AN OPTICAL SWITCH

by

NICKOLAS PEPPINO VLANNES

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Bachelor of Science
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1975

Signature of Author

Department of Electrical Engineering
and Computer Science, May 9, 1975

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Theses
Title: An Optical Switch
Author: Nickolas P. Vlannes
Thesis Supervisor: Professor Hermann A. Haus
Department of Electrical Engineering
and Computer Science

This paper is a discussion of how one can obtain the power transfer and coupling length of an optical switch by using coupling of modes theory and a dispersion relation. Once the parameters of a single switch are found, the design for multiple switches and construction of a switch are considered.
ACKNOWLEDGEMENTS

It is with great appreciation that I thank Professor Hermann A. Haus of the Department of Electrical Engineering and Computer Science for the many hours he spent guiding me towards the completion of this thesis. Without his advice and guidance, I would have neither started nor completed this work. I also wish to express my gratitude for the patience, time and consideration of Professor Malcolm W. P. Strandberg in reading this dissertation for the Department of Physics. Further I want to thank Mr. Robert Fontana, Jr. for my many discussions with him about electro-optic materials and electro-optic devices, and also Mr. Walter Legowski, Mr. Alan Sopelak, and Mr. George Young for their conversations with me about computer convergence routines for finding zeros of functions. Lastly, I wish to acknowledge the love and devotion of my family, and the encouragement and support which has motivated and sustained my efforts.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Notation</td>
<td>5</td>
</tr>
<tr>
<td>Introduction</td>
<td>7</td>
</tr>
<tr>
<td>Coupling of Modes Theory</td>
<td>9</td>
</tr>
<tr>
<td>Dispersion Relation Theory</td>
<td>16</td>
</tr>
<tr>
<td>Optical Switch</td>
<td>33</td>
</tr>
<tr>
<td>Multi-Switch Design</td>
<td>46</td>
</tr>
<tr>
<td>Construction of an Optical Switch</td>
<td>59</td>
</tr>
<tr>
<td>Conclusion</td>
<td>61</td>
</tr>
<tr>
<td>Appendix I</td>
<td>64</td>
</tr>
<tr>
<td>Appendix II</td>
<td>71</td>
</tr>
<tr>
<td>References</td>
<td>78</td>
</tr>
</tbody>
</table>
GENERAL NOTATION

\( A_i, A'_i \) Coefficients used in defining the electric field amplitudes of the Dispersion Relation Theory

\( A_{1,2}(z) \) Coefficients of electric and magnetic fields defined by equations (1) and (2)

\( a_i(z) \) Defined by equation (5)

\( a_i \) Defined by equation (8)

\( c_{1,2} \) Coupling constants of equations (3) and (4)

\( c_{12}, c_{21} \) Coupling of Modes Theory coupling constants

\( d \) One half the distance between two waveguides (meter)

\( E \) Total complex electric field of radiation (volt/meter)

\( E \) Complex electric field amplitude of radiation (volt/meter)

\( E_a \) Electric field applied to electro-optic materials (volt/meter)

\( F \) Coupling of modes power factor

\( F_i, G_i \) Defined by equations AI.19 through AI.22

\( H \) Total complex magnetic field of radiation \([\text{weber}/(\text{meter})^2]\)

\( H \) Complex magnetic field amplitude of radiation \([\text{weber}/(\text{meter})^2]\)

\( h \) Ideal capacitor plate separation

\( i \) Subscript

\( j \) \( \sqrt{-1} \)

\( L \) Coupling length for full power transfer (meter)
\( l \)  
Coupling length (meter)

\( n \)  
Index of refraction

\( p \)  
\[ p = (\omega^2 \mu \varepsilon - \beta^2)^{\frac{1}{2}} \] (1/meter)

\( R \)  
Electrical resistance (ohms)

\( r_{13}, r_{33} \)  
Elements of electro-optic tensor of LiNbO\(_3\) (meter/volt)

\( x, y \)  
Cartesian coordinates (meter)

\( \hat{y} \)  
Unit vector in y-direction

\( Z_0 \)  
Impedance of transmission line

\( z \)  
Direction of propagation of radiation (meter)

\( [ ] \)  
Matrix

\( \alpha \)  
\[ \alpha = (\beta^2 - \omega^2 \mu \varepsilon)^{\frac{1}{2}} \] (1/meter)

\( \beta \)  
Propagation constant (1/meter)

\( \varepsilon \)  
Dielectric constant

\( \theta \)  
Thickness of waveguides

\( \mu \)  
Magnetic permeability constant

\( \tau \)  
Capacitor rise time

\( \omega \)  
Radial frequency of radiation
INTRODUCTION

With the development of low loss optical fibers or dielectric waveguides, optical signal processing has gained technical interest. In order to perform signal processing, it is necessary to be able to modulate and switch an optical signal. The problem is to develop an optical switch with the additional advantages of small size and the capability to integrate the switch on a single chip.

Dielectric waveguides have a characteristic which is a natural switch and this is coupling between two adjacent waveguides; however, to be useful, one must be able to determine the amount of coupling and to control it. One approach to this is to examine the coupling of modes of two waveguides. Coupling of modes theory provides two features. These are the amount of power that will be transferred between two adjacent waveguides and the distance over which the power is transferred. The goal is to determine these two variables from the physical parameters of the switch such as dielectric constants, permeability constants, distance between the waveguides, and the thickness of the waveguides.

One approach to finding the variables that determine switch size and power transfer is through the dispersion relation of the coupled system. Once the dispersion relation
is found, one can derive the coupling of modes form from a Taylor expansion around the characteristics of the single waveguides. With the coupling of modes equation determined, those terms that give power transfer and coupling length for a two waveguide switch, control of the switch, extension to multi-switch design, and construction of the switch can be discussed.
COUPLING OF MODES THEORY

The electromagnetic field distributions of an isolated dielectric waveguide can be represented as:

\[ E = E_0 e^{j(\omega t - \beta z)} \]
\[ H = H_0 e^{j(\omega t - \beta z)} \]

When the two guides are closely spaced, the two waveguides influence each other and the field amplitudes may change with distance. Thus one can express the total fields as:

1. \[ E = A_1(z)E_1 + A_2(z)E_2 \]
2. \[ H = A_1(z)H_1 + A_2(z)H_2 \]

Where the subscripts 1 and 2 represent the appropriate value for each waveguide. Substituting these expressions into Faraday's and Ampere's electromagnetic field laws, one can derive two differential coupled equations for \( A_1(z) \) and \( A_2(z) \):

3. \[ \frac{\partial A_1(z)}{\partial z} = j c_1 A_2(z) e^{j(\beta_1 - \beta_2)} \]
4. \[ \frac{\partial A_2(z)}{\partial z} = j c_2 A_1(z) e^{j(\beta_1 - \beta_2)} \]

Here \( c_1 \) and \( c_2 \) are the coupling constants and can be obtained from the functions that represent the electromagnetic field. However, one does not usually know the field distribution in the propagation or z-direction, or the calculation of the
coupling constants is difficult, and thus the coupling constants would remain unknown. Further, the coupled wave equations (3) and (4) are not exact, since in the derivation of the equations, certain second order terms are ignored and only two modes are considered.¹

The coupled wave equations (3) and (4) can be written in a different form by introducing wave amplitudes:

\[(5) \quad A_i(z) = a_i(z)e^{j\beta_i z} \quad i=1,2\]

and substituting this into equations (3) and (4). This results in the form of the coupled mode equations:

\[(6) \quad \frac{d a_1(z)}{d z} = -j\beta_1 a_1(z) + j\epsilon_1 a_2(z)\]
\[(7) \quad \frac{d a_2(z)}{d z} = -j\beta_2 a_2(z) + j\epsilon_2 a_1(z)\]

Though this discussion is limited to two waveguides and two modes, the coupled mode formalism for optical fiber power transfer also holds for many fibers and multi-modes.²

Solutions of the coupled mode equations (6) and (7) are of the form:

\[(8) \quad a_i(z) = a_i e^{-j\beta_i z}\]

Substituting this into equations (6) and (7), one has the results:

\[-j\beta a_1 = -j\beta_1 a_1 + \epsilon_1 a_2\]
\[-j\beta a_2 = -j\beta_2 a_2 + \epsilon_2 a_1\]
where \( c_{12} = j c_1 \) and \( c_{21} = j c_2 \). Rewriting the above equations, one has:

\[
\begin{align*}
(9) \quad j(\beta - \beta_1)a_1 + c_{12}a_2 &= 0 \\
(10) \quad j(\beta - \beta_2)a_2 + c_{21}a_1 &= 0
\end{align*}
\]

In matrix form, equations (9) and (10) appear as:

\[
[B][a] = \begin{bmatrix}
 j(\beta - \beta_1) & c_{12} \\
 c_{21} & j(\beta - \beta_2)
\end{bmatrix}
\begin{bmatrix}
 a_1 \\
 a_2
\end{bmatrix} = 0
\]

In order that equations (9) and (10) have non-trivial solutions, the \( \det[B] = 0 \). Thus,

\[
-(\beta - \beta_1)(\beta - \beta_2) - c_{12}c_{21} = 0
\]

\[
\beta^2 - \beta(\beta_1 + \beta_2) + \beta_1\beta_2 + c_{12}c_{21} = 0
\]

Solving for \( \beta \) and regrouping some terms, one has:

\[
(11) \quad \beta = \beta_1 + \frac{\beta_2}{2} \pm \frac{1}{2}((\beta_1 - \beta_2)^2 - 4c_{12}c_{21})^{\frac{1}{2}}
\]

With the coupled mode approximation, the total average power is given approximately by:

\[
(12) \quad |a_1(z)|^2 \pm |a_2(z)|^2 = \text{constant}
\]

where the + sign is used if the group velocities of the modes are in the same direction, and − sign is taken for group velocities in the opposite direction. Since (12) represents the total average power,

\[
(13) \quad c_{12} = \overline{c_{21}}^*
\]
Thus equation (11) can be written as:

$$\beta = \frac{\beta_1 + \beta_2}{2} \pm \left( \left( \frac{\beta_1 - \beta_2}{2} \right)^2 + |c_{12}|^2 \right)^{\frac{1}{2}}$$

Throughout the rest of this paper, only modes with group velocities in the same direction will be discussed, hence:

$$\beta = \frac{\beta_1 + \beta_2}{2} \pm \left( \left( \frac{\beta_1 - \beta_2}{2} \right)^2 + |c_{12}|^2 \right)^{\frac{1}{2}}$$

If initially the total power is put on one waveguide, the power in each waveguide, normalized with respect to the total power, is given as:

$$P_1(z) = 1 - F \sin^2 (\beta_b z)$$

$$P_2(z) = 1 - P_1(z) = F \sin^2 (\beta_b z)$$

where,

$$\beta_b = \left( \left( \frac{\beta_1 - \beta_2}{2} \right)^2 + |c_{12}|^2 \right)^{\frac{1}{2}}$$

$$F = \left( \frac{1}{\beta_b} \right)^2$$

$P_1(z)$ is the power in line 1, the waveguide the power starts on, and $P_2(z)$ is the power in line 2, the second waveguide.

