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A FEASIBILITY STUDY OF HELICOPTER VIBRATION REDUCTION BY SELF-OPTIMIZING HIGHER HARMONIC BLADE PITCH CONTROL

by

John Shaw, Jr.
B.S.E. Princeton University
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Certified by _____
Thesis Supervisor

Accepted by _____
Chairman, Departmental Committee on Graduate Students

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ABSTRACT

The concept of eliminating most helicopter vibration by reducing certain rotor harmonic airloads and blade motions with harmonic pitch control is discussed in detail. A simple method of obtaining the required pitch harmonics with the swashplate is presented. Attention is given to the origin and characteristics of four problems expected in the application of the harmonic pitch control concept: shed wake attenuation, high blade bending moments, high control loads, and inflight adjustment. Results of calculations are given for the effect of third harmonic pitch control on cockpit vibration caused by a three bladed, fully articulated rotor at moderate forward speed. The calculations indicate that, in this case, the optimal harmonic pitch control will be one which greatly reduces first mode flatwise blade bending excitation. Control adjustment strategy to decrease bending excitation is demonstrated, as well as the extremely adverse effects of the shed wake on the effectiveness of the pitch changes. Finally, the calculations give reason to believe that inflight adjustment by steepest-descent type search will be feasible.

Thesis Supervisor: Rene H. Miller

Title: Slater Professor of Flight Transportation

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PRINCIPAL SYMBOLS

V	helicopter forward speed, ft/sec
Ω	rotor speed, rad/sec
GW	gross weight, lb
I_{fus}	pitching moment of inertia of fuselage about its c. g., slug-ft^2
S	normal hub shear, lb
$\bar{S}$	chordwise hub shear, lb
$\bar{\bar{S}}$	longitudinal inplane hub shear, lb
R	rotor radius, ft
e	radial location of lag hinge, ft
m	blade mass per unit span, slug/ft
I	blade moment of inertia about flapping hinge, slug-ft^2
$\bar{I}$	blade moment of inertia about lagging hinge, slug-ft^2
L_N	blade Lock number $\frac{2\pi c \rho R^4}{I}$
ω	natural frequency of first mode bending, rad/sec
$\bar{\omega}$	natural frequency of rigid blade lagging motion, rad/sec
ψ	blade azimuthal location, measured from downwind position positive in direction of blade motion

L	blade elemental lift, lb
H	blade elemental chordwise force, lb, positive aft
c	blade chord, ft
σ	rotor solidity; ratio of blade area to disk area
Θ	blade pitch relative to tip path plane
q	dynamic pressure, lb/ft^2
U_p	component of wind velocity relative to blade normal to no feathering plane
U_T	component of wind velocity relative to blade in chordwise direction in no feathering plane
c_ℓ	two-dimensional lift coefficient
c_d	two-dimensional drag coefficient
δ	angle between blade elemental lift vector and shaft direction
z	blade element displacement above hub plane, ft
y	blade element displacement aft of radial line, ft
$\overline{EI}$	blade inplane stiffness
ζ	lag angle of blade, rad, positive aft from radial direction
β	flapping angle of blade, rad, positive up from hub plane
ξ	magnitude of first bending mode
ϕ	shape of first bending mode
η	ratio of radial location to rotor radius

η_H	value of η at lagging hinge

Superscripts

$\cdot$	time derivative
$'$	radial derivative

Subscripts

3	third harmonic component

CHAPTER 1

INTRODUCTION

An investigation of helicopter vibration reduction may be put in context by answering several background questions. First, how severe is vibration in today's helicopters? How important are the reasons for wanting to reduce vibration? Second, what vibration reduction methods are currently employed? How effective are they? Finally, how does the method examined in this paper differ from current methods? Why is it being considered now?

Vibration, to today's helicopter designers, is a serious problem whose importance is continually increasing. In the past, vibration has often had extremely objectionable effects on comfort and component fatigue life. These effects were tolerated out of necessity, because they were already reduced as far as possible with existing methods. Now, however, at a time when great value is placed on increasing the maximum speed of helicopters for both military and civilian applications, better methods of vibration reduction are required. The higher flight speeds now being reached aggravate vibratory conditions to intolerable levels, in spite of efforts to control them. If the practical

speed range of the helicopter is to be increased, vibration must be controlled more effectively. The problem is intensified for manufacturers by the continually rising expectations of helicopter purchasers with regard to passenger comfort and maintenance cost. Low speed vibration levels which were merely annoying before are becoming unacceptable. Manufacturers are aware of the sharp increase in the market for short-haul intercity air transportation; they realize the extent to which vibration-caused discomfort disqualifies current helicopters from filling such needs. Competitive VTOL vehicles will soon be available, so the development of an adequate way to reduce vibration is of great importance to the helicopter industry.

The magnitude of the vibration problem in the current generation of helicopters may be inferred from several statements reflecting different points of view. First, it is not unusual during transition from hover to cruise or during flight at high speeds to experience periodic variations of rotor hub force (the force supporting the fuselage) of twenty per cent about the mean value. During maneuvers such as the landing flare, these variations may be significantly higher. Second, some helicopters are limited as much as twenty per cent in maximum flight speed by intolerable vibration levels. Third, the exposure of all helicopter components to continual vibratory loads necessitates frequent replacement of many of them after flight times which are short

by the standards of the rest of the aircraft industry. Number of hours between major overhauls is roughly twenty per cent of the normal for fixed-wing aircraft. The parts and labor involved contribute substantially to the cost of operating the helicopter.

Present methods of vibration reduction are based on a variety of approaches. None of these approaches is as fundamental as the one discussed in this paper, which acts directly on the major source of the vibration. Some of them are quite successful in particular cases, but none of them has been entirely adequate at high speeds or even at all low speeds. All of them are normally applied to a design after the flight of a prototype, since they are all essentially "fixes". One approach concentrates on the natural vibratory modes of the helicopter. An effort is made to detune these modes, which depend on the structure of the blades and fuselage, from the principal exciting frequencies of the rotor unsteady airloads. The detuning procedure reduces the magnitude of the dynamic response. Another approach exploits the importance of the fuselage mode shapes relative to the pilot's location. Through careful distribution of the fuselage mass, or by addition of mass at the pilot's seat, the cockpit is caused to lie at a node of the most undesirable vibratory mode. The fuselage may then vibrate strongly while the pilot remains largely unaware of the motion. A third approach attempts to absorb the energy of excitation in masses

other than the fuselage. Heavily damped masses are added to the rotor system or the fuselage, dynamically isolated from the fuselage, and tuned to resonate at the primary exciting frequencies. The strong damping absorbs large amounts of vibratory energy. Still another approach curtails the effect of excitation forces by dynamically isolating large sections of the helicopter. For example, isolation mounts (spring-damper devices) may be used to connect the rotor system to the fuselage, or to connect the pilot's seat and instrument panel to the cockpit floor. All of these approaches are fundamentally similar, since they all suggest modifications of the helicopter's dynamic response to vibratory inputs. By contrast, the approach examined in this paper is concerned with the vibratory inputs themselves.

This analytical investigation of helicopter vibration reduction takes advantage of significant advances in the understanding of rotor blade unsteady airloads.[1] These unsteady airloads are known to be the primary cause of helicopter vibration. High speed digital computers have permitted the analysis of the cause of the unsteady airloads, the irregular airflow through and about the rotor, as the net effect of a complex vortex pattern caused both by steady and changing lift on the blades. Earlier investigations, performed without digital computers, were forced to rely on the approximate predictions of propeller momentum theory, combined blade element-momentum theory, or

simplified closed-form propeller vortex theory. The approximations used by these theories to make the airloads problem tractable were often successful for performance problems, but were highly inaccurate for predictions of vibratory loads, especially in forward flight. This is not true of predictions by current vortex analysis methods. Using the vortex theory of rotors in its present form, implemented on the high-speed digital computer, it is possible to determine accurately the feasibility of greatly reducing unsteady airloads and the resulting vibration with a suitable blade pitch control system.

Vibration has been seen to be a problem of some severity in current helicopters, limiting performance, decreasing pilot and passenger comfort, and increasing cost of operation. Requirements of helicopter users for higher flight speeds, greater comfort, and lower costs have caused considerable pressure on manufacturers to find an adequate solution. Several successful "fixes" have been used, but none of them is entirely adequate to meet the stringent demands now confronting the industry. This paper discusses the feasibility of a solution which is essentially different from current approaches, concentrating on the origin and control of unsteady rotor airloads. The analysis of such a vibration control has only recently been made possible by the application of high-speed digital computation to the complex rotor airloads problem.

CHAPTER 2

CHARACTERISTICS OF THE VIBRATION PROBLEM AND THE PROPOSED SOLUTION

2.1 The Major Cause of Helicopter Vibration

Vibrations in a helicopter fuselage are caused almost entirely by high frequency components of rotor hub shear force. Hub shear force has unique characteristics related to the fundamental nature of the helicopter rotor. The rotor hub and the shaft to which it is attached exert both a force and a moment on the fuselage. The force is the vector sum of the forces applied to the hub by the individual blades; the sum depends only on the beam shear forces at the blade roots, since the radial forces at the roots have a negligible vector sum at all times. Hub force is therefore approximated by the total hub shear force, often called simply the hub shear. Hub shear is the combined effect of blade airloads and inertial reaction to various types of blade motion relative to the hub: flapping, lagging, flapwise bending, and chordwise bending in the most general case of a fully hinged, flexible rotor. It has a component parallel to the shaft, called the normal hub shear, and a component perpendicular to the shaft, called the inplane hub shear. The mean value of the sum supports the fuselage against the force of gravity and pulls it through the air against the force of drag.

In any steady flight condition, both blade airloads and blade motions relative to the hub are periodic, with a period equal to that of the rotor in its rotation about the shaft. The hub shears are therefore also periodic. All these periodic variables may be analyzed in Fourier harmonics of the rotational frequency. They contain many higher harmonics in the general case, because the rotor airflow, which causes the airloads and blade motions, is highly irregular. The trend, however, is to lower magnitudes for higher harmonics. A typical hub shear harmonic analysis is given below.

Vertol CH46-A "Chinook" tandem helicopter
three-bladed, fully hinged rotors

V=140 knots; Ω = 264 rpm

$(F_z)_f$ = normal hub shear, forward rotor

$$\triangleq \sum_{k=0}^{\infty} \alpha_k \cos(k\psi + \phi_k)$$

$\alpha_0 = 9760$

$\alpha_1 = 530$	$\alpha_6 = 105$
$\alpha_2 = 420$	$\alpha_7 = 15$
$\alpha_3 = 1565$	$\alpha_8 = 15$
$\alpha_4 = 270$	$\alpha_9 = 10$
$\alpha_5 = 30$	$\alpha_{10} = 10$

(all units: pounds)

source: harmonic analysis of Reference 2 data, page 5. 0-97

In this data, the third and sixth harmonics are considerably larger than the harmonics surrounding them. This calls attention to the mechanical "filtering" characteristic of the rotor. All the blades, being indistinguishable, have the same hub shear dependence on azimuth in a steady flight condition. If one blade is arbitrarily designated as an azimuth reference, the total normal hub shear of a three-bladed rotor, for example, is given by

$$F_z(\psi) = F_{z_b}(\psi) + F_{z_b}(\psi+120°) + F_{z_b}(\psi+240°) \tag{2.1}$$

where $F_{z_b}(\psi)$ is the normal hub shear of a single blade. The contributions of the harmonic components of $F_{z_b}(\psi)$ to $F_z(\psi)$ may be considered individually. Straightforward trigonometry shows that only F_{z_b} harmonics whose orders are multiples of the number of blades (namely the zeroth, or mean value, the third, sixth, ninth, and higher three-multiple harmonics, for a three-bladed rotor) have a non-zero effect on total normal hub shear $F_z(\psi)$. The total normal hub shear for a three-bladed rotor must then be composed only of zeroth, third, sixth, etc. harmonics. At these harmonics, the blades act together, while at other harmonics they cancel each other's effects. The typical data above show that this is not precisely true. Harmonics other than the three-multiples have finite magnitudes on a real rotor because the blades are not completely indistinguishable. Neither the blade airloads

nor the blade motions are exactly the same for all three blades; the motion is termed slightly "out of track", because the blades do not exactly "track", or follow, each other. The rotor, however, strongly attenuates all harmonic components of hub shear whose orders are not integral multiples of the number of blades. The most objectionable vibratory input to the fuselage from a three-bladed rotor is therefore the third harmonic hub shear. The third is the lowest harmonic which is strengthened, rather than attenuated, by the rotor, so its magnitude is usually much larger than that of any other harmonic (except the zeroth, or mean value). As an example of the frequency involved, the third harmonic hub shear shakes the fuselage at 12.5 cps for a typical rotor speed of 250 rpm.

Inplane hub shears, even more than hub shears in general, depend on the unique nature of the helicopter rotor. Hub shears for a single blade are measured or calculated in the plane perpendicular to the blade root, that is, in the normal and chordwise directions. The chordwise direction rotates continually as the blade itself rotates. In other words, hub shears are measured or calculated in a rotating coordinate system. Hub shear vibratory effects, however, are observed in a fixed direction on the aircraft: longitudinal, lateral, or normal. Since calculated inplane hub shears refer to a rotating direction, their components along fixed directions must be taken to

determine their effect on the fuselage. No transformation is necessary for normal hub shears, since the normal direction (parallel to the shaft) is the same in both rotating and fixed coordinate systems.

Converting the inplane hub shears from rotating to fixed coordinates changes their harmonic content. For example, if the inplane hub shear for a blade is measured to be

$$F_{rot} = A \sin 5\psi \tag{2.2}$$

(a pure fifth harmonic sine), its longitudinal component is

$$F_x = A \sin 5\psi \sin\psi = \frac{A}{2} (\cos 4\psi - \cos 6\psi) \tag{2.3}$$

and its lateral component is

$$F_y = A \sin 5\psi \cos\psi = \frac{A}{2} (\sin 4\psi + \sin 6\psi) \tag{2.4}$$

That is, a fifth harmonic chordwise variation has components in fixed directions which vary as combinations of fourth and sixth harmonics.

