14.581 MIT PhD International Trade
—Lecture 15: Gravity Models (Theory)—

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Introduction to 'Gravity Models'

- Recall that in this course we have so far seen a wide range of trade models:
  - Neoclassical:
    - Ricardo; basic, DFS (1977), Eaton and Kortum (2002), and Costinot, Donaldson and Komunjer (2011).
    - Ricardo-Viner; we saw general version; but easy to imagine a 'gravity' version that would be CDK (2011) with > 1 factor of production and some factors immobile across sectors.
    - Heckscher-Ohlin; we saw general version; but again, easy to imagine 'gravity version' as CDK (2011) with > 1 factor of production and all factors mobile across sectors.
  - Monopolistic Competition:
    - Krugman (1979, 1980)
    - Melitz (2003)
    - Extensions of Melitz (2003) like Bernard, Redding and Schott (2007), Chaney (2008) or Arkolakis (2011)

Introduction to 'Gravity Models'

- A surprising number of these models generate effectively the same 'gravity equation' prediction for trade flows.
- In this lecture we will:
  -
  - Define the statement 'gravity equation'
  - Discuss which of the above models do and do not deliver 'gravity'; we'll call these 'gravity models'
  - Discuss other features that are common to these 'gravity models'.
- In the next lecture we will discuss empirical estimation of gravity equations (and in particular their use for inferring the magnitude of trade costs).

What Do We Mean by 'Gravity Equation'?

- Short answer: When predicted trade flows (expenditures) can be written in the following form:

$$\ln X_{ij}^{k}(\boldsymbol{\tau}, \mathbf{E}) = A_i^k(\boldsymbol{\tau}, \mathbf{E}) + B_j^k(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^k \ln \tau_{ij}^k \tag{1}$$

- Where:
  - i is the exporting country, j is the importing country, and k is the industry.
  - τ_{ij}^k is some measure of bilateral trade costs.
  - The terms $A_i^k(\boldsymbol{\tau}, \mathbf{E})$ and $B_j^k(\boldsymbol{\tau}, \mathbf{E})$ are terms that vary only at the ik and jk levels. That is, they are not bilateral. However, they may depend on the full set of bilateral objects (ie the full matrix of bilateral trade costs $\boldsymbol{\tau}$).
  - Note that the $A_i^k(\boldsymbol{\tau}, \mathbf{E})$ and $B_j^k(\boldsymbol{\tau}, \mathbf{E})$ terms are (at least potentially) endogenous (they depend on the vector of equilibrium total expenditures $\mathbf{E}$). So the above expression for trade flows is not closed-form.
  - Note, equivalently, that the parameter ε^k only captures the 'partial equilibrium' (ie holding $A_i^k(\boldsymbol{\tau}, \mathbf{E})$ and $B_j^k(\boldsymbol{\tau}, \mathbf{E})$ constant) effect of τ_{ij}^k on $\ln X_{ij}^k$.

What Do We Mean by ‘Gravity Equation’?

- Short answer: When predicted trade flows (expenditures) can be written in the following form:

$$\ln X_{ij}^k(\boldsymbol{\tau}, \mathbf{E}) = A_i^k(\boldsymbol{\tau}, \mathbf{E}) + B_j^k(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^k \ln \tau_{ij}^k \qquad (2)$$

- Clearly this definition incorporates the ‘simple (naive)’ gravity equation we have discussed in this course so far:

$$\ln X_{ij}^k = \alpha \ln Y_i^k + \beta \ln E_j^k + \varepsilon \ln \tau_{ij}^k$$

- Tinbergen (1962) is often credited as the first empirical exploration of an expression like this.

What Do We Mean by 'Gravity Equation'?

$\ln X_{ij}^k(\boldsymbol{\tau}, \mathbf{E}) \quad A_i^k(\boldsymbol{\tau}, \mathbf{E}) + B_j^k(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^k \ln \tau_{ij}^k$

- Anderson (1979), and Anderson and van Wincoop (AER, 2003) highlight how this 'simple' gravity equation lacks theoretical justification:

    1. It does not respect market clearing (that is, the output produced in i needs to equal the sum of purchases of these goods: $Y_i^k = \sum_j X_{ij}^k$).
    2. It does not incorporate fact that consumers may view goods as substitutes. In particular, if appealing to a CES preference system (which begins to nicely justify the constant coefficient ε^k in front of $\ln \tau_{ij}^k$) then one should also include a price index that involves the prices of <u>all</u> countries' goods (ie the substitues for country i's goods.)