A graphical representation of equations (15) and (16) is shown in Fig. 1, page 13. The distance $z = l$ over which maximum power is transferred is:

$$l_{\beta_b} = \frac{\pi}{2}$$

$$l = \frac{\pi/2}{\beta_b}$$

Hence the coupling length $l$ and power factor $F$ are
Fig. 1

Graphical Representation of Power Transfer Between Waveguides
dependent upon $(\beta_1 - \beta_2)$ and $|c_{12}|$. By changing $(\beta_1 - \beta_2)$ and/or $|c_{12}|$, power transfer between the waveguides and coupling length change. When $\beta_1 = \beta_2$, $F = 1$, and full power is transferred in a coupling length:

$$L = \frac{\pi}{2|c_{12}|} \tag{20}$$

If $\beta_1 \neq \beta_2$, then $F < 1$, and the coupling length $\ell$ is not equal to $L$. A graphical comparison of the two cases $\beta_1 = \beta_2$, and $\beta_1 \neq \beta_2$ is shown in Fig. 2, page 15. Fig. 2 indicates that $\ell$ will decrease when $\beta_1 - \beta_2 \neq 0$. However, it is possible that by changing $(\beta_1 - \beta_2)$, $|c_{12}|$ will also change, resulting in $\ell$ increasing or remaining the same. Modulation and switching can therefore be controlled by changing $(\beta_1 - \beta_2)$ and/or $|c_{12}|$ so that power transferred is changed, coupling length is changed, or both. Hence, one must obtain expressions for $(\beta_1 - \beta_2)$ and $|c_{12}|$. One approach is through a dispersion relation.
Graphical Comparison of Power Transfer for $\beta_1 = \beta_2$ and $\beta_1 \neq \beta_2$
Fig. 3, page 17, illustrates two slab waveguides as seen through their cross-section, and the electric field of a TE mode is also drawn. In regions I, II, III, IV, and V, the electric fields for a TE mode are:

\[ E_i = E_i e^{j(\omega t - \beta z)} \]

where, \( E_i \) are given as:

I. \(-d < x < -d/2\)
   \[ E_{I} = (A_1 \cos[p_1(x+\frac{\theta}{2}+d)] + A_1 \sin[p_1(x+\frac{\theta}{2}+d)])y \]

II. \(d < x < d+\theta_2\)
   \[ E_{II} = (A_2 \cos[p_2(x-\frac{\theta}{2}-d)] + A_2 \sin[p_2(x-\frac{\theta}{2}-d)])y \]

III. \(x \leq -d\)
    \[ E_{III} = (A_3 e^{\alpha_3 x})y \]

IV. \(-d < x < d\)
   \[ E_{IV} = (A_4 e^{-\alpha_4 x} + A_4 e^{\alpha_4 x})y \]

V. \(d+\theta_2 < x\)
   \[ E_{V} = (A_5 e^{-\alpha_5 x})y \]

From Maxwell's Equations:

\[ \nabla \times E_i = -j\omega \mu_0 H_i \]
\[ \frac{j}{\omega \mu_0} \frac{dE_i}{dx} = H_{zi} \]

and letting \( \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu \), the z-components of the magnetic field in regions I, II, III, IV, and V are:

I. \(-d < x < -d/2\)
   \[ H_{zI} = \frac{ip_1}{\omega \mu} (-A_1 \sin[p_1(x+\frac{\theta}{2}+d)] + A_1 \cos[p_1(x+\frac{\theta}{2}+d)]) \]

II. \(d < x < d+\theta_2\)
    \[ H_{zII} = \frac{ip_2}{\omega \mu} (-A_2 \sin[p_2(x-\frac{\theta}{2}-d)] + A_2 \cos[p_2(x-\frac{\theta}{2}-d)]) \]
Fig. 3

Cross-Section of
Coupled Slab Waveguides

Waveguide I

Waveguide II

TE mode

Waveguide III

Waveguide IV

Waveguide V
III. \( x \leq -d-\theta_1 \)
\[ H_{III} = \frac{i}{\omega \mu} (A_3 e^{\alpha_3 x}) \]

IV. \( -d \leq x \leq d \)
\[ H_{IV} = \frac{i}{\omega \mu} (-A_4 e^{-\alpha_4 x} + A'_4 e^{\alpha_4 x}) \]

V. \( d+\theta_1 \leq x \)
\[ H_{V} = \frac{i}{\omega \mu} (-A_5 e^{-\alpha_5 x}) \]

At the boundaries between the waveguides and the cladding, the electric field and the z-component of the magnetic field are continuous. Matching boundary conditions at:

\( x = -d-\theta_1 \):

\[ E_{III} = E_I \]
\[ A_3 e^{\alpha_3 (-d-\theta_1)} = A_1 \cos(p_1 \frac{-\theta_1}{2}) + A'_1 \sin(p_1 \frac{-\theta_1}{2}) \]

(21) \[ A_3 e^{-\alpha_3 (d+\theta_1)} - A_1 \cos(p_1 \frac{\theta_1}{2}) + A'_1 \sin(p_1 \frac{\theta_1}{2}) = 0 \]

\[ H_{III} = H_{II} \]
\[ \frac{i}{\omega \mu} [A_3 e^{\alpha_3 (-d-\theta_1)}] = \frac{i}{\omega \mu} (-A_1 \sin(p_1 \frac{-\theta_1}{2}) + A'_1 \cos(p_1 \frac{-\theta_1}{2})) \]

(22) \[ A_3 e^{-\alpha_3 (d+\theta_1)} - A_1 p_1 \sin(p_1 \frac{\theta_1}{2}) - A'_1 p_1 \cos(p_1 \frac{\theta_1}{2}) = 0 \]

\( x = -d \):

\[ E_I = E_{IV} \]
\[ A_1 \cos(p_1 \frac{\theta_1}{2}) + A'_1 \sin(p_1 \frac{\theta_1}{2}) = A_4 e^{\alpha_4 d} + A'_4 e^{-\alpha_4 d} \]

(23) \[ A_1 \cos(p_1 \frac{\theta_1}{2}) + A'_1 \sin(p_1 \frac{\theta_1}{2}) - A_4 e^{\alpha_4 d} - A'_4 e^{-\alpha_4 d} = 0 \]
\[ H_{zI} = H_{zIV} \]

\[ \frac{j\omega}{\mu} [ -A_1 \sin(p_{12}^0) + A_1 \cos(p_{12}^0) ] = \frac{j}{\omega} [ -A_4 e^{\alpha_4 d} + A_4 e^{-\alpha_4 d} ] \]

(24) \[ A_1 p_1 \sin(p_{12}^0) - A_1 p_1 \cos(p_{12}^0) - A_4 e^{\alpha_4 d} + A_4 e^{-\alpha_4 d} = 0 \]

\[ x = d: \]

\[ E_{IV} = E_{II} \]

\[ A_4 e^{-\alpha_4 d} + A_4 e^{\alpha_4 d} = A_2 \cos[p_2 (\frac{-\theta_2}{2})] + A_2 \sin[p_2 (\frac{-\theta_2}{2})] \]

(25) \[ A_4 e^{-\alpha_4 d} + A_4 e^{\alpha_4 d} - A_2 \cos(p_{22}^0) + A_2 \sin(p_{22}^0) = 0 \]

\[ H_{zIV} = H_{zII} \]

\[ \frac{1}{\omega} [ -A_4 e^{-\alpha_4 d} + A_4 e^{\alpha_4 d} ] = \frac{j\omega}{\mu} [ -A_2 \sin[p_2 (\frac{-\theta_2}{2})] + A_2 \cos[p_2 (\frac{-\theta_2}{2})] ] \]

(26) \[ A_4 e^{-\alpha_4 d} - A_4 e^{\alpha_4 d} + A_2 p_2 \sin(p_{22}^0) + A_2 p_2 \cos(p_{22}^0) = 0 \]

\[ x = d + \theta_2: \]

\[ E_{II} = E_{V} \]

\[ A_2 \cos(p_{22}^0) + A_2 \sin(p_{22}^0) = A_2 e^{-\alpha_5(d+\theta_2)} \]

(27) \[ A_2 \cos(p_{22}^0) + A_2 \sin(p_{22}^0) - A_2 e^{-\alpha_5(d+\theta_2)} = 0 \]
\[ H_{zII} = H_{zV} \]

\[
\frac{j}{\omega \mu} [-A_2 \sin(\frac{p_2 \theta_2}{2}) + A_2 \cos(\frac{p_2 \theta_2}{2})] = \frac{j}{\omega \mu} [-A_2 \alpha_5 e^{-\alpha_5 (d+\theta_2)}] \]

(28) \[ A_2 p_2 \sin(\frac{p_2 \theta_2}{2}) - A_2 \cos(\frac{p_2 \theta_2}{2}) - A_2 \alpha_5 e^{-\alpha_5 (d+\theta_2)} = 0 \]

On page 21, equations (21) - (28) are rewritten in matrix form. In order that the equations have a non-trivial solution, \( \det[C] = 0 \). The determinental equation that results from this matrix is the dispersion relation and is shown below:

(29) \[
[\alpha_4 \alpha_5 - p_4^2 + p_1 (\alpha_4 + \alpha_5) \cot(p_1 \theta_1)] [\alpha_4 \alpha_5 - p_4^2 + p_2 (\alpha_4 + \alpha_5) \cot(p_2 \theta_2)] \\
- e^{-4\alpha_4 d} [\alpha_4 \alpha_5 + p_4^2 + p_1 (\alpha_4 - \alpha_5) \cot(p_1 \theta_1)] [\alpha_4 \alpha_5 + p_4^2 + p_2 (\alpha_4 - \alpha_5) \cot(p_2 \theta_2)] = 0
\]

