EXPERIMENTAL INVESTIGATION OF VARIABLES AFFECTING REGENERATIVE PUMP PERFORMANCE

BY

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Secretary of the Faculty  
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Dear Dr. Millard:

In accordance with the requirements for the degree of Master of Science in Mechanical Engineering, I herewith submit this thesis entitled "Experimental Investigation of Variables Affecting Regenerative Pump Performance."

Yours very truly,

Miguel Angel Santalo  

MAS/mbs
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LIST OF SYMBOLS

a  cross sectional area of impeller vane.
A  cross sectional area of flow passage, area in general.
b, c, d,  pump dimensions, Figure 3.
f, h  pump dimensions, Figure 3.
D  impeller diameter.
g  acceleration of gravity.
H  head and head loss.
k  head loss coefficient.
K  constant.
p  pressure.
Q  flow.
r  radius, Figure 3.
R  radius to centre of gravity of impeller vane.
T  torque.
W  work.
u  internal energy.
U  velocity of impeller.
V  absolute velocity.
Z  number of blades.

SUBSCRIPTS

c  circulatory flow.
**SUBSCRIPTS (Continued)**

- **G**: center of gravity of flow passage.
- **t**: tangential or through flow
- **l**: leakage.
- **i**: impeller.
- **o**: pump flow, passage.
- **s**: solid body rotation.

**GREEK SYMBOLS**

- **α**: ratio \( V_t/U \) at point 1, Figure 3.
- **η**: efficiency.
- **θ**: angle measured from pump inlet, in radians.
- **ρ**: density, mass per unit volume
- **τ**: ratio \( V_t/U \) at point 2, Figure 3.
- **φ**: dimensionless flow coefficient, \( Q/ωD^3 \).
- **ψ**: dimensionless head and head loss coefficient, \( gH/ω^2D^2 \).
- **ψ**: dimensionless power, \( W/ ω^3D^5 \).
- **ω**: angular speed in radians per second.
The term "regenerative pump" will be used in this thesis to denote a hydrodynamic unit often referred to in the literature as periphery pump, turbulence pump, friction pump, turbine pump etc. The main feature of this unit is that it develops in a single rotor very high heads at very low flow rates. Self-priming is also an important operating characteristic. Although its efficiency is not impressive it is very good compared to other hydrodynamic pumps of comparable specific speeds and has found many applications in industry for a number of years. The characteristics of the regenerative pump have made it particularly attractive for lubrication, control, filtering and booster systems.

A research program was started at M.I.T. in September, 1952 to analyse the operation of the regenerative pump and to determine the main factors influencing the performance of this unit. This thesis reports only one phase of this investigation. Other phases have been reported previously and will be referred to in the introduction.

The author hereby wishes to acknowledge the support received from Worthington Corporation for this project, to express his gratitude to Professor William A. Wilson for his help and valuable advice, and to thank Mr. John A. Oelrich for his cooperation in the experimental work and in discussing several phases of this thesis.
ABSTRACT

The regenerative pump considered has a single impeller with a large number of radial blades at its periphery. The average flow inside the pump follows approximately helical streamlines. John A. Oelrich presented in June, 1953 a thesis developing a theoretical analysis based on a simplified model reflecting this flow pattern.

The theoretical analysis introduced empirical parameters and loss coefficients. In this thesis, these parameters are rationalized on the basis of the flow pattern described and previous work done on losses in other hydrodynamic units. To simplify the analysis the internal performance is considered separately from the overall performance, the latter including the effects and losses due to inlet and exhaust ports.

An experimental unit with 19" impeller diameter was designed and built. The pump performance characteristics were then determined for six different flow passage geometries. An empirical correlation of the internal flow parameters as functions of the two major flow passage dimensions is presented.

A better understanding of the influence of these variables has been achieved permitting the qualitative explanation of most of the phenomena so far encountered and the prediction of possible means of improving the regenerative pump performance.
1. INTRODUCTION

The regenerative pump studied in this thesis consists of an impeller with a large number of radial blades in its periphery, as shown schematically in Figure 1. Flow enters the pump at point "1" and circulates repeatedly between the blades and the pump flow passage. When the fluid reaches point "11" it is not permitted to proceed back to the inlet. At that point a "stripper! reduces the flow passage area nearly to zero. Fluid is then discharged at point 2.

To assist the reader to understand the operation of the pump Figure 2 reproduces a typical experimental curve of pressure variation of the fluid as it circulates through the pump. This curve suggests three different regions in the pump operation:

a. Inlet region, from 1 to 6.

b. Linear region, from 6 to 10. Note that here the pressure rise per unit of angle, that is, the slope $dp/d\theta$, is essentially constant.

c. Exhaust region, from 10 to 2.

Working in the linear region, Lazo and Hopkins \(^{(1)}\) and Lutz \(^{(2)}\) conducted experiments with a small thread probe to determine the direction of the velocity at different points in a section of the pump flow passage. In spite of the turbulence of the flow they were able to corroborate that in this region the fluid follows helical streamlines

* Numbers in parenthesis refer to references listed in the Bibliography.
as shown schematically in Figure 1a. One of these streamlines is projected on the cross-sectional drawing, Figure 1b. This flow pattern is taken herein as being the flow pattern existing inside the theoretical model of the pump.

Senoo (3) and Iverson (4) in their papers analysed the regenerative pump on the basis of turbulent friction. Their approach, however, fails to explain the physical nature of the flow as observed by Lazo, Hopkins, and Lutz. The analysis presented by Oelrich (5) and described in section 2.2 is believed to represent a closer picture of the behaviour of the regenerative pump, and is taken as a starting point for this report.

This thesis will be concerned with a study of the nature of the parameters and losses introduced in Oelrich's analysis. In particular the dependence of these losses and parameters on the geometry of the flow passage is considered. An experimental unit with a 19 inch diameter impeller was built. Test results obtained on this unit are analysed on the basis of the assumed nature of the losses. This analysis leads to some conclusions with respect to the basic factors limiting the regenerative pump performance.

* It should be remarked that Senoo's experiments were made on flow passage geometries rather different to the ones considered in this thesis. His dimensions d (Figure 3) were very small and in most tests his c/d ratio was much higher than ours.
2. ANALYSIS OF OPERATION

2.1 Similitude laws and dimensionless parameters

The regenerative pump is a fluid dynamic unit and follows the similitude laws of fluid machinery. This was confirmed by Lazo and Hopkins\(^{(1)}\) who showed that for seven different speeds* and with air as the working fluid the performance characteristics on dimensionless basis were the same as the ones reported by the manufacturer for water. Their curves also showed that the Reynolds number effect was very small; it is neglected in this thesis.

For a family of geometrically similar pumps the similitude laws say that the functional relation

\[
\frac{gH}{N^2D^2} = f\left(\frac{Q}{ND^3}\right)
\]

is independent of speed, diameter and fluid pumped.

