THE USE OF PIEZO-CERAMICS AS DISTRIBUTED ACTUATORS IN FLEXIBLE SPACE STRUCTURES

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ABSTRACT

Distributed segmented piezo-electric actuators bonded to an elastic substructure and embedded inside a laminated composite material are modeled. Static mechanical models for various actuator geometries exciting both extension and flexure in the substructural material are developed. The flexural static models for both surface bonded and embedded cases are then integrated into a simple dynamic model for a cantilevered Bernoulli-Euler beam. This integration leads to the ability to predict, a priori, the response of the structural member to an excitation voltage applied to the piezo-electric. These models are experimentally verified for the first several modes of an aluminum cantilevered beam with eight piezo-ceramics bonded on the surface near the root and for a cantilevered glass/epoxy beam with four embedded piezo-ceramics. A scaling analysis demonstrates that the effectiveness of piezo-electrics scales with the structure, and allows the evaluation of candidate piezo-electric materials.

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NOMENCLATURE

\( a \) = Nondimensional piezo-electric center, normalized by \( \ell \)

\( A \) = Cross-sectional area of cantilevered beam

\( A_0 \) = Cross-sectional area at root

\( b \) = Width of the beam and of the piezo-electric

\( B \) = Modal damping of the beam

\( d_{31} \) = Piezo-electric constant

\( E_h \) = Young's modulus of the substructure

\( E_c \) = Young's modulus of the piezo-electric

\( \ell_{\text{max}} \) = Maximum possible applied piezo-electric field

\( F \) = Force applied at the pins.

\( F_k \) = Lengthwise varying force resultant

\( F_k' \) = Value of \( F_k \) at the right edge of actuator

\( F_k'' \) = Value of \( F_k \) at the left edge of actuator

\( F_1 \) = Force applied at the lower pin of imbedded actuator

\( F_2 \) = Force applied at the upper pin of imbedded actuator

\( \Delta F \) = \( F_2 - F_1 \)

\( \bar{F} \) = \( F_1 + F_2 \)

\( G \) = Shear modulus of the bonding layer

\( I \) = Moment of inertia

\( X \) = Non-dimensional stiffness

\( K \) = Modal stiffness of the beam

\( K_{\text{PIEZO}} \) = Modal piezo-electric stiffness

\( \ell \) = Length of cantilevered beam

\( L \) = Length of the piezo-electric

\( \ell \) = Nondimensional piezo-electric length

\( m \) = Mass per unit length of cantilevered beam

\( \ell \) = Nondimensional piezo-electric half-length

\( M \) = Modal mass of cantilevered beam

\( M_k \) = Lengthwise varying moment resultant

\( M_k' \) = Value of \( M_k \) at the right edge of actuator
\[ M' = \text{Value of } M_k \text{ at the left edge of actuator} \]
\[ M = \text{Non-dimensional modal mass} \]
\[ q = \text{Generalized modal coordinate} \]
\[ Q = \text{Modal force} \]
\[ t_b = \text{Thickness of the substructure} \]
\[ t_c = \text{Thickness of the piezo-electric} \]
\[ t_s = \text{Thickness of the bonding layer} \]
\[ \tilde{t}_s = \text{Nondimensional bonding layer thickness} \]
\[ u_b = \text{Displacement of the substructure} \]
\[ u_c = \text{Displacement of the piezo-electric} \]
\[ V = \text{Applied voltage across the piezo-electric} \]
\[ w = \text{Beam displacement in } z \text{ direction} \]
\[ x = \text{Piezo-electric centered coordinate} \]
\[ \tilde{x} = \text{Nondimensional coordinate } (2x/L) \]
\[ \bar{x} = \text{Non-dimensional coordinate from beam root} \]
\[ z = \text{Thickness coordinate} \]
\[ z_i = \text{Coordinate of lower edge of imbedded actuator} \]
\[ z_m = \text{Coordinate of mid-plane of imbedded actuator} \]
\[ \gamma = \text{Modulus ratio of beam to piezo-electric} \]
\[ \tau_s = \text{Shear strain in the bonding layer} \]
\[ \Gamma_b = \text{Non-dimensional bending shear transfer parameter} \]
\[ \Gamma_e = \text{Non-dimensional extensional shear transfer parameter} \]
\[ \epsilon_b = \text{Strain in the substructure} \]
\[ \epsilon_b^+ = \text{Substructure strain at the right edge of the actuator} \]
\[ \epsilon_b^- = \text{Substructure strain at the left edge of the actuator} \]
\[ \epsilon_b^s = \text{Strain on the surface of the beam} \]
\[ \epsilon_b^{s+} = \text{Surface strain at the right edge of actuator} \]
\[ \epsilon_b^{s-} = \text{Surface strain at the left edge of actuator} \]
\[ \epsilon_c = \text{Strain in the piezo-electric} \]
\[ \zeta = \text{Damping ratio} \]
\[ \theta_b = \text{Thickness ratio of substructure to piezo-electric} \]
\[ \theta_s = \text{Thickness ratio of bonding layer to piezo-electric} \]
\[ \theta_z = \text{Thickness ratio of } z_m \text{ to piezo-electric} \]
\( \lambda_v \) = Piezo-electric linear feedback gain
\( \Lambda \) = Piezo-electric strain term \( (= \frac{d_{31}V}{t_c}) \)
\( \rho \) = Density of cantilevered beam
\( \sigma_b \) = Stress in the substructure
\( \sigma_b^s \) = Stress on the surface of the substructure
\( \sigma_c \) = Stress in the piezo-electric
\( \tau \) = Stress in the bonding layer
\( \tau^* \) = Non-dimensional shear stress \( (\tau/G) \)
\( \phi \) = Mode shape
\( \phi \) = Effective stiffness ratio
\( \omega_0 \) = Natural beam frequency
CHAPTER ONE: INTRODUCTION

The missions envisioned for large space structures include a functional requirement for control of both rigid body and elastic deformations. In the past, control of spacecraft has been largely limited to the control of rigid body modes, since the elastic vibrations were small as compared to the overall rigid body motion, and at sufficiently high frequency to be outside the bandwidth of the controller. Sometimes, as in the case of launch vehicles, structural bending modes could fall within the controller bandwidth. However, this could be accounted for in the design of the controller by notch-filtering around the bending mode frequency. Since future large space structures will be assembled in orbit, their design does not have to account for the large stresses encountered during launch from earth. Therefore, these structures can be very large and flexible and would have a large number of elastic modes at low frequencies. At the same time, however, many of the missions envisioned for these structures, such as large communication antennas or deep space observation platforms, require a large degree of pointing accuracy. Therefore, any control scheme that is used on these spacecraft must be able to control all the elastic vibrations that interfere with the performance of the mission and not just the rigid body motion.

In the traditional approach to the control of flexible
structures, the detection and control of these elastic vibrations would be carried out by a small number of sensors and actuators whose placement is usually predetermined by design considerations other than those relevant to the control of the structure. This approach requires that the modes of the structure be known to a high degree of accuracy in order to provide high authority control of the controlled modes yet avoid spillover of control into the modes not modelled by the controller. As space structures become larger and more complex, the dynamic behavior becomes more difficult to predict. The problem is compounded by the difficulty in performing an accurate and complete ground test program for a large structure. These difficulties can lead to on-orbit open loop behavior that differs substantially from pre-flight ground test measurements or analytical predictions.

If the choice of locations for the small number of sensors and actuators were made based on erroneous predictions or ground-test measurements, on-orbit performance of the actuators and sensors could be greatly diminished, or possibly even destabilizing. With a small number of actuators the ability to reconfigure an on-orbit system with software may be limited. However, if a highly distributed actuator network were used, software adjustments to modify and tune the closed loop behavior could be easily implemented. In addition, the failure of an individual sensor or actuator would not be critical, due to the large number of sensors and actuators that would be present.
Since the actuators in such a system must be highly distributed, it is desirable that they not modify extensively the passive mechanical and dynamic properties of the system. Piezo-electric materials, which exhibit mechanical deformation when an electric field is applied to them, could be used for this purpose. Bonding these materials to the surface of structural elements, or embedding them in a laminated structure, would allow the application of localized stresses and strains through which the deformations of the structural elements could be modified and controlled. In addition, the application of these localized stresses and strains allow these actuators to directly control the local deformation of the structure to which they are attached without applying rigid body torques and forces.

Since piezo-electrics can also produce a voltage when a mechanical deformation is applied to them, they have been used extensively in the past in transducers such as strain gauges and accelerometers. PZT ceramics, a type of piezo-electric material, has been used experimentally for structural excitation of turbomachinery\cite{1}\cite{2} and other components \cite{3}\cite{4} and for the experimental damping of simple dynamic structures \cite{5}. More recently, piezo-electric film, a low modulus piezo-electric material, has been used as an actuator \cite{6} and as a sensor\cite{7}. However, an analytic model of the mechanical coupling of bonded segmented piezo-electric actuators to the dynamics of the structural member has not been formulated. The term segmented
actuator is defined to be an actuator that can provide different forcing levels at different locations along the length of the structure. For example, a continuous actuator bonded along the surface of a cantilevered beam would only be able to apply a constant strain everywhere along the beam. Segmented actuators on the surface of the same beam could apply varying forcing levels, depending on their location. As will be seen in Chapter Four, the motion produced by a disturbance exciting various modes in the cantilevered beam cannot be damped effectively by a continuous actuator. A segmented actuator however, can efficiently damp out the higher vibrational modes.

Both an analytic and experimental investigation of this concept have been conducted, leading to the capability to predict a priori the motion of the structural member in response to a specified excitation voltage across the piezo-electric actuator. Initially, in order to establish the fundamental elastic relationships of the piezo-electric-substructure system, static analyses were performed for a variety of actuator configurations. In Chapter Two, complete static shear lag analyses for surface-bonded extensional and moment piezo-electric actuators are developed along with simplified models for these two systems that assume a perfect bond between the actuator and the substructure. In addition, embedded extensional and moment actuators are also modelled by assuming a perfect bond between the actuator and the material in which it is embedded. Shear lag models for these
embedded systems were not developed since the interlaminar bonding layer between the actuator and the surrounding material is thin enough such that the resulting shear can be neglected with a small loss in accuracy. In Chapter Three, both the surface and embedded moment actuator models were coupled to a dynamic structural model of a Bernoulli-Euler cantilevered beam. Using this analysis, the behavior of the beam under loading from the piezo-electric actuators could be predicted. Two experimental specimens were constructed and tested. These specimens consisted of an aluminum beam with piezo-ceramics bonded near the root and a glass/epoxy composite beam with embedded piezo-ceramics. The manufacturing and testing procedure for both these beams, along with the experimental results, are shown in Chapter Four. Finally, a scaling analysis examining how the effectiveness of a piezo-electric actuator changes with the scale of the structure is performed in Chapter Five for both force and moment actuators.
Chapter Two: Static Elastic Analysis of Bonded and Embedded Actuators

In this chapter, static elastic analyses for surface bonded and embedded actuators which excite extension and bending in the substructure will be presented. For the surface bonded cases, two models will be derived. In the first, simpler model, the actuators will be assumed to be perfectly bonded to the substructure or surrounding material. This is equivalent to assuming the strain in the actuator is equal to the strain in the substructure at the interfaces. In the second, more complete model, a complete elasticity solution will be performed which incorporates elastic and finite thickness bonding layer properties. The results of this more complete analysis will be shown to reduce to the results obtained from the perfectly bonded cases as the characteristic length of the shear lag in the bonding layer goes to zero. For the embedded cases, only the simple, perfectly bonded models will be derived. It is assumed that the interlaminar bonding layer between the actuators and the surrounding material is sufficiently thin such that neglecting the shear will not introduce any significant errors to the final model.
2.1 Surface-Bonded Piezo-Electric Actuators In Extension

The simplest application of piezo-electric actuators is to bond them to an elastic substructure and drive them so as to excite extension. In this section, two models will be discussed. First, the piezo-electric will be assumed to be perfectly bonded to the substructure. This is the case where the displacement of the substructure is equal to that of the piezo-electric, as if the piezo-electric were pinned at its free edges. Second, a more exact model incorporating a shear layer of a known finite thickness will be presented. This more exact model will be shown to reduce to the perfectly bonded model when the shear lag in the bonding layer is reduced to zero.

2.1.1 Perfectly Bonded Extensional Model

Fig. 2.1 shows a pair of piezo-electric actuators perfectly bonded to the surface of a bar. An externally applied, lengthwise varying stress resultant \( F_k \) is assumed to act on the bar. The load \( F \) is assumed to be transferred from the piezo-electric actuator to the bar by "pins" at its edges. By using the equilibrium, stress-strain and compatibility relationships for this system, the strains in the piezo-electric and in the substructure will be determined. The equilibrium relationships
for the piezo-electric ceramic and the bar are

\[ \sigma_c = \frac{F}{bt_c} \]  

(2.1)

\[ \sigma_b = -\frac{2F}{bt_b} - \frac{F_k}{bt_b} \]  

(2.2)

where \( F \) is the force on the pins. The factor of two in the first term of Eq. 2.2 is due to the assumption that two actuators are bonded to the bar on its upper and lower surfaces.

The stress-strain relationship for a piezo-electric is similar to that of a material under thermal loading\[^8\] , the thermal strain term \( \alpha \Delta T \) term being replaced by the piezo-strain term \( \frac{d_{31}V}{t_c} \). The term \( d_{31} \) is the piezo-electric constant relating \( V \), the applied voltage across the piezo-electric of thickness \( t_c \) to the strain produced. Therefore, the stress-strain relationship for the piezo-electric is

\[ \varepsilon_c = \frac{\sigma_c}{E_c} + \frac{d_{31}V}{t_c} = \frac{\sigma_c}{E_c} + \Lambda \]  

(2.3)

where \( \frac{d_{31}V}{t_c} = \Lambda = \) the piezo-electric strain

For the beam, the stress-strain relationship is

\[ \varepsilon_b = \frac{\sigma_b}{E_b} \]  

(2.4)
The strain-displacement relationships for the piezo-electric and the bar are

\[ \varepsilon_c = \frac{du_c}{dx} \]  \hspace{2cm} (2.5)
\[ \varepsilon_b = \frac{du_b}{dx} \]  \hspace{2cm} (2.6)

Finally, under these ideal conditions, the displacement of the piezo-electric and of the beam is the same. Therefore,

\[ u_c = u_b \]  \hspace{2cm} (2.7)

The seven equations (Eq. 2.1 through 2.7) governing the perfectly bonded model have now been derived. These can now be solved for the seven unknown variables \( \sigma_c, \sigma_b, \varepsilon_c, \varepsilon_b, u_c, u_b, \) and \( F \). The known quantities are the externally applied force \( F_k \), including the values of this force at the edges \( F^+ \) and \( F^- \). Differentiating Eq. 2.7, inserting Eq. 2.5 and 2.6 and combining Eq. 2.1 through 2.4, \( \frac{F}{b} \) is found to be

\[ \frac{F}{b} = \frac{-1}{2 + \varphi} \left[ \frac{F_k}{b} \right] - \left[ \frac{A}{2 + \varphi} \right] E_b t_b \]  \hspace{2cm} (2.8)

where the parameter \( \varphi \) represents the relative extensional stiffness of the substructure and the piezo-electric, and is equal to

- 22 -
\[ \psi = \frac{E_b t_b}{E_c t_c} \]

Inserting Eq. 2.8 into either Eq. 2.5 or Eq. 2.6, the strain in the piezo-electric and the beam is

\[ \varepsilon_b = \varepsilon_c = \frac{-\psi}{2 + \psi} \left[ \frac{F_k}{E_b t_b b} \right] + \left[ \frac{2\alpha}{2 + \psi} \right] \quad (2.9) \]

where the strain has been expressed as a continuous function in \( x \) of the externally applied force resultant \( F_k \). Note that this force resultant is the only quantity which varies over the length of the piezo-electric actuator. When the exact details of the distribution of this force are not known, it is often useful to express the force \( F_k \) as a function of \( F^+ \) and \( F^- \), where \( F^+ \) and \( F^- \) are the values of the force at the right edge of the actuator and at the left edge, respectively. In order to accomplish this, \( F_k \) can be approximated by,

\[ F_k = F_0 + F_1 \bar{x} + F_2 \bar{x}^2 + F_3 \bar{x}^3 + \ldots \quad (2.10) \]

Where \( \bar{x} \) is the non-dimensional coordinate centered on the piezo-electric actuator segment (Fig. 2.2). With only two boundary values of \( F_k \), only the first two terms in the series will be used. Evaluating \( F_k \) at the ends of the segment, \( \bar{x} = 1 \) and \( \bar{x} = -1 \), and using the definitions given above for \( F^+ \) and \( F^- \),
\[ F^+ = F_0 + F_1 \]  
(2.11)  
\[ F^- = F_0 - F_1 \]  
(2.12)

or, solving these two equations for \( F_0 \) and \( F_1 \)

\[ F_0 = \frac{F^+ + F^-}{2} \]  
(2.13)  
\[ F_1 = \frac{F^+ - F^-}{2} \]  
(2.14)

Inserting Eqs. 2.13 and 2.14 into Eq. 2.10, an expression for the force \( F_k \) as a function of the value of the force at the boundaries of the actuator can be obtained. Substituting into Eq. 2.9, an expression for the strains as a function of the forces at the edges of the bar and of the piezo-electric strain term \( \Delta \) is obtained,

\[ \varepsilon_k = \varepsilon_c = \frac{-\varphi}{2 + \varphi} \left[ \frac{F^+ + F^-}{2B_b t_b} + \frac{F^+ - F^-}{2B_b t_b} x \right] + \left[ \frac{2\Delta}{2 + \varphi} \right] \]  
(2.15)

Eq. 2.15 can also be rewritten,

\[ \varepsilon_k = \varepsilon_c = \frac{\varphi}{2 + \varphi} \left[ \frac{\varepsilon_k^+ + \varepsilon_k^-}{2} + \frac{\varepsilon_k^+ - \varepsilon_k^-}{2} x \right] + \left[ \frac{2\Delta}{2 + \varphi} \right] \]  
(2.16)

where \( \varepsilon_k^+ \) and \( \varepsilon_k^- \) are the values of the strain due to the force \( F_k \) at the right and left edges of the actuator. From Eq. 2.2 and 2.4
these are equal to,

\[ \varepsilon_b^+ = -\frac{F^+}{E_b t_b} \]
\[ \varepsilon_b^- = -\frac{F^-}{E_b t_b} \]  \hspace{1cm} (2.17)

\[ \varepsilon_b = \varepsilon_c = \left[ \frac{2A}{2 + \psi} \right] \hspace{1cm} (2.18) \]

\[ \varepsilon_b = \varepsilon_c = \left[ \frac{2A}{2 + \psi} \right] \hspace{1cm} (2.19) \]

If \( \varepsilon_b^+ \) and \( \varepsilon_b^- \) are set to zero, i.e., the externally applied distributed force \( F_k \) is set to zero, Eq. 2.16 reduces to,

\[ \varepsilon_b = \varepsilon_c = \left[ \frac{2A}{2 + \psi} \right] \hspace{1cm} (2.19) \]

Eq. 2.19 implies that the amount of strain in the beam \( \varepsilon_b \), depends directly on the piezo-electric strain \( A \), and indirectly on the relative stiffness and thickness of the beam to the piezo-electric. A small value for the relative stiffness parameter \( \psi \), corresponding to a high-modulus or thick piezo-electric bonded to a relatively thin or low-modulus beam would produce strain in the beam which approaches the free piezo-electric strain. Higher modulus substructures will produce larger values of \( \psi \), thus reducing the effective transfer of strain into the substructure.