This equation is derived for a TE mode; however, the procedure can also be used for TM modes. One can obtain expressions for \( (\beta_1 - \beta_2) \) and \( |c_{12}| \) by developing equation (29) into the form of equation (14), knowing that for the weak coupling situation, the variables \( \alpha, \beta, \) and \( p \) deviate by only small perturbations from the single waveguide case. With only small variances in these terms, one can do a Taylor expansion around the single waveguide variables and substitute the expanded terms. The expansion is done around both waveguides so that when substituting into equation (29) the variables associated with a given waveguide...
\[
0 = [C] [A] = \\
\begin{bmatrix}
    e^{-\alpha_3(d+\theta_1)} & -\cos(p_1 \theta_1) & \sin(p_1 \theta_1) & 0 & 0 & 0 & 0 \\
    \alpha_3 e^{-\alpha_3(d+\theta_1)} & -p_1 \sin(p_1 \theta_1) & -p_1 \cos(p_1 \theta_1) & 0 & 0 & 0 & 0 \\
    0 & \cos(p_1 \theta_1) & \sin(p_1 \theta_1) & -e^{\alpha_4 d} & -e^{-\alpha_4 d} & 0 & 0 \\
    0 & p_1 \sin(p_1 \theta_1) & -p_1 \cos(p_1 \theta_1) & -\alpha_4 e^{\alpha_4 d} & \alpha_4 e^{-\alpha_4 d} & 0 & 0 \\
    0 & 0 & 0 & e^{-\alpha_4 d} & e^{\alpha_4 d} & -\cos(p_2 \theta_2) & \sin(p_2 \theta_2) \\
    0 & 0 & 0 & \alpha_4 e^{-\alpha_4 d} & -\alpha_4 e^{\alpha_4 d} & p_2 \sin(p_2 \theta_2) & p_2 \cos(p_2 \theta_2) \\
    0 & 0 & 0 & 0 & 0 & \cos(p_2 \theta_2) & \sin(p_2 \theta_2) & -e^{-\alpha_5(d+\theta_2)} \\
    0 & 0 & 0 & 0 & 0 & p_2 \sin(p_2 \theta_2) & -p_2 \cos(p_2 \theta_2) & -\alpha_5 e^{-\alpha_5(d+\theta_2)} \\
\end{bmatrix}
\]
will be placed into that part of equation (29) that is readily identified with a given waveguide.

Appendix I contains the derivation of the coupled mode equation from the determinental equation (29). Again, it is noted that the derivation is for a TE mode, but the same procedure can be used for TM modes. The result of the derivation is:

\[
\beta = \frac{\beta_1 + \beta_2}{2} \pm \left( \frac{\beta_1 - \beta_2}{2} \right)^2 + \frac{G_1 G_2}{F_1 F_2} e^{-2(\alpha_{41} + \alpha_{42})d} \frac{1}{2}
\]

where,

\[
F_1 = \frac{\beta_1}{p_01 \alpha_{41} \alpha_{03}} \left[ p_01^2 + a_4^2 \right] \left[ p_01 + a_0^2 \right] \left[ 1 + \frac{\alpha_{41} \alpha_{03} \theta_1}{(\alpha_{41} + \alpha_{03})} \right]
\]

\[
F_2 = \frac{\beta_2}{p_02 \alpha_{42} \alpha_{05}} \left[ p_02^2 + a_4^2 \right] \left[ p_02 + a_0^2 \right] \left[ 1 + \frac{\alpha_{42} \alpha_{05} \theta_2}{(\alpha_{42} + \alpha_{05})} \right]
\]

\[
G_1 = \alpha_{41} \alpha_{03} + p_01 + (\alpha_{41} - \alpha_{03}) \left[ -\frac{p_01 - \alpha_{41} \alpha_{03}}{(\alpha_{41} + \alpha_{03})} \right]
\]

\[
G_2 = \alpha_{42} \alpha_{05} + p_02 + (\alpha_{42} - \alpha_{05}) \left[ -\frac{p_02 - \alpha_{42} \alpha_{05}}{(\alpha_{42} + \alpha_{05})} \right]
\]

Equation (30) is identical in form to equation (14), the coupled mode equation. One can identify \(|c_{12}|\) as:

\[
|c_{12}| = \left( \frac{G_1 G_2}{F_1 F_2} \right)^{1/2} e^{-2(\alpha_{41} + \alpha_{42})d}
\]

The variables \(\alpha_{41}, \alpha_{03}, p_01, \beta_1,\) and \(\alpha_{42}, \alpha_{05}, p_02, \beta_2\) are
the single waveguide terms related through:

Waveguide I: \[
\begin{align*}
\alpha_{o3} &= (\beta_1^2 - \omega^2 \mu_3 \varepsilon_3)^{\frac{1}{2}} \\
\alpha_{41} &= (\beta_1^2 - \omega^2 \mu_4 \varepsilon_4)^{\frac{1}{2}} \\
 p_{o1} &= (\omega^2 \mu_1 \varepsilon_1 - \beta_1^2)^{\frac{1}{2}} \\
\tan(p_{o1} \theta_1) &= \frac{p_{o1}(\alpha_{41} + \alpha_{o3})}{(p_{o1}^2 - \alpha_{41} \alpha_{o3})}
\end{align*}
\]

Waveguide II: \[
\begin{align*}
\alpha_{o5} &= (\beta_2^2 - \omega^2 \mu_5 \varepsilon_5)^{\frac{1}{2}} \\
\alpha_{42} &= (\beta_2^2 - \omega^2 \mu_4 \varepsilon_4)^{\frac{1}{2}} \\
 p_{o2} &= (\omega^2 \mu_2 \varepsilon_2 - \beta_2^2)^{\frac{1}{2}} \\
\tan(p_{o2} \theta_2) &= \frac{p_{o2}(\alpha_{42} + \alpha_{o5})}{(p_{o2}^2 - \alpha_{42} \alpha_{o5})}
\end{align*}
\]

For the case of \( \theta_1 = \theta_2 = \theta, \varepsilon_1 = \varepsilon_2, \varepsilon_3 = \varepsilon_4 = \varepsilon_5, \) and \( \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5: \)

\[
\begin{align*}
\alpha_{o3} &= \alpha_{41} = \alpha_{42} = \alpha_{o5} = \alpha \\
p_{o1} &= p_{o2} = p \\
\beta_1 &= \beta_2 \\
(34) \beta &= \beta_1 \pm |c_{12}|
\end{align*}
\]

and \( |c_{12}| \) is one half the difference between the propagation constants derived from equation (30). Then:
(35) \[ |c_{12}| = \frac{a^2 p^2 e^{-2ad}}{\beta_1 (1 + \frac{a0}{2}) (a^2 + p^2)} \]

This is identical to one half the difference in propagation constants found from coupled mode theory by D. Marcuse with appropriate symbols changed.

In order to examine the accuracy of equation (30) in determining \( \beta \), to the \( \beta \)'s found from equation (29), a FORTRAN IV computer program, OP, was written to make a numerical comparison. The computer program assumes the case of equations (34) and (35). Under these conditions, equation (29) can be written as:

(36) \[ [a^2 - p^2 + 2ap \cot(p\theta)]^2 - (a^2 + p^2)^2 e^{-4ad} = 0 \]

Equation (36) can be factored into two simpler equations:

(37) \[ a^2 - p^2 + 2ap \cot(p\theta) - (a^2 + p^2) e^{-2ad} = 0 \]

(38) \[ a^2 - p^2 + 2ap \cot(p\theta) + (a^2 + p^2) e^{-2ad} = 0 \]

Since the cladding materials of the waveguides are identical, the following dispersion relation for single waveguides can be used to find \( \beta_1 \) and \( \beta_2 \):

(39) \[ a = p \tan\left(\frac{p0}{2}\right), \text{ or} \]

(40) \[ a - p \tan\left(\frac{p0}{2}\right) = 0 \]

This equation is used rather than the odd TE dispersion relation:
\[ \alpha + p \cot \left( \frac{P_2}{2} \right) = 0, \]

because it is the dispersion relation for the lowest order TE mode and only single mode operation is considered.

Equations (37), (38), and (40) form the basis for the computer program OP. Equations (37) and (38) are used to find the \( \beta \)'s that satisfy the determinental equation (29) of the coupled system. After solving equation (40) for \( \beta_1 \) and then \( |c_{12}| \) from equation (35), the \( \beta \)'s of the Taylor expansion (30) can be found from equation (34).

A listing of the program OP can be found in Appendix II, page 71. The parameters used in the program are:

\[ \begin{align*}
\theta &= 2 \text{ microns and 4 microns} \\
d &= 1 \text{ micron and 2 microns} \\
\lambda &= 6328 \text{ Angstroms and 1.06 microns} \\
\mu &= \mu_0 
\end{align*} \]

The dielectric constant \( \epsilon_1 \) was varied from 2\( \epsilon_0 \) to 8\( \epsilon_0 \), with \( \epsilon_1 - \epsilon_2 = \Delta \epsilon \) varying from 0.005\( \epsilon_0 \) to 0.1\( \epsilon_0 \). The program also calculates the coupling lengths for full power transfer.

One noteworthy result from the calculations is the ratio of the difference between the \( \beta \)'s of the Taylor expansion (34) and the dispersion relation equations (37) and (38), to the \( \beta \)'s of equations (37) and (38). It was found that the ratio has a maximum of two parts in 10^4, which indicates a close
agreement between the results of the Taylor expansion and the original determinental equation. It should be noted that the accuracy of the variables in program OP are calculated to the significant figure FORTRAN IV double precision can obtain.

The term identified with $|c_{12}|$ in equation (31) is the coupling constant of equation (14). Several results of the program for $|c_{12}|$ are illustrated in Graph 1, Graph 2, and Graph 3.

Graph 1, page 27, gives the relationships of $|c_{12}|$ plotted against $\Delta \varepsilon /\varepsilon_0$, for $\varepsilon_1 /\varepsilon_0 = 2$ and 8, and for both wavelengths and waveguide thicknesses. This graph shows several significant points. First, for increasing $\varepsilon_1$, the coupling constant decreases for the same $\Delta \varepsilon$, and for a constant $\varepsilon_1$ and increasing $\Delta \varepsilon$, the coupling constant also decreases. Further, the shorter the wavelength, the smaller the coupling constants. Lastly, the term identified with $|c_{12}|$ in equation (31) decreases after a $\Delta \varepsilon /\varepsilon_0$ at which there is a maximum. In this region to the left of the maximum, the Taylor expansion equation (34) continues to follow the dispersion relation; however, it is no longer valid to draw the analogy between equation (30) and the coupled mode equation (14).