The trigonometric identities

$$\sin n\psi \sin\psi = \frac{1}{2}[\cos(n-1)\psi - \cos(n+1)\psi] \tag{2.5}$$

$$\sin n\psi \cos\psi = \frac{1}{2}[\sin(n-1)\psi + \sin(n+1)\psi] \tag{2.6}$$

indicate that, in general, a harmonic in the rotating system is equivalent to a linear combination of the harmonics of order one higher and one lower in the fixed system. For this reason, the objectionable third harmonic longitudinal and lateral inplane hub shears of a three-bladed rotor are caused not by third harmonic, but by second and fourth harmonic airloads and blade motions. The third harmonic airloads and blade motions, while entirely responsible for third harmonic normal hub shear, cause only second and fourth harmonic longitudinal and lateral hub shears. These are strongly attenuated by the three-bladed rotor.

Third harmonic hub shear, which is the primary cause of fuselage vibration in helicopters with three-bladed rotors, is therefore a combined effect of second, third, and fourth harmonic airloads and blade motions. Its normal component is caused by third harmonic blade effects, while its longitudinal and lateral inplane components are caused by second and fourth harmonic blade effects.

2.2 The higher harmonic pitch control concept

Because so much helicopter vibration, perhaps as much as eighty per cent of the total, is caused by just three harmonics of blade motion and airloads, a large vibration reduction could certainly be made by reducing the magnitudes of these three harmonics. For a three-bladed rotor in particular, this requires control of the second, third, and fourth harmonic components of blade motion and airloads. Such control would succeed if it greatly attenuated these harmonics; it could also succeed by adjusting them to work against each other in their effects on fuselage vibration.

Only one method of rotor control is available on current helicopters: blade pitch relative to the hub is adjustable. Helicopter pilots presently control the zeroth and first harmonics of blade pitch variation through the collective and cyclic "sticks" in the cockpit. No other harmonics of pitch variation with azimuth are used. Blade pitch affects airloads directly, since lift and drag depend on local flow angle relative to the chord line. Thus, if second, third, and fourth harmonic pitch variations were used, they would affect the corresponding airload harmonics. These three airload harmonics in turn would influence the second, third, and fourth harmonics of blade motion, and finally, the modified motions and airloads together would change the

third harmonic hub shear. This is the basis for the type of helicopter vibration control considered in this paper. For the specific case of a three-bladed rotor, the control concept is to create second, third, and fourth harmonic blade pitch variations with magnitudes and phases adjusted to minimize some measurable or calculable index of fuselage vibration level. The most elementary index, and the one used in this feasibility study, is the third harmonic normal vibration amplitude at the pilot's location. (As in the case of hub shears, "normal" refers to the shaft direction).

A practical implementation of higher harmonic blade pitch changes is available with existing hardware. The present means of pitch control is the swashplate, a tilted, non-rotating disk pierced at its center by the rotor shaft and located just below the blades. A vertical rod, called a pitch link, connects the trailing edge of each blade root to the point directly below it on the circumference of the swashplate. As the blades rotate, the lower ends of the pitch links move around the swashplate perimeter, going up and down periodically because of the swashplate tilt. The resulting vertical motion of the upper ends of the pitch links rotates the blades in pitch about a pivot near the quarter-chord. The pilot's collective stick adjusts the height of the swashplate on the shaft, and thus the mean, or zeroth harmonic, pitch angle. His cyclic stick adjusts the magnitude and direction of

swashplate tilt , and thus the magnitude and phase of the first harmonic pitch variation. The swashplate is adjusted from below by two moveable vertical rods. One rod tilts the disk fore-and-aft, determining the sine component of first harmonic pitch variation. The second rod tilts the swashplate laterally, determining the cosine component of first harmonic pitch. Motion of the rods together changes the swashplate height and thus the mean pitch. (The pitch links are 90° of azimuth behind the blades they control, because of their particular means of attachment to the trailing edge of the blade roots. It is for this reason that a fore-and-aft swashplate tilt causes a side-to-side variation of blade pitch or, in other words, that a cosine-phased variation of swashplate height causes a sine-phased variation of pitch.)

Higher harmonic pitch control can be applied if there is some mechanical means for superimposing external swashplate adjustments on those of the pilot. One practical method of superposition is to make the pilot's swashplate-positioning rods hydraulically extensible. Changing the length of the rods with the pilot's end fixed is equivalent to moving the pilot's end while the length is fixed. The pilot and the external control thus position the swashplate additively, the pilot by moving the lower ends of the control rods with the cockpit sticks, and the external control by changing their length. This mechanism is

already used in some current helicopters as part of an electronic stability augmentation system.

When external additive control of the swashplate is available, second, third, and fourth harmonic pitch variations may be obtained completely independently of the pilot's use of zeroth and first harmonics to control the flight path of the helicopter. For example, a third harmonic may be added to the pitch variation with azimuth by varying the swashplate height as a third harmonic about the height set by the pilot. Swashplate height variation can be used to pitch the blades collectively in this case because, for a third harmonic, all the blades pitch in the same time phase. This is not true of second or fourth harmonic variations; for these harmonics, the blade pitch changes are out of phase.

The mechanization, however, is equally simple for second and fourth harmonics; the control logic is more complicated. If the swashplate fore-and-aft tilt is varied as a third harmonic about the tilt set by the pilot, the result is a sum of second and fourth harmonic pitch variations. The change in pitch is

$$\Delta\theta(\psi) = A \cos \psi \tag{2.7}$$

where A varies as a third harmonic. If A is sine-phased, of amplitude a, the pitch change is

$$\Delta\theta(\psi) = a \sin 3\psi \cos\psi = \frac{a}{2}[\sin 2\psi + \sin 4\psi] \tag{2.8}$$

If A is cosine-phased, the pitch change is

$$\Delta\theta(\psi) = a \cos 3\psi \cos\psi = \frac{a}{2}[\cos 2\psi + \cos 4\psi] \tag{2.9}$$

Similarly, sine- and cosine-phased third harmonic lateral tilts cause pitch changes

$$\Delta\theta(\psi) = a \sin 3\psi \sin\psi = \frac{a}{2}[\cos 2\psi - \cos 4\psi] \tag{2.10}$$

and

$$\Delta\theta(\psi) = a \cos 3\psi \sin\psi = \frac{a}{2}[\sin 4\psi - \sin 2\psi] \tag{2.11}$$

By adjusting the amplitudes and phases of a combination of third harmonic fore-and-aft and lateral swashplate tilts, any magnitude and phase combination of second and fourth harmonic pitch variation is possible. For example, if a pitch change

$$\Delta\theta(\psi) = \sin 2\psi + 3 \cos 2\psi + 7 \sin 4\psi + \cos 4\psi \tag{2.12}$$

were desired, it could be obtained by tilting the swashplate fore-and-aft with angle

$$\Delta b_1(\psi) = 8 \sin 3\psi + 4 \cos 3\psi \tag{2.13}$$

and laterally with angle

$$\Delta a_1(\psi) = 2 \sin 3\psi + 6 \cos 3\psi \tag{2.14}$$

These functions satisfy the relation

$$\Delta \Theta(\psi) = \Delta a_1(\psi) \sin\psi + \Delta b_1(\psi) \cos\psi \tag{2.15}$$

(This example makes the arbitrary assumption that one degree of swashplate tilt causes a first harmonic pitch variation of one degree amplitude. The ratio of swashplate angle to blade angle is not necessarily one, since it depends on the chordwise point of attachment of the pitch links to the blades. If it were not one, the equations would simply be modified by a constant factor. Deltas are used in all the above equations to indicate that these angle variations are added to the variations already commanded by the pilot.)

This discussion may be summarized briefly. For three-bladed rotors, control of second, third, and fourth harmonic airloads and blade motions is a promising method of vibration reduction. The most direct way of obtaining control of harmonic airloads and blade motions is to create and control corresponding harmonics of blade pitch. Fortunately, this may be accomplished easily with the swashplate pitch control system found on most current helicopters. All possible combinations of second, third, and fourth harmonic pitch variations may be obtained with third

harmonic swashplate height and tilt changes. Third harmonic swashplate control can be superimposed on the pilot's zeroth and first harmonic swashplate control with a mechanical adding system already used successfully on many helicopters for other purposes.

2.3 Problems associated with vibration reduction by higher harmonic pitch control

Higher harmonic pitch control is almost completely untested and unproven. To the author's knowledge, only one flight test program has been conducted up to this time.[3] (That test, for reasons which are far from completely understood, showed only very limited and qualified improvement in fuselage vibration due to second harmonic control of a two-bladed rotor.) A number of problems lie ahead, therefore, in the practical application of this new control concept. Some of these, of course, will be resolved quickly, while others are extremely troublesome. It is unknown at this time which are the difficult and which are the simple problems; they are presented here as points deserving equal attention in future research and development efforts.

The first two possible problems involve the success of the control concept itself: whether harmonic pitch variations are practically effective as controls of the corresponding harmonic airloads and blade motions. More specifically, these problems rest on two types of aerodynamic difficulty; first, that complex rotor wake effects may make hub shears virtually insensitive to harmonic pitch changes; and second, that pitch control which is adjusted to minimize hub shears may cause spanwise blade load distributions which are structurally dangerous. These very important possibilities deserve dis-

cussion in some detail.

The first possibility, that of detrimental wake effects, is suggested by the existing approximate applications of classical unsteady aerodynamics to helicopter rotors. The point may be developed from the most fundamental result of unsteady aerodynamics, the starting vortex. When a lifting wing begins to move through the air, establishment of the bound circulation associated with lift is accompanied by the creation of a starting vortex. This vortex remains at the initial location of the wing, lies parallel to the wing, and has circulation equal and opposite to that of the bound vortex. A similar effect is observed when the lift of an already moving wing is changed abruptly, by an increased angle of attack, for example. At the location where the lift changed and the bound circulation changed accordingly, a starting vortex also remains. This is called a "shed vortex"; its circulation is equal and opposite to the change in bound circulation. As an extension of these effects, a continuous sheet of shed vorticity appears behind a wing whose angle of attack is changed sinusoidally. The strength, or linear density, of vorticity in the sheet varies sinusoidally with distance, just as the circulation and lift of the wing varied when it created the sheet.

The sheet of shed vorticity behind the wing has an associated velocity field which changes the wing's angle of attack. That is, the shed wake in general creates a net upwash or downwash at the wing.

The wing moves with respect to the shed wake, which is left behind it, so the effect on angle of attack is time-varying. Qualitatively, the sheet of shed vorticity reduces the wing's angle of attack at all times and thus attenuates the sinusoidal lift changes. It also causes the attenuated sinusoidal lift to lag behind the sinusoidal pitch. The attenuating and lagging effect of the shed wake on unsteady lift was described quantitatively by Theodorsen[4] in a paper which is now a cornerstone of unsteady aerodynamics.

The sinusoidal shed wake effect, with modifications, is observed on helicopter rotors, because rotor blades are simply rotating wings. Blade lift varies periodically (as a sum of sinusoidal harmonics), so each blade leaves along its curved path a sheet of shed vorticity. The strength of vorticity in the sheet varies periodically with distance behind the blade (measuring distance along the curved path). The shed vortex sheet is "blown", or convected, downward by the rotor airflow and forms a distorted corkscrew below the rotor. The axis of the corkscrew points down and somewhat aft, depending on forward speed. Spirals from the different blades are coaxial and interlocked, since they are created simultaneously.

Blade rotation and airflow through the rotor establish the basic helical form of the wake. Forward speed, as compared to disk-averaged flow velocity through the rotor (which determines the average

downward convection speed), establishes the skewedness of the spirals. Distortion of the spirals is caused by the interaction of their various parts and also, more significantly, by the strong vortices which trail continuously from the blade tips.

The blade tip speed due to rotation is usually at least two and a half times the speed of the aircraft. As a result, several turns of each shed wake spiral are close beneath the rotor as it moves forward, except at very high speeds. Since the lift variation is periodic with the same frequency as the rotor, these nearby turns are simply "stacked" repetitions of each other. That is, maximum, minimum, and zero vorticity occur at points along the wake which are vertically aligned below the rotor. A blade is influenced, therefore, not only by the shed wake behind it, but also by one or more repetitions of that wake spaced evenly below it. If blade pitch were varied harmonically, the modified shed wake directly behind the blade would be sufficient alone to attenuate the effects on lift; it is possible that under some flight conditions the additional presence of several turns of returning shed wake, all in phase below the blade, would eliminate the lift changes entirely.

Some research has already been done in this regard. The analytical results have been extensions of Theodorsen's work. Theodorsen formulated the shed wake effect on wings as a "lift deficiency

function", which he called C. Using Theodorsen's approach, sinusoidal lift due to sinusoidal wing pitch can be found by multiplying the lift calculated without the shed wake (that is, by using the simple slope of the lift curve) by the complex number C. The magnitude of C is the factor by which the shed wake attenuates the lift (always between 0.5 and 1.0). The angle of C is the phase by which the lift oscillation leads the pitch oscillation (always negative, indicating a lag in every case). Theodorsen showed that C depends only on k, the "reduced frequency" of the wing motion, where reduced frequency is defined as

$$k = \frac{\omega c}{2V} \tag{2.16}$$

c is the wing chord, V is the wing velocity, and ω is the radian frequency of the pitch oscillation. Thus k is 2π times the number of cycles of shed wake which lie in a semi-chord length of the wing. As an example of the lift deficiency function, C = .501-.019i when exactly one cycle of shed wake fits into a semi-chord length (k = 6.28). This indicates a lift attenuation factor of .502 and a phase lag of 2.1°. C(k) is tabulated for the entire range of k in Reference 5.

Loewy[6] adopted Theodorsen's formulation and found values for C', the lift deficiency function applying to a wing whose shed wake also appears at regular intervals below the plane of its motion. He

allowed for the possibility of additional shed wake below the wing caused by preceding wings, so the result could be applied directly to helicopter rotors. C', like C, depended on reduced frequency, but it also depended on the spacing of the shed wake layers below the wing and on the relative phase of those layers. Loewy expressed C' as a function of the rotor parameters which determine these variables. He applied his results to the calculation of two blade parameters which affect the possibility of destructive flutter: aerodynamic flap damping and aerodynamic pitch damping. He found, for example, that flap damping (the negative flapping moment due to positive flapping velocity) decreased to a small fraction of its ordinary value when blade motion and the accompanying aerodynamics occurred at any frequency equal or nearly equal to a harmonic of the rotor frequency. The decrease was especially pronounced in cases of low mean rotor inflow, when the shed wake is convected downward relatively slowly and its layers are closely spaced. This is a situation in which the shed wake clearly reduces aerodynamic effects caused by a particular type of harmonic blade motion. Ham et al.[7] verified this and many other of Loewy's results experimentally with wind tunnel tests. Substantial shed wake attenuation effects may therefore be fully expected to occur when higher harmonic pitch control is used for vibration reduction. This may result in a fundamental limit on the effectiveness of the pitch

control.

