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AN APPRAISAL OF ALTERNATIVE PLANNING MODELS*

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Certainly one of the most powerful incentives to the clarification of theoretical frameworks must be the potentiality of their application. Discussion of multi-sectoral, inter-temporal economic planning models in a particular context may not have led to a resolution of the conceptual problems but it has provided a strong stimulus.

In this paper we report some results of such stimulation. We attempt an appraisal of various types of consistency models, including the one discussed in the preceding paper, through an explicit comparison with planning models with optimization features. Our notions of what is involved in the use of fully optimizing models are based on the theoretical literature on this subject as well as our independent work and the suggestions of our colleagues.[1] As will become clear, we make different judgments as to the actual or potential availability of various types of empirical information. For the most part, except where on a priori grounds the strictly computational problems appear to preclude some particular theoretical formulation, we abstract from such problems.

Both the consistency models and the optimizing approach to planning entail a common framework of production involving many sectors with interconnecting

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1. See R. Dorfman, P.A. Samuelson and R. Solow, Linear Programming and Economic Analysis, 1958, esp. Chaps. 9 to 12; S. Chakravarty, "Optimal Savings with Finite Planning Horizons", forthcoming in International Economic Review, September 1962; R. S. Eckaus and L. Lefeber, "Capital Formation and Economic Growth: A Theoretical and Empirical Analysis", in Review of Economics and Statistics, May 1962.

flows. The conditions of production are those of the usual Leontief-type models and are described by two matrices: one of "flow" coefficients for current inputs and one of "capital" coefficients for additions to capacity. It is not necessary that these matrices be the same in every period but in both types of the models, they are specified exogenously and the models are not required to choose the most appropriate coefficients. It is possible to embody technological changes through variation of the flow and capital coefficients. This can be accomplished by making the input-output dependences linear rather than proportional, or when working out a solution by an iterative process to change the coefficients exogenously, perhaps depending on the level of output achieved. Thus, "fixed" coefficients are not a necessary feature of this type of planning framework. Technological change can be embodied in a similar way to the extent that it is known and put into effect. Outputs are divided among intermediate flows and final uses in private and government consumption, capital formation and exports. For the purposes of this note we abstract from special considerations relating to exports and government consumption.

I. A Consistency Model

The intent of this approach as it has been formulated in the previous paper is to decide if there exists and, if so, to determine for each year of the planning period, a pattern of output and investment which is consistent with a particular "target" vector of terminal production levels and specified rates of growth of consumption during the plan period. The "free" variables which may adjust to permit the solution of the problem are the initial levels of consumption. The working of the system is constrained on the production

side, however, by the specification of the initial levels of capacity in each sector and the conditions of production as described by the two Leontief matrices.

The whole object of the framework is to find an intertemporally consistent set of outputs, investments and consumptions within the planning period. There is no presumption that the path so determined is an optimal one. Strictly speaking, the concept of an "optimum path" cannot be defined within the limits of a single consistency model. It is, however, always possible to derive alternative consistent paths of development by varying the targets and other exogenous conditions which are set. The planners may then choose the one particular consistent path which they consider most satisfactory. This mechanism of selection is not included within the model itself.

In the "target" version of the consistency model the construction of the planning framework proceeds in the following way: (1) the first step is to establish the terminal conditions; these are the targets to be achieved at the end of the planning period. Since the initial conditions are conveniently specified in terms of the full capacity levels of production prevailing at the beginning of the planning period, the terminal conditions are also set in the same dimensions. From the mathematical point of view it is only necessary that these conditions be in some way established, whether it be done by picking a set of numbers from a table of random digits or by a set of independent calculations. From an economic point of view there is unavoidably some arbitrariness in confining our forward vision in a planning model to, say, five or fifteen years and stating the goals to be achieved then without looking beyond. A method is suggested which at least makes explicit the nature of the arbitrary decisions which are involved, as they always are in any planning model with a finite horizon.

For the end of the planning period an independent estimate is made of target levels of consumption of each item. The method of estimation is left as an independent procedure at this stage of model building. Desired per capita levels of consumption of different items might be used with population projections. Or an aggregate annual average growth in per capita consumption can be projected and then divided among consumption items by a combination of Engel relations and stipulation based on exogenous considerations. Even when terminal consumption levels have been specified that establishes only one of the target, final "bill-of-goods vectors" and, by itself, does not take account of the desire to have economic growth continuing beyond the planning period. In order to ensure this it can, for example, be specified that consumption of all items be able to grow at some particular rate, r, beyond the terminal year. Having already established the final consumption levels, the specification of a growth rate in consumption to be achieved beyond the final period permits the derivation of terminal production levels in each sector, along the lines discussed in the previous paper.

