Situational Awareness for a Navy Unmanned Undersea Vehicle

By

Andrew Peter Mierisch

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Author .............................................
Department of Electrical Engineering and Computer Science

William Kreamer
The Charles Stark Draper Laboratory, Inc.
Technical Supervisor

Certified by ..........................................
Professor Leslie Pack Kaelbling
Professor of Computer Science and Engineering, MIT
Thesis Supervisor

Accepted by .........................................
Arthur C. Smith
Chairman, Department Committee on Graduate Students
Situational Awareness for a Navy Unmanned Undersea Vehicle

by

Andrew Peter Mierisch

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Abstract

This thesis presents a new approach for Navy unmanned undersea vehicles (UUVs) to determine if a contact in the vehicle’s operating environment has counter-detected it. This approach uses contact tracking information from an Extended Kalman Filter to analyze the behavior of a single contact and determine if the contact has counter-detected the UUV. Basic laws of probability and probabilistic models of potential contact behavior are used to determine the likelihood that counter-detection has occurred at fixed time intervals. Different scenarios are tested through computer-simulations to demonstrate how various behaviors that the contact displayed affected the UUV’s belief that it had been counter-detected. Also, a maneuver decision aid will be presented that can enable the UUV to make intelligent decisions with respect to quickly determining that a contact has been alerted to the presence of the UUV. Various scenarios are tested to show the effectiveness of the maneuver decision aid and its potential value to a UUV during an intelligence/surveillance/reconnaissance (ISR) mission.

Technical Supervisor: William Kreamer
Title: Member of the Technical Staff, The Charles Stark Draper Laboratory, Inc.

Thesis Supervisor: Leslie Pack Kaelbling
Title: Professor of Computer Science and Engineering, MIT
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Andrew Mierisch
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Chapter 1

Introduction

The objective of this thesis is to develop maneuver decision aid algorithms that will enable a United States Navy unmanned undersea vehicle (UUV) to track contacts in its operating environment and determine if counter-detection has occurred while performing intelligence/surveillance/reconnaissance (ISR) missions. The work in this thesis will focus on simulating a realistic tracking algorithm that can be used by the maneuver decision aid algorithms to estimate the state of surrounding contacts and using the laws of probability to develop an effective maneuver decision aid. Various computer-simulated test cases will be investigated to determine the usefulness of the maneuver decision aid algorithms.

1.1 Problem Motivation

Recently, rapid advances in technology have helped produce a budding industry of autonomous vehicles. An autonomous system can perform its designated tasks without the help of a human or other intelligent operator. The appeal of autonomous systems to the military is that they provide humans with a safer and more efficient existence by performing tasks that are either too dangerous, difficult, or tedious for humans. A recent and popular example of an unmanned system is the United States government's use of the Predator, an unmanned airplane. It has been used to perform extended surveillance missions over hostile territory. The Predator has performed well in recent years by providing valuable data and services to commanders in the field of operations, while performing many missions that are very dangerous and too long for a human pilot to carry out. Although it requires a human to remotely operate it, the Predator shows the impact that unmanned vehicles can have on the military [2].
The Navy needs stealthy and unmanned systems to gather information and engage targets in areas where traditional forces are denied entry. Various military threats, as well as diplomatic constraints or rules of engagement may prevent the early entry of overt maritime forces into important areas. Assets are needed that avoid counter detection by the enemy to allow sustained independent operations in these denied regions. With these types of options, military commanders can keep other forces out of harm’s way during the beginning phases of conflict while still being able to prepare and shape the battle space [1].

In the future the United States Navy plans on using unmanned undersea vehicles (UUVs) to play important roles in the battle space. Vital missions including intelligence, surveillance, reconnaissance, mine countermeasures, tactical oceanography, communications, navigation, and anti-submarine warfare can be addressed with UUVs. UUVs are advantageous in these types of missions because they can increase performance, lower cost, and reduce risk to manned systems. UUVs are able to provide these improvements because of their ability to put the following operational advantages to use:

- **Sensor Deployment.** UUVs have the ability to put sensors in an excellent position in both the vertical and horizontal dimensions.

- **Autonomy.** The ability of a UUV to operate independently for extended periods creates a force multiplier that allows manned systems to extend their reach and focus on more difficult tasks. Reduced costs are also a result when sensors and weapons are operated from smaller platforms like UUVs.

- **Risk.** Since UUVs are unmanned, there is a reduced threat to personnel from a harsh sea or enemy combatants.

- **Deployability.** UUVs can be designed as flyaway packages or pre-positioned in forward areas. They can be launched from a wide variety of platforms including ships, submarines, aircraft, and shore facilities. Also, they do not have to be recovered from the same craft they were launched from. Recoveries may be delayed or abandoned because of the expendability created by a UUV’s low cost.
Environmental Adaptability. UUVs can operate in a diverse range of environments including deep to shallow water, foul weather and seas, and tropical or arctic conditions [1].

An unmanned undersea vehicle is defined as a self-propelled submersible whose operation is either fully autonomous or under minimal supervisory control and is untethered except for data links such as a fiber optic cable [1]. The work in this thesis will contribute to the autonomy of an untethered UUV. Humans have traditionally performed tasks such as mission planning in the Navy. The difficulty in creating an effective autonomous system is to translate the thoughts and actions of a human operator into a set of rules and behaviors for an autonomous system to follow.
A study team for the Navy UUV Master Plan [1] established a long-term vision for UUVs by listing priorities for near-term acquisition programs and technology investment while laying a foundation for long-term applications. There was no initial consideration of technical, operational, or fiscal constraints. They first generated a comprehensive list of potential UUV missions by taking advantage of a wide range of current and potential UUV users through field surveys, expert panels, and analysis.

The study team found the number one priority mission to be Intelligence/Surveillance/Reconnaissance (ISR). ISR missions include collection and delivery of various types of data products such as intelligence collection, target detection and mapping data. UUVs are well suited for these types of missions because of their ability to operate at long standoff distances, remain on station for long periods of time, operate independently, and provide a level of stealth unparalleled by other naval platforms. As mentioned previously, UUVs can provide the ability to access previously denied areas and provide information at a lower risk to personnel or higher valued units. Potential ISR missions include:

- Intelligence collection
- Battle damage assessment
- Bio-chemical or nuclear detection and defense
- Ship escort: extended “eyes and ears”
- Search and recover to full ocean depth
- Deployment of leave-behind sensors or sensor arrays
- Underwater security: divers, mines, etc.[1]

After the list of missions was put together, each mission was analyzed for technical feasibility, political acceptability, and operational desirability. Table 1.1 shows the missions that were prioritized by the study team. The feasible and appropriate missions were then grouped according to operational and technological requirements to four signature capabilities. The maritime reconnaissance signature capability, which addresses the ISR missions, was determined to be the top priority capability [1]. The focus of this thesis will be on Maritime Reconnaissance.
1.2 Problem Statement

The objective of Maritime Reconnaissance is to collect multidisciplinary intelligence data across the entire electromagnetic spectrum while remaining undetected by the enemy. UUVs can be launched from safe distances to accomplish these missions in high-risk areas or water that is too shallow for larger, more conventional platforms. Using UUVs for intelligence/surveillance/reconnaissance (ISR) collection can offset reduction in the overall size of the fleet, increase coverage rate and tactical reach, and provide acceptable risk in a hostile area with dynamic threats [1].

1.2.1 Concept of Operations

A general ISR mission for a Navy UUV would begin with the vehicle being launched from a platform such as a submarine, surface ship, aircraft or shore facility. It would then maneuver to a designated observation area, to collect information over a predetermined amount of time. During the mission the UUV could reposition itself to collect additional data or avoid threats. The information collected could immediately be transmitted to appropriate parties or the vehicle could carry the information back to its host platform. While transiting to and from the observation area and collecting data, the
UUV must be able to track, recognize, and avoid mobile threats such as enemy ships in its operating environment [1].

1.2.2 Situational Awareness and Assessment

Charles Stark Draper Laboratory is currently developing a Maritime Reconnaissance Demonstration system. This system will integrate automated situation awareness (SA) with closed-loop planning and control aboard in-water systems. The system will maintain SA for use by the vehicle’s planning system and use it as data that can be drawn upon as needed. This thesis will concentrate on an element of this situation awareness and assessment subsystem known as the SA Assessor (SAA). Situation assessment is the process of determining the impact of the current situation as provided by situation awareness on the vehicle’s plans. The SAA must interpret data received from its sensors to track dynamic obstacles, perform tactical maneuvers, detect hostile actions by dynamic objects, and manage and improve the SA picture it maintains. Of particular interest in this thesis is the UUV’s ability to track contacts in its area and determine if any potentially hostile contacts have been alerted to its presence there (also called counter-detection).

1.3 Problem Approach

A common approach used by military units to accomplish many of its missions is known as the OODA loop. The OODA loop organizes actions performed by the unit into one of four blocks: observe, orient, decide, and act. The “observe” block refers to the ability of the unit to correctly understand information about its state and environment. The “orient” block determines how possible future actions will affect the current situation given by the “observe” block. The “decide” block, using information given by the “orient” block, creates a plan of action. Once the “decide” block generates a plan, the
“act” block executes the details of the plan. As the plan is executed and the situation changes, the “observe” block can detect the need for a new plan and initiate the entire loop again [2] [3].

The OODA loop can also be applied to a UUV maintaining its SA picture with respect to determining if counter-detection has occurred. The UUV must be able to observe the states of contacts in its area and analyze their behavior to detect potentially hostile intent. Using this information, the UUV must then be able to understand what impact various actions will have on its situational awareness and mission accomplishment, decide which actions are best, and then execute its plans. This thesis

![Figure 1.2 OODA Loop](image)

will concentrate on contributing to the observe, orient, and decide phases of Charles Stark Draper Laboratory’s Maritime Reconnaissance Demonstration Situational Awareness Assessor.

The work of this thesis will contribute to the Situational Awareness Assessor by developing a maneuver decision aid that will enable the UUV to determine if it has been counter-detected by contacts in its operating environment. The aid will determine appropriate course and speed commands for the UUV to determine more accurately if surrounding contacts have been alerted to its presence. First a realistic tracking system will be simulated that will estimate the position, course, and speed of contacts in the area

25
of the UUV. Then maneuver decision aids will be developed that will use probabilistic models of hostile and non-hostile contacts to determine if counter-detection has occurred and make course recommendations to improve the SAA knowledge of any possible counter-detection occurrences, while also taking into the consideration the need to arrive at the designated observation area in a timely fashion. This approach potentially can enable the UUV to behave in an intelligent matter. Various test cases will be investigated through computer simulation to evaluate the performance of the maneuver decision aid in combination with realistic tracking algorithms.

1.4 Contributions

The work in this thesis will contribute to future advancement of the maritime reconnaissance signature capability discussed earlier. These contributions are listed below:

1) Artificial Intelligence in Navy UUVs
   - Very little work has been done with any sort of artificial intelligence for Navy UUVs
   - Probabilistic framework will enable UUV to make intelligent decisions for situational awareness.

2) Potential Use in Fleet
   - Using realistic tracking algorithms in simulated test cases will show that the maneuver decision aid algorithms developed in this thesis have the potential to be used in real world applications.

3) Contribution to Draper Laboratory Maritime Reconnaissance Demonstration Project
   - The ideas behind the developed maneuver decision aid algorithms can be used in future integration of automated situation awareness with closed-loop planning and control aboard in-water systems.
1.5 Organization

The remainder of this thesis is divided into five chapters. Chapter 2 investigates the use of Extended Kalman Filtering techniques to develop realistic tracking algorithms. Chapter 3 introduces an algorithm that analyzes the motion of a contact in a UUV’s operating environment to determine if it has been counter-detected. Chapter 4 presents UUV maneuver-decision aids that enable the UUV to quickly determine if a contact has counter-detected it. Chapters 3 and 4 will be the major contributions made by this thesis. Chapter 5 will provide examples of how the algorithm presented in Chapter 3 and the maneuver-decisions aids of Chapter 4 can be used by a UUV during an ISR mission. Chapter 6 provides conclusions and suggestions for future work.
Chapter 2

Extended Kalman Filter Tracking Techniques

In the Maritime Reconnaissance role a UUV must be able to estimate the states of surrounding contacts and know how accurate those estimates are before any maneuver decision aid can determine if the UUV has been counter-detected by any potentially hostile contacts. In the context of this chapter a contact’s state is its position, heading, and speed. Many successful implementations of target tracking algorithms use an Extended Kalman Filter (EKF) for state estimation. This chapter will review EKF algorithms with sonar measurements that will be used in computer-simulated tests of the maneuver decision aid algorithms discussed later in this thesis. For a more detailed description of EKFs refer to Bar-Shalom or Gelb [4] [5].

2.1 Sonar

Sonar is the primary tool used by U.S. Navy undersea platforms to detect and track contacts in their operating environments. The two types of sonar used are active and passive. Active sonars operate by generating a directed sound pulse and measuring the travel time of the reflected return off objects in the environment. The distance to the object, range, can be calculated using knowledge of the speed of sound and the pulse duration. The direction of the object relative to the sensor, bearing, can be determined by knowing where the sound pulse was directed. Inaccuracies can occur in these measurements due to factors such as angular uncertainty created by a finite beam width or a pulse reflecting off multiple surfaces. In contrast to active sonar, passive sonar does not generate sound pulses. It relies on being able to detect acoustic signals created by other objects. Thus passive sonar is only able to produce bearing measurements of surrounding contacts [6].
2.2 EKF Contact Tracking With Active Sonar

An EKF is a minimum mean squared-error recursive filter. The filter uses linearized mathematical models for both the measurement process and target state dynamics to maintain an estimated state vector and a covariance matrix that represents the uncertainties of the estimates and the correlations of those uncertainties. An EKF attempts to minimize the mean squared estimation error in the target's state [7].

