Design of a 460 GHz Second Harmonic Gyrotron Oscillator for use in Dynamic Nuclear Polarization

by

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Abstract

Dynamic nuclear polarization (DNP) is a promising technique for sensitivity enhancement in the nuclear magnetic resonance (NMR) of biological solids. Resolution in the NMR experiment is related to the strength of the static magnetic field. For that reason, high resolution DNP NMR requires a reliable millimeter/sub-millimeter coherent source, such as a gyrotron, operating at high power levels. In this work we report the design of a gyrotron oscillator for continuous-wave operation in the $TE_{06}$ mode at 460 GHz at a power level of up to 50 W to provide microwave power to a 16.4 T 700 MHz NMR system and 460 GHz EPR/DNP unit. The gyrotron design is closely modeled on a previous successful design of a 25 W, 250 GHz CW gyrotron oscillator. The present design will operate at second harmonic of the cyclotron frequency, $\omega \approx 2\omega_c$. Second harmonic operation requires careful analysis in order to optimize the oscillator design under conditions of moderate gain, high ohmic loss, and competition from fundamental ($\omega \approx \omega_c$) modes. In addition, an internal mode converter will be used. The output radiation will propagate through a window and perpendicular to the magnetic field through a cross-bore pipe in the magnet. The radiation will then travel through a transmission line to arrive at the probe used for NMR experiments. In addition, the transmission line of a 140 GHz gyrotron oscillator currently operating in DNP-NMR experiments, including an external serpentine mode converter which converts the $TE_{01}$ to $TE_{11}$ mode, is analyzed experimentally and theoretically.

Thesis Supervisor: Richard J. Temkin
Title: Senior Research Scientist, Department of Physics
Acknowledgments

This masterwork is exactly that: a work. Moreover, it flows from the wells of continuity upon giants' shoulders much like a droplet, in hopes of becoming a splash. Let me now recount to you a passage from Kahlil Gibran's "The Prophet" [1].

You work that you may keep pace with the earth and the soul of the earth. For to be idle is to become a stranger unto the seasons, and to step out of life's procession, that marches in majesty and proud submission towards the infinite. When you work you are a flute through whose heart the whispering of the hours turns to music. Which of you would be a reed, dumb and silent, when all else sings together in unison? Always you have been told that work is a curse and a labor, a misfortune. But I say to you that when you work you fulfil a part of earth's furthest dream, assigned to you when that dream was born, And in keeping yourself with labor you are in truth loving life, And to love life through labor is to be intimate with life's inmost secret. ... Work is love made visible. And if you cannot work with love but only with distaste, it is better that you should leave your work and sit at the gate of the temple and take alms of those who work with joy. For if you bake bread with indifference, you bake a bitter bread that feeds but half man's hunger. And if you grudge the crushing of the grapes, your grudge distills a poison in the wine. And if you sing though as angels, and love not the singing, you muffle man's ears to the voices of the day and the voices of the night.

Many shoulders were scrambled upon in the completion of this work, like a tower of tumbling skydivers standing strongly in the four winds upon their imaginary framework that only they and Don Quixote can see. For it is the giants who must believe the existence of the windmill. And the rest of us to prove it. And would for we should never stop dreaming lest all should come crashing down? And who are those skyclimbers who support with their minds and hearts?

In impressionable and impressive times, both Sigrid McAfee and Evangelia Tzanakou (like ebb and flow) wove a seesaw hodgepodge patchwork girl into a yarn of her choosing. Thank you Professors for speaking with wisdom, steering with stories, and not looking up to count the countless pink hourglass sands fallen whilst we forgot the worries whisking past the open door.

In the uncertain days, Jagadishwar Sirigiri showed me his Lab when I was still known as a “Teaching Assistant”. Ken Kreischer started me on this Work, and tried
to teach me all he knew in record time. Only I think my ears had a lot of wax in
them. And thus Richard Temkin became my Mentor. Thank you, Rick, Ken, and
Jags. And that was only the Beginning. Now I have come to know and depend on
many more, as numerous as the Stars in a Night Lake:

Michael Shapiro without whom a lot of Theoretical
Things worldwide would not be possible
Bob Griffin for inspiring me to have a Couch in my office
Jim Anderson my Officemate, for answering Stupid
Questions that ought’nt’ve been asked
Steve Korbly for his help on EGUN simulations
Vik Bajaj for knowing alot about Chemistry
Melanie Rosay for taking care of the Gyrotron
the other Graduate Students for Reasons only they know
Jeff Vieregg for his work on the Controls
Simplicious for Library Things
Paul “Mr. Magnet” Thomas for bringing meaning to the Result
Catherine Fiore &
Amanda Hubbard for the Women’s Lunches
my Mother and my Father for Much More
my Three Sisters for Sisterly Things
Toto (not the gyrotron),
Oskar, & family for the support given by Dogs & Family

And until the end, after thanking my Good Friends, some of whom I call Sisters
and Brothers in Arms (don’t scold me for not writing Your Names; there is a magic
in the Unspoken!), should we awake to realize that we are but the dream of a mad
knight errant, let the lesson be in not grudging the essence of our daily lives but in
knowing the luck of oneday joining the ranks of the giants in the sky.
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Chapter 1

Introduction

The gyrotron oscillator, sometimes referred to as the “electron cyclotron resonance maser” or “gyromonotron”, is a high-power high-frequency source of coherent electromagnetic radiation \[7\]. It is a fast-wave device, one of the latest additions to the microwave tube family, which can operate with efficiencies as high as 60\% \[8\]. Gyrotrons can operate throughout the microwave and millimeter-wave spectra, inching into the sub-millimeter-wave spectra.

In the microwave regime (3–30 GHz), conventional microwave tubes such as klystrons, traveling wave tubes (TWTs), and backward wave oscillators (BWOs), all slow-wave devices, are fierce competitors. The dimensions of the interaction structures of these slow-wave devices scale down accordingly with increasing frequency of operation; their decreasing structure size intrinsically limits these devices to the microwave regime due to increasingly high power levels and ohmic heating concerns. Thus, the millimeter-wave band (30–300 GHz) witnesses the inherent dominance of gyrotrons in generating high power, supplanting the pre-eminence of the conventional microwave tubes.

In the sub-millimeter-wave regime (>300 GHz), the need to operate at harmonics of the cyclotron frequency due to limitations of the superconducting solenoids that provide the magnetic field leave this range at a draw, perhaps acquiescing to free-electron-lasers (FELs) or still gyrotrons with a clever design \[9\]. This work endeavors to demonstrate that gyrotrons can also occupy this burgeoning niche.
The gyrotron is a source of high power coherent radiation. It consists of a magnetron injection gun, which generates an annular electron beam which is focussed into an open cavity resonator along an axial magnetic field, created by a superconducting magnet. In the cavity, the RF field interacts with the cyclotron motion of the electrons in the beam and converts the transverse kinetic energy into an RF beam which may then be internally converted into a Gaussian beam. The spent electron beam leaves the cavity and propagates to the collector where it is collected.

The development of gyrotrons has split in two main directions. High power millimeter wave gyrotrons are needed as the power sources of electron cyclotron heating (ECH) of plasmas, electron cyclotron current drive of tokamaks [10], radar [11], and for ceramic sintering [12]. Medium power millimeter to submillimeter wave gyrotrons are required for plasma scattering measurements [13], and more recently electron spin resonance (ESR) experiments [14] and nuclear magnetic resonance (NMR) signal enhancement by dynamic nuclear polarization (DNP) [15, 16, 17, 18, 19, 20, 21]. This paper will discuss the design of a gyrotron for the latter purpose.

Due to small nuclear Zeeman energy splittings and correspondingly small bulk magnetization at thermal equilibrium, NMR spectroscopy is plagued by low sensitivity. Relaxation processes in the solid state further compromise these experiments, with the result that solid state NMR experiments have two or three orders of magnitude lower sensitivity in time than comparable experiments in the solution state. Dynamic nuclear polarization is a technique for sensitivity enhancement in these studies. Previously, its application has been restricted to low magnetic fields, a regime in which the resolution of typical NMR spectra is not sufficient for structure determination of large macromolecules. This restriction was due mostly to the lack of robust millimeter wave sources required for the DNP experiment at high fields; available sources relied on slow-wave structures which are fragile and cannot support sustained operation at the power levels required for DNP experiments. The cyclotron resonance maser, or gyrotron, is a fast-wave, millimeter/sub-millimeter wavelength device developed for high power (megawatt-range) plasma heating. Because it is a fast-wave device, it is not dimensionally constrained to the order of a wavelength, and it can
Table 1.1: Progress in DNP gyrotrons at MIT

<table>
<thead>
<tr>
<th>Year</th>
<th>Frequency [GHz]</th>
<th>Cyclotron Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>140</td>
<td>1</td>
</tr>
<tr>
<td>1998</td>
<td>250</td>
<td>1</td>
</tr>
<tr>
<td>2001</td>
<td>460</td>
<td>2</td>
</tr>
</tbody>
</table>

be easily scaled to the lower powers and higher duty cycles required for DNP.

In this work, we present the design of a high power second harmonic gyrotron oscillator for use in dynamic nuclear polarization experiments at 460 GHz in conjunction with a 700 MHz, 16.4 T NMR system. The design is based upon a continuous-wave 250 GHz gyrotron oscillator which is already used in DNP experiments. A DNP-NMR spectrometer operating with a 140 GHz gyrotron has also been successfully implemented and achieved signal enhancements.

Some relevant theory of gyrotrons and DNP is presented in Chapter 2. Chapter 3 describes the design parameters and simulations of the 460 GHz gyrotron in detail. A related experiment of the transmission line, including a snake mode converter, of a 140 GHz DNP-gyrotron will be presented in Chapter 4. Finally, chapter 5 contains some concluding remarks.
Chapter 2

Principles and Theory

2.1 Gyrotron theory

This is merely an introduction to relevant topics in gyrotron, nuclear magnetic resonance, and dynamic nuclear polarization theory. A more complete description of gyrotron theory can be found through various references in the bibliography [7, 8, 9, 22]. A useful introduction to NMR theory can be found in [23]. This chapter presents some of the more immediately useful gyrotron equations, most importantly including a discussion of the mechanism through which the electron beam couples with the RF field and transfers its energy to the resonant cavity mode.

The gyrotron, as with all microwave sources, is based on the conversion of electron beam energy into radiation using a resonant structure. An annular electron beam is produced by a magnetron injection gun and travels through a resonant cavity, located at the center of the superconducting magnet. The magnetic field causes the electrons to gyrate and thus emit radiation. If the magnetic field and cavity are tuned to match the beam parameters, the radiation will couple into a resonant cavity TE mode. When the interaction is completed, the spent beam leaves the cavity and propagates to the collector. The microwaves are transmitted from the cavity to a mode converter, where they are transformed into a Gaussian beam that then exits the gyrotron through a vacuum window. Past the window, the beam can then be propagated down a transmission line, usually a cylindrical waveguide, to its place of
2.1.1 Electron beam

First, let’s begin from the electron beam. The electron beam is generated by an electron gun at the cathode and accelerated towards the anode. In the region between the cathode and the anode, the electric and magnetic fields are more or less constant, with the electric fields lines and magnetic field lines perpendicular to each other. The electrons have a constant $\vec{E} \times \vec{B}$ drift velocity $v_{\perp K}$ given by

$$v_{\perp K} = \frac{\vec{E} \times \vec{B}}{B^2}$$  \hspace{2cm} (2.1)

$$v_{\perp K} = \frac{E_{\perp K}}{B_K}$$  \hspace{2cm} (2.2)

As the electrons reach the end of the cathode-anode region, the electric field lines change from mostly perpendicular to parallel to the magnetic field lines. This change in $E_{\perp K}$ at the cathode over a gyro-period implies a non-adiabatic transition.