The post-terminal evaluation guaranteed by this type of calculation is characterized by equiproportional growth in all consumption items. However, as demonstrated in the preceding paper, it is possible to derive a set of conditions on the terminal capital stocks which will permit growth of consumption in specified, non-proportional rates in the first post-terminal period.

Following the time path directed by the model up to and including the terminal period conditions derived as above will make possible further growth in the post-terminal period at rates which can be specified in advance.

Certainly there is some arbitrariness in asking the planners to stipulate in advance the post-terminal rate or rates of growth in consumption which are to be achieved. But the procedure provides one, explicit method of treating such arbitrariness which is always involved in planning models with finite time horizons.[2]

There are other ways of specifying terminal conditions. They may in some part be set by a "political" process: so many steel mills, so much aluminium capacity, self-sufficiency in particular lines of production. One way or another, however, they must in this type of model be established.

(2) The second step is to specify the planning period. There is no analytic device which will permit one to pick the "best" planning period on a priori grounds. Some of the relevant considerations have been discussed briefly in the paper entitled, "The Choice Elements in Inter-temporal Planning". These, however, are not completely economic in character.[3]

(3) In the third step a method of determining the level of consumption of the various items within the planning period must be settled upon. The method used in our earlier paper requires the exogenous determination of the rate of growth of consumption in each sector. This can be a single number,

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2. For a simple illustration of the logical intricacies involved in dealing with planning models over an infinite horizon see the paper by S. Chakravarty, "The Existence of an Optimum Savings Program", Econometrica, January, 1962.

3. In this, as in the planning frameworks to be discussed later, application of the model over different planning periods raises different problems of estimation of empirical parameters and of projection of terminal conditions. The logic of the models does not change with the planning period so long as it remains finite. It is true, however, that the quantitative results will be sensitive to changes in the length of the horizon.

so that consumption of all items is guaranteed to grow at the same rate or different consumption growth rates can be specified for each sector. The determination of the different growth rates is a matter outside the analytical boundaries of the model itself. If this procedure appears arbitrary then consumption can be just as well treated endogenously by means of consumption-income relations for the various sectors. In the latter procedure the model becomes "closed" with respect to consumption and the path of development is constrained by private consumption patterns. In this case, however, we lose our freedom to posit any set of terminal conditions and the terminal positions must be worked out from the model.

It is equally possible, and, perhaps, closer to reality to regard some consumption patterns, e.g., in food grains, textiles as, in fact, constraining the pattern of development and the consumption of other items, e.g., automobiles and housing, as subject to the exogenous determination and control of specific policies. The existence of endogenous consumption-income relationships of a sub-set of commodities and sectors would limit the number of terminal conditions which can be stipulated in advance.

(4) Finally, after all the previous steps have been taken the "problem" can be solved. The problem is to determine the initial levels of consumption and subsequent levels of output, consumption and investment in each sector in each period which are consistent with the decisions taken in the first three steps relating to final targets, the planning period and consumption behavior.

Solutions for the differential as well as the difference equation formulation of the model have been worked out, that is for continuous as well as

discrete time progression, and for uniform as well as non-uniform exogenously specified rates of growth of consumption.[4] The analytic framework presented in the preceding paper shows that, given the targets, the planning period and the intermediate consumption behavior, we can find initial and subsequent levels of consumption, investment and output which are consistent.

It should be clear, however, that in this formulation the burden of adjustment necessary to achieve consistency is thrown on the initial levels of consumption. These are not "givens"; they are not taken from what has been observed at the start of the planning period. They are derived as a consequence of the choice of the other elements which are made. Will that "burden" of adjustment be a difficult one? Would the changes required be "politically" feasible? If they are not, what changes in the other elements would produce initial consumption levels which are realistic? These, most significant questions cannot be answered by a priori reasoning. They depend not just on the structure of the analytic framework but on the numerical values of the parameters, as well as the values given to the choice elements. The answers must wait, therefore, on the numerical trial of this model.