The first cause for concern is therefore the interdependence of harmonic airloads and the shed wake associated with them. Because of this interdependence, third harmonic pitch changes, for example, might at times cause only negligible changes in third harmonic airloads and blade motion. This would indicate that, in those flight conditions, harmonic pitch control caused shed wake modifications which made the net effect on lift almost zero. An important goal of the present feasibility study was to provide more information about this possibility.

The second possible difficulty with the higher harmonic pitch control concept concerns blade structural limitations. Hub shears due to the various blades are the integrated effects of variable spanwise airload and motion distributions. Pitch control, however, can only be applied to all spanwise stations of a blade at once, equally. The spanwise distribution of its effects on airloads and motion cannot be changed. It would only be coincidental if this distribution were the same as the distribution of the effects which cause objectionable hub shears. The effort to minimize third harmonic hub shears by pitch control may therefore lead to highly irregular, even destructive, airload and inertial load distributions along the blades. For example,

minimization of third harmonic hub shear might, in one flight condition, involve the cancellation of a large third harmonic lift peak near the blade tip with a large (negative) third harmonic lift peak near the mid-point of the blade. The two lift peaks together would minimize the integrated effect at the root as desired, but they might also fatigue and structurally destroy the blade. Blade bending moments must therefore be monitored carefully in early flight tests of harmonic pitch control.

Reference 3, in describing flight test results for second harmonic control of a two-bladed rotor, reports peak-to-peak oscillatory flatwise blade bending moments at 15 per cent radius as high as 160 per cent of normal. The unusually high 160 per cent level was experienced in flight at 80 knots, using 1.14° second harmonic pitch phased to be maximum positive at 100° and 280° azimuth. Results of the present investigation, as described in following chapters, show that this is an extremely disadvantageous control phasing. Nevertheless, harmonic pitch of 1.14° is a relatively moderate control, so this flight test measurement indicates the possible severity of the modified blade load distribution.

Even if harmonic pitch control proves to be both effective and safe in reducing vibratory hub shear, its benefits could be curtailed by an unfortunate characteristic of the swashplate hardware. Blade

pitching moments cause pitch link forces on the swashplate, which in turn cause forces on the fuselage through the swashplate supporting links. The blade pitching moments in the presence of harmonic pitch control can easily be large enough to cause significant fuselage vibratory inputs in this manner. For example, in presenting flight test measurements for second harmonic control, Reference 2 reports peak-to-peak oscillatory pitch link loads as high as 650 pounds. The 650 pound load level occurred in flight at 100 knots, while using approximately 0.3° of second harmonic pitch phased to be maximum positive at 140° and 320° azimuth. The same rotor experienced a 370 pound peak-to-peak pitch link load at 100 knots with no harmonic pitch control at all, so the 650 pound level represents a 75 per cent increase over the normal condition. For this reason, Reference 3 theorizes that increased control loads were at least partly responsible for the discouraging results of the flight tests it describes. Control loads 75 per cent higher than normal undoubtedly could have worked against any beneficial effects of harmonic pitch control in reducing fuselage vibration.

It is important to remark that the flight tests reported in Reference 3 were made on a two-bladed teetering rotor with built-in coning angle. Such a rotor may experience somewhat higher blade pitching moments and pitch link loads than a fully hinged rotor when

harmonic pitch control is applied. The blades of the teetering rotor are rigidly connected and rock freely in flapping, like a seesaw. Viewed from the side, their coning angle makes them form a shallow V, rather than a continuous straight line. These blades can only be pitched in an equal and opposite way because of their rigid connection; pitch changes are obtained by tilting the entire shallow V about a horizontal axis. Blade inertial pitching moments depend on blade inertia about the pitching axis; blade aerodynamic pitching moments depend to a large extent on the moment arm of the blade drag about the pitching axis. Moments of both types for a fully hinged rotor blade thus depend primarily on blade bending, since the pitching axis tilts with the blade in flapping. When it is not bent away from the pitching axis, the blade has a very low moment of inertia about that axis and the aerodynamic forces have very short moment arms. By contrast, the shallow V of the teetering rotor has a rather large moment of inertia about its pitching axis even in its unloaded, unbent condition. In this condition, too, the drag already has a significant moment arm about the pitching axis. The fundamental difference between the two rotor types is that the pitching axis of the hinged blade moves with the blade as it flaps, whereas the pitching axis of the teetering blade always remains perpendicular to the shaft. This is a consequence of the mechanical design of the hub. Thus, the teetering blade is farther

away from its pitching axis than the hinged blade for many of the same blade motions. Identical blade motions and identical drag may therefore cause higher pitch link forces for a teetering blade than for a hinged blade.

Flight test results indicate that the difference is not important in the absence of harmonic pitch control. For example, Reference 2 data shows a 600 pound peak-to-peak oscillatory pitch link load for a three-bladed fully hinged rotor at 100 knots. Rotor thrust had the same order of magnitude as for the corresponding zero pitch control case in Reference 3, mentioned above, where the peak-to-peak control load for the teetering rotor was 370 pounds. Thus, for identical flight velocity and similar thrust, pitch link loads were higher for the fully hinged rotor. The control load difference, of course, involves blade mass and aerodynamic differences, as well as differences in number of blades and in point of attachment of the pitch links to the blades. The conclusion is simply that pitch link loads have the same order of magnitude for the two rotors in the absence of harmonic control. Unfortunately, no flight test data is available for higher harmonic control of the hinged rotor, so no conclusions can be made concerning possible disadvantages of the teetering rotor in the case of most interest. Restated, the present understanding is this: flight tests show that control loads rise significantly when harmonic pitch control is applied to

a teetering rotor. Comparison of the teetering rotor to the fully hinged rotor shows that, in general, control loads are likely to be worse on the teetering rotor for the same flight condition. This implies that control load increases due to harmonic control may not be so severe on the fully hinged rotor, even though flight test data show little difference between the two rotor types in the no-control case. Because of the present lack of data, further investigation of control loads will clearly be necessary to the development of higher harmonic pitch control by means of the swashplate.

High control loads are not only objectionable because their fuselage inputs may work against hub shear reduction to defeat the purpose of harmonic pitch control. They also cause component fatigue and failure,, raising maintenance cost. They require component redesign in many cases, increasing control system weight and size. For several important reasons, then, increased control loads may be a serious problem associated with vibration reduction by higher harmonic pitch control.

One possible solution to the control load problem is to regulate blade pitch with small blade trailing edge flaps, rather than with pitch links. Only the flaps, instead of the entire blades, would be mechanically connected to the swashplate. Control forces would be considerably smaller because of the low inertia of the flaps and the relatively small

aerodynamic forces on them. Such a blade pitch control system has already been used in the past on production model helicopters. It is worth serious consideration in the future in cases where conventional pitch links experience forces of magnitudes comparable to those of the vibratory hub shears.

A final important problem associated with harmonic pitch control is the inflight adjustment of that control to obtain minimum vibration at some point in the fuselage. Rotor airflow changes significantly with flight condition, causing accompanying changes in blade airloads, blade motions, and hub shears. A harmonic pitch control which is beneficial in one flight condition may therefore be less effective, or even disadvantageous, in another flight condition. As an indication of the way vibration level changes with flight condition, Reference 2 shows a continual increase in second harmonic normal hub shear from 50 pounds amplitude in hover to 450 pounds amplitude at 100 knots. This is flight test data for the two-bladed teetering rotor without higher harmonic pitch control. In view of these large changes, it is almost certain that no single harmonic pitch adjustment could be satisfactory in minimizing vibration over the entire flight range.

Given the conclusion that control adjustment will be necessary as flight condition changes, it may be possible to make the required adjustment on the basis of measured flight condition. That is, har-

monic pitch control might be programed as a function of measured flight speed or mean normal hub shear, for example. Several flight variables, however, would have to be measured simultaneously to establish the rotor flow condition accurately; during maneuvers the rotor airflow may change almost independently of flight speed or mean hub shear, to consider the two examples of measured flight variables mentioned above. Flight condition, therefore, is not the most sensitive measure of pitch control requirements. A more direct method of harmonic pitch adjustment is the one suggested by the fundamental idea of feedback control: measure the error between the controlled variable and its desired level and use some function of this error as a system input which decreases the error. In terms of the vibrating helicopter, this amounts to measuring the vibration level at some point in the fuselage or at the rotor hub and using the measured value to determine what change, if any, is necessary in the harmonic pitch control input. There is a very basic difference between this method of adjustment and preprogramed adjustment dependent on measured flight condition as described above: one is an open-loop and the other a closed-loop control system. In general, closed-loop control is far more versatile and reliable than open-loop control. Unfortunately, it is inherently more complicated to implement.

The closed-loop control system just described is far from simple; strictly speaking, it is an adaptive control system. The "plant transfer function", that is, the functional relationship between vibration level (output) and harmonic pitch control (input) is unknown. It is certainly not linear, and cannot even be approximated as linear. Besides being unknown, it changes with flight condition. These characteristics of the adjustment problem imply that a continuous inflight search will be necessary: a search among the infinitely many possible harmonic pitch inputs for that one which minimizes vibration at each instant of the flight.

The closed-loop control task is reduced to manageable proportions by two facts. First, it has been observed that only three harmonics of pitch control are necessary; this means that only six input variables are involved: the magnitude and phase of each of the three harmonics. If the three critical pitch harmonics had not been identified, the field of search would have been infinite, or very large, rather than six-dimensional. Second, the effect of harmonic pitch control on vibration is partially understood; the helicopter is not entirely a "black box" in this respect. Some characteristics of the "plant transfer function" are known. Another goal of the present investigation was to develop and expand this knowledge: to obtain some of the physical insight necessary to an efficient, automatic inflight search for the op-

timal control adjustment. These points are therefore discussed in Chapters 4 and 5, which concern the results and conclusions of this investigation.

In summary, four important problems are expected in the development of higher harmonic pitch control for vibration reduction. First, the effects of pitch control on blade airloads and the accompanying motion will be strongly dependent on a unique characteristic of the helicopter, that each rotor blade, or rotating wing, passes continually over its own wake and over the wakes of the other blades. The wake modifications caused by higher harmonic pitch control are expected to attenuate the effects of the pitch control on hub shears, and therefore on vibration. Second, harmonic pitch control, in the amount necessary for significant vibration reduction, may lead to highly irregular load distributions along the blades. The resulting bending moments are likely to cause blade fatigue, and could be large enough to be destructive. Third, harmonic pitch control may significantly raise the control loads transmitted to the fuselage, increasing vibration, causing fatigue of control system components, and even necessitating some linkage design changes. Fourth, inflight adjustment of the harmonic pitch control will be necessary to compensate for large changes in rotor airflow with flight condition and during maneuvers. The adjustment will require a continuous search for the optimal combination of six input variables:

the magnitude and phase of each of three pitch harmonics. A carefully designed adaptive control system will be necessary for this relatively complex control task. The present investigation considered two of these four problems in particular: the first and the fourth.

CHAPTER 3

THE METHOD OF ANALYSIS

3.1 Simplification of the Problem

Helicopter fuselage vibration is, from one point of view, the output of a complex physical feedback system whose input is the airflow through and across the rotor. There is, in other words, a cause-effect relationship between rotor airflow and fuselage vibration which is the result of a sequence of more elementary physical dependences. Rotor airflow determines blade lift and drag; simultaneously, the bound circulation associated with blade lift creates new additions to the rotor wake, which in turn has a major influence on the rotor airflow. Here, then, is a small feedback loop: airflow to loads to wake, back to airflow. This first loop is followed by a second, similar one; airloads cause blade motions which in turn modify the airloads. Blade motions and airloads, determined in these two loops, are inputs to the next physical variable in the chain, the hub shear. The fuselage responds to hub shear in bending and translation, permitting the hub, which is at-

tached to it, to move. Hub motion, however, affects the rotor, changing both blade motion and hub shear. Thus, two more loops are established; blade motions and airloads create hub shear, which causes fuselage motion, which feeds back to blade motions and hub shear. Figure 1 shows these relationships schematically; it adapts Figure 2 of Reference 1 to the present discussion.

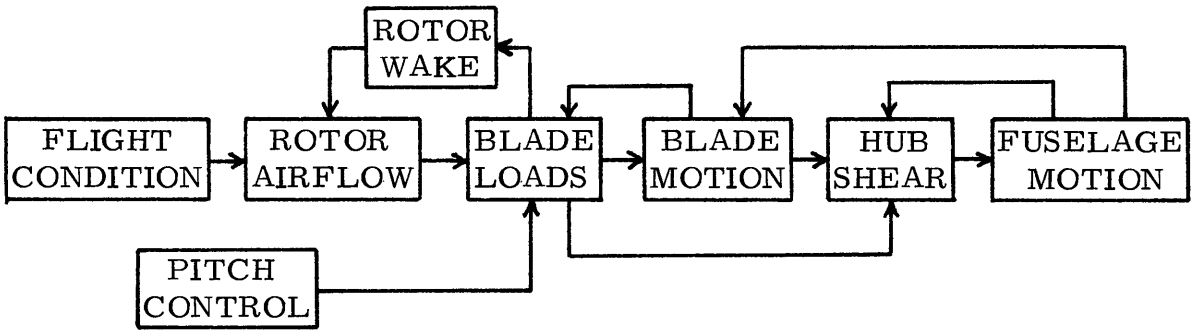


Figure 1. Physical Interactions which Affect Vibration

The interdependences shown in Figure 1, when applied without simplification to the analysis of vibration in a particular flight condition, lead to calculations which are extremely long even when performed by current high-speed digital computers. This investigation, therefore, was confined to the analysis of a somewhat simplified system, both to use the available time efficiently and to concentrate on the central matter of interest, avoiding irrelevant complexity. The central point is

this: the success of harmonic pitch control in reducing vibration depends on its effectiveness in reducing the vibratory hub shear due to the rotor. If that hub shear can be reduced, it is certain that the fuselage response will be reduced. The nature of the fuselage dynamic response is only secondary in an early feasibility study. The interactions essential to this analysis were therefore those which relate to the origin of vibratory hub shear, rather than to fuselage response. Thus the fuselage dynamics were simplified, and the feedback of fuselage motion to hub shear shown in Figure 1 was neglected. Fuselage vibratory motion is very small compared to blade flapping and bending motion, so the effects of hub displacements on rotor behavior were also neglected. In other words, all feedback effects of fuselage motion were eliminated from Figure 1. The resulting approximate system which was actually analyzed is shown in Figure 2. Fuselage motion is observed to affect no other physical variables in the simplified system. Clearly, it could have also been eliminated, leaving hub shear as the final result of the calculation. However, it was of interest to consider some simple way in which normal and inplane hub shears might combine in their effects on cockpit vibration. Thus the fuselage was imagined to exhibit rigid body response (translational and rotational) to the unmodified hub shears from the rotor.

