likelihood of liquidation worsens the entrepreneur’s incentives because it raises the chances of the states of nature in which the final payoff to the entrepreneur is unrelated to the action chosen in period 0.
4.1.6 Equilibrium

For each entrepreneur \( z \), the equilibrium determines whether the entrepreneur implements the project, the payment \( d(z) \) from the foreign investors, the threshold \( \tilde{a}(z) \) for the aggregate shock \( a \) below which the project is forced into liquidation, and the contingent payment \( R_E(z) \).

Note first that if the IC constraint (4.8) is binding, the liquidation rule (4.3) and the IC constraint itself imply that the liquidation threshold and the contingent payment do not depend on the entrepreneur's wealth \( w(z) \), i.e. \( \tilde{a}(z) = \tilde{a} \) and \( R_E(z) = R_E \). This result implies in turn that the pledgeable income (4.6) that entrepreneur \( z \) can pledge to the foreign investors does not depend on the entrepreneur's own wealth, i.e. \( y(R_E(z), \tilde{a}(z)) = y \).

We are interested in an equilibrium in which the entrepreneurs and the investors behave as atomistic agents, i.e. they do not take into account the externality that they may exert on other agents through their liquidation decisions. The atomistic behavior is consistent with the interpretation of the model in which there is a continuum of mass one of entrepreneurs of each type \( z \), and in which the competitive investors treat the entrepreneurs of the same type symmetrically. Thus, we define a competitive equilibrium as follows:

1) entrepreneurs of type \( z \) and the potential investors in their projects believe that the projects of type \( z \) will be liquidated if the aggregate shock is below the threshold \( \tilde{a}^c(z) \);

2) given \( \tilde{a}^c(z) \), the contract between the entrepreneurs and the investors maximize the entrepreneurs' surplus, subject to the investors' break even condition and the incentive constraint;

3) given the contract between the entrepreneurs of type \( z \) and the investors in projects \( z \), the threshold \( \tilde{a}(z) \) that is ex-post optimal coincides with their prior belief, i.e. \( \tilde{a}^c(z) = \tilde{a}(z) \).

Suppose that at a competitive equilibrium, the IC constraint on each entrepreneur is binding, given a certain expected threshold \( \tilde{a}^c(z) \). From the observation that we made at the beginning of this subsection, the threshold and the contingent payment do not depend on the entrepreneur's wealth, i.e. \( \tilde{a}^c(z) = \tilde{a} \) and \( R_E(z) = R_E \). From the definition of the pledgeable income in (4.6), pledgeable income is decreasing in the contingent payment. Pledgeable income is thus maximized by setting the contingent payment at the level for which the IC constraint is binding. Therefore, at a competitive equilibrium in which pledgeable income is maximized, the IC constraint is binding, as conjectured. Provided that an entrepreneur who implements the
project receives a nonnegative surplus – and the current argument implies that this surplus is independent of her initial wealth – while an entrepreneur who cannot implement her project receives zero, maximizing pledgeable income is equivalent to maximizing the entrepreneurs' surplus, subject to the IC constraint and the investors' break even condition. A competitive equilibrium requires that the prior belief is confirmed, \( \tilde{a} = \tilde{a} \).

This logic implies that a competitive equilibrium \((R_E, \tilde{a}, y)\) satisfies the following conditions,

\[
\tilde{a} = \frac{R_E}{1 - \lambda} \tag{4.9}
\]

\[
[1 - F(\tilde{a})] \Delta_p R_E = B \tag{4.10}
\]

\[
y = p_H E[a] - p_H \frac{B}{\Delta_p} - (1 - \lambda) p_H E[a|a < \tilde{a}] F(\tilde{a}) \tag{4.11}
\]

Given \( y \), all the entrepreneurs with wealth below \( 1 - y \) cannot raise enough external funds to implement their projects. Thus we can define the threshold \( \tilde{z} \) such that the entrepreneurs \( z < \tilde{z} \) do not implement their projects and the entrepreneurs \( z \geq \tilde{z} \) implement their projects. Given the wealth function \( w(.) \), the threshold \( \tilde{z} \) is defined as

\[
w(\tilde{z}) = 1 - p_H E[a] + p_H \frac{B}{\Delta_p} + (1 - \lambda) p_H E[a|a < \tilde{a}] F(\tilde{a}). \tag{4.12}
\]

As expected, the threshold \( \tilde{z} \) depends positively (meaning that fewer entrepreneurs can raise enough external finance to implement their project) on the private benefit from shirking (the third term on the right-hand side) and on the expected loss from liquidation (the last term). Since the size of the projects is fixed and normalized to one, aggregate investment coincides with the mass \( 1 - H(\tilde{z}) \) of entrepreneurs who implement their projects.

### 4.1.7 Multiple equilibria and equilibrium selection

The equilibrium conditions (4.9) and (4.10) define two upward sloping loci in the \((\tilde{a}, R_E)\) space, and thus in principle multiple equilibria are possible, as shown in Figure 2. The number of intersections between the two loci depends on the shape of the distribution function \( F(.) \).

Even though the two quantities have to be mutually consistent in equilibrium, logically the IC constraint (4.10) yields a contingent payment \( R_E \) given \( \tilde{a} \) (in period 0 at the time of
contracting, agents have rational expectations over $\hat{a}$, while the liquidation rule (4.9) yields $\hat{a}$ in terms of $R_E$ (when liquidation decisions are made in period 1, the contracts have already been signed in period 0). This logic implies that an equilibrium in which the equilibrium locus defined by (4.9), the line L in the figure, intersects the equilibrium locus defined by (4.10), the curve IC in the figure, from below is unstable, and is stable if the opposite happens.\footnote{Formally, a competitive equilibrium is a fixed point to the following dynamic system:}

$$R_E^{(i)} = \frac{B}{[1 - F(\hat{a}^{e(t-1)})][\Delta r]}$$

$$\tilde{a}^{(i)} = \frac{R_E^{(i)}}{1 - \lambda}$$

$$\tilde{a}^{e(t)} = \tilde{a}^{(i)}$$

where $R_E^{(i)}$, $\tilde{a}^{(i)}$ and $\tilde{a}^{e(t)}$ are the contingent payment, the threshold and the expected threshold at the $i$-th iteration of the system, subject to an initial condition $\tilde{a}^{e(0)} = \hat{a}^0$. The first equation is the IC that gives the updated contingent payment for the $i$-th iteration, given the existing belief $\tilde{a}^{e(t-1)}$. The second equation gives the threshold $\tilde{a}^{(i)}$ that in period 1 is consistent with the contingent payment $R_E^{(i)}$ agreed in period 0. The last equation updates the belief $\tilde{a}^{(i)}$ for the next iteration consistently with the outcome of the current iteration.

Figure 4-2: Competitive equilibria
\((R_E = B/\Delta_p)\). The previous logic implies that the equilibrium corresponding to point B is stable, while the equilibria corresponding to A and C are unstable. From the inspection of the two equilibrium conditions, it is clear that if there exists at least an interior equilibrium \(0 < \tilde{\alpha} < 1\), then there exists also an interior stable equilibrium, corresponding to the smallest value for \(\tilde{\alpha}\) at which the two equilibrium conditions are satisfied (the point B in the figure). We focus on this equilibrium in the following.

**Remark 1.** The competitive equilibria can be Pareto-ranked: the lower the equilibrium threshold for liquidation, the higher the surplus of all the entrepreneurs who implement their projects, and the larger the fraction of entrepreneurs who implement their projects (See (4.7) and (4.12)). Given that the investors break even in equilibrium, an equilibrium with low liquidation Pareto dominates an equilibrium with high liquidation.

**Remark 2.** If the investors could commit to long-term finance, then the optimal contract would require the investors to leave their investment in the domestic firms until the maturity of the project. This is the competitive equilibrium that corresponds to point C in the Figure.

**Example: Uniform distribution.** Suppose that the aggregate shock is uniformly distributed on \([\underline{\alpha}, \overline{\alpha}]\), with \(\overline{\alpha} = \underline{\alpha} + 1\). Then, the equilibrium threshold \(\tilde{\alpha}\) is a solution to the following quadratic equation,

\[
\tilde{\alpha}^2 - \overline{\alpha} \tilde{\alpha} + \frac{B}{\Delta_p(1 - \lambda)} = 0.
\]

This equation has real roots if \(\overline{\alpha}^2 > \frac{4B}{\Delta_p(1 - \lambda)}\), in which case the stable equilibrium is given by

\[
\tilde{\alpha} = \frac{1}{2} \overline{\alpha} - \frac{1}{2} \left[ \overline{\alpha}^2 - \frac{4B}{\Delta_p(1 - \lambda)} \right]^{\frac{1}{2}}. \quad \Box
\]

### 4.2 Institutions

In the previous section we have derived the competitive equilibria of an economy in which a plausible form of contractual incompleteness leads to excessive liquidation of socially profitable projects. In this section we investigate how different institutions can mitigate this excessive liquidation problem.

Institutions play a role at different stages.
First, the degree of development of institutions in developing countries, e.g. institutions for the enforcement of property rights, disclosure and corporate governance, determines the severity of the agency problem between the domestic entrepreneurs and the foreign investors, i.e. the magnitude of the private benefit $B$ in the model.

