Distributed Measures of Solution Existence and Its Optimality in Stationary Electric Power Systems: Scattering Approach

by

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Abstract
Currently, problems of large scale power systems are defined mainly in the voltage-current domain. This thesis introduces an alternate formulation for power systems based on the scattering matrix network formulation. In particular, the load flow problem is defined in the scattering domain. This formulation yields a geometric representation of the equilibrium solution which allows for the determination of more than one solution to the load flow and bounds on system inputs for which a solution exists.

However, as deregulation takes place there is a need for more decentralized computing/decision making. Thus the basic approach to load flow computation is changing because of the lack of centralized information. A byproduct of this scattering formulation is the ability, through local measurements at one bus, to obtain information (i.e. voltage and current) about another bus, a valuable tool in competitive power systems. The procurement of this information is done in a completely decentralized manner.

Given the move toward competitive electricity markets, transmission capacity measures, which tell whether power transfers can be made on the system, have become increasingly important. In the thesis, the scattering versions of LBTC (Load Bus Transmission Capacity) and GBTC (Generator Bus Transmission Capacity) are defined. Proof of concepts were illustrated on a three bus system.

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Dedication

This thesis is dedicated in loving memory of

HELEN A. MILLIS

MARY B. KUSTKA

DR. CYNTHIA D. MILLIS

Although my Mother, Aunt and Sister all died before this thesis was finished, their encouragement and understanding (especially my sister’s) made this endeavor possible. I owe them a great deal of thanks for their love and support.
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Chapter 1

Introduction

The operation of today’s electric power system requires that generation supplies the time-varying load plus transmission losses, bus voltage magnitudes remain close to their rated values, generators operate within specified real and reactive power limits and transmission lines transfer power within their thermal and stability limits. However, an electric power system is never at its equilibrium. Power system dynamics in normal conditions are driven primarily by changes in demand. Typical demand varies on a daily, weekly, monthly and yearly basis and is approximately as predictable as the weather. To meet the anticipated demand, economic dispatch and unit commitment, compensation for transmission losses and meeting static operating constraints are services provided by all generating units. Small random deviations from anticipated demand are handled by automatic generation control which involves changing the set point of governors on some generators.

The main objectives of real-time operation of the electric power system are to schedule maintenance of power plants, turn them on and off and change generation outputs as required to supply the predicted system demand. Since generators cannot be instantly turned on and produce power, generating units must be scheduled in advance in order to meet system demand and have an adequate reserve margin should generation or transmission line outages or demand exceed the anticipated amounts. The units are chosen such that the expected total cost over a specified time horizon is optimized. Necessary parameters used in determining the turn on and turn off times for scheduling generation include start up
costs, rates of response and must-run times. In order to meet these objectives, a system operator uses a variety of static network and generation modeling tools which include load flow studies, economic dispatch simulations and optimal power flow (OPF) analyses. Each of these tools assumes that the demand is known which is only true within the accuracy of the forecasted variables, in particular load and unit outage statistics. Typically the power system operates in a stationary condition with slowly varying loads (on the order of hours) and a minimal random load fluctuation, typically under 1%. Thus most of the time dynamics play no role and can be disregarded in stationary analyses. This thesis is mainly concerned with an alternate load flow model. In particular, the problem will be reformulated in the scattering domain and argued that this domain is a more natural basis in which to define power transfer processes.

In this chapter, the conventional load flow problem will be revisited in the voltage-current domain. Depending on how the load flow equations are interpreted, whether as a set of nonlinear equations, finding the zeros of a function or as a mapping, leads to a variety of approaches and techniques to solve the equations. A number of the techniques and approaches will be reviewed along with their strengths and weaknesses. With this as a background, the motivation behind why a scattering formulation was proposed will be discussed.

1.1 Stationary Analysis- Conventional Load Flow

Load flow (stationary) analysis is primarily concerned with the problem that generation must supply the load and accompanying transmission losses. Since the load is generally specified in terms of real and reactive power and the generators are power sources as opposed to independent voltage or current sources, the load flow equations are formulated as a set of non-linear algebraic equations. As demand and/or unexpected equipment contingencies occur, the basic question is whether the system can sustain its operations under the new set of conditions, i.e. does an equilibrium under the new conditions exist. This question is typically answered by performing a load flow study which answers the question of solution existence of the new equilibrium point. This question of solution existence of
1.1. Stationary Analysis- Conventional Load Flow

A new equilibrium and its properties as contingencies occur is a major concern in power systems operations. For example, if a particular generator or transmission line goes out, a load flow analysis can be used to determine a priori where the problems will occur and thus can be used to develop strategies to overcome the problems. Load flow analyses are also important in planning.

The complexity of obtaining a solution to the load flow equations arises because of different types of data specified at the buses. In particular, loads can be described either as constant impedances or constant real and reactive power (PQ) loads; generators can be modeled as a voltage source (slack bus) or by its real power and voltage magnitude (PV bus). This mix of variables, admittance, voltage, real and reactive power implies that the load flow equations be written in what will be defined as the conventional voltage, current (V-I) domain in order to distinguish from other approaches that will be discussed. The load flow equations are nothing more than the power balance equations written at each bus with the exception of the slack bus. Thus the power balance equations at bus $i$ are as follows:

$$
P_i = \sum_{j=1}^{n} |V_i||V_j|[g_{ij}\cos(\delta_i - \delta_j) + b_{ij}\sin(\delta_i - \delta_j)]$$  \hspace{1cm} (1.1)

$$
Q_i = \sum_{j=1}^{n} |V_i||V_j|[g_{ij}\sin(\delta_i - \delta_j) - b_{ij}\cos(\delta_i - \delta_j)]$$  \hspace{1cm} (1.2)

for all $i = 1, \ldots, n$ where $P_i$ is the real power at bus $i$, $Q_i$ is the reactive power, $|V_i|$ is the bus voltage magnitude, $g_{ij}$ and $b_{ij}$ are the line admittance parameters and $\delta_i$ is the bus voltage angle. Since the important variables at each bus in an electric power system are voltage, magnitude and angle, real and reactive power, the solution to the load flow yields the voltage magnitude and angle at a PQ bus and the angle and reactive power $Q$ at a PV bus.

A typical approach to obtaining a solution to the load flow equations is to apply a numerical technique [26]. One of the numerical algorithms used in solving the conventional load flow is the Newton-Raphson method. Since the load flow equations defined in equations (1.1) and (1.2) can be written as $F(x) = 0$, the Newton-Raphson algorithm involves
a Taylor series expansion of $F$ around an operating point:

$$x^{k+1} = x^k - J(x^k)^{-1}F(x^k)$$

where $x^k$ and $x^{k+1}$ are the values of $x$ at the $k$th and $(k+1)$th steps respectively, $J(x^k)^{-1}$ is the inverse of the Jacobian $\frac{\partial F}{\partial x}$ at $x^k$. Because it is a numerical algorithm, the Newton-Raphson method may fail to converge to a solution if the Jacobian is singular, [27] the initial guess is outside the region of attraction of the solution or there is a limit cycle problem. It is noted that if the algorithm does not converge, there is no way of knowing whether there is no solution or one of the previous conditions exists.

Since the computation time of the Newton-Raphson algorithm is on the order of $n^3$, use of this methodology would not be practical for very large power systems [2]. However properties of the electric power system are exploited in order to reduce the computation time. The structure of a typical power system is such that it has fewer than three lines connected to each bus [2]. As such, the bus admittance matrix of the transmission system, $Y_{BUS}$, has a small number of nonzero elements, i.e. it is sparse. Many load flow programs employ sparse matrix techniques to reduce the time to solution convergence.

1.2 Stationary Analysis - Alternative Approaches

One way to view the load flow equations in (1.1) and (1.2) is that they are a set of nonlinear algebraic equations. However, as previously detailed, they can also be interpreted as a set of equations $F(x) = 0$ where the objective is to find the zeros of the function $F(x)$. A third interpretation is that the function $F(x)$ is a mapping of $\mathbb{R}^n \rightarrow \mathbb{R}^n$. Each of these interpretations leads to different approaches in solving the load flow and there is extensive literature detailing these approaches [22, 23, 24, 25]. Simplifying assumptions based on characteristics and properties of the electric power system that lead to simpler models are what will be described as modeling approaches. These include the decoupled and DC load flow problems. Finding the zeros of $F(x)$ is usually accomplished via iterative techniques, i.e. Newton-Raphson, Gauss-Seidel. Viewing $F(x)$ as a mapping leads to continuation
1.2 Stationary Analysis - Alternative Approaches

There have been a number of approaches taken to simplify and solve these equations either from the perspective of modeling assumptions or from the choice of numerical technique. This section will present a brief summary of some of the common modeling and numerical techniques that are used.

1.2.1 Modeling Approaches

From the electrical characteristics and inherent properties of the electric power system in normal operation, many simplifying assumptions can be made to obtain either simpler models to be used for computing the load flow or a simpler set of load flow equations. In this section a number of approaches, namely the decoupled real power/angle load flow problem, the decoupled reactive power/voltage load flow problem and the nonlinear resistor network interpretation of the real power/angle load flow problem will be reviewed [28]. Exhaustive and extensive descriptions of other approaches including applications of the above approaches in the deregulated industry are found in [1]. However, the main purpose of all of these approaches is to find a solution to the load flow equations. Due to the nonlinearity of the equations and potential difficulties in applying standard numerical techniques such as Newton-Raphson discussed above, attempts have been made to simplify the equations through modeling assumptions about the physical characteristics and inherent properties of the electric power system. In each of these approaches, certain assumptions are made about the state of the electric power system (i.e. transmission system, voltage angles and magnitudes and the modeling of loads) in order to simplify the equations. Simplification of the equations allows the establishment of conditions (i.e. bounds on system input and topology changes) under which solutions are guaranteed to exist. Use of a specific numerical algorithm only detects that a solution is not computable.

Since contingencies are a major concern in power system operations, it is necessary to know what power flow changes will occur due to a particular generator or transmission line outage. This contingency information obtained in real time can be used to anticipate problems caused by such outages and to develop strategies to alleviate the problem. Decoupled load flow algorithms have been developed that improve computational efficiency and reduce
data storage requirements. The decoupled load flow approach is based on the premise that most transmission lines are predominantly inductive (the ratio of reactance to resistance is on the order of 10 to 1) and that under normal operations, the angle differences between buses tend to be less than 10 degrees, and voltages are close to 1 per unit [3]. This leads to two observations:

- Changes in the voltage angle $\delta$ at a bus primarily affect the flow of real power $P$ in the transmission lines while the flow of reactive power $Q$ is relatively unchanged ($P-\delta$ decoupled load flow problem).

- Changes in the voltage magnitude $|V|$ at a bus primarily affect the flow of reactive power $Q$ in the transmission lines while the flow of real power $P$ is relatively unchanged ($Q-V$ decoupled load flow problem).

Under these assumptions, the load flow equations are defined as two lower order sets of decoupled load flow equations because the voltage angle corrections $\Delta\delta$ are calculated using only real power mismatches $\Delta P$ while the voltage magnitude corrections are calculated using only $\Delta Q$ mismatches. These approximations are used to simplify elements in the Jacobian matrix and hence the dimensionality of the computation is reduced. The result is that convergence is faster than that of the Newton-Raphson based computing approach of the coupled load flow problem. However it is important to point out that the use of a decoupled load flow generally results in some loss of accuracy in analytical or numerical conclusions of interest. This is of particular noteworthy if the system is subject to significant changes.

If in addition to the decoupling assumptions all the bus voltages are assumed constant at nominal values of 1 per unit, the result is the DC load flow model. If, in addition, the resistive part of the transmission lines is neglected, this formulation can be interpreted as a linear resistive network whose branch parameters are a function of the susceptance of the bus admittance matrix $Y_{BUS}$ and whose currents and voltages are replaced by real power flows and phase angles respectively. This interpretation is important since positive resistive
networks with independent voltage and current sources always have a unique solution. This guarantees existence of the solution for phase angles under small changes in input power.

There are also properties of the electric power system that are exploited in the implementation of some of these approaches. One property in particular is the localized response property of the P-δ power network which is the localized response of a transmission network to changes in real power input [29, 30]. Many algorithms for fast computation of the effects of changes in real power propagation in a large system are based on this property. The localized response property of a transmission network to changes in real power input is based on the fact that changes in voltage phase angles decrease monotonically as the electrical distance from a triggering event decreases. This property is often used by a system operator in deciding which corrective actions to take in the case of a contingency or contingencies.

To further assist the power system operator in the evaluation of possible contingencies is the use of an approximate method known as the distribution factors method. Great precision is not required since the primary interest is in knowing whether or not an insecure or vulnerable condition exists in the steady state following an outage. The distribution factors method is a linear method based on the DC load flow model and is used for assessing the effect of changes in real power line flows. This method uses sensitivities of power flow in the line of interest with respect to the power injection at bus i.

### 1.2.2 Numerical Approaches

Whereas the previous section discussed modeling assumptions and simplifications, this section will present a brief summary of some of the numerical tools used in obtaining solutions to the load flow equations. These include continuation methods, embedding algorithms and homotopy methods [22, 24]. The issues involved in choosing the appropriate numerical method are the nature of the nonlinearity, conditions under which the method will converge to the required solutions, the rate of convergence of the iterative scheme, computational speed and computer storage.

The load flow equations in (1.1) and (1.2) can be rewritten as $F(x) = 0$ where $F$ is a nonlinear function. The only way to find the zeros of the nonlinear function $F$ is to use an
iterative technique [23]. An iterative approach involves starting with an initial guess and calculating successive approximations to the solution by applying an appropriate iterative technique. However, iterative approaches may converge slowly (as in the case of Newton-Raphson) and suffer from numerical instabilities and bifurcations.

Rather than viewing the load flow equations as finding the zeros of a nonlinear function, $F(x)$ can be thought of as a mapping of $\mathbb{R}^n \rightarrow \mathbb{R}^n$. Given this interpretation, an alternative to the conventional numerical techniques is a continuation algorithm. The basic idea of the continuation method is the idea of gradually deforming a simple (usually linear) problem to a nonlinear one. In the case of the load flow equations, the solution to $F(x) = 0$ is desired. Since $F(x)$ is a mapping of $\mathbb{R}^n \rightarrow \mathbb{R}^n$, the continuation method involves deforming an arbitrary mapping with a known solution into the mapping $F(x)$ whose solution is $x^*$ by varying a parameter. This continuation parameter, defined as $\lambda$, $\lambda \in [0, 1]$, is used to define a new mapping $H$ (i.e. $\mathbb{R}^n \rightarrow \mathbb{R}^n$) where

$$H(x, \lambda) = (1 - \lambda)G(x) + \lambda F(x) = 0$$

and where $G$ is a trivial map with known solution $x^0$. By gradually varying $\lambda$ from 0 to 1, the continuation method deforms a trivial map $G(x)$ with known solution to the desired problem $F(x)$, whose solution is $x^*$. Thus starting at $\lambda = 0$, $(H(x^0, 0) = G(x^0) = 0)$ and increasing $\lambda$ in positive fixed steps results in $H(x^*, 1) = F(x^*) = 0$ which is the desired result. Note that the solution space is a curve which is parametric in $\lambda$, i.e. $x = x(\lambda)$ and this continuous curve in $\mathbb{R}^n$ starts at the initial point $(x^0, 0)$ and ends at the required solution $(x^*, 1)$.

Advantages of the continuation method are that they may increase the domain of convergence of a solution and may also be used to bring the initial guess closer to the solution and within the region of convergence. These advantages are due to the global convergence and convergence to multiple solutions properties of the algorithm.

If the interval is partitioned into $k$ equal subintervals, $\delta = \frac{1}{k}$, then this continuation algorithm is known as the embedding algorithm. Thus starting at the known solution points, the solution space defined by $H(x, \lambda_i + \delta \lambda) = 0$ for $i = 1, \ldots k$ is solved for $x$ using
1.3. *Scattering Approach to Power Systems - Motivation*

In the previous sections, both the modeling approaches and numerical techniques for solving the load flow equations are defined in the voltage, current domain. The reason for this is that loads are modeled as either constant impedances or as constant real and reactive power

an iterative method which applies the \( x^{i-1} \) solution as a starting approximation to the \( i \)th problem. Under the assumption that \((\lambda_{i+1} - \lambda_i)\) is sufficiently small, \( x^{i-1} \) should be a good approximation to \( x^i \) so that convergence occurs. However, this embedding algorithm does not work well in general.

To overcome these inherent difficulties, the homotopy method is used [25]. This method is generally more robust and exhibits numerically stable characteristics. The reason is that the homotopy curves are always smooth, well-behaved paths. In particular bifurcation points and other ill-conditioned cases may be avoided. This involves a number of restrictions on the mapping \( H(x, \lambda) \). The first is to ensure that at least one path exists that will connect the initial solution of the mapping \( (\lambda = 0) \) to the final solution \( (\lambda = 1) \). This involves determining whether \( H^{-1} \) consists of well-behaved paths that do not intersect themselves. The second issue is to ensure that if the solution exists, the path will always reach a solution and will never tend to infinity. There are many other issues applying the homotopy method to obtain the solution of \( F(x) = 0 \) including ways of embedding a homotopy parameter, in particular the free-running case, the forced case and globally convergent probability-one homotopy methods.

Continuation methods have been successfully applied in obtaining the solution to the load flow equations. A detailed description of the results is given in [1]. However the key points in applying the homotopy method to power systems are that:

- Generation of multiple solutions is dependent on the type of homotopy mapping.
- Starting point on the path determines whether single or multiple solutions will be generated.
- The method only converges to stable operating points.

1.3 Scattering Approach to Power Systems - Motivation

In the previous sections, both the modeling approaches and numerical techniques for solving the load flow equations are defined in the voltage, current domain. The reason for this is that loads are modeled as either constant impedances or as constant real and reactive power
loads, generators are modeled as voltage sources or in terms of their specified constant real power and voltage magnitude and the transmission system is described by its bus admittance matrix. Thus the load flow equations, which are the power balance equations at each bus (excluding the slack bus) are a mix of voltage magnitude and angle, real and reactive power and admittance parameters. It is this mix of variables that results in the complicated nonlinear algebraic equations that constitute the load flow problem since the common basis in which to represent voltage and power is the voltage, current basis.

In this thesis an alternate approach to modeling a power system is considered, namely the scattering approach. In the most general terms, the scattering approach is concerned with the interconnection of lumped elements or “obstacles” within a flow type process in which flows are partially transmitted, reflected or absorbed at a system boundary. In the scattering domain, the basis variables are the incident and reflected variables which are linear combinations of voltage and current appropriately scaled by a normalizing impedance such that the units of these basis variables are (volt-amperes)$^{1/2}$. The scattering parameter, defined as the ratio of the reflected and the incident variables, can describe the performance of a system under a specified set of terminating conditions. In particular, questions of power transfer from a finite impedance generator to a load are frequently best handled by scattering relations and it is possible to deduce general theorems for these situations which are hidden when impedance or admittance characterizations are used [4].

The application of the scattering formalism is particularly important because of the non-uniform characterization of loads, generation and transmission in the voltage, current domain. Using the basis variables of incident and reflected waves, load, generation and transmission can be uniquely characterized in terms of scattering parameters and/or a scattering matrix for a specified normalization. The conventional load flow equations represent the power balance at each bus, namely that power injected into the transmission system at bus $i$ must match the flows through the transmission links connected to bus $i$. In the scattering formulation of the problem, generation and load represent boundary conditions at each of the ports of the transmission network. At the boundary, instead of matching power flows, the incident and reflected waves of the transmission network and its ports must
be matched.

Furthermore, use of the scattering approach introduces an important property which is the idea of separability based on a decomposition of the system into the transmission network and its ports, generation and loads. The transmission network can be thought of as a network constraint whereas load and generation can be viewed as terminal constraints. This separability establishes a mechanism by which local measurements at a bus can be used to deduce information about another bus. This leads to a decentralized approach to the load flow. As deregulation takes place and competition increases, less information will be made public and the basic approach to computation of the load flow will change. In the new deregulated environment, there will be a need for more decentralized computing and decision making and alternative approaches such as the scattering approach will become necessary.

1.4 Contribution of Thesis

This thesis is essentially a proof of concept on the application of a new approach in solving some problems in the electric power system. The contributions of this thesis fall into three sections corresponding to three problems in the electric power industry that were investigated.

The first is the casting of the coordinated load flow problem in the scattering domain. The coordinated load flow is the same as the conventional load flow which is used for feasibility and reliability studies. Since the scattering literature is devoted to linear applications, this is the first attempt to separate the linear portion of the load flow problem from the nonlinear port constraints. One of the contributions is to define the nonlinear constraints, i.e. PQ loads, in terms of finding the appropriate intersection points of real and reactive power circles. This leads to the main contribution of a geometric display of the solution at each and every bus which allows for the determination of more than one solution. In contrast to the traditional approach to solving load flow, a numerical algorithm yielding only one solution per simulation, this approach yields additional information about how close the system is to not being solvable. The separation of the two solution points is a
metric for how close that bus and the system comes to being solvable. Thus, in the case where the two solution points are very close at a particular bus, doing a power transfer from some other point on the system would not be possible.

The second contribution is a formulation of a decentralized load flow problem which involves obtaining information in a decentralized manner. The assumptions made in defining the decentralized load flow problem are that there is a step change in one or more than one of the load parameters and the system settles to a new equilibrium and that transmission and generation (i.e. the slack bus) are known by all players on the system.

The contribution in the decentralized load flow approach is the exploitation of the fact that if the loads and generation are defined in terms of scattering parameters which are \( a \) and \( b \), the incident and reflected voltages respectively, the transmission system is defined as \( b = Sa \), where \( S \) is the scattering matrix of the transmission system, the problem is transformed into the scattering domain and solved explicitly in the scattering domain. In contrast to the previously defined approach, the nonlinear port characterizations are transformed into an \( a, b \) port characterization. This approach exploits the linear input/output description \( b = Sa \). The nonlinearity of the loads is transformed by characterizing the loads in terms of the incident and reflected powers. Since the problem is now linear, a Gauss approach is used to determine the scattering parameter, \( \Gamma \), at the other bus (in a three bus example).

The third contribution is the calculation of the transfer capacity in the power network using the scattering domain approach. While the conventional approach is to do a DC load flow to approximate the solution, the scattering approach exploits the geometry of the problem by obtaining the tangency points of power circles since the premise is that as the real power is varied, the maximum amount of power that can be transferred to a particular bus is the value of the real power such that the two circles (in a three bus example) are at their tangency point.

The contribution in the calculation of the transfer capacity in the power network is that this is a geometric approach to obtain the load bus transmission capacity as opposed to a straight numerical algorithm.
1.5 Thesis Overview

Chapter 1 reviews the literature on solving stationary analysis (load flow) problems including both the modeling approaches and numerical techniques that are currently known. This review serves as a starting point and establishes the motivation behind why the scattering approach was proposed in this thesis.

In Chapter 2, the scattering parameter and the scattering matrix are revisited using both simple one-port and two-port examples. In particular, the different types of normalization, real and complex are reviewed as this becomes an issue in the case of voltage and reactive power relations. However, the methodology on applying scattering parameters/matrices to power problems is extended in this chapter to include loads that are not simple impedance type loads. The approach in handling loads which have a nonlinear port characterization in the voltage-current domain, namely PQ loads, is detailed for both the one-port and the two-port examples. This defines part of the approach taken in solving multi-bus examples. The approach used to describe these nonlinear loads leads to a novel geometric display of the solution and is illustrated in examples for both the one-port and two-port. This serves as the starting point to characterize the multi-bus coordinated load flow problem in Chapter 3.

Chapter 3 lays the structure for defining the coordinated load flow problem for the multi-bus system. A three bus example is used to illustrate the results. Three varieties of the load flow formulation in the scattering domain, terminal/network constraint load flow formulation, scattering voltage and current load flow formulation and geometric load flow formulations were defined and simulated. The setup of the terminal/network constraint load flow formulation and the scattering voltage and current load flow formulation are similar since both are cast as $F(X) = 0$ type problems. The geometric load flow formulation exploits the geometry of the problem. Rather than using a Newton-Raphson approach as was done in the previously mentioned formulations, the idea is to generate the intersection points at each iteration in the simulation. Multiple three bus examples verify that each of these formulations yields the same solution. A five bus example consisting of a single generator and four PQ loads is shown to illustrate a multi-bus example. Problems in modeling PV
elements (generators) are discussed.

In Chapter 4, the decentralized load flow problem is defined. This problem involves obtaining information about other buses on the system using local measurements. Deregulation is changing the basic approach to load flow because of the lack of systemwide information. Thus there is a requirement for more decentralized computing/decision making. The assumptions made in defining the decentralized load flow problem are that there is a step change in one or more than one of the load parameters and the system settles to a new equilibrium and that transmission and net generation (i.e the slack bus) are known by all system users. In this thesis only a single generator multiple load system examples are studied. Generalizations for multi-generator systems requires new modeling of PV elements.

The problem is posed and solved in the scattering domain. This approach exploits the linear input/output map \( b = Sa \). The nonlinearity of the loads is transformed by characterizing the loads in terms of the incident and reflected powers. Since the problem is now linear, a Gauss approach is used to determine the scattering parameter \( \Gamma \) at the other bus (in the three bus example). In the case of the three bus example in the thesis, which is used as proof of concept, it is assumed that characteristics of one of the loads are known. Since it is known, it can be characterized by \( a, b \) and hence \( \Gamma \). The solution to the problem is to solve for \( \Gamma \) at the other load such \( b = Sa \) is valid for the entire system. Three examples are used to illustrate the approach. Since this approach has applications in the competitive electric power industry, several examples are discussed in detail as to the usefulness of this approach.

Chapter 5 deals with the transfer capacity in the power network. It exploits the fact that solutions to the load flow in the scattering domain can be obtained from the intersection points of circles. Using this geometric approach to the problem, the transfer capacity in the power network is determined by obtaining the tangency points of power circles since the premise is that as the real power is varied, the maximum amount of power that can be transferred to a particular bus is the value of the real power such that the two circles (in a three bus example) are at their tangency point. The geometry of the problem and two three bus examples illustrate this approach.
Chapter 6 summarizes the contributions of this thesis and discusses further areas of research.
Chapter 2

Review of Scattering Parameters/ Matrices

In this chapter, the basic definitions of scattering parameters and matrices will be reviewed for both real and complex normalizations. Given these definitions, expressions for the real and reactive power delivered to a load will be derived. It will be shown that these expressions can be represented geometrically as circles in the scattering plane. Thus, solutions to the network problem will be obtained from the intersection points of the circles. If there are no intersection points, then there is no solution. In particular, examples for two topologies, a one-port and a two-port, will illustrate the application of scattering parameters for obtaining solutions to the network. For the case of the one-port, two different load models, a constant impedance and a constant real and reactive power model, will be explored. In the case of the two-port example, only the real and reactive power model will be illustrated. The cases will serve as a basis to extend the model to three buses which is the content of the next chapter.

2.1 Scattering Approach

Historically, the literature concerning scattering parameters and/or matrices has been divided into two areas: applications to microwave problems and applications to specific net-
Chapter 2. Review of Scattering Parameters/Matrices

Represent by Thevenin Equivalent

\[ V_{G1} \]
\[ Z_{G1} \]
\[ \text{GENERATOR} \]
\[ \text{TRANSMISSION NETWORK} \]
\[ \text{LOAD} \]

**ONE PORT DECOMPOSITION: THEVENIN EQUIVALENT**

GENCO

TRANSCO

LSE

**TERMINAL CONSTRAINT, NETWORK CONSTRAINTS DECOMPOSITION**

Figure 2-1: One-Port and Terminal and Network Constraints Configurations
work problems. Scattering theory has its origin in microwave theory. A review of scattering parameters in microwave theory is given in Appendix A. Many typical problems are concerned with wave propagation on transmission lines with specified terminations. In microwave circuits, the electric and magnetic fields inside a termination are uniquely determined by either the voltage or the current at the terminals. Rather than representing these fields in terms of their voltage and current, the natural representation is the amplitude of the incident and reflected waves. The amplitude and phase of the transverse component of the electric field in the incident wave are designated by “a” such that $\frac{1}{2} aa^*$ represents the average incident power where $a^*$ is the complex conjugate of $a$. The amplitude and phase of the reflected wave are designated by “b”. For any incident or reflected wave, the fields are uniquely defined. Given the definitions of $a$ and $b$, a reflection coefficient, $\Gamma = \frac{b}{a}$, is defined for a particular reference plane or terminal pair. Since there is a connection between the electric field and the magnetic field and the terminal voltage and current, there is likewise a connection between currents and voltages and incident and reflected waves. If “$e$” is a measure of the total transverse electric field, and “$i$”, is a measure of the magnetic field, it can be shown that $e = Ka(1 + \Gamma)$ and $i = \frac{a(1-\Gamma)}{K}$ where $K$ is a proportionality factor since $a$ and $b$ are normalized with respect to power. Thus for microwave circuits, scattering parameters are a natural representation for the fields of the incident and reflected waves of a transmission line terminated in a specified structure [5].

Given the characterization of microwave circuits in terms of fields (voltage and current) and scattering parameters, there is a similar analogy between characterizing linear networks in terms of voltages and currents or scattering parameters. The corresponding scattering variables are termed incident and reflected voltages or incident and reflected currents. These are linear relations between the scattering variables and the true voltages and currents in the circuit. Since the scattering parameters of a network describe the performance of a network under any specified terminating conditions, every linear passive, time-invariant network has a scattering matrix although it may not have an impedance or admittance matrix [10].

The scope of this thesis is the use of scattering parameters in the solution of specified network problems. In particular, scattering parameters have been used to describe the per-
formance of a network under specified termination conditions. They have also been used to obtain characterizations of networks since every linear passive, time-invariant network has a scattering matrix. It is noted that the scattering matrix of an ideal transformer exists while its admittance/impedance does not. Particular applications of how scattering parameters and matrices have been used include calculation of the noise figure in negative-resistance amplifiers [6] and classification of lossless three-ports [7]. Much of the scattering parameter/matrix literature is devoted to problems of physical realizability and the synthesis of ideal transformer networks and also applications of scattering parameters to network synthesis. There are also a number of specific examples in the literature [9, 10] where scattering parameters/matrices are used in determining the power transfer from a complex generator (defined as a voltage source with source impedance) to an impedance load. The methodology is to match-terminate at each port, i.e. choosing a normalizing impedance such that there is no incident wave at that port. For example, in the case of a two-port, described by a scattering matrix \( S \), match terminating at ports 1 and 2 means that there are no incident waves at ports 1 and 2. Under these conditions, \( |S_{21}|^2 \), is interpreted as the forward transducer power gain which is the ratio of actual load power to the maximum power available from the generator when both ports are matched. Since match-termination is defined for the specified load, if the load changes the port has to be match-terminated again. Rather than resolving the entire problem, Youla and Patterno [8], developed a methodology to modify the elements of the scattering matrix. However, these approaches can not be directly applied to power systems since in most cases the loads are not defined as constant impedance but as constant real and reactive power (PQ) loads. These approaches do serve as a starting point to gain intuition in defining electric power problems in terms of scattering parameters and matrices.