What Do We Mean by ‘Gravity Equation’?

$\ln X_{ij}^k(\boldsymbol{\tau}, \mathbf{E}) \quad A_i^k(\boldsymbol{\tau}, \mathbf{E}) + B_j^k(\boldsymbol{\tau}, \mathbf{E}) + \varepsilon^k \ln \tau_{ij}^k$

- Anderson (1979), and Anderson and van Wincoop (AER, 2003) derive the following system of equations which incorporates the above two points:

$$
\begin{aligned}
X_{ij}^k &= \frac{E_j^k Y_i^k}{Y^k} \left(\frac{\tau_{ij}^k}{P_j^k \Pi_i^k} \right)^{1-\epsilon^k} \\
(\Pi_i^k)^{1-\varepsilon^k} &= \sum_j \left(\frac{\tau^k}{P_j^k} \right)^{1-\varepsilon^k} \frac{E_j^k}{Y^k} \\
(P_j^k)^{1-\varepsilon^k} &= \sum_i \left(\frac{\tau^k}{\Pi_i^k} \right)^{1-\varepsilon^k} \frac{Y_i^k}{Y^k}
\end{aligned}
$$

- Clearly this, too, fits into our general definition.

Which Models Generate a 'Gravity Equation'?

- Neoclassical:
  - Eaton and Kortum (2002) with one industry. Then gravity equation describes aggregate trade flows.
  - Models like Costinot, Donaldson and Komunjer (2011) which feature EK (2002) set-up within each of multiple industries. Then gravity equation relates to each industry one industry at a time.
  - Could also add multiple factors of production easily (and retain gravity) but the Frechet-distributed productivity shock (if EK or CDK) needs to be Hicks-neutral.
- Monopolistic Competition:
  - Krugman (1980)
  - Melitz (2003) with Pareto-distributed productivities (as in Chaney (2008)).

Why Do These (and Only These) Models Generate 'Gravity'?

- One answer due to Deardorff (1998):
  - Gravity will arise whenever you have complete specialization, homothetic CES preferences, and iceberg trade costs.
- Similar answer in Anderson and van Wincoop (2004):
  - Gravity will arise whenever you have:
    - CES preferences
    - Iceberg trade costs
    - And a 'trade separable' set-up: in which the decision of how much of a good category to consume is separable from the decision about where to buy it from (two-stage budgeting); and a similar condition holds on the supply side.

What Is to Like About Models Featuring the 'Gravity Equation'?

1. As we shall see in the next lecture, these models fit the data well.
    - Though exactly how well, and how many degrees of freedom are used up in that good fit, are typically not mentioned. (There are a lot of unspecified fixed effects in the above definition. And direct data on τ_{ij}^k is very hard to get.)

2. There is a very strong correspondence between the set of models that generate a gravity equation, and the set of models that are particularly tractable (when asked to include real-world features like multiple countries, multiple industries, and trade costs.)

- Note that every model we've seen in this course that can handle these features is a gravity model.

What Else is Implied by ‘Gravity Models’?

- Arkolakis, Costinot and Rodriguez-Clare (AER 2011) introduce the phrase ‘gravity models’ to refer to models that (in addition to a few other conditions that we will see shortly) generate a gravity equation.

What Else is Implied by 'Gravity Models'?

- ACRC (2011) then show that, for any model satisfying these conditions, a number of additional features are common to all of these models. Conditional on the trade flows we observe in the world today, and one observed parameter:
  - **Weak ex-ante equivalence:** The 'gains from trade' (GT) in the model (that is, the losses that would obtain if a country in the model went to autarky) are the same. (Title: "New Trade Models, Same Old Gains?")
  - **Strong ex-ante equivalence:** Under (somewhat) stronger conditions, the response of any endogenous variable to a change in any exogenous variable will be the same in all models.
  - **Weak ex-post equivalence:** If we see a country's trade flows change between two equilibria, we can back out the welfare change associated with this change, and it will be the same in all models.
- We now go through this in detail.

