14.581 Spring 2011
Problem Set 4: Gravity Models, and the Estimation of Trade Costs
Solutions

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1. (15 marks) This question and the two that follow ask you to work through some of the results in Arkolakis, Costinot and Rodriguez-Clare (2011), henceforth ACRC. Consider first the Armington model. There are I countries i, each with fixed labor endowment L_i. Each good is produced with a production function $Y_i = T_i L_i$, where Y_i is the amount of output produced by country i. A large number of perfectly competitive firms in country i have access to this technology. Make the 'Armington assumption' on country technologies, which is to say that there are I different goods in the world and the only good that country i can make is 'good i'. Suppose all consumers in the world have the same preferences, which are CES preferences over each good with elasticity of substitution between any two goods equal to σ. Variable (iceberg) trade costs between any two countries are $\tau_{ij} \geq 1$ and $\tau_{ii} = 1$. Assume trade balance.

(a) Write down the 'import demand system' in this economy, i.e. $\frac{\partial ln(X_{ij}/X_{jj})}{\partial ln\tau_{ik}}$ Does it satisfy 'R3' in ACRC? Can you think of another assumption about preferences around the world that would satisfy R3?

Solution: Letting $\epsilon_j^{ik} \equiv \frac{\partial ln(X_{ij}/X_{jj})}{\partial ln\tau_{ik}}$, R3 states that the import demand system is such that for any importer j and any pair of exporters $i \neq j$ and $i' \neq j$, $\epsilon_j^{ik} = \epsilon < 0$ if $i = k$, and zero otherwise.

In words, R3 states that the change in the relative demand for the good from country i, X_{ij}/X_{jj}, only depends on the changes in τ_{ij}, i.e. trade costs between the exporter i and the importer j. Intuitively, any change in a third country's trade costs, eg τ_{ki}, must have the same proportional impact on X_{ij} and X_{jj} for R3 to be satisfied.

From the CES utility function and the Armington assumption (j only imports good i from country i), we know that the value of j's imports from country i is given by:

$$X_{ij} = p_{ij}x_{ij} = \left(\frac{\tau_{ij}w_i}{P_j T_i}\right)^{1-\sigma} w_j L_j,$$

where $p_{ij} = \frac{w_i \tau_{ij}}{T_i}$; $x_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{-\sigma} X_j$ and finally, $P_j = \left(\sum_i (\frac{\tau_{ij} w_i}{T_i})^{1-\sigma}\right)^{1/(1-\sigma)}$. Using $\tau_{jj} = 1$, we have:

$$\frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{ij} w_i}{w_j}\right)^{1-\sigma}$$

Taking logs and the first derivative, we notice that this satisfies R3:

$$\frac{\partial ln(X_{ij}/X_{jj})}{\partial ln\tau_{ik}} = \begin{cases} 1-\sigma & \text{if } k = j \\ 0 & \text{otherwise} \end{cases}$$

Any preferences that generate a 'CES import demand system' would satisfy R3. The term 'CES import demand system' refers to the fact that changes in the relative demand for two goods k and l, (C_k/C_l), satisfies the following conditions: (i) $\frac{\partial ln(C_k/C_l)}{\partial lnp_{k'}} = 0$ if $k' \neq k, l$ and (ii) $\frac{\partial ln(C_k/C_l)}{\partial lnp_k} = \frac{\partial ln(C_{k'}/C_l)}{\partial lnp_{k'}} \neq 0$ for all $k, k' \neq l$. These conditions are analogous to those often used to define a CES demand system.

(b) Describe the best possible empirical paper you could write that would test R3.

Solution: In order to test R3, one would need to show that the change in the relative demand in country j for the good from country i, X_{ij}/X_{jj}, only depends on the changes in τ_{ij}. A second testable restriction in R3 is the fact that the effect of τ_{ij} on X_{ij}/X_{jj} is the same for all i and j in the sample.

Ideally one would find panel data where for each country j and its respective trade partner $\forall\ i \neq j$ through time t, one can find: (i) X_{ij} as the imports of country j from country i for trading partner of $i \neq i$; (ii) X_{jj} as country j's gross production minus its net exports (note that getting gross output, especially by sector, is hard for many countries and time periods); (iii) trade costs, τ_{ik}, for all $k \neq j$, i.e. for each trade partner i of j. The greatest challenge here is getting data on trade costs. Three possibilities are: First, one could proxy the bilateral costs through one of its observable components (as in the literature surveyed by Anderson and Van Wincoop (2004, JEL)). A second strategy would be to find a natural experiment in the fashion of Feyrer (2009) (or via alternatives such as wars, railroads, transportation related technological advances, trade agreements, and so on) where it can be plausibly argued that trade costs fell, even though trade costs may not actually be observed. A third approach would be to estimate trade costs using price dispersion and then use these measures in the above tests.

Depending on the method, the strategy would be to either regress X_{ij}/X_{jj} on τ_{ij} and τ_{ik} for all $k \neq j$ with country and time fixed effects or compare the average X_{ij}/X_{jj} before and after a shock that affects the trade cost between i and k, $\forall\ k \neq j$ as well as the trade cost i and j.

(c) Now consider any arbitrarily large change in trade costs around the world (except that domestic trade costs, τ_{ii}, do not change). Show that the proportional

change in welfare of consumers in country j can be written as $\hat{W}_j = \hat{\lambda}^{1/(1-\sigma)}$, with $\lambda_{ij} = X_{ij}/X_j$, and where we use the notation that for any variable $\hat{\nu}$, $\hat{\nu} = \nu'/\nu$, where ν is the starting value and ν' is the end value. Explain the intuition for this result as well as the intuition behind any intermediate steps you use in its derivation.