Second, the domestic government may be able to intervene in the interim period with the aim of preventing liquidation, at a cost that depends on the availability of international financial safety nets. The government may or may not be able to commit ex-ante to certain intervention policies; the government’s ability to pre-commit is influenced by the existence of external institutions that set “pre-conditions” on the government’s intervention, e.g. limits to the possible size of the intervention. If the domestic entrepreneurs do not bear the full cost of the intervention, government intervention can give rise to net transfers to the domestic entrepreneurs.

Third, institutions that limit the investors ability to demand the early repayment of their claims in certain states of nature can be designed as a way to limit the contractual incompleteness between the entrepreneurs and the foreign investors. Recent proposals of institutions for international workouts move in this direction.

In this section we discuss these issues.

### 4.2.1 Bailouts

We extend the model to allow for government intervention. After the aggregate shock $a$ has been realized and before the foreign investors make their liquidation decisions, the government can take an action $g(a)$, possibly contingent on realization of the aggregate shock $a$. The action $g$ raises the payoff of all the projects in the economy in case of success from $a$ to $a + g$, at a cost of $\Psi(g)$; the ex-ante expected total cost of the government intervention is thus $E[\Psi(g)]$. For simplicity, we assume that the cost of the action $g$ is linear, i.e. $\Psi(g) = \psi g$, where $\psi$ is a positive parameter. Below we will discuss two interpretations of the government intervention and its cost.

The specific institutional arrangement in which the government makes its decisions determines the choice of $g$, and its possible dependence on $a$. For example, the government may not have the ability to commit ex-ante to a certain course of action, giving rise to time-consistency
problems, or the action $g$ may not be observable, giving rise to moral hazard problems with regard to the government's adjustment policy. In this chapter we do not study the moral hazard problem that arises when the government action is not observable. We consider however the issue of time-consistency for the government intervention.

**Ex-post intervention policy**

Suppose that in period 1 the aggregate shock $a$ is realized. Given $a$, the government chooses $g(a)$ to minimize the aggregate cost of liquidation net of the cost of implementing $g(a)$, that is, we assume that the government is benevolent. To determine the government's optimal policy, we need to work backwards from the investors' optimal liquidation policy in the presence of the government intervention.

Thus, suppose that following the aggregate shock $a$, the government chooses $g$. Given the aggregate shock and the government's intervention, the agents in the economy know that the projects' payoff in case of success will be $a + g$. An argument similar to the argument in the previous section implies that the investors in project $z$ will demand the repayment of their claims in period 1 if\footnote{\footnotesize In this section we do not emphasize the dependence of the contingent payment $R_E(x)$ on $x$ because in equilibrium the contingent payment will not depend on the entrepreneur's own wealth.}

$$p_H(a + g - R_E) \leq \lambda p_H(a + g).$$

Note that this inequality assumes that the government intervention cannot be contingent on the investors' liquidation decisions. Equivalently, liquidation occurs if the value of the entrepreneur's claim exceeds the cost of liquidation,

$$R_E \geq (1 - \lambda)(a + g). \quad (4.13)$$

When choosing $g$, the government anticipates that the investors will liquidate their investment if the inequality (4.13) holds. Define $g(a)$ to be the minimum (nonnegative) intervention that prevents liquidation of the project,

$$g(a) = \max \left\{ \frac{R_E}{1 - \lambda} - a, 0 \right\}. \quad (4.14)$$
If the government chooses \( g(a) \), the project is not forced into liquidation, and the economy saves \((1 - \lambda)p_Ha\) in terms of liquidation cost. To implement \( g(a) \), the government incurs the cost \( \psi g(a) \). Since the government’s benefit from intervention is increasing in the aggregate shock \( a \) while its cost is decreasing, there exists a threshold \( \tilde{a} \) such that if \( a < \tilde{a} \) the intervention that prevents liquidation is too costly relative to its benefit, while intervention is optimal if \( a \geq \tilde{a} \). The threshold \( \tilde{a} \) is defined by

\[
\psi g(\tilde{a}) = p_H g(\tilde{a}) + (1 - \lambda)p_H \tilde{a}.
\]

The left-hand side represents the cost of implementing the minimum intervention \( g \) required to prevent liquidation, when the aggregate shock is at the threshold \( \tilde{a} \); the magnitude of this cost depends on the parameter \( \psi \). The right-hand side is the benefit of intervention from the perspective of the government: intervention raises the payoff of the project in case of success (the first term); furthermore, the intervention prevents the loss of the fraction \((1 - \lambda)\) of the project’s value that would occur if the government did not intervene. By using (4.14), the threshold \( \tilde{a} \) is given by

\[
\tilde{a} = \frac{\psi - p_H}{\psi - \lambda p_H} \frac{R_E}{1 - \lambda}.
\]

Clearly, \( \tilde{a} > a \), i.e. the government does not intervene in all the states of nature, only if \( \psi \) is sufficiently larger than \( p_H \). We assume this to be the case, i.e. \( \psi \gg p_H \) (see the Appendix for the details). Under this assumption, the government’s intervention will be justified exclusively by the purpose of preventing costly liquidation: whenever the government finds it profitable to intervene, it chooses the smallest intervention size \( g(a) \).

The government’s optimal intervention policy, denoted by \( g(a) \), is as follows:

- In \([a, \tilde{a}]\), there is no intervention, \( g(a) = 0 \). The projects are forced into liquidation; the investors receive the liquidation value \( \lambda a \), and the entrepreneur receive zero.

- In \([\tilde{a}, \frac{R_E}{1 - \lambda}]\), the government intervention is \( g(a) = g(a) \), where \( g(.) \) is given by (4.14). The government intervention prevents liquidation; given the intervention \( g(a) \), the payoff of the project in case of success is \( \frac{R_E}{1 - \lambda} \), and the payoffs to the entrepreneur and investors are \( R_E \) and \( \frac{\lambda}{1 - \lambda} R_E \) respectively.
Figure 4-3: The post-intervention payoff of the projects in case of success

- In \([\frac{R_E}{1-\lambda}, \bar{a}]\), no intervention is necessary to prevent liquidation, \(g(a) = 0\). The payoffs to the entrepreneurs and investors in case of success are \(R_E\) and \(a - R_E\) respectively. If \(\frac{R_E}{1-\lambda} > \bar{a}\), only the first two regions occur.

The payoff of a typical project in case of success as a function of the aggregate shock \(a\), given the government’s optimal policy, is shown in Figure 3. The figure shows that in the interval \([\bar{a}, \frac{R_E}{1-\lambda}]\) the government intervention raises the payoff of the projects in case of success to \(\frac{R_E}{1-\lambda}\), the minimum level that prevents liquidation given the contingent payment \(R_E\) determined in the financial contracts.

We assume that \(\psi E[g(a)]\), the expected cost of saving a project, has to be incurred ex-ante. It is convenient to think that in period 0 the government levies a tax on the domestic entrepreneurs to cover this cost. To rule out implicit transfers from the low wealth agents (who will not be able to implement their projects) to the high wealth agents (who will be able to implement their projects), we assume that the cost of the intervention is borne by an entrepreneur if she implements her project, while no tax is levied on the entrepreneur if she does not implement the project. However, we will allow for the possibility that the entrepreneurs
expect to be charged only a fraction $\eta$ of the expected cost of intervention, i.e. that the tax is $T = \eta \psi E[g(a)]$. If $\eta < 1$, the entrepreneurs receive a transfer through the government intervention. Below we will highlight some implications of this transfer.

In order to prevent socially inefficient liquidation, the government transfers resources from period 0 to the period 1’s states of nature in which the aggregate shock is low. The parameter $\psi$ determines the efficiency of the technology through which this transfer takes place. The magnitude of the total cost of intervention depends on the contingent payment $R_E$ specified in the contracts between the entrepreneur and the investors, because $R_E$ determines the minimum size of the intervention that is needed to prevent liquidation.

**Example 1: International credit line.** Suppose that the government draws from a credit line granted by an international lending institution and makes a transfer $S(a)$ equal to $p_H g(a)$ to each entrepreneur following the aggregate shock $a$. The government is charged a gross interest rate $r > 1$ on any withdrawal from the credit line. The credit line is repaid by imposing a tax on the projects of the entrepreneurs;\textsuperscript{10} this tax reduces the expected income that the entrepreneurs can pledge ex-ante to the investors by an amount equal to the expected value of the tax. In terms of the previous formulation, $\psi = rp_H$. The expected value of the tax is $T = r E[S(a)] = rp_H E[g(a)]$. □

**Example 2: Costly adjustment programs.** Because the government can implement a policy that offsets the aggregate shock on the return of the domestic firms, the action $g$ admits the interpretation of an adjustment policy, and the cost $\Psi(g)$ of the political cost of reform. The magnitude of the intervention cost is influenced by the existence of international financial safety net; for instance, a support package provided by IMF could alleviate the burden of adjustment and thus reduce the cost of intervention. □

\textsuperscript{10}Since we have normalized the payoff of the project in case of failure to zero, the tax could be paid only in case of success. In this economy of risk-neutral agents, however, what matters is the effect of the tax on the expected pledgeable income. If the payoff in case of failure were not zero, then the tax could be paid for sure even if levied in period 2.
The agents’ surplus