This section will discuss how an Extended Kalman Filter can be used with active sonar to track a contact. The contact motion model, observation model, and EKF steps using these models will be reviewed. Work in this section and the remainder of this thesis will be restricted to two-dimensional motion for convenience.

2.2.1 Contact Motion Model

The state estimate for a contact in this implementation can be represented by

\[
X_c[k] = \begin{bmatrix}
x_c[k] \\
y_c[k] \\
\theta_c[k] \\
v_c[k]
\end{bmatrix},
\]

(2.1)

storing the Cartesian north and east coordinates in a locally flat earth reference frame as well as the heading and speed of the contact. There is a command input, \( u_c \), at time \( k \), with time \( \Delta t \) between updates, such that

\[
u_c[k] = \begin{bmatrix}
\varphi_c[k] \\
\gamma_c[k]
\end{bmatrix},
\]

(2.2)
where $\phi_c[k]$, is the commanded heading and $\gamma_c[k]$ is the commanded speed.

A discrete time dynamic model describes the state transition of the contact from time $k$ to time $k+\Delta t$. It can be defined as

$$X_c[k+\Delta t] = f(X_c[k], u_c[k], \Delta t) + w_c[k]. \quad (2.3)$$

The function $f$ represents a nonlinear system that takes as input the contact state, $X_c[k]$, and the command input, $u_c[k]$, at time $k$ and produces the contact state at time $k+\Delta t$, $X_c[k+\Delta t]$, if there was no noise in the contact’s motion. This propagation process can be written as

$$f(X_c[k], u_c[k], \Delta t) = \begin{bmatrix}
x_c[k] + (\Delta t \times \gamma_c[k] \times \cos(\phi_c[k])) \\
y_c[k] + (\Delta t \times \gamma_c[k] \times \sin(\phi_c[k])) \\
\phi_c[k] \\
\gamma_c[k]
\end{bmatrix}. \quad (2.4)$$

Noisy components of the contact’s motion are included in the random vector, $w_c$, which is a $4 \times 1$ vector. This vector is assumed to consist of uncorrelated zero mean Gaussian white noise [6] [7] with covariance

$$Q = E[w_c w_c^T] = \begin{bmatrix}
x_0 & 0 & 0 & 0 \\
0 & y_0 & 0 & 0 \\
0 & 0 & \varphi_0 & 0 \\
0 & 0 & 0 & \gamma_0
\end{bmatrix}. \quad (2.5)$$
2.2.2 Observation Model

An observation model is used to describe measurement of the contact’s position relative to the UUV. In this example using active sonar, the measurements are the range and bearing of the contact relative to the UUV. At time $k$ a measurement of the contact produces

$$z[k] = \begin{bmatrix} r[k] \\ \beta[k] \end{bmatrix},$$

which consists of the relative range and bearing respectively. A model of the observation function is given by

$$z[k] = h(X_v[k], X_c[k]) + w_o,$$

where $X_c[k]$ is the state of the contact at time $k$ and

$$X_v[k] = \begin{bmatrix} x_v[k] \\ y_v[k] \\ \theta_v[k] \\ v_v[k] \end{bmatrix},$$

is the state of the UUV at time $k$. The function $h$ gives the range and bearing.
measurements if no noise is in the observation and can be written as

\[
h(X_v[k], X_c[k]) = \begin{bmatrix} \sqrt{(x_v[k] - x_c[k])^2 + (y_v - y_c[k])^2} \\ \theta_v[k] - \arctan \left( \frac{y_v[k] - y_c[k]}{x_v[k] - y_c[k]} \right) \end{bmatrix}.
\] (2.9)

Noisy components of the measurements are included in the random vector, \( w_0 \), which is a 2 X 1 vector. This vector is assumed to consist of uncorrelated zero mean Gaussian white noise with covariance

\[
R = \begin{bmatrix} \tau_w & 0 \\ 0 & \theta_w \end{bmatrix}.
\] (2.10)
R will be referred to as the observation error covariance matrix where $\tau$ and $\theta$ are the variances of the range and bearing measurements respectively. These values can be chosen properly with knowledge of how the modeled sensors perform in real world environments [6] [7].

### 2.2.3 EKF Algorithm

Using the contact motion and observation models presented in the sections 2.2.1 and 2.2.2, an EKF can maintain a single state vector representing the estimates of the contact’s position, course, and speed. An associated covariance matrix, which contains the uncertainties and correlations of the estimated states, is also maintained. This section will review the EKF algorithm steps in tracking a single contact with active sonar.

We can assume that at time $k$ there is an initial estimate of the contact’s state, $\hat{x}_c[k]$, and a covariance matrix $P[k]$. The “hat” denotes that a term is an estimate. The state estimate is of the form

$$\hat{X}_c[k] = \begin{bmatrix} \hat{x}_c[k] \\ \hat{y}_c[k] \\ \hat{\theta}_c[k] \\ \hat{v}_c[k] \end{bmatrix}. \quad (2.11)$$

The estimated covariance matrix can be expanded as

$$\hat{P}[k] = \begin{bmatrix} \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 \\ \sigma_x^2 & \sigma_y^2 & \sigma_y^2 & \sigma_y^2 \\ \sigma_y^2 & \sigma_y^2 & \sigma_y^2 & \sigma_y^2 \\ \sigma_y^2 & \sigma_y^2 & \sigma_y^2 & \sigma_y^2 \end{bmatrix}. \quad (2.12)$$
The terms on the main diagonal of the covariance matrix represent the filter’s estimate of the variance of the error in each state variable estimate. Each off-diagonal element is the covariance of the respective state variable estimation errors. Maintaining estimates of the covariance values is important for two reasons. First, information gained on one state variable can improve estimates of other variables. Also, the covariance estimates keep the state estimates from becoming overconfident. If all the assumptions and models used in the EKF are valid, it is guaranteed that the estimates of the state estimate errors are consistent with the actual errors [6]. At time \( k+\Delta t \), a new estimate of the current state and covariance matrix can be formed using the measurement produced at that time and the previous state estimate and covariance matrix produced at time \( k \). The steps involved in forming these new estimates are described next.

The first step in updating the state estimate at time \( k+\Delta t \) is to predict what the contact state will be based on the previous estimate. In this review, the new state is predicted assuming the contact will continue on the previously estimated course and speed. The contact state can be predicted according to the equation

\[
\hat{X}_c[k + \Delta t] = f(\hat{X}_c[k], \hat{u}_c[k], \Delta t),
\]

(2.13)

where the \( f \) is the equation referred to in Equation 2.4 and \( \hat{u}_c[k] \) is the estimated contact command input that can be expanded as

\[
\hat{u}_c[k] = \begin{bmatrix} \hat{\theta}_c[k] \\ \hat{v}_c[k] \end{bmatrix}.
\]

(2.14)

The superscript "-'" in Equation 2.13 and later equations will serve as a reminder that the term is the best estimate prior to a measurement being taken.

A predicted covariance matrix, \( P[k+\Delta t] \) is also produced according to the form
\[
P^{-}[k + \Delta t] = F_c[k]\hat{P}[k](F_c[k]^T) + Q .
\]

\(F_c[k]\) is the Jacobian of the discrete time dynamic model function, \(f\) (Equation 2.4), with respect to the estimated contact state at time \(k\), and is defined as

\[
F_c[k] = \left[ \begin{array}{cccc}
1 & 0 & -\Delta t \times \hat{v}_c[k] \times \sin(\hat{\theta}_c[k]) & \Delta t \times \cos(\hat{\theta}_c[k]) \\
0 & 1 & \Delta t \times \hat{v}_c[k] \times \cos(\hat{\theta}_c[k]) & \Delta t \times \sin(\hat{\theta}_c[k]) \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array} \right] .
\] (2.16)

\(Q\) is the contact motion noise covariance matrix defined in Equation 2.5.

The last prediction made by the filter is a predicted measurement of the contact’s relative range and bearing with respect to the UUV, \(z^{-}[k + \Delta t]\). Equation 2.9 can be used to predict the measurement according to the form

\[
z^{-}[k + \Delta t] = h(\hat{X}_c[k + \Delta t], X_c[k + \Delta t]) .
\] (2.17)

Once these predictions are made, the filter can use the actual measurement produced, \(z[k + \Delta t]\), according to Equation 2.7 to update the estimated contact state and covariance matrix. This process starts by computing the measurement residual, \(r[k + \Delta t]\), which is the difference between the predicted and actual measurements, according to the equation

\[
r[k + \Delta t] = z[k + \Delta t] - z^{-}[k + \Delta t] .
\] (2.18)
The residual covariance, $S[k+\Delta t]$, is then found according to the equation

$$S[k + \Delta t] = H[k + \Delta t]P^{-1}[k + \Delta t](H[k + \Delta t]^T) + R ,$$

where $R$ is the observation error covariance matrix defined in Equation 2.10. $H[k+\Delta t]$ is the Jacobian of the observation model function, $h$ (Equation 2.9), with respect to the predicted contact state, $\hat{X}_c[k + \Delta t]$. It can be computed by the equation

$$H[k + \Delta t] = \left. \frac{\partial h}{\partial \hat{X}_c} \right|_{\hat{X}_c[k + \Delta t]} = \begin{bmatrix}
\frac{\hat{x}_c - x_v}{\sqrt{(\hat{x}_c - x_r)^2 + (\hat{y}_c - y_v)^2}} & \frac{\hat{y}_c - y_v}{\sqrt{(\hat{x}_c - x_r)^2 + (\hat{y}_c - y_v)^2}} & 0 \\
\frac{\hat{y}_c - y_v}{\sqrt{(\hat{x}_c - x_r)^2 + (\hat{y}_c - y_v)^2}} & \frac{\hat{x}_c - x_v}{\sqrt{(\hat{x}_c - x_r)^2 + (\hat{y}_c - y_v)^2}} & 0 \\
\frac{(\hat{x}_c - x_r)^2 + (\hat{y}_c - y_v)^2}{(x_c - x_r)^2 + (y_c - y_v)^2} & \frac{(x_c - x_r)^2 + (y_c - y_v)^2}{(x_c - x_r)^2 + (y_c - y_v)^2} & 0 
\end{bmatrix} .$$

Next the Kalman gain, $K[k+\Delta t]$, for the update of the estimated contact state is defined by

$$K[k + \Delta t] = P^{-1}[k + \Delta t](H[k + \Delta t]^T)S^{-1}[k + \Delta t] .$$

When updating the contact state estimate and covariance matrix, this Kalman gain will minimize the mean squared estimation error [7]. Using the Kalman Gain and the residual, the updated contact state estimate is determined to be

$$\hat{X}_c[k + \Delta t] = \hat{X}_c[k + \Delta t] + K[k + \Delta t]r[k + \Delta t] .$$
The covariance matrix is updated using the Joseph form covariance update to maintain the symmetric nature of $P$. It is defined as

$$\hat{P}[k + \Delta t] = C[k + \Delta t]P[k + \Delta t](C[k + \Delta t]^T) + K[k + \Delta t]R(K[k + \Delta t]^T), \quad (2.23)$$

where $C[k + \Delta t]$ is

$$C[k + \Delta t] = I - K[k + \Delta t]H[k + \Delta t], \quad (2.24)$$

and $I$ is the identity matrix.

### 2.2.4 EKF Example With Active Sonar

Figures 2.2, 2.3, 2.4, 2.5, and 2.6 show the computer-simulated results of a UUV using the EKF algorithm discussed in Section 2.2.3 with active sonar measurements to track a single contact. It was assumed that the state of the UUV was determined by some shipboard inertial navigation system during the 2500-second simulation. Gaussian noise was added to the bearing and range measurements with variances of $(.034 \text{ radians})^2$ and $(25 \text{ meters})^2$ respectively. Noise was introduced to the contact’s motion by adding Gaussian noise to the contact’s heading and speed with variances of $(.017 \text{ radians})^2$ and $(.4 \text{ meters/second})^2$ respectively. These values were chosen because they seemed to be reasonable estimates of the noise values. If more was known of the performance of the contact or UUV’s sensors, then these values could modeled to be more consistent with their actual values. The initial $x$ and $y$ coordinates of the UUV and contact were $(0 \text{ meters}, 0 \text{ meters})$ and $(10,000 \text{ meters}, 5,000 \text{ meters})$ respectively. The filter was given an accurate initial estimate of the contact’s state.

Figure 2.2 shows that the EKF provided an accurate method to estimate the position of the contact. There was some error when the contact maneuvered, but the
estimate quickly converged back to the true state after a maneuver. The errors in the estimates when the contact maneuvered occur because the filter’s model of the contact motion expected the contact to follow a straight trajectory. Figures 2.3 – 2.6 show the errors in the individual contact state variable estimates as well as the 3 sigma or 99% confidence bound for the errors provided by the covariance matrix. The estimation errors were within the bounds except for immediately after maneuvers by the contact occurred. These errors also quickly returned to within the bounds, showing that the covariance matrix produced by the filter provided good estimates of the accuracy of the contact state estimates.