Solution: First, let the domestic wage be the numeraire, w_j be the numeraire. Notice that the welfare (indirect utility) is simply the real wage, $W_j = \frac{w_j L_j}{P_j}$. Hence, given w_j is the numeraire and L_j is constant, we have:

$$dlnW_j = -dln(P_j)$$

Remember that $P_j = \left(\sum_i (\frac{\tau_{ij} w_i}{T_i})^{1-\sigma}\right)^{1/(1-\sigma)}$, i.e. the price index. Hence,

$$dln(P_j) = (1/(1-\sigma))((1-\sigma)\frac{\sum_i (\frac{\tau_{ij} w_i}{T_i})^{-\sigma}(\tau_{ij} dw_i + d\tau_{ij} w_i)/T_i}{P_j^{1-\sigma}}) = \tag{1}$$

$$\sum_i (\frac{\tau_{ij} w_i}{T_i P_j})^{1-\sigma}(\frac{dw_i}{w_i} + \frac{d\tau_{ij}}{\tau_{ij}}) = \sum_i (\frac{\tau_{ij} w_i}{T_i P_j})^{1-\sigma}(dln(w_i) + dln(\tau_{ij})) = \tag{2}$$

$$\sum_i \lambda_{ij}(dln(w_i) + dln(\tau_{ij})) = \sum_i \lambda_{ij}(dln(\lambda_{ij}) - dln(\lambda_{jj})/(1-\sigma)) \tag{3}$$

$$\sum_i \lambda_{ij}(dln(\lambda_{ij})/(1-\sigma)) - dln(\lambda_{jj})/(1-\sigma) = -dln(\lambda_{jj})/(1-\sigma) \tag{4}$$

where we use $dT_i = 0$, $\frac{X_{ij}}{w_j L_j} = \frac{X_{ij}}{X_j} = (\frac{\tau_{ij} w_i}{T_i P_j})^{1-\sigma}$, $dln(\lambda_{ij}) = (1-\sigma)(dln\tau_{ij} + dlnw_i - dlnP_j)$, $dln(\lambda_{jj}) = (1-\sigma)(dlnw_j - dlnP_j)$)(where $dw_j = 0$ because it is the numeraire), $\sum_i \lambda_{ij} = 1$ and $\sum_i \lambda_{ij} dln(\lambda_{ij}) = 0$. Hence, $\sum_i \lambda_{ij}(dln(\lambda_{ij})/(1-\sigma)) = 0$

Hence, we have:

$$dlnW_j = dln(\lambda_{jj})/(1-\sigma)$$

Given for any variable $\hat{\nu}$, $\hat{\nu} = \nu'/\nu$, where ν is the starting value and ν' is the end value:

$$\hat{W}_j = \hat{\lambda}^{1/(1-\sigma)}$$

2. (25 marks) Now consider a more general Ricardian model than the particularly stark Ricardian model (the Armington model) assumed in Question 1 above. There is still one factor, labor. Now there is a fixed set of goods indexed by ω, of measure N. All consumers have CES preferences (with elasticity of substitution σ) over these goods. Country i requires $\alpha_i(\omega)$ units of labor to produce one unit of good ω. Assume there are many potential producers of each good ω in each country i; hence there is perfect competition. Let $G(\alpha_1, ..., \alpha_n)$ denote the share of goods ω such that $\alpha_i(\omega) \leq \alpha_i$ for all i, and let $g(\alpha_1, ..., \alpha_n)$ denote its associated density function.

(a) Derive an expression for aggregate exports from country i to country j (denoted X_{ij}) as a function of $c_{ij} \equiv w_i \tau_{ij}$ and the function $g_i(\alpha_i, c_{1j}, ..., c_{nj})$ which is the density of goods with unit labor requirements α_i in country i such that country i is the lowest cost supplier of these goods to country j.

Solution: From the CES utility function, the expenditure on good ω in country j is defined as $X_j(\omega) = \left(\frac{p_j(\omega)}{P_j}\right)^{1-\sigma} w_j L_j$. Remember that due to perfect competition, the consumers will buy the good from the cheapest source: $p_j(\omega) = \min_i(\alpha_i c_{ij})$. Let $g_i(\alpha_i, c_{1j}, ...c_{nj})$ be the cdf of the goods in i such that i is the lowest cost supplier and remember that country i will provide the good if it is the least cost supplier, the expenditure in country j from country i is given by:

$$X_{ij} = (P_j)^{\sigma-1} w_j L_j \int_0^{\infty} (\alpha_i c_{ij})^{1-\sigma} g_i(\alpha_i, c_{1j}, ...c_{nj}) d\alpha_i$$

(b) Hence derive an expression for the import demand system in this model (i.e. $\frac{\partial ln(X_{ij}/X_{jj})}{\partial ln \tau_{ik}}$). Feel free to use the notation

$$\gamma_{ij}^{i'} \equiv \frac{\partial \ln[\int_0^{\infty} \alpha_i^{1-\sigma} g_i(\alpha_i, c_{1j}, ...c_{nj}) d\alpha_i]}{\partial \ln c_{i'j}}, \tag{5}$$

but be sure to explain and interpret this term if you do so.