In later pages the terms flow, \(Q\), and head, \(H\), are used only in the intermediate steps of the derivations and the results will be reported in terms of dimensionless coefficients. Thus to indicate flow the term,

\[
\phi = \frac{Q}{WD^3}
\]

is used, and similarly for head and head loss:

\[
\psi = \frac{\rho \cdot \Delta p}{\omega^2D^2} = \frac{gH}{\omega^2D^2}
\]

* 1200 to 4800 RPM on TH-7 pump, (Sta-Rite).
2.2 Theoretical analysis of performance in the linear region.

Oelrich\(^{(5)}\) developed a theoretical analysis of the performance of the pump in the linear region. He describes the helical motion of the fluid inside the pump by means of two perpendicular components of the velocity at each point: the tangential component \(V_t\) and the component \(V_c\) in the plane perpendicular to \(V_t\). Two terms are then introduced:

a. Tangential or through flow, defined as \(Q = \int V_t dA\) where \(A\) is the cross sectional area of the flow passage.

b. Circulatory or meridional flow \(Q_c\) which is that associated with the component \(V_c\) of the velocity, in cubic feet per second per radian of arc length.

To develop a theoretical analysis of this complicated three-dimensional fluid motion, Oelrich used a simplified model based on the following assumptions:

1. Flow may be considered steady if time averages are used for pressures and velocities.

2. Fluid is incompressible.

3. There is no internal leakage.

4. All processes within the pump are adiabatic.

5. Characteristic flow is one dimensional in each major direction (radial, tangential and axial).

6. Tangential pressure gradient is independent of radius.
To apply the assumption of one dimensionality of the flow in each direction, he further assumed that all the circulatory flow leaves the impeller at the tip of the blade and enters at a radius $r_1$. See Figure 3.

Oelrich's equations have been rederived (see Appendix) in simplified form. The application of the angular momentum relations to the control volume of Figure 4a, lead to an expression for the tangential pressure rise:

$$
\frac{1}{9} \frac{\partial p}{\partial \theta} = \frac{Q_c \omega (\sigma r_2^2 - \alpha r_1^2)}{r_G A} - gH_t
$$

(1)

where $\alpha$, $\sigma$, and $Q_c$ are defined by:

$$
\alpha \equiv \frac{V_{t1}}{U_1} \quad \quad Q_c \equiv \frac{V_c}{r_2}.b \quad \text{in cubic feet per radian arc length.}
$$

$$
\sigma \equiv \frac{V_{t2}}{U_2} \quad \quad H_t = \text{tangential flow losses.}
$$

Dividing by $\omega^2D^2$ to non-dimensionalize:

$$
\frac{d \psi}{d \Theta} = \frac{1}{4} \frac{\sigma - \left( \frac{r_2}{r_1} \right)^2 \alpha}{r_G A/D^3} \frac{\phi_c}{\psi_t} - \psi
$$

(1a)

Angular momentum, or Euler's pump and turbine equation, applied to the control volume of Figure 4b leads to the work done by the impeller on the circulatory flow:
The last term represents work input to raise the pressure of the fluid which is ultimately trapped in the blades and carried back from the exhaust to inlet (11 to 4 in Figure 1). This work is recoverable in the form of turbine work through the stripper.

Dividing by \( \omega^3 D^5 \) to non-dimensionalize, the net work input is found:

\[
\frac{dW_{IN}}{d\theta} = \frac{1}{4} \left( \phi_c - \frac{r_1^2}{r_2^2} \right) \frac{\omega}{\sqrt{r_2^2 - r_1^2}} \alpha \quad (2a)
\]

The steady flow energy equation is applied to the control volumes of Figure 4. Combining the results with equations (1) and (2) leads to an expression for the total losses in the circulatory flow:

\[
gH_c = \omega^2 \left( \phi - \frac{r_1^2}{r_2^2} \right) \left( 1 - \frac{Q}{rG^A\omega} \right) \quad (3)
\]

Non-dimensionalizing:

\[
\psi_c = \frac{1}{4} \left( \phi_c - \frac{r_1^2}{r_2^2} \right) \left( 1 - \frac{\phi}{rG^A} \right) \frac{r_1^2}{r_2^2} \quad (3a)
\]

Variation of \( V_t \) in the flow passage may be derived if the mean streamline is further assumed to be as shown by the dotted line 1-2-3-4-5-1 in Figure 3. Remembering that the flow

\[
\phi = \frac{Q}{\omega D^3} = \frac{1}{\omega D^3} \int_A V_t \cdot dA
\]
repeated application of the dynamic equations will lead to:

\[ \phi = \frac{A}{4D^2} \left( K_1 \mathcal{C} + \frac{r_1^2}{r_2^2} K_2 \alpha \right) \]  

(4)

where \( K_1 \) and \( K_2 \) are dimensionless coefficients depending only on the geometry of the flow passage:

\[ K_1 = K_3 + 2 \frac{A_t}{A} - \frac{C}{r_3} \frac{A_2}{A} - \frac{C}{D} \frac{A_1}{A} \]

\[ K_2 = -K_3 + \frac{D}{r_1} \frac{A_s}{A} - \frac{A_2}{A} \frac{4}{6r_1^2} \left( 5 - 2 \frac{r_3}{r_1} \right) \]

\[ K_3 = \frac{r_2}{r_G} \left[ \frac{A_s^2}{A^2} - \frac{A_t^2}{A^2} + \frac{C}{r_3} \frac{A_1}{A} - \frac{A_2}{A} \frac{A_3}{A} + \frac{1}{3r_1^2} \left( A_s - f_d \frac{r_3}{r_1} \right) \right] \]

Four equations have been derived for the hypothetical model of the regenerative pump. In these equations the independent variable may be taken to be the flow \( \phi \) or the parameter \( \alpha \) since both are related by Equation 4 once the factor \( \mathcal{C} \) is known. For simplicity \( \alpha \) is used as the independent variable remembering that the desired final result is the pressure rise \( \frac{d\psi}{d\phi} \) as a function of the flow \( \phi \).

Equation (1a) gives the pressure rise in terms of the circulatory flow \( \phi_c \), the variable \( \alpha \) and the tangential flow losses \( \psi_t \). It is expected that the circulatory flow losses \( \psi_c \) of equation (3a) will depend on \( \phi_c \) and that if they are given the proper form they will permit the evaluation of the circulatory flow itself.
In the next section the losses and parameters introduced in these equations will be discussed in more detail.