Figure 2.2 Example UUV, True Contact, and Estimated Contact Positions With Active Sonar
Figure 2.3 Example X-Position Error and 3 Sigma Confidence Bound

Figure 2.4 Example Y-Position Error and 3 Sigma Confidence Bound

Figure 2.5 Example Heading Error and 3 Sigma Confidence Bound
2.3 EKF Contact Tracking with Passive Sonar

This section will discuss the difficulties in using passive sonar to track a contact using Cartesian coordinates. It will also discuss a different approach, using modified polar coordinates, that provides improved results when compared to Cartesian coordinates.

2.3.1 EKF Contact Tracking with Passive Sonar and Cartesian Coordinates

Figures 2.7, 2.8, 2.9, 2.10, and 2.11 show the computer-simulated results of a UUV using the EKF algorithm discussed in Section 2.2.3, without range measurements, and passive sonar to track a single contact in Cartesian coordinates. The conditions of the simulation were the same as the conditions used in Section 2.2.4 with active sonar. The filter was initially successful in tracking the contact until the contact’s first maneuver. Once the contact maneuvered at 500 seconds, the estimate of the contact’s state quickly diverged. The filter completely failed in providing an accurate estimate of the contact’s state.
Figure 2.7 Example UUV, True Contact, and Estimated Contact Positions With Passive Sonar and Cartesian Coordinates

Figure 2.8 Example X-Position Error and 3 Sigma Confidence Bound
Figure 2.9 Example Y-Position Error and 3 Sigma Confidence Bound

Figure 2.10 Example Heading Error and 3 Sigma Confidence Bound

Figure 2.11 Example Speed Error and 3 Sigma Confidence Bound
This failure by the Cartesian-coordinate bearings-only EKF can easily be explained. The x-position and y-position components of the contact’s state vector are not observable when only bearing measurements are provided. One bearing observation can be produced by an infinite number of contact locations. As illustrated in Figure 2.12, several different contact trajectories can produce the same series of bearing measurements. This makes it very difficult for the bearings-only EKF to converge on the correct state estimate.

![Figure 2.12 Difficulties In Bearings-Only EKF](image-url)
2.3.2 EKF Contact Tracking with Passive Sonar and Modified Polar Coordinates

Using modified polar coordinates for the state estimate in an EKF with passive sonar provides more observability. A modified polar coordinate contact state vector, \( \hat{X}_{mp}[k] \) consists of bearing rate, range rate divided by range, bearing, and the reciprocal of range. All of these components, except for the reciprocal of range, are observable in bearing measurements. The reciprocal of range is observable if the UUV maneuvers while the contact maintains course and speed. Whenever the contact maneuvers, the UUV must also maneuver to make the reciprocal of range observable again [8].

The modified polar coordinate state of the contact can be estimated using an EKF similar to the one discussed in Section 2.2.3. Given the UUV state, defined by Equation 2.8, and the MPC state estimate of the Contact, \( \hat{X}_{mp}[k] \), the Cartesian coordinates of the contact state can be determined according to the equation

\[
\hat{X}_c[k] = \begin{bmatrix}
\hat{x}_c[k] \\
\hat{y}_c[k] \\
\hat{\theta}_c[k] \\
\hat{v}_c[k]
\end{bmatrix} = \begin{bmatrix}
x_v[k] + \frac{\sin(B[k])}{1/r[k]} \\
y_v[k] + \frac{1}{\cos(B[k])} \\
\tan^{-1}\left(\frac{\text{contyspeed}}{\text{contxspeed}}\right) \\
\sqrt{(\text{contxspeed})^2 + (\text{contyspeed})^2}
\end{bmatrix},
\]
where

\[
\text{contxspeed} = \left( \frac{r[k] \times \sin(B[k])}{1} \right) \left( \frac{\dot{r}[k] \times \cos(B[k])}{1} \right) + (v, [k] \times \cos(\theta, [k]))
\] (2.27)

and

\[
\text{contyspeed} = \left( \frac{r[k] \times \cos(B[k])}{1} \right) \left( \frac{\dot{r}[k] \times \sin(B[k])}{1} \right) + (v, [k] \times \sin(\theta, [k]))
\] (2.28)

Figures 2.13 through 2.21 show the computer-simulated results of a UUV using the EKF algorithm discussed in Section 2.2.3, without range measurements, and passive sonar to track a single contact in modified polar coordinates. The conditions of the simulation were the same as the conditions used in Section 2.3.1, except that the UUV zigzagged to keep all four components of the MPC contact state vector observable. The MPC component of the contact state estimates were transformed to Cartesian coordinates according to Equation 2.26 with the estimated trajectory shown on Figure 2.13. Figures 2.14 through 2.17 show the 3 sigma bounds and estimate errors in modified polar coordinates, while Figures 2.18 through 2.21 show the state estimate errors in Cartesian coordinates. These results show an improvement in performance compared to using Cartesian coordinates in an EKF with passive sonar, but still leave much to be desired in state estimate accuracy.
True Position of Contact, Estimated Position of Contact, and Position of UUV With Passive Sonar

Figure 2.13 Example UUV, True Contact, and Estimated Contact Positions With Passive Sonar and Modified Polar Coordinates

Figure 2.14 Example Bearing Rate Error and 3 Sigma Confidence Bound
Figure 2.15 Example Range Rate/Range Error and 3 Sigma Confidence Bound

Figure 2.16 Example Bearing Error and 3 Sigma Confidence Bound

Figure 2.17 Example 1/Range Error and 3 Sigma Confidence Bound
Figure 2.18 Example X-Position Error

Figure 2.19 Example Y-Position Error

Figure 2.20 Example Heading Error
In conclusion, an EKF can provide a UUV with the capability to estimate the state of a contact and the uncertainty of those estimates. This information can then be used by a UUV to analyze the motion of a contact to determine if counter-detection has occurred. The EKF algorithms discussed in this chapter will be used to provide contact state estimates for the counter-detection belief state (presented in Chapter 3) and maneuver decision aids developed later in this thesis.
Chapter 3

Counter-Detection Recognition

Many possible Maritime Reconnaissance missions for Navy UUVs will require them to avoid counter-detection in regions to which an enemy wants to deny them access, as discussed in Chapter 1. Counter-detection of the UUV by a potentially dangerous contact may cause the mission to fail if the UUV is required to abort the mission upon detection or if the contact engages and captures or destroys the UUV. Thus it is imperative that a UUV is able to accurately determine if counter-detection has occurred and have an estimate of the confidence in that determination. This chapter will present an approach that can be used by a maneuver decision aid in tandem with tracking algorithms like the ones discussed in Chapter 2 to determine if a UUV has been counter-detected by any potentially dangerous contacts. The work in this chapter will be restricted to constant speed motion for the contact and UUV.

3.1 Defining Behaviors

Before one can develop algorithms to recognize counter-detection, one must know what kind of behaviors to look for in contacts that have counter-detected a UUV (also referred to in this thesis as hostile contacts) and ones that have not been alerted to the presence of a UUV (passive contact) in their operating environment. Probabilistic models of hostile and passive contacts can represent these behaviors to determine the likelihood that counter-detection has occurred.

In the work of this thesis a hostile contact will be viewed as a contact that is actively tracking the UUV and thus presents a threat to the UUV. There are three key behaviors that a hostile contact can display. They are:
- Contact continuously changes course towards UUV in response to UUV’s course changes.
- Contact remains in UUV’s baffles (Area directly behind the UUV).
- Contact suddenly changes course towards the UUV after detecting the UUV.

The first two are typical of a contact that has been alerted to a UUV’s presence for some time and is pursuing it. The trajectories of the contact and UUV shown in Figure 3.1 illustrate these two behaviors.

![Position of Contact and UUV](image)

**Figure 3.1 Example #1 of Hostile Behavior**

In this example the contact turns to in response to the UUV maneuvers so that it can continue to follow the UUV. Also the contact remains behind the UUV, revealing the second behavior listed. The behavior illustrated in this example clearly shows that counter-detection has occurred.
The third behavior is typical of a contact that has recently been alerted to a UUV's presence and has changed course to pursue the UUV. This type of behavior is illustrated in Figure 3.2. In this example the contact is moving away from the UUV until it suddenly turns toward the UUV as if it has recently determined there is a suspicious presence in the area.

On the other hand a passive contact in the context of this thesis is a contact that is not actively tracking the UUV and thus does not present a threat to the UUV. A passive contact is not likely to make sudden maneuvers towards the UUV or try to follow the vehicle. It will tend to keep a constant course for extended periods of time, with some random course changes. These course changes will not be affected by the location of the UUV, since the contact is unaware of its presence. This type of behavior is illustrated in Figure 3.3. In this example there is no behavior that indicates that the contact has been
alerted to the UUV’s presence. The contact makes several course changes, but never moves towards the UUV.

### 3.2 Counter-Detection Belief State Update

The intentions of a contact are not completely observable to the UUV. The UUV can only make observations of the contact’s physical behavior and attempt to estimate the likelihood that counter-detection has occurred. Using its understanding of how both hostile and passive contacts behave, the UUV can try to infer the contact’s intent. Also, the UUV needs some method to summarize its counter-detection observations of the contact that will allow it to define and periodically update the UUV’s uncertainty of whether counter-detection has occurred. Maintaining a counter-detection belief state can accomplish this.
The counter-detection belief state is the UUV's estimate of the probability that a contact has counter-detected it. It is updated periodically after a fixed time frame passes, given the estimated physical states of the contact at the beginning and end of the time frame provided by tracking algorithms such as the ones discussed in Chapter 2. The belief state can be updated, using Bayes’ rule [9], as

\[
b_{k+w} = \frac{b_k \times f_{\text{hostile}}(\hat{X}_c(k+w) | \hat{X}_c(k), X_v[k])}{(b_k \times f_{\text{hostile}}(\hat{X}_c(k+w) | \hat{X}_c(k))) + ((1-b_k) \times f_{\text{passive}}(\hat{X}_c(k+w) | \hat{X}_c(k)))},
\]

where \(b_k\) is the belief state at time \(k\) and \(w\) is the fixed time frame that passes between updates. \(f_{\text{hostile}}\) is the probability density function (PDF for short) of the estimated contact state at time \(k+w\) seconds, \(\hat{X}_c[k+w]\), given the estimated contact state at time \(k\), \(\hat{X}_c[k]\), and assuming the contact is hostile. \(f_{\text{passive}}\) is the probability density function of the estimated contact state at time \(k+w\) seconds, given the estimated contact state at time \(k\) and assuming the contact is passive. These functions will be defined in Sections 3.3 and 3.4. Using Bayes’ rule in this application allows the UUV to infer the contact’s counter-detection state, by observing its movements. If the hostile and passive PDFs are modeled accurately, then this provides the best method to estimate the likelihood that counter-detection has occurred.

### 3.3 Hostile PDF Model

The function \(f_{\text{hostile}}\) provides the joint PDF for a contact’s Cartesian coordinates and heading at time \(k+w\) seconds, \(x_c[k+w], y_c[k+w], \) and \(\theta_c[k+w]\), given the contact is hostile and the estimates at time \(k\), \(\hat{x}_c[k], \hat{y}_c[k], \) and \(\hat{\theta}_c[k]\). Speed is not taken into account since we are only dealing with constant speed motion in this thesis.
This joint PDF is a nonnegative function that represents the likelihood of the contact’s state at time $k+w$ if the contact is hostile. For a more detailed description of PDFs refer to [9].

The PDF, $f_{\text{hostile}}$, assumes that the contact will make at most one course change over the two minute interval. It attempts to capture the hostile behaviors discussed in Section 3.1 by assuming there is a high probability, $P(\text{host}_\text{man})$, of maneuvering towards the UUV. There is also a lower probability, $P(\text{random}_\text{man})$, of making some random maneuver. $f_{\text{hostile}}$ is defined according to Equation 3.2 by using the Total Probability Theorem, to account for the possibility that the contact can maneuver at any time during the time frame of length $w$, and conditioning on the event of a hostile or random maneuver [9].

$$f_{\text{hostile}}(\hat{X}_c[k+w] | \hat{X}_c[k], \hat{X}_v[k]) =$$

$$\sum_{t=0}^{t=w-1} (f_{x_c, y_c | \theta_c, k+t} (\hat{X}_c[k+w] | \hat{X}_c[k], \hat{X}_v[k]) \times f_{\hat{\theta}_c[k+w] | \hat{\theta}_c[k], \hat{X}_c[k], \hat{X}_v[k]} (\hat{\theta}_c[k+w], \hat{X}_c[k], \hat{X}_v[k]) \times (P(\text{host}_\text{man})/w))$$

$$+ \sum_{t=0}^{t=w-1} (f_{x_c, y_c | \theta_c, k+t} (\hat{X}_c[k+w] | \hat{X}_c[k], \hat{X}_v[k]) \times f_{\hat{\theta}_c[k+w] | \hat{\theta}_c[k], \hat{X}_c[k], \hat{X}_v[k]} (\hat{\theta}_c[k+w], \hat{X}_c[k], \hat{X}_v[k]) \times (P(\text{random}_\text{man})/w))$$

is the PDF of the estimated $x$ and $y$ Cartesian coordinates of the contact at time $k+w$ seconds, given the contact’s estimated state vector at time $k$, the contact’s estimated heading at time $k+w$, and assuming the contact changed course at time $k+t$. $f_{\hat{\theta}_c, \text{host}_\text{att} \text{at} \text{t}, k+t}$ is the PDF of the contact’s estimated heading at time $k+w$, given that the contact chooses to make a hostile course change at time $k+t$. $f_{\hat{\theta}_c, \text{rand}_\text{att} \text{at} \text{t}, k+t}$ is the PDF of the contact’s estimated heading at time $k+w$, given that the contact chooses to make a random course change at time $k+t$. These distributions are defined in Sections 3.3.1, 3.3.2, and 3.3.3.
### 3.3.1 Hostile Course Change PDF