Solution: First from the previous result and the fact that w_{jj} is the numeraire and $\tau_{jj} = 1$, we know that

$$X_{ij}/X_{jj} = \frac{\int_0^{\infty} (\alpha_i c_{ij})^{1-\sigma} g_i(\alpha_i, c_{1j}, ...c_{nj}) d\alpha_i}{\int_0^{\infty} (\alpha_j g_i(\alpha_j, c_{1j}, ...c_{nj}) d\alpha_j}$$

Hence,

$$\ln(X_{ij}/X_{jj}) = (1-\sigma)\ln(c_{ij}) + \ln(\int_0^{\infty} (\alpha_i c_{ij})^{1-\sigma} g_i(\alpha_i, c_{1j}, ...c_{nj}) d\alpha_i) - \ln(\int_0^{\infty} \alpha_j^{1-\sigma} g_i(\alpha_j, c_{1j}, ...c_{nj}) d\alpha_j)$$

Differentiating the expression with respect to $\ln(\tau_{ik})$:

$$\frac{\partial \ln(X_{ij}/X_{jj})}{\partial ln \tau_{ik}} = \begin{cases} 1 - \sigma + \gamma_{ij}^k - \gamma_{jj}^k & \text{if } k = j \\ \gamma_{ij}^k - \gamma_{jj}^k & \text{otherwise} \end{cases}$$

Here, γ_{ij}^k represents the elasticity of the extensive margin, i.e. the percentage change in the set of products exported from country j to country j as a response to a percentage change in the production costs of exporters k. A change in trade costs affect the markets a country is exporting a good to, which influences the average productivity as well as the average costs.

(c) Does the import demand system in this model necessarily satisfy R3 in ACRC? Does R3 imply perfect specialization? Does this model necessarily imply perfect specialization? Explain what would have to be true in this model if it were to satisfy R3 (be sure to explain both the math and the economics).

Solution: One can easily see that the import demand system does not necessarily satisfy R3. To satisfy R3, one would need: (i) $\gamma_{ij}^k - \gamma_{jj}^k = c$ for $k = i$ and some constant c, and $\gamma_{ij}^k = \gamma_{jj}^k$ for $k \neq i$. R3 implies perfect specialization under perfect competition. If perfect specialization were not true, then a least one ϵ_j^{ik} would be equal to infinity.

(d) Consider an arbitrarily large change in trade costs around the world (except that domestic trade costs, τ_{ii}, do not change). Show that R3 in ACRC implies that $\hat{W}_j = \hat{\lambda_{jj}}^{1/\epsilon}$ for some constant, ϵ.

Solution: As in question 1, we have
$ln(W_j) = -\ln(P_j)$, where $\ln(P_j) = (1/(1-\sigma))\ln(\sum_{i=1}^N \int_0^\infty (c_{ij}\alpha_i(\omega))^{1-\sigma} g_i(\alpha_i, c_{1j}, ..., c_{nj})d\alpha_i)$.
Hence, differentiating we have:

$$d\ln P_j = \frac{\frac{1}{1-\sigma}(\sum_{i=1}^N \int_0^\infty (1-\sigma)(c_{ij}\alpha_i(\omega))^{-\sigma}\alpha_i(\omega))dc_{ij} g_i(\alpha_i, c_{1j}, ..., c_{nj})d\alpha_i)}{P_j^{1-\sigma}} \quad (6)$$

$$= \sum_{i=1}^N \frac{dc_{ij}}{c_{ij}} \int_0^\infty (\frac{c_{ij}\alpha_i(\omega)}{P_j})^{1-\sigma} g_i(\alpha_i, c_{1j}, ..., c_{nj})d\alpha_i \quad (7)$$

$$= \sum_{i=1}^N \frac{dc_{ij}}{c_{ij}} \frac{X_{ij}}{X_j} = \sum_{i=1}^N d\ln c_{ij}\lambda_{ij} \quad (8)$$

Where $\lambda_{ij} = \frac{X_{ij}}{X_j}$; Furthermore, notice that as earlier: $d\ln\lambda_{ij} - d\ln\lambda_{jj} = (1-\sigma+\gamma_{ij}^i - \gamma_{jj}^i)d\ln c_{ij} + \sum_{i'\neq i,j}(\gamma_{ij}^{i'} - \gamma_{jj}^{i'})d\ln c_{i'j}$. Hence solving for $d\ln c_{ij}$,

$$d\ln P_j = \sum_{i=1}^N \lambda_{ij} \frac{d\ln\lambda_{ij} - d\ln\lambda_{jj} - \sum_{i'\neq i,j}(\gamma_{ij}^{i'} - \gamma_{jj}^{i'})d\ln c_{i'j}}{(1-\sigma+\gamma_{ij}^i - \gamma_{jj}^i)} \quad (9)$$

$$= \sum_{i=1}^N \lambda_{ij} \frac{d\ln\lambda_{ij} - d\ln\lambda_{jj} - \sum_{i'\neq i,j}(\gamma_{ij}^{i'} - \gamma_{jj}^{i'})d\ln c_{i'j}}{\epsilon} \quad (10)$$

$$= \sum_{i=1}^N \lambda_{ij} \frac{d\ln\lambda_{ij} - d\ln\lambda_{jj}}{\epsilon} = -\frac{d\ln\lambda_{jj}}{\epsilon}, \quad (11)$$

where from R3, we know that $(1-\sigma+\gamma_{ij}^i - \gamma_{jj}^i) = \epsilon$ for $i \neq j$ and $\gamma_{ij}^{i'} - \gamma_{jj}^{i'} = 0$ for $i' \neq j, i$. The remaining is identical to question (1.c) and hence:

$$\hat{W}_j = \hat{\lambda_{jj}}^{1/\epsilon} \text{for some constant, } \epsilon$$

(e) Now suppose that the particular Ricardian model we are working with is that in Eaton and Kortum (2002). Derive the density $g(\alpha_1, ..., \alpha_n)$ in this model. Hence show that this model satisfies R3. Explain why R3 is satisfied in both the Armington and Eaton and Kortum (2002) models. Can you explain what feature of the Frechet distribution in Eaton and Kortum (2002) allows the model to satisfy R3? Can you think of another distribution that would allow the model to satisfy R3 in general?

