14.581 Spring 2011Problem Set 3: Increasing Returns to Scale and Heterogeneous Firms *Solutions (Preliminary and Incomplete)*

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1. (25 marks)The objective of this first exercise is to revisit Krugman's (1980) Home-market effect. Consider a world economy with two countries, Home and Foreign, each endowed with one factor of production, labor. L and L^* denote the endowments of labor in the two countries and w and w^* denote the associated wages.

There are two sectors, agriculture (A) and manufacturing (M). The agricultural sector produces a homogeneous good one-for-one for labor under perfect competition, whereas the manufacturing sector produces a large number of varieties under increasing returns to scale and monopolistic competition. Production of $q(\omega)$ units of a given variety ω requires labor

$$l(\omega) = f + q(\omega)$$

where $f > 0$ is an overhead fixed cost.

The preferences of a representative consumer can be represented by

$$U = C_A^{1-\beta} C_M^{\beta}$$

where $\beta \in (0,1)$ is the share of expenditure on manufacturing goods and the manufacturing aggregate is given by

$$C_M = \left[\int_0^N [c(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega + \int_0^{N^*} [c^*(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where N and N^* are the endogenous number of Home and Foreign varieties and $\sigma > 1$ is the elasticity of substitution between varieties, respectively.

(1.a) For now, suppose that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. In order to sell 1 unit of a given variety in the other country, domestic firms must ship $\tau > 1$ units.

(i) Show that

$$\frac{N}{N^*} = \frac{L/L^* - \tau^{1-\sigma}}{1 - (L/L^*)\tau^{1-\sigma}}$$

Solution: Let $p_A = 1$. As a consequence, given that both countries are trading, they share the same technology and there is no iceberg cost in the agricultural sector, $w = w^* = 1$ (From the maximization problem of the firm in each country).

First let's derive the optimal expenditure for each sector at home (for foreign the problem is symmetric):

$$\begin{aligned} A &: \quad C_A = (1-\beta) L \\ M &: \quad C_M p_M = \beta L \end{aligned}$$

where p_M is a composite price index of the manufacturing good. Furthermore, we have for the demand of a manufacturing good:

$$c(\omega) : c(\omega) = C_M (\lambda p(\omega))^{-\sigma}$$

This should hold for all ω. Hence, one could show that:

$$\lambda = P^{-1} = \left(\int_0^N p(\omega)^{-(\sigma-1)} d\omega + \int_0^{N^*} p^*(\omega)^{-(\sigma-1)} d\omega \right)^{\frac{1}{\sigma-1}}$$

Hence, the individual demand is given by:

$$c(\omega) = C_M \left(\frac{p(\omega)}{P} \right)^{-\sigma}$$

Now turn to the supply side of the economy. Assume that the home and the foreign market are perfectly segmented (the firms can separately choose the prices they charge in the two markets.) Each firm in the home country for instance is going to behave monopolistically given the demand they are facing:

$$\max_{p,p^*} c(p) p + c^*(p^*) p^* - ((c(p) + \tau c^*(p^*)))$$

FOC:

$$p = \sigma/(\sigma - 1)$$

and

$$p^* = p\tau = \tau\sigma/(\sigma - 1)$$

Each firm will have the same price on their own market and the same export price in each country (there is perfect symmetry in this problem).

Finally, in order to find the equilibrium number of variety in each differentiated good, first define the equilibrium output for each variety by the zero profit condition:

$$x = (\sigma - 1) f$$

Furthermore, consider the two market clearing conditions home and abroad for each good ω:

$$q = \frac{\beta L}{P}\left(\frac{p}{P}\right)^{-\sigma} + \tau\frac{\beta L^*}{P^*}\left(\frac{p\tau}{P^*}\right)^{-\sigma}$$

where $P^* = p\left(N + N^*\tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ and $P = p\left(N\tau^{1-\sigma} + N^*\right)^{\frac{1}{1-\sigma}}$. Hence, we have:

$$\begin{aligned} q &= \frac{\beta L}{P}\left(\frac{p}{P}\right)^{-\sigma} + \tau\frac{\beta L^*}{P^*}\left(\frac{p\tau}{P^*}\right)^{-\sigma} \\ &\Leftrightarrow \\ \sigma f &= \frac{\beta L}{(N + N^*\tau^{1-\sigma})} + \frac{\beta L^*\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)} \end{aligned}$$

For the foreign country, we have:

$$\sigma f = \beta L\frac{\tau^{1-\sigma}}{(N + N^*\tau^{1-\sigma})} + \beta L^*\frac{1}{(N\tau^{1-\sigma} + N^*)}$$

Suppose N and N^* are positive, we have:

$$\beta L\frac{\tau^{1-\sigma}}{(N + N^*\tau^{1-\sigma})} + \beta L^*\frac{1}{(N\tau^{1-\sigma} + N^*)} = \frac{\beta L}{(N + N^*\tau^{1-\sigma})} + \frac{\beta L^*\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)}$$

$$\frac{N}{N^*} = \frac{L/L^* - \tau^{1-\sigma}}{1 - (L/L^*)\,\tau^{1-\sigma}}$$

(ii) Using (i), show that Home is a net exporter of the manufacturing good if and only if $L > L^*$.

Solution: Suppose that $L > L^*$, then it is easy to show that $N/N^* \geq 1$. Hence, it is easy to argue that home is a net exporter of the manufacturing good. One way to see this is by looking at the home trade balance in the manufacturing sector (good produced home consumed foreign-good produced foreign consumed home):

$$T_b = Np^*c^* - N^*pc$$

where $Np^*c^* = \beta L^*N\left(\frac{p\tau}{P^*}\right)^{1-\sigma} = \beta L^*N\frac{\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)}$ and $N^*pc = \beta LN^*\left(\frac{p\tau}{P}\right)^{1-\sigma} = \beta LN^*\frac{\tau^{1-\sigma}}{(N + \tau^{1-\sigma}N^*)}$. Substituting it, we get:

$$\begin{aligned} T_b &= \beta L^*\left(\left(\frac{N\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)} - \frac{(L/L^*)\,N^*\tau^{1-\sigma}}{(N + \tau^{1-\sigma}N^*)}\right)\right) \\ &= \beta L^*\left(\frac{\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)}\left(N - N^*\frac{(N\tau^{1-\sigma} + N^*)\,(L/L^*)}{(N + N^*\tau^{1-\sigma})}\right)\right) \\ &= \beta L^*\frac{\tau^{1-\sigma}}{(N\tau^{1-\sigma} + N^*)}\,(N - N^*) \end{aligned}$$

where I used $L/L^* = \dfrac{\left(N + N^*\tau^{1-\sigma}\right)}{\left(N\tau^{1-\sigma} + N^*\right)}$ from the market clearing conditions.