The function \( f_{\theta_{H\text{host}_t\text{at}_{k+w}}} \) is the PDF of the estimated contact heading at time \( k+w \), given that the contact maneuvered in an aggressive manner at time \( k+t \). It is a triangular shaped distribution centered on the course that would point the contact directly at the UUV at time \( k \), which will also be referred to as the hostile heading or \( \theta_H \). The distribution is defined such that the contact’s heading at time \( k+w \) would point the contact within approximately 3000 meters of the UUV (illustrated in Figure 3.4) or within an angle with a width of 1.57 radians (90 degrees) centered around \( \theta_H \), whichever creates a smaller angular width. This angular distribution width will be referred to as \( \theta_E \). It can be explicitly defined as

\[
\theta_E = \begin{cases} 
\arctan(3000/\text{Range}) & \text{if } \arctan(3000/\text{Range}) < 1.57 \text{ Radians (90 Degrees)}, \\
1.57 \text{ Radians (90 Degrees)} & \text{otherwise}
\end{cases}
\]  

(3.3)

![Figure 3.4 Illustration of \( \theta_E \) for Hostile Course Change PDF](image)
The function, $f_{\theta_{H|host_{alt+1}}}^{\theta_{E|host_{alt+1}}}$, can be defined as

$$f_{\theta_{H|host_{alt+1}}}^{\theta_{E|host_{alt+1}}} = \begin{cases} \frac{1}{\theta_{E}} \left( \frac{1}{\theta_{E}} \right)^{\left( \theta_{E} \right)} \left( \frac{\theta_{E} - \theta_{H}}{\theta_{E}} \right) & \text{if } |\theta_{E} - \theta_{H}| < \theta_{E} \cdot \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \quad (3.4)$$

Figure 3.5 shows the shape of $f_{\theta_{H|host_{alt+1}}}^{\theta_{E|host_{alt+1}}}$, with $\theta_{E} = 0.785$ radians (45 degrees) and $\theta_{H} = 0$ radians. This graph shows that $\theta_{H}$ is the most likely course for a hostile contact, with the likelihood of a subsequent course decreasing the further it is from $\theta_{H}$.

![Plot of Hostile Course Change PDF](image)

**Figure 3.5 Hostile Course Change PDF With $\theta_{E} = 0.785$ Radians (45 Degrees) and $\theta_{H} = 0$ Radians**
3.3.2 Hostile Random Course Change PDF

The function \( f_{\theta_{h}, \text{rand at } k+1} \) is the PDF of the estimated contact heading at time \( k+w \), given that the contact maneuvered in a random manner at time \( k+t \). It accounts for the possibility of a hostile contact not maneuvering directly towards the UUV over the fixed time frame. This PDF can be viewed as a two-tiered uniform distribution, centered on \( \theta_h \), which is illustrated in Figure 3.6 and defined as

\[
f_{\theta_{h}, \text{rand at } k+1}(\theta_{h}, \text{rand at } k+1, \hat{X}_{h}, X_{h}, [k]) = \begin{cases} 
0.796178 & \text{if } -\pi + \theta_h \leq \hat{\theta}_h [k+120] \leq \left( -\frac{\pi}{2} \right) + \theta_h \\
3 \times 0.796178 & \text{if } \left( -\frac{\pi}{2} \right) + \theta_h \leq \hat{\theta}_h [k+120] \leq \left( \frac{\pi}{2} \right) + \theta_h \\
0.796178 & \text{if } \left( \frac{\pi}{2} \right) + \theta_h \leq \hat{\theta}_h [k+120] \leq \pi + \theta_h
\end{cases}
\]  

(3.5)

Defining \( f_{\theta_{h}, \text{rand at } k+1} \) in this manner means that if a hostile contact decides to make a maneuver that is not directly pointed at the UUV, it is three times more likely to move towards the general vicinity of the UUV than away from it during the time interval.

![Figure 3.6 Distribution of Hostile Random Course Change With \( \theta_h=0 \) Radians](image)

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3.3.3 Position Distribution

The contact’s expected state at time k+w seconds, \( \hat{X}_c[k + w] \), and expected covariance matrix, \( P^{-}[k + w] \), can be determined given the following values:

- The estimated contact state vector at time k, \( \hat{X}_c[k] \).
- The contact state estimate error covariance matrix, \( P[k] \).
- The contact changed course at time k+t.
- The course the contact changed to at time k+t, which is assumed to be \( \hat{\theta}_c[k+120] \).

The following series of equations will illustrate this. First, the contact’s expected state immediately before changing course at time k+t, \( \hat{X}_c[k + t] \) is determined according to the equation

\[
\hat{X}_c[k + t] = f \left( \hat{X}_c[k], \begin{bmatrix} \hat{\theta}_c[k] \\ \hat{\nu}_c[k] \end{bmatrix}, t \right),
\]  

(3.6)

where \( f \) is the discrete time dynamic model function defined in Section 2.2.1 and Equation 2.4. The expected covariance matrix at time k+t can be calculated as

\[
P^{-}[k + t] = F_c[k, t] P[k][F_c[k, t]]',
\]  

(3.7)

\( F_c[k, t] \) is the Jacobian of the discrete time dynamic model function and is similar to the Jacobian discussed in Section 2.2.3 and Equation 2.16. It can be defined as
In equation 3.9, the (w-t) term is used because that is the amount of time between when the contact changes course at time k+t and reaches its end state in the time frame at time k+w. The expected covariance matrix at time k+w can be calculated as

\[ P^-[k+w] = \hat{F}_c[k+w](w-t)P^-[k+t](\hat{F}_c[k+w](w-t))^T, \quad (3.10) \]

where \( \hat{F}_c[k+w](w-t) \) is defined as
\[ F_c[k + w](w - t) = \frac{\partial f}{\partial \hat{X}_c} \hat{X}_c[k + w] \]  

\[
\begin{bmatrix}
1 & 0 & -(w - t)\hat{v}_c[k + w]\times\sin(\hat{n}_c[k + w]) & (w - t)\times\cos(\hat{n}_c[k + w]) \\
0 & 1 & (w - t)\hat{v}_c[k + w]\times\cos(\hat{n}_c[k + w]) & (w - t)\times\sin(\hat{n}_c[k + w]) \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]

\( \hat{X}_c[k + w] \) and \( P^{-}[k + w] \) can be described as the expected contact state and state estimate error covariance matrix at time \( k+w \) if it maintains course \( \hat{n}_c[k + w] \) and speed \( \hat{v}_c[k + w] \) from time \( k+t \) until time \( k+w \).

Figures 3.7 through 3.11 display the expected contact state and 3 sigma or 99% confidence region for a contact’s expected x and y Cartesian coordinates provided by the expected state estimate error covariance matrix at time \( k+w \) given the initial state estimate error covariance matrix listed below each figure, the time frame, \( w \), is 120 seconds, and that the estimated contact motion follows the shown trajectory. It is worth noting how various changes in the initial state estimate error covariance matrix at time \( k \) produce corresponding changes in the expected covariance matrix at time \( k+w \). In each of these figures the initial estimated contact position is at the origin.
Figure 3.7 Expected Contact X and Y Coordinates 3 Sigma Confidence Region With

\[
P[k] = \begin{bmatrix} 4000 & 0 & 0 & 0 \\ 0 & 4000 & 0 & 0 \\ 0 & 0 & \left(1 \times \frac{\pi}{180}\right)^2 & 0 \\ 0 & 0 & 0 & (4)^2 \end{bmatrix}
\]

Figure 3.8 Expected Contact X and Y Coordinates 3 Sigma Confidence Region With

\[
P[k] = \begin{bmatrix} 5 \times 4000 & 0 & 0 & 0 \\ 0 & 4000 & 0 & 0 \\ 0 & 0 & \left(1 \times \frac{\pi}{180}\right)^2 & 0 \\ 0 & 0 & 0 & (4)^2 \end{bmatrix}
\]

Figure 3.9 Expected Contact X and Y Coordinates 3 Sigma Confidence Region With

\[
P[k] = \begin{bmatrix} 4000 & 0 & 0 & 0 \\ 0 & 5 \times 4000 & 0 & 0 \\ 0 & 0 & \left(1 \times \frac{\pi}{180}\right)^2 & 0 \\ 0 & 0 & 0 & (4)^2 \end{bmatrix}
\]

Figure 3.10 Expected Contact X and Y Coordinates 3 Sigma Confidence Region With

\[
P[k] = \begin{bmatrix} 4000 & 0 & 0 & 0 \\ 0 & 4000 & 0 & 0 \\ 0 & 0 & 5 \times \left(1 \times \frac{\pi}{180}\right)^2 & 0 \\ 0 & 0 & 0 & (4)^2 \end{bmatrix}
\]
The contact's estimated x and y Cartesian coordinates at time $k+w$ can be assumed to have a bivariate normal (or Gaussian) probability distribution function, given the contact's estimated state at time $k$, the contact changed course at time $k+t$, and the contact's estimated heading at time $k+w$. The expected contact state and state estimate error covariance matrix derived above are also used to define this distribution. For more information on bivariate normal PDFs refer to [10]. This distribution of the contact's estimated x and y Cartesian coordinates at time $k+w$ is defined as

$$f_{x, y|\hat{x}_k, \hat{y}_k, k+t}(\hat{X}_d[k+w], \hat{\theta}_d[k+w], \hat{\theta}_e[k]) = \frac{1}{(2\pi)^\frac{3}{2}\sqrt{|G|}} \times \exp \left[ -\frac{1}{2} (r-\mu)^T \times G^{-1} \times (r-\mu) \right]. \quad (3.12)$$
where

\[
G = \begin{bmatrix}
\left(\sigma_{\hat{x}_c, x_c}^2\right)^2 & \left(\sigma_{\hat{y}_c, x_c}^2\right)^2 \\
\left(\sigma_{\hat{x}_c, y_c}^2\right)^2 & \left(\sigma_{\hat{y}_c, y_c}^2\right)^2
\end{bmatrix}
\] (3.13)

contains the expected state estimate error covariance values of the contact’s x and y coordinates provided by the expected covariance matrix \( P^{-}[k + w] \),

\[
\mu = \begin{bmatrix}
\hat{x}_c[k + w] \\
\hat{y}_c[k + w]
\end{bmatrix}
\] (3.14)

is a 2 x 1 vector containing the x and y coordinates of the expected contact state at time \( k+w \), and

\[
r = \begin{bmatrix}
\hat{x}_c[k + w] \\
\hat{y}_c[k + w]
\end{bmatrix}
\] (3.15)

is the 2 x 1 vector containing the x and y coordinates of the estimated contact state at time \( k+w \).
3.3.4 Example Plot of Hostile PDF

Figure 3.12 represents the likely resulting Cartesian coordinates of a hostile contact at time $k+w$ if $w = 120$ seconds and the contact's state at time $k$ is

$$X_c[k] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ \pi/2 \text{ Radians} \\ 12 \text{ Meters/Second} \end{bmatrix},$$

and the UUV's state at time $k$ is

$$X_s[k] = \begin{bmatrix} 8000 \text{ Meters} \\ 0 \text{ Meters} \\ 0 \text{ Radians} \\ 10 \text{ Meters/Second} \end{bmatrix}.$$
The figure gives some idea of how the hostile pdf, \( f_{\text{hostile}} \), developed earlier in this section is structured. The plot illustrates that the most likely resulting Cartesian coordinates of a hostile contact in this case, represented by the peak in the graph, are the ones where the contact turns toward the UUV during the two minute interval.

### 3.4 Passive PDF Model

The function \( f_{\text{passive}} \) provides the joint PDF for a contact’s Cartesian coordinates and heading at time \( k+w \) seconds, \( x_c[k+w], y_c[k+w], \) and \( \theta_c[k+w] \), given the contact is passive and the estimates at time \( k \), \( \hat{x}_c[k], \hat{y}_c[k], \) and \( \hat{\theta}_c[k] \). The passive
PDF model is built in a similar manner to the hostile PDF model, except that it attempts to capture the passive behaviors described in Section 3.1. The reason for the similarities is so that the hostile and passive pdf values determined after each time interval of length $w$ can easily be compared using Bayes' Rule, as in Equation 3.1, to determine what sort of behavior the contact is following. Like in the hostile PDF model, speed is not taken into account since we are only dealing with constant speed motion in this thesis. This joint PDF is a nonnegative function that represents the likelihood of the contact's estimated state at time $k+w$ if the contact is passive.