2.1.2 Shear Lag Extensional Model

Fig. 2.2 shows two piezo-electric elements bonded by a finite
thickness bonding layer to an elastic substructure. The geometry and elastic properties of the piezo-electric, bonding layer and substructure are assumed to be known. The piezo-electrics are polarized so that when electric fields of equal magnitudes are applied in the directions of the double arrows, the piezo-electrics expand in the x direction, causing extension of the substructure. Taking advantage of the symmetry of the problem about the x axis which lies on the centroid of the elastic substructure, the differential element outlined by the dashed area A in Fig. 2.2 is examined (Fig. 2.3).

A one dimensional elasticity solution will be performed. It is assumed that:

- The bonding layer carries only shear stresses.
- The beam and the piezo-electric carry only normal stresses.

The elasticity relations for the element will be presented. They will then be reduced to a pair of uncoupled fourth order differential equations for the strain in the piezo-electric \( \varepsilon_c \) and the strain in the substructure \( \varepsilon_b \). The differential equations will be solved and appropriate boundary conditions will be applied in order to obtain a complete solution.

The strain-displacement relationships for the piezo-electric, beam and shear layer are

\[
\varepsilon_c = \frac{d u_c}{d x} = u_c',
\] (2.20)
Balancing the forces on the isolated differential piezo-electric and elastic substructure elements leads to the following equations

\[(\sigma_c + d\sigma_c) b_t c - \sigma_c b_t c - \tau b dx = 0\]
\[(\sigma_b + d\sigma_b) b_t b - \sigma_b b_t b + 2 \tau b dx = 0\]

where again the presence of the factor of two in the last term of the substructure equilibrium equation is due to the assumption that two actuators are bonded on the upper and lower surfaces of the substructure. These equations reduce to the two differential equations of equilibrium for the piezo-electric and the substructure

\[\frac{d\sigma_c}{dx} - \frac{\tau}{t_c} = 0\]  \hspace{1cm} (2.23)
\[\frac{d\sigma_b}{dx} + \frac{2\tau}{t_b} = 0\]  \hspace{1cm} (2.24)

The stress-strain relationships for the piezo-electric, the substructure, and the bonding layer are

\[\sigma_c = E_c (\epsilon_c - \frac{d_{31}\gamma}{t_c}) = E_c (\epsilon_c - \Lambda)\]  \hspace{1cm} (2.25)
\( \sigma_b = E_b \varepsilon_b \) \hspace{1cm} (2.26)
\( \tau = G \gamma_s \) \hspace{1cm} (2.27)

Eqs. 2.20 through 2.27 are the eight governing elastic equations. The eight unknowns in these equations are the stresses and strains in the piezo-electric, the substructure, and the bonding layer, \( \sigma_c, \sigma_b, \tau, \varepsilon_c, \varepsilon_b, \gamma_s \), and the displacements in the piezo-electric and the substructure, \( u_c \) and \( u_b \). These equations can now be reduced to a pair of fourth order differential equations.

Differentiating and combining the equilibrium equations (Eqs. 2.22, and 2.23) and the bonding layer equations (Eqs. 2.24 and 2.27), and substituting in the remaining equations (Eqs. 2.20, 2.21, 2.25 and 2.26), the following two equations are obtained

\[ \varepsilon_c'''' - \frac{G}{E_c t_c t_s} (\varepsilon_c - \varepsilon_b) = 0 \] \hspace{1cm} (2.28)
\[ \varepsilon_b'''' + \frac{2G}{E_b t_b t_s} (\varepsilon_c - \varepsilon_b) = 0 \] \hspace{1cm} (2.29)

These are the two governing differential equations for the strain in the system. In order to non-dimensionalize Eqs. 2.28 and 2.29 the geometric and elastic constants are non-dimensionalized by the piezo-electric properties, resulting in the following parameters

\[ x = \frac{x}{L/2} \]
\[ \theta_b = \frac{t_b}{t_c} \]
\[ \gamma = \frac{E_b}{E_c} \]
\[ G = \frac{G}{E_c} \]
\[ t_s = \frac{t_s}{L/2} \]
\[ \theta_s = \frac{t_s}{t_c} \]
Substituting into Eqs. 2.28 and 2.29, this leads to a pair of non-dimensional, coupled, second-order differential equations:

\[ \dddot{\varepsilon}_c - \frac{G\theta_s}{t_s^2} (\dddot{\varepsilon}_c - \dddot{\varepsilon}_b) = 0 \]  \hspace{1cm} (2.30)

\[ \dddot{\varepsilon}_b + \frac{2G\theta_s}{\gamma \theta_b t_s^2} (\dddot{\varepsilon}_c - \dddot{\varepsilon}_b) = 0 \]  \hspace{1cm} (2.31)

where the overbar refers to differentiation with respect to \( \bar{x} \).

Eqs. 2.30 and 2.31 can be further reduced to a single fourth order differential equation by solving for \( \dddot{\varepsilon}_c \) as a function of \( \dddot{\varepsilon}_b \) in Eq. 2.28:

\[ \dddot{\varepsilon}_c = \dddot{\varepsilon}_b - \frac{\gamma \theta_b t_s^2}{2G\theta_s} \dddot{\varepsilon}_b \]  \hspace{1cm} (2.32)

Differentiating Eq. 2.32 twice and substituting into Eq. 2.30 yields a single fourth order equation in \( \dddot{\varepsilon}_b \):

\[ \dddot{\varepsilon}_b - \frac{G\theta_s}{t_s^2} \left[ \frac{2 + \theta_b \gamma}{\theta_b \gamma} \right] \dddot{\varepsilon}_b = 0 \]  \hspace{1cm} (2.33)

Alternatively, Eq. 2.29 can be solved for \( \dddot{\varepsilon}_b \), differentiated and substituted into Eq. 2.31 to obtain a single fourth order equation in \( \dddot{\varepsilon}_c \).
These two fourth order equations are now uncoupled and can be solved independently. In order to do this, a set of four boundary conditions in \( \varepsilon_b \) would be necessary to find the solution to Eq. 2.33. Alternatively, a set of four boundary conditions in \( \varepsilon_c \) would be required to solve Eq. 2.34. However, boundary conditions are routinely expressed in terms of a mixture of the variables \( \varepsilon_c \) and \( \varepsilon_b \). Therefore, it is convenient to solve these two equations simultaneously, even though they appear to be independent.

Solutions to this set of equations are of the form

\[
\varepsilon_b = B_1 + B_2 x + B_3 \sinh \Gamma_\phi x + B_4 \cosh \Gamma_\phi x
\]

\[
\varepsilon_c = C_1 + C_2 x + C_3 \sinh \Gamma_\phi x + C_4 \cosh \Gamma_\phi x
\]

where the parameter \( \Gamma_\phi \) is defined by

\[
\Gamma_\phi^2 = \frac{\bar{G} \gamma_s}{t_s^2} \left[ \frac{2 + \theta_b \gamma}{\theta_b \gamma} \right]
\]

It appears that there are eight independent constants. However, since each of the original governing equations (Eqs. 2.20 and 2.27) are only second order, only four constants should be necessary to specify the solution. Therefore, some of the constants \( C_i \) and \( B_i \) must be related. In order to reduce these
eight constants to four, Eq. 2.35 is substituted into Eq. 2.32

\[ \varepsilon_c = B_1 + B_2 \bar{x} + B_3 \sinh \Gamma_e \bar{x} + B_4 \cosh \Gamma_e \bar{x} \]

\[ - \frac{\gamma \theta_b t^2}{2 \bar{G} \theta_s} \left[ B_3 \Gamma_e \sinh \Gamma_e \bar{x} + B_4 \Gamma_e \cosh \Gamma_e \bar{x} \right] \]

Substituting for \( r^2 \) from Eq. 2.37 and comparing term by term with Eq. 2.36, the constants \( C_i \) are related to the constants \( B_i \) and the following solution set is obtained

\[
\begin{bmatrix}
\varepsilon_c \\
\varepsilon_b
\end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} B_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} B_2 \bar{x} + \begin{bmatrix} \frac{\phi}{2} \\ 1 \end{bmatrix} B_3 \sinh \Gamma_e \bar{x}
\]

\[
+ \begin{bmatrix} \frac{\phi}{2} \\ 1 \end{bmatrix} B_4 \cosh \Gamma_e \bar{x}
\]

The governing equations for the strain in the beam and in the piezo-electric (Eq. 2.20 and 2.27) have now been solved and the boundary conditions can now be applied in order to determine the constants \( B_i \). It should be noted that the piezo-electric strain term \( A \) does not appear explicitly in Eq. 2.39. However, it will be shown to enter into the equations through the boundary conditions.

For any particular geometry, the \( B_i \) constants can be evaluated by applying the appropriate boundary conditions, which depend on the manner in which the piezo-electrics are bonded to
the elastic substructure. It should be noted that a segmented actuator on the surface of the substructure always implies that the edges of the piezo-electric are stress free. Therefore, from Eq. 2.25 it is seen that to satisfy this boundary condition \( \varepsilon_c \) must equal the piezo-electric strain term \( \frac{d_{31}V}{t_c} \). This is not required for the edges of the substructure. Therefore, non-homogeneous boundary conditions on \( \varepsilon_b \) are possible. The boundary conditions are therefore

\[
\begin{align*}
\theta \bar{x} = +1 & \quad \varepsilon_c = \frac{d_{31}V}{t_c} = \Lambda & \quad \varepsilon_b = \varepsilon_b^+ \\
\theta \bar{x} = -1 & \quad \varepsilon_c = \frac{d_{31}V}{t_c} = \Lambda & \quad \varepsilon_b = \varepsilon_b^-
\end{align*}
\]

where \( \varepsilon_b^+ \), \( \varepsilon_b^- \) are known strain values at the locations of extreme left (-) and right (+) ends of the piezo-electric segment.

Substituting these boundary conditions into Eq. 2.39 the constants \( B_i \) are

\[
B_1 = \frac{\varphi \left[ \frac{\varepsilon_b^+ + \varepsilon_b^-}{2} \right] + 2\Lambda}{\varphi + 2} \quad (2.40)
\]

\[
B_2 = \frac{\varphi}{\varphi + 2} \left[ \frac{\varepsilon_b^+ - \varepsilon_b^-}{2} \right] \quad (2.41)
\]

\[
B_3 = \frac{\varphi}{(\varphi + 2) \sinh F_0} \left[ \frac{\varepsilon_b^+ - \varepsilon_b^-}{2} \right] \quad (2.42)
\]
\[ B_4 = \frac{2}{(\varphi + 2) \cosh \Gamma_0} \left[ \frac{\bar{\varepsilon}_b^+ + \bar{\varepsilon}_b^-}{2} - 1 \right] \] (2.43)

The complete solution for the strains in terms of the geometric and elastic properties of the substructure and the piezo-electric has now been determined. It is instructive to examine the strain free substrate case. This corresponds to the boundary conditions on the substructure, \( \bar{\varepsilon}_b^+ \) and \( \bar{\varepsilon}_b^- \), being set to zero. In this case, the solution for the strains reduces to

\[
\begin{bmatrix}
\bar{\varepsilon}_e \\
\bar{\varepsilon}_b
\end{bmatrix}
= \begin{bmatrix}
1 \\
1
\end{bmatrix}
\begin{bmatrix}
\frac{2}{\varphi + 2} \\
\frac{2}{\varphi + 2}
\end{bmatrix}
\begin{bmatrix}
\frac{\varphi}{2} \\
\frac{2}{\varphi + 2}
\end{bmatrix}
\cosh \Gamma_0 x
\cosh \Gamma_0
\] (2.44)

The appropriate boundary conditions have been applied and the total solution for the strains in the piezo-electric and in the substructure has been found. It is now possible to examine Eq. 2.44 in more detail in order to understand how variations in the two parameters \( \varphi \) and \( \Gamma_0 \) affect the strain in both the piezo-electric and the substructure.

These equations are plotted in Fig. 2.4 for various values of \( \Gamma_0 \) and \( \varphi = 14.5 \), which is a typical value for a piezo-electric bonded to an aluminum bar. The \( \Gamma_0 \) parameter, which from Eq. 2.37 is equal to
\[ r_s = \frac{G \theta_s}{t_s^2} \left[ \frac{2 + \theta_b \gamma}{\theta_b \gamma} \right] = \frac{G \theta_s}{t_s^2} \left[ \frac{2 + \varphi}{\varphi} \right] \]

is a measure of the effectiveness of the shear transfer. It is influenced by the stiffness and thickness of the bonding layer and the relative stiffnesses and thicknesses of the piezo-electric to the elastic substructure. As the shear modulus G of the bonding layer increases or the thickness \( t_s \) decreases, shear lag becomes less prominent. In the limit, with an infinitely stiff bonding layer, there would be a discontinuous jump in the shear stress at the edge of the piezo-electric, indicating that the strain is transferred between the piezo-electric and the substructure over an infinitesimal distance near the edges of the piezo-electric.

As \( r_s \) approaches infinity, the shear lag present in the bonding layer approaches zero. Therefore, the non-homogeneous substructure boundary condition case (Eqs. 2.39 through 2.43) reduces to

\[
\begin{bmatrix}
\varepsilon_c^+ \\
\varepsilon_b^+
\end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left( \frac{\varphi}{\varphi + 2} \right) \left[ \frac{\varepsilon_b^+ + \varepsilon_b^-}{2} \right] + 2\Delta \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left( \frac{\varphi}{\varphi + 2} \right) \left[ \frac{\varepsilon_b^+ - \varepsilon_b^-}{2} \right]
\]

and the homogeneous case (Eq. 2.44) reduces to

\[ (2.45) \]
Therefore, for both the non-homogeneous and homogenous substructure boundary condition cases, $\bar{\epsilon}_c$ is equal to $\bar{\epsilon}_b$ in the limiting case of no shear lag. Note that Eq. 2.45 and 2.46 are the same solutions obtained in section 2.1.1 when the piezo-electric was modeled as being perfectly bonded to the substructure.

The $\gamma$ parameter, which is equal to the product of the modulus and thickness ratios, determines the maximum theoretical non-dimensional strain level present in both the piezo-electric and the substructure when the boundary conditions on the substructure are homogeneous, if the shear lag in the bonding layer is zero. In any real application $\gamma$ is not infinite, and therefore shear lag is present. However, the strains in the piezo-electric and the substructure can still be equal away from the edges, provided $\gamma$ is large.

For the case of homogeneous substructure boundary conditions with no shear lag and $\gamma = 0$, the strain in the substructure and in the piezo-electric would be equal to the piezo-electric strain $\Delta$. This is the case of a very compliant or a very thin substructure (as compared to the piezo-electric) and it indicates that the

\[
\begin{bmatrix}
\bar{\epsilon}_c \\
\bar{\epsilon}_b
\end{bmatrix} =
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\frac{2\Delta}{\gamma + 2} \quad \text{(2.46)}
\]
substructure would deform the same amount as the piezo-electric. Conversely, a large $\Phi$, corresponding to a thick, high modulus substructure, implies that the strain in both the piezo-electric and the substructure would equal zero. In the limit of an extremely stiff substructure, the substructure and the piezo-electric would not deform at all when a voltage is applied across the actuator.

The analysis in this section shows that the known distributed force $F_k$ of the previous section corresponds to the non-homogenous boundary conditions that were applied in this section. By applying $F_k$, the strain at the $x$ location of the pins in the model of section 2.1.1 was set to a known, non-zero value. This force enters the problem through the equilibrium equation for the bar. Table 2.1 compares the equations and solutions of sections 2.1.1 and 2.1.2. The similarities between the two solutions are readily apparent.

The exact static solution for the case of a pair of piezo-electric actuators bonded to an elastic substructure and exciting extension is now complete. It has been shown to reduce to the much simpler case of a piezo-electric pinned to the substructure for situations where there is small or negligible shear lag due to the bonding layer. For many practical cases where the bonding layer thickness is very small, as compared to the thickness of the piezo-electric, the shear lag effect is small enough that the use of the simpler, perfectly bonded model
introduces very little error. In the following sections, similar models will be presented for various other actuator geometries. With some very slight variations, the procedures will be the same as the ones presented above.

2.2 Surface Bonded Piezo-Electric Actuators in Bending

In section 2.1, a surface bonded piezo-electric actuator was used to excite extension in the substructure. However, by simply phasing the voltage applied across the top piezo-electric by 180° with respect to the voltage applied to the bottom piezo-electric, the same actuator used in section 2.1 could be used to excite bending in the substructure. In a practical sense, this allows the use of one actuator for two different purposes: exciting extension and bending.