2.3. Evaluation of Losses and parameters in the linear region.

2.3.1. Tangential flow losses.

It is believed that most of the losses occurring in the flow passage may be accounted for in terms of wall friction and elbow or turning losses. From previous experience these losses would be proportional to the square of the absolute velocity, namely proportional to \((V_c^2 + V_t^2)\). The velocity varies both in magnitude and direction from point 2 to point 1 (Figure 3) so that an integration in terms of a single constant becomes quite involved. When, as an approximation, average velocities are introduced with losses of the form \(k_t \phi^2\) and \(k_c \phi^2\), the constants \(k_t\) and \(k_c\) would in general be different. The treatment of \(k_t\) and \(k_c\) as independent of each other may be justified if the losses in the tangential flow are considered small compared with the losses due to the circulatory motion of the flow. The tangential pressure drop associated with the \(k_t \phi^2\) losses will be approximated by a pipe loss of the form \(4fL \frac{V^2}{D_h 2g}\). The term \(\psi_t\) in equation (1a) will then be expressed as \(k_t \phi^2\) referring to losses per radian of arc in the tangential direction. To determine the order of magnitude of these losses the following values will be used:

\[ L = r_G \text{ (or } r_G \theta \text{ where } \theta = 1 \text{ rad.}) \]

\[ D_h = \frac{4A}{P}, \text{ hydraulic diameter} \]
$P = \text{wetted perimeter}$

$s = h + 2d + c + b$

$V = \text{average velocity of the tangential flow} = \frac{Q}{A}$

$4f = \text{friction coefficient} \approx 0.01$

With these approximations the tangential losses in dimensionless form are taken to be:

$$\psi_t \approx k_t \phi^2 \approx \frac{1}{\omega^2 D^2} \left(4f \frac{L}{D} \frac{V^2}{2} \right) \approx 4f \frac{r_G P D^4}{8A^3} \left(\frac{Q}{\omega D^3}\right)^2$$

or

$$k_t \approx 0.01 \frac{r_G P D^4}{8A^3} \quad (5)$$

For the pumps tested this expression gives values of the loss $\psi_t$ which are very small and are neglected for simplicity in the computations of $\frac{d\psi}{d\theta}$.

2.3.2. Circulatory flow losses.

The circulatory flow losses, $\psi_c$ in equation (3a), will be considered as made up of two terms:

a. Blade entrance loss. This loss will arise when the velocity relative to the rotor at point 1 (Figure 3) differs from purely axial and has a tangential component:

$$U_1 - V_{t1} = (1 - \alpha)U_1$$

These losses will be evaluated according to the form suggested by Spanhacke $^{(6)}$: 
\[ (1 - \alpha)^2 \frac{U_1^2}{2} \]

b. Other losses in the circulatory flow which may include friction losses, elbow type losses in the outside passage, turbulence losses due to separation inside the impeller and at the tip of the impeller. All these losses are considered to be proportional to \( V_c \). Taking the velocity at the tip of the impeller as characteristic of the flow in the pump passage we may write the losses as:

\[
  k_c \left( \frac{1}{\omega^2 D^2} \cdot \frac{Q_c^2}{r_2^2 b^2} \right) = k_c \frac{D^4}{r_2^2 b^2} \phi_c^2
\]

where \( k_c \) will be a function of the flow passage geometry. An analytical expression for \( k_c \) could not be derived due to the complexity of the losses it represents. An empirical evaluation of this loss coefficient will be considered in section 4.

Summarizing we may then write for the total circulatory flow losses:

\[
  \psi_c = \frac{1}{8} (1 - \alpha)^2 \frac{r_1^2}{r_2^2} + k_c \frac{D^4}{r_2^2 b^2} \phi_c^2 \tag{6}
\]

2.3.3. Departure from perfect blade guidance (factor \( \sigma \))

The factor \( \sigma \) was introduced to account for the lack of perfect guidance to be expected from a finite number of blades. It is defined as the ratio of the average absolute tangential velocity of the fluid
leaving the impeller to the tip speed of the impeller:

\[ \mathcal{S} = \frac{V_{t2}}{U_2} \]

Busemann \(^{(7)}\) evaluated the factor \( \mathcal{S} \) considering the potential two-dimensional flow between the blades of a centrifugal impeller. For the case of purely radial blades, when \( \frac{r_1}{r_2} \to 0 \) and the number of blades \( Z \) is large his results can be approximated by the expressions:

\[ \mathcal{S} \approx 1 - \frac{2}{Z} \quad 40 > Z > 8 \]

\[ \mathcal{S} \approx 1 - \frac{2.5}{Z} \quad Z > 40. \]

He also showed that \( \mathcal{S} \) becomes a strong function of \( \frac{r_1}{r_2} \) as this ratio approaches unity, Figure (5a), becoming zero at \( \frac{r_1}{r_2} = 1 \). In this figure the curve for \( Z = 40 \) has been approximated based on the calculated values of:

\[ \mathcal{S} \approx 0.944 \text{ for } \frac{r_1}{r_2} = 0 \]

\[ \mathcal{S} \approx 0.938 \text{ for } \frac{r_1}{r_2} = 0.9 \]

In Figure (5b) are compared the results from Busemann with those suggested by Stodola \(^{(8)}\) and by Stanitz and Ellis \(^{(9)}\).

The flow between the blades of the regenerative pump differs from the case of radial blades considered by Buseman mainly
because: the tip clearance is finite, the inlet flow is not radial, irreversibilities are present. Busemann's result of $\frac{9}{Z} = 0.94$ for $Z = 40$ will only be taken as a limiting value.

2.3.4. Circulatory flow streamline parameter, $r_1$.

In the derivation of the dynamic equations for the simplified model, $r_1$ was chosen arbitrarily at an unknown distance $\frac{f}{2}$ from the inner radius of the flow passage. With all the flow leaving at the tip of the blade, $r_1$ was defined as the radius at which the assumed mean streamline of the circulatory flow enters the blade.

It is to be expected that $r_1$ is a function of the flow passage geometry. $r_1$ might also vary for a given geometry as the flow through the pump is changed. A further simplification is made in the analysis taking this radius to be a constant for a given geometry; consequently the results may be expected not to agree with experiment throughout the range of operation of the pump, from shut-off to maximum flow.

2.4. Effects due to inlet and discharge.

So far only the performance in the linear region has been considered. This region has been defined as that in which the pressure rise $\frac{d\psi}{d\theta}$ is constant and independent of $\theta$. Consequently, $\phi, \chi$ and all other parameters will remain constant from section to section.

Returning to Figure 2 it is noticed that these conditions may not be expected to prevail beyond and before the linear region. A brief
hypothesis of the phenomena occurring there will now be stated using Figure 2 as a reference.

2.4.1. Inlet region.

Fluid enters the impeller at a mean point located approximately at 4*. The flow from 1 to 4 might be simulated by a series of pipes and elbows. The pressure drop will be difficult to evaluate. It may be stated however, that this pressure drop will probably be of the form \( k \phi^2 \)**, where \( k \) depends on the specific geometry of the inlet.

2.4.2. Acceleration region.

At approximately point 4 the flow will enter what might be called the "working section" of the pump with a velocity direction dependent largely on the inlet port. Until the flow reaches the steady pattern mentioned for the linear region it will undergo an acceleration. During this acceleration the circulatory flow will increase.

Continuity or conservation of mass through the control volumes of Figure 4 requires that for a given section of width \( d\Theta \), the circulatory flow \( \phi, \alpha \) and consequently all the other parameters and loss coefficients have fixed values. The acceleration region can then be considered as that in which \( \phi \) varies continuously from a value determined by inlet geometry to the value given by equation 4 for the linear region.

* Notice that the boundaries of the mentioned regions of operation are not clearly defined.