The PDF, $f_{\text{passive}}$, assumes that the contact will make at most one course change over the time interval. It attempts to model the passive behaviors discussed in Section 3.1 by assuming there is a high probability, $P(\text{no\_man})$, of the contact making no significant course change over the time interval. There is also a relatively lower probability, $P(\text{maneuver})$, of making some random maneuver. $f_{\text{passive}}$ is defined according to Equation 3.16 by using the Total Probability Theorem, to account for the possibility that the contact can maneuver at any time during the fixed time frame, and conditioning on the events of the contact making no maneuver or a random maneuver [9].

\[
\begin{align*}
  f_{\text{passive}}(\hat{X}_c[k+w]|\hat{X}_c[k],X_\gamma[k]) &= \\
  \sum_{t=0}^{w-1} (f_{\hat{X}_c[k+w]|\hat{X}_c[k],X_\gamma[k]}(\hat{X}_c[k+w]|\hat{X}_c[k],X_\gamma[k]) &\times f_{\hat{\theta}_c[k+w]|\hat{\theta}_c[k],\hat{X}_c[k],X_\gamma[k]}(\hat{\theta}_c[k+w]|\hat{\theta}_c[k],\hat{X}_c[k],X_\gamma[k]) &\times P(\text{maneuver}/w)) \\
  &+ f_{\hat{X}_c[k+w]|\hat{\theta}_c[k+w],\hat{X}_c[k]}(\hat{X}_c[k+w]|\hat{\theta}_c[k+w],\hat{X}_c[k]) &\times f_{\hat{\theta}_c[k+w]|\hat{\theta}_c[k+w],\hat{X}_c[k],X_\gamma[k]}(\hat{\theta}_c[k+w]|\hat{\theta}_c[k+w],\hat{X}_c[k],X_\gamma[k]) &\times P(\text{no\_man})
\end{align*}
\]

$f_{\hat{x}_c,\hat{y}_c|\hat{\theta}_c,\hat{X}_c[k]}$, defined in Section 3.3.3, is the PDF of the estimated $x$ and $y$ Cartesian coordinates of the contact at time $k+w$ seconds, given the contact's estimated state vector at time $k$, the contact changes course at time $k+t$, and the contact's estimated heading at time $k+w$. $f_{\hat{\theta}_c|\text{man\_at\_t+k}}$ is the PDF of the contact's estimated heading at time $k+w$, given that the contact chooses to makes a significant random course change at...
time \( k+t \). \( f_{\hat{\theta}_c | \text{no_maneuver}} \) is the PDF of the contact's estimated heading at time \( k+w \), given that the contact chooses not to make a significant course change during the time frame, \( w \). The latter two distributions are defined in Sections 3.4.1 and 3.4.2.

### 3.4.1 Passive Random Course Change PDF

The function \( f_{\hat{\theta}_c | \text{man_at}_k+t} \) is the PDF of the estimated contact heading at time \( k+w \), given that the contact maneuvered in a random manner at time \( k+t \). Like the hostile random course change PDF defined in Section 3.3.2, this PDF can be viewed as a two-tiered uniform distribution, centered on the estimated contact heading at time \( k \), \( \hat{\theta}_c[k] \), which is illustrated in Figure 3.13 and defined as

\[
f_{\hat{\theta}_c | \text{man_at}_k+t} (\hat{\theta}_c[k + w], \hat{X}_c[k], \hat{X}_c[k]) = \begin{cases} 
0.0796178 & \text{if } -\pi + \hat{\theta}_c[k] \leq \hat{\theta}_c[k + w] \leq \left( \frac{\pi}{2} \right) - \hat{\theta}_c[k] \\
3 \times 0.0796178 & \text{if } -\pi + \hat{\theta}_c[k] \leq \hat{\theta}_c[k + w] \leq \left( \frac{3\pi}{2} \right) + \hat{\theta}_c[k] \\
0.0796178 & \text{if } \left( \frac{3\pi}{2} \right) + \hat{\theta}_c[k] \leq \hat{\theta}_c[k + w] \leq \pi + \hat{\theta}_c[k]
\end{cases}
\]  
(3.17)

Defining \( f_{\hat{\theta}_c | \text{man_at}_k+t} \) in this manner means that if a passive contact decides to make a maneuver, it is three times more likely to move in the same direction as before than turn completely around during the time frame of length \( w \).
3.4.2 Non-Significant Course Change PDF

The function $f_{\theta_c,\text{no-man}}(\theta_c)$ is the PDF of the estimated contact heading at time $k+w$, given that the passive contact did not significantly change course during the time frame of length $w$. It is a triangular shaped distribution centered on the contact’s estimated course at time $k$, $\hat{\theta}_c[k]$. Similarly to the hostile course change PDF, the width of this distribution, $\theta_E$, is defined as in Section 3.3.1. The function $f_{\theta_c,\text{no-man}}(\theta_c)$, can be defined as

$$f_{\theta_c,\text{no-man}}(\theta_c[k+w],\hat{X}_c[k],X_c[k]) = \begin{cases} \frac{1}{\theta_E} \left( \frac{1}{\theta_E} \right)^\top \left( \hat{\theta}_c[k+w] - \hat{\theta}_c[k] \right) & \text{if } |\hat{\theta}_c[k+w] - \hat{\theta}_c[k]| < \theta_E \cr 0 & \text{otherwise} \end{cases}$$ (3.18)
3.4.3 Example Plot of Passive PDF

Figure 3.14 represents the likely resulting Cartesian coordinates of a passive contact at time $k+w$ if $w = 120$ seconds and the contact’s state at time $k$ is

$$X_c[k] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ \frac{\pi}{2} \text{ Radians} \\ 12 \text{ Meters/Second} \end{bmatrix}.$$
The figure gives some idea of how the passive pdf, \( f_{\text{passive}} \), is structured. The plot illustrates that the most likely resulting Cartesian coordinates of a passive contact in this case are a result of the contact not deviating significantly from his original course at time \( k \) during the two minute interval.

### 3.5 Example Cases

The following example simulations illustrate how a UUV can maintain the counter-detection belief state using the formulation presented in Sections 3.2, 3.3, and 3.4. The values in the hostile pdf model, defined in Equation 3.2, for \( P(\text{host\_man}) \) and \( P(\text{rand\_man}) \) were .73 and .27 respectively. The values in the passive pdf model, defined in Equation 3.16, for \( P(\text{maneuver}) \) and \( P(\text{no\_man}) \) were .61 and .39 respectively. Each example simulated the UUV and a single contact maneuvering in an open area for 2500 seconds. The belief state was updated every 120 seconds \((w = 120)\) and the initial belief state was \( b_0 = .5 \). Also, in each simulation the speed of the UUV was 10 meters/second and the speed of the contact was 12 meters/second. The initial values for the courses of the UUV and contact are measured counter-clockwise from the positive x-axis. If a passive contact was simulated, then the contact was free to maneuver randomly. In the case of a hostile contact, the contact moved towards the position of the UUV. The UUV was given random course commands in all of the simulations. All of the examples simulated the UUV tracking the contact with active sonar and the EKF described in Section 2.2. The asterisks in the trajectory plots are spaced 120 seconds apart to give a better understanding of the locations of the UUV and contact at various times in the simulation.
3.5.1 Example Simulation 3.1 (Passive Contact)

Figures 3.15 through 3.18 show the results of a simulation with a passive contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad X_c[0] = \begin{bmatrix}
5000 \text{ Meters} \\
5000 \text{ Meters} \\
3.92 \text{ Radians (225 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}.
\]

For the first few 120 second intervals the belief state stayed around .5 because of the difficulty in determining if counter-detection has occurred in the initial moments of the simulation. This difficulty arose because the contact’s initial course pointed it directly towards the UUV. In this situation it becomes difficult to differentiate between hostile and passive behavior because the contact can continue on its original course and still pursue the UUV. Figure 3.16 shows that as the UUV was able to move away from the contact, the belief state dropped when the contact did not maneuver to follow the UUV. The belief state went to 0 when the contact turned away from the UUV. Quick analysis of Figure 3.17 shows that the passive pdf values were higher when the contact maintained course and dropped the two times that the contact changed course. Figure 3.18 shows that the hostile pdf values initially were similar to the passive values as the contact moved towards the UUV, but dropped as it moved away, causing the belief state to correctly go to zero.
Figure 3.15 Example Simulation 3.1 UUV and Contact Positions

Figure 3.16 Example Simulation 3.1 Belief States

Figure 3.17 Example Simulation 3.1 Passive PDF Values

Figure 3.18 Example Simulation 3.1 Hostile PDF Values
3.5.2 Example Simulation 3.2 (Hostile Contact)

Figures 3.19 through 3.22 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix},
X_c[0] = \begin{bmatrix}
-18000 \text{ Meters} \\
2000 \text{ Meters} \\
0 \text{ Radians} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

In this simulation, the belief state gradually rose to approximately .75 as the contact maneuvered to its right to initially pursue the UUV. Once the contact was forced to turn completely around to follow the UUV, the belief state quickly rose to 1. Of note here is how a drastic course change by the hostile contact caused the belief state to move more quickly to 1 when compared to the initial gradual course changes of the contact.
3.5.3 Example Simulation 3.3 (Initially Passive Contact With Counter-Detection Occurring During Simulation)

Figures 3.23 through 3.26 show the results of a simulation with initial UUV and contact states being

\[ X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} 8000 \text{ Meters} \\ -4000 \text{ Meters} \\ 1.57 \text{ Radians (90 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix}. \]
Figure 3.23 Example Simulation 3.3 UUV and Contact Positions

Figure 3.24 Example Simulation 3.3 Belief States

Figure 3.25 Example Simulation 3.2 Passive PDF Values

Figure 3.26 Example Simulation 3.2 Hostile PDF Values
In this simulation the contact was initially passive, until it switched to hostile behavior after 1000 seconds to simulate the occurrence of counter-detection. The belief state was given a floor of .01 in this simulation. Initially, the UUV was very confident that the contact was passive. At the 1000 second mark the belief state jumped to .4 after the contact suddenly turn toward the UUV. After that point the belief state continued to gradually rise as the contact continued to maneuver toward the UUV.

3.5.4 Example Simulation 3.4 (Hostile Contact)

Figures 3.27 through 3.30 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
\begin{bmatrix}
0 & \text{Meters} \\
0 & \text{Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 & \text{Meters/Second}
\end{bmatrix}, \quad
\begin{bmatrix}
4000 & \text{Meters} \\
1000 & \text{Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 & \text{Meters/Second}
\end{bmatrix}.
\]

In this simulation the contact started on a course where it could trail the UUV without changing course, making it difficult initially for the UUV to determine if counter-detection had occurred. The belief state remained close to 0.5 until the hostile contact had to turn sharply to follow the UUV. Once the contact changed course significantly the belief state jumped to approximately 0.9 and eventually went to 1.
Figure 3.27 Example Simulation 3.4 UUV and Contact Positions

Figure 3.28 Example Simulation 3.4 Belief States

Figure 3.29 Example Simulation 3.4 Passive PDF Values

Figure 3.30 Example Simulation 3.4 Hostile PDF Values
3.5.5 Example Simulation 3.5 (Passive Contact)

Figures 3.31 through 3.34 show the results of a simulation with a passive contact and initial UUV and contact states being

\[
x_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix},
\]

\[
x_c[0] = \begin{bmatrix}
4000 \text{ Meters} \\
1000 \text{ Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 \text{ Meters/ Second}
\end{bmatrix}.
\]

The initial conditions for this simulation were the same as for Example Simulation 3.4, except that the contact was passive in this example. Similarly to Example 3.4 the contact started on a course where it could trail the UUV without changing course, making it difficult initially for the UUV to determine if counter-detection had occurred. The belief state remained close to 0.5 until the UUV turned and the contact did not maneuver to follow the UUV. As the contact maneuvered away from the UUV, the belief state went to zero.
Plot of UUV and Contact Position

- UUV
- Contact

Figure 3.31 Example Simulation 3.5 UUV and Contact Positions

Figure 3.32 Example Simulation 3.5 Belief States

Figure 3.33 Example Simulation 3.5 Passive PDF Values

Figure 3.34 Example Simulation 3.5 Hostile PDF Values
3.5.6 Example Simulation 3.6 (Hostile Contact)

Figures 3.35 through 3.38 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad
X_c[0] = \begin{bmatrix}
10000 \text{ Meters} \\
10000 \text{ Meters} \\
3.93 \text{ Radians (225 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}.
\]

The contact’s initial course was directly towards the UUV. During the simulation the contact was able to stay on a near constant course with some slight maneuvering towards the end of the 2500 seconds. Because of this the belief state slowly drifted upwards and was slightly above .8 after 2500 seconds. Because the contact made no significant course changes, it was difficult for the UUV to determine whether the contact was passive or hostile.
Figure 3.35 Example Simulation 3.6 UUV and Contact Positions

Figure 3.36 Example Simulation 3.6 Belief States

Figure 3.37 Example Simulation 3.6 Passive PDF Values

Figure 3.38 Example Simulation 3.6 Hostile PDF Values
3.6 Summary

The example simulations shown in Section 3.5 show how the counter-detection belief state formulation discussed in Sections 3.2 through 3.4 can enable a UUV to determine if a contact has counter-detected it. How the belief state behaves is determined by the structure of the passive and hostile probabilistic models. In the context of this thesis, if a contact does not maneuver towards the UUV, then the belief state will quickly go to zero. Conversely, the belief state goes towards one when a contact maneuvers towards the UUV. The behavior of the contact affects how quickly the belief state goes to zero or one. In situations like Example Simulations 3.4, 3.5, and 3.6, where the contact could maintain its initial course and still pursue the UUV, the UUV was slow in determining if counter-detection occurred. The belief state moved quickly to zero or one only when the UUV turned and forced the contact to significantly change course to follow or observed a passive contact continue on its original course. In other simulations where the occurrence of counter-detection was not so ambiguous, the UUV was able to quickly assess whether counter-detection had occurred or not. The belief state went to zero quickly for a passive contact that never maneuvered towards the UUV, as in Example Simulation 3.1. Sudden hostile maneuvers, such as in Example Simulation 3.2 and 3.3, caused the belief state to quickly move to one.

