Design of Survivable IP-over-WDM Networks:
Providing Protection and Restoration at the
Electronic Layer

by

Daniel Dao-Jun Kan

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of Master of Science in Computer Science and Engineering at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2003

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Author

Department of Electrical Engineering and Computer Science

May 27, 2003

Certified by

Eytan Modiano
Assistant Professor, Department of Aeronautics and Astronautics
Thesis Supervisor

Certified by

Dr. Aradhana Narula-Tam
Staff, MIT Lincoln Laboratory
Thesis Supervisor

Accepted by

Arthur C. Smith
Chairman, Department Committee on Graduate Students
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Abstract

We focus on providing protection and restoration for IP-over-WDM networks at the electronic layer against single fiber link failures. Protection at the electronic layer requires overprovisioning the network so that the spare capacity available after the fiber link failure can be utilized for rerouting disrupted traffic. The routing of the lightpaths on the WDM network can significantly affect the capacity requirement for network survivability. We address the problem of achieving network survivability using the minimum amount of network capacity. The problem involves the joint optimization of lightpath routing and capacity allocation. We establish important criteria for the routing of the lightpaths and the corresponding capacity requirement. Two routing strategies are developed for reducing capacity requirement. We also compare link restoration and end-to-end restoration in IP-over-WDM networks. Lastly, we present a method for computing the minimum capacity requirement for general survivable IP networks against single link failures.

Thesis Supervisor: Eytan Modiano
Title: Assistant Professor, Department of Aeronautics and Astronautics

Thesis Supervisor: Dr. Aradhana Narula-Tam
Title: Staff, MIT Lincoln Laboratory
Acknowledgments

I would like to thank my parents, Mr. Jiaxi Kan and Mrs. Jingzhi Fan, for their great love and care of me. Without them, none of this would be possible.

I would like to thank my supervisors, Prof. Modiano and Dr. Narula-Tam, for their continual guidance and ideas. I would like to especially thank Dr. Narula-Tam for her great patience on teaching me about researching and writing.

I thank all my friends, especially Jen Alltop, Dan Maynes-Aminzade, Tonya Drake, Sonia Jain, Matt Everett, and Anand Srinivas, for making MIT such a friendly and fun place to work in.

Lastly, I dedicate this thesis to my nephew, Calton, who is turning one in July.
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Chapter 1

Introduction

Ensuring the continuation of services during network component failures is essential in WDM networks. The interruption of high-speed optical connections operating at bit rates such as 10Gb/s, even for a few seconds, leads to a huge loss of data. Node failures can occur from power outages or equipment failures. Links can fail because of fiber cuts. In almost all cases, protection mechanisms are designed to protect against a single failure event. This assumes that the network is designed well enough that simultaneous failures are very rare. Alternatively, we may assume that it is unlikely that we will have a failure event occur while we are trying to restore service from another earlier failure event. Moreover in practice, node failures are less frequent than link failures. Consequently in this thesis, we consider the problem of providing protection and restoration for WDM networks against any single fiber link failure.

1.1 Overview of IP over WDM Networks

The explosive growth of Web-related services over the Internet is creating a growing demand for bandwidth. Traditional fiber optics communication systems use a single high-speed optical channel with electronic time division multiplexing (TDM) for data transmission. The limitations in the electronic components have prevented these systems from sustaining bit rates beyond 40Gb/s. Wavelength division multiplexing (WDM) has emerged as the solution to the bandwidth problem by using
multiple wavelengths to transmit data. With the advent of erbium-doped fiber amplifiers (EDFA), up to 160 wavelengths can be transmitted simultaneously on a single fiber. At a bit rate of 40Gb/s per wavelength, the new WDM technology allows the transmission of over 6 terabits of data on a single fiber in one second. WDM has scaled well over time, providing more wavelengths per fiber and higher bit rates per wavelength. Thus WDM networks are used to carry the fast growing Internet protocol (IP) traffic.

1.1.1 IP-over-WDM Network Model

In this thesis, we assume a simplified IP-over-WDM network model as shown in Figure 1-1. For the IP network to be directly overlaid on the WDM network, an enhanced IP layer and WDM layer are necessary. The enhanced layers may be responsible for load balancing and reconfiguration, protection and restoration, optical flow switching, cross-layer optimization, and network management [1]. The exact division of these functionalities between the IP and WDM layers is still an ongoing debate.

Under this network model, the WDM network consists of optical cross connect (OXC) nodes interconnected by fiber links (see Figure 1-1(b)). The fiber links are pairs of fibers, where fibers in a pair carry signals in opposite directions. An OXC can switch the optical signal on a WDM channel from an input port to an output port.
without requiring the signal to undergo any optoelectronic conversion. Each OXC node is equipped with tunable transceivers. The WDM network provides lightpaths between pairs of OXC nodes. A lightpath is a high-speed optical layer connection that starts at a transmitter and ends at a receiver, and is composed of a sequence of wavelengths that are cross-connected at intermediate OXCs [2]. Each OXC node may also be equipped with wavelength converters. In the absence of wavelength conversion, a lightpath must be assigned the same wavelength on all the links along its route. With wavelength conversion, it may be assigned different wavelengths on different links along its route.

1.1.2 Network Topology

We introduce the concepts of physical and logical topologies to model IP-over-WDM networks. The physical topology is the network topology seen by the WDM layer. The nodes in this topology correspond to the OXCs, and two nodes are connected by a link if there is a fiber link. The logical topology is the network topology seen by the IP layer. The nodes in this topology correspond to the IP routers, and a link in this topology represents an IP link realized by a lightpath that has been established between the corresponding nodes. In the logical topology, the lightpath not only carries the direct traffic between the nodes it interconnects but also the
traffic between the nodes that are not directly connected in the logical topology using electronic packet switching at the intermediate nodes [3, 4]. The electronic packet-switching functionality is provided by the IP routers. Figure 1-2 shows the physical and logical topologies for the network in Figure 1-1(b). For convenience, we often refer to the lightpath as the logical link, and the fiber link as the physical link.

1.2 Protection for IP-over-WDM Networks

For an IP-over-WDM network, protection can be offered at the optical layer or the electronic layer [5, 6]. Each has its own advantages and disadvantages. Protection at the optical layer protects the lightpaths by establishing protection lightpaths. Protection at the electronic layer requires overprovisioning the network so that the spare capacity available after the physical link failure is utilized for rerouting the disrupted traffic. By providing protection at the optical layer, recovery of the failed lightpaths is transparent to the higher layers. Recovery also tends to be much faster at the optical layer [7]. However, the advantages comes at the cost of hardware complexity at the optical layer, where components are typically more expensive than those at the electronic layer.

1.2.1 Protection at the WDM Layer

Two protection mechanisms are available at the optical layer: path protection and link protection. In path protection, during the establishment of a lightpath, a protection lightpath is set up as well. In the event of a physical link failure, all the traffic on the primary lightpath can be diverted to the protection lightpath. Path protection comes in two forms:

1. Dedicated path protection: At the time of call setup, for each primary path, a fiber-disjoint\textsuperscript{1} protection path and wavelength are dedicated to that call. The protection wavelengths reserved on the links of the protection path are dedicated

\textsuperscript{1}By fiber-disjoint, we mean that the protection path for a connection has no fiber links in common with the primary path for that connection.
to that call and are not shared with other protection paths. Figure 1-3(a) shows an example of dedicated path protection for two lightpaths (1, 2) and (5, 6). Note that different wavelengths are used for the two protection lightpaths because the protection lightpaths share physical link (3, 4).

2. Shared path protection: At the time of call setup, for a primary path, a fiber-disjoint protection path and wavelength are reserved. However, the protection wavelength on the links of the protection path may be shared with other protection paths. As a result, protection channels are shared among different failure scenarios, and therefore shared-path protection is more capacity efficient when compared with dedicated path protection. Figure 1-3(b) illustrates shared path protection. Both primary lightpaths use wavelength $\lambda_1$ and their corresponding protection lightpaths use wavelength $\lambda_2$. Because the primary lightpaths are fiber disjoint, the protection lightpaths can share wavelength $\lambda_2$ on physical link (3, 4).

In link protection, each fiber link is protected. The protection mechanism reacts to a failure by diverting all the lightpaths on the failed link to an alternate path, thus bypassing the damaged fiber as shown in Figure 1-4. Link protection can be implemented using various techniques such as generalized loopback and ring loopback [8, 9].
However, shared path protection is more capacity efficient than link protection because it avoids “backhauls”, and distributes protection lightpaths over a wider region of the network [10].

### 1.2.2 Protection at the Electronic Layer

Protection at the electronic layer requires provisioning the network with sufficient spare capacity. After a physical link failure, IP routers can reroute the disrupted traffic on alternate paths, which must have enough spare capacity to support the additional traffic. Otherwise, buffer overflow will occur in the IP routers, which results in the dropping of IP packets from both working and rerouted traffic. IP routers can dynamically find an alternate path between the end-nodes of the failed link by employing an interior gateway protocol (IGP) for exchanging routing information. An example of an IGP employed by routers within an autonomous system (AS) is the open shortest path first (OSPF) protocol, which has a mechanism for periodically updating paths between nodes in the network [11].

Various rerouting strategies exist at the electronic layer. In order to reduce costs, the spare capacity dedicated to protection can be shared by backup routes under different failure scenarios. We focus on two shared rerouting strategies: link restoration and end-to-end restoration. In link restoration, all of the traffic carried on the failed
A fiber cut occurs in the WDM network. Backup route for link (1,4) Backup route for link (2,4)

(a) A fiber cut occurs in the WDM network. (b) Physical topology

(c) Logical topology

Figure 1-5: Protection at the electronic layer

lightpath is rerouted together using the same backup route to bypass the failed lightpath. Since multiple lightpaths may fail from a single physical link failure, multiple link restorations may be necessary to restore all of the disrupted traffic.

We use an example to illustrate how the electronic layer can recover from multiple lightpath failures using link restoration. In Figure 1-5, a fiber cut results in the failure of link (1,4) in the corresponding physical topology. Because logical links (1,4) and (2,4) are routed on the same fiber, both links fail simultaneously from the physical link failure. To restore the network, traffic carried on failed logical links
(1, 4) and (2, 4) can be rerouted using backup routes \{(1, 3), (3, 4)\} and \{(2, 3), (3, 4)\}, respectively.

An alternative rerouting strategy is the end-to-end restoration. Each disrupted end-to-end traffic is rerouted individually between the source and destination using a backup route that is link-disjoint from the failed lightpaths. Although end-to-end restoration is more capacity efficient than link restoration, it is more complex than link restoration because it requires finding backup routes between numerous source-destination pairs, as opposed to finding backup routes between the end-nodes of the failed lightpaths.

1.3 Problem Description

In this thesis, we focus on providing protection for IP-over-WDM networks at the electronic layer because it is more efficient in utilizing capacity due to the sharing of the spare capacities\(s\). Furthermore, it is less costly compared to protecting every individual lightpath. Low network cost is important for cost-sensitive Internet users. Although restoration time is longer on the order of seconds rather than milliseconds, the delays may be tolerable to many IP users, who primarily use the Internet for web browsing, email, and downloading and uploading.

1.3.1 Network Survivability

We define network survivability for an IP-over-WDM network as the ability of the network to recover from any single physical link failure. Two criteria must be satisfied in order to ensure network survivability: 1) the source and destination of every traffic demand must remain connected after any physical link failure, and 2) the spare capacity in the network must be sufficient to support all of the disrupted traffic. The first criterion can be satisfied if the logical topology remains connected after any physical link failure. Since every pair of nodes in the logical topology is connected, there always exist backup routes between the source and destination of the disrupted traffic demand. However, the network must still have sufficient spare capacity to reroute
Figure 1-6: An example of disconnected logical topology from a single physical link failure.

The routing of the logical links on the physical topology plays a significant role in the connectivity of the logical topology after a physical link failure. We illustrate this idea using a simple example. In Figure 1-6, the failure of physical link (1, 2) results in the failures of both logical links (2, 3) and (2, 4). As a result, node 2 is disconnected from the rest of the logical topology. If node 2 were a source or destination of any traffic, network survivability would not be met. However, if logical link (2, 3) had been routed on physical link (2, 3) instead of links {(2, 1), (1, 3)}, the logical topology would always remain connected regardless of which physical link failed.

The problem of finding a routing of the logical links on the physical topology so that the logical topology remains connected after any physical link failure was first proposed by Crochat and Le Boudec [12]. However, they did not find an efficient algorithm that solved the problem exactly. Instead, they used a tabu search heuristic to find suboptimal solutions. The problem has been shown to be NP-complete [13, 14, 15]. Modiano and Naraula-Tam found an integer linear programming formulation of the problem, which can be solved exactly using mathematical software packages [14]. Deng, Sasaki, and Su later formulated the problem as a mixed integer linear program with the number of constraints growing as a polynomial with
Figure 1-7: An example of topologies for which no survivable routing exists.

the size of the network [16].

We call a routing survivable\(^2\) if any physical link failure leaves the logical topology connected. A routing cannot possibly be survivable if the underlying logical topology is not 2-connected\(^3\). Even if the logical topology is 2-connected, a survivable routing may not exist [15]. Consider the case where both the physical and logical topologies are 4-node rings shown in Figure 1-7. No matter how we route the logical links; there always exists a physical link shared by at least two logical links. Thus a physical link failure will result in the failures of two logical links. Since the logical topology is only 2-connected, the removal of any two links would disconnect the topology. Therefore, no survivable routing exists.

Determining where to place spare capacity in the network and how much spare capacity must be allocated to guarantee the restoration of the network against single link failures is called the spare capacity allocation problem. A considerable amount of effort has been put into solving this problem for different networks like SONET/SDH [17, 18, 19], ATM [10, 18, 20], WDM [5, 21, 22], and IP/MPLS [23]. However, all these works do not consider the notion of a higher layer (i.e. logical topol-

---

\(^2\)We differentiate routing survivability from network survivability. The former refers to the connectivity of the logical topology after any physical link failure. The latter refers to the restorability of the network.

\(^3\)The logical topology is 2-connected if the removal of any logical link does not cause the topology to be disconnected.
ogy) being embedded on a lower layer (i.e. physical topology). In an IP-over-WDM network, the routing of the logical links can affect the spare capacity requirement because a single physical link failure can result in the failures of multiple logical links.

1.3.2 Problem Objective

In this thesis, we consider the problem of achieving network survivability for an IP-over-WDM using the minimum amount of capacity. The problem requires the joint optimization of routing and capacity allocation. The goal is to obtain insights on the capacity requirement for providing protection at the electronic layer.

Since network cost is dominated by electronic processing, we want to reduce the bandwidths of the lightpaths required to support all the traffic while still maintaining network survivability. The bandwidth of the lightpath is directly related to the rate of the corresponding transceivers. Since current electronics limit the bandwidth of the lightpath to 40Gb/s, multiple lightpath connections may be established to meet bandwidth requirements beyond 40Gb/s. We define the capacity of a lightpath as the bandwidth associated with it. By defining the capacity in such a way, minimizing capacity at the electronic layer minimizes the rate requirement of the transceivers, the number of transceivers, and the number of wavelengths, thus reducing the cost at the WDM layer as well. Consequently, we only consider network capacity at the electronic layer.

The rest of the thesis is organized in the following way. In Chapter 2, we investigate how the routing of the logical links on the physical topology can affect the capacity requirement for network survivability. We develop two routing strategies for reducing capacity requirement. In Chapter 3, we study the joint optimization problem of routing the logical links on the physical topology and routing traffic on the logical topology. We compare capacity requirement under link restoration and end-to-end restoration. In Chapter 4, we study capacity requirement for protecting general survivable networks against single link failures. Finally, we conclude the studies in Chapter 5.
1.4 Notations and Assumptions

For convenience, we define some general notations and assumptions that are used throughout the rest of the thesis.

Let \((N_P, E_P)\) denote the physical topology, which consists of a set of nodes \(N_P = \{1 \ldots |N_P|\}\) and a set of links \(E_P\) where link \((i, j)\) is in \(E_P\) if a fiber link exists between node \(i\) and \(j\). We assume a bidirectional physical topology, where if link \((i, j)\) is in \(E_P\) so is link \((j, i)\). Furthermore, we assume that a failure (fiber cut) of link \((i, j)\) will also result in a failure of link \((j, i)\). This assumption stems from the fact that the physical fiber carrying the link from \(i\) to \(j\) is typically bundled together with that from \(j\) to \(i\). In some systems, the same fiber is used for communicating in both directions. Let \(E'_P = \{(i, j) \in E_P : i > j\}\) denote the set of bidirectional physical links. Lastly, we assume that each physical link is capable of supporting \(W\) wavelengths.

Let \((N_L, E_L)\) denote the logical topology. The logical topology can be described by a set of nodes \(N_L\) and a set of links \(E_L\), where \(N_L\) is a subset of \(N_P\) and link \((s, t)\) is in \(E_L\) if both \(s\) and \(t\) are in \(N_L\) and there exists a logical link, or a lightpath, between them. We also assume a bidirectional logical topology, where if link \((s, t)\) is in \(E_L\) so is link \((t, s)\).

Given a logical topology, we want to route every logical link on the physical topology. We refer to this process as the routing of the logical topology. In order to route logical link \((s, t)\) on the physical topology, we must establish a corresponding lightpath on the physical topology between node \(s\) and \(t\). Such a lightpath consists of a set of physical links connecting nodes \(s\) and \(t\) as well as wavelengths along those links. If wavelengths converters are available then any wavelength can be used on any link. However, without wavelength converters, the same wavelength must be used along the route. We assume that either wavelength converters are available or that the number of wavelengths available exceeds the number of lightpaths. Let \(f_{ij}^{st} = 1\) if logical link \((s, t)\) is routed on physical link \((i, j)\), and 0 otherwise. We denote the routing of the logical topology by the assignment of values to the variables \(f_{ij}^{st}\) for all physical links \((i, j)\) and logical links \((s, t)\). We assume that logical link \((s, t)\)
will not be routed on both physical links \((i, j)\) and \((j, i)\), thus \(f_{ij}^{st} + f_{ji}^{st} \leq 1\).

Every logical link \((s, t)\) is associated with a capacity denoted by \(C_{st}^{s}\). We ignore the modularity requirement (e.g. OC-4, OC-12, etc), which is studied in [21, 24]. The capacity of each logical link \((s,t)\) is divided into working capacity and spare capacity denoted by \(\beta_{st}^{s}\) and \(\mu_{st}^{s}\), respectively. The working capacity is dedicated to carry working traffic, while the spare capacity is dedicated to carry rerouted traffic in case of failures. We define the total working capacity as the sum of working capacity on all logical links. Similarly, we define the total spare capacity as the sum of spare capacity on all logical links. Lastly, we define the total capacity as the sum of total working capacity and total spare capacity.