** For simplicity \( k \) will also include exit losses.
It may be concluded that the duration of the accelerating period will depend both on the inlet geometry and on the operating flow. The latter effect may be observed from the curves in Figure 17.

An important conclusion that will be used later can be derived from the foregoing statements. Since \( \alpha \) and \( \phi_c \) may be thought of as having a definite value for each section \( d\Theta \) in the accelerating region, Equations (1a) and (2) will still be valid if the appropriate value for is used. If the term \( \psi_t \) is considered negligible according to the statements made in section 2.3, in the accelerating region, as well as in the linear region:

\[
\frac{d\psi}{dW} = \frac{D^3}{rGA} \tag{7}
\]

2.4.3. Deceleration region.

The pressure distributions seem to show the existence of a region (approximately 10 to 11) where a deceleration probably occurs and some of the kinetic energy of the circulatory flow is recovered as a pressure rise. Again this may be thought of as a region of varying \( \phi_c \) where relation (7) holds.

2.4.4. Exhaust region.

In this region (approximately 11 to 2) a reasoning similar to that given for the inlet region may be made. A loss proportional to \( \phi^2 \) may
then be expected which will be incorporated in the $k \phi^2$ term mentioned in paragraph 2.4.1.

2.4.5. Leakage.

Leakage plays an important role in the performance of the regenerative pump. Internal leakage $\phi_t$ from outlet to inlet through the clearance between the rotor and the stripper is believed to be the most important. It causes the flow $\phi_o$ in the internal passage of the pump to be larger than the flow $\phi$ measured at either inlet or outlet:

$$\phi = \phi_o - \phi_t$$

To evaluate the order of magnitude of $\phi_t$, this leakage flow will be assumed to be the superposition of two flows, Figure 6a:

a. Fluid dragged by the impeller at the blade speed

$$Q_{t1} = r_2 \omega (b.e) + \frac{r_o + r_2}{2} \omega (he)$$

b. Flow due to the pressure gradient $P_3 - P_{11}$ across the stripper. Flow past each blade may be thought of as flow past an orifice. The assumption of equal pressure drop past each blade would lead to:

$$Q_{t2} = (h + b) e.C \sqrt{2g \frac{P_3 - P_{11}}{Z_s \cdot \rho g}}$$

where $C \approx 0.85$, orifice coefficient

$Z_s$ = average number of blades in the stripper at one time.
\[ \frac{P_3 - P_{11}}{\rho g} = \Theta_P \frac{dH}{d\theta} \]

Adding the two flows and non-dimensionalizing:

\[ \phi_i = \frac{(h+b)e}{2D^2} \left[ 1 + 1.70 \sqrt{\frac{2.0\Theta_P}{Z_s}} \sqrt{\frac{d\psi}{d\theta}} \right] - \frac{he}{4D^2} \left( 1 - 2 \frac{r_o}{D} \right) \] \tag{9}

In this expression, \( \Theta_P \) may be defined as an "equivalent working section" of the pump, in radians. It will be such that \( \Theta_P \) times the pressure rise \( \frac{d\psi}{d\theta} \) in the linear region will give the actual pressure rise inside the pump including the acceleration and deceleration regions mentioned before.

Figure 6b shows the results obtained from Equation (9) using conservative values for the unit tested, i.e. \( \Theta_P = 5.42 \) radians, \( Z_s = 3 \), \( e = 0.020 \) inches.

2.5. Overall performance and efficiency.

The total head developed by the pump (from 1 to 2 in Figure 2) might be expressed as

\[ \psi = \Theta_P \frac{d\psi}{d\theta} - k\phi^2 \] \tag{10}

\( k\phi^2 \) was defined in paragraph 2.4.1 as the inlet and exit losses. \( \Theta_P \) was defined in paragraph 2.4.5 as the equivalent working section of the pump.
If mechanical losses are assumed negligible the total work input to the pump will be according to Equation 7,

\[ \mathcal{W} = \theta_p \frac{d\mathcal{W}}{d\theta} = \theta_p \frac{r_G A}{D^3} \frac{d\psi}{d\theta} \]

Combining with Equation (10):

\[ \mathcal{W} = \frac{r_G A}{D^3} (\psi + k \phi^2) \]  \hspace{1cm} (11)

A more rigorous analysis will show that \( k \phi^2 \) will also include the tangential flow losses inside the pump so that \( k \) may be expected to depend not only on the inlet construction but also on flow passage geometry.

The efficiency of a pump is generally defined as

\[ \eta = \frac{Q:\Delta P}{W_{in}} = \frac{\phi \cdot \psi}{\mathcal{W}} \]

Substituting (10) and (11)*,

\[ \eta = \frac{\phi}{r_G A/D^3} \left[ \frac{1}{1 + k \phi^2/\psi} \right] \]  \hspace{1cm} (12)

3. DESCRIPTION OF APPARATUS AND TEST PROCEDURE.

3.1. Preliminary design considerations.

Determinations of the flow characteristics inside the pump have been necessary to substantiate the hypotheses made in the theoretical analysis and to correlate this analysis with the actual pump performance.

* Note that \( \frac{r_G A}{D^3} \) is the dimensionless expression for the flow \( r_G A \omega \) at solid body rotation, i.e. when all the fluid has a tangential velocity \( r \omega \).
Previous experimental work at M.I.T. \((1)\)\(^{(2)}\) was carried out with a commercial unit, Sta-Rite TH-7 pump having an impeller diameter of 5.4 inches. The small dimensions of this pump made measurements in the flow passage extremely difficult and limited in nature and accuracy.

It was proposed to build a larger model that would facilitate these measurements and permit easy changes in some of the geometrical characteristics of the pump. In addition to special provision for changes in dimensions \(c\) and \(d\) of the pump flow passage, (see Figure No. 3) other features were incorporated in the design of this larger model. Some of these features are,

a. Flow passage large enough to permit the introduction of measuring probes.

b. Accessibility and visibility of the flow passage from inlet to discharge.

c. Removable rotor blades to permit some changes in the shape and the number of blades.

In the preliminary design it was decided to build a unit whose dimensions would be geometrically similar to those of the Sta-Rite TH-7\(^*\) pump. Making use of the laws of dynamic similarity maximum power consumption and head rise were estimated for several pump sizes.

Practical limitations in size determined a unit with an 18.9 inch impeller diameter (3.5 times larger than TH-7 pump) for operation with air.

---

\(^*\) Inlet ports, however, are radically different in design.
and to be driven by a 5 H. P. - D. C. dynamometer at speeds up to 2000 RPM.

3.2. Description of the experimental unit.

Referring to Figure 7, the test unit finally designed consisted of:

a. Aluminum impeller mounted on the dynamometer shaft.
   Forty radial blades are held into slots in the impeller by means of two brass retaining rings.

b. Aluminum back cover supporting the rest of the casing.
   This back cover was fastened to the base plate through a rigid L-shaped steel support.

c. Outer casing and front cover, both made from transparent plastic (Lucite).

d. Thrust bearing to prevent axial displacements of the impeller.