(1.b) Like in 1.a, we assume that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. However, we assume that in order to sell 1 *unit in the Foreign country, Home firms must now ship* $\tau^* > 1$ *units. In order to sell* 1 *unit in the Home country, Foreign firms must still ship* $\tau > 1$ *units.*

(i) Express $\frac{N}{N^*}$ *as a function of* τ *and* τ^*

Solution: Note that this question builds on Ossa (JPE 2011). This problem is identical to the previous one to the exception that $\tau \neq \tau^*$. Notice that the price set by each firm and the supply remains unchanged. The main difference comes from the market clearing conditions:

The home market good clearing condition:

$$q = \frac{\beta L}{P}\left(\frac{p}{P}\right)^{-\sigma} + \tau^* \frac{\beta L^*}{P^*}\left(\frac{p\tau^*}{P^*}\right)^{-\sigma}$$

where $P^* = p\left(N\tau^{*1-\sigma} + N^*\right)^{\frac{1}{1-\sigma}}$ and $P = p\left(N + \tau^{1-\sigma}N^*\right)^{\frac{1}{1-\sigma}}$. Hence, we have:

$$\sigma f = \frac{\beta L}{\left(N + \tau^{1-\sigma}N^*\right)} + \frac{\beta L^* \tau^{*1-\sigma}}{\left(N\tau^{*1-\sigma} + N^*\right)}$$

The foreign good market clearing condition:

$$\sigma f = \beta L \frac{\tau^{1-\sigma}}{\left(N + \tau^{1-\sigma}N^*\right)} + \beta L^* \frac{1}{\left(N\tau^{*1-\sigma} + N^*\right)}$$

$$\beta L \frac{\left(\tau^{1-\sigma} - 1\right)}{\left(N + \tau^{1-\sigma}N^*\right)} = \frac{\beta L^*\left(\tau^{*1-\sigma} - 1\right)}{\left(N\tau^{*1-\sigma} + N^*\right)}$$

$$\begin{aligned}
\left(N/N^*\tau^{*1-\sigma} + 1\right) &= \left(N/N^* + \tau^{1-\sigma}\right)\frac{L^*\left(\tau^{*1-\sigma} - 1\right)}{L\left(\tau^{1-\sigma} - 1\right)} \\
N/N^* &= \frac{\tau^{1-\sigma}\frac{L^*\left(\tau^{*1-\sigma}-1\right)}{L\left(\tau^{1-\sigma}-1\right)} - 1}{\left(\tau^{*1-\sigma} - \frac{L^*\left(\tau^{*1-\sigma}-1\right)}{L\left(\tau^{1-\sigma}-1\right)}\right)} \\
N/N^* &= \frac{\frac{\left(\tau^{1-\sigma}-1\right)}{\left(\tau^{*1-\sigma}-1\right)}L/L^* - \tau^{1-\sigma}}{\left(1 - \frac{\left(\tau^{1-\sigma}-1\right)}{\left(\tau^{*1-\sigma}-1\right)}L/L^*\tau^{*1-\sigma}\right)}
\end{aligned}$$

Finally, notice that:

$$N^* = \frac{\beta}{\sigma f}\left(\frac{L^*}{\left(1 - \tau^{1-\sigma}\right)} - \frac{\tau^{*1-\sigma}L}{\left(1 - \tau^{*1-\sigma}\right)}\right)$$

and

$$N=\frac{\beta}{\sigma f}\left(\frac{L}{(1-\tau^{*1-\sigma})}-\frac{L^{*}\tau^{1-\sigma}}{(1-\tau^{1-\sigma})}\right)$$

(ii) Suppose that the Home government can choose the level of iceberg trade costs $\tau > 1$, perhaps by imposing product standards or other technical barriers to trade. What is the optimal level of $\tau > 1$ (i.e. the level that maximizes Home welfare)?

Solution: The government needs to pick τ such that C_M is maximized. This is equivalent to minimizing $P = p\left(N+\tau^{1-\sigma}N^{*}\right)^{\frac{1}{1-\sigma}}$ which is equivalent to maximizing $\left(N+\tau^{1-\sigma}N^{*}\right)$:

$$\begin{aligned} N+\tau^{1-\sigma}N^{*} &= \frac{\beta}{\sigma f}\left(\frac{L}{(1-\tau^{*1-\sigma})}-\frac{L^{*}\tau^{1-\sigma}}{(1-\tau^{1-\sigma})}\right)+\tau^{1-\sigma}\frac{\beta}{\sigma f}\left(\frac{L^{*}}{(1-\tau^{1-\sigma})}-\frac{\tau^{*1-\sigma}L}{(1-\tau^{*1-\sigma})}\right) \\ &= \frac{\beta L}{\sigma f}\left(\frac{\tau^{1-\sigma}\tau^{*1-\sigma}-1}{\tau^{*1-\sigma}-1}\right) \end{aligned}$$

The welfare of the home country is increasing in τ for any level of τ^{*}. Hence, $\tau \to \infty$.

(iii) Suppose now that the Home and Foreign governments choose τ and τ^{*} simultaneously in order to maximize welfare in their respective country. What is the volume of trade in Nash equilibrium?

Solution: Trivially, we can see that the only Nash equilibrium is autarky.

(iv) Can you think of institutional arrangement(s) that would lead to Pareto improvements compared to the Nash equilibrium?

Solution: The Nash equilibrium is inefficient. One could think of a trade agreements where the principle of reciprocal trade liberalization is present (i.e. the reciprocity principle in the GATT). Both country could agree to set their tariff below the Nash level.

(1.c) Like in 1.A, we assume that $\tau = \tau^{*}$, but we now relax the assumption that the agricultural good is freely traded, whereas manufacturing goods are subject to iceberg trade costs. In order to sell 1 unit of the agricultural goods in the other country, domestic firms must now ship $\gamma > 1$.

(i) Show that if $\gamma = \tau$, then

$$\frac{N}{N^{*}}=\frac{L}{L^{*}}$$

Solution: This question builds on the work of Davis AER (1998). Before we considered an equilibrium where $P_A = w = 1$ in both countries. Assume that $L > L^*$, if the numeraire is taken to be a unit of Y_A available in the small country (foreign country), then demand for Y_A in the small country is simply given by $(1-\beta)\, w^* L^*$. In the home country,the demand is given as $(1-\beta)\, wL/P_A$. If the home country imports the agricultural good, then $P_A = \gamma$. On the other hand, if the large country exports the agricultural good, then $P_A = 1/\gamma$. If the agricultural good is not traded in equilibrium, we must have $(1-\beta)$ share of the labor force in each country used for the production of the agricultural good, and the residual to manufactures. In addition, the condition on the wage is given by: $w/w^* > 1/\gamma$ and $w/w^* < \gamma$.

Suppose that in equilibrium, the homogeneous good is not being traded. Then one can show that $\dfrac{N}{N^*} = \dfrac{L}{L^*}$ given that in each country we have:

$$L - (1-\beta)\, L = N\,(f + x)$$

and

$$L^* - (1-\beta)\, L^* = N^*\,(f + x)$$

where it follows from the equilibrium production in each differentiated good is equal accoss countries and differentiated good (x) as above and the fact that each country needs to assign $(1-\beta)$ of its labor force to the agricultural good. From the free entry conditions and the maximization problem of the firms we have:

$$p = w\sigma/(\sigma - 1)$$

and

$$\begin{aligned} xp - (x - f)\, w &= 0 \\ &\Leftrightarrow \\ x &= f\,(\sigma - 1) \end{aligned}$$

In the next part, we are going to show that in fact in equilibrium the homogeneous good is not traded.

(ii) What does that tell us about the Home-market effect? Can you think of necessary and sufficient conditions on τ and γ for the Home-market effect to occur?

Solution: The home market effect depends on the tariffs in the homogeneous good. In particular, it depends on the fact that the agricultural not be traded which will have $(1-\beta)$ share of the labor force in each country used for the production of the agricultural good. We need to show conditions on τ and γ such that the homogeneous good won't be traded in equilibrium. In equilibrium, if the homogeneous good is not traded, we know that the trade balance in the homogeneous good must be in equilibrium. In other words, we must have:

$$\begin{aligned}
Np\frac{\beta L^* w^*}{P^*}\left(\frac{p}{P^*}\right)^{-\sigma} &= N^* p^* \frac{\beta L w}{P}\left(\frac{p^*}{P}\right)^{-\sigma} \\
NL^* w^* \frac{w^{1-\sigma}}{N\left(w\tau\right)^{1-\sigma}+\left(w^*\right)^{1-\sigma}N^*} &= N^* L w \frac{w^{*1-\sigma}}{\left(w\right)^{1-\sigma}N+\left(w^*\tau\right)^{1-\sigma}N^*} \\
N/N^* \frac{\left(w/w^*\right)^{-\sigma}}{N/N^*\left(w/w^*\tau\right)^{1-\sigma}+1} &= L/L^* \frac{1}{\left(w/w^*\right)^{1-\sigma}N/N^*+\tau^{1-\sigma}}
\end{aligned}$$

Furthermore, we know from above that $\dfrac{L/L^*}{N/N^*}=1$. Hence,

$$\left(\left(w/w^*\right)^{1-\sigma}N/N^*+\tau^{1-\sigma}\right)=\left(N/N^* w/w^* \tau^{1-\sigma}+\left(w/w^*\right)^{\sigma}\right)$$

We need to have: $w/w^*>1/\gamma$ and $w/w^*<\gamma$. We need to show under which condition $w/w^*>1/\gamma$ and $w/w^*<\gamma$. In which case, the equilibrium with non traded homogeneous good holds.