The traffic demand for source-destination pair \((u, v)\) is the amount of traffic that must be delivered from node \(u\) to \(v\), where both \(u\) and \(v\) are in \(N_L\). We assume a multi-hop network, where a traffic demand may have to hop through zero or more intermediate logical nodes. Lastly, we assume the IP routers can perform load sharing. If there are multiple routes from the a source to a destination, traffic can be bifurcated over these multiple routes.
Chapter 2

Lightpath Routing and Capacity Assignment

In an IP-over-WDM network, a single physical link failure may result in the failures of multiple lightpaths. We assume failure recovery at the electronic layer thus all of the traffic carried on the failed lightpaths must be rerouted using the remaining lightpaths. In order to support the additional traffic, lightpaths should be provisioned with sufficient spare capacity to protect against failures. The routing of the lightpaths on the WDM network can significantly affect the amount of capacity required for network survivability. In this chapter, we investigate how to route the logical topology in a manner that reduces network capacity. In Section 2.1, we show that a quantity, called the load factor, is a good criterion for routing the logical topology. We present an algorithm for finding a routing that maximizes the load factor. In Section 2.2, we are also given the working capacity on each logical link. We want to find a routing that minimizes the total spare capacity required for network survivability. The motivation is that the network should use the minimum amount of spare capacity to protect all of the working capacities against any physical link failure.
2.1 Lightpath Routing and Maximum Load Factor

Given the physical topology and the logical topology, we want to develop a criterion for routing the logical topology. We associate a routing with a quantity between 0 and 1, called the load factor. Assume each logical link has capacity $C$. Given the routing and the corresponding load factor denoted by $\alpha$, network survivability is guaranteed if the traffic on every logical link does not exceed $\alpha C$. The spare capacity on every logical link required for carrying disrupted traffic in the event of any physical link failure is at most $C(1 - \alpha)$. Consequently, we want to find a routing that maximizes the load factor. We call this the lightpath routing and maximum load factor (LRMLF) problem. We will show that the load factor is indeed a good criterion for routing the logical topology and present a method to solve this problem.

2.1.1 Defining the Load Factor

For a given routing of the logical topology, we can allocate $\beta^{st}$ amount of working capacity on each logical link $(s, t)$ in a manner that the spare capacity, $C - \beta^{st}$, is sufficient to protect all of the working capacities against any physical link failure. We call an allocation valid if network survivability is satisfied. Let $\alpha^{st} = \frac{\beta^{st}}{C}$ denote the normalized working capacity for logical link $(s, t)$. The normalized capacity is clearly between 0 and 1. Without the loss of generality, we assume $C = 1$ because it is only a scaling factor.

We define $\alpha_{min} = \min_{(s,t) \in E_L} \alpha^{st}$ as the minimum working capacity over the set of all links in the logical topology. We show that $\alpha_{min}$ is an important quantity for measuring the quality of working capacity allocation. Figure 2-1 shows two valid allocations of working capacity for a given routing of the logical topology on the physical topology shown in Figure 2-1(a). It is easy to check that all of the working capacities are protected against any physical link failure. In Figure 2-1(b), the allocation with $\alpha_{min} = 0$ does not allow any traffic between source-destination pairs (1, 3) and (2, 4). On the other hand, the allocation with $\alpha_{min} = \frac{1}{3}$ allows traffic between any source-destination pair. In general, a network should be able to send traffic between any
(a) Routing of Logical Topology
(b) $\alpha_{\text{min}} = 0$
(c) $\alpha_{\text{min}} = \frac{1}{3}$

Figure 2-1: Two valid allocations for working capacity resulting from routing the logical topology on the 4-node ring physical topology.

A good criterion for measuring the quality of a routing should be unique for a given routing. One such quantity is the maximum total working capacity allowed over the set of all valid allocations for a given routing. It is unique because the total working capacity is always bounded. However, this quantity does not reveal any information about the working capacity on each logical link. In the previous example, the maximum total working capacity for the given routing is 2. Although both allocations have the same maximum total working capacity, the allocation with $\alpha_{\text{min}} = \frac{1}{3}$ is preferred over the one with $\alpha_{\text{min}} = 0$. Thus we define another quantity $\alpha$, called the load factor, which is the maximum $\alpha_{\text{min}}$ achievable over the set of all valid allocations for a given routing. Given the load factor $\alpha$, we know that the working capacity on every logical link is at least $\alpha$. If the traffic on each logical link does not exceed $\alpha$, network survivability is guaranteed. Furthermore, the corresponding spare capacity required for protecting the traffic on each logical link is at most $1 - \alpha$. The load factor directly gives us a measure for network redundancy required for network survivability for a given routing.

We now show an interesting consequence of the definition of the load factor. We
first define some new notations. A cut is a partition of the set of nodes \( N \) into two parts: \( S \) and \( N - S \). Associated with each cut is a set of links, where each link has one node in \( S \) and the other node in \( N - S \). We refer to this set of links as the cut-set associated with the cut \( \langle S, N - S \rangle \), or simply \( CS(S, N - S) \). Let \( |CS(S, N - S)| \) equal the size of the cut-set \( \langle S, N - S \rangle \); that is, the number of links in the cut-set. Furthermore, assume each link has an associated capacity. We define the capacity of the cut as the sum of the capacities on all links in the associated cut-set. Let \( C(S) \) denote the capacity of the cut \( S \).

Assume all the working capacity is dedicated to a single traffic demand. Let \( F_{uv} \) be the maximum demand allowed for source-destination pair \((u, v)\). The following theorem gives a lower bound on \( F_{uv} \) for any source-destination pair \((u, v)\).

**Theorem 2.1.1** Given the load factor \( \alpha \) and the logical topology \((N_L, E_L)\),

\[
F_{uv} \geq \min_{S \subseteq N_L} \alpha |CS(S, N_L - S)| \quad \forall u, v \in N_L, \ u \neq v. \tag{2.1}
\]

**Proof** Since traffic demand can be bifurcated, we can treat the demand as a network flow. The maximum-flow minimum-cut theorem states that the maximum flow \( F_{uv} \) for source-destination pair \((u, v)\) must be equal to the capacity of the minimum cut \( C(S) \), where \( u \in S \) and \( v \in N_L - S \) [25]. Thus, we have the following:

\[
F_{uv} = \min_{S \subseteq N_L : u \in S, v \in N_L - S} C(S) \geq \min_{S \subseteq N_L} C(S).
\]

The inequality holds because the minimum over a set must be less than or equal the minimum over a subset. Since the working capacity on every logical link is at least \( \alpha \), \( C(S) \geq \alpha |CS(S, N_L - S)| \). Combining this with the inequality above, we have Eq. (2.1).

Although having a single traffic demand is not a likely scenario, Theorem 2.1.1 shows that the load factor, a single quantity, can affect the amount of traffic that can sent be across the network.
2.1.2 Determining the Load Factor

In this section, we first establish a necessary condition on the load factor and the corresponding routing using the maximum-flow minimum-cut theorem. We then derive a formula for the exact value of the load factor for any given routing. Finally, we obtain an upper bound on the load factor for any given physical and logical topologies.

Given a routing of the logical topology denoted by the assignment of values to the variables $f_{st}^{ij}$ for all physical links $(i, j)$ and logical links $(s, t)$, the following lemma gives a necessary condition on the load factor.

**Lemma 2.1.2** For every cut-set $CS(S, NL - S)$ of the logical topology and every physical link failure $(i, j)$, the load factor must satisfy the following inequality:

$$\sum_{(s,t) \in CS(S,NL-S)} (f_{st}^{ij} + f_{st}^{ji})\alpha \leq \sum_{(s,t) \in CS(S,NL-S)} \{1 - (f_{st}^{ij} + f_{st}^{ji})\}(1 - \alpha).$$  \hfill (2.2)

**Proof** As mentioned in Section 1.4, we assume a physical link failure corresponds to the failure of a bidirectional link. Therefore, $f_{st}^{ij} + f_{st}^{ji} = 1$ implies that the logical link $(s, t)$ will be broken if physical link $(i, j)$ fails. Likewise, $f_{st}^{ij} + f_{st}^{ji} = 0$ implies that the logical link $(s, t)$ will remain intact. The above condition states that the amount of working capacity lost on the broken logical links in the cut-set (given in the lhs. of Eq. (2.2)) due to a physical link failure must be less than or equal to the amount of spare capacity on the remaining logical links in the cut-set (given in the rhs. of Eq. (2.2)). This condition must hold for every cut-set of the logical topology and every single physical link failure. This follows directly from the maximum-flow minimum-cut theorem. \hfill \blacksquare

Consider a cut-set $CS(S, NL - S)$, $|CS(S, NL - S)|$ is the total number of logical links in the cut-set and $\sum_{(s,t) \in CS(S,NL-S)} f_{st}^{ij} + f_{st}^{ji}$ is the total number of logical links lost in the cut-set after physical link $(i, j)$ fails. Thus \[ \frac{|CS(S,NL-S)| - \sum_{(s,t) \in CS(S,NL-S)} f_{st}^{ij} + f_{st}^{ji}}{|CS(S,NL-S)|} \] is the fraction of the number of logical links that remain intact after the failure over the size of the cut-set. For a given routing of the logical topology, we show that the load factor $\alpha$ is the minimum of such fractions over all cut-sets and all possible single
physical link failures. Recall that $E'_p$ is the set of bidirectional physical links, whereas $E_p$ is the set of all unidirectional physical links.

**Theorem 2.1.3** Given the routing denoted by the set of variables $f_{ij}^t$,

$$
\alpha = \min_{S \subseteq N_L, (i,j) \in K} \frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}.
$$

**Proof** Let’s rearrange Eq. (2.2) from Lemma 2.1.2 as follows:

$$
\sum_{(s,t) \in CS(S, N_L - S)} (f_{ij}^t + f_{ji}^t) \alpha \leq \sum_{(s,t) \in CS(S, N_L - S)} (1 - (f_{ij}^t + f_{ji}^t))(1 - \alpha) + \sum_{(s,t) \in CS(S, N_L - S)} 1 - \alpha
$$

$$
\sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t \leq (1 - \alpha)|CS(S, N_L - S)|
$$

$$
\alpha|CS(S, N_L - S)| \leq |CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t
$$

$$
\alpha \leq \frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}, \quad \forall S \in N_L, (i, j) \in E'_p.
$$

Since the load factor is defined to be the maximum value that satisfies the inequality above for every cut-set and every single physical link failure for the given routing,

$$
\alpha = \min_{S \subseteq N_L, (i,j) \in K} \frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}.
$$

We define the quantity, $\frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}$, as the normalized residual capacity associated with cut-set $CS(S, N_L - S)$ and physical link failure $(i, j)$. Theorem 2.1.3 states that the load factor must equal the minimum normalized residual capacity over all cut-sets and all single physical link failures. This condition is a consequence of the maximum-flow minimum-cut theorem. Intuitively, the load factor corresponds to the maximum-flow and the minimum normalized residual capacity corresponds to the capacity of the minimum-cut.

In order to determine the load factor for a given routing, Theorem 2.1.3 considers
every cut-set of the logical topology and every single physical link failure. Computing
the exact value of the load factor may be difficult because the number of cut-sets grows
exponentially with the size of $N_L$ [26]. Therefore, it is useful to establish an upper
bound on the the load factor that can be easily computed. Let $P_k$ denote the degree
of physical node $k$ in $N_P$. Let $L_k$ denote the degree of logical node $k$ in $N_L$.

**Corollary 2.1.4** Given a physical topology $(N_P, E_P)$ and a logical topology $(N_L, E_L)$,

$$\alpha \leq 1 - \max_{k \in N_L} \frac{L_k}{P_k}$$  \hspace{1cm} (2.4)

**Proof** The upper bound directly results from Theorem 2.1.3 when we only consider
cuts of a single node. More precisely, we consider cut-sets $CS(S, V - S)$, where
$S = \{k : k \in N_L\}$. Furthermore, $|CS(\{k\}, V - \{k\})| = L_k$ because the cut-set only
contains links incident to node $k$. Now, we show how to derive the upper bound.
According to Theorem 2.1.3,

$$\alpha = \min_{S \subseteq N_L} \frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{st} + f_{ts}^{st}}{|CS(S, N_L - S)|}$$

By considering only single node cuts,

$$\alpha \leq \min_{k \in N_L} 1 - \frac{\sum_{(s,t) \in CS(\{k\}, N_L - \{k\})} f_{st} + f_{ts}^{st}}{|CS(\{k\}, N_L - \{k\})|}.$$

Since the cut-set of a single node only contains links incident to node, the inequality
above is equivalent to

$$\alpha \leq 1 - \max_{k \in N_L} \frac{\max_{(i,j) \in E_P} \sum_{l, s.t.(k,l) \in E_L} f_{ij}^{kl} + f_{ji}^{kl}}{L_k}.$$

Let $M_{ij}^k$ denote the quantity, $\sum_{l, s.t.(k,l) \in E_L} f_{ij}^{kl} + f_{ji}^{kl}$. It is the number of broken
logical links incident to node $k$ if physical link $(i, j)$ fails. The key observation is
$max_{(i,j) \in E_P} M_{ij}^k \geq \lceil \frac{L_k}{P_k} \rceil$ regardless of how we route the logical topology. Consider a
node $k$, the optimal routing of the incident logical links that minimizes max_{(i,j) \in E_P} M_{ij}^k
is one that evenly divides the logical links among the physical links incident to node $k$. Since $L_k$ may not be divisible by $P_k$, some physical links incident to node $k$ should have at least $\lceil \frac{L_k}{P_k} \rceil$ number of logical links. Thus the maximum number of broken logical links incident to node $k$ after any physical link failure is at least $\lceil \frac{L_k}{P_k} \rceil$. Since $\max_{(i,j)\in E_P} M_{ij}^k \geq \lceil \frac{L_k}{P_k} \rceil$, 

$$\alpha \leq 1 - \max_{k\in N_L} \frac{\max_{(i,j)\in E_P} M_{ij}^k}{L_k} \leq 1 - \max_{k\in N_L} \frac{\lceil \frac{L_k}{P_k} \rceil}{L_k}.$$ 

Corollary 2.1.4 not only provides an easy way to obtain an upper bound on the load factor, it also provides some interesting insights on how the physical and logical topologies can inherently affect the load factor. We examine the following two special cases:

1. If $L_k \leq P_k$ for all $k$ in $N_L$, then $\lceil \frac{L_k}{P_k} \rceil \leq 1$ and $\alpha \leq 1 - \frac{1}{\min_{k\in N_L} L_k}$. 

The minimum degree of the logical node over the set $N_L$ limits the load factor.

2. If $L_k > P_k$ for all $k$ in $N_L$, $\alpha \leq 1 - \max_{k\in N_L} \frac{\lceil \frac{L_k}{P_k} \rceil}{L_k} \leq 1 - \max_{k\in N_L} \frac{\frac{L_k}{P_k}}{L_k} = 1 - \frac{1}{\min_{k\in N_L} E'_P}$. 

The minimum degree of the physical node over the set $N_L$ limits the load factor.

The sparser topology limits the load factor. Furthermore, if either the physical topology or logical topology is a ring, the load factor is at most $\frac{1}{2}$.

### 2.1.3 Routing that Maximizes the Load Factor

We have established some important relationships between the load factor and the routing of the logical topology in the previous section. Now we want to find a routing that maximizes the load factor. Let $\{f_{ij}^{rt}\}^*$ denote such a routing, and $R$ denote the
set of all possible routings of the logical topology. Using Theorem 2.1.3,

\[
\{f_{ij}^t\}^* = \underset{\{f_{ij}^t\} \in R}{\arg \max} \alpha \frac{|CS(S, N_L - S)| - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}
\]

\[
= \underset{\{f_{ij}^t\} \in R}{\arg \max} \min_{S \subseteq N_L (i,j) \in E_p} \frac{1 - \sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}
\]

Finding a routing that maximizes the load factor is equivalent to finding one that minimizes the maximum quantity, \(\frac{\sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|}\), which can be interpreted the fraction of the number of broken logical links in the cut-set over the size of the cut-set. The cut-set with the largest fraction of broken links limits the load factor. Intuitively, if the fraction of broken logical links in every cut-set is small, the logical topology remains well-connected after the physical link failure. This implies that even if the broken logical links had carried a large amount of working traffic, there exist enough diverse backup routes to reroute the disrupted traffic. Therefore, the load factor can also be interpreted as a quantity for measuring the disjointness of the corresponding routing. The larger the load factor, the more disjoint the routing is.

The LRMLF problem is directly related to the survivable routing problem discussed in Section 1.3.1. Solving this problem also solves the survivable routing problem, which has been proven to be NP-complete. Given the load factor \(\alpha^*\) and the routing \(\{f_{ij}^t\}^*\) are the optimal solution to the LRMLF problem, we make the following two observations, which are sufficient to show that the LRMLF problem is NP-hard.

1. If \(\alpha^* = 0\), \(\{f_{ij}^t\}^*\) is not survivable.

2. If \(\alpha^* > 0\), \(\{f_{ij}^t\}^*\) is survivable.

We first prove the first statement. From Eq. (2.3), the load factor is zero if and only if \(\max_{S \subseteq N_L (i,j) \in E_p} \frac{\sum_{(s,t) \in CS(S, N_L - S)} f_{ij}^t + f_{ji}^t}{|CS(S, N_L - S)|} = 1\). This implies that there exists at least
one cut-set, in which all of the logical links are broken after some physical link failure. Thus the logical topology is disconnected, and the routing is not survivable. Similarly, to prove the converse, the load factor is nonzero if and only if 
\[
\max_{S \subseteq N_p} \sum_{(i,j) \in E_p} \frac{f_{ij}^s + f_{ji}^t}{|CS(S,N_P - S)|} < 1.
\]
This implies every cut-set has at least one logical link intact after any physical link failure. Thus the logical topology is always connected, and the routing is survivable. The two observations together imply that the load factor is nonzero if and only if the routing is survivable.

2.1.4 Mixed Integer Linear Program Formulation

Based on the criteria established in the previous section, we now formulate the LRMLLF problem as a mixed integer linear program (MILP). Given a physical topology and a logical topology, we want to find a routing of the logical topology on the physical topology that maximizes the load factor.