Coupled mode theory would predict that as $\Delta \varepsilon$ decreased, the coupling constant would continue to increase. Coupled mode theory is based on small coupling between the waveguides in which "full power" is transferred from one guide to the
Graph 1

$|c_{12}|$ vs. $\Delta \varepsilon / \varepsilon_0$

$\theta = 2$ microns

$\lambda: 1.06$ microns

$\lambda: 6328$ Å

$d = 1$ micron
other. However, at a given $\Delta \varepsilon/\varepsilon_0$, the decay of the electro-
magnetic fields is small, and a large percentage of the power
remains in both guides, and though coupling takes place, there
is not a complete transfer. As an example, take the cases of
$\lambda = 1.06$ microns, $\varepsilon_1/\varepsilon_0 = 2$, $\theta = 2$ microns, and $\Delta \varepsilon/\varepsilon_0 = .005$
and .1. With a single waveguide, the largest electric field
for the lowest order TE mode is in the middle of the wave-
guide, and the TE field has a $\cos(px)$ dependence where $x=0$ is
in the center of the waveguide. At the boundary of the clad-
ding, the electric field has been reduced by a factor of .54
for $\Delta \varepsilon/\varepsilon_0=.1$ and .93 for $\Delta \varepsilon/\varepsilon_0=.005$. At a distance of 2
microns from the waveguide, the field is reduced by a factor
of .023 for $\Delta \varepsilon/\varepsilon_0=.1$ and .67 for $\Delta \varepsilon/\varepsilon_0=.005$. There is more
than 29 times as much electric field for $\Delta \varepsilon/\varepsilon_0=.005$ than for
$\Delta \varepsilon/\varepsilon_0=.1$. Further, since power per unit area is proportion-
al to electric field squared, there is over 800 times more
power available to couple to the second waveguide for $\Delta \varepsilon/\varepsilon_0=
.005$. The Taylor expansion equation (30) still follows the
dispersion relation (29) to an accuracy of two parts in $10^4$,
but it can no longer be considered analogous to the coupled
mode equation (14), and thus a limitation is imposed on the
correspondence between equation (30) and equation (14).
Equation (31) is valid only in the region the coupled mode
approximations are appropriate, and this is in the region
to the right of the maximum of the curves in Graph 1.
Graph 2, page 30 is a comparison of the $|c_{12}|$ of equation (31) for $\epsilon_1/\epsilon_0=8$ with $\theta = 2$ microns and $\theta = 4$ microns. As $\theta$ is increased, the $|c_{12}|$ decrease and the maximum shifts to the left. As mentioned earlier, $|c_{12}|$ is one half of the difference between the propagation constants derived from equation (34). For a comparison, plotted with $|c_{12}|$ is the difference between the $\beta$'s found from the determinental equation ($\Delta \beta$) divided by two. It can be seen from Graph 2, $\Delta \beta/2$ follows the curve of $|c_{12}|$ until the region the strong coupling begins to occur or near the maximum of each curve.

Another series of calculations were made to examine the effect of different distances between the waveguides. Graph 3, page 31, illustrates this for $d = 1$ micron and $d = 2$ microns. When $d$ is increased, the coupling constants decrease, and the $d = 2$ microns curve does not show a maximum as the $d = 1$ micron curve for the values of $\Delta \epsilon/\epsilon_0$ shown in the graph.

For the case that the waveguides are identical, the coupling length is inversely proportional to the coupling constants. The value of some of the coupling constants that comprise the graphs give values for coupling length of centimeters, millimeters, and submillimeters. These sizes are acceptable for developing a switch and being able to integrate it on a single chip a few centimeters long.

Knowing the characteristics of the coupled system for
Graph 2

\[ \Delta \beta/2 \text{ & } |c_{12}| \text{ vs. } \Delta \varepsilon/\varepsilon_0 \]

- \[ d = 1 \text{ micron} \]
- \[ \lambda = 1.06 \text{ microns} \]
- \[ \frac{\varepsilon_1}{\varepsilon_0} = 8 \]

- \[ \Delta \beta/2 : \hdashline \]
- \[ |c_{12}| : \hdashline \]

\[ \theta = 2 \text{ microns} \]
\[ \theta = 4 \text{ microns} \]
Graph 3

$|c_{12}|$ vs. $\Delta \varepsilon / \varepsilon_0$

$\lambda = 6328$ Å

$\theta = 2$ microns

d = 1 micron

d = 2 microns
various physical parameters, one can consider design and construction of a controllable device.
Equation (30) provides the information necessary to find power transfer and coupling length, and therefore device size and configurations can be determined. The two primary variables that can be changed are the power factor $F$ and the coupling length $l$. Both $l$ and $F$ are dependent upon $(\beta_1 - \beta_2)$ and $|c_{12}|$, which in turn are functions of $\omega$, $\varepsilon_1$, $\mu_1$, $d$, and $\theta_1$, and these are the physical variables of the switch. It is assumed that the switch is designed so that $d$ and $\theta_1$ are fixed, and that this is not a parametric device, hence $\omega$ is constant. Thus the two physical variables that can be changed are $\varepsilon_1$ and $\mu_1$. The dielectric constants, $\varepsilon_1$, can be varied using the electro-optic effect, and the permeability constants, $\mu_1$, can be changed by applying the magneto-optic effect. This discussion will be concerned with the electro-optic effect because of the primary interest in the electro-optic materials to construct modulators, switches, and waveguides, and because the magneto-optic effect is orders of magnitude smaller than the electro-optic effect. Therefore it is assumed that the $\mu_1$ do not change and that $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_0$.

With electro-optic materials, the $\varepsilon_1$ are modified by applying an electric field to that part of the switch that is comprised of these materials, thereby changing $(\beta_1 - \beta_2)$ and/or $|c_{12}|$. This can be done as shown in Fig. 4, Fig. 5,
and Fig. 6. Fig. 4a, page 35, has the electrodes on one of the waveguides, and Fig. 4b and Fig. 4c shows the electrodes over a part of the cladding. Another option is to place electrodes over several portions of the switch as shown in Fig. 4d. An additional method is to have the geometries as shown in Fig. 5 and Fig. 6, page 36. These two examples have the electrodes on one surface of the switch. Fig. 5 is an example of the electrodes on either side of the waveguides. Fig. 6 shows a bulk-slab waveguide configuration in which the electrodes are on one of the waveguides. The methods presented in Fig. 4, Fig. 5, and Fig. 6, accomplish the effect of changing $\varepsilon_1$ by having one or both waveguides changed by the electric field, and also if necessary the cladding. For the case of Fig. 6, the radiation is confined in the region of the waveguide between the electrodes. In all cases, the mode propagates in the z-direction and has a TE field with the electric field of the mode pointing in the y-direction. Any of these techniques can be used to enhance or impede coupling.

By varying the dielectric constants of the waveguides and/or cladding, both $F$ and $l$ can change, thus modulating power in terms of the maximum amount that will transfer in a given coupling length $l$, and the amount that will transfer in a fixed switch length. As an example, let the physical length of the switch be $L$ and $\varepsilon_2=\varepsilon_4=\varepsilon_5$, and let the
Fig. 4

Slab Waveguide Switch with Different Electrode Placement

Fig. 4a

electrodes

waveguides

cladding

Fig. 4b

Fig. 4c

Fig. 4d
Fig. 5

Slab Waveguide Switch with Electrodes on One Surface

![Diagram of Slab Waveguide Switch with Electrodes on One Surface]

Fig. 6

Bulk-Slab Switch

![Diagram of Bulk-Slab Switch]
power initially be all on waveguide 1. Then change the dielectric constant of one of the waveguides so that $\varepsilon_1 \neq \varepsilon_2$ and $l = L/2$. Since $(\beta_1 - \beta_2)$ is not zero, $F$ is reduced. At $l = L/2$ a percentage of the total power will be transferred out of waveguide 1. However, since the total length of the switch is $L$, the power will transfer out of the second waveguide. Hence, there is no power in line two once the signal is out of the switch, and thus no signal is switched. A second example is the case where $\varepsilon_1 = \varepsilon_2$, $\varepsilon_3 = \varepsilon_5$, and $\varepsilon_4$ is varied. $|c_{12}|$ is dependent upon $\alpha_4$, and $\alpha_4$ changes with $\varepsilon_4$. With this case, $F$ is not altered since $\beta_1 = \beta_2$, but $|c_{12}|$ is modified and thus $l$. Therefore, if the switch length is $L$, and $|c_{12}|$ is changed so that $l \neq L$, the signal will leave the switch so that power is not fully transferred to line two, or the power in line two is starting to couple back to line one. Thus one can modulate a signal by varying $F$ and/or $l$.

In addition to analog modulation as described above, a primarily on-off switch can be considered in which concern over changes in coupling length can be ignored. If $\varepsilon_1$ differs from $\varepsilon_2$ by a sufficiently large amount, $(\beta_1 - \beta_2)$ is large and $F$ can be made orders of magnitude smaller than one. Hence coupling is so small that essentially all the power remains on the line it starts with.

To examine the changes in power transfer and coupling length by varying the difference in dielectric constants of
the two waveguides ($\delta \varepsilon$), another FORTRAN IV computer program OPT was written. A listing of this program can be found in Appendix II, page 74. Because the literature for electro-optic materials uses index of refraction (n), the ensuing discussion will be in terms of n rather than dielectric constants. The values of n used in the program in the following examples are based on n=2.29 for the two waveguides when they are identical. This is the index of refraction of the ordinary ray axis of LiNbO$_3$, and is also the index of refraction of KTa$_{1-x}$Nb$_x$O$_3$ with x=.35. LiNbO$_3$ is a linear electro-optic material, while KTa$_{1-x}$Nb$_x$O$_3$ is a quadratic electro-optic material. Other parameters of the program are:

- $\theta = 2$ microns
- $d = 1$ micron
- $\lambda = 6328$ Angstroms
- $\varepsilon_3 = \varepsilon_4 = \varepsilon_5$
- $\left(\varepsilon_3/\varepsilon_0\right)^{\frac{1}{2}} = n_3 = 2.28$

The results of the program are plotted in Graph 4, page 39, and Graph 5, page 40, as functions of the absolute value of the change in index of refraction, $|\Delta n|$, of one of the waveguides. The other waveguide is held fixed at n=2.29. The data plotted is for, $+\Delta n$, an increase in index of refraction of one of the waveguides, and $-\Delta n$, a decrease in the index of refraction. Graph 4 shows coupling length versus
Graph 4

Coupling Length $\ell$ vs. $|\Delta n|$
Graph 5

Power Factor $F$ vs. $|\Delta n|$
$|\Delta n|$ and Graph 5 is a plot of $F$ versus $|\Delta n|$. The results demonstrate that coupling length and power factor decrease as $|\Delta n|$ increases. With the case of $-\Delta n$, the coupling of modes theory analogy of equations (30) and (31) may not be completely valid at a certain point because one of the waveguides index of refraction is beginning to approach that of the cladding. The electric field of this waveguide may no longer be proportionally small at the boundary of the cladding and the other waveguide when compared to the electric field inside of the changeable waveguide. However, for $-\Delta n$, coupling is still asynchronous and power is not fully transferred as is shown in Graph 5.