Both inlet and outlet ports were made radial as shown schematically in Figure 1. This design of the inlet port and the stripper was mostly influenced by the desire to make the flow passage easily accessible. Photographs of the inlet port and of the entire unit may be found in the Appendix.

The flow passage itself was designed large enough to accommodate six lucite fillers: three flat disks and three wide rings, Figure 8. With these removable fillers sixteen different geometries of the flow passage

---

* At 2000 RPM pumping water this unit would require about 3500 HP and develop some 2000 psi pressure rise at no flow!

** Description of inlet ports in Sta-Rite TH-7 pump may be found in references 1 and 5.
may be obtained, as listed in Table 1.

**Table 1**

**Possible Configurations of the Pump Flow Passage**

<table>
<thead>
<tr>
<th>Configuration No.</th>
<th>C (Inches)</th>
<th>d (Inches)</th>
<th>c/d</th>
<th>A (Square Inches)</th>
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<tr>
<td>1</td>
<td>1.130</td>
<td>1.700</td>
<td>0.665</td>
<td>8.00</td>
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<tr>
<td>2</td>
<td>0.875</td>
<td>1.700</td>
<td>0.515</td>
<td>7.38</td>
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<tr>
<td>3</td>
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<td>1.700</td>
<td>0.365</td>
<td>6.75</td>
</tr>
<tr>
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<td>1.700</td>
<td>0.215</td>
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<tr>
<td>5</td>
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<td>1.410</td>
<td>0.802</td>
<td>6.80</td>
</tr>
<tr>
<td>6</td>
<td>0.875</td>
<td>1.410</td>
<td>0.621</td>
<td>6.15</td>
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<td>7</td>
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<td>1.410</td>
<td>0.440</td>
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<td>1.410</td>
<td>0.259</td>
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<td>1.065</td>
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<td>1.130</td>
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<td>1.485</td>
<td>4.10</td>
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<tr>
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<td>0.760</td>
<td>1.150</td>
<td>3.68</td>
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<td>0.760</td>
<td>0.816</td>
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<td>16</td>
<td>0.365</td>
<td>0.760</td>
<td>0.480</td>
<td>2.89</td>
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</tbody>
</table>

* Configuration tested. Results are referred to these configuration numbers.
Preliminary tests on the pump showed that the Lucite front cover deflected more than had been predicted by calculations when subjected to the pressure in the flow passage. This flexibility of the Lucite was subsequently used to permit a better control of the clearance between the front head and the impeller and thus reduce the amount of internal leakage. A rigid structure mounted on the base plate with three bolts was used to press on the outside of the front cover. (Photograph No. 5 in Appendix)

3.3 Instrumentation and test procedure.

The instrumentation installed in the experimental unit consisted mainly of:

a. Venturi at inlet, for the measurement of flow.

b. Static pressure taps connected to a water manometer, to measure the pressure rise at inlet and exhaust and at nine points at the same radius and various circumferential positions in the internal flow passage. The location of these pressure taps is shown on Figures 1 and 2.

c. 5 H.P. direct-current cradled dynamometer equipped with a scale for the measurement of torque. The friction in the thrust bearing shown in Figure 7 was an appreciable fraction of the torque required to drive the pump with air as working fluid. No effort was made to correct this for this investigation as head-flow characteristics were the major results desired.

* The Mechanical losses in a commercial unit are a low percentage of the power input. Efficiencies determined are thus considered to be of little value for comparison purposes.
d. A commercial "Strobotac" was used for the measurement of speeds.

Although some check runs were made at 2000 RPM, all the experimental data was obtained with a rotor speed of 1000 RPM. For each of the six geometries indicated in Table 2, in Appendix, data was recorded to determine the overall and internal pressure-flow characteristics of the pump. As the pressure changed in the flow passage (for different flow rates) the clearance between the front cover and the impeller was kept to a minimum of the order of 0.010 inches by means of the rig described in section 3.2. In the stripper section clearance was harder to control and was measured to be of the order of 0.020 inches.

4. **ANALYSIS OF EXPERIMENTAL RESULTS.**

4.1. **Internal performance.**

4.1.1. **Influence of the two major flow passage dimensions, c and d.**

Several attempts were made to relate the internal performances of the six pumps tested through the analysis described earlier in this thesis. In these attempts the values of the loss coefficients and flow parameters were chosen according to some hypothesis that would agree with the general ideas discussed in sections 2.2 and 2.3.

Equations (1a), (2a), (4), and (6) were used with $k_c$, $\delta$ and $r_1$ as the parameters to be determined. Only the most successful of these
attempts is reported here.

Reference to paragraph 2. 3. 4 and inspection of Figure 3 suggest the following limiting conditions for \( r_1 \).

a. As \( c/d \to \infty \), \( r_1 \to r_2 \), \( f \to 0 \).

b. As \( c/d \to 0 \), \( r_1 \to r_0 \), \( f \to 0 \). This second condition is assumed to hold for the model used in the analysis although it is recognized that for actual pumps the limiting value of \( r_1 \) might be larger than \( r_0 \).

Between these limiting cases a value of \( c/d \), say \((c/d)^*\), will exist for which \[ r_1 = \frac{r_0 + r_2}{2} \] and the flow enters over the whole height of the blade, \( f = h \).

Satisfying these limiting conditions \( r_1 \) was assumed to vary according to the relations:

\[
\begin{align*}
    r_1 &= r_0 + \frac{f}{2} ; \quad f = h \frac{c/d}{(c/d)^*} \quad \text{if } c/d \leq (c/d)^* \\
    r_1 &= r_2 - \frac{f}{2} \quad ; \quad f = h \frac{c/d^*}{c/d} \quad \text{if } c/d \geq (c/d)^*
\end{align*}
\]

which are plotted in Figure 9a. The value of \((c/d)^*\) is believed to be mainly a function of the impeller geometry. For the rotor geometry common to the six pumps tested the most appropriate value was found to be \((c/d)^* = 0.8\).

Using these expressions for \( r_1 \), values of \( \sigma \) and \( k_c \) were obtained that would give the best agreement between the analysis and

\* According to the one-dimensional analysis of the pump the mean velocity of the circulatory flow entering the blades at \( l \) is:

\[ V_{cl} = \frac{Q}{r_1 f} \].  See Figure 4.
experimental data. Figures 9b and 9c show these results.

The variation of $\bar{v}$ does not contradict the statements made in paragraph 2.3.3. Figure 9b shows that the influence of the tip clearance, $c$, is particularly important.

As would be expected from what was said in paragraph 2.3.2, $k_c$ appears to be a complicated function of the flow passage geometry. If we imagine that the losses in the circulatory flow in the outside passage will be made up mainly of elbow-type losses it seems reasonable that these should have a minimum at some critical value of $d$ or $c$ as Figure (9c) indicates. The values used for $\bar{v}$, $k_c$ and $r_1$ are recorded in Table 2. (in Appendix)

Figures 10 to 15 show the match that is obtained with the measured internal performance by using the values of Table 2. Leakage as discussed in paragraph 2.4.5 has been considered in these graphs. It will be recalled that the experimental data is based on the flow measured at the pump inlet which is smaller than the flow in the pump passage because of leakage.