It is straightforward to modify the entrepreneurs’ and investors’ surplus to take the government intervention into account. Taking into account the optimal intervention policy, the surplus (4.5) of the investors in a project, modified to include the expected value of the government intervention $E[g(a)]$, can be written as

$$V_I = p_z \{E[a] + E[g(a)]\} - (1 - \lambda)p_z E[a|a < \tilde{a}]F(\tilde{a}) - p_z R_E(1 - F(\tilde{a})) - d \quad (4.16)$$

that is, the investors receive the expected value of the project (now inclusive of the expected government’s intervention), minus the expected liquidation cost, the payment to the entrepreneur, and the cost of capital. With regard to the entrepreneurs, their surplus in (4.4) is modified by the tax $T$ that is levied on each entrepreneur who implements her project,

$$V_E = [1 - F(\tilde{a})]p_z R_E - \bar{w} - T. \quad (4.17)$$

Since the entrepreneur’s tax liability does not depend on the outcome of her project, the incentive constraint in (4.8) still applies.\(^{11}\) The break even condition on the investors allows us to define the pledgeable income,

$$y(R_E, \tilde{a}) \equiv p_z \{E[a] + E[g(a)]\} - p_z[1 - F(\tilde{a})]R_E - (1 - \lambda)p_z E[a|a < \tilde{a}]F(\tilde{a}) \quad (4.18)$$

and to write the net surplus of an entrepreneur (4.7) as

$$V_E = p_z \{E[a] + E[g(a)]\} - (1 - \lambda)p_z E[a|a < \tilde{a}]F(\tilde{a}) - 1 - T \quad (4.19)$$

that is, the entrepreneur receives the expected value of the government intervention, $p_z E[g(a)]$, against which she is charged the tax $T$.

\(^{11}\)Here the assumption that the tax is levied in period 0 rather than in period 2 (conditional on the project’s success) matters. This difference, however, matters only because we have normalized the projects’ payoff in case of failure to zero, implying that no tax could be paid by the entrepreneur when the project fails. If the tax could be paid also in case of failure (because the project’s payoff is positive even in this case), then the tax would not affect the IC constraint even if levied in period 2 on the projects’ payoffs.
Equilibrium with bailouts

The competitive equilibrium \((R_E, \tilde{a}, y)\) satisfies the following conditions,

\[
\tilde{a} = \frac{\psi - p_H}{\psi - \lambda p_H} \frac{R_E}{1 - \lambda} \tag{4.20}
\]

\[
[1 - F(\tilde{a})] \Delta_p R_E = B \tag{4.21}
\]

\[
y = p_H \{ E[a] + E[g(a)] \} - p_H \frac{B}{\Delta_p} - (1 - \lambda)p_H E[a|a < \tilde{a}]F(\tilde{a}) \tag{4.22}
\]

Given \(y\), all the entrepreneurs with wealth below \(1 + T - y\) cannot raise enough external funds to implement their projects. Thus we can define the threshold \(\hat{z}\) such that the entrepreneurs \(z < \hat{z}\) do not implement their projects and the entrepreneurs \(z \geq \hat{z}\) implement their projects. The threshold \(\hat{z}\) is defined as

\[
w(\hat{z}) = 1 - p_H E[a] + p_H \frac{B}{\Delta_p} + (1 - \lambda)p_H E[a|a < \tilde{a}]F(\tilde{a}) + (\eta \psi - p_H) E[g(a)]. \tag{4.23}
\]

As before (see (4.12)), the threshold \(\hat{z}\) depends positively on the private benefit from shirking (the third term on the right-hand side) and on the loss from liquidation (the fourth term). Furthermore, the impact of the expected government intervention \(E[g(a)]\) depends on the fraction \(\eta\) of the cost of intervention \(\psi E[g(a)]\) that is borne by the entrepreneurs. If \(\eta < \frac{p_H}{\psi}\), the entrepreneurs bear a cost that is below the increase in the expected value of the projects, i.e. the entrepreneurs receive a net transfer through the government intervention. The higher the expected government intervention, the lower the threshold \(\hat{z}\). The opposite happens if \(\eta > \frac{p_H}{\psi}\).

**Example: Uniform distribution.** Suppose that the aggregate shock is uniformly distributed on \([g, \bar{a}]\), with \(\bar{a} = a + 1\). Then, the equilibrium threshold \(\tilde{a}\) is a solution to the following quadratic equation,

\[
\tilde{a}^2 - \tilde{a} + \frac{\psi - p_H}{\psi - \lambda p_H} \frac{B}{\Delta_p(1 - \lambda)} = 0.
\]

This equation has necessarily real roots if there exists an interior stable equilibrium when there
is no intervention. Then, the stable equilibrium is given by

\[
\hat{a} = \frac{1}{2} \bar{a} - \frac{1}{2} \left[ \bar{a}^2 - \frac{\psi - p_H}{\psi - \lambda p_H \Delta_p (1 - \lambda)} \frac{4B}{\Delta_p (1 - \lambda)} \right]^{\frac{1}{2}}.
\]

Denote by \( \Delta \) the width of the interval in which intervention takes place at the competitive equilibrium \((R_E, \hat{a})\), i.e. \( \Delta \equiv \frac{R_E}{1 - \lambda} - \hat{a} = \frac{\psi - p_H}{\psi - \lambda p_H} \frac{R_E}{1 - \lambda} \). The size of the expected government intervention depends only on \( \Delta \), i.e. \( E[g(a)] = \frac{1}{2} \Delta^2 \). Some algebra shows that intervention raises welfare in equilibrium relative to the equilibrium without intervention studied in the previous section if

\[
1 - \left( \frac{\psi - p_H}{\psi - \lambda p_H} \right)^{\frac{1}{3}} < \frac{\hat{a}_D - \hat{a}}{\hat{a}_D}
\]

where \( \hat{a}_D \) is the threshold in the competitive equilibrium without intervention. The left-hand side is a measure of the cost of financing the intervention with an inefficient tax (that is, of having \( \psi > p_H \)). The cost of financing the intervention decreases with \( \psi \), because the size of the intervention shrinks as its cost increases. The right-hand side is the percentage reduction in liquidation due to intervention, and represents the benefit of intervention; the benefit also falls with \( \psi \) because the threshold \( \hat{a} \) approaches \( \hat{a}_D \) as intervention becomes more costly. We show in the Appendix that this inequality holds for sufficiently small values of the cost parameter \( \psi \).\(^{12}\)

The role of domestic institutions for corporate governance and property rights

Consider the effect on the competitive equilibrium of an increase in the entrepreneurs’ private benefit \( B \) from shirking, e.g. due to a worsening in the domestic institutions for corporate governance and property rights. The increase in \( B \) raises both the threshold \( \bar{a} \) and the contingent payment to the entrepreneurs \( R_E \). Intuitively, the increase in \( B \) worsens the entrepreneurs’ incentives, raising the minimum contingent payment \( R_E \) that must be pledged to the entrepreneurs in case of success. At each realization of the aggregate shock \( a \), the increase in \( R_E \) raises the government’s cost of intervention, since the minimum size of the intervention increases with \( R_E \). The benefit of intervention must be higher in order for intervention to be justified, i.e. the

\(^{12}\)

This result is derived under an additional parameter restriction.
minimum threshold $\hat{a}$ at which intervention occurs has to increase. The increase in $\hat{a}$ in turn worsens the entrepreneurs’ incentives further, since it increases the chances of those states of nature in which the projects are forced into liquidation regardless of the entrepreneurs’ initial action.

With regard to the effect on pledgeable income (4.22) and the threshold $\tilde{z}$ for the entrepreneurs who implement their projects (4.23), the increase in $B$ reduces pledgeable income directly (because it worsens the entrepreneurs’ incentives) and indirectly through the increase in $\hat{a}$, the likelihood of liquidation. There is an additional effect on the threshold $\tilde{z}$ due to the net transfer through the government intervention. Even if the interval in which intervention takes place widens following the increase in $R_E$, the effect on the expected government intervention is in general ambiguous, because it depends on the distribution of the aggregate shock; it is positive, for example, in the uniform distribution case. Assuming that the expected government intervention increases following the increase in $B$, it is in principle possible that if the cost of the government intervention borne by the entrepreneurs is sufficiently smaller than the value of the transfer from the government, i.e. $\eta$ is sufficiently smaller than $\frac{E\psi}{\psi}$, pledgeable income could increase with the private benefit $B$, thus raising aggregate investment. Therefore, this model implies that economies with poorer institutions for corporate governance can have higher ex-ante investment rates relative to economies with better institutions. These economies are however more vulnerable to ex-post liquidation, and when liquidation occurs, it is larger and involves larger capital outflows.\(^{13}\)

**The cost of government intervention**

Consider an increase in the cost of government intervention $\psi$; this exercise is interesting because the magnitude of this cost depends on the availability of institutions for international lending of last resort. The increase in $\psi$ raises both the threshold $\hat{a}$ and the contingent payment to the entrepreneurs $R_E$. Intuitively, since the increase in $\psi$ raises the cost of intervention, the benefit of intervention must be higher in order to justify it, i.e. the minimum threshold $\hat{a}$ at which intervention occurs has to increase (recall that the benefit of intervention coincides with

\(^{13}\)In the sense that liquidation is more likely (since $\hat{a}$ is higher), and when it occurs it is larger (because aggregate initial investment is larger).
the loss due to the liquidation of a project, which is increasing in the aggregate shock $a$). The increase in $\bar{a}$ in turn worsens the entrepreneurs' incentives, because it raises the chances of the states of nature in which the projects are liquidated regardless of her initial action. The two effects reinforce each other, and in the new equilibrium both $\bar{a}$ and $R_E$ are higher.

