Stabilization of Passively Mode locked
Solid-state Lasers
by use of Electronic Feedback

by
Karen E. Robinson

Submitted to the
Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of
Master of Engineering in Electrical Engineering and Computer Science
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2003

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Abstract

Undesired self-Q-switching is an instability that plagues many modelocked solid-state laser systems, especially those with higher upper-state lifetimes. Electronic feedback control is a powerful technique to suppress this Q-switching instability. In this research we determine the design requirements for an electronic feedback control loop to stabilize general modelocked systems. We then discuss a system stabilized with an electronic control loop: a Nd:YAG laser modelocked with a repetition rate of 83 MHz. We show the broad range of operating powers at which stable mode locked operations is possible, with electronic feedback.

Thesis Supervisor: Franz X. Kärtner
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Acknowledgments

This project was part of a collaboration with High-Q Lasers, GmbH, in Hohenems, Austria. My personal support came from MIT EECS. The idea to use feedback to stabilize these lasers is one Thomas Schibli had over his cornflakes, he says. I was fortunate to be able to work with him on it in Karlsruhe, Germany, and to continue working on it in this thesis work.

The people who introduced me to ultrafast optics were Thomas, and Professor Franz Kaertner, when I worked with them in Karlsruhe. Now, my time in Karlsruhe was wonderful, thanks to the people in the lab there. Wolfgang Seitz and Alexander Blecken helped me most with German. From Wolfgang, Richard Ell, and Phillip Wagenblast I learned the art (and patience) of aligning a laser resonator. And, the people in the Karlsruhe lab all made it a pleasant working and learning environment.

After returning to MIT, I went through Jim Fujimoto’s Nonlinear Optics course with Jade Wang and Juhi Chandalia. This was a good class, and studying with Jade and Juhi made it better – and the intellectual and personal relationships we formed over NLO are invaluable.

Professors Hermann Haus and Erich Ippen are inspiring people to work near, personally and professionally. I hold them both up as personal role models.

Thanks to the Kaertner group, and also to Juhi, Jade, Mihai Ibanescu, Nate Fitzgerald, and Laura Nichols for listening to dry runs for my thesis presentations and offering helpful suggestions. Thanks to Scott Johnston, Aaron Mihalik, and Anne Hunter for the good advice: “Don’t take it personally” and “Turn it in this term.”

Life would be different if I couldn’t zephyr -c help – the people on this class helped me learn about makefiles, latex for slides, pdflatex, gnuplot, advanced matlab, graphics conversion, and other tools I needed to write and present this thesis.

And none of this would be happening, were it not for my mom Pamela J. Robinson, who comes to visit and encourages me personally, and my dad John A. Robinson who first taught me the value of being able to explain science to anyone, and sparked my interest in the world around me with his explanations.
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Chapter 1

Introduction

This work expands on some work done by Thomas R. Schibli in his doctoral work at the University of Karlsruhe, to stabilize solid-state lasers with electronic feedback [6]. A similar problem was explored in Ref. [5]. The goal of this project is to learn how to stabilize mode locked solid-state lasers, using electronic feedback. The experiment are carried out on an Nd:YAG laser, passively mode locked with a semiconductor saturable absorber, operating at wavelength 1064 nm.

The output of the mode locked laser is short optical pulses, each pulse about ten picoseconds in duration. As is often the case with mode locked solid-state lasers, the mechanism used to mode lock the laser (a saturable Bragg reflector) leads to a much longer modulation, in the form of passive Q-switching, over a train of mode locked pulses. The Q-switching is viewed as an instability which we remove by applying an electronic feedback loop to the laser current source.

The idea of applying feedback to stabilize an unstable system is well-understood, but what makes this project interesting from an electrical standpoint is the high speed required to suppress Q-switching and high current required to modulate the laser current source. (See 1.1)

This stabilization allows the laser to be operated at power levels that would otherwise be unstable. It is cheaper to implement than are optical stabilization schemes, and because feedback stabilization is a well-understood concept, it will in general be more straightforward to implement, to a wider range of lasers.
Figure 1-1: A minimal laser resonator, consisting of mirrors and a gain medium.

We anticipate that electronic feedback, like the scheme outlined here, will be used to build various stable mode locked lasers. These lasers, and pulsed lasers in general, are excellent for cutting and boring in a number of materials. Because of the short duration of each pulse, heat does not dissipate far from the bore site, so heat damage to surrounding material is reduced. Pulsed infrared lasers, similar to the one used in these experiments, will also find applications in industry as well as in medicine, possibly for corneal transplants or for drilling hard dental tissue.

1.1 Optical Background

A laser cavity is formed by two end mirrors, between which the laser light resonates. The resonator consists of at least these end mirrors and the gain medium; a minimal resonator is shown in Figure 1-1. In addition to the end mirrors, the resonator studied here includes a gain medium (an optical amplifier), and other optical elements, including other high-reflectivity mirrors (both curved and flat), a saturable absorber to induce mode locking, and an output coupler, a mirror which allows one percent of
incident light to pass through. Each time light makes a round-trip back and forth in
the cavity it passes through the gain medium and is amplified. Some light is coupled
out of the cavity into the output beam on each round trip when it impinges on the
output coupler.

This research concerns solid-state lasers, in which the gain medium is a solid-state
crystal. In this case the solid-state material is Yttrium-Aluminum-Garnet doped with
Neodymium (Nd:YAG).