A parallel development to that of section 2.1 will be followed here. First, a perfectly bonded model will be presented and the strains in both the piezo-electric and the substructure will be determined. Secondly, a more exact model incorporating a bonding layer between the piezo-electric and the substructure will be presented. The solution for the strains in this model will be shown to reduce to that of the perfectly bonded model when the shear lag in the bonding layer is reduced to zero.
2.2.1 Perfectly Bonded Bending Model

Fig. 2.5 shows the same piezo-electric actuators from Fig. 2.1 with the voltage between the top and bottom piezo-electric phased by 180°. In this case, a lengthwise-varying externally applied moment $M_k$, analogous to the distributed force $F_k$ of section 2.1.1, is applied to the Bernoulli-Euler beam. The equilibrium relationship for the piezo-electric is the same as in section 2.1.1. However, equilibrium for the substructure is obtained by equating the moment produced by the forces applied at the pins and the distributed moment $M_k$ to the integral of the stress in the substructure. If a Bernoulli-Euler behavior is assumed, i.e., the stress varies linearly through the thickness, the equilibrium relationship for the substructure is

$$\sigma_s = -\frac{Mt_b}{2I} - \frac{M_k t_b}{2I} = -\frac{6F}{bt_b} - \frac{M_k t_b}{2I} \quad (2.47)$$

Since the stress and strain in the substructure are not constant with thickness, the "s" superscript refers to the quantity at the surface of the substructure ( $z = \frac{t_b}{2}$. The stress-strain and compatibility relationships are the same as in section 2.1.1 (Eq. 2.3, 2.4 and 2.7). The solution for the force $F$ at the pin is found by equating Eq. 2.2 and 2.4 and substituting in Eq. 2.1, and then substituting into the new beam equilibrium relationship, Eq. 2.47, which gives

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\[
\frac{F}{b} = -\frac{1}{6 + \varphi} \left[ \frac{M_k t_b^2}{2I} \right] - \left[ \frac{A}{6 + \varphi} \right] E_b t_b
\] (2.48)

The strain in the piezo-electric and on the surface of the beam is

\[
\varepsilon_b = \varepsilon_c = -\frac{\varphi}{6 + \varphi} \left[ \frac{M_k t_b}{2E_b} \right] + \left[ \frac{6A}{6 + \varphi} \right]
\] (2.49)

Eq. 2.49 expresses the strain in terms of the continuously varying stress resultant \( M_k \). Since the moment \( M_k \) is analogous to the force \( F_k \) in section 2.1.1, a linearly varying expression for \( M_k \) will also be used here. Therefore, the moment \( M_k \) is approximated by,

\[
M_k = M_0 + M_1 x
\] (2.50)

Evaluating \( M^+ \) and \( M^- \), the values of the moment \( M_k \) at the right and left edges of the beam under the piezo-electric, \( M_0 \) and \( M_1 \) are evaluated

\[
M_0 = \frac{M^+ + M^-}{2}
\] (2.51)

\[
M_1 = \frac{M^+ - M^-}{2}
\] (2.52)

and the moment \( M_k \) is approximated by,
\[ M_k = \frac{M^+ + M^-}{2} + \frac{M^+ - M^-}{2}\alpha \]  

(2.53)

Finally, the moments \( M^+ \) and \( M^- \) can be evaluated in terms of the surface strain in the substructure since, from the Bernoulli-Euler assumption,

\[ \varepsilon_b^+ = \frac{-M^+ t_b}{2IE_b} \]  

(2.54)

\[ \varepsilon_b^- = \frac{-M^- t_b}{2IE_b} \]  

(2.55)

Solving for \( M^+ \) and \( M^- \) in Eq. 2.54 and 2.55, inserting into Eq. 2.53 to find \( M_k \) as a function of the strains \( \varepsilon_b^+ \) and \( \varepsilon_b^- \) and inserting into Eq. 2.49, the strains are found to be,

\[ \varepsilon_b = \varepsilon_c = \frac{\phi}{6 + \phi} \left( \frac{\varepsilon_b^+ + \varepsilon_b^-}{2} + \frac{\varepsilon_b^+ - \varepsilon_b^-}{2} \alpha \right) + \left[ \frac{6A}{6 + \phi} \right] \]  

(2.56)

and, if the non-homogeneous strain terms \( \varepsilon_b^+ \) and \( \varepsilon_b^- \) are set to zero, Eq. 2.56 reduces to,

\[ \varepsilon_b^s = \varepsilon_c = \left[ \frac{6A}{6 + \phi} \right] \]  

(2.57)

Eqs. 2.56 and 2.57 are the final expressions relating the strain in the piezo-electric and on the surface of the substructure to the applied piezo-electric strain term \( A \) for both
the non-homogeneous and homogeneous substructure boundary condition cases. These are exactly the same expressions as Eqs. 2.16 and 2.19, the only difference being that the factor 2, which arises in Eq. 2.2 from the assumption that the substructure is undergoing pure extension, and appears in Eq. 2.47 as a 6, due to the assumption of Bernoulli-Euler bending. This indicates a strong parallel between the extension and bending cases which will often appear throughout this chapter.

2.2.2 Shear Lag Bending Model

As was done with the perfectly bonded model, the extensional actuator shear lag model will now be modified to represent bending of the substructure by exciting the two piezo-electrics 180° out of phase. From Fig. 2.2 it is seen that if an electric field is applied across the top piezo-electric in the direction of the double arrow and across the bottom piezo-electric in the direction opposite the double arrow, then the net effect is to deform the substructure in pure bending. In order to examine this problem, the differential element outlined by the dashed region in Fig. 2.2 is shown in Fig. 2.6. Again, it is assumed that:

- The bonding layer carries only shear stresses.
- The beam and the piezo-electric carry only normal stresses.
In addition, it is assumed:

- The stress in the substructure is distributed linearly throughout its thickness, i.e., the substructure behaves as a Bernoulli-Euler beam.
- The thickness of the piezo-electric is small as compared to the substructure, and therefore the stress $\sigma_c$ is assumed constant.

The strain-displacement and stress-strain relationships for the piezo-electric, substructure, and shear layer, and the equilibrium relationship for the piezo-electric, are the same as for the previous case of the piezo-electrics exciting extension in the substructure of section 2.1.2. For the substructure, the equilibrium equation is obtained by summing the moments about point A.

$$
\begin{align*}
\int_{-t_b/2}^{t_b/2} b \tau \tau_{tb} \, dx - \int_{-t_b/2}^{t_b/2} b \sigma_{tb}^s \left\{ \frac{2z}{t_b} \right\} z \, dz + \int_{-t_b/2}^{t_b/2} b \left( \sigma_{tb}^s + d\sigma_{tb}^s \right) \left\{ \frac{2z}{t_b} \right\} z \, dz &= 0 \\
(2.58)
\end{align*}
$$

Integrating and simplifying, this reduces to the differential equation of equilibrium for the substructure.
The eight governing equations have now been derived and once again it is observed that they are similar to the relation for the shear lag extensional case. Therefore, the same type of selection procedure that was carried out previously can be performed. The following are the non-dimensional, coupled, second order differential equations for the strain in the piezo-electric and in the substructure:

\[
\frac{d^2 \xi_b}{dx^2} + \frac{6 \tau}{t_b} = 0 \quad (2.59)
\]

Again, these can be written as a pair of uncoupled, fourth order differential equations:

\[
\dddot{\varepsilon}_c - \frac{\bar{G} \theta_s}{t_s^2} (\dddot{\varepsilon}_c - \dddot{\varepsilon}_b) = 0 \quad (2.60)
\]

\[
\dddot{\varepsilon}_b + \frac{6 \bar{G} \theta_s}{\gamma \theta_b t_s^2} (\dddot{\varepsilon}_c - \dddot{\varepsilon}_b) = 0 \quad (2.61)
\]

Solutions to this set of equations are of the form:

\[
\dddot{\varepsilon}_b = B_1 + B_2 x + B_3 \sinh \Gamma_b x + B_4 \cosh \Gamma_b x \quad (2.64)
\]
\[ \bar{e}_c = C_1 + C_2 \bar{x} + C_3 \sinh \Gamma_b \bar{x} + C_4 \cosh \Gamma_b \bar{x} \]  

(2.65)

where now:

\[ \Gamma_b = \sqrt{\frac{G \theta_s}{t_s^2} \left[ \frac{6 + \nu}{\nu} \right]} \]  

(2.66)

Relating the \( B_i \) and \( C_i \) constants as before, the following solution set is obtained:

\[
\begin{bmatrix}
\bar{e}_c \\
\bar{e}_b
\end{bmatrix} =
\begin{bmatrix}
1 \\
1
\end{bmatrix} B_1 +
\begin{bmatrix}
1 \\
1
\end{bmatrix} B_2 \bar{x} +
\begin{bmatrix}
-\frac{\nu}{6} \\
1
\end{bmatrix} B_3 \sinh \Gamma_b \bar{x}
\]

(2.67)

\[ + \begin{bmatrix}
-\frac{\nu}{6} \\
1
\end{bmatrix} B_4 \cosh \Gamma_b \bar{x} \]

The governing equations for the strain in the substructure and in the piezo-electric have now been solved and the boundary conditions can now be applied in order to determine the constants \( B_i \). Once again, the concept of a segmented actuator dictates a stress-free boundary condition for the piezo-electric, while the boundary conditions for the strain in the substructure can be set to a given strain value. This case corresponds to the boundary conditions:
\[ @x = + 1 \quad \varepsilon_c = \Lambda \quad \varepsilon_b = \varepsilon_b^+ \]
\[ @x = - 1 \quad \varepsilon_c = \Lambda \quad \varepsilon_b = \varepsilon_b^- \]

where \( \varepsilon_b^+ \) and \( \varepsilon_b^- \) are known strain values on the surface at the locations of extreme left ( - ) and right ( + ) ends of the piezo-electric segment.

Applying these boundary conditions and solving for the constants \( B_i \):

\[
B_1 = \frac{\varphi \left[ \frac{-\varepsilon_b^+ + \varepsilon_b^-}{2} \right]}{\varphi + 6} + 6\Lambda \quad (2.68)
\]

\[
B_2 = \frac{\varphi}{\varphi + 6} \left[ \frac{-\varepsilon_b^+ - \varepsilon_b^-}{2} \right] \quad (2.69)
\]

\[
B_3 = \frac{\varphi}{(\varphi + 6) \sinh \Gamma_b} \left[ \frac{-\varepsilon_b^+ - \varepsilon_b^-}{2} \right] \quad (2.70)
\]

\[
B_4 = \frac{6}{(\varphi + 6) \cosh \Gamma_b} \left[ \frac{-\varepsilon_b^+ + \varepsilon_b^-}{2} - \Lambda \right] \quad (2.71)
\]

Note that the \( B_1 \) and \( B_4 \) terms account for the "transmission" of the piezo-electric strain \( \Lambda \), while the \( B_2 \) and \( B_3 \) account for the non-symmetry of the boundary conditions on the substructure.

Inserting the expressions for the four constants \( B_i \) into Eq. 2.67, the final solution expressing the strains in both the piezo-electric and the substructure as a function of the
piezo-electric strain term \( A \) is obtained.

Finally, in order to facilitate coupling of this static analysis to the dynamic analysis in the next chapter, it is desirable to have an explicit expression for the shear stress \( \tau \) in the bonding layer. Substituting Eq. 2.67 into the stress-strain relationships (Eqs. 2.20 and 2.21) to obtain expressions for \( \bar{u}_c' \) and \( \bar{u}_b' \), and integrating:

\[
\begin{bmatrix}
\bar{u}_c \\
\bar{u}_b
\end{bmatrix} =
\begin{bmatrix}
1 \\
1
\end{bmatrix}
B_1 \bar{x} + 
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\frac{B_2 \bar{x}^2}{2} + 
\begin{bmatrix}
-\frac{\Phi}{6} \\
1
\end{bmatrix}
\frac{B_3}{\Gamma_b} \cosh \Gamma_b \bar{x} +
\begin{bmatrix}
-\frac{\Phi}{6} \\
1
\end{bmatrix}
\frac{B_4}{\Gamma_b} \cosh \Gamma_b \bar{x} 
\]

Substituting Eq. 2.22 into Eq. 2.27,

\[
\bar{\tau} \equiv \frac{\tau}{E_c} = \frac{G}{t_s} \left[ \frac{\bar{u}_b - \bar{u}_c}{t_s} \right] 
\]

Finally, substituting Eq. 2.72 into Eq. 2.73 yields the desired expression for the shear stress

\[
\bar{\tau} = \frac{G}{t_s \Gamma_b} \left[ \frac{\bar{\epsilon}_b - \bar{\epsilon}_c}{2} \frac{\cosh \Gamma_b \bar{x}}{\sinh \Gamma_b} + \left\{ \frac{\bar{\epsilon}_b + \bar{\epsilon}_c}{2} - A \right\} \frac{\sinh \Gamma_b \bar{x}}{\cosh \Gamma_b} \right] 
\]
Eq. 2.74 represents the complete solution of the non-dimensional shear stress \( \tau \), incorporating the classic shear load transfer analysis and the piezo-electric strain term \( A \). In order to examine the behavior of Eq. 2.67, along with Eq. 2.74, they will be simplified by setting \( \varepsilon^+_{b} = \varepsilon^-_{b} = 0 \). This corresponds to a stress-free boundary condition at the edges of the substructure. In this case, Eqs. 2.67, and 2.74 reduce to

\[
\begin{align*}
\frac{\varepsilon_c}{\lambda} & = \frac{6}{\varphi + 6} + \frac{\varphi}{(\varphi + 6) \cosh \Gamma_b x} \cosh \Gamma_b x \\
\frac{\varepsilon^-_{b}}{\lambda} & = \frac{6}{\varphi + 6} - \frac{6}{(\varphi + 6) \cosh \Gamma_b x} \cosh \Gamma_b x \\
\frac{\tau}{\lambda} & = \frac{G}{\Gamma_b} \frac{\sinh \Gamma_b x}{\cosh \Gamma_b x}
\end{align*}
\]  

(2.75)  

(2.76)  

(2.77)

The Eqs. 2.75 and 2.76 are plotted in Fig. 2.7 for various values of \( \Gamma_b \) and for \( \varphi = 14.5 \), a typical value for a piezo-electric bonded to an aluminum beam. This figure is the same as Fig. 2.4 except for the fact that as the shear lag parameter \( \Gamma_b \) goes to infinity, the non-homogeneous substructure boundary condition case (Eqs. 2.67 through 2.71) reduces to

\[
\begin{align*}
\begin{bmatrix} \varepsilon_c \\ \varepsilon^-_{b} \end{bmatrix} & = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{\varphi}{\varphi + 6} \left[ \frac{\varepsilon^+_{b} + \varepsilon^-_{b}}{2} \right] + 6A \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{\varphi}{\varphi + 6} \left[ \frac{\varepsilon^+_{b} - \varepsilon^-_{b}}{2} \right] \cosh \Gamma_b x \\
\end{align*}
\]  

(2.78)
and the homogeneous case (Eqs. 2.75 and 2.76) reduces to

\[
\begin{bmatrix}
\epsilon_c \\
\epsilon_b
\end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{6A}{\varphi + 6}
\]

(2.79)

which is the same solution found in section 2.2.1. Eq. 2.75 and 2.76 indicate that more piezo-electric strain gets transferred into the substructure when the piezo-electrics excite bending than when they excite extension. By being located on the surface of the substructure away from the centerline, when the actuators are used in bending, the piezo-electric strain is applied at the point where it is most effective, i.e., on the surface of the substructure. This is the area where the bending strain is highest. When the actuators are used in extension, the strain is constant throughout the thickness of the substructure and the effectiveness of the piezo-electrics is reduced. Therefore, for given piezo-electric and substructure properties, surface mounted piezo-electrics are more effective when used to excite bending than when used to excite extensional motion.

Fig. 2.8 plots Eq. 2.77, normalized by the \( \tau \) evaluated at the boundary, for various values of \( \Gamma_b \). It shows very clearly that as \( \Gamma_b \) increases, there is only shear stress in the bonding layer near the boundaries; away from the boundaries, \( \tau = 0 \). In the limit, if \( \Gamma_b = \infty \), \( \tau \) would be zero everywhere except at an
infinitesimally small distance near the boundaries.

The analysis for the surface bonded piezo-electric actuator exciting extension and bending in an elastic substructure is now complete. It has been shown that for cases where the shear lag is small, the perfectly bonded model for the piezo-electric actuator is adequate for predicting the strain in the piezo-electric and the substructure. For relative thick bonding layers however, the shear lag terms must be taken into account. The results from this section, along with the results for the perfectly bonded model in section 2.2.1, are summarized in Table 2.1.

2.3 Perfectly Bonded Embedded Extension Model

In the previous sections, the piezo-electric actuators were bonded to the surface of an elastic substructure and used to excite either extension or bending. Another possible geometry would be to insert a piezo-electric actuator inside a laminated structure. In this way, the actuators now form an integral part of the structure. By appropriately matching the elastic properties of the piezo-electric and the surrounding material, the combined structure would behave passively much as if the actuators were not present. In a space application, embedding the actuators provides the additional benefit of protecting them from the exterior environment.

In this section, embedded extensional actuators will be
investigated following a similar procedure to the one used in section 2.1. First, the actuator will be assumed to be perfectly bonded to the surrounding material. A solution for the strains in the model will be found. In this section however, a solution incorporating the shear present in the interlaminar bonding layer between the actuator and the surrounding material will not be developed, since for most applications this bonding layer is thin enough so that the shear present in this layer is negligible. For example, if the piezo-electric were embedded inside a graphite/epoxy laminated structure, the bonding layer thickness would be on the order of two graphite fiber diameters [9].

Fig. 2.9 shows two piezo-electrics embedded along the centerline inside an elastic material. Two piezo-electrics are used in order to allow comparison with the previous surface-bonded case, where a piezo-electric was bonded on both the top and bottom surfaces of the bar. The piezo-electric is assumed to be perfectly bonded to the surrounding material and a known distributed force $F_k$ is assumed to be applied to the surrounding material. Under these assumptions, the equilibrium relationships for the piezo-electric and the material are

$$\sigma_c = \frac{F}{bt_c}$$
$$\sigma_b = \frac{F}{bt} - \frac{F_k}{bt}$$

where $t$ is the thickness of the material above the
piezo-electrics.

The stress-strain relationships are the same as in section 2.1.1. The displacements of the piezo-electric and the surrounding material is assumed to be the same, and therefore the strains are equal. Once again, using the same procedure as in section 2.1.1, the force $F$ at the pins is found to be

$$\frac{F}{b} = \left[ \frac{-A}{1 + \frac{\gamma t}{t_c}} \right] E_b t_b - \left[ \frac{1}{1 + \frac{\gamma t}{t_c}} \right] \left[ \frac{F_k}{b} \right]$$

(2.82)

As in section 2.1.1, the expression for $F$, Eq. 2.82, is now used to determine the strains in the elastic material and the piezo-electric

$$\varepsilon_b = \varepsilon_c = \left[ \frac{A}{1 + \frac{\gamma t}{t_c}} \right] + \left[ \frac{\gamma t}{t_c} \right] \left[ \frac{\varepsilon_b^+ + \varepsilon_b^-}{2} + \frac{\varepsilon_b^+ - \varepsilon_b^-}{2} \right]$$

(2.83)

If the force resultant $F_k$ is set to zero, Eq. 2.83 reduces to

$$\varepsilon_b = \varepsilon_c = \left[ \frac{A}{1 + \frac{\gamma t}{t_c}} \right]$$

(2.84)

Eq. 2.83 is similar to the surface bonded equation, Eq. 2.16. However, in order to compare the two, the parameter $t$ must be
defined in more detail. The question arises as to whether the embedding of the piezo-electrics was accomplished by removing material from the substructure, or whether the entire assembly was just made thicker. In the first case, the thickness \( t \) of the substructure would be given by

\[
t = \frac{t_b - 2t_c}{2} \quad (2.85)
\]

In the second case, the thickness \( t \) would simply equal half the original substructure thickness

\[
t = \frac{t_b}{2} \quad (2.86)
\]

Substitution of Eq. 2.86 into Eq. 2.83 would only yield the same strain solution as in section 2.1.1. This reaffirms the obvious conclusion that for perfectly bonded actuators exciting extension, their location through the thickness, i.e., along the \( z \) axis, does not affect their effectiveness. It is more realistic to assume that when the actuators were embedded, material from the substructure was removed. Therefore, placing Eq. 2.85 in Eq. 2.83, the strains are found to be,

\[
\varepsilon_b = \varepsilon_c = \left[ \frac{2\Delta}{2 + \gamma(\theta_b - 2)} \right]
\]
and with the force resultant $F_k$ set to zero, Eq. 2.87 reduces to the homogeneous relationship

$$\varepsilon_b = \varepsilon_c = \frac{2A}{2 + \gamma(\theta_b - 2)}$$  \hspace{1cm} (2.88)

Eqs. 2.87 and 2.88 are the final relations between the strains in the piezoelectric and the surrounding material and the piezoelectric strain term $A$ when the actuator is used to excite extension. They are similar to the equations derived in section 2.1.1 with the only differences being due to the location of the piezoelectrics along the thickness of the elastic material.