Start with a Simple Example

- Consider first a simple example: the Armington model (as formalized by Anderson (1979) and Anderson and van Wincoop (2003)):
  - Countries produce unique goods, by assumption. (The only country that can produce 'French goods' is France.) "Armington differentiation."
  - Consumers have CES preferences over all of these different country-specific goods.
- Notes:
  - Specialization in this model is completely by assumption (and is therefore very boring).
  - But this modeling trick is of great help, since now one only has to solve for where the goods will end up.
  - "Armington" is often thought of as something to do with preferences. But I find it more natural to think of "Armgington" as a supply-side restriction, where countries have extremely different sets of relative productivities across all goods in the world. In this sense, Armington is just an extreme Ricardian model.

The Armington Model: Equilibrium

- Labor endowments

$$L_i \text{ for } i = 1, ...n$$

- Dixit-Stiglitz preferences $\Rightarrow$ consumer price index

$$P_j^{1-\sigma} = \sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}$$

- Aggregate bilateral demand

$$X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j} \right)^{1-\sigma} Y_j$$

- Labor market equilibrium

$$\sum_i X_{ji} = w_j L_j$$

- Trade shares and real income

$$\begin{aligned} \lambda_{ij} &\equiv X_{ij}/Y_j \\ W_i &\equiv Y_i/P_i \end{aligned}$$

The Armington Model: Weak **Ex-Post** Welfare Result

Step 1: changes in real income depend on changes in ToT ($c_{ij} \equiv w_i \tau_{ij}$)

$$d \ln W_j = d \ln Y_j - d \ln P_j = - \sum_{i=1}^{n} \lambda_{ij} \left(d \ln c_{ij} - d \ln c_{jj} \right).$$

Step 2: changes in relative imports depend on changes in ToT

$$d \ln \lambda_{ij} - d \ln \lambda_{jj} = (1 - \sigma) \left(d \ln c_{ij} - d \ln c_{jj} \right).$$

Step 3: combining these two equations yields

$$d \ln W_j = - \frac{\sum_{i=1}^{n} \lambda_{ij} \left(d \ln \lambda_{ij} - d \ln \lambda_{jj} \right)}{1 - \sigma}.$$

Step 4: noting that $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ then

$$d \ln W_j = \frac{d \ln \lambda_{jj}}{1 - \sigma}.$$

Step 5: integration yields ($\hat{x} = x'/x$)

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/(1-\sigma)}.$$

The Armington Model: Weak **Ex-Ante** Welfare Result

- We showed that, for any change in trade flows $(\widehat{\lambda}_{jj})$, the change in welfare in this model is: $\widehat{W_j} = \widehat{\lambda}_{jj}^{1/(1-\sigma)}$
- To show the 'weak ex-ante welfare result', just note that if we are interested in the Gains From Trade (ie losses of going to autarky) this can be computed by evaluating $\widehat{\lambda}_{jj} = \lambda_{jj} - 1$ since $\lambda_{jj} = 1$ under autarky.

General Results

- We now step way back and (following ACRC, 2011) consider a much more general model that will be sufficient to derive results, and is general enough to encompass many widely-used trade models.
- The approach in ACRC (2011) was to:
  - Consider a 'micro structure' that is extremely broad. The idea here is that the vast majority of microfoundations that (trade) economists use will fit in here.
  - And then introduce 3 'macro restrictions' that are sufficient to generate their results. Note, though, that not all of the above microfoundations will always satisfy these macro restrictions (ie the macro restrictions do restrict!)

Preferences and Endowments

- **Dixit-Stiglitz preferences**
  - Consumer price index,

$$P_i^{1-\sigma} = \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega,$$

- **One factor of production:** labor
  - $L_i \equiv$ labor endowment in country i
  - $w_i . \equiv$ wage in country i

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = \underbrace{q w_i \tau_{ij} \alpha_{ij}(\omega)\, t^{\frac{1}{1-\sigma}}}_{\text{variable cost}} + \underbrace{w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega)\, m_{ij}(t)}_{\text{fixed cost}},$$

q : quantity,
τ_{ij} : iceberg tansportation cost,
$\alpha_{ij}(\omega)$: good-specific heterogeneity in variable costs,
ξ_{ij} : fixed cost parameter,
$\phi_{ij}(\omega)$: good-specific heterogeneity in fixed costs.