Of course, there are other formulations of this basic idea of a consistency model which would adapt themselves to the political fact, such as it is, that initial levels of consumption cannot be changed, or cannot be changed much or, more precisely, can only be changed by amounts which the planner could specify. If initial consumption levels become "givens", then the "free" variables must be either the intermediate growth rates in consumption or the terminal conditions. To the extent that consumption of certain commodities is determined by income elasticities or Engel equations these items are

4. If there are extended and different gestation lags of investment in the various sectors the analytical and computational problems become somewhat more involved but are not essential barriers to a solution.

not completely free to adjust if initial and terminal conditions are fixed. The entire burden of adjustment then is thrown on just those "free" consumption items, say, automobiles and housing. Again, however, it remains a practical question which could be decided only by numerical trials as to whether this would provide sufficient flexibility.

There is another question regarding this latter formulation as to the feasibility of solution. When the unknowns in the problem are each of the intermediate consumption growth rates, the mathematical difficulties seem to be substantially increased. This, however, requires still further exploration.

Still another, alternative formulation is to let the terminal conditions be the "free" variables with initial conditions taken at whatever they happen to be and intermediate consumption levels determined by endogenous relations or exogenous specification, whichever is considered most realistic and tractable. In this formulation the planning framework becomes a "projection" model. The purpose of working out the whole thing is to see what these specifications imply about the future. The "targets" for the end of the planning period cannot be set exogenously. They are an implication of the model which must be worked out in order to see where the economy is going and where it will be at the end of the planning period. The terminal consumption levels and investment levels and the post-terminal growth rates are not chosen before hand but are implications of the working out of the system itself. They are calculated after everything else is set.

In this as in the other formulations of the "consistency" model there can be trials to determine the implications of alternative stipulations of conditions

for the model. In the "projection" version these trials would demonstrate the alternative patterns of growth in each sector and final levels of production which would be achieved. The planners could then choose that option which is considered most satisfactory.

In each of these formulations of a "consistency" model, the generalization from a single sector or aggregated one good version to a many sector framework is relatively straightforward. It is for many purposes quite possible to think in terms of the aggregated version and generalize the conclusions to many sectors. Naturally, the generalization is not always obvious and may miss certain essential problems.

In the many sector version the question arises as to whether the condition of "non-negativity" is preserved for all the variables. Mathematical consistency which requires that some sectors operate at negative output levels or some capital stocks be run down below zero is not acceptable economics. It is not necessary and it may not even be desirable to impose the condition that all investment levels be non-negative as well. This would commit the economy to maintain capital stocks in every sector even when its preferences may have changed.

Non-negativity of output and capital stock variables cannot always be guaranteed in this type of consistency model although situations in which negativity troubles will not arise can be specified in advance. Moreover, once all the necessary numbers are specified it can be ascertained whether or not non-negativity is preserved by actual computations. If negativity is a danger, an intervening period of adjustment to a safe set of initial conditions can be imagined. What a "safe" set of initial conditions would be, whether or not they would also be "politically feasible" are questions which must be answered by actually working out alternative solutions.

There are limits as to how far one can go in a priori specification of a "consistency" model, which can be guaranteed to be satisfactory to a planner. Though the details have not been worked out here and some problems require further work the boundaries have been established. The only way of going beyond them is to begin to actually work the model and to analyze the results.

Suppose, for example, using the initial formulation of the "consistency" model, terminal conditions were to be set for consumption and capital stock vectors. Then the behavior of intermediate consumption in each sector is accounted for by stipulation in some sectors and Engel equations in other sectors. Doing all this implies that a considerable amount of empirical work has already been done on estimation of parameters before one ever tries to work the planning model. It is not easy work but feasible, given enough effort and resources. The next step is to solve the problem and find the initial conditions and path of output, consumption and investment implied. Only at this point can it be determined whether the condition of non-negativity, for example, is met. Maybe it will be and it is not even a close question. It is tempting to say that would be lucky, but it is not a matter of luck. The answer is all there in the numbers and planning framework and just has to be worked out. Once past that hurdle the initial consumption conditions implied are examined and compared with those actually in existence at the beginning of the planning period. The comparison may be close, so that this implication of the plan is judged to be "politically" feasible and the other implications can be inspected. Even if it all "passed" on political as well as economic feasibility grounds a re-run would be ordered immediately in order to determine whether a change in the terminal or intermediate conditions would lead to results which were both feasible and politically even more desirable.

Of course the results might not "pass". There may be troubles due to negativity and the derived initial levels of consumption may be impossibly different from the prevailing ones. So again the terminal conditions would have to be adjusted and a new, perhaps more acceptable, solution found. The trials and re-trials would not have to be "blind". A program of experimentation might be developed to explore alternatives. But the alternatives would have to be explored empirically as purely deductive efforts alone would not yield full answers to the empirical questions asked. Though, logically, the amount of empirical exploration possible is endless, the actual amount necessary for planning purposes need not turn out to be so great. The problem though a big one seems well within the capacity of modern computing machines to solve many times, relatively quickly. That is what must be done in order to develop a useful planning framework.