When the piezoelectrics are embedded inside the laminate, the reduction in the amount of substructure material increases the amount of strain that is transferred to the substructure. It is important to note that this effect is due totally to the removal of some quantity of elastic substructural material in order to embed the actuators. It is not due to the position of the actuators. Once the effect of the removal of the elastic material has been accounted for, Eq. 2.87 indicates that the actuators may be located anywhere within the thickness of the elastic substructure without affecting their effectiveness.

The perfectly bonded analysis for embedded actuators exciting
extension in an elastic material has now been completed, and similar results to those of section 2.1.1 have been obtained. As explained earlier, the corresponding shear lag model such as the one presented in section 2.1.2 will not be developed, due to the small effect the interlaminar bonding layer has on the final solution for the strains. In the next section, the same actuators will be phased so as to produce bending in the surrounding material. Once again, the shear in the bonding layer will be ignored and the actuators will be assumed to be perfectly bonded.

2.4 Perfectly Bonded Embedded Bending Model

In order to complete the static analysis of embedded piezo-electric actuators, the case for a pair of actuators exciting bending in the surrounding material will be analyzed. As in the previous section, when the embedded actuators were used to excite extension in the surrounding material, only the perfectly bonded model will be developed. Fig. 2.10 shows a pair of piezo-electric actuators embedded in an elastic material. The actuators are excited $180^\circ$ out of phase with each other and are assumed to be pinned at the edges. Finally, both the actuators and the surrounding material are assumed to be in Bernoulli-Euler bending about the centerline of the entire assembly. The same reduction procedure that was used in section 2.2.1 will be used here. A known, lengthwise varying moment $M_k$ is applied to the
Referring to Fig. 2.10, the forces $F_1$ and $F_2$ are the forces applied at the top and bottom pins, respectively. In addition, the following definitions will be useful:

- $\Delta F = \text{The difference between the two pin forces.} = F_2 - F_1$
- $\bar{F} = \text{The sum of the pin forces.} = F_1 + F_2$
- $z_i = \text{The distance from the centerline of the entire section to the lower face of the piezo-electric.}$
- $z_m = \text{The distance from the centerline of the entire section to the centerline of the piezo-electric.}$
- $M = \text{The moment applied at the pins}$

$$M = 2 \left \{ F_1 z_i + F_2 (z_i + t_c) \right \} = 2 \left \{ \Delta F \frac{t_c}{2} + \bar{F} z_m \right \}$$

Using the above definitions the equilibrium relations for the material and the piezo-electric are

$$\sigma_b = - \frac{M z}{I} - \frac{M_k z}{I}$$

$$\sigma_c = \frac{6\Delta F}{b t_c^2} (z - z_m) + \frac{\bar{F}}{b t_c}$$

The stress-strain relationships are the same as the ones in section 2.1.1. Since the actuator is assumed to be perfectly pinned, the displacements of the material and the piezo-electric
at the top and bottom pins are equal, and therefore the strains are also equal. Equating Eqs. 2.3 and 2.4 at \( z = z_m - \frac{t_c}{2} \) for the lower pin and \( z = z_m + \frac{t_c}{2} \) for the upper pin, and inserting Eqs. 2.89, 2.90, and 2.91, the following two equations are found

\[
\theta z = z_m - \frac{t_c}{2} \nonumber
\]

\[
\left[ \frac{-2}{E_b I} \right] \left\{ \Delta F \left\{ \frac{t_c}{2} + \bar{F} \right\} z_m \right\} - \frac{M_k}{E_b I} \right\} \right] \left[ z_m - \frac{t_c}{2} \right] = \frac{-3\Delta F + \bar{F}}{E_c bt_c} + \Lambda 
\]

(2.92)

\[
\theta z = z_m + \frac{t_c}{2} \nonumber
\]

\[
\left[ \frac{-2}{E_b I} \right] \left\{ \Delta F \left\{ \frac{t_c}{2} + \bar{F} \right\} z_m \right\} - \frac{M_k}{E_b I} \right\} \right] \left[ z_m + \frac{t_c}{2} \right] = \frac{3\Delta F + \bar{F}}{E_c bt_c} + \Lambda 
\]

(2.93)

Solving Eqs. 2.92 and 2.93 for \( \Delta F \) and \( \bar{F} \), and substituting for the moment of inertia \( I \), \( I = \frac{bt_b^3}{12} \)

\[
\frac{\bar{F}}{b} = \frac{- (2E_c t_c + E_b t_b \theta_h^2) \Lambda}{24 \theta_z^2 + \theta_h^2 + 2} + \frac{12 M_k}{t_c b \theta_z} \nonumber
\]

(2.94)

\[
\frac{\Delta F}{b} = \frac{4E_c t_c \theta_2 \Lambda}{24 \theta_z^2 + \theta_h^2 + 2} + \frac{2 M_k}{t_c b} \nonumber
\]

(2.95)

where the parameter \( \theta_z \) is defined as

\[
\theta_z = \frac{z_m}{t_c} 
\]
Inserting these equations into either of the equilibrium relations, Eqs. 2.90 and 2.91, and using Eqs. 2.23, 2.24 and 2.89, and expressing the moment $M_k$ as a function of the surface strains $e_{h}^{+}$ and $e_{h}^{-}$ as was done in section 2.2.1, the strain in the actuator and in the surrounding material is found to be

$$
\varepsilon_{b} = \varepsilon_{c} = \frac{24 \, \varepsilon_{2} \, \frac{z}{t_{c}}}{24 \theta_{a}^{2} + \nu \theta_{b}^{2} + 2} \Lambda + \frac{2 \nu \theta_{b} \, \frac{z}{t_{c}}}{24 \theta_{a}^{2} + \nu \theta_{b}^{2} + 2} \left[ \frac{\varepsilon_{h}^{+} + \varepsilon_{h}^{-}}{2} + \frac{\varepsilon_{b}^{+} + \varepsilon_{b}^{-}}{2} \right] x
$$

(2.96)

Finally, if the thickness of the actuator $t_{c}$ is small compared to the thickness of the surrounding material $t_{b}$, and the beam boundary conditions are set to zero, then Eq. 2.96 reduces to

$$
\varepsilon_{b} = \varepsilon_{c} = \frac{24 \, \varepsilon_{2} \, \frac{2}{t_{c}}}{24 \theta_{a}^{2} + \nu \theta_{b}^{2}} \Lambda
$$

(2.97)

This is, of course, the case where a very thin actuator is inserted in a very thick elastic material.

The equations derived in this section allow the calculation of the forces applied by a pair of piezo-electric actuators embedded in an elastic material, assuming that there is no shear layer present between the actuator and the surrounding structure.
Once again, an example where this no-shear lag situation exists is in the case of a laminated structure where the piezo-electrics are the same thickness as the plies making up the structure. For this case, the above analysis would provide very accurate results.

2.5 Summary

The static solution for surface bonded and embedded actuators exciting extension and bending in the substructural material have been derived. The relevant equations along with the solutions for the homogeneous strains in the actuator and in the substructure are summarized in Table 2.1. As is readily apparent, the six sets of solutions derived in this chapter have nearly identical forms. The differences arise from the geometry of the various actuator systems. In the next chapter, these static models will be coupled to a dynamic model for the substructure and will be used to predict the dynamic behavior of the substructure under piezo-electric loading.
CHAPTER THREE: COUPLING OF THE ACTUATORS TO STRUCTURAL DYNAMIC MODELS

In Chapter Two, the detailed static models for interaction between surface-bonded and embedded actuators and the host structure were analysed. In this chapter, the actuator systems will be assumed to be part of a larger dynamic model of the host structure. The mass of the actuator will be limited to a small fraction of the total mass of the system and the actuator resonant frequencies will be assumed to be much higher than the resonant frequencies of the associated dynamic model. Under these conditions, the solutions obtained in the previous chapter to the static equations of elasticity can be integrated into any appropriate dynamic model. For example, the embedded, perfectly bonded model of section 2.3 could be combined with an extensional dynamic model of a rod with embedded piezo-electrics. Or similarly, the surface bonded shear lag model of section 2.2.1 can be used in a dynamic model for a simply supported plate with surface bonded actuators.

In this chapter, two examples of the integration technique will be presented in detail:

° The integration of surface-mounted piezo-electric actuators into a dynamic model of a cantilevered beam in bending.
° The integration of embedded piezo-electric actuators
into a dynamic model of a laminated cantilevered beam in bending.

These cases were chosen for two reasons. First, by using a finite bonding layer model in the first case and a perfectly bonded model in the second, the integration of both types of models into a dynamic model can be demonstrated. Second, as will be seen in the next chapter, these two cases are representative of experiments that were performed, and are also representative of typical applications of piezo-electric actuators.

3.1 Dynamic Analysis for Surface-Bonded Piezo-Electric Actuators Bonded to a Cantilevered Beam.

In this section, a cantilevered beam with a pair of surface bonded actuators will be modeled. A continuous model for the beam will be used. A Rayleigh-Ritz analysis will be used to calculate the beam modal mass and stiffness. A shear is applied to a limited area of the surface of the beam underneath the piezo-electric. The shear distribution will then be inserted into the expression for the modal force $Q$ in the Rayleigh-Ritz model. This modal force $Q$ will have both a homogeneous and a non-homogeneous part. The homogeneous part is in effect a passive added stiffness due to the bonding of the actuators to the beam. The non-homogeneous part will be a function of the applied voltage across the piezo-electrics. With the modal force thus determined,
the response of the beam as a function of the piezo-electric voltage can be determined.

Fig. 3.1 shows a cantilevered beam with shear loading applied over a localized area near the root. The loading is applied such that the shear loading on the top surface of the beam is opposite to that on the bottom. The net effect of a force couple applied in this manner is to excite bending in the beam. The shear loading is applied by a pair of piezo-electric actuators bonded onto the surface of the beam. In order to perform the analysis, two models will be needed. First, a model for the applied shear force is necessary. Here, the static model for surface-bonded actuators in bending developed in section 2.2.2 will be used. Second, a dynamic model for the cantilevered beam is necessary. Any dynamic model could be used. For example, a model based on Galerkin's method, or a Finite Element Analysis could be appropriate. In this case, the cantilevered beam will be assumed to behave as a Bernoulli-Euler beam and will be modelled using a one mode Rayleigh-Ritz analysis.

The coupling analysis will begin by using a one mode Rayleigh-Ritz model in which the displacement \( \tilde{w}(x,t) \) of the beam is

\[
\tilde{w} = \Psi(x)q(t)
\]  

(3.1)

where \( \Psi(x) \) is an assumed mode shape satisfying the geometric
boundary conditions. The modal mass and stiffness terms for this one mode model are simply

\[ M = \int_0^1 m \rho^2 \, dx \tag{3.2} \]

\[ K = \int_0^1 E I (\rho')^2 \, dx \tag{3.3} \]

Since the shearing force on the top surface of the beam is phased 180° out of phase with the force on the bottom, it can be modelled as applying a pure distributed moment on the beam. For the general case of a beam under a distributed moment loading condition, the virtual work term \( \delta W \) is equal to

\[ \delta W = Q \delta (q) = \int M \left( \delta \frac{\partial \psi}{\partial x} \right) \, dx \tag{3.4} \]

In this case, the applied moment is due to the shear loading applied over a finite length of the beam. Therefore,

\[ M = \tau \cdot t_b \cdot b \tag{3.5} \]

Eqs. 3.4 and 3.5 are combined and appropriate limits are applied to the integration in order to reflect that the loading is
only applied over a finite segment of the beam. Since the loading is applied by a pair of piezo-electrics of non-dimensional length \( \sim 2m \) and centered at \( \sim a \), the integration is performed from \( \sim a - m \) to \( \sim a + m \). The modal force \( Q \) is therefore

\[
Q = \int_{a-m}^{a+m} r \cdot b \cdot t_b \cdot \mathbf{P}'(x) \, dx
\]

where \( \sim a \) is the normalized center of the applied shear loading and \( \sim m \) is half the normalized distance over which the loading is applied (Fig. 3.1). The root-centered \( x \) coordinate has now been normalized by \( \ell \), the length of the beam.

Eq. 3.6 expresses the modal force \( Q \) as a function of the applied shear stress \( \tau \). It is now necessary to find an expression for the shear stress \( \tau \) in this equation. As was mentioned earlier, this will be accomplished by using Eq. 2.74, the solution for the shear stress found from the static model of the piezo-electrics bonded to a beam in section 2.2.2. Inserting this equation into the dynamic model for the cantilevered beam can only be done if the axial displacement of the piezo-electrics is ignored. This is consistent with ignoring the shear deformation of the beam in the simple Bernoulli-Euler model. This degree of freedom can be ignored, since the axial resonance frequency of the piezo-electrics is two orders of magnitudes greater than the
bending frequencies of the modes of the beam under consideration

Under these assumptions, Eq. 2.74, the static solution for \( r \), can be used. It should be noted that \( r \) in this equation was derived with respect to the non-dimensional \( \bar{x} \) coordinate, which centered on the piezo-electric and normalized by its length. In order to substitute into Eq. 3.6, it is necessary to transform the coordinate system to the root-centered system used in this chapter. The coordinate transformation that is required is

\[
\bar{x} = \frac{2}{L} (x - a)
\]

where \( \bar{L} \) is the length of the piezo-electric normalized by the length of the cantilevered beam. Substituting this into Eq. 2.74, \( r \) is found to be:

\[
\tau = \frac{G}{\bar{t}_b \bar{r}_b} \left[ \frac{-\bar{s}^+ - \bar{s}^-}{2} \right] \cdot \frac{\cosh \left\{ \frac{2}{L} \bar{r}_b (\bar{x} - \bar{a}) \right\}}{\sinh \bar{r}_b} \left[ \frac{\sinh \left\{ \frac{2}{L} \bar{r}_b (\bar{x} - \bar{a}) \right\}}{\cosh \bar{r}_b} \right] + \left\{ \frac{-\bar{s}^+ + \bar{s}^-}{2} - \Lambda \right\} \cdot \frac{\cosh \left\{ \frac{2}{L} \bar{r}_b (\bar{x} - \bar{a}) \right\}}{\sinh \bar{r}_b}
\]

where \( \tau \) is now expressed in the reference system of the beam. Eq. 3.7 can now be combined with Eq. 3.6 to obtain an expression for the modal force \( Q \) in the Rayleigh-Ritz analysis.

The static solution from section 2.2.2 has been successfully
coupled to the dynamic analysis. However, when Eq. 2.74 was derived, the boundary conditions on the substructure, $\varepsilon_b^+$ and $\varepsilon_b^-$, were assumed to be known. The actuators will used to excite bending in the cantilevered beam. Therefore, in order to assure compatibility with the host system, these boundary conditions must be equal to the strains normally present in the cantilevered beam when bending in one of its natural mode shapes. From Bernoulli-Euler beam theory, the strain on the surface of a beam is equal to:

$$\varepsilon_b = -\frac{t_b}{2} \cdot \frac{\partial^2 w}{\partial x^2} = -\frac{t_b}{2} \cdot \frac{\partial^2 \psi}{\partial x^2} q$$

(3.8)

Using this equation to find $\varepsilon_b^+$ and $\varepsilon_b^-$ in Eq. 3.7 and substituting for $\tau$ in Eq. 3.6 yields the complete form for the modal force $Q$

$$Q = \frac{-G b}{\bar{t}_b \Gamma_b} \frac{t_b^2}{4 \ell^2} \left[ \frac{\psi''(a+m) - \psi''(a-m)}{\sinh \Gamma_b} \right] I_c$$

$$+ \frac{\psi'''(a+m) + \psi'''(a-m)}{\cosh \Gamma_b} I_s q$$

$$- \frac{G \cdot t_b \cdot b}{\bar{t}_s \Gamma_b} \left[ \frac{\Lambda}{\cosh \Gamma_b} I_s \right]$$

(3.9)

where:
This is now the final form expressing the modal force in terms of both the motion of the beam and the applied voltage. It should be noticed that the first bracketed term of Eq. 3.9 is homogeneous in $q$ and therefore will contribute to the modal stiffness. This term arises from the presence of non-homogeneous boundary conditions on the beam and can be considered an added passive stiffness that arises from bonding the piezo-electrics to the beam. As can be seen from Eq. 3.9, the passive stiffness depends the value of the strain around the locations where the piezo-electrics are bonded. For example, if the piezo-electrics are bonded near the tip of a cantilevered beam, the added stiffness would be much less than if they were bonded near the root, since the strain on a cantilevered beam is higher near the root.

The equation of motion for the single-mode model of the piezo-beam system is therefore
\[ M\ddot{q} + B\dot{q} + \left[ K + K_{PIEZO} \right] q = -\frac{G t_b b}{t_s I_b} \left[ \frac{d_{31} V}{t_c \cosh \Gamma_b} I_s \right] \]  
\[ (3.12) \]

where \( A \) has been replaced by \( \frac{d_{31} V}{t_c} \) (see Eq. 2.3). \( B \) is an assumed modal damping to be determined and \( K_{PIEZO} \) is the added stiffness due to the bonding of the piezo-electrics to the beam. From Eq. 3.9, this is equal to

\[ K_{PIEZO} = \frac{G b}{t_s I_b} \frac{t_b^2}{4\ell^2} \left[ \frac{\varphi''(\tilde{a}+\tilde{m}) - \varphi''(\tilde{a}-\tilde{m})}{\sinh \Gamma_b} I_c \right. \\
\left. + \frac{\varphi''(\tilde{a}+\tilde{m}) + \varphi''(\tilde{a}-\tilde{m})}{\cosh \Gamma_b} I_s \right] \]  
\[ (3.13) \]

The coupling of the static solution to the dynamic model for a cantilevered beam under loading from a pair of piezo-electric actuators bonded on the surface has now been completed. The static solution for the shear stress \( \tau \) was integrated into a one mode dynamic model for a cantilevered beam. Boundary conditions consistent with the assumption of a Bernoulli-Euler beam were evaluated and inserted into the expression for \( \tau \). The coupled piezo-beam system has been reduced to a single degree of freedom, second order equation of motion for the generalized displacement \( q(t) \). Solutions to this equation can easily be obtained and used to obtain the displacement of the beam for any voltage level \( V \) that is applied to the piezo-electrics.
3.2 Dynamic Analysis for an Embedded Piezo-Electric Actuator Bonded to a Cantilevered Beam.