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)$$

where $m_{ij}(t)$ is the cost for endogenous, destination specific technology choice, t,

$$t \in [\underline{t}, \overline{t}], \;\; m_{ij}' > 0, \;\; m_{ij}'' < 0$$

Technology

- **Linear cost function:**

$$C_{ij}(\omega, t, q) = q w_i \tau_{ij} \alpha_{ij}(\omega) t^{\frac{1}{1-\sigma}} + w_i^{1-\beta} w_j^{\beta} \xi_{ij} \phi_{ij}(\omega) m_{ij}(t)$$

- Heterogeneity across goods drawn from CDF:

$$G_j(\alpha_1, ..., \alpha_n, \phi_1, ..., \phi_n) \equiv \{\omega \in \Omega \mid \alpha_{ij}(\omega) \leq \alpha_i,\ \phi_{ij}(\omega) \leq \phi_i,\ \forall i\}$$

Market Structure

- **Perfect competition**
  - Firms can produce any good.
  - No fixed exporting costs.
- **Monopolistic competition**
  - Either free entry: firms in i can pay $w_i F_i$ for monopoly power over a random good.
  - Or fixed entry: exogenous measure of firms, $\overline{N}_i < \overline{N}$, receive monopoly power.
- Let N_i be the measure of goods that can be produced in i
  - Perfect competition: $N_i = \overline{N}$
  - Monopolistic competition: $N_i < \overline{N}$

Macro-Level Restriction (I): Trade is Balanced

Bilateral trade flows are, by definition:

$$X_{ij} = \int_{\omega \in \Omega_{ij} \subset \Omega} x_{ij}(\omega)\, d\omega$$

R1 *For any country j,*

$$\sum_{i \neq j} X_{ij} = \sum_{i \neq j} X_{ji}$$

Note that this is trivial if perfect competition or $\beta = 0$. But non-trivial if $\beta > 0$.

Macro-Level Restriction (II): Profit Share is Constant

- **R2** *For any country* j,

$$\Pi_j / \left(\sum_{i=1}^{n} X_{ji} \right) \textit{ is constant}$$

- Where Π_j : aggregate profits gross of entry costs, $w_j F_j$, (if any).
    - Trivial under perfect competition.
    - Direct from Dixit-Stiglitz preferences in Krugman (1980).
    - Non-trivial in more general environments.

Macro-Level Restriction (III): CES Import Demand System

- *Import demand system* defined as

$$(\mathbf{w}, \mathbf{N}, \tau) \rightarrow \mathbf{X}$$

- **R3**

$$\varepsilon_j^{ii'} \equiv \partial \ln \left(X_{ij} / X_{jj} \right) / \partial \ln \tau_{i'j} = \begin{cases} \varepsilon < 0 & i = i' \neq j \\ 0 & \textit{otherwise} \end{cases}$$

Note: symmetry and separability.

Macro-Level Restriction (III): CES Import Demand System

- Note that the *trade elasticity* ε is an *upper-level* elasticity: it combines
  - $x_{ij}(\omega)$ (*intensive margin*)
  - Ω_{ij} (*extensive margin*).

- Note that R3 $\implies$ complete specialization.

- Also note that R1-R3 are not necessarily independent
  - Eg, if $\beta = 0$ then R3 $\implies$ R2.

Macro-Level Restriction (III'): Strong CES

- **R3'** The IDS satisfies,

$$X_{ij} = \frac{\chi_{ij} M_i \left(w_i \tau_{ij}\right)^{\varepsilon} Y_j}{\sum_{i'=1}^{n} \chi_{i'j} M_{i'} \left(w_{i'} \tau_{i'j}\right)^{\varepsilon}}$$

  where χ_{ij} is independent of $(\mathbf{w}, \mathbf{M}, \tau)$.

- Same restriction on $\varepsilon_j^{ii'}$ as R3 but, but additional structural relationships.

Welfare results

State of the world economy:

$$\mathbf{Z} \equiv (\mathbf{L}, \tau, \xi)$$

Foreign shocks: a change from $\mathbf{Z}$ to $\mathbf{Z}'$ with no domestic change.

Welfare results

Proposition 1: *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}.$$

Implication: 2 sufficient statistics for welfare analysis $\widehat{\lambda}_{jj}$ and ε

New margins affect structural interpretation of ε

...and composition of gains from trade (GT)...

... but size of GT is the same.