We first develop a set of constraints which must be satisfied by every routing of the logical topology. To route logical link \((s, t)\) on the physical topology, we must find a corresponding path on the physical topology between nodes \(s\) and \(t\). When the logical links are bidirectional, finding a route from \(s\) to \(t\) also implicitly gives a route from \(t\) to \(s\) that follows the same physical links in the opposite direction. Let 
\[ f_{ij}^s = 1 \text{ if logical link } (s, t) \text{ is routed on physical link } (i, j) \text{ and } 0 \text{ otherwise.} \]
Using the standard network flow formulation, finding a route from \(s\) to \(t\) is equivalent to routing one unit of flow from node \(s\) to node \(t\) [27]. This can be expressed by the following set of constraints on variables \(f_{ij}^s\) associated with logical link \((s, t)\):

\[
\sum_{j \in N_p} f_{ij}^s - \sum_{j \in N_p} f_{ji}^s = \begin{cases} 
1, & \text{if } s = i \\
-1, & \text{if } t = i \\
0, & \text{otherwise}
\end{cases}, \quad \forall i \in N_p.
\]

(2.5)

The set of constraints above are the flow conservation constraints for routing one unit of flow from node \(s\) to node \(t\). Equation (2.5) requires that equal amounts of flow due to lightpath \((s, t)\) enter and leave each node that is not the source or destination.
of \((s, t)\). Furthermore, node \(s\) has an exogenous input of one unit of flow that has to find its way to node \(t\). There are many possible combinations of values that can satisfy Eq. (2.5). Any feasible solution has a route from \(s\) to \(t\) embedded in it. To find a routing of the logical topology, we must find a route for every logical link \((s, t)\) in \(E_L\).

If the number of wavelengths on a fiber is limited to \(W\), a wavelength capacity constraint can be imposed as follows:

\[
\sum_{(s,t)\in E_L} f_{st}^W \leq W, \quad \forall (i, j) \in E_P. \tag{2.6}
\]

The routing of logical topology, denoted by \(\{f_{ij}^W\}^*\), that maximizes the load factor must satisfy the following condition established in the previous section:

\[
\{f_{ij}^W\}^* = \arg \min_{\{f_{ij}^W\} \in \mathbb{R}} \max_{(i,j) \in E_P} \frac{\sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^W + f_{ji}^W}{CS(S,N_L-S)}. \tag{2.7}
\]

This directly translates to the following objective function, which must be minimized:

\[
\max_{\sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^W + f_{ji}^W \leq CS(S,N_L-S)} \frac{\sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^W + f_{ji}^W}{CS(S,N_L-S)}. \tag{2.7}
\]

The objective function has the form \(\max_{i=1,\ldots,m} c^T x\). It is piecewise linear and convex rather than linear [27]. Mixed integer linear programming does not allow such objective functions. Problems with piecewise linear convex objective functions can be solved by solving an equivalent MILP problem. Note that \(\max_{i=1,\ldots,m} c^T x\) is equal to the smallest number \(f\) that satisfies \(f \geq c^T x\) for all \(i\). For this reason, the LRMLF problem is equivalent to the following MILP problem:

\[
\text{minimize} \quad f
\]

Subject to:

1. Load factor constraints:

\[
f \geq \frac{\sum_{(s,t)\in CS(S,N_L-S)} f_{ij}^W + f_{ji}^W}{CS(S,N_L-S)}, \quad \forall S \subseteq N_L, \forall (i, j) \in E'_P.
\]
2. Connectivity constraints:

\[
\sum_{s.t. (i,j) \in E_p} f_{ij}^{st} - \sum_{s.t. (j,i) \in E_p} f_{ji}^{st} = \begin{cases} 
1, & \text{if } s=i \\
-1, & \text{if } t=i \\
0, & \text{otherwise}
\end{cases}, \quad \forall i \in \mathbb{N}_p, \forall (s,t) \in E_L.
\]

3. Wavelength capacity constraints:

\[
\sum_{(s,t) \in E_L} f_{ij}^{st} \leq W, \quad \forall (i,j) \in E_p.
\]

4. Integer flow constraints: \( f_{ij}^{st} \in \{0,1\} \).

Let \( f^* \) and \( \{f_{ij}^{st}\}^* \) be the optimal solution to the MILP problem assuming the problem is feasible. Using Theorem 2.1.3, the maximum load factor \( \alpha^* = 1 - f^* \). Furthermore, \( f^* \geq \max_{k \in \mathbb{N}_L} \frac{|f_k^P|}{L_k} \) according to Corollary 2.1.4. The corresponding routing of the logical topology for the load factor is embedded in \( \{f_{ij}^{st}\}^* \). The set of physical links used to route each logical link \((s,t)\) is given by \( \{(i,j) \in E_p : f_{ij}^{st} = 1\} \). The route may contain cycles because \( \{f_{ij}^{st}\}^* \) with cycles always satisfy the connectivity constraints (i.e. flow conservation constraints) [27]. Cycles can be avoided by modifying the original objective function to the following, where \( c \) is a large constant:

\[
\text{minimize} \quad cf + \sum_{(s,t) \in E_L} f_{ij}^{st}.
\]

Changing the objective function does not affect the feasibility of the original optimal solution \( f^* \) and \( \{f_{ij}^{st}\}^* \) because the constraints are not changed. Thus \( f^* \) and \( \{f_{ij}^{st}\}^* \) remain feasible in the modified problem. Let \( f' \) and \( \{f_{ij}^{st}\}' \) be the optimal solution to the modified problem. We show by contradiction that if \( c \) is sufficiently large (i.e. \( c \gg |E_P||E_L| \geq \sum_{(s,t) \in E_L} f_{ij}^{st} \)), then \( f' = f^* \).

**Proof** Let \( C' \) be the objective value associated with \( f' \) and \( \{f_{ij}^{st}\}' \) and \( C^* \) be the objective value associated with \( f^* \) and \( \{f_{ij}^{st}\}^* \). Assume \( f' > f^* \). We must show that
C' > C*, which would contradict the assumption that C' is the optimal objective value in the modified problem. Using the modified objective function, we can express C' > C* as

\[ cf' + \sum_{(s,t) \in E_L} f_{ij}^{st} > cf^* + \sum_{(s,t) \in E_L} f_{ij}^{st*} \]

\[ c(f' - f^*) > \sum_{(s,t) \in E_L} f_{ij}^{st*} - f_{ij}^{st'} \]

Since |E_P| |E_L| > \sum_{i,j \in E_L} f_{ij}^{st*} - f_{ij}^{st'}, it is sufficient to show \( c(f' - f^*) > |E_P| |E_L| \).

According to Eq. (2.7), both f' and f* can only take on values from a finite set of rational numbers, which is a subset of \( \{ \frac{m}{n} : 0 \leq m \leq n \leq K \} \), where m and n are integers and \( K = \max_{S \in \mathcal{N}_L} |CS(S, N_L - S)| \). Since f' and f* are rational, f' - f* must also be rational. Because we assumed f' > f*, the minimum nonzero difference is: \( f' - f^* \geq \frac{1}{K(K-1)} \). If c is sufficiently large such that \( \frac{c}{K(K-1)} > |E_P| |E_L| \), then C' > C*. This would contradict the assumption that C' is the optimal objective value. Therefore, f' = f*.

For this reason, we can add the term \( \sum_{(s,t) \in E_L} f_{ij}^{st} \) to the objective function, which can be considered as the cost of total flows associated with routing the logical topology. Since any unnecessary flow will be avoided with the new objective, the routing embedded in variables \( \{f_{ij}^{st}\}' \) must not contain any cycles.

The above MILP can now be solved using a variety of techniques. We use the AMPL modeling language to implement the MILP and the CPLEX solver to solve it [28]. The CPLEX solver uses the simplex method along with branch and bound techniques for solving MILPs [29]. However, the CPLEX solver takes a very long time to optimally solve our MILP. We find that if we use “\( \text{minimize} \sum_{(s,t) \in E_L} f_{ij}^{st} \)” as the objective function, the solver can quickly find a feasible solution for a given value of f. Let MILP-LF(f) denote this modified MILP. By solving MILP-LF(f) for different values of f, we can obtain the minimum value of f for which MILP-LF(f) generates a feasible solution.
We now present an iterative algorithm that finds a routing with $f$ arbitrarily close to the optimal value $f^*$. Let $f'$ and $\{f^*_{ij}\}'$ denote the solution returned by the algorithm. Let $\{f^*_{ij}\}$ denote the optimal solution to MILP-LF$(f)$ for a given value of $f$. Let $\delta$ be a small number ($\sim 0.01$). Let $\epsilon$ be the maximum allowed gap between $f'$ and $f^*$.

**Algorithm 2.1.5** *Given the physical topology $(N_P, E_P)$ and the logical topology $(N_L, E_L)$, we obtain an $\epsilon$-close optimal solution to the LRMLF problem using the following procedures:*

1. If MILP-LF$(f)$ with $f = 1 - \delta$ is infeasible, then 
   return no survivable routing exists.

2. Set $f_{ib} := \max_{k \in N_L} \frac{L_k}{L_k}$, $f_{ub} := \max_{s \subset N_L} \frac{\sum_{(i, j) \in E_P} CS(s, N_L - s) f^*_{ij} + f^*_{ij}}{|CS(s, N_L - s)|}$

3. Repeat
   - If MILP-LF$(f)$ with $f = \frac{f_{ib} + f_{ub}}{2}$ is feasible, then
     set $f' := \max_{s \subset N_L} \frac{\sum_{(i, j) \in E_P} CS(s, N_L - s) f^*_{ij} + f^*_{ij}}{|CS(s, N_L - s)|}$, $\{f^*_{ij}\}' := \{f^*_{ij}\}$, $f_{ub} = f'$
   - else
     set $f_{ib} := f$

4. Until $(f_{ub} - f_{ib}) \leq \epsilon$

5. Return $f'$ and $\{f^*_{ij}\}'$.

Algorithm 2.1.5 first determines whether or not any survivable routing exists for the given physical and logical topologies (step 1). Note that MLP-LF$(f)$ with $f = 1 - \delta$ is precisely the ILP formulation of the survivable routing problem presented in [14]. If no survivable routing exists, the load factor can only be zero. Otherwise, the algorithm iteratively searches for the routing that minimizes $f$. The optimal value $f^*$ must be bounded between $f_{ib}$ and $f_{ub}$ (step 2). The key to finding the optimal value is to “squeeze” the two bounds as tightly as possible. During each iteration (step 3), we solve an instance of MILP-LF$(f)$ by choosing $f$ as the middle point between $f_{ib}$
and $f_{ub}$. If the chosen $f$ yields a feasible solution, we use Eq. (2.7) to compute $f'$ associated with the routing $\{f_{ij}^{st}\}$. Note that $f'$ will be always be less than $f$. We also lower $f_{ub}$ to $f'$ because any feasible $f'$ is an upper bound on the optimal value $f^*$. If the instance of MILP-LF($f$) for the chosen $f$ is infeasible, we increase $f_{lb}$ to $f$. The gap between the two bounds strictly decreases after every iteration. The algorithm terminates when the gap is less than $\epsilon$ (step 4). At this point, $f'$ is at most $\epsilon$ away from $f^*$, and the corresponding routing is given by $\{f_{ij}^{st}\}'$ (step 5).

2.1.5 Simulation Results

We attempted to embed random 14-node and 10-node logical topologies of degree $k$ ($k = 3, 4, 5...$) on the NSFNET and the 10-node degree 5 physical topologies shown in Figure 2-2, respectively. We define a topology of degree $k$ to be a topology where every node has degree $k$. The NSFNET is a sparse and asymmetric topology with nodes of degrees 2, 3, and 4. On the other hand, the 10-node degree 5 topology is a dense and symmetric topology.

![Figure 2-2: Physical topologies used in simulations.](image)

For each degree $k$, we generated 100 random logical topologies and ran Algorithm 2.1.5 to find the routing that maximizes the load factor. Since we are mainly concerned with the routing, the simulations ignored the wavelength capacity constraint (i.e. we assume no wavelength restriction). Obviously, if needed, the con-
straints can be easily incorporated into the solution.

To illustrate how much impact the routing of the logical topology has on the load factor, we compare the maximum load factor for the routing returned by Algorithm 2.1.5 with the load factor for the routing returned by the survivable routing algorithm (denoted by SR-MN) presented in [14], which does not take load factor into account. We also compare both load factors with the upper bound obtained from Corollary 2.1.4. The upper bound illustrates how the degrees of the nodes in these topologies affect the load factor.

![Graph](graph.png)

Figure 2-3: Average load factor for embedding random 14-node logical topology of degree $k$ on the NSFNET

The results for the NSFNET are shown in Figure 2-3. The three curves in the figure are the result of averaging the load factors over 100 random logical topologies for each degree $k$. The maximum load factors are on average 80% higher than the SR-MN load factors. This result confirms that the routing of the logical topology can significantly affect the load factor. The figure also shows that the upper bound and the maximum load factor are almost identical. The upper bound seems to be a good
estimator for the maximum load factor without explicitly computing the routing.

The figure shows that the maximum load factor never exceeds 0.5. The reason is that the physical topology is inherently limiting the load factor; the NSFNET (see Figure 2-2(a)) has two nodes of degree 2. The upper bound on the load factor is, therefore, given by $1 - \frac{\lceil \frac{1}{2} \rceil}{k} \leq 0.5$. Intuitively, if either one of the two physical links incident to the node of degree 2 fails, all of the disrupted traffic must be diverted onto the other physical link. Thus for all logical links routed through the node, at most 50% of the capacity can be used to carry traffic.

It is also interesting to see that the maximum load factors for logical topologies of odd degree $k$ dip below 0.5. Again, this follows directly from the upper bound equation. When $k$ is odd, the ceiling function in $\lceil \frac{k}{2} \rceil$ becomes active; the upper bound reduces to $1 - \frac{k/2+0.5}{k} = 0.5 - \frac{1}{2k}$. For small odd values of $k$, the fraction $\frac{1}{2k}$ is relatively large. For large odd values of $k$, the fraction diminishes. The simulation results do indeed agree with this analysis. The load factor approaches closer to 0.5 as $k$ (odd) increases.

The results for the 10-node degree 5 physical topology are shown in Figure 2-4. As expected the maximum load factors are much higher than the SR-MN load factors. Because the underlying physical topology is highly connected, the maximum load factors are much higher than those for the NSFNET. For comparison, we averaged the maximum load factors over all degrees for both physical topologies. The average maximum load factor is 0.7 for the 10-node degree 5 physical topology, while it is only 0.43 for the NSFNET. Using the 10-node degree 5 physical topology, 70% of the capacity on every logical link is guaranteed working capacity, while only 43% of the capacity on every logical is guaranteed working capacity using the NSFNET.

There are several interesting phenomenon in Figure 2-4. First, the maximum load factor takes a dip when $k = 6$. This is exactly the point where the logical topology becomes denser than the physical topology. For $k \leq 5$, the upper bound is given by $1 - 1/k$. For $k > 5$, the upper bound is given by $1 - \frac{\lceil \frac{k}{2} \rceil}{k}$. According to these equations, the upper bound for $k = 6$ is indeed lower than that for $k = 5$. Furthermore, for $k > 5$, the upper bound approaches 0.8. As shown in the figure, the maximum load factor
Figure 2-4: Average load factor for embedding random 10-node logical topology of degree $k$ on the 10-node degree 5 physical topology

indeed approaches 0.8 as $k$ increases. This result follows from our earlier analysis that the physical topology limits the load factor as the logical topology becomes denser.

Another interesting phenomenon is that the maximum load factor is identical to the upper bound for $k > 5$, however, it is consistently below the upper bound for $k \leq 5$. Recall that the routing that maximizes the load factor minimizes $f = \max_{S \subseteq N_L} \frac{\sum_{(i,j) \in E_P} \frac{f_{ij}^L + f_{ij}^P}{|CS(S,N_L - S)|}}{f_L}$. Also recall that the derivation of the upper bound only included single node cuts. For $k > 5$, the simulation results imply that cut-sets associated with single node cuts have the largest fraction of broken logical links. Intuitively, smallest cut-sets are most vulnerable to failures. However, when $k \leq 5$, one more effect comes into play; all links in the logical topology may be routed disjointly on the physical topology of degree 5. The maximum load factor can achieve the upper bound, $1 - 1/k$, if all of the logical links are routed using disjoint physical links. When deriving the upper bound, we assumed that if the degree of the logical node is less than or equal to the degree of the corresponding physical node, each
incident logical link can be routed on a distinct incident physical link. For any given logical and physical topologies, complete disjoint routing is not always possible especially when the logical topology is randomly generated. Thus the maximum load factor cannot meet the upper bound.

2.1.6 Conclusion

We showed that the load factor, the minimum fraction of working capacity on each logical link, is an important criterion for routing the logical topology. Assume each logical link has capacity $C$. Given a routing with load factor $\alpha$, network survivability is guaranteed if the traffic carried on every logical link does not exceed $\alpha C$. This implies if the working capacity on every logical link is less than or equal to $\alpha C$, the spare capacity allocated on every logical link is sufficient to protect all of the working capacities. Intuitively, the load factor measures the disjointness of the routing. A large load factor implies that the logical topology will always remain well-connected after any physical link failure. We presented an iterative algorithm based on the MILP formulation to find a routing that maximizes the load factor. Given a physical topology and a logical topology, our results showed that the load factor can be increased significantly if Algorithm 2.1.5 is used. However, the degrees of nodes in the physical and logical topologies ultimately limit the load factor.

2.2 Lightpath Routing and Spare Capacity Assignment

In this section, we investigate the design of a survivable network including lightpath routing and spare capacity assignment from a slightly different perspective. Here we assume that in addition to the logical topology, we are also given the working capacity $\beta^{st}$ associated with each logical link, corresponding to the amount of working traffic that must be carried on each lightpath. Furthermore, we assume that $\beta^{st}$ is strictly greater than zero for every logical link $(s, t)$. The goal is to further route the logical
topology and assign spare capacity to each logical link in a manner that minimizes the total spare capacity required for network survivability. The motivation is to use the minimum amount of total spare capacity to protect all of the working capacity requirements. We call this the lightpath routing and spare capacity assignment (LRSCA) problem. We present a heuristic algorithm to solve this problem.

2.2.1 Lower Bound on the Total Spare Capacity

We first establish a lower bound on the total spare capacity required for network survivability for a given physical topology and logical topology. The lower bound is derived by looking at the spare capacity requirement for each node independently. There are two cases to consider for each node \( k \) depending on the degrees of the corresponding physical node and logical node, denoted by \( P_k \) and \( L_k \).

We first study the case when \( L_k > P_k \). In Figure 2-5(a), each logical link \((k, t)\) incident to node \( k \) is labeled with \( \beta^{kt} \) and \( \mu^{kt} \), which denote the working capacity and spare capacity on the logical link, respectively. Figure 2-5(b) shows the corresponding physical topology for node \( k \). Each physical link \((k, j)\) incident to node \( k \) is labeled with \( x_{kj} \) and \( y_{kj} \), which denote the working capacity and spare capacity on the physical link, respectively. The capacity on the physical links results from the capacity of the logical links that are routed on them. When a physical link fails, the lost working traffic must be diverted to the remaining physical links, which must have enough spare capacity to support the additional traffic. For ease of explanation, we derive

Figure 2-5: Spare capacity requirement for node \( k \) when \( L_k \geq P_k \)
the lower bound on the spare capacity required for the example in Figure 2-5. We then generalize the result.