In the previous section, the static model for a surface bonded piezo-electric actuator was combined with a Rayleigh-Ritz analysis for a cantilevered beam. In this section, a similar procedure will be followed for a pair of actuators embedded inside a cantilevered beam. The perfectly bonded static model of an embedded actuator derived in section 2.4 will be used. Since the previous section showed how the shear lag models of Chapter Two are incorporated into a dynamic analysis, this section will show how perfectly bonded models are used, and will therefore allow the two methods to be contrasted. As was mentioned in Chapter Two, many applications of embedded actuators will involve placing the actuators inside a laminated structure as an integral part of one of the plies of the laminate. In this situation, the interlaminar bonding layer is very small, usually on the order of two fiber diameters, if the material is a fiberous composite such as graphite/epoxy. For this case, the shear lag will be negligible and the perfectly bonded solution of section 2.4 will provide an adequate model.

Fig. 3.2 shows a cantilevered beam with a pair of embedded piezo-electric actuators near the root. If a Rayleigh-Ritz model is used as in the previous section, the expressions for the modal
mass and stiffness are given in Eqs. 3.2 and 3.3. The expression for the virtual work however, is now obtained by multiplying $M$, the moment applied at the left and right edges of the piezo-electric, by the slope of the beam at each pin which is equal to $\delta \left( \frac{\partial W}{\partial X} \right)$. Therefore, the virtual work is equal to

$$
\delta W = \left[ M \left( \frac{\partial W}{\partial X} \right) \right]_+ - \left[ M \left( \frac{\partial W}{\partial X} \right) \right]_-
$$

$$
= \left[ \left\{ \frac{M}{e} \varphi' \right\}_+ - \left\{ \frac{M}{e} \varphi' \right\}_- \right] \delta q = Q \delta q \tag{3.14}
$$

where the "-" and "+" subscripts refer to the value of the mode shape evaluated at the left and right edges of the actuator, respectively. The length of the beam $\ell$ appears in the equation because the mode shapes are expressed, as in the previous section, with respect to the non-dimensional coordinate $\tilde{x}$, normalized by the length of the beam $\ell$. It should be noted that in Eq. 3.14, the moment $M$, and not a distributed moment $m$, is used since the generalized force is applied only at the edges of the actuator and not along its entire length. Therefore, no integration along the length of the actuator need be performed in calculating the virtual work. Inserting the expression for the moment from section 2.4 (Eq. 2.89) the modal force $Q$ is found to be

$$
Q = \frac{2}{\ell} \left\{ \left[ \Delta F \frac{t_c}{2} + \bar{F} z_m \right] \varphi' \right\}_+ - \frac{2}{\ell} \left\{ \left[ \Delta F \frac{t_c}{2} + \bar{F} z_m \right] \varphi' \right\}_-
$$
Substituting Eqs. 2.94 and 2.95 for $\Delta F$ and $\bar{F}$, the final form for the modal force $Q$ is

$$Q = -\frac{2}{\ell} \left[ \left( \frac{E_b t_b \theta^2 b}{24 \theta_0^2 + \theta_0^2 + 2} \right) \Lambda \left( \phi^\prime \right)_+ - \phi^\prime \right] + \left\{ (M_k \phi^\prime)_+ - (M_k \phi^\prime)_- \right\} \frac{12 \theta_0^2 + 1}{24 \theta_0^2 + \theta_0^2 + 2} (3.16)$$

From the Bernoulli-Euler bending assumption,

$$M_k = \frac{-2IE_b}{t_b} \epsilon_{kk} \quad \text{(3.17)}$$
and

$$\epsilon_{kk} = -\frac{t_b}{2} \frac{\sigma^2 \gamma}{\sigma x^2} q \quad \text{(3.18)}$$

Substituting these two equations into Eq. 3.16, the modal force is found to be,

$$Q = \frac{-E_b t_b \theta^2 b}{6 \theta^2 (24 \theta_0^2 + \theta_0^2 + 2)} \left\{ \phi^\prime (a+m) \cdot \phi^\prime (a+m) - \phi^\prime (a-m) \cdot \phi^\prime (a-m) \right\} q$$

$$- \frac{2 E_b t_b \theta^2 b \cdot z_m}{\ell (24 \theta_0^2 + \theta_0^2 + 2)} \left\{ \phi^\prime (a+m) - \phi^\prime (a-m) \right\} \Lambda \quad \text{(3.19)}$$
where the locations for the right and left hand edges of the actuators, $a + \delta$ and $a - \delta$ were also inserted.

The similarities between Eq. 3.19 and Eq. 3.9 from the previous section are obvious. Once again, the modal force consists of two parts: a homogeneous term which appears in the equation of motion as an added passive stiffness term and a non-homogeneous term which is proportional to the voltage applied at the actuators. Eq. 3.19 can now be inserted into the single mode equation of motion

\[
\ddot{q} + \dot{B}q + (K + K_{\text{PIEZO}})q = - \frac{2 \varepsilon t_b \delta_b b z_m}{\varepsilon(24\theta^2 + \theta^2_b + 2)} \left\{ \left(\varphi^\prime(a+\delta) - \varphi^\prime(a-\delta) \right) \right\} A
\]

where $K_{\text{PIEZO}}$ is the added passive stiffness term due to embedding the actuators inside the beam and, from Eq. 3.19 is equal to,

\[
K_{\text{PIEZO}} = \frac{E_b b t_b^3(12\theta^2 + 1)}{6\varepsilon(24\theta^2 + \theta^2_b + 2)} \left\{ \left(\varphi^\prime(a+\delta) \cdot \varphi^\prime(a+\delta) - \varphi^\prime(a-\delta) \cdot \varphi^\prime(a-\delta) \right) \right\}
\]

Once again, the modal damping $B$ in Eq. 3.20 is yet to be determined. It should be noted $K_{\text{PIEZO}}$, as before, reflects the added passive stiffness due to the presence of the piezo-electrics. However, in this embedded case, material had to
be removed from the beam in order to insert the actuators. Removing this material decreased the stiffness of the beam. This decrease must be taken into account by the expression for the modal stiffness used in this analysis, Eq. 3.3. If the piezo-electrics have similar elastic properties to those of the beam, then $K_{PIEZO}$ (Eq. 3.21) would be approximately equal to the stiffness that is removed from the beam by embedding the actuators. This question will again be addressed in the next chapter.

The perfectly bonded embedded static model has now been successfully coupled to the dynamic model of a cantilevered beam. Eq. 3.20 can be used to find the solution for the time dependence of the generalized displacement $q(t)$ for any appropriate mode shape $\Psi$. The two models presented in this chapter can now be used to predict the behavior of cantilevered beams with surface bonded or embedded actuators. In the following chapter, these two models will be used to predict the behavior that of an aluminum beam with surface bonded actuators and of a glass/epoxy composite beam with embedded piezo-electrics used to excite the first and second bending modes of the beam. However, it should be noted that by simply choosing the appropriate mode shape $\Psi(x)$, the models derived above can be used for any Bernoulli-Euler beam with any type of boundary conditions.
In Chapter Two, static models for various configurations of surface bonded and embedded piezo-electric actuator models were examined. In Chapter Three, it was assumed that the actuators were part of a larger dynamic system. The static models were incorporated into a Rayleigh-Ritz model for the overall structure. In this chapter, these models will be used to predict the behavior of actual test articles.

Two test articles were constructed. The first consisted of an aluminum cantilevered beam with eight piezo-ceramics bonded near the root. The second was a cantilevered glass/epoxy beam with four piezo-ceramics embedded near the root. The exact locations of the actuators will be discussed in section 4.1. Each specimen presented some unique manufacturing difficulties which will be described in detail in this chapter. In particular, embedding the piezo-electrics in a glass/epoxy laminate involved some unique manufacturing problems. The high temperatures and pressures involved in the manufacture of glass/epoxy laminates imposed certain criteria on the type of piezo-electric material used. These criteria will be explained in section 4.2.1.

The test specimens were excited by driving the various actuators at the first and second bending resonant frequencies. The results from these tests are presented in section 4.3. The analysis from Chapter Three was then used to predict the behavior
of the specimens when excited by the piezo-electrics. The models developed in Chapter Three require a knowledge of the damping, and therefore a series of transient ringdown tests were carried out in order to determine the damping in the system. These tests are also explained in section 4.3.

In addition, since embedding piezo-electrics in a laminated structure necessarily affects the elastic properties of the structure, a series of static tests of glass/epoxy laminates with embedded actuators were performed. The data from these tests indicate that the embedded actuators reduce the ultimate strength of the glass/epoxy laminate, while the elastic modulus is not affected significantly.

Finally, since a realistic application for piezo-electric actuators would be to increase the damping in a structure as part of an active control system, a simple rate-feedback control experiment was performed on the aluminum test specimen. In this experiment, the aluminum beam was excited and the actuators were driven with a voltage $180^\circ$ out of phase with the velocity of the beam. Results from these tests show that with a small increase in mass, the damping in the system can be substantially increased.

4.1 Design of Dynamic Test Specimens

The aluminum and glass/epoxy test specimens are shown in Fig. 4.1 and 4.2. The piezo-ceramics in both specimens were arranged
so as to form Piezo-electric Moment Actuators. Here, a Piezo-electric Moment Actuator (PMA) is defined to consist of two piezo-ceramics, bonded or embedded at opposite but equal distances from the centerline of the beam, and driven $180^\circ$ out of phase. In this way, the net effect of a PMA is to impart a moment to the beam. In this section the choice of locations for the PMAs for both specimens will be discussed.

It was desired to excite the first and second cantilevered bending modes for both specimens. According to Meirovitch \textsuperscript{10}, the analytical expressions for these modes are:

\[ \Phi(x) = \sin(\lambda x) - \sinh(\lambda x) + \left[ \frac{\cos(\lambda) + \cosh(\lambda)}{\sin(\lambda) - \sinh(\lambda)} \right] \left[ \cos(\lambda x) - \cosh(\lambda x) \right] \]

where: \( \lambda = 1.875 \) for the first mode
\[ = 4.694 \] for the second mode

In order to excite the modes effectively, the PMAs must be placed in locations where their effect is greatest. Intuitively, since piezo-electrics deform the elastic substructure to which they are bonded, it would seem that they should be placed in regions of high average strain. This is confirmed by the theoretical analysis of Chapter 3. From Eq. 3.14, the modal force \( Q \) is equal to
If the length of the piezo-electric is reduced, the difference between the first derivative of the mode shapes approaches the value of the second derivative at the center of the piezo-electric. Since strain is proportional to the second derivative, Eq. 4.2 indicates that the piezo-electrics should be placed in regions of high average strain and away from areas of no strain. These "strain nodes" are determined by differentiating Eqs. 4.1 twice and finding the zero-crossing points for these functions. Performing this differentiation leads to the following strain shapes for the cantilever bending modes,

\[
\ddot{\tilde{r}}' = -\lambda^2 \left\{ \sin(\lambda \tilde{x}) + \sinh(\lambda \tilde{x}) + \left[ \frac{\cos(\lambda) + \cosh(\lambda)}{\sin(\lambda) - \sinh(\lambda)} \right] \cos(\lambda \tilde{x}) \right. \\
+ \left. \cosh(\lambda \tilde{x}) \right\} 
\] (4.3)

Eq. 4.3 is plotted in Fig. 4.3 for the first cantilevered bending mode and in Fig. 4.4 for the second mode. The first mode has no strain nodes. Therefore, if this were the only mode to be excited, the actuators could be placed anywhere along the beam and always be in a region of non-zero strain. Of course, for maximum effectiveness, they would be placed as near to the root as possible. Fig. 4.4 shows that the second mode has a strain node at \( \tilde{x} = 0.216 \). Since the actuators have a finite non-dimensional length \( \tilde{2m} \), the location of the center of the piezo-electric \( \tilde{a} \) must
satisfy either of the following conditions:

\[ a < .216 - \sim \quad (4.4) \]

or

\[ \sim > .216 + m \quad (4.5) \]

The exact locations for the actuators for each specimen will be described in the following section.

Fig. 4.4 also indicates why it is necessary to independently control the driving voltage that is applied to each PMA. Piezo-electrics apply a strain to the structure to which they are bonded. Since the strain shape for the second bending mode of a cantilevered beam has a strain node, a PMA located at \( \sim < .216 \) must be driven 180° out of phase with a PMA located at \( \sim > .216 \). If they were in phase, one set of actuators would, in effect, be inhibiting the motion of the beam. Therefore, it is necessary to independently control the driving voltage applied to each PMA. As will be seen in the next section, this requirement will introduce some manufacturing difficulties for both the aluminum and the composite test specimen.

4.2 Manufacturing Procedure

In this section, the manufacturing procedure for the two test specimens will be described in detail. First however, the choice
of the piezo-electric material will be discussed, along with the criteria considered in selecting the material used in the design of the test specimens.

4.2.1 Selection of the Piezo-Electric Actuator

A wide variety of piezo-electric materials, ranging from piezo-electric film to piezo-electric bimorph elements, are currently available for use as actuators. In the selection of the one used in the manufacture of the dynamic test specimens, certain criteria had to be considered, particularly in the design of the composite cantilevered beam.

A surface bonded actuator can be composed of almost any type of piezo-electric material. Since the actuator is bonded on the surface of the beam, there is no degradation of the passive elastic properties of the beam. It is, of course, desirable to use a piezo-electric material which has a high piezo-electric constant $d_{33}$, since this indicates a small amount of voltage applied across the terminals produces a large amount of strain. Besides this one obvious requirement, the design of the aluminum test specimen was not an important driver in the choice of piezo-electric material. The design of the composite test specimen, however, requires that the choice of actuator material satisfy two criteria:
The piezo-electric material must have an elastic modulus comparable to the modulus of a typical advanced composite laminate, because in a realistic application, when the piezo-electric is not being used as an actuator it must be able to carry stresses due to the passive loading of the composite. Also, when the PMA is embedded, glass/epoxy material must be removed to make room for the actuator. By inserting a material with a comparable modulus, the degradation of the passive properties of the composite would be minimized.

The Curie temperature (the temperature above which the piezo-electric permanently loses its piezo-electric properties) of the piezo-electric material must be higher than the curing temperature of the composite. If this were not the case, the piezo-electric would be destroyed during the curing process for the composite.

The two criteria above eliminated piezo-electric film as a candidate for the actuator material, since it has both a low elastic modulus and a low Curie temperature. The choice was then reduced to various varieties of piezo-electric ceramics. A piezo-electric ceramic manufactured by Piezo Electric Products, G-l195, was found to satisfy all these properties. The properties of this material are summarized in Table 4.1. In addition, G-l195 is readily available in flat sheets of various sizes with a
thickness of 10 mils. This is also the nominal thickness for a ply of a glass/epoxy laminate. Therefore, this material could be easily embedded inside a glass ply.

4.2.2 Manufacture of the Aluminum Test Specimen

After the selection of the piezo-electric material, the manufacture of the aluminum and glass/epoxy test specimens could now proceed. In this section, the manufacture of the aluminum test specimen will be described. This test specimen consisted of a 40 x 3.81 x .318 cm 2024 aluminum beam (Fig. 4.1). The actual physical length of the aluminum was 50 cm, but as will be seen in the next section, 10 cm was inserted inside the clamp in order to insure a cantilevered boundary condition. Therefore, only 40 cm of the beam remained outside to clamp. Four PMAs were bonded to the beam. As can be seen from Fig. 4.1, the PMAs were arranged to form two sets of actuators. The dimensions of each G-1195 ceramic were 38.1 mm long x 15.24 mm wide x .25 mm thick. The rear set was bonded with the center of the actuators located at \( x = 0.06 \) and the front set at \( x = 0.3 \). These locations satisfy Eqs. 4.4 and 4.5, and therefore the actuators were located away from the second mode strain node.

Each ceramic had electrodes over the top and bottom faces and were poled so that a voltage applied through the thickness produced displacement along the length (Fig. 4.5). However, in
order to apply voltage across the thickness of each piezo-electric, it is necessary to make electrical contact with the top and bottom electrodes of each piezo-electric individually. As was discussed in the previous section, this is an important requirement, and will allow the actuators to be used effectively. Once the actuators are bonded to the beam, however, it is impossible to make electrical contact with the electrode of the actuators in contact with the beam. Two solutions to this problem were considered. First, conducting epoxy could be used to bond the piezo-ceramics. Extreme care would have to be taken in order to insure that none of the conducting epoxy flowed around the edges of the piezo-electric, thus shorting it electrically. The entire beam could then be used as an electrical bus connection. Grounding the beam electrically would ground all the electrodes of the piezo-ceramics in contact with the beam. Positive or negative voltages, with respect to the electrical ground, could then be applied across the actuators, depending on their location along the beam and the mode being excited. This would require either two separate power supplies to be used to drive the actuators, or a single supply capable of delivering a positive and a negative voltage.

A second option would be to use a non-conductive epoxy and drill small holes underneath each actuator in order to make electrical contact directly with each electrode. This option has the advantage that a single AC power supply could be used to drive
all the actuators. The phase of the driving voltage could be set to either 0° or 180° by simply switching the positive and negative terminals on the incoming driving voltage before reaching the actuator electrodes. This is the option that was used in manufacturing the aluminum specimen. In order to achieve this, before the piezo-electrics were bonded to the beam, small access holes, \( \frac{1}{32}'' \) diameter and .3" deep, were drilled at the side of the beam at the center of the locations where the PMA were to be bonded (Fig. 4.6). In addition, \( \frac{1}{32}'' \) holes were drilled through the thickness at the geometric center of each PMA location such that these intersected the holes that were drilled previously. In this way, wires could be inserted through these holes to contact the electrodes on the underside of each piezo-electric. Finally, in order to avoid any possibility of a short-circuit through the beam, the entire beam was anodized.

The manufacturing of the aluminum beam is now complete and the piezo-electrics may now be applied. The piezo-electrics were bonded to the beam using EPY-150, a 24 hour room temperature curing epoxy manufactured by BLH Electronics. The properties of this epoxy are given in Table 4.2. Emerson & Cuming IG-101 microballoons of a maximum diameter of 175 microns were mixed into the epoxy. In this way, when pressure is applied during the cure, the thickness of the bonding layer decreases until it is equal to the maximum diameter of the microballoons.