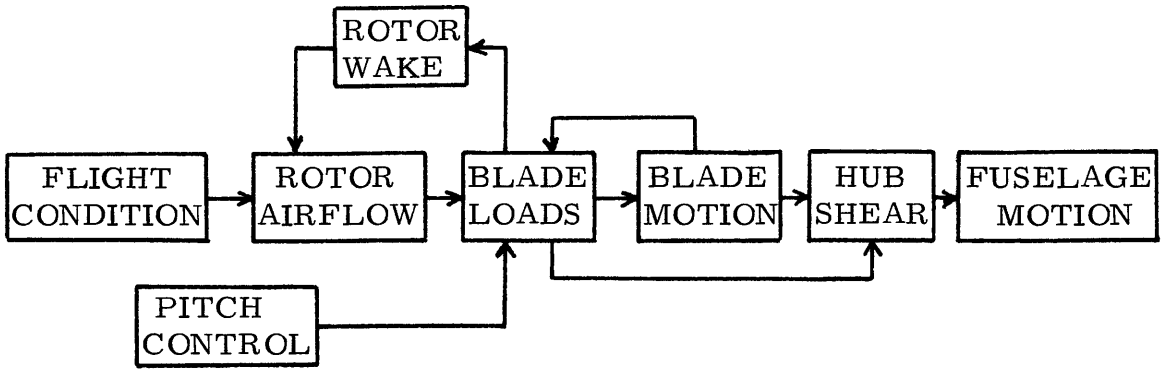


Figure 2. Physical Interactions Included in the Present Analysis

Fuselage vibratory dynamics, in fact, have an important influence on hub shear and cockpit vibration. The fuselage is not even nearly a rigid body, particularly at the frequencies of the oscillatory hub shear. Many helicopters have fuselage vibratory modes which are nearly in resonance with the strong blade-multiple harmonics of rotor hub shear. Such modes exhibit large vibratory responses in many flight conditions. The resulting hub accelerations change the hub shears as much as 100 per cent, usually amplifying them. This effect depends on the phase and amplitude of fuselage vibratory response, which in turn depends on which elastic modes are near resonance with the hub excitation. In the unrealistic case of a rigid fuselage, translational accelerations would attenuate hub shear forces somewhat. This unusual reduction, however, would only be by the ratio of blade mass to total aircraft mass, normally about five per cent. This small effect was also ne-

glected in the present investigation.

To summarize briefly, this feasibility study was primarily concerned with the effect of harmonic pitch control on rotor-induced hub shears, and devoted little attention to fuselage response. Hub shears were calculated on the basis of the rotor airflow and were not modified by fuselage dynamics. The fuselage response was roughly approximated as that of a rigid body with mass and moment of inertia, suspended under the rotor. Elimination of fuselage motion "feedback" effects from the physical system simplified the calculations; this omission affected the calculated rotor behavior only slightly but caused significant errors in predicted vibratory response of the hub and fuselage. Such errors, however, had little bearing on the real aim of the calculation, which was to determine the effectiveness of harmonic pitch control in decreasing fuselage excitation by the rotor.

3.2 Computational Methods

The simplified system in Figure 2, even though the fuselage feedback effects have been neglected, still contains the element of helicopter vibration which is hardest to analyze: the relationship of harmonic airloads to flight condition and harmonic pitch control. Engineering experience has been applied to the formulation of approximate analytical methods, which are still evolving as computational experience

accumulates. The evolution has led to increasing success in isolating the essence of the physical situation.[1] Present methods thus show good agreement with flight test results in many instances. Correlation is best at moderate forward speeds; at these speeds, wake effects are most easily modeled mathematically and blade airloads may be determined by well-established aerodynamics. Low forward speeds and high forward speeds involve physical effects which are difficult to analyze, such as wake suck-up through the rotor at low speeds and extensive dynamic blade stall and reverse flow at high speeds. Vibration correlation is still weak in these flight regimes. Reference 2, page vii, demonstrates the state-of-the-art as it has just been described; in comparing measured and predicted hub shears for tandem three-bladed rotors, zero prediction error is indicated for each fixed-direction shear component of each rotor somewhere in the speed range 75-110 knots. Errors at speeds other than the individual zero-error speeds are generally near 20 per cent, although some are as high as 100 per cent.

The analytical approach used in this feasibility study was based on present computational experience at moderate flight speeds, where accuracy is best. Airloads and blade flapping and bending motion were computed by a digital computer program developed over the past six

years at MIT.[8] This program, as mentioned above, is still evolving. Its development is described in References 1, 9, and 10. The solution takes advantage of the fact that physical "feedback loop" performance is most naturally evaluated iteratively, when a convergent iteration can be devised. That is, the input is applied and the resulting output is computed, then the output is fed back to revise the input, a revised output is computed, and so on until the revisions are negligible and a consistent picture has been obtained. In this solution, airloads are assumed, then the wake circulation is calculated. Next the wake-induced downwash is computed and the airloads revised. Then blade flapping and bending motions are calculated and the airloads revised still further. Finally the wake circulation is recalculated and the procedure is repeated. Repetition continues until the iteration converges. The wake is assumed to consist of trailing vortices from the tip and mid-span of each blade, with discrete lines of shed vorticity laid down between each pair of trailing vortices at intervals of 15° in azimuth. Each segment of wake remains at the location where the blade created it, except that it is assumed to be convected downward, normal to the rotor disk, at the mean rotor inflow velocity calculated by simple momentum theory. (This is the so-called "rigid wake" assumption.) The rotor downwash field is calculated as a sum of the effects of many short straight-line vortices which, when taken together, approximate the actual vortex

spirals. For the first iteration, bound circulation is assumed constant around the azimuth with strength such that the corresponding blade lift supports a specified helicopter gross weight. This causes constant strength trailing vortices and no shed vorticity at all. As an example of the way in which computational experience has isolated essential effects, early investigations[1, 9] showed that, at least at moderate forward speeds, the rotor downwash calculated from this very simple wake has practically the same harmonic content as the downwash obtained after many iterations. That is, the constant-strength wake due to mean lift is the major contributor to downwash harmonics and the accompanying harmonic blade loading. The next most important contributor to downwash harmonics has been found to be the shed wake due to the corresponding lift harmonics; that is, the shed wake caused by n^{th} harmonic lift has a greater effect than the rest of the shed wake on n^{th} harmonic downwash. This is far from clear by inspection because of the geometrical complexity of the skewed wake spirals. A full explanation is given in References 1 and 9. The important shed wake contribution to harmonic downwash is largely accounted for in the second iteration of the program, so in most flight conditions the calculation is virtually complete after two iterations.

Blade flapping and first mode bending motion are obtained in the program by Runge-Kutta solution of two coupled second order flapping

and bending equations. These equations may be found in Reference 11. They are analogous to the equations of motion of two coupled spring-mass-damper systems, driven in two different ways by the lift distribution along the blade. The shape of the assumed first mode bending displacement is given by equation 3.1:

$$\phi(\eta) = \eta - \frac{\pi}{3} \sin \pi \eta \tag{3.1}$$

Inclusion of first mode bending motion was particularly important because this elastic mode is nearly in resonance with third harmonic airloads, which were of fundamental interest in this investigation.

The known existence of stall and reverse flow which affect flapping and bending led to a modification of the conventional Runge-Kutta solution for coupled second order systems with damping. Blade angle of attack is due primarily to wake-induced downwash and to any tilt of the rotor disk into the wind, but it is modified by blade motion. This is the source of aerodynamic damping in both flapping and bending. The program calculates the modified angle of attack and compares it to a preset value to make a rough check for dynamic stall. It then finds the lift coefficient from the modified angle, assuming that the lift curve simply flattens off above dynamic stall and that the blade is symmetrical front and back for reverse flow. This approximate treatment of extreme aerodynamic conditions makes it convenient to place

the damping forces on the right side, or driving side, of the differential equations. Such a change, however, requires that the Runge-Kutta solution be iterated, updating the damping-modified lift distribution after every iteration. Thus the program solves the flapping and bending equations as though they had no aerodynamic damping but were driven by the "residual" lift including blade motion effects. The predicted residual lift and blade motion are made consistent by iteration. This solution method is not at all inconvenient, since iteration is necessary in any case to find bending and flapping solutions which are continuous at their end points, that is, which are the same at 0° and 360° azimuth.

For the purposes of the present investigation, the equations of blade lagging motion, of normal and inplane hub shear, and of fuselage rigid body response to hub shear were added to the program. These equations, with the exception of the one for fuselage response, are easily derived from general forms found in Reference 11. The basic lagging equation used in the program is this:

$$\bar{I}\,\ddot{\zeta} + \bar{I}\bar{\omega}^2\,\zeta = \int_e^R (r - e)\,dH \tag{3.2}$$

for which $\bar{\omega}$ is calculated as

$$\bar{\omega} = \left(\frac{3}{2}\,\frac{e}{R-e}\right)^{1/2}\Omega \tag{3.3}$$

Equation 3. 2 is based on the assumptions that blade mass is uniformly distributed in the spanwise direction and that no chordwise bending occurs. The H forces which drive the lagging equation (3. 2) are aerodynamic drag, centrifugal loading, and coriolis forces due to flapping, bending, and lagging velocities. Lagging motion is so small that its effect on airloads is negligible, as well as its coriolis effect on flapping and bending. The program therefore solves the flapping and bending equations first, independently of the lagging equation (including full iteration for consistent residual airloads); then residual airloads and flapping and bending coriolis forces are used as inputs to the lagging equation. A complete derivation of the exact form of the lagging equation used in the program, including expressions for the various H forces, is given in Appendix A.

Normal and chordwise hub shears are found by the program using the following equations:

$$S = \int_0^R dL - \int_0^R \ddot{z}\, dm = \int_0^R dL - \frac{m(\Omega R)^2}{2}\left(\frac{d^2\beta}{d\psi^2} - \frac{1}{3}\frac{d^2\xi}{d\psi^2}\right) \qquad (3.4)$$

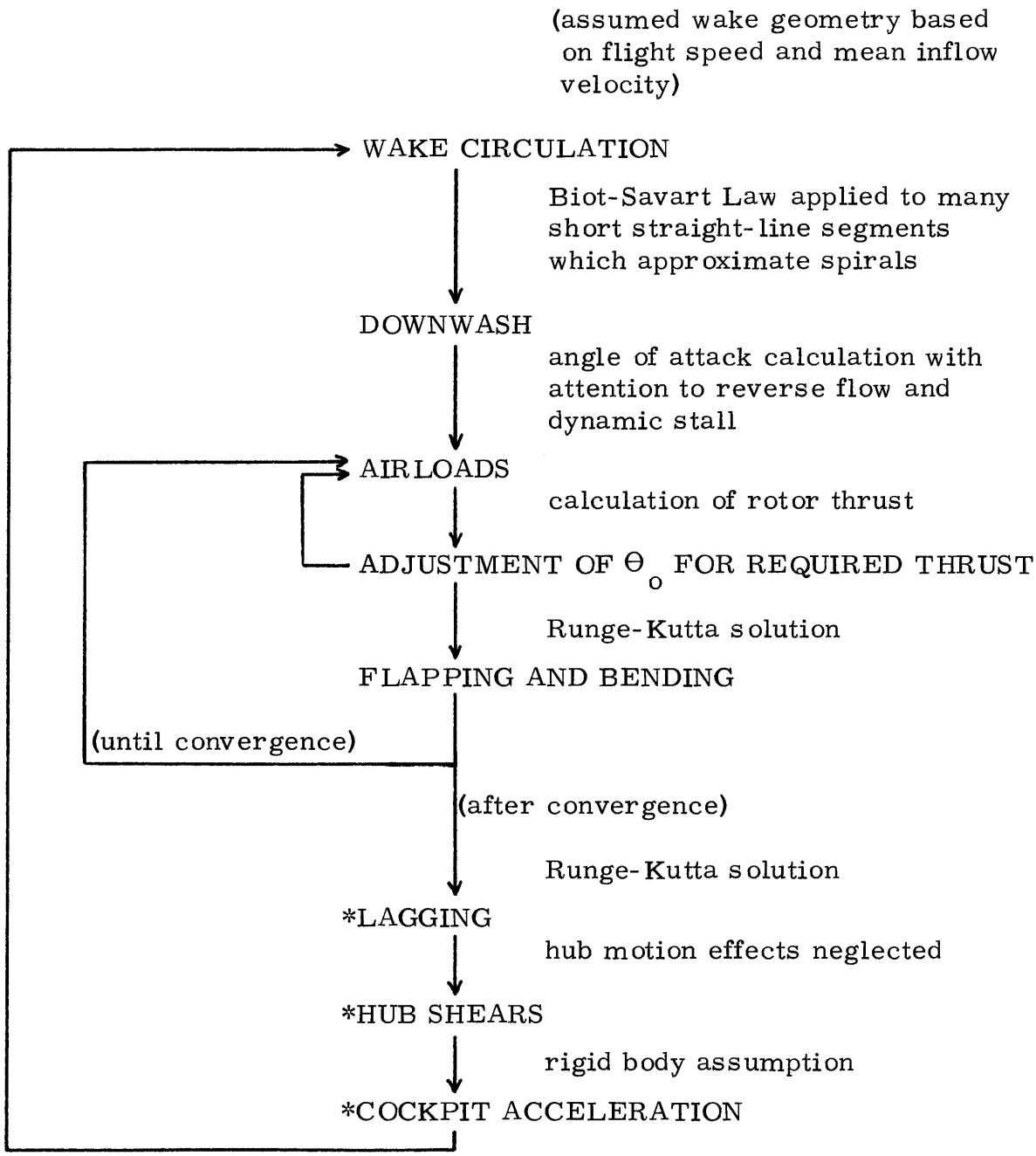
$$\bar{S} = \int_e^R dH - \int_e^R \ddot{y}\, dm + \Omega^2 \int_e^R y\, dm = \int_e^R dH + \frac{m\Omega^2(R-e)^2}{2}\left(\zeta - \frac{d^2\zeta}{d\psi^2}\right) \qquad (3.5)$$

These equations are also derived in Appendix A. They are used by the program to determine single-blade hub shears after all blade motion and residual airloads have been tabulated as a function of azimuth. Longitudinal inplane hub shear is then obtained as the component of chordwise hub shear in the fore-and-aft direction. At this point, the program is specialized to the case of a three-bladed rotor. Third harmonic components of the normal and inplane hub shears are calculated, then tripled to account for the synchronized effects of the three blades. Finally, cockpit third harmonic normal acceleration is found from the following equation:

$$(\ddot{z}_p)_3 = 3F_v S_3 + 3F_h \bar{\bar{S}}_3 \tag{3.6}$$

Here, F_v and F_h denote, respectively, the "vertical acceleration factor" and "horizontal acceleration factor" appropriate to the rigid body assumptions of the analysis. These two factors depend in a very simple way on the geometry, gross weight, and pitching inertia of the fuselage. Their actual form is discussed in Appendix A.