In terms of a device, the applied electric field ($E_a$) is critical. Dielectric breakdown must be avoided, and if $\Delta n$ is not large enough to effect a change in $l$ or $F$, the externally controlled optical switch based on the electro-optic effect is not feasible. In one case for LiNbO$_3$:

$$\Delta n = \frac{1}{2}(n_0^3 r_{13}) E_a$$

$n_0 = 2.29$ \(^4\)

$r_{13} = 8.6 \times 10^{-12}$ meter/volt

$$\Delta n = (5.16 \times 10^{-11}$ meter/volt) $E_a$$

For $\Delta n = .0001$, $E_a = 2 \times 10^6$ volts/meter. If the waveguide and electrodes of Fig. 4a are treated as an ideal capacitor, with an electrode separation of 2 microns, a potential of 4
volts is needed. If the radiation is polarized along the extraordinary index of refraction \((n_e=2.21)^4\) axis,

\[ \Delta n = \frac{1}{2}(n_e^2 r_{33}) E_a \]

\[ r_{33} = 30.8 \times 10^{-12} \text{ meter/volt}^4 \]

\[ \Delta n = (1.64 \times 10^{-10} \text{ meter/volt}) E_a \]

and a larger \(\Delta n\) is obtained for the same field. Of course for this case \(n_3\) must be less than 2.21. If \(\Delta n\) is to be on the order of 0.005, the applied electric field must be from 30 \(\times\) \(10^6\) volts/meter to 100 \(\times\) \(10^6\) volts/meter. This gives a potential on the waveguide of 60 volts to 200 volts. By this point, dielectric breakdown will occur, and for a device a few millimeters long and approximately 10 microns thick, these voltages are impractical. However, by using materials such as \(\text{KTa}_{1-x}\text{Nb}_x\text{O}_3\) one can cause changes of as much as \(\Delta n=0.005\) with electric fields of \(10^6\) volts/meter.

From a device point of view, electro-optic materials exist that can be used to create modulators and switches. This is true of analog switches and from Graph 5, \(\Delta n=0.005\), \(F=7 \times 10^{-5}\), which indicates that digital switches mentioned earlier can be constructed.

In addition to modulation and device size, speed of switching is also a consideration in the switch. If one treats the combination of waveguide with plates as a capacitor, and attached to a voltage source, one has a trans-
mission line as shown in Fig. 7, page 44. With the transmission line, one has the problems of added inductances and capacitances in the line caused by bends and varying dielectrics of the transmission line to the waveguide. However, if pulse time of greater than a nanosecond is intended, then the problems of additional reactance due to design and construction of the switch can be ignored as is done with integrated circuits. Given this, one has at the capacitor end of the transmission line an RC circuit equivalent as is illustrated in Fig. 8, page 44. The rise time of an ideal RC circuit is $\tau = RC$. Assuming the plates and waveguide represent an ideal parallel plate capacitor with a coupling length of one millimeter, a plate separation (h) of 2 microns, a waveguide thickness of 10 microns, and a DC dielectric constant of $\varepsilon = 10^5 \varepsilon_0$ for $KTa_{1-x}Nb_xO_3$:

$$C = \frac{\varepsilon \ell \theta}{h} = 443 \text{ picofarads},$$

$$R = \left( \frac{\mu_0}{\varepsilon_0} \right)^\frac{1}{2} = 370 \text{ ohms, therefore}$$

$$\tau = RC = .16 \text{ microseconds}$$

This ideal rise time is sufficiently fast to permit microsecond modulation. If $\varepsilon$ is the DC dielectric constant of $\text{LiNbO}_3$, then $\tau = 4.3 \times 10^{-12}$ seconds, and sub-microsecond modulation can be done.

From this single switch discussion, one can conclude
Fig. 7
Transmission Line Model

\[ Z_0 = \frac{1}{\sqrt{\mu_0}} \]

Fig. 8
RC Circuit at Capacitor

\[ R = Z_0 \]
that materials exist for construction of the optical switch and since full coupling length can be one millimeter or less, several switches can be made on a chip on the order of 2 centimeters. Thus one can extrapolate this discussion to multi-switch design and construction.
MULTI-SWITCH DESIGN

Knowing something of the characteristics of a single switch, multiple switching can be examined. Fig. 9, page 47, and Fig. 10, page 48, show two possible configurations for multi-switches when viewed from the top. Fig. 9 illustrates a random entrance into the multi-switch, in that the signal enters through any of the waveguides and can be switched to any of the other waveguides. Fig. 10 shows the signal brought into the switch through one waveguide and switched at the appropriate waveguide.

The basic multi-switch design presented in Fig. 9, has the capability to switch or modulate anywhere within the structure. There are several ways for switching and modulation to be done. It is assumed for argument that full coupling can occur, and that the application of an electric field to an electro-optic waveguide causes a reduction in coupling.

Let the signal initially start in guide 1 as is shown in Fig. 11a, page 49. In this illustration, an electric field has not been applied to any part of the switch, the dielectric constants of the waveguides are identical, and the cladding is all the same. After a distance z=L, power is transferred to guide 2, but before z=L, the signal has already begun to couple into guide 3 as shown in Fig. 11b.
Fig. 9

Random Entrance Multi-Switch
Fig. 10

Single Entrance Multi-Switch

waveguides

1

2d

3

4

θ

z

x

y
Fig. 11

Passive Multi-switch

Fig. 11a

Fig. 11b

at \( z = 0 \)

at \( z < L \)
With the situation of Fig. 11, the signal will be coupled to several waveguides upon leaving the switch. If a method of external control is not built into this multi-switch, the switch becomes a passive coupler and coupling will be to more than one waveguide.

If controlled coupling is intended, then several parts of the switch must be made of *electro-optic* materials so that upon application of an electric field the dielectric constants change. One example is to let the waveguides be electro-optic. As discussed earlier, if the difference in dielectric constants of the waveguides, $\delta\varepsilon$, is sufficiently large, the power factor $F$ can be reduced to $10^{-3}$ or less. Thus very little power is transferred between the waveguides. Again let the signal start in guide 1 and initially the waveguides are identical. If no transfer is intended, then an electric field can be applied to guide 2 of Fig. 11a. This creates a difference between guide 1 and guide 2 so that the signal remains in guide 1, or very little couples to guide 2 and can thus be ignored. Also a difference in dielectric occurs between guide 2 and guide 3, insuring that coupling is not permitted to guide 3. If full power is to be transferred to guide 2, then an electric field is applied to guide 3 and part of guide 1 as illustrated in Fig. 12a, page 51. From $z=0$ to $z=L$, full power is shifted to guide 2, and because of the $\delta\varepsilon$
Fig. 12

Electro-optic Waveguides

Fig. 12a

Fig. 12b

Area Electric Field is applied
between guide 2 and guide 3, no power or negligible power will transfer to guide 3. From \( z=L \) to \( z=2L \), the signal remains in guide 2 because of the dissimilar dielectrics of guide 2 and guide 1, and guide 2 and guide 3. Hence upon leaving the switch, the signal is in guide 2. For complete transfer to guide 3, an electric field is applied to part of guide 1 and part of guide 3 as shown in Fig. 12b, page 51. From \( z=0 \) to \( z=L \), the signal is switched to guide 2, and from \( z=L \) to \( z=2L \), the signal switches to guide 3, while nothing is returned to guide 1 because of \( \delta \epsilon \) between guide 2 and guide 1. Thus at \( z=2L \), the signal is in guide 3.

Besides making the waveguides electro-optic, the cladding can also be made of electro-optic materials, and thus a varying dielectric constant of the cladding can be utilized. As discussed before, by varying \( \epsilon_4 \) the coupling length \( l \) is modified. Hence, by making the material between the waveguides electro-optic, the dielectric \( (\epsilon_4) \) can be changed, and the device size can be reduced from \( 2L \) to \( L \). Assuming the waveguides are initially identical, and the signal is to be switched from guide 1 to guide 2, Fig. 13a, page 53, shows the application of an electric field to guide 3. At \( z=L \), the power has transferred to guide 2, but due to \( \delta \epsilon \) caused by the electric field, negligible power is switched to guide 3. If the signal is to be sent to guide 3, Fig. 13b, pictures the areas an electric field is imposed. A
Electro-optic Waveguides and Cladding

Fig. 13

Area Electric Field is applied

Fig. 13a

Fig. 13b
change in the dielectric of the cladding between the waveguides can reduce the difference in cladding and waveguide dielectrics ($\Delta \varepsilon$) which will enhance coupling and reduce the coupling length for full power transfer. With an appropriate decrease of $\Delta \varepsilon$, the full power coupling length can be reduced by half. As shown in Fig. 13b, an electric field is applied to the cladding between guide 1 and guide 2 in the region $z=0$ to $z=L/2$, and in the same distance, an electric field is put on guide 3. The dielectric of the cladding with the applied field permits full coupling from guide 1 to guide 2 in a coupling length of $L/2$ instead of $L$. From $z=0$ to $z=L/2$, no effective power transfers from guide 2 to guide 3 because of the $\delta \varepsilon$ between the two guides. By applying an electric field to guide 1 from $z=L/2$ to $z=L$, nothing returns from guide 2 to guide 1 since these guides are no longer identical. However, because no field is applied to guide 3, the signal will couple to guide 3 from guide 2, and with an electric field on the cladding in the region $z=L/2$ to $z=L$ between these two guides, the coupling length is also $L/2$. Hence at $z=L$, all the signal is in guide 3. Therefore, switch size can be reduced by taking advantage of the cladding. This case demonstrates how reduction in $\Delta \varepsilon$ can improve coupling; however, if $\Delta \varepsilon$ is increased, coupling is reduced. Thus increasing $\Delta \varepsilon$ and creating $\delta \varepsilon$, both can be used to prevent power transfer.
In the previous discussion, the electric fields applied to the waveguides can increase or decrease the dielectric. In either situation, the coupling factor $F$ decreases. One case of reducing the dielectric of a waveguide is to make the waveguide's dielectric constant equal to or less than that of the cladding. To illustrate this, return to Fig. 13a, page 53. An electric field applied to guide 3 can reduce the dielectric of that waveguide so that it no longer is a waveguide, and what was guide 3 is now part of the cladding. Thus a signal transferring to guide 2 from guide 1, will not couple out of guide 2 before $z=L$, and upon leaving the switch the signal remains in guide 2.