1.1.1 Gain and Gain Saturation

The energy level in the laser is set by the interaction of the gain from the gain medium
and cavity loss. The gain dynamics can be described by:

\[
\frac{dg}{dt} = -\frac{g - g_0}{\tau_L} - \frac{E_P}{E_{sat,L}T_R}g,
\]

(1.1)

where \(g\) is the saturated gain, \(g_0\) is the unsaturated gain (also called the “small signal
gain”), \(\tau_L\) is the spontaneous lifetime of the gain, \(E_{sat,L}\) is the saturation energy of the
gain, \(T_R\) the time required for light to make one round-trip in the resonator (“round-
trip time”), and \(E_P\) is the energy in the laser resonator. In stable operation we assume
that the energy in the resonator is not changing, and also set the derivative of the
saturated gain to zero. Then the energy \(E_P\) and the saturated gain \(g\) are related by:

\[
g = \frac{g_0}{1 + \frac{E_P}{E_{sat,L}T_R}\tau_L}
\]

(1.2)

Further, in steady state the saturated gain must have the same value as the sum of
the losses in the resonator. Then the cavity energy can be determined:

\[
E_P = \left(\frac{g_0}{l} - 1\right) \frac{E_{sat,L}T_R}{\tau_L}
\]

(1.3)

The gain in the gain medium is provided by a laser diode beam which optically
pumps the solid-state crystal. The small-signal gain parameter \(g_0\) is, to a reasonable
approximation, proportional to the current of the laser diode. In this experiment, this current will be 17-30A.

1.1.2 Coupled Rate Equations

If we do not assume stable operation, we must consider the rate of change in energy in the resonator, \( \frac{dE_p}{dt} \). This quantity is a function of \( g \), so we have coupled rate equations:

\[
T_R \frac{dE_p}{dt} = [g - l - q_p(E_p)]E_p, \\
T_R \frac{dg}{dt} = -T_R \frac{g - g_0}{\tau_L} - \frac{E_p}{E_{sat,L}}g
\]  

The saturable absorption \( q \) in this set of coupled differential equations leads to pulsed behavior of the laser. The second equation in 1.4 is also written using the parameter

\[ r = 1 + \frac{E_p}{E_{sat,L} \tau_L} \]

so \( g = g_0/r \).

We use \( G(g, E_p) \) to represent the net gain of the system, \( [g - l - q_p(E_p)] \). We will need the partial derivatives of net gain by both gain and pulse energy. Now, \( \frac{dG}{dE_p} = \frac{dg}{dE_p} - \frac{dq}{dE_p} \), but the gain saturates slowly after a change in energy, much more slowly than the saturable absorption, or saturable loss. Thus, when we assume a stable system, we use:

\[
\frac{dG}{dE_p} = \frac{dq}{dE_p}.
\]

As for \( \frac{dG}{dg} \), it is approximately 1 for a laser with small bandwidth. Only when the optical spectrum is comparable to the spectrum of the gain transition is \( \frac{dG}{dg} \) different from 1, because then gain filtering, a component of loss \( l \), changes with \( g \). This is important when soliton-like pulse shaping effects are present.

It is instructive to look at a phase portrait of Q-switching, in the gain-energy plane and in the time domain. This is shown in Figure 1-2. On the left sub-figure, the gain rises nearly linearly (which is clear from 1.4 when \( E_p \) is small), then falls

\footnotetext{The pump parameter \( r \) can also be written in terms of power: \( r = 1 + P/P_{sat} \), where \( P = E_p/T_H \) and \( P_{sat} = F_{sat,L}/\tau_L \) is the pump parameter used for non-mode locked lasers.}
suddenly when the energy rises and falls in a quick Q-switched pulse.

To make analysis of these equations more tractable, one usually linearizes the coupled rate equations 1.4 around a stable operating point, power $E_P^0$ and gain $g^0$. Then with the substitutions $E_P = E_P^0 + E_p$, $g = g^0 + g$, the system is (written in matrix form):

$$
T_R \frac{\partial}{\partial T} \begin{pmatrix}
\Delta E_P \\
\Delta g
\end{pmatrix}
= \begin{pmatrix}
-\frac{\partial g}{\partial E_P} E_{P_s} & E_{P_s} \\
-\frac{1}{E_{sat,L}} - \frac{T_R}{\tau_L} - \frac{E_{P_s}}{E_{sat,L}} & -\frac{T_R}{\tau_L}
\end{pmatrix}
\begin{pmatrix}
\Delta E_P \\
\Delta g
\end{pmatrix}
= \mathbf{A}
\begin{pmatrix}
\Delta E_P \\
\Delta g
\end{pmatrix}
$$

(1.6)

This becomes more transparent when we note that $A_{22}$ can be written as $-T_R/\tau_L' = -T_R/(\tau_L/r)$, and $\tau_L' = \tau_L/r$ is the stimulated lifetime of the gain medium (“upper-state lifetime”).

In writing this linearization, one assumes that the products $\Delta g \cdot \Delta E_P$, $(\Delta g)^2$, and $(\Delta E_P)^2$ are small enough to be negligible, and that second derivatives of $G$ with respect to $g$ and $E_P$ are also negligible.
1.1.3 Relaxation Oscillations

When a laser is first turned on or when it is disturbed, the power fluctuates in what are called relaxation oscillations. We can find the frequency of these oscillations in a laser without a saturable absorber, and use this frequency as the upper bandwidth of the gain-energy dynamics. We call the relaxation oscillation frequency $\omega_s$.

The relaxation oscillation frequency is found in Ref. [12] in terms of laser parameters. Here, we follow [8] to find it as the natural frequency of the matrix 1.6 with no saturable absorption - that is, when $dq/dE_P = 0$. These are the zeros of:

$$\begin{pmatrix}
    p & -E_P \\
    \frac{g_s}{E_{sat,L}} & p + \frac{T_R}{\tau_L}
\end{pmatrix},$$

which lie at

$$p = -\frac{1}{2} \frac{T_R}{\tau_L} \pm \sqrt{\frac{4 \tau^2}{\tau_L^2} - \frac{g_s E_P}{E_{sat,L}}}.
$$

We introduce the parameter $\tau_e = T_R/l$, the decay time of photons in the cavity, and since the stable value of the gain $g_s$ equals the loss $l$, we can write $g_s = l = \frac{T_R}{\tau_e}$. Further, $r = 1 + \frac{E_P}{T_R E_{sat,L}}$, so $\frac{E_P}{E_{sat,L}} = (r - 1) \frac{T_R}{\tau_e}$. With these substitutions, we can write the zeros of $p$ as $-\frac{1}{2} \frac{T_R}{\tau_L} \pm \sqrt{\frac{4 \tau^2}{\tau_L^2} - \frac{(r-1) T_R}{\tau_L \tau_e}}$. In lasers prone to Q-switching - those to which this research would be applicable - $\tau_L$ is typically much greater than $T_R$. Then we can neglect the first term inside the square root with respect to the second. The natural frequency is the imaginary part of the zeros of $p$, so

$$\omega_s = \sqrt{\frac{r - 1}{\tau_e \tau_L}},$$

and this agrees with the result in [12] as well.