In this example, if physical link \((k, 1)\) fails, a necessary condition on the amount of spare capacity on physical links \((k, 2)\) and \((k, 3)\) is: \(y_{k2} + y_{k3}\) should be at least \(x_{k1}\). Associated with each physical link failure is a corresponding inequality. Thus we express the capacity criteria on the physical links as follows:

\[
\begin{align*}
x_{k1} & \leq y_{k2} + y_{k3} \\
x_{k2} & \leq y_{k1} + y_{k3} \\
x_{k3} & \leq y_{k1} + y_{k2}.
\end{align*}
\] (2.8)

Summing the inequalities above yields

\[
\frac{x_{k1} + x_{k2} + x_{k3}}{2} \leq y_{k1} + y_{k2} + y_{k3}.
\] (2.10)

Note that independent of the routing, the total working capacity on all physical links must equal the total working capacity on all logical links. Similarly, the total spare capacity on all physical links must equal the total working capacity on all logical links. These two conditions can be expressed as follows:

\[
\begin{align*}
x_{k1} + x_{k2} + x_{k3} & = \beta^{k1} + \beta^{k2} + \beta^{k3} + \beta^{k4} \\
y_{k1} + y_{k2} + y_{k3} & = \mu^{k1} + \mu^{k2} + \mu^{k3} + \mu^{k4}.
\end{align*}
\] (2.11)

Combining Equations (2.10) and (2.11) yields

\[
\frac{\beta^{k1} + \beta^{k2} + \beta^{k3} + \beta^{k4}}{2} \leq \mu^{k1} + \mu^{k2} + \mu^{k3} + \mu^{k4}.
\] (2.12)

Thus in this example, the total spare capacity on all logical links incident to node \(k\) must be at least half of the total working capacity on all incident logical links. For ease of explanation, let \(W\) denote the total working capacity and \(S\) denote the total spare capacity. Note that independent of the routing, in one of the three possible single
physical link failure scenarios, at least one third of the total capacity, or $\frac{1}{3}(W + S)$, will be lost. This implies at most $\frac{2}{3}(W + S)$ amount of total capacity remains to support $W$ amount of working traffic. In other words, $\frac{2}{3}(W + S) \geq W$, which yields $S \geq \frac{W}{2}$.

In Equations (2.8), there is a capacity criterion associated with each physical link. The number of equations in (2.8) equals the number of physical links incident to node $k$. Thus the denominator in the fraction on the lhs. of Eq.(2.12) is related to the degree of the physical node $k$. When $L_k > P_k$, the spare capacity requirement for node $k$ can be expressed as the following inequality:

$$\sum_{l \text{ s.t.} (k,l) \in E_L} \beta^{kl} \leq \sum_{l \text{ s.t.} (k,l) \in E_L} \mu^{kl}. \quad (2.13)$$

We now examine the case when $L_k < P_k$ shown in Figure 2-6. Each of the two logical links incident to node $k$ can be routed on a distinct incident physical link. Thus one of the physical links does not have any working capacity and spare capacity. In this scenario, we can directly express the capacity criteria on the logical links incident to node $k$ as follows:

$$\beta^{k1} \leq \mu^{k2}$$

$$\beta^{k2} \leq \mu^{k1}. \quad (2.14)$$
Summing the equations above yields

$$\beta^{k1} + \beta^{k2} \leq \mu^{k1} + \mu^{k2}.$$ (2.15)

Generalizing these constraints for any node $k$ with $L_k < P_k$ yields

$$\frac{\sum_{l \text{ s.t.}(k,l) \in E_L} \beta^{kl}}{L_k - 1} \leq \sum_{l \text{ s.t.}(k,l) \in E_L} \mu^{kl}. \quad (2.16)$$

Equations (2.13) and (2.16) must hold for every node $k$ in $N_L$. Combining the two equations and summing up over all nodes in $N_L$ gives

$$\sum_{k \in N_L} \frac{\sum_{l \text{ s.t.}(k,l) \in E_L} \beta^{kl}}{\min(P_k, L_k) - 1} \leq \sum_{k \in N_L} \sum_{l \text{ s.t.}(k,l) \in E_L} \mu^{kl} = \sum_{(s,t) \in E_L} \mu^{st}. \quad (2.17)$$

The lower bound is a function of $P_k$, $L_k$, and $\beta^{st}$. Thus the total spare capacity requirement is directly dependent on the degrees of the nodes in the physical and logical topologies. Let $W = \sum_{(s,t) \in E_L} \beta^{st}$ and $S = \sum_{(s,t) \in E_L} \mu^{st}$ denote the total working capacity and total spare capacity, respectively. We examine the lower bound in the following three special cases:

1: If $L_k = P_k = D$, $\forall k \in N_L$,
the lower bound corresponds to $S \geq \frac{\sum_{k \in N_L} \sum_{l \text{ s.t.}(k,l) \in E_L} \beta^{kl}}{D - 1} = \frac{W}{D - 1}$. If the physical and logical topologies are identical and regular (i.e. all nodes have the same degree), the condition above is satisfied and the lower bound is equivalent to that presented in [10]. Furthermore, if $D = 2$, $S \geq W$. This agrees with the spare capacity requirement for the ring topology, which requires 100% capacity redundancy for protection.

2: If $L_k \leq P_k$, $\forall k \in N_L$,
the lower bound corresponds to $S \geq \sum_{k \in N_L} \frac{\sum_{l \text{ s.t.}(k,l) \in E_L} \beta^{kl}}{L_k - 1}$.

3: If $L_k \geq P_k$, $\forall k \in N_L$,
the lower bound corresponds to $S \geq \sum_{k \in N_L} \frac{\sum_{l \text{ s.t.}(k,l) \in E_L} \beta^{kl}}{P_k - 1}$.
In cases 2 and 3, the sparser of the two topologies determines the lower bound on the total spare capacity. Unlike the load factor, the degree of every node influences the spare capacity requirement, not just the minimum degree. Note this is because the upper bound on the load factor is determined by the worst case node.

2.2.2 Criteria for Spare Capacity Assignment

We develop some important conditions required for network survivability on the routing of the logical topology and the corresponding spare capacity assignment. We first establish a necessary condition required for network survivability for the routing denoted by variables \( f_{ij}^{st} \) and the corresponding spare capacity assignment denoted by variables \( \mu_{ij}^{st} \).

**Lemma 2.2.1** Given the working capacity on every logical link \((s, t)\) is \( \beta_{ij}^{st} \), the routing of the logical topology and the corresponding spare capacity assignment must satisfy

\[
\sum_{(s,t) \in CS(S, NL - S)} \left( f_{ij}^{st} + f_{ji}^{st} \right) \beta_{ij}^{st} \leq \sum_{(s,t) \in CS(S, NL - S)} (1 - (f_{ij}^{st} + f_{ji}^{st})) \mu_{ij}^{st},
\]

\[\forall S \subseteq NL, (i, j) \in E_p.\]  

**Proof** The above condition requires the amount of working capacity lost on the broken logical links in the cut-set due to a physical link failure to be less than or equal to the amount of spare capacity on the remaining logical links in the cut-set. This condition must hold for every cut-set of the logical topology and every single physical link failure. This follows directly from the maximum-flow minimum-cut theorem. 

We say that a spare capacity assignment is feasible if Lemma 2.2.1 is satisfied for some routing. We now establish an important requirement on the routing with the following theorem.

**Theorem 2.2.2** A feasible spare capacity assignment always exists if the routing of logical topology is survivable.
Proof If the routing of the logical topology is survivable, every node in the logical topology is still connected after any physical link failure. Thus all of the disrupted traffic can be rerouted as long as sufficient amount of spare capacity is assigned to each logical link. The condition in Lemma 2.2.1 is satisfied by definition.

Theorem 2.2.2 implies that if no survivable routing exists for the given topologies or the routing is not survivable, a feasible spare capacity assignment may not exist.

2.2.3 Heuristic Approach using Decomposition

We attempted to solve the LRSCA problem by formulating it as one entire MILP (see Appendix A). However, the CPLEX solver is not able to solve the problem for any nontrivial networks. In this section, we develop a heuristic to solve the problem.

Theorem 2.2.2 states that if a survivable routing is found, a feasible spare capacity assignment always exists. Therefore, we decompose the LRSCA problem into two sub-problems: routing and spare capacity assignment, where each sub-problem can be solved quickly. We first find a survivable routing. Based on this routing, we make a spare capacity assignment that minimizes total spare capacity. The main drawback with decomposition is that the solution is sub-optimal. The routing of the logical topology may significantly affect the spare capacity assignment. The key to overcome the drawback is to find a “good” survivable routing that can give us a “good” spare capacity assignment. We first show how to obtain the spare capacity assignment that minimizes the total spare capacity given a survivable routing.

Given the survivable routing, the goal is to find a corresponding spare capacity assignment that minimizes the total spare capacity. The problem can be formulated as the following linear program (LP), which can be solved by the simplex method [27].

\[
\text{minimize } \sum_{(s,t) \in E_L} \mu_{st}
\]

Subject to:
1. Spare capacity constraints:
\[
\sum_{(s,t) \in CS(S,N_L-S)} (f_{st}^t + f_{ji}^t) \beta_{st}^t \leq \sum_{(s,t) \in CS(S,N_L-S)} \{1 - (f_{st}^t + f_{ji}^t)\} \mu_{st}^t, \\
\forall S \subseteq N_L, (i, j) \in E'.
\]

2. Nonnegativity constraints: \( \mu_{st}^t \geq 0 \quad \forall (s, t) \in E_L. \)

Let LP-SCA denote the above LP. The objective is to minimize the total spare capacity. Note that the spare capacity constraints are precisely the condition given by Lemma 2.2.1. Since the given routing is survivable, the LP must be feasible according to Theorem 2.2.2. Furthermore, the spare capacity assignment is bounded by the nonnegativity constraints. Thus an optimal solution always exists.

We now show how the survivable routing can implicitly provide an upper bound on the optimal total spare capacity for the corresponding LP-SCA problem. Consider a cut-set \( CS(S, N_L-S) \), \( \frac{\sum_{(s,t) \in CS(S,N_L-S)} (f_{st}^t + f_{ji}^t) \beta_{st}^t}{\sum_{(s,t) \in CS(S,N_L-S)} \beta_{st}^t} \) is the fraction of the working capacity lost in the cut-set due to physical link failure \((i, j)\) over the total working capacity in the cut-set. Given any survivable routing, if the fraction is bounded by \( f \) for all cut-sets and all single physical link failures, the following theorem provides an upper bound on the fraction of total spare capacity over the total capacity required in the network. For convenience, let \( S^* \) be the optimal total spare capacity to the LP-SCA problem corresponding to the given routing. Let \( W = \sum_{(s,t) \in E_L} \beta_{st}^t \), which is the total working capacity.

**Theorem 2.2.3** Given a quantity \( f \), where \( 0 \leq f < 1 \), if the survivable routing satisfies the following condition:
\[
\frac{\sum_{(s,t) \in CS(S,N_L-S)} (f_{st}^t + f_{ji}^t) \beta_{st}^t}{\sum_{(s,t) \in CS(S,N_L-S)} \beta_{st}^t} \leq f, \quad \forall S \subseteq N_L, (i, j) \in E', \tag{2.19}
\]
then the fraction of the optimal total spare capacity over total capacity is bounded by
\[
\frac{S^*}{S^* + W} \leq f. \tag{2.20}
\]
Equivalently, the optimal total spare capacity is bounded by

\[ S^* \leq \frac{fW}{1 - f}. \]  

Proof We use duality theory to prove the theorem [27]. To facilitate the proof, we convert Eq. (2.19) and LP-SCA to matrix forms. Let \( M \) be the number of inequalities in Equations (2.19), i.e. the product of the number of cut-sets and the number of bidirectional physical links. Note that \( M \) also equals the number of the spare capacity constraints in LP-SCA. Let \( N \) equal the size of the set \( E_L \), i.e. \( |E_L| \). Let \( \beta \) be a \( N \times 1 \) vector, where each entry denotes \( \beta_s \). Let \( \mu \) be a vector, where each entry denotes \( \mu_s \). Let \( 1 \) be a \( N \times 1 \) vector, where each entry is 1. Let \( Q \) and \( C \) be a \( M \times N \) matrix. Because the proof does not require knowledge about the exact structure of \( Q \) and \( C \), we assume these two matrices can always be constructed. Therefore, we rewrite Eq. (2.19) as \( C\beta \leq f Q\beta \). Similarly, we express LP-SCA in the following matrix form:

\[
\begin{align*}
\text{minimize} & \quad 1'\mu \\
\text{subject to} & \quad (Q - C)\mu \geq C\beta \\
& \quad \mu \geq 0.
\end{align*}
\]

The LP problem above is called the primal problem. Using duality theory, the corresponding dual problem is as follows:

\[
\begin{align*}
\text{maximize} & \quad p'C\beta \\
\text{subject to} & \quad p'(Q - C) \leq 1' \\
& \quad p \geq 0,
\end{align*}
\]

where \( p \) is the \( M \times 1 \) vector of dual variables corresponding to the \( M \) spare capacity constraints in the primal problem. The primal problem must have an optimal solution \( \mu^* \) because the given routing is survivable. Using strong duality, the dual problem must also have an optimal solution \( p^* \), and the respective optimal costs are equal [27].
In other words, $1' \mu^* = p'' C \beta$. Consider the dual constraint

$$p''(Q - C) \leq 1'$$

(2.24)

at the optimal solution $p^*$. The matrix $Q - C$ only contains nonnegative entries in the form of $1 - (f_{ij}^{st} + f_{ji}^{st})$. Since $p''(Q - C)$ and $1$ are nonnegative vectors, multiplying both sides of Eq. (2.24) by $\beta'$ preserves the inequality

$$p''(Q - C)\beta' \leq 1'\beta'$$

(2.25)

$$p''Q\beta' \leq p''C\beta + 1\beta'.$$

(2.26)

Since we are given $C\beta \leq f Q\beta$, multiplying both sides by $p^*$ preserves the inequality because $p^* \geq 0$. Combining $p''C\beta \leq f p''Q\beta$ with Eq. (2.26) yields

$$\frac{p''C\beta'}{f} \leq p''Q\beta' \leq p''C\beta + 1\beta'$$

$$\frac{p''C\beta'}{p''C\beta + 1\beta'} \leq f.$$

By strong duality, $1' \mu^* = p'' C \beta$. Finally, we have

$$\frac{1' \mu^*}{1' \mu^* + 1' \beta} \leq f$$

$$\frac{S^*}{S^* + W} \leq f.$$

The theorem above states that if the survivable routing satisfies Eq. (2.19) for a given $f$, $\frac{fW}{1 - f}$ gives an upper bound on the total spare capacity required for network survivability. Because we assumed $\tilde{\beta}^{st} > 0$ for every logical link $(s, t)$, $f$ must be strictly less than 1. Otherwise, there exists at least one cut-set in which all of the working capacity is lost after some physical link failure. This implies that all of the logical links in the cut-set are lost, and therefore contradicting the assumption that the routing is survivable.

For a given routing, the upper bound can be tighter by choosing $f$ to be the
minimum value that satisfies Eq. (2.19) using the following equation:

\[
f = \max_{(i,j) \in E_P} \frac{\sum_{(s,t) \in CS(S,N_L-S)} (f_{ij}^{st} + f_{ji}^{st}) \beta_{st}}{\sum_{(s,t) \in CS(S,N_L-S)} \beta_{st}}. \tag{2.27}
\]

We call \( f \) the spare factor associated with the corresponding routing. For a given routing, the spare factor is the maximum fraction of working capacity lost in the cut-set due to a physical link failure over all cut-sets and all single physical link failures. The smaller the spare factor \( f \), the tighter the upper bound, \( \frac{fW}{1-f} \), is on the minimum total spare capacity for the corresponding routing. Thus the spare factor can be used as a criterion for routing the logical topology. We conjecture that a routing with a smaller spare factor requires less total spare capacity than a routing with a larger spare factor. Consequently, we want to find a routing that minimizes the spare factor. Let \( \{f_{ij}^{st}\}^* \) denote such a routing given by

\[
\{f_{ij}^{st}\}^* = \arg\min_{\{f_{ij}^{st}\} \in \mathbb{R}} \max_{(i,j) \in E_P} \frac{\sum_{(s,t) \in CS(S,N_L-S)} (f_{ij}^{st} + f_{ji}^{st}) \beta_{st}}{\sum_{(s,t) \in CS(S,N_L-S)} \beta_{st}}. \tag{2.28}
\]

As mentioned earlier, \( \frac{\sum_{(s,t) \in CS(S,N_L-S)} (f_{ij}^{st} + f_{ji}^{st}) \beta_{st}}{\sum_{(s,t) \in CS(S,N_L-S)} \beta_{st}} \) is the fraction of the working capacity lost in the cut-set due to a physical link failure over the original working capacity in the cut-set. Minimizing the spare factor requires minimizing the maximum of such fractions over all cut-sets and all single physical link failure scenarios. Intuitively, to reduce the fraction of working capacity lost in the cut-set, logical links with large working capacity should be routed on disjoint physical links. This implies that the routing should evenly distribute working capacities among physical links. Note that when \( \beta^{st} = \beta \) for every logical link \( (s, t) \), finding a routing that minimizes the spare factor corresponds to finding a routing that maximizes load factor. In this scenario, the minimum spare factor \( f^* \) is related to the maximum load factor \( \alpha^* \) by: \( f^* = 1 - \alpha^* \). Since the working capacity on every logical is the same, it only matters that the logical links are evenly routed on the physical links.