There are two effects on the expected government intervention $E[g(a)]$:

- the direct effect of the increase in $\psi$ is to reduce the magnitude of the expected intervention because it raises the minimum threshold $\bar{a}$ at which intervention occurs;

- however, the private agents’ response (the increase in $R_E$) raises the magnitude of the expected intervention, since the intervention is larger whenever it occurs.

We show in the Appendix that if the aggregate shock is uniformly distributed, there exists a threshold $a^{**}$ such that if the equilibrium threshold $\bar{a}$ is initially below $a^{**}$ the direct effect prevails, and the increase in $\psi$ reduces the magnitude of the expected government intervention. However, if $\bar{a}$ is initially above $a^{**}$ the magnitude of the intervention increases in spite of the increase in its cost.

If $\eta$ is sufficiently smaller than $\frac{\mu}{\psi}$, i.e. the entrepreneurs receive a net transfer through the government intervention, and $\bar{a}$ is initially above $a^{**}$, it is thus possible that the increase in the cost of intervention raises the net transfer to the entrepreneurs and thus aggregate investment. The implication of this observation is that if the objective of the policy makers is to alleviate the “moral hazard problem” on the private sector, i.e. the net transfer to the entrepreneurs through the government intervention, which occurs in the model if $\eta < \frac{\mu}{\psi}$, a critical element is the overall shape of the institutions, in particular those limiting ex-post intervention. If the policy makers lack the ability to precommit to ex-post intervention policies, the private sector's response to the partial institutional reform can interact with the contractual incompleteness and the government’s ex-post incentives, and defeat the initial purpose of the reform.

**The role of asset market liquidity**

Consider an increase in $\lambda$, the liquidation value of the projects. This exercise captures an improvement in the liquidity of the resale market of the entrepreneurs' assets. An increase
in $\lambda$ raises both the equilibrium threshold $\tilde{a}$ and the equilibrium contingent payment to the entrepreneurs $R_E$.

Intuitively, the increase in liquidation value has two effects on the threshold $\tilde{a}$ in (4.20):

- It raises the minimum government intervention that is required to prevent liquidation; the reason is that the investors hold a put option on the project with a strike price of $R_E$, the entrepreneur's claim to the project's payoff. *Ceteris paribus*, the increase in liquidation value $\lambda$ raises the private return to the investors of exercising the put option. Hence, the minimum government intervention $q$ that prevents liquidation at any aggregate shock $a$ is larger, and so is its cost $\psi q$.

- It lowers the social return from preventing liquidation at any aggregate shock $a$, since the social loss from transferring the projects (the fraction $1 - \lambda$ of the projects' value) falls when the asset market becomes more liquid. In other words, the increase in liquidity reduces the benefit from government intervention.

These two effects raise the threshold $\tilde{a}$, given $R_E$. In turn, the increase in the threshold $\tilde{a}$ worsens the entrepreneurs' incentives, and thus raises the contingent payment $R_E$ that has to be pledged to the entrepreneurs. These two effects reinforce each other.

The net effect of the increase in $\lambda$ on welfare and aggregate investment depends on the relative strength of its direct effect (*ceteris paribus*, an increase in the liquidation value reduces the loss from liquidation) and its indirect effects through the increase in the threshold $\tilde{a}$ (raising loss from liquidation) and the response in expected government intervention. In general, the net effect is ambiguous.

The main implication of this exercise is that an increase in market liquidity raises ex-post liquidation, because the improvement in liquidity interacts with the contractual incompleteness in raising the private return from liquidation. The liquidity structure of the entrepreneurs' liabilities deserves some more attention; in the next section we will allow for two types of sources of external claims with different liquidity properties.
The value of pre-commitment

The government’s decision to intervene in order to prevent the liquidation of the projects depends on the contracts between the entrepreneurs and the foreign investors that have already been signed at the time the decision to intervene is made. In particular, the contingent payment to the entrepreneurs in case of success, \( R_E \), raises the minimum intervention \( g \) (see (4.14)), and thus the cost of intervention. Since the private agents do not take into account the impact of their contractual decisions on the government’s intervention policy, we conjecture that the government’s inability to commit to a certain liquidation policy is costly. We study in which sense this conjecture is true.

Suppose that the government announces that intervention will be at most \( g^* \) and that within this bound intervention will be as small as required to prevent liquidation, and that this announcement is credible. Given the bound \( g^* \), the contingent payment \( R_E^* \) specified in the contracts between the entrepreneurs and the investors pins down the threshold \( a^* \) for the aggregate shock \( a \) below which liquidation takes place,

\[
a^* + g^* = \frac{R_E^*}{1 - \lambda}.
\]

The bound \( g^* \) and the threshold \( a^* \) imply that the government’s intervention policy \( g^*(a) \) is as follows,

\[
g^*(a) = \begin{cases} 
0 & a \in [a, a^*] \\
 a^* + g^* - a & a \in [a^*, a^* + g^*] \\
0 & a \in [a^* + g^*, \overline{a}] 
\end{cases}
\]

That is, under the announced policy, intervention takes place in the interval \([a^*, a^* + g^*] \), and in this interval intervention declines one for one with the aggregate shock.

At the competitive equilibrium with pre-commitment, \((R_E^*, a^*)\) satisfies (4.24) and the incentive constraint (4.21) (obviously, with \( \overline{a} = a^* \)). At \((R_E^*, a^*)\), the pledgeable income and the fraction of the entrepreneurs who implement their projects are given respectively by (4.22) and (4.23).

The question that we pose is whether government intervention with pre-commitment dominates intervention without pre-commitment, in the following sense. By pre-committing to a
certain intervention policy (described by the upper bound $g^*$), can the government reduce liquidation, i.e. the threshold $\hat{\alpha}$, while at the same time reducing the magnitude of its (costly) expected intervention? Or, can the government reduce liquidation only by expanding its expected intervention? For simplicity, we address this question under the assumption that the aggregate shock is distributed uniformly over $[\underline{a}, \overline{a}]$, with $\overline{a} = \underline{a} + 1$.\footnote{\textsuperscript{14}In this case the expected government intervention under pre-commitment depends only on the bound $g^*$, i.e. $E[g^*(a)] = \frac{1}{3} (g^*)^2$.}

The following proposition answers this question. We denote again by $\Delta$ the width of the interval in which intervention takes place at the competitive equilibrium $(R_E, \hat{\alpha})$ without pre-commitment, i.e. $\Delta \equiv \frac{R_E}{1-\lambda} - \hat{\alpha}$. In deriving this proposition, we assume that the equilibrium conditions (4.24) and (4.26) have the same stability properties as the equilibrium conditions without pre-commitment; this is the case if $g^*(1-\lambda) < B/\Delta_p$.\footnote{\textsuperscript{15}In particular, provided that $g^*(1-\lambda) < B/\Delta_p$, if there exists a competitive equilibrium with pre-commitment, there exists necessarily a stable equilibrium.} In this proposition, we compare the pre-commitment competitive equilibrium with the lowest threshold for liquidation to the competitive equilibrium without pre-commitment with the lowest liquidation.

**Proposition 1.** Assume that the aggregate shock is distributed uniformly over $[\underline{a}, \overline{a}]$, with $\overline{a} = \underline{a} + 1$, and that $g^*(1-\lambda) < B/\Delta_p$. The threshold $\alpha^*$ for the aggregate shock in the competitive equilibrium with pre-commitment is below the threshold $\hat{\alpha}$ in the competitive equilibrium without pre-commitment if and only if $g^* > \Delta$.

Starting from a competitive equilibrium without commitment, i.e. $g^* = \Delta$, a marginal increase in the bound $g^*$ is Pareto-improving. $\square$

This result can be understood intuitively from Figure 4. Suppose that we start from the equilibrium without pre-commitment (point A in the figure); the same allocation obtains in the equilibrium with commitment by setting $g^* = \Delta$. Suppose that the government wishes to lower the threshold $\alpha^*$ below $\hat{\alpha}$, holding the maximum intervention at $g^* = \Delta$. In the figure, this attempt corresponds to a movement along the P line to point B. However, this movement is not consistent with the entrepreneurs' incentives, since in point B the IC constraint is violated. Therefore, $R^*_E$ has to increase to point C. The increase in $R^*_E$, however, raises the investors' return from liquidating the projects for any realization of the aggregate shock. As a result, the
government has to commit to a larger intervention in order to reduce $a^*$ below $\hat{a}$, i.e. $g^*$ has to increase above $\Delta$.