Chapter 4 will present various algorithms that will recommend courses for the UUV to follow that will enable the UUV to quickly determine whether counter-detection has occurred. The algorithms will exploit the knowledge of how passive and hostile contacts behave, as defined in this chapter, to force a contact to display clear hostile or passive behavior.
Chapter 4

Counter-Detection Maneuver Decision Aids

Examples from Chapter 3 showed that there are cases where it can difficult for the UUV to determine if counter-detection has occurred if the contact can pursue the UUV without making significant course changes. This chapter will present maneuver-decision aids that will enable a UUV to quickly determine if a contact has counter-detected it. These aids will use the knowledge of how passive and hostile contacts behave, as defined in Chapter 3, to move the belief state quickly to zero or one.

4.1 Entropy

Before an attempt can be made to develop a maneuver-decision aid to enable a UUV to quickly determine if counter-detection has occurred, one must be able quantify uncertainty. A common method to accomplish this in information theory is to determine a belief state’s entropy. In the context of this thesis entropy can be looked at as a measure of the uncertainty in the UUV’s counter-detection belief state. If the belief state at time $t$ is $b_t$, then its entropy can be defined as

$$
\text{entropy } (b_t) = \text{g}(b_t) = -b_t \times \ln(b_t) - (1 - b_t) \times \ln(1 - b_t).
$$

The graph of this function is shown in Figure 4.1. In this case the natural logarithm was used, but it is also common for $\log_2$ to be used.
As can be seen in Figure 4.1 the entropy of the belief state goes to zero as the belief approaches zero or one. As the belief state approaches .5, the most uncertain value for the belief state, the entropy approaches a maximum value of .7. The maneuver decision aids presented in the next section will utilize the entropy function by attempting to minimize the entropy of the belief state with each action. By minimizing the entropy, a UUV will also try to push the belief state to zero or one and thus become certain of whether or not counter-detection has occurred. For a more detailed explanation of entropy, refer to [11],[12].
4.2 Maneuver Decision Aid Formulation

4.2.1 Decision Space

Since this thesis works with constant-speed motion, the counter-detection maneuver decision aids presented in this chapter will give a recommended course and amount of time to remain on that course. The recommended course and time duration will be the ones deemed to minimize the objective function \( f_{\text{ENT}} \), which will be defined in Section 4.2.2, among all potential course and time durations examined. The suggested action can also be viewed as the action that the UUV believes will minimize the counter-detection belief state’s entropy in the future.

4.2.2 Objective Function

A potential action, \( a_i(k) \), that can be taken by the UUV at time \( k \) can be defined as

\[
a_i(k) = \begin{bmatrix} \theta_i[k] \\ t_i[k] \end{bmatrix},
\]

where \( \theta_i[k] \) is the potential course and \( t_i[k] \) is the potential time duration of that course command. The expected UUV state after this action is taken, \( X_{\text{future},i} \), can be calculated according to the equation

\[
X_{\text{future},i} = f(X_v[k], \begin{bmatrix} \theta_i[k] \\ v_v[k] \\ t_i[k] \end{bmatrix}),
\]

(4.3)
where $f$ is the discrete time state transition dynamic model function defined in Section 2.2.1 and Equation 2.4, $X_v[k]$ is the state of the UUV at time $k$, and $v_v[k]$ is the speed of the UUV at time $k$.

At time $k$ the contact can also choose to follow course $\theta_c$ and its expected state at time $k+w$ ($w$ is fixed time frame defined in Chapter 3) given that chosen course can be calculated as

$$X_{\text{future}} = f(X_c[k], \begin{bmatrix} \theta_c \\ v_c[k] \end{bmatrix}, w), \quad (4.4)$$

where $\hat{X}_c[k]$ is the estimated state of the contact at time $k$, and $v_c[k]$ is the speed of the contact at time $k$. Given that the UUV chose action $a_i(k)$ and the contact assumes course $\theta_c$ at time $k$, the expected value of the belief state at time $k+w$ can be calculated as

$$E[b_{k+w}] = b_{\text{update}}(X_{\text{future}}, b_i, \hat{X}_c[k], X_{\text{future}}), \quad (4.5)$$

using the expected UUV and contact states given by Equations 4.3 and 4.4 respectively. $b_{\text{update}}$ is the counter-detection belief-state update equation defined in Section 3.2 and equation 3.1. The entropy of the expected belief state can be determined according to the equation

$$g(b_{\text{update}}(X_{\text{future}}, b_i, \hat{X}_c[k], X_{\text{future}})), \quad (4.6)$$

where $g$ is the entropy function defined in Section 4.1 and Equation 4.1.
Using the above formulations, an objective function can be used to evaluate the usefulness of the UUV choosing action $a_i(k)$ in regards to pushing the belief state to zero or one. This objective function, $f_{\text{ENT}}$, can be evaluated according to the equation

$$
\begin{align*}
f_{\text{ENT}} & \left( a_i \bigg| b_t, X_v[k], \hat{X}_e[k] \right) = \\
& b_t \int_{\theta_e} f_{\theta_e, \text{host}} \left( \theta_e \bigg| \hat{X}_e[k], X_{\text{vfuture},i} \right) \times g(b_{\text{update}} ((X_{\text{cfuture},i}, b_t) \bigg| \hat{X}_e[k], X_{\text{vfuture},i})) \, d\theta_e \\
& + (1 - b_t) \int_{\theta_e} f_{\theta_e, \text{pass}} \left( \theta_e \bigg| \hat{X}_e[k], X_{\text{vfuture},i} \right) \times g(b_{\text{update}} ((X_{\text{cfuture},i}, b_t) \bigg| \hat{X}_e[k], X_{\text{vfuture},i})) \, d\theta_e
\end{align*}
$$

where $b_t$ is the belief state at time $t$, $f_{\theta_e, \text{host}}$ is the distribution of a hostile contact's course as defined in Section 3.3, and $f_{\theta_e, \text{pass}}$ is the distribution of a passive contact's course as defined in Section 3.4. The objective function evaluated for $a_i$ can be interpreted as the expected entropy value at time $k+w$ if the UUV's state was $X_{\text{vfuture},i}$ at time $k$. By evaluating $f_{\text{ENT}}$ for various actions, a maneuver decision aid can analyze how various positions that it could maneuver to would influence the counter-detection belief state. The maneuver decision aids presented in the next section will minimize this objective function over discrete action spaces. By doing this, the decision aids will tell the UUV to maneuver in such a way as to reduce subsequent belief states' entropy and thus quickly determine if counter-detection has occurred. It's expected that the objective function presented above would recommend maneuvers that would enable to the UUV to force hostile contacts to make significant course changes or quickly see that a passive contact has no intentions of pursuing the UUV.
Figure 4.2 illustrates what the function $f_{\text{ENT}}$ looks like. In this particular example the UUV state was

$$X_i[k] = \begin{bmatrix} 0 \\ 0 \\ \pi \text{ Radians (180 Degrees Measured From Positive X - Axis)} \\ 10 \text{ Meters / Second} \end{bmatrix}.$$ 

the estimated contact state was

$$\hat{X}_i[k] = \begin{bmatrix} 4000 \text{ Meters} \\ 10 \text{ Meters} \\ \pi \text{ Radians (180 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix}.$$ 

and the belief state was $b_k=0.5$. The function, $f_{\text{ENT}}$, was evaluated for several values of $a_i$ such that

$$a_i(k) = \begin{bmatrix} n \times 0.085 \text{ Radians (n \times 5 Degrees)} \\ k \times 120 \text{ Seconds} \end{bmatrix} \quad n = 0, 1, 2, ..., 72 \\
\quad \quad \quad k = 1, 2, 3, 4, 5.$$ 

The contours in the plot correspond to the locations that the sets of actions evaluated would have moved the UUV to. These locations were plotted against the expected entropy values, given by the function $f_{\text{ENT}}$, for the corresponding action. The innermost contour represents the shortest time duration of 120 seconds and the outermost contour represents the longest time duration of 600 seconds. From analyzing the figure one can see that the highest values for the expected entropy occur if the UUV chooses to continue
on its original course. This would be expected since the contact could follow on its original course and it would still not be clear whether or not it had counter-

![Plot of Expected Entropy Values](image)

**Figure 4.2 Illustration of f_{ENT}**

detected the UUV. The lowest values for the expected entropy occur if the UUV maneuvers to its right or left or moves to a point directly behind where the contact was headed. In these cases a hostile contact would have to make significant maneuvers in order to pursue the UUV or a passive contact would no longer be following the UUV, thus reducing the uncertainty in the occurrence of counter-detection. The figure shows that the more the UUV can cause a hostile contact to deviate from its original course, the lower the expected entropy will be and the more desirable that action will be for the UUV.
4.3 One Time Horizon Look Ahead Example Simulations

The following sets of example simulations have the same conditions as in the examples in Chapter 3. The belief state was updated after every 120 second time window. After each update, the UUV followed the course recommendations of a maneuver decision aid that minimized the objective function presented in Section 4.2 and expressed in Equation 4.7 over a discrete set of action choices consisting of a course command and a length of time to maintain the commanded course such that

\[ a_i(120 \times k) = \begin{cases} 
  n \times 0.085 \text{ Radians} & n = 0, 1, 2, ..., 72 \\
  120 \text{ Seconds} & k = \text{integer}
\end{cases} \]

In other words, every 120 seconds the belief state was updated and the maneuver decision aid provided a course command that was a multiple of 5 degrees and would be maintained until the next belief state update at the end of the subsequent 120 second time horizon. The results of these simulations will be compared to some of the simulations from Chapter 3 to help determine if the maneuver decisions aids provide course recommendations that are effective in quickly determining if counter-detection has occurred. One would expect the tested maneuver-decision aids to avoid some of the ambiguous situations shown in Chapter 3 by forcing a contact to make significant course changes if the contact wished to pursue the UUV.
4.3.1 Example Simulation 4.1 (Hostile Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.3 and 4.4 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad
X_c[0] = \begin{bmatrix}
-18000 \text{ Meters} \\
2000 \text{ Meters} \\
0 \text{ Radians} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

Figure 4.3 Example Simulation 4.1 UUV and Contact Positions

Figure 4.4 Example Simulation 4.1 Belief States

Figure 4.5 Example Simulation 3.2 UUV and Contact Positions

Figure 4.6 Example Simulation 3.2 Belief States
This simulation had the same initial conditions as Example Simulation 3.2, which is shown in Figures 4.5 and 4.6. In Example Simulation 3.2 the UUV continued on its original course for several minutes and the belief state never reached .75 before 900 seconds. The belief state went to 1 quickly after 1000 seconds because the contact turned around to follow the UUV after closing the initial range from the contact to the UUV. In Example Simulation 4.1 the maneuver decision aid forced the hostile contact to maneuver to its left, pushing the belief state to .95 after 900 seconds. The belief state went to 1 before 1500 seconds when the contact made a loop in following the UUV. Example Simulation 4.1 illustrates how the maneuver decision aid forced the contact to show its hostile intentions early on in the simulation. The contact was forced to significantly deviate from its original course in order to continue to pursue the UUV. Also of note is how the UUV zigzagged at the end of the simulation to force the hostile contact to zigzag with it.

4.3.2 Example Simulation 4.2 (Hostile Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.7 and 4.8 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad
X_c[0] = \begin{bmatrix}
4000 \text{ Meters} \\
1000 \text{ Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

This simulation had the same initial conditions as Example Simulation 3.4, which is shown in Figures 4.9 and 4.10. The maneuver decision aid clearly provided the UUV with a course of action that avoiding any ambiguity in determining if counter-detection occurred. The maneuver decision aid quickly determined that if it could maneuver to its right it would easily be able to identify either clear hostile or passive behavior. If the
contact had remained on its original course after the UUV maneuvered, then it would have easily been determined to be passive. In this case the contact had to basically make a "u-turn" to follow the UUV, making it clear that the contact was hostile and pushing the belief state to 1 after approximately 500 seconds, instead of over 2000 seconds in Example Simulation 3.4.

Figure 4.7 Example Simulation 4.2  
UUV and Contact Positions

Figure 4.8 Example Simulation 4.2  
Belief States

Figure 4.9 Example Simulation 3.4  
UUV and Contact Positions

Figure 4.10 Example Simulation 3.4  
Belief States
4.3.3 Example Simulation 4.3 (Passive Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.11 and 4.12 show the results of a simulation with a passive contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \\
X_c[0] = \begin{bmatrix}
4000 \text{ Meters} \\
1000 \text{ Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

Figure 4.11 Example Simulation 4.3 UUV and Contact Positions

Figure 4.12 Example Simulation 4.3 Belief States

Figure 4.13 Example Simulation 3.5 UUV and Contact Positions

Figure 4.14 Example Simulation 3.5 Belief States
This simulation had the same initial conditions as Example Simulation 3.5, which is shown in Figures 4.13 and 4.14. Similarly to Example Simulation 4.2, the maneuver decision aid quickly decided to turn and see if the contact would follow it. In this case the contact was passive and continued on its original course. By avoiding the unclear events shown in Example Simulation 3.5, the maneuver decision aid enabled the UUV to make a much quicker decision that the contact was passive in Example Simulation 4.3 when compared to Example Simulation 3.5.