When the electrons acquire a velocity perpendicular to the axial magnetic field, they experience a Lorentz force,

$$F = e \frac{v_{\perp}}{c} \times B_0$$  \hspace{2cm} (2.3)

resulting in the creation of a circular motion with angular frequency

$$\omega_c = \frac{eB_0}{m \gamma}$$  \hspace{2cm} (2.4)

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$  \hspace{2cm} (2.5)

is the relativistic mass factor. The result of the superposition of the circular motion on axial motion is a helically gyrating trajectory about a fixed guiding center, where
the radius of the path, the Larmor radius,

\[ r_L = \frac{v_\perp}{\omega_c} \]  

(2.6)

is small compared to the radius of the beam, so that the beam remains annular in its propagation.

The magnetic field at the cathode, \( B_K \), increases with increasing axial distance until the cavity where it reaches its maximum value of \( B_0 \). The emitted electrons follow the magnetic field lines to the cavity where the interaction with the RF field takes place and microwave radiation is generated. In this region, the electromagnetic fields vary slowly in comparison with a gyro-period, hence the adiabatic electron magnetic moment \( \mu_a \), where \( m \) is the electron’s mass and \( v \) is its velocity, given by

\[ \mu_a = \frac{1}{2} mv_\perp^2 / B \]  

(2.7)

is conserved.

The conservation of \( \mu_a \) allows the perpendicular velocity of the beam in the cavity, \( v_{\perp0} \) to be written in terms of the perpendicular velocity of the beam in the cathode \( v_{\perp K} \), as

\[ v_{\perp0} = v_{\perp K} \sqrt{\frac{B_0}{B_K}} \]  

(2.8)

where \( B_0/B_K \) is known as the magnetic compression of the beam.

Combining Eqs. 2.2 and 2.8 gives

\[ v_{\perp0} = \frac{E_{\perp K}}{B_K} \sqrt{\frac{B_0}{B_K}} \]  

(2.9)

If \( v_{\perp0} < v_0 \) then the beam will reach the cavity with the perpendicular velocity defined above. However if \( v_{\perp0} > v_0 \), then the electrons will be reflected back toward the cathode, causing the electrons to become trapped, leading to charge buildup and arcing. The total velocity \( v_0 \) is given by the kinetic energy conservation equation,

\[ T(v) = e(V_K - V_{dep}) = mc^2(\gamma - 1) \]  

(2.10)
\[
1 \approx \frac{1}{2} m_e v_0^2 \\
= \frac{1}{2} m_e (v_{||0}^2 + v_{\perp 0}^2)
\]

(2.11)

(2.12)

where \( m_e \) is the mass of an electron, \( V_K \) is the cathode voltage, and \( V_{dep} \) is the space charge voltage depression.

The beam radius, \( r \), is determined by the invariance of magnetic flux,

\[
\Phi = \pi r^2 B = \text{constant.}
\]

(2.13)

With a beam radius \( r_K \) and magnetic field \( B_K \) at the cathode, we find that the beam radius in the cavity, \( r_{e0} \) (with a cavity magnetic field \( B_0 \)) is

\[
r_{e0} = r_K \sqrt{\frac{B_K}{B_0}}
\]

(2.14)

**Adiabatic theory**

"Adiabatic" is defined as "occurring without loss or gain". The adiabatic approximation describes the basic equations of the electron beam. Adiabatic theory requires that the electron magnetic moment \( \mu_a \) of Eq. 2.7 be held constant.

However, our approximation excludes the region of the gun, since the approximation is only valid if the scale length of the variations of the electric and magnetic fields are small compared to the electron gyro-motion.

\[
\left| \frac{\partial^2 \{B, E\}}{\partial z^2} \right| \ll \frac{\{B, E\}}{z_L^2}
\]

(2.15)

\[
\left| \frac{\partial \{B, E\}}{\partial z} \right| \ll \frac{\{B, E\}}{z_L}
\]

(2.16)

where \( z_L \) is the axial distance that the electron propagates during one cyclotron period.

Therefore the adiabatic theory does not provide accurate results for the transverse velocity of the electrons in the vicinity of the gun.
Self-consistent simulations

To follow the trajectories of the electrons in this locality, numerical simulations are often used to more accurately calculate this information. An example of one of these codes, a modified version of the electron optics code EGUN developed by Herrmannsfeldt at Stanford Linear Accelerator [24], was used to design the electron gun for the present gyrotron design. The beam characteristics generated by this code have been demonstrated to have superior accuracy to the theoretical calculations approximated from adiabatic theory.

As inputs, the code needs the surface geometry of the cathode, anode, gun, and gyrotron tube (as far as will need to be modeled) and also the $B$-field of the magnet. The code in turn calculates the ratio of transverse to axial velocity $\alpha$, the spreads in both the transverse and axial velocities, the beam radius and width, and the trajectories of the electrons being simulated. The code accounts for space charge, self magnetic fields, and relativistic effects. The program is a 2 1/2 dimension code; the fields are 2-dimensional while the particle trajectories are 3-dimensional.

To solve for the electron trajectories, first Poisson’s equation (with no space charge) is solved by the method of finite differences, then potentials are differentiated from the electrostatic fields. Magnetic fields are specified externally, and the beam self magnetic field and electron trajectories are calculated for the $E$ and $B$-fields. Poisson’s equation is then solved again with inclusion of the beam charge and the electron trajectories are re-solved accounting for space charge. The iterations persist until they reach a convergence and provide self-consistent results.

The EGUN code can be used to elegantly design an electron gun, or to merely propagate the electron beam to assist in the design of the device.

Magnetron Injection Guns

The source of the annular electron beam used in gyrotrons, this “electron gun” that we have assumed until now, is usually a magnetron injection gun (MIG). A typical triode MIG configuration, a single cathode and two anodes, is shown in Fig. 2-1 [2].
The electron beam is produced by an annular strip of thermionic material which, when heated by a current, releases electrons from its surface partially perpendicular to the magnetic field. To accelerate the ejected electrons toward the anode, this emitting cathode is biased to a large negative voltage ($10 - 100$ kV) relative to it.

The gun in Fig. 2-1 shows the cathode emitter and two anodes. The accelerating anode serves to accelerate the beam toward the interaction cavity while the gun anode finely adjusts the transverse velocity of the emitted electrons.

Guns are also designed in the “diode configuration” with only one anode instead of two. In this case, the gun anode is omitted because with proper simulations of the electron beam and effects of the electrode geometry, there is no longer a need to fine-tune the beam.

2.1.2 Principles of interaction between electrons and field

Now with a basic understanding of beam dynamics, we can proceed to the cavity where we can create an understanding of the electromagnetic waves.
Cavity fields

Gyrotron cavities are typically tapered cylinders. This tells us that we can use basic cylindrical waveguide theory to simplify our analysis. Even though these cavities can support many resonant electromagnetic modes, we can simplify still further; gyrotrons are operated close to cutoff with \( k_1 \gg k_c \), so transverse magnetic (TM) modes are suppressed in favor of transverse electric (TE) modes. Therefore only the TE modes of the cavity RF field need to be calculated to understand the gyrotron interaction.

The fields of a cylindrical circular metallic waveguide cavity in a \( TE_{mpq} \) mode can be calculated as follows. Starting with Maxwell’s equations in differential vector form [25];

\[
\begin{align*}
\nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\
\nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} + \vec{J} \\
\n\nabla \cdot \vec{B} &= 0 \\
\n\nabla \cdot \vec{D} &= \rho
\end{align*}
\]

we assume a source free environment (\( \vec{J} = \rho = 0 \), time harmonic form \( (e^{i\omega t}) \), \( \vec{B} = \mu_0 \vec{H} \), and \( \vec{D} = \epsilon_0 \vec{E} \);

\[
\begin{align*}
\nabla \times \vec{E} &= -i\omega \mu_0 \vec{H} \\
\n\nabla \times \vec{H} &= i\omega \epsilon_0 \vec{E} \\
\n\nabla \cdot \vec{H} &= 0 \\
\n\nabla \cdot \vec{E} &= 0
\end{align*}
\]

By taking the curl of Eq. 2.22, substituting in the curl of Eq. 2.21, and using \( \epsilon_0 \mu_0 = 1/c^2 \), we derive the wave equation,

\[
\nabla^2 \vec{H} + \frac{\omega^2}{c^2} \vec{H} = 0
\]

25
Writing the z-component of the wave equation in cylindrical coordinates,

\[
\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \varphi^2} + \frac{\omega^2}{c^2} H_z + \frac{\partial^2 H_z}{\partial z^2} = 0
\]  

(2.26)

and assuming that \( H_z \) is of the form \( H_z = B(r, \varphi)f(z) \), we find that our generating equation \( H_z \) is a solution to Eq. 2.26.

\[
H_z = J_m \left( \frac{\nu_m}{a} r \right) e^{im\varphi} f(z)
\]  

(2.27)

Writing Maxwell’s equations in terms of each component \((r, \varphi, z)\), with \( E_z = 0 \) since we are solving for the \( TE_{mpq} \) case, we can solve for the remainder of the \( E \) and \( H \) components. First we formulate an equation for \( E_r \) in terms of \( H_z \),

\[
\frac{\partial^2 E_r}{\partial z^2} + \frac{\omega^2}{c^2} E_r = -i\omega\mu_0 \frac{1}{r} \frac{\partial H_z}{\partial \varphi}
\]  

(2.28)

Substituting Eq. 2.27 into Eq. 2.28 we solve for \( E_r \),

\[
E_r = \frac{m}{\omega\epsilon_0 c} \frac{1}{r} J_m \left( \frac{\nu_m}{a} r \right) e^{m\varphi} f(z)
\]  

(2.29)

Now we can also solve for \( H_\varphi \) using

\[
H_\varphi = -\frac{1}{\omega\mu_0} \frac{\partial E_r}{\partial z}
\]  

(2.30)

\[
= \frac{imc^2}{\omega^2} \frac{1}{r} J_m \left( \frac{\nu_m}{a} r \right) e^{im\varphi} f'(z)
\]  

(2.31)

\( E_\varphi \) follows from

\[
\frac{\partial^2 E_\varphi}{\partial z^2} - i\omega\mu_0 \frac{\partial H_z}{\partial r} = -\omega^2 \epsilon_0\mu_0 E_\varphi
\]  

(2.32)

where we can ignore the first term since \( k_\parallel \ll k_\perp \). Substituting in Eq. 2.27,

\[
E_\varphi = i \sqrt{\frac{\mu_0}{\epsilon_0}} J'_m \left( \frac{\nu_m}{a} r \right) e^{im\varphi} f(z)
\]  

(2.33)
where \( \omega_c \simeq \nu_{mn}/a \). Finally, \( H_r \) can be solved from

\[
H_r = \frac{1}{i \omega \mu_0} \frac{\partial E_\varphi}{\partial z} = \frac{1}{\omega c J_m' \left( \frac{\nu_{mn}}{a} r \right)} e^{im\varphi} f'(z) \tag{2.35}
\]