Before bonding each PMA, wires were inserted through the
holes in the aluminum beam and carefully soldered to the electrodes of the piezo-ceramic. Epoxy was applied to the piezo-ceramic electrode. As the ceramic was set on the beam, the wire soldered to the electrode was pulled through the hole so that it remained taut (Fig. 4.6). In this way, no slack accumulated underneath the piezo-ceramic. This is very important since the ceramics are extremely brittle. If the wire was not pulled completely back into the hole in the beam, the actuators would break when pressure was applied for bonding. This procedure was performed twice for each actuator since two piezo-cermics are necessary to make up one PMA. Finally, a compressive rotary spring was applied in order to maintain a constant pressure on the piezo-electrics throughout the 24 hour curing time.

In order to measure the response of the beam, an Endevco 222C accelerometer was bonded to the tip and a steel target for a Bentley-Nevada 7200 proximity transducer was bonded at mid-span. The transducer was then aimed at the target so as to permit displacement measurements at mid-span. The use of these devices will be explained in more detail in section 4.3.

Having completed the manufacture of the aluminum test specimen, the manufacture of the composite specimen will now be described. Because this second specimen was manufactured out of an advanced composite material, the manufacturing techniques used were completely different.
4.2.3 Manufacture of the Composite Test Specimen

The dynamic composite test specimen consisted of a 32 × 3.81 × .254 cm glass/epoxy beam. Once again, the actual length of the glass/epoxy beam was 37 cm, however 5 cm were inserted inside the clamp in order to insure a cantilevered boundary condition. Four G1195 piezo-ceramics, forming two PMAs, were embedded inside the beam. The dimensions of these piezo-ceramics were the same as the ones used in the manufacture of the aluminum beam. Each ceramic had a metallic lead soldered to the electrode on each face (Fig. 4.7). The rear PMA was embedded at \( x = .06 \) and the front PMA was at \( x = .3 \). Once again, these locations satisfy Eqs. 4.4 and 4.5, insuring that the actuators are located away from the second mode strain node.

The laminate consisted of 10 plies of glass/epoxy material. Each ply of this laminate had a nominal thickness of .254 mm. This is equal to both the nominal thickness of the piezo-ceramic and the thickness of the attached leads. Therefore, by cutting out holes of the exact dimensions of the piezo-ceramics and the leads, they could be inserted inside the laminate without causing any change in the nominal thickness. The glass/epoxy beam layup was [0/90/0/90/0]_{s}. This corresponds to the fibers in the top ply of the beam being at a 0° angle, the second ply being at a 90° angle, and so on. The " s " subscript in the layup indicates that the layup sequence is repeated symmetrically about the centerline.
The piezo-ceramics were embedded in the third and seventh ply. This layup was chosen primarily to simplify the manufacturing procedure. In laying-up glass/epoxy prepreg, it is much easier to cut either perfectly perpendicular to a fiber or along a fiber than to cut at an angle. Therefore, by only using plies whose fibers ran along the length or the width of the laminate, cutting out a rectangular hole in one of the plies in order to insert a piezo-electric would be greatly simplified. In addition, slits for the leads bonded to the piezo-electrics also had to be cut. By having a 90° ply on either side of the embedded piezo-ceramic, the slits could be cut without cutting any fibers, and thus not significantly reducing the strength of the composite.

Having completed the description of the laminate, the lay-up and curing procedure for the glass/epoxy laminate will now be summarized. The curing procedure followed was the standard one used by the Technology Laboratory for Advanced Composites at M.I.T. It is based on recommendations from the manufacturer plus additional information obtained from actual testing performed to determine the optimal curing cycle for various advanced composite materials. The glass/epoxy prepreg was cut and laid up in a clean room environment. The piezo-lead assemblies were set in the prepreg during the layup procedure. Teflon was wrapped around the leads once they were outside the laminate in order to keep any excess epoxy that might flow during the curing process off the leads. The laminate was then covered in peel-ply and set on an
aluminum cure plate. It was surrounded by a cork dam assembly to keep it from shifting during the curing process. Various sheets of teflon and bleeder were placed on top of the laminate in order to absorb the excess epoxy. Finally, an aluminum top plate cut to the exact dimensions of the laminate was placed on top. The entire assembly was vacuum-bagged and inserted in the autoclave. Once inside, a vacuum was drawn, pressure was applied and the temperature was elevated. The vacuum, pressure and temperature cure cycle profiles are shown in Fig. 4.8.

After curing, the laminate was removed from the curing assembly. In order to verify the conditions of the piezo-electrics, a voltmeter was attached to the leads to check for any short-circuit through the thickness of the ceramics. All the ceramics were found to be in working order. Finally, an Endevco 222C accelerometer was attached at the tip, in order to measure the response of the beam.

In addition to the dynamic test specimen, static test specimens were constructed in order to test how the presence of the embedded actuator affected the static properties of the laminate in which it was embedded. The dimensions of these specimens are shown in Fig. 4.9. They were constructed in the same manner as the dynamic one, except that only one PMA, located in the center of the specimen, was embedded. In addition, glass tabs were subsequently bonded at the ends of the specimens in order to allow clamping in the static testing machine. Strain
gauges were attached in the region of the PMA and in the far field in order to measure the Young's modulus of the material and the effect of the piezo-electric on the local strain distribution. The location of these strain gauges, along with a description of the static tests themselves, will be described in section 4.6. Finally, static test specimens without any embedded PMAs were constructed and tested in order to obtain control values for the modulus and ultimate strength of the material.

The manufacture of the dynamic composite laminate and the associated static specimens is now complete. However, it should be noted that before the glass/epoxy laminate was manufactured, several attempts were made to manufacture a similar composite beam using graphite/epoxy. A similar manufacturing procedure to the one described above was used. In all cases, the actuators shorted out because the electrodes made contact with the surrounding graphite. Various attempts were made to insulate the piezo-electrics from the graphite. In particular, the piezo-electrics were coated with M-Coat C, a polyurethane insulator, before embedding them into the graphite/epoxy laminate. This was unsuccessful and the ceramic shorted through the graphite. This specimen was then dissected and close examination revealed that the insulator had withstood the elevated curing temperature, but the high pressure applied during the cycle allowed the graphite fibers to tear the insulating layer, thus shorting the ceramic. Another insulating coating tested was M-Bond 610, which
is normally used as a strain gauge adhesive. This coating, however, could not survive the high curing temperatures and debonded from the ceramic during the cure. Another attempt was made using 3501-6 epoxy. However, this too failed because the shrinkage of the epoxy as it cooled shattered the ceramics. Experimentation continues in this area, and it is expected that this insulation problem will be solved eventually.

4.3 Experimental Procedure for Dynamic Tests

In this section, the dynamic testing procedure for both test specimens will be described along with the technique used to measure the damping present in the system. This is necessary since a knowledge of the damping is required to correctly predict the behavior of the system.

Both test specimens were vertically clamped to a 50 × 50 × 15 cm steel base test stand. The clamp consisted of two 10 × 20 × 5 cm aluminum blocks bolted to the test stand (Fig. 4.10). The entire test stand sat on a 1/2 " thick rubber sheet in order to dynamically isolate the test specimen from the surrounding laboratory vibrations. The clamping assembly was effective in providing an accurate cantilevered condition since the ratio of the mass of the aluminum beam to the clamp was 2 %. The same ratio for the glass beam was 0.7 %. In addition, the actual 1st mode bending frequency measured for the aluminum beam was
16.35 Hz. The theoretical first mode frequency obtained by assuming a cantilevered Bernoulli-Euler beam \[^{[10]}\] is

\[
\omega = 1.875^2 \sqrt{\frac{EI}{ml}}
\]  \hspace{1cm} (4.6)

Inserting the dimensions of the aluminum test specimen into Eq. 4.6, the theoretical natural frequency is found to be 16.5 Hz. Therefore, the differences between the actual and theoretical frequency is less than 2%. The same comparison cannot be made for the composite test specimen since the natural frequency of composite materials is normally 10-20% less than the value predicted by Eq. 4.6, independently of how effectively the cantilevered boundary condition is applied \[^{[11]}\].

The excitation and measurement systems for the dynamic tests will now be described. These systems are shown in block diagram form in Figs. 4.11 and 4.12. The test specimen was excited sinusoidally using a Wavetek Model 184 signal generator. This signal was amplified using an Altec 1590C 200 W amplifier. Since piezo-electrics are high impedance devices, the amplifier was configured for a high impedance load. In this configuration, the amplifier can be modelled as a voltage source.

Before being sent to the PMAs, the amplified signal was sent through a switching box. This box had a dual purpose:
It separated the input into two or four separate signals of equal magnitude, depending on how many actuators were on the specimen being tested. In this way all the PMAs can be driven separately while using only one amplifier and the beam could be excited using any combination of the four PMAs.

It controlled the phasing of the driving voltage that is used to drive the PMAs. Once the incoming signal was separated, it could be sent to each PMA with a relative phase of either $0^\circ$ or $180^\circ$. This is accomplished by switching the positive and negative terminals on the incoming signal. Fig. 4.13 represents the electrical schematic for two PMAs being driven by an AC amplifier.

During each test, the outputs from the proximity sensor and the accelerometer were recorded on a Nicolet Digital Oscilloscope. The input voltage from the amplifier was determined using a digital multi-meter.

In order to test for the effective coupling of the ceramics to the beam, sinusoidal excitation tests were made at first and second resonance for both test specimens. The response to this excitation will be compared to the predicted response for both specimens. When driven at resonance, the second order system represented by Eq. 3.12 or Eq. 3.20 responds with amplitude
Amplitude = $\frac{Q}{2(K + K_{\text{PIEZ}})\zeta}$ (4.7)

where $Q$, $K$, and $K_{\text{PIEZ}}$ were found in Chapter Three. However, for the embedded actuator case, the beam modal stiffness $K$ (Eq. 3.3) does not account for the reduction in modal stiffness due to the removal of material from the beam when the actuators were embedded. Therefore, Eq. 3.3 actually computes the stiffness for a cantilevered beam without any embedded actuators. In order to compensate for this, the piezo-electric stiffness term $K_{\text{PIEZ}}$ is set to zero when using Eq. 4.7 to predict the response of the glass/epoxy beam. Of course for the aluminum beam, bonding the actuators on the surface does not imply a reduction of beam modal stiffness and $K_{\text{PIEZ}}$ calculated in section 3.1 (Eq. 3.13) will be used.

For Eq. 4.7 to correctly predict the amplitude of response, a knowledge of the damping $\zeta$ must be at hand. In order to accomplish this, a series of ringdown tests were performed for both specimens at both the first and second bending frequencies. These tests were used to determine the damping in each system as a function of amplitude. The specimen was excited using the PMA and after steady-state had been reached, the excitation was stopped. The resulting exponentially decaying signal from the accelerometer was recorded on the oscilloscope. This data was later transferred to an IBM PC-XT and an exponentially decaying curve was fitted
using a least squares algorithm. These experimentally determined functions of damping vs. amplitude were then inserted into Eq. 4.7 to find the response of the test specimens. Using these fitted curves, it was determined that the damping in the system varies with the amplitude of the beam. This was postulated to be partly due to the effects of aerodynamic damping, since the tests were not conducted in a vacuum, and partly to the friction present in the clamp. The actual measured damping and the beam response at each excitation level for various tests are given in Tables 4.3 through 4.8 for the aluminum beam and in Tables 4.10 through 4.15 for the glass/epoxy beam. As can be seen from these tables, the variation in damping was more pronounced for the aluminum test specimen than for the composite one, due to the larger excitation amplitudes.

With the damping thus determined, the response of the specimens at resonance as a function of the applied voltage could be determined. This was done for both the first and second bending frequencies using various combinations of actuators. These predicted responses were then compared with the measured responses of various sine dwell tests performed on the two specimens. These results and the way the response was predicted for both specimens will be discussed in more detail in the following two sections.
4.4 Dynamic Test Results - Aluminum Beam

A series of sine dwell tests were done on the aluminum beam in order to determine its response when excited by the PMAs. In this section, the results of these tests will be presented and the method of predicting the response of the beam will be described in detail.

Using Eq. 4.7 along with the definitions for the terms in these equations derived in section 3.1, the amplitude of the beam as a function of the voltage applied across a PMA can be predicted. The relevant equations will not be rewritten here, but the procedure for predicting the response will be outlined. Eq. 3.9 is used to find the modal force $Q(V)$ as a function of the voltage applied across the PMA. In order to accomplish this, a mode shape $\psi$ must be assumed for the beam. For these cantilevered bending tests, the mode shape assumed is the exact solution for a Bernoulli-Euler beam given by Eq. 4.1. The assumed mode shape is also used in determining the modal stiffness $K$ and the piezo-electric stiffening term $K_{PIZZO}$ from Eq. 3.3 and Eq. 3.13 respectively. The values obtained for $Q(V)$, $K$, and $K_{PIZZO}$ are then inserted into Eq. 4.7. This leads to a value for the amplitude at resonance of the generalized displacement $q$ as a function of the applied voltage and the experimentally measured damping $\zeta$. The damping is then determined as described in the
previous section and an expression for the \( q \) at resonance as a function of the applied voltage is obtained. Finally, this is inserted into

\[
w = \mathcal{P}(\bar{x})q(t)
\]  

(4.8)

By setting \( \bar{x} = 1 \), the tip amplitude for the aluminum test specimen is determined as a function of the voltage. Using the above procedure, the response of the aluminum beam can be predicted for any PMA driving either the first or the second bending mode. Having predicted the response of the beam, sine dwell tests were performed on the test specimen and the response was compared to the predicted behavior.

The beam was excited in its first and second mode using first the rear actuator set, then the front set, then both sets simultaneously. For this last case, the voltages applied to the front and rear actuator set was always equal in magnitude, however it was only equal in phase for the first mode tests; the driving voltages were 180° out of phase for the second mode tests due to the presence of the strain node between the actuator locations.

Figs. 4.14, 4.15, and 4.16 show the results of the sine dwell tests for the first mode. Figs. 4.17, 4.18, and 4.19 show the results for the second mode tests. The actual measured tip amplitude of the beam, as well as the predicted amplitude based on the analysis described above is shown. These results are also
shown in Tables 4.3 through 4.8 along with the experimentally measured damping at the response amplitudes. As was mentioned earlier, the actual measured damping varied with amplitude and this accounts for the non-linearity of the predicted response. As can be seen from the graphs, this is in agreement with the non-linearity observed in the actual test results.

It should be noted that as predicted, the rear PMAs, which for the first bending mode were located in regions of higher strain than the forward PMAs, excite a greater response in the beam. It is also observed that when all PMAs are equally excited, their responses add linearly. This is not surprising since the analytical model is linear, and therefore the principal of superposition is valid.

4.5 Dynamic Test Results - Glass Beam

A series of sine dwell tests were performed on the glass/epoxy specimen. These tests were similar to the ones performed on the aluminum test specimen. The coupled, embedded actuator-dynamic model of section 3.2 was used to predict the results of these tests. In this section, the results of these tests will be presented. In addition, the method for predicting the response of the beam will be outlined in detail.

As mentioned earlier, the embedded, perfectly bonded model was used to predict the response of this test specimen due to the
small amount of shear present in the bonding layers between the actuator and the glass/epoxy. Using Eq. 4.7 and 4.8, the response of the glass specimen when driven by either PMA can be predicted. The definitions for the terms in these two equations were derived in Chapter Three. The solution procedure for determining the beam displacement as a function of the applied voltage is similar to the one used for the aluminum test specimen, however the expression for the modal force Q(V) as a function of the applied voltage is now given in Eq. 3.19. The mode shapes assumed in deriving this expression are the same as the ones used for the aluminum test specimen (Eq. 4.1 with \( \lambda = 1.875 \) for the first mode or \( \lambda = 4.694 \) for the second). This mode shape is also used in finding the modal stiffness \( K \) (Eq. 3.3). The elastic properties of the glass/epoxy beam are given in Table 4.9. These were obtained using L.A.S.P., a laminated plate analysis computer routine used in T.E.L.A.C. This routine calculates the elastic properties of each ply in the laminate, given the material properties and the ply orientation. The program then uses these ply properties to solve the laminated plate equations exactly. The laminate values for the various elastic constants are thus obtained [12].

Having obtained the modal stiffness \( K \) and the modal force \( Q(V) \) as a function of the voltage, these are then inserted into Eq. 4.7 along with the experimentally determined function for the damping, to obtain an expression for the generalized amplitude as
a function of the voltage. Finally, by inserting this result into Eq. 4.8, an expression for the beam amplitude at resonance as a function of the voltage applied across the PMA is obtained.

With the beam amplitude as a function of the voltage determined, a series of sine dwell tests were performed on the glass/epoxy specimen. The beam was first excited at its first cantilevered bending mode frequency, then at its second cantilevered bending frequency. Three series of tests at both frequencies were performed: first the rear actuator alone was used to drive the beam, then the front, and finally both simultaneously. Once again, for this last series the actuators were kept in phase with each other for the first mode tests but were phased by 180° in the second mode tests, due to the presence of a strain node (Fig. 4.4).

Figs. 4.20, 4.21, 4.22 show the results of the sine dwell tests for the first mode. Figs. 4.23, 4.24, and 4.25 show the results for the second mode tests. The actual measured tip amplitude of the beam, as well as the predicted response based on the above analysis, is shown. These results are also shown in Tables 4.10 through 4.15, along with the experimentally measured damping at the various amplitudes. The variation of the damping with amplitude accounts for the slight non-linearity observed in the responses. As can be seen from the graphs, the measured and predicted responses are in excellent agreement for both the first and second mode tests. This is not surprising even though a very
simple pinned-model was used in the prediction of the response, since the small interlaminar bonding layer reduces the shear lag in the system to negligible levels. Also, it can be observed from the graphs that the effectiveness of the PMAs add linearly, i.e., the principle of superposition is valid. The response obtained from using both actuators simultaneously is equivalent to that obtained by adding the responses of the rear and front actuators when driving the beam individually.

4.6 Static Testing Procedure and Results

When the piezo-electrics were embedded inside the glass/epoxy laminate, their presence caused a discontinuity in the fibers of the ply in which they were embedded. It is reasonable to assume that the presence of this discontinuity would have some effect on the passive elastic static properties of the laminate. In particular, the discontinuity should decrease the ultimate strength of the laminate. In order to obtain a quantitative measure of the effect of the embedded actuator on the laminate properties, a series of static tests were carried out. Six static test specimens were manufactured, as described in section 4.2.3. These consisted of: two specimens without any piezo-ceramics embedded inside, for the purposes of obtaining experimental unflawed laminate properties; two specimens with a PMA embedded inside in the same manner that was embedded in the dynamic
specimen; two specimens with a G-1195 bimorph embedded along the centerline. A bimorph consists of two G-1195 ceramics bonded together in such a way that a voltage applied across the assembly causes the bimorph to bend. In this way, even though it is located along the centerline of the laminate, it can still cause bending when driven by a voltage source. Of course, bimorphs are not as effective as the PMAs that were used for the dynamic tests, since by having two ceramics directly bonded to each other, the moment arm of the actuator is effectively reduced to zero, thus reducing the modal force \( Q \) (Eq. 3.19). Nevertheless, embedding the bimorphs produces only one discontinuity in the laminate, unlike embedding a standard PMA which produces two separate discontinuities. Therefore, embedding the bimorphs in a static specimen would provide an indication as to whether it is desirable to have one large discontinuity or two small ones. As will be seen shortly, the behavior of all the flawed laminates was identical and independent of whether a bimorph or a PMA was embedded.