The frequency of the relaxation oscillations can be considered as the bandwidth of the gain dynamics.

However, it is difficult to know precisely how much loss $l$ is in the laser cavity, or to know precisely the pump parameter $r$. Thus, when we use this expression experimentally, we regard it as only an approximation.
1.1.4 Saturable Absorption

Saturable absorption is loss in the laser resonator which is lower for higher cavity energies. This leads to laser pulsing, as will be described below. We use the following model of saturable absorption:

\[ q = q_0 \frac{1 - e^{-E_p/E_{sat,A}}}{E_p/E_{sat,A}}. \]  

(1.10)

We can use this relationship because the saturable absorbers we use, from High-Q Lasers, GmbH, Vorarlberg, Austria, have saturation time comparable to the spontaneous lifetime of Nd:YAG. Further, we use a fairly small pump parameter, \( r < 5 \), so that the spontaneous and stimulated emission times are of comparable length.

1.2 Brief Introduction to Passive Mode locking

In continuous-wave (cw) operation, the light is evenly distributed over the space in the resonator. When a laser operates in mode locked operation, the light in the resonator is concentrated into a short pulse that travels through the resonator. The pulse contributes to the output each time it is reflected off the output coupler.

For a laser to favor mode locked operation, the losses in the resonator must be lower for a pulse of light than for other spatial distributions of energy in the resonator, such as multiple pulses or continuous light. In practice this is achieved by modulating (in time) the loss or gain of some element in the resonator. In solid-state lasers it is almost always the loss that is modulated, as the gain dynamics are much slower than the duration of a mode locked pulse.

It is possible to mode lock some systems by modulating gain or loss with an external (usually electric or optic) signal; this is referred to as active mode locking. Active mode locking has decreased effectiveness (a falling pulse shortening rate) for shorter pulses, and the modulation frequency must be nearly matched to that of the laser resonator. In passive mode locking schemes, the modulation is a result of the laser pulse itself, rather than an external signal.
There are several schemes to accomplish passive mode locking. This research concerns lasers mode locked by use of a saturable absorber. Under normal circumstances the absorber absorbs incident light, so that very little is reflected. When the incident light has sufficient energy to saturate the absorption, it becomes transparent. This favors laser operation involving high-intensity light, and the result is a pulse of high intensity rather than low-intensity light distributed throughout the resonator.

The Q-switching instability we will suppress accompanies saturable-absorber mode locking. Other mode locking schemes include additive pulse mode locking, which is an older technique often used in fiber lasers, and Kerr lens mode locking, which is used to produce shorter pulses with higher peak intensity than those in the lasers considered here.

1.3 The Self-Q-Switching Instability

If a laser is mode locked, there is one pulse traveling in the cavity, coupled into output once every round-trip. The output of a Q-switched laser is also pulsed, but because the average energy in the cavity is rising and falling. Q-switched pulses are on a time scale much larger than the round-trip time of the resonator.

The quality factor, or Q-factor of a laser cavity is proportional to the energy stored in the cavity, and inversely proportional to cavity losses. Q-Switching is a technique to achieve higher peak energy from a laser than would be possible continuously by switching on and off some of the cavity loss, thus switching the Q-factor low and high. When the loss is high, it exceeds the gain $g$ so no light resonates in the cavity. This allows gain in the gain medium to rise toward its unsaturated value. At some point the loss is reduced below the gain, and the energy in the cavity grows with $e^{g-t}$ until the gain saturates. While the gain is saturating, cavity energy grows beyond its steady-state value.

Considering Q-switching and mode locking, there are four regimes of laser operation, as shown in Figure 1. While mode locked pulses’ pulse durations are in the femtosecond-picosecond range, Q-switched pulses are a few microseconds wide and
occur with a frequency of tens of kilohertz. When a laser is operating in the Q-switched and mode locked regime, the power of the mode locked pulses is modulated by the Q-switching envelope. This research will begin with a laser in Q-switched mode locked regime, and force it to operate in the cw-mode locked regime. In some cases, such as with a saturable absorber mirror, a laser may be self-Q-switched. That is, the saturable absorber dynamics cause the laser to Q-switch erratically. To understand this we must look at the saturable absorber dynamics.

Saturable loss is a decreasing function of pulse energy. If the derivative of this curve is large enough, it may be that the pulses grow to have higher energy than the steady-state value – this is the rise into a Q-switched pulse. This is possible because the gain saturates slowly. But once the higher pulse energies have saturated the saturable loss, the energy rise in the cavity is proportional to $e^{\delta_0}$, which quickly becomes large enough to saturate the gain. Gain saturates below its steady-state value, net gain becomes negative, and intracavity energy falls. In the absence of energy gain rises, and the cycle repeats.

Self-Q-switching can be seen as an exacerbation of the relaxation oscillations, the natural frequency of the system with $dG/dE_p \neq 0$. This is a lower frequency than the relaxation oscillations, and the Q-switching frequency approaches the relaxation oscillations.

Figure 1-3: Four regimes of operation for lasers with saturable absorbers.
oscillation frequency as the laser approaches stability.

Often in undesirable self-Q-switching, the gain saturates very deeply, because the energy in the resonator was very large during the end of the exponential rise. If the saturation is deep enough the energy in the resonator will decrease to zero — no photons will be left resonating — after the Q-switched pulse. When the resonator energy disappears between pulses, Q-switched pulsing is erratic.