4.3.4 Example Simulation 4.4 (Hostile Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.15 and 4.16 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[ X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} 10000 \text{ Meters} \\ 10000 \text{ Meters} \\ 3.93 \text{ Radians (225 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix} \]

This simulation had the same initial conditions as Example Simulation 3.6, which is shown in Figures 4.17 and 4.18. At the end of Example Simulation 3.6 the UUV was only approximately 80% certain that the contact was hostile, because the contact made no significant course changes while pursuing the UUV. The maneuver decision aid in Example Simulation 4.4 clearly provided better performance by avoiding that situation and forcing the hostile contact to constantly maneuver to its right while pursuing the UUV and then zigzag once it closed range with the UUV.
Figure 4.15 Example Simulation 4.4 UUV and Contact Positions

Figure 4.17 Example Simulation 3.6 UUV and Contact Positions

Figure 4.16 Example Simulation 4.4 Belief States

Figure 4.18 Example Simulation 3.6 Belief States
4.3.5 Example Simulation 4.5 (Hostile Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.19 and 4.20 show the results of a simulation with a hostile contact and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad X_c[0] = \begin{bmatrix}
-8000 \text{ Meters} \\
-8000 \text{ Meters} \\
.785 \text{ Radians (45 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

![Plot of UUV and Contact Position](image1)

**Figure 4.19 Example Simulation 4.5 UUV and Contact Positions**

![Plot of Belief That Contact is Hostile](image2)

**Figure 4.20 Example Simulation 4.5 Belief States**

The simulation provides another example of the maneuver decision aid forcing a hostile contact to show its intentions clearly. The UUV forced the contact to maneuver to
its left and pushed the belief state to approximately .85 once the contact closed its range to the UUV. Then the UUV made the contact turn completely around, pushing the belief state to one.

4.3.6 Example Simulation 4.6 (Passive Contact, 1 Time Horizon Maneuver Decision Aid)

Figures 4.21 and 4.22 show the results of a simulation with a passive contact and initial UUV and contact states being

\[
x_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad x_c[0] = \begin{bmatrix} -8000 \text{ Meters} \\ -8000 \text{ Meters} \\ .785 \text{ Radians (43 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix}
\]

![Figure 4.21 Example Simulation 4.6 UUV and Contact Positions](image1)

![Figure 4.22 Example Simulation 4.6 Belief States](image2)
In this simulation the maneuver decision aid attempted to see if the contact would maneuver to its left to pursue the UUV. When the contact did not move towards the UUV and continued on its original course for several minutes the belief state quickly went to zero. Another interesting behavior shown here is once the contact turned away from the UUV, the UUV always maneuvered to a point directly behind the contact to see if it would reverse its course and pursue the UUV.

4.4 1-5 Time Horizon Look Ahead Example

Simulations

The following sets of example simulations have the same conditions as the previous ones, with one difference. After each belief state update, the UUV followed the course recommendations of a maneuver decision aid that minimized the objective function presented in Section 4.2 and expressed in Equation 4.7 over a discrete set of action choices consisting of a course command and a length of time to maintain the commanded course such that

\[ a_i(120 \times k) = \begin{cases} n \times 0.085 \text{ Radians} (n \times 5 \text{ Degrees}) \\ 120 \times j \text{ Seconds} \end{cases}, \quad n = 0, 1, 2, ..., 72 \\
\quad j = 1, 2, 3, 4, 5 \\
\quad k = \text{integer} \]

In other words, every 120 seconds the belief state was updated and the maneuver decision aid either provided a course command that was a multiple of 5 degrees and would be maintained for one through five 120 second time horizons or ordered the previous course command if it was not completed. The results of these simulations will be compared to some of the previous simulations of this chapter. The reason for running the next set of simulation with longer time duration commands was to see if allowing the UUV to plan further into the future would improve the belief state performance. For example, by looking further ahead the UUV might be able to recognize situations where it could force
a hostile contact to make more significant course changes than it could by looking ahead for only a single 120 second commanded course time duration.

4.4.1 Example Simulation 4.7 (Hostile Contact and 1-5 Time Horizon Maneuver Decision Aid)

Figures 4.23 and 4.24 show the results of a simulation with the one to five time horizon aid, a hostile contact, and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad X_c[0] = \begin{bmatrix}
4000 \text{ Meters} \\
1000 \text{ Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

Figure 4.23 Example Simulation 4.7 UUV and Contact Positions

Figure 4.24 Example Simulation 4.7 Belief States

Figure 4.25 Example Simulation 4.2 UUV and Contact Positions

Figure 4.26 Example Simulation 4.2 Belief States
This simulation had the same initial conditions as Example Simulation 4.2, which is shown in Figures 4.25 and 4.26. In Example Simulation 4.7 the UUV chose to maneuver to a point directly behind the contact and force it to turn around. This quickly pushed the belief state to one but there was no improvement over the one time horizon decision aid. At every decision point in Example Simulation 4.7 the maneuver decision aid chose the longest commanded course time duration of five time horizons, but the advantage of maneuvering to locations where the UUV felt it could get a better understanding of the contact's intentions appear to be negated by the time it took to transit to those locations. It is still interesting to note how the UUV forced the contact to turn around twice and then sharply turn twice to continue to pursue it.

4.4.2 Example Simulation 4.8 (Passive Contact and 1-5 Time Horizon Maneuver Decision Aid)

Figures 4.27 and 4.28 show the results with the one to five time horizon aid, a passive contact, and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} 4000 \text{ Meters} \\ 1000 \text{ Meters} \\ 3.26 \text{ Radians (187 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix}
\]
This simulation had the same initial conditions as Example Simulation 4.3, which is shown in Figures 4.29 and 4.30. Once again the one to five time horizon look ahead aid chose the longest possible course command duration at every decision point. Similarly to Example Simulation 4.7 the one to five time horizon decision aid initially attempted to maneuver to a point directly behind the contact to see if it would completely turn around in pursuit of the UUV. In this case the contact moved away from the UUV and the belief state quickly went to zero. Although the one to five time horizon decision aid showed effective behavior in quickly determining if counter-detection occurred, there was once again no improvement over the one time horizon maneuver decision aid.
4.4.3 Example Simulation 4.9 (Hostile Contact, 1-5 Time Horizon Maneuver Decision Aid)

Figures 4.31 and 4.32 show the results with the one to five time horizon aid, a hostile contact, and initial UUV and contact states being

\[ X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} 10000 \text{ Meters} \\ 10000 \text{ Meters} \\ 3.93 \text{ Radians (225 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix} \]

This simulation had the same initial conditions as Example Simulation 4.4, which used a one time horizon look ahead decision aid and is shown in Figures 4.33 and 4.34. Once again there were no improvements in performance provided by the longer look time horizon aid. For a few time steps the 1-5 time horizon maneuver decision aid showed poor performance as the belief state dropped significantly. During this time the UUV was stuck executing a command for an extended amount of time that increased the belief state’s entropy instead of lowering it. Because of the longer time durations, the UUV was unable to quickly adjust and fix the problem.
4.4.4 Example Simulation 4.10 (Passive Contact, 1-5)

Time Horizon Maneuver Decision Aid

Figures 4.35 and 4.36 show the results with the one to five time horizon aid, a passive contact, and initial UUV and contact states being

\[ x_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad x_c[0] = \begin{bmatrix} -8000 \text{ Meters} \\ -8000 \text{ Meters} \\ 0.785 \text{ Radians (43 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix} \]
This simulation had the same initial conditions as Example Simulation 4.6, which used a one time horizon look ahead decision aid and is shown in Figures 4.37 and 4.38. As in the other comparisons of this section, there were no improvements in performance provided the longer look ahead aid.

4.5 Summary

The results shown in this chapter show how the maneuver decision aids presented can use their knowledge of how hostile and passive contacts behave to intelligently maneuver and quickly determine if a contact has counter-detected it by attempting to minimize the entropy of the counter-detection belief state. In the context of this thesis
that knowledge was provided by the probabilistic models of both hostile and passive contacts that were presented in Chapter 3. In all of the simulations of this chapter the maneuver decision aids enabled the UUV to maneuver in such a way that it quickly became clear whether the contact was hostile or passive. By minimizing the entropy of the belief states at each decision point the UUV forced hostile contacts to make significant course changes while pursuing the UUV or saw that passive contacts that continued on their original courses were no longer maneuvering towards the UUV.

Contrasting simulations were run where in some cases the commanded durations of the UUV course commands only lasted for a single 120 second time horizon and in others the durations lasted from 1 to 5 120 second time horizons. It was found that the simulations with the longer duration commands offered no improvements over the single time horizon duration simulations because the advantage of better locations that were farther away from the UUV were negated by the amount of time it took to reach those locations. Also, in one case the performance of the longer duration simulations was poorer because the UUV made a course change that did not lower the belief state entropy and could not adjust to a better course until the time of the commanded duration passed. Chapter 5 will present a way that the maneuver decision aid developed in this chapter can be used in a potential Maritime Reconnaissance mission and provide some examples.
Chapter 5

Intelligence/Surveillance/
Reconnaissance (ISR) Simulation Results

This chapter will present a slightly modified version of the maneuver decision aids presented in Chapter 4 that will show how the counter-detection belief state update algorithm presented in Chapter 3 and the maneuver decision aids presented in Chapter 4 can be used by a UUV’s situational awareness and assessment subsystem (SA Assessor) during an ISR mission. The results of several simulations will be shown to help illustrate their usefulness.

5.1 ISR Concept of Operations

Section 1.2.1 discussed a general ISR mission for a Navy UUV. This mission involved a UUV being launched from a host platform and then maneuvering to a designated observation area to collect information. While transiting to that area, the UUV must be able to track and recognize mobile threats. One part of recognizing mobile threats that pertains to the work in this thesis would be determining if a potentially dangerous contact has counter-detected the UUV. This would be one of the things accomplished by the UUV’s SA Assessor (SAA). As mentioned in Chapter 1 the SAA must interpret data received from its sensors to track dynamic obstacles, perform tactical maneuvers, detect hostile actions, and manage and improve the Situational Awareness picture it maintains. It must be able to do this while also maneuvering to the designated observation area. Of particular interest in this chapter is how the SAA can balance the need to perform tactical maneuvers to determine if counter-detection has occurred with the requirement to transit to the observation area in a timely manner. The maneuver decision aid presented in the next section and the simulations using this aid will illustrate
how the algorithms presented in Chapters 3 and 4 can be used by the SAA to accomplish this balance.

5.2 ISR Maneuver Decision Aid

In order to develop a maneuver decision aid that can balance the requirements of recognizing counter-detection and quickly transiting to an observation point, one must be able to quantify how potential actions can delay the UUV from reaching the observation point as quickly as possible. This can be done in the following manner. Given the Cartesian coordinates of the observation point,

$$X_{OBS} = \begin{bmatrix} x_{OBS} \\ y_{OBS} \end{bmatrix}, \quad (5.1)$$

and the state of the UUV at time $k$,

$$X_v[k] = \begin{bmatrix} x_v[k] \\ y_v[k] \\ \theta_v[k] \\ v_v[k] \end{bmatrix}, \quad (5.2)$$

the minimum time required to transit to the observation point, given that the UUV maintains its speed can be defined as

$$t_{min} = \frac{\sqrt{(x_{OBS} - x_v[k])^2 + (y_{OBS} - y_v[k])^2}}{v_v[k]}, \quad (5.3)$$
Similarly to Chapter 4, a potential action, \( a_i(k) \), that can be taken by the UUV at time \( k \) is defined as
\[
a_i(k) = \begin{bmatrix} \theta_i[k] \\ t_i[k] \end{bmatrix}.
\] (5.4)

The expected state at time \( k + t_i[k] \) if the UUV chooses action \( a_i(k) \) can be calculated according to the equation
\[
X_{\text{future},i} = f(X_{i}[k], \begin{bmatrix} \theta_i[k] \\ V_i[k] \end{bmatrix}, t_i[k]),
\] (5.5)

where \( f \) is the discrete-time state-transition dynamic model function defined in Section 2.2.1 and Equation 2.4. The minimum time required to transit to the observation point, given that the UUV first executes action \( a_i(k) \) and maintains its speed can be defined as
\[
t_{a_i(k)} = t_i[k] + \sqrt{(x_{\text{OBS}} - x_{\text{future},i}[k + t_i[k]])^2 + (y_{\text{OBS}} - y_{\text{future},i}[k + t_i[k]])^2} / v_i[k].
\] (5.6)

A function that can quantify how the potential action can delay the UUV from reaching the observation point as quickly as possible is
\[
f_{\text{time}}(a_i(k), X_{\text{OBS}}, X_{i}[k]) = 0.7 \times \left( 1 - \frac{t_{\text{min}}}{t_{a_i(k)}} \right).
\] (5.7)
The reason for multiplying by .7 is to make the possible values of the function range from 0 to .7, similarly to the entropy objective function presented in Chapter 4. Figure 5.1 shows a graph of the function $f_{time}$ versus possible UUV courses where the Cartesian coordinates of the UUV and observation point are

$$
\begin{bmatrix}
x_v \\
y_v
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad X_{OBS} = \begin{bmatrix} -4000 \\ 0 \end{bmatrix},
$$

and the commanded time duration of the action is fixed at 120 seconds. The minimum of the function occurs at 3.14 radians, which is the course that would move the UUV directly towards the observation point. The function increases as the potential course of the UUV takes it further from the observation point.