Summarizing these results,

\[
E_\varphi = i \sqrt{\frac{\mu_0}{\epsilon_0}} J_m' \left( \frac{\nu_{mn}}{a} r \right) e^{im\varphi} f(z) \tag{2.36}
\]

\[
E_r = \frac{1}{\omega \epsilon_0} J_m \left( \frac{\nu_{mn}}{a} r \right) e^{im\varphi} f(z) \tag{2.37}
\]

\[
H_\varphi = i \sqrt{mc^2} \frac{1}{\omega^2 r^2} J_m \left( \frac{\nu_{mn}}{a} r \right) e^{im\varphi} f'(z) \tag{2.38}
\]

\[
H_r = \frac{1}{\omega c} J_m' \left( \frac{\nu_{mn}}{a} r \right) e^{im\varphi} f'(z) \tag{2.39}
\]

\[
H_z = J_m \left( \frac{\nu_{mn}}{a} r \right) e^{im\varphi} f(z) \tag{2.40}
\]

where \( \omega \) is the angular resonant frequency, \( r \) is the radial cavity position, \( a \) is the cavity radius, \( k_\perp = \nu_{mp}/r \) is the transverse wave number, \( \nu_{mp} \) is the \( p^{th} \) zero of \( J_m' \), \( J_m \) is a Bessel function of the ordinary type, \( m \) is the azimuthal mode index, and \( p \) is the radial mode index. The above equations satisfy the boundary conditions of no tangential \( E \)-field or perpendicular \( H \)-field at the wall at \( r = a \). For fixed end walls of the resonator (at \( z = 0, L \)), the function \( f(z) \) is satisfied by:

\[
f(z) = \sin \left( \frac{q \pi}{L} z \right) \tag{2.41}
\]

for \( q = \text{integer} > 0 \). For an open resonator, we can use an approximation, justified by detailed numerical simulation, that

\[
f(z) \approx e^{-4z^2/L^2} \tag{2.42}
\]

The boundary conditions, along with Eqs. 2.36-2.40, identify constraints on the
frequency $\omega$ and the wave number $k$, given by;

$$\omega^2 = c^2 k^2$$  \hspace{1cm} (2.43)

$$= c^2 (k^2 + k_{||}^2)$$  \hspace{1cm} (2.44)

$$k_{\perp} = \frac{\nu_{mp}}{r_0}$$  \hspace{1cm} (2.45)

$$k_{||} = \begin{cases} \frac{2a}{L} & \text{for } f(z) \propto e^{-k_{||}^2 z^2} \\ \frac{2\pi}{L} & \text{for } f(z) \propto \sin(k_{||}z) \end{cases}$$  \hspace{1cm} (2.46)

where $r_0$ is the cavity radius, $L$ is the effective cavity interaction length, and $q$ is the axial mode number (the number of maxima in the axial field profile) which is usually just one. Since a gyrotron operates close to cutoff ($k_{\perp} \gg k_{||}$), we can approximate the resonant frequency by the cutoff frequency;

$$\omega \approx c \frac{\nu_{mp}}{r_0}$$  \hspace{1cm} (2.47)

In order for electrons to couple to the RF field and transfer their energy, they must have approximately the same frequency as the mode. Accounting for the Doppler shift of the wave due to the axial velocity of the electrons ($k_{||}v_{||}$), we get the resonance condition for exciting the cyclotron instability,

$$\omega - k_{||}v_{||} = n\omega_c$$  \hspace{1cm} (2.48)

with the electron cyclotron frequency $\omega_c = eB_0/\gamma m$, the harmonic number $n$, and the relativistic factor $\gamma$.

**Dispersion diagrams**

The uncoupled dispersion diagram, that is, with no beam/wave coupling, can be obtained ($\omega$ versus $k_{||}$) by plotting Eqs. 2.44 and 2.48. An intersection of the two curves indicates a resonance of the beam with the cavity mode.

A general dispersion diagram is shown in Fig. 2.1.2. The three beam lines corre-
The waveguide modes correspond to the fundamental, first, and second harmonics of the cyclotron mode. The intersection of the fundamental beam line and waveguide mode represents the fundamental resonance. Similarly, the intersection of the second harmonic beam line and waveguide mode represents the second harmonic resonance.

Mode excitation

If we transform the electric field from Eqs. 2.36 and 2.37 to the beam’s guiding center, the coupling of a $TE_{mp}$ mode wave is given by the coupling coefficient $C_{mp}$,

$$C_{mp} = \frac{J_{m+n}^2(\kappa_{\perp r_0})}{(\nu_{mp}^2 - m^2)J_m^2(\nu_{mp})}$$  \hspace{1cm} (2.49)

Using this, we can find the optimum radius of the beam if it is to have maximum coupling to our chosen mode.
CRM interaction

Now that we understand more about the cavity fields and coupling between the beam and mode, we can delve deeper and try to understand the underlying mechanism which transfers the energy from the beam to the field, creating the sought-after RF power.

As we have learned, the gyrotron interaction (or CRM interaction) occurs in the cavity region of the gyrotron. First we will discuss the fundamental mode interaction, ignoring variations in the electric field across the Larmor radius. Fig. 2-3(a) shows a cross-section of the electron beam at the beginning of the interaction region. As the beam progresses into the cavity, the transverse electric field will exert a $F_{RF} = -qE_{RF}$ force that will cause some electrons to accelerate and others to decelerate, depending on the relative phase of the electric field. In Fig. 2-3(b), electrons 2, 3, and 4 are decelerated, while electrons 6, 7, and 8 are accelerated, and electrons 1 and 5 are undisturbed. This perturbation results in a change of energy. If an electron gains energy, the relativistic factor $\gamma$ increases, which decreases the electron cyclotron frequency $\omega_c$ and increases the Larmor radius $r_L$. On the other hand, if an electron loses energy, $\gamma$ decreases which causes $\omega_c$ to increase and $r_L$ to decrease. After a few cycles, the electrons that gained energy lag in phase and the electrons that lost energy advance in phase. Soon, the electrons have formed a bunch (Fig. 2-3(c)). If the frequency of the electric field is exactly equal to the electron cyclotron frequency, the bunch will not gain or lose energy. In order to extract power from the beam, the bunch must be formed at a field maximum. If the axial magnetic field is tuned such that the RF frequency $\omega$ is slightly greater than the cyclotron frequency $\omega_c$, then the bunches will orbit in phase and transfer their rotational energy to the TE field mode, increasing the field strength and encouraging more bunching. This domino effect results in a rapid amplification of the dominant mode.
Figure 2-3: Gyrotron phase bunching: (a) initial condition, where the electrons are uniformly distributed on a beamlet (b) new positions of the electrons immediately after the change in energy (c) formation of the electron bunch [4]
Harmonic CRM interaction

The fundamental interaction is not the only possible interaction that can occur to transfer energy from the beam to RF field. If the electric field varies across the Larmor radius, it is possible to have a harmonic interaction. A quadrapole electric field is required for a second harmonic interaction, as shown in Fig. 2-4, where the electrons gain energy in the \( \dot{\gamma} > 0 \) region and lose energy in the \( \dot{\gamma} < 0 \) region. A hexapole field would be required for a third harmonic interaction, etc. Two bunches of electrons may be formed during a second harmonic interaction. Up to \( n \) bunches may be formed when operating at the \( n^{th} \) harmonic. The resonance condition is now \( \omega \approx n\omega_e \) for operation at the \( n^{th} \) harmonic.

In order to excite a second harmonic interaction, a stronger second harmonic than fundamental electric field must be experienced by the beam. There are two methods by which this can be accomplished. The first method relies on the beam being positioned where the coupling to the second harmonic field is stronger. Otherwise, a stronger second harmonic electric field is necessary, if the coupling to the second harmonic and fundamental modes is comparable [4].

Weibel instability

The gain mechanism for gyrotrons, the CRM interaction described in this section, is caused by the azimuthal bunching of electrons. There is also a mechanism which causes axial bunching of electrons, a Lorentz force interaction between the electrons and the RF magnetic field, known as the Weibel instability [26]. The Weibel instability and the CRM interaction compete with each other, but one usually dominates [27]. In a fast-wave device, the CRM interaction (also know as the gyrotron interaction) is the dominant one. Since gyrotrons operate near cutoff, \( k_\perp \gg k_\parallel \). And \( k_\parallel v_\parallel \) is small, so the fast-wave condition of the phase velocity,

\[
v_\phi = \frac{\omega}{k_\parallel} \gg c
\]

holds true.
2.1.3 Non-linear theory

To develop a non-linear theory for a gyrotron oscillator, we begin with the equations of motion for an electron moving in an electromagnetic field,

\[
\frac{d\mathcal{E}}{dt} = -e\mathbf{\bar{v}} \cdot \mathbf{\bar{E}}
\]

\[
\frac{d\mathbf{\bar{p}}}{dt} = -e\mathbf{\bar{E}} - \frac{e}{c}\mathbf{\bar{v}} \times \mathbf{\bar{B}}
\]

where \( \mathcal{E} = \gamma m_0 c^2 \) is the electron energy, \( \gamma = (1 - v^2/c^2)^{-1/2} \), and \( |\mathbf{\bar{p}}| = \gamma m_0 c^2 \) is the electron momentum.

Normalized parameters

Assuming that the electric field is a \( TE \) cavity mode, we can write the transverse efficiency \( \eta_\perp \), the fraction of transverse power that has been transferred to the RF.
field, in terms of four normalized parameters \[5,\]

\[
F = \frac{E_0}{B_0} \beta_{10}^{-4} \left( \frac{n^{n-1}}{n12^{n-1}} \right) J_{m+n}(k_{10} r_{e0}) \tag{2.53}
\]

\[
\mu = \pi \left( \frac{\beta_{10}^2}{\beta_{||0}} \right) \left( \frac{L}{\lambda} \right) \tag{2.54}
\]

\[
\Delta = \frac{2}{\beta_{10}^2} \left( 1 - \frac{n\omega_c}{\omega} \right) \tag{2.55}
\]

\[
\beta_{10} = \frac{v_{10}}{c} \tag{2.56}
\]

where \( F \) is the normalized field amplitude, \( \mu \) is the normalized cavity interaction length, \( \Delta \) is the detuning between the wave frequency and the electron cyclotron frequency, \( \omega_c = eB/\gamma m \) (magnetic field parameter), and \( \beta_{10} \) is the normalized transverse velocity at the entrance to the cavity.

In addition, a normalized energy variable,

\[
u = \frac{2}{\beta_{10}^2} \left( 1 - \frac{\gamma}{\gamma_0} \right) \tag{2.57}
\]

and a normalized axial position variable,

\[
\zeta = \pi \left( \frac{\beta_{10}}{\beta_{||0}} \right) \left( \frac{z}{\lambda} \right) \tag{2.58}
\]

have been defined.