THE COMPUTER CALCULATION



*has no effect on main airload - downwash loop; calculated in each iteration only as an intermediate, approximate result

Figure 3. The Computer Calculation

CHAPTER 4

RESULTS

This investigation considered specifically the effects of third harmonic pitch control on the third harmonic cockpit acceleration caused by unsteady airloads on a three-bladed rotor. The results are only preliminary in terms of a complete evaluation of harmonic pitch control; first, because they apply to only one flight condition and rotor configuration; and second, because even for that single case, they raise questions about shed wake effects and about the simultaneous adjustment of three pitch control harmonics which can only be answered by further research. Nevertheless, they permit several significant conclusions about the feasibility and characteristics of a harmonic pitch control system.

The helicopter parameters used in the program do not apply to a particular production model, but are typical of relatively large single-rotor aircraft. They are listed, together, with the flight condition variables, in Table 1, which presents the computer input data. The most significant flight condition variables, as far as establishing the type of wake effects to be expected, are these:

1. Advance ratio $\mu = 0.2$ This indicates that the forward speed of the aircraft is approximately one fifth of the blade tip speed due to rotation. The helicopter therefore travels only slightly farther than one rotor radius during a complete revolution of the rotor, and the wake spirals are not extremely skewed. Almost half of the first turn of each spiral lies directly under the rotor disk.

2. Inflow ratio $\lambda = .059$ This indicates that the mean inflow velocity, by which the spirals are assumed to be convected downward, is about six per cent of the blade tip speed due to rotation. Thus a spiral travels downward less than half a rotor radius during a complete rotor revolution.

3. Blade loading $C_T/\sigma = 0.1$ This indicates that the mean blade lift coefficient is approximately six tenths. This is a rather high mean lift, which causes relatively strong trailing vortices. Together, the fact that half of the first spiral turn lies directly below the rotor, less than half a radius away, and the fact that the vortices are relatively strong indicate that strong wake effects may be expected in this flight condition.

In considering the quantitative results of the investigation, it is appropriate to look first at some intermediate results of the main computer downwash-airloads iteration, for the demonstration of rotor wake effects and program convergence speed they provide. Six iterations of

Rotor parameters	Fuselage parameters
3 blades	gross weight ≐ 17,150#
$R = 25'$	pitching moment of inertia about c.g. = 17,150 slug-ft^2
$c = 2'$	hub position with respect to c.g.: 0.5' forward, 6.0' above
$\sigma = .075$	
$L_N = 10$	
total blade weight = 855#	pilot position with respect to c.g.: 5.0' away along a line pointing forward and 20° up from horizontal
$m = .35$ slug/ft	

proportionality factors between pounds hub shear and cockpit g's:

$$F_v = 6.26 \times 10^{-5}$$

$$F_h = 5.13 \times 10^{-5}$$

Blade dynamics

first mode flatwise bending natural frequency: $2.85\,\Omega$

lag hinge at 5% span, causing lag natural frequency $\bar{\omega} = .28\Omega$

Flight condition parameters

steady flight	$\lambda = .059$	$C_T = .0075$
$\mu = 0.2$	$-\tan i = 0.2$ ($i = 11°$)	$C_T/\sigma = 0.1$
$\Omega R = 700$ ft/sec	β_o(approx) = .15 (9°)	Θ_o(approx) = .19(11°)

Table 1. Computer Input Data

third harmonic downwash at three spanwise locations are shown in Table 2, for the case of no harmonic pitch control. As explained in Chapter 3, the first iteration is based on the constant strength trailing vortices created by the mean blade lift, and experience has shown that these vortices are the primary contributors to harmonic downwash. The second iteration includes the next most important effect, that of third harmonic shed wake on the third harmonic downwash. Subsequent iterations show rapid convergence, but it is clear that the results of the second iteration are an excellent approximation to the fully converged solution. Table 3 shows five successive approximations to third harmonic flapping and bending, obtained during the iterative solution which gave the downwash values in Table 2. These variables, of course, converge in the same way as the downwash; the convergence is slightly slower because of their dependence on both the magnitude and spanwise distribution of downwash.

After six iterations, resulting in practically complete convergence of all predicted variables, second, third, and fourth harmonic downwash were plotted against spanwise location in Figure 4. These downwash harmonics, as explained in Chapter 2.1, are the major source of vibration in this helicopter, since it has a three-bladed rotor. They are the origin of the lift and blade motion harmonics which cause third harmonic hub shear and the accompanying strong third harmonic fuselage

Iteration	Third Harmonic Downwash		
	95% span mag., phase	85% span mag., phase	60% span mag., phase
1	.0114, 166°	.0142, 180°	.0032, 106°
2	.0086, 174°	.0133, 169°	.0092, 78°
3	.0083, 176°	.0122, 173°	.0078, 66°
4	.0090, 174°	.0127, 175°	.0071, 72°
5	.0087, 173°	.0127, 174°	.0073, 72°
6	.0085, 174°	.0125, 174°	.0072, 71°

Table 2. Results of six successive iterations for third harmonic downwash with no higher harmonic pitch control

Notes for Table 2: "Downwash" here denotes rotor inflow velocity divided by blade tip speed. "Phase" of the third harmonic refers to the angle at which this harmonic becomes maximum positive (maximum downward through the disk); that is,

$$\lambda_3 = A \cos (3\psi - \phi)$$

where λ_3 denotes third harmonic downwash and ϕ denotes its "phase" as listed above. To find the rotor azimuth angles corresponding to maximum λ_3, divide ϕ by three for the first and add 120° and 240° to this result for the second and third.

Iteration	Flapping, β_3 mag., phase	Bending, ξ_3 mag., phase
1	.00063, 159°	.00473, 157°
2	.00072, 132°	.00483, 154°
3	.00059, 137°	.00509, 157°
4	.00063, 139°	.00511, 156°
5	.00064, 138°	.00504, 156°

Table 3. Results of five successive iterations for third harmonic flapping and bending with no higher harmonic pitch control

vibration. Table 4 gives the numerical values of the lift, motion, and shear harmonics they cause, together with the resulting third harmonic normal acceleration at the cockpit. These values, since they refer to the case with no higher harmonic pitch control, provide a description of the nature of the vibration control task: the goal is to modify these particular values of third harmonic lift, flapping, bending, and lagging in such a way as to decrease significantly the .22 g's third harmonic normal acceleration felt by the pilot.

Table 4 shows that calculated third harmonic normal hub shear is thirty times larger than third harmonic inplane hub shear. This is certainly not the case in all flight conditions or for all rotors. The

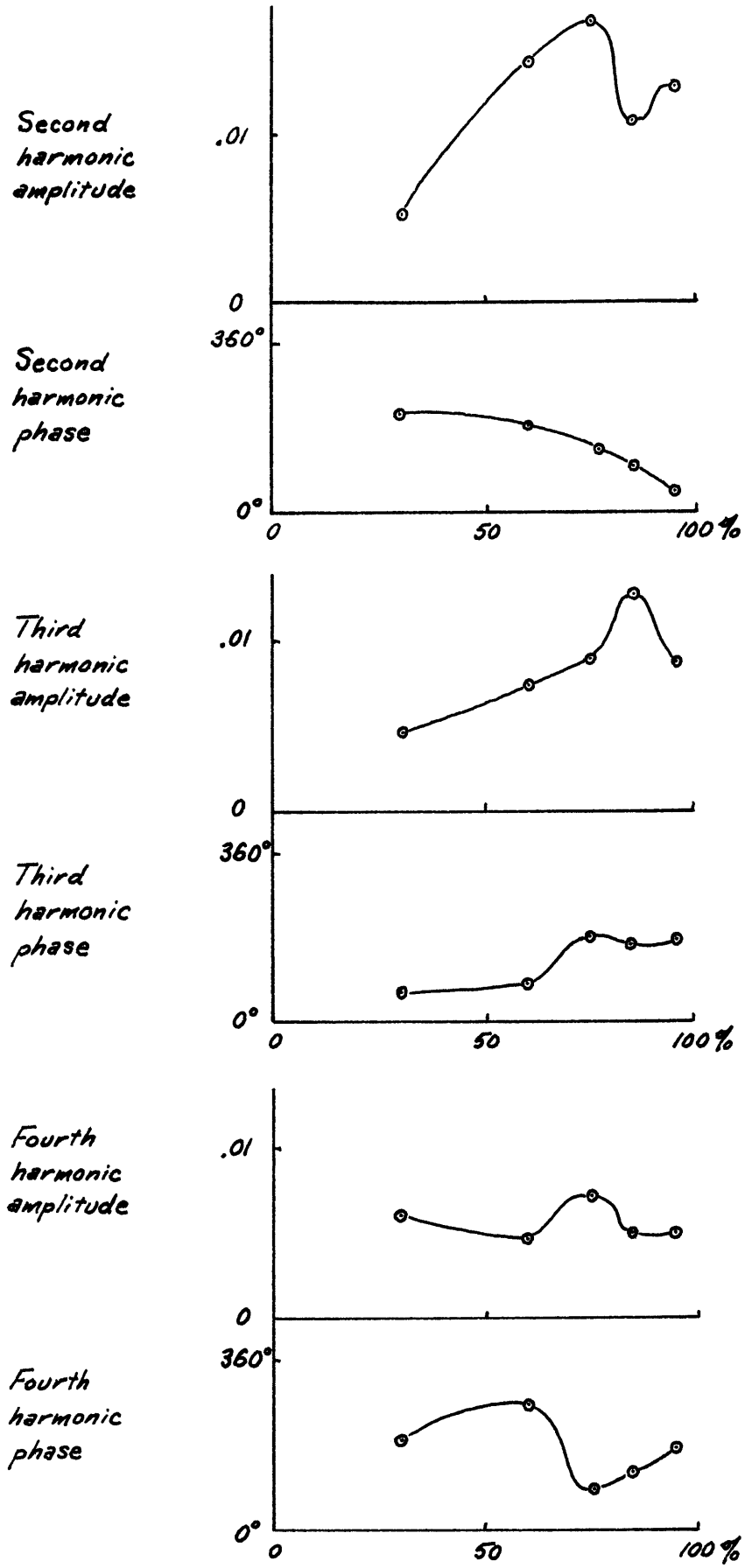


Figure 4. Calculated second, third, and fourth harmonic downwash as a function of spanwise location, with no harmonic pitch control

Flapping β_3	=	.00064, 137°
Bending ξ_3	=	.00504, 156°
Lagging ζ_2	=	.00196, 214°
ζ_4	=	.00015, 254°
Hub shear		
Normal_3	=	3512#, -16° (sum of three blades)
Inplane_3	=	108#, 102° (sum of three blades)
Cockpit normal g's		
		.22, -15°

Table 4. Calculated blade motion and hub shear quantities for case of no harmonic pitch control

problem, however, in this flight condition, is to reduce the third harmonic normal hub shear without incurring a large penalty in the magnitude of the inplane hub shear. It would of course be possible to discover any harmonic pitch control inputs which accomplished this goal by conducting an exhaustive search over all the possibilities. Table 5 is useful at this point because it leads to some educated guesses for the starting point and direction of search, making exhaustive search unnecessary. This table shows a breakdown of the net normal hub shear into the components due to various sources.

Third harmonic normal hub shear per blade	
Net =	1171#, -16°
Residual lift component =	396#, -36°
Flapping reaction component =	533#, 142°
Bending reaction component =	1295#, -19°

Table 5. Breakdown of net normal hub shear into components due to various sources

Figure 5 presents the data of Table 5 in graphical form. The net third harmonic normal hub shear and its individual components are represented by vectors, which in this application are sometimes called "phasors". A phasor is imagined to rotate counter-clockwise at the frequency of the corresponding sinusoid, giving the value of the sinusoid in time by its component in some fixed direction, usually the horizontal. The phasor is drawn in its initial position (zero time or, in this case, zero azimuth) to show the phase of the sinusoid. Thus, a phasor of length A pointing to the right represents a cosine of amplitude A, while a phasor of length B pointing 45° downward to the left represents a sinusoid of amplitude B lagging a pure cosine by 135°. A pure sine points downward, a negative cosine points to the left, and a negative sine points upward. The virtue of the notation is that it permits visualization of the manner in which sinusoids of the same frequency but

of different amplitude and phase combine to form a net sinusoid, without recourse to trigonometric identities.

Table 5 and Figure 5 call attention to two important facts about normal hub shear. First, the hub shear components due to residual lift and to flapping reaction largely oppose and cancel each other. Second, the hub shear component due to bending reaction is a strong source of vibration. These facts are well understood and deserve explanation in some detail.