With these discussions, the application of an electric field has been used to cause total transfer to one waveguide or completely hinder transfer. The application of the electric fields can be arranged so that part of the signal in one waveguide can be coupled to a number of other guides, or mixing of signals among the waveguides can be done.

The second multi-switch geometry is shown in Fig. 14, page 56. Again the waveguides can be made so that either coupling is prevented or enhanced. In this case the signal is brought in along guide 1 and then totally or partially switched at the appropriate waveguide or waveguides. The shaded areas are the regions of the waveguides that coupling would occur. The electrode configuration of Fig. 14 will
Fig. 14

Single Entrance Multi-Switch with Electro-optic Waveguide
change the dielectric in the coupling areas of guide 1; however, other electrode placement can be considered.

Previously it has been assumed that the switch is planar or of a slab configuration; however, a multi-switch could be "three dimensional." Rather than slab-waveguides, circular guides can be used. Analysis of cross-talk for circular waveguides has been done using coupling of modes. An example of what this switch could look like is given in Fig. 15, page 58. One disadvantage is that it would be difficult to make because of its geometry. Also there is the problem that the electrodes can interact with the external fields of the waveguide signal, thereby distorting the electric fields and introducing losses due to the resistivity of the plates.
Multi-Switch with Circular Waveguides
CONSTRUCTION OF AN OPTICAL SWITCH

As mentioned earlier, a few comments will be directed to construction of a switch. Several possibilities exist for fabrication of the waveguides. R. V. Schmidt and I. P. Kaminov have diffused transition metals such as Ti, V, and Ni into LiNbO$_3$ to form low-loss TE and TM mode waveguides, and LiNbO$_3$ is an electro-optic material. W. E. Martin and D. B. Hall reported fabrication of optical waveguides by diffusion in II-VI compounds, and with H. F. Taylor and V. N. Smiley have made "single-crystal semiconductor optical waveguides." Another means of making the switch is ion implantation that has been demonstrated by S. Namba, H. Aritome, T. Nishimura, and K. Masuda. Their report indicated H$^+$, H$_2^+$, Li$^+$, and B$^+$ ions were implanted in fused quartz to form waveguides. A third method of manufacturing the waveguides of the switch is an electron beam technique developed by D. B. Ostrowsky, M. Papuchon, A. M. Roy, and J. Trotel. This system has been used to make curved waveguides and couplers. In addition to these three methods that use crystalline materials, V. Ransaswamy and H. P. Weber have authored an article on "Low-Loss Polymer Films with Adjustable Refractive Index." Each of these methods have reported waveguide size of two microns thickness or less, and waveguide separation on the order of one micron.
Beyond the problem of making the waveguides, are the difficulties of constructing the total switch and eventually integrating it. These problems include fabrication of electrodes and insulation of the leads of the electrodes from each other. This problem is especially acute in the construction of multi-switches.
CONCLUSION

Coupled mode theory provides information concerning coupling length and power factor. The approach in determining these variables has been through the dispersion relation of the coupled two waveguide system. From this dispersion relation a Taylor expansion is made about each waveguide to find the propagation constants. Once the Taylor expansion is made, the result is in a form identical to the coupled mode equation. From this the coupling constant can be identified as is found in equation (31).

A numerical comparison of the β's of the Taylor expansion and the dispersion relation gives a limit to the correspondence between the results of the Taylor expansion and coupled mode theory. This occurs when coupling is no longer weak and the term identified as the coupling constant in equation (31) reaches a maximum as the difference between waveguide and cladding dielectric constants is reduced. Coupling of modes theory would indicate that the coupling constant would increase indefinitely as the difference in dielectric decreased. However, coupling of modes is not valid for strong coupling and the analogy between the Taylor expansion and coupled mode theory is not appropriate. The maximum of the Taylor expansion is the limiting point of coupled mode theory and the analogy. The Taylor expansion
still follows the curve of the dispersion relation, but the error begins to increase in the region of strong coupling.

In regard to device size and configuration, coupling lengths calculated from the coupling constants indicate that the optical switch can be made in millimeter and submillimeter lengths. This permits integration of several switches on a single chip centimeters in length. Further materials exist that permit electro-optic control of switching and modulation, thereby giving one an externally controlled device. By understanding the single switch, multi-switches can be designed in a number of configurations and electro-optic control schemes. Finally methods exist for construction of the switch; however, materials and construction are areas to further be explored.

There are limitations to the theory and discussion presented here. The model of slab waveguides is an idealization with perfect boundaries and materials. Further, devices would most likely be made of rectangular cross-section waveguides which introduce problems of boundary matching at the corners of the rectangle. Also, switch design does not consider difficulty in constructing the switch, or loss problems associated with different materials.

The approach taken in this thesis demonstrates that the coupled mode expression can be derived from the dispersion relation of the coupled system, and that it is feasible to
consider the construction of optical switches. This paper provides one approach to optical switches and modulators, and the eventual objective of developing integrated optics.
APPENDIX I

Derivation of Coupled Mode Equation from the Determinental Equation

Rewriting the determinental equation, one has:

\[ \begin{align*}
\text{AI.1} \\
\left[ \alpha_4 \alpha_5 - p_1^2 + p_1 (\alpha_4 + \alpha_5) \cot(p_1 \theta_1) \right] \\
\left[ \alpha_4 \alpha_5 - p_2^2 + p_2 (\alpha_4 + \alpha_5) \cot(p_2 \theta_2) \right] - \\
e^{-2\alpha_4 d} \left[ \alpha_4 \alpha_5 + p_1^2 + p_1 (\alpha_4 - \alpha_5) \cot(p_1 \theta_1) \right] \\
e^{-2\alpha_4 d} \left[ \alpha_4 \alpha_5 + p_2^2 + p_2 (\alpha_4 - \alpha_5) \cot(p_2 \theta_2) \right] = 0
\end{align*} \]

At this point one can do a Taylor expansion of the above determinental equation around the single waveguides knowing that for weak coupling, \( \alpha_1, \beta_1, \) and \( p_1 \) vary by only a small amount from the single waveguide values. The expansion terms for each waveguide are:

\[ \begin{align*}
\text{Waveguide I:} \\
\alpha_5 &= \alpha_0 + \delta \alpha_3 = (\beta^2 - \omega^2 \mu_3 \varepsilon_3)^{\frac{1}{2}} \\
\alpha_4 &= \alpha_4 + \delta \alpha_4 = (\beta^2 - \omega^2 \mu_4 \varepsilon_4)^{\frac{1}{2}} \\
p_1 &= p_0 + \delta p_1 = (\omega^2 \mu_1 \varepsilon_1 - \beta^2)^{\frac{1}{2}} \\
\beta &= \beta_1 + \delta \beta_1 \\
\alpha_0 &= (\beta_1^2 - \omega^2 \mu_3 \varepsilon_3)^{\frac{1}{2}} \\
\alpha_4 &= (\beta_1^2 - \omega^2 \mu_4 \varepsilon_4)^{\frac{1}{2}}
\end{align*} \]
\[ p_{01} = (\omega^2 \mu_1 \epsilon_1 - \beta_1^2)^{\frac{1}{2}} \]

\[ \alpha_3 = (\frac{\partial \alpha_3}{\partial \beta_1^2}) \delta \beta_1 = \frac{\beta_1}{\alpha_3} \delta \beta_1 \]

\[ \alpha_4 = (\frac{\partial \alpha_4}{\partial \beta_1^2}) \delta \beta_1 = \frac{\beta_1}{\alpha_4} \delta \beta_1 \]

\[ p_1 = (\frac{\partial p_1}{\partial \beta_1^2}) \delta \beta_1 = \frac{\beta_1}{p_{01}} \delta \beta_1 \]

**Waveguide II:**

\[ \alpha_5 = \alpha_{05} + \delta \alpha_5 = (\beta_2^2 - \omega^2 \mu_5 \epsilon_5)^{\frac{1}{2}} \]

\[ \alpha_4 = \alpha_{42} + \delta \alpha_4 = (\beta_2^2 - \omega^2 \mu_4 \epsilon_4)^{\frac{1}{2}} \]

\[ p_2 = p_{02} + \delta p_2 = (\omega^2 \mu_2 \epsilon_2 - \beta_2^2)^{\frac{1}{2}} \]

\[ \beta = \beta_2 + \delta \beta_2 \]

\[ \alpha_{05} = (\beta_2^2 - \omega^2 \mu_5 \epsilon_5)^{\frac{1}{2}} \]

\[ \alpha_{42} = (\beta_2^2 - \omega^2 \mu_4 \epsilon_4)^{\frac{1}{2}} \]

\[ p_{02} = (\omega^2 \mu_2 \epsilon_2 - \beta_2^2)^{\frac{1}{2}} \]

\[ \alpha_5 = (\frac{\partial \alpha_5}{\partial \beta_2^2}) \delta \beta_2 = \frac{\beta_2}{\alpha_{05}} \delta \beta_2 \]

\[ \alpha_4 = (\frac{\partial \alpha_4}{\partial \beta_2^2}) \delta \beta_2 = \frac{\beta_2}{\alpha_{42}} \delta \beta_2 \]

\[ p_2 = (\frac{\partial p_2}{\partial \beta_2^2}) \delta \beta_2 = \frac{\beta_2}{p_{02}} \delta \beta_2 \]

Further,

\[ \cot(p_{i1}) = \cot(p_{01} \theta_i) - \delta p_{i1} \csc^2(p_{01} \theta_i), \text{ where } i=1,2 \]
Also from this discussion:

\[ e^{-2\alpha_4 d} = e^{-2(\alpha_{41} + \delta\alpha_{41}) d}; \]

however, since this is for weak coupling \( \alpha_{41} \gg \delta\alpha_{41} \), and

\[ e^{-2\alpha_4 d} = e^{-2\alpha_{41} d} \]