If we return to the interpretation of the government intervention as the access to an international credit line, then $g^*$ represents the size of the credit line. A change in expectations of the private agents to a lower liquidation equilibrium has to be backed by an increase in the size of the credit line. The proposition therefore shows that there is a trade-off between the benefit of limiting liquidation through pre-commitment and the size of the intervention that is required to do so. Rewrite for convenience the expected value of an entrepreneur who implements her project (see (4.19)),

$$V_E = p_H E[a] - 1 - (\psi - p_H) E[g^*(a)] - (1 - \lambda)p_H E[a|a < a^*]F(a^*)$$

The increase in $g^*$ lowers the loss from liquidation (the last term), but requires a larger tax. However, starting from $g^* = \Delta$, i.e. from the situation in which the competitive equilibrium with pre-commitment coincides with the competitive equilibrium without pre-commitment, the reduction in liquidation at the margin more than compensates the larger tax that is required to finance the government intervention.
4.2.2 Orderly Workout

Krueger (2001) has argued in favor of an institution that could trigger a temporary standstill on the repayment of claims issued by borrowers in developing countries to foreign investors in order to facilitate their restructuring "when a country finds itself shouldering a truly unsustainable debt burden".\textsuperscript{16} The motivation for this institution lies in the logistical and legal difficulties of coordinating the numerous and dispersed holders of claims traded in financial markets, and in the excessive political and economic disruption that is caused by the failure of achieving such coordination and restructuring. Our model provides us with a well defined notion of the disruption that is caused by the lack of coordination among claimholders in terms of the (excessive) liquidation of socially viable projects. Thus, we can study the aggregate implications of a mechanism that facilitates the restructuring of the investors' claims (that is, a mechanism for "orderly workouts"), and to compare it with the mechanism based on the ex-post government intervention.

A simple workout mechanism

In the model considered so far, all the projects are identical ex-ante and ex-post (i.e. conditional on the realization of the aggregate shock). As a result, all the investors always make the same liquidation or continuation decision. We modify the model slightly in order to study a workout mechanism based on a majority rule.

Suppose that the payoff of project \( z \) in case of success is \( a(1 + \epsilon(z)) \), where \( \epsilon(z) \) is a zero mean random variable, independent and identically distributed across the projects. The random variable \( \epsilon(z) \) represents the idiosyncratic component of the project, and we assume that this component is lost if the project is liquidated (i.e. if the entrepreneur is forced to transfer her project to a liquidity supplier). We consider an institutional mechanism according to which the demand of the repayment of a claim by an investor is allowed only if a fraction \( \delta \) of claimholders agree to demand the repayment as well. Under this mechanism, it is possible to define the threshold \( \alpha_{\delta} \) for the aggregate shock such that liquidation occurs if \( a < \alpha_{\delta} \), where

\[
p_H(\alpha_{\delta} + \epsilon_{\alpha_{\delta}} - R_E) = \lambda p_H \alpha_{\delta}
\]

\textsuperscript{16}To be precise, the proposal focuses on sovereign claims.

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and \( \overline{\varepsilon} \) is defined by \( \Pr(\varepsilon \leq \overline{\varepsilon}) = \delta \), the requirement that a fraction of the projects’ idiosyncratic components below \( \overline{\varepsilon} \) is exactly \( \delta \). Simple algebra allows us to rewrite the threshold \( \widehat{a}_\delta \) as

\[
\widehat{a}_\delta = \frac{1}{1 - \theta \lambda} R_E,
\]

(4.25)

where we have defined \( \theta \equiv 1 - \frac{\overline{\varepsilon}}{\lambda} \), and assumed that \( 0 < \theta < 1 \). From the comparison between (4.9) and (4.20), it is clear that by making liquidation more difficult, the workout mechanism reduces the threshold for the aggregate shock at which liquidation takes place at any contingent payment \( R_E \).

**Remark.** For the aggregate shocks just below the threshold \( \widehat{a}_\delta \), the value of the claims of the investors in the project with high idiosyncratic components exceed the project’s liquidation value.\(^{17}\) To avoid the inessential complication of heterogeneous liquidation decisions across investors, we consider the simple case in which the mechanism treat all the projects symmetrically.\(^{18}\) With this simplification, the workout mechanism is equivalent to a mechanism in a world without heterogeneity in which investors can demand liquidation only if the private benefit from liquidation is sufficiently larger than their continuation value. That is, the mechanism specifies a parameter \( 0 \leq \theta \leq 1 \) such that investors can demand liquidation only if \( p_H(a - R_E) \leq \theta \lambda p_{Ha} \). This equivalence shows that the workout mechanism introduces a wedge between the value of the investors’ claims and their return from exercising the option of liquidating their claims, and therefore reduces the strike price at which they exercise their option.

**Workouts vs bailouts**

The competitive equilibrium \((R_E, \widehat{a}_\delta)\) satisfies (4.21) and (4.25). Given \((R_E, \widehat{a}_\delta)\), the pledgeable income \( y \) and the threshold \( \widehat{\alpha} \) for the marginal entrepreneur who implements her projects are determined from (4.11) and (4.12). It is easy to show that in the competitive equilibrium with the workout mechanism, the threshold for liquidation and the contingent payment are smaller than in the competitive equilibrium without intervention.

---

\(^{17}\)By definition, there is at most a fraction \( 1 - \delta \) of such investors.

\(^{18}\)That is, no investors liquidate their claims if less than a fraction \( \delta \) of investors wish to liquidate, and all the investors liquidate if a fraction \( \delta \) of investors wish to liquidate.
The determination of the liquidation threshold and the contingent payment in a competitive equilibrium with the workout mechanism is formally equivalent to their determination in a competitive equilibrium with government intervention. It is easy to show that for any cost of intervention $\psi$ there exists a parameter $\theta(\psi)$ for the workout mechanism that yields the same competitive equilibrium for $R_E$ and $\bar{a}_8$, and that $\theta'(\psi) > 0$, i.e. the higher the cost of government intervention, the easier for investors to demand liquidation under the workout mechanism.\(^{19}\) The critical difference between the two institutional setups, however, is that the workout mechanism does not give rise to the loss of welfare that is required to finance the costly government intervention. If $\psi > p_H$, the tax that is required to finance the government intervention imposes a deadweight loss in terms of the surplus of each entrepreneur who implement her project and of the mass of entrepreneurs who manage to raise enough external finance to implement their project. On the other hand, the workout mechanism operates entirely as a coordination device for the agents’ expectations (i.e. it is a “catalyst for voluntary agreements”, as Krueger (2001) describes it) and as such it does not introduce a similar distortion. Therefore, the workout mechanism dominates the regime with government intervention because it does not rely on the costly reallocation of resources across periods and states of nature that underpins the government intervention.

4.3 Hot money vs foreign direct investment

In our model the inefficiency (excessive liquidation) arises from the specific type of contractual incompleteness that we have assumed, the inability of foreign investors to commit to long term finance. We believe that this feature captures an important element of short-term portfolio capital flows, which have increased substantially in the last decade with the capital account liberalization in developing countries. Yet, the last decade has also seen a marked increase in the importance (in relative and absolute terms) of portfolio equity and FDI flows to developing countries. By definition, equity claims and FDI have a long maturity (coinciding with the life

\[ \theta(\psi) = \frac{1}{\lambda} - \frac{(1 - \lambda)(\psi - \lambda p_H)}{(1 - \lambda)(\psi - p_H)}. \]
of the firm/project). In this section, we interpret FDI as the private sector’s potential response to the problem of committing to long-term finance, and we ask whether the private sector can take care by itself of the contractual incompleteness by choosing the right composition of claims issued to the foreign investors, and if not whether there tends to be over- or under-investment in the more stable source of finance.

To address these issues, we extend our model to allow the domestic entrepreneurs to issue two types of external claims. First, domestic entrepreneurs can rely on foreign investors for short-term financing (“hot money”); for simplicity, we refer to the foreign investors who supply this type of finance as “bondholders”. Second, domestic entrepreneurs can bring in a more stable source of finance (FDI); we refer to the investors who provide this source of finance as “FDI investors”. We stress the following salient features of FDI:

1. FDI investors can supply long-term finance by committing to leave their funds in the project until maturity;

2. FDI investors are more closely involved with the management of the entrepreneurs’ activities; as a result, FDI mitigates the moral hazard problem between the entrepreneur on one side and the external investors on the other;\(^{20}\)

3. however, because it requires a more direct involvement by the investors, FDI is more costly to provide relative to the short-term source of finance.

As before, we denote by \(R_E\) the contingent payment to the entrepreneur in case of success, by \(\tilde{a}\) the threshold below which bondholders choose to demand the repayment of their claims, and by \(d\) the fraction of the initial (unit) investment cost financed by the bondholders. We need to introduce some more notation. We denote by \(i\) the fraction of the initial investment cost of the investment financed by the FDI investors. We assume that the claims of the FDI investors are junior relative to the claims of bondholders, and we denote by \(R_F\) the payment

\(^{20}\)This feature is consistent with the empirical findings in Hausmann and Fernández (2000), Albuquerque (2000) and Lane and Milesi-Ferretti (2000) that the share of FDI in developing countries’ total private capital inflows and stock of liabilities are relatively higher in countries with lower (absolute and per capita) GDP and higher credit risk. Gordon and Bovenberg (1996) and Razin, Sadka, Yuen (1998) present models in which the foreign investors are less informed than the domestic investors (and both are less informed than the domestic entrepreneurs), i.e. adverse selection is more severe with foreign investors.
to the FDI investors in case the project succeeds in period 2 provided the project is not forced into liquidation in period 1.