The solution to the equation is given by:

$$F\left(\omega\right)=0$$

where $F\left(.\right)$ is a continuous function in ω. Furthermore, $F\left(0\right)=+\infty$ and $\lim_{\omega\to\infty}F\left(\omega\right)=-\infty$; $F'\left(\omega\right)=\left(1-\sigma\right)\left(w/w^*\right)^{-\sigma}N/N^*-\left(N/N^*\tau^{1-\sigma}+\sigma\left(w/w^*\right)^{\sigma-1}\right)\leq 0$. Hence, we just need to show conditions under which $F\left(1/\gamma\right)>0$ and $F\left(\gamma\right)<0$. In other words, we need to have:

$$\tau^{1-\sigma}>\gamma^{\sigma}\frac{\left(\gamma^{1-2\sigma}-N/N^*\right)}{\left(\gamma-N/N^*\right)}$$

and

$$\tau^{1-\sigma}<\gamma^{\sigma}\frac{\left(1-\gamma^{1-2\sigma}N/N^*\right)}{\left(1-N/N^*\gamma\right)}$$

Notice that if $\tau=\gamma$: The equilibrium with non traded homogeneous good holds.

One needs to have the tarriff in the homogeneous good small enough and the tarriff in the differentiated good relatively big in order to have the homogeneous good non traded because of the wage difference in the two countries. In other words, one needs to have the wage difference between the two countries to not be too large.

2. (25 marks) The objective of this second exercise is to revisit Melitz and Ottaviano's (2008) results. Like in Exercise 1, we consider a world economy with two countries, Home and Foreign, each endowed with one factor of production, *labor. L and L^* denote the endowments of labor in the two countries and w and w^* denote the associated wages.*

Again, there are two sectors, agriculture (A) and manufacturing (M). The agricultural sector produces a homogeneous good one-for-one for labor under perfect competition, whereas the manufacturing sector produces a large number of varieties under increasing returns to scale and monopolistic competition. Firms are heterogeneous in terms of their productivity φ, which is randomly drawn from a Pareto distribution $G(\varphi) = 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^{\theta}$ for $\varphi \geq \underline{\varphi}$. In order to production $q(\omega)$ units of a given variety ω, a firm with productivity φ requires labor:

$$l(\omega) = f_e + q(\omega)/\varphi$$

where $f_e > 0$ is an fixed entry cost paid before *firms know their productivity φ.*

The preferences of a representative consumer can be represented by

$$U = C_A + C_M$$

where the manufacturing aggregate is given by

$$C_M = \alpha_{\omega \in \Omega} c(\omega)\, d\omega - \frac{1}{2}\gamma_{\omega \in \Omega} [c(\omega)]^2\, d\omega - \frac{1}{2}\eta\, [_{\omega \in \Omega} c(\omega)\, d\omega]^2$$

(2.a) We start by analyzing the Home country under autarky.

(i) Show that total demand for a given variety is given by

$$q(\omega) = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p(\omega) + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \overline{p}$$

where N is the measure of consumed varieties and $\overline{p} \equiv \frac{1}{N}{}_{\omega \in \Omega} p(\omega)\, d\omega$ is the average price

Solution: First notice that as before, I will set $P_A = w = 1$. As in the previous example the maximization problem of the consumer has two steps:

$$\max_{C_A, C_M} C_A + C_M$$
$$st$$
$$C_A + \int_{\omega \in \Omega} c(\omega)\, p(\omega)\, d\omega = 1$$
$$C_M = \alpha \int_{\omega \in \Omega} c(\omega)\, d\omega - \frac{1}{2}\gamma \int_{\omega \in \Omega} [c(\omega)]^2\, d\omega - \frac{1}{2}\eta \left[\int_{\omega \in \Omega} c(\omega)\, d\omega\right]^2$$

The problem of consumer for each good is given by: The FOC of the problem of the consumer is given by:

$$\alpha - \gamma c(\omega) - \eta \left[\int_{\omega \in \Omega} c(\omega)\, d\omega\right] = \lambda p(\omega)$$

where $\lambda = 1$, if we assume that in equilibrium, they are consuming a positive amount of goods in both sectors. Suppose that there is a measure N of consumed varieties in the economy, one can redefine $\int_{\omega \in \Omega} c(\omega)\, d\omega$, solving for the relation:

$$Q = \frac{-\int_{\Omega*} p(\omega)\, d\omega + N\alpha}{(\gamma + \eta N)}$$

Hence,

$$\begin{aligned} c(\omega) &= \frac{\alpha - p(\omega) - \eta \left[\dfrac{-\int_{\Omega*} p(\omega)\, d\omega + \alpha}{(\gamma + \eta N)} \right]}{\gamma} \\ &= \frac{\alpha}{(\gamma + \eta N)} + \frac{\eta N}{\gamma(\gamma + \eta N)} \frac{\int_{\Omega*} p(\omega)\, d\omega}{N} - \frac{p(\omega)}{\gamma} \end{aligned}$$

Furthermore, given there are L consumers in our economy, the aggregate demand is given by:

$$q(\omega) = Lc(\omega) = \frac{\alpha L}{\eta N + \gamma} - \frac{L}{\gamma} p(\omega) + \frac{\eta N}{\eta N + \gamma} \frac{L}{\gamma} \overline{p}$$

where $\overline{p} = \dfrac{\int_{\Omega*} p(\omega)\, d\omega}{N}$ if the demand is positive, i.e. if $p(\omega) \leq \frac{\alpha\gamma}{\eta N + \gamma} + \frac{\eta N}{\eta N + \gamma}\overline{p}$.

(ii) Let $\varphi^* = \inf\left\{\varphi \geq \underline{\varphi} | \pi(\varphi) \geq 0\right\}$ where $\pi(\varphi)$ are the profits of a firm with productivity φ. Show that the mark-up $m(\varphi) \equiv \dfrac{p(\varphi) - w/\varphi}{p(\varphi)}$ of a firm with productivity $\varphi \geq \varphi^*$ satisfies

$$m(\varphi) = \frac{1/\varphi^* - 1/\varphi}{1/\varphi^* + 1/\varphi}$$

Solution: Define the profit of a firm φ:

$$\pi(\varphi) = p(q)\, q - q/\varphi$$

where $p(q)$ is defined as above. Notice that we are in a competitive monopolistic case. As a consequence, each firm φ, will set the quantity produced and the price such that

$$\max_q \; p(q)\, q - q/\varphi$$

The FOC is given by:

$$q = \frac{L}{\gamma}(p - 1/\varphi)$$

and

$$p = \frac{1}{2}\left(1/\varphi + \frac{\alpha\gamma}{\eta N + \gamma} + \frac{\eta N}{\eta N + \gamma}\overline{p}\right)$$

Furthermore, the profit is defined as:

$$\pi(\varphi) = \frac{L}{\gamma}\left(p(\varphi) - 1/\varphi\right)^2$$

φ^* reference to the cost of the firm who is just indifferent about remaining in the industry, i.e. $\pi(\varphi^*) = 0$. This firm earns zero profit as its price is driven down to its marginal cost, $p = 1/\varphi^*$ and its demand level is driven to zero. In other words, it is the firm such that $1/\varphi^* = \dfrac{\alpha\gamma}{\eta N + \gamma} + \dfrac{\eta N}{\eta N + \gamma}\overline{p}$ as one can derive from the optimal price above. As a consequence, one can express the price of a firm φ, $p = \frac{1}{2}(1/\varphi + 1/\varphi^*)$ and the markup can be rewritten as $m(\varphi) \equiv \dfrac{-1/\varphi + 1/\varphi^*}{1/\varphi + 1/\varphi^*}$.