The problem of finding a routing that minimizes the spare factor has the exact
same structure as the LRMLF problem. Equation (2.28) directly translates to the following objective function:

\[
\max_{(i,j) \in E_p} \sum_{(s,t) \in CS(s, N_L - S)} \frac{(f_{ij}^{st} + f_{ji}^{st}) \beta_{st}}{\sum_{(s,t) \in CS(s, N_L - S)} \beta_{st}}, \quad \forall S \subset N_L, \forall (i, j) \in E_p'.
\] (2.29)

The objective function above is piecewise linear and convex. Using the same technique as before, we formulate this routing problem as the following equivalent MILP:

\[\text{minimize} \quad f\]

Subject to:

1. Spare factor constraints:

\[f \geq \sum_{(s,t) \in CS(s, N_L - S)} \frac{(f_{ij}^{st} + f_{ji}^{st}) \beta_{st}}{\sum_{(s,t) \in CS(s, N_L - S)} \beta_{st}}, \quad \forall S \subset N_L, \forall (i, j) \in E_p'.\]

2. Connectivity constraints:

\[
\sum_{j.t.(i, j) \in E_p} f_{ij}^{st} - \sum_{j.s.(j, i) \in E_p} f_{ji}^{st} = \begin{cases} 
1, & \text{if } s=i \\
-1, & \text{if } t=i \\
0, & \text{otherwise}
\end{cases}, \quad \forall i \in N_p, \forall (s, t) \in E_L.
\]

3. Wavelength capacity constraints:

\[
\sum_{(s,t) \in E_L} f_{ij}^{st} \leq W, \quad \forall (i, j) \in E_p.
\]

4. Integer flow constraints: \(f_{ij}^{st} \in \{0, 1\}\).

Note that constraints 2-4 are exactly the same as those for the LRMLF problem. We solve the MILP using the same techniques used for solving the LRMLF problem. We change the objective function to “\(\text{minimize} \sum_{(s,t) \in E_L} f_{ij}^{st}\),” and find a feasible solution for a given value of \(f\). Let MILP-SF(f) denote the modified MILP. We obtain the minimum value of \(f\) for which MILP-SF(f) generates a feasible solution. Using
Figure 2-7: Physical topology and summary of working capacity generation.

Given the routing that minimizes the spare factor, we can solve for LP-SCA to obtain the minimum spare capacity requirement for the corresponding routing.

2.2.4 Simulation Results

In our simulations, we used the 10-node degree 5 topology shown in Figure 2-2(b) and the Sprint OC-48 network shown in Figure 2-7(a) as the underlying physical topologies. The Sprint OC-48 network is a sparse 12-node 18-link topology with nodes of degrees 2, 3, 4, and 5. Thus we tested our heuristic performance under two vastly different physical topologies. We generated 50 random 10-node and 12-node logical topologies of degree $k$ for the 10-node degree 5 topology and the Sprint OC-48 network, respectively. Associated with each random logical topology, we generated two sets of working capacities, uniform and random. For the uniform set, the working capacity on every logical link is 1. For the random set, the working capacity on every logical link is a random real number between 1 and 5 inclusively.

Given a physical topology, a logical topology, and a set of working capacities, we compute the spare capacity requirement corresponding to the routing obtained under each of the following three routing strategies.

1. SR-MN: Survivable routing algorithm presented in [14].
2. SR-MLF: Survivable routing that maximizes the load factor.

3. SR-MSF: Survivable routing that minimizes the spare factor.

If a survivable routing exists for the given topologies, each routing strategy always returns a survivable routing. For each routing, we solve LP-SCA to determine the spare capacity requirement. As mentioned earlier, the SR-MSF and SR-MLF routing strategies are identical when the working capacity on each logical link is the same. For the uniform working capacity case, we only determine spare capacity requirements under the SR-MN and SR-MSF routing strategies. For comparison convenience, we normalize the spare capacity requirement by expressing it in terms of network redundancy, which we define as the ratio of total spare capacity to total capacity. This normalization also allows us to directly compare network redundancy with the spare factor. Since we do not have a mechanism to compute the optimal spare capacity requirement for the given topologies and working capacities, we compare the spare capacity requirement under each routing strategy with the lower bound derived in Section 2.2.1.

The average network redundancy required to embed 50 random logical topologies with uniform working capacity on the 10-node degree 5 physical topology is shown in Figure 2-8. The figure shows that for degrees less than or equal to 5, network redundancy for SR-MSF is about 0.14 less than that for SR-MN. For degrees greater than 5, the difference is about 0.05. SR-MSF routing strategy clearly outperforms the SR-MN in terms of the spare capacity requirement. Network redundancy for SR-MSF also closely follows the minimum spare factor, differing by no more than 0.04. Furthermore, network redundancy for SR-MSF comes within 0.05 of the lower bound except for degree 6. This suggests that network redundancy for SR-MSF cannot be far away from the optimal network redundancy.

In general, network redundancy should decrease as the degree of the logical topology increases because a dense logical topology always remains well-connected after any physical link failure. The spare capacity can be shared by many diverse backup routes, thus spare capacity requirement is reduced. However, network redundancy
Figure 2-8: Average network redundancy for embedding random 10-node logical topologies of degree $k$ with uniform working capacity on the 10-node degree 5 physical topology for SR-MSF peaks at degree 6 after decreasing initially. The reason is that under uniform working capacity, SR-MSF reduces to finding a routing that maximizes the load factor. As mentioned in Section 2.1.5, the reason that the maximum load factor dips at degree 6 for the 10-node degree 5 physical topology is that logical topology of degree 6 cannot be evenly routed on the physical topology of degree 5. For the exact same reason, the minimum spare factor peaks at degree 6. Since the network redundancy for SR-MSF closely follows the trend of the minimum spare factor, it also peaks at degree 6. Intuitively, for each node, 6 incident logical links have to be routed on only 5 incident physical links. Thus one physical link must carry 2 logical links. Additional spare capacity must be dedicated to protect that physical link from failure. As a result, network redundancy increases. As the degree of the logical topology continues to increase, the additional spare capacity can be shared more evenly. As shown in the figure, network redundancy for SR-MSF indeed decreases at degree 7.
Figure 2-9: Average network redundancy for embedding random 10-node logical topologies of degree $k$ with random working capacity on the 10-node degree 5 physical topology.

We now discuss the average results for embedding random topologies with random working capacity on the 10-node degree 5 physical topology. In this case, all three routing strategies are considered. Figure 2-9 shows that both SR-MLF and SR-MSF consistently outperform SR-MN by a significant amount. It is interesting to see that SR-MLF requires slightly less network redundancy than SR-MSF for degrees less than 6. This suggests that when the physical topology is denser than or just as dense as the logical topology, it is important to route the logical links on a more disjoint set of physical links. As long as the routing is relatively disjoint, only one logical link will fail under most physical link failure scenarios, thus the logical topology always remain well connected. Even if the working capacity on the failed logical link is large, the lost working capacity is protected by many diverse backup routes. When the degree of the logical topology is greater than 5, SR-MSF outperforms SR-MLF by a reasonable margin because it incorporates working capacity into routing. SR-MSF
routes the logical topology in a manner that distributes the working capacities evenly among the physical links. When any physical link fails, the lost working capacity will not be large. For example, for degree 6, 6 logical links incident must be routed on 5 physical links. SR-MSF will not route the two logical links with large working capacity on the same physical link. On the other hand, SR-MLF indiscriminately routes any two logical links on the same physical link because it essentially assumes the working capacities on all links are equal. Figure 2-9 again shows that the SR-MSF curve closely follows the trend for the minimum spare factor. Because the working capacity is random, network redundancy for SR-MSF decreases steadily as the degree of the logical topology increases, whereas SR-MLF is still plagued by the problem mentioned above for the logical topology of degree 6.

![Network Redundancy Graph](image)

Figure 2-10: Average network redundancy for embedding random 12-node logical topologies of degree $k$ with uniform working capacity on the Sprint OC-48 network.

Figure 2-10 shows results of embedding random logical topologies with uniform working capacity on the Sprint OC-48 network. As expected, network redundancy for SR-MSF is lower than that for SR-MN. One major difference from the previous
results is that the SR-MSF curve does not follow the minimum spare factor curve. The reason is that the underlying physical topology, Sprint OC-48 network (see Figure 2-7(a)), has 3 nodes of degree 2, which immediately give an upper bound of 0.5 on the load factor, or equivalently a lower bound of 0.5 on the spare factor. Recall from the previous section, for logical topologies of odd degree, the upper bound on the load factor is even lower than 0.5, since more than half the logical links have to be routed on one physical link. The figure indeed shows that all of the minimum spare factors are above 0.5. For \( k = 5 \) and \( k = 7 \), network redundancy requirement can be much lower than the minimum spare factor because the logical topology is highly meshed compared to the physical topology. Similar to the NSFNET, embedding random logical topologies on the Sprint OC-48 network requires high network redundancy due to the degree 2 nodes. We see that even the lower bound on network redundancy is around 0.4.

Figure 2-11: Average network redundancy for embedding random 12-node logical topologies of degree \( k \) with random working capacity on the Sprint OC-48 network

Figure 2-11 shows results of embedding random logical topologies with random
working capacity. The Sprint OC-48 network consists of 18 bidirectional links, while the randomly generated logical topologies consist at least 18 bidirectional links. The figure shows that the SR-MSF curve is always below the SR-MLF curve. This result confirms the earlier analysis that SR-MSF outperforms SR-MLF when the logical physical is denser than the physical topology. With random working capacity, the minimum spare factors form a smoother curve, which is always above 0.5 because of the degree 2 nodes in the physical topology.

The simulation results in each of the 4 figures show that the SR-MSF and SR-MLF are much better routing strategies than SR-MN for reducing network redundancy. The SR-MLF routing strategy developed in Section 2.1 performs well considering it does not incorporate working capacity into routing. However, when the logical topology is denser than the physical topology, network redundancy can further be reduced if the SR-MSF routing strategy is used.

### 2.2.5 Conclusion

Given the set of working capacities associated with the logical topology, survivable routing ensures that the corresponding spare capacity assignment can always achieve network survivability. We showed that the routing can implicitly provide an upper bound, the spare factor, on the network redundancy requirement. By routing the logical topology in a manner that minimizes the spare factor, we can significantly reduce the total spare capacity required for network survivability. Such a routing strategy evenly distributes working capacities among physical links so that only small amount of working capacity is lost after any physical link failure. The results also showed that the routing that maximizes the load factor is a good routing strategy for reducing spare capacity requirement even without incorporating working capacities into routing. This routing strategy is ideal when the working capacity requirements may change or be unknown.
Chapter 3

Lightpath Routing and Joint Capacity Assignment

In the previous chapter, we formulated routing and spare capacity criteria for network survivability based on the cut-sets concept. In this chapter, we present an alternative flow-based approach to determine capacity requirement for network survivability [10]. Here we assume that in addition to the physical and logical topologies, we are also given a set of traffic demands. Our goal is to route the traffic on the logical topology, route the logical topology on the physical topology, and assign working and spare capacities to each logical link in a manner that network survivability is achieved and total capacity required is minimized. We call this the lightpath routing and joint capacity assignment (LRJCA) problem. Minimizing the total capacity ensures that the network can be designed using the minimal amount of resources. Other possible objectives may include minimizing the load on each logical link so the congestion on the link can be reduced. Our formulation allows the flexibility of easily changing the objective. In this chapter, we are mainly concerned with investigating the minimum capacity requirement for network survivability.

In an IP-over-WDM network, a single physical link failure may result in the failures of multiple logical links, which leads to disrupted traffic demands. We consider and compare two options for restoring the lost traffic demands: link restoration and end-to-end restoration. In link restoration, all of the traffic carried on each broken logical
link is rerouted on the same backup route. In end-to-end restoration, each disrupted traffic demand is individually rerouted. We will study capacity requirements under both rerouting strategies.

### 3.1 Motivation for Flow-based Approach

In the cut-set approach, the survivable routing guarantees the existence of backup routes between the source and destination of traffic after any physical link failure. The sufficiency of spare capacity is achieved if for every cut-set, the amount of spare capacity remaining in the cut-set after the failure is greater than or equal to the amount of working capacity lost in the cut-set. In the flow-based approach, each disrupted traffic is considered a flow that must find its way from its source to its destination. End-to-end backup routes must exist and must have enough spare capacity to support the flow. If these two conditions hold for every physical link failure, network survivability is achieved in the flow-based approach.

Solving the LRJCA problem using the flow-based approach is preferable to the cut-set approach because it simultaneously computes both the backup routes and the capacity assignments. The cut-set approach only determines the optimal capacity requirements. Backup routes for each traffic demand must be computed explicitly and subsequently. Moreover, solving the spare capacity assignment problem for a given set of working capacity requirements using the cut-set approach is complex because the number of cut-sets grows exponentially with the size of the network. The exact same problem can be solved using the flow-based approach with the number of constraints growing as a polynomial with the size of the network. Lastly, the cut-set approach can only determine the minimum capacity requirement under link restoration, while the flow-based approach can determine the minimum capacity requirements under both link restoration and end-to-end restoration.
3.2 Heuristic Approach using Decomposition

In addition to routing the logical topology and assigning spare capacity, we must also jointly route the traffic on the logical topology and allocate working capacity. We attempted to formulate the problem as one complete MILP (see Appendix B). However, the CPLEX solver is not able to solve the problem for any nontrivial networks. Therefore, we need to develop a good heuristic to solve the problem.

The main reason the MILP is hard to solve is that the lightpath routing problem is difficult due to its discrete nature. If the routing of the logical topology is given, the MILP reduces a linear program (LP), which can be easily solved by the simplex method. Thus we again decompose the problem into two sub-problems. We divide the LRJCA problem into 1) lightpath routing and 2) joint traffic routing and capacity assignment (JTRCA). By Theorem 2.2.2, if the routing of the logical topology is survivable, network survivability is guaranteed as long as sufficient spare capacity is assigned. We first obtain a survivable routing and then solve the LP to route the traffic on the logical topology and assign capacity to each logical link. Both link restoration and end-to-end restoration are considered. We first present the LP formulation for the JTRCA problem assuming the survivable routing is given. We then discuss how the lightpath routing problem can be solved to reduce the capacity requirements for the traffic routing and capacity assignment stage.

3.2.1 Joint Traffic Routing and Capacity Assignment Problem

The LP formulation for the JTRCA problem consists of three components: 1) routing the traffic on the logical topology, 2) rerouting the disrupted traffic after a physical link failure, and 3) assigning working and spare capacities to each logical link. The objective is to minimize the total capacity required for network survivability for a given routing of the logical topology.

When a physical link fails, multiple logical links may fail. The electronic layer sees a new logical topology, which only contains links that remain intact after the
failure. The backup routes used to reroute the disrupted traffic must only consist of links that are intact. Let $G_{ij}$ denote the set of logical links that remain intact after physical link $(i, j)$ fails. Recall that $E_L$ is the set of all logical links. Clearly, $G_{ij} \subseteq E_L$, where equality holds if no logical links are routed on physical links $(i, j)$ and $(j, i)$. The routing of the logical topology is denoted by variables $f_{ij}^{st}$. If logical link $(s, t)$ is routed on either physical links $(i, j)$ or $(j, i)$, $f_{ij}^{st} + f_{ji}^{st} = 1$. Since physical link failure is bidirectional, $f_{ij}^{st} + f_{ji}^{st} = 1$ implies that logical link $(s, t)$ will be broken if physical link $(i,j)$ fails. Likewise, $f_{ij}^{st} + f_{ji}^{st} = 0$ implies that logical link $(s, t)$ will remain intact. Given the routing of the logical topology, $\{(s, t) \in E_L : f_{ij}^{st} + f_{ji}^{st} = 0\}$ is precisely the set of logical links that remain intact after physical link $(i,j)$ fails. Thus $G_{ij} = \{(s, t) \in E_L : f_{ij}^{st} + f_{ji}^{st} = 0\}$.

We formulate the routing of the traffic demands as a multicommodity flow problem, where each traffic demand corresponds to a distinct commodity [27]. Let $T = \{(u, v) : u, v \in N_L, u \neq v\}$ denote the set of all possible node pairs in the logical topology. Let $\lambda_{uv}^w$ be the traffic demand for source-destination pair $(u,v)$. For convenience, we refer to traffic demand for source-destination pair $(u,v)$ as traffic demand $(u,v)$. We introduce flow variables $\lambda_{st}^{uv}$ indicating the amount of traffic with source $u$ and destination $v$ that traverse logical link $(s,t)$. Routing traffic demand $(u,v)$ is equivalent to routing $\lambda_{uv}^w$ amount of flow from node $u$ to node $v$. We use the standard multicommodity flow formulation to express the following set of constraints on flow variables $\lambda_{st}^{uv}$ associated with $\lambda_{uv}^w$:

$$\sum_{t \text{ s.t.}(s,t) \in E_L} \lambda_{st}^{uv} - \sum_{t \text{ s.t.}(t,s) \in E_L} \lambda_{ts}^{uv} = \begin{cases} \lambda_{uv}^w, & \text{if } u=s \\ -\lambda_{uv}^w, & \text{if } v=s \\ 0, & \text{otherwise} \end{cases},$$

(3.1)

$\forall s \in N_L, (u,v) \in T$.

The set of constraints above are the flow conservation constraints for routing $\lambda_{uv}^w$ amount of flow from node $u$ to node $v$. Equation (3.1) requires equal amounts of flow due to traffic demand $(u,v)$ to enter and leave each node that is not the source.
or destination of \((u, v)\). Furthermore, we have an exogenous net flow of \(\lambda^u v\) and 
\(-\lambda^v u\) at the source \(u\) and destination \(v\), respectively. To route all of the traffic, we
must find routes for every source-destination pair \((u, v)\) in \(T\). The flow variables \(\lambda^u v_{st}\) implicitly contain the routes for the traffic demands. Let \(R^u v\) be the set of
logical links used to route traffic demand \((u, v)\). For each traffic demand \((u, v)\),
\(R^u v = \{(s, t) \in E_L : \lambda^u v_{st} > 0\} \). We call \(R^u v\) the working routes for traffic demand
\((u, v)\).

Given the routing of the traffic, the working traffic on each logical link is given by
the following set of constraints:

\[
\beta^s t = \sum_{(u, v) \in T} \lambda^u v_{st}, \quad \forall (s, t) \in E_L. \tag{3.2}
\]

The amount of working traffic on each logical link \((s, t)\) is the aggregation of flow from
all the traffic demands traversing the link. This is precisely the minimum amount of
working capacity required on the link.

**Link Restoration**

In link restoration, for each failed logical link \((s, t)\), \(\beta^s t\) is the amount of disrupted
traffic that must be rerouted using backup routes that connect node \(s\) and node \(t\). Given the routing of the logical topology denoted by variables \(f^s t_{ij}\), the amount of
disrupted traffic on logical link \((s, t)\) due to physical link failure \((i, j)\) is denoted by
\(\beta^s t_{ij}\), which equals \((f^s t_{ij} + f^s t_{ij})\beta^s t\). Note that \(\beta^s t_{ij}\) is restricted to either \(\beta^s t\) or 0. Moreover,
the disrupted traffic on each failed logical link must be rerouted only on logical links
in \(G_{ij}\), which is the set of logical links that remain intact after physical link \((i, j)\)
fails. We model the disrupted traffic on each failed logical link as a commodity. We
associate flow variables \(\gamma^s t_{kl}(ij)\) with the commodity \(\beta^s t_{ij}\), where for a physical link
failure \((i, j)\), \(\gamma^s t_{kl}(ij)\) is the amount of rerouted traffic with source \(s\) and destination \(t\)
that traverses logical link \((k, l)\). We use the multicommodity flow formulation to give
the following set of constraints:

\[
\sum_{l \text{ s.t.}(k,l) \in G_{ij}} \gamma_{kl}^{st}(ij) - \sum_{l \text{ s.t.}(l,k) \in G_{ij}} \gamma_{lk}^{st}(ij) = \begin{cases} 
(f_{ij}^s + f_{ji}^t)\beta_{st}, & \text{if } s = k \\
-(f_{ij}^s + f_{ji}^t)\beta_{st}, & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}, \quad (3.3)
\]

\forall k \in N_L, (s,t) \in E_L, (i,j) \in E'_P.