The dimensions and strain gauge locations of the static test specimens are shown in Fig. 4.26. As can be seen from this figure, strain gauges were applied in the far-field away from the actuator, as well as in the region surrounding the PMA. The locations of the gauges were picked in order to obtain the behavior under loading of the laminate near the actuator, as well as far away where the effect of the PMA would be reduced.
Therefore, strain gauges were placed directly over the PMA, as well as close to it near the edge and over the leads. For the exact locations of the strain gauges, please refer to Fig. 4.26. Additionally, the distribution of the strain around the actuator was determined by using a photoelastic coating on some of the specimens. By taking photographs of the coating through a polarizing glass during the static test, the strain distribution could be visualized.

A schematic of the static test set-up is shown in Fig. 4.27. The testing procedure consisted of mounting the specimens on the MTS-810 testing machine and attaching the strain gauges to a DEC PDP 11-23 computer. The loading machine then began applying a monotonic loading under stroke control at a rate of .04 in/min, until failure in the specimen was detected. The loading and strain gauge data is automatically collected by the computer and is immediately available for analysis.

A load vs. strain plot using the strain readings from all four longitudinal gauges for a specimen with no embedded actuators is shown in Fig. 4.28. The labels on this plot indicate that the strain gauges on this specimen are located at the locations as those on the specimens with embedded actuators. Fig. 4.29 shows typical load vs. strain data for a specimen with an embedded PMA. As can be seen from these graphs, the behavior of the curves is similar for both specimens and the presence of the ceramics does not seem to affect the slope of the curves. By fitting a straight
line using a least squares routine to the initial part of the load-strain curves, numerical values for the elastic modulus of all the specimens based on the strain data from each gauge were obtained and are shown in Table 4.16. The theoretical value for the elastic modulus obtained from the LASP program (Section 4.5) is 27.8 GPa. Therefore, the presence of the ceramic does not appear to have an effect on the modulus of the laminate.

Table 4.17 shows the value of the stress at which the laminates failed catastrophically. The predicted value was obtained using stress interaction criteria [12] assuming the laminate could still carry load after first ply failure. These values agree very well with the failure stresses of the unflawed laminates. However, the flawed laminates failed at a stress 20% less than the predicted value. Therefore, the presence of the piezo-electrics, while not affecting the modulus of the laminate has a noticeable effect on the ultimate strength. This result is not surprising when the stress-distribution around the ceramic is observed using the photoelastic coating. Fig. 4.32 shows the area around the ceramic under loading. The lighter areas correspond to regions of high stress. It is clear that the entire area around the PMA has numerous stress-concentrations. Therefore, the stress in these areas reaches a failure level before the far-field stress does. This is, of course, not the case for a typical unflawed composite which, theoretically does not have any stress concentrations.
4.7 Active Control Experiments

Until now, the physical models developed have been used only to predict the response of the system under excitation from the piezo-electric actuators. However, in many real applications, the actuators would be integrated into an active control system, and would be used to damp out any motion resulting from external excitations on the structure. In order to physically demonstrate this capability on our test specimen, a simple active control experiment was performed on the aluminum test specimen.

For this test, the output from the accelerometer was connected to the analog to digital channel of a Labtech microcomputer. Using a simple numerical integration routine, the accelerometer signal was integrated and amplified. The output was then inputted into the PMA. This is equivalent to using a simple rate-feedback control algorithm.

The aluminum test specimen was excited at its first mode bending frequency using all four PMAs as described in section 4.3. After steady state had been reached, the excitation signal from the signal generator was turned off and replaced by the output of the controller. Since this output is proportional to the velocity of the beam, this causes the PMAs to apply a strain $180^\circ$ out of phase with the motion, thus causing the beam motion to be damped out very rapidly.
The decay of the beam using the PMAs as active controllers is shown in Fig. 4.31. For comparison, the natural decay of the beam without any active control is shown in Fig. 4.32. As can be seen from these two figures, there is a large increase in damping when the ceramics are used. The natural damping in the system is .4%. Using the ceramics for active control increases the damping to 4%. It should be remembered that this result is obtained using a very simple rate feedback control algorithm and, more importantly, with a negligible mass increase, since the mass of the aluminum beam is considerably greater than the mass of the ceramics. By adding more ceramics, or improving the control algorithm, the results in this section could be improved.

4.8 Summary

In this chapter, two experimental systems with piezo-electric actuators were constructed and tested. The concept of a Piezo-Electric Moment Actuator was explained. The analytical models derived in Chapters Two and Three were used to predict the behavior of both systems. However, because the dynamic equations require the damping in the system to be known, the damping was measured before each dynamic test and this experimental value for the damping was incorporated into the analytical model. Using this model, the behavior of both test specimens was very accurately predicted.
Since flaws are introduced into the composite specimen by embedding the actuators, a static test program to determine how the static elastic values of the composite are affected by the presence of the PMA was carried out. The results of these tests show that while the modulus of the composite is not affected by the actuator presence, the ultimate strength is reduced by 20%. This decrease in strength is large enough to require that it be taken into account in designing structures with embedded actuators. However, due to the small amounts of strain that piezo-electric produce, they normally would not be used in any application which requires high strength. Of course, this reduction in ultimate strength is not present in any structure with surface actuators, since no flaws are introduced into the structure by simply bonding the piezo-electrics to the surface.

Finally, a simple control experiment demonstrated the effectiveness of piezo-electrics when used as actuators to increase the damping in a structural system. On the aluminum test specimen, an increase in the damping of an order of magnitude was observed when the actuators were used as part of a simple rate-feedback control loop. Of course, this has not been proved as a general result. That is, it might not work as well on another structure with different dimensions and different elastic properties. A more general analysis of this question will be addressed in Chapter 5. There, the effects of scale on the amount of damping that a piezo-electric actuator can produce when used in a rate feedback control loop will be examined.
A valid and reasonable question remains as to the effectiveness of piezo-electrics in the control of realistic structures. While it has been shown in the previous chapters that coupled static-dynamic models are adequate in predicting the behavior of structures with distributed control systems, the experimental verification of these models was performed on small laboratory test specimens. Therefore, in order to demonstrate the validity of the concept in larger, more realistic space structures, the capability of distributed actuators to scale with the structure to which they are attached must be analyzed.

In this chapter, two cases will be analyzed, corresponding to a beam in extension with a pair of surface mounted piezo-electrics and to a beam in bending with one piezo-electric moment actuator bonded on the surface. For the purpose of this analysis, the perfectly bonded surface mounted actuator models from section 2.1.1 and section 2.2.1 will be used. Using these models, and assuming a simple rate feedback control law, analytical expressions for the effective active damping produced by the actuators are obtained. The damping therefore serves as a non-dimensional measure of the effectiveness of the piezo-electric actuator.
5.1 Extensional Scaling Analysis

In this section, the appropriate scaling relationships of a bar with a pair of extensional surface bonded piezo-electric actuators are derived. Fig. 5.1 shows a bar with two perfectly bonded piezo-electric actuators bonded on the top and bottom surfaces. For the purposes of this analysis, the boundary conditions on the bar need not be specified. For this system, the modal mass and modal stiffness terms are given by:

\[
M = \int_{0}^{1} m \ell \varphi^2 \, \dd x = \rho A_0 \ell \int_{0}^{1} \frac{A}{A_0} \varphi^2 \, \dd x = \rho b t_b \ell M
\]

\[
K = \int_{0}^{1} \frac{E A (\varphi')^2}{\ell} \, \dd x = \frac{E A_0}{\ell} \int_{0}^{1} \frac{E A}{E A_0} (\varphi')^2 \, \dd x = \frac{E b t_b}{\ell} K
\]

where \(M\) and \(K\) are non-dimensional constants that depend only on the assumed axial mode shape \(\varphi\) and a rectangular cross section has been assumed for the bar. In the above two equations, the axial displacement \(u(x,t)\) has been assumed to be:

\[
u(x,t) = \varphi(x)q(t)
\]

For this case, the modal force is equal to:

\[
Q = 2F \cdot \left[ \varphi \big|_{-} - \varphi \big|_{+} \right]
\]
where $F$ is the force applied at the edges of the piezo-electric. The '−' and '+' subscripts refer to the values at the left hand side and the right hand side of the piezo-electric.

From section 2.1.1 (Eq. 2.8) the force $F$ applied by the piezo-electric actuator in the absence of other externally applied forces is equal to

$$\frac{F}{b} = \left[ \frac{A}{2 + \varphi} \right] E_b t_b \quad \text{(5.5)}$$

In order to observe how the effectiveness of the piezo-electric scales, it is assumed that a simple rate feedback law has been implemented to increase the modal damping. In such a linear rate feedback system, the excitation voltage is assumed to be proportional to the bar extensional velocity:

$$V = \lambda_v \dot{q} \quad \text{(5.6)}$$

and therefore the applied piezo-strain is:

$$\lambda = \frac{d_{31} V}{t_c} = \frac{d_{31} \lambda_v}{t_c} \dot{q} \quad \text{(5.7)}$$

Eqs. 5.6 and 5.7 are now inserted into Eq. 5.5, to obtain an expression for the modal force $Q$, proportional to the axial velocity of the beam.
It is observed that since the modal force $Q$ was assumed to be proportional to the bar velocity, when Eq. 5.8 is inserted into the one degree of freedom equation of motion,

$$M \ddot{q} + 2\zeta \omega_0 \dot{q} + \omega_0^2 M q = Q \quad (5.9)$$

the modal force will increase the modal damping. In order to measure the increase in damping ratio $\zeta$ due to the actuator, Eq. 5.8 is equated to the modal damping term,

$$2\zeta \omega_0 \dot{q} = Q = 2 \cdot b \cdot \frac{d_3}{t_c} \cdot \lambda_v \left[ \frac{E_b t_b}{2 + \varphi} \right] \cdot \left[ \rho \left| \rho_- - \rho_{+} \right| \right] \dot{q} \quad (5.10)$$

Solving Eq. 5.10 for the effective modal damping $\zeta$,

$$\zeta = \frac{b}{M \omega_0} \cdot \frac{d_3}{t_c} \cdot \lambda_v \left[ \frac{E_b t_b}{2 + \varphi} \right] \cdot \left[ \rho_{\text{lhs}} - \rho_{\text{rhs}} \right] \quad (5.11)$$

This is the damping obtained from the rate feedback of the piezo-electric actuator.

It is clear that as the gain $\lambda_v$ is increased, the damping due to the rate feedback on the actuators is also increased.
Therefore, in order to determine a maximum value of $\zeta$ it is necessary to impose a maximum value of $\lambda_v$, which is set by the actuator saturation limits. In this case, the appropriate upper limit is the field necessary to depole or destroy the piezo-electric properties of the actuator. The maximum field $\zeta_{\text{max}}$ that can be applied is related to the gain as:

$$\lambda_v = \frac{V_{\text{max}}}{q_{\text{max}}} = \frac{\zeta_{\text{max}} t_c}{q_{\text{max}}}$$  \hspace{1cm} (5.12)$$

It is now assumed that the motion of interest is at the resonant frequency of the bar and therefore, $\dot{q} = \omega_0 q$. Substituting this into Eq. 5.12, the maximum gain that can be applied is

$$\lambda_v = \frac{\zeta_{\text{max}} t_c}{\omega_0 q_{\text{max}}}$$  \hspace{1cm} (5.13)$$

Substituting Eq. 5.13, into Eq. 5.11, the effective modal damping is

$$\zeta = \frac{b}{q_{\text{max}}} \frac{d_{31} \zeta_{\text{max}}}{K} \left[ \frac{E_h t_b}{2 + \varphi} \right] \left[ \varphi \left| - \varphi \right|_{\cdot} \right]$$  \hspace{1cm} (5.14)$$

where the relationship between the modal mass and stiffness

$$K = \omega_0^2 M$$  \hspace{1cm} (5.15)$$
was substituted. Finally, inserting Eq. 5.15, the final expression for the effective modal damping due to the piezo-electric actuators becomes

\[
\zeta = \left[ \frac{\ell}{q_{\text{max}}} \right] (d_{31} c_{\text{max}}) \left[ \frac{1}{2 + \varphi} \right] \left[ \frac{\varphi - \varphi}{\kappa} \right]
\]  

(5.16)

Eq. 5.16 is the expression for the non-dimensional damping of a bar due to surface bonded extensional actuators. A discussion of this equation will be presented in the following section, since it will be shown that Eq. 5.16 is very similar to the expression that will be derived in section 5.2 for surface bonded actuators exciting bending in a cantilevered beam.

5.2 Bending Scaling Analysis

The bending scaling analysis of this section is very similar to the extensional analysis of the previous section. As before, the perfectly bonded model developed in Chapter Two will be used. For this system, the modal mass and stiffness terms are

\[
M = \int_0^1 m \ell \varphi^2 \, dx = \rho A_0 \ell \int_0^1 \frac{A}{A_0} \varphi^2 \, dx = \rho b t_b \ell M
\]  

(5.17)
where the displacement $w(x,t)$ is assumed to be:

$$w(x,t) = \hat{r}(x)q(t)$$  \hspace{0.5cm} (5.19)

For this case, the modal force due to the force $F$ applied at each pin in the absence of any externally applied moments is

$$Q = \frac{F t_b}{\ell} \left[ \hat{r}'|_+ - \hat{r}'|_- \right]$$  \hspace{0.5cm} (5.20)

From section 2.2.1, the force $F$ applied at the pins is equal to,

$$\frac{F}{b} = \left[ \frac{A}{b + \alpha} \right] E_b t_b$$  \hspace{0.5cm} (5.21)

The same procedure used in the previous section for the extensional actuators will be used here. Again, using the rate feedback law from the previous section (Eq. 5.7) will be assumed and substituted into Eq. 5.21. This will then be substituted into the expression for the modal force $Q$, Eq. 5.20, giving a relation between $Q$ and the beam velocity.
\[ Q = \frac{bt_b}{\ell} \frac{d_{31} \lambda v}{t_c} \left[ \frac{E_b t_b}{6 + \varphi} \right] \left[ \frac{r'_t - r'_s}{\varphi} \right] q \] (5.22)

As before, this equation is equated to the modal damping term in Eq. 5.9 and the effective modal damping \( \zeta \) is found to be

\[ \zeta = \frac{d_{31} \lambda v}{2M_{\omega_0} t_c \ell} \frac{E_b t_b^2}{6 + \varphi} \left[ \frac{r'_t |_{\text{lhs}} - r'_s |_{\text{rhs}}}{} \right] \] (5.23)

Finally, assuming a maximum value of \( \lambda v \) determined by the actuator saturation limits (Eq. 5.13) and assuming that the motion of interest is at the resonance frequency of the beam, the damping \( \zeta \) obtained from the rate feedback of the piezo-electric actuator is

\[ \zeta = \frac{t_b}{q_{\text{max}}} \left[ \frac{t_{\text{max}} d_{31}}{\ell} \right] \left[ \frac{6}{6 + \varphi} \right] \left[ \frac{r'_t |_{\text{ lhs}} - r'_s |_{\text{ rhs}}}{} \right] \left[ \frac{\ell}{t_b} \right]^2 \] (5.24)

For reference, the result for the extensional case, Eq. 5.16 is repeated here

\[ \zeta = \left[ \frac{\ell}{q_{\text{max}}} \right] (d_{31} t_{\text{max}}) \left[ \frac{1}{2 + \varphi} \right] \left[ \frac{r |_{-} - r |_{+}}{} \right] \] (5.16)

As can be readily observed, Eq. 5.16 is very similar to Eq. 5.24. Now that a measure of the nondimensional effectiveness of the actuator has been determined, it is instructive to examine...
how the effectiveness changes as the structure is increased in size. The first term in both equations is the inverse of the normalized displacement amplitude. This nondimensional deflection remains constant as the dimensions of the system are scaled. The second term is just the maximum piezo-strain that can be obtained from the actuators, and is independent of scale. The third depends on the stiffness ratio of the beam and the piezo-electric and stays constant if the thickness of the piezo-electric is increased as the beam is scaled. The fourth term contains factors involving the non-dimensional mode shape, and stays constant if the ratio of the length of the piezo-electric actuator to the beam remains constant. This could be done either by increasing the length of a single piezo-electric actuator, or using more piezo-electrics whose total length is a fixed fraction of the structural length. This term also indicates where to optimally place the piezo-electric actuator along the beam, i.e., they should span regions of high average strain. Finally, Eq. 5.24 contains an extra term that is not present in the extensional case. This term is the slenderness ratio of the beam, squared. It arises in Eq. 5.24 and not in Eq. 5.16 due to the differences in the expression of the modal stiffnesses, Eqs. 5.2 and 5.18, and would not change with scale.

Eq. 5.16 and 5.24 indicate that as the system is scaled in three dimensions, the damping that is obtained through rate feedback to the piezo-electric actuators remains constant if the
ratio of actuator dimensions to the beam dimensions remains constant. Therefore, the effectiveness of piezo-electric actuators when used to excite either extension or bending in a substructure remains constant as the structure is scaled.

5.3 Comparison of Piezo-Ceramics and PVDF Film

The scaling laws in Eq. 5.16 and 5.24 serve as a useful guide as to what properties are desirable in a piezo-electric actuator material. The significant properties are identifiable as material piezo-electric properties, $\varepsilon_{\text{max}}$, $d_{31}$, and the material elastic properties $E_c t_c$. Increases in both the maximum allowable field and the piezo-electric constant $d_{31}$ are obviously advantageous. It is also desirable to have a piezo-electric with a high modulus, so that the third term in Eqs. 5.16 and 5.24 is large as possible.

To constrain the effectiveness of two available piezo-electric materials, Table 5.1 summarizes the pertinent properties of Gl195 piezo-ceramics and PVDF piezo-electric film. The Curie temperature above which piezo-electric properties are destroyed is also included. The fifth line, labelled "effectiveness", is the total amount of strain that can be transferred into the material. It is the product of the second and third terms in Eq. 5.16 for the extension case and in Eq. 5.14 for the bending case. It was calculated assuming an aluminum substructure whose thickness is 10 times the piezo-electric
thickness. Finally, the last line in the Table is the effectiveness per field, i.e., the amount of strain that can be transferred into the substructure at a given electric field. This item allows the comparison of the two materials when an equal electric field is applied to both and should be used for comparisons when the amount of voltage that can be supplied to an actuator in a real application must be held at a given level.