(iii) Let $\overline{m}$ denote the average mark-up. What is the relationship between $\overline{m}$ and L? Explain.

Solution: Given that ex-ante a firm could draw any cost φ from the pareto distribution, $G(\varphi) = 1 - \left(\frac{\underline{\varphi}}{\varphi}\right)^{\theta}$ for $\varphi \geq \underline{\varphi}$,[1] $1/\varphi$ has the following distribution with upper bound $\underline{\varphi}$:
simplify some of the ensuing analysis, we use a specific parametrization for this distribution.

$$G(1/\varphi) = \left(\frac{\varphi}{\underline{\varphi}}\right)^{\theta}, \; 1/\varphi \leq 1/\underline{\varphi}$$

As any truncation of the cost distribution from above will retain the same distribution function and shape parameter θ. The productivity distribution of surviving firms will therefore also be Pareto with shape θ, and the truncated distribution will be given by:

$$G(1/\varphi) = \left(\frac{\varphi}{\varphi^*}\right)^{\theta}, \; 1/\varphi \leq 1/\varphi^*$$

The ex ante free entry condition implies that:

$$\int_0^{1/\varphi^*} \left(1/\varphi - 1/\varphi^*\right)^2 dG(1/\varphi) = \gamma 4 f_e / L$$

Now, we have all the ingredients to derive the the cut-off level by solving the free entry condition for $1/\varphi^*$:

$$1/\varphi^* = \left(\frac{2(\theta+1)(\theta+2)\gamma\left(1/\underline{\varphi}\right)^{\theta} f}{L}\right)^{1/(\theta+2)}$$

[1] I assume that $\varphi^* > \underline{\varphi}$

Hence, the average mark-up is given by: $\int_0^{1/\varphi^*} m(\varphi)\, dG(1/\varphi) = \int_0^{1/\varphi^*} .\dfrac{1/\varphi^* - 1/\varphi}{1/\varphi + 1/\varphi^*} dG(1/\varphi)$.
For the sake of the intuition, let us focus on $p - \frac{1}{\varphi} = \frac{1}{2}(1/\varphi + 1/\varphi^*) - \frac{1}{\varphi}$. In which case, the average markup is given by:

$$\frac{1}{2(\theta+1)\varphi^*} = \frac{1}{2(\theta+1)} \left(\frac{2(\theta+1)(\theta+2)\gamma\left(1/\underline{\varphi}\right)^{\theta} f}{L} \right)^{1/(\theta+2)}$$

As L increases, i.e. larger market, competition becomes 'tougher' because more firms compete. Average prices are smaller. A firm with cost $1/\varphi$ responds to this tougher competition by setting a lower markup.

(2.b) Suppose now that all goods can be freely traded between Home and Foreign.

(i) Show that the utility of the representative agent in country c can be written as

$$U^c = 1 + \frac{1}{2}\left(\eta + \frac{\gamma}{N^c}\right)^{-1} (\alpha - \overline{p}^c)^2 + \frac{1}{2}\frac{N^c}{\gamma}\left(\sigma_p^c\right)^2,$$

where $\left(\sigma_p^c\right)^2 = \frac{1}{N^c}\int_{\omega\in\Omega} \left[p(\omega) - \overline{p}^c\right]^2 d\omega$

Solution: We know from above that:

$$U = C_A + \alpha \int_{\omega\in\Omega} c(\omega)\, d\omega - \frac{1}{2}\gamma \int_{\omega\in\Omega} [c(\omega)]^2 d\omega - \frac{1}{2}\eta \left[\int_{\omega\in\Omega} c(\omega)\, d\omega\right]^2$$

where $C_A = 1 - \int_{\omega\in\Omega^*} c(\omega)\, p(\omega)\, d\omega$ (I assume it being positive). Hence, we have:

$$U = C_A + \alpha Q - \frac{1}{2}\gamma \int_{\omega\in\Omega} [c(\omega)]^2 d\omega - \frac{1}{2}\eta [Q]^2$$

where $Q = \dfrac{N(\alpha - \overline{p})}{(\gamma + \eta N)}$.

$$U = C_A + \alpha Q - \frac{1}{2}\gamma \int_{\omega\in\Omega} [c(\omega)]^2 d\omega - \frac{1}{2}\eta [Q]^2$$

Furthermore, from above we know that:

$$\begin{aligned}
[c(\omega)]^2 &= \left(\frac{\alpha}{\eta N + \gamma} - \frac{1}{\gamma} p(\omega) + \frac{\eta N}{\eta N + \gamma}\frac{1}{\gamma}\overline{p}\right)^2 \\
&= \left(\frac{\alpha - \overline{p}}{\eta N + \gamma} - \frac{1}{\gamma}(p(\omega) - \overline{p})\right)^2 \\
&= \left(\frac{\alpha - \overline{p}}{\eta N + \gamma}\right)^2 + \left(\frac{1}{\gamma}(p(\omega) - \overline{p})\right)^2 - \frac{2}{\gamma}(p(\omega) - \overline{p})\frac{\alpha - \overline{p}}{\eta N + \gamma}
\end{aligned}$$

Substituting in the utility function:

$$
\begin{aligned}
U &= C_A + \alpha Q \\
&\quad -\frac{1}{2}\gamma\left[N\left(\frac{\alpha-\overline{p}}{\eta N+\gamma}\right)^2 + \int_{\omega\in\Omega}\left(\frac{1}{\gamma}\left(p\left(\omega\right)-\overline{p}\right)\right)^2 d\omega - \frac{\alpha-\overline{p}}{\eta N+\gamma}\frac{2}{\gamma}\int_{\omega\in\Omega}\left(p\left(\omega\right)-\overline{p}\right)d\omega\right] \\
&\quad -\frac{1}{2}\eta\left[Q\right]^2 \\
&= C_A + \alpha Q - \frac{1}{2}\gamma N\left(\frac{\alpha-\overline{p}}{\eta N+\gamma}\right)^2 - \frac{N}{2\gamma}\left(\sigma_p^c\right)^2 - \frac{1}{2}\eta\left[Q\right]^2 \\
&= C_A + \alpha\frac{N\left(\alpha-\overline{p}\right)}{\left(\gamma+\eta N\right)} - \frac{N}{2\gamma}\left(\sigma_p^c\right)^2 - \frac{1}{2}\frac{N}{\left(\gamma+\eta N\right)}\left(\alpha-\overline{p}\right)^2
\end{aligned}
$$

where $\left(\sigma_p^c\right)^2 = \frac{1}{N^c}\int_{\omega\in\Omega}\left[p\left(\omega\right)-\overline{p}^c\right]^2 d\omega$.
Finally, notice that

$$
C_A = 1 - \int_{\omega\in\Omega^*} c\left(\omega\right)p\left(\omega\right)d\omega
$$

where

$$
\begin{aligned}
\int_{\omega\in\Omega^*} c\left(\omega\right)p\left(\omega\right)d\omega &= \int_{\omega\in\Omega^*}\left(\frac{\alpha}{\eta N+\gamma}p\left(\omega\right) - \frac{1}{\gamma}p^2\left(\omega\right) + \frac{\eta N}{\eta N+\gamma}\frac{1}{\gamma}\overline{p}p\left(\omega\right)\right)d\omega \\
&= \frac{\alpha N}{\eta N+\gamma}\overline{p} - \frac{1}{\gamma}\int_{\omega\in\Omega^*}p^2\left(\omega\right)d\omega + \frac{\eta N^2}{\eta N+\gamma}\frac{1}{\gamma}\overline{p}^2 \\
&= \frac{\left(\alpha-\overline{p}\right)N}{\eta N+\gamma}\overline{p} - \frac{N}{\gamma}\left(\sigma_p^c\right)^2
\end{aligned}
$$