Figure 16 reproduces for one configuration the measured pressure variation inside the pump from inlet to discharge. Curves for four different flows are shown. This graph indicates clearly that the assumption of existence of a linear region of operation is quite justified.
4.1.2. Influence of other geometric factors

Most commercial units like Sta-Rite TH-7 pump have rounded corners in the pump flow passage and it was believed of interest to investigate their influence on the pump performance. Figure 17 shows the results from a test made to compare the internal head-flow characteristics of the pump with and without a rounded corner. This test was made before the front clearance e could be controlled properly. (These results may not therefore be compared directly with graphs reported in paragraph 4.1.1.) With reference to the analysis presented in section 2 these two curves may be compared if approximately the same value of $\bar{u}$ and $r_1$ is used for both tests. The higher head developed with the rounded corner may then be attributed, according to Equations 1, 3a, and 6, to a decrease in the cross-sectional area of the flow passage, and to a decrease in the loss coefficient $k_c$. For the same pump flow $\phi$ this would mean an increase in the circulatory flow $\phi_c$ and in the pressure rise $\frac{d\psi}{d\theta}$ as observed from experimental results in Figure 17.

4.2. Overall performance

The overall performance curves obtained experimentally for the six geometries tested are shown in Figure 18. Because of the uncertainty in the power measurements mentioned in paragraph 3.3.6 the efficiency curves are not recorded on this graph.
No attempt has been made to match the overall head-flow characteristics by use of Equation 10 so that no information has been obtained on the influence of the flow passage geometry on the parameters $\Theta_p$ and $k$.

To check the validity of the analysis made on the overall performance in sections 2.4 and 2.5 a set of manufacturer's curves were used. With Equation 12 the value of the loss coefficient $k$ was evaluated at the maximum efficiency point. Using this value of $k$ and the head-flow characteristic the complete efficiency curve was computed. Two such calculations are shown in Figures 19 and 20 for Sta-Rite's TG 8 and TH-4 regenerative pumps. A very good agreement may be observed.
5. CONCLUSIONS AND RECOMMENDATIONS

Previous work has shown that the hypothesis of operation reflects the actual mechanics of the internal flow of the regenerative pump. Investigation of a limited number of pumps has permitted qualitative rationalization of the parameters and flow coefficients introduced in the theoretical analysis. One empirical correlation of these parameters in terms of the two major dimensions in the pump flow passage is presented.

Losses in the internal pump are considered as comprising three elements: (a) tangential flow losses proportional to $Q^2$ due mostly to wall friction; (b) circulatory flow losses in the form of a shock loss at blade entrance evaluated according to Spannhacke; and (c) other circulatory flow losses proportional to $Q_c$. The latter presumably include mostly elbow-type or turning losses associated with the secondary motion of the fluid.

The slip factor $\sqrt{r}$ is seen to agree generally with values predicted for other radial flow hydrodynamic runners. Small tip clearances between the impeller and the casing (dimension $c$) seem to decrease this factor appreciably. The parameter $r_1$ relates the hypothetical model to the actual pump and is believed to be a strong function of the impeller geometry and of the ratio, $c/d$, of dimensions of the flow passage.
From the nature of these losses some recommendations may be made on the possibilities for improving and changing the head-flow characteristics of the pump in the linear region. The tangential losses probably have a small influence on this performance. The head rise according to Equation (1a) may be increased by increasing the circulatory flow $Q_c$. Consequently an increase in head at a given flow might be attained in two ways:

a. Reducing the blade entrance loss. This may be accomplished at some flows and would result in an increase of the loss elsewhere. For example, backward leaning blades (lower $f$) will decrease the shock loss at low flows (Equations 4 and 6) but probably increase it at high flows. Curving of the blades at the sides might also be used to shift the flow for which shock loss is minimum (zero according to the analysis).

b. Reducing the loss coefficient $k_c$ by properly designing the flow passage. This was discussed in section 4.1.

The overall head-flow characteristics of the pump may be raised according to Equation (9) mainly by:

a. Increasing the pressure rise in the linear region as described above.

b. Designing the inlet ports so that the flow is introduced with a motion and velocity as close to that described for the linear region as possible. This would reduce the extent

* Calculations have shown that maximum flow generally occurs at values of $\alpha$ slightly greater than unit.
of the accelerating region. A pressure drop would be associated with this change so that this solution appears not to be entirely compatible with the following proposal.

c. By reducing the pipe and elbow-type losses at the inlet, namely, "k." This would increase the head at high flows and increase the maximum pump flow.

The efficiency of the pump is seen from Equation (12) and Figures 19 and 20 to be limited at a given flow to the value $Q/Q_s$

$\left( Q_s = r_G A \omega \right)$. High values of $Q/Q_s$ at zero head, large heads at high flows and low values of the inlet and exhaust loss coefficient $k$, are seen to indicate means of improving the maximum efficiency of the regenerative pump. It is believed, however, that efficiencies much higher than those of present day commercial pumps will probably not be attained. It should be remarked that the regenerative pump is a unit of very low specific speed. According to conventional plots (Reference 10) the efficiency compares quite favorably with other pumps. For example, for the TG 8 pump, Figure 20,

$$N_s = \frac{RPM \sqrt{GPM}}{H^{3/4}} = 183.$$  

This value falls beyond the graph and extrapolation indicates efficiencies of about 35%. Note that this pump has a maximum efficiency of 48%.

Substantiation of the above conclusions open a large number of research possibilities for the improvement of the performance of
of the regenerative pump. To permit predictions of the performance characteristics over wide ranges of geometric parameters further research on the internal flow is recommended. The latter would provide more solid foundations for the evaluation of the loss coefficients and flow parameters corresponding to a larger number of pump geometry factors than those considered in this thesis.
Figure 1. - Schematic drawing of the regenerative pump.
Figure 2 - Pressure variation inside the pump.

Points 1 to 11 refer to Figure 1-A.
\[ A_1 = \frac{bc}{2} \]
\[ A_2 = \frac{b+d}{2} \cdot c \]
\[ A_3 = (r_3 - r_1) \cdot d \]
\[ A_4 = \frac{f \cdot d}{2} \]

\[ A_t = A_1 + A_2 = \left( b + \frac{d}{2} \right) \cdot c \]
\[ A_s = A_3 + A_4 = (r_3 - r_0) \cdot d \]
\[ A = A_t + A_s \]
\[ a = \text{Blade area} \]

**Figure 3.** - Flow passage and impeller dimensions.
FIGURE 4. - BASIC CONTROL VOLUMES USED IN ANALYSIS.
A) - BUSEMANN'S RESULTS

B) - COMPARISON OF RESULTS FOR $r_1/r_2 = 0$

FIGURE 5 - DEPARTURE FROM PERFECT BLADE GUIDANCE - "SLIP FACTOR" $\phi$. 
A) - STRIPPER SECTION.