Welfare results

- Proposition 1 is an *ex-post* result... a simple *ex-ante* result:
- **Corollary 1:** *Suppose that R1-R3 hold. Then*

$$\widehat{W}_j^A = \lambda_{jj}^{-1/\varepsilon}.$$

Welfare results

A stronger ex-ante result for **variable trade costs** under R1-**R3'**:

Proposition 2: *Suppose that R1-R3' hold. Then*

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

where

$$\widehat{\lambda}_{jj} = \left[\sum_{i=1}^{n} \lambda_{ij} \left(\hat{w}_i \hat{\tau}_{ij} \right)^{\varepsilon} \right]^{-1},$$

and

$$\widehat{w}_i = \sum_{j=1}^{n} \frac{\lambda_{ij} \hat{w}_j Y_j \left(\hat{w}_i \hat{\tau}_{ij} \right)^{\varepsilon}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j} \right)^{\varepsilon}}.$$

ε and $\{\lambda_{ij}\}$ are sufficient to predict $\widehat{W}_j$ (ex-ante) from $\hat{\tau}_{ij}$, $i \neq j$.

Taking Stock

- We have considered models featuring:
  - (*i*) Dixit-Stiglitz preferences;
  - (*ii*) one factor of production;
  - (*iii*) linear cost functions; and
  - (*iv*) perfect or monopolistic competition;
- with three macro-level restrictions:
  - (*i*) trade is balanced;
  - (*ii*) aggregate profits are a constant share of aggregate revenues; and
  - (*iii*) a CES import demand system.
- Equivalence for ex-post welfare changes, under R3' equivalence carries to ex-ante welfare changes

Taking Stock

- Examples that (one can show) fit into the above framework:
  - Armington model (Anderson, 1979)
  - Krugman (1980)
  - Eaton and Kortum (2002)
  - Anderson and Van Wincoop (2003)
  - Variations and extensions of Melitz (2003) including Chaney (2008), Arkolakis (2009), and Eaton, Kortum and Kramarz (2010).

An example: Melitz (2003)

- Now consider Melitz (2003) as a special case.
- We will see how the general Melitz (2003) model does not fit into the above framework, but how very the very commonly used Pareto parameterization of Melitz (2003) does.

Basics

- To simplify, here we assume $\underline{t} = \overline{t} = 1$ and $\phi = 1$ for all i, j, ω.

- Let $c_{ij} \equiv w_i \tau_{ij}$. Monopolistic competition implies

$$p_j(\omega) = \frac{\sigma}{\sigma - 1} c_{ij} \alpha_{ij}(\omega) \text{ for } \omega \in \Omega_{ij}$$

with

$$\Omega_{ij} = \left\{ \omega \in \Omega | \alpha_{ij} \left(\omega \right) \leq \alpha^*_{ij} \right\}$$

The import demand system

- Dixit-Stiglitz preferences imply:

$$X_{ij} = \frac{N_i \int_0^{\alpha_{ij}^*} \left[c_{ij}\alpha\right]^{1-\sigma} g_i(\alpha)\, d\alpha}{\sum_{i'=1}^{n} N_{i'} \int_0^{\alpha_{i'j}^*} \left[c_{i'j}\alpha\right]^{1-\sigma} g_{i'}(\alpha)\, d\alpha} Y_j.$$

The elasticity of the *extensive margin* is

$$\gamma_{ij} \equiv \frac{d \ln \left(\int_0^{\alpha_{ij}^*} \alpha^{1-\sigma} g_i(\alpha)\, d\alpha \right)}{d \ln \alpha_{ij}^*}$$

We now have

$$\frac{\partial \ln X_{ij}/X_{jj}}{\partial \ln \tau_{i'j}} = \varepsilon_j^{ii'} = \begin{cases} 1 - \sigma - \gamma_{ij} + (\gamma_{jj} - \gamma_{ij}) \left(\frac{\partial \ln \alpha_{jj}^*}{\partial \ln \tau_{ij}} \right) & \text{for} \quad i' = i \\ (\gamma_{jj} - \gamma_{ij}) \left(\frac{\partial \ln \alpha_{jj}^*}{\partial \ln \tau_{i'j}} \right) & \text{for} \quad i' \neq i \end{cases}$$

The logic behind Proposition 1

- Recall the result for Armington

$$d \ln W_j = d \ln Y_j - d \ln P_j = d \ln Y_j - \sum_{i=1}^{n} \lambda_{ij} d \ln c_{ij}$$

Now, in Melitz (2003), we have

$$\begin{aligned} d \ln W_j &= d \ln Y_j - d \ln P_j = d \ln Y_j - \sum_{i=1}^{n} \lambda_{ij} d \ln c_{ij} \\ &\quad + \sum_{i=1}^{n} \lambda_{ij} \left[\frac{d \ln N_i + \gamma_{ij} d \ln \alpha^*_{ij}}{1 - \sigma} \right]. \end{aligned}$$

But $d \ln N_i + \gamma_{ij} d \ln \alpha^*_{ij}$ related to $d \ln \lambda_{ij} - d \ln \lambda_{jj}$ ...