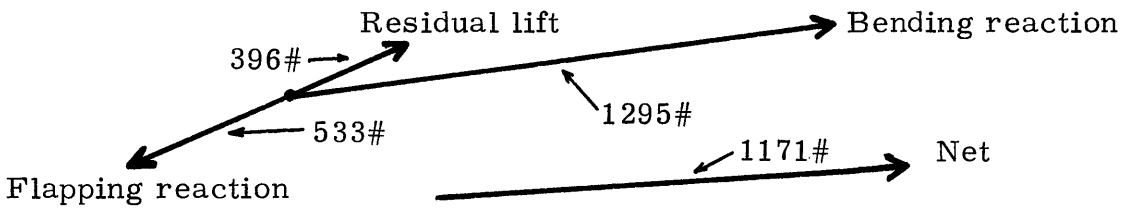


Figure 5. Net third harmonic normal hub shear and its components (phasor notation)

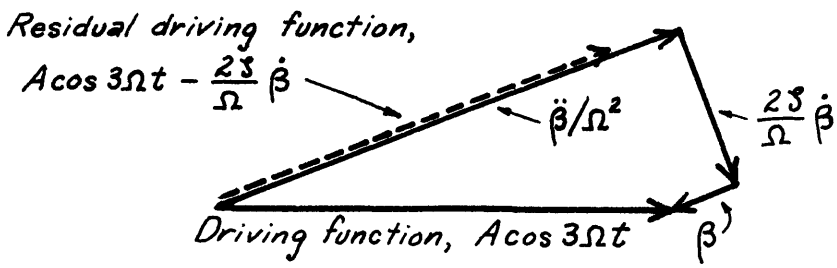
First, the hub shears due to residual lift and flapping acceleration are nearly equal and opposite. Third harmonic flapping motion occurs at three times the blade natural frequency in flapping. The damping ratio of the flapping motion is predicted by well-established rotor analysis to be one-sixteenth of the blade Lock number; in this case the damping ratio is thereby found to be slightly more than 0.6. When

the excitation frequency and damping ratio are known in this way, certain approximate characteristics of the sinusoidal response may be predicted by simple dynamic reasoning. In particular, Figure 6, a phasor diagram of flapping response to a third harmonic driving function, shows that flapping acceleration is in phase with the residual driving function -- that phasor obtained by subtracting the damping term from the actual driving function. This means, in other words, that flapping acceleration is in phase with the flapping moment due to residual lift. Figure 6 shows, furthermore, that in calculating third harmonic flapping acceleration, the "spring term", or centrifugal force effect (shown as "β" in the Figure), may be neglected as a good approximation. Denoting residual lift by L_R, the approximate equation (4. 1) may be written for flapping acceleration.

$$\ddot{\beta} \cong \frac{1}{I} \int_0^R r \, dL_R \tag{4. 1}$$

These blades have a constant spanwise mass distribution, so equation (4. 1) leads to equation (4. 2) in which the flapping moment of inertia is written explicitly.

$$\ddot{\beta} \cong \frac{3}{mR^2} \int_0^R r \, dL_R \tag{4. 2}$$



$$\frac{\ddot{\beta}}{\Omega^2} + \frac{2(.6)}{\Omega}\dot{\beta} + \beta = A\cos 3\Omega t$$

Figure 6. Flapping response to a third harmonic driving function for a damping ratio of 0.6

Equation (3.4) indicated that the hub shear due to flapping acceleration is given by equation (4.3).

$$S_{\ddot{\beta}} = -\frac{mR^2}{2}\ddot{\beta} \tag{4.3}$$

Equations (4.2) and (4.3) may be combined to obtain another form of the equation for hub shear due to flapping acceleration, given as equation (4.4)

$$S_{\ddot{\beta}} \cong -\frac{3}{2}\int_0^R r\, dL_R \tag{4.4}$$

$r_{c.l.}$ may be used to denote the spanwise center of residual lift. Using this parameter, equation (4.4) takes the more interesting form (4.5).

$$S_{\ddot{\beta}} \cong -\frac{3}{2} r_{c.l.} \int_0^R dL_R \tag{4.5}$$

The hub shear due to residual lift is given by equation (4.6).

$$S_{L_R} = \int_0^R dL_R \tag{4.6}$$

Equations (4.5) and (4.6) make it clear that if the residual lift is centered at 67% span, the hub shears due to flapping and residual lift will be equal and opposite, to the order of the dynamic approximations already made. If the residual lift is centered outboard of 67% span, flapping will more than offset the hub shear due to residual lift, but not by a large amount. Figures 4 and 5 show that a slight over-cancellation is exactly what has occurred in the present case.

The second fact observed in Figure 5 (the breakdown of third harmonic normal hub shear into its components) is that first mode blade bending causes a substantial amount of hub shear -- so much, in fact, that it is the main source of vibration. The details of the creation of so much hub shear by first mode bending are clear. First of all, this mode of bending is one third as effective as flapping in causing hub shear, as shown by equation (3.4). Shear due to first mode bending is given by equation (4.7), taken from equation (3.4).

$$S_{\ddot{\xi}} = \frac{mR^2}{6} \ddot{\xi} \tag{4.7}$$

First mode bending motion is a "bowing" of the blade, curving down at the center and sloping upward at the tip. During positive acceleration

in this mode, the downward acceleration of inboard and center-span mass elements more than offsets the upward acceleration of outboard mass elements, and the net effect is an upward reaction force (d'Alembert force) on the hub. Positive flapping acceleration, on the other hand, causes upward acceleration of all blade mass elements and a downward reaction force on the hub. Because all mass elements accelerate in the same direction in flapping, this acceleration is more effective than bending in causing hub shear. Nevertheless, the factor of three between flapping and bending effectiveness is not extremely important when it is realized that third harmonic bending may easily be much larger than third harmonic flapping: it is nearly in resonance with the driving frequency and has a relatively low damping ratio, typically about 0.4. Table 4 shows that in the present case bending motion is about eight times as large as flapping motion. Both the flapping and bending mode shapes have unit displacement (one radius) at the tip: thus, in other words, third harmonic bending causes eight times as much tip displacement as third harmonic flapping (0.5% radius compared to .06% radius, or 1 1/2 inches compared to 3/16 inch). The final result is that first mode bending is responsible for about eight-thirds, or 2.7, times as much hub shear as that caused by flapping motion. This has been seen in Figure 5, the phasor diagram of hub shear components due to three sources.

A final point about third harmonic bending remains to be made. Even though the first mode bending response is greatly amplified by its near-resonance with the third harmonic driving frequency, the real cause of the motion is the third harmonic lift excitation. The origin of the driving function will be examined.

If lift were proportional to radius, its distribution would correspond to the shape of the flapping mode. The blade would flap, but no bending would occur. Any "bowing" motion of a blade with constant spanwise mass distribution requires a loading distribution out of proportion to radius. Third harmonic lift has such a loading distribution because of the fundamental characteristics of the rotor wake. Lift, in the absence of pitch control, depends on downwash variation with span and azimuth, and this in turn depends on the wake. The largest variation of rotor downwash with azimuth is from smaller at the front of the disk to larger at the rear. This accounts for a large first harmonic component added to the mean value. The transition from high at the front to low at the rear is rather sharp, however. The largest changes occur in the vicinity of 90° and 270° azimuth, when the blade passes over a trailing vortex from the preceding blade. The resulting intense downwash variations make the front-rear variation more like a square wave than a sinusoid. But the main characteristics of a square wave are obtained by adding a negative third harmonic to the basic first

harmonic. Thus the third harmonic downwash component accounts for the sharper-than-sinusoidal changes from front to rear of the rotor. Here, incidentally, is a verification of the conclusion presented in Chapter 3 and illustrated in Table 2: the harmonic content of the downwash is primarily determined by the mean-lift trailing vortices. Significantly, the trailing vortex from the preceding blade does not pass under all spanwise stations simultaneously, but "sweeps" along the blade from tip to root. Thus the sharp increase and decrease in downwash are not simultaneous at all spanwise stations, and the third harmonic component of downwash at inboard stations is out of phase with that at the outboard stations, as may be seen clearly in Figure 4. This has a considerable effect on third harmonic bending excitation.

Figure 7 is the first step in an approximate approach to an understanding of third harmonic bending excitation. It shows a representative third harmonic "blade tip downwash" phasor and a representative third

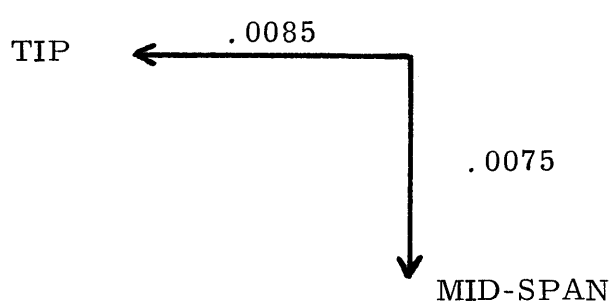


Figure 7. Representative tip and mid-span third harmonic downwash phasors

harmonic "mid-span downwash" phasor, obtained by inspection of Figure 4. Mid-span and outboard downwash are observed to have comparable magnitudes but a 90° difference in phase. Figure 8 is the second step, showing two phasors representing "downwash effect on lift". These are the Figure 7 phasors with the "tip" phasor doubled to account for the higher dynamic pressure due to rotation at the tip, and with the directions reversed to account for the correspondence between positive lift and negative downwash. The difference between the Figure 8 phasors (the dashed phasor running from the head of the

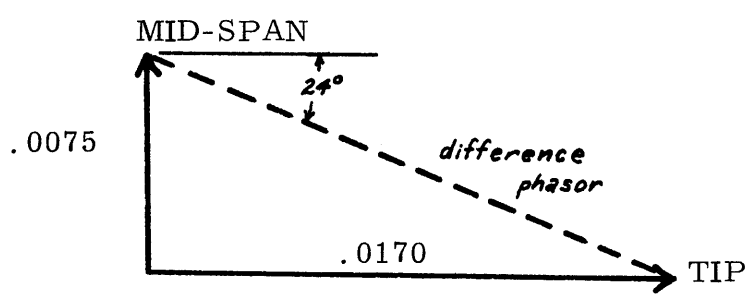


Figure 8. Representative tip and mid-span third harmonic downwash effect on lift

mid-span phasor to the head of the tip phasor) accounts for third harmonic bending excitation, insofar as the "effects on lift" at mid-span and tip are not proportional to radius. The angle of this phasor gives the approximate phase of the third harmonic bending excitation, about 24°. The motion is driven at $\frac{3}{2.85}$, or 1.05 times its natural frequency

with a damping ratio of about 0.4; thus simple dynamics predicts that the response lags the excitation by about 135°. First mode bending can therefore be expected to have a phase of about 24° plus 135°, or 159°. Table 4 shows that the computer-calculated phase is 156°. This favorable comparison provides some verification of the validity of the approximate approach.

Figure 8 permits some important conclusions about the effects of third harmonic pitch control, and actually suggests a control adjustment strategy. The Figure 8 phasors may be used to represent mid-span and tip blade lift excitation. Any third harmonic pitch input will change these phasors, both directly and as a result of wake modifications which change the downwash. The wake modifications will be ignored for the moment and discussed in some detail later. Pitch inputs act with the same phase at all spanwise stations, moving the heads of both Figure 8 phasors in the same direction. Because of the difference in mid-span and tip dynamic pressure, the "tip" phasor will move about twice as fast as the "mid-span" phasor. (More specifically, "tip" is taken as about 90% span, while "mid-span" refers to about 60% span.) For example, a cosine-phased third harmonic pitch input of amplitude .01 radians would move the head of the "tip" phasor .02 to the right while moving the head of the "mid-span" phasor .01 to the right. The

harmonic pitch control will greatly reduce normal hub shear if it reduces first mode bending excitation. The goal, then, is to apply pitch control with such a phase and magnitude that the resulting Figure 8 phasors will be aligned, with the tip phasor 3/2 the length of the mid-span phasor. At this point, the lift distribution will be proportional to radius; no bending will occur, and flapping acceleration will exactly compensate for lift in their effects on hub shear (lift will be centered at 67% span). This, of course, is idealized, but it is firmly based on the essence of the physical situation. Inspection of Figure 8 indicates that the required phasor alignment and amplitude adjustment is possible. If harmonic pitch control is applied in the direction opposite to that of the difference phasor, the tip phasor will "chase" the mid-span phasor, moving twice as fast in the same direction, and will finally overtake it. The tip and mid-span lift excitation will be equal when the pitch control has an amplitude of about .0170 radians, or just under one degree. In that case, the mid-span phasor would have moved a distance of .0170 in the direction opposite to the difference phasor while the tip phasor moved .0340 in the same direction. The two phasors would then be identical. This is shown in Figure 9. After equalization of mid-span and outboard lift, additional pitch control could be applied, in the phase common to both lifts, with a magnitude which doubled the mid-span lift. The tip lift would change twice as much, thereby tripling its

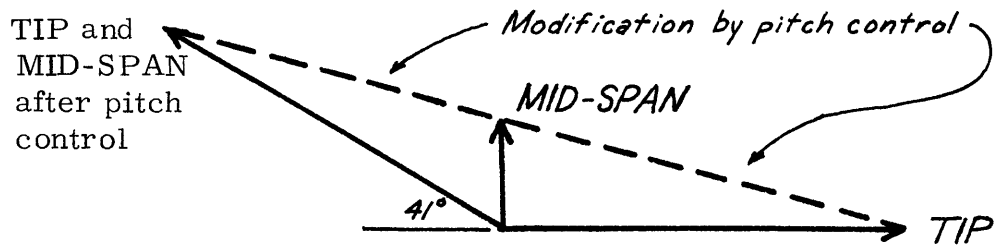


Figure 9. Phasor illustration of the equalization of mid-span and outboard lift by harmonic pitch control, with wake modification effects neglected

length and attaining a magnitude 3/2 times that of the mid-span lift. Thus bending excitation would be eliminated and normal hub shear would be greatly reduced. Tip lift would be three times as great as it was originally (but almost 180° out of phase with the original), and mid-span lift would be two-thirds that high, or almost six times as large as it was originally. Flapping could therefore be expected to have increased to five or six times its original value. However, because of the lift-flapping opposition in hub shear effects, net hub shear would still be very small. Furthermore, flapping itself would still be very small, since it was mentioned earlier that flapping with no pitch control causes only 3/16 inch tip displacement. The amplitude of the total pitch control would be approximately two degrees.