B) - ESTIMATED LEAKAGE FOR PUMP TESTED.

FIGURE 6 - LEAKAGE FLOW THROUGH STRIPPER.
Figure 7. - Cross-section of experimental unit.
FIGURE 8. - FLOW PASSAGE IN THE WORKING SECTION AND MAJOR DIMENSIONS OF THE EXPERIMENTAL UNIT.
Figure 9 - Variation of internal flow parameters with passage geometry.
CONFIGURATION No. 2. (TABLE 1)

--- CALCULATED CURVE
--- CALCULATED CURVE DEDUCTING END LEAKAGE.

* EXPERIMENTAL POINTS - 1000 RPM.

**FIGURE 10.** COMPARISON OF CALCULATED INTERNAL PERFORMANCE WITH EXPERIMENTAL RESULTS.
Figure II. - Comparison of calculated internal performance with experimental results.

Configuration No. 4 (Table I)

--- Calculated Curve

--- Calculated Curve deducting end leakage.

△ Experimental points - 1000 RPM.
CONFIGURATION NO. 5. (TABLE I)

--- CALCULATED CURVE

--- CALCULATED CURVE DEDUCTING END LEAKAGE.

+ EXPERIMENTAL POINTS - 1000 RPM.

Figure 12. - Comparison of calculated internal performance with experimental results.
CONFIGURATION No. II (TABLE 1)

- - - - CALculated CURve
- - - - CALculated CURve DEDUCTING END LEAKAGE.

Δ EXPERIMENTAL POINTS. 1000 RPM.

FIGURE 13. - COMPARISON OF CALCULATED INTERNAL PERFORMANCE WITH EXPERIMENTAL RESULTS.
CONFIGURATION No. 14 (TABLE 1)

- - - - CALCULATED CURVE

- - - - CALCULATED CURVE DEDUCTING END LEAKAGE.

○ EXPERIMENTAL POINTS - 1000 RPM

FIGURE 14: - COMPARISON OF CALCULATED INTERNAL PERFORMANCE WITH EXPERIMENTAL RESULTS.
Configuration No 16. (Table 1)

--- Calculated Curve

--- Calculated Curve deducting end leakage.

Experimental points - 1000 RPM.

Figure 15: Comparison of calculated internal performance with experimental results.

Dimensionless head $h_0$

Dimensionless flow $\phi$

$0 \quad 1 \quad 2 \quad 3 \times 10^{-3}$
CONFIGURATION No. II.
1000 RPM
(REFER TO FIG. 1 AND 2)
1 - INLET
2 - DISCHARGE

FIGURE 16. - PRESSURE DISTRIBUTION IN THE PUMP AT VARIOUS FLOWS.
CONFIGURATION No. II
(Large front clearance: e)

- 1000 RPM
- 1000 RPM with rounded corner... R = 2 inches.

**Figure 17.** Comparison of internal performance with and without rounded corner.
OVERALL PERFORMANCE - 1000 RPM

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>A</th>
<th>c/d</th>
</tr>
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<tbody>
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<td>No. 2</td>
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<td>.515</td>
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<tr>
<td>16</td>
<td>2.89</td>
<td>.430</td>
</tr>
</tbody>
</table>

**FIGURE 18.** - OVERALL PERFORMANCE OF THE SIX PUMPS TESTED
STA-RITE TH-4 PUMP

\[ c = 0.282'' \quad r_2 = 2.596'' \]
\[ d = 0.197'' \quad A = 0.399 \text{ in}^2 \]
\[ b = 0.151'' \quad Q = \rho \cdot A \cdot \omega = 54.2 \text{ GPM} \]
\[ h = 0.719'' \quad 1750 \text{ RPM} \]

\[ \eta = \frac{Q}{Q_2} \]

**FIGURE 19.** DETERMINATION OF EFFICIENCY FROM HEAD CURVE WITH A SINGLE CONSTANT RESULTING FROM MATCH AT MAXIMUM EFFICIENCY.
STA-RITE TG-8 PUMP

Manufacturer's curves

Calculated from head curve

\[ r_2 = 2.150'' \quad A = 0.457 \text{ in}^2 \]
\[ c = 0.193'' \quad Q_{3.5} \text{gpm} = 45.4 \text{gpm} \]
\[ d = 0.266'' \quad 1750 \text{ rpm} \]
\[ b = 0.182'' \]
\[ h = 0.625'' \]

\[ \eta = \frac{Q}{Q_3} \]

**Figure 20.** Determination of efficiency from head curve with a single constant resulting from match at maximum efficiency.
APPENDIX

Contents:

Development of the analytical expressions for the linear region.

Table 2. Values of parameters used in the analysis of the experimental data.

Photographs of the experimental unit.
APPENDIX A-1

Development of the analytical expressions for the linear region.

With the six basic assumptions made by Oelrich and stated in section 1.2, the four expressions (1) to (4) will be derived here in more detail.

A. Tangential pressure rise.

Applying angular momentum relations to the control volume of Figure 4a.,

\[ \int_{A} \rho r dA - \int_{A} (\rho + \frac{dp}{d\theta} d\theta) r dA - S \cdot r_m = \rho Q_c d\theta (\alpha U_1 r_1 - \sigma U_2 r_2) \]

where \( S \cdot r_m \) denotes the moment of the shear forces at the wall.

Simplifying and dividing by \( r_0 A \),

\[ \frac{1}{\rho} \frac{dp}{d\theta} = \frac{Q_c \omega (\sigma r_2^2 - \alpha r_1^2)}{r_0 A} - \frac{S \cdot r_m}{\rho r_0 A} \]

(1)

In the body of the report the last term was replaced by the tangential flow losses \( gH_t \).

B. Circulatory flow losses.

Applying the steady flow energy equation to the same control volume of Figure 4a:
simplifying according to the original assumptions and defining:

\[ Q = \int_A V_e \, dA \]

\[ \frac{du}{d\theta} = -gH_t \]

tangential losses

\[ u_1 - u_2 = gH_c \]

circulatory flow losses in pump flow passage.

\[ 0 = \varphi Q \left( \frac{1}{g} \frac{dp}{d\theta} + gH_t \right) + \varphi Q_c \left( gH_c + \frac{Q_c^2}{2r_1^2} - \frac{Q_c^2}{2r_2^2} - \frac{\alpha^2 U_t^2}{2} - \frac{\sigma^2 U_i^2}{2} + \frac{P_1 - P_2}{g} \right) \]

regrouping and substituting \( \frac{dp}{d\theta} \) from Equation 1,

\[ \frac{P_2 - P_1}{g} = \frac{Q \omega (\sigma r_2^2 - \alpha r_1^2)}{r_0 A} + \frac{Q_c^2}{2r_1^2} - \frac{Q_c^2}{2r_2^2} + \frac{\alpha^2 U_t^2}{2} - \frac{\sigma^2 U_i^2}{2} + gH_c \]

(a)

Applying the angular momentum relation to the control volume of Figure 4b:

\[ dT = \varphi Q_c d\theta \left( r_2 \tau U_2 - r_1 \alpha U_1 \right) + Ra \frac{dp}{d\theta} \, d\theta \]

Since \( \omega dT = dW_{\text{in}} \),

\[ \frac{dW_{\text{in}}}{d\theta} = \varphi Q_c \left( \sigma U_t^2 - \alpha U_i^2 \right) + Ra \omega \frac{dp}{d\theta} \] (4)

* For an incompressible, one-dimensional adiabatic flow, with no work, the head loss between two sections is defined as

\[ H_{\text{loss}} \equiv \Delta \left( \frac{P}{g} + \frac{V^2}{2} \right) \]

Steady flow energy equation:

\[ Q = 0 = \Delta \left( u + \frac{P}{g} + \frac{V^2}{2} \right) \]

Combining:

\[ \Delta u = -H_{\text{loss}}. \]
as indicated in section 2.2.