In this thesis, scattering parameters/matrices will be applied in a different context, namely using what will be described as a scattering approach to obtain steady state solutions (equilibria) in electric power networks. To understand the ramifications of this undertaking, the results of applying the scattering approach to simple electric circuits need to be studied in order to link the V-I domain to the scattering domain. In this chapter, the scattering
parameter and matrix is reviewed and applied to both one- and two-port examples.

The scattering parameter approach involves "transforming" the circuit from the V-I domain to the scattering domain via the scattering transformation. Circuit parameters expressed in a V-I context are mapped into a scattering parameter formulation from which real and reactive power and voltage can be extracted. In the one-port example, the approach involves solving the problem at a specified point in the circuit, i.e. at the load or at the generator and assuming a Thevenin equivalent as seen by that point. For the purposes of this discussion, it is assumed that the real and reactive power delivered to the load and the voltage across the load are desired so that the point of reference will be the load.

In this and the next chapter, how the network topology is decomposed, either as a one-port structure or separated by loads and generators from the transmission network, will play an important role not only in the interpretation of the scattering parameter but also in the formulation of the problem. As shown in Figure 2-1, the network consisting of one generator, one load and a transmission network can be modeled either as a Thevenin equivalent driving the load or the generator and load can be viewed as terminal constraints on the transmission network which itself is viewed as the network constraints. It is noted that in the current deregulated vernacular, the generator is defined as a GENCO, the transmission system as a TRANSCO, and the load as a Load Serving Entity (LSE) as shown in Figure 2-1. This decomposition can be extended to multi-ports, however decomposing the network into the terminal and network constraints decomposition lends itself to a more manageable structure.

For the purposes of illustration, the advantages of the one-port structure will be reviewed and two examples using different load models will be shown. With this type of topology, the scattering parameter has a physical interpretation. This can be contrasted to the terminal/network decomposition where the scattering parameter of each port may or may not have a physical interpretation. In particular, the scattering parameter at each port can be interpreted as being a local measure as opposed to the one-port structure where the scattering parameter contains both system and local information. It is this attribute that makes the scattering parameter such a powerful approach in solving these problems. As will be detailed in the sections that follow, system information in the form of the Thevenin
impedance is encoded into the scattering parameter for the one-port formulation. If the problem is defined in a Thevenin structure, the scattering parameter also captures local information, namely the voltage and real and reactive powers at the point of interest. Thus, two independent local variables can be extracted from the scattering parameter via predetermined expressions. To illustrate the advantage of this approach, consider the circuit in Figure 2-2 and assume that the load is described by an impedance $Z_L$. $Z_L$ is defined as the ratio of voltage to current so knowledge of one of these variables is necessary to calculate the real and reactive power delivered to $Z_L$. The voltage across the load is obtained via a voltage divider which is a direct result of the Thevenin structure of the problem. However, the voltage or real and reactive powers cannot be obtained strictly from knowledge of $Z_L$ and likewise knowledge of the voltage does not imply knowledge of $Z_L$ or the real and reactive powers. $Z_L$, voltage, real and reactive powers are all local variables in the V-I domain and contain no system information. By using the scattering approach in the one-port formulation, local information such as the voltage across the load and real and reactive powers delivered to the load is captured in a single parameter for which there is no analog in the V-I domain. Since the scattering parameter formalization depends on the type of load model used, i.e. constant impedance or constant PQ, two different strategies for solving the problem will be derived. For the case of a constant impedance load, a standard approach of applying the definitions of the scattering parameter is used to calculate the real and reactive power delivered to the load and the voltage across the load. For the PQ load, specifying the real and reactive power delivered to the load defines the scattering parameter(s) through which the voltage(s) across the load can be determined. If no scattering parameter can be constructed, there is no solution.

### 2.2 Review of the Scattering Parameter

The scattering parameter is by definition a ratio of reflected voltage “b” to an incident voltage “a” where the reflected and incident “voltages” are linear combinations of the currents and voltages that have been normalized by a normalizing impedance. Depending on the type of normalization used, real ($Z_o = R_o$) or complex ($Z_o = R_o + jX_o$), two different
"definitions" and uses of the scattering parameter result. For each type of normalization, a set of equations describing the relationship between the scattering parameter and real and reactive power and voltage will be derived. For these derivations it is assumed that the load is a constant impedance and that the resulting expressions can be applied to a PQ load, the details of which are given in the following section.

In the following two sections, the scattering parameter for a one-port, assuming real and complex normalization, will be derived assuming the network configuration shown in Figure 2-3.

2.2.1 Real Normalization

The network shown in Figure 2-3 consists of a one-port driven by a Thevenin equivalent. The incident and reflected voltages designated as a and b in the figure are defined as:

\[ a = \frac{1}{2} \left[ \frac{V_L}{\sqrt{R_o}} + I_L \sqrt{R_o} \right] \]
Figure 2-3: One-Port Structure

\[ b = \frac{1}{2} \left( \frac{V_L}{\sqrt{R_o}} - I_L \sqrt{R_o} \right) \]

where \( V_L \) and \( I_L \) are the actual voltage across and current into the load. For the case where the one-port is modeled as a constant impedance \( Z_L \), the scattering parameter \( s \) is defined to be

\[ s = \frac{b}{a} = \frac{Z_L - Z_o}{Z_L + Z_o} = \Gamma. \]

It is noted that since \( Z_o \) is real, the above expression is the same as the reflection coefficient \( \Gamma \) for transmission lines. \( \Gamma \) is used in determining the voltage and current at any point along a transmission line modeled with distributed parameters. In the literature, the above definition is the most common and in many cases \( R_o \) is taken to be one. Given the network configuration in Figure 2-3, the normalization \( Z_o \) is set equal to \( Z_{GEN} \).

Solving the above equations of \( a \) and \( b \) for \( V_L \) and \( I_L \) yields

\[ V_L = \frac{R_o}{\sqrt{R_o}} \left[ a + b \right] \]

and

\[ I_L = \frac{|a - b|}{\sqrt{R_o}}. \]
Since $b = sa$ the above expressions for $V_L$ and $I_L$ become

$$V_L = \frac{a}{\sqrt{R_o}} [1 + s]$$

and

$$I_L = \frac{a}{\sqrt{R_o}} [1 - s].$$

The complex power delivered to the load is $V_L I_L^* = P_L + jQ_L$ where * denotes the complex conjugate. Using the above expressions for the voltage and the current, the relationships for the real and reactive power as a function of the scattering parameter are

$$P_L = P_o (1 - |s|^2)$$

and

$$Q_L = 2 P_o s_y$$

where $s_y$ is the imaginary part of $s$ and $P_o$ is the power delivered to the load under matched conditions, that is when the load impedance equals the complex conjugate of the Thevenin impedance as seen by the load [11]. The equation relating $P_L$ to the scattering parameter is a circle centered at the origin with radius equal to $\sqrt{1 - \frac{P_L}{P_o}}$ while the relationship between $Q_L$ and the scattering parameter is a horizontal line in the scattering plane. Since the normalization is the real part of the Thevenin impedance, the voltage across the load is

$$V_L = \frac{V_{GEN}}{2\sqrt{R_o}} (1 + s).$$

### 2.2.2 Complex Normalization

Assuming the network configuration in Figure 2-3, the definitions for the incident and reflected voltages, assuming complex normalization are as follows:

$$a = \frac{1}{2} \left[ \frac{V_L}{\sqrt{R_o}} + \frac{I_L Z_o}{\sqrt{R_o}} \right]$$

and

$$b = \frac{1}{2} \left[ \frac{V_L}{\sqrt{R_o}} - \frac{I_L Z_o^*}{\sqrt{R_o}} \right]$$
where $V_L$ and $I_L$ are the actual voltage across and current into the load. The scattering parameter for a constant impedance $Z_L$ is

$$s = \frac{b}{a} = \frac{Z_L - Z_o^*}{Z_L + Z_o}.$$ 

This definition of the scattering parameter incorporates the concept of maximum power transfer. Under complex normalization, the scattering parameter measures the degree of mismatch from maximum power transfer. If $Z_o = Z_{GEN}$, then $s = 0$ represents maximum power transfer [11]. Both definitions result in slightly different methods for solving power problems. Rewriting the above expressions for $V_L$ and $I_L$ in terms of $b$ and $a$ yields

$$V_L = \frac{Z_o^*a + Z_o b}{\sqrt{R_o}}$$

and

$$I_L = \frac{a - b}{\sqrt{R_o}}.$$ 

Since $b = sa$, the above expressions for $V_L$ and $I_L$ become

$$V_L = \frac{a[Z_o^* + Z_os]}{\sqrt{R_o}}$$

and

$$I_L = \frac{a[1 - s]}{\sqrt{R_o}}.$$ 

Now $V_L I_L^* = P_L + jQ_L$ therefore

$$P_o(1 -|s|^2) = P_L \quad (2.3)$$

$$(s_x - 1)^2 + (s_y - c)^2 = c^2 - \frac{Q_Lc}{P_o} \quad (2.4)$$

where $c = \frac{R_o}{X_o}$ and $s_x$ and $s_y$ are the real and imaginary parts respectively of $s$. The expression for the voltage across the load is

$$V_L = \frac{a}{\sqrt{R_o}}(Z_o^* + Z_os).$$ 

The equation relating $P_L$ to the scattering parameter is the same as that obtained for the
real normalization case whereas the relationship between \( Q_L \) and the scattering parameter is now a circle centered at \((1, c)\) with radius \( \sqrt{c^2 - \frac{Q_L}{P_0}} \).

2.3 Solution Existence - Geometric Approach Via Scattering Parameters

As derived in the previous section, the expressions for the real and reactive power delivered to the load and also the voltage across the load map into circles and lines in the scattering plane depending on the normalization. Thus a problem previously described in the V-I domain can now be portrayed by a geometric display in the scattering plane where existence of solutions is determined by points of intersections of circles.

The expression for real power delivered to the load is independent of the normalization as evidenced by equations (2.1) and (2.3) and in both cases is represented as a circle centered at the origin in the scattering plane with a radius of \( \sqrt{1 - \frac{P_L}{P_0}} \). It is noted that \( P_L \) must be less than \( P_0 \) which implies that the load be passive. The expression for reactive power delivered to the load, however, depends on the type of normalization. In the case of real normalization, \( Q_L \) is proportional to a horizontal line in the s plane (equation (2.2)). If the line of reactive power is tangent to the real power circle or lies inside it, there will be one or two solutions respectively. If the reactive power line lies outside the circle, there is no solution. This geometric interpretation also establishes bounds, either from the perspective of the real power circle or from the perspective of the reactive power line. For complex normalization, \( Q_L \) is related to a circle centered at \((1, c)\) with radius \( \sqrt{c^2 - \frac{Q_L}{P_0}} \) (equation (2.4)). The existence of solutions is determined by overlapping circles and points of tangency. Again bounds for existence can be established either from the perspective of the real power circle or from the perspective of the reactive power circle.

To incorporate the voltage across the load into the geometric presentation, the expression for the voltage in the case of real normalization is rewritten as

\[
|V_L|^2 = P_0|1 + s|^2
\]
which is a circle centered at \((-1, 0)\) with radius \(|V_L|^2 / P_o\). In the case of complex normalization,

\[ |V_L|^2 = P_o |Z_o^* + Z_o s|^2 \]

which is a circle centered at \(\left(\frac{Z_o^*}{Z_o}, 0\right)\) with radius \(|V_L|^2 / P_o|Z_o|^2\).

By mapping the problem from a V-I framework to a scattering framework, a geometric perspective is obtained which indicates whether a solution exists or not. In the case of existence, solution(s) are points of intersection which define the scattering parameters and for the given normalization allow the determination of the real and reactive power(s) and load voltage(s). If a solution does not exist, the previous derivations give insight as to which parameters can be changed to obtain points of intersection resulting in solutions to the problem.

The aforementioned results apply directly to constant impedance loads. For the case of the PQ load, the real and reactive power circles will be obtained via the complex normalization case.

### 2.4 One-Port Scattering Parameter Examples

To illustrate the scattering parameter approach in solving electric power networks, the scattering parameter approach will be applied to a simple electric power network. Two different types of load models, a constant impedance model and a constant PQ load, will be considered. A network of one generator and one load as shown in Figure 2-2 will be used assuming a complex normalization. The transmission line connecting the generator to the load has a line resistance of \(R_o\) and reactance of \(X_o\). The generator is modeled as an ideal voltage source whose magnitude \(V_G\) and angle are fixed. Solutions for both the constant impedance and constant PQ loads will be derived. In both cases a generator of 1 volt and angle zero and a transmission line of \(1 + j10\) is assumed. Thus \(V_G = 1, R_o = 1, X_o = 10\) and \(c = 0.1\). Thus \(Z_o = Z_{GEN} = 1 + j10\).

- **Constant Impedance Load (Complex Normalization)** \(Z_L = 1 - j6\)
2.4. One-Port Scattering Parameter Examples

- Method of Solution: For a given $Z_L$ compute $s$ at the load

$$ s = \frac{b}{a} = \frac{Z_L - Z_o^*}{Z_L + Z_o} = 0.8 + j0.4 $$

- Given $s$ and the normalization $Z_o$, calculate $P_L, Q_L$ and $V_L$ directly

$$ P_o(1 - |s|^2) = P_L = 0.05 $$

$$ \frac{P_o}{c}(c^2 - (s_x - 1)^2 - (s_y - c)^2) = Q_L = -0.3 $$

$$ \frac{a}{\sqrt{R_o}}(Z_o^* + Z_o s) = V_L = -1.1 - j0.8 $$

The resulting circles relating $P_L, Q_L,$ and $V_L$ are shown in Figure 2-4.

- Constant PQ Load (Complex Normalization) $P_L = 0.05$ $Q_L = -0.3$

- Method of Solution: Given $P_L, Q_L$ and the normalization, construct the real and reactive power circles and determine the intersection points.

$$ P_o(1 - |s|^2) = P_L = 0.05 $$

$$ \frac{P_o}{c}(c^2 - (s_x - 1)^2 - (s_y - c)^2) = Q_L = -0.3 $$

- There are two intersection points, $A$ and $B$, of the $P$ and $Q$ circles as shown in Figure 2-5.

$$ s_A = 0.8 + j0.4 $$

$$ s_B = 0.8633663 - j0.2336634 $$

$$ \frac{a}{\sqrt{R_o}}(Z_o^* + Z_o s_A) = V_{LA} = -1.1 - j0.8 $$
Chapter 2. Review of Scattering Parameters/Matrices

Figure 2-4: One-Port Example: Constant Impedance Load

Figure 2-5: One-Port Example: Constant PQ Load
\[
\frac{a}{\sqrt{R_o}}(Z_o^* + Z_o s_B) = V_{LB} = 2.1 - j0.8
\]

The resulting circles relating \( P_L, Q_L, \) and \( V_L \) are shown in Figure 2-5.

### 2.5 Two-Port Structure

In this section, the scattering formulation for a specified two-port structure will be derived. For the one-port examples, the Thevenin structure of the problem was exploited which resulted in a simple, straightforward formulation of the problem in the scattering parameter domain. The natural choice for normalization was the Thevenin impedance. However, for the two-port case, a Thevenin structure is no longer assumed. The two-port structure can be viewed as consisting of two parts, the network constraints, characterized by the scattering matrix of the transmission network and the terminal constraints characterized by the generator and for the example chosen, a PQ load. The objective of this section is to derive the expressions for the real and reactive power at port 2 as a function of the parameters of the network as was done in the one-port case. This approach differs from the one-port case in that the reflected voltage at port 2 is no longer a function of the scattering parameter at port 2 and the incident voltage at port 2. It will be shown that the reflected voltage is not only a function of its own scattering parameter but also the transmission network along with the maximum power delivered to the network. Drawing an analogy with the one-port case, the characteristics of the transmission network were “summarized” by using a Thevenin structure whereas in the two-port case, the reflected voltage at port 2 will be an explicit function of the individual elements of the scattering matrix.

In the two-port formulation, the transmission network is described by a scattering matrix. The scattering matrix for an impedance or admittance description of the network is derived in the next section.
2.5.1 Derivation of the Scattering Matrix

In this section, the scattering matrix of a network, either described by an impedance or an admittance characterization will be derived. The formalization of the two-port example is fundamentally different from that of the one-port and can be extended to multiports as will be shown in the next chapter for the three bus power network example. In the two-port case, the transmission network represents the network constraints described by \( b = Sa \) where \( b \) is a vector of reflected voltages at ports 1 and 2, \( a \) is a vector of incident voltages at ports 1 and 2 and \( S \) is the scattering matrix. The load at port 2 is viewed as a terminal constraint as is the generator at port 1. Figure 2-6 illustrates the network.

The scattering matrix is a linear transformation between the reflected and incident voltages at each of the ports, i.e. \( b = Sa \). The derivation for \( S \) is as follows:

- Augment the n-port with impedances \( Z_{o1} \ldots Z_{on} \) and voltage sources \( V_{G1} \ldots V_{Gn} \)
• Define $\tilde{Y}_A$, the admittance matrix of this augmented networks as $I = \tilde{Y}_AV_G$ ($I$ and $V_G$ are column vectors of port currents and generator voltages).

• From KVL, $V_G = V + Z_o I$ where $Z_o$ is a diagonal matrix defined as $Z_o = \text{diag}(Z_{o1} \ldots Z_{on})$

• Define $a$ and $b$

$$2R_o^{1/2}a = V + Z_o I$$
$$2R_o^{1/2}b = V - Z_o^* I$$

where $R_o$ is a diagonal matrix defined as $R_o = \text{diag}(\text{Real}(Z_{o1})^{1/2} \ldots \text{Real}(Z_{on})^{1/2})$

• Rewrite $V - Z_o^* I$:

$$V - Z_o^* I = V + Z_o I - 2RI$$
$$= (E - 2R_o\tilde{Y}_A)(V + Z_o I)$$

where $E$ is the identity matrix.

• Rewrite the above expression for $b$:

$$b = R_o^{-1/2}[E - 2R_o\tilde{Y}_A]R_o^{1/2}a$$

• Finally, $S = E - 2R_o^{1/2}\tilde{Y}_A R_o^{1/2} = E - 2Y_A$ where $Y_A$ is the normalized augmented admittance matrix.

2.5.2 Two-Port Formulation

The structure of the two-port example is shown in Figure 2-7. As was the case for the one-port, the incident and reflected voltages at port $k$ are defined as

$$a_k = \frac{1}{2}\left[\frac{V_k}{\sqrt{R_{ok}}} + \frac{I_k Z_{ok}}{\sqrt{R_{ok}}}\right]$$

(2.5)
where $Z_{ok}$ is the normalization impedance which in general is different at each port. Because of the Thevenin structure of the one-port example, the normalization impedance was chosen to be the Thevenin impedance. In this chapter, for the two-port example, the normalization impedances will be chosen arbitrarily. At port 1, the generator is modeled as a voltage source with a series impedance. The terminal constraint is $V_1 = V_{G1} - Z_{G1} I_1$. Substituting $V_1 = z\frac{a_1}{2\sqrt{R_{ok}}} + \frac{b_1}{\sqrt{R_{ok}}}$ and $I_1 = \frac{a_1}{\sqrt{R_{oz}}} + \frac{b_1}{\sqrt{R_{oz}}}$ into the terminal constraint yields the scattering parameter representation of the generator at port 1, $a_1 = \Gamma_1 b_1 + a_{S1}$ where $\Gamma_1 = \frac{Z_{G1} - Z_{G1}}{Z_{G1} + Z_{G1}}$ and $a_{S1} = \frac{V_{G1} \sqrt{R_{ok}}}{Z_{G1} + Z_{G1}}$. Note that $\Gamma_1$ is not the same as the $\Gamma$ described for the real normalization. If $Z_{o1} = Z_{G1}$, then $\Gamma_1 = 0$, $a_1 = a_{S1} = \frac{V_{G1}}{2\sqrt{R_{oz}}}$ and $|a_1|^2 = \frac{|V_{G1}|^2}{4R_{oz}} = P_0$. Note that the maximum power the generator at port 1 can deliver to the network is $\frac{|V_{G1}|^2}{4R_{G1}}$. (Maximum power transfer occurs when $Z_{IN} = Z_{G1}^*$ [11]). Under this constraint, the maximum power transfer is $\frac{|V_{G1}|^2}{4R_{G1}}$. Interpreting this in terms of the scattering parameter, there is no reflected
wave. At port 2, the load is modeled as a PQ bus. For the purposes of the derivation of the terminal constraint, assume an equivalent load impedance $Z_{L2}$ at port 2. As shown in Figure 2-7, since the load is the terminal constraint, the reference directions for the incident and reflected voltages and the current and voltage at port 2 are defined relative to the two-port, i.e. the transmission network. The current at port 2 is defined as going into the transmission network, not into the load. Thus $V_2 = -I_2 Z_{L2}$. Substituting $V_2 = \frac{Z_{o1} a_2 + Z_{o2} b_2}{\sqrt{R_o}}$ and $I_2 = \frac{a_2 - b_2}{\sqrt{R_o}}$ into the terminal constraint yields the scattering parameter representation of the load at port 2,

$$\Gamma_2 = \frac{a_2}{b_2} = -\frac{Z_{L2} - Z_{o2}}{Z_{L2} + Z_{o2}}.$$  \hfill (2.7)

The network constraints are obtained from the scattering matrix of the transmission network. The scattering matrix of the two-port transmission network is defined as follows:

$$b_1 = S_{11} a_1 + S_{12} a_2$$ \hfill (2.8)

$$b_2 = S_{21} a_1 + S_{22} a_2.$$ \hfill (2.9)

To compute the scattering matrix of the transmission network shown in Figure 2-7, normalizing impedances of $Z_{o1}$ and $Z_{o2}$ are assumed for ports 1 and 2 respectively. It can be shown that the augmented matrix for the transmission network shown in the figure is

$$\hat{Y}_A = \begin{bmatrix}
\frac{Z_{o2} + Z_1}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})} & -\frac{Z_1}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})} \\
-\frac{Z_1}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})} & \frac{Z_{o2} + Z_1}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})}
\end{bmatrix}$$

and since

$$R_o = \begin{bmatrix}
R_{o1} & 0 \\
0 & R_{o2}
\end{bmatrix}$$

the scattering matrix of the transmission network, $S = E - 2 R_o^{1/2} \hat{Y}_A R_o^{1/2}$, is

$$S = \begin{bmatrix}
1 - \frac{2 R_{o1} (Z_{o2} + Z_1)}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})} & -\frac{2 \sqrt{R_{o1} R_{o2}}}{} \\
-\frac{2 \sqrt{R_{o1} R_{o2}}}{} & 1 - \frac{2 R_{o2} (Z_{o1} + Z_1)}{Z_{o1} Z_{o2} + Z_1 (Z_{o1} + Z_{o2})}
\end{bmatrix}.$$
The relationship between $b_2$ and $a_1$ (where $|a_1|^2 = P_o$ is the maximum power that the generator can deliver to the network) is obtained by substituting equation (2.7) into equation (2.9). The result is

$$b_2 = \frac{S_{21}a_1}{1 - S_{22}\Gamma_{L2}}. \quad (2.10)$$

Given this expression for $b_2$, expressions can be derived for the real and reactive power at port 2 in terms of the scattering parameter at port 2, elements of the scattering matrix and the maximum power that the generator can deliver to the network. To compute the real and reactive power delivered to the load at port 2, $V_2(-I_2)^* = P_2 + jQ_2$ needs to be computed. Since $a_2 = \Gamma_{L2}b_2$, the expressions for $V_2$ and $-I_2$ become

$$V_2 = \frac{b_2[Z_{o2}\Gamma_{L2} + Z_{o2}]}{\sqrt{R_{o2}}},$$

and

$$-I_2 = \frac{b_2[1 - \Gamma_{L2}]}{\sqrt{R_{o2}}}.$$

Thus the expressions for the real and reactive power at port 2 are

$$P_2 = |b_2|^2(1 - |\Gamma_{L2}|^2) \quad (2.11)$$

and

$$Q_2 = |b_2|^2\left(\frac{X_{o2}}{R_{o2}}(1 + |\Gamma_{L2}|^2) + 2\frac{Im(Z_{o2}\Gamma_{L2})}{R_{o2}}\right). \quad (2.12)$$

Substituting equation (2.10) into equations (2.11) and (2.12) yields

$$P_2 = \frac{|S_{21}|^2P_o(1 - |\Gamma_{L2}|^2)}{|1 - S_{22}\Gamma_{L2}|^2}$$

and

$$Q_2 = \frac{|S_{21}|^2P_o\left(\frac{X_{o2}}{R_{o2}}(1 + |\Gamma_{L2}|^2) + 2\frac{Im(Z_{o2}\Gamma_{L2})}{R_{o2}}\right)}{|1 - S_{22}\Gamma_{L2}|^2}.$$

Rewriting the above equations in terms of the real and imaginary parts of $\Gamma_{L2}$, it can be shown that the resulting expressions are circles in the $\Gamma_{L2}$ plane. Thus,

$$(\Gamma_{x2} - a_2P)^2 + (\Gamma_{y2} - b_2P)^2 = r_{2P}^2.$$
and \((\Gamma_{x_2} - a_2Q)^2 + (\Gamma_{y_2} - b_2Q)^2 = r_2Q^2\)

where

\[
P_2' = \frac{P_2}{|S_{21}|^2 P_o} \quad (2.13)
\]

\[
Q_2' = \frac{Q_2}{|S_{21}|^2 P_o} \quad (2.14)
\]

\[
a_{2P} = P_2'\left[\frac{Re(S_{22})}{|S_{22}|^2 P_2' + 1}\right] \quad (2.15)
\]

\[
b_{2P} = -P_2'\left[\frac{Im(S_{22})}{|S_{22}|^2 P_2' + 1}\right] \quad (2.16)
\]

\[
r_{2P} = \sqrt{\frac{(1 - P_2')}{|S_{22}|^2 P_2' + 1} + a_{2P}^2 + b_{2P}^2} \quad (2.17)
\]

\[
a_{2Q} = \left[\frac{X_{R_{02}} - Q_2' Re(S_{22})}{|S_{22}|^2 Q_2' - \frac{X_{R_{02}}}{R_{02}}}\right] \quad (2.18)
\]

\[
b_{2Q} = \frac{-1}{|S_{22}|^2 Q_2' - \frac{X_{R_{02}}}{R_{02}}} \quad (2.19)
\]

\[
r_{2Q} = \sqrt{\frac{X_{R_{02}} - Q_2'}{|S_{22}|^2 Q_2' - \frac{X_{R_{02}}}{R_{02}}} + a_{2Q}^2 + b_{2Q}^2}. \quad (2.20)
\]

There are two observations to be made concerning the above equations. The first is that the above two-port example could have been reduced to a one-port structure by computing the Thevenin equivalent as seen by the load. However, this separation into network constraints and terminal constraints becomes important as the number of ports increases. This concept of separation clearly shows what effect the transmission network has on the real and reactive power at port 2 as evidenced by equations (2.13) through (2.20). This effect will become even more dominant in the three bus case of the next chapter. The second observation is that the real and reactive power circles at port 2 are functions only of its local variables (the real and reactive powers), the network constraints and the maximum power the generator can deliver to the network. This does not extend to the multi-port case. As will be shown in the next chapter for a three bus power system, the real and
reactive power circles at bus 2 are indeed a function of the scattering parameter at bus 3. The system is coupled, i.e. changes at bus 3 will affect bus 2 and vice versa and may result in no solution to the system. The three bus power system will be formalized and studied in the next chapter.

2.6 Two-Port Example: Constant PQ Load

A numerical example for the two-port network as shown in Figure 2-7 assuming complex normalization was simulated. The data is as follows:

\[ P_L = 0.0043 \quad Q_L = 0.0022 \quad Z_{o1} = 1 + j10 \quad Z_{o2} = 1 + j1 \quad Z_1 = 1 + j2 \quad Z_{LOAD} = 1 + j0.5 \]

\[
\tilde{Y}_A = \begin{bmatrix}
0.0132 - j0.0916 & -0.0152 + j0.0554 \\
-0.0152 + j0.0554 & 0.1674 - j0.2637
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
0.9736 + j0.1833 & 0.0303 - j0.1107 \\
0.0303 - j0.1107 & 0.6651 + j0.5274
\end{bmatrix}
\]

- Method of solution: Given \( P_L, Q_L \), the above scattering matrix and the normalization, construct the real and reactive power circles and determine the intersection points.

\[
\begin{align*}
a_{2P} &= 4.490432e-01 \\
b_{2P} &= -3.560303e-01 \\
r_{2P} &= 4.084710e-01 \\
a_{2Q} &= 1.069136e+00 \\
b_{2Q} &= -1.241152e+00 \\
r_{2Q} &= 1.425651e+00
\end{align*}
\]
2.6. Two-Port Example: Constant PQ Load

There are two intersection points, A and B, of the P and Q circles as shown in Figure 2-8.

\[ s_A = 4.688512e - 01 + j5.196021e - 02 \]
\[ s_B = 5.882353e - 02 - j2.352941e - 01 \]
\[ |V_{LA}| = 0.1216 \]
\[ |V_{LB}| = 0.0736 \]

The resulting circles relating \( P_L \) and \( Q_L \) are shown in Figure 2-8.
2.7 Two-Port Example: Constant Impedance Load - Match Terminated

The use of scattering parameters/matrices in calculating the power transfer from complex generator to impedance load was alluded to in the first section of this chapter. To illustrate this concept of match terminating each port such that the power to the load is \( P_{LOAD} = |S_{21}|^2 P_0 \), the previous example will be reworked. To understand where this relationship came from, the scattering relationships of the two-port are revisited. For a two-port, \( b = Sa \) is

\[
\begin{align*}
    b_1 &= S_{11}a_1 + S_{12}a_2 \\
    \text{and } b_2 &= S_{21}a_1 + S_{22}a_2.
\end{align*}
\]

It is imperative to keep in mind that the scattering matrix is a function of the normalizations so that different normalizations yield different scattering matrices. However, for a specific normalization, the elements of the scattering matrix allow for an intuitive physical interpretation and a simplified solution. Under the case of match termination which means no incident wave into the two-port or no reflected wave from the one-port (see Figure 2-7), the elements of the scattering matrix can be interpreted as follows: \( S_{11} = \frac{b_1}{a_1}|_{a_2=0} \) \( S_{12} = \frac{b_1}{a_2}|_{a_1=0} \) \( S_{21} = \frac{b_2}{a_1}|_{a_2=0} \) \( S_{22} = \frac{b_2}{a_2}|_{a_1=0} \). Examining the expression \( S_{21} = \frac{b_2}{a_1}|_{a_2=0} \), the scattering coefficient \( S_{21} \) is the ratio of the reflected wave into port 2 to the incident wave into port 2 under the constraint that there is no incident wave into port 1. The impedance analogy of this is \( Z_{21} = \frac{V_2}{I_1} \) where \( I_2 = 0 \), i.e. port 2 is an open circuit. However from a power transfer interpretation, all the power injected at port 1 is transferred to port 2 since none of it is reflected back \( (a_2 = 0) \). The expression for the incident wave at port 2 is \( a_2 = \frac{1}{2}[\frac{V_2}{\sqrt{R_{a_2}}} + \frac{b_2 Z_{02}}{\sqrt{R_{a_2}}}] \). If the normalization is chosen equal to the load impedance, then \( V_2 = -I_2 Z_{02} \) and \( a_2 = 0 \). Note that since the currents are defined as positive into the two-port, \( V_2 = -I_2 Z_2 \). From the analysis in Section 2.5.2, it can be shown that if \( Z_{01} = Z_{GEN} \), then \( a_1 = a_{s1} = \frac{V_{GEN}}{2\sqrt{R_{01}}} \) and \( |a_1|^2 = \frac{|V_{GEN}|^2}{4R_{01}} = P_0 \). Since the interest is in power transfers from the complex
generator to the load, \( a_2 = 0 \). Thus, \( b_1 = S_{11}a_1 \) and \( b_2 = S_{21}a_1 \). Now \( |S_{21}|^2 = \frac{|b_2|^2}{|a_1|^2} |a_2=0 \) and \( b_2 = \frac{1}{2} \left[ \frac{V_2}{\sqrt{R_2}} - \frac{I_2 Z_{22}^*}{\sqrt{R_2}} \right] \). Since port 2 is match-terminated, \( Z_{o2} = Z_2 \), it can be shown that \( b_2 = I_2 \sqrt{R_{o2}} = I_2 \sqrt{R_2} \) and that \( |b_2|^2 = |I_2|^2 R_2 = P_2 \). Thus, \( |S_{21}|^2 = \frac{P_2}{P_0} \) or \( P_2 = |S_{21}|^2 P_0 \). These results can also be obtained from equation (2.11) by setting \( \Gamma_2 = 0 \) and \( b_2 = S_{21}a_1 \).