II. The Fully Optimizing Approach

A fully optimizing model has an appeal which cannot be matched by any less ambitious framework. It attempts to find the "best" possible pattern of resource allocation over time and among sectors. The "best" is understood in the sense of maximization of a stipulated preference function involving the relevant dated variables. So, if an optimal program can, be established, the attempt to do as well as possible has succeeded; the goal has been achieved and the planner can sit back with a justified sense of satisfaction.

In attaining all this a fully optimizing model will also eliminate those worries about the possible negativity of solutions which may bother a planner using a consistency model. This will be a result of the inequalities which

which fully fulfil the investment and which keep the system at least at non-negative levels of output in all the sectors. As regards initial conditions, these can be included in the set of constraints which characterize the model. Or, alternatively, they can be included among the unknowns which will be determined by the solution to the intertemporal maximization problem.

Without giving a fully optimizing model in detail an outline will be presented of the main steps involved in constructing such a planning framework.

(a) The first step is to decide on what it is which will be optimized and into what type of preference function these variables will enter. Any variety of things could be optimized though not all choices are equivalent in terms of the economic opportunities they provide. Just two possibilities will be considered here.

(a) Given some terminal capital stocks as well as initial capital stocks, the planning framework might be asked to optimize with respect to some specified utility function which includes all the various consumption items in each year of the planning period. The terminal capital stocks could be determined in such a way as to guarantee a particular rate of equi-proportional growth or a vector of growth rates for the different sectors in the post-terminal period. The planned initial levels of consumption could be chosen now to be at just those actually prevailing, or at other levels chosen by the planner. Likewise it would be possible to specify exogenous policy parameters, consumption of particular items at predetermined levels and to maximize the utility to be gained from all other consumption goods. For analytic and computational reasons ideally the utility function chosen should have a linear form. However,

this, economically, is quite a restrictive assumption. It implies a constancy of the marginal utility of consumption with respect to each consumption item for any period of time.

(b) Or it would be possible to specify some linear be avior relations for consumption goods and for given initial levels of capacity. Optimization could then be carried out with respect to some preference function of terminal capital stocks. This function likewise ought to be linear in form to insure analytic and computational feasibility. In this case the terminal condition targets and the potential post-terminal growth rates could not be known until there were a solution to the planning problem.

(c) Most inter-temporal linear programing models are bound to rely on the choice of one or other set of constant preference coefficients which, of course, cannot avoid being arbitrary. It is logically possible to formulate the planning problem as a non-linear problem, where the implied preference function may have a greater degree of intuitive significance. If we formulate the planning problem as one minimizing the time needed to transform a given set of initial conditions to a desired set of terminal goals, then the problem assumes a non-linear character. Thus, if the initial conditions are written as a vector X(0) and the desired terminal conditions as a vector X(T) the rate of change of X with respect to time t is

$$\frac{dX}{dt} = f\left[\bar{X}(t),\ y\ (t),\ t\bar{\ }\right] \quad ,$$

where y(t) is a set of decision variables referring to time t and is a vector valued function of vectors. With a model of constant coefficients type t will not figure as an independent variable.

Thus, for a dynamic model of the Leontief type

$$B \frac{dX}{dt} = (I - A) X(t) - C(t)$$

or

$$\frac{dX}{dt} = B^{-1} (I - A) X(t) - B^{-1} C(t)$$

where B is assumed to be irreducible and B^{-1} exists. The problem, then, is to choose C(t) in such a way as to minimize the time needed to transform X(0) into X(T). Additional restrictions on the non-negativity of the solution or inequalities of the form $C(t) \geq \overline{C}$ may be imposed. This type of optimization problem with inequalities added as a side condition, however, has a number of special difficulties in obtaining a solution.

(2) Having stipulated the form of the model and the constraints desired, the next step is to work out the solution. Inter-sectoral and inter-temporal consistency are automatically assured from the method of setting up and solving the problem and the result is sure to be the "best" possible one.