Pitch control adjustment by this control strategy was tested with the computer program described in Chapter 3. The downwash predicted

Third harmonic pitch control output	Third harmonic normal hub shear per blade			
	bending reaction	flapping reaction	residual lift integral	total
0	1295, -19°	533, 142°	396, -36°	1171, -16°
.0170, 190°	693, -6°	600, 20°	720, 194°	559, -5°
.0170, 200°	661, -15°	684, 33°	778, 207°	554, -27°
.0170, 220°	707, -34°	867, 55°	910, 229°	625, -37°
.0170, 240°	870, -44°	1048, 73°	1046, 249°	808, -47°
.0190, 220°	645, -37°	968, 51°	1022, 227°	572, -42°
.0210, 220°	585, -41°	1072, 48°	1135, 225°	524, -48°
.0230, 220°	529, -46°	1178, 46°	1250, 223°	483, -55°
.0250, 220°	478, -53°	1287, 44°	1365, 222°	450, -63°
.0270, 220°	434, -60°	1397, 42°	1481, 220°	428, -72°
.0290, 220°	399, -70°	1508, 41°	1598, 220°	318, -82°
.0270, 210°	311, -40°	1308, 31°	1416, 210°	267, -61°
.0270, 200°	292, -6°	1225, 19°	1356, 198°	173, -21°
.0320, 200°	124, 19°	1508, 16°	1652, 197°	20, 216°
.0350, 205°	35, 224°	1718, 20°	1857, 201°	175, 212°

Table 6. Effects of various third harmonic pitch control inputs, wake modifications neglected, showing steady hub shear improvement based on step-by-step search

after six iterations for the zero pitch control case was used, and downwash modifications due to pitch control were temporarily neglected. In other words, only the first iteration with pitch control was calculated, and the downwash was not revised to account for the changes in wake circulation. A steady improvement to almost zero third harmonic bending excitation and almost zero third harmonic hub shear was obtained with a number of successive control inputs, each based on the results of the one preceding it. The initial adjustment was based on the approximate control effect calculations discussed above. The results of the various inputs are listed in Table 6.

The approximate lift phasor method just described was a great aid in the process illustrated in Table 6: finding the third harmonic pitch control input necessary to minimize normal hub shear. As pitch control amplitude and phase were changed, the variations in hub shear due to bending, flapping, and residual lift were almost exactly as predicted by the simple approximations. For example, the discussion above led to the expectation that, neglecting wake modifications, first mode bending excitation would finally be eliminated by third harmonic pitch control of amplitude about two degrees and phase about 210°. The last line of Table 6 shows that computer-calculated first mode bending excitation was minimized by the input .0350, 205°;

that is, by 1.99 degrees amplitude at 205° phase. This excellent correlation between approximate predictions and calculated results shows that the representative lift phasor approach is a powerful aid to understanding hub shear excitation.

When calculated with the computer program, bending and normal hub shear were not minimized exactly simultaneously as predicted in the earlier discussion. The last line of Table 6 shows that, when bending was virtually eliminated, flapping hub shear slightly undercompensated for hub shear due to residual lift, by 144 pounds per blade. To minimize net normal hub shear, the control input was modified slightly to cause a small amount of bending with the amplitude and phase necessary to make up the difference between flapping and residual lift hub shears. The final result was, as shown in the next-to-last line of Table 6, a third harmonic control input of .0320 radians (1.83 degrees) phased at 200°.

The "optimum" control input just described is a third harmonic blade pitching which is maximum positive at 67°, 187°, and 307° azimuth, and maximum negative at 7°, 127°, and 247° azimuth. This control has an amplitude large enough to reverse the natural phase of the third harmonic lift component at the tip and a phase such that it synchronizes the third harmonic lift at the mid-span with the new variation at the tip. It is clear by inspection of Figure 8 that this control makes

the only type of normal hub shear improvement possible with full spanwise blade pitch control. Full spanwise pitch control with any other magnitude or phase cannot possibly be as effective as this one in reducing first mode bending excitation, for the simple reasons described earlier. In other words, it is clear that a unique "optimum" has been found.

During description of the vibration control task in the early part of this chapter, it was shown that it would be necessary to reduce normal hub shear significantly in the present case without incurring a large penalty in the inplane hub shear, in order to achieve the ultimate goal of reducing cockpit vibration. Table 7, which compares computer-calculated blade motion, hub shear, and vibration values for zero control and for the above "optimum" control (next-to-last line in Table 6), indicates that this "optimum" control actually has the desired effect. The critical harmonics of blade lagging motion and inplane hub shear are changed very little in magnitude by the application of the "optimum" control for reducing third harmonic normal hub shear. The net result is a very large decrease in third harmonic cockpit vibration to less than 2% of its original value.

Wake modifications due to harmonic pitch control have been neglected until this point. These wake modifications are an extremely important physical effect of pitch control; because they are so important,

Third harmonic values	Zero control case	"Optimum" control case
Normal hub shear (3 blades)	3511, -16°	60, 216°
First mode bending	.0448, -19°	.0043, 19°
Flapping	.0061, -38°	.0173, 196°
Inplane hub shear (3 blades)	108, 103°	121, -2°
Lagging		
2nd harmonic	.0019, -35°	.0020, -57°
4th harmonic	.0001, -112°	.0001, 107°
Cockpit normal vibration	.21 g's, -15°	.0039 g's, -38°

Table 7. Comparison of blade motion, hub shear and normal vibration values for zero and "optimum" third harmonic pitch control

Note for Table 7: "Optimum" here refers to that pitch control which minimizes normal hub shear when wake modifications are neglected. Subsequent results show the neglect of wake modifications to be a very inaccurate approximation.

the results shown in Table 6, which neglect them, are much more useful as a means of understanding control adjustment concepts than as an accurate estimate of control effectiveness. It is for this reason that the word "optimum", when applied to the best control for normal hub shear reduction shown in Table 6, has continually been placed in quotation marks. When wake modifications are considered, this control does not have the same effect as shown in Table 6. It therefore does not reduce normal hub shear so much, and it is not the real optimum. Table 8 shows a comparison between the effects of the Table 6 "optimum" control on normal hub shear with and without wake modifications considered in the calculation. The difference can be explained easily in terms of the "representative lift phasor" approximations presented earlier in the chapter. Shed wake attenuation of lift changes due to pitch control has prevented the tip phasor from catching up to the mid-span phasor and overtaking it to eliminate the first mode bending excitation. It might be expected that the application of an increasingly larger amplitude pitch control phasor in the same direction as the Table 6 "optimum" would eventually overcome the shed wake attenuation effects and arrive at a new, accurate optimum which actually eliminated first mode bending. Several calculations were made to investigate this possibility; the results are shown in Table 9. It is observed that even

Third harmonic values	Without wake modifications	Including wake modifications
Normal hub shear (per blade)	20, 216°	662, -22°
Bending reaction component	124, 19°	812, -19°
Flapping reaction component	1508, 16°	1160, -7°
Residual lift	1652, 196°	1316, 173°

Table 8. Effects of the Table 6 "optimum" pitch control on normal hub shear with and without wake modification effects included

Third harmonic pitch control	Normal hub shear due to first mode bending
0	1295, -19°
.0320, 200° ("optimum")	812, -19°
.0500, 200°	776, -27°
.0800, 200°	476, -43°

Table 9. Change of hub shear component due to bending when pitch control amplitude is increased to compensate for shed wake attenuation

when third harmonic pitch amplitude was increased to the impractical level of .08 radians (4.6 degrees) the tip phasor had still not overtaken the mid-span phasor; 35% of the zero control first mode bending excitation was still present. This calls attention to an important fact: if the tip phasor moves twice as fast as the mid-span phasor when wake changes are not considered, but nevertheless overtakes the mid-span phasor extremely slowly when these changes are included, the conclusion is that wake attenuation effects are strongest in the outboard region. The large returning wake effects predicted at the beginning of the chapter are observed here; if there were no returning wake, Theodorsen's results would indicate greater lift attenuation in the mid-span and inboard regions where the reduced frequency is highest. This suggests the interesting possibility that harmonic airload distributions are inherently resistant to change. When harmonic blade pitch is employed to change the natural spanwise distribution of harmonic lift, the wake becomes rearranged to counteract the effect very strongly. Wake modification effects accompanying harmonic pitch control are sufficiently complex to require very detailed investigation; it is necessary to examine carefully the separate contributions to harmonic downwash of the trailing and shed vortices from each blade, and to devise a clear explanation of the essential physical effects involved. Time did not permit such detailed investigation before the time of this writing. The present study, however, will be continued

in exactly this direction.

One important possibility will be considered first when the present study is continued. The computer program was originally written with the trailing vortex sheet behind the blades approximated by two discrete trailing vortices, one at the blade tip and one at mid-span. This is an excellent approximation in the ordinary case, when it is not too inaccurate to assume blade lift constant over the outer half span. However, when harmonic pitch control is applied in such a way as to influence the relation between tip and mid-span lift, the validity of the approximation is questionable. Several more trailing vortices will therefore be added to the framework of the computer analysis to account for critical differences in lift loading along the span.

CHAPTER 5

CONCLUSIONS

The conclusions of this study apply in particular to vibration control for helicopters with three-bladed fully hinged rotors, in moderate speed flight conditions where third harmonic normal hub shear is the major cause of vibration. The first conclusion is that large third harmonic normal hub shear is primarily due to first mode bending motion of the blades. Third harmonic normal hub shear is initially caused by third harmonic airloads, but the hub shear due to the integral of these airloads along the blade is almost exactly cancelled by the relief due to flapping acceleration. In fact, if a third harmonic lift distribution centered at 67% span is applied to a blade with constant spanwise mass distribution, the d'Alembert force on the hub due to flapping acceleration precisely cancels the force on the hub due to lift. The cancellation in most actual cases, if not complete, is very nearly complete. Large hub shear still occurs, however, because the third harmonic lift distribution excites first mode bending motion. This motion tends to be large because of the low damping ratio and near-resonance of the mode with the third harmonic exciting frequency. The corresponding blade acceleration in the first bending mode causes significant d'Alembert forces

on the hub (normal hub shear).

The second conclusion is that third harmonic first mode bending excitation is largely a result of the phase difference between third harmonic lift at the "tip" (say, 90% span) and the "mid-span" (say, 60% span) of the blade, rather than a result of a magnitude difference between the third harmonic lifts at these locations. If third harmonic lift at 90% span and 60% span were in phase and had amplitudes in the ratio three to two, there would be very little third harmonic first mode bending. (Such a 90% – 60% span comparison indicates a radial lift distribution corresponding at least approximately to the flapping mode; this mode is orthogonal to first mode bending, so the bending would not be excited.) Third harmonic lift at 90% and 60% span are not in phase, however, and this is an important cause of third harmonic first mode bending. The phase difference is uniquely related to the origin of the third harmonic component of downwash. Downwash varies most basically over the rotor disk from small at the front to large at the rear. This is a first harmonic variation. The changes from small to large and large to small are abrupt, however. They occur as the blade passes over the strong tip vortex from the preceding blade, in the vicinity of 90° and 270° in azimuth. This makes the azimuthal variation of downwash more like a square wave than a sinusoid. The third harmonic component of downwash

accounts for the sharp rise and fall from front to rear and rear to front of the disk; adding a negative third harmonic to a first harmonic makes it look more like a square wave. The phase difference between tip and mid-span third harmonics occurs because the abrupt rise and fall of downwash near 90° and 270° is not simultaneous at all spanwise stations. The trailing vortex from the preceding blade "sweeps" under the blade from tip to root, making the sharp change in downwash occur earlier at the tip than at the mid-span.

The third conclusion is that blade pitch control could only be successful in reducing third harmonic normal hub shear by changing the third harmonic spanwise lift distribution, making it approach a simple distribution which does not excite first mode bending: lift proportional to radius. The control phase and magnitude which cause such a change in spanwise lift distribution may be deduced from a simple approximation. In this approximation, third harmonic outboard lift is replaced by a representative "tip" phasor and third harmonic center-span lift is replaced by a representative "mid-span" phasor. The two phasors may be drawn from the same point and the difference between them (the phasor connecting their heads) taken as an indication of first mode bending excitation. When they are aligned and the "tip" phasor is 1.5 times as long as the "mid-span" phasor, there will be little bending excitation, for the reason mentioned above. Application of pitch control

changes the magnitude and direction of both phasors. Because pitch variations occur with the same phase all along the blade, pitch control moves the heads of both phasors in the same direction; the "tip" phasor moves about twice as fast as the "mid-span" phasor because of the higher dynamic pressure in the outboard region. If the wake modifications and downwash changes due to pitch control are neglected, it is not difficult to utilize the representative phasor approximation in conjunction with computer calculations to find the third harmonic pitch control which completely eliminates third harmonic first mode bending and thereby greatly reduces third harmonic normal hub shear. Pitch adjustment to this optimal value may be considered to consist of several steps. First, pitch control is applied with the phase and magnitude which make the tip phasor change from its original value to the original value of the mid-span phasor. Then the control amplitude is doubled. The net result is to make the new tip and mid-span phasors identical. (This may be observed in the simple phasor diagram of Figure 9 in Chapter 4.) Next, additional pitch control is applied in the same direction as the new phasors, with a magnitude which doubles the length of the new mid-span phasor. Twice as much change occurs in the length of the tip phasor, and the final result is that the tip phasor is 1.5 times as long as the mid-span phasor and aligned with it. Thus first mode bending is practically eliminated. The accuracy of this reasoning was verified in Chapter 4.

The fourth conclusion is that outboard blade pitch control by trailing edge flap would be far more efficient than full-span pitch control in accomplishing the necessary reduction in first mode bending excitation. The problem is to control the relationship between tip and mid-span lift; this is more easily accomplished when one can be changed while the other remains approximately constant. It may be observed in Figure 9 that the same reduction in first mode bending excitation as obtained by full-span pitch control could be obtained by outboard flaps with one-quarter the change in tip lift and no change at all in mid-span lift. As an incidental benefit, it would hardly be necessary to increase flapping at all as a consequence of eliminating bending excitation. Furthermore, partial span pitch control by trailing edge flaps has the additional advantage of not causing large increases in pitch link forces transmitted to the fuselage.