Hence, rearranging the terms we get:

$$
U^c = 1 + \frac{1}{2}\left(\eta+\frac{\gamma}{N^c}\right)^{-1}\left(\alpha-\overline{p}^c\right)^2 + \frac{1}{2}\frac{N^c}{\gamma}\left(\sigma_p^c\right)^2,
$$

where $\left(\sigma_p^c\right)^2 = \frac{1}{N^c}\int_{\omega\in\Omega}\left[p\left(\omega\right)-\overline{p}^c\right]^2 d\omega$.

(ii) Show that the measured of consumed varieties in country c is given by

$$
N^c = \frac{2(\theta+1)\left(\alpha\varphi_c^*-1\right)\gamma}{\eta}
$$

Solution:

The number of variety is determind by the free entry condition and $1/\varphi^* = \dfrac{\alpha\gamma+\eta N\overline{p}}{\eta N+\gamma}$, to solve for N.

$$
N^c = \frac{2(\theta+1)\left(\alpha\varphi_c^*-1\right)\gamma}{\eta}
$$

where I used the fact that $\overline{p} = \frac{\int_{\Omega *} \frac{1}{2} \left(1/\varphi + 1/\varphi^*\right) d\omega}{N}$.

(iii) Using your results in (i) and (ii), show that the utility of the representative agent can be written as

$$U^c = 1 + \frac{1}{2\eta}\left(\alpha - \frac{1}{\varphi_c^*}\right)\left(\alpha - \frac{\theta+1}{\theta+2}\frac{1}{\varphi_c^*}\right)$$

Solution: We know from above that:

$$U^c = 1 + \frac{1}{2}\left(\eta + \frac{\gamma}{N^c}\right)^{-1}(\alpha - \bar{p}^c)^2 + \frac{1}{2}\frac{N^c}{\gamma}\left(\sigma_p^c\right)^2,$$

Substituting N^c in the indirect utility function:

$$U^c = 1 + \frac{1}{2\eta}\left(1 + \frac{1}{2(\theta+1)\left(\alpha\varphi_c^* - 1\right)}\right)^{-1}(\alpha - \bar{p}^c)^2 + \frac{(\theta+1)\left(\alpha\varphi_c^* - 1\right)}{\eta}\left(\sigma_p^c\right)^2$$

where $\left(\sigma_p^c\right)^2 = E\left(\left(p\left(\varphi\right) - \bar{p}^c\right)^2 | \varphi \geq \varphi^*\right) = \frac{1}{4\varphi^{*2}}\frac{\theta}{\left(\theta+1\right)^2\left(\theta+2\right)}$ and $\bar{p}^c = \frac{1}{2\varphi^*}\frac{1+2\theta}{1+\theta}$. Substituting in the indirect utility function:

$$\begin{aligned}
U^c &= 1 + \frac{1}{2\eta}\left(1 + \frac{1}{2(\theta+1)\left(\alpha\varphi_c^* - 1\right)}\right)^{-1}(\alpha - \frac{1}{2\varphi^*}\frac{1+2\theta}{1+\theta})^2 + \frac{\left(\alpha\varphi_c^* - 1\right)}{\eta 4\varphi^{*2}}\frac{\theta}{(\theta+1)(\theta+2)} \\
U^c &= 1 + \frac{1}{2\eta}\left(\alpha - \frac{1}{\varphi_c^*}\right)(\alpha - \frac{(2\theta+1)(\theta+2) - \theta}{(\theta+2)(\theta+1)2\varphi_c^*}) \\
U^c &= 1 + \frac{1}{2\eta}\left(\alpha - \frac{1}{\varphi_c^*}\right)\left(\alpha - \frac{\theta+1}{\theta+2}\frac{1}{\varphi_c^*}\right)
\end{aligned}$$

3. (25 marks) This question concerns the empirical implications of the 'home market effect' (HME), and ways in which it can be estimated.

(a) Describe what is meant by the HME and the intuition behind it.

Solution: Economies characterized by the presence of (i) increasing returns, (ii) monopolistic competition, and (iii) trade costs exhibit a more-than-proportional relationship between a countrys share of world production of a good and its share of world demand for the same good. This relation between a countrys market size and industrial specialization has been highlighted by Krugman (1980) and Helpman and Krugman (1985) as the Home Market Effect.

(b) Davis and Weinstein (JIE 2003) argue that the HME is empirically powerful: it is a prediction made by increasing returns to scale mod- els of trade,

but not made by neoclassical (comparative advantage) models of trade. Explain this argument.

Solution: The following from DW (2003) puts it well: Consider a positive shock to the home demand structure for a good. Will this call forth additional local supply, and if so will supply move more than one-for-one (as required for the home market effect)? If the production set is strictly convex, additional supply of the good will be forthcoming only if its relative price rises. But then, provided the foreign export supply curve has the conventional positive slope, this will also call forth additional net exports from abroad. In such a case, the idiosyncratic demand will be partly met by additional local supply and partly by higher imports. Local supply, then, moves less than one-for-one with the idiosyncratic demand. In this conventional comparative advantage world, there is no home market effect.

(c) Davis and Weinstein implement a test for the HME. Explain how they do this. What do they find? Do you believe their answer?

Solution: See DW (2003).

(d) Can you think of a natural experiment that would enable you to test the HME more directly?

Solution: The key challenge would be to find a clean shock to a countrys demand for a good that does not affect its supply of the good. One idea would be to consider a shock (of any kind) to a downstream industry that demands inputs from the industry in question. Another idea could exploit demographic shifts like those used by DellaVigna and Pollet (AER 2007), who study how, Cohort size fluctuations produce forecastable demand changes for age-sensitive sectors, such as toys, bicycles, beer, life insurance, and nursing homes. Armed with such a demand shock one explore whether this shock leads the country to export more of the good in question.

(e) Suppose you had access to a consumer-level scanner dataset (like that used by Broda and Weinstein (2008, Understanding International Price Differences Using Barcode Data), Gopinath, Gourin- chas, Hseih and Li (2010), or Burstein and Jaimovich (2008)) that contains the prices of identical goods (identified with their barcode or UPC) at various points in space. (Or alternatively, consider any dataset you can dream up that contains very high quality price data across regions or countries.) Is there a way to use this dataset to test for the HME?

Solution: The general idea here was to see if it is possible to move away from quantity-based tests of the HME towards price-based tests. Are there differential implications across these models for the response of prices, terms of trade, etc, to the relative labor endowment of two economies?

4. (10 marks) Bernard, Redding and Schott (ReStud, 2007) describe a 2-by-2 Heckscher-Ohlin model with increasing returns to scale (a la Helpman and Krugman, 1995), and intra-industry heterogeneity with fixed costs of exporting (a la Melitz, 2003). Proposition 11 states the implications of this model for the HOV equations (i.e. the factor content of trade) in this model relative to those in the baseline Helpman-Krugman model.

(a) Explain the intuition behind Proposition 4 in BRS (2007) and the reason why it is different from Proposition 2.