Because FDI investors are involved with the management of the entrepreneur's activities (Feature 2), we assume that the entrepreneur's private benefit from shirking depends negatively (and for simplicity linearly) on the size of FDI; the provate benefit is thus given by $\beta(1 - i)$, where $\beta > 0$. The incentive compatibility constraint that induces the entrepreneur to choose the high probability of success becomes

$$[1 - F(\tilde{a})]R_E \geq \frac{\beta}{\Delta_p}(1 - i).$$

(4.26)

An increase in $i$ reduces the private benefit from shirking and thus loosens the IC constraint, e.g. because of monitoring embedded in FDI. We assume that FDI is more costly to provide than short-term finance (Feature 3). In order to install $i$ units of FDI, the FDI investors have to incur the installation cost $\Theta(i)$. We assume that $\Theta(0) = 0$, and that the cost is increasing and convex, $\Theta' > 0$ and $\Theta'' > 0$.

Since bondholders have priority over the proceeds from liquidation, they will demand the repayment of their claims whenever the expected value of their claims, given the aggregate shock $a$, falls short of the project's liquidation value. In evaluating the continuation value, bondholders take into account that the entrepreneur and the FDI investors have claims of $R_E + R_F$ on the project in case of success. By repeating the analysis of the previous sections, the threshold $\tilde{a}$ below which the bondholders demand the repayment of their claims is given by

$$\tilde{a} = \frac{R_E + R_F}{1 - \lambda}.$$

The fact that the liquidation decision by the short-term investors depends on the total value of the long term claims on the project's payoff (the claims owned by the entrepreneur and the FDI investors) will have important welfare implications.

The surplus of an entrepreneur who invests $\tilde{w}$ in her project is still given by (4.4) if the entrepreneur can implement the project, and zero otherwise. The surplus (at the high probability
of success $p_H$) of the bondholders investing $d$ can be written as

$$V_I = p_H E[a] - (1 - \lambda)p_H E[a|a < \tilde{a}]F(\tilde{a}) - p_H(1 - F(\tilde{a}))(R_E + R_F) - d.$$

This expression allows us to define $y$, the income pledgeable to the bondholders,

$$y = p_H E[a] - (1 - \lambda)p_H E[a|a < \tilde{a}]F(\tilde{a}) - p_H(1 - F(\tilde{a}))(R_E + R_F)$$

Finally, the expected value of the investment of the FDI investors investing $i$ is given by

$$V_F = [1 - F(\tilde{a})]p_H R_F - i - \Theta(i),$$

that is, the difference between the expected payment in case of success and the cost of investment; the latter consists of the opportunity cost $i$, and the cost $\Theta(i)$ of installing $i$ units of investment.

The break even condition on the bondholders, $V_I = 0$, implies that $d = y$. Thus, given $i$ and $y$, the threshold $\tilde{z}$ for the entrepreneur who has enough own wealth to implement a project is given by $w(\tilde{z}) + i + d = 1$, that is,

$$w(\tilde{z}) = 1 - p_H E[a] - i + (1 - \lambda)p_H E[a|a < \tilde{a}]F(\tilde{a}) + p_H(1 - F(\tilde{a}))(R_E + R_F)$$

(4.27)

### 4.3.1 The market for FDI

Suppose that the market for FDI is competitive, in the sense that there is a large pool of potential, identical FDI investors. We consider the following two stage contractual environment among the entrepreneurs, the FDI investors, and the bondholders:

- in the first stage, the entrepreneur and an FDI investor enter into a contract $(i, R_F)$ determining the investment $i$ from the FDI investor and the contingent payment $R_F$ to the FDI investor in case of success;

- in the second stage, the entrepreneur negotiates with the bondholders the contract $(d, R_E)$

\footnote{Again, for simplicity we do not index the contract to the entrepreneur, since in equilibrium all the contracts will be equal.}

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determining the investment $d$ from the bondholders and the payment $R_E$ to the entrepreneur in case of success.

In the first stage, the entrepreneur and the FDI investor take into account that their contract will affect the contract that the entrepreneur will be able to negotiate at the subsequent stage with the bondholders, since the size of the FDI investment affects the severity of the agency problem between the entrepreneur and the external investors. Specifically, they anticipate that the entrepreneur will be subject to the IC constraint,

$$[1 - F(\hat{a})] R_E = \frac{\beta}{\Delta_p} (1 - i) \quad (4.28)$$

whose position depends on the FDI investment; at the first stage the entrepreneur and the FDI investor take this dependence into account. However, we assume as before that the entrepreneur and the FDI investor are atomistic with regard to the determination of the threshold $\hat{a}$, i.e. they take $\hat{a}$ as given.

Competition among the potential FDI investors implies that an FDI investor breaks even on her investment $i$,

$$[1 - F(\hat{a})] p_H R_F = i + \Theta(i) \quad (4.29)$$

and that the equilibrium first-stage optimal contract maximizes the surplus of the marginal entrepreneur who can implement her project, subject to the IC constraint on the entrepreneurs and the break-even conditions on the bondholders’ and FDI investor’s investments. Provided that in equilibrium the entrepreneurs who implement their project receive a strictly positive surplus, this equilibrium requirement is equivalent to minimizing the threshold $\tilde{z}$ subject to the IC constraint and the break-even conditions. Using (4.28) and (4.29) in (4.27), the threshold $\tilde{z}$ becomes

$$w(\tilde{z}) = 1 - p_H E[a] + (1 - \lambda)p_H E[a < \hat{a}] F(\hat{a}) + p_H \frac{\beta}{\Delta_p} (1 - i) + \Theta(i). \quad (4.30)$$

The difference with the model without FDI (see (4.12)) is that FDI reduces the private benefit and thus the minimum threshold $\tilde{z}$; however, the installation cost $\Theta(i)$ that is required to install $i$ units of FDI raises the threshold, since the FDI investors’ break even condition implies that
the entrepreneurs ultimately bear it. The minimization of the threshold with respect to \(i\) then yields

\[
p_H \frac{\beta}{\Delta_p} = \Theta'(i),
\]

that is, the marginal reduction in the entrepreneur’s private benefit made possible by FDI equals the marginal cost of installing FDI.

In the second stage of contracting, \(i\) and \(R_F\) are given. At this stage, the issue is whether the entrepreneur has enough own wealth (given \(i\), the FDI that has been secured in the previous stage) to incur the initial cost of the project (taking into account that \(R_F\) has already been pledged to the FDI investor). The threshold \(\tilde{z}\) in (4.30), where \(i\) satisfies (4.31), determines the entrepreneurs who have enough own wealth to start their projects.

### 4.3.2 Equilibrium

A competitive equilibrium \((i, R_E, R_F, \tilde{a}, \tilde{z})\) is a solution to the following equations (rewritten for convenience):

\[
p_H \frac{\beta}{\Delta_p} = \Theta'(i) \tag{4.32}
\]

\[
\tilde{a} = \frac{R_E + R_F}{1 - \lambda} \tag{4.33}
\]

\[
[1 - F(\tilde{a})] R_E = \frac{\beta}{\Delta_p} (1 - i) \tag{4.34}
\]

\[
p_H [1 - F(\tilde{a})] R_F = i + \Theta(i) \tag{4.35}
\]

\[
w(\tilde{z}) = 1 - p_H E[a] + (1 - \lambda) p_H E[a | a < \tilde{a}] F(\tilde{a}) + p_H \frac{\beta}{\Delta_p} (1 - i) + \Theta(i) \tag{4.36}
\]

### 4.3.3 Constrained efficiency

Next we perform the following thought experiment. Suppose that FDI is perturbed at the margin from its competitive equilibrium level, and that the FDI investors are compensated for the change in their investment in order to meet the break even condition (thus ensuring that no net transfer from the FDI investors to the entrepreneurs takes place). We are interested in this exercise because intuition might suggest that an increase in FDI, the more stable source of finance, should mitigate the inefficiency in the economy arising from the inability of the
bondholders to commit to long-term finance. However, the following proposition states that this intuition is not correct.

**Proposition 2.** In the neighborhood of a stable competitive equilibrium, a positive perturbation \( d_i \) to the level of FDI raises the threshold \( \hat{a} \) for the aggregate shock below which the projects are liquidated. \( \Box \)

Intuitively, when FDI increases, the FDI investors have to be compensated for their larger investment, i.e. \( R_F \) has to increase. Given a contingent payment \( R_E \) to the entrepreneurs, the increase in \( R_F \) raises the threshold \( \hat{a} \) for the aggregate shock that is required by the bondholders not to demand the repayment of their claims (see (4.33)). On the other hand, the increase in FDI mitigates the agency problem between the entrepreneurs and the external investors because it reduces the entrepreneurs’ private benefit from shirking. The resulting improvement in incentives reduces the contingent payment \( R_E \) that has to be retained by the entrepreneurs for incentive reasons. Given a contingent payment \( R_F \) to the FDI investor, this reduction in \( R_E \) reduces the threshold \( \hat{a} \).\(^{22}\) However, the proposition states that at a stable competitive equilibrium the increase in FDI always raises the stake \( R_E + R_F \) of the long-term agents (the entrepreneur and the FDI investor), and this in turn exacerbates the excessive liquidation problem.