Solution: First let's derive the distribution $g(\alpha_1, ..., \alpha_n)$. Following EK (2002), we know that $1/\alpha_i(\omega)$ are drawn from i.i.d Frechet distribution. Hence,

$$F(\alpha_i) = P(x \leq \alpha_i) = P(1/\alpha_i \leq 1/x) = 1 - P(1/\alpha_i \geq 1/x) \tag{12}$$

$$= 1 - \exp^{-T_i(\alpha_i)^\theta} \tag{13}$$

$$\tag{14}$$

Hence, the pdf is defined as $f(\alpha_i) = \theta\alpha_i^{\theta-1}T_i \exp^{-T_i\alpha_i^\theta}$. Given the independence assumption, we know that:

$$g(\alpha_1, ..., \alpha_n) = \prod_{i=1}^{N} \theta\alpha_i^{\theta-1}T_i \exp^{-T_i\alpha_i^\theta}$$

. One can easily check that this satisfies the restriction we derived above on $\gamma_{ij}^{i'}$ required for a perfect competition Ricardian model with CES preferences to satisfy R3: $\gamma_{ij}^k - \gamma_{jj}^k = c$ for $k = i$ and some constant c, and $\gamma_{ij}^k = \gamma_{jj}^k$ for $k \neq i$. Note that this requires $g()$ to take a power law form, which is provided in the Frechet model and also in the Armington model (which can be thought of as the case of a degenerate productivity distribution); no other general distribution could work with CES preferences to satisfy R3.

(f) Discuss the extent to which the Armington model and the Eaton and Kortum (2002) model are 'isomorphic' with respect to one another.

Solution: From the perspective of gains from trade, conditional on data on λ_{jj} and a value of the trade elasticity ϵ, the two models are isomorphic (ie observationally equivalent). This is the point of ACRC (2010) of course. However, if one had outside data on σ or θ one could (in principle) test between these models by checking whether the observed trade elasticity is $-\theta$ or $(1-\sigma)$. Likewise, one could see how much some observed measure of welfare, like real income, rose as λ_{jj} changed and estimate whether the proportional change is closer to $\frac{-1}{\theta}$ or $\frac{1}{1-\sigma}$. Another distinction between the models is that the EK2002 model has micro-heterogeneity and makes predictions about the exit of certain technologies (ie specialization) as trade costs fall.

3. (20 marks) Finally, consider a similar model to that in Question 2 but where we now assume monopolistic competition. Again, there is one factor, *labor. There is an infinitely large number of goods ω that could potentially be produced. All consumers have CES preferences (with elasticity of substitution σ) over these goods. The cost of a firm in country i producing $q_{ij}(\omega)$ units of good ω and selling them in country j is given by: $\alpha_i(\omega)q_{ij}(\omega)\tau_{ij}w_i + f_{ij}w_i^{\mu}w_j^{1-\mu}$, where w_k is the wage in any country k. Once a firm starts producing a good ω it obtains monopoly rights over that good, but otherwise there are no barriers to entry. Let $G(\alpha_1, ..., \alpha_n)$ again denote the share of goods ω such that $\alpha_i(\omega) \leq \alpha_i$ for all i, and let $g(\alpha_1, ..., \alpha_n)$ denote its associated density function.*

(a) Derive an expression for X_{ij} as a function of c_{ij}, the total number of varieties made by country i (denoted N_i), and $g_i(\alpha_i)$ which is the marginal distribution of α_i.

Solution: First, under monopolistic competition with Dixit-Stiglitz preferences, firms charge a constant markup, $\sigma/(\sigma-1)$, over marginal costs. The profits of a producer of good ω in country i selling in country j are thus given by

$$\pi_{ij}(\omega) = (\sigma c_{ij}\alpha_{ij}(\omega)/(\sigma-1)P_j)^{1-\sigma}(X_j/\sigma) - f_{ij}w_i^{\mu}w_j^{1-\mu}.$$

Denoting by α_{ij}^* the cutoff determining the entry of firms from country i in country j, $\alpha_i(\omega) < \alpha_{ij}^*$, ie such that $\pi_{ij}(\alpha_{ij}^*)$=0, we have:

$$\alpha_{ij}^* = \frac{\sigma-1}{\sigma}\frac{P_j}{c_{ij}}(\frac{f_{ij}w_i^{\mu}w_j^{1-\mu}\sigma}{X_j})^{1/(1-\sigma)}$$

Now, using the CES demand formula,

$$X_{ij} = \frac{X_j}{P_j^{1-\sigma}}\int_0^{\alpha_{ij}^*}(\frac{\sigma}{\sigma-1}c_{ij}\alpha_i)^{1-\sigma}N_i g_i(\alpha_i)d\alpha_i,$$

where P_j is the price index given by $P_j = (\sum_{i'=1}^{n} N_i^{'} \int_0^{\alpha_{ij}^*}(\frac{\sigma}{\sigma-1}c_{i'j}\alpha_{i'})^{1-\sigma}N_{i'}g_{i'}(\alpha_{i'})d\alpha_{i'})^{1-\sigma}$.