If the electron beam is weakly relativistic and \( n_0 \beta_{10}^2 \ll 1 \), then \( \eta_\perp \) is reduced to being a function of only \( F, \mu, \) and \( \Delta \). The beam current can be related to the field amplitude \( F \) by an energy balance equation. The total cavity \( Q, Q_T \), can be written in terms of the total stored energy \( U \) and the power dissipated \( P \),

\[
Q_T = \frac{\omega U}{P} \tag{2.59}
\]

where the dissipated power \( P \) is given by

\[
P = \eta I_A V, \tag{2.60}
\]

34
\( I_A \) is the beam current, and \( V \) is the cathode voltage. Evaluating the stored energy with a Gaussian axial field profile, the energy balance equation is derived as

\[
F^2 = \eta_{\perp} I,
\]

(2.61)

where the normalized current parameter \( I \) is defined by

\[
I = 0.238 \times 10^{-3} \left( \frac{Q_T I_A}{\gamma_0} \right) \beta_{0,0}^2 (n-3) \times \left( \frac{\lambda}{L} \right) \left[ \eta_n \right]^2 \frac{J_m^{2} (k_{\perp} r_{e0})}{(\nu_{mp}^2 - \nu^2) J_m^{2} (\nu_{mp})}
\]

(2.62)

### Efficiency

Fig. 2-5 shows contour curves of \( \eta_{\perp} \), at second harmonic operation, as a function of \( F \) and \( \mu \), optimized with respect to the magnetic field parameter \( \Delta \), with the optimum value at \( \Delta_{opt} \). The curves are shown for second harmonic operation, since that is the design being presented in this work. For the second harmonic interaction, peak perpendicular efficiencies of over 70% are theoretically possible.

The total efficiency also accounts for voltage depression and parallel energy. \( \eta_{el} \) is the fraction of beam power in the perpendicular direction and \( \eta_{Q} \) is the reduction due to ohmic losses.

\[
\eta_T = \eta_{el} \times \eta_{Q} \times \eta_{\perp}
\]

(2.63)

\[
= \frac{\beta_{0,0}^2}{2(1 - \gamma_0^{-1})} \times \frac{Q_{OHM}}{Q_D + Q_{OHM}} \times \eta_{\perp}
\]

(2.64)

\[
= \frac{P_{out}}{IV}
\]

(2.65)

where \( I \) is the beam current and \( V \) is the beam voltage,

\[
Q_T = \frac{Q_{OHM} Q_D}{Q_{OHM} + Q_D}
\]

(2.66)

is the total \( Q \),

\[
Q_{OHM} = \frac{\gamma_0}{\delta} \left( 1 - \frac{m^2}{\nu_{mp}^2} \right)
\]

(2.67)
Figure 2-5: Transverse efficiency contour $\eta_\perp$ (solid line) as a function of the normalized field amplitude $F$ and normalized effective interaction length $\mu$ for optimum detuning $\Delta$ (dashed line) and second harmonic $n = 2$ [5]
is the ohmic $Q$, $\delta$ is the skin depth, $Q_D$ is the diffractive $Q$,
\[
Q_D = 4\pi \frac{(L/\lambda)^2}{1 - |R_{1,2}|}\]
(2.68)
and $R_{1,2}$ is the wave reflection coefficient of the input and output cross-sections of a resonator.

Starting current

The starting current $I_{st}$ can be numerically calculated as a function of magnetic field for a given mode. Two codes that are used to calculate the starting current, LINEAR [28] and CAVRF [29] will be discussed.

The diffractive $Q$ from Eq. 2.68 and the effective interaction length from Eq. 2.54 need to be calculated numerically. These quantities can be calculated by the code CAVRF developed by A. Fliflet at the Naval Research Laboratory, which solves for the eigenmodes of a cold gyrotron cavity (in the absence of an electron beam). The effective interaction length is defined as the axial distance between which the RF field amplitude is greater than $1/e$ times the maximum RF field amplitude.

Now using the total $Q$ and the effective interaction length determined by CAVRF, we can calculate the starting current with the code LINEAR developed by K. Kreischer at MIT. LINEAR calculates the starting current for a Gaussian or sinusoidal RF field profile in a cylindrical open resonator, assuming that adiabatic theory is valid in the gun region, and the electron beam is monoenergetic annular azimuthally symmetric with no radial thickness or velocity spread. To calculate the linear characteristics, several device parameters must be user-specified: beam and anode voltages, cavity and cathode magnetic fields, mode and harmonic numbers, cavity radius, effective cavity interaction length, cathode/anode distance, ohmic $Q$, diffractive $Q$, and the cathode radius. Using this data, the starting current is calculated as a function of the magnetic field.
2.1.4 Second harmonic challenges

The excitation of the second harmonic can be quite challenging. At lower frequencies, harmonic modes are observed when there is a gap in the fundamental spectrum, [30] however the fundamental spectrum becomes dense at frequencies above 300 GHz [4].

Foremost, the starting current of the second harmonic is at least 1.6 times higher than that of the fundamental modes, resulting in the suppression of harmonic modes in favor of the fundamental. In the linear regime, the normalized starting current becomes

\[ I_{st} = \frac{4}{\pi \mu^2} \frac{e^{2x^2}}{\mu x - n} \]  

(2.69)

where \( x = \mu \Delta / 4 \). Using the normalized current parameter Eq. 2.62, and assuming the coupling coefficient \( C_{mp} \) from Eq. 2.49 is proportional to the square of the wavelength and the linear normalized starting current \( I_{st} \) from Eq. 2.69 is proportional to \( 1/\mu^3 \), we find an expression for the ratio between starting currents for the \( n^{th} \) harmonic and the fundamental [31],

\[ \frac{I_n}{I_1} \approx \frac{Q T_1 \beta_{10}^{2(1-n)}}{Q T_n} \]  

(2.70)

This expression tells us that we can lower the starting current by either raising the cavity \( Q \) or the velocity ratio \( \alpha \) (by raising the normalized perpendicular velocity \( \beta_{10} \)). Since in this experiment an existing gun design is being used, increasing \( \beta_{10} \) can only be achieved to a certain degree. However it is possible to design for a higher cavity \( Q \).

Secondly, in order to reduce ohmic losses, the highly overmoded cavities required make mode competition more severe; the mode density increases as the cavity size becomes larger.

Thirdly, a thick beam can couple simultaneously to several different modes. When operating at a second harmonic mode, the beam can effectively couple to the design mode as well as one or two fundamental modes and several other harmonic modes.

In order to excite a second harmonic mode, we can clearly see that the fundamental modes need to be suppressed. In this thesis, this has been attempted through clever
design of lowering the starting current for the desired second harmonic mode and selecting a mode that is relatively isolated from the fundamentals.

### 2.2 Nuclear magnetic resonance

The principal motivation for this gyrotron design is in high-field DNP NMR studies. In order to better understand this application, we present a brief overview of the theoretical foundations of this experiment.

Nuclear magnetic resonance is a powerful and routine spectroscopic technique for the study of structure and dynamics in condensed phases and, in particular, of biological macromolecules. Its principal limitation is low sensitivity; the small nuclear Zeeman energy splittings result in correspondingly small nuclear spin polarization at thermal equilibrium [23]:

\[
\frac{N_m}{N} = \exp\left(-\frac{E_m}{k_BT}\right) / \sum_{m=-I}^{I} \exp\left(-\frac{E_m}{k_BT}\right) = \exp\left(-\frac{m\gamma B_0}{k_BT}\right) / \sum_{m=-I}^{I} \exp\left(-\frac{m\gamma B_0}{k_BT}\right) \approx \left(1 + \frac{m\gamma B_0}{k_BT}\right) / \sum_{m=-I}^{I} \left(1 + \frac{m\gamma B_0}{k_BT}\right) \\
\approx \left(1 + \frac{m\gamma B_0}{k_BT}\right) / (2I + 1)
\]

where \(N_m\) is the number of nuclei in the \(m^{th}\) state \((-\frac{1}{2} \text{ or } \frac{1}{2} \text{ for a spin-}\frac{1}{2} \text{ nucleus})\), \(N\) is the total number of spins, \(T\) is the absolute temperature, \(k_B\) is the Boltzmann constant, \(E_m = -m\gamma B_0\) is the nuclear Zeeman energy, \(\hbar\) is Planck's constant divided by \(2\pi\), \(B_0\) is the static magnetic field, \(I\) is the nuclear spin angular momentum, and we have taken the high temperature limit in Eq. 2.74. For example, protons at room temperature exhibit a spin polarization of less than 0.01% in a field of 5 T [15]. Though both solution and solid state NMR suffer from this poor sensitivity, relaxation processes in the solid state further compromise the time-averaged sensitivity of these experiments by two or three orders of magnitude. The low sensitivity of SSNMR
complicates the study of biological systems, where sample amounts are limited and spectra are complex.

2.2.1 Dynamic nuclear polarization

Dynamic Nuclear Polarization (DNP) is a magnetic resonance technique used to enhance the polarization of nuclei through interactions with the electron spin population. It occurs through a variety of mechanisms, all involving irradiation of the electron spins at or near their Larmor frequency. The effect was first observed in 1956 by Carver and Slichter [32] and later in 1958 by Abragam and Proctor [33]. Historically, it has been used to enhance the polarization of targets in nuclear scattering experiments, in sensitivity enhancement in the NMR of amorphous solids, and, recently, for sensitivity enhancement in high resolution NMR spectroscopy. There are three principal polarization transfer mechanisms: the solid effect, thermal mixing, and the Overhauser effect.

Solid effect

The solid effect, also known as the solid state effect, occurs in solids with fixed paramagnetic centers where the time-averaged value of the anisotropic hyperfine interaction is not zero [34]. In these systems, the spatial part of the hyperfine interaction can be described by a stationary Hamiltonian; as a result, the electron-nuclear spin system is no longer described by pure tensor product states alone, and we must admit a small admixture of states in the electron-nuclear wave function. The consequence of this admission is that so-called “forbidden transitions” involving simultaneous nuclear and electron spin flips can occur with small probability if the system is irradiated near $\omega = \omega_e \pm \omega_n$. These transitions give rise to polarization enhancements of the nuclear spin population, where the enhancement is given by [35]

$$\epsilon_{SE} = \alpha \frac{\gamma_e N_e}{\gamma_n b^3 \delta} \left( \frac{B_1}{B_0} \right)^2 T_{1n}$$

(2.75)
where the electron and nuclear gyromagnetic ratios $\gamma_e$ and $\gamma_n$ respectively are defined by the ratio of the frequency $f$ to the static magnetic field $B_0$,

$$\gamma_{p,e} = \frac{f_{p,e}}{B_0}$$

$$= 42.6 \text{ MHz/T for protons} \quad (2.77)$$

$$= 28.0 \text{ GHz/T for electrons} \quad (2.78)$$

$\alpha$ contains physical constants, $N_e$ is the density of unpaired electrons, $\delta$ is the EPR linewidth, $b$ is the nuclear spin diffusion barrier, $B_1$ is the microwave field strength, and $T_{1n}$ is the nuclear spin-lattice relaxation time.

**Thermal mixing**

The most useful mechanism of polarization enhancement in these high-field DNP studies has been thermal mixing. Thermal mixing occurs in systems with fixed paramagnetic centers at high concentration, such that the ESR line is homogeneously broadened. Under these conditions, a thermodynamic and separable treatment of electron-electron and electron-nuclear interactions is possible. In this treatment, three thermal reservoirs corresponding to the Zeeman system, the spin-spin interaction system, and the lattice are at thermal equilibrium [36]. Irradiation near the electron Larmor frequency can produce nuclear polarization enhancements through a variety of mechanisms, with the enhancement approximately given by [35]

$$\epsilon_{TM} = \alpha' \frac{\gamma_e}{\gamma_n} \frac{N_e^2}{\delta^2} \left( \frac{B_1^2}{B_0} \right) T_{1n} T_{1e}$$

(2.79)

where $\alpha'$ contains physical constants and $T_{1e}$ is the electronic nuclear relaxation time. So in principle, signal enhancements on the order of $\gamma_e/\gamma_n$ can be obtained. This corresponds to a factor of 657 for $^1\text{H}$ nuclei and 2615 for $^{13}\text{C}$ nuclei.