Assuming match termination, the previous two-port example was recalculated.

Constant Impedance Load (Complex Normalization)

\[
Z_{o1} = 1 + j10 \quad Z_{o2} = Z_{LOAD} = 1 + j0.5 \quad Z_1 = 1 + j2
\]

\[
\hat{Y}_A = \begin{bmatrix}
0.0143 - j0.0932 & -0.0234 + j0.0615 \\
-0.0234 + j0.0615 & 0.2201 - j0.2825
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
0.9714 + j0.1863 & 0.0468 - j0.1231 \\
0.0468 - j0.1231 & 0.5598 + j0.5650
\end{bmatrix}
\]

Results: \( P_0 = 0.2500, P_2 = P_{LOAD} = 0.0043 \) and \( Q_2 = Q_{LOAD} = .0022 \).

In comparing this example to the previous one, a number of issues become apparent. First of all, both the augmented admittance matrix \( \hat{Y}_A \) and the scattering matrix \( S \) are different in each example. This is because of the different normalizations used in each example. However, the final results are the same. A second issue has to do with properties of the scattering matrix. The scattering matrix is a property of the n-port, exclusive of its terminations. However in the special case when the ports are terminated in impedances equal to the port normalizations, where the port normalizations were used in calculating both the augmented admittance and scattering matrices, the absolute values of the elements of the scattering matrix can be interpreted in terms of power transfers. In the first example of the constant PQ load, the normalization was chosen arbitrarily. As indicated in Figure 2-8, there are two solutions corresponding to two equivalent impedances. However, to use the methodology of match terminating the load, the system would have to be solved for the
voltages and currents at the load to determine an effective impedance of the load. This defeats the purpose of applying scattering parameters/matrices in solving these types of problems. The second example, a constant impedance load, lends itself to the framework of match terminating the network to solve for the power transferred to the load. Hence the choice of normalization is straightforward.

This use of match-termination pervades the scattering parameter literature as a means of interpreting the absolute values of elements of the scattering matrix in terms of power. This approach is only valid if the loads can be modeled as constant impedances or if measurements were made on a system that cannot be easily characterized by an augmented admittance matrix and in turn a scattering matrix. For the application of scattering parameters/matrices to power systems which are generally characterized by constant real and reactive loads, this methodology is not readily applicable.

Comparison of these two simple two-port network examples brings up an important issue in the application of the scattering approach to power systems. That is, the issue of the “optimal” choice of normalization. Since the incident and reflected waves can be measured locally at each bus in the electric power system, the local scattering parameters computed at each bus would convey information as to how far a bus is from its maximum power transfer point. However, as previously discussed, the question is how to choose the normalization correctly such that the scattering parameter is a valid metric for this information. This issue will be addressed in a later chapter.
Chapter 3

Coordinated Load Flow Problem: Scattering Approach

As discussed in Chapter 1, one of the basic questions in operating and planning electric power systems is concerned with conditions under which a stationary solution exists. For a two bus power system topology closed form solutions do exist, however there are no known closed form solutions for three bus or greater topologies. Thus numerical methods must be employed to compute system equilibria. Many of these numerical approaches i.e. Newton-Raphson, Gauss-Seidel, homotopy etc. assume an initial guess with the assumption that the algorithm converges to a solution. If the solution does not exist, many of the algorithms provide no insight as to why the solution does not exist or which parameters on the system to change to effect a solution.

In this chapter, the first application of scattering parameters to power systems is the centralized load flow, defined as the load flow problem in the literature. It is defined as the centralized load flow problem since all the information about the system is known. This is in contrast to the decentralized load flow problem, described in the next chapter, which solves a load flow type problem with only partial information about the system. The centralized load flow problem will be defined and tested on a three bus example. In particular, three different approaches to solving the load flow in the scattering domain will be derived and verified on a three bus example. In the course of deriving these approaches, a number of
advantages to recasting the power system network problem via scattering parameters and matrices will become apparent. In the case of the centralized load flow formulation, the scattering formulation allows for a geometric representation of the solution space which is not possible for either an admittance or impedance formulation of the problem. Extensions to multi-bus systems will be described.

3.1 General Formulation of Load Flow Problem

The previous chapter reviewed the scattering parameter formulation for both real and complex normalizations and for both a single port and a two-port. This chapter will extend those results to obtain the formulation for the three bus system.

For a power system described by generators, PQ, PV loads and a transmission network, the solution to the system is obtained from a load flow analysis. The load flow is a numerical algorithm that obtains the solutions to the power balance equations defined at each bus in the power system with the exception of the slack bus. That is,

\[
P_i = \sum_{j=1}^{n} |V_i||V_j|\left[g_{ij}\cos(\delta_i - \delta_j) + b_{ij}\sin(\delta_i - \delta_j)\right]
\]  
\[
Q_i = \sum_{j=1}^{n} |V_i||V_j|\left[g_{ij}\sin(\delta_i - \delta_j) - b_{ij}\cos(\delta_i - \delta_j)\right]
\]

where \(P_i\) is the real power at each bus, \(Q_i\) is the reactive power, \(|V_i|\) is the bus voltage magnitude, \(g_{ij}\) and \(b_{ij}\) are the line admittance parameters and \(\delta_i\) is the angle at bus \(i\).

The solution to the load flow yields the voltage magnitude and angle at a PQ bus and the reactive power \(Q\) at a PV bus. However, even though the power balance equations are quadratic in nature, the load flow calculates only one solution. Furthermore since the load flow is a numerical algorithm a solution may or may not be obtained. If a solution does not exist, there is no mechanism other than trial and error to change a load quantity to obtain a solution. Results from the previous chapter suggest that the scattering parameter/matrix formulation is a natural framework in which to cast the problem. Based on the geometric nature of the solution, the scattering/matrix approach yields not only all the solutions to
the load flow but also a mechanism to obtain bounds on the solutions. The result is a unique geometric representation of the solutions to the load flow. At each load bus, either PQ or PV, use of scattering parameters results in circles whose parametric dependence is either the real and reactive power or the real power and the voltage. The intersections of these circles, i.e. none, one or two, give not only the solution to the load flow at each bus but also the bounds on the solution.

There is another approach to formulating the load flow equations in the voltage-current domain. In the voltage-current domain, the port characterizations of the loads are

\[ f_i(I_i, V_i, \text{Parameters}_i) = 0 \text{ for all } i. \]  

(3.3)

The transmission system is characterized by

\[ I = Y_{BUS}V. \]  

(3.4)

The load flow equations are obtained from solving equation (3.3) for \( I_i = f_i^{-1}(V_i, \text{Parameters}_i) \) and then substituting this into equation (3.4) yielding the load flow equations,

\[ \text{diag}(f_i^{-1}(V_i, \text{Parameters}_i))1 = Y_{BUS}V. \]  

(3.5)

The scattering formulation in the multi-bus formulation separates the system into two parts, network constraints and terminal constraints. The terminal constraints are the constraints imposed by the devices at each of the ports, namely generators, PQ, PV or constant impedance loads. The network constraint is defined as the rest of the system minus the ports, which is the transmission network, and is described by its scattering matrix while the terminal constraints are defined in terms of the scattering parameters at each port. The scattering parameter at each port, \( \Gamma_i \), is defined as

\[ \Gamma_i = \frac{Z_{Li} - Z_{oi}}{Z_{Li} + Z_{oi}^*}, \]  

(3.6)

where \( Z_{Li} \) and \( Z_{oi} \) are the load or equivalent load impedance and normalizing impedance.
respectively at each port i. For the purposes of this chapter, the normalizing impedance will be arbitrarily defined as discussed in another section. \( \Gamma_i \) relates the incident and reflected voltages, \( a_i \) and \( b_i \) respectively. Since the voltage and current at each port are functions of the incident and reflected voltages, expressions for the real and reactive power and voltage will be functions of \( \Gamma_i \) at every port in the system, entries of the scattering matrix of the transmission system and the maximum power that a generator can deliver to that port.

### 3.2 Scattering Formulation for a Three Bus System

To illustrate the scattering approach for a multi-bus system, the equations relating the scattering parameters at each port to either a PQ or a PV load will be obtained for a three bus network. In particular, a three bus example consisting of one generator and two PQ loads will be explored.

However, in deriving the load flow formulation, a parallel can be drawn with the voltage-current domain. In the scattering domain, the port characterization in a-b space is \( a = \Gamma b + a_s \), \( a_i = \Gamma_i b_i \), for all \( i \), \( \Gamma = \text{diag}(\Gamma_i) \). For the transmission system, the description is \( b = S a \) and the system characterization: \( b = (I - S \Gamma)^{-1} S a_s \). As in the voltage-current domain, the coupled port-system characterization in a-b space is needed since the unknowns are \( b \) and \( \Gamma \). Thus, the port characterization has the form \( f(\Gamma_i, b_i, \text{Parameters}_i) = 0 \). In particular, the PQ bus characterization of \( f_i(\Gamma_i, b_i, P_i, Q_i) = 0 \) is

\[
P_i = |b_i|^2(1 - |\Gamma_i|^2)
\]

\[
Q_i = |b_i|^2(\frac{X_{ai}}{R_{ai}}(1 + |\Gamma_i|^2) + 2\text{Im}(Z_{ai}^* \Gamma_i))
\]

and the PV bus characterization is

\[
P_i = |b_i|^2(1 - |\Gamma_i|^2)
\]

\[
V_i = \frac{b_i(Z_{ai}^* \Gamma_i + Z_{ai})}{\sqrt{R_{ai}}}
\]

for all \( i \) except the slack bus. The result is a geometric display of the solution to the load
flow represented by the intersection point or points in the scattering domain at each bus in the system.

The previously described equations showing the structure of the load flow in the scattering domain will be derived in the next group of sections. The point of the comparison is to show the similarity of the structure of the load flow equations, namely the separation of the port characterizations and the transmission characterization.

Consider the network shown in Figure 3-1. The transmission network is driven by a generator modeled by a Thevenin equivalent and two loads which can be modeled either by their real and reactive powers or voltage and real power. The scattering parameter/matrix formulation of the problem was initially defined in the previous chapter and will be extended for the three bus case. The transmission network defines the network constraints on the system and the real and reactive power or voltage and real power at buses 2 and 3 define the terminal constraints.

In terms of the scattering parameter/matrix representation, the terminal constraints are
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

defined as \( a = \Gamma b + a_s \) and the network constraints are defined as \( b = Sa \) where \( a \) and \( b \) are the incident and reflective voltages and \( a_s \) is related to the generator voltage. Thus an expression for \( b \) in terms of \( a_s \) is obtained as

\[
b = (I - S\Gamma)^{-1}Sa_s.
\] (3.7)

The transmission network is characterized by \( Y_{BUS} \) which is defined as follows for the three bus example:

\[
Y_{BUS} = \begin{bmatrix}
Y_{12} + Y_{13} & -Y_{12} & -Y_{13} \\
-Y_{12} & Y_{12} + Y_{23} & -Y_{23} \\
-Y_{13} & -Y_{23} & Y_{13} + Y_{23}
\end{bmatrix}
\]

where \( Y_{ij} \) is admittance between node \( i \) and node \( j \) and \( Y_{ii} \) is the negative sum of the admittance connected to node \( i \). The augmented admittance matrix of the transmission network is \( \tilde{Y}_A = (I + Y_{BUS}Z_o)^{-1}Y_{BUS} \) where \( Z_o \) is the normalization matrix and \( I \) is the identity matrix. Thus, the scattering matrix of \( Y_{BUS} \) is \( S = I - 2R_o\tilde{Y}_AR_o \) where \( R_o \) is the square root of the real part of the normalization matrix \( Z_o \).

For the particular three bus example the above equation yields the following expressions for reflected voltage at buses 2 and 3 as a function of the incident voltage \( a_1 \) at bus 1:

\[
b_2 = \frac{S_{21}(1 - S_{33}\Gamma_{L3}) + S_{23}S_{31}\Gamma_{L3}}{(1 - S_{22}\Gamma_{L2})(1 - S_{33}\Gamma_{L3}) - S_{23}S_{32}\Gamma_{L3}\Gamma_{L2}}a_1
\] (3.8)

and

\[
b_3 = \frac{S_{31}(1 - S_{22}\Gamma_{L2}) + S_{32}S_{21}\Gamma_{L2}}{(1 - S_{22}\Gamma_{L2})(1 - S_{33}\Gamma_{L3}) - S_{23}S_{32}\Gamma_{L3}\Gamma_{L2}}a_1.
\] (3.9)

Note that the above equations show that this is a coupled system; \( b_2 \) and \( b_3 \) are functions of both \( \Gamma_{L2} \) and \( \Gamma_{L3} \).

Rewriting these equations to show their explicit dependence on their respective scattering parameters

\[
b_2 = \frac{K_1}{K_A - K_C\Gamma_{L2}}a_1
\] (3.10)

\[
b_3 = \frac{K_3}{K_D - K_C\Gamma_{L3}}a_1
\] (3.11)
3.2. Scattering Formulation for a Three Bus System

where

\[ K_1 = S_{21} + \Gamma_{L3}(S_{23}S_{31} - S_{21}S_{33}) \]
\[ K_A = 1 - S_{33}\Gamma_{L3} \]
\[ K_B = S_{23}S_{32}\Gamma_{L3} \]
\[ K_C = K_B + S_{22}K_A \]
\[ K_3 = S_{31} + \Gamma_{L2}(S_{32}S_{21} - S_{31}S_{22}) \]
\[ K_D = 1 - S_{22}\Gamma_{L2} \]
\[ K_F = S_{23}S_{32}\Gamma_{L2} \]
\[ K_G = K_F + S_{33}K_D. \]

In Figure 3-1, the port currents are defined as flowing into the network, so the expressions for the voltage and current at ports 2 and 3 are as follows:

\[ V_2 = \frac{Z_{ao2}a_2 + Z_{o3}b_2}{\sqrt{R_{o2}}} \quad I_2 = \frac{b_2 - a_2}{\sqrt{R_{o2}}} \quad V_3 = \frac{Z_{ao3}a_3 + Z_{o3}b_3}{\sqrt{R_{o3}}} \quad I_3 = \frac{b_3 - a_3}{\sqrt{R_{o3}}} \]

where \( Z_{o2} \) and \( Z_{o3} \) are the normalizations at ports 2 and 3 and \( R_{o2} \) and \( R_{o3} \) are the real parts of the normalizations. The above expressions for voltage and current can be written explicitly as a function of \( \Gamma_{Li} \):

\[ V_i = \frac{b_i(Z_{oi}\Gamma_{Li} + Z_{oi})}{\sqrt{R_{oi}}} \quad (3.12) \]
\[ \text{and} \quad I_i = \frac{b_i(1 - \Gamma_{Li})}{\sqrt{R_{oi}}}. \quad (3.13) \]

Since \( V_iI_i^* = P_i + jQ_i \) and \( a_i = \Gamma_i b_i \) then

\[ P_i = |b_i|^2(1 - |\Gamma_{Li}|^2) \quad (3.14) \]
\[ \text{and} \quad Q_i = |b_i|^2\left(\frac{X_{oi}}{R_{oi}}(1 + |\Gamma_{Li}|^2) + \frac{2}{R_{oi}}\text{Im}(Z_{oi}^*\Gamma_{Li})\right) \quad (3.15) \]

where \( i \) is the port number and for this example \( i = 2, 3 \). Substituting equations (3.10) and (3.11) which relate \( b_i \) to \( a_i \) into the above equations for \( P_i \) and \( Q_i \), (equations (3.14) and (3.15)) the following expressions are obtained for \( P, Q \) and \( V \) at each port as a function of the scattering matrix of the transmission network, the scattering parameter at each port.
and the maximum deliverable power of the generator to the network \( P_o \):

\[
P_2 = \frac{|K_1|^2 P_{o21}(1 - |\Gamma_{L2}|^2)}{|K_A - K_G \Gamma_{L2}|^2} \tag{3.16}
\]

\[
Q_2 = \frac{|K_1|^2 P_{o21}(\frac{X_{o2}}{R_{o2}}(1 + |\Gamma_{L2}|^2) + \frac{2}{R_{o3}} Im(Z_{o2} \cdot \Gamma_{L2}))}{|K_A - K_G \Gamma_{L2}|^2} \tag{3.17}
\]

\[
|V_2|^2 = \frac{|K_1|^2 P_{o21}|Z_{o2} \cdot \Gamma_{L2} + Z_{o2}|^2}{R_{o2}|K_A - K_G \Gamma_{L2}|^2} \tag{3.18}
\]

\[
P_3 = \frac{|K_3|^2 P_{o31}(1 - |\Gamma_{L3}|^2)}{|K_D - K_G \Gamma_{L3}|^2} \tag{3.19}
\]

\[
Q_3 = \frac{|K_3|^2 P_{o31}(\frac{X_{o3}}{R_{o3}}(1 + |\Gamma_{L3}|^2) + \frac{2}{R_{o3}} Im(Z_{o3} \cdot \Gamma_{L3}))}{|K_D - K_G \Gamma_{L3}|^2} \tag{3.20}
\]

\[
|V_3|^2 = \frac{|K_3|^2 P_{o31}|Z_{o3} \cdot \Gamma_{L3} + Z_{o3}|^2}{R_{o3}|K_D - K_G \Gamma_{L3}|^2}. \tag{3.21}
\]

These equations can easily be shown to be circles in their respective \( \Gamma \) planes. Rewriting the above equations in terms of the real and imaginary parts of \( \Gamma \), \( \Gamma_x \) and \( \Gamma_y \), the following equations describe circles with radius “r” located at “a,b” in the \( \Gamma \) plane:

\[
(\Gamma_{x2} - a_2 P)^2 + (\Gamma_{y2} - b_2 P)^2 = r_2 P^2
\]

\[
(\Gamma_{x2} - a_2 Q)^2 + (\Gamma_{y2} - b_2 Q)^2 = r_2 Q^2
\]

\[
(\Gamma_{x2} - a_2 V)^2 + (\Gamma_{y2} - b_2 V)^2 = r_2 V^2
\]

\[
(\Gamma_{x3} - a_3 P)^2 + (\Gamma_{y3} - b_3 P)^2 = r_3 P^2
\]

\[
(\Gamma_{x3} - a_3 Q)^2 + (\Gamma_{y3} - b_3 Q)^2 = r_3 Q^2
\]

\[
(\Gamma_{x3} - a_3 V)^2 + (\Gamma_{y3} - b_3 V)^2 = r_3 V^2
\]

where

\[
P_2' = \frac{P_{o21}}{|K_1|^2 P_{o21}} \quad Q_2' = \frac{Q_{o21}}{|K_1|^2 P_{o21}} \quad V_2' = \frac{V_{o2}}{|K_1|^2 P_{o21}}
\]

\[
P_3' = \frac{P_{o31}}{|K_3|^2 P_{o31}} \quad Q_3' = \frac{Q_{o31}}{|K_3|^2 P_{o31}} \quad V_3' = \frac{V_{o3}}{|K_3|^2 P_{o31}}
\]

\[
a_2 P = P_2' \left[ \frac{Re(K_C)Re(K_A) + Im(K_A)Im(K_C)}{|K_C|^2 P_2' + 1} \right]
\]
3.2. Scattering Formulation for a Three Bus System

\[ b_{2P} = -P_2' \frac{\text{Re}(K_A)\text{Im}(K_C) - \text{Im}(K_A)\text{Re}(K_C)}{|K_C|^2 P_2' + 1} \]
\[ r_{2P} = \sqrt{\frac{(1 - P_2'|K_A|^2)}{|K_C|^2 P_2' + 1} + a_{2P}^2 + b_{2P}^2} \]
\[ a_{2Q} = \frac{\frac{\bar{X}_{o2}}{R_{o2}} - Q_2'(\text{Re}(K_A)\text{Re}(K_C) + \text{Im}(K_A)\text{Im}(K_C))}{|K_C|^2 Q_2' - \frac{\bar{X}_{o2}}{R_{o2}}} \]
\[ b_{2Q} = \frac{-Q_2'(\text{Re}(K_A)\text{Im}(K_C) - \text{Im}(K_A)\text{Re}(K_C))}{|K_C|^2 Q_2' - \frac{\bar{X}_{o2}}{R_{o2}}} \]
\[ r_{2Q} = \sqrt{\frac{|\bar{X}_{o2}|^2 - V_2'|K_A|^2}{|K_C|^2 Q_2' - \frac{\bar{X}_{o2}}{R_{o2}}} + a_{2Q}^2 + b_{2Q}^2} \]
\[ a_{2V} = \frac{V_2'[\text{Re}(K_A)\text{Re}(K_C) + \text{Im}(K_A)\text{Im}(K_C)] - (R_{o2}^2 - X_{o2}^2)}{|K_C|^2 V_2' - |Z_{o2}|^2} \]
\[ b_{2V} = \frac{V_2'[\text{Re}(K_A)\text{Im}(K_C) - \text{Im}(K_A)\text{Re}(K_C)] - 2X_{o2}R_{o2}}{|K_C|^2 V_2' - |Z_{o2}|^2} \]
\[ r_{2V} = \sqrt{\frac{|Z_{o2}|^2 - V_2'|K_A|^2}{|K_C|^2 V_2' - |Z_{o2}|^2} + a_{2V}^2 + b_{2V}^2} \]
\[ a_{3P} = -P_3' \frac{\text{Re}(K_D)\text{Re}(K_C) + \text{Im}(K_D)\text{Im}(K_C)}{|K_C|^2 P_3' + 1} \]
\[ b_{3P} = -P_3' \frac{\text{Re}(K_D)\text{Im}(K_C) - \text{Im}(K_D)\text{Re}(K_C)}{|K_C|^2 P_3' + 1} \]
\[ r_{3P} = \sqrt{\frac{(1 - P_3'|K_D|^2)}{|K_C|^2 P_3' + 1} + a_{3P}^2 + b_{3P}^2} \]
\[ a_{3Q} = \frac{\frac{\bar{X}_{o3}}{R_{o3}} - Q_3'(\text{Re}(K_D)\text{Re}(K_G) + \text{Im}(K_D)\text{Im}(K_G))}{|K_G|^2 Q_3' - \frac{\bar{X}_{o3}}{R_{o3}}} \]
\[ b_{3Q} = \frac{-Q_3'(\text{Re}(K_D)\text{Im}(K_G) - \text{Im}(K_D)\text{Re}(K_G))}{|K_G|^2 Q_3' - \frac{\bar{X}_{o3}}{R_{o3}}} \]
\[ r_{3Q} = \sqrt{\frac{|\bar{X}_{o3}|^2 - V_3'|K_D|^2}{|K_G|^2 Q_3' - \frac{\bar{X}_{o3}}{R_{o3}}} + a_{3Q}^2 + b_{3Q}^2} \]
\[ a_{3V} = \frac{V_3'[\text{Re}(K_D)\text{Re}(K_G) + \text{Im}(K_D)\text{Im}(K_G)] - (R_{o3}^2 - X_{o3}^2)}{|K_G|^2 V_3' - |Z_{o3}|^2} \]
\[ b_{3V} = \frac{V_3'[\text{Re}(K_D)\text{Im}(K_G) - \text{Im}(K_D)\text{Re}(K_G)] - 2X_{o3}R_{o3}}{|K_G|^2 V_3' - |Z_{o3}|^2} \]
\[ r_{3V} = \sqrt{\frac{|Z_{o3}|^2 - V_3'|K_D|^2}{|K_G|^2 V_3' - |Z_{o3}|^2} + a_{3V}^2 + b_{3V}^2} \]
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

It is noted that equations (3.16) through (3.21) can be rewritten to take advantage of the structure in equations (3.8) and (3.9). In particular, for bus \( i \), equations (3.8) and (3.9) can be written in general as

\[
b_i = \frac{S_{i,igen} + \Gamma_j \det M_{igen,i}}{1 - S_{j,j} \Gamma_j - \Gamma_i (S_{i,i} - \Gamma_j \det M_{i,1})} a_1
\]

where \( i = i_{bus} \), the bus of interest, \( igen \) is the generator bus, \( j \neq i \neq igen \), \( M_{igen,i} \) is a minor of the scattering matrix \( S \) formed by deleting row \( igen \) and column \( i \) of \( S \) and \( M_{i,1} \) is also a minor of the scattering matrix \( S \) formed by deleting row \( i \) and column 1 of \( S \). Now assume a specific form for the real and reactive power expressions at bus \( i \), namely

\[
P_i = \frac{|N_1|^2 P_{oii}(1 - |\Gamma_{Li}|^2)}{|D_1 - D_2 \Gamma_{Li}|^2}
\]

and \( Q_i = \frac{|N_1|^2 P_{oii}(X_{oii}^2(1 + |\Gamma_{Li}|^2) + \frac{2}{R_{oii}} Im(Z_{oii} \Gamma_{Li}))}{|D_1 - D_2 \Gamma_{Li}|^2} \).

Then based on the structure of equation (3.22),

\[
N_1 = S_{i,igen} + \Gamma_j \det M_{igen,i}
\]

\[
D_1 = 1 - S_{j,j} \Gamma_j
\]

\[
D_2 = S_{i,i} - \Gamma_j \det M_{i,1}
\]

where it is noted that \( N_1, D_1 \) and \( D_2 \) are not functions of either \( \Gamma_j \) or \( \Gamma_i \), but are numbers. For the given \( \Gamma_j \) and scattering matrix \( S \), the expressions for \( N_1, D_1 \) and \( D_2 \) yield numerical values.