Solution: Proposition 4 states that if trade is costly, opening up to trade has a positive impact on the steady-state zero profit productivity cut-off and the average productivity of all the industries. On the other hand, Proposition 2 states that if trade is costless, opening up to trade leaves the steady-state zero profit productivity and the average productivity remain unchanged.

The intuition behind Proposition 4 is that given trade is costly, opening up to trade has a differential effect on firms with different productivity. Overall, as the country opens up to trade, more firms want to enter the industry as the expected value of entry increases. This increases the competition within the industry and decrease profit to the zero profit condition in the steady-state. However, as this happens, the least productive firms drop out of the industry as they don't meet anymore they fixed production cost. Hence, opening up to trade is accompanied by a structural change within the industry where the industry is on average more productive.

On the other hand, if trade is costless (Proposition 2), trade affects all the firms identically as all firms will export. As such, there is no impact on the productivity distribution within the industry as all firms are affected symmetrically.

(b) Proposition 11 of BRS (2007) states the implications of this model for the HOV equations (ie the factor content of trade) in this model relative to those in the baseline Helpman-Krugman model. Explain the intuition behind this result. Does this proposition rationalize any of the empirical failures of the HOV predictions that we have seen in this course? Describe an empirical paper that you could write that would build on this model to explore how the presence of intra-industry heterogeneity (and fixed trade costs) alters our under- standing of how factor endowments affect trade.

Solution: It is not surprising that the HOV equations do not hold in this BRS (2007) model: the existence of variable trade costs alone will mean that FPE cannot hold, and hence the HOV equations can- not hold. (Recall that this is for two reasons: (a) the A(w) matrix will now depend on each countrys w vector; and (b), the iceberg trade costs mean that for every unit of a good demanded by final consumers, ? units need to be shipped). In addition, there are fixed trade costs which also affect FPE and hence A(w) and hence the HOV equations. Finally, selection into exporting (due to fixed trade costs) will alter the productivities of active firms in each economy and this will further violate the HOV assumption that (even at constant w) countries use the same A(w) matrix of technologies. We should expect (though BRS (2007) do not prove this) that all of these forces will lead to there being missing [net factor content of] trade, as Trefler (1995) put it. Davis and Weinstein (2001 AER) and Helpman (1998 JEP) already emphasize how non-FPE will bias the NFCT downwards, and of course iceberg trade costs will do so even further (because they not only induce non-FPE but also diminish trade flows). A final point is that there will now (in the BRS (2007) model) be positive factor content to intra-industry trade flows since varieties within the same industry may be produced with different factor intensities. This is something that Davis and Weinstein (2004) find support for (though they do not relate their findings to a Melitz (2003)-style model).

(c) Consider the BRS (2007) model with costly trade. Write down the problem of a social planner who wishes to maximize the value of total output in the economy subject to the economys resource constraints, and while holding fixed the same variables that monopolistically competitive firms (with a continuum of firms) are assumed to take as given. (When a small, monopolistically competitive firm from country H is active in industry i of the domestic market, call the variable that is the composite of all the variables the firm takes as given, Aid; call the equivalent in the export market Aix.) Show that the solution of this problem is identical to the equilibrium conditions in the monopolistically competitive economy. Hence show that the economy admits a revenue function (of sorts) and characterize the properties of this function. (The revenue function can be written as $R(A_{1d}, A_{1x}, A_{2d}, A_{2x}; V)$, where V is the vector of factor endowments.)

Solution: The solution to this problem is a proposition in Feenstra and Kee (2008, JIE). The social planner problem is given by maximizing total GDP in the economy (sum of the revenue over the sectors from selling in the domestic R_{id} and the export market R_{ix}):

$$R = \sum_{i=1}^{N} R_{id} + R_{ix}$$

subject to the total resource constraints for the economy, which are defined as (i) the total resources used for domestic production

$$\int_{\psi_i^*}^{\infty} h_i[v_i(\psi)]\mu_i(\psi_i)d\psi_i = \int_{\psi_i^*}^{\infty} M_i[\frac{q_i(\psi_i)}{\psi_i} + f_i]\mu_i(\psi_i)d\psi_i$$

(ii) the total resources for the exporting production

$$\int_{\psi_{ix}^*}^{\infty} h_i[v_{ix}(\psi)]\mu_i(\psi_i)d\psi_i = \int_{\psi_{ix}^*}^{\infty} M_i[\frac{q_{ix}(\psi_i)}{\psi_i} + f_{ix}]\mu_i(\psi_i)d\psi_i$$

and (iii) the factor market clearing condition

$$\sum_{i=1}^{N}\left[\int_{\psi_i^*}^{\infty}[v_i(\psi)]\mu_i(\psi_i)d\psi_i+\int_{\psi_{ix}^*}^{\infty}[v_{ix}(\psi)]\mu_i(\psi_i)d\psi_i\right]=V-\sum_{i=1}^{N}M_{ie}v_{ie}$$

where M_i is the mass of firms in sector i and $\mu_i(.)$ is the distribution of productivities for firms in sector i. Defining more each terms and equation, we have:

(a) The revenue from domestic and export sales in sector i is given by:

$$R_{id}\equiv M_iA_{id}\int_{\psi_i^*}^{\infty}q_i(\psi_i)^{\frac{\sigma_i-1}{\sigma_i}}\mu_i(\psi_i)d\psi_i$$

and

$$R_{ix}\equiv M_iA_{ix}\int_{\psi_{ix}^*}^{\infty}q_{ix}(\psi_i)^{\frac{\sigma_i-1}{\sigma_i}}\mu_i(\psi_i)d\psi_i$$

The total revenue comes from the revenue earned by a home firm selling in the domestic sector ($r_i(\psi_i)$) given the CES utility function and its equivalent price index (P_i^H) and the revenue earned by the exporter ($r_{ix}(\psi_i)$):

$$r_i(\psi_i)=p_i(\psi_i)q_i(\psi i)=\frac{p_i(\psi_i)}{P_i^H})^{1-\sigma_i}E_i^H,\ \sigma_i>1.$$

which is derived from the maximization problem of the firm as well as the CES utility specification demand function. One can solve the price as a function of the quantity from the condition just stated and plug it back in the revenue function to get:

$$r_i(\psi_i)=A_{id}(q_i(\psi_i))^{\frac{\sigma_i-1}{\sigma_i}}$$

where $A_{id}\equiv P_i^H\left(\frac{E_i^H}{P_i^H}\right)^{\frac{1}{\sigma_i}}$ defines all the givens for the firm. One can retrieve the revenue of the exporter identically, where $A_{ix}\equiv\frac{P_i}{\tau_i}^F\left(\frac{\tau_iE_i^F}{P_i^F}\right)^{\frac{1}{\sigma_i}}$ and τ_i are the iceberg cost faced by the exporter.

(b) The total ressource cost is given by the fact that there are M_i firms producing $q_i(\psi_i)$ for the domestic market and $q_{ix}(\psi_i)$ for the export market and facing respectively:

$$h_i[v_i(\psi_i)]=M_i[\frac{q_i(\psi_i)}{\psi_i}+f_i]$$

and

$$h_i[v_{ix}(\psi_i)]=M_i[\frac{q_{ix}(\psi_i)}{\psi_i}+f_{ix}]$$

where f_i and f_{ix} are respectively the fixed cost of production and the additional fixed cost of exporters; $v_.(\psi_i)$ is a k-dimensional vector of factor demand; and h_i is a homogeneous of degree one and strictly quasi-concave mapping from the vector of factor demands to a scalar.