(b) Hence derive an expression for the import demand system in this model. Feel free to use the notation $\gamma_{ij} \equiv d\ln\int_0^{\alpha_{ij}^*}\alpha^{1-\sigma}g_i(\alpha)d\alpha/d\ln\alpha_{ij}^*$ but again explain and interpret this term if you do so.

Solution: As before, we have $\ln(X_{ij}/X_{jj}) = (1-\sigma)\ln(c_{ij}) + \ln(\int_0^{\alpha_{ij}^*}(\alpha_i)^{1-\sigma}g_i(\alpha_i)d\alpha_i) - \ln(\int_0^{\alpha_{jj}^*}(\alpha_j)^{1-\sigma}g_j(\alpha_j)d\alpha_j) + \ln N_i - \ln N_j$. This is the same expression as before but note now that we have to account for the endogenous varieties, N.

Hence,

$$\frac{\partial\ln(X_{ij}/X_{jj})}{\partial\ln\tau_{ik}} = \begin{cases} 1-\sigma-\gamma_{ij}+(\gamma_{ij}-\gamma_{jj})\frac{\partial\ln\alpha_{ij}^*}{\partial\ln\tau_{kj}} & \text{if } k=j \\ (\gamma_{ij}-\gamma_{jj})\frac{\partial\ln\alpha_{ij}^*}{\partial\ln\tau_{kj}} & \text{otherwise} \end{cases}$$

where γ_{ij} is the counterpart of the extensive margin derived above, i.e. change in the average productivity as a result of the cutoff good changing.

(c) Does the IDS necessarily satisfy R3 in ACRC? Explain what would have to be true in this model if it were to satisfy R3 (be sure to explain both the math and the economics).

Solution: In general, it does not hold. For R3 to hold, we need $\gamma_{ij} = \gamma_{jj} = \gamma$, for all i, j. In words, the first equality implies that any trade cost changes imply a symmetric effect on the extensive margin for both importers and exporters. The second equality implies the elasticity is the same for all α's.

(d) Consider a small change in trade costs around the world (except that domestic trade costs, τ_{ii}, do not change). Derive an expression for the resulting change in welfare in country j, W_j. Explain in what respects this expression is similar to, and different from, that in the above perfectly competitive case in Questions 1 and 2 above.

Solution: Again, as earlier we know:

$$d \ln W_j = -d \ln P_j$$

Where $d \ln P_j = \sum_{i=1}^{n} \lambda_{ij}(d \ln c_{ij} + \frac{d \ln N_i + \gamma_{ij} d \ln \alpha^*_{ij}}{1-\sigma})$

Substituting the definition for α^*_{ij} from the zero profit condition and $d \ln \alpha^*_{ij} = d \ln P_j - d \ln(c_{ij}) + (1/(1-\sigma))(d \ln f_{ij} + \mu d \ln w_i)$:

$$d \ln P_j = \sum_{i=1}^{n} \frac{\lambda_{ij}}{1-\sigma-\gamma_j}(d \ln c_{ij}(1-\sigma-\gamma_{ij}) + \frac{\gamma_{ij}}{1-\sigma}(d \ln f_{ij} + \mu d \ln w_i) + d \ln N_i)$$

Using $d \ln \alpha^*_{jj} = d \ln P_j$ and $d \ln \alpha^*_{ij} = d \ln \alpha^*_{jj} - d \ln(c_{ij})(+(1/(1-\sigma))(d \ln f_{ij} + \mu d \ln w_i)$, in the relative trade flows, i.e. $d \ln \lambda_{ij} - d \ln \lambda_{jj}$:

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = (d \ln c_{ij}(1-\sigma-\gamma_{ij}) + \frac{\gamma_{ij}}{1-\sigma}(d \ln f_{ij} + \mu d \ln w_i) + d \ln N_i) - d \ln N_j + (\gamma_{ij} - \gamma_{jj}) d \ln \alpha^*_{jj}$$

Hence,

$$dnP_j = \sum_{i=1}^{n} \frac{\lambda_{ij}}{1-\sigma-\gamma_j}(d \ln \lambda_{ij} - d \ln \lambda_{jj} - (\gamma_{ij} - \gamma_{jj}) d \ln \alpha^*_{jj} + d \ln N_j).$$

This is similar to the expression derived above (in Question2) apart from the addition of the last two terms. The first term involves $d \ln \alpha^*_{jj}$, namely that a change in trade costs in the model could introduce selection (ie a change in the exit productivity cutoff); this term is unique to models of heterogeneous firms with fixed exporting and production costs. The second term, $d \ln N_j$, accounts for a change in the number of varieties available to consumers in country j; this term is common to all models with an endogenous number of varieties (eg Krugman (1979)).

(e) Is R3 sufficient to guarantee that, for an arbitrarily large change in trade costs around the world, $\hat{W}_j = \hat{\lambda}_{jj}^{1/\epsilon}$ *for some constant,* ϵ*? If not, what other restrictions would guarantee this result?*

Solution: R3 is not sufficient to generate this result; one would need additional restrictions, perhaps such as R1 and R2 in ACRC. R1 and R2 imply that aggregate profits are a constant share of total expenditure, which guarantees that the measure N_j of goods that can be produced in country j is not affected by foreign shocks.