The expanded terms of AI.2, AI.5, AI.8, and AI.9 derived above can be substituted into equation AI.1 so that the expanded terms that are derived from a given single waveguide are placed in that part of the determinental equation that can be associated with a specific waveguide. The result is:

\[ e^{-2(\alpha_{41} + \delta\alpha_{41})} \] [(\( \alpha_{41} + \delta\alpha_{41} \))(\( \alpha_{03} + \delta\alpha_{3} \))-(\( p_{01} + \delta p_{1} \))²+(\( p_{01} + \delta p_{1} \))(\( \alpha_{41} + \delta\alpha_{41} + \alpha_{03} + \delta\alpha_{3} \))

(cot(\( p_{01} \theta_{1} \))-\( \delta p_{1} \theta_{1} \)csc²(\( p_{01} \theta_{1} \)))][(\( \alpha_{42} + \delta\alpha_{42} \))(\( \alpha_{05} + \delta\alpha_{5} \))-(\( p_{02} + \delta p_{2} \))²+(\( p_{02} + \delta p_{2} \))(\( \alpha_{42} + \delta\alpha_{42} + \alpha_{05} + \delta\alpha_{5} \))\( (\cot(\( p_{02} \theta_2 \))-\( \delta p_{2} \theta_2 \)csc²(\( p_{02} \theta_2 \)))\]

\[ - e^{-2(\alpha_{41} + \delta\alpha_{41})} \] [(\( \alpha_{41} + \delta\alpha_{41} \))(\( \alpha_{03} + \delta\alpha_{3} \))-(\( p_{01} + \delta p_{1} \))²+(\( p_{01} + \delta p_{1} \))(\( \alpha_{41} + \delta\alpha_{41} + \alpha_{03} + \delta\alpha_{3} \))

(cot(\( p_{01} \theta_{1} \))-\( \delta p_{1} \theta_{1} \)csc²(\( p_{01} \theta_{1} \)))][(\( \alpha_{42} + \delta\alpha_{42} \))(\( \alpha_{05} + \delta\alpha_{5} \))+(\( p_{02} + \delta p_{2} \))²+(\( p_{02} + \delta p_{2} \))(\( \alpha_{42} + \delta\alpha_{42} + \alpha_{05} + \delta\alpha_{5} \))

(cot(\( p_{02} \theta_2 \))-\( \delta p_{2} \theta_2 \)csc²(\( p_{02} \theta_2 \))) = 0

Keeping only those terms that are first order in \( \delta \), and using the approximation for \( e^{-2\alpha_4 d} \) in equation AI.9, one has:
The term $e^{-2\alpha_4 i d} < 1$, and $\delta p_1$ and $\delta a_1$ are also small compared to $p_{01}$ and $a_1$. Therefore multiplying $e^{-2\alpha_4 i d}$ by $\delta p_1$ and $\delta a_1$ gives even smaller variables and can be ignored when compared to other terms in the equation. Further it is assumed that the system operates sufficiently far from cutoff that the cosecant and cotangent terms do not contribute significantly to offset the small values of $\delta p_1$ and $\delta a_1$. Upon eliminating those terms that are products of $e^{-2\alpha_4 i d}$ and $\delta p_1$ or $\delta a_1$, equation AI.11 becomes:
AI.12

\[ [\alpha_4 \alpha_{03} + \alpha_{03} \delta \alpha_4 + \alpha_4 \delta \alpha_3 - p_0^2 \delta p_1 + p_0 (\alpha_4 \alpha_{03}) \cot (p_0 \theta_1) - \\
\rho_1 (\alpha_4 \alpha_{03}) \delta p_1 \csc^2 (p_0 \theta_1) + p_0 (\delta \alpha_4 + \delta \alpha_3) \cot (p_0 \theta_1) + (\alpha_4 + \\
\alpha_{03}) \delta p_1 \cot (p_0 \theta_1)] \times [\alpha_4 \alpha_{05} + \alpha_{05} \delta \alpha_4 + \alpha_4 \delta \alpha_5 - p_0^2 \delta p_2 + \\
p_0 (\alpha_4 \alpha_{05}) \cot (p_0 \theta_2) - p_0 (\alpha_4 \alpha_{05}) \delta p_2 \csc^2 (p_0 \theta_2) + p_0 \delta \alpha_4 + \\
\delta \alpha_5) \cot (p_0 \theta_2) + (\alpha_4 \alpha_{05}) \delta p_2 \cot (p_0 \theta_2)] = e^{-2\alpha_{41} d} \times \alpha_4 \alpha_{03} + \\
p_0^2 + p_0 (\alpha_4 - \alpha_{03}) \cot (p_0 \theta_1)] \times e^{-2\alpha_{42} d} \times \alpha_4 \alpha_{05} + p_0^2 + p_0 (\alpha_4 - \alpha_{05}) \cot (p_0 \theta_2) = 0

Substituting the terms in AI.4 and AI.7 for \( \delta p_1 \) and \( \delta a_1 \), and grouping terms in \( \delta \beta_1 \), equation AI.12 changes to:

AI.13

\[ [\alpha_4 \alpha_{03} - p_0^2 + p_0 (\alpha_4 \alpha_{03}) \cot (p_0 \theta_1) + \beta_1 \delta \beta_1 \left( \frac{\alpha_{03}}{\alpha_4} + \frac{\alpha_{03}}{\alpha_4} \right) + 2 + \\
(\alpha_4 \alpha_{03}) \theta_1 \csc^2 (p_0 \theta_1) + p_0 (\frac{1}{\alpha_4} + \frac{1}{\alpha_{03}}) \cot (p_0 \theta_1) - (\alpha_4 \alpha_{03}) \cot (p_0 \theta_01)]] \times [\alpha_4 \alpha_{05} - p_0^2 + p_0 (\alpha_4 \alpha_{05}) \cot (p_0 \theta_2) + \beta_2 \delta \beta_2 \\
(\frac{\alpha_{05}}{\alpha_4} + \frac{\alpha_{05}}{\alpha_4} \csc^2 (p_0 \theta_2) + p_0 \delta \alpha_4 + \frac{1}{\alpha_{05}}) \cot (p_0 \theta_2) - (\alpha_4 \alpha_{05}) \frac{1}{p_0^2} \cot (p_0 \theta_2)] = e^{-2(\alpha_{41} \alpha_{42}) d} \\
[\alpha_4 \alpha_{03} + p_0 (\alpha_4 \alpha_{03}) \cot (p_0 \theta_1)] \times [\alpha_4 \alpha_{05} + p_0 (\alpha_4 \alpha_{05}) \cot (p_0 \theta_2)] = 0

- 68 -
Since the approximations are based on a single waveguide, one can use the dispersion relation for a single waveguide with different cladding to determine $\cot(p_{01} \theta_1)$ and $\csc^2(p_{01} \theta_1)$:

**Waveguide I:**
$$\tan(p_{01} \theta_1) = \frac{p_{01}(\alpha_{41} + \alpha_{03})}{(p_{01}^2 - \alpha_{41} \alpha_{03})}$$

**Waveguide II:**
$$\tan(p_{02} \theta_2) = \frac{p_{02}(\alpha_{42} + \alpha_{05})}{(p_{02}^2 - \alpha_{42} \alpha_{05})}$$

Trigonometrically, one has the following relations for:

\begin{align*}
\text{AI.14} & \ csc^2(p_{01} \theta_1) = \frac{(p_{01}^2 + \alpha_{41}^2)(p_{01}^2 + \alpha_{03}^2)}{p_{01}^2 (\alpha_{41} + \alpha_{03})^2} \\
\text{AI.15} & \ csc^2(p_{02} \theta_2) = \frac{(p_{02}^2 + \alpha_{42}^2)(p_{02}^2 + \alpha_{05}^2)}{p_{02}^2 (\alpha_{42} + \alpha_{05})^2} \\
\text{AI.16} & \ cot(p_{01} \theta_1) = \frac{p_{01} - \alpha_{41} \alpha_{03}}{p_{01} (\alpha_{41} + \alpha_{03})} \\
\text{AI.17} & \ cot(p_{02} \theta_2) = \frac{p_{02} - \alpha_{42} \alpha_{05}}{p_{02} (\alpha_{42} + \alpha_{05})}
\end{align*}

Substituting these expressions into equation AI.13 and simplifying, one has:

\begin{align*}
\text{AI.18} & \ \delta \beta_1 \left[ \frac{\beta_{1}}{p_{01} \alpha_{41} \alpha_{03}} \left( \frac{p_{01}^2 + \alpha_{41}^2}{p_{01}^2 + \alpha_{03}^2} \right) \left( 1 + \frac{\alpha_{41} \alpha_{03} \beta_1}{\alpha_{41} + \alpha_{03}} \right) \right] \\
& \ \delta \beta_2 \left[ \frac{\beta_{2}}{p_{02} \alpha_{42} \alpha_{05}} \left( \frac{p_{02}^2 + \alpha_{42}^2}{p_{02}^2 + \alpha_{05}^2} \right) \left( 1 + \frac{\alpha_{42} \alpha_{05} \beta_2}{\alpha_{42} + \alpha_{05}} \right) \right]
\end{align*}
\[ e^{-2(a_{41} + a_{42})d} \left[ a_{41}a_{03} + p_0^2 + (a_{41} - a_{03}) \left( -\frac{p_0^2}{a_{41} + a_{03}} \right) \right] \]

\[ [a_{42}a_{05} + p_0^2 + (a_{42} - a_{05}) \left( -\frac{p_0^2}{a_{42} + a_{05}} \right) = 0] \]