It is useful to look at the Q-switched mode locking and cw mode locking regimes in the Fourier domain, shown in Figure 1-4. In the microwave spectrum, mode locking appears as a comb of pulses in the frequency domain; one peak in the frequency comb is shown on the bottom left in Fig. 1-4. When the laser is Q-switched and mode locked, the Q-switching frequency appears as side bands around the mode locked repetition rate.

Stabilization was viewed experimentally on an RF analyzer in the microwave spectrum, and these results are shown in Sec. 3.2.

Figure 1-4: Microwave spectrum of stable (cw) and unstable (Q-switched) mode locking. The frequency axis is linear, centered at $1/T_R$, with width approximately 30 times $\omega_s$, and less than $T_R$. 
1.4 Stability Conditions

We use the linearized rate equation 1.6 to determine the condition for self-Q-switching. This analysis can be found, in more detail, in Ref. [2]. The linearized rate equation is repeated below, in the Fourier domain, where $\tilde{E}_P$ represents the Fourier transform of $E_P$, and likewise for other variables.

$$pT_R \begin{pmatrix} \tilde{E}_P \\ \tilde{g} \end{pmatrix} = \begin{pmatrix} -\frac{g_s}{E_{P_s}} E_{P_s} & E_{P_s} \\ -g_s \frac{1}{E_{sat,L}} & -\frac{T_R}{\tau_L} \end{pmatrix} \begin{pmatrix} \tilde{E}_P \\ \tilde{g} \end{pmatrix} = A \begin{pmatrix} \tilde{E}_P \\ \tilde{g} \end{pmatrix} \quad (1.11)$$

We know from system theory (See Refs [10, 11]) that, for this system to be stable, the matrix $A$ must satisfy both $\det A > 0$ and $\text{Tr} A < 0$. Or,

$$\frac{dq}{dE_P} \frac{T_R}{\tau_L} + \frac{g_s}{E_{sat,L}} > 0 \quad (1.12)$$

Remember that saturable loss decreases with increasing energy, so $dq/dE_P$ is a negative quantity.

The first condition in Eq. 1.12 ("determinant condition") is nearly always satisfied; the challenge is presented by the first, or "trace condition". To see that the determinant condition is trivially satisfied, we rewrite it:

$$\frac{dq}{dE_P} + g_s \frac{\tau_L}{\tau_T E_{sat,L}} > 0. \quad (1.13)$$

Now, it is interesting to differentiate the equation for the stable value of the saturated gain $g_s$, Eq. 1.2, repeated here:

$$g_s = \frac{g_0}{1 + \frac{E_{Ps}}{E_{sat,L}} \frac{T_R}{\tau_L}}$$

$$\frac{dg_s}{dE_{Ps}} = g \frac{\tau_L}{\tau_T E_{sat,L}}. \quad (1.14)$$
Then the determinant condition is just that

\[ \left| \frac{dq}{dE_P} \right| < \left| \frac{dg}{dE_P} \right| \]  \hspace{1cm} (1.15)

This is satisfied for the simple reason that \( g_0 \) must be greater than \( q_0 \), in order for the laser to ever reach threshold, and they must cross in order for there to be an operating point. Saturation curves of \( g_s \) and \( (q + l) \), versus intracavity energy, are shown in Figure 1-5. The Figure shows that Equation 1.15 is satisfied.

In evaluating the trace condition, we will use the derivative of Eq. 1.10:

\[ \frac{dq}{dE_P} = \frac{q_0 e^{-E_P/E_{sat,A}}}{E_P} \left[ 1 - \frac{E_{sat,A}}{E_P} \right] - \frac{q_0 E_{sat,A}}{E_P^2} \approx -\frac{q_0 E_{sat,A}}{E_P^2}. \]  \hspace{1cm} (1.16)

The approximation is valid for \( E_P \gg E_{sat,A} \). The lasers from High-Q Lasers, GmbH typically operate with \( E_P/E_{sat,A} \) between 4 and 6, so the error introduced by using this approximation is less than that due to experimental uncertainties.

One way to build a stable mode locked laser is to tweak the saturable absorber parameters so that the trace condition in Eq. 1.12 is satisfied. This research explores a different approach: we modify Equation 1.11 by using electronic feedback to control the gain \( g \) as a function of the intracavity energy \( E_P \).
Chapter 2

Feedback Requirements and Parameters

2.1 Feedback Stabilization

2.1.1 Linearized Rate Equations

In previous work, linearization of the coupled rate equations 1.4 has often treated $g_0$ as a constant. If we instead assume that $\dot{g}_0 \neq 0$, the linearization is:

\[
pT_R \tilde{E}_P = -\frac{dq}{dE_P} \bigg|_{E_P} \tilde{E}_P + E_P \dot{\tilde{g}}_P
\]

\[
pT_R \tilde{g} = g_s \frac{E_{sat,L}}{E_P} \tilde{E}_P - \frac{T_R}{\tau_L} \tilde{g} + \frac{T_R}{\tau_L} \tilde{g}_0
\]  \hspace{1cm} (2.1)

2.1.2 Feedback Model

This development is mathematically similar to the development in Ref. [6], but more intuitive from a feedback point of view. We write the entire laser system as a system with feedback, as in Figure 2-1.

We now stop using $\sim$ to denote the Fourier transforms, for the sake of brevity. It should be clear from context where the Fourier transform is implied.
To simplify the system representation in Figure 2-1, we will use Black’s formula:

\[ F = \frac{F_F}{1 - F_FF_R}. \]  \hspace{1cm} (2.2)

By applying Eq. 2.2 to the system in Figure 2-1 twice, we find the transfer function \( E_P/g_0 \):

\[ F_F = \frac{E_P}{g_R} = \frac{T_R E_P}{\tau_L p^2 T_R^2 - p^2 T_R^2 \text{Tr}A + \text{det}A}. \]  \hspace{1cm} (2.3)

We can now apply feedback around the system shown in Figure 2-1, to stabilize \( E_P \). This is shown in the diagram in Figure 2-2, simplified in Figure 2-3, and studied below. The feedback approach was first worked on in Ref. [2].