Given the above form for the real and reactive power, then regardless of the number of buses on the system,

\[
a_{iP} = \frac{P_i'[Re(D_2)Re(D_1) + Im(D_1)Im(D_2)]}{|D_2|^2 P_i' + 1}
\]

\[
b_{iP} = -P_i'[Re(D_1)Im(D_2) - Im(D_1)Re(D_2)] \quad (3.26)
\]
3.2. Scattering Formulation for a Three Bus System

\[ r_{iP} = \sqrt{\frac{(1 - P'_i|D_1|^2)}{|D_2|^2P'_i + 1} + a_{iP}^2 + b_{iP}^2} \quad (3.27) \]
\[ a_{iQ} = -\frac{\{X_{ai} - Q'_i(Re(D_1)Re(D_2) + Im(D_1)Im(D_2))\}}{|D_2|^2Q'_i - \frac{X_{ai}}{R_{ai}}} \quad (3.28) \]
\[ b_{iQ} = \frac{-Q'_i(Re(D_1)Im(D_2) - Im(D_1)Re(D_2))}{|D_2|^2Q'_i - \frac{X_{ai}}{R_{ai}}} \quad (3.29) \]
\[ r_{iQ} = \sqrt{\frac{X_{ai} - Q'_i|D_1|^2}{|D_2|^2Q'_i - \frac{X_{ai}}{R_{ai}}} + a_{iQ}^2 + b_{iQ}^2} \quad (3.30) \]

As the number of buses increases, the complexity of the expressions for \( b_i \) increases. Since the three bus system may seem too simplistic, let us consider a four bus system. As will be derived in Section 3.5.5, the expressions for \( b_2/a_1 \) and \( b_3/a_1 \) for a four bus system are

\[ b_2 = \frac{S_{21}(1 - S_{33} \Gamma_{L3} - S_{44} \Gamma_{L4}) + S_{31} S_{23} \Gamma_{L3}(1 - S_{44} \Gamma_{L4}) + S_{41} S_{24} \Gamma_{L4}(1 - S_{33} \Gamma_{L3})}{(1 - \Gamma_{L2} S_{22})(1 - \Gamma_{L3} S_{33})(1 - \Gamma_{L4} S_{44}) - S_{23} S_{24} \Gamma_{L2} \Gamma_{L3} - S_{24} S_{42} \Gamma_{L2} \Gamma_{L4} - S_{34} S_{43} \Gamma_{L3} \Gamma_{L4}} \]
\[ a_1 = \frac{S_{31}(1 - S_{22} \Gamma_{L2} - S_{44} \Gamma_{L4}) + S_{21} S_{32} \Gamma_{L2}(1 - S_{44} \Gamma_{L4}) + S_{41} S_{34} \Gamma_{L4}(1 - S_{22} \Gamma_{L2})}{(1 - \Gamma_{L2} S_{22})(1 - \Gamma_{L3} S_{33})(1 - \Gamma_{L4} S_{44}) - S_{23} S_{24} \Gamma_{L2} \Gamma_{L3} - S_{24} S_{42} \Gamma_{L2} \Gamma_{L4} - S_{34} S_{43} \Gamma_{L3} \Gamma_{L4}} \]
\[ b_3 = \frac{S_{31}(1 - S_{22} \Gamma_{L2} - S_{44} \Gamma_{L4}) + S_{21} S_{32} \Gamma_{L2}(1 - S_{44} \Gamma_{L4}) + S_{41} S_{34} \Gamma_{L4}(1 - S_{22} \Gamma_{L2})}{(1 - \Gamma_{L2} S_{22})(1 - \Gamma_{L3} S_{33})(1 - \Gamma_{L4} S_{44}) - S_{23} S_{24} \Gamma_{L2} \Gamma_{L3} - S_{24} S_{42} \Gamma_{L2} \Gamma_{L4} - S_{34} S_{43} \Gamma_{L3} \Gamma_{L4}} \]

As in the case of the three bus system, the structure of the expressions for \( b_i \) can be generalized and the resulting expressions for \( N_1, D_1 \) and \( D_2 \) become

\[ N_1 = S_{i,igen} + \sum_{k \in C, \neq i} (S_{k,igen} S_{i,k} - S_{i,igen} S_{k,k}) \Gamma_k - \sum_{j \in C, \neq i} \Gamma_j \Gamma_k S_{k,igen} S_{i,k} S_{k,k} \quad (3.31) \]
\[ D_1 = 1 - \sum_{j \in C, \neq i} S_{j,j} \Gamma_j - \sum_{j \in C, \neq i} \Gamma_j \Gamma_k (S_{k,j} S_{j,k} - S_{k,k} S_{j,j}) \quad (3.32) \]
\[ D_2 = S_{i,i} \sum_{k \in C, \neq i} \Gamma_k (S_{i,k} S_{k,i} - S_{i,i} S_{k,k}) - \prod_{j \in C, \neq i} \Gamma_j S_{j,j} \quad (3.33) \]

where \( C \) is the set of all nongeneration buses, i.e. \( C = [2, 3] \). Thus, the real and reactive power for buses in \( C \) are obtained from equations (3.25) through (3.30) with equations (3.31), (3.32) and (3.33) yielding the values for \( N_1, D_1 \) and \( D_2 \).
3.3 Existence of Solutions Via Scattering Parameters

The previously obtained expressions for voltage in equation (3.12) and real and reactive power in equations (3.14) and (3.15) are functions of \( \Gamma_{Li} \). For a specified \( \Gamma_{Li} \), the voltage, real and reactive power can be determined. To establish the idea of solution existence and bounds, the equations are reinterpreted as follows: given the real and reactive power at port \( i \) or the voltage and real power at port \( i \), obtain the values of \( \Gamma_{Li} \) that satisfy both constraints. If only one value of \( \Gamma_{Li} \) satisfies both constraints, this defines bounds on either \( P \) or \( Q \), whereas if no values of \( \Gamma_{Li} \) satisfy both constraints, \( \Gamma_{Li} \) is undefined, i.e., it does not exist. The assumption is that existence of \( \Gamma_{Li} \) at a particular port implies that a solution to the load flow exists whereas non-existence of \( \Gamma_{Li} \) implies that a solution to the load flow equations does not exist. In particular, consider equations (3.14) and (3.15); if the \( P \) and \( Q \) circles intersect, the point or points of intersection define a unique solution set for \( \Gamma_{Li} \). That is, the \( P \) and \( Q \) circles intersect at either one or two points in the \( \Gamma_{Li} \) plane which defines \( \Gamma_{Li} \) at that port. For the given \( \Gamma_{Li} \), the voltage(s) at that port can be determined using equation (3.12). Thus equations (3.14) and (3.15) constitute a parametric description of all possible solutions in terms of the real and reactive power at each port. This establishes a direct link between variations in the real and reactive power and solution existence/non-existence. Thus if there is no intersection of the \( P \) and \( Q \) circles, no \( \Gamma_{Li} \) exists and hence no solutions to the load flow exist. The same argument applies to a PV load.

3.4 Choice of Normalization

As previously stated in this chapter, the normalization will be chosen arbitrarily. All of the equations that have been derived so far, i.e. port voltages and currents and real and reactive powers and the scattering matrix of the transmission network are functions of the normalization. This works both for and against the application of scattering parameters in obtaining solution existence and bounds. The definition of \( \Gamma_{Li} \) in equation (3.6) involves the port impedance and a port normalizing impedance. However, the bus constraints are defined in terms of either \( P \) and \( Q \) or \( P \) and \( V \) and neither representation allows for the
direct calculation of an equivalent port impedance. If the port impedance were known, then for a specified normalization impedance, \( \Gamma \) at that port could be calculated directly. In the one and two-port examples in the previous chapter, there was no need to calculate an equivalent port impedance; the solution could be obtained directly from the intersection of the \( P \) and \( Q \) circles. However, the reason this could be done was the simplicity of the topology. There was no coupling to the other ports. In the three bus formulation, the system is coupled, i.e. the expressions for \( P_2 \) and \( Q_2 \) are not only functions of \( \Gamma_{L_2} \) but also \( \Gamma_{L_3} \) (this is because \( b_2 \) and \( b_3 \) in equations (3.10) and (3.11) are functions of both \( \Gamma_{L_2} \) and \( \Gamma_{L_3} \)). Thus \( \Gamma_{L_3} \) affects the solution existence at port 2 and \( \Gamma_{L_2} \) affects the solution existence at port 3.

In this chapter the normalization will be chosen using the prespecified data. For the load flow formulation a “base case” will define the normalization. Rather than assigning an arbitrarily defined normalization, a base case which specifies the real and reactive power at each of the load buses will serve as a “nominal reference point”, with the normalization chosen such that \( \Gamma_{L_2} \) and \( r_{L_3} \) are both zero. Further numerical examples will be defined in terms of deviations of the loads from this base case.

### 3.5 Load Flow Formulations in the Scattering Domain

In the V-I domain, (voltage, current space), a solution to the load flow equations is obtained numerically via a Newton-Raphson approach. The power balance equations are obtained at each bus (except the slack bus) and are written in the form \( F(X) = 0 \). The roots \( X \) of this equation are the solutions to the load flow equations. In this section the load flow equations in terms of scattering parameters will be derived and solved for a specified case. However, an important distinction exists between the scattering domain and the V-I formulations. In the V-I domain, the unknown quantities at a PQ bus are the voltage magnitude and angle while at a PV bus, the unknowns are the reactive power and the voltage angle. In the scattering representation, the unknown is always \( \Gamma \) regardless of whether it is a PQ or PV bus. Thus the \( X \)'s are always the real and imaginary parts of \( \Gamma \) at each bus.

As in the V-I domain, the structure of the load flow equations in the scattering domain
can be easily extended for higher order systems. It will be shown in another section how the basic form of the load flow equations is structurally the same regardless of the number of buses. However, as the number of buses increases, the detailed structure of the load flow equations is easily obtained from the linear transformation $b = Sa$.

The load flow equations in the scattering domain are equations (3.14) and (3.15). Since the three bus system shown in Figure 3-2 has two PQ loads and a slack bus, the load flow equations for buses 2 and 3 written as a function of $\Gamma_{L2}$ and $\Gamma_{L3}$ and elements of the scattering matrix $S$ are

$$P_2 = P_{21} \frac{|S_{21} + \Gamma_{L3}K_1|^2(1 - |\Gamma_{L2}|^2)}{|1 - S_{22}\Gamma_{L2} - S_{33}\Gamma_{L3} + K_2\Gamma_{L2}\Gamma_{L3}|^2}$$  \hspace{1cm} (3.34)

$$Q_2 = Q_{21} \frac{|S_{21} + \Gamma_{L3}K_1|^2(X_{sc2}^2(1 + |\Gamma_{L2}|^2) + 2Im(\Gamma_{L2}) - 2X_{sc2}Re(\Gamma_{L2}))}{|1 - S_{22}\Gamma_{L2} - S_{33}\Gamma_{L3} + K_2\Gamma_{L2}\Gamma_{L3}|^2}$$  \hspace{1cm} (3.35)

$$P_3 = P_{31} \frac{|S_{31} + \Gamma_{L2}K_3|^2(1 - |\Gamma_{L3}|^2)}{|1 - S_{22}\Gamma_{L2} - S_{33}\Gamma_{L3} + K_2\Gamma_{L2}\Gamma_{L3}|^2}$$  \hspace{1cm} (3.36)

$$Q_3 = Q_{31} \frac{|S_{31} + \Gamma_{L2}K_3|^2(X_{sc3}^2(1 + |\Gamma_{L3}|^2) + 2Im(\Gamma_{L3}) - 2X_{sc3}Re(\Gamma_{L3}))}{|1 - S_{22}\Gamma_{L2} - S_{33}\Gamma_{L3} + K_2\Gamma_{L2}\Gamma_{L3}|^2}$$  \hspace{1cm} (3.37)

where $K_1$, $K_2$ and $K_3$ are as previously defined. Subtracting the right hand side from the left hand side, these equations are of the form $F(X) = 0$. The solution to these equations is the values of $\Gamma_{L2}$ and $\Gamma_{L3}$ that satisfy all four equations.

In this section, three different formulations of the load flow equations will be derived. The first two formulations are Newton-Raphson based while the third formulation exploits the geometry of the scattering domain.

### 3.5.1 Approach I: Terminal/Network Constraint Load Flow Formulation

The previous sections discussed the formulation of the load flow equations in the scattering domain. This formulation parallels the load flow equations in the V-I domain, namely that at each bus the expressions for real and reactive power are written as a function of local variables, voltage and angle, and system variables which are the line parameters. The scattering analogy is that the local variables are the port scattering parameters and normalizations and the system variables are elements of the scattering matrix which describe
3.5. Load Flow Formulations in the Scattering Domain

the transmission network. These equations form the basis for the Newton-Raphson approach to solving the problem.

In the first of the two Newton-Raphson approaches to solving the load flow in the scattering domain, the natural separation into system variables and local variables is exploited. This approach is based on writing the load flow equations as a function of the reflected voltages and the real and reactive powers at each bus. (i.e. equations (3.7), (3.14), and (3.15)). For Newton-Raphson approaches, the problem is cast as $F(X) = 0$ or in the scattering domain, $F(\Gamma) = 0$. Starting with an initial guess for $\Gamma$, the reflected voltages at each bus are calculated from $b = (I - ST)^{-1}Sa$. Since $P_i = |b_i|^2(1 - |\Gamma_i|^2)$ and $Q_i = |b_i|^2(\frac{X_{ao}}{R_{ao}}(1 + |\Gamma_i|^2)) + \frac{2}{R_{ao}}Im(Z_{ao} \cdot \Gamma_i)$, then $F(\Gamma) = 0$ for a three bus example this formulation becomes

$$P_2 - |b_2|^2(1 - |\Gamma_2|^2) = 0$$
$$Q_2 - |b_2|^2(\frac{X_{a2}}{R_{a2}}(1 + |\Gamma_2|^2)) + \frac{2}{R_{a2}}Im(Z_{a2} \cdot \Gamma_2) = 0$$
$$P_3 - |b_3|^2(1 - |\Gamma_3|^2) = 0$$
$$Q_3 - |b_3|^2(\frac{X_{a3}}{R_{a3}}(1 + |\Gamma_3|^2)) + \frac{2}{R_{a3}}Im(Z_{a3} \cdot \Gamma_3) = 0.$$

This is the generalized scattering domain version of equations (3.1) and (3.2).

In this and the next approach, only one solution is generated, i.e. typically the high voltage solution, $(\Gamma_{L2}^o, \Gamma_{L3}^o)$. To obtain all solutions to the load flow, the intersection points of the $P$ and $Q$ circles at each bus are computed. For bus 2, for the given $\Gamma_{L3}^o$, the intersection points of the $P_2$ and $Q_2$ circles are calculated. Likewise, for bus 3, for the given $\Gamma_{L2}^o$, the intersection points of the $P_3$ and $Q_3$ circles are calculated. Simulations illustrating this approach are shown in Section 3.5.4. It is noted that these intersection points can be obtained directly using the third approach.
3.5.2 Approach II: Scattering Voltage and Current Load Flow Formulation

In the second of the two Newton-Raphson approaches to solving the load flow equations in the scattering domain, the load flow equations can be written as $P + jQ = V^T I^*$ or $P + jQ - V^T I^* = 0$. For a three bus system using the definitions of $V_i$ and $I_i$ in equations (3.12) and (3.13) the following is obtained

$$\begin{bmatrix} P_2 \\ P_3 \end{bmatrix} + j \begin{bmatrix} Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} \frac{b_2(\bar{Z}_{o2}^* r_{L2} + Z_{o2})}{\sqrt{R_{o2}}} & 0 \\ 0 & \frac{b_3(\bar{Z}_{o3}^* r_{L3} + Z_{o3})}{\sqrt{R_{o3}}} \end{bmatrix} \begin{bmatrix} \frac{b_2^*(1-\Gamma_{L2})}{\sqrt{R_{o2}}} & 0 \\ 0 & \frac{b_3^*(1-\Gamma_{L3})}{\sqrt{R_{o3}}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} P_2 \\ P_3 \end{bmatrix} + j \begin{bmatrix} Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} \frac{|b_2|^2(\bar{Z}_{o2}^* r_{L2} + Z_{o2})}{R_{o2}}(1-\Gamma_{L2}) & 0 \\ 0 & \frac{|b_3|^2(\bar{Z}_{o3}^* r_{L3} + Z_{o3})}{R_{o3}}(1-\Gamma_{L3}) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$ 

Thus as in the previous approach, start with an initial guess for $\Gamma_i$ and calculate the reflected voltages at each bus calculated from $b = (I - S \Gamma)^{-1} S a_s$. Simulations illustrating this approach are shown in Section 3.5.4. As with the previous approach, to obtain all solutions to the load flow, the intersection points of the $P_i$ and $Q_i$ circles are obtained for bus $i$ assuming $\Gamma_j = \Gamma_j^o$ where $i \neq j$.

3.5.3 Approach III: Geometric Load Flow Formulation

In this formulation the geometry of the problem is exploited. At each bus, the real and reactive powers can be represented as circles in the scattering plane and as discussed earlier, the intersection points of these circles define the solutions to the load flow equations. Rather than computing a solution to the load flow equations using a Newton-Raphson approach and then generating the other solutions to the load flow by computing the intersections of these circles, the idea is to generate the intersection points in real time. In the derivation of the algorithm assume that there are always two intersection points. In the case of the three bus example, equations (3.34) and (3.35) can be represented as $P_2 = P_2(\Gamma_{L2}, \Gamma_{L3})$.
and \( Q_2 = Q_2(\Gamma_{L2}, \Gamma_{L3}, Z_{o2}) \). For a given \( \Gamma_{L3} \) and \( Z_{o2} \), the intersection points of the \( P_2 \) and \( Q_2 \) are defined as \((\Gamma_{L2}^p, \Gamma_{L2}^n)\) where \( p \) designates the positive solution to the quadratic and \( n \) designates the negative solution. Likewise for bus 3, equations (3.36) and (3.37) can be represented as \( P_3 = P_3(\Gamma_{L3}, \Gamma_{L2}) \) and \( Q_3 = Q_3(\Gamma_{L3}, \Gamma_{L2}, Z_{o3}) \). For a given \( \Gamma_{L2} \) and \( Z_{o3} \), the intersection points of the \( P_3 \) and \( Q_3 \) are defined as \((\Gamma_{L3}^p, \Gamma_{L3}^n)\). The idea behind this geometric approach is at each iteration \( k \), a solution set is constructed. That is, pair the positive solution of \( \Gamma_{L2} \) with both the positive and negative solutions for \( \Gamma_{L3} \) and pair the negative solution of \( \Gamma_{L2} \) with both the positive and negative solutions for \( \Gamma_{L3} \). Thus a solution set exists that contains four points: \([(\Gamma_{L2}^k p, \Gamma_{L3}^k p), (\Gamma_{L2}^k p, \Gamma_{L3}^k n), (\Gamma_{L2}^k n, \Gamma_{L3}^k p), (\Gamma_{L2}^k n, \Gamma_{L3}^k n)]\) or in a more compact notation,

\[
\Gamma_{SS}^k = \begin{bmatrix}
\Gamma_{L2}^{k,p} & \Gamma_{L3}^{k,p} \\
\Gamma_{L2}^{k,p} & \Gamma_{L3}^{k,n} \\
\Gamma_{L2}^{k,n} & \Gamma_{L3}^{k,p} \\
\Gamma_{L2}^{k,n} & \Gamma_{L3}^{k,n}
\end{bmatrix}
\]

At the next iteration, \( k + 1 \), for each point in the solution set, i.e. \((\Gamma_{L2}^{k,p}, \Gamma_{L3}^{k,p})\), again compute the intersection points of the \( P \) and \( Q \) circles at each bus. Thus for the first point in the solution set, \((\Gamma_{L2}^{k,p}, \Gamma_{L2}^{k,n})\), compute the intersections points of the \( P_2 \) and \( Q_2 \) circles, \((\Gamma_{L2}^{k+1,p}, \Gamma_{L2}^{k+1,n})\) assuming \( \Gamma_{L3} = \Gamma_{L3}^{k,p} \) and compute the intersections points of the \( P_3 \) and \( Q_3 \) circles, \((\Gamma_{L3}^{k+1,p}, \Gamma_{L3}^{k+1,n})\) assuming \( \Gamma_{L2} = \Gamma_{L2}^{k,p} \). If either bus or both buses have no intersection points, that solution point is discarded. However, if there exists intersection points for both buses 2 and 3, then the first “point” in the solution set at iteration \( k + 1 \) is

\[
\Gamma_{SS}^{k+1} = \begin{bmatrix}
\Gamma_{L2}^{k+1,p} & \Gamma_{L3}^{k+1,p} \\
\Gamma_{L2}^{k+1,p} & \Gamma_{L3}^{k+1,n} \\
\Gamma_{L2}^{k+1,n} & \Gamma_{L3}^{k+1,p} \\
\Gamma_{L2}^{k+1,n} & \Gamma_{L3}^{k+1,n}
\end{bmatrix}
\]
The algorithm is terminated when the absolute difference between sequential solution sets is less than some set tolerance, i.e. $|\Gamma_{SS}^{k+1} - \Gamma_{SS}^k| \leq \text{Tolerance}$. At that point the entries in $\Gamma_{SS}^k$ are checked to see if they satisfy the P and Q criteria at each bus. Any point that does not satisfy the criteria is removed from the solution set. Thus the remaining points are the solutions to the load flow equations. Simulations illustrating this approach are shown in Section 3.5.4.

### 3.5.4 Load Flow Examples in the Scattering Domain

In this section, the load flow using the scattering domain formulations as formulated in the previous sections will be computed. Consider the three bus system as shown in Figure 3-2. The generator $G_1$ is a slack bus with $|V_{GEN}| = 1$ and $\angle V = 0$ and the loads $L_2$ and $L_3$ are PQ buses with $P_2 = 1.1$, $Q_2 = -0.07$ and $P_3 = 1.3$, $Q_3 = -0.4$. The transmission line data is specified in Table 3.1

As detailed in a previous section, the choice for normalization in this chapter will be
3.5. Load Flow Formulations in the Scattering Domain

<table>
<thead>
<tr>
<th>FROM BUS</th>
<th>TO BUS</th>
<th>R</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.00450</td>
<td>0.0450</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.00000</td>
<td>0.1000</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.00000</td>
<td>0.0833</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.00000</td>
<td>0.0667</td>
</tr>
</tbody>
</table>

Table 3.1: Base Case Transmission Line Data

arbitrary. Given that, the above data comprises what will herein be defined as the Base Case. As discussed in the previous chapter, the normalization at the generator bus is chosen to be its own generator impedance, \( Z_{GEN} \) and thus \( Z_{o1} = 0.0045 + j0.0450 \). In the case of the load buses, one possible choice for normalization is the equivalent impedance at each bus. Since the above data at each bus only consist of real and reactive power, the load flow in the V-I domain was run to determine an equivalent impedance since \( Z = \frac{|V|^2}{P - jQ} \) and the voltages at buses 2 and 3 are unknown. Thus the normalization at each bus is set equal to the equivalent load impedance. The load flow results for the Base Case are shown in Figure 3-3. Thus for bus 2 the normalization is \( Z_{o2} = 0.9119 - j0.0580 \) and the normalization for bus 3 is \( Z_{o3} = 0.7173 - j0.2207 \). For these normalizations, \( \Gamma_{L2} \) and \( \Gamma_{L3} \) are identically zero via equation (3.6).

Given the normalization for the system, \( Z_o \), the augmented admittance matrix of the transmission network is computed as \( \tilde{Y}_A = (I + Y_{BUS}Z_o)^{-1}Y_{BUS} \) and the scattering matrix of \( Y_{BUS} \) is \( S = I - 2R_o\tilde{Y}_AR_o \). The augmented admittance matrix and scattering matrix were computed for the Base Case and the results are shown in Figure 3-4. Since the normalization, scattering matrix and \( \Gamma \)'s have been determined, the \( \Gamma_{L2} = 0 \) and \( \Gamma_{L3} = 0 \) solutions were verified to yield the same results as those obtained from the load flow (refer to Figure 3-4).

To compute the load flow using scattering parameters, the Base Case values were changed as follows: \( Q2 \) changed from -.07 to -.02 and \( Q3 \) changed from -.4 to .07. Using the normalization and scattering matrix computed from the Base Case, in Approaches I and II, the solution to the load flow problem was computed using a Newton-Raphson approach via a MATLAB routine FSOLVE. The initial guess for both \( \Gamma_{L2} \) and \( \Gamma_{L3} \) for the
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

Load Flow INPUT file for Base Case:

**THE 4 BUS SYSTEM** 1.0

% The Load flow data in IEEE common format

**BUS DATA FOLLOWS** 4 ITEMS

% Number Name type V(p.u) Delta(d) Pl Ql Pg Qg

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Type</th>
<th>V (p.u)</th>
<th>Delta</th>
<th>P1</th>
<th>Q1</th>
<th>Pg</th>
<th>Qg</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2bus</td>
<td>1</td>
<td>0.980</td>
<td>0.0000</td>
<td>1.1</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>3bus</td>
<td>1</td>
<td>1.100</td>
<td>0.0000</td>
<td>1.30</td>
<td>-0.4</td>
<td>0.00</td>
<td>0.00</td>
</tr>
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<td>1bus</td>
<td>1</td>
<td>1.000</td>
<td>0.0000</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
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<td>0.0000</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

-999

**BRANCH DATA FOLLOWS** 4 ITEMS

% from to #circuits R X B R_Tap I_tap

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<thead>
<tr>
<th>% from to</th>
<th>#circuits</th>
<th>R</th>
<th>X</th>
<th>B</th>
<th>R_Tap</th>
<th>I_tap</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 2 0 0 1 0</td>
<td>0.00000</td>
<td>0.1000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2 3 0 0 1 0</td>
<td>0.00000</td>
<td>0.0833</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>4 3 0 0 1 0</td>
<td>0.00000</td>
<td>0.0667</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>1 4 0 0 1 0</td>
<td>0.00450</td>
<td>0.0450</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Load Flow INPUT file for Base Case:

**FAST DECOUPLED LOAD FLOW**

**SYSTEM TITLE**

**SYSTEM STUDY**

**CONVERGED IN** 3 ITERATIONS

******************************************************************************

**NODE RESULTS**

**BUS TYPE** V-MAG V-ANGLE P GEN Q GEN P LOAD Q LOAD P UNB Q UNB

<table>
<thead>
<tr>
<th>Bus</th>
<th>Type</th>
<th>V-Mag</th>
<th>V-Angle</th>
<th>P Gen</th>
<th>Q Gen</th>
<th>P Load</th>
<th>Q Load</th>
<th>P Unb</th>
<th>Q Unb</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>PQ</td>
<td>1.00359</td>
<td>-12.07682</td>
<td>0.00006</td>
<td>0.00006</td>
<td>1.10000</td>
<td>-0.07000</td>
<td>-0.00006</td>
<td>-0.00006</td>
</tr>
<tr>
<td>3</td>
<td>PQ</td>
<td>1.01032</td>
<td>-11.62346</td>
<td>0.00007</td>
<td>-0.00019</td>
<td>1.30000</td>
<td>-0.40000</td>
<td>-0.00007</td>
<td>0.00019</td>
</tr>
<tr>
<td>4</td>
<td>PQ</td>
<td>0.99362</td>
<td>-6.30048</td>
<td>-0.00005</td>
<td>0.00063</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00005</td>
<td>-0.00063</td>
</tr>
<tr>
<td>1</td>
<td>SB</td>
<td>1.00000</td>
<td>0.00000</td>
<td>2.42641</td>
<td>0.03206</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.00116</td>
<td>-0.03206</td>
</tr>
</tbody>
</table>

******************************************************************************

**LINE POWER FLOWS**

**FROM** **TO** P Send Q Send P Rec Q Rec P Loss Q Loss

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>P Send</th>
<th>Q Send</th>
<th>P Rec</th>
<th>Q Rec</th>
<th>P Loss</th>
<th>Q Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>0.09631</td>
<td>-0.08204</td>
<td>-0.09631</td>
<td>-0.08073</td>
<td>0.00000</td>
<td>0.00131</td>
</tr>
<tr>
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<td>2</td>
<td>1.00362</td>
<td>-0.04847</td>
<td>-1.00362</td>
<td>0.15073</td>
<td>0.00000</td>
<td>0.10226</td>
</tr>
<tr>
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<td>3</td>
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<td>-1.39624</td>
<td>0.31796</td>
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<td>0.13399</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2.42641</td>
<td>0.03255</td>
<td>-2.39992</td>
<td>0.23243</td>
<td>0.02650</td>
<td>0.26498</td>
</tr>
</tbody>
</table>

******************************************************************************

**GENERAL SYSTEM PARAMETERS**

P GEN. = 2.4265  P LOAD = 2.4000
Q GEN. = 0.0326  Q LOAD = -0.4700
TOTAL P LOSSES = 0.0265  TOTAL Q LOSSES = 0.5026

Figure 3-3: Base Case: LOAD FLOW INPUT and OUTPUT FILES
3.5. Load Flow Formulations in the Scattering Domain

SCATTERING RESULTS FOR BASE CASE:

\[ Z_0 = 0.0045 + 0.0450i \]
\[ 0.9119 - 0.0580i \]
0
0
0
0.7173 - 0.2207i

\[ Y_{BUS} = 0 + 24.9925i \]
0 +10.0000i
0 -22.0048i
0 +12.0048i
0 -26.9973i

\[ Y_A = 2.4265 - 0.0326i \]
-1.0864 + 0.1611i
1.0800 - 0.0553i
0.0064 - 0.1058i
1.3337 + 0.2343i

\[ S = 0.9782 + 0.0003i \]
0.1392 - 0.0206i
0.1523 + 0.0146i
0.1392 - 0.0206i
-0.9699 + 0.1009i
-0.0103 + 0.1711i
0.1523 + 0.0146i
-0.0103 + 0.1711i
-0.9133 - 0.3362i

LOAD DATA: \( P_2 = 1.100000e+000 \), \( Q_2 = -7.000000e-002 \), \( V_2M = 1.003590e+000 \)
\( P_3 = 1.300000e+000 \), \( Q_3 = -4.000000e-001 \), \( V_3M = 1.010320e+000 \)

Since the normalization is chosen to be equal to the equivalent impedance, then \( \gamma_{12} = 0 \), \( \gamma_{13} = 0 \)

Check the \( \gamma \) equals zero solution:

\( P_2 = 1.099999e+000 \) \( Q_2 = -6.999992e-002 \) \( V_2M = 1.003589e+000 \) \( V_2ANG = -1.207740e+001 \)
\( P_3 = 1.300005e+000 \) \( Q_3 = -4.000016e-001 \) \( V_3M = 1.010322e+000 \) \( V_3ANG = -1.162402e+001 \)

Figure 3-4: Base Case Scattering Results
The numerical load flow results for Approaches I and II are shown in Figure 3-6. The geometrical display of the intersections of the real and reactive circles for bus 2 are shown in Figures 3-7 and 3-8. The results for bus 3 are shown in Figures 3-9 and 3-10. These results were verified using another program which computes the load flow in the conventional manner and they are shown in Figure 3-5. It is important to note that the scattering approach yields two solutions to the load flow equations whereas the conventional load flow yields only one.

The numerical results for Approach III are shown in Figures 3-11, 3-12, 3-13 and 3-14 for a tolerance value of $1e^{-06}$. It took six iterations for the condition $|\Gamma_{SS}^{k+1} - \Gamma_{SS}^k| \leq 1e^{-06}$ to be satisfied. The resulting solutions to the load flow equations are shown in Figure 3-14. These three solutions are identical to those obtained using Approaches I and II in Figure 3-6. It is noted that in Figure 3-6, the solution $(\Gamma_{L2}, \Gamma_{L3}) = (-0.0391 + 0.0253i, 0.0256 + 0.1770i)$ is the common intersection point of buses 2 and 3 and is the solution obtained using the Newton-Raphson based approaches.

### 3.5.5 Structure of the Load Flow Equations in the Scattering Domain

In this section, the structure of the load flow equations will be examined and in particular, how the equations change as the number of buses increases. It will be shown how the structure of the equations can be easily obtained from the linear transformation “$b = Sa$”.