Let,

\[ F_1 = \frac{\beta_1}{p_0^2a_{41}a_{03}} \left[ p_0^2 + \alpha_{41}^2 \right] \left[ p_0^2 + \alpha_{03}^2 \right] \left[ 1 + \frac{a_{41}a_{03}p_0^2}{a_{41} + a_{03}} \right] \]

\[ F_2 = \frac{\beta_2}{p_0^2a_{42}a_{05}} \left[ p_0^2 + \alpha_{42}^2 \right] \left[ p_0^2 + \alpha_{05}^2 \right] \left[ 1 + \frac{a_{42}a_{05}p_0^2}{a_{42} + a_{05}} \right] \]

\[ G_1 = a_{41}a_{03} + p_0^2 + (a_{41} - a_{03}) \left( -\frac{p_0^2}{a_{41} + a_{03}} \right) \]

\[ G_2 = a_{42}a_{05} + p_0^2 + (a_{42} - a_{05}) \left( -\frac{p_0^2}{a_{42} + a_{05}} \right) \]

and knowing that \( \delta \beta_1 = \beta - \beta_1 \), equation AI.18 can be written as:

\[ F_1F_2(\beta - \beta_1)(\beta - \beta_2) - G_1G_2 e^{-2(a_{41} + a_{42})d} = 0 \]

\[ (\beta - \beta_1)(\beta - \beta_2) - \frac{G_1G_2}{F_1F_2} e^{-2(a_{41} + a_{42})d} = 0 \]

\[ \beta = \frac{\beta_1 + \beta_2}{2} \pm \left( \left( \frac{\beta_1 - \beta_2}{2} \right)^2 + \frac{G_1G_2}{F_1F_2} e^{-2(a_{41} + a_{42})d} \right)^{\frac{1}{2}} \]

Equation AI.23 is in the same form as the coupled mode equation (14), where \( |c_{12}| \) can be identified as:

\[ |c_{12}| = \left( \frac{G_1G_2}{F_1F_2} \right)^{\frac{1}{2}} e^{-2(a_{41} + a_{42})d} \]
APPENDIX II

Program OP

00010 C PROGRAM OP BY NICK VLANNES
00020 DOUBLE PRECISION C, D, WL, PI, W, PM, E1, E3, E, TH, BE
00030 DOUBLE PRECISION COM, COM2, P, A, E, EX, G, FUNC, BT
00040 DOUBLE PRECISION DIS, COTPBACL, C12, BOLD, BNEW, B2
00050 DIMENSION BT(2), BE(2), CL(5), C12(5)
00060 C = 2.997925D8
00070 D = 2.D-6
00080 WL = 6.328D-7
00090 PI = 3.141592654D0
00100 W = 2.*PI*C/WL
00110 PM = PI*4.D-7
00120 E = 1./(PM*C*C)
00130 TH = 2.D-6
00140 DO 600 K = 2, 8, 6
00150 EE1 = K
00160 WRITE(6, 130) EE1
00170 130 FORMAT('EE1 = ', F6.3, /)
00180 DO 570 I = 1, 5
00190 EE3 = EE1 - .02D0*I
00200 WRITE(6, 170) EE3
00210 170 FORMAT('EE3 = ', F6.3)
00220 E1 = EE1*E
00230 E3 = EE3*E
00240 COM = W*W*E1*PM - (PI/TH)**2
00250 COM2 = W*W*PM*E3
00260 IF (COM .LT. COM2) COM = COM2
00270 M = -1
DO 550 L=1,2
IF(L.EQ.2)M=1
BOLD=DSQRT(COM)
BNEW=W*DSQRT(PM*E1)
IC=0
52 B=(BOLD+BNEW)/2.
IC=IC+1
B2=B*B
P=DSQRT(W*W*E1*PM-B2)
A=DSQRT(B2-COM2)
COTP=DCOS(P*TH)/DSIN(P*TH)
EX=M*DEXP(-2.*A*D)
G=A*A+P*P
DIS=A*A-P*P+2.*A*P*COTP
FUNC=DIS+EX*G
IF(IC-40)51,440,440
51 IF(FUNC)53,440,54
53 BOLD=B
GO TO 52
54 BNEW=B
GO TO 52
BE(L)=B
IF(L-2)400,550,550
400 BOLD=DSQRT(COM)
BNEW=W*DSQRT(PM*E1)
IC=0
300 B=(BOLD+BNEW)/2.
IC=IC+1
520 B2=B*B
P=DSQRT(W*W*E1*PM-B2)
A=DSQRT(B2-COM2)
TAN=DSIN(P*TH/2.)/DCOS(P*TH/2.)
FUNC=A-P*TAN
IF (IC-40) 490, 480, 490
IF (FUNC) 444, 480, 470
BOLD=B
GO TO 300
BNEW=B
GO TO 300
G=A*A+P*P
F=B*G*G*(1.+A*TH/2.)/(P*P*A*A)
C12(I)=(G/F)*DEXP(-2.*A*D)
CL(I)=PI/(2.*C12(I))
BA=B
WRITE(6, 41) BA, C12(I), CL(I)
BT(L)=BA+M*C12(I)
IF (BE(1)-BE(2)) 594, 594, 560
B=BE(1)
BE(1)=BE(2)
BE(2)=B
WRITE(6, 595)
FORMAT(' ', 8X, 'BETA DISP.', 23X, 'BETA CM')
WRITE(6, 706) BE(1), BE(2), BT(1), BT(2)
FORMAT(F14.4, F14.4, 4X, F14.4, F14.4, 4X)
CONTINUE
WRITE(6, 610)
FORMAT(/ /)
CONTINUE
STOP
END
Program OPT

00010  C     PROGRAM OPT BY NICK VLANNES
00020  D     DOUBLE PRECISION C,D,PM,WL,PI,EP,TH,E,G1,G2,F1,F2,
          C     COM,W,A4
00030  D     DOUBLE PRECISION B,P,A,FUNC,BOLD,BNEW,XN,XM,Q,R,P2,
          D     FM,FN,PHI
00040  D     DOUBLE PRECISION BBB,BW,BA,C12,F,SC12,COTPCSC2,LEFT,CL
00050  D     DIMENSION E(5),COM(5),P(2),A(7)
00060  D     DIMENSION BA(2),BW(2),A4(2),GP(2)
00070  D     C=2.997925D8
00080  D     PHI=(DSQRT(5.0DO)-1.)/2.
00090  D     D=1.2D-6
00100  D     WL=6.328D-7
00110  D     PI=3.141592654D0
00120  D     FM=PI*4.D-7
00130  D     EP=1./(PM*C*C)
00140  D     TH=2.*D-6
00150  D     W=2.*PI*C/WL
00160  D     WRITE(6,10)
00170  10    FORMAT('RN')
00180  D     READ(5,12) RN
00190  12    FORMAT(F8.4)
00200  D     E(1)=5.2441*EP
00210  D     E(2)=RN*RN*EP
00220  D     E(3)=5.1984*EP
00230  D     E(4)=5.1984*EP
00240  D     E(5)=5.1984*EP
00250  D     COM(1)=W*W*PM*E(1)-(PI/TH)**2
00260  D     COM(2)=W*W*PM*E(2)-(PI TH)**2
00270  D     DO 100 J=3,5
00280  100   COM(J)=W*W*PM*E(J)
00290  D     DO 110 J=3,4
00300 IF(COM(1).LT.COM(J)) COM(1)=COM(J)
00310 IF(COM(2).LT.COM(J+1)) COM(2)=COM(J+1)
00320 110 CONTINUE
00330 DO 650 J=1,2
00340 BOLD=DSQRT(COM(J))
00350 BNEW=W*DSQRT(PM*E(J))
00360 XM=BOLD+PHI*(BNEW-BOLD)
00370 LCK=1
00380 B=XM
00390 GO TO 520
00400 294 FM=FUNC
00410 LCK=2
00420 IC=0
00430 300 XN=BNEW+PHI*(BOLD-BNEW)
00440 IC=IC+1
00450 B=XN
00460 520 P(J)=DSQRT(W*W*PM*E(J)-B*B)
00470 P2=P(J)*P(J)
00480 COS=DCOS(P(J)*TH)
00490 A(J)=DSQRT(B*B-W*W*PM*E(J+2))
00500 A(J)=DSQRT(B*B-W*W*PM*E(J+3))
00510 CSC2=(1./COS)**2
00520 COTP=COS/DSIN(P(J)*TH)
00530 FUNC=COTP+(A(J)*A(J+1)-P2)/(P(J)*(A(J)+A(J+1)))
00540 FUNC=DABS(FUNC)
00550 IF(FUNC-1.D-7)655,655,490
00560 490 IF(LCK-1)294,294,495
00570 495 IF(IC-34)500,655,655
00580 500 Q=IDINT(10.*BNEW+.5)
00590 R=IDINT(10.*BOLD+.5)
00600 IF(Q-R)550,655,550
00610 550 IF(FM-FUNC)440,470,470
00620 440 BOLD=BNEW
00630  BNEW=XN
00640  GO TO 300
00650  470  BNEW=XM
00660  FM=FUNC
00670  XM=XN
00680  GO TO 300
00690  655  F1=B*(P2+A(J)*A(J))*(P2+A(J+1)*A(J+1))/(P2*A(J)*A(J+1))
00700  F2=1.+A(J)*A(J+1)*TH/(A(J)+A(J+1))
00710  K=1
00720  A4(J)=A(J+1)
00730  IF(J.EQ.2)A4(J)=A(J)
00740  IF(J.EQ.2)K=3
00750  G1=(A4(J)-A(J))*(P2-A(J)*A(J+1))/(A(J)+A(J+1))
00760  G2=P2+A(J)*A(J+1)
00770  GF(J)=(G1+G2)/(F1*F2)
00780  650  BW(J)=B
00790  SC12=GF(1)*GF(2)*DEXP(-2.*(A4(1)+A4(2))*D)
00800  C12=DSQRT(SC12)
00810  BB=.5*DSQRT((BW(1)-BW(2))**2+4.*SC12)
00820  BA(1)=(BW(1)+BW(2))/2.-BB
00830  BA(2)=(BW(1)+BW(2))/2.+BB
00840  WRITE(6,710) BW(),BW(2),BB,C12
00850  710  FORMAT(' BI=',F10.0,4X,'B2=',F10.0,4X,'BB=',F6.0,4X,'C12=',F6.0,/)  
00860  WRITE(6,770)
00870  770  FORMAT(' COUPLING OF MODES')
00880  WRITE(6,790) BA(1),BA(2)
00890  790  FORMAT(' SMALLEST BETA=',F10.0,4X,'LARGEST BETA=',F10.0,/)  
00900  F=(C12/BB)**2
00910  CL=PI/(2.*BB)
00920  WRITE(6,830) F,CL
00930  830  FORMAT(' POWER FACTOR=',D12.4,'COUPLING LENGTH=',D12.4,/)  

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>00940</td>
<td>GO TO 151</td>
<td></td>
</tr>
<tr>
<td>00950</td>
<td>END</td>
<td></td>
</tr>
</tbody>
</table>

- 77 -
REFERENCES