Though these mechanisms all play roles in enhancing the sensitivity, studies show that thermal mixing is the predominant effect. Using a 140 GHz gyrotron, we have
previously demonstrated that signal enhancements of several orders of magnitude (100 – 400) are achievable at a magnetic field of 5 T [15, 16, 17, 18, 19, 37]. However, to obtain higher resolution spectra, it is desirable to perform DNP at higher field strengths (9 – 18 T), where NMR is commonly employed today.

There are several problems encountered when performing DNP at high fields. First, the enhancement decreases as $1/B_0^2$ with increasing static field strength for the solid effect and as $1/B_0$ for thermal mixing as indicated by Eqs. 2.75 and 2.79. Second, relaxation mechanisms responsible for the DNP effect are fundamentally different at higher fields. These problems at high field can be overcome, and significant signal enhancements obtained, by using high radical concentrations and high microwave driving powers. Furthermore, the enhancements scale with the square of the microwave driving field and only inversely with the applied magnetic field. Therefore, large signal enhancements can be achieved, even at high fields (9 - 18 T) if sufficient microwave power (1 - 10 W) is available to drive the polarization transfer.
Chapter 3

460 GHz Gyrotron Design

Other millimeter wave sources, such as the EIO (extended interaction oscillator) or BWO (backward wave oscillator) rely on fragile slow-wave structures to generate microwave radiation, and thus at the high power levels required for DNP experiments have limited operating lifetimes. Consequently gyrotrons are the only feasible choice for generating such high microwave powers at high frequencies (100 – 1000 GHz). This has been the motivation for the 250 GHz gyrotron [21] recently constructed at the Plasma Science and Fusion Center which allows DNP-NMR experiments to be performed in a routine manner and the presented design of a 460 GHz gyrotron.

A schematic of the 460 GHz gyrotron is depicted in Fig. 3-1. Starting from the bottom of the picture, the electron beam is generated by the electron gun. The magnetic field of the gun region is adjusted using the gun coil. The beam is compressed by an axial magnetic field provided by the superconducting magnet. The cavity, located in the center of the magnetic field, is where the electron beam energy is extracted. The electron beam is collected at the collector. The RF beam is launched into free space in the mode converter where it becomes a Gaussian beam and is reflected out a side vacuum window.
Figure 3-1: Schematic of the 460 GHz gyrotron for DNP
Table 3.1: Comparison of important 250 and 460 GHz gyrotron design parameters

<table>
<thead>
<tr>
<th></th>
<th>460 GHz</th>
<th>250 GHz</th>
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</thead>
<tbody>
<tr>
<td>Mode</td>
<td>$TE_{061}$</td>
<td>$TE_{031}$</td>
</tr>
<tr>
<td>Harmonic number</td>
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<td>1</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>460</td>
<td>250</td>
</tr>
<tr>
<td>Magnetic Field (T)</td>
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<td>9.06</td>
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<tr>
<td>Diffractive $Q$</td>
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<td>4,950</td>
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<tr>
<td>Total $Q$</td>
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<td>3,400</td>
</tr>
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<td>Cavity Radius (mm)</td>
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<td>1.94</td>
</tr>
<tr>
<td>Cavity Length (mm)</td>
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<td>18</td>
</tr>
<tr>
<td>Cavity Beam Radius (mm)</td>
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<td>1.02</td>
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<tr>
<td>Beam Voltage (kV)</td>
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<td>12</td>
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<tr>
<td>Beam Current (mA)</td>
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<td>40</td>
</tr>
<tr>
<td>Beam Velocity Ratio $\alpha$</td>
<td>2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

### 3.1 250 GHz gyrotron

Our design is based upon the 250 GHz gyrotron used for DNP studies, designed by K. Kreischer [21]. A summary of the design parameters of the tube can be found in [20] and Table 3.1. The gyrotron operates in the fundamental $TE_{031}$ mode at a frequency of 250 GHz and magnetic field of about 9 T. An output power of 25 W at continuous wave operation can be generated.

The present design incorporates the same electron gun as the previous design, limiting certain factors, such as the beam velocity ratio $\alpha$. The frequency chosen matches NMR magnetic field 16.4 T.

### 3.2 Cavity design

The gyrotron cavity is where the electron beam transfers energy to the transverse electric field mode. With a good design, the microwave radiation can be extracted at high efficiency levels. The design of the optimal cavity has been determined through the use of several codes. There are several constraining parameters for the design of the cavity that are presented, emanating from NMR and second harmonic considerations. The final cavity design including the RF field profile is pictured in Fig. 3.2.
Table 3.2: Gyrotron cavity design parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>$TE_{061}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>460</td>
</tr>
<tr>
<td>Magnetic Field (T)</td>
<td>8.39</td>
</tr>
<tr>
<td>Diffractive $Q$</td>
<td>37,770</td>
</tr>
<tr>
<td>Total $Q$</td>
<td>12,950</td>
</tr>
<tr>
<td>Cavity Radius (mm)</td>
<td>2.04</td>
</tr>
<tr>
<td>Cavity Length (mm)</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3.3: Beam extraction parameters

| Cavity Beam Radius (mm) | 1.03 |
| Beam Voltage (kV)       | 12   |
| Beam Current (mA)       | 97   |
| Beam Velocity Ratio $\alpha$ | 2    |
| Power (W)               | 50   |
| Net Efficiency $\eta_T$ (%) | 4.3 |

using the design parameters from Table 3.2 and 3.3.

3.2.1 NMR considerations

A 17 T, high-resolution, NMR magnet corresponding to a proton Larmor frequency of 700 MHz (16.4 T) has been acquired for use with a DNP NMR spectrometer. The DNP NMR experiment requires a matching of proton and electron fields. The ratio of the proton frequency to the gyrotron frequency is thus equal to the ratio of their respective gyromagnetic ratios:

$$f_e = f_p \frac{\gamma_e}{\gamma_p}$$

$$= 700 \times 10^6 \frac{28.0 \times 10^9}{42.6 \times 10^6}$$

$$= 460 \text{ GHz}$$

The corresponding frequency needed to be generated by the gyrotron is 460 GHz.

Previous dynamic nuclear polarization experiments driven by 140 and 250 GHz
freq=459.953 GHz
Q=37708.61
L/λ= 31.9
Peak ohm= .05
Leak= .000%
F= .007
μ=38.22

Figure 3-2: Cavity design and RF field profile
gyrotrons indicate that a CW power capability of 10-100 W is sufficient to extend DNP into higher fields [15, 16, 17, 20]. A design power of 50 W has been chosen to meet this need.

### 3.2.2 Second harmonic

In section 2.1.4, we discussed the challenges of realizing the second harmonic, such as a higher starting current and coupling of the beam of finite thickness to multiple modes. These theoretical issues may evolve into design issues that need to be addressed.

#### Operating mode

In order to operate at a higher frequency, we are selecting a second harmonic mode of operation. Fundamental modes have much lower starting currents, thus the mode we select must be sufficiently free from such fundamental modes and also other second harmonic modes, as seen in Sec. 2.1.3.

We must ensure that there is a gap in the frequency spectrum in which our mode is situated. Recall from Eq. 2.47 that the frequency $f$ of a $TE_{mp}$ mode can be approximated by

$$f = \frac{c\nu_{mp}}{2\pi r_0}$$

and from the resonance condition Eq. 2.48 that $\nu_{mp1} \simeq \frac{1}{2} \nu_{mp2}$, where 1 denotes the fundamental and 2 the second harmonic modes. We see from Eq. 3.4 that gaps in the frequency spectrum are a direct result of spacing of the $TE$ mode indices, $\nu_{mp}$. To avoid interference from fundamental modes, we must choose a second harmonic mode where half of the value of its index, $\frac{1}{2} \nu_{mp2}$, is in a sizable gap between consecutive fundamental mode indices $\nu_{mp1}$. After examining the mode spectrum in Fig. 3-3 we chose $TE_{06}$, whose mode index is located between the fundamental modes $TE_{81}$ and $TE_{23}$.

The gap in mode indices where our second harmonic mode is located should translate into a region in the magnetic field where our mode can be excited without competition from the fundamental. Fig. 3-4 shows a plot of the starting currents of modes.
in the vicinity of our design $TE_{06}$ mode versus cavity magnetic field, as obtained by the code LINEAR. The starting current depicted for the design mode is about 75 mA at a cavity magnetic field of 8.4 T. The fundamental modes shown are the $TE_{31}$, $TE_{23}$, and the $TE_{03}$. The remaining modes are second harmonic. The $TE_{06}$ mode was chosen because it is sufficiently far away from the fundamental modes. The width of the curves are directly related to the $Q$ of the cavity.

**Starting current**

Another technique to suppress the fundamental modes comes from lowering the starting currents needed to excite the mode of operation. We learned from Eq. 2.70 that in order to lower the starting current of second harmonic modes, we must raise the cavity $Q$. To do this, we see from Eq. 2.66 that we must raise the diffractive $Q$ (Eq. 2.68) by building a longer cavity (increasing the effective cavity interaction length). The side-effect of raising the $Q_D$ can be seen in Eq. 2.64; the efficiency is lowered by almost the square of the effective cavity length! But from closely examining the energy balance equation, 2.61, and the normalized current parameter in Eq. 2.62, we find that the perpendicular component of the efficiency scales proportionally with the effective cavity length! In order to optimize these factors to obtain the maximum total efficiency, simulations of the code CAVRF were run. We found that the efficiency was optimized for a cavity length of 25 mm.

**Beam radius**

Our design mode must allow a beam with proper radius to interact with one of its maxima. In Fig. 3-5 we can see the 6 radial maxima of the $TE_{06}$ mode in the gyrotron cavity simulated by CAVRF. Our beam has been designed to couple with the third maxima, with a radius of 1.03 mm, which is approximately the same as the beam radius used in the 250 GHz gyrotron. The beam radius is relatively fixed based on what the gun is capable of producing.
Figure 3-3: Chart of the $TE$ mode indices up to $\nu_{mp} = 21$
Figure 3-4: Starting current for modes in the vicinity of the $TE_{06}$ design mode, with the design parameters of Table 3.2
Figure 3-5: Normalized field amplitude versus beam radius for the $TE_{06}$ mode
3.3 Superconducting magnet

A large axial magnetic field of 8.4 T is required to operate the gyrotron. A magnetic field of up to 9 T is provided by a superconducting solenoid manufactured by Cryomagnetics, with a bore diameter of 3 in. This diameter is half an inch larger than that of the 250 GHz gyrotron in order to provide ample room for alignment of the tube. The cross-bore of the magnet will be axially positioned a few centimeters further than in the previous design. The placement and design of the internal mode converter are affected by this due to an increased beam radius at its location.

3.4 Electron gun

We know from Sec. 2.1.1 that magnetron injection guns are often used to provide the annular electron beam in gyrotron oscillators. The electron gun incorporated into the 460 GHz gyrotron design is the same magnetron injection gun as was used in the 250 GHz gyrotron, designed by Dr. Kenneth Kreischer. Instead of using the more complex triode configuration described in Sec. 2.1.1, a more compact diode configuration using only one anode was selected. The electrode dimensions were designed using the electron trajectory program EGUN [24]. The designed gun has been tested at currents of up to 50 mA at a maximum voltage of 14 kV. The perpendicular velocity spread was found to be 3.4% at these conditions. The EGUN simulations showed that the gun is able to perform well over a range of cathode voltages and magnetic fields. A gun coil will also be used to fine-tune the beam parameters.