### 2.1.3 Feedback Strength

We begin by assuming that a proportional-differential (PD) controller will stabilize the system, which is believable because the gain medium acts as an integrator. Thus, we use \( F_R = a + bp \).

In order for the system to be stable:

\[
E_P/T_L + (a + pT_R(b/T_R)) \left( p^2 T_R^2 - pT_R \text{Tr}A + \text{det}A \right) \neq 0
\]  \hspace{1cm} (2.4)
Figure 2-2: Feedback model, including electronic feedback $a + bp$

Figure 2-3: Simplified system model, with electronic feedback $a + bp$
for \( p \) with positive real part. This leads to the conditions:

\[
-a < \frac{\tau_L \det A}{T_R E_P} \\
b > \frac{\tau_L \text{Tr} A}{E_P}
\]  

(2.5)

The condition on \( a \) is actually satisfied even for \( a = 0 \) because, as we saw following Eq. 1.12, the determinant of \( A \) is always positive. Thus the circuit required is a pure differentiator.

The coefficient \( b \) must be fairly large. We used Eq. 2.5 for an approximate value of the circuit gain required, then used as much gain in the circuit as was possible without driving other instabilities.

### 2.1.4 Bandwidth of Feedback Loop

In this analysis we have neglected delays in the feedback circuit. In order for delays to be negligible, the bandwidth of the differentiator should be significantly higher than the bandwidth of the laser without feedback. We use the relaxation oscillation frequency as the bandwidth of the laser system. Thus for a feedback circuit which is a differentiator, we set the cutoff frequency of the differentiation to be at least a factor of three higher than the relaxation oscillation frequency.

Feedback circuits will often be applied to a laser that always operates in the Q-switched mode locked regime. The relaxation oscillation frequency is difficult to know precisely, but it is always larger than the Q-switching frequency. To design the feedback control loop for the Nd:YAG, we assumed that the relaxation oscillation frequency was within a factor of 2 of the Q-switching frequency.

This is one feedback circuit specification which becomes more difficult to fulfill as the repetition rate of the laser increases, because the relaxation frequency increases with the square root of the repetition rate.
2.1.5 Modulation Depth of the Feedback Loop

This circuit parameter is the most difficult to grasp analytically, because the clipping that results when the modulation depth is exceeded is highly nonlinear. Thus it has not been included in any detail in previous theoretical calculations. The best way to determine whether a system has high enough modulation depth is by simulation of the coupled rate equations, with feedback.

2.2 Parameters of the Existing System

We tested feedback stabilization on an Nd:YAG laser, passively mode locked with a semiconductor saturable absorber, operating at wavelength 1064 nm, with repetition rate 83 MHz.

We had at our disposal two appropriate saturable absorbers, one with \( q_0 = 2.5\% \), and one with \( q_0 = 0.5\% \). The saturation fluence \( F_{sat,A} \) was quoted by High-Q Lasers as being 0.1 mJ/cm\(^2\), and the cavity was designed with a spot size on the absorber so that \( E_p/area = 0.5 \text{ mJ/cm}^2 \), so \( E_{sat,A} = F_{sat,A} \times area \approx E_p/5 \).

The output coupler has transmissivity 1%, and light encounters it twice per round-trip. We use 230\( \mu \text{s} \) for the upper-state lifetime of Nd:YAG, from Ref. [12].

The output power of the Nd:YAG laser is typically 1 W to 1.5 W, which translates to a pulse energy of \( E_p = power \times time = power/f_{rep} \approx 12nJ \).

2.2.1 Power Curve of the Pump Diode

The power from the laser diode was measured, as a function of diode current, and the result is shown in Figure 2-4. The measurement was done using the Melles-Griot power meter, with only a fraction of the power from the diode falling on the active area of the meter to avoid saturating the meter.

From this plot we see that the threshold current of the diode, \( I_{th,D} \), is 8.6A. If we knew what fraction of the diode power we were measuring, or if we had measured the full power, we could find the slope efficiency from this plot as well.
We did not measure how the pump beam changes with current, which would affect
the beam overlap in the crystal and thus the transfer of energy from the laser diode
to gain.

2.2.2 Conversion of Current to Gain

We assume that after some threshold, there is a linear relationship between supply
current and small-signal gain. That is, we assume constant slope efficiency of
the power curve. This linear relationship is implied by Equation 1.3 and was seen
experimentally, shown in Fig.s 3-7 and 3-8.

We assume that the small signal gain $g_0$ is proportional to the pump energy in the
crystal. Some energy from the diode is lost due to imperfect overlap between the pump
mode and the laser mode, but we assume the usable pump energy is proportional
to the diode energy, which is in turn proportional to the current above the diode
threshold current. Thus $g_0 \propto (I_D - I_{th,D})$.

For the laser to operate, the cavity gain must equal the total cavity loss — this is
clear from the first equation in Eq. 1.4, if not intuitively. If the small signal gain $g_0$
is greater than the cavity loss, the gain saturates according to Eq. 1.2; if $g_0$ is smaller
than the loss the laser does not operate. The laser threshold is the point at which
\( g_0 \) just equals the loss:

\[
g_0 = l. \tag{2.6}
\]

We can find the laser threshold experimentally, so we can use this threshold condition to find the proportionality constant \( \alpha \) between current and laser energy. At threshold:

\[
g_0 = \alpha(\text{\textit{I}}_{\text{th}} - \text{\textit{I}}_{\text{th},D}) = l. \tag{2.7}
\]

The loss \( l \) is the sum of loss \( l_{OC} \) from the output coupler, loss \( l_m \) from the many cavity mirrors, and non-saturable loss \( l_q \) from the saturable absorber. This system uses a 1% output coupler off which the light reflects twice, so \( l_{OC} = 0.02 \). We estimate \( l_m \) to be 0.001 per high-reflectivity mirror, or 0.007. The saturable absorber introduces approximately as much non-saturable loss as saturable loss, so \( l_q = q_0 \).

