13 Experimental attractors

In this brief lecture we show examples of strange attractors found in experiments.

13.1 Rayleigh-Bénard convection

$\boxed{\text{BPV, Figure VI.25}}$

Two dynamical variables, (ΔT) and $(\Delta T)'$, represent time-dependent thermal gradients measured measured by the refraction of light, inside the convecting system.

A 3-D phase space is defined by the coordinates

$$(\Delta T), (\dot{\Delta T}), \text{ and } (\Delta T)'.$$

The Poincaré section in the plane $(\Delta T), (\dot{\Delta T})$ is obtained by strobing the system at the dominant frequency of the fluctuations of one of the dynamical variables.

Like the Hénon attractor, the points are arranged in a "complex but well-defined structure" (BPV, p. 137).

13.2 Belousov-Zhabotinsky reaction

See Strogatz, Section 8.3.

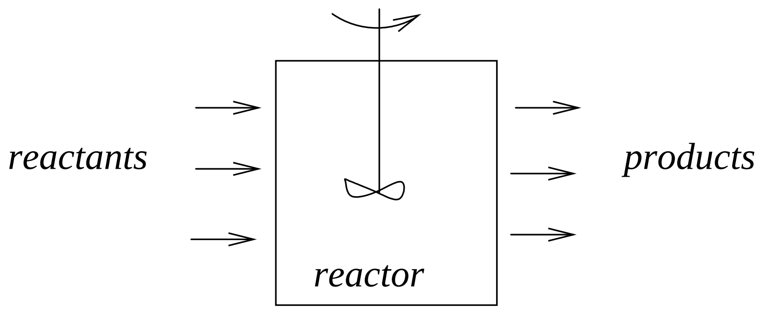
The B-Z reaction is a particularly well studied chemical reaction that is of interest for

- dynamics, because it is oscillatory; and
- pattern formation and nonlinear waves.

An example of the pattern formation is

$\boxed{\text{Strogatz, Plate 1, Section 8.3}}$

For us the dynamical aspects are of greater interest. This chemical reaction, like many others, can be envisioned as a flux of reactants into a reactor, and a flux of products out of the reactor:



Nonlinearities arise from chemical reactions like

$$A + B \to C$$

which yield terms like

$$\frac{\mathrm{d}C}{\mathrm{d}t} = AB$$

The control parameter is typically the reactant flux.

The dynamical variable $X(t)$ measures the concentration of a particular chemical species within the reactor.

Phase portraits are obtained in the 3-D phase space formed by

$$X(t), X(t+\tau), X(t+2\tau).$$

The periodic regime (limit cycle) is evident in the 2-D projection given by

$\boxed{\text{BPV, Figure VI.26}}$

The chaotic regime is evident in the 2-D projection given by

$\boxed{\text{BPV, Figure VI.27}}$

A Poincaré section reveals evidence of strong dissipation, as seen in a plot of intersections with a plane perpendicular to $X(t), X(t+\tau)$:

$\boxed{\text{BPV, Figure VI.28}}$

The straightness of the line is insignificant, but its thinness indicates strong dissipation.

Due to dissipation, a nice 1-D first-return map can be formed by plotting X_{k+1} vs. X_k, where X_k is the kth observation of $X(t)$:

$\boxed{\text{BPV, Figure VI.29a}}$

That all the points essentially lie on a simple curve implies *deterministic order* in the system.

Attraction is demonstrated by inducing a small perturbation and then observing the re-establishment of the first return map:

$\boxed{\text{BPV, Figure VI.29b}}$

Sensitivity to initial conditions is observedby plotting the distribution of points that results after passage close to the same point:

$\boxed{\text{BPV, Figure VI.30}}$

Stretching and folding may also be observed. Plotting the data with a different choice of τ:

$\boxed{\text{BPV, Figure VI.31}}$

Nine different slices through the system reveal stretching (from 9 to 1) and folding (between 2 and 8):

$\boxed{\text{BPV, Figure VI.32}}$