Once having that result, however, the planner can, by no means rest on his laurels, but must immediately begin recomputing for alternative specifications of the consumption utility function or the terminal stock preference function, depending on the formulation used. There is no way of knowing what the coefficients of a national utility or preference function are but we can be sure that the planning results will be sensitive to those coefficients. For example, if all the relevant relationships are linear, as they are nearly always assumed to be for computational tractability, the set of solutions satisfying all the inequalities for each time period span a solution space of very high dimensionality with a lot of "corners". Any optimal solution will be at one of the corners or will be a convex combination of some of the corner conditions. Changing one or more

of the coefficients of the utility or preference function is like shifting the weights on a board resting on an irregular pile of rocks. The board will move and come to rest some place else. What will be the new solution? It is not possible to provide an answer on a priori grounds. One must just try and see what the results are.

Trying the model for many different combinations of the coefficients of the utility or preference functions is a big job which will yield information about the sensitivity of the results to changes in these coefficients. Suppose the results are not sensitive to "small" changes and are sensitive to "big" changes. What then? There is still no basis for a firm idea as to what these coefficients are. Perhaps by taking a "fresh look" at the different results a planner might decide which he liked best.

In the fully optimizing model as in the consistency model, the framework, the data requirements and character of the results are closely related. If it is the utility of consumption over the entire planning period which is optimized with stipulated terminal stocks, all the problems of stipulating the terminal stocks emerge as in the previous planning frameworks and there is the additional requirement of specifying a utility function which includes all the different consumption items. If it is a preference function of terminal capital stocks which is optimized with consumption stipulated exogenously or by means of Engel equations, or both, then as in the projection version of the consistency model the terminal state of the economy and the potential post-terminal growth rates will be determined only in the process of solving the model.

In fully optimizing models we are not committed to a policy of full utilization of capacity in each sector in each period. However, it is doubtful whether an optimal solution will deviate significantly from the no-excess capacity pattern provided that we assume final demand for each sector to increase and that a meaningful solution exists on the no-excess capacity assumption.

III. Summary

The difficulties in applying multi-sector, inter-temporal planning frameworks are essential difficulties which arise not from the model approach but from the inherent nature of the planning problem. These difficulties are only concealed or ignored in the usual procedures which plan with an aggregate income concept or ignore the requirements of inter-sectoral consistency or are restricted to static consistency requirements. Certainly any opinion which regards the difficulties of applying multi-sector, inter-temporal planning models as faults rather than virtues is obscurantist.

In turn this does not imply that the proper planner must instantly embrace the most sophisticated and fully developed, optimizing framework to establish himself firmly on the side of human progress and enlightenment. There are tactics of planning as well as grand strategy. What are the criteria for a tactical choice among the alternative approaches described above? The following two are suggested:

(1) The planning framework must be understandable to those persons on whom the burden of economic decisions falls. This, to be sure, is not a clear-cut criterion. The least common denominator in understanding should not be controlling; neither should the professional economist specialist in planning techniques dictate the terms in which planning is to be done.

The latter must lead but not by so far as to lose contact with those for whom he is working. This can only be a matter of judgment. It is a matter which deserves careful thought because planning models whose general structure is not understandable to decision makers are, for them, a waste of time and have no chance of application.

(2) The planning framework suggested must have "good" chances of success. That is it should offer results which represent additions to knowledge as compared to the results achieved by the currently used techniques. In addition these results must also be attainable within a "reasonable" span of time. It is no good offering results only after "perfect" data and "perfect" planning models are available.

This criterion should not be so hard to meet. The explicit, multi-sectoral inter-temporal planning models should have at least the data and sense of any less sophisticated method. Even if the results of a fully specified multi-sector planning model only verified currently used, less sophisticated methods, that in itself would represent an addition to knowledge which would be worth the effort unless someone could predict with certainty that outcome in advance. Certainly the results should not lead to error and represent a subtraction from wisdom. That could happen if the quality of information and judgment used in an explicit planning framework were less good than otherwise available and practiced and yet the former were allowed to prevail. In a planning organization open to diverse points of view that seems to be an unlikely outcome.

The conclusion to be drawn from this survey seems to be that none of the types of models considered can be so fully and adequately specified in advance

that they can be fed into a computer which in turn will proceed to grind out the next Five or Twenty Year Plan. A series of actual trials with alternative numbers for targets, preference functions, consumption behavior relations, initial conditions and so on must be an essential part of the process of developing a planning framework, whatever formal planning structure is used. This may be a disappointment but it does not imply that inter-temporal multi-sector planning cannot be done or that it is useless if attempted. It does mean that there is another important step in the development of planning models between writing down an analytical framework and obtaining results which can be implemented.