Applying the steady flow energy equation to the control volume of Figure 4b:

\[ 0 = -dW_{in} + \oint \dot{Q} \, d\theta \left( \frac{r^2 u_z^2}{2} + \frac{Q_c^2}{2r_i^2 b} + u_z + \frac{P_e}{\dot{Q}} - \frac{\alpha u_z^2}{2} - \frac{Q_c^2}{2r_i^2 b^2} - u_i - \frac{P_i}{\dot{Q}} \right) + \oint \rho R \omega \left( \frac{1}{\dot{Q}} \frac{d\rho}{d\theta} \right) d\theta. \]

Substituting \( dW_{in} \) from (2), and defining

\[ u_z - u_i = g H_{ci} \]

circulatory flow losses through the impeller

and rearranging

\[ \frac{P_e - P_i}{\dot{Q}} = \sigma U_z^2 - \alpha U_i^2 + \frac{\alpha^2 U_i^2}{2} + \frac{Q_c^2}{2r_i^2 b^2} - \frac{Q_c^2}{2r_i^2 b^2} - g H_{ci}. \]  

(b)

Equating expressions (a) and (b) and simplifying:

\[ -Q \frac{\omega \left( \sigma r_z^2 - \alpha r_i^2 \right)}{r_c A} + \left( \sigma U_z^2 - \alpha U_i^2 \right) = g \left( H_{ci} + H_{co} \right). \]

Calling \( H_c = H_{ci} + H_{co} \) the total loss in the circulatory flow and rearranging:

\[ g H_c = \omega^2 \left( \sigma r_z^2 - \alpha r_i^2 \right) \left( 1 - \frac{Q}{r_c A \omega} \right). \]  

(3)

C. Tangential Flow Equation.

The detailed derivation of these expressions is quite laborious and may be found in Oelrich's thesis. Only the outline and simplifications made in obtaining Equation (4), will be indicated here.

The variation of the tangential velocity \( V_t \) in the outside passage may be derived if the mean streamline in the outside passage is assumed to be of the form shown in Figure 3. At point 2, \( V_{t2} = \sigma U_2 \)
and at point 1, \( V_{t1} \equiv \alpha U_{1} \). Using these values and matching the velocities at points 3 and 5, successive applications of the angular momentum relations yield:

\[
V_{t(2-3)} = \frac{r_{2}}{r_{1}} \int U_{2} - \frac{1}{\oint U_{c}} \frac{dp}{d\theta} \frac{b}{2} \left( \frac{r_{2}^{2} - r_{1}^{2}}{r_{1}^{2}} \right) \\
V_{t(3-4)} = \frac{r_{2}}{r_{3}} \int U_{2} - \frac{1}{\oint U_{c}} \frac{dp}{d\theta} \left[ \frac{b}{2} \frac{(r_{3}^{2} - r_{2}^{2})}{r_{3}} - c \right] \\
V_{t(1-5)} = \alpha U_{1} + \frac{1}{\oint U_{c}} \frac{dp}{d\theta} f \quad Z \\
V_{t(5-4)} = \alpha U_{1} \frac{r_{2}}{r_{1}} + \frac{1}{\oint U_{c}} \frac{dp}{d\theta} \left[ f \frac{r_{2}}{r_{1}} + \frac{r_{2}^{2} - r_{1}^{2}}{r_{1}} \right]
\]

The total flow will be found performing the integrals:

\[
Q = \int_{A} V_{t} \cdot dA \\
= \int_{r_{2}}^{r_{3}} V_{t(2-3)} b \, dr + \int_{0}^{r_{3}} V_{t(3-4)} \cdot c \, dz + \int_{r_{1}}^{r_{3}} V_{t(5-4)} d \, dr + \int_{0}^{f} V_{t(5-1)} f \, dz
\]

with the result:

\[
Q = r_{2} \int U_{2} \left( b \ln \frac{r_{3}}{r_{2}} + \frac{A_{2}}{r_{3}} \right) + r_{1} \alpha U_{1} \left( d \ln \frac{r_{3}}{r_{1}} + \frac{A_{4}}{r_{1}} \right) + \\
+ \frac{1}{\oint U_{c}} \frac{dp}{d\theta} \left[ \frac{b^{2}r_{2}^{2}}{2} \ln \frac{r_{3}}{r_{2}} - \frac{A_{1}b(r_{3}+r_{2})}{4} - \frac{A_{1}A_{2}(r_{3}+r_{2})}{2r_{3}} - \frac{A_{2}^{2}}{2} + \\
+ r_{1}A_{4}d \ln \frac{r_{3}}{r_{1}} + \frac{A_{3}d(r_{3}+r_{1})}{4} - \frac{r_{1}^{2}d^{2}}{2} \ln \frac{r_{3}}{r_{1}} + \frac{A_{4}^{2}}{2} \right]
\]

From the series expansion:

\[
\ln x = (x-1) - \frac{1}{2} (x-1)^{2} + \frac{1}{3} (x-1)^{3} - \ldots \quad \quad 0 < x < 2
\]
the logarithmic terms were replaced by three terms of the series when
\[ x = \frac{r_3}{r_1} \]
and by two terms of the series when \[ x = \frac{r_3}{r_2} \]. This series
expansions were stopped in each case when terms became less than 1% of
the other terms in the equation, for the geometries considered.

Making these substitutions and replacing \( \frac{de}{d\theta} \) from equation
(1) leads to equation (4) in section 2.2.
TABLE 2
PARAMETERS USED IN THE ANALYSIS OF THE EXPERIMENTAL DATA

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<td>.875</td>
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</table>
Photograph No. 1

General view of experimental set up for the Regenerative pump project showing 19" diameter model on the left and Sta-Rite's commercial unit (5.4" diameter) in the center.
Photograph No. 2

Assembly of large experimental unit showing flow control valve at exhaust, venturi at inlet, and dynamometer control panel and scale.
Photograph No. 3

Large experimental unit with front cover off. Notice part of stripper and inlet port attached to the front cover.
Photograph No. 4

Detail of stripper showing inlet (at left) and exhaust ports.
Photograph No. 5

Entirely assembled unit showing pressure taps on front cover and rigid structure to control clearance between the front cover and the rotor.