The load flow equations in the scattering domain have the same basic form for $n \geq 2$ buses where $n$ is the number of PQ and PV load in the system. Consider the expressions for voltage and current in equations (3.12) and (3.13). The voltage and current relationships are functions of $b_i$, the reflected voltage, and $\Gamma_{Li}$ and the normalization. In particular, the port voltage and current can be viewed as consisting of two parts, a “system” part, $b_i$ and a local “part”, $\Gamma_{Li}$ and $Z_{ai}$. The same observations can be made for the real and reactive power in equations (3.14) and (3.15). The “system” part is $|b_i|^2$ and the “local” part contains $\Gamma_{Li}$ and $Z_{ai}$. Thus as the number of buses increases what changes is the structure of $b_i$. 
3.5. Load Flow Formulations in the Scattering Domain

THE 4 BUS SYSTEM

% The Load Flow data in IEEE common format
BUS DATA Follows
% Number Name type V(p.u) Delta(d) Pl Ql Pg Qg
2 2bus 1 1 0 0 0.980 0.0000 1.1 -0.02 0.00 0.00
3 3bus 1 1 0 0 1.100 0.0000 1.30 0.07 0.00 0.00
1 ibus 1 1 0 0 1.000 0.0000 0.00 0.000 0.00 0.00
4 4bus 1 1 0 0 0.000 0.0000 0.00 0.000 0.00 0.00

BRANCH DATA Follows
% from to #circuits R X B R.Tap I.tap
4 2 0 0 1 0 0.00000 0.1000 0.000 0 0 0 0 0 0.0 0.0
2 3 0 0 1 0 0.00000 0.0833 0.000 0 0 0 0 0 0.0 0.0
4 3 0 0 1 0 0.00000 0.0667 0.000 0 0 0 0 0 0.0 0.0
1 4 0 0 1 0 0.0450 0.0450 0.000 0 0 0 0 0 0.0 0.0

FAST DECOUPLED LOAD FLOW SYSTEM TITLE

SYSTEM STUDY

CONVERGED IN 3 ITERATIONS

NODE RESULTS

NAME BUS TYPE V-MAG V-ANGLE P GEN Q GEN P LOAD Q LOAD P UNB Q UNB
2 PQ 0.96276 -12.51819 0.00071 0.00056 1.10000 -0.02000 -0.00074 -0.00058
3 PQ 0.96074 -12.04235 0.00082 0.00095 1.30000 0.07000 -0.00086 -0.00099
4 PQ 0.96883 -6.32004 -0.00157 0.00191 0.00000 0.00000 0.00162 -0.00197
1 SB 1.00000 0.00000 2.42809 0.57725 0.00000 0.00000 -0.00096 -0.00099

LINE POWER FLOWS

FROM TO P SEND Q SEND P REC Q REC P LOSS Q LOSS
3 2 0.09222 -0.02301 -0.09222 0.02382 0.00000 0.00082
4 2 1.00707 0.11325 -1.00707 -0.00384 0.00000 0.10942
4 3 1.39140 0.18705 -1.39140 -0.04699 0.00000 0.14006
1 4 2.42809 0.58078 -2.40004 -0.30030 0.02805 0.28048

GENERAL SYSTEM PARAMETERS

P GEN. = 2.4280 P LOAD = 2.4000
Q GEN. = 0.5807 Q LOAD = 0.0500
TOTAL P LOSSES = 0.0280 TOTAL Q LOSSES = 0.5307

Figure 3-5: LOAD FLOW INPUT and OUTPUT FILES
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

SCATTERING RESULTS: CHANGED Q2 FROM -.07 to -.02 AND Q3 FROM -.4 to .07
LOAD FLOW RESULTS:
P2 = 1.1 Q2 = -.02 V2 = 0.96276 V2ANG = -12.51819
P3 = 1.3 Q3 = .07 V3 = 0.96074 V3ANG = -12.04235

Newton-Raphson Results using Fsolve: gamma_1 = -0.0391 + 0.0253i 0.0256 + 0.1770i
P2 = 1.100000e+000 Q2 = -2.000000e-002 V2M = 9.627430e-001 V2ANG = -1.252235e+001
P3 = 1.300000e+000 Q3 = 7.000000e-002 V3M = 9.607147e-001 V3ANG = -1.204614e+001

BUS 2 Results:
Intersection points of the P2 & Q2 circles: Gamma2
Positive solution: xp=-9.648473e-001,yp=1.228273e-001
Negative solution: xn=-3.905931e-002,yn=2.530145e-002

gamma_1 = -0.9648 + 0.1228i 0.0256 + 0.1770i
P2 = 1.0999999e+000 Q2 = -2.000000e-002 V2M = 1.182234e-001 V2ANG = -8.351127e+001
P3 = 1.525656e-001 Q3 = 8.215068e-003 V3M = 3.291175e-001 V3ANG = -1.525998e+001

BUS 3 Results:
Intersection points of the P3 & Q3 circles: Gamma3
Positive solution: xp=-8.007215e-001,yp=5.464182e-001
Negative solution: xn=2.557024e-002,yn=1.769589e-001

gamma_1 = -0.0391 + 0.0253i -0.8007 + 0.5464i
P2 = 8.499121e-002 Q2 = -1.545295e-003 V2M = 2.676092e-001 V2ANG = -2.153202e+001
P3 = 1.300005e+000 Q3 = 7.000000e-002 V3M = 1.261628e-001 V3ANG = -7.810187e+001

Figure 3-6: Approach I Scattering Results
Figure 3-7: Three-Port Example: Bus 2 P and Q Circles

Figure 3-8: Three-Port Example: Bus 2 P and Q Circles: Blowup
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

Figure 3-9: Three-Port Example: Bus 3 P and Q Circles

Figure 3-10: Three-Port Example: Bus 3 P and Q Circles: Blowup
3.5. Load Flow Formulations in the Scattering Domain

Iteration No. = 1

\[ \gamma_1 = 0 \quad 0 \]

Intersection points of the P2 & Q2 circles: \( x_p = -9.654926e-001, y_p = 1.229203e-001 \)
\( x_n = -2.924703e-003, y_n = 2.293526e-002 \)
Intersection points of the P3 & Q3 circles: \( x_p = -8.008140e-001, y_p = 5.464731e-001 \)
\( x_n = 3.075651e-002, y_n = 1.753273e-001 \)

\[ \Gamma_{SS} = -0.9655 + 0.1229i \quad -0.8008 + 0.5465i \]
\[ -0.9655 + 0.1229i \quad 0.0308 + 0.1753i \]
\[ -0.0029 + 0.0229i \quad -0.8008 + 0.5465i \]
\[ -0.0029 + 0.0229i \quad 0.0308 + 0.1753i \]

Iteration No. = 2

\[ \gamma_1 = -0.9655 + 0.1229i \quad -0.8008 + 0.5465i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.9655 + 0.1229i \quad 0.0308 + 0.1753i \]
Intersection points of the P2 & Q2 circles: \( x_p = -9.648557e-001, y_p = 1.228285e-001 \)
\( x_n = -3.866296e-002, y_n = 2.527493e-002 \)
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0029 + 0.0229i \quad -0.8008 + 0.5465i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: \( x_p = -8.007609e-001, y_p = 5.464416e-001 \)
\( x_n = 2.724837e-002, y_n = 1.764302e-001 \)

\[ \gamma_1 = -0.0029 + 0.0229i \quad 0.0308 + 0.1753i \]
Intersection points of the P2 & Q2 circles: \( x_p = -9.648557e-001, y_p = 1.228285e-001 \)
\( x_n = -3.866296e-002, y_n = 2.527493e-002 \)
Intersection points of the P3 & Q3 circles: \( x_p = -8.007609e-001, y_p = 5.464416e-001 \)
\( x_n = 2.724837e-002, y_n = 1.764302e-001 \)

\[ \Gamma_{SS} = -0.9649 + 0.1228i \quad -0.8008 + 0.5464i \]
\[ -0.9649 + 0.1228i \quad 0.0272 + 0.1764i \]
\[ -0.0387 + 0.0253i \quad -0.8008 + 0.5464i \]
\[ -0.0387 + 0.0253i \quad 0.0272 + 0.1764i \]

Figure 3-11: Approach III Scattering Results
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

Iteration No. = 3
\[ \gamma_1 = -0.9649 + 0.1228i -0.8008 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.9649 + 0.1228i 0.0272 + 0.1764i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648500e-001, y_p=1.228277e-001 \]
\[ x_n=-3.893087e-002, y_n=2.529287e-002 \]
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0387 + 0.0253i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007220e-001, y_p=5.464185e-001 \]
\[ x_n=2.559163e-002, y_n=1.769522e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

---

Iteration No. = 4
\[ \gamma_1 = -0.9649 + 0.1228i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.9649 + 0.1228i 0.0256 + 0.1770i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

\[ \gamma_1 = -0.0389 + 0.0253i \]
Intersection points of the P2 & Q2 circles: \[ x_p=-9.648473e-001, y_p=1.228273e-001 \]
\[ x_n=-3.905844e-002, y_n=2.530141e-002 \]
Intersection points of the P3 & Q3 circles: \[ x_p=-8.007217e-001, y_p=5.464184e-001 \]
\[ x_n=2.557851e-002, y_n=1.769564e-001 \]

Gamma_SS = \[ -0.9649 + 0.1228i -0.8007 + 0.5464i \]
\[ -0.9649 + 0.1228i 0.0256 + 0.1770i \]
\[ -0.0389 + 0.0253i -0.8007 + 0.5464i \]
\[ -0.0389 + 0.0253i 0.0256 + 0.1770i \]

---

Figure 3-12: Approach III Scattering Results
3.5. Load Flow Formulations in the Scattering Domain

Iteration No. = 5

\[ \gamma_1 = -0.9648 + 0.1228i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.9648 + 0.1228i 0.0256 + 0.1770i \]
Intersection points of the P2 & Q2 circles: \( xp=-9.648473e-001, yp=1.228273e-001 \)
\( xn=-3.905945e-002, yn=2.530148e-002 \)
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0391 + 0.0253i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: \( xp=-8.007216e-001, yp=5.464183e-001 \)
\( xn=2.557226e-002, yn=1.769583e-001 \)

\[ \gamma_1 = -0.0391 + 0.0253i 0.0256 + 0.1770i \]
Intersection points of the P2 & Q2 circles: \( xp=-9.648473e-001, yp=1.228273e-001 \)
\( xn=-3.905993e-002, yn=2.530151e-002 \)
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0391 + 0.0253i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: \( xp=-8.007216e-001, yp=5.464183e-001 \)
\( xn=2.557226e-002, yn=1.769583e-001 \)

\[ \gamma_1 = 0.1228i 0.0256 \]
\[ -0.8007 + 0.5464i \]
\[ -0.0391 + 0.0253i -0.8007 + 0.5464i \]
\[ -0.0391 + 0.0253i 0.0256 + 0.1770i \]

-------------------------------

Iteration No. = 6

\[ \gamma_1 = -0.9648 + 0.1228i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.9648 + 0.1228i 0.0256 + 0.1770i \]
Intersection points of the P2 & Q2 circles: \( xp=-9.648473e-001, yp=1.228273e-001 \)
\( xn=-3.905993e-002, yn=2.530151e-002 \)
Intersection points of the P3 & Q3 circles: NONE

\[ \gamma_1 = -0.0391 + 0.0253i -0.8007 + 0.5464i \]
Intersection points of the P2 & Q2 circles: NONE
Intersection points of the P3 & Q3 circles: \( xp=-8.007216e-001, yp=5.464183e-001 \)
\( xn=2.557221e-002, yn=1.769584e-001 \)

\[ \gamma_1 = -0.0391 + 0.0253i 0.0256 + 0.1770i \]
Intersection points of the P2 & Q2 circles: \( xp=-9.648473e-001, yp=1.228273e-001 \)
\( xn=-3.905993e-002, yn=2.530151e-002 \)
Intersection points of the P3 & Q3 circles: \( xp=-8.007216e-001, yp=5.464183e-001 \)
\( xn=2.557221e-002, yn=1.769584e-001 \)

Figure 3-13: Approach III Scattering Results
Tolerance of 1e-06 has been reached

Compute P and Q for each point in Gamma_SS

\[
\text{gamma}_11 = -0.9648 + 0.1228i \quad -0.8007 + 0.5464i
\]

\[
P2 = 3.080811e-001 \quad Q2 = -5.601475e-003 \quad V2M = 6.256625e-002
\]

\[
P3 = 5.530296e-001 \quad Q3 = 2.977851e-002 \quad V3M = 8.228714e-002
\]

\[
\text{gamma}_12 = -0.9648 + 0.1228i \quad 0.0256 + 0.1770i
\]

\[
P2 = 1.100000e+000 \quad Q2 = -2.000000e-002 \quad V2M = 1.182235e-001
\]

\[
P3 = 1.525656e-001 \quad Q3 = 8.215070e-003 \quad V3M = 3.291176e-001
\]

\[
\text{gamma}_13 = -0.0391 + 0.0253i \quad -0.8007 + 0.5464i
\]

\[
P2 = 8.499108e-002 \quad Q2 = -1.545292e-003 \quad V2M = 2.676090e-001
\]

\[
P3 = 1.300000e+000 \quad Q3 = 7.000000e-002 \quad V3M = 1.261623e-001
\]

\[
\text{gamma}_14 = -0.0391 + 0.0253i \quad 0.0256 + 0.1770i
\]

\[
P2 = 1.100000e+000 \quad Q2 = -2.000000e-002 \quad V2M = 9.627430e-001
\]

\[
P3 = 1.300000e+000 \quad Q3 = 7.000000e-002 \quad V3M = 9.607147e-001
\]

Point 1 in Gamma_SS does not satisfy P and Q criteria. Point is removed from solution set.

Solutions to the load flow:

\[
\text{Gamma}_SS_P = -0.9648 + 0.1228i \quad 0.0256 + 0.1770i
\]

\[
-0.0391 + 0.0253i \quad -0.8007 + 0.5464i
\]

\[
-0.0391 + 0.0253i \quad 0.0256 + 0.1770i
\]

Figure 3-14: Approach III Scattering Results
3.5. Load Flow Formulations in the Scattering Domain

As previously indicated, \( b = (I - S\Gamma)^{-1}Sa \). However, this expression does not lend much insight as to how the structure changes as the number of buses increases. A much more intuitive approach is to revisit the linear transformation, \( b = Sa \), from the point of view of signal flow graphs and apply the terminal constraints. Consider the signal flow graph of the three-port system as shown in Figure 3-15 and described by the linear transformation \( b = Sa \). That is,

\[
\begin{align*}
    b_1 &= S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \\
    b_2 &= S_{21}a_1 + S_{22}a_2 + S_{23}a_3 \\
    b_3 &= S_{31}a_1 + S_{32}a_2 + S_{33}a_3.
\end{align*}
\]

This mapping is shown as solid lines in the figure. If the terminal constraints are added (dashed lines), Mason’s rules can be used to determine the expressions for \( b_2 \) and \( b_3 \) as a...
function of $a_1$. Mason’s Rule states that

$$\frac{b_i}{a_1} = \frac{\sum P_k \Delta_k}{\Delta}$$

(3.38)

where $P_k =$ Forward Path Gain, $\Delta_k = 1 -$ Loop gains of loops that do not touch $P_k$ and

$\Delta = 1 - \sum$ of all loop gains + $\sum$ of gain products of all possible combinations of two nontouching loops. In particular, to determine $\frac{b_2}{a_1}$, it can be shown from the figure that

$$P_1 = S_{21}$$
$$P_2 = S_{31} S_{23} \Gamma L_3$$
$$\Delta = 1 - \Gamma L_2 S_{22} - \Gamma L_3 S_{33} - S_{23} \Gamma L_2 S_{32} \Gamma L_3 + \Gamma L_2 S_{22} \Gamma L_3 S_{33}$$
$$\Delta_1 = 1 - \Gamma L_3 S_{33}$$
$$\Delta_2 = 1$$

where in determining $\Delta_1$ and $\Delta_2$ all loops that touch the forward path are set to zero. Thus

$$\sum P_k \Delta_k = P_1 \Delta_1 + P_2 \Delta_2 = S_{21}(1 - S_{33} \Gamma L_3) + S_{31} S_{23} \Gamma L_3.$$  

(3.39)

Therefore, from Mason’s Rules,

$$\frac{b_2}{a_1} = \frac{S_{21}(1 - S_{33} \Gamma L_3) + S_{31} S_{23} \Gamma L_3}{(1 - \Gamma L_2 S_{22})(1 - \Gamma L_3 S_{33}) - S_{23} S_{32} \Gamma L_2 \Gamma L_3}.$$  

(3.40)

Note that in computing $\frac{b_2}{a_1}$, the only thing that changes is the numerator $\sum P_k \Delta_k$. In this case,

$$P_1 = S_{31}$$
$$P_2 = S_{21} S_{32} \Gamma L_2$$
$$\Delta_1 = 1 - \Gamma L_2 S_{22}$$
$$\Delta_2 = 1.$$
From Mason's Rules, the following is obtained
\[
\frac{b_3}{a_1} = \frac{S_{31}(1 - S_{22}\Gamma_L) + S_{21}S_{32}\Gamma_{L2}}{(1 - \Gamma_{L2}S_{22})(1 - \Gamma_{L3}S_{33}) - S_{23}S_{32}\Gamma_{L2}\Gamma_{L3}}. \tag{3.41}
\]

From applying Mason's Rules to the signal flow graph in Figure 3-15, the expressions for \(b_2\) and \(b_3\) as a function of \(a_1\) can be interpreted by how the system affects the point to point “power transfers”. In particular, the transfer from \(a_1\) to \(b_2\) is directly affected by \(S_{21}, S_{33}, S_{31}\) and \(S_{23}\).

Because this is a linear transformation, this structure can be easily extended as the number of buses increases. However, the signal flow graph is no longer planar but becomes a polyhedron \([12]\). Nonetheless, the structure can be easily deduced given the structure of the three bus case. For the two bus case in the previous chapter, \(\frac{b_2}{a_1} = \frac{S_{21}}{1 - S_{22}\Gamma_L}\). Because there was only one load, the forward path gain is \(S_{21}\) and \(\Delta = 1 - S_{22}\Gamma_L\). This result can be obtained from the expression derived for the three bus case by setting \(\Gamma_{L3} = 0\). Thus for a four bus example, i.e. extending the three bus example by adding another load bus, the expressions for \(\frac{b_2}{a_1}\) and \(\frac{b_3}{a_1}\) become
\[
\begin{align*}
\frac{b_2}{a_1} &= \frac{S_{21}(1 - S_{33}\Gamma_{L3} - S_{44}\Gamma_{L4}) + S_{31}S_{23}\Gamma_{L3}(1 - S_{44}\Gamma_{L4}) + S_{41}S_{24}\Gamma_{L4}(1 - S_{33}\Gamma_{L3})}{(1 - \Gamma_{L2}S_{22})(1 - \Gamma_{L3}S_{33})(1 - \Gamma_{L4}S_{44}) - S_{23}S_{32}\Gamma_{L2}\Gamma_{L3} - S_{24}S_{42}\Gamma_{L2}\Gamma_{L4} - S_{34}S_{43}\Gamma_{L3}\Gamma_{L4}} \\
\frac{b_3}{a_1} &= \frac{S_{31}(1 - S_{22}\Gamma_{L2} - S_{44}\Gamma_{L4}) + S_{21}S_{32}\Gamma_{L2}(1 - S_{44}\Gamma_{L4}) + S_{41}S_{34}\Gamma_{L4}(1 - S_{22}\Gamma_{L2})}{(1 - \Gamma_{L2}S_{22})(1 - \Gamma_{L3}S_{33})(1 - \Gamma_{L4}S_{44}) - S_{23}S_{32}\Gamma_{L2}\Gamma_{L3} - S_{24}S_{42}\Gamma_{L2}\Gamma_{L4} - S_{34}S_{43}\Gamma_{L3}\Gamma_{L4}}
\end{align*}
\]

which reduce to the previously derived three bus expressions by setting \(\Gamma_{L4} = 0\). The scattering signal flow graph is shown in Figure 3-16.

3.5.6 Multi-Bus Example

In the previous sections there have been a number of approaches suggested to obtain the solution as the number of buses increases. These approaches exploited the structure in the previous section, namely the signal flow graph, and the application of Mason’s rules to obtain the expressions for \(b_i\) for \(i > 3\). However, there is a simpler, more straightforward
Figure 3-16: Scattering Signal Flow Graph of a 4 Bus System
approach. The previous approach focused on the specified structure. In this section, the basic definition of $\Gamma_i$, namely that, $\Gamma_i = \frac{Z_{Li} - Z_{ol}}{Z_{Li} + Z_{ol}}$, will be exploited.

The definition of $\Gamma_i$ is a function of the equivalent load impedance, $Z_{Li}$, which is fixed and $Z_{ol}$, the normalization, which is arbitrarily chosen. The approach taken in this section is to use the normalization to greatly simplify the expressions for real and reactive power which are functions of all the reflected voltages $b_i$ on the system. As indicated previously in this chapter, $b_i$ is a function of elements of the scattering matrix $S$ and the incident voltages $a_i$, which in turn are related to $b_i$ through the relationship $a_i = \Gamma_i b_i$. The important point is that $S$, $b_i$, $\Gamma_i$ and $a_i$ depend on the normalization chosen. Thus, if the normalization is chosen such that at the solution point, $\Gamma'_i = 0$, then $a'_i$ is zero and the expression for $b'_i$ is greatly simplified. The idea is to then choose the normalization at the solution point such that $\Gamma'_i$ is zero for all $i$, i.e. transform the problem into another "normalization space". Thus, since $Z_{Li}$ is a property of the system, choosing $Z'_{ol} = Z_{Li}$ results in $\Gamma'_i = \frac{Z_{Li} - Z'_{ol}}{Z_{Li} + Z'_{ol}} = 0$. Given this new normalization, the equations for the reflected voltages are greatly simplified. This in turn simplifies and standardizes the equations for the real and reactive power circles regardless of the number of buses. In this new "normalization space", i.e. the primed normalization space, the intersection points of the real and reactive power circles are calculated. To obtain the intersection points in the original normalization space, the solution is untransformed.

To illustrate the above process, the expressions for the reflected voltages at buses 2 and 3 will be reviewed. Given the structure of these equations, the general structure of $b_i$ can be deduced and thus extensions to the multi-bus case are straightforward.

Once the solution to the load flow has been obtained using the Newton-Raphson approach, the voltage and the real and reactive powers at each bus are known. The idea is to repeat what was done initially which is to calculate a normalizing impedance such that $\Gamma'_i$ is zero. In this case, $Z'_{ol} = \frac{|V_i|^2}{P_i - jQ_i} = Z_{Li} = \text{Equivalent Impedance of the load}$. Since the normalization has been changed, the scattering matrix has to be recalculated using this new normalization. The recalculted scattering matrix is defined as $S'$. In the primed space, $\Gamma'_i = 0$ for all $i$. The derivation of the equations for the circles of real and reactive power
requires the reflected voltages at each bus. Thus,

\[ b_2' = S_{21}'a_{s1}' + S_{22}'\Gamma_2'b_2' \]
\[ b_3' = S_{31}'a_{s1}' + S_{33}'\Gamma_3'b_3' \]

since \( a_i' = \Gamma_i'b_i' \). Solving for \( b_i' \) as a function of elements of the scattering matrix and \( \Gamma_i' \) yields

\[ b_2' = \frac{S_{21}'}{1 - S_{22}'\Gamma_2'} \]
\[ b_3' = \frac{S_{31}'}{1 - S_{33}'\Gamma_3'} \]

Thus in general,

\[ b_i' = \frac{S_{i1}'}{1 - S_{ii}'\Gamma_i'} \]

The general expressions for \( P_i \) and \( Q_i \) in the primed normalization space now become

\[ P_i = |b_i'|^2(1 - |\Gamma_i'|^2) \]
\[ Q_i = |b_i'|^2(\frac{X_i'}{P_i}1 + |\Gamma_i'|^2) + 2Im(Z_i' \star \Gamma_i') \]

where it is noted that \( P_i \) and \( Q_i \) are the real and reactive powers of the load at bus \( i \). It can be shown that the equation for \( P_i \) is a circle in the \( \Gamma_i' \) plane such that

\[ (\Gamma_x - x_P)^2 + (\Gamma_y - y_P)^2 = r_P^2 \]

\[ x_P = \frac{P'\text{real}(S_{ii}')}{P'|S_{ii}'|^2 + 1} \]
\[ y_P = -\frac{P'\text{imag}(S_{ii}')}{P'|S_{ii}'|^2 + 1} \]
\[ r_P = \sqrt{\frac{1 - P'}{P'|S_{ii}'|^2 + 1} + x_P^2 + y_P^2} \]
\[ P' = \frac{P_i}{|S_{ii}'a_{s1}'|^2} \]
Likewise, it can be shown that the equation for $Q_i$ is a circle in the $\Gamma'_i$ plane such that

$$
(\Gamma_x - x_Q)^2 + (\Gamma_y - y_Q)^2 = r_Q^2
$$

$$
x_Q = \frac{Q'_\text{real}(S'_{hi})}{Q'|S'_{hi}|^2 - c'}
$$

$$
y_Q = \frac{-Q'_\text{imag}(S'_{hi}) - 2}{Q'|S'_{hi}|^2 - c'}
$$

$$
r_Q = \sqrt{\frac{c' - Q'}{Q'|S'_{hi}|^2 - c'} + x_Q^2 + y_Q^2}
$$

$$
Q' = \frac{Q_i}{|S'_{hi}a_1|^2}.
$$

The solutions are obtained from the intersection points of the circles in the primed space. It is noted that one of the intersection points is the point $\Gamma' = 0$ by definition of the problem since the normalization is chosen such that $\Gamma'_i = 0$. To get the original solutions, the solutions in the original space (i.e. unprimed space), the primed space solutions are transformed back to the original space. This is accomplished by first computing the equivalent impedance at that bus, namely, solving for $Z_{Li}$ in the expression

$$
\Gamma'_i = \frac{Z_{Li} - Z'_{oi}}{Z_{Li} + Z'_{oi}}
$$

the result being that

$$
Z_{Li} = \frac{\Gamma'_i Z'_{oi} + Z'_{oi}}{1 - \Gamma'_i}
$$

which for the case of $\Gamma'_i = 0$ must be the same as $Z'_{oi}$. To compute the solutions in the unprimed space,

$$
\Gamma_i = \frac{Z_{Li} - Z_{oi}}{Z_{Li} + Z_{oi}^*}.
$$

To illustrate this approach, a five bus system consisting of four PQ loads and a slack bus was run. To define a normalization, the five bus bus case as shown in Figure 3-17 was run using a standard load flow program The results are shown in Figure 3-18.
THE 5 BUS SYSTEM

% The Load flow data in IEEE common format

BUS DATA follows 6 ITEMS

% Number Name type V(p.u) Delta(d) P1 Q1 Pg Qg
4 4bus 1 1 0 0 1.000 0.0000 1.1 0.09 0.00 0.00
5 5bus 1 1 0 0 1.000 0.0000 1.30 0.07 0.00 0.00
2 2bus 1 1 0 0 1.000 0.0000 0.8 0.11 0.80 0.00
3 3bus 1 1 0 0 1.000 0.0000 0.8 0.15 0.80 0.00
1 1bus 1 1 0 0 1.000 0.0000 0.0 0.000 0.80 0.00
6 6bus 1 1 0 0 0.000 0.0000 0.0 0.000 0.00 0.00

-999

BRANCH DATA follows 7 ITEMS

% from to #circuits R X B R.Tap I.tap
6 2 0 0 1 0 0.0005 0.0056 0.000 0 0 0 0 0 0 0 0.0 0.0
6 4 0 0 1 0 0.0009 0.0090 0.000 0 0 0 0 0 0 0 0.0 0.0
3 2 0 0 1 0 0.0007 0.0070 0.000 0 0 0 0 0 0 0 0.0 0.0
2 4 0 0 1 0 0.0008 0.0088 0.000 0 0 0 0 0 0 0 0.0 0.0
4 5 0 0 1 0 0.0007 0.0067 0.000 0 0 0 0 0 0 0 0.0 0.0
3 5 0 0 1 0 0.0004 0.0035 0.000 0 0 0 0 0 0 0 0.0 0.0
1 6 0 0 1 0 0.00450 0.0450 0.000 0 0 0 0 0 0 0 0.0 0.0

Figure 3-17: 5 Bus Case: Base Case: LOAD FLOW INPUT FILE

results were then used as an input file (bus5datc) for the scattering approach. The real and reactive powers were changed from this base case (Figure 3-18) and the results are shown in Figure 3-20. These are the results that the scattering approach must verify.

Since the scattering version of the problem requires the power injected into each bus, (i.e. the net power injected into the load), the net injection into each bus is calculated using the data in Figure 3-18). The net injection into the bus is \( PLOAD - PGEN \) and is calculated for each bus. This net injection is the "P" and "Q" referred to in the derivation of the equations and also the output from the simulation. The scattering results do indeed verify the standard load flow results.