(c) Finally, on the right of the factor market condition, V is the vector of factor endowment for the economy and $\sum_{i=1}^{N} M_{ie} v_{ie}$ the factor necessary from the entry into each sector, where M_{ie} is defined as the mass of entering firms. Notice that given that in a stationary equilibrium, the mass of entering firms is equal to the mass of firms exiting, i.e. $[1 - G_i(\psi_i^*)]M_{ie} = \delta M_i$, where G is the cdf of ψ. Hence, (iii) can be rewritten as:

$$\sum_{i=1}^{N} \left[\int_{\psi_i^*}^{\infty} [v_i(\psi)]\mu_i(\psi_i)d\psi_i + \int_{\psi_{ix}^*}^{\infty} [v_{ix}(\psi)]\mu_i(\psi_i)d\psi_i \right] = V - \sum_{i=1}^{N} \delta M_i[1 - G_i(\psi_i^*)v_{ie}$$

Solution of the Problem:

A) Lagrange multiplier: Given that A's and V are the only variables taken as given, we already know that the revenue function from the maximization problem will be a function of A's and V. Lets find the solution in order to make the statement more precise and show the equivalence with the monopolistic competition problem. First, notice that the Lagrangian can be written as follows:

$$L = \sum_{i=1}^{N} R_{id} + R_{ix}$$

$$+ \sum_{i=1}^{N} m_i \left(\int_{\psi_i^*}^{\infty} h_i[v_i(\psi)]\mu_i(\psi_i)d\psi_i - \int_{\psi_i^*}^{\infty} M_i[\frac{q_i(\psi_i)}{\psi_i} + f_i]\mu_i(\psi_i)d\psi_i \right)$$

$$+ \sum_{i=1}^{N} m_{ix} \left(\int_{\psi_{ix}^*}^{\infty} h_i[v_{ix}(\psi)]\mu_i(\psi_i)d\psi_i - \int_{\psi_{ix}^*}^{\infty} M_i[\frac{q_{ix}(\psi_i)}{\psi_i} + f_{ix}]\mu_i(\psi_i)d\psi_i \right)$$

$$+ w' \left(V - \sum_{i=1}^{N} \delta M_i[1 - G_i(\psi_i^*)v_{ie} - \sum_{i=1}^{N} \left[\int_{\psi_i^*}^{\infty} [v_i(\psi)]\mu_i(\psi_i)d\psi_i + \int_{\psi_{ix}^*}^{\infty} [v_{ix}(\psi)]\mu_i(\psi_i)d\psi_i \right) \right.$$

where m_i, m_{ix} and w are respectively the lagrange multipliers of the three sets of constraints.

B) Regroup the Lagrange multiplier into different terms depending on a specific variable of interest (q, v, ψ):

$$L = \sum_{i=1}^{N} M_i \int_{\psi_i^*}^{\infty} (A_{id} q_i(\psi_i)^{\frac{\sigma_i - 1}{\sigma_i}} - m_i[\frac{q_i(\psi_i)}{\psi_i} + f_i])\mu_i(\psi_i)d\psi_i$$

$$+ \sum_{i=1}^{N} M_i \int_{\psi_{ix}^*}^{\infty} (A_{ix} q_{ix}(\psi_i)^{\frac{\sigma_i - 1}{\sigma_i}} - m_{ix})[\frac{q_{ix}(\psi_i)}{\psi_i} + f_{ix}])\mu_i(\psi_i)d\psi_i$$

$$+ \sum_{i=1}^{N} \int_{\psi_i^*}^{\infty} (m_i h_i[v_i(\psi)] - w' v_i(\psi))\mu_i(\psi_i)d\psi_i$$

$$+ \sum_{i=1}^{N} \int_{\psi_{ix}^*}^{\infty} (m_{ix} h_i[v_{ix}(\psi)] - w' v_{ix}(\psi))\mu_i(\psi_i)d\psi_i$$

$$+w'\left(V-\sum_{i=1}^{N}\delta M_i[1-G_i(\psi_i^*)v_{ie}\right)$$

C) Differentiate wrt q

The first order condition with respect to q_i and q_{ix} is given by differentiating the first and the second term given in B. Using the fact that $p_i(\psi_i) = A_{id}q_i(\psi_i)^{-1/\sigma_i}$ and $p_{ix}(\psi_i) = A_{ix}q_{ix}(\psi_i)^{-1/\sigma_i}$, as a solution, we get:

$$\frac{1-\sigma_i}{\sigma_i}p_i(\psi_i) = \frac{m_i}{\psi_i}$$

and

$$\frac{1-\sigma_i}{\sigma_i}p_{ix}(\psi_i) = \frac{m_{ix}}{\psi_i}$$

D) Differentiate wrt v

To Solve for v_i and v_{ix}, we can use the third and fourth integral:

$$m_i\frac{\partial h_i(v_i(\psi_i)}{\partial v_i(\psi_i)} = w$$

and

$$m_{ix}\frac{\partial h_i(v_{ix}(\psi_i)}{\partial v_{ix}(\psi_i)} = w$$

Given the function hi is assumed to be strictly quasi-concave, it follows from the first order condition that the ratio of demand for factors k and l are identical in domestic and export use: $\frac{\partial h_i(v_i(\psi_i)}{\partial v_{il}(\psi_i)}/\frac{\partial h_i(v_i(\psi_i)}{\partial v_{ik}(\psi_i)} = \frac{\partial h_i(v_{ix}(\psi_i)}{\partial v_{ixl}(\psi_i)}/\frac{\partial h_i(v_{ix}(\psi_i)}{\partial v_{ixk}(\psi_i)} = \frac{w_l}{w_k}$. Therefore, the values of vi and vix are multiples of each other, $v_i = \lambda_i v_{ix}$. But since h_i homogeneous of degree one, its first derivative is homogeneous of degree zero, so any solution $v_i = \lambda_i v_{ix}$ gives the same value for the derivatives. It follows that the equalities will hold only if the lagrange multipliers are identical. Hence, multiplying the two conditions by v_i and v_{ix} in both side, we get $[m_ih_i(v_i) - w'v_i] = [m_{ix}h_i(v_{ix}) - w'v_{ix}] = 0$.

E) Substitute all the information in the Lagrangian

$$L = \sum_{i=1}^{N} M_i\int_{\psi_i^*}^{\infty}(\frac{r_i(\psi_i)}{\sigma_i} - m_if_i)\mu_i(\psi_i)d\psi_i$$

$$+\sum_{i=1}^{N} M_i\int_{\psi_{ix}^*}^{\infty}(\frac{r_{ix}(\psi_i)}{\sigma_i} - m_{ix}f_i)\mu_i(\psi_i)d\psi_i$$

$$+w'\left(V-\sum_{i=1}^{N}\delta M_i[1-G_i(\psi_i^*)v_{ie}\right)$$

where I used the last equality in D and from C, I used the fact that the profit from the export and domestic sale is give by $\frac{r_i(\psi_i)}{\sigma_i}$ and $\frac{r_{ix}(\psi_i)}{\sigma_i}$.

F) Differentiate wrt export cutoff ψ_{ix}^*

Using the new Lagrangian, we can show that the profits earned by the marginal exporter should exactly cover the fixed costs:

$$\frac{r_{ix}(\psi_i^*)}{\sigma_i} = m_i f_{ix}$$

G) Differentiate wrt M_i

One can see that the expected discounted profits equal the fixed costs of entry.

$$\frac{[1 - G_i(\psi_i^*)}{\delta} \left(\int_{\psi_i^*}^{\infty} (\frac{r_i(\psi_i)}{\sigma_i} - m_i f_i)\mu_i(\psi_i)d\psi_i + \int_{\psi_{ix}^*}^{\infty} (\frac{r_{ix}(\psi_i)}{\sigma_i} - m_{ix} f_i)\mu_i(\psi_i)d\psi_i \right) = w' v_{ie}$$

H) Differentiate wrt domestic cutoff ψ_i^*

Using the free entry condition we just derived and the fact that by definition, the pdf μ is a function of ψ_i^* as it is divided by it, we obtain the following condition:

$$\frac{r_i(\psi_i^*)}{\sigma_i} = m_i f_i$$

Notice that from C,D,F,G,H, one can retrace the equilibrium conditions in the monopolistically competitive economy.[2]

(d) Now assume that firms in industry i draw their productivities from a Pareto distribution whose CDF is: $G_i(\psi_i) = 1 - \psi^{-\theta_i}$, with $\theta_i > \sigma - 1$. Under this restriction, derive a simplified form for the revenue functions dependence on A_{1d}, A_{1x}, A_{2d} and A_{2x}.