R3 alone allows the following: let $\epsilon = 1 - \sigma - \gamma_j$, so the above expression simplifies to

$$dnP_j = (-d\ln\lambda_{jj} + d\ln N_j)/\epsilon$$

Finally, note that R1 and R2 together imply together that profits cannot change as a result of the trade costs change. This then implies that $\Pi_j = N_j F_j$ is proportional to X_j and $d\ln X_j = 0$. Hence, due to free entry, $d\ln N_j = 0$. So we have the same result as before:

$$\hat{W}_j = \hat{\lambda_{jj}}^{1/\epsilon}$$

(f) Suggest two prominent restrictions on $g_i(\alpha_i)$ *that would ensure that R3 is satisfied.*

Solution: R3 will be satisfied under the restriction that firm-productivity levels are randomly drawn from a Pareto distribution.

$$g_i(\alpha_i) = \frac{\theta\alpha_i^{\theta-1}}{\bar{\alpha_i}^{\theta}}, \text{for all } 0 \leq \alpha_i \leq \bar{\alpha_i}$$

One can easily show that under the Pareto distribution, $\int_0^{\alpha_{ij}^*} \alpha^{1-\sigma} g_i(\alpha)d\alpha = \frac{\theta}{\bar{\alpha}^{\theta}} \int_0^{\alpha_{ij}^*} \alpha^{\theta-\sigma} d\alpha = \frac{\theta}{\bar{\alpha}^{\theta}(\theta-\sigma+1)} \alpha_{ij}^{*(\theta-\sigma+1)}$ and

$$\gamma_{ij} = d\ln \int_0^{\alpha_{ij}^*} \alpha^{1-\sigma} g_i(\alpha)d\alpha / d\ln\alpha_{ij}^* = \theta - \sigma + 1$$

which satisfies R3. Once again, a power law form for $g_i(\alpha_i)$, coupled with CES preferences, seems essential for generating R3.

A second prominent restriction on $g_i(\alpha_i)$ is that it is degenerate, as in Krugman (1979).

(g) Can you explain why the Frechet-distributed productivities ensures R3 under perfect competition but not under monopolistic competition?

Solution: R3 requires strong functional form in order for the intensive and extensive margin elasticities to behave in the same way. As we have seen, this requirement (when coupled with CES preferences) is that the productivity distribution of draws actually used, $g_i(\alpha_i)$, is of the power law form. With monopolistic competition, consumers have a love of variety and will consume all varieties that can be (weakly) profitably sold. So the only generic distribution $g_i(\alpha_i)$ of draws that get produced that is power law is created by a MC model with power law-distributed (ie Pareto distributed) exogenous productivities. Under perfect competition consumers will only buy from the lowest cost supplier. Extreme value distributions (of which the Frechet is a leading example) have the special characteristic that extreme values (such as the minimum) drawn from them will be power law-distributed.

4. (20 marks) A large literature, surveyed in Anderson and van Wincoop (JEL 2004), uses estimates from the gravity model of trade to shed light on the nature of trade costs.

(a) Explain this methodology precisely along with the assumptions that authors make when using it.

Solution: This method measures the full extent of (variable) trade costs by inferring trade costs from trade flows. Implicitly, they are comparing the amount of trade we see in the real world to the amount we'd expect to see in a frictionless world where the 'difference' in trade between these two worlds is assigned to trade costs. One specification of the gravity model predicts a linear equation of the amount of bilateral exports (X_{ij}^k) on exporter-times-commodity and importer-times-commodity fixed effects as well as trade costs proxied for by the distance between the 2 countries (among other variables). There are mainly three assumptions made:

1. The model must be 'trade separable'.

- Any demand side assumptions you want can determine E_j^k (Expenditure of good k in country j). But this is a separable decision from the decision about where to buy your total E_j^k. (Two-stage budgeting.)

- Any supply side assumptions you want can determine Y_i^k. But there is an analogous supply-side restriction about cross-country separability.

- Conditional on E_j^k and Y_i^k, we can derive trade flows as a bilateral (ie ij) function of trade costs. (Supply must equal demand).

2. Within good class k, demand is CES (with parameter ϵ^k) across varieties.

3. Trade costs τ_{ij}^k are ad valorem and don't depend on X_{ij}^k.

(b) Under these maintained assumptions from part (a) above: Are trade costs identified by this methodology? Are the effects of observable determinants of trade costs identified? Are the relative effects of observable determinants of trade costs (i.e. which determinants impede trade relatively more) identified?

Solution: There are different approaches: The first approach is the residual one. As long as one can measure internal trade and the demand elasticity parameter ϵ^k, it is possible to identify the trade cost by this methodology. However, this method often sets $\tau_{ii}^k = 1$, so it identifies international trade costs only up to the normalization that intra-national trade costs are zero. Another approach to measuring trade costs consists of parameterizing the measure by observables. In this method, there is no attempt to identify the full extent of trade costs, but instead to look at the observable dimensions of trade costs, e.g. assuming that trade cost is a given parametrized function of distance. In both cases the identification of the trade costs (or parameters mapping observables to trade costs) is achieved only if the elasticity of substitution σ^k is identified from some other study.

(c) Write down a form of taste differences across countries that would not be separately identified from trade costs in the model developed by these authors.

Solution: A reasonable change in the preference consists in introducing home bias:

$$U_j = (\sum_i \omega_{ij} c_{ij}^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

Where $\omega_{ij} < \omega_{jj}$, for all $i \neq j$. In which case, one can not distinguish the weights ω from trade costs (ie this model reduces to the 'standard' model but with trade costs changed to $\tau_{ij}\omega_{ij}$).