The above equation requires flow be conserved at each node other than the source and destination of the disrupted traffic. To achieve network survivability, all of the disrupted traffic must be rerouted under every physical link failure scenario. Because the routing of the logical topology is survivable, the logical topology is always connected after any physical link failure. Thus backup routes always exist.

The final step is to assign spare capacity to each logical link. Consider the quantity \(\sum_{(s,t) \in E_L} \gamma_{kl}^{st}(ij)\). It is the amount of additional traffic resulting from the disrupted traffic rerouted on logical link \((k,l)\) after physical link \((i,j)\) fails. Let \(\mu_{kl}^{st}\) be the amount of spare capacity required on logical link \((k,l)\). It must satisfy the following set of constraint:

\[
\mu_{kl}^{st} \geq \sum_{(s,t) \in E_L} \gamma_{kl}^{st}(ij), \quad \forall(k,l) \in E_L, (i,j) \in E'_P.
\]

The constraints above require that each logical link \((k,l)\) have enough spare capacity to support the additional traffic under every physical link failure scenario.

**End-to-end Restoration**

To reroute the disrupted traffic under end-to-end restoration, we must first determine the amount of traffic lost by each source-destination pair. We use an example to show how the same source-destination pair can lose different amounts of traffic under different failure scenarios. Figure 3-1 shows that traffic demand \((1,5)\) with 1 unit of traffic is routed on two working routes connecting nodes 1 and 5, where each route carries 0.5 units of traffic. In Figure 3-1(a), source-destination pair \((1,5)\) loses 1 unit
Figure 3-1: An example of how the same source-destination pair can lose different amounts of traffic under different failure scenarios.

of traffic after logical links (1, 2) and (1, 7) both fail from some physical link failure. In Figure 3-1(b), both logical links (7, 6) and (6, 5) are broken, but source-destination pair (1, 5) only loses 0.5 units of traffic.

Determining the traffic lost by each source-destination pair after a physical link failure is nontrivial. We approach the problem from a different perspective by considering the uninterrupted traffic for each source-destination pair after the failure. Consider traffic demand \((u, v)\) and its corresponding working routes \(R_{uv}\). After physical link \((i, j)\) fails, \(G_{ij}\) is the set of logical links that remain intact. Physical link failure \((i, j)\) disrupts traffic demand \((u, v)\) if and only if \(R_{uv} \cap G_{ij} \neq R_{uv}\), which implies that at least one logical link in \(R_{uv}\) is broken. Let \(\beta_{ij}^{uv}\) be the amount of traffic lost by source-destination pair \((u, v)\) after physical link \((i, j)\) fails. The amount of uninterrupted traffic for source-destination pair \((u, v)\) is \(\lambda_{uv} - \beta_{ij}^{uv}\), which we model as a commodity. We introduce flow variables \(\alpha_{ij}^{uv}(ij)\) indicating the amount of traffic for source-destination pair \((u, v)\) that is still carried on logical link \((s, t)\) after physical link \((i, j)\) fails. We use the multicommodity flow formulation to express the following
sets of constraints on variables $\beta_{ij}^{uv}$ and $\alpha_{st}^{uv}(ij)$:

$$\alpha_{st}^{uv}(ij) \leq \lambda_{st}^{uv}, \quad \forall (s,t) \in E_L, (u,v) \in T, (i,j) \in E_p.' \tag{3.5}$$

$$0 \leq \beta_{ij}^{uv} \leq \lambda_{uv}^{uv}, \quad \forall (u,v) \in T, (i,j) \in E_p.' \tag{3.6}$$

$$\sum_{t \text{ s.t.}(s,t) \in G_{ij}} \alpha_{st}^{uv}(ij) - \sum_{t \text{ s.t.}(t,s) \in G_{ij}} \alpha_{ts}^{uv}(ij) = \begin{cases} 
\lambda_{uv}^{uv} - \beta_{ij}^{uv}, & \text{if } u=s \\
-(\lambda_{uv}^{uv} - \beta_{ij}^{st}), & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}, \tag{3.7}$$

$$\forall s \in N_L, (u,v) \in T, (i,j) \in E_p.'$$

Note that variables $\lambda_{st}^{uv}$ are the routing of the traffic demands before the physical link failure. The set of constraints (3.5) forces the uninterrupted portion of each traffic demand stay on the original routing after the failure. The constraints also ensure that the flow on each logical link $(s,t)$ associated with traffic demand $(u,v)$ after the failure cannot not exceed the original flow $\lambda_{st}^{uv}$. In LP literatures, constraints (3.5) are called the forcing constraints [27]. The second set of constraints (3.6) requires the amount of traffic lost by each source-destination pair $(u,v)$ to be bounded between 0 and $\lambda_{uv}^{uv}$. The third set of constraints (3.7) are the flow conservation constraints associated with routing $\lambda_{uv}^{uv} - \beta_{ij}^{uv}$ amount of flow from source $u$ to destination $v$ after physical link $(i,j)$ fails. The amount of traffic lost by each source-destination pair must satisfy all three sets of constraints. One may argue to reroute every traffic demand on the new logical topology after the failure. However, restoration time would be much longer since every traffic demand could be potentially rerouted. Thus rerouting every traffic demand after a physical link failure is not practical.

After a physical link $(i,j)$ fails, we must reroute $\beta_{ij}^{uv}$ amount of traffic for each source-destination pair $(u,v)$ using logical links only in $G_{ij}$. Let $\gamma_{st}^{uv}(ij)$ be the corresponding flow variable indicating the amount of traffic rerouted on logical link $(s,t)$ with source $u$ and destination $v$ after physical link failure $(i,j)$. We use the the following multicommodity flow formulation to express a set of constraints on variables
\( \gamma_{st}(ij) \) associated with commodities \( \beta_{ij}^{uv} \):

\[
\sum_{t \; s.t. (s,t) \in G_{ij}} \gamma_{st}(ij) - \sum_{t \; s.t. (t,s) \in G_{ij}} \gamma_{ts}(ij) = \begin{cases} 
\beta_{ij}^{uv}, & \text{if } u=s \\
-\beta_{ij}^{uv}, & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}
\tag{3.8}
\]

\( \forall s \in N_L, (u, v) \in T, (i, j) \in E'_P. \)

The flow conservation constraints above are very similar to Eq. (3.3) for link restoration. The only difference is that end-to-end restoration individually reroutes the disrupted traffic for every source-destination pair while link restoration jointly reroutes the total disrupted traffic on every failed logical link. The capacity efficiency gained from using end-to-end restoration comes at the cost of complexity.

The final step is to assign spare capacity to each logical link. The total amount of rerouted traffic on logical link \((s, t)\) due to physical link failure \((i, j)\) is \(\sum_{(u, v) \in T} \gamma_{uv}^{st}(ij)\). The spare capacity assigned to each logical link must be sufficient to support the additional traffic after any physical link failure. Therefore, the spare capacity on each logical link must satisfy the following set of constraints:

\[
\mu_{st} \geq \sum_{(u, v) \in T} \gamma_{uv}^{st}(ij), \quad \forall (s, t) \in E_L, (i, j) \in E'_P.
\tag{3.9}
\]

**Complete LP Formulations**

Given the routing of the logical topology on the physical topology, we want to find the minimum capacity requirement for network survivability. Let LP-JTRCA\(_{LR}\) denote the following LP formulation for the JTRCA problem under link restoration:

\[
\text{minimize} \quad \sum_{(s,t) \in E_L} \beta_{st} + \mu_{st}
\]

Subject to:
1. Traffic routing constraints:

\[
\sum_{t \text{ s.t.} (s,t) \in E_L} \lambda_{st}^{uv} - \sum_{t \text{ s.t.} (t,s) \in E_L} \lambda_{ts}^{uv} = \begin{cases} 
\lambda_{uv}, & \text{if } u=s \\
-\lambda_{uv}, & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}, \\
\forall s \in N_L, (u,v) \in T.
\]

2. Working capacity assignment constraint:

\[
\beta_{st} = \sum_{(u,v) \in T} \lambda_{st}^{uv}, \quad \forall (s,t) \in E_L.
\]

3. Traffic rerouting constraints:

\[
\sum_{l \text{ s.t.} (k,l) \in G_{ij}} \gamma_{kl}^{st}(ij) - \sum_{l \text{ s.t.} (l,k) \in G_{ij}} \gamma_{lk}^{st}(ij) = \begin{cases} 
(f_{ij}^{st} + f_{ji}^{st})\beta_{st}, & \text{if } s = k \\
-(f_{ij}^{st} + f_{ji}^{st})\beta_{st}, & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}, \\
\forall k \in N_L, (s,t) \in E_L, (i,j) \in E'_p.
\]

4. Spare capacity assignment constraints:

\[
\mu_{kl}^{st} = \sum_{(s,t) \in E_L} \gamma_{kl}^{st}(ij), \quad \forall (k,l) \in E_L, (i,j) \in E'_p.
\]

5. Nonnegativity constraints:

\[
\lambda_{st}^{uv} \geq 0, \quad \forall (u,v) \in T, (s,t) \in E_L. \\
\gamma_{kl}^{st}(ij) \geq 0, \quad \forall (s,t), (k,l) \in E_L, (i,j) \in E'_p.
\]

Note that if constraints 1-2 and variables $\lambda_{st}^{uv}$ are removed, and variables $\beta_{st}$ are given as a set of working capacity requirements, the modified LP-JTRCA$_{LR}$ is an
alternative flow-based formulation for the spare capacity assignment (SCA) problem, which was formulated using the cut-set approach in Section 2.2.

For end-to-end restoration, constraints 3-5 in LP-JTRCALR are replaced by the following four sets of constraints:

3. Disrupted traffic constraints:

\[
\alpha_{st}(ij) \leq \lambda_{st}, \quad \forall(s, t) \in E_L, (u, v) \in T, (i, j) \in E_p'.
\]

\[
0 \leq \beta_{ij}^{uv} \leq \lambda^{uv}, \quad \forall(u, v) \in T, (i, j) \in E_p'.
\]

\[
\sum_{t \text{ s.t. } (s, t) \in G_{ij}} \alpha_{st}^{uv}(ij) - \sum_{t \text{ s.t. } (t, s) \in G_{ij}} \alpha_{ts}^{uv}(ij) = \begin{cases} 
\lambda^{uv} - \beta_{ij}^{uv}, & \text{if } u=s \\
-(\lambda^{uv} - \beta_{ij}^{st}), & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}
\]

\forall s \in N_L, (u, v) \in T, (i, j) \in E_p'.

4. Traffic rerouting constraints:

\[
\sum_{t \text{ s.t. } (s, t) \in G_{ij}} \gamma_{st}^{uv}(ij) - \sum_{t \text{ s.t. } (t, s) \in G_{ij}} \gamma_{ts}^{uv}(ij) = \begin{cases} 
\beta_{ij}^{uv}, & \text{if } u=s \\
-\beta_{ij}^{uv}, & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}
\]

\forall s \in N_L, (u, v) \in T, (i, j) \in E_p'.

5. Spare capacity assignment constraints:

\[
\mu^{st} \geq \sum_{(u, v) \in T} \gamma_{st}^{uv}(ij), \quad \forall(s, t) \in E_L, (i, j) \in E_p'.
\]

6. Nonnegativity constraints:

\[
\lambda_{st}^{uv} \geq 0, \quad \forall(u, v) \in T, (s, t) \in E_L.
\]

\[
\alpha_{st}^{uv}(ij), \quad \gamma_{st}^{uv}(ij) \geq 0, \quad \forall(u, v) \in T, (s, t) \in E_L, (i, j) \in E_p'.
\]
Let LP-JTRCA\(_{ER}\) denote the LP formulation for the JTRCA problem under end-to-end restoration. LP-JTRCA\(_{ER}\) requires significantly more constraints and variables than LP-JTRCA\(_{LR}\). Let \(|N_P|\) and \(|E_P|\) denote the number of nodes and links in the physical topology, respectively. Similarly, let \(|N_L|\) and \(|E_L|\) denote the number of nodes and links the logical topology, respectively. Table 3.1 summarizes the complexity for each formulation in terms of the number of constraints and variables. It is easy see that LR-JTRCA\(_{ER}\) requires \(O\left(\frac{|N_L|^2}{|E_L|}\right)\) times more constraints and variables than LR-JTRCA\(_{LR}\). When the logical topology is a fully connected, end-to-end restoration is the same as link restoration because each traffic demand is routed on a distinct logical link. As expected \(O\left(\frac{|N_L|^2}{|E_L|}\right)\) reduces to \(O(1)\) in this case.

<table>
<thead>
<tr>
<th></th>
<th>No. of constraints</th>
<th>No. of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-JTRCA(_{LR})</td>
<td>(O(</td>
<td>E_P</td>
</tr>
<tr>
<td>LP-JTRCA(_{ER})</td>
<td>(O(</td>
<td>E_P</td>
</tr>
</tbody>
</table>

Table 3.1: Complexity comparison between LP-JTRCA\(_{LR}\) and LP-JTRCA\(_{ER}\).

### 3.2.2 Lightpath Routing Problem

Given the physical topology, the logical topology, and the traffic demands, we want to find a good survivable routing for the JTRCA problem. Ideally, traffic demands should be incorporated into the routing. However, this will make the routing problem more complex. Furthermore, the LP-JTRCA formulation already jointly optimizes the routing of the traffic and capacity assignment for a given routing. In the previous chapter, we have shown that maximizing the load factor (SR-MLF) is a good routing strategy for reducing spare capacity requirement for network survivability compared to the SR-MN routing strategy. As long as the logical topology remains well-connected after any physical link failure, the disrupted traffic can be rerouted on a diverse set of backup routes. This promotes a greater sharing of the spare capacity, thus reducing the capacity requirement for network survivability. We have also developed the SR-MSF routing strategy, which minimizes the spare factor. However, we were not able to design a suitable set of working capacity requirements from the traffic demands for
the SR-MSF routing strategy. We attempted using the following heuristic to obtain a set of working capacity requirements:

1. Route the logical topology using the SR-MLF routing strategy.
2. Solve the LP-JTRCA problem for the obtained routing.
3. Extract the working capacity requirements from the solution to the LP-JTRCA problem.

However, the SR-MSF routing strategy is not able to find a routing that requires a lower total capacity requirement than the SR-MLF routing strategy. The reason is that LP-JTRCA routes the traffic demands and assigns working capacities that are tailored for the given routing. The SR-MSF routing strategy cannot find a better routing based on the working capacities that are optimized for another routing. Thus we select the SR-MLF routing strategy to find a suitable routing for the LP-JTRCA problem.

### 3.3 Simulation Results

We generated 50 random 10-node logical topologies of degree $k$ to embed on the 10-node degree 5 and 10-node degree 3 physical topologies shown in Figure 3-2. We
assumed a uniform set of traffic demands, i.e. the traffic demand between every node pair is 1. For each logical topology and the corresponding physical topology, we route the logical topology using both SR-MN (for comparison) and SR-MLF routing strategies. For each routing, we solve LP-JTRCA_{LR} and LP-JTRCA_{ER} to obtain the total capacity requirements under link restoration and end-to-end restoration.

![Graph showing capacity requirements for different logical topologies and routing strategies](image)

Figure 3-3: Average total capacity requirement for embedding random 10-node logical topologies of degree $k$ on the 10-node degree 5 physical topology

Figure 3-3 shows the average total capacity requirement for embedding random logical topologies of degree $k$ on the 10-node degree 5 physical topology. The capacity requirement is significantly reduced using the SR-MLF routing strategy. As expected, end-to-end restoration is more capacity efficient than link restoration. It is interesting to see that SR-MLF under link restoration has about the same capacity requirement as SR-MN under end-to-end restoration. This suggests that it is not always necessary to use the more complex end-to-end restoration to reduce capacity requirement. By routing the logical topology wisely, the same capacity requirement can be achieved using the simpler link restoration scheme. The figure also shows that total capac-
ity requirement decreases as the degree of the logical topology increases. Since we have a 10-node logical topology with uniform traffic, total traffic demand is 90 for all logical topologies. As the degree of the logical topology increases, more source and destination are directly connected by lightpaths. Since traffic can be routed on shorter routes, the overall working capacity is reduced. The total spare capacity required to protect all of the working capacities also decreases. Thus the total capacity requirement decreases as the degree of the logical topology increases.

Figure 3-4: Average total capacity requirement for embedding random 10-node logical topologies of degree $k$ on the 10-node degree 3 physical topology.

Figure 3-4 shows that the average total capacity requirement for the 10-node degree 3 physical topology is much higher than that for the 10-node degree 5 physical topology. Because the underlying physical topology is much more sparse, each physical link has to support more logical links. Thus more traffic must be rerouted after a physical link failure, which results in a higher total capacity requirement. The figure again shows that the SR-MLF routing strategy requires significantly less capacity than the SR-MN routing strategy. For degrees greater than 4, SR-MLF under link
restoration still requires less capacity than SR-MN under end-to-end restoration. For degrees less than or equal to 4, it requires more capacity. When the physical topology and the logical topology are equally sparse, the logical topology will not remain well-connected regardless of routing. In this case, link restoration will not be able to reroute disrupted traffic efficiently because of "backhauls".

<table>
<thead>
<tr>
<th>Phys. top.</th>
<th>10-node degree 5</th>
<th>10-node degree 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log. top. degree</td>
<td>Link</td>
<td>End-to-end</td>
</tr>
<tr>
<td>3</td>
<td>17.56</td>
<td>12.55</td>
</tr>
<tr>
<td>4</td>
<td>12.95</td>
<td>7.91</td>
</tr>
<tr>
<td>5</td>
<td>10.47</td>
<td>6.17</td>
</tr>
<tr>
<td>6</td>
<td>4.32</td>
<td>3.77</td>
</tr>
</tbody>
</table>

Table 3.2: Average capacity saving of SR-MLF over SR-MN for corresponding Figures 3-3 and 3-4

To examine the capacity improvement obtained from using the SR-MLF routing strategy more in detail, we introduce a new comparison quantity called the capacity saving. For a given logical topology and physical topology, let x and y be the total capacities corresponding to the SR-MN and SR-MLF routing strategies, respectively. We define capacity saving of SR-MLF over SR-MN for the given topologies as the percentage of the total capacity saved by using the SR-MLF routing strategy over the SR-MN routing strategy, i.e. \( \frac{x-y}{x} \times 100 \).