The logic behind Proposition 1

- Change in welfare

$$
\begin{aligned}
d \ln W_j \quad = \quad & d \ln Y_j \\
& - \sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} \right] \\
& - \sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[- \left(\gamma_{ij} - \gamma_{jj} \right) d \ln \alpha_{jj}^{*} + d \ln N_j \right].
\end{aligned}
$$

The logic behind Proposition 1

- R1 and R2 $\Longrightarrow d \ln Y_j = d \ln N_j = 0$ and hence

$$
\begin{aligned}
d \ln W_j &= 0 \\
&\quad - \sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} \right] \\
&\quad - \sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j} \right) \cdot \left[-\left(\gamma_{ij} - \gamma_{jj} \right) d \ln \alpha_{jj}^{*} + 0 \right].
\end{aligned}
$$

The logic behind Proposition 1

- R1 and R2 $\Longrightarrow d \ln Y_j = d \ln N_j = 0$ and hence

$$
\begin{aligned}
d \ln W_j &= -\sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} \right] \\
&\quad -\sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1-\sigma-\gamma_j} \right) \cdot \left[-\left(\gamma_{ij} - \gamma_{jj} \right) d \ln \alpha_{jj}^{*} \right].
\end{aligned}
$$

The logic behind Proposition 1

- R3 $\Longrightarrow \gamma_{ij} = \gamma_{jj}$ and $1 - \sigma - \gamma_j = \varepsilon$ and hence

$$
\begin{aligned}
d \ln W_j &= -\sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} \right] \\
&\quad - \sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{1 - \sigma - \gamma_j} \right) \cdot \left[-\left(\gamma_{ij} - \gamma_{jj} \right) d \ln \alpha_{jj}^{*} \right] \\
&= -\sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{\varepsilon} \right) \cdot \left[d \ln \lambda_{ij} - d \ln \lambda_{jj} \right]
\end{aligned}
$$

The logic behind Proposition 1

- $\sum_i \lambda_{ij} = 1 \Longrightarrow \sum_i \lambda_{ij} d \ln \lambda_{ij} = 0$ and hence

$$\begin{aligned} d \ln W_j &= -\sum_{i=1}^{n} \left(\frac{\lambda_{ij}}{\varepsilon} \right) \cdot [d \ln \lambda_{ij} - d \ln \lambda_{jj}] \\ &= \frac{d \ln \lambda_{jj}}{\varepsilon} . \end{aligned}$$

The logic behind Proposition 1

- We thus have the local result

$$d \ln W_j = \frac{d \ln \lambda_{jj}}{\varepsilon}$$

- R3 $\Longrightarrow$ ε constant across equilibria,

$$\widehat{W}_j = \widehat{\lambda}_{jj}^{1/\varepsilon}$$

The Pareto density implies R1-R3

- Productivity distributed Pareto,

$$g_i\left(\alpha_1, ..., \alpha_n\right) = \prod_{i'} \alpha_{i'}^{\theta}$$

- Pareto + Free Entry $\Longrightarrow$ R1 + R2
- Pareto $\Longrightarrow \gamma_{ij} = \gamma_{jj} = \theta - (\sigma - 1) \Longrightarrow$ R3,

$$\frac{\partial \ln X_{ij}/X_{jj}}{\partial \tau_{i'j}} = \begin{cases} 1 - \sigma - \gamma_{ij} + \left(\gamma_{jj} - \gamma_{ij}\right)\left(\frac{\partial \ln \alpha_{jj}^*}{\partial \ln \tau_{ij}}\right) = -\theta & \text{if} \quad i' = i \\ \left(\gamma_{jj} - \gamma_{ij}\right)\left(\frac{\partial \ln \alpha_{jj}^*}{\partial \ln \tau_{i'j}}\right) = 0 & \text{if} \quad i' \neq i \end{cases}$$

How about R3'?