Obviously, since the Curie temperature of PZT ceramics is $260^\circ$ higher than the PVDF temperature, the ceramics can be used in many applications requiring a wider operating temperature range where it would be impossible to use the polymer film. In addition, the 2 to 1 ratio in the effectiveness also indicates that currently available piezo-ceramics are preferable over polymer films for use as actuators. This is even more pronounced when the two materials are compared at a given voltage. Therefore, it is usually preferable to use piezo-ceramics instead of polymer film. However, in certain applications, the ability of the polymer to conform to non-planar surface shapes may prove advantageous.
Segmented piezo-electric actuators have been shown to be a viable concept for dynamic vibration and shape control of structures. Because of their small size and high modulus, piezo-electrics bonded or embedded in a structural member can be joined together in a highly distributed actuator network with a minimal effect on the passive structural properties of the member.

Static analyses for various actuator geometries were presented, leading to complete analytical solutions of the governing equations for each case. For the surface bonded cases, both perfectly bonded and finite bonding layer models were derived and compared. For this second case, as the shear lag in the bonding layer is decreased, the solution converges to that of the perfectly bonded case. For the embedded models, only the perfectly bonded case was presented, since the shear lag present in the thin interlaminar bonding layer is negligible.

The flexural static models for both surface bonded and embedded actuators were integrated into dynamic models for a cantilevered Bernoulli-Euler beam. The forcing applied by the actuators was shown to consist of a homogeneous component, which accounts for the additional passive stiffness present in the system due to the actuator, and a non-homogeneous component which relates to the ability of the piezo-electric actuator to impart a force on the structure. This coupling of the static and dynamic
models led to the ability to predict the behavior of the beam under excitation from the piezo-electric actuators.

Two dynamic experimental apparatuses were built and tested. The first consisted of eight piezo-ceramic actuators bonded to an aluminum cantilevered beam. The second test specimen consisted of four piezo-ceramics embedded in a glass/epoxy test specimen. Using the experimentally measured damping values, excellent agreement between the experimentally measured and analytically predicted amplitude of vibration was obtained for both specimens. In addition, a series of static tests using glass/epoxy specimens with embedded actuators showed no significant reduction in the elastic modulus of the specimen with only a 20% reduction in ultimate strength. Therefore, for stiffness applications, piezo-electric actuators can be embedded in a composite laminate without significantly affecting the passive elastic properties.

A simple rate feedback experiment demonstrated the effectiveness of piezo-ceramics as control actuators. The effective modal damping ratio was increased from its inherent passive damping value to 4% using the four PMAs bonded near the root of the aluminum cantilevered beam test article.

A scaling analysis was conducted for piezo-electrics bonded to a beam exciting either extension or bending. Actuator saturation through the mechanism of piezo-electric depoling was included in the analysis. For both the extensional and bending cases, the analysis indicates that the nondimensional
effectiveness of the piezo-electric actuator remains constant if the dimensions of the piezo-electric increase in scale with those of the structure. Finally, the scaling analysis also provides criteria for ranking candidate piezo-electric materials by a comparison of the material and piezo-electric properties. It is shown that materials with a high modulus and a high piezo-electric constant $d_{31}$, such as piezo-ceramics, are better suited for use as actuators than materials with a low modulus and low piezo-electric constants, such as piezo-electric film. Even if the film can produce much higher strain levels than ceramics, due to their high $\ell_{\text{max}}$, their low modulus prevents that strain to be transferred to the sub-structure, thus limiting their effectiveness.

In conclusion, all the tools necessary to use segmented piezo-electric actuators in embedded or surface bonded geometries in bar and beam-like structures, and the ability to predict $a priori$ the performance of these actuators has been developed and experimentally verified.
REFERENCES


<table>
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<tr>
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<th>SHEAR LAG</th>
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<td>[ \frac{d\sigma_c}{dx} = \frac{F}{bt_c} ]</td>
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<tr>
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<td>[ \frac{\sigma_c}{b} = \frac{F}{bt_c} ]</td>
</tr>
<tr>
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<td>[ \frac{d\sigma_c}{dx} = \frac{F}{bt_c} ]</td>
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<td>[ \frac{\sigma_c}{b} = \frac{F}{bt_c} ]</td>
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**Notes:**
- \( E = \text{Young's modulus} \)
- \( t = \text{thickness} \)
- \( A = \text{cross-sectional area} \)
- \( b = \text{width} \)
- \( c = \text{length} \)
- \( d = \text{distance} \)
- \( F = \text{force} \)
- \( t = \text{time} \)
- \( \theta = \text{angle} \)
- \( \rho = \text{density} \)
- \( \lambda = \text{Lambdta} \)
- \( \mu = \text{shear modulus} \)
- \( \nu = \text{poisson's ratio} \)
Table 4.1 Properties of G-1195 Piezo-Ceramic with Nickle Electrodes

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Dielectric Constant, $k_3$</td>
<td>2000</td>
</tr>
<tr>
<td>Loss Tangent</td>
<td>0.022</td>
</tr>
<tr>
<td>Curie Temperature</td>
<td>360 °C</td>
</tr>
<tr>
<td>$d_{31}$</td>
<td>$270 \cdot 10^{-12}$ m/V</td>
</tr>
<tr>
<td>$e_{31}$</td>
<td>$11 \cdot 10^{-3}$ V/m</td>
</tr>
<tr>
<td>$k_{31}$</td>
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</tr>
<tr>
<td>Density</td>
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<td>Elastic Modulus</td>
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Table 4.2 Properties for Epoxy EPY-150

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<th>Property</th>
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<td>Shear Modulus</td>
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<td>Ult. Tensile Strength</td>
<td>$72 \cdot 10^6$ Pa</td>
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Manufactured by BLH Electronics
Table 4.3 Measured and Predicted Response: Aluminum Beam, Rear PMAs, First Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>Voltage (VAC)</th>
<th>Tip Ampl./Beam Thick.</th>
<th>Measured Damping Ratio $^\dagger$</th>
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<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7.20</td>
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<td>13.93</td>
<td>0.93</td>
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<tr>
<td>21.52</td>
<td>1.31</td>
<td>1.35</td>
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<td>28.30</td>
<td>1.64</td>
<td>1.64</td>
</tr>
<tr>
<td>35.43</td>
<td>1.94</td>
<td>1.91</td>
</tr>
<tr>
<td>42.47</td>
<td>2.24</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 4.4 Measured and Predicted Response: Aluminum Beam, Front PMAs, First Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>Voltage (VAC)</th>
<th>Tip Ampl./Beam Thick.</th>
<th>Measured Damping Ratio $^\dagger$</th>
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<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
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<td>0.42</td>
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<tr>
<td>14.09</td>
<td>0.65</td>
<td>0.73</td>
</tr>
<tr>
<td>21.30</td>
<td>0.91</td>
<td>1.00</td>
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<tr>
<td>28.47</td>
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<td>1.34</td>
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<td>41.92</td>
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Table 4.5  Measured and Predicted Response: Aluminum Beam, All PMAs, First Mode, Experimentally Determined Damping

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<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING RATIO</th>
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<td>1.33</td>
<td>1.46</td>
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<td>21.16</td>
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<td>1.92</td>
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<td>28.52</td>
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<td>2.36</td>
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Table 4.6  Measured and Predicted Response: Aluminum Beam, Rear PMAs, Second Mode, Experimentally Determined Damping

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<th>MEASURED DAMPING RATIO</th>
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<td>35.50</td>
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<td>42.19</td>
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<td>49.70</td>
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Table 4.7 Measured and Predicted Response: Aluminum Beam, Front PMAs, Second Mode, Experimentally Determined Damping

<table>
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<td>PREDICTED</td>
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Table 4.8 Measured and Predicted Response: Aluminum Beam, All PMAs, Second Mode, Experimentally Determined Damping

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<th>VOLTAGE (VAC)</th>
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Table 4.9 Laminate Engineering Constants for [0/90/0/90/0]s Laminate

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<thead>
<tr>
<th>Property</th>
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<th>Value</th>
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<tr>
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<tr>
<td>Transverse Modulus</td>
<td>$E_T$</td>
<td>$22.0 \cdot 10^9$</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>$G_{LT}$</td>
<td>$5.5 \cdot 10^9$</td>
</tr>
<tr>
<td>Major Poisson's Ratio</td>
<td>$\nu_{LT}$</td>
<td>$.122$</td>
</tr>
<tr>
<td>Minor Poisson's Ratio</td>
<td>$\nu_{TL}$</td>
<td>$.0966$</td>
</tr>
</tbody>
</table>

Table 4.10  Measured and Predicted Response: Glass Beam, Rear PMA, First Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
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<tbody>
<tr>
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<td>PREDICTED</td>
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<td>0.00</td>
<td>0.00</td>
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<tr>
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<td>41.96</td>
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<td>1.83</td>
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</table>

Table 4.11  Measured and Predicted Response: Glass Beam, Front PMA, First Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
</tr>
</thead>
<tbody>
<tr>
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<td>PREDICTED</td>
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<td>0.00</td>
<td>0.00</td>
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Table 4.12 Measured and Predicted Response: Glass Beam, Both PMAs, First Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
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<tr>
<td>42.45</td>
<td>2.28</td>
<td>2.80</td>
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</table>

Table 4.13 Measured and Predicted Response: Glass Beam, Rear PMA, Second Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
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<tr>
<td></td>
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<td>0.00</td>
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<td>0.00</td>
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<tr>
<td>7.88</td>
<td>0.02</td>
<td>0.03</td>
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<tr>
<td>14.04</td>
<td>0.04</td>
<td>0.05</td>
</tr>
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<td>21.23</td>
<td>0.07</td>
<td>0.08</td>
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<tr>
<td>28.86</td>
<td>0.09</td>
<td>0.11</td>
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<tr>
<td>34.65</td>
<td>0.11</td>
<td>0.13</td>
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<tr>
<td>41.78</td>
<td>0.14</td>
<td>0.16</td>
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Table 4.14 Measured and Predicted Response: Glass Beam, Front PMA, Second Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MEASURED</td>
<td>PREDICTED</td>
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<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>7.45</td>
<td>0.03</td>
<td>0.03</td>
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<tr>
<td>14.54</td>
<td>0.05</td>
<td>0.06</td>
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<tr>
<td>20.93</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>28.51</td>
<td>0.10</td>
<td>0.12</td>
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<tr>
<td>35.68</td>
<td>0.13</td>
<td>0.15</td>
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<tr>
<td>43.54</td>
<td>0.16</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 4.15 Measured and Predicted Response: Glass Beam, Both PMAs, Second Mode, Experimentally Determined Damping

<table>
<thead>
<tr>
<th>VOLTAGE (VAC)</th>
<th>TIP AMPL./BEAM THICK.</th>
<th>MEASURED DAMPING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MEASURED</td>
<td>PREDICTED</td>
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<tr>
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<td>0.00</td>
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<tr>
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<tr>
<td>13.94</td>
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<td>0.10</td>
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<td>21.34</td>
<td>0.14</td>
<td>0.16</td>
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<td>28.57</td>
<td>0.19</td>
<td>0.21</td>
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<tr>
<td>35.53</td>
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<td>43.42</td>
<td>0.25</td>
<td>0.31</td>
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### Table 4.16 Static Moduli at Various Gauge Locations

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>TYPE OF ACTUATOR</th>
<th>MODULUS, IN GPA, AT GAUGE:</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>CON 1A</td>
<td>NONE</td>
<td>31</td>
</tr>
<tr>
<td>CON 1B</td>
<td>NONE</td>
<td>27</td>
</tr>
<tr>
<td>SCE 1A</td>
<td>EXTENSOR</td>
<td>33</td>
</tr>
<tr>
<td>SCE 1B</td>
<td>EXTENSOR</td>
<td>32</td>
</tr>
<tr>
<td>SCB 1A</td>
<td>BIMORPH</td>
<td>33</td>
</tr>
<tr>
<td>SCB 1B</td>
<td>BIMORPH</td>
<td>30</td>
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</table>

### Table 4.17 Failure Stresses

<table>
<thead>
<tr>
<th>PREDICTION</th>
<th>TYPE OF ACTUATOR</th>
<th>FAILURE STRENGTH, MPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON 1A</td>
<td>NONE</td>
<td>615</td>
</tr>
<tr>
<td>CON 1B</td>
<td>NONE</td>
<td>634</td>
</tr>
<tr>
<td>SCE 1A</td>
<td>EXTENSOR</td>
<td>505</td>
</tr>
<tr>
<td>SCE 1B</td>
<td>EXTENSOR</td>
<td>522</td>
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<tr>
<td>SCB 1A</td>
<td>BIMORPH</td>
<td>548</td>
</tr>
<tr>
<td>SCB 1B</td>
<td>BIMORPH</td>
<td>513</td>
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Table 5.1 Typical Values of Piezo Film and Ceramic

<table>
<thead>
<tr>
<th></th>
<th>CERAMIC</th>
<th>FILM</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(PZT G1195)</td>
<td>(PVDF)</td>
</tr>
<tr>
<td>Curie Temp., °C</td>
<td>360</td>
<td>100</td>
</tr>
<tr>
<td>ε_{max}, V/m</td>
<td>600 · 10³</td>
<td>40 · 10⁶</td>
</tr>
<tr>
<td>d_{31}, m/V</td>
<td>270 · 10^{-12}</td>
<td>23 · 10^{-12}</td>
</tr>
<tr>
<td>E_{c}, N/m²</td>
<td>63 · 10⁹</td>
<td>3 · 10⁹</td>
</tr>
<tr>
<td>EFFECTIVENESS</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EXTENSION</td>
<td>12 · 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>BENDING</td>
<td>55 · 10^{-6}</td>
</tr>
<tr>
<td>EFFECTIVENESS</td>
<td></td>
<td></td>
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<tr>
<td>PER FIELD</td>
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<td></td>
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<tr>
<td></td>
<td>EXTENSION</td>
<td>20 · 10^{-12}</td>
</tr>
<tr>
<td></td>
<td>BENDING</td>
<td>92 · 10^{-12}</td>
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</tbody>
</table>
Fig. 2.1 Perfectly Bonded Actuators Exciting Extension In the Substructure.

Fig. 2.2 Two Piezo-Electrics Bonded to an Elastic Substructure.
Fig. 2.3  Differential Section A : Model of a Piezo-Electric Exciting Extension in the Substructure.
Fig. 2.4 Beam and Actuator Strains for Various Values of $\Gamma_\circ$. 

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Fig. 2.5 Perfectly Bonded Actuators Exciting Bernoulli-Euler Bending in the Substructure

Fig. 2.6 Differential Section $\mathcal{B}$ : Model of Two Piezo-Electrics Exciting Bending on the Substructure.
Fig. 2.7  Beam and Actuator Strains for Various Values of $\Gamma_b$. 
Fig. 2.8 Bonding Layer Shear Stress for Various Values of $\Gamma_b$. 
Fig. 2.9 Embedded Piezo-Electrics Bonded Inside a Laminated Structure

Fig. 2.10 Embedded Piezo-Electrics Exciting Bending in the Surrounding Material
Fig. 3.1 Cantilevered Beam under Localized Shear Loading Applied at the Surface

Fig. 3.2 Cantilevered Beam with Embedded Actuators
Fig. 4.1 Aluminum Test Specimen
Fig. 4.2 Glass/Epoxy Test Specimen
Fig. 4.3 Second Derivative of the First Cantilevered Bending Mode
Fig. 4.4 Second Derivative of the Second Cantilevered Bending Mode.

Note the strain node present at $x = 0.216$. 
Fig. 4.5 G-1195 Piezo-Ceramic With Nickle Electrodes. An electric field in the direction of the arrow produces displacement as shown.
Fig. 4.6 Cross Section of Aluminum Test Specimen

Fig. 4.7 Piezo-Electric with Leads Attached for Embedding in Glass/Epoxy Test Specimen
Fig. 4.8 Temperature, Pressure and Vacuum Profiles for Standard T.E.L.A.C. Cure Cycle
Fig. 4.9 Static Test Specimen with Embedded Piezo-Ceramic Actuator
Fig. 4.10 Clamping Assembly for Dynamic Tests
Fig. 4.11 Block Diagram of the Driving Circuit for the PMAs
Fig. 4.12 Block Diagram For Data Acquisition System
Fig. 4.13 Two PMAs in Parallel with a Voltage Source. Double arrows indicate direction of electric field required for positive strain.
Fig. 4.14 First Mode Bending Test, Aluminum Beam, Rear PMAs, Experimental and Analytical Results

Fig. 4.15 First Mode Bending Test, Aluminum Beam, Front PMAs, Experimental and Analytical Results
Fig. 4.16 First Mode Bending Test, Aluminum Beam, All PMAs, Experimental and Analytical Results

Fig. 4.17 Second Mode Bending Test, Aluminum Beam, Rear PMAs, Experimental and Analytical Results
Fig. 4.18 Second Mode Bending Test, Aluminum Beam, Front PMAs, Experimental and Analytical Results

Fig. 4.19 Second Mode Bending Test, Aluminum Beam, All PMAs, Experimental and Analytical Results
Fig. 4.20 First Mode Bending Test, Glass/Epoxy Beam, Rear PMA, Experimental and Analytical Results

Fig. 4.21 First Mode Bending Test, Glass/Epoxy Beam, Front PMA, Experimental and Analytical Results
Fig. 4.22 First Mode Bending Test, Glass/Epoxy Beam, All PMAs, Experimental and Analytical Results

Fig. 4.23 Second Mode Bending Test, Glass/Epoxy Beam, Rear PMA, Experimental and Analytical Results
Fig. 4.24 Second Mode Bending Test, Glass/Epoxy Beam, Front PMA, Experimental and Analytical Results

Fig. 4.25 Second Mode Bending Test, Glass/Epoxy Beam, All PMAs, Experimental and Analytical Results
Fig. 4.26 Static Test Specimen Showing Gauge Placement
Fig. 4.27 Schematic of Static Test Set-Up
Fig. 4.28 Load vs. Strain Plot for Static Specimen with no Embedded Actuator
Fig. 4.29 Load vs. Strain Plot for Static Specimen with Embedded Actuator
Fig. 4.30 Loaded Static Test Specimen with Photoelastic Coating
Fig. 4.31  First Bending Mode Decay of Aluminum Specimen with Rate Feedback to all Actuators

Fig. 4.32  First Bending Mode Free Decay of Aluminum Specimen
Fig. 5.1 Two Perfectly Bonded Piezo-Electric Actuators Bonded to a Dynamic Substructure. The Boundary Conditions are not Specified.