Table 3.2 summarizes the average capacity saving of SR-MLF over SR-MN corresponding to the SR-MLF and SR-MN curves in Figures 3-3 and 3-4. The results show that SR-MLF consistently requires less capacity than SR-MN under link restoration and end-to-end restoration. The reason is that the SR-MLF routing strategy ensures the logical links are evenly routed on the physical links, while the SR-MN routing strategy may result in many logical links sharing the same physical link.

In the case of the 10-node degree 5 physical topology, the capacity saving consistently decreases as the degree of the logical topology increases. When the logical topology is sparse, it is essential that logical links are routed using disjoint physical links so that any physical link failure does not leave the logical topology poorly connected. Because the underlying physical topology is dense, the SR-MLF rout-
ing strategy is able to route the sparser logical topology on more disjoint physical links than the SR-MN routing strategy. Thus the capacity saving is much higher for smaller degrees. As the logical topology becomes more dense, it always remains well-connected after any physical link failure. Thus the capacity saving is much lower for larger degrees.

In the case of the 10-node degree 3 physical topology, the same pattern does not occur. The results show that the capacity saving for degrees 5 and 6 are much higher than that for degrees 3 and 4. Because the underlying physical topology is sparse, the diversity of good survivable routings is limited when the logical topology is equally sparse. Even though SR-MLF is still a better routing strategy, most single physical link failures will still leave the logical topology poorly connected. However, as the degree of the logical topology increases, the diversity of survivable routings increases as well. The SR-MLF routing strategy is able to find a good survivable routing that distributes logical links evenly on the physical links, whereas the SR-MN routing strategy is still plagued by the problem of many logical links sharing the same physical link. Thus we observe a greater capacity saving for larger degrees.

<table>
<thead>
<tr>
<th>Phys. top.</th>
<th>10-node degree 5</th>
<th>10-node degree 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log. top. degree</td>
<td>SR-MN</td>
<td>SR-MLF</td>
</tr>
<tr>
<td>3</td>
<td>13.16</td>
<td>7.88</td>
</tr>
<tr>
<td>4</td>
<td>11.13</td>
<td>5.97</td>
</tr>
<tr>
<td>6</td>
<td>6.64</td>
<td>6.10</td>
</tr>
</tbody>
</table>

Table 3.3: Average capacity saving of end-to-end restoration over link restoration for corresponding Figures 3-3 and 3-4

We define capacity saving of end-to-end restoration over link restoration as the percentage of total capacity saved by using end-to-end restoration over link restoration for a given physical topology and logical topology.

Table 3.3 summarizes the average capacity saving of end-to-end restoration over link restoration corresponding to the link restoration and end-to-end restoration curves in Figures 3-3 and 3-4. Under single logical link failures, end-to-end restoration is more capacity efficient than link restoration. Under multiple logical link failures,
multiple links are restored using the same mechanisms. Thus end-to-end restoration should be even more capacity efficient than link restoration by the multiplicative factor. The results confirm this analysis. In the case of 10-node degree 5 physical topology, the capacity saving for SR-MLF is much lower than the rest. As mentioned earlier, because the underlying physical topology is dense, the SR-MLF routing strategy is able to route logical links disjointly on the physical links. As a result, most single physical link failures result in a single logical link failure. Thus the capacity saving is not as large as the rest.

The results also show that the efficiency of end-to-end restoration decreases as the degree of the logical topology increases. The reason is that as the logical topology becomes more dense, less traffic is carried on each logical link. In other words, each logical link is not carrying much more traffic than a single traffic demand. Furthermore, because the logical topology is denser, the spare capacity on each logical link can be shared by many more diverse backup routes that connect the end-nodes of the failed logical links. Therefore, the capacity requirement for link restoration will converge to that for end-to-end restoration. In the limiting case when the logical topology is fully connected, each traffic demand is carried on a distinct logical link, in which the capacity requirement for link restoration and end-to-end restoration is identical.

3.4 Conclusion

In this chapter, we presented a heuristic algorithm for solving the LRJCA problem, which involved the the lightpath routing problem and joint traffic routing and capacity assignment problem. We used the routing strategy that maximizes the load factor developed in the previous chapter to route the logical topology. We formulated the JTRCA problem under both link restoration and end-to-end restoration as linear programs. The results showed that the total capacity requirement for network survivability can be significantly reduced by using this routing strategy over an arbitrary survivable routing strategy. The results also showed that end-to-end restoration
can be much more capacity efficient than link restoration in IP-over-WDM networks, where many logical links may fail from a single physical link failure.
Chapter 4

Capacity Assignment in Survivable IP Networks

Thus far we have considered embedding random logical topologies on given physical topologies. In this chapter, we assume that physical and logical topologies are identical. Thus each logical link is routed on a distinct physical link, and any physical link failure causes exactly one bidirectional failure in the corresponding logical link. With this assumption, the LRJCA problem reduces to the JTRCA problem, since there is no lightpath routing problem. Given a network topology and a traffic demand, we want to jointly route the traffic and assign working and spare capacities to each link in a manner that minimizes total capacity required for network survivability. We call this the survivable network capacity assignment (SNCA) problem. It is applicable to general IP networks, not only to IP-over-WDM networks. In this chapter, we first present a simplified LP formulation of the JTRCA problem under both link restoration and end-to-end restoration for the SNCA problem. We then analyze capacity requirement for network survivability for three real world networks.

4.1 Computing Eligible Routes

There has been a considerable amount of research in the area of joint optimization of working capacity and spare capacity assignments to ensure the survivability of
networks against single link failures [10, 21, 30]. All previous algorithms, however, require the set of eligible routes to be precomputed. For each source-destination pair \((u, v)\), the eligible routes are the set of all possible paths connecting nodes \(u\) and \(v\), which are also called the set of elementary paths for node pair \((u, v)\). Because the number of elementary paths for any node pair grows exponentially with the size of the network, the number of eligible routes has to be restricted. Typically, elementary paths longer than the “hop limit” are eliminated from the set of eligible routes [10, 17]. However, this approach causes the solution to be suboptimal. In order to determine the minimum capacity requirement, the set of all eligible routes should be considered.

We approach the problem from a different perspective. The SNCA problem is a special case of the LTRCA problem, where both physical and logical topologies are identical and every logical link is routed on the corresponding physical link. Given the routing and the traffic demands, we can solve LP-JTRCA to determine minimum capacity requirement without ever explicitly computing the eligible routes. The only inputs to the JTRCA problem are the traffic demands and the routing. Because there is no routing of the logical topology in the SNCA problem, we can remove extraneous variables and constraints associated with routing from the LP-JTRCA formulation to obtain a much simpler LP formulation for both link restoration and end-to-end restoration.

4.2 Linear Program Formulation

We first introduce some notations and assumptions. Let \((N, E)\) denote the network topology, where \(N\) is the set of nodes and \(E\) is the set of links. The network topology must be at least 2-connected, otherwise, a single link failure can cause the network to become disconnected. We assume a bidirectional topology, where if link \((s, t)\) is in \(E\) so is link \((t, s)\). We also assume link failure is bidirectional; if link \((s, t)\) fails, so does link \((t, s)\). We define \(E'\) as the set of bidirectional links, where \(E' = \{(s, t) \in E : s > t\}\). Let \(T = \{(u, v) : u, v \in N, u \neq v\}\) denote the set of all possible node pairs in the
network topology. Lastly, the traffic demand for source-destination pair \((u, v)\) is given by \(\lambda_{uv}\).

We formulate the routing of the traffic demands as a multicommodity flow problem, where each traffic demand corresponds to a distinct commodity. We introduce flow variables \(\lambda_{st}^{uv}\) indicating the amount of traffic with source \(u\) and destination \(v\) that traverses link \((s, t)\). We use the same multicommodity flow formulation to express the following set of constraints on flow variables \(\lambda_{st}^{uv}\):

\[
\sum_{t \text{ s.t. } (s,t) \in E} \lambda_{st}^{uv} - \sum_{t \text{ s.t. } (t,s) \in E} \lambda_{ts}^{uv} = \begin{cases} 
\lambda_{uv}, & \text{if } u = s \\
-\lambda_{uv}, & \text{if } v = s \\
0, & \text{otherwise}
\end{cases}, \quad \forall s \in N; \ (u, v) \in T.
\]

The working traffic on each link \((s, t)\) is the aggregation of flow of all the traffic demands traversing the link:

\[
\beta^{st} = \sum_{(u,v) \in T} \lambda_{st}^{uv}, \quad \forall (s, t) \in E.
\]

### 4.2.1 LP Formulation for Link Restoration

After link \((s, t)\) fails, the set of links that remain intact is \(G_{st} = E - \{(s, t), (t, s)\}\). Under link restoration, for any link failure \((s, t)\), \(\beta^{st}\) and \(\beta^{ts}\) must be rerouted only on the links in \(G_{st}\). We introduce flow variables \(\gamma_{kl}^{st}\) to indicate the amount of rerouted traffic with source \(s\) and destination \(t\) that traverses link \((k, l)\) after the failure of link \((s, t)\). We model the disrupted traffic \(\beta^{st}\) as a commodity. We use the multicommodity
flow formulation to express the following set of constraints on the flow variables $\gamma_{kl}^{st}$:

$$
\sum_{l \text{ s.t.}(k,l) \in G_{st}} \gamma_{kl}^{st} = \begin{cases} 
\beta_{st}, & \text{if } s = k \\
-\beta_{st}, & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}
$$

(4.3)

$\forall k \in N, (s, t) \in E$.

The set of constraints above are the flow conservation constraints for routing $\beta_{st}$ amount of flow from node $s$ to node $t$. Equation (4.3) requires that equal amounts of flow due to disrupted traffic $\beta_{st}$ enter and leave each node that is not the source or destination of $(s, t)$. Furthermore, the routes used by the flow must only contain links in $G_{st}$.

Let $\mu^{kl}$ denote the amount of spare capacity required on link $(k, l)$. Since the spare capacity on each link must be sufficient to support the additional traffic after any link failure, it must satisfy the following constraints:

$$
\mu^{kl} \geq \gamma_{kl}^{st} + \gamma_{lk}^{st}, \quad \forall (k, l) \in E, (s, t) \in E'.
$$

(4.4)

4.2.2 LP Formulation for End-to-end Restoration

Since only one bidirectional link fails at a time, finding and rerouting the disrupted traffic demands in end-to-end restoration is much easier than before. For each source-destination pair $(u, v)$, $\lambda_{ul}^{uv} + \lambda_{ls}^{uv}$ is precisely the amount of traffic lost by source-destination pair $(u, v)$ after link $(s, t)$ fails. We model this quantity as a commodity. We introduce flow variables $\gamma_{kl}^{uv}(st)$ to indicate the amount of rerouted traffic on link $(k, l)$ with source $u$ and destination $v$ after link failure $(s, t)$. We use the following multicommodity flow formulation to express the following set of constraints on
variables $\gamma_{kl}^{uv}(st)$:

$$\sum_{l \in \mathcal{L}, (k,l) \in G_{st}} \gamma_{kl}^{uv}(st) - \sum_{l \in \mathcal{L}, (l,k) \in G_{st}} \gamma_{lk}^{uv}(st) = \begin{cases} 
\lambda_{st}^{uv} + \lambda_{ts}^{uv}, & \text{if } s = k \\
-(\lambda_{st}^{uv} + \lambda_{ts}^{uv}), & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}, \quad (4.5)$$

$\forall k \in \mathcal{N}, (u,v) \in \mathcal{T}, (s,t) \in \mathcal{E}'$.

The set of constraints above are the flow conservation constraints for routing $\lambda_{st}^{uv} + \lambda_{ts}^{uv}$ amount of flow from source $u$ to destination $v$ after link failure $(s,t)$. To ensure network survivability, we must reroute disrupted traffic for every source-destination pair after any link failure.

The amount of additional traffic on each link $(k,l)$ after link failure $(s,t)$ is $\sum_{(u,v) \in \mathcal{T}} \gamma_{kl}^{uv}(st)$. The spare capacity on each link $(k,l)$ must satisfy the following set of constraints:

$$\mu_{kl}^{st} \geq \sum_{(u,v) \in \mathcal{T}} \gamma_{kl}^{uv}(st), \quad \forall (k,l) \in \mathcal{E}, (s,t) \in \mathcal{E}''. \quad (4.6)$$

### 4.2.3 Complete LP formulations for the SNCA problem

Given the network topology $(\mathcal{N}, \mathcal{E})$ and the traffic demands, we want to find the minimum capacity requirement for network survivability. Let LP-SNCA$_{LR}$ denote the following LP formulation for the SNCA problem under link restoration:

minimize $\sum_{(s,t) \in \mathcal{E}_L} \beta^{st} + \mu^{st}$

Subject to:
1. Traffic routing constraints:

\[ \sum_{(s,t) \in E} \lambda_{st}^{uv} - \sum_{(s,t) \in E} \lambda_{ts}^{uv} = \begin{cases} 
\lambda_{uv}, & \text{if } u = s \\
-\lambda_{uv}, & \text{if } v = s \\
0, & \text{otherwise}
\end{cases} \]

\( \forall s \in N_L, (u,v) \in T. \)

2. Working capacity assignment constraint:

\[ \beta_{st}^{st} = \sum_{(u,v) \in T} \lambda_{st}^{uv}, \quad \forall (s,t) \in E. \]

3. Traffic rerouting constraints:

\[ \sum_{l \text{ s.t.}(k,l) \in G_{st}} \gamma_{kl}^{st} - \sum_{l \text{ s.t.}(k,l) \in G_{st}} \gamma_{lk}^{st} = \begin{cases} 
\beta_{st}^{st}, & \text{if } s = k \\
-\beta_{st}^{st}, & \text{if } t = k \\
0, & \text{otherwise}
\end{cases} \]

\( \forall k \in N, (s,t) \in E. \)

4. Spare capacity assignment constraint:

\[ \mu_{kl}^{st} \geq \gamma_{kl}^{st} + \gamma_{lk}^{ts}, \quad \forall (k,l) \in E, (s,t) \in E'. \]

5. Nonnegativity constraints:

\[ \lambda_{st}^{uv} \geq 0, \quad \forall (u,v) \in T, (s,t) \in E. \]
\[ \gamma_{kl}^{st} \geq 0, \quad \forall (k,l), (s,t) \in E. \]

For end-to-end restoration, constraints 3-5 in LP-SNCA_{LR} are replaced by the following three sets of constraints:
3 Traffic rerouting constraints:

$$
\sum_{l \in G_{st}} \gamma_{kl}^{uv}(st) - \sum_{l \in G_{st}} \gamma_{lk}^{uv}(st) = \begin{cases} 
\lambda_{st}^{uv} + \lambda_{ls}^{uv}, & \text{if } s = k \\
-(\lambda_{st}^{uv} + \lambda_{ts}^{uv}), & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}
$$

$$
\forall k \in N, (u, v) \in T, (s, t) \in E'.
$$

4 Spare capacity assignment constraints:

$$
\mu_{kl} \geq \sum_{(u,v) \in T} \gamma_{kl}^{uv}(st) \quad \forall (k, l) \in E, (s, t) \in E'.
$$

5 Nonnegativity constraints:

$$
\lambda_{st}^{uv} \geq 0, \quad \forall (u, v) \in T, (s, t) \in E.
$$

$$
\gamma_{kl}^{uv}(st) \geq 0, \quad \forall (u, v) \in T, (k, l) \in E, (s, t) \in E'.
$$

<table>
<thead>
<tr>
<th></th>
<th>No. of constraints</th>
<th>No. of variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP-SNCA_{LR}</td>
<td>(O(</td>
<td>E</td>
</tr>
<tr>
<td>LP-SNCA_{ER}</td>
<td>(O(</td>
<td>E</td>
</tr>
</tbody>
</table>

Table 4.1: Complexity comparison between LP-SNCA_{LR} and LP-SNCA_{ER}.

Let LP-SNCA_{ER} denote the LP formulation for the SNCA problem under end-to-end restoration. Let \(|N|\) and \(|E|\) denote the number of nodes and links in the network topology \((N, E)\). The complexity in terms of the number of variables and constraints for both formulations is summarized in Table 4.1. As expected, LR-SNCA_{ER} requires more variables and constraints than LR-SNCA_{LR}. However, even when the network topology is fully connected, LR-SNCA_{ER} requires \(O(|N|)\) times more constraints and \(O(|N|^2)\) times more variables than LR-SNCA_{LR}. Note that independent of the routing of the traffic demands, link restoration only reroutes the lost traffic, \(\beta^{st}\) and \(\beta^{st}\), after any link failure \((s, t)\). Thus for each link failure, the rerouting of disrupted traffic was formulated as a 2-commodity flow problem. In contrast, end-to-end restoration
reroutes each disrupted traffic demand individually. For any link failure, the disrupted traffic demands are not known in advance since they depend on the routing of the traffic demands. Thus for each link failure, the formulation for end-to-end restoration must consider traffic lost by every source-destination pairs.

### 4.3 Simulation Results

![Network Topologies](image)

(a) 14-Node, 21-link NSFNET
(b) 12-Node, 18-link Sprint OC-48 Network
(c) 11-Node, 23-link New Jersey LATA Network

Figure 4-1: Real world network topologies used in simulations.

In our simulations, we used the NSFNET, Sprint OC-48 network, and New Jersey LATA network topologies shown in Figure 4-1. We used two types of traffic demands:
uniform and random. In the uniform case, every traffic demand $(u, v)$ is 1. In the random case, every traffic demand $(u, v)$ is randomly chosen from a uniform interval $[1, 5]$. We generated 100 sets of traffic demands in the random case. For each of the three network topologies, we solved LP-SNCA$_{LR}$ and LP-SNCA$_{ER}$ to obtain minimum capacity requirements for network survivability under link restoration and end-to-end restoration. Similar to before, we define capacity saving as the percentage of total capacity saved by using end-to-end restoration over link restoration.

<table>
<thead>
<tr>
<th>Network</th>
<th>Avg. node degree</th>
<th>Tot. traffic demand</th>
<th>Tot. cap. LR</th>
<th>Tot. cap. ER</th>
<th>Cap. saving of ER over LR(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSFNET</td>
<td>3</td>
<td>182</td>
<td>618</td>
<td>551.25</td>
<td>10.80</td>
</tr>
<tr>
<td>Sprint</td>
<td>3</td>
<td>132</td>
<td>434</td>
<td>412.6</td>
<td>4.93</td>
</tr>
<tr>
<td>LATA</td>
<td>4.18</td>
<td>110</td>
<td>303.33</td>
<td>303.33</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.2: Average total capacity requirement for link restoration and end-to-end restoration with uniform traffic demands

The results for minimum total capacity requirement with uniform traffic demands are summarized in Table 4.2. As expected, end-to-end restoration is more capacity efficient than link restoration. The results confirm the earlier analysis that the capacity saving of end-to-end restoration over link restoration diminishes as the topology becomes more dense. Even though both the NSFNET and Sprint OC-48 network have the average node degree of 3, the NSFNET is more sparse because it contains more nodes.