Fig. 3-6 shows a simulation of the magnetron injection gun section (anode and cathode) using the code EGUN. “Mu” represents a normalized length scale in mesh units. The plot shows the geometry of the gun, the equipotential lines, the increasing magnetic field, and the electron beam.
3.5 Quasi-optical transmission line

The purpose of the design of this gyrotron is to perform DNP experiments. To this end, the microwave power needs to be transmitted through the gyrotron output window to the spectrometer magnet and into the NMR probe. The first step is to convert the $TE_{06}$ radiation generated in the cavity into an easily transmittable Gaussian beam. The next step is to actually transmit the beam through an output waveguide to the NMR probe itself.

3.5.1 Internal mode converter

To design an internal mode converter, we have used a quasi-optical approach, in which the beam is manipulated while propagating through free space (in a visible optics manner) instead of while confined in a waveguide.

The first element in the quasi-optical transmission line is the quasi-optical antenna. Located internal to the gyrotron, its function is to efficiently convert the $TE_{06}$ mode into a Gaussian beam which will then be transmitted out of the vacuum tube to the DNP unit through a transmission line.

The quasi-optical mode converter consists of a circular waveguide with a step-cut, a cylindrical parabolic reflector, and a flat steering mirror. First, the antenna, a Vlasov slotted waveguide launcher [40], converts the gyrotron output into a linearly polarized beam. Using geometrical optics, the Brillouin angle $\theta_B$ is determined by

$$\sin \theta_B = \frac{k_\perp}{k} \frac{\nu_{mp}}{\nu_{06}}$$

(3.5)

where $a_{wg}$ is the waveguide radius, $\nu_{mp} = \nu_{06}$ is the mode index, and $k_\perp = \nu_{mp}/a_{wg}$.

Thus the length of the slot $L_v$ is determined by

$$L_v = 2a \cot \theta_B$$

(3.7)

Then the parabolic reflector molds the beam into a Gaussian shape. Lastly the
steering mirror directs the beam through the gyrotron output window.

The mode converter has been optimized for efficiency and space (such that it fits inside the bore of the magnet and is not within the beam radius).

### 3.5.2 Transmission line

The transmission line waveguide has not yet been developed for this current design, but in the next section we will discuss the characteristics of the transmission line in a 140 GHz gyrotron used in DNP experiments.

### 3.6 Discussion

The 460 GHz gyrotron design presented will be based upon the 250 GHz gyrotron designed by Dr. Kenneth Kreischer, with changes including the operating mode, cavity, mode converter, and a higher axial magnetic field. Its main feature is its potential to operate at the second harmonic of the cyclotron frequency. Second harmonic design requires careful analysis, namely, every trick in the book. It is extremely necessary to isolate a TE cavity mode sufficiently free from both fundamental modes and other harmonic modes, especially from the fundamental modes. In addition, it is important to have a lower starting current in the chosen second harmonic mode than in the neighboring fundamentals. To obtain a lower starting current, the cavity $Q$ needs to be designed higher, which brings problems of ohmic heating. All factors need to be thoroughly evaluated in order to come to a balance that will hopefully result in a second harmonic excitation and fundamental suppression.
Figure 3-7: Schematic of the quasi-optical internal mode converter showing calculated design parameters (a) side view (b) front view [6]
Figure 3-8: Drawing of the internal mode converter (including launcher and mirrors) indicating the paths of the electron and RF beams, and output window
Chapter 4

Transmission Line

In this chapter we switch focus from the design of the 460 GHz gyrotron oscillator to the transmission line of the existing 140 GHz gyrotron oscillator, the first of the joint series of Francis Bitter Magnet Laboratory and Plasma Science and Fusion Center DNP gyrotron collaborations. We have analyzed the transmission line of the 140 GHz gyrotron system in order to determine if it is indeed transmitting the microwave power at optimal efficiency.

The 140 GHz gyrotron generates power in the $TE_{03}$ mode. Located internally is a $TE_{03}$-$TE_{01}$ mode converter. The gyrotron output window has a diameter of 1.27 cm, followed by the snake external $TE_{01}$-$TE_{11}$ mode converter, a miter bend, a vertical pipe, another miter bend, horizontal pipe, and a downtaper to fundamental waveguide, a 90° bend and a circular to rectangular transition before entering the NMR magnet from the top (Fig. 4-2). The snake shown in Fig. 4-1 is an external mode converter that is an asymmetrical periodically perturbed pipe which should convert the $TE_{01}$ mode to the $TE_{11}$ mode. It has been fabricated from a piece of pipe almost a meter long that has been perturbed periodically and these perturbations are held in place with clamps.

To test the key features of the transmission system, first we measured the radiation pattern from the output window of the gyrotron to determine that it is generating the correct mode. Secondly, we measured the radiation pattern from the snake mode converter to determine that it is converting the radiation to the proper mode. Lastly
Figure 4-1: (a) Schematic of the $TE_{01} - TE_{11}$ snake mode converter, where $a$ is the waveguide radius, $\delta$ is the perturbation, $d$ is a period, and $L$ is the total length (b) close-up of one period.
Figure 4-2: Schematic of the 140 GHz transmission line
we calculated the mode losses due to the miter bends and waveguide.

The results and methods of this experiment are useful for the design of the transmission line of the 460 GHz gyrotron as well as correcting any problems with the existing transmission line.

4.1 Snake mode converter experiment

4.1.1 Setup

A data acquisitioning system was used for the field scans of the gyrotron output and snake output [41]. The apparatus consists of an oscilloscope, a PC, a motorized attenuator, and a Millitech diode. The device features a four axes positioning system with two translational and two rotational axes. The sensor consists of a diode and a motorized attenuator. Due to the nonlinearity of the diode, the attenuation level must be adjusted until the diode signal reaches a preset level, in our case, 10 mV. The diode signal is read on the oscilloscope and sent to the PC, which then adjusts the attenuator. The gyrotron was set at a 2 Hz repetition rate. This factor, along with the scanning repetitions due to the instability of the signal, results in a very long scanning time.

At the gyrotron window, a miter bend was placed followed by a short pipe of 1.27 cm radius. (The miter bend was necessary due to the wall parallel to the gyrotron). In the second experiment, the short pipe was replaced with the snake $TE_{01}-TE_{11}$ mode converter. The scanner was placed about 5.08 cm away from the end of the open pipe/snake.

The scanner only reads in one polarization at a time, so each scan must be repeated in order to take data on both horizontal and vertical polarizations.

Since the relation

$$\frac{ka^2}{2z} > 1 \quad (4.1)$$

holds true, we know that we are operating in the near field, where $a$ is the radius of the aperture (6.35 mm), $z$ is the distance from the aperture (5.08 cm), and the
frequency \( f = \frac{ck}{2\pi} = 140 \text{ GHz} \).

### 4.1.2 Radiation pattern measurements

**Gyrotron radiation pattern**

In the first experiment, we measured the radiation pattern from the gyrotron output using the apparatus described in the previous section in order to verify that it is indeed in a \( TE_{01} \) mode. We obtained the data plotted in Figs. 4-3 and 4-4.

**Snake radiation pattern**

In the second experiment, we measured the radiation pattern from the snake using the apparatus described in Section 4.1.1. in order to verify that it is indeed in a \( TE_{11} \) mode. We obtained the data plotted in Figs. 4-5 and 4-6.

### 4.1.3 Snake analysis

**Data analysis**

First we added together the horizontal and vertical components of the gyrotron radiation patterns, likewise with the snake radiation patterns, so we can examine the composite pattern. These can be seen in Fig. 4-7.

Since the horizontally polarized component of the snake radiation pattern, Fig. 4-5(a), strongly resembles the theoretical horizontal component of the \( TE_{01} \) pattern, Fig. 4-9(a), instead of the theoretical \( TE_{11} \), Fig. 4-10(a), we can first assume that the snake is not converting at 100\% efficiency. Secondly we can assume that the horizontal polarization comes solely from the \( TE_{01} \) mode,

\[
P_h = \frac{P_{01}}{2} \tag{4.2}
\]

\[
P_v = P_{11} + \frac{P_{01}}{2} \tag{4.3}
\]

where \( P_h \) is the horizontal power from the snake radiation pattern, \( P_v \) is the vertical
Figure 4-3: (a) Horizontal and (b) vertical polarizations of the gyrotron output at $z = 5.08$ cm; normalized dB contour plot.
Figure 4-4: Gyrotron output at $z = 5.08$ cm, (a) $y = 0$; (b) $x = 0$; the solid line represents the total intensity, the dotted line the vertical polarization, and the dashed line the horizontal polarization.
Figure 4-5: (a) Horizontal and (b) vertical polarizations of the snake output at $z = 5.08$ cm; normalized dB contour plot.
Figure 4-6: Snake output at $z = 5.08$ cm, (a) $y = 0$; (b) $x = 0$; the solid line represents the total intensity, the dotted line the vertical polarization, and the dashed line the horizontal polarization.
power component, $P_{01}$ is the total power in the $TE_{01}$ mode, and $P_{11}$ is the total power in the $TE_{11}$ mode.

From this, we find that the $TE_{01}$-$TE_{11}$ conversion efficiency of the snake is

$$\frac{P_{11}}{P_{11} + P_{01}} = 0.59 \quad (4.4)$$

**Radiation from a waveguide**

Let us compare our scanned radiation patterns with the theoretically predicted patterns. First we will radiate the $TE_{01}$ and $TE_{11}$ mode patterns out of a pipe.

Starting with the equations for the $TE_{01}$ mode,

$$E_\phi = J_1 \left( \frac{\nu_{01}}{a} r \right)$$

$$E_r = 0 \quad (4.5)$$

and the $TE_{11}$ mode,

$$E_\phi = -J'_1 \left( \frac{\nu_{11}}{a} r \right) \cos(\phi) \quad (4.7)$$

$$E_r = \frac{a}{\nu_{11} r} J_1 \left( \frac{\nu_{11}}{a} r \right) \sin(\phi) \quad (4.8)$$

we radiate them from the end of a cylindrical waveguide using the cylindrical Fresnel diffraction integral [42, 43].