Then we can find \( \alpha \) for the different absorbers:

\[
\alpha = \frac{0.0027 + q_0}{\text{\textit{I}}_{\text{th}} - 8.6}, \tag{2.8}
\]

where \( q_0 \) is the saturable loss of the absorber being used, and the laser threshold \( \text{\textit{I}}_{\text{th}} \) is easily measurable.

### 2.2.3 Frequency Response of the Pump Diode

For designing this feedback system and for designing future feedback controls, it is useful to know the frequency response of the laser diode. We measured the optical response to a voltage sine wave at the gate of the FET module, and the result is shown in Figure 2-5. The sine wave was provided by the SRS function generator, with a DC bias of 4.2 V and wave amplitude of 1.6V, peak-to-peak. This voltage was connected to the gates of the 4 FETs in the FET / laser diode module.

Note that the diode behaves approximately like a low-pass filter, with cutoff frequency just below 1 MHz. This is above the designed cutoff frequency of the differentiator, so we neglect it calculations in this thesis. It would be significant for higher repetition rates.
2.2.4 Observed Q-Switching Frequency

We observed the Q-switching frequency, as a function of average cavity power. The observations are graphed in Fig. 2-6 and Table 2.1. The highest observed Q-Switching frequency was slightly over 100 kHz.

<table>
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<td>86</td>
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<td>.26</td>
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<tr>
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<td>.87</td>
<td>88</td>
<td>18</td>
<td>.12</td>
<td>61.6</td>
</tr>
</tbody>
</table>

Table 2.1: Q-Switching frequency, variance with operating point.

Figure 2-5: Bode plot of frequency response of the pump diode, with pump current 27A.
2.3 Feedback Implementation: Current Shunt

The mechanism for modulating the current through the pump diode is shown in Figure 2-7. Current is provided by a commercially available laser diode driver (a modified version of a Lambda 6-33), and some is shunted away from the laser diode by one or more MOSFETs, in parallel. The feedback loop controls the current through the MOSFET shunt.

In this configuration, the depth of the current modulation depends on the current-carrying capacity of the MOSFET shunt.

It is also possible to build feedback stabilization for a circuit by building a controllable current source with bandwidth on the order of 3 MHz [2]. The shunt configuration was chosen primarily for two reasons. First, it is safer for the diode, because the FET cannot source current and thus is unlikely to either send reverse current through the diode or to give the diode current high than its specified maximum current. Then, the voltage signal transmitted between the feedback circuit and the MOSFETs is relatively low-power, so does not require the RF shielding that we would need around a high-power signal.
2.4 Q-switching Transients

It is interesting to ask whether a laser system with feedback circuit will Q-switch when it is turned on, before entering the cw mode locking regime. If the cavity is blocked, the gain builds up in the laser crystal, and when the laser turns on this is dumped into at least one Q-switched pulse. If, on the other hand, the laser is turned on by a slow enough current ramp, the laser will mode lock without ever entering the Q-switched mode locked regime. The current supply we have worked with, Lambda 6-33, takes approximately one second to change current levels, which is much slower than any other dynamics in the system. Thus, we would not expect to see Q-switched transients when the laser is turned on or when the current level is changed. We verify this experimentally in 3.4.

2.5 Feedback Parameters

To design feedback loops, we need to write Eq. 2.5 in terms of the electrical gain of the feedback loop and the cutoff frequency \( f_c \) of the differentiation.

The input to the circuit is many times attenuated compared to the optical energy in the resonator. We use quite a number of proportional relationships to estimate
this attenuation.

We used the Silicon amplified photodiode PDA55 from ThorLabs. The PDA55 needs only a small amount of photocurrent, so we used a stray beam which passed through a high-reflectivity mirror ($R \approx 99.9\%$). To avoid saturating the PDA55, we reflected the beam off a glass plate ($R \approx 4\%$), at which point it was approximately 10 times larger, in area, than the photodiode area. Thus the power on the photodiode $P_D$ we approximate as:

$$P_D = (10^{-3})(.04)(0.1)P_{intra} = 4 \times 10^{-6}f_{rep}E_p,$$

where $P_{intra} = E_p \cdot f_{rep}$ is the intracavity power.

The current $I_{PD}$ produced by the PDA55 is, according to its datasheet, $I_{PD} = (0.2A/W)P_D$ at 1064 nm. The transimpedance amplifier in the PDA55 amplifies according to $V_{PD} = (15 \times 10^3V/A)I_{PD}$. The input to the feedback circuit is $V_{in} = V_{PD}$, so

$$V_{in} = (15 \times 10^3V/A)(0.2A/W)(4 \times 10^{-6})f_{rep}E_p = (0.12V/W)f_{rep}E_p$$

The feedback circuit is several amplifiers (amplification $A$), an RC differentiator, and shunt transistors. The voltage transfer function up to the shunt transistors is:

$$\frac{V_{out}}{V_{in}} = A_{p+RC} \frac{p}{f_c+1} = A_{p+RC} \frac{f_c}{f_c+1},$$

where $f_c$ is the cutoff frequency of the differentiator. Then, including the transimpedance $K_Q$ of the transistors, the current modulation $\Delta I_D$ in the diode is

$$\Delta g_0 = \alpha \Delta I_D = \alpha K_Q A \frac{p}{f_c+1} (0.12V/W)f_{rep}E_p$$

In Sec. 2.1.3, we calculated a value for $b$, the differential gain of the feedback from $E_p$ to $g_0$. Now, using Eq. 2.11, we can translate this to the differential gain required from $E_p$ to current modulation $I_D$. 

30
Chapter 3

Experimentation and Results

3.1 Circuit Design

3.1.1 Circuit Parameters

The objective was to build a circuit with cutoff frequency at least half an order of magnitude greater than the highest relaxation frequency $\omega_s$ observed, and with differential gain $b$ satisfying Eq. 2.5.

Shown in Fig. 2-6 and Table 2.1, the highest Q-switching frequency observed was 100.8 kHz. We set the cutoff frequency at 300 kHz. A higher frequency may be more appropriate, but we chose this low cutoff frequency for better stability, to minimize high-frequency noise, and to allow use of lower high-frequency gain (see Sec. 2.5).