The scattering simulation was run for the four PQ loads and a slack bus. Three input files are necessary to run the simulation. The first input file (bus5in) contains the net P and net Q at each bus. This is the data input for which the load flow solution is to be calculated. The next input file (line5) contains the line data, i.e. the transmission data. The last input
3.5. Load Flow Formulations in the Scattering Domain

FAST DECOUPLED LOAD FLOW OUTPUT OF RESULTS

SYSTEM TITLE
SYSTEM STUDY
CONVERGED IN 3 ITERATIONS

******************************************************************************

NODE RESULTS

<table>
<thead>
<tr>
<th>NAME</th>
<th>TYPE</th>
<th>V-MAG</th>
<th>V-ANGLE</th>
<th>P GEN</th>
<th>Q GEN</th>
<th>P LOAD</th>
<th>Q LOAD</th>
<th>P UNB</th>
<th>Q UNB</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>PQ</td>
<td>0.95861</td>
<td>-6.99759</td>
<td>0.00047</td>
<td>0.00076</td>
<td>1.10000</td>
<td>0.09000</td>
<td>-0.00049</td>
<td>-0.00075</td>
</tr>
<tr>
<td>5</td>
<td>PQ</td>
<td>0.95751</td>
<td>-7.22703</td>
<td>0.00067</td>
<td>0.00096</td>
<td>1.30000</td>
<td>0.07000</td>
<td>-0.00070</td>
<td>-0.00100</td>
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<tr>
<td>2</td>
<td>PQ</td>
<td>0.95920</td>
<td>-6.74638</td>
<td>0.79989</td>
<td>0.00018</td>
<td>0.80000</td>
<td>0.11000</td>
<td>0.00011</td>
<td>-0.00019</td>
</tr>
<tr>
<td>3</td>
<td>PQ</td>
<td>0.95773</td>
<td>-7.06441</td>
<td>0.79988</td>
<td>0.00019</td>
<td>0.80000</td>
<td>0.15000</td>
<td>0.00013</td>
<td>-0.00020</td>
</tr>
<tr>
<td>6</td>
<td>PQ</td>
<td>0.96138</td>
<td>-6.33640</td>
<td>-0.00083</td>
<td>-0.00009</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00087</td>
<td>0.00009</td>
</tr>
<tr>
<td>1</td>
<td>SB</td>
<td>1.00000</td>
<td>0.00000</td>
<td>2.43240</td>
<td>0.74346</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.00201</td>
<td>-0.74346</td>
</tr>
</tbody>
</table>

******************************************************************************

LINE POWER FLOWS

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
<th>P SEND</th>
<th>Q SEND</th>
<th>P REC</th>
<th>Q REC</th>
<th>P LOSS</th>
<th>Q LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>-0.55876</td>
<td>-0.09716</td>
<td>0.55900</td>
<td>0.09951</td>
<td>0.00025</td>
<td>0.00235</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.46023</td>
<td>0.02317</td>
<td>-0.46005</td>
<td>-0.02114</td>
<td>0.00018</td>
<td>0.00203</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.74081</td>
<td>-0.02507</td>
<td>-0.74057</td>
<td>0.02717</td>
<td>0.00024</td>
<td>0.00210</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-0.74093</td>
<td>-0.12491</td>
<td>0.74136</td>
<td>0.12922</td>
<td>0.00043</td>
<td>0.00431</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>1.19992</td>
<td>0.18272</td>
<td>-1.19849</td>
<td>-0.16837</td>
<td>0.00143</td>
<td>0.01435</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1.20253</td>
<td>0.27158</td>
<td>-1.20170</td>
<td>-0.26237</td>
<td>0.00082</td>
<td>0.00921</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>2.43240</td>
<td>0.74553</td>
<td>-2.40328</td>
<td>-0.45428</td>
<td>0.02913</td>
<td>0.29126</td>
</tr>
</tbody>
</table>

******************************************************************************

GENERAL SYSTEM PARAMETERS

<table>
<thead>
<tr>
<th>P GEN.</th>
<th>Q GEN.</th>
<th>TOTAL P LOSSES</th>
<th>TOTAL Q LOSSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0325</td>
<td>0.7455</td>
<td>0.0325</td>
<td>0.3255</td>
</tr>
</tbody>
</table>

Figure 3-18: 5 Bus Case: Base Case: LOAD FLOW OUTPUT FILE
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

THE 5 BUS SYSTEM

% The Load flow data in IEEE common format
BUS DATA FOLLOWS

% Number Name     type V(p.u) Delta(d) Pl Ql Pg Qg
4 4bus    1 1 0 0 1.000 0.0000 1.1 0.07 0.00 0.00
5 5bus    1 1 0 0 1.000 0.0000 1.30 0.05 0.00 0.00
2 2bus    1 1 0 0 1.000 0.0000 0.8 0.2 0.80 0.00
3 3bus    1 1 0 0 1.000 0.0000 0.8 0.1 0.80 0.00
1 1bus    1 1 0 3 1.000 0.0000 0.00 0.000 0.80 0.00
6 6bus    1 1 0 0 0.000 0.0000 0.00 0.000 0.00 0.00

BRANCH DATA FOLLOWS

% from to #circuits R X B R.Tap I.tap
6 2 0 0 1 0 0.0005 0.0056 0.000 0 0 0 0 0 0.0 0.0
6 4 0 0 1 0 0.0009 0.0090 0.000 0 0 0 0 0 0.0 0.0
3 2 0 0 1 0 0.0007 0.0070 0.000 0 0 0 0 0 0.0 0.0
2 4 0 0 1 0 0.0008 0.0088 0.000 0 0 0 0 0 0.0 0.0
4 5 0 0 1 0 0.0007 0.0067 0.000 0 0 0 0 0 0.0 0.0
3 5 0 0 1 0 0.0004 0.0035 0.000 0 0 0 0 0 0.0 0.0
1 6 0 0 1 0 0.00450 0.0450 0.000 0 0 0 0 0 0.0 0.0

Figure 3-19: 5 Bus Case: LOAD FLOW INPUT FILE
### Load Flow Formulations in the Scattering Domain

**FAST DECOUPLED LOAD FLOW**

**OUTPUT OF RESULTS**

**SYSTEM TITLE**

**SYSTEM STUDY**

**CONVERGED IN 3 ITERATIONS**

<table>
<thead>
<tr>
<th>NODE RESULTS</th>
<th>NAME</th>
<th>TYPE</th>
<th>V-MAG</th>
<th>V-ANGLE</th>
<th>P GEN</th>
<th>Q GEN</th>
<th>P LOAD</th>
<th>Q LOAD</th>
<th>P UNB</th>
<th>Q UNB</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>PQ</td>
<td>0.95875</td>
<td>-6.99830</td>
<td>0.00048</td>
<td>0.00072</td>
<td>1.10000</td>
<td>0.07000</td>
<td>-0.00050</td>
<td>-0.00075</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>PQ</td>
<td>0.95780</td>
<td>-7.22852</td>
<td>0.00068</td>
<td>0.00092</td>
<td>1.30000</td>
<td>0.05000</td>
<td>-0.00071</td>
<td>-0.00096</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>PQ</td>
<td>0.95911</td>
<td>-6.74588</td>
<td>0.79985</td>
<td>0.00033</td>
<td>0.80000</td>
<td>0.20000</td>
<td>0.00016</td>
<td>-0.00034</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>PQ</td>
<td>0.95801</td>
<td>-7.06600</td>
<td>0.79991</td>
<td>0.00011</td>
<td>0.80000</td>
<td>0.10000</td>
<td>0.00009</td>
<td>-0.00012</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>PQ</td>
<td>0.96138</td>
<td>-6.33635</td>
<td>-0.00083</td>
<td>-0.00008</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00087</td>
<td>0.00009</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>SB</td>
<td>1.00000</td>
<td>0.00000</td>
<td>2.43239</td>
<td>0.74334</td>
<td>0.00000</td>
<td>0.00000</td>
<td>-0.00204</td>
<td>-0.74334</td>
<td></td>
</tr>
</tbody>
</table>

**LINE POWER FLOWS**

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
<th>P SEND</th>
<th>Q SEND</th>
<th>P REC</th>
<th>Q REC</th>
<th>P LOSS</th>
<th>Q LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>-0.55879</td>
<td>-0.07731</td>
<td>0.55903</td>
<td>0.07964</td>
<td>0.00024</td>
<td>0.00232</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.46021</td>
<td>-0.00170</td>
<td>-0.46003</td>
<td>0.00373</td>
<td>0.00018</td>
<td>0.00203</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.74077</td>
<td>-0.02522</td>
<td>-0.74053</td>
<td>0.02732</td>
<td>0.00024</td>
<td>0.00210</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-0.74086</td>
<td>-0.07477</td>
<td>0.74128</td>
<td>0.07900</td>
<td>0.00042</td>
<td>0.00423</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>1.19996</td>
<td>0.16766</td>
<td>-1.19853</td>
<td>-0.15336</td>
<td>0.00143</td>
<td>0.01429</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1.20247</td>
<td>0.28653</td>
<td>-1.20164</td>
<td>-0.27727</td>
<td>0.00083</td>
<td>0.00926</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>2.43239</td>
<td>0.74541</td>
<td>-2.40326</td>
<td>-0.45416</td>
<td>0.02912</td>
<td>0.29125</td>
</tr>
</tbody>
</table>

**GENERAL SYSTEM PARAMETERS**

<table>
<thead>
<tr>
<th>P GEN</th>
<th>P LOAD</th>
<th>Q GEN</th>
<th>Q LOAD</th>
<th>TOTAL P LOSSES</th>
<th>TOTAL Q LOSSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0325</td>
<td>4.0000</td>
<td>0.7453</td>
<td>0.4200</td>
<td>0.0325</td>
<td>0.3253</td>
</tr>
</tbody>
</table>

Figure 3-20: 5 Bus Case: LOAD FLOW OUTPUT FILE
file, busdatc, contains a load flow solution from which the normalization is calculated.

**SCATTERING RESULTS:**

kldflow
Enter bus data input filename (w/o .dat ext): bus5in

**COMPUTE LOAD FLOW FOR THE FOLLOWING DATA:**

<table>
<thead>
<tr>
<th>Bus No</th>
<th>PL</th>
<th>QL</th>
<th>PG</th>
<th>QG</th>
<th>VM</th>
<th>VANG</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.000000e-001</td>
<td>2.000000e-001</td>
<td>-7.998500e-001</td>
<td>-3.300000e-004</td>
<td>9.591100e-001</td>
<td>-6.745880e+000</td>
</tr>
<tr>
<td>3</td>
<td>8.000000e-001</td>
<td>1.000000e-001</td>
<td>-7.999100e-001</td>
<td>-1.100000e-004</td>
<td>9.580100e-001</td>
<td>-7.066000e+000</td>
</tr>
<tr>
<td>4</td>
<td>1.100000e+000</td>
<td>7.000000e-002</td>
<td>-4.800000e-004</td>
<td>-7.200000e-004</td>
<td>9.587500e-001</td>
<td>-6.998300e+000</td>
</tr>
<tr>
<td>5</td>
<td>1.300000e+000</td>
<td>5.000000e-002</td>
<td>-6.800000e-004</td>
<td>-9.200000e-004</td>
<td>9.578000e-001</td>
<td>-7.228520e+000</td>
</tr>
<tr>
<td>1</td>
<td>0.000000e+000</td>
<td>0.000000e+000</td>
<td>-2.432390e+000</td>
<td>-7.433400e-001</td>
<td>1.000000e+000</td>
<td>0.000000e+000</td>
</tr>
</tbody>
</table>

Data = Bus No. Type Net P Net Q
3.5. Load Flow Formulations in the Scattering Domain

\[
\begin{array}{cccc}
2.0000 & 0 & 0.0002 & 0.1997 \\
3.0000 & 0 & 0.0001 & 0.0999 \\
4.0000 & 0 & 1.0995 & 0.0693 \\
5.0000 & 0 & 1.2993 & 0.0491 \\
\end{array}
\]

Enter network data input filename: line5

\[
Y = 1.0e+002 * \\
0.1582 - 1.7716i \\
0.1100 - 1.1001i \\
0.1025 - 1.1270i \\
0.1414 - 1.4144i \\
0.3223 - 2.8203i \\
0.1543 - 1.4764i
\]

Enter loadflow output filename: bus5datc

INPUT LOAD FLOW FILE: USE FOR NORMALIZATION

Bus No = 2: PL = 8.000000e-001 QL = 1.100000e-001 PG = -7.998900e-001
Bus No = 2: QG = -1.800000e-004 VM = 9.592000e-001 VANG = -6.746380e+000
NET INJECTIONS: PLOAD = 1.100000e-004 QLOAD = 1.098200e-001

Bus No = 3: PL = 8.000000e-001 QL = 1.500000e-001 PG = -7.998800e-001
Bus No = 3: QG = -1.900000e-004 VM = 9.577300e-001 VANG = -7.064410e+000
NET INJECTIONS: PLOAD = 1.200000e-004 QLOAD = 1.498100e-001

Bus No = 4: PL = 1.100000e000 QL = 9.000000e-002 PG = -4.700000e-004
Bus No = 4: QG = -7.600000e-004 VM = 9.586100e-001 VANG = -6.997590e+000
Chapter 3. Coordinated Load Flow Problem: Scattering Approach

NET INJECTIONS: 
\[ P_{LOAD} = 1.099530e+000 \quad Q_{LOAD} = 8.924000e-002 \]

Bus No = 5: 
\[ \begin{align*} 
    P &= 1.300000e+000 \\
    Q &= 7.000000e-002 \\
    G &= -6.700000e-004 \\
    M &= 9.575100e-001 \\
    V &= -7.227030e+000 \\
    \end{align*} \]

Bus No = 5: 
\[ \begin{align*} 
    Q &= -9.600000e-004 \\
    V &= 9.575100e-001 \\
    G &= -7.227030e+000 \\
    \end{align*} \]

NET INJECTIONS: 
\[ P_{LOAD} = 1.299330e+000 \quad Q_{LOAD} = 6.904000e-002 \]

Data = Bus No. | Type | Net P | Net Q
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0000</td>
<td>0</td>
<td>0.0002</td>
<td>0.1997</td>
</tr>
<tr>
<td>3.0000</td>
<td>0</td>
<td>0.0001</td>
<td>0.0999</td>
</tr>
<tr>
<td>4.0000</td>
<td>0</td>
<td>1.0995</td>
<td>0.0693</td>
</tr>
<tr>
<td>5.0000</td>
<td>0</td>
<td>1.2993</td>
<td>0.0491</td>
</tr>
</tbody>
</table>

\[ Zo = \begin{bmatrix} 
    0.0045 + 0.0450i \\
    0 \\
    0 \\
    0 \\
    0 \\
\end{bmatrix} \]

Columns 1 through 4

\[ \begin{bmatrix} 
    0.0049 + 0.61227i \\
    0 \\
    0 \\
    0 \\
    0 \\
\end{bmatrix} \]

Column 5

\[ \begin{bmatrix} 
    0.7036 + 0.0374i \\
\end{bmatrix} \]
3.5. Load Flow Formulations in the Scattering Domain

YBUS = 1.0e+002 *

Columns 1 through 4

\[
\begin{array}{cccc}
0.2682 - 2.8717i & -0.1582 + 1.7716i & 0 & -0.1100 + 1.1001i \\
-0.1582 + 1.7716i & 0.4021 - 4.3131i & -0.1414 + 1.4144i & -0.1025 + 1.1270i \\
0 & -0.1414 + 1.4144i & 0.4638 - 4.2347i & 0 \\
-0.1100 + 1.1001i & -0.1025 + 1.1270i & 0 & 0.3667 - 3.7036i \\
0 & 0 & -0.3223 + 2.8203i & -0.1543 + 1.4764i \\
\end{array}
\]

Column 5

\[
\begin{array}{c}
0 \\
0 \\
-0.3223 + 2.8203i \\
-0.1543 + 1.4764i \\
0.4766 - 4.2967i \\
\end{array}
\]

S = Columns 1 through 4

\[
\begin{array}{cccc}
0.9781 + 0.0067i & -0.0002 - 0.0014i & -0.0002 - 0.0015i & 0.1378 - 0.0284i \\
-0.0002 - 0.0014i & 1.0000 + 0.0020i & -0.0000 - 0.0000i & 0.0010 - 0.0003i \\
-0.0002 - 0.0015i & -0.0000 - 0.0000i & 1.0000 + 0.0016i & 0.0011 - 0.0004i \\
0.1378 - 0.0284i & 0.0010 - 0.0003i & 0.0011 - 0.0004i & -0.9439 + 0.2684i \\
0.1505 - 0.0273i & 0.0012 - 0.0003i & 0.0013 - 0.0004i & 0.0434 + 0.1150i \\
\end{array}
\]

Column 5
0.1505 - 0.0273i
0.0012 - 0.0003i
0.0013 - 0.0004i
0.0434 + 0.1150i
-0.9479 + 0.2424i

Original Data:

Bus 2: \( P = 1.100226 \times 10^{-4} \) \( Q = 1.098425 \times 10^{-1} \) \( \text{VM} = 9.592984 \times 10^{-1} \) \( \text{VANG} = -6.745520 \times 10^{0} \)
Bus 3: \( P = 1.200259 \times 10^{-4} \) \( Q = 1.498424 \times 10^{-1} \) \( \text{VM} = 9.578335 \times 10^{-1} \) \( \text{VANG} = -7.063581 \times 10^{0} \)
Bus 4: \( P = 1.099768 \times 10^{0} \) \( Q = 8.925934 \times 10^{-2} \) \( \text{VM} = 9.587139 \times 10^{-1} \) \( \text{VANG} = -6.996752 \times 10^{0} \)
Bus 5: \( P = 1.299632 \times 10^{0} \) \( Q = 6.905604 \times 10^{-2} \) \( \text{VM} = 9.576212 \times 10^{-1} \) \( \text{VANG} = -7.226219 \times 10^{0} \)

\( x_o = 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \)

LOAD FLOW SOLUTION:

\( x = 1.0000 \quad -0.0045 \quad 1.0000 \quad 0.0032 \quad 0.0017 \quad -0.0089 \quad 0.0012 \quad -0.0076 \)

\( \text{Zeq} = 0 \quad 0.0035 + 4.6082i \quad 0.0083 + 9.1902i \quad 0.8329 + 0.0525i \quad 0.7052 + 0.0266i \)

Bus 2: \( P = 1.500000 \times 10^{-4} \) \( Q = 1.996700 \times 10^{-1} \) \( \text{VM} = 9.592279 \times 10^{-1} \) \( \text{VANG} = -6.743397 \times 10^{0} \)
Bus 3: \( P = 9.000000 \times 10^{-5} \) \( Q = 9.989000 \times 10^{-2} \) \( \text{VM} = 9.581295 \times 10^{-1} \) \( \text{VANG} = -7.063464 \times 10^{0} \)
Bus 4: \( P = 1.099520 \times 10^{0} \) \( Q = 6.928000 \times 10^{-2} \) \( \text{VM} = 9.588714 \times 10^{-1} \) \( \text{VANG} = -6.995775 \times 10^{0} \)
Bus 5: \( P = 1.299320 \times 10^{0} \) \( Q = 4.908000 \times 10^{-2} \) \( \text{VM} = 9.579178 \times 10^{-1} \) \( \text{VANG} = -7.225958 \times 10^{0} \)

\( \text{ZoPrime} = 0.0045 + 0.0450i \quad 0 \quad 0 \quad 0 \quad 0 \)
3.5. Load Flow Formulations in the Scattering Domain

\[
\begin{align*}
&0 & 0.0035 + 4.6082i & 0 & 0 \\
&0 & 0 & 0.0083 + 9.1902i & 0 \\
&0 & 0 & 0 & 0.8329 + 0.0525i \\
&0 & 0 & 0 & 0 \\
\end{align*}
\]

Column 5

\[
\begin{align*}
0 & \\
0 & \\
0 & \\
0 & \\
0.7052 + 0.0266i & \\
\end{align*}
\]

\[
S_{\text{Prime}} =
\]

Columns 1 through 4

\[
\begin{align*}
0.9781 + 0.0067i & -0.0002 - 0.0016i & -0.0002 - 0.0013i & 0.1383 - 0.0259i \\
-0.0002 - 0.0016i & 1.0000 + 0.0015i & -0.0000 - 0.0000i & 0.0012 - 0.0004i \\
-0.0002 - 0.0013i & -0.0000 - 0.0000i & 1.0000 + 0.0018i & 0.0010 - 0.0003i \\
0.1383 - 0.0259i & 0.0012 - 0.0004i & 0.0010 - 0.0003i & -0.9530 + 0.2341i \\
0.1509 - 0.0249i & 0.0014 - 0.0004i & 0.0011 - 0.0003i & 0.0395 + 0.1163i \\
\end{align*}
\]

Column 5

\[
\begin{align*}
0.1509 - 0.0249i & \\
0.0014 - 0.0004i & \\
0.0011 - 0.0003i & \\
0.0395 + 0.1163i & \\
\end{align*}
\]
-0.9549 + 0.2132i

Using ZoPrime and Sprime, verify P, Q, V and VANG at each bus:

<table>
<thead>
<tr>
<th>Bus</th>
<th>P (W)</th>
<th>Q (Var)</th>
<th>VM (p.u.)</th>
<th>VANG (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.50000e-004</td>
<td>9.996700e-001</td>
<td>9.592279e-001</td>
<td>-6.743397e+000</td>
</tr>
<tr>
<td>3</td>
<td>9.000000e-005</td>
<td>9.989000e-002</td>
<td>9.581295e-001</td>
<td>-7.063464e+000</td>
</tr>
<tr>
<td>4</td>
<td>1.099520e+000</td>
<td>6.928000e-002</td>
<td>9.588714e-001</td>
<td>-6.995775e+000</td>
</tr>
<tr>
<td>5</td>
<td>1.299320e+000</td>
<td>4.908000e-002</td>
<td>9.579178e-001</td>
<td>-7.225958e+000</td>
</tr>
</tbody>
</table>

In primed space: compute intersection points at each bus

**INTERSECTION POINTS BUS 2:** Positive solution: xp=2.724325e-010, yp=9.377715e-009

Negative solution: xn=9.999989e-001, yn=-1.502640e-003

**INTERSECTION POINTS BUS 3:** Positive solution: xp=-9.738888e-011, yp=7.456081e-009

Negative solution: xn=9.999984e-001, yn=-1.802036e-003

**INTERSECTION POINTS BUS 4:** Positive solution: xp=-1.691783e-013, yp=-5.585063e-014

Negative solution: xn=-9.853629e-001, yn=-1.242379e-001

**INTERSECTION POINTS BUS 5:** Positive solution: xp=-2.172767e-013, yp=-1.108632e-013

Negative solution: xn=-9.865458e-001, yn=-7.423759e-002

\[
\text{gamma_prime_array} =
\begin{bmatrix}
0.0000 + 0.0000i & 0 & 0 & 0 \\
1.0000 - 0.0015i & 0 & 0 & 0 \\
0 & 0.0000 + 0.0000i & 0 & 0 \\
\end{bmatrix}
\]
3.5. Load Flow Formulations in the Scattering Domain

\[
\begin{array}{cccc}
0 & 1.0000 - 0.0018i & 0 & 0 \\
0 & 0 & -0.0000 - 0.0000i & 0 \\
0 & 0 & -0.9854 - 0.1242i & 0 \\
0 & 0 & 0 & -0.0000 - 0.0000i \\
0 & 0 & 0 & -0.9865 - 0.0742i \\
\end{array}
\]

\[\text{gamma_array} = \]

\[
\begin{array}{cccc}
1.0000 - 0.0045i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
1.0000 - 0.0020i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
1.0000 - 0.0045i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
1.0000 - 0.0045i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
1.0000 - 0.0045i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
1.0000 - 0.0045i & 1.0000 + 0.0032i & 0.0017 - 0.0089i & 0.0012 - 0.0076i \\
\end{array}
\]

\[\text{Gamma} = 1.0000 - 0.0045i \quad 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i\]

Bus 2: \( P = 1.500000e-004 \) \( Q = 1.996700e-001 \) \( VM = 9.592279e-001 \) \( VANG = -6.743397e+000 \)

Bus 3: \( P = 9.000000e-005 \) \( Q = 9.989000e-002 \) \( VM = 9.581295e-001 \) \( VANG = -7.063464e+000 \)

Bus 4: \( P = 1.099520e+000 \) \( Q = 6.928000e-002 \) \( VM = 9.58714e-001 \) \( VANG = -6.995775e+000 \)

Bus 5: \( P = 1.299320e+000 \) \( Q = 4.908000e-002 \) \( VM = 9.579178e-001 \) \( VANG = -7.225958e+000 \)

---

\[\text{Gamma} = 1.0000 - 0.0020i \quad 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i\]

Bus 2: \( P = 1.500004e-004 \) \( Q = 1.996701e-001 \) \( VM = 9.592279e-001 \) \( VANG = -5.321602e+000 \)

Bus 3: \( P = 9.000000e-005 \) \( Q = 9.989000e-002 \) \( VM = 9.581295e-001 \) \( VANG = -7.063464e+000 \)

Bus 4: \( P = 1.099520e+000 \) \( Q = 6.928000e-002 \) \( VM = 9.58714e-001 \) \( VANG = -6.995775e+000 \)

Bus 5: \( P = 1.299320e+000 \) \( Q = 4.908000e-002 \) \( VM = 9.579178e-001 \) \( VANG = -7.225958e+000 \)
### Chapter 3. Coordinated Load Flow Problem: Scattering Approach

\[
\begin{align*}
\text{Gamma} & = 1.0000 - 0.0045i \\
& = 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i
\end{align*}
\]

<table>
<thead>
<tr>
<th>Bus 2:</th>
<th>( P = 1.500000e-004 )</th>
<th>( Q = 1.996700e-001 )</th>
<th>( VM = 9.592279e-001 )</th>
<th>( VANG = -6.743397e+000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3:</td>
<td>( P = 9.000000e-005 )</td>
<td>( Q = 9.989000e-002 )</td>
<td>( VM = 9.581295e-001 )</td>
<td>( VANG = -7.063464e+000 )</td>
</tr>
<tr>
<td>Bus 4:</td>
<td>( P = 1.099520e+000 )</td>
<td>( Q = 6.928000e-002 )</td>
<td>( VM = 9.58714e-001 )</td>
<td>( VANG = -6.995775e+000 )</td>
</tr>
<tr>
<td>Bus 5:</td>
<td>( P = 1.299320e+000 )</td>
<td>( Q = 4.908000e-002 )</td>
<td>( VM = 9.579178e-001 )</td>
<td>( VANG = -7.225958e+000 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Gamma} & = 1.0000 - 0.0045i \\
& = 1.0000 + 0.0089i \quad 0.0012 - 0.0076i
\end{align*}
\]

<table>
<thead>
<tr>
<th>Bus 2:</th>
<th>( P = 1.162124e-006 )</th>
<th>( Q = 1.546942e-003 )</th>
<th>( VM = 9.581295e-001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3:</td>
<td>( P = 9.999991e-005 )</td>
<td>( Q = 9.989000e-002 )</td>
<td>( VM = 9.581295e-001 )</td>
</tr>
<tr>
<td>Bus 4:</td>
<td>( P = 8.153324e-003 )</td>
<td>( Q = 5.137353e-004 )</td>
<td>( VM = 9.588714e-001 )</td>
</tr>
<tr>
<td>Bus 5:</td>
<td>( P = 1.433104e-003 )</td>
<td>( Q = 5.413352e-005 )</td>
<td>( VM = 9.579178e-001 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Gamma} & = 1.0000 - 0.0045i \\
& = 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i
\end{align*}
\]

<table>
<thead>
<tr>
<th>Bus 2:</th>
<th>( P = 1.500000e-004 )</th>
<th>( Q = 1.996700e-001 )</th>
<th>( VM = 9.592279e-001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3:</td>
<td>( P = 9.000000e-005 )</td>
<td>( Q = 9.989000e-002 )</td>
<td>( VM = 9.581295e-001 )</td>
</tr>
<tr>
<td>Bus 4:</td>
<td>( P = 1.099520e+000 )</td>
<td>( Q = 6.928000e-002 )</td>
<td>( VM = 9.58714e-001 )</td>
</tr>
<tr>
<td>Bus 5:</td>
<td>( P = 1.299320e+000 )</td>
<td>( Q = 4.908000e-002 )</td>
<td>( VM = 9.579178e-001 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Gamma} & = 1.0000 - 0.0045i \\
& = 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i
\end{align*}
\]

<table>
<thead>
<tr>
<th>Bus 2:</th>
<th>( P = 1.081661e-006 )</th>
<th>( Q = 1.439834e-003 )</th>
<th>( VM = 9.581295e-001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3:</td>
<td>( P = 4.514270e-007 )</td>
<td>( Q = 5.010338e-004 )</td>
<td>( VM = 9.58714e-001 )</td>
</tr>
<tr>
<td>Bus 4:</td>
<td>( P = 1.099520e+000 )</td>
<td>( Q = 6.928000e-002 )</td>
<td>( VM = 9.58714e-001 )</td>
</tr>
<tr>
<td>Bus 5:</td>
<td>( P = 5.522449e-003 )</td>
<td>( Q = 2.086028e-004 )</td>
<td>( VM = 9.579178e-001 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Gamma} & = 1.0000 - 0.0045i \\
& = 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad 0.0012 - 0.0076i
\end{align*}
\]

<table>
<thead>
<tr>
<th>Bus 2:</th>
<th>( P = 1.500000e-004 )</th>
<th>( Q = 1.996700e-001 )</th>
<th>( VM = 9.592279e-001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus 3:</td>
<td>( P = 9.000000e-005 )</td>
<td>( Q = 9.989000e-002 )</td>
<td>( VM = 9.581295e-001 )</td>
</tr>
<tr>
<td>Bus 4:</td>
<td>( P = 1.099520e+000 )</td>
<td>( Q = 6.928000e-002 )</td>
<td>( VM = 9.58714e-001 )</td>
</tr>
<tr>
<td>Bus 5:</td>
<td>( P = 1.299320e+000 )</td>
<td>( Q = 4.908000e-002 )</td>
<td>( VM = 9.579178e-001 )</td>
</tr>
</tbody>
</table>
3.5. *Load Flow Formulations in the Scattering Domain*

Bus 5: \( P = 1.299320e+000 \) \( Q = 4.908000e-002 \) \( VM = 9.579178e-001 \) \( VANG = -7.225958e+000 \)

\[
\begin{align*}
\text{Gamma} &= 1.0000 - 0.0045i \quad 1.0000 + 0.0032i \quad 0.0017 - 0.0089i \quad -0.9838 - 0.1044i \\
\text{Bus 2: } P &= 2.318822e-006 \quad Q = 3.086661e-003 \quad VM = 1.192641e-001 \quad VANG = -3.20584e+001 \\
\text{Bus 3: } P &= 6.252719e-007 \quad Q = 6.939823e-004 \quad VM = 7.986149e-002 \quad VANG = -5.721664e+001 \\
\text{Bus 4: } P &= 1.328761e-002 \quad Q = 8.372433e-004 \quad VM = 1.054101e-001 \quad VANG = -3.773312e+001 \\
\text{Bus 5: } P &= 1.299320e+000 \quad Q = 4.908000e-002 \quad VM = 7.018744e-002 \quad VANG = -7.829394e+001
\end{align*}
\]
Chapter 4

Decentralized Load Flow: Scattering Approach

As the electric power industry restructures, deregulation takes place and competition increases. This results in less information being made available and the basic approach to computation of the load flow will change. Computation of the conventional (centralized) load flow requires total knowledge of the system: load, generation and transmission. However in the new deregulated environment there will be a need for more decentralized decision making and computing. In this chapter, the second application of scattering parameters to power systems, a decentralized approach to the load flow, will be introduced. A decentralized approach involves taking local measurements at one bus in order to determine the state of the system. These local measurements at bus \(i\) are translated into local incident and reflected voltages. From the input output characterization of the transmission system in the scattering domain and the local measurements at bus \(i\), the scattering parameter at bus \(j\) is obtained using a numerical approach. Given the scattering parameter and normalization at bus \(j\), it is straightforward to compute real and reactive power and the voltage.

Proof of concept of a decentralized approach to the load flow is proposed and illustrated on a three bus system consisting of one generator and two PQ loads. To validate the algorithm, three cases were run to simulate measurements at the different buses and to show the effect of initial conditions on the robustness of the algorithm.
Chapter 4. Decentralized Load Flow: Scattering Approach

To illustrate the importance and usefulness of a decentralized approach to the load flow, two examples involving the California system are described. These examples are just a few among many in which obtaining information about the state of the system will be crucial in the new deregulated environment.

4.1 Decentralized Load Flow Approach

In the coming/current deregulated environment, determining information about what other players on the system are doing will be key in devising appropriate gaming strategies to optimize a player's profit. Currently, however, there is no methodology to obtain information about what other players are doing other than data mining. It will be shown in this chapter that for a simple three bus network, in particular, information about bus 3, namely voltage and real and reactive power, can be obtained from local measurements at bus 2. This proof of concept will be illustrated on a three bus network consisting of one generator and two PQ loads.

A proposed solution is to cast the problem in the scattering domain using the formulation \( b = Sa \). This formulation can be interpreted as either an input output relationship between the incident voltages, \( a \), (input) and the reflected voltages, \( b \), (output) or as a set of network constraints as a function of the incident and reflected voltages. The local measurements, namely voltage and current, can be translated into reflected and incident voltages and via the network constraint relationships, the reflected and incident voltages at another bus can be obtained numerically using a Gauss approach. This approach in obtaining information about another bus using local measurements at a particular bus will be herein defined as the decentralized load flow approach. It is defined as a decentralized approach because the method used in obtaining information about other buses on the network is not done in a coordinated manner as it is in computing the centralized load flow solution.