We find that for the $TE_{01}$ mode,

$$E_\phi = -\frac{k}{z} e^{-ikz} \int_0^a dr' r' J_1 \left( \frac{\nu_{01}}{a} r' \right) \exp \left( -ik \frac{r^2 + r'^2}{2z} \right) J_1 \left( \frac{krr'}{z} \right) \quad (4.9)$$

and for the $TE_{11}$ mode,

$$E_\phi = -\frac{k}{z} e^{-ikz} \cos \phi \int_0^a r' dr' \frac{1}{2} \left[ J_0 \left( \frac{\nu_{11}}{a} r' \right) J_0 \left( \frac{krr'}{z} \right) + J_2 \left( \frac{\nu_{11}}{a} r' \right) J_2 \left( \frac{krr'}{z} \right) \right] \exp \left( -ik \frac{r^2 + r'^2}{2z} \right)$$

68
Figure 4-7: Sum of horizontal and vertical polarizations at $z = 5.08$ cm of (a) gyrotron output; (b) snake output; normalized dB contour plot
Figure 4-8: (a) $TE_{01}$ and (b) $TE_{11}$ theoretically radiated at $z = 5.08$ cm; normalized dB contour plot.
Figure 4-9: (a) Horizontal and (b) vertical polarizations of $TE_{01}$ intensity theoretically radiated at $z = 5.08$ cm; normalized dB contour plot
Figure 4-10: (a) Horizontal and (b) vertical polarizations of $TE_{11}$ intensity theoretically radiated at $z = 5.08$ cm; normalized dB contour plot
\begin{align*}
E_r &= i \frac{k}{z} e^{-ikz} \sin \phi \int_0^a r' dr' \frac{1}{2} \left[ J_0 \left( \frac{\nu_{11}}{a} r' \right) J_0 \left( \frac{r r'}{z} \right) - J_2 \left( \frac{\nu_{11}}{a} r' \right) J_2 \left( \frac{r r'}{z} \right) \right] \\
&\quad \exp \left( -ik \frac{r^2 + r'^2}{2z} \right)
\end{align*}

Recalling that
\begin{align*}
E_x &= E_r \cos(\phi) + E_\phi \sin(\phi) \\
E_y &= E_r \sin(\phi) - E_\phi \cos(\phi)
\end{align*}

we can rewrite them in Cartesian terms, to match for our horizontal and vertical polarizations.

Now we can compare our theoretically radiated mode patterns with our scanned data. The theoretical patterns can be seen in Figs. 4-8 – 4-10. The theoretical $TE_{01}$ mode patterns match well with the gyrotron output radiation patterns.

However, the $TE_{11}$ mode patterns do not agree with the snake radiation patterns. So we have experimentally added together different percentages and phases of $TE_{11}$ and $TE_{01}$ radiation patterns. We found that for 60% $TE_{01}$ and 40% $TE_{11}$ and a relative phase of 33.75° that the vertical polarization of the radiation patterns match well to our scanned snake data. An intensity of 65% $TE_{01}$ and 35% $TE_{11}$ with a relative phase of −75° also matched the vertical polarization of the scanned data. These patterns can be seen in Figs. 4-11 and 4-12. However the corresponding horizontal polarization radiation patterns do not match so well, so this is relegated to a sidenote possibility.

### 4.1.4 Mode conversion theory

**Two-mode approach**

This section contains the design summary of a $TE_{01}$ to $TE_{11}$ converter using the two-mode approach described by C. Moeller [44].

Conversion of a circular $TE_{01}$ mode to the $TE_{11}$ mode can be achieved by an asymmetrically periodically pipe, perturbed in one plane. This serpentine structure
Figure 4-11: Intensity of 60% $TE_{01}$ and 40% $TE_{11}$ (33.75°) is shown for (a) vertical and (b) horizontal polarizations; radiated at $z = 5.08$ cm; normalized dB contour plot.
Figure 4-12: Intensity of 65% $TE_{01}$ and 35% $TE_{11}$ ($-75^\circ$) is shown for (a) vertical and (b) horizontal polarizations; radiated at $z = 5.08$ cm; normalized dB contour plot.
is often referred to as a “snake”. (See Fig. 4-1).

The radius of this snake can be written as

$$r(z, \varphi) = a + \delta(z, \varphi)$$

(4.12)

where $a$ is the waveguide radius and $\delta$ is the perturbation. The perturbation can be written in terms of a sinusoid,

$$\delta(z, \varphi) = a\varepsilon_1 \cos(\frac{2\pi}{d} \cdot z - \varphi)$$

(4.13)

with $\frac{2\pi}{d} = |k_{z01} - k_{z11}|$, $k_{z01} = \sqrt{k^2 - \nu_{01}^2/a^2}$, $k_{z11} = \sqrt{k^2 - \nu_{11}^2/a^2}$, $\nu_{01} = 3.8317$, and $\nu_{11} = 1.8412$.

We define a coupling coefficient $K_{01}$ in the $TE_{01}$ to $TE_{11}$ conversion,

$$K_{01} = \frac{1}{R} \left[ \frac{g_1(k_a)^2 - h_1}{\left(k_{z01}ak_{z11}a\right)^{\frac{3}{2}}} + g_1\left(k_{z01}ak_{z11}a\right)^{\frac{3}{2}} \right]$$

(4.14)

where $R$ is the radius of curvature, $k_{zmp}$ is the propagation constant, $\nu_{mp}$ is the $p^{th}$ zero of $J'_m$, and $g_1$ and $h_1$ are defined by

$$g_1 = 2\frac{3}{2} \nu_{01} \nu_{11}^{\frac{3}{2}}(\nu_{11}^2 - 1)^{-\frac{3}{2}}(\nu_{01}^2 - \nu_{11}^2)^{-2}$$

(4.15)

$$h_1 = g_1 \frac{\nu_{01} + \nu_{11}}{2}$$

(4.16)

The coupling coefficient simplifies to

$$K_{01} = -K_{10}^* = i \frac{1}{2\pi^2} \frac{\nu_{01}^2 \nu_{11}^2/a^2}{\nu_{01}^2 k_{z01}k_{z11} \sqrt{\nu_{11}^2 - 1} \nu_{01}^2} \varepsilon_1 \pi$$

(4.17)

Considering the two modes of interest and their respective amplitudes $A_0$ and $A_1$ we obtain

$$\frac{dA_0}{dz} = K_{01}A_1$$

(4.18)

$$\frac{dA_1}{dz} = -K_{01}^*A_0$$

(4.19)
Table 4.1: Inputs to the two-mode approach

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the snake $L$ (m)</td>
<td>0.93</td>
</tr>
<tr>
<td>Waveguide radius $a$ (mm)</td>
<td>6.35</td>
</tr>
<tr>
<td>Frequency $f$ (GHz)</td>
<td>140</td>
</tr>
</tbody>
</table>

We find that

$$A_0(z) = \cos |K_{01}|z$$ (4.20)

$$A_1(z) = \sin |K_{01}|z$$ (4.21)

Now we can find the length of the period $d$ as

$$d = \frac{2\pi}{|k_{z01} - k_{z11}|}$$ (4.22)

the number of periods $N$ from

$$N = \frac{L}{d}$$ (4.23)

From

$$\sin(|K_{01}|Nd) = 1$$ (4.24)

$$|K_{01}|Nd = \frac{\pi}{2}$$ (4.25)

we can finally solve for $\varepsilon_1$,

$$\varepsilon_1 = \frac{\pi}{Nd} \frac{\sqrt{k_{z01}k_{z11}}}{} \frac{\nu_{11}^2 - 1}{\nu_{01}^2 \nu_{11}^2}$$ (4.26)

With a waveguide radius of a quarter of an inch (6.35 mm) and total snake length of 0.93 m, we find an optimized period of 13.0 cm and about 7 periods. The perturbation $a\varepsilon_1$ is found to be 0.3 mm. The optimized parameters are listed in Table 4.2.
Table 4.2: Optimum two-mode approach snake parameters

<table>
<thead>
<tr>
<th>Period length $d$ (cm)</th>
<th>13.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of periods $N$</td>
<td>7.16</td>
</tr>
<tr>
<td>Perturbation $a\varepsilon_1$ (mm)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 4.3: Measured snake parameters

<table>
<thead>
<tr>
<th>Length of the snake $L$ (m)</th>
<th>0.93</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waveguide radius $a$ (mm)</td>
<td>6.35</td>
</tr>
<tr>
<td>Period length $d$ (cm)</td>
<td>13.3</td>
</tr>
<tr>
<td>Number of periods $N$</td>
<td>7</td>
</tr>
</tbody>
</table>

Multi-mode approach

Since multiple modes are too difficult to keep track of analytically, simulations must be performed in order to completely analyze the mode conversion [45]. Using a mode conversion code [46] to analyze multiple modes on our measured snake parameters, a conversion efficiency of 93% is determined. The other modes present at the end of the snake are 5% $TE_{12}$, 0.3% $TE_{01}$, and 0.6% $TE_{21}$. If we increase the number of periods on the snake from 7 to 8, our conversion efficiency will increase from 93 to 96%.

Table 4.4: Conversion efficiency of the snake in the multi-mode approach

<table>
<thead>
<tr>
<th>Mode</th>
<th>Efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{11}$</td>
<td>93</td>
</tr>
<tr>
<td>$TE_{12}$</td>
<td>5</td>
</tr>
<tr>
<td>$TE_{21}$</td>
<td>0.6</td>
</tr>
<tr>
<td>$TE_{01}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>
4.2 Transmission line losses

Diffraction losses

The miter bends in the transmission line can be modeled as gaps in a multimode waveguide and can be described by using both waveguide and optical properties. The analytical solution for the total losses $1 - |A_0|^2$ is derived in D. Wagner [47], where $A_0$ is the normalized modal amplitude of the incident mode coupled to the waveguide.

For $TE_{01}$, $TE_{11}$, and $HE_{11}$ respectively,

\[
1 - |A_{01}|^2 = \frac{2}{\sqrt{\pi}} \frac{\nu_{01}^2}{3} \left( \frac{L}{k_0 a^2} \right)^{3/2}
\]

\[
1 - |A_{11}|^2 = \frac{2}{\sqrt{\pi}} \left[ \frac{1}{\nu_{11}^2 - 1} \sqrt{\frac{L}{k_0 a^2}} + \frac{\nu_{11}^2 (\frac{L}{k_0 a^2})^{3/2}}{3} \right]
\]

\[
1 - |A_{HE_{11}}|^2 = \frac{2}{\sqrt{\pi}} \frac{\nu_{HE_{11}}^2}{3} \left( \frac{L}{k_0 a^2} \right)^{3/2}
\]

where $A_{mp}$ is the modal amplitude, $k_0$ is the wave number in free space, $a$ is the waveguide radius, $L$ is the gap length, $\nu_{mp}$ is the Bessel function root of a $TE_{mp}$ mode, and $\nu_{HE_{11}}$ is the root of the characteristic equation of the $HE_{11}$ mode.

With $k_0 = 2\pi f/c$, the frequency $f$ is 140 GHz, $a$ is 6.35 mm, and $L$ is twice the waveguide radius, the diffraction losses in the $TE_{01}$ mode are 0.8 dB, 0.9 dB in the $TE_{11}$ mode, and 0.33 dB in the $HE_{11}$ mode.

Ohmic losses

In addition to the diffraction losses in the miter bends, there are ohmic losses in the circular waveguide. These losses, $\alpha$ are governed by the equation,

\[
\alpha = \frac{R_s}{\eta_0} \frac{\nu_{\omega}^2}{k^2 a^2} + \frac{1}{\nu_{mp}^2 - 1} \frac{1}{\sqrt{1 - \frac{\nu_{mp}^2}{k^2 a^2}}}
\]

where the skin depth $\delta_s = \sqrt{2/\omega \mu_0 \sigma}$ and the resistance $R_s = 1/\sigma \delta_s$.

In a waveguide of radius $a$, the ohmic losses are 0.015 dB/m in the $TE_{01}$ mode,
Table 4.5: 140 GHz transmission line losses

<table>
<thead>
<tr>
<th>Mode</th>
<th>Diffraction losses [dB]</th>
<th>Ohmic losses [dB/m]</th>
<th>Total losses [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{01}$</td>
<td>0.8</td>
<td>0.015</td>
<td>-</td>
</tr>
<tr>
<td>$TE_{11}$</td>
<td>0.9</td>
<td>0.16</td>
<td>4.8</td>
</tr>
<tr>
<td>$HE_{11}$</td>
<td>0.33</td>
<td>0.008</td>
<td>3.1</td>
</tr>
</tbody>
</table>

0.16 dB/m in the $TE_{11}$ mode, and 0.008 dB/m in the $HE_{11}$ mode.