(d) Do you believe the estimates that emerge from these studies? If not, explain which of the methodology's maintained assumptions you find most troublesome. Can you suggest an empirical test for your confounding story?

Solution: One of the troublesome assumptions made is trade separability – under this assumption there is no scope for production or consumption to colocate, perhaps to avoid/minimize trade costs (as would happen, for example, in Brainard's (1997) model of horizontal FDI). In principle, a model of horizontal FDI could be nested into the estimation of the gravity equation to 'test' this confounding story. The other problematic assumption is the constant elasticity of substitution within a sector/industry. In fact, one could think that it is also a function of the location the good has been produced, i.e. the elasticity of substitution between two American cars might be different from the elasticity of substitution between a French and an American car. One could literally test the second assumption by estimating the elasticity of substitution and the extent in which they differ by countries. It would be interesting to see to what extent one can relax these assumptions.

(e) Rossi-Hansberg (AER, 2004, "A Spatial Theory of Trade") outlines one explanation for a 'border effect' in the data when one does not exist in the trade costs function. Explain the argument here and the intuition behind it.

Solution: Note: This question said 'when one does not exist in the trade costs function', but that was wrong. In Rossi-Hansberg (2004) there is indeed a 'border-crossing cost', a tariff. The key insight in R-H (2004) with respect to this question is that the elasticity of trade flows with respect to this tariff will be large (i.e. the 'border effect' in the language of McCallum (1995) will be large even when the underlying border-crossing cost is small.) The explanation for the border effect outlined by Rossi-Hansberg (2004) suggests an explanation for the border effect which is related to the idea that integrated supply chains choose to locate near each other. His theory naturally delivers a high elasticity. The intuition behind the model is that there is a discontinuity in relative prices implied by tariffs at borders, affecting the specialization patterns. This effect is amplified in equilibrium by both agglomeration effects and transport costs as (i) since bigger clusters of firms affect the productivity of nearby firms, and (ii) firm's clusters supply goods mainly for the domestic market. As a consequence, international trade has been dampened (and a high elasticity obtains).

(f) Discuss the implications of the findings in Bronnenberg, Dhar and Dube (JPE, 2009) for the existence, nature, and estimation of trade costs.

Solution: BDD (2009) show support in favor of persistence in early entry advantage looking at brands in 34 consumer packaged goods industries across the 50 largest U.S. cities. They document that in the US, the share of the demand is higher in cities where a brand entered first. The key finding is that current market shares are higher in markets closest to a brand's historic city of origin than in those farthest. An important first question is what this finding implies: does it imply the existence of high fixed entry Costs (that have prevented even prosperous US brand names from spreading throughout the US), location based on tastes (but the result holds even within narrowly defined commodity groups), or the long-lived effects of marketing (which act to alter tastes in the areas where marketing was applied). However, the separation of these points isn't that clear-cut because one can think of marketing as a fixed entry costs. In this case, the fixed entry costs are quite nuanced and interesting. For example, it could be the case that the first entrant has a cheaper ability to 'alter tastes' than a following entrant – in which case the fixed entry costs varies across firms in an endogenous manner. To take another example, the fact that even leading brands continue to spend money on advertising suggests that the marketing fixed entry costs is not truly fixed and paid once and for all, but is something that needs to be invested in (a sort of 'brand capital' in each market). Fundamentally, this paper (written by economists in the marketing department of a business school) shows us that geographic elements of marketing are interesting and powerful even within the US – so this is probably even more true internationally.

(g) How is it that empirical researchers employing this methodology are able to avoid bias due to general equilibrium 'spillovers' across their units of observation (i.e. the fact that export flow X_{ij} from country i to country j is likely to depend on both trade costs within this diad, τ_{ij}, and on trade costs within any other diad, τ_{lm})? Which assumptions enable this?

Solution: The main reason is that the general equilibrium spillovers, due to the assumption of the model stated above (separability), are accounted for by the fixed effects. This arises in gravity models due to the three assumptions made above. First, trade separability puts endogenous objects (E_j^k and Y_i^k) that will respond, in general equilibrium, to any change in any country's economy, into the fixed effect terms. Second, the assumption of CES preferences means that all 'competitive' effects across countries (i.e. that if the price of my neighbor's good falls, then a third country will buy less from me) are subsumed into the CES price index. This is true of a wider class of preferences than just CES preferences (a class that Diewert refers to as 'generalized mean preferences'). And finally, the assumption that trade costs don't depend on trade flows obviously shuts down any general equilibrium 'feedback' in the regression.

(h) Is there any evidence from Anderson and van Wincoop (AER 2003) that these general equilibrium spillovers actually matter (i.e. that failing to control for them introduces significant econometric bias)?

Solution: McCallum (1995) exhibits much higher estimates of the distance and border effects than Anderson and van Wincoop (2003), yet the latter simply re-runs McCallum's regression but with appropriate fixed effects added (or with a NLS routine that solves for the non-linear price index terms). This is evidence for GE effects mattering, but it can hardly be considered strong evidence, since there are other possible causes of the change in coefficients when one goes from OLS to fixed effects. None of the model's predicted GE effects is directly tested.

5. (20 marks) Much of the attention to the estimation of trade costs (e.g. the entire content of Anderson and van Wincoop's 2004 survey of 'Trade Costs') has been concerned with estimating variable trade costs. This question asks you to discuss approaches to estimating fixed trade (exporting) costs

(a) Explain what is meant by a fixed exporting cost (FEC). What is an example of such a cost?