In the previous chapter, the network constraints as a function of all the incident and reflected voltages for a three bus system are as follows:

\[
b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 \quad (4.1)
\]
where the values of the scattering matrix, $S_{ij}$ are known quantities. In the course of deriving the equations to implement the Gauss approach, assume that information about bus 3 is known and that information about bus 2 is to be determined. If local measurements of voltage $V_{L3}$ and current $I_{L3}$ are made, then

\begin{align*}
    a_3 &= \frac{1}{2} \left[ \frac{V_{L3}}{\sqrt{R_{03}}} + I_{L3} \sqrt{R_{03}} \right] \\
    b_3 &= \frac{1}{2} \left[ \frac{V_{L3}}{\sqrt{R_{03}}} - I_{L3} \sqrt{R_{03}} \right] \\
    \Gamma_{L3} &= \frac{a_3}{b_3}
\end{align*}

It is noted that PQ loads are defined in terms of their real and reactive power. Knowing the voltage, it is straightforward to show that $I = \frac{(P-jQ)}{V}$. These relationships, coupled with equations (4.2) and (4.3), form the basis for the Gauss approach in obtaining information about another bus. (The results obtained from numerical simulation are for both buses. In this section, only the equations for bus 3 will be derived. The equations for bus 2 can be obtained in a similar manner.) The assumptions made in this section are that the values of the scattering matrix, $S_{ij}$, and the bus normalizations $Z_{oi}$ are known by everyone on the system. The $S_{ij}$'s are the elements of the scattering matrix of the transmission network as defined in the previous chapter. At bus 3, $a_3$, $b_3$ and $\Gamma_{L3}$ are locally measurable quantities while $a_2$, $b_2$ and $\Gamma_{L2}$ are unknowns. At the generator bus, the only information needed is $a_{s1}$. Since a Gauss approach will be used to determine $a_2$, $b_2$ and $\Gamma_{L2}$ iteratively, the notation used is as follows: a measured quantity has the superscript $m$, while a calculated quantity at time step $k$ has the superscript $k$.

The objective is to calculate $\Gamma_{L2}$ given the measurements at bus 3. Once $\Gamma_{L2}$ is determined, it is straightforward to calculate the voltage and real and reactive power at bus 2 using the expressions for $P_2$ and $Q_2$ derived in the previous chapters. Starting with
equation (4.3),

\[ b_3^m = S_{31}a_{s1} + S_{32}a_2^k + S_{33}a_3^m \]

Since \( a_2 = \Gamma_{L2}b_2 \),

\[ b_3^m = S_{31}a_{s1} + S_{32}\Gamma_{L2}b_2^k + S_{33}a_3^m \]

Solving for \( \Gamma_{L2}^k \) yields

\[ \Gamma_{L2}^k = \frac{b_3^m - S_{31}a_{s1} - S_{33}a_3^m}{b_2^kS_{32}} \]

Rewriting equation (4.2) in terms of measured and calculated values, an expression for \( b_2^k \) is as follows:

\[ b_2^k = S_{21}a_{s1} + S_{22}a_2^k + S_{23}a_3^m \]

where the initial value of \( a_2^k \) may or may not be zero. To determine if the correct solution has been reached, \( \Gamma_{L3}^k \) is calculated using the expressions for \( \Gamma_{L2}^k \) and \( b_2^k \). An error term using the difference from that value in comparison with \( \Gamma_{L3}^{k-1} \) is defined to determine whether the solution has converged. In particular,

\[ \Gamma_{L3}^k = \frac{b_2^k - S_{21}a_{s1} - S_{22}a_2^k}{b_3^mS_{23}} \]

\[ \text{Error in } \Gamma_{L3} = |\Gamma_{L3}^k - \Gamma_{L3}^{k-1}| \]

If the error in \( \Gamma_{L3} \) is within some prespecified tolerance, i.e. \( |\Gamma_{L3}^k - \Gamma_{L3}^{k-1}| \leq 10^{-d} \) then the solution has been reached and no more iterations are needed. If, however, the solution has not been reached, \( \Gamma_{L3}^k \) is used in determining \( \Gamma_{L2}^{k+1} \) for the next iteration \( k \) and the expression for \( a_2^{k+1} \) is as follows:

\[ \Gamma_{L2}^{k+1} = \Gamma_{L2}^k + k_1(\Gamma_{L3}^k - \Gamma_{L3}^{k-1}) \]
and \( a_{2}^{k+1} = \Gamma_{L2}^{k+1} b_{2}^{k} \)

where \( k \) is an acceleration factor, arbitrarily chosen to be 0.1, with the constraint that \( |k_{1}| < 1 \). \( k \) is an acceleration factor which greatly reduces the number of iterations required to reach the specified accuracy. It is noted that the Gauss method with the use of acceleration factors with \( k \leq 1 \) is known as underrelaxed Gauss iterations [14]. Before the next time step, \( \Gamma_{L3}^{k} \) is set equal to the \( \Gamma_{L3}^{k+1} \). Iteration is carried out until the error in \( \Gamma_{L3} \) is within the prespecified tolerance.

The equations for local measurements at bus 2 can be determined in a similar manner and are summarized as follows:

\[
\begin{align*}
    b_{3}^{k} &= S_{31}a_{s1} + S_{32}a_{s2} + S_{33}a_{s3} \\
    \Gamma_{L3}^{k} &= \frac{b_{2}^{m} - S_{21}a_{s1} - S_{22}a_{s2}}{b_{2}^{m}S_{23}} \\
    a_{3}^{k} &= \Gamma_{L3}^{k} b_{3}^{k} \\
    \Gamma_{L2}^{k} &= \frac{b_{2}^{m} - S_{21}a_{s1} - S_{22}a_{s2}}{b_{2}^{m}S_{22}} \\
    \text{Error in } \Gamma_{L2} &= |\Gamma_{L2}^{k} - \Gamma_{L2}^{k-1}| 
\end{align*}
\]

where the initial value of \( a_{3}^{k} \) may or may not be zero. For the next iteration \( k \), \( a_{3}^{k+1} \) is defined as follows:

\[
\begin{align*}
    \Gamma_{L3}^{k+1} &= \Gamma_{L3}^{k} + k_{2}(\Gamma_{L2}^{k} - \Gamma_{L2}^{k-1}) \\
    a_{3}^{k+1} &= \Gamma_{L3}^{k+1} b_{3}^{k} \\
\end{align*}
\]

and \( \Gamma_{L2}^{k} \) is set equal to \( \Gamma_{L2}^{k+1} \).

Using the above derived equations, a MATLAB program was written to simulate the Gauss approach. The basic hypothesis is that each bus would make local measurements of their incident and reflected voltages to extract information about the state of the system. For the purposes of illustration, the previously described Base Case is used to define the initial state of the system. This establishes both the normalization and the scattering matrix.
of the transmission system and this information is available to all players on the system. To simulate how this Gauss approach would be applied, both $Q_2$ and $Q_3$ were changed to effect changes in the loads on the system. $Q_2$ was changed from -.07 to -.02 and $Q_3$ was changed from -.4 to .07 and the load flow was run to obtain the voltages at buses 2 and 3 to simulate the local voltage measurements. However, it is important to stress that only bus 2 knows how much $Q_2$ has changed and what the voltage is at bus 2. The same approach is applicable to bus 3.

Given these local voltage measurements, the currents at buses 2 and 3 were determined from the fact that $I = \frac{(P-jQ)}{V}$. So at each bus, the measured voltage and current are used in computing the local $a$, $b$ and $\Gamma$. Thus at bus 2, $a_2$, $b_2$ and $\Gamma_L^2$ are the measurable quantities and at bus 3, $a_3$, $b_3$ and $\Gamma_L^3$ are the measurable quantities. Assuming the initial values of both $a_3^k$ and $a_2^k$ are zero, two cases were run. The first case uses measurements at bus 3 to determine the data at bus 2 and the second case uses measurements at bus 2 to determine the data at bus 3. The numerical results are shown in Figures 4-1 and 4-2 respectively. To illustrate the robustness of the technique, the initial value of $a_2^k$ was chosen arbitrarily assuming measurements at bus 3. The results are shown in Figure 4-3. The initial value of $a_2^k$ or $a_3^k$ is not crucial in having the algorithm converge to a solution. These results were comparable to those obtained by calculating $\Gamma_L^2$ and $\Gamma_L^3$ from the load flow data as indicated in each of these examples.

4.2 Applications of Decentralized Approach in Competitive Electric Power Industry

In this section, two examples will be described for which a decentralized load flow approach would be applicable. The first example is concerned with estimating or predicting the bid curves of competitors in order to game the energy market. Expanding on the first example, the second example is concerned with issues of compliance from the perspective of the independent system operator (ISO). Both examples involve the California system. The system and its players will be described and defined in order to set the context of the
EXAMPLE I: DECENTRALIZED LOADFLOW APPROACH: CHANGED Q2 FROM -.07 to -.02 AND Q3 FROM -.4 to .07:

Calculated Gammas from LoadFlow Results:
\[ \Gamma_2 = -3.904238e-002 + j 2.530033e-002 \]
\[ \Gamma_3 = 2.559654e-002 + j 1.769507e-001 \]

Using Above Gammas:
\[ P_2 = 1.099967e+000 \quad Q_2 = -1.999940e-002 \quad V_2M = 9.627456e-001 \]
\[ P_3 = 1.299939e+000 \quad Q_3 = 6.999670e-002 \quad V_3M = 9.607174e-001 \]

GAUSS ITERATION SCHEME TO OBTAIN GAMMA VIA LOCAL MEASUREMENTS:

<table>
<thead>
<tr>
<th>Iteration no.</th>
<th>( \Gamma_2_{k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-3.926922e-002 + j 2.731147e-002)</td>
</tr>
<tr>
<td>2</td>
<td>(-3.877484e-002 + j 2.523318e-002)</td>
</tr>
<tr>
<td>3</td>
<td>(-3.856596e-002 + j 2.524247e-002)</td>
</tr>
<tr>
<td>4</td>
<td>(-3.866937e-002 + j 2.525264e-002)</td>
</tr>
<tr>
<td>5</td>
<td>(-3.867347e-002 + j 2.523705e-002)</td>
</tr>
<tr>
<td>6</td>
<td>(-3.867090e-002 + j 2.524168e-002)</td>
</tr>
<tr>
<td>7</td>
<td>(-3.867182e-002 + j 2.524227e-002)</td>
</tr>
<tr>
<td>8</td>
<td>(-3.867167e-002 + j 2.524205e-002)</td>
</tr>
<tr>
<td>9</td>
<td>(-3.867161e-002 + j 2.524209e-002)</td>
</tr>
<tr>
<td>10</td>
<td>(-3.867163e-002 + j 2.524209e-002)</td>
</tr>
<tr>
<td>11</td>
<td>(-3.867162e-002 + j 2.524208e-002)</td>
</tr>
</tbody>
</table>

RESULTS:

Iterated \( \Gamma_2 = -3.867162e-002 + j 2.524208e-002 \)

Using Iterated Solution for Bus 2 and Measured Data from Bus 3

\[ P_2 = 1.099201 \quad Q_2 = -0.02005896 \quad |V_2| = 0.9627714 \quad V_2ANG = -12.51688 \]
\[ P_3 = 1.300000 \quad Q_3 = 0.07 \quad |V_3| = 0.9607400 \quad V_3ANG = -12.04235 \]

Figure 4-1: Example I Decentralized Approach
EXAMPLE II: DECENTRALIZED LOADFLOW APPROACH

Using measured data at Bus 2, compute data at Bus 3

CHANGED Q2 FROM -.07 to -.02 AND Q3 FROM -.4 to .07 :

Calculated Gammas from LoadFlow Results:
Gamma_2 = -3.904238e-002 + j 2.530033e-002
Gamma_3 = 2.559654e-002 + j 1.769507e-001

Using Above Gammas:
P2 = 1.099967e+000 Q2 = -1.999940e-002 V2M = 9.627456e-001
P3 = 1.299939e+000 Q3 = 6.999670e-002 V3M = 9.607174e-001

GAUSS ITERATION SCHEME TO OBTAIN GAMMA VIA LOCAL MEASUREMENTS:

Iteration no. = 1 Gamma3_k = 5.748583e-002 + j 1.733075e-001
Iteration no. = 2 Gamma3_k = 2.588785e-002 + j 1.768959e-001

RESULTS:
Iterated Gamma3 = 2.588785e-002 + j 1.768959e-001

Using Iterated Solution for Bus 3 and Measured Data from Bus 2

P2 = 1.099998 Q2 = -.01999996 |V2| = .9627591 V2ANG = -12.51853
P3 = 1.299179 Q3 = .07005088 |V3| = .9607302 V3ANG = -12.04122

Figure 4-2: Example II Decentralized Approach
EXAMPLE III: DECENTRALIZED LOADFLOW APPROACH
Using measured data at Bus 2, compute data at Bus 3
CHANGED Q2 FROM -.07 to -.02 AND Q3 FROM -.4 to .07:

Calculated Gammas from LoadFlow Results:
\[ \Gamma_2 = -3.904238\times10^{-2} + j2.530033\times10^{-2} \]
\[ \Gamma_3 = 2.559654\times10^{-2} + j1.769507\times10^{-1} \]
\[ P_2 = 1.099967e+000 \quad Q_2 = -1.999940e-002 \quad V_2M = 9.627456e-001 \]
\[ P_3 = 1.299939e+000 \quad Q_3 = 6.999670e-002 \quad V_3M = 9.607174e-001 \]

GAUSS ITERATION SCHEME TO OBTAIN GAMMA VIA LOCAL MEASUREMENTS:

DECENTRALIZED APPROACH:
\[ P_2 = 1.099998e+000 \quad Q_2 = -1.999996e-002 \quad V_2M = 9.627591e-001 \quad V_2ANG = -1.251853e+001 \]
\[ P_3 = 1.299179e+000 \quad Q_3 = 7.005088e-002 \quad V_3M = 9.607302e-001 \quad V_3ANG = -1.204122e+001 \]

Non-zero initial condition for \( \Gamma_k \)
Initial Value of \( a_{2,k} = -3.570934e-002 + j3.332962e-002 \)

Using measured data at Bus 3, compute data at Bus 2
Iteration no. = 1 \( \Gamma_{2,k} = -3.866016e-002 + j2.522875e-002 \)
Iteration no. = 2 \( \Gamma_{2,k} = -3.867081e-002 + j2.524248e-002 \)
Iteration no. = 3 \( \Gamma_{2,k} = -3.867245e-002 + j2.524173e-002 \)
Iteration no. = 4 \( \Gamma_{2,k} = -3.867161e-002 + j2.524199e-002 \)
Iteration no. = 5 \( \Gamma_{2,k} = -3.867163e-002 + j2.524213e-002 \)
Iteration no. = 6 \( \Gamma_{2,k} = -3.867163e-002 + j2.524209e-002 \)
Iteration no. = 7 \( \Gamma_{2,k} = -3.867162e-002 + j2.524208e-002 \)
Iteration no. = 8 \( \Gamma_{2,k} = -3.867162e-002 + j2.524208e-002 \)

RESULTS:
Iterated \( \Gamma_{2,k} = -3.867162e-002 + j2.524208e-002 \)

Using Iterated Solution for Bus 2 and Measured Data from Bus 3
\[ P_2 = 1.099201e+000 \quad Q_2 = -2.005896e-002 \quad V_2M = 9.627714e-001 \quad V_2ANG = -1.251688e+001 \]
\[ P_3 = 1.300000e+000 \quad Q_3 = 7.000000e-002 \quad V_3M = 9.607400e-001 \quad V_3ANG = -1.204235e+001 \]

Figure 4-3: Example III Decentralized Approach
The California energy market\cite{13} consists of three categories of participants: the bidders, the scheduling coordinators and the independent system operator. The bidders submit load and generation bid curves, i.e. the load bid curve consists of the quantities of MW of the load and the prices it is willing to pay and the generation bid curve consists of the quantity of MW the generator or portfolio of generators is willing to supply and the price.

The function of the scheduling coordinator is to match the load and generation bids in order to submit a balanced transaction to the system operator. The scheduling coordinator aggregates the bids on the supply side (generation) to produce a cumulative supply function and on the demand side (load) to produce a cumulative demand function. The price at which the cumulative supply and demand functions intersect is defined as the market clearing price which is the price demanded by the most expensive bid accepted. This price is awarded to all accepted bids. Moreover, the clearing price does not reflect the bid price for most of the generators. In fact, information about the generation bid curves is confidential. However, use of the decentralized load flow approach could be used to estimate power injection of generators using available/historical data. Thus one could produce more accurate bid curves for individual competitors to better game the market.

The second example is concerned with issues of market compliance from the perspective of the independent system operator (ISO). The ISO is responsible for maintaining the reliability of the transmission system and providing equitable and open access to the transmission system for transactions between buyers and sellers of electricity. Continuing the California example, the ISO is responsible for scheduling transactions across the grid and for ensuring that the accepted schedules for the use of the transmission system are feasible. The submitted load/generation schedules must be balanced. If the proposed schedules are not feasible and would cause congestion, the ISO would conduct a congestion management protocol to efficiently allocate access to the grid. Since the ISO is responsible for maintaining the reliability of the system, it is responsible for providing sufficient ancillary services as use of the system changes over time. The ancillary services include various types of reserves (spinning, non-spinning and replacement), regulation (automatic load balancing through
automatic generation control) and voltage support/reactive power.

Since the ISO is responsible for operating the transmission system in real time, the ISO has to ensure that generation equals the total loads, trades and losses. The scheduling coordinator must submit balanced schedules to the ISO. If the actual transactions differ from these schedules, the ISO has to balance the system by purchasing energy from generators providing ancillary services and the costs of purchase is charged to those market participants. However, the ISO has no way of monitoring what the generators are injecting into the system. The ISO does not know whether generator i has the required spinning reserves or whether generator j is supplying the agreed upon energy into the system. In fact, the ISO relies on the generators to monitor themselves. Use of the decentralized load flow approach could be used to measure the actual injections in real time in order to ensure market compliance. This is of particular interest to the FERC (Federal Energy Regulatory Commission) as an enforcement tool to ensure that generators are complying to agreed upon commitments.
Chapter 5

Transmission Capacity In Power Networks

In this chapter some measures of feasible power transfers from generation to load in terms of the scattering parameters of the network will be investigated. One of the reasons for posing the power system problem in terms of scattering parameters and matrices is that the scattering parameters, and in turn the incident and reflected waves, can be measured at each and every port. Since these can be measured in real time, they should provide information on what is happening in the current system. The objective in this chapter is to define a set of measures/metrics which are functions of the scattering matrix and scattering parameters that will allow us to deduce information about the transfer of power from a generator to a load.

To gain some insight into what measures are useful, the two-port example defined in an earlier chapter will be revisited. Given some insight from this example, formulations for the three bus example will be derived.

5.1 Introduction

Understanding the transmission capacity of an electric power system has become increasingly important due to three main issues:
Environmental constraints severely limit expansion of transmission grids.

The system inputs such as generation and input are significantly different in the deregulated versus regulated industry.

Recent technological advances allow for variation in transmission characteristics.

In the deregulated industry, the commercial success of an open and competitive electric power market depends on accurate, up to date information about the capacity of the transmission network to accommodate power transfers requested by the market participants [16]. In order to characterize the capacity of the transmission network, there are a set of possible definitions for power networks of arbitrary topologies [15]. Some of these definitions will be reviewed in this section and then reposed in terms of scattering parameters and matrices in other sections.

One way to characterize the capacity of the transmission network is in terms of ATC (available transfer capability) of a network. This is a measure of the transfer capability remaining in the physical transmission subnetwork for further commercial activity over and above already committed uses. ATC is defined as the Total Transfer Capability (TTC), less the Transmission Reliability Margin (TRM), less the sum of existing transmission commitments and the Capacity Benefit Margin (CBM). ATC is posted on a system called OASIS which is available on the Internet. The purpose of calculating ATC and posting it on OASIS is to further the open-access of the transmission system by providing a signal of its capability and availability to deliver energy. ATC is particularly useful to parties doing trading of electricity since it provides information as to the extent to which they may inject or receive power from the network without violating technical operating constraints. However, the key challenge to more efficient utilization of the transmission network is the inability of a single ATC to adequately capture all the information necessary for each power transfer across a network. Since each power transfer on a network reduces the available transfer capability of other potential transfers, ATC of a network must be continuously updated. Since power is not transmitted point to point but distributed along all transmission lines, information about the entire connected network is necessary to properly calculate ATC.
Currently, ATC is posted as a single network value that measures the ability of the interconnected electric power system to reliably transfer power from one subnetwork to another over all transmission lines interconnecting the subnetworks for specified line flows in these interconnecting lines. For a simple network, this form of ATC can be viewed as the maximum additional power, over some base condition, that can be received by all load buses from all generator buses according to a set of constraints. However, this viewpoint of ATC is that it is really a multi-dimensional property of the network. The maximum transmission capacity available to the generators and loads of a network is not unique for every set of power transfers. In some sense, ATC can be regarded as an index of how congested the network is at a particular time even though it does not provide an accurate estimate of the capacity available to individual bilateral or multilateral power transfers by market participants.

There are other various notions of transfer capability used in the industry that will be reviewed and also reposed in terms of scattering parameters and matrices. In particular, they are the characterizations of bus transmission capacity, namely LBTC (load bus transmission capacity) and GBTC (generator bus transmission capacity). LBTC is the maximum real power received at a given network bus, incremental above a given operating point, that can be securely transmitted through the network from all system generation buses. \( LBTC_i \) defines the maximum possible incremental load that can be supplied by the network to a particular bus \( i \). No assumptions are made about where this power is generated, and all generators may vary within their individual limits of operation. However, all incremental loads are set to zero except for bus \( i \). LBTC is defined as:

\[
LBTC_i = \max_{S} \Delta P_{di} \tag{5.1}
\]

\[
\Delta P_{dj} = 0; \ j \neq i. \tag{5.2}
\]

Analogously, a generation bus transmission capacity may be defined as the maximum power that can be sent from a given bus to all the load buses. GBTC is defined as:

\[
GBTC_i = \max_{S} \Delta P_{gi} \tag{5.3}
\]
In general, LBTC and GBTC are not equal even if calculated at the same bus since, in a system with transmission limits, the ability of a bus to send and receive power may differ.

LBTC and GBTC define upper bounds on the capacity of a given bus to receive or send power from or to other buses in the network regardless of the consequences that operation at this limit may have on the capacity to send or receive power at other network buses. Although it would not be normal to operate at these limits because of the sharp reduction in the transmission capacity of other buses, these definitions do provide a measure of the extent to which a given bus may participate in power interchanges. In particular, LBTC and GBTC can serve as upper bounds on any potential incremental interchange from or into the network, as represented by

\[-LBTC_i \leq \Delta P_i \leq GBTC_i\]

where $\Delta P_i$ is defined as the real power injection into bus $i$.

In this chapter, LBTC and GBTC will be reposed in terms of scattering parameters and matrices. A simple topology of only one generator bus and multiple load buses will be assumed. However, it is noted that information about the state of the system, in particular, by how much a load deviates from its nominal operating point and other measures can be derived in terms of scattering parameters and matrices.

To understand LBTC and GBTC in the right context, the maximum power transfer theorem for a very simple network will be reviewed. For the simple two bus system with a voltage source $V_G$ connected to an impedance load $Z_L$ by a transmission line $Z_o = R_o + jX_o$, as depicted in Figure 2-2 in Chapter 2, maximum (real) average power occurs when $Z_L = Z_o^*$. Under that constraint, the following applies:

\[
\begin{align*}
\text{Voltage Source: } P_G &= \frac{|V_G|^2}{2R_o} \quad Q_G = 0 \\
\text{Transmission Line: } P_{Z_o} &= \frac{|V_G|^2}{4R_o} = \frac{P_G}{2} \quad Q_{Z_o} = \frac{|V_G|^2 X_o}{4R_o^2}
\end{align*}
\]
5.2 Scattering Formulations of LBTC and GBTC

\[ \text{Load: } P_L = \frac{|V_G|^2}{4R_o} = \frac{P_G}{2}, \quad Q_L = -\frac{|V_G|^2 X_o}{4R_o^2} \]

\[ |V_L| = \frac{\sqrt{R_o^2 + X_o^2} |V_G|}{2R_o} \]

One way to think about the constraint \( Z_L = Z_o^* \) is that \( P_L \) at this constraint is a global maximum since the voltage source and transmission line are a fixed topology. However, this is not what LBTC measures. As will be shown in the two-port example, given a nominal operating point specified by \( P_2^0 \) and \( Q_2^0 \), LBTC answers the question, what is the maximum power that can be transferred to port 2 for the specified \( Q_2^0 \) and topology of the network. The answer is not the global maximum provided by the maximum power transfer theorem [11] but is a local maximum stipulated by the value of \( Q_2^0 \).

It is also noted that for a typical transmission line, the reactance of the line is approximately a factor of ten greater than the resistance of the line. Under the assumption that \( X_o = 10R_o \), then \( |V_L| = \frac{\sqrt{11}|V_G|}{2} \). For a generator voltage of 1 p.u., the voltage at the load is approximately 1.66 p.u. which is outside the operating range of \( .95 \leq V_L \leq 1.05 \).

5.2 Scattering Formulations of LBTC and GBTC

In the derivations of the scattering versions of LBTC and GBTC, a nominal operating point of the system is defined since this is presupposed by the definitions of LBTC and GBTC. Given that nominal operating point, the normalization at each load port on the system is chosen as the equivalent impedance of that load bus, i.e. \( Z_{oi} = Z_{iBASE} \). As a first example, consider the two-port topology. This can easily be extended in the multiport case with one generator bus and multiple load buses. If the normalization \( Z_{o2} = Z_{2BASE} \) at port 2 is chosen, then by definition \( \Gamma_2 = 0 \). The normalization at the generator is chosen so that a matched generator results. Recall from Chapter 2 that \( a_1 = \Gamma_1 b_1 + a_{s1} \) where \( \Gamma_1 = \frac{Z_{o1} - Z_{d1}}{Z_{o1} + Z_{d1}} \) and \( a_{s1} = \frac{\sqrt{R_o}}{Z_{G1} + Z_{o1}} \). Choosing \( Z_{o1} = Z_{GEN} \), then \( \Gamma_1 = 0 \), \( a_1 = a_{s1} = \frac{|V_{G1}|^2}{2\sqrt{R_o}} \) and \( |a_1|^2 = \frac{|V_{G1}|^2}{4R_o} = P_0 \). Note that the maximum power the generator at port 1 can deliver to the network is \( \frac{|V_{G1}|^2}{4R_{G1}} \). (Maximum power transfer occurs when \( Z_{in} = Z_{G1}^* \). Under this constraint, the maximum power transfer is \( \frac{|V_{G1}|^2}{4R_{G1}} \). This is by definition GBTC. Thus
As defined in Chapter 2,

\[ P_2 = \frac{|S_{21}|^2 P_o (1 - |\Gamma_2|^2)}{|1 - S_{22} \Gamma_2|^2}. \]

Since at the nominal operating point \( \Gamma_2 = 0 \), then \( P_o^o = P_o |S_{21}|^2 \). Thus

\[
\Delta P_2 = P_2^{\text{max}} - P_o^o \\
\Delta P_2(\Gamma_2) = P_o |S_{21}|^2 \frac{(1 - |\Gamma_2|^2) - |1 - S_{22} \Gamma_2|^2}{|1 - S_{22} \Gamma_2|^2} \\
\Delta P_2(\Gamma_2^T) = \Delta P_d = \text{LBTC}_2
\]

where \( P_2^{\text{max}} \) is the maximum power that can be delivered to the load for the specified \( Q_2 \). \( P_2^{\text{max}} \) is obtained from the point of tangency \( (\Gamma_2^T) \), i.e. when the \( P_2 \) and \( Q_2 \) circles at bus 2 intersect at one and only one point. As was noted in a previous chapter, if there is no intersection point, there is no solution to the load flow. Thus the maximum power that can be delivered to bus 2 for a two bus system for a specified \( Q_2 \) is determined from the tangency point of the \( P_2 \) and \( Q_2 \) circles. The geometry of the problem is illustrated in Figure 5-1. For the specific example, the tangency point is interior to the \( Q_2 \) circle. The objective is to find the value of \( P_2 \) such that the right triangle in the figure is formed. This yields the maximum value of \( P_2 \) that can be delivered to port 2 for the specified value of \( Q_2 \). From the figure, \( r_1 \), \( c \) and \( d \) are functions of \( P_2 \) and \( r_2 \), \( a \) and \( b \) are functions of \( Q_2 \), which in this case is given and fixed. Thus the objective is to find the value of \( P_2 \) such that

\[
(r_2(Q_2) - r_1(P_2))^2 = (c(P_2) - a(Q_2))^2 + (d(P_2) - b(Q_2))^2.
\]

The two-port example in Chapter 2, Figure 2-7, will be used to determine \( \text{LBTC}_2 \). A program was written in MATLAB using the function FSOLVE to iterate \( P_2 \) until \( (r_2(Q_2) - r_1(P_2))^2 - (c(P_2) - a(Q_2))^2 + (d(P_2) - b(Q_2))^2 = 0 \). The results are plotted in Figure 5-2 and the numerical results are shown in Figure 5-3.

In the multiport case, the problem is posed in terms of finding the tangency point of
5.2. Scattering Formulations of LBTC and GBTC

Figure 5-1: Two-Port: Tangency Point Geometry, $P_2$ Circle Inside $P_3$

Figure 5-2: Two Bus Example: Tangency Point at Bus 2 ($P_2$ and $Q_2$ circles)
\[ V_g = 1.0 \]

**Normalization Data:**
\[ Z_g = Z_o1 = 1 + j1 \quad Z_o2 = 1 + j1 \]

**Transmission Data:**
\[ Z_l = 1 + j2 \]

**Load Data:**
\[ Z_{load} = 1.0 + j0.5 \]

\[ Y_A = \begin{bmatrix} 0.0132 - 0.0916i & -0.152 + 0.0554i \\ -0.152 + 0.0554i & 0.1674 - 0.2637i \end{bmatrix} \]

\[ P_{2 \ max} = 4.815645e-003 \quad \text{LBTC}_2 = 4.828895e-004 \]

**TANGENCY POINT:**
\[ \gamma_{max} = 2.610209e-001 \quad \gamma_{may} = -6.666014e-002 \]

\[ P_2 = 0.0048, \quad Q_2 = 0.0022 \quad |V_2| = 0.0988 \quad \text{V2ANG} = -38.5855 \]

Figure 5-3: Two Bus Example: Tangency Points of \( P_2 \) and \( Q_2 \) circles

Real power circles. \( LBTC \) is defined with the constraint that \( \Delta P_{dj} = 0; \quad j \neq i \). That is, the power at bus \( j \neq i \) is fixed and does not change. For a given nominal operating point the normalization at each bus (port) is chosen to be the equivalent impedance at that port. Thus \( \Gamma = 0 \) at each bus. To compute \( LBTC_i \) the maximum power that can be transferred to bus \( i \) for the specified \( P_j \)'s where \( j \neq i \) needs to be determined. It is proposed that this be done by finding the tangency point of the \( P_i \) and \( P_j \) circles.