Total theoretical losses

Adding everything together, we get the total losses in the transmission line. For the $TE_{11}$ mode, the losses are as follows;

$$TE_{11} \text{ mode losses} = 0.31 \text{ dB (snake efficiency)} + 0.16 \text{ dB/m } \times 1 \text{ m (snake ohmic)} + 0.16 \text{ dB/m } \times 2.5 \text{ m (waveguide ohmic)} + 0.9 \text{ dB } \times 2 \text{ (miter bends)} + 1 \text{ dB (downtaper ohmic)} = 3.7 \text{ dB}$$

For the $HE_{11}$ mode, the losses are as follows;

$$HE_{11} \text{ mode losses} = 0.31 \text{ dB (snake efficiency)} + 0.16 \text{ dB/m } \times 1 \text{ m (snake ohmic)} + 0.5 \text{ dB (efficiency of } TE_{11} - HE_{11} \text{ mode converter)} + 0.008 \text{ dB/m } \times 2.5 \text{ m (waveguide ohmic)} + 0.33 \text{ dB } \times 2 \text{ (miter bends)} + 1 \text{ dB (downtaper ohmic)} = 2.6 \text{ dB}$$
The $TE_{11}$ mode is currently used as the mode of propagation through the waveguide and miter bends. If we switched to an $HE_{11}$ mode, we could see an improvement of about 1 dB, as calculated above.

**Experimental losses**

With a calorimeter we experimentally measured the losses in the transmission line. Before the downtaper, a loss of 2.5 dB was measured. In the downtaper to the fundamental, a loss of 3.9 dB was measured. This totals to a loss of 6.4 dB after the downtaper.

### 4.3 Discussion

In this section, we discussed the transmission line of the 140 GHz gyrotron used in DNP studies, focusing on the losses of the snake external mode converter. First we presented the experimental $TE_{06}$ radiation patterns from both polarizations of the gyrotron in Figs. 4-3 and 4-4. We determined that it was emitting the proper mode because they strongly resembled the theoretical radiation patterns shown in Figs. 4-8a and 4-9.

Next we presented the radiation patterns from the snake $TE_{01} - TE_{11}$ external mode converter (Figs. 4-5 and 4-6). We compared these with theoretical radiation patterns in Figs. 4-8b and 4-10. We concluded that since the horizontal polarization of the snake output strongly resembled that of the $TE_{01}$ mode instead of the $TE_{11}$ mode that it must not be functioning at optimum efficiency and that since this component should be very small in comparison to the vertical polarization that it must come mostly from the $TE_{01}$ mode. From this, we calculated in Eqs. 4.2 – 4.4 that the snake is operating at about 60% efficiency.

To verify this, we added together varying percentages and phases of the $TE_{01}$ and $TE_{11}$ radiation patterns. We found two phase/ratios where the snake output radiation pattern resembled our simulated patterns for the vertical polarization (Figs. 4-11(a) and 4-12(a)), however the horizontal polarization did not match as well (Figs. 4-11(b)
and 4-12(b)).

Next we calculated the theoretical efficiency of the mode converter if it were functioning properly using a two-mode and multi-mode approach. For the two-mode approach we found that the snake was approximately accurately designed and for the multi-mode approach we found the snake was designed at an astounding 93% efficiency! The only way to increase its efficiency was found to be by increasing the number of periods from 7 to 8, but due to experimental concerns, the 3% increase in efficiency would not be noticeable.

Upon examination of the snake, we noticed that it was not manufactured in a robust manner and that the fabrication was most likely the problem with our low efficiency; the forced perturbations have relaxed over time such that their amplitudes are not uniform. A design that has been discussed is to fabricate a serpentine groove cut from a block and to insert a cylindrical pipe into the groove. In a design like this, there are no clamps to release their hold, so the snake would retain its shape over extremely long periods of time.

We also analyzed the total losses in the transmission line in order to optimize the design and yield more power to the sample. We found that if we transmit the $HE_{11}$ mode down the transmission line instead of the $TE_{11}$ mode we can reduce the losses. We found that instead of obtaining the theoretical loss of 3.7 dB in our system, we measured 6.4 dB. Part of this can be accounted for by the additional 1.9 dB loss in the snake mode converter. The additional loss in the taper is most likely due to the fact that there are additional modes created by the snake mode converter that do not pass through the downtaper to the $TE_{11}$ fundamental waveguide.
Chapter 5

Conclusions

This thesis reports the design of the 460 GHz gyrotron to be used in dynamic nuclear polarization NMR studies. This gyrotron is based on the design by Dr. Kenneth Kreischer of a 250 GHz gyrotron used in DNP. The main feature of the design is its potential capability to operate at the second harmonic of the cyclotron frequency. Several fundamental mode suppression techniques have been incorporated into the design.

Firstly, a second harmonic mode index $\nu_{mp}$ has been selected that is sufficiently isolated from both fundamental modes and other harmonic modes. This gap in the fundamental spectrum should translate into a region of isolation in the magnetic field, where there is an absence of fundamental modes. Other techniques for suppressing the fundamental modes include lowering the starting current for the second harmonic mode. This has been accomplished by designing the cavity for higher $Q$ and operating at a higher beam velocity ratio $\alpha$.

Other considerations include designing the gyrotron to meet the needs for NMR. This fixed the frequency at 460 GHz due to the existing NMR magnet and DNP NMR spectrometer. Because the sample needs sufficient power in order to excite the DNP mechanisms that in turn enhance the NMR sensitivity, a power level of 50 W was chosen.

The gyrotron will be constructed in early 2002 and is expected to be ready for DNP experiments later that year.
The transmission line of a 140 GHz gyrotron used in dynamic nuclear polarization experiments is analyzed. First the output of a snake $TE_{01} - TE_{11}$ mode converter was examined experimentally and analyzed with several different theories. From experimental results, we find that the snake is probably operating with 60% efficiency. From theory, we find that the snake should be operating at 93% efficiency. Upon close examination of the snake itself, we conclude that the snake mode converter has relaxed its shape over the 10 years or so since it was built and needs to be re-fabricated in a more lasting manner. Secondly the total losses of the transmission line were theoretically examined in order to optimize the line such that we can obtain maximum power at the sample. If we propagate the $HE_{11}$ mode instead of the $TE_{11}$ mode we can avoid about 1 dB of losses.

The analysis of the 140 GHz transmission line should result in creating a more efficient system that will deliver more power to the sample for DNP NMR studies. The fabrication of the 460 GHz gyrotron will bring the realization of dynamic nuclear polarization as a routine experiment one step closer.
Appendix A

Glossary

Acronyms

BWO  backward wave oscillator
\(^{13}\)C  carbon
CAVRF  a cold cavity code written by A. Fliflet [29]
CRM  cyclotron resonance maser
CW  continuous wave
DNP  dynamic nuclear polarization
ECRH  electron cyclotron resonance heating
EGUN  an electron optics and gun design program by W. B. Herrmannsfeldt [24]
FEL  free-electron laser
\(^{1}\)H  hydrogen
LINEAR  a linear code written by K. E. Kreischer [28]
MIG  magnetron injection gun
MIT  Massachusetts Institute of Technology
NMR  nuclear magnetic resonance
RF  radio frequency (also refers to oscillations)
SSNMR  solid-state nuclear magnetic resonance
TE  transverse electric
TM  transverse magnetic
TWT  traveling wave tube
Variables

\( \alpha \)  
beam velocity ratio

\( b \)  
nuclear spin diffusion barrier

\( C_{mp} \)  
coupling coefficient

\( E \)  
electric field

\( \mathcal{E} \)  
electron energy

\( B \)  
magnetic field

\( B_0 \)  
static magnetic field

\( B_1 \)  
microwave field strength

\( \beta \)  
normalized velocity

\( \delta \)  
EPR linewidth

\( \Delta \)  
detuning

\( \epsilon_{SE} \)  
DNP enhancement factor due to solid effect

\( \epsilon_{TM} \)  
DNP enhancement factor due to thermal mixing

\( f(z) \)  
a function

\( F \)  
Lorentz force

\( \phi, r, z \)  
aximuthal, radial, and axial cylindrical coordinates

\( F \)  
normalized field amplitude

\( \Phi \)  
magnetic flux

\( \gamma \)  
relativistic factor

\( \gamma_{e,n} \)  
electron and nuclear gyromagnetic ratio

\( \eta \)  
efficiency

\( \eta_{el} \)  
electron efficiency

\( \eta_{\perp} \)  
perpendicular efficiency

\( \eta_Q \)  
Q efficiency

\( I \)  
normalized current parameter

\( I_a \)  
beam current

\( I_{st} \)  
starting current

\( J_m() \)  
Bessel function of the ordinary type

\( k \)  
wave number

\( L \)  
effective cavity interaction length
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$m, p, q$</td>
<td>azimuthal, radial, and axial mode indices</td>
</tr>
<tr>
<td>$\mu$</td>
<td>normalize cavity interaction length</td>
</tr>
<tr>
<td>$\mu_a$</td>
<td>electron magnetic moment</td>
</tr>
<tr>
<td>$n$</td>
<td>harmonic number</td>
</tr>
<tr>
<td>$N_e$</td>
<td>density of unpaired electrons</td>
</tr>
<tr>
<td>$\nu_{mp}$</td>
<td>the $p^{th}$ zero of $J_m$</td>
</tr>
<tr>
<td>$p$</td>
<td>electron momentum</td>
</tr>
<tr>
<td>$P$</td>
<td>power dissipated</td>
</tr>
<tr>
<td>$Q$</td>
<td>the cavity $Q$</td>
</tr>
<tr>
<td>$Q_D$</td>
<td>diffractive $Q$</td>
</tr>
<tr>
<td>$Q_{OHM}$</td>
<td>ohmic $Q$</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>total $Q$</td>
</tr>
<tr>
<td>$r_e$</td>
<td>beam radius</td>
</tr>
<tr>
<td>$r_K$</td>
<td>cathode radius</td>
</tr>
<tr>
<td>$r_0$</td>
<td>cavity radius</td>
</tr>
<tr>
<td>$r_L$</td>
<td>Larmor radius</td>
</tr>
<tr>
<td>$R_{1,2}$</td>
<td>wave reflection coefficient</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T_{1n}$</td>
<td>nuclear spin-lattice relaxation time</td>
</tr>
<tr>
<td>$u$</td>
<td>normalized energy</td>
</tr>
<tr>
<td>$U$</td>
<td>total stored energy</td>
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<tr>
<td>$v$</td>
<td>electron velocity</td>
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<tr>
<td>$V$</td>
<td>beam voltage</td>
</tr>
<tr>
<td>$V_{dep}$</td>
<td>space charge voltage depression</td>
</tr>
<tr>
<td>$V_K$</td>
<td>cathode voltage</td>
</tr>
<tr>
<td>$\omega$</td>
<td>wave frequency</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cyclotron frequency</td>
</tr>
<tr>
<td>$z_L$</td>
<td>axial distance propagated by an electron in one cyclotron period</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>normalized axial position</td>
</tr>
</tbody>
</table>
Subscripts

0    cavity region
K    cathode region
⊥    perpendicular
∥    parallel
φ, r, z cylindrical coordinates
Bibliography