FDI is excessive in the competitive equilibrium because it generates a negative externality through the contractual incompleteness associated with the bondholders’ short-term investment. The contractual incompleteness implies that the bondholders hold a put option on the firms’ projects at a strike price equal to the stake of the long-term agents in the projects. The increase in the stake of these agents makes the states of nature in which short-term investors find it convenient to exercise their option more likely (that is, the option is more likely to be in the money in the interim period). On the other hand, while FDI raises the return to the bondholders of exercising their option, FDI does not mitigate the disruption that is caused by the withdrawal of the short-term investors, the socially costly liquidation of the projects. In a

\(^{22}\)Furthermore, the IC constraint (4.34) implies that the higher \( \hat{a} \), the stronger the effect of an increase in FDI in lowering the contingent payment \( R_E \) to the entrepreneurs, that is, the higher the marginal return of FDI-related monitoring.
competitive equilibrium, the entrepreneur’s and the FDI investors’ failure to take this negative external effect into account implies excessive FDI.

This model has an interesting implication for the relationship between the severity of the moral hazard problem between the entrepreneurs and the external investors on one side and the volatility of short-term capital flows on the other. An increase in $\beta$ worsens the moral hazard problem, because *ceteris paribus* it raises the entrepreneurs’ private benefit from shirking. Since the FDI-related return from monitoring increases with $\beta$, FDI is higher in a competitive equilibrium (see (4.32)). In turn, the resulting increase in the stake in the projects of the long-term agents raises the threshold $\tilde{a}$, i.e. it makes liquidation more likely. The severity of the moral hazard problem is related to the development of the domestic financial institutions. Then, if we interpret the likelihood of liquidation as a measure of the volatility of the short-term capital flows, this model predicts that countries with less developed financial institutions should rely more on FDI and at the same time should experience more volatility in the short-term capital inflows. Fernández-Arias and Haussmann (2000b) present evidence that is broadly consistent with the prediction that FDI raises the vulnerability to short-term flows: the share of non-FDI flows in total capital flows raises the probability of currency crises, but only in developing countries.

Controls on short-term capital inflows are often proposed as an effective policy to reduce the vulnerability of developing countries to the volatility of short-term speculative capital flows, citing the experience of Chile in the 1990’s (see, for example, Rodrik and Velasco (1999)). The underlying idea is that by discouraging short-term inflows, the domestic borrowers are induced to raise a more stable source of external finance. The implicit, however obvious, assumption is that the substitution across maturities should not be prevented by any form of market incompleteness. The importance of this assumption is evident in our model with a single source of finance. Suppose, for example, that the foreign investors are subject to a reserve requirement of $l$ percent bearing an interest rate below the zero competitive rate of return. Given the reserve requirement, a domestic entrepreneur can raise at most $d = \frac{y}{1+(1-\gamma)} < y$, where $\gamma < 1$ is the gross rate of return on the reserve requirement, and $y$ is the equilibrium pledgeable income (4.11). Since this policy has no effect on the equilibrium threshold and contingent payment in (4.9) and (4.10) but reduces the mass of entrepreneurs who can implement their
projects, it unambiguously reduces welfare. However, Proposition 2 implies that this policy is not necessarily welfare improving even if there exists a stable source of finance; indeed, in our model this policy magnifies the negative welfare implications of the contractual incompleteness associated with the short-term flows.

Remark. A key assumption for the result in Proposition 2 is that FDI is supplied competitively; it is possible, however, that if there are significant barriers to the FDI flows, a policy of encouraging a substitution towards FDI can improve welfare in the recipient country in spite of the contractual incompleteness associated with the short-term flows.  

4.4 Conclusions

In this chapter we have provided a model of international lending based on a basic form of contractual incompleteness: foreign investors cannot commit costlessly to provide long-term or state-contingent finance to domestic agents. We believe that this contractual environment captures a fundamental feature of capital flows to emerging markets, and that the change in the composition of capital flows in the last decade has magnified its importance. In this environment, there is excessive liquidation of socially viable projects in the competitive equilibrium that emerges in decentralized markets. As a result, institutions that manage to limit liquidation have the potential to improve welfare.

We have first compared an institutional arrangement based on the bailout of the domestic entrepreneurs through ex-post government intervention with a workout mechanism based on a majority rule in the basic model with short term claims only. As regard bailouts, we have shown there are welfare gains from the government's ability to commit ex-ante to a bailout policy. However, a credible pre-commitment has to be backed by the willingness to implement a larger intervention. The workout mechanism is based on the simple majority rule that a foreign investor is allowed to demand the early repayment of her claims only if a sufficiently

\[ -p_H \frac{\Delta^r}{\Delta^*} + 1 + \Theta' < 0, \quad \text{that is,} \quad \Theta' < -p_H \frac{\Delta^r}{\Delta^*}. \]

That is, an increase in FDI (subject to the FDI investors' break even condition) raises welfare if the marginal benefit of FDI in alleviating the moral hazard problem between the entrepreneurs and the external investors is much larger than its installation cost. In this case, the increase in the contingent payment \( R_F \) that is required to compensate the FDI investors is smaller than the reduction in the continent payment \( R_E \) to the entrepreneurs, reducing the long-term agents' overall stake in the projects.

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large majority of investors choose to do the same. Under the plausible assumption that financing government intervention is costly, the workout mechanism is a better institution than ex-post bailouts to deal with the excessive liquidation problem. The reason is that the workout mechanism operates directly as a coordination device for the agents’ expectations and does not require the costly transfer of resources across periods and states of nature that underpins the government bailout.

We have then extended the model to allow for two types of external claims, short-term “hot money” and a more stable source of finance (FDI) that is not subject to the contractual incompleteness. In a competitive equilibrium FDI is excessive, because the entrepreneurs and the FDI investors fail to take into account that FDI affects the value of the short-term investors’ option to demand the early repayment of their claims. We believe that this result deserves attention because it contrasts with the stated objectives of controls on short-term capital inflows as a way to reduce a country’s vulnerability to the volatility of short-term flows.

An important final caveat is that in this chapter we have abstracted from the role of liquidation as an incentive device; this role is likely to be important when the agency problems between domestic agents and external investors are severe.

Financial markets in developing countries seem to be very incomplete; understanding the implications of such incompleteness is key for the design of institutions for international lending.

4.5 Appendix

4.5.1 The model with government intervention

The minimum cost of intervention

In general, the value of the minimum cost of intervention $\psi$ that the threshold $\widehat{a}$ in (4.15) is at the lower bound of the support of the aggregate shock, i.e. $\widehat{a} = a$ depends on the distribution function $F(.)$ for the aggregate shock $a$, since $\widehat{a}$ depends on the contingent payment $R_F$. If the aggregate shock is distributed uniformly over $[\underline{a}, \overline{a}]$, with $\overline{a} = a + 1$, it is easy to see that $\widehat{a} \geq a$ if $\psi \geq \psi$, where $\psi$ is given by

$$\psi \equiv \frac{1 - \lambda a \Delta \Pi (1-\lambda)}{1 - \frac{a \Delta \Pi (1-\lambda)}{B} P_H}.$$
The minimum cost parameter $\psi$ is larger than $p_H$ (implying that ex-post intervention is less than complete) if and only if and only if $\frac{B}{\Delta_p(1-\lambda)} > \bar{a} \equiv \bar{a} - 1$. Recalling that real roots occur if $\frac{B}{\Delta_p(1-\lambda)} < \frac{\bar{a}^2}{4}$, the two inequalities are satisfied simultaneously if $\bar{a} - 1 < \frac{B}{\Delta_p(1-\lambda)} < \frac{\bar{a}^2}{4}$. This interval is nonempty, since if $\frac{\bar{a}^2}{4} - \bar{a} + 1 = \left(\frac{\bar{a}}{2} - 1\right)^2 > 0$. Therefore, provided that $\bar{a} - 1 < \frac{B}{\Delta_p(1-\lambda)} < \frac{\bar{a}^2}{4}$, intervention is less than complete ($\bar{a} \geq \underline{a}$) if and only if $\psi \geq \psi$.

Is ex-post intervention welfare-improving?