The Pareto density also implies

$$X_{ij} = \frac{N_i w_i^{-\theta+(1-\beta)[1-\theta/(\sigma-1)]} \tau_{ij}^{-\theta}}{\sum_{i'} N_{i'} w_{i'}^{-\theta+(1-\beta)[1-\theta/(\sigma-1)]} \tau_{i'j}^{-\theta}} Y_j.$$

R3' is satisfied iff $\beta = 1$. Otherwise, need β and σ for counterfactuals.

Extensions, and Estimation

- ACRC (2011) then go on to discuss 2 extensions:
    1. Multiple sectors/industries.
    2. Tradable intermediate goods.

- They also discuss how different models, which will have different implications for exactly *what* the trade elasticity parameter ε is composed of, will nevertheless all have the feature that this parameter can be estimated in the same way.

Multiple Sectors

- **Multiple sectors:** Goods $\omega \in \Omega$ are separated into $s = 1, ..., S$ sectors
    - Country j spends a constant share η_j^s of their income on sector s
    - ε^s : trade elasticity of that sector

Multiple Sectors

- Under PC changes in real income are given by

$$\hat{W}_j =_{s=1}^{S} \left(\hat{\lambda}_{jj}^{s}\right)^{\eta_j^s / \varepsilon^s} - 1$$

- Under MC with free entry changes in real income are given by

$$\hat{W}_j =_{s=1}^{S} \left(\hat{\lambda}_{jj}^{s} / \hat{L}_s\right)^{\eta_j^s / \varepsilon^s} - 1$$

where L_s is total employment in sector s.

- Reallocations across sectors imply $\widehat{N}_j^s \neq 0$
  - Equivalence **between** PC and MC no longer holds.
  - This is due to a general result (in, eg, Dixit and Stiglitz, 1977) that the MC model with CES is allocatively efficient iff the economy sector faces inelastic factor supply.

Intermediate Inputs

- **Tradable intermediate goods:**
  - Variable production cost of good ω in country i is equal to

$$c_i(\omega) = \frac{w_i^{\beta} P_i^{1-\beta}}{z(\omega)}$$

  - Under MC, firms from country i must incur:
    (i) a fixed entry cost, $w_i F_i$ in order to produce in country i
    (ii) a fixed marketing cost , $w_i^{\beta} P_i^{1-\beta} \xi_{ij}$, in order to sell in country j

Intermediate Inputs

- Under PC, changes in real income are:

$$\hat{W}_j = \left(\hat{\lambda}_{jj}\right)^{1/(\beta\varepsilon)} - 1$$

- Under MC, changes in real income are

$$\hat{W}_j = \left(\hat{\lambda}_{jj}\right)^{1/\left[\beta\varepsilon + (1-\beta)\left(\frac{\varepsilon}{\sigma-1}+1\right)\right]} - 1$$

- Thus, sizes distribution of firms also matters, through $\varepsilon/\left(\sigma-1\right)$

Estimation of the trade elasticity

- If models satisfy

$$X_{ij} = \frac{\chi_{ij} \cdot N_i \cdot w_i^{\eta} \tau_{ij}^{\varepsilon} \cdot Y_j}{\sum_{i'=1}^{n} \chi_{i'j} \cdot N_{i'} \cdot w_{i'}^{\eta} \tau_{i'j}^{\varepsilon}},$$

  with χ_{ij} being orthogonal to $\tau_{i'j'}$ for any i, i', j, j' then ε can be estimated from a gravity OLS regression of $\ln X_{ij}$ on $\ln \tau_{ij}$ and fixed effects.

Some Numbers

- Consider Belgium (a very open economy).
- What do the trade data say?
  1. Share of domestic expenditure: $\lambda_{BEL} = 0.73$
  2. Trade elasticity: $\overline{\varepsilon} = -5$
- How large are the gains from trade?
  - **Example 1**: *Gravity trade models:* $\alpha = \beta = 0$
    - $GT \equiv (0.73)^{-1/5} - 1 \simeq 6.5\%$
  - **Example 2**: *Models* with $\beta = 0.5$:
    - GT under PC and MC$\equiv (0.73)^{-1/(0.5\times 5)} - 1 \simeq 13\%$