Solution: Let $\Phi_i(A_{id}, A_{ix})$ be a CES aggregator of (A_{id}, A_{ix}) such that $\Phi_i(A_{id}, A_{ix}) = \left[(A_{id}^{\frac{\theta_i \sigma_i}{\sigma_i - 1}} + A_{ix}^{\frac{\theta_i \sigma_i}{\sigma_i - 1}})(f_{ix}/f_i)^{1 - \frac{\theta_i}{\sigma_i - 1}} \right]$

One can show that R is a function of Φ:

From the revenue function for two different productivity ψ, we know that: $\frac{r_i(\psi_i')}{r_i(\psi_i'')} = (\frac{p_i(\psi_i')}{p_i(\psi_i'')})^{1-\sigma_i}$. Given the price is given by $\frac{1-\sigma_i}{\sigma_i} p_i(\psi_i) = \frac{m_i}{\psi_i}$, the relative revenue of two different productivities is simply given by $\frac{r_i(\psi_i')}{r_i(\psi_i'')} = (\frac{\psi_i'}{\psi_i''})^{1-\sigma_i}$. In particular, we have $r_i(\psi_i) = (\frac{\psi_i}{\psi_i^*})^{1-\sigma_i} r_i(\psi_i^*)$, for any ψ_i relative to ψ_i^*, i.e. the cutoff productivity.. Finally, remember that $r_i(\psi_i) = A_{id} q_i(\psi_i)^{\frac{\sigma_i - 1}{\sigma_i}}$. Hence, $r_i(\psi_i^*) = A_{id} q_i(\psi_i^*)^{\frac{\sigma_i - 1}{\sigma_i}}$. Now, one can substitute it in the total revenue function generated in the domestic and the export market:

$$R_{id} = M_i A_{id} q_i(\psi_i^*)^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\psi_i^a(\psi_i^*)}{\psi_i^*} \right)^{\sigma_i - 1}$$

[2] Notice that I solved the problem for a more general case. One can easily go back to the special case with 2 sectors

and

$$R_{ix} = \chi_i M_i A_{ix} q_{ix}(\psi_{ix}^*)^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\psi_{ix}^a(\psi_{ix}^*)}{\psi_{ix}^*} \right)^{\sigma_i - 1}$$

where ψ_i^a and ψ_{ix}^a are respectively given from the integral in the total revenue function and are defined by $\left(\int_{\psi_i^*}^{\infty} \psi_i^{\frac{\sigma_i - 1}{\sigma_i}} \mu_i(\psi_i) d\psi_i \right)^{1/(\sigma_i - 1)}$ and ψ_i^a and ψ_{ix}^a are respectively given from the integral in the total revenue function and are defined by $\left(\int_{\psi_{ix}^*}^{\infty} \psi_i^{\frac{\sigma_i - 1}{\sigma_i}} \frac{g_i(\psi_i)}{1 - G_i(\psi_{ix}^*)} d\psi_i \right)^{1/(\sigma_i - 1)}$. Finally, χ is simply defined as the range of export varieties relative to domestic varieties, i.e. $\frac{1 - G(\psi_{ix}^*)}{1 - G(\psi_i^*)}$.

Notice that given the pareto distribution, one can show that ψ_i^a and ψ_{ix}^a are equal and given by $\theta_i/(\theta_i - \sigma_i + 1)$ where $\theta_i > \sigma_i - 1$. Hence, taking the ratio between R_{id} and R_{ix}, one can find

$$\frac{R_{id}}{R_{ix}} = \frac{A_{id} q_i(\psi_i^*)^{\frac{\sigma_i - 1}{\sigma_i}}}{\chi_i A_{ix} q_{ix}(\psi_{ix}^*)^{\frac{\sigma_i - 1}{\sigma_i}}}$$

Using the fact that from the zero cutoffs profits conditions, we have $r_i(\psi_i^*)/\sigma_i = m_i f_i)$ and $r_i(\psi_{ix}^*)/\sigma_i = m_i f_{ix})$.

$$\frac{R_{id}}{R_{ix}} = \frac{f_i}{\chi_i f_{ix}}$$

Finally, noticing that $\chi = \frac{1 - G(\psi_{ix}^*)}{1 - G(\psi_i^*)} = (\frac{\psi_i^*}{\psi_{ix}^*})^{\theta_i}$, one can use one more time the definition of the relative revenue $(\frac{r_{ix}(\psi_{ix}^*)}{r_i(\psi_i^*)} = \left(\frac{\psi_{ix}^* P_i^F / \tau_i}{P_i^H \psi_i^*} \right)^{\sigma_i - 1} \frac{E_i^F}{E_i^H})$ at the two cutoffs to get after using the definition of the A's:

$$\frac{A_{ix}}{A_{id}} = \chi_i^{(\sigma_i - 1)/\sigma_i \theta_i} (\frac{f_{ix}}{f_i})^{1/\sigma_i}$$

and

$$\frac{R_{ix}}{R_{id}} = (\frac{A_{ix}}{A_{id}})^{\sigma_i \theta_i / (\sigma_i - 1)} (\frac{f_{ix}}{f_i})^{1 - \frac{\theta_i}{\sigma_i - 1}}$$

where $0 < (\sigma_i - 1)/\sigma_i \theta_i < 1$ and I used the relation between and the relative A's to substitute A's in the relative total revenue. As a consequence of this last relation, we know that the parameters (A_{id}, A_{ix}) in each sector is weakly separable from all other shift parameters as well as the endowments in the GDP. As such, the GDP can be written as a function of a function of (A_{id}, A_{ix}). Furthermore, one can easily show from the last relation that the function of $\Phi(A_{id}, A_{ix})$ is a CES.

(e) Describe an empirical paper that would build on this model to explore how the presence of intra-industry heterogeneity (and fixed trade costs) alters our understanding of how factor endowments affect trade.

Solution: One clear testable implication of the BRS (2007) model is that, within industries, the factor content of firm-level exporting will differ across firms. Measuring this empirically could be interesting because all HOV equation tests to date have applied one industry- wide technology matrix to the production of both domestic-selling and exporting firms, something that this model tells us is inappropriate. (In particular, the egregious assumption made by HOV empirical work, in the light of BRS (2007), is to assume that the factor content of exporting (which we dont observe) is equal to the factor content of total production (which we do observe, from IO tables)). To look at this one would need a firm-level production dataset that contains data on factor usage, sales by exporting and domestic use separately, and (ideally) a measure of inputs purchased (so that one can compute both the firms A(.) matrix and its B(.) matrix, where this notation follows from the lectures). From this information one could describe simple features of the factor content of each firms ex- ports, and compare this to the HOV assumption that every firm has the same factor content of production/exports. With access to this data from two major trading partners one could compare the NFCT computed by the standard HOV methodology to the actual NFCT measured at the firm-level in each of these countries. A second possibility here would be to specify and estimate a structural model that features trade costs and heterogeneous firms, and compare its predictions for NFCT with that seen in the data. The model could be a multi-country and multi-sector version of BRS (2007). Or it could be a H-O version of the model in Costinot, Donaldson and Komunjer (2010). In the latter case, a particularly interesting extension would be to use a model of production that would allow firms to use different combinations of factors depending on their productivity draw. Burstein and Vogel (2009) specify a CES- style production function that would deliver this.

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