To answer this question, we need to compare $(\psi - p_H) E[g(a)] + (1 - \lambda) p_0 E[a|a < \bar{a}] F(\bar{a})$ with $(1 - \lambda) p_0 E[a|a < \bar{a}] F(\bar{a})$, where $\bar{a}$ is the competitive equilibrium without any intervention (decentralized equilibrium). Assume for simplicity that the aggregate shock is uniformly distributed. Using the equilibrium conditions, the expected government intervention is given by

$$E[g(a)] = \frac{1}{2} \left( \frac{R_E}{1-\lambda} - \bar{a} \right)^2 = \frac{1}{2} \left( \frac{p_H(1-\lambda)}{\psi - p_H} \right)^2 \bar{a}^2.$$ 

Therefore,

$$(\psi - p_H) E[g(a)] + (1 - \lambda) p_0 E[a|a < \bar{a}] F(\bar{a}) = \frac{1}{2} \frac{p_H^2(1-\lambda)^2}{\psi - p_H} \bar{a}^2 + (1 - \lambda) p_0 \frac{1}{2} \left( \bar{a}^2 - \bar{a}^2 \right)$$

to be compared with

$$(1 - \lambda) p_0 \frac{1}{2} (\bar{a}^2 - \bar{a}^2).$$

Some algebra shows that intervention is welfare improving if and only if

$$\frac{\psi - \lambda p_H}{\psi - p_H} \frac{1}{2} \bar{a} < \bar{a}.$$ 

Using the equilibrium values for $\bar{a}$ and $\bar{a}$,

$$\bar{a} - \bar{a} = \frac{1}{2} \left[ \bar{a}^2 - \frac{\psi - p_H}{\psi - \lambda p_H} \Delta_p(1-\lambda) \right]^{\frac{1}{2}} - \frac{1}{2} \left[ \bar{a}^2 - \frac{4B}{\Delta_p(1-\lambda)} \right]^{\frac{1}{2}}. \quad 145$$
That is, intervention is welfare improving if and only if

\[
\left[ \left( \frac{\psi - \lambda p_H}{\psi - p_H} \right)^{\frac{1}{2}} - 1 \right] \delta_D < \frac{1}{2} \left[ \frac{\bar{a}^2}{\psi - \lambda p_H} - \frac{4B}{\Delta_p(1 - \lambda)} \right]^{\frac{1}{2}} - \frac{1}{2} \left[ \frac{\bar{a}^2}{\Delta_p(1 - \lambda)} \right]^{\frac{1}{2}}
\]

We show that this inequality holds for sufficiently small values of \( \psi \). If this inequality holds at \( \psi \), then by continuity it holds for sufficiently close values.

Suppose first that \( \bar{a} < 1 \). From the definition of \( \psi \),

\[
\frac{\psi - \lambda p_H}{\psi - p_H} = \frac{B}{\bar{a} \Delta_p(1 - \lambda)},
\]

which allows us to rewrite the above inequality as

\[
\frac{\bar{a} - B}{\Delta_p(1 - \lambda)} < \frac{(\bar{a} - \epsilon)^2}{4}
\]

where we have defined \( \epsilon \equiv \frac{1}{2} \left[ \frac{\bar{a}^2}{4} - \frac{B}{\Delta_p(1 - \lambda)} \right]^{\frac{1}{2}} \). Provided that \( \epsilon \) is sufficiently small, this inequality is satisfied, since \( \frac{B}{\Delta_p(1 - \lambda)} < \frac{\bar{a}^2}{4} \) and \( \bar{a} < 1 \). If \( \bar{a} > 1 \), the inequality can be written as

\[
\frac{1}{\bar{a}} \frac{B}{\Delta_p(1 - \lambda)} < \frac{(\bar{a} - \epsilon)^2}{4}
\]

which is satisfied if \( \epsilon \) is sufficiently small.

The comparative statics of the cost of intervention

Differentiating (4.20) and (4.21),

\[
d\bar{a} = \frac{\psi - p_H}{\psi - \lambda p_H} \frac{1}{1 - \lambda} dR_E + \frac{(1 - \lambda)p_H}{[\psi - \lambda p_H]^2} \frac{R_E}{1 - \lambda} d\psi
\]

\[
d\bar{a} = \frac{[1 - F(\bar{a})]}{f(\bar{a}) R_E} dR_E
\]

which in turn imply

\[
\frac{dR_E}{d\psi} = \frac{(1 - \lambda)p_H R_E}{[\psi - \lambda p_H] f(\bar{a}) R_E} - \frac{\psi - p_H}{\psi - \lambda p_H}
\]

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Thus, $\frac{dR_E}{d\psi} > 0$ and $\frac{da}{d\psi} > 0$ if the stability condition holds,

$$\frac{1 - F(\tilde{a})}{f(\tilde{a})} > \frac{\psi - p_H}{\psi - \lambda p_H (1 - \lambda)} \frac{R_E}{(1 - \lambda)}.$$

Under this condition, some algebra shows that the interval $\Delta$ shrinks if and only if

$$\frac{R_E}{1 - \lambda} < \frac{1 - F(\tilde{a})}{f(\tilde{a})}.$$

If the shock $a$ is uniformly distributed on $[\underline{a}, \bar{a}]$ (and for notational simplicity $\bar{a} - \underline{a} = 1$), this condition simplifies to

$$R_E < (1 - \lambda)(\bar{a} - \tilde{a}).$$

This condition is not implied necessarily by the stability condition, since the latter implies that

$$R_E < \frac{\psi - \lambda p_H}{\psi - p_H} (1 - \lambda)(\bar{a} - \tilde{a}).$$

Therefore, it is possible to divide the space $(\tilde{a}, R_E)$ into two regions:

- if $R_E < (\bar{a} - \tilde{a})(1 - \lambda)$, then around a stable equilibrium an increase in $\psi$ reduces $E[g(a)]$;
- if $R_E > (\bar{a} - \tilde{a})(1 - \lambda)$, then around a stable equilibrium an increase in $\psi$ raises $E[g(a)]$.

Furthermore, it is possible to describe these two regions entirely in terms of the equilibrium threshold $\tilde{a}$. To do so, define $a^{**}$ as the (unique) value at which the IC constraint intersects the boundary of the two regions,

$$\frac{B}{(\bar{a} - a^{**}) \Delta_p} = (\bar{a} - a^{**})(1 - \lambda).$$

that is,

$$a^{**} = \bar{a} - \left[ \frac{B}{(1 - \lambda)\Delta_p} \right]^\frac{1}{2}.$$

The threshold $a^{**}$ has the property that

- if $\tilde{a} < a^{**}$, then around a stable equilibrium an increase in $\psi$ reduces $E[g(a)]$;
- if $\tilde{a} > a^{**}$, then around a stable equilibrium an increase in $\psi$ raises $E[g(a)]$.  

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4.5.2 Proofs

Proof of Proposition 1. From the inspection of Figure 4, it is clear that if $g^*(1 - \lambda) < B/\Delta_p$, if there is an intersection between the IC locus and the locus corresponding to (4.24) – the loci P and P' in the figure– then the intersection with the lowest threshold is stable, in the sense of the previous discussion. Then, the result follows immediately by noting that if we set $g^* = \Delta$, then $a^* = \widehat{a}$ (this equilibrium corresponds to the intersection between P and IC). Only by setting $g^* > \Delta$ it is possible that $a^* < \widehat{a}$.

The marginal effect of an increase in $g^*$ on the surplus of a typical entrepreneur who implements the project is

$$\frac{dV_E}{dg^*} = -(\psi - p_H)g^* - (1 - \lambda)p_Ha^* \frac{da^*}{dg^*}.$$  
From the differentiation of (4.24) and (3.23),

$$\frac{da^*}{dg^*} = -\frac{1}{1 - \frac{1}{1 - \lambda} \frac{R_E}{[\widehat{a} - a^*]}} < 0$$

where the inequality follows from the stability condition. Then, $\frac{dV_E}{dg^*} > 0$ if and only if

$$(1 - \lambda)p_Ha^* \frac{1}{1 - \frac{1}{1 - \lambda} \frac{R_E}{[\widehat{a} - a^*]}} > (\psi - p_H)g^*$$  

$$(1 - \lambda)p_Ha^* > (\psi - p_H)g^* \left[1 - \frac{1}{1 - \lambda} \frac{R_E}{[\widehat{a} - a^*]} \right]$$

Use the fact that at the initial equilibrium, $g^* = \frac{R_E}{1 - \lambda} - \widehat{a} = \frac{p_H(1 - \lambda)}{(\psi - p_H)}a^*$, this inequality becomes

$$\frac{1}{1 - \lambda} \frac{R_E}{[\widehat{a} - a^*]} > 0.$$  
Thus, $\frac{dV_E}{dg^*} > 0$. The same type of argument implies that pledgeable income increases with a marginal increase in $g^*$, the mass on entrepreneurs who can implement their project increases as well. □

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Proof of Proposition 2. Differentiate (4.33), (4.34), and (4.35):

\[ \frac{d\tilde{a}}{d\tilde{a}} = \frac{dR_E + dR_F}{1 - \lambda} \]

\[ [1 - F(\tilde{a})] dR_E - f(\tilde{a})R_E d\tilde{a} = -\frac{\beta}{\Delta_p} di \]

\[ p_H [1 - F(\tilde{a})] dR_F - p_H f(\tilde{a})R_F d\tilde{a} = [1 + \Theta'(i)] di \]

The second and third equations imply that

\[ p_H [1 - F(\tilde{a})] (dR_E + dR_F) - p_H f(\tilde{a}) (R_E + R_F) d\tilde{a} = di \]

where we have used (4.32). By substituting \( d\tilde{a} \) out,

\[ p_H \left[ [1 - F(\tilde{a})] - f(\tilde{a}) \frac{(R_E + R_F)}{1 - \lambda} \right] (dR_E + dR_F) = di. \]

Thus, the sign of \( \frac{d\tilde{a}}{di} \) is equal to the sign of

\[ \frac{1 - F(\tilde{a})}{f(\tilde{a})} - \frac{(R_E + R_F)}{1 - \lambda} + \frac{1 - F(\tilde{a})}{f(\tilde{a})} - \tilde{a} \]

where we have used (4.33), the equilibrium condition for \( \tilde{a} \). However, a stability argument similar to the one used in Section 2 applied to \( (R_E, R_F) \) implies that \( \frac{d\tilde{a}}{di} > 0 \) at a stable equilibrium. \( \square \)
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