3.1.2 Schematic

A block diagram of the laser system with feedback is shown in Figure 3-1.

The first stage of the circuit, for amplification and differentiation, is shown in Figure 3-2. It consists of a tenfold amplifier, an RC differentiator, and another tenfold amplifier. The cutoff frequency of the differentiation is set by adjusting the value of capacitor $C_{diff}$. For $C_{diff}$, we used 27 $\mu$F. Schottky diodes should be placed at the input of the first stage, to protect against voltage spikes from the photodetector. The 50 $\Omega$ resistor at the input is for impedance matching to the BNC cable used to connect
Figure 3-1: Block diagram of experimental setup

Figure 3-2: First Stage: amplification and differentiation
between the photodetector and circuit.

The second stage of the circuit is shown in Figure 3-3. The bottom two amplifier IC's in the circuit set the circuit bias: The second half of the THS6012 IC is a voltage buffer, and the OP1177 is connected as an integrator. The integration frequency is set by $C_{\text{int}}$ and should be several orders of magnitude below the Q-switching frequency. The low pass filter formed by $R_{\text{fil}}$ and $C_{\text{fil}}$ should block any high-frequency signal picked up by the wire between the Laser diode module and the circuit.

The output of the bias branch is a voltage, matched by the voltage $\text{ctrl}$. We used two resistors and a potentiometer to set the value of $\text{ctrl}$, and adjust it during experimentation.

The 100-$\Omega$ and 200-$\Omega$ resistors form an adder, so that the tenfold amplifier built on the THS6012 IC sees a combination of the small signal from the first stage and the bias signal from the lower branch on this schematic. The 27-$\mu$F capacitor is to limit high-frequency noise, and its value was determined experimentally. The 10-$\Omega$ resistor could be called a "good luck resistor," and prevents the impedance of the FET, as seen by the circuit, from being negative. With the input capacitance of the MOSFETs it forms a low-pass filter, but with cutoff frequency high enough to make the filter negligible (30 MHz).
The current shunt is done by FETs which are connected in parallel with the diode inside the laser diode housing. We used the BLF246B FETs, each of which has input capacitance of 125 pF. We used four FETs in parallel, to shunt up to 12A of current. The 30-mΩ resistor is to sense the current passing through the FETs, and the voltage across it is used by the bias setting branch of the second stage of the circuit (Figure 3-3). The connections inside the laser diode housing are shown in Figure 3-4.

3.1.3 Technical Details

The boards were designed using OrCAD with PSpice, were laid out using Eagle, freeware from CadSoft in Pleiskirchen, Germany. They were and printed by PCBexpress, a division of ECD, Milwaukie, OR.

The FETs and the 30-mΩ resistor were screwed to a small slab of aluminum, with indium foil sandwiched between the device and the aluminum. The 30-mΩ resistor was made from many 1.8-Ω .25-Watt SMD resistors, soldered in parallel between two pieces of copper foil. The shielded cable from the circuit to the diode housing was tied to the aluminum slab at two points, providing mechanical stability for the connection between this cable and the gates of the FETs. The aluminum slab was then screwed to the water-cooled laser diode housing, again with indium foil in the junction.

The high-frequency rolloff seen in Fig. 3-5 is caused by the low slew rate of the THS6012 for output voltage steps of a few volts, which is what we require.
3.1.4 Measured Frequency Response

The frequency response of the feedback circuit is shown in Figure 3-5. The bandwidth to a phase shift of $\pi/4$ is 300 kHz, as shown by the dotted line.

3.2 RF Spectra

Figure 3-6 shows suppression of the laser when the HQL24A absorber (with 2.5% saturable absorption) was in the cavity, at a supply current of 28A. In Appendix C are spectra taken at other supply currents and with a different absorber – the HQL5, with saturable absorption $q_0 = 0.5\%$.

3.3 Output Power

Using electronic feedback, we expected to be able to stabilize the laser over its entire operating range. This was in fact the case, for both absorbers tested. Shown in Figs 3-7 and 3-8 are the output powers of the laser versus input power, with dots at points with corresponding spectra in Sec. 3.2.

Figure 3-5: Bode plot of frequency response of the feedback circuit, with $C_{\text{diff}}=27\mu\text{F}$. 

\[ \text{Current [A]} \]

\[ \text{Phase [rad.]} \]

\[ 0.001 \quad 0.01 \quad 0.1 \quad 1 \quad 10 \]

\[ 0.001 \quad 0.01 \quad 0.1 \quad 1 \quad 10 \]

\[ -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]
Experimental Results: 2.5% Absorber

Figure 3-6: Microwave spectrum showing stabilization of the mode locked laser. (Compare to Fig. 1-4)
Figure 3-7: Laser output power vs. input power, with absorber HQL5 ($q_0 = 0.5$). The best-fit line is shown as $f(x)$, and microwave spectra taken at each of the measured points is in Appendix C.

Figure 3-8: Laser output power vs. input power, with absorber HQL24A ($q_0 = 2.5$). Again, $f(x)$ is the best-fit line. Spectra associated with these measured points are not in the appendix, but are similar to those associated with Fig. 3-7.
3.4 Startup Transients

As described in Sec. 2.4, we did not expect to see Q-switching transients when we changed the laser supply current. We tested this by watching the output of the Melles-Griot fast (1.5 GHz) photodiode on the LeCroy oscilloscope in single trigger mode. The oscilloscope triggered when the cavity power rose. We observed stable transitions from currents below the diode threshold to the maximum diode current, and the oscilloscope trace from this test is shown in Figure 3-11.

For reference are oscilloscope traces of stable cw mode locking (Figure 3-9) and of Q-switched mode locking with the feedback circuit off (Figure 3-10).
Figure 3-10: LeCroy oscilloscope trace: Q-switched mode locking

Figure 3-11: LeCroy oscilloscope trace during laser turn-on
Chapter 4

Conclusions and Future Work

4.1 Conclusions

In this research, we have shown that electronic feedback successfully stabilizes the Nd:YAG laser described in Sec. 2.2, over its entire operating range, with two different saturable absorbers. This robustness suggests that feedback is a powerful technique that can be used to stabilize a wide range of lasers which are prone to self-Q-switching.