<table>
<thead>
<tr>
<th>Network</th>
<th>Cap. saving of ER over LR</th>
<th>Standard deviation$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSFNET</td>
<td>11.93</td>
<td>0.73</td>
</tr>
<tr>
<td>Sprint</td>
<td>5.09</td>
<td>0.74</td>
</tr>
<tr>
<td>LATA</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.3: Average capacity saving of end-to-end link restoration over link restoration over 100 sets of random traffic demands.

Table 4.3 shows that the capacity saving of end-to-end restoration over link restoration follows the same trend even with random traffic demands. The results are almost identical to those for the uniform traffic demands. Furthermore, the standard

$^1$The standard deviation is calculated using the “biased” method: $\sqrt{\frac{n}{n-1} \left( \sum x^2 - \left( \sum x \right)^2 \right)}$. 
deviation is very small, which suggests that the efficiency of end-to-end restoration is inherent to the topology. Surprisingly, the capacity requirements under end-to-end restoration and link restoration in the New Jersey LATA network are always identical. Understanding this result is an area for future work.

We also examined the distribution of the capacity assignments in each of the networks. We found that a significant amount of capacity is required on links incident to nodes of degree 2. This is not surprising considering if either of the two links fails, all of the traffic must be diverted to the other link. According to Table 4.4, the capacity (working and spare) assigned to the links incident to degree 2 nodes accounts for majority of the network’s total capacity in the Sprint OC-48 and New Jersey LATA networks. Recall that network redundancy is the ratio of total spare capacity to total capacity. Even though the New Jersey LATA network is very dense, network redundancy remains high because of the 3 degree 2 nodes, whose incident links account for 40% of the network’s total capacity.

<table>
<thead>
<tr>
<th>Network</th>
<th>No. of nodes of degree 2</th>
<th>Deg. 2 cap. LR (in %)</th>
<th>Deg. 2 cap. ER (in %)</th>
<th>Net. redun. LR (in %)</th>
<th>Net. redun. ER (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSFET</td>
<td>2</td>
<td>21.8</td>
<td>21.8</td>
<td>37</td>
<td>29</td>
</tr>
<tr>
<td>Sprint</td>
<td>4</td>
<td>47</td>
<td>47.5</td>
<td>38</td>
<td>35</td>
</tr>
<tr>
<td>LATA</td>
<td>3</td>
<td>39.6</td>
<td>39.6</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 4.4: Impact of degree 2 nodes on network redundancy.

4.4 Conclusion

In this chapter, we formulated the SNCA problem as a linear program under both link restoration and end-to-end restoration. Unlike previous approaches, our formulation does not require the explicit computation of working and backup routes. The input to the LP only consists of the topology and the traffic demands. As result, we are able to compute the theoretical minimum capacity requirement for network survivability against single link failure. We studied three real world networks under link restoration and end-to-end restoration. The results confirm that end-to-end restoration is more capacity efficient than link restoration. Finally, we showed that the capacity on links
incident to the nodes of degree 2 accounts for large portion of network’s total capacity. In order to reduce network redundancy, nodes of degree 2 should be avoided when designing networks.
Chapter 5

Conclusion

5.1 Summary

In this thesis, we studied the problem of providing protection for IP-over-WDM networks at the electronic layer against single physical link failures. Protection at the electronic layer requires provisioning each lightpath with sufficient spare capacity to support all of rerouted traffic after the physical link failure. The routing of the logical topology on the physical topology significantly affects the amount of capacity required for network survivability. We established necessary and sufficient conditions on the routing of the logical topology and the corresponding capacity requirement for achieving network survivability.

Based on these conditions, we developed two criteria for measuring the quality of a survivable routing: the load factor and the spare factor. The load factor measures the disjointness of the routing. A routing with a large load factor ensures that any physical link failure always leaves the logical topology well-connected. As a result, disrupted traffic can be rerouted on a diverse set of backup routes. This promotes a greater sharing of the spare capacity, thus reducing the capacity requirement. Finding a routing that maximizes the load factor is a good general routing strategy when the working capacity requirements may change or be unknown. For a given set of working capacity requirements, we showed that the spare factor associated with a routing provides an upper bound on the corresponding spare capacity requirement.
Consequently, minimizing the spare factor also minimizes the upper bound. Intuitively, the spare factor measures how evenly the working capacities are routed on the physical topology. A routing with a small spare factor ensures that only a small fraction of working capacity will be lost after any physical link failure. We formulated the SR-MLF and SR-MSF routing strategies as mixed integer linear programs. Our results showed that both routing strategies can significantly reduce the capacity requirement for network survivability when compared to an arbitrary survivable routing strategy.

We investigated the joint optimization problem of routing the logical topology and routing traffic. We used a heuristic approach consisting of decomposing the problem into two sub-problems: lightpath routing and joint traffic routing and capacity assignment. We used the routing strategy that maximizes the load factor to find a suitable routing, and then solved the joint traffic routing and capacity assignment problem by solving an equivalent linear program. The results showed that our routing strategy again requires much less capacity compared to an arbitrary survivable routing strategy. The results also showed that end-to-end restoration can be much more capacity efficient than link restoration in an IP-over-WDM network, where multiple logical link failures may occur simultaneously.

Lastly, we developed a method for computing the minimum capacity requirement for protecting general IP networks against single link failures. We analyzed capacity requirements for three real world networks: the NSFNET, the Sprint OC-48 network, and the New Jersey LATA network. The results showed that the network topology inherently limits the effectiveness of end-to-end restoration over link restoration. For the New Jersey LATA network, link restoration requires the same capacity requirement as end-to-end restoration. We also showed that the capacity on the links incident to degree 2 nodes accounts for majority of the total capacity. Degree 2 nodes should be avoided when designing networks.
5.2 Future Work

The mixed integer linear programming formulations for the two routing strategies require a number of constraints that grows exponentially with the size of the network. Heuristics should be developed to solve the MILPs efficiently for large networks. The design of a suitable set of working capacity requirements from the traffic demands for the SR-MSF routing strategy remains to be found. Different objective functions other than minimizing total capacity should also be explored, such as minimizing the load on each lightpath. Furthermore, we only focused on reducing network capacity at the electronic layer. In practice, the cost associated with routing each lightpath on the WDM network includes the number of physical links used and the number of wavelengths required. Future work should incorporate these factors as well as the modularity requirement of the lightpath. Because IP routers use their own routing algorithm for routing traffic, the convergence of lost traffic to backup routes with sufficient spare capacity after a physical link failure may take a long time. Analysis should be done on the restoration time for the network to recover from failures. Lastly, further analysis of link restoration versus end-to-end restoration capacity requirements for different topologies is needed. This will lead to key insights on how the network topology affects the performance of both rerouting strategies.
Appendix A

MILP Formulation for the LRSCA Problem

Given the physical topology \((N_P, E_P)\), the logical topology \((N_L, E_L)\), and the set of working capacity requirements \(\beta^{st}\), the LRSCA problem can be formulated as the following MILP:

\[
\begin{align*}
\text{minimize} & \quad \sum_{(s,t) \in E_L} \mu^{st} \\
\text{Subject to:} & \\
1. & \quad \text{Connectivity constraints:} \\
& \quad \sum_{j \in \mathcal{N}_i \setminus \{(s,t) \in E_P}} f_{ij}^{st} - \sum_{j \in \mathcal{N}_i \setminus \{(s,t) \in E_P}} f_{ji}^{st} = \begin{cases} 
1, & \text{if } s = i \\
-1, & \text{if } t = i \\
0, & \text{otherwise}
\end{cases} \\
& \quad \forall i \in N_P, (s, t) \in E_L
\end{align*}
\]

2. Spare capacity forcing constraints:

\[
\zeta^{st}_{ij} \leq k \{1 - (f_{ij}^{st} + f_{ji}^{st})\}, \quad \forall (s, t) \in E_L, (i, j) \in E'_{\mu}.
\]
3. Spare capacity criteria constraints:

\[
\sum_{(s,t) \in CS(s,N_L-s)} (f_{ij}^{st} + f_{ji}^{st}) \beta_{st} \leq \sum_{(s,t) \in CS(s,N_L-N)} \zeta_{ij}^{st}, \quad \forall S \subset N_L, (i,j) \in E_p.
\]  

(A.3)

4. Spare capacity assignment constraints:

\[
\mu_{ij}^{st} \geq \zeta_{ij}^{st}, \quad \forall (s,t) \in E_L, (i,j) \in E_p.
\]  

(A.4)

5. Nonnegativity constraints:

\[
\zeta_{ij}^{st} \geq 0, \quad \forall (s,t) \in E_L, (i,j) \in E_p.
\]  

(A.5)

6. Integer flow constraints: \( f_{ij}^{st} \in \{0, 1\} \).

Constraints (A.1) are the flow conservation constraints for routing the logical topology on the physical topology. Let \( \zeta_{ij}^{st} \) be the amount of spare capacity available on logical link \( (s,t) \) for carrying rerouted traffic after physical link \( (i,j) \) fails. Constraints (A.2) force \( \zeta_{ij}^{st} \) to be zero if physical link failure \( (i,j) \) results in the failure of logical link \( (s,t) \). The parameter \( k \) is a large constant (i.e. \( k \geq \sum_{(s,t) \in E_L} \beta_{st} \)) that ensures no spare capacity limit is placed on the logical link if the link remains intact after the physical link failure. Constraints (A.3) require the amount of working capacity lost in the cut-set due to the physical link failure to be less than or equal to the amount of spare capacity remaining in the cut-set for all cut-sets and all single physical link failures. Constraints (A.4) ensures the spare capacity on each logical link is sufficient to reroute all of the disrupted traffic after any physical link failure. If the MILP above is feasible, an optimal solution exists. The optimal spare capacity assignments are given by variables \( \mu^{st} \) and the corresponding routing is given by variables \( f_{ij}^{st} \).
Appendix B

MILP Formulation for the LRJCA Problem

Given the physical topology \((N_P, E_P)\), the logical topology \((N_L, E_L)\), and the set of traffic demands \(\lambda_{uv}\), the LRJCA problem under link restoration can be formulated as the following MILP:

\[
\text{minimize} \quad \sum_{(s,t) \in E_L} \beta_{st} + \mu_{st}
\]

Subject to:

1. Traffic routing constraints:

\[
\sum_{t \text{ s.t.} (s,t) \in E_L} \lambda_{st}^{uv} - \sum_{t \text{ s.t.} (t,s) \in E_L} \lambda_{ts}^{uv} = \begin{cases} 
\lambda_{uv}, & \text{if } u=s \\
-\lambda_{uv}, & \text{if } v=s \\
0, & \text{otherwise}
\end{cases} \quad (B.1)
\]

\(\forall s \in N_L, (u, v) \in T.\)

2. Working capacity assignment constraint:

\[
\beta_{st} = \sum_{(u,v) \in T} \lambda_{st}^{uv}, \quad \forall (s,t) \in E_L. \quad (B.2)
\]
3. Connectivity constraints:

\[
\sum_{j \text{s.t. } (i,j) \in E_p} f_{ij} - \sum_{j \text{s.t. } (j,i) \in E_p} f_{ji} = \begin{cases} 
1, & \text{if } s=i \\
-1, & \text{if } t=i \\
0, & \text{otherwise}
\end{cases}, \quad \forall i \in N_P, (s,t) \in E_L. 
\]  

4. Disrupted traffic constraints:

\[
\alpha_{ij}^{st} \leq k f_{ij}^{st}, \quad \forall (s,t) \in E_L, (i,j) \in E_P. 
\]

\[
\sum_{j \text{s.t. } (i,j) \in E_P} \alpha_{ij}^{st} - \sum_{j \text{s.t. } (j,i) \in E_P} \alpha_{ji}^{st} = \begin{cases} 
\beta_{st}, & \text{if } s=i \\
-\beta_{st}, & \text{if } t=i \\
0, & \text{otherwise}
\end{cases}, \quad \forall i \in N_P, \forall (s,t) \in E_L. 
\]  

5. Traffic rerouting constraints:

\[
\gamma_{kl}^{st}(ij) \leq k \left\{ 1 - (f_{ij}^{st} + f_{ji}^{st}) \right\}, \quad \forall (s,t) \in E_L, (k,l) \in E_L, (i,j) \in E'_P. 
\]

\[
\sum_{l \text{s.t. } (k,l) \in E_{L'}} \gamma_{kl}^{st}(ij) - \sum_{l \text{s.t. } (l,k) \in E_{L'}} \gamma_{lk}^{st}(ij) = \begin{cases} 
\alpha_{ij}^{st} + \alpha_{ji}^{st}, & \text{if } s = k \\
-(\alpha_{ij}^{st} + \alpha_{ji}^{st}), & \text{if } t = k \\
0, & \text{otherwise}
\end{cases}, \quad \forall k \in N_L, (s,t) \in E_L, (i,j) \in E'_L. 
\]  

6. Spare capacity assignment constraints:

\[
\mu_{kl}^{st} \geq \sum_{(s,i) \in E_L} \gamma_{kl}^{st}(ij), \quad \forall (k,l) \in E_L, (i,j) \in E'_P. 
\]
7. Nonnegativity constraints:

\[ \lambda_{st}^{uv} \geq 0, \quad \forall (u,v) \in T, \ (s,t) \in E_L. \]
\[ \alpha_{ij}^{st} \geq 0, \quad \forall (s,t) \in E_L, \ (i,j) \in E'_P \]
\[ \gamma_{kl}^{st}(ij) \geq 0, \quad \forall (s,t), \ (k,l) \in E_L, \ (i,j) \in E'_P. \] (B.9)

8. Integer flow constraints: \( f_{ij}^{st} \in \{0,1\}. \)

Constraints (B.1) are the flow conservation constraints for routing the traffic demands on the logical topology. Constraints (B.2) state that the working traffic on each logical link is the aggregate of flows from all the traffic demands traversing the link. Constraints (B.3) are the flow conservation constraints for routing the logical topology on the physical topology. Let \( \alpha_{ij}^{st} \) be the amount of working traffic on logical link \((s,t)\) traversing physical link \((i,j)\). Constraints (B.4) ensures that the working traffic on logical link \((s,t)\) can only traverse physical link \((i,j)\) if logical link \((s,t)\) is routed on physical link. The parameter \( k \) is a large constant (i.e. \( k \geq \sum_{(u,v)\in T} \lambda_{uv}^{uv} \)) that ensures the working traffic traversing the physical link is not bounded by 1. Constraints (B.4) and (B.5) together require that the working traffic on the logical link to traverse the same set of the physical links that are used to route the logical link. After physical link \((i,j)\) fails, \( \alpha_{ij}^{st} + \alpha_{ji}^{st} \) is precisely the amount of working traffic lost on logical link \((s,t)\). Let \( \gamma_{kl}^{st}(ij) \) be the amount of rerouted traffic with source \( s \) and destination \( t \) that traverses logical link \((k,l)\) after physical link \((i,j)\) fails. Constraints (B.6) force the rerouted traffic to use only logical links that remain intact after the physical link failure. Constraints (B.7) are the flow conservation constraints for rerouting disrupted traffic on each failed logical link. Constraints (B.8) require the spare capacity on each logical to be sufficient to support the rerouted traffic after any physical link failure.

For end-to-end restoration, constraints 4-7 are replaced by the following four sets of constraints:

4. Disrupted traffic constraints:

\[ \alpha_{st}^{uv}(ij) \leq k \{1 - (f_{ij}^{st} + f_{ji}^{st})\}, \quad \forall (s,t) \in E_L, \ (k,l) \in E_L, \ (i,j) \in E'_P. \] (B.10)
\[ \alpha_{st}^{uv}(ij) \leq \lambda_{st}^{uv}, \quad \forall (s,t) \in E_L, (u,v) \in T, (i,j) \in E'_{EP}. \]  \tag{B.11}

\[ 0 \leq \beta_{ij}^{uv} \leq \lambda^{uv}, \quad \forall (u,v) \in T, (i,j) \in E'_{EP}. \]  \tag{B.12}

\[ \sum_{t \text{ s.t.}(s,t) \in E_L} \alpha_{st}^{uv}(ij) - \sum_{t \text{ s.t.}(t,s) \in E_L} \alpha_{ts}^{uv}(ij) = \begin{cases} 
\lambda^{uv} - \beta_{ij}^{uv}, & \text{if } u=s \\
-(\lambda^{uv} - \beta_{ij}^{st}), & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}, \]  \tag{B.13}

\[ \forall s \in N_L, (u,v) \in T, (i,j) \in E'_{EP}. \]

5. Traffic rerouting constraints:

\[ \gamma_{st}^{uv}(ij) \leq k \{ 1 - (f_{ij}^{st} + f_{ji}^{st}) \}, \quad \forall (s,t) \in E_L, (k,l) \in E_L, (i,j) \in E'_{EP}. \]  \tag{B.14}

\[ \sum_{t \text{ s.t.}(s,t) \in E_L} \gamma_{st}^{uv}(ij) - \sum_{t \text{ s.t.}(t,s) \in E_L} \gamma_{ts}^{uv}(ij) = \begin{cases} 
\beta_{ij}^{uv}, & \text{if } u=s \\
-\beta_{ij}^{st}, & \text{if } v=s \\
0, & \text{otherwise}
\end{cases}, \]  \tag{B.15}

\[ \forall s \in N_L, (u,v) \in T, (i,j) \in E'_{EP}. \]

6. Spare capacity assignment constraints:

\[ \mu^{st} \geq \sum_{(u,v) \in T} \gamma_{uv}^{st}(ij), \quad \forall (s,t) \in E_L, (i,j) \in E'_{EP}. \]  \tag{B.16}

7. Nonnegativity constraints:

\[ \lambda_{st}^{uv} \geq 0, \quad \forall (u,v) \in T, (s,t) \in E_L. \]  \tag{B.17}

\[ \alpha_{st}^{uv}(ij), \gamma_{st}^{uv}(ij) \geq 0, \quad \forall (u,v) \in T, (s,t) \in E_L, (i,j) \in E'_{EP}. \]

The explanation for these sets of constraints are the same as the one provided in Section 3.2.1. The only differences are the additional forcing constraints (B.10) and (B.14), which require the traffic after any physical link failure to be routed only on logical links that remain intact.
Bibliography