Solution: A fixed exporting cost is defined as a cost that an exporting firm needs to incur only once to access the other market. It does not depend on the amount exported (in contrast to a tariff). Exporting in another country might require the exporter to find customers, learn a new language, new manners, new market behavior and often administrative forms are required to access the other market.

(b) Why would the existence of FECs matter for trade theory and for policy?

Solution: The fixed cost is a barrier on the decision of a firm to export within the industry which is more dependent on the size of the firm and the amount of export once they will be exporting contrary to the variable of trade such as tariff. For instance, a very productive but small firm might be deterred from starting to export because of the existence of such fixed costs. Further, the dynamic responses of trade flows etc will differ in the presence of FECs – once these costs are sunk they don't matter, but ex ante they do matter. Baldwin and Krugman (1989) coin these 'beachhead effects'. Finally, one might imagine that some FECs like 'finding customers' have an element of non-rivalry within an industry, implying a potential role for government intervention.

(c) Discuss the implications of Chaney's (AER 2008) theoretical work (on gravity models with FECs) for the method of estimating variable trade costs that Anderson and van Wincoop survey.

Solution: Chaney (AER, 2008) studies the implication of selection of heterogeneous firms into export markets for trade volumes. His Proposition 1 states that the gravity equation takes the form:

$$X_{ij}^k = \mu^k \frac{Y_i Y_j}{Y} (\frac{w_i \tau_{ij}^k}{P_j^k})(f_{ij}^k)^{-\gamma^k/(\sigma^k-1)-1}$$

where the notation follows that in the lecture slides. Here, both variable trade costs (τ_{ij}^k) and FECs (f_{ij}^k) affect trade flows. There are two consequences of this for estimating the effect of variable trade costs on trade flows. First, if we had a good proxy for variable trade costs (eg tariffs) then the coefficient on that variable in the gravity equation (in logs) should be, in Chaney (2008), interpreted as $-\gamma^k$, rather than $(1 - \sigma^k)$ as Anderson and van Wincoop (2003 and 2004) do. Second, the use of distance as a typical proxy for variable trade costs is now called into question because distance might enter the FEC too. In this case, again, the interpretation of coefficients a la Anderson and van Wincoop (2004) would be incorrect.

(d) Roberts and Tybout (AER 1997), Das, Roberts and Tybout (Ecta 2008), and Eaton, Kortum and Kramarz (2011) all provide estimates of FECs. Pick one of these papers and describe: how the authors estimate FECs, what assumptions are made in order to identify the FECs, the estimate of FECs that the authors arrive at, and the extent to which you believe the answer.

Solution: Robert and Tybout (1997) try to test the sunk-cost explanation for hysteresis in trade flows. They develop a dynamic discrete- choice model of exporting behavior, which discriminate between the role played by profit heterogeneity and sunk entry costs in explaining plants' exporting status. They use this model to analyze plant-level exporting decisions for consistency with the theory. Using panel data on Colombian manufacturing plants in four exporting industries, they estimate the model to analyze plant-level exporting decisions. They reject the null hypothesis that entry costs are empirically irrelevant. Sunk costs are found to be significant. Furthermore, they show that having a prior export experience tend to increase the probability of exporting by as much as 60 percentage points, which is evidence for the existence of FECs.

(e) Ciliberto and Tamer (Ecta 2009) develop new tools for estimating 'entry games' – the interacting strategic decisions made by firms about whether to enter a market. Do these tools hold any promise for estimating FECs? What would be the attraction of applying these tools relative to the existing literature (eg the papers in part (d) above)?

Solution: Papers such as Roberts and Tybout (1997), and Das, Roberts and Tybout (2007) are superficially about the 'decision to export' but are really about a firm's decision to export a secondary market. Early rounds of the 'entry games' literature in IO (usually associated with Bresnahan and Reiss, 1990 and 1991) were typically concerned with whether an entrepreneur (e.g. a dentist) would enter a single market or not (not with the 'exporting' entry decision, of whether a firm with an established base in country A will decide to enter country B). But more recent papers in this literature (including Ciliberto and Tamer (2009) and Jia (2008) concern strategically interacting firms making decisions about which market(s) to enter.) One attraction of applying the tools in CT (2009) to the exporting decision is that strategic interaction among firms could actually be taken seriously (as opposed to being modeled through monopolistic competition). A second, key, attraction of CT (2009) is that only weak assumptions need to be made about the precise game that strategically interacting firms are playing with one another (and even which equilibrium gets played in the case of multiple equilibria). The set-up in CT (2009) is thus far more general that that in RT (1997) or DRT (2007). The 'cost' of making only weak assumptions in CT (2009), however, is that parameters can only be bounded ('set identified') instead of point identified as we are used to in econometrics. But in CT (2009)'s empirical application they find that the bounds are still useful – that is, reasonably tight. One application of CT (2009) to the exporting decision could focus on simply estimating (i.e. putting bounds on) the FEC. Some problems in doing this might be adding a dynamic element to CT (2009), so that DRT (2007) is truly nested inside CT (2009), and the fact that CT (2009) might (I'm not sure) become computationally challenging with many firms (their application is to the US airline industry, which has very few firms). A final challenge might be in finding data (though I suspect this might not be truly necessary) on firm-level behavior in both the exporting country (this is easy – that's what DRT (2007) do) and on domestic firms in the importing country. Perhaps Canadian, US and/or Mexican firms would work here in the context of NAFTA/CUSFTA, or perhaps intra-Mercosur countries (Brazil and Argentina?) would work.

Update: see the work of Morales, Sheu and Zahler (2011) for a paper along the lines of the above.