![Figure 5.1 Plot of $f_{time}$ for Fixed Time Duration Command of 120 Seconds and UUV and Observation Point Cartesian Coordinates Being (0,0) and (-4000,0) Respectively](image-url)
An overall objective function that evaluates how potential action $a_i(k)$ satisfies the requirements of recognizing counter-detection and quickly transiting to an observation point given the estimated contact state at time $k$, the UUV state at time $k$, the belief state at time $k$, the observation point, and a weighting parameter, $p$, is defined as

$$f_{\text{obj}}\left(a_i(k), X_v[k], b, X_{\text{OBS}}, p\right) = p \times f_{\text{sim}}(a_i(k), X_{\text{OBS}}, X_v[k]) + f_{\text{ENT}}\left(a_i[k], X_v[k], X_c[k]\right),$$  \hspace{1cm} (5.8)

where $f_{\text{ENT}}$ is the entropy-driven objective function presented in Chapter 4. The weighting parameter, $p$, can be interpreted as an adjustable parameter that determines the importance of each of the goals of recognizing counter-detection and quickly transiting to the observation point. A maneuver decision aid that minimizes $f_{\text{obj}}$ with a low value for $p$ would weight its decisions more on how they could influence the counter-detection belief state. Conversely, a high value for $p$ would put more emphasis on quickly moving to the observation point. This value could be determined by factors such as how important it is for the UUV to get to the observation point quickly or how much of a threat the contact poses if it does counter-detect the UUV.

Figures 5.2 and 5.3 show plots of the ISR Maneuver Decision Aid Objective function, $f_{\text{obj}}$, with $p$ values of .1 and .5 respectively. In this particular example the UUV state was

$$X_v[k] = \begin{bmatrix} 0 \\ 0 \\ \pi \text{ Radians (180 Degrees Measured From Positive X - Axis)} \\ 10 \text{ Meters / Second} \end{bmatrix},$$
the estimated contact state was

\[
\hat{X}_c[k] = \begin{bmatrix}
4000 \text{ Meters} \\
10 \text{ Meters} \\
\pi \text{ Radians (180 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix},
\]

and the observation point was

\[
X_{\text{obs}} = \begin{bmatrix}
-4000 \text{ Meters} \\
10 \text{ Meters}
\end{bmatrix}.
\]

The commanded time duration of the course command was fixed at 120 seconds. The contours in the plot correspond to the location that a potential course command would have moved the UUV to. These locations were plotted against the objective function, \( f_{\text{obj}} \), defined in this section. Of note in these plots is how with a lower \( p \) value of .1, the objective function favored recommending that the UUV turn left or right to see if the contact would follow it. With the higher \( p \) value of .5 the objective function favored heading more towards the observation point.
Figure 5.2 Plot of ISR Maneuver Decision Aid Objective Function With $p=0.1$

Figure 5.3 Plot of ISR Maneuver Decision Aid Objective Function With $p=0.5$
5.3 1 Time Horizon ISR Maneuver Decision Aid Simulation Results

The following sets of example simulations have the same motion conditions, tracking conditions, and counter-detection belief state update procedures as in the examples in Chapters 3 and 4, but tests an ISR maneuver decision aid that uses the objective function developed in this chapter. The belief state was updated after every 120 second time window. After each update, the UUV followed the course recommendations of the maneuver decision aid that minimized the objective function, $f_{\text{obj}}$, presented earlier in Equation 5.8 over a discrete set of action choices consisting of a course command and a length of time to maintain the commanded course such that

$$a_i(120 \times k) = \begin{cases} \text{n} \times 0.085 \text{ Radians (n x 5 Degrees)} & \text{n} = 0,1,2,\ldots,72 \\ 120 \text{ Seconds} & \text{k = integer} \end{cases}$$

In other words, every 120 seconds the belief state was updated and the maneuver decision aid provided a course command that was a multiple of 5 degrees and would be maintained until the next belief state update at the end of the subsequent 120 second time horizon. Various weighting parameter values were used to see how the changes affected the behavior of the ISR maneuver decision aid. For all of the simulations the Cartesian coordinates of the observation point were (-15000,0). It would be expected that for small values of the weighting parameter, $p$, the UUV would pay little attention to moving towards the observation point until it was certain of whether the contact had counter-detected it or not. As $p$ became larger the UUV would become less concerned with determining the intentions of the contact and move more directly to the observation point. For intermediate values of $p$ there should be some balance between the two requirements.
5.3.1 Example Simulation Set 5.1 (Hostile Contact and 1 Time Horizon ISR Maneuver Decision Aid)

Figures 5.4 through 5.13 show the results of 5 simulations with the one time horizon ISR aid, a hostile contact, and initial UUV and contact states being

\[
\begin{bmatrix}
0 \\
0 \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad \begin{bmatrix}
4000 \text{ Meters} \\
1000 \text{ Meters} \\
3.26 \text{ Radians (187 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

The values used for the weighting parameter, \( p \), were .1, .5, 1, 2, and 10.

Figure 5.4 Example Simulation Set 5.1 UUV and Contact Positions
\( p = .1 \)

Figure 5.5 Example Simulation Set 5.1 Belief States
\( p = .1 \)

Figure 5.6 Example Simulation Set 5.1 UUV and Contact Positions
\( p = .5 \)

Figure 5.7 Example Simulation Set 5.1 Belief States
\( p = .5 \)
The results of these simulations are similar to what would be expected of them. For low values of $p$, which would cause the UUV to be more concerned with determining if counter-detection occurred, the UUV seemed to maneuver without regard for where the
observation point was initially and tried to force the contact to maneuver and follow it. Once the UUV was certain that the contact had counter-detected it, the UUV proceeded to the observation point. As p took on larger values, the UUV became more concerned with reaching the observation point quickly and it took the belief state longer to reach one than for the cases with lower p values. One interesting behavior shown when the value of p was 2, was how the UUV compromised between forcing the contact to make significant maneuvers to follow and still trying to head to the observation point. The UUV did not travel as far up the positive y-axis, as in the simulations for lower values of p.

5.3.2 Example Simulation Set 5.2 (Hostile Contact and 1 Time Horizon ISR Maneuver Decision Aid)

Figures 5.14 through 5.23 show the results of 5 simulations with the one time horizon ISR aid, a hostile contact, and initial UUV and contact states being

\[
X_v[0] = \begin{bmatrix}
0 \text{ Meters} \\
0 \text{ Meters} \\
3.14 \text{ Radians (180 Degrees)} \\
10 \text{ Meters/Second}
\end{bmatrix}, \quad X_c[0] = \begin{bmatrix}
10000 \text{ Meters} \\
10000 \text{ Meters} \\
3.97 \text{ Radians (225 Degrees)} \\
12 \text{ Meters/Second}
\end{bmatrix}
\]

The values used for the weighting parameter, p, were again .1, .5, 1, 2, and 10.
Figure 5.16 Example Simulation Set 5.2
UUV and Contact Positions
\( p = 0.5 \)

Figure 5.17 Example Simulation Set 5.2
Belief States
\( p = 0.5 \)

Figure 5.18 Example Simulation Set 5.2
UUV and Contact Positions
\( p = 1 \)

Figure 5.19 Example Simulation Set 5.2
Belief States
\( p = 1 \)

Figure 5.20 Example Simulation Set 5.2
UUV and Contact Positions
\( p = 2 \)

Figure 5.21 Example Simulation Set 5.2
Belief States
\( p = 2 \)
The results of these simulations show the tradeoffs in choosing a value for $p$. The lower the value of $p$, the more the UUV maneuvered away from the observation point to force the contact to maneuver in response to it. In the case of $p = .1$ the UUV did not even reach the observation point by the end of the simulation. As $p$ became higher, it took less time for the UUV to get to the observation point, but longer for the belief state to reach one.

5.3.3 Example Simulation Set 5.3 (Passive Contact and 1 Time Horizon ISR Maneuver Decision Aid)

Figures 5.24 through 5.33 show the results of 5 simulations with the one time horizon ISR aid, a passive contact, and initial UUV and contact states being

$$X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} 4000 \text{ Meters} \\ 1000 \text{ Meters} \\ 3.26 \text{ Radians (187 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix}.$$

The values used for the weighting parameter, $p$, were again .1, .5, 1, 2, and 10.
The results of these simulations show similar behavior to the previous set of simulation. There appears to be no difference in the behaviors of Example Simulation Set 5.2 for p values of .1, .5, and 1. The UUV turned to see if the contact would follow it. When the contact continued on its original course and the UUV was certain that counter-detection had not occurred, the UUV proceeded to the observation point. For p = 2 the UUV initially turned to see if the UUV would follow, but didn’t turn as sharply away from the observation point as for the lower values of p. When p was 10 the UUV transited straight to the observation point and it took longer for the UUV to determine the contact was passive.
5.3.4 Example Simulation Set 5.4 (Passive Contact and 1 Time Horizon ISR Maneuver Decision Aid)

Figures 5.34 through 5.43 show the results of 5 simulations with the one time horizon ISR aid, a passive contact, and initial UUV and contact states being

\[ X_v[0] = \begin{bmatrix} 0 \text{ Meters} \\ 0 \text{ Meters} \\ 3.14 \text{ Radians (180 Degrees)} \\ 10 \text{ Meters/Second} \end{bmatrix}, \quad X_c[0] = \begin{bmatrix} -8000 \text{ Meters} \\ -8000 \text{ Meters} \\ .785 \text{ Radians (45 Degrees)} \\ 12 \text{ Meters/Second} \end{bmatrix} \]

The values used for the weighting parameter, \( p \), were again .1, .5, 1, 2, and 10.

![Figure 5.34 Example Simulation Set 5.4 UUV and Contact Positions \( p = .1 \)](image1)

![Figure 5.35 Example Simulation Set 5.4 Belief States \( p = .1 \)](image2)

![Figure 5.36 Example Simulation Set 5.4 UUV and Contact Positions \( p = .5 \)](image3)

![Figure 5.37 Example Simulation Set 5.4 Belief States \( p = .5 \)](image4)
Figure 5.38 Example Simulation Set 5.4
UUV and Contact Positions
p = 1

Figure 5.39 Example Simulation Set 5.4
Belief States
p = 1

Figure 5.40 Example Simulation Set 5.4
UUV and Contact Positions
p = 2

Figure 5.41 Example Simulation Set 5.4
Belief States
p = 2

Figure 5.42 Example Simulation Set 5.4
UUV and Contact Positions
p = 10

Figure 5.43 Example Simulation Set 5.4
Belief States
p = 10
These results show similar behaviors to the previous simulations in this chapter. Although there was little difference in how quickly the belief state went to zero for the different p values, the UUV initially maneuvered more directly towards the observation point as the value of p rose.

5.4 Summary

The results in this chapter illustrate how the algorithms developed in Chapters 3 and 4 could be used in a potential ISR mission while transiting to an observation point. The ISR maneuver decision aid presented in this chapter was able to balance the need to transit to the observation point while also looking to see if a contact has counter detected it. How much balance there is can be determined by the setting the weighting parameter, p, to an appropriate value. That value could be set according to how urgent it was to reach the observation point or how much of a threat is posed by the contact.
Chapter 6

Conclusions and Future Research

This chapter summarizes the contributions of this thesis and presents suggestions for future research related to this thesis.

6.1 Thesis Contributions

This thesis presented decision-making algorithms that can assist a Navy UUV in accomplishing an Intelligence/Surveillance/Reconnaissance (ISR) type mission where it must transit to an observation point to collect intelligence. Chapter 2 reviewed Extended Kalman Filter (EKF) tracking algorithms that could be used in computer simulated-tests to model realistic tracking of contacts in the vicinity of a UUV. An algorithm that periodically updates a UUV’s counter-detection belief state was presented in Chapter 3. The counter-detection belief state is the probability that a contact has detected the presence of the UUV. The algorithm used basic laws of probability to define probabilistic models of hostile (counter-detection occurring) and passive (no counter-detection) contact behavior, analyze the behavior of a contact and update the belief state using Bayes’ rule. The algorithm recognized hostile behavior whenever a contact maneuvered directly towards the UUV and recognized passive behavior whenever the contact appeared to mind to its own business and not move towards the UUV. Chapter 4 presented a maneuver decision aid that enabled the UUV to make intelligent maneuver decisions and quickly recognize if counter-detection had occurred. This was accomplished by recommending maneuvers for the UUV that would force a contact to quickly show its intentions. Finally, Chapter 5 provided examples of how the maneuver decision aid presented in Chapter 4 could be used in an ISR mission for a UUV.
6.2 Future Research

There are several possible avenues for future research related to the work of this thesis. One is to model other potentially hostile behaviors into the probabilistic model of a hostile contact presented in Chapter 3. The algorithms presented in this thesis only looked at whether a contact was moving directly towards it in order to determine if counter-detection has occurred. A hostile contact can pursue the UUV in other manners not investigated in this thesis. Figures 6.1 and 6.2 illustrate this shortcoming. In this example, the contact paralleled the course of the UUV without moving directly towards the UUV. Quick analysis of this situation would probably lead one to suggest that the contact was trailing the UUV, but the UUV failed to recognize this as hostile behavior.

![Figure 6.1 Example Simulation UUV and Contact Locations](image1)

![Figure 6.2 Example Simulation Belief States](image2)

To evaluate the utility of the decision aids presented in this thesis one could interview people with submarine experience in the Navy to see how the UUV maneuvers in the simulations compare to what someone with real world experience would do in those situations if a submarine were involved, instead of a UUV. Other possibilities for future work include investigating how the algorithms presented in this thesis perform in simulations in an environment with obstacles instead of the open environments used in this thesis. One could also look into how the work presented in this thesis could apply to scenarios with multiple contacts present. Another area of interest would be to apply...
alternate objective functions in the maneuver decision aids presented in Chapters 4 and 5. Also, one could look into how combinations of future actions would affect the counter-detection belief state instead of looking at only one leg of a trajectory as was done in this thesis.
Bibliography