The fifth conclusion is that returning shed wake attenuates the effect of full-span third harmonic pitch on third harmonic first mode bending excitation by a factor as high as four. In the present study an attempt was made to eliminate first mode bending by increasing pitch control amplitude farther and farther in the optimum direction on the phasor diagram. Bending decreased continually, but reached only 37% of its original value when third harmonic pitch amplitude had increased to over four and a half

degrees. (It had been possible to eliminate first mode bending entirely with less than two degrees control amplitude when wake effects were neglected.) If no returning wake were present, Theodorsen's results indicate that pitch control would be less effective at mid-span than at the tip, because the reduced frequency is higher at mid-span than at the tip. Yet, in terms of the simple representative phasor approximation, failure to eliminate first mode bending indicates failure of the tip phasor to overtake the mid-span phasor and attain 1.5 times its magnitude. Thus the returning wake is responsible for a greater attenuation of pitch control effects at the tip than at mid-span. Understanding of the exact manner in which the attenuation occurs will require detailed inspection of the individual contributions of the trailing and shed vorticity from each blade, aimed at isolating the most important physical effects. Such an understanding is the first goal for a continuation of this study.

The remaining conclusions of the present study concern the automatic inflight adjustment of harmonic pitch control to minimize cockpit vibration. These conclusions are, in fact, only broad implications of the calculated results of this investigation, which apply to only one flight condition; they will require considerable substantiation by further study. Nevertheless, they establish several important facts to consider

in such further study. First, it is significant that third harmonic pitch control effects on first mode bending excitation are so well described by the representative tip and mid-span lift phasor approximation. This approximation makes it almost certain that the variation of third harmonic normal hub shear with third harmonic pitch control amplitude and phase is completely smooth, with a unique minimum. The minimum could easily be found by trial and error during flight, with a small on-board hybrid computer. Such a computer could make small changes alternately in amplitude and phase of third harmonic pitch on the basis of small test changes to either side of the present value. This would be an approximate steepest-descent search method. Since it is a search for a minimum of third harmonic normal hub shear, it would require measurement of normal acceleration at the hub as an index of this hub shear. An important ultimate goal, however, is minimization of cockpit vibration; this will certainly require simultaneous consideration of inplane hub shear.

Another conclusion based on the calculated results is that third harmonic pitch control, adjusted to minimize third harmonic normal hub shear, will typically affect third harmonic inplane hub shear by less than ten per cent. The third harmonic inplane hub shear depends primarily on second harmonic effects in the rotating system, and these effects are not changed sufficiently by third harmonic pitch to make

large changes in the inplane hub shear. The amount of coupling between third harmonic lift and motion effects on normal hub shear and control of inplane hub shear by second and fourth harmonic pitch remains to be investigated. The control of normal and inplane hub shear can probably be accomplished independently, to a large extent. It would be possible to decrease cockpit vibration significantly by first adjusting third harmonic pitch to minimize measured normal hub vibration and then adjusting second harmonic pitch to minimize measured inplane hub vibration. (Second harmonic inplane effects are at least ten times as large as fourth harmonics, so fourth harmonic control will not be very useful). On the other hand, better results might be obtained at the cockpit by adjusting second and third harmonic pitch simultaneously to minimize normal vibration measured there. In this case, the minimization might be accomplished by making the normal and inplane hub shears work against each other. Such beneficial opposition probably would not occur if the two shear components were minimized separately with no regard for net effect at the cockpit. But if cockpit vibration alone were minimized, the routine trial-and-error search might result in very large values of third harmonic normal and inplane hub shear, working against each other in their effects on cockpit vibration. It would clearly be better to minimize a weighted sum of third harmonic normal acceleration at the hub, third harmonic inplane acceleration at the hub, and third harmonic normal acceleration at the cockpit.

This, however, would involve problems with local minima, making steepest-descent type minimization impossible. No minimization scheme very different from steepest-descent is practical, though, because only small test changes in control input away from the present value can be made during flight. That is, only "smooth" control input adjustment would be satisfactory. Any minimization scheme must be based only on "local" criteria: whether or not a small change of input makes the variable of interest increase or decrease. The most practical approach appears to be the one mentioned first: minimizing normal and inplane third harmonic hub acceleration alternately, over and over again during flight. The repetition would allow for change of flight condition and, during steady flight, it would effectively be an iteration to eliminate minimization errors due to interaction of the two adjustments. This would, unfortunately, eliminate the possibility of establishing beneficial opposition of the two hub shear components in their effects on cockpit vibration, except by coincidence.

A further conclusion is that, because of the strong dependence of hub shear on wake effects, no extremely rapid change of adjustment will be possible after a change in flight condition. Search for a minimum depends on testing the effect of small changes in control input. The effect of a change cannot be determined accurately until a new wake pattern is established. This would probably require one or two rotor revolutions,

or about one quarter of a second. Assuming, optimistically, that only twelve test changes were necessary to find the new minimum, three seconds would be the time required for the adjustment. This is undesirable in the particular case of entering a landing flare, when vibration becomes much larger very quickly, and rapid pitch control adjustment would be extremely beneficial.

Finally, since wake attenuation effects are so strong that large control amplitudes will be required, but structural factors will establish limitations on control amplitude, it is possible that any automatic adjustment computer would continually run up against the amplitude "stops". That is, an automatic vibration minimization system might always call for the preset maximum amplitude pitch control. The search would then be, in fact, a search for the best phase at a fixed amplitude. If this is actually the case, and if the structurally established maximum amplitude is large enough to make pitch control practical, the complexity of adjustment would be halved.

It is clear that much remains to be learned about inflight adjustment of higher harmonic pitch control. Much of the necessary information will be obtained incidentally during investigation of pitch control effects. It may be expected that this will be a result of the continuation of this study.

APPENDIX A

DERIVATION OF THE EQUATIONS ADDED TO EXISTING COMPUTER PROGRAM[8] FOR THE SPECIFIC PURPOSES OF THIS INVESTIGATION

I. Equation for Inplane Blade Motion

1. General form from Reference 11, page 19, for completely flexible blade:

$$m(r)\ddot{y}(r,t) + [\overline{EI}(r)\, y''(r,t)]'' + r\Omega^2 m(r) y'(r,t)$$
$$-\Omega^2 y''(r,t)\int_r^R m(\xi)\xi d\xi - m(r)\Omega^2 y(r,t) = H'(r,t) \qquad \text{(A. 1)}$$

2. Assumptions:

(a) constant spanwise mass distribution

$$m(r) = m$$
$$\therefore \bar{I} = \frac{m(R-e)^3}{3}$$

(b) negligible chordwise bending

$$y = (r-e)\zeta$$
$$\therefore y' = \zeta$$
$$y'' = 0$$

3. General form A. 1 with above assumptions, multiplied on both sides by (r-e):

$$m(r-e)^2\ddot{\zeta} + me(r-e)\Omega^2\zeta = (r-e)H'(r,t) \tag{A. 2}$$

4. Form A. 2 integrated over r from e to R:

$$\bar{I}\ddot{\zeta} + \frac{3}{2}\bar{I}\frac{e}{R-e}\Omega^2\zeta = \int_e^R (r-e)dH(t) \tag{A. 3}$$

5. Definition of symbol $\bar{\omega}$ for blade natural frequency in lagging:

$$\bar{\omega} = \left(\frac{3}{2}\frac{e}{R-e}\right)^{1/2}\Omega \tag{A. 4}$$

6. Form A. 3 using symbol $\bar{\omega}$:

$$\bar{I}\ddot{\zeta} + \bar{I}\bar{\omega}^2\zeta = \int_e^R (r-e)\,dH(t) \tag{A. 5}$$

7. Inplane driving forces[11]:

(a) coriolis force due to flapping and bending velocities:

$$dH_{cor_1} = -2m\Omega\dot{z}\frac{dz}{dr}dr \tag{A. 6}$$

(b) coriolis force due to lagging velocity:

$$dH_{cor_2} = -2m\Omega\dot{y}\frac{dy}{dr}dr \tag{A. 7}$$

(c) aerodynamic force due to induced and parasite drag:

$$dH_{aero} = cq(r)[c_\ell \sin\delta(r) + c_d(r)]\, dr \qquad \text{(A. 8)}$$

8. Additional assumption:

blade vertical motions include only flapping and first mode blade bending

$$z(r,t) = r\beta(t) + R\phi(r)\xi(t)$$

$$\therefore \quad \dot{z} = r\dot{\beta} + R\phi\dot{\xi}$$

$$\frac{dz}{dr} = \beta + \xi R \frac{d\phi}{dr}$$

9. Inplane driving forces, including the above assumption and the previous ones:

(a) $$dH_{cor_1} = -2m(\Omega R)^2 \left(\eta \frac{d\beta}{d\psi} + \phi \frac{d\xi}{d\psi}\right)\left(\beta + \xi \frac{d\phi}{d\eta}\right) d\eta \qquad \text{(A. 9)}$$

(b) $$dH_{cor_2} = -2m(\Omega R)^2\, \zeta \frac{d\zeta}{d\psi} (\eta - \eta_H)\, d\eta \qquad \text{(A. 10)}$$

(c) $$dH_{aero} = L_N \frac{I\Omega^2}{2R} \frac{U_p^2 + U_T^2}{(\Omega R)^2} \frac{c_\ell \sin\delta + c_d}{2\pi} d\eta \qquad \text{(A. 11)}$$

10. Form A. 5 of the inplane motion equation including expressions A. 9, A. 10, A. 11 for the driving forces and rearranged for Runge-Kutta solution:

$$\frac{d^2\zeta}{d\psi^2} = -\left(\frac{\bar{\omega}}{\Omega}\right)^2\zeta - \frac{6}{(1-\eta_H)^2}\int_{\eta_H}^{1}(\eta-\eta_H)\left(\eta\frac{d\beta}{d\psi} + \phi\frac{d\xi}{d\psi}\right)\left(\beta + \xi\frac{d\phi}{d\eta}\right)d\eta$$

$$-2\zeta\frac{d\zeta}{d\psi} + \frac{L_N}{2(\Omega R)^2}\frac{1}{(1-\eta_H)^3}\int_{\eta_H}^{1}(\eta-\eta_H)(U_p^2 + U_T^2)\frac{c_\ell \sin\delta + c_d}{2\pi}d\eta \qquad \text{(A. 12)}$$

A. 12 is the form of the inplane motion equation solved in the program by the Runge-Kutta method.

II. Equation for Normal Hub Shear

1. Form obtained from elementary physical considerations:

$$S = \int_0^R dL - \int_0^R \ddot{z}\,dm \qquad \text{(per blade)} \qquad \text{(A. 13)}$$

2. Expression for z obtained using assumptions stated in above derivation of lagging equation:

$$\ddot{z} = r\ddot{\beta} + R\phi\ddot{\xi} \qquad \text{(A. 14)}$$

3. Definition of ϕ, the assumed shape of the first bending mode:

$$\phi(\eta) = \eta - \frac{\pi}{3}\sin \pi\eta \qquad \text{(A. 15)}$$

4. Form A. 13, including expressions A. 14 and A. 15, with the acceleration integral evaluated:

$$S = \int_{o}^{R} dL - \frac{m(\Omega R)^2}{2} \left(\frac{d^2\beta}{d\psi^2} - \frac{1}{3} \frac{d^2\xi}{d\psi^2}\right) \qquad \text{(A. 16)}$$

A. 16 is the form used in the program to calculate normal hub shear per blade from tabulated values of lift and flapping and bending acceleration as a function of azimuth. Flapping and bending acceleration are found directly in the course of Runge-Kutta solution of the equations for flapping and bending displacement.

III. Equation for Inplane Hub Shear

1. Form obtained from elementary physical considerations:

$$\bar{S} = \int_{e}^{R} dH - \int_{o}^{R} \ddot{y}\, dm + \Omega^2 \int_{o}^{R} y dm \qquad \text{(per blade)} \qquad \text{(A. 17)}$$

2. Expression for y assuming negligible chordwise bending:

$$y = (r - e)\zeta \qquad \text{(A. 18)}$$

3. Form A. 17 with substitution of A. 18 and change to derivative with respect to ψ:

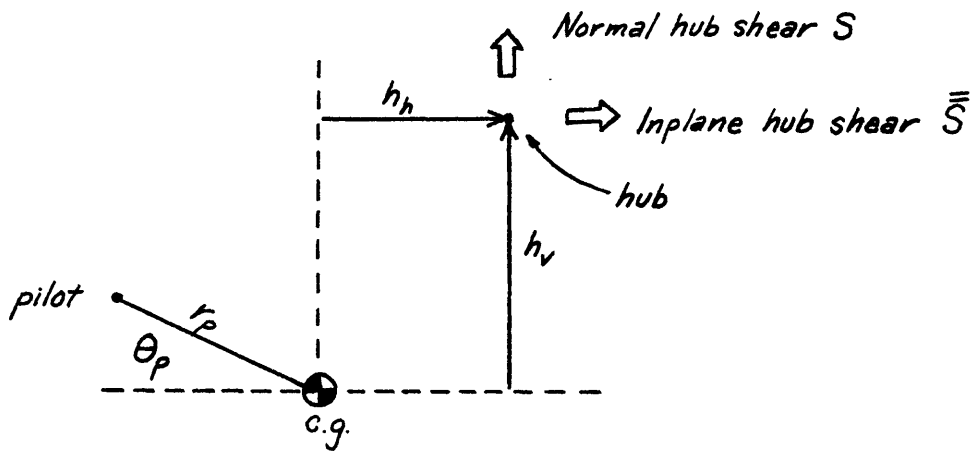
$$\bar{S} = \int_{e}^{R} dH + \frac{m\Omega^2}{2} (R - e)^2 \left(\zeta - \frac{d^2\zeta}{d\psi^2}\right) \qquad \text{(A. 19)}$$

A. 19, with expressions A. 9, A. 10, and A. 11 for the three H forces, is the form used in the program to calculate chordwise hub shear per

blade from tabulated values of lift, flapping, bending, and lagging as a function of azimuth. "Inplane hub shear" refers to the component of chordwise hub shear in the fore-and-aft direction, obtained by multiplying $\bar{S}$ by $\sin\psi$.

IV. Fuselage rigid body dynamics

1. Definition of fuselage dimensions



2. Pilot's third harmonic normal acceleration:

$$g's = S\left(\frac{1}{GW} - \frac{h_h\, r_p}{g\, I_{fus}} \cos\Theta_p\right) + \bar{S}\left(\frac{h_h\, r_p}{g\, I_{fus}} \cos\Theta_p\right) \qquad \text{(A. 20)}$$

3. Definition of parameter combinations:

$$F_v = \frac{1}{GW} - \frac{h_h \times r_p}{g \times I_{fus}} \cos \Theta_p$$

$$F_h = \frac{h_v \times r_p}{g \times I_{fus}} \cos \Theta_p$$

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