As an example to illustrate this approach, the three bus example used in the previous chapters is revisited. In Chapter 3, the real power at buses 2 and 3 as a function of \( \Gamma_{L2} \) and \( \Gamma_{L3} \) are

\[
\begin{align*}
P_2 &= P_{o21} \frac{|S_{21}(1-S_{33}\Gamma_{L3}) + S_{31}S_{23}\Gamma_{L3}|^2(1-|\Gamma_{L2}|^2)}{|(1-S_{22}\Gamma_{L2})(1-S_{33}\Gamma_{L3}) - S_{23}S_{32}\Gamma_{L2}\Gamma_{L3}|^2} \\
P_3 &= P_{o31} \frac{|S_{31}(1-S_{22}\Gamma_{L2}) + S_{21}S_{32}\Gamma_{L2}|^2(1-|\Gamma_{L3}|^2)}{|(1-S_{22}\Gamma_{L2})(1-S_{33}\Gamma_{L3}) - S_{23}S_{32}\Gamma_{L2}\Gamma_{L3}|^2}.
\end{align*}
\]

The above equations can be interpreted as \( P_2 = P_2(\Gamma_{L2}, \Gamma_{L3}) \) and \( P_3 = P_3(\Gamma_{L2}, \Gamma_{L3}) \). To compute \( LBTC_2 \), for a fixed \( P_3 \), what is required is the largest value of \( P_2 \) such that the \( P_2 \)
and $P_3$ circles touch at one and only one point in the $\Gamma_{L2}$ plane, i.e. the tangency point. This value of $P_2$ is defined as $P_2^{\text{max}}$ and is the maximum power that can be delivered to bus 2 for a fixed $P_3$. Thus, $LBTC_2 = P_2^{\text{max}} - P_2^0$, where $P_2^0$ is the value of $P_2$ at the initial operating point. Since the normalization at bus 3 has been chosen such that $\Gamma_{L3} = 0$, then $P_2 = P_2(\Gamma_{L2})$ and $P_3 = P_3(\Gamma_{L2})$. Thus the above expressions for $P_2$ and $P_3$ reduce to

\[
P_2 = P_{o21} \frac{|S_{21}|^2(1 - |\Gamma_{L2}|^2)}{|1 - S_{22}\Gamma_{L2}|^2}, \quad P_3 = P_{o31} \frac{|S_{31} + \Gamma_{L2}(S_{32}S_{21} - S_{31}S_{22})|^2}{|1 - S_{22}\Gamma_{L2}|^2}.
\]

$P_2$ and $P_3$ can be described by circles in the $\Gamma_{L2}$ plane,

\[
(G_{L2}x - a_{P_2})^2 + (G_{L2}y - b_{P_2})^2 = r_{P_2}^2
\]

\[
(G_{L2}x - a_{P_3})^2 + (G_{L2}y - b_{P_3})^2 = r_{P_3}^2
\]

where

\[
a_{P_2} = \frac{P_2'ReS_{22} - (ReS_{31}ReK_1 + ImS_{31}ImK_1)}{D_1},
\]

\[
b_{P_2} = -\frac{P_2' ImS_{22} - (ImS_{31}ReK_1 - ReS_{31}ImK_1)}{D_1},
\]

\[
r_{P_2} = \sqrt{\frac{|S_{31}|^2 - P_2'^2}{D_1} + a_{P_2}^2 + b_{P_2}^2},
\]

\[
a_{P_3} = \frac{P_3'ReS_{33} - (ReS_{32}ReK_2 + ImS_{32}ImK_2)}{D_2},
\]

\[
b_{P_3} = -\frac{P_3' ImS_{33} - (ImS_{32}ReK_2 - ReS_{32}ImK_2)}{D_2},
\]

\[
r_{P_3} = \sqrt{\frac{|S_{21}|^2 - P_2'^2}{D_2} + a_{P_3}^2 + b_{P_3}^2},
\]

\[
P_2' = \frac{P_2}{P_{o21}},
\]

\[
P_3' = \frac{P_3}{P_{o31}},
\]

\[
D_1 = P_2' |S_{22}|^2 - |K_1|^2
\]
The geometry of the problem is illustrated in Figure 5-4. In this figure, it is assumed that the tangency point is exterior to the $P_3$ circle whereas in Figure 5-5 the tangency point is interior to the $P_3$ circle. It is not possible to tell apriori which situation exists. Increasing $P_2$ by a fractional amount to determine whether the $P_2$ circle moves to the exterior or interior of the $P_3$ circle will determine which scenario is valid.

Assuming that Scenario 1 is valid, the objective is to find the value of $P_2$ such that the right triangle is formed. This yields the maximum value of $P_2$ that can be delivered to bus 2 for the specified value of $P_3$. It is noted that $r_1$, $c$ and $d$ are functions of $P_2$ and that $r_2$, $a$ and $b$ are functions of $P_3$, which in this case is given and fixed. Thus, the value of $P_2$ that satisfies

\[(r_1(P_2) + r_2(P_3))^2 = (c(P_2) - a(P_3))^2 + (d(P_2) - b(P_3))^2\]

is the maximum power that can be delivered to bus 2 for a fixed $P_3$. For Scenario 2 in Figure 5-5, the objective is to find the value of $P_2$ such that

\[(r_2(P_3) - r_1(P_2))^2 = (c(P_2) - a(P_3))^2 + (d(P_2) - b(P_3))^2.\]

In general, to find the maximum power that can be delivered to bus $i$ assuming the power at bus $j$, $j \neq i$, is fixed, then either of the following two equations must be satisfied:

**Scenario 1:**
\[(r_1(P_i) + r_2(P_j))^2 = (c(P_i) - a(P_j))^2 + (d(P_i) - b(P_j))^2 \quad (5.5)\]

**Scenario 2:**
\[(r_2(P_j) - r_1(P_i))^2 = (c(P_i) - a(P_j))^2 + (d(P_i) - b(P_j))^2 \quad (5.6)\]
5.2. Scattering Formulations of LBTC and GBTC

In either of the two scenarios,

\[ LBTC_i = P_i^{\text{max}} - P_i^0. \]  (5.7)

5.2.1 Three Bus Examples

In this section, the three bus example of the previous two chapters will be revisited and the LBTC’s of buses 2 and 3 will be determined.

In the first example, LBTC is computed at each bus. Hence the maximum power for bus 2 is determined by iterating \( P_2 \) until equation (5.5) is satisfied. Note that \( i = 2 \) and \( j = 3 \). A program was written in MATLAB using the function FSOLVE and iterating \( P_2 \) until \( (r_1(P_2) + r_2(P_3))^2 - (c(P_2) - a(P_3))^2 + (d(P_2) - b(P_3))^2 = 0 \). The results for bus 2 are plotted in Figures 5-6 and 5-7. Likewise for bus 3, the maximum power for bus 3 is determined by iterating \( P_3 \) until equation (5.5) is satisfied. Note that \( i = 3 \) and \( j = 2 \). The results for bus 3 are plotted in Figures 5-8 and 5-9. The numerical results are shown in Figure 5-10 and a summary of statistics is shown in Table 5.1. It is noted in Table 5.1 that the voltages for buses 2 and 3 are greater than 1.05 p.u. at the maximum power transfer point. This is consistent with an earlier result shown on a two bus system in [17] that for typical parameters of transmission line, the bus load voltages at the maximum power transfer level are unacceptably high. This result is based on the maximum power transfer theorem. As has been shown earlier, for a simple two bus system with a voltage source \( V_G \) connected to an impedance load \( Z_L \) by a transmission line \( Z_0 \), maximum (real) average power occurs when \( Z_L = Z_0^* \). Under that constraint, the magnitude of the voltage

| BUS | POINT  | P   | Q    | |V|    | AV
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>NOMINAL</td>
<td>1.1</td>
<td>-.07</td>
<td>1.0036</td>
<td>-12.077</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>MAXIMUM</td>
<td>11.7746</td>
<td>-24.322</td>
<td>1.5057</td>
<td>-101.48</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>NOMINAL</td>
<td>1.3</td>
<td>-.04</td>
<td>1.0132</td>
<td>-11.6235</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Summary Statistics
Chapter 5. Transmission Capacity In Power Networks

Figure 5-4: Scenario One: $P_2$ Circle Outside $P_3$ Circle

Figure 5-5: Scenario Two: $P_2$ Circle Inside $P_3$ Circle
at the load is \( |V_L| = \frac{\sqrt{R_0^2 + X_0^2}}{2R_0} |V_G| \) where \( R_0 \) is the resistance and \( X_0 \) is the reactance of the transmission line \( Z_0 \). At maximum power transfer, the source does not deliver any net reactive power to the combination of \( Z_L \) and \( Z_0^* \) since the series combination is purely resistive, \( 2R_0 \). Thus the source is capable of delivering maximum power when its reactive power output is zero. In terms of operating power systems, the “better” reactive power compensation, the more real power can be transferred from the source to the network.

The results shown in Table 5.1 differ from those shown in [17] which were obtained for a two bus network. Given the difference in the size of the systems, the maximum power that the load could receive was calculated in [17] assuming that \( \frac{Q_L}{P_L} \) is given and assumed constant. However, in the three bus example, as indicated in Table 5.1, \( \frac{Q_L}{P_L} \) is not constant. For example, in calculating the maximum power transferred to bus 2, \( P_3 \) and \( Q_3 \) are constant while \( Q_2 \) is not. Furthermore, at the nominal point, \( \frac{Q_2}{P_2} = -0.0636 \) and at the maximum power transfer point, \( \frac{Q_2}{P_2} = -2.0656 \).
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Figure 5-6: Three Bus Example: Tangency Point at Bus 2 (P2 and P3 circles)

Figure 5-7: Three Bus Example: Tangency Point at Bus 2 (P2 and P3 circles) - Blowup
5.2. Scattering Formulations of LBTC and GBTC

Figure 5-8: Three Bus Example: Tangency Point at Bus 3 (P3 and P2 circles)

Figure 5-9: Three Bus Example: Tangency Point at Bus 3 (P3 and P2 circles) - Blowup
Chapter 5. Transmission Capacity In Power Networks

\[
Z_0 = \begin{bmatrix}
0.0045 + 0.0450i & 0 & 0 \\
0 & 0.9119 - 0.0580i & 0 \\
0 & 0 & 0.7173 - 0.2207i
\end{bmatrix}
\]

\[
Y_{BUS} = \begin{bmatrix}
0 & -24.9925i & 0 +10.0000i & 0 +14.9925i \\
0 +10.0000i & 0 & -22.0048i & 0 +12.0048i \\
0 +14.9925i & 0 +12.0048i & 0 & -26.9973i
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
0.9782 + 0.0003i & 0.1392 - 0.0206i & 0.1523 + 0.0146i \\
0.1392 - 0.0206i & -0.9699 + 0.1009i & -0.0103 + 0.1711i \\
0.1523 + 0.0146i & -0.0103 + 0.1711i & -0.9133 - 0.3362i
\end{bmatrix}
\]

INPUT LOAD DATA:

P2 = 1.100000e+000, Q2 = -7.000000e-002, V2M = 1.003590e+000 
P3 = 1.300000e+000, Q3 = -4.000000e-001, V3M = 1.010320e+000

PORT 1 STATISTICS:

Pgen = 2.426504e+000 P dissipated in Zgen = 2.650043e-002 
P2 = Power injected into Port 1 = 2.400004e+000 P2 + P3 = 2.400000e+000

BUS 2:

P2 max = 1.177459e+001 LBTC2 = 1.067459e+001

TANGENCY POINT: \( \text{gamma}_x = -9.222624e-001 \) \( \text{gamma}_y = -3.542869e-002 \)

\( \gamma_1 = [-0.9223 - 0.0354i, 0] \)

P2 = 1.177459e+001 Q2 = -2.432202e+001 V2M = 1.505687e+000 V2ANG = -1.014816e+002 
P3 = 1.300000e+000 Q3 = -4.000000e-001 V3M = 1.010320e+000 V3ANG = -8.804329e+001

BUS 3:

P3 max = 1.197462e+001 LBTC3 = 1.067462e+001

TANGENCY POINT: \( \text{gamma}_x = -8.258745e-001 \) \( \text{gamma}_y = 3.808031e-001 \)

\( \gamma_1 = [0, -0.8259 + 0.3808i] \)

P2 = 1.100000e+000 Q2 = -7.000000e-002 V2M = 1.003590e+000 V2ANG = -8.849666e+001 
P3 = 1.197462e+001 Q3 = -2.138437e+001 V3M = 1.337344e+000 V3ANG = -9.786370e+001

Figure 5-10: Example I: Tangency Points of P2 and P3 circles
Chapter 6

Conclusions

In this chapter, the main results and contributions of this thesis will be summarized and directions for future research will be outlined.

The initial thrust of this thesis was a proof concept of applying scattering parameters/matrices in solving various types of problems in the electric power industry. Initially, the objective was to define the load flow problem in the scattering domain and compare the results obtained to those obtained using conventional approaches. The main reason for describing the problem in the scattering domain is that this domain is a natural basis in which to describe power type problems. In the process of doing this, a number of advantages and attributes of describing power system problems in the scattering domain became clear. This lead to two other applications, namely a decentralized load flow approach and the calculation of the transfer capacity in power networks.

6.1 Contributions of Thesis

6.1.1 Coordinated Load Flow Problem

The first application of scattering parameters/matrices was defining the load flow problem in the scattering domain which is the coordinated load flow problem. This is the same as the conventional load flow which is used for feasibility and security studies. The motivation behind the scattering approach was to exploit the superposition property of the reflected
power. In particular, in the voltage-current description of a transmission system, \( I = YV \). This is a linear map and superposition holds for \( V \). In the scattering domain, where the variables of interest are \( a \), the incident power, and \( b \), the reflected power, the scattering description of the transmission system is \( b = Sa \). This is also a linear map and superposition holds for \( a \), the reflected power.

The conventional approach to load flow is to describe the loads, generators and transmission system in the voltage, current domain. In that domain loads are described as PQ loads and generation is either described as a PV bus or a slack bus, modeled as a constant voltage source. In the scattering domain, all the loads and generation (with the exception of the slack bus) are characterized by the scattering parameter \( \Gamma \). The slack bus is characterized by \( \Gamma \) and \( a_S \), which is a function of the voltage and internal impedance of the generator. The transmission system, as alluded to previously, is described by a linear map \( b = Sa \).

The structure of the load flow equations in the scattering domain has a direct parallel to that of the load flow equations in the voltage, current domain. In the voltage, current domain, the port characterizations of the loads are

\[
fi(I_i, V_i, Parameters_i) = 0 \text{ for all } i. \tag{6.1}
\]

The transmission system is characterized by

\[
I = Y_{BUS}V. \tag{6.2}
\]

The load flow equations are obtained from solving equation (6.1) for \( I_i = f_i^{-1}(V_i, Parameters_i) \) and then substituting this into equation (6.2) yielding the load flow equations,

\[
diag(f_i^{-1}(V_i, Parameters_i)) \mathbb{1} = Y_{BUS}V. \tag{6.3}
\]

The parallel in the scattering domain is that the port characterization in a-b space is \( a = \Gamma b + a_g \quad a_i = \Gamma_i b_i \), for all \( i \), \( \Gamma = diag(\Gamma_i) \). For the transmission system, the description
is $b = S_a$ and the system characterization is $b = (I - ST)^{-1} S_a$. As in the voltage-current space, the coupled port-system characterization in a-b space is needed since the unknowns are $b$ and $\Gamma$. Thus, the the port characterization has the form $f(\Gamma_i, b_i, \text{Parameters}_i) = 0$. In particular, the PQ bus characterization of $f_i(\Gamma_i, b_i, P_i, Q_i) = 0$ is

$$P_i = |b_i|^2(1 - |\Gamma_i|^2)$$

$$Q_i = |b_i|^2\left(\frac{X_{oi}}{R_{oi}}(1 + |\Gamma_i|^2) + 2Im(Z_{oi}^*\Gamma_i)\right)$$

and the PV bus characterization is

$$P_i = |b_i|^2(1 - |\Gamma_i|^2)$$

$$V_i = \frac{b_i(Z_{oi}^*\Gamma_i + Z_{oi})}{\sqrt{R_{oi}}}$$

for all $i$ except the slack bus. The result is a geometric display of the solution to the load flow represented by the intersection point or points in the scattering domain at each bus in the system. The solutions obtained from the scattering domain approach were the same as those obtained from the conventional approach, namely voltages in the range of .95 to 1.05 p.u. However, this approach also yields the low voltage solution. Perhaps, the most striking feature of a solution display is that the separation of the two solution points is a metric of how close the solution is to the edge, i.e. how close a bus of interest and the system come to not having a solution. Thus, in the case where the two solution points are very close at a particular bus, doing a power transfer from some other point on the system would not be possible.

The main contributions for the centralized load flow problem in the scattering domain are the geometric representation of the equilibrium solution, the geometric representation that allows for the determination of more than one solution and the ability to put bounds on the system inputs for which a solution exists.
6.1.2 Decentralized Load Flow Formulation

A decentralized load flow problem formulation involves obtaining information about other buses on the system using local measurements. Because of deregulation, the basic approach to load flow is changing due to a lack of systemwide information. There is a need for more decentralized computing/decision making. The assumptions made in posing a decentralized load flow problem are that there is a step change in one or more of the load parameters and the system settles to a new equilibrium and that transmission and generation (i.e. the slack bus) are known by all players on the system.

A proposed decentralized load flow approach capitalizes on the fact that if the loads and generation are defined in terms of scattering parameters and the transmission system is defined as \( b = Sa \), the problem is transformed into the scattering domain and solved explicitly in the scattering domain. In the case of a three bus example in the thesis, which was used as proof of concept, it is assumed that characteristics of one of the loads is known. Since it is known, it can be characterized by \( a \), \( b \) and hence \( F \). The solution to the problem is to solve for \( F \) at the other load such that \( b = Sa \) is valid for the entire system simultaneously.

This approach exploits the linear input/output map \( b = Sa \). The nonlinearity of the loads is transformed by characterizing the loads in terms of the incident and reflected powers. Since the problem is now linear at each iteration, a Gauss approach was used to determine \( F \) at the other bus (in the three bus example). In particular if the data is known at bus 3, the network constraints are solved for expressions that relate \( F_2 \) as a function of generation, transmission and the incident and reflected waves at bus 2. The algorithm is then iterated until a prespecified error criterion is reached. As stated in the previous chapter, a decentralized approach proposed has many potential applications in the deregulated environment as a mechanism to obtain information about the state of the system through local measurements.

The main contribution is that the scattering description of the problem establishes a methodology in which information can be obtained in a decentralized manner.
6.1.3 Transmission Capacity In Power Networks

Since the solutions to the load flow in the scattering domain can be obtained from the intersection points of circles, this geometric approach was applied in the calculation of the transfer capacity in the power network. The conventional approach is to do a DC load flow to obtain the solution. The scattering approach exploits the geometry of the problem by obtaining the tangency points of power circles since the premise is that as the real power is varied, the maximum amount of power that can be transferred to a particular bus is the value of the real power such that the two circles (in a three bus example) are at their tangency point.

The load bus transmission capacity is defined as the maximum real power received at a given network bus, incremental \(^1\) above a given operating point, that can be securely transmitted through the network from all system generation buses. The load bus transmission capacity of bus \(i\) defines the maximum possible incremental load that can be supplied by the network to a particular bus \(i\). Thus, for a given operating point, i.e the real power specified and fixed at all load buses \(i \neq j\), what is the maximum real power that can be transferred to bus \(i\) above the current operating point such that there is still a solution to the load flow? This is readily translatable into finding the first tangency point of the real power circle of bus \(i\) to all the other real power circles for buses \(i \neq j\).

The main contribution is that this is a geometric approach to obtain the load bus transmission capacity as opposed to a straight numerical algorithm. Also the method is iterative in a way which requires separation between system and individual port characteristics.

6.2 Suggestions for Future Research

The idea behind this thesis was to provide proof of concept for formulating and solving certain electric power system problems that are not easily or readily solved in the voltage and current domain in the scattering domain. This thesis has just started the journey toward understanding the implications of what the scattering domain representations have

\(^1\)Increment is not necessarily infinitesimally small; it is a finite increment.
to offer in the way of geometrical representations of solutions and simplified algorithms to solve the problems in complex large-scale electric power systems. With that in mind, the basic centralized load flow in the scattering domain needs further extensions to multi-bus systems. That work has been started in Chapter 3. The algorithms for multi-bus systems are discussed in that chapter. However, modelling issues concerning PV buses need to be resolved. Since the PV bus is by nature an active device, it is injecting power into the system and the model of the PV bus can not simply be captured by $\Gamma$ as it can in the case of PQ loads. At a snap shot in time, the PV bus has to be modeled similarly as a slack bus, namely assigning a voltage and an impedance. The characterization of the voltage source is straightforward, however, the characterization of the impedance is not. This needs further investigation.

The scattering algorithms by which the load flow is solved can be defined in other ways. In particular, they can be partitioned in other ways. These algorithms need to be investigated such that the convergence rate of the load flow is improved. In particular, how these algorithms compare to the current load flow algorithms needs to be benchmarked.

Another issue is that line power flow constraints are not incorporated into the current algorithms for the scattering version of the load flow.

In the case of a decentralized load flow formulation, multi-bus extensions also need to be made. Multi-bus extensions need to be made for the previously defined formulation which assumes a step change in a load parameter leading to a stable equilibrium, but a new formulation of a decentralized load flow problem needs to be characterized in the situation where the system is changing all the time. Perhaps a Kalman type filter approach that uses samples of information up to time step $k$ would be appropriate for learning about the current state of the system.

Lastly, the scattering formulation needs to be applied to other optimality measures, i.e., ATC, etc. In this case, line power flow constraints also need to be accounted for.
Appendix A

Scattering Parameters in Microwave Theory

In this appendix the role of a scattering parameter and/or a scattering matrix in microwave theory is reviewed. In particular, the basic concepts of transmission line theory (TEM: Transverse Electromagnetic wave only) are reviewed [20] and the analogies between field analysis and basic circuit theory are drawn by reviewing the expressions for power transfer.

A.1 An Overview of Transmission Line Theory

The criteria to determine whether a lumped or distributed circuit model is used, are the circuit dimensions and the operating frequency. If the wavelength is large compared to the dimension of the circuit, i.e., there is negligible phase change from one point to another or if it is a low-frequency circuit, then lumped passive or active components with unique voltages and currents defined at any point are used. The circuit that satisfies the low-frequency approximation must possess the concept of an impedance element [19]. That is, the element has two terminals and that charge may be transferred to the element only by means of the terminals. If a current flows into one terminal, an equal current must flow out of the other. Moreover, between the terminals of an impedance element there exists an electric field. The potential difference, or voltage, between the two terminals is
defined as the line integral of the electric field from one to the other. For the low-frequency approximation, this line integral is independent of the path between two terminals.

On the other hand, the circuit with dimensions in fractions of a wavelength, or many wavelengths, is a distributed-parameter network, where voltages and currents can vary in magnitude and phase over its length. Hence, circuit theory cannot be applied to analyze a distributed-parameter network. Field analysis and Maxwell's equations can be used to analyze this network to obtain quantities that are directly related to a circuit or transmission line parameter. This allows for the transmission line or waveguide to be treated as a distributed component characterized by its length, propagation constant, and characteristic impedance [20]. However, the short piece of line of length $\Delta z$ can be modeled as a lumped-element circuit, where resistance, conductance, inductance, and capacitance are per unit length quantities.

For example, a lossless TEM transmission line can be characterized by its inductance $L$ and capacitance $C$ per unit length, where the governing differential equations are

$$\frac{\partial V(z)}{\partial z} = -j\omega LI(z)$$
$$\frac{\partial I(z)}{\partial z} = -j\omega CV(z)$$

(A.1)

and $R = G = 0$. The circuit representation of the transmission line [21] is shown in Figure 1.

![Figure A-1: Incremental approximate equivalent circuits for TEM transmission lines](image)

The difference equations for $V(z)$ and $I(z)$ in the lumped-element circuit are

$$V(z + \Delta z) - V(z) = \Delta V(z) = -j\omega L \Delta z I(z)$$
A.1. An Overview of Transmission Line Theory

\[ I(z + \Delta z) - I(z) = \Delta I(z) = -j\omega C \Delta z V(z). \] \hspace{1cm} (A.2)

Equation (A.1) suggests that as \( \Delta z \to 0 \), the models become equivalent. Dividing equation (A.2) by \( \Delta z \) and letting \( \Delta z \to 0 \) yields the differential equation (A.1). However, the approximation of a TEM transmission line by discrete (lumped) elements is good only if \( \Delta z \ll \lambda \). Therefore, a finite length of transmission line can be viewed as a cascade of sections and are characterized by equations (A.1) and (A.2).

In addition, the equations and phenomenon of plane waves and transmission lines are similar since both are cases of basic wave phenomenon as derived from Maxwell’s equations. Thus, concepts such as reflection etc. developed for one system can be equally applied to the other.

A.1.1 A Transmission Line

Given the following definitions,

- \( R \): series resistance per unit length
- \( L \): series inductance per unit length
- \( G \): shunt conductance per unit length
- \( C \): shunt capacitance per unit length,

a TEM transmission line is generally characterized by the following differential equations:

\[ \frac{\partial V(z)}{\partial z} = -(R + j\omega L)I(z) \] \hspace{1cm} (A.3)

and

\[ \frac{\partial I(z)}{\partial z} = -(G + j\omega C)V(z) \] \hspace{1cm} (A.4)

or

\[ \frac{\partial^2 V(z)}{\partial z^2} = (R + j\omega L)(G + j\omega C)V(z). \] \hspace{1cm} (A.5)
The solution to this wave equation is

\[ V(z) = V_+ e^{-jkz} + V_- e^{jkz} \quad (A.6) \]

and

\[ I(z) = Y_o(V_+ e^{-jkz} - V_- e^{jkz}) \quad (A.7) \]

where

\[ k^2 = -(R + j\omega L)(G + j\omega C) \]

and

\[ Y_o = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \quad \text{and} \quad Y_o^{-1} = Z_o. \quad (A.8) \]

\( Y_o \) is defined as the characteristic admittance and \( Z_o \) is the characteristic impedance of the TEM line. Note that if the transmission line of interest is lossless \( (G, R = 0) \), then \( Z_o = \sqrt{\frac{k}{C}} \).

Equation (A.7) can be rewritten as

\[ I(z) = I_+ e^{-jkz} + I_- e^{jkz}. \quad (A.9) \]

Hence,

\[ \frac{V_+}{I_+} = Z_o = -\frac{V_-}{I_-}. \quad (A.10) \]

Substituting this relationship into equation (A.9) yields

\[ I(z) = \frac{V_+}{Z_o} e^{-jkz} - \frac{V_-}{Z_o} e^{jkz}. \quad (A.11) \]
Equations (A.6) and (A.7) can be rewritten in the following form ([18]) as

\[ V(z) = Ae^{-jkz} + Be^{+jkz} \]  \hspace{1cm} (A.12)

\[ I(z) = Z_o^{-1}(Ae^{-jkz} - Be^{+jkz}) \]  \hspace{1cm} (A.13)

where \( A \) and \( B \) are complex numbers. In equations (A.12) and (A.13), the voltage and current at every point on the line are automatically determined if the following conditions are specified:

- Both voltage and current are given at one point along the line.
- Voltages are specified at two different points a distance \( l \) apart where \( kl \neq n\pi \) (\( n \) : integer).
- Currents at these two points are given.
- The ratio of the voltage to the current at one point and either the voltage or current at another are specified.

Applying a linear transformation to \( V(z) \) and \( I(z) \) results in

\[ a(z) = \frac{1}{2} Z_o^{-1/2} \{V(z) + Z_o I(z)\} \quad b(z) = \frac{1}{2} Z_o^{-1/2} \{V(z) - Z_o I(z)\}. \]  \hspace{1cm} (A.14)

For \( Z_o \) fixed, both \( a(z) \) and \( b(z) \) are defined if \( V(z) \) and \( I(z) \) are given. Conversely, if \( a(z) \) and \( b(z) \) are given both \( V(z) \) and \( I(z) \) can be obtained from:

\[ V(z) = Z_o^{1/2} \{a(z) + b(z)\} \quad I(z) = Z_o^{-1/2} \{a(z) - b(z)\}. \]  \hspace{1cm} (A.15)

Hence, the transmission line can be studied in terms of \( V(z) \) and \( I(z) \) or in terms of \( a(z) \) and \( b(z) \). From the above equations, it can be shown that:

\[ a(z) = Z_o^{-1/2} Ae^{-jkz} \quad b(z) = Z_o^{-1/2} Be^{+jkz}. \]  \hspace{1cm} (A.16)
Appendix A. Scattering Parameters in Microwave Theory

It is clear that the magnitudes of \( a(z) \) and \( b(z) \) are constant and that the phases are directly proportional to \( z \). Equation (A.16) shows that \( a(z) \) represents a wave moving toward the positive \( z \)-direction with a constant velocity \( v_p \), the phase velocity, which is defined as

\[
v_p = \frac{\omega}{k}.
\]

Similarly, \( b(z) \) is a wave moving toward the negative \( z \)-direction with constant velocity \( v_p \) as well. \( a(z) \) and \( b(z) \) are traveling waves. The net power flowing toward the positive \( z \) direction can be determined as

\[
P = \text{Re}\{V(z)I^*(z)\} = \text{Re}\{[a(z) + b(z)]a^*(z) - b^*(z)]\}
\]

\[
P = |a(z)|^2 - |b(z)|^2.
\]

Equation (A.17)

The wave \( a(z) \) moving in the positive \( z \)-direction carries the power \(|a(z)|^2 \) [18] and similarly the wave \( b(z) \) carries the power \(|b(z)|^2 \) toward the negative \( z \)-direction. The net power toward the positive \( z \)-direction is therefore given by the difference in the two powers as shown by equation (A.17)

The ratio of \( a(z) \) and \( b(z) \) is defined as the reflection coefficient \( \Gamma \)

\[
\Gamma = \frac{b(z)}{a(z)} = \frac{(B/A)e^{2kz}}.(A.18)
\]

If \( a(z) \) is considered to be an incident wave, \( b(z) \) represents the reflected wave and \( \Gamma \) expresses the magnitude and phase of the reflected wave relative to the incident wave.

The standing wave ratio (SWR) is defined as

\[
\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}
\]

(A.19)

where \( \Gamma_L \) is the reflection coefficient at the load. From Figure 2 [20] the total voltage and current at the load are related by the load impedance, so at \( z = 0 \) the result is
Figure A-2: A transmission line terminated in a load impedance $Z_L$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_+ + V_-}{V_+ - V_-} Z_0$$  \hspace{1cm} (A.20)$$

and thus from equation (A.18),

$$\Gamma_L = (B/A) = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}.$$  \hspace{1cm} (A.21)$$

This is a bilinear-transformation which maps the $Z_L$-space or normalized $Z_L/Z_0$-space into the $\Gamma$-space. This leads to the *Smith chart.*
Bibliography