With the current ramp provided by the Lambda 6-33 current supply, the laser power can be changed while the mode locking remains stable, and the laser can be turned on without the laser Q-switching.

4.2 Future Work

The most immediate interesting work would be to design a circuit with variable gain and variable differentiation cutoff frequency, to determine the optimum frequency response for a feedback circuit. Ref. [16] suggests helpful current-feedback amplifier circuits: the Bourne potentiometer they suggest (#3262) may be useful in making a variable-cutoff differentiation stage. The “Adjustable Gain using a FET” circuit (duplicated in Fig. 4-1) and the “Output Current Booster” — a push-pull output stage — may be useful almost as suggested in the application note, with different devices.
References [15] and [17] are instructive in evaluating current feedback amplifier circuits.

The Nd:YAG laser studied in this thesis can be operated with a repetition rate of approximately 300 MHz, as well as 83 MHz. It should be possible to stabilize this cavity, with not much more work electronically than has already been done. Then, we could determine what feedback gain $b$ and what differentiation cutoff frequency $f_c$ is necessary for this repetition rate, as well as for 83 MHz.

The bandwidth of this feedback control scheme is ultimately limited by the bandwidth of the gain, the inverse of the upper-state lifetime of the gain material. To avoid this limitation, some systems will need feedback which controls the intercavity loss, rather than the intercavity gain. This is a problem being worked on by others in Franz Kaertner’s research group.
Appendix A

List of Symbols

round-trip time: time required for light to make one round-trip in the resonator $T_R$

Saturated gain $g$

Unsaturated / small-signal gain $g_0$

saturation energy of the gain $E_{\text{sat},L}$

loss $l$

saturated loss $q$

saturable loss $q_0$

saturation energy of the saturable loss $E_{\text{sat},A}$

net gain: $g - l - q$ $G$

spontaneous lifetime of the gain $\tau_L$

normalized spontaneous gain lifetime: $\tau_L/T_R$ $T_L$

stimulated lifetime of the gain $\tau'_L$

normalized stimulated gain lifetime: $\tau'_L/T_R$ $T'_L$

lifetime of photons in the cavity $\tau_c$

Pulse energy / the energy in the laser resonator $E_P$

pump rate: $1 + (E_P \tau_L)/(E_{\text{sat},L} T_R)$ $r$

Relaxation oscillation frequency $\omega_s$

Q-Switching Frequency $\omega_{QS}$

independent variable in the Laplace domain $p$

forward transfer function (of the laser) $F_F$
feedback transfer function $F_R$
proportional circuit gain $a$
differential circuit gain $b$
current from the laser current supply $I_S$
loss due to output coupler $l_{OC}$
nonsaturable loss due to saturable absorber $l_q$
loss due to high-reflecting mirrors $l_m$
current through the pump diode $I_D$
slope proportionality of unsaturated gain $g_0$ to diode current $I_D$ $\alpha$
threshold current of the pump diode $I_{th,D}$
threshold current of the Nd:YAG laser $I_{th}$
current from the photodiode $I_{PD}$
voltage output from the photodiode transimpedance amplifier $V_{PD}$
input voltage to the circuit $V_{in}$
cutoff frequency of the differentiation $f_c$
transimpedance of the FET $K_Q$

Conventions

steady-state value of $x$ $x_s$
Fourier transform of $x$ $\tilde{x}$
change in $x$ $\Delta x$
Appendix B

Test Equipment

Below is a list of test equipment used in these experiments and mentioned in the text.

- Melles-Griot power meter: Melles-Griot 30W Broadband Power/Energy Meter. The head has serial number 46935.
- SRS function generator: Stanford Research Systems Model DS345 30 MHz Synthesized Function Generator
- LeCroy Oscilloscope: LeCroy LC684DXL 1.5 GHz Oscilloscope
- RF Analyzer: Agilent E4407B 9kHz - 26.5 GHz ESA-E series Spectrum Analyzer

For information about the printed circuit boards, please see Sec. 3.1.3 in the main text.
Appendix C

Microwave Spectra

Following are spectra taken of the output light from the Nd:YAG laser. These plots were taken with various values of the current supply $I_s$: The supply current is noted on each figure. The saturable absorber used during the taking of these plots was the HQL5 from HighQ Lasers, which has saturation depth $q_0=0.5\%$. One set of figures (feedback on, feedback off) is included for each point on the curve in figure 3-7.

Figure 1-4 repeated for comparison with experimentally observed microwave spectra: cw mode locking and Q-Switched mode locking, in the Fourier domain.
HQL5 (qD=0.5%), 28A, feedback off
Modelocked Repetition Rate: 83.31 MHz

sidebands:
Q-Switched Repetition Rate

HQL5 (qD=0.5%), 28A, feedback on
Modelocked Repetition Rate: 83.31 MHz
Modelocked Repetition Rate: 83.31 MHz

QL5 (q0=0.5%), 26.5A, feedback off

Modelocked Repetition Rate: 83.31 MHz

QL5 (q0=0.5%), 26.5A, feedback on
HQL5 (q=0.5%), 24A, feedback off

Modelocked Repetition Rate: 83.31 MHz

sidebands:
Q-Switched Repetition Rate

HQL5 (q=0.5%), 24A, feedback on

Modelocked Repetition Rate: 83.31 MHz
HQL5 (q=0.5%), 22.5A, feedback off

Modelocked Repetition Rate: 83.31 MHz

sidebands
Q-Switched Repetition Rate

HQL5 (q=0.5%), 22.5A, feedback on

Modelocked Repetition Rate: 83.31 MHz
HQL5 (q0=0.5%), 21A, feedback off

Modelocked Repetition Rate: 83.31 MHz

sidebands:
Q-Switched Repetition Rate

HQL5 (q0=0.5%), 21A, feedback on

Modelocked Repetition Rate: 83.31 MHz
Bibliography


