Simulation of Ionospheric Plasma Heating Experiments in the Versatile Toroidal Facility

by

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ABSTRACT

Remote sensing techniques employed to diagnose ionospheric modification experiments are intrinsically ambiguous, uncorrelated with "ground truth." To overcome this limitation, laboratory experiments are performed in the model ionosphere of the Versatile Toroidal Facility (VTF). The VTF contains a thermonically produced, weakly magnetized (\(\omega_{ce} < \omega_{pe}\)) background plasma of either hydrogen or argon. The HF "pump" wave of ionospheric experiments is modeled by 2.45 GHz microwaves, launched perpendicular to the magnetic field and the density gradient of the VTF in the ordinary mode. The peak plasma density is several times greater than the critical density (\(n_e \equiv 7.4 \times 10^{16} \text{ m}^{-3}\)), and the microwaves reflect, forming a standing wave Airy pattern. Wave spectra produced near reflection are measured using a miniature double probe and microwave receiver along with a fast oscilloscope. This combination is capable of simultaneously measuring spectra in two 250 MHz bands, one near DC and the other near the 2.45 GHz pump, to \(\mu\text{s}\) resolution. In addition, absolute electric field strengths and wavenumber spectra can be estimated. To explore the extent to which the VTF experiments simulate ionospheric heating, similarity rules are derived from the governing equations and applied to the two plasmas. A set of ten dimensionless parameters results, six of which match satisfactorily between the two plasmas. Three others can be neglected, leaving only one unmatched parameter: the ratio \(T_e/T_i\), which in the VTF is about 12 and in the ionosphere is near unity. Consideration of boundary conditions limits the scope of the simulation to the first Airy maximum. The main observational results of VTF heating experiments are: (1) Langmuir wave sidebands both up- and down-shifted from the pump frequency that decrease monoionically to the noise floor in tens of MHz, (2) lower hybrid waves in a broad band from 35 - 150 MHz, with maximum power occurring at 50 - 90 MHz, (3) both Langmuir and lower hybrid waves appear in bursts of duration and period in the 2 - 100 ms range, depending upon radius, (4) Langmuir and lower hybrid bursts are anti-correlated at the edge of the plasma but become uncorrelated in the core, and (5) the electric field, both of the pump and the plasma sidebands, varies by a factor of 100 in a burst period, from 1.3 to 130 kV/m for the pump (expected: 10.8 kV/m). The main features of ionospheric heating were reproduced in these experiments: down- and up-shifted high frequency sidebands, extreme time-variability of electric field amplitude, large pump wave absorption, and significant electron heating. The observed spectral bursts suggest the concentration of electric field into small time-varying regions. The periods and parameter dependencies of the bursts resemble results of three-
dimensional simulations of Langmuir turbulence. However, the upshifted Langmuir waves predicted by strong Langmuir turbulence (SLT) and nonlinear scattering theory are not observed in the VTF. A consistent account of the VTF observations is obtained by combining the caviton collapse cycle of SLT and the parametric production of lower hybrid waves by energetic Langmuir waves. As the high frequency electric field concentrates in cavitons, the threshold for the Langmuir decay instability is exceeded, generating lower hybrid waves in anti-correlated bursts. Because of the similarity of the VTF experiments to ionospheric heating, the observation of lower hybrid wave production during heating may also be borne out by future field experiments with diagnostics capable of viewing field-aligned modes.

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Chapter 1

Introduction

"The characteristics of the remote ionosphere are both dynamic and complex, and... are not completely known."
—Utlaut (1970)

This dissertation explores the extent to which the Versatile Toroidal Facility (VTF) can be used to simulate ionospheric modification experiments at Arecibo, Puerto Rico. Ionospheric heating by powerful radio waves results in increased electron temperature, production of turbulence at various length scales, and in some cases, gross modifications to the density profile in the heated region. Heating experiments in the VTF have produced similar results (see Moriarty (1996), Lee et al. (1997), and Lee et al. (1998c)). However, a robust set of connections between this laboratory plasma and the ionosphere have not yet been established, and a detailed comparison of results from both plasmas has not been performed. It is the intent of this dissertation to establish these connections and thus to shed light on the physical processes responsible for plasma frequency heating, and more generally, on the process underlying laboratory simulation.

1.1 Motivation

In the most general sense, it is the intrinsic ambiguity of remote sensing that drove the experiments presented in this thesis. Normally, one tries to join remote sensing with experimental "ground truth" to validate the data extraction and interpretation algorithms. For example, in NASA experiments to measure the water content of the ground using an airborne radiometer, soil samples from several locations were measured for water content in
the laboratory. The observed moisture profiles in different fields were compared with radiometric data and with theory to establish practical relationships between remotely sensed quantities and the water content of the soil (Burke et al. (1979)). This is the ideal way to do remote sensing. It cannot be done in ionospheric heating experiments.

*In situ* measurements in the ionosphere are extremely difficult to impossible to perform—sounding rockets have difficulty hitting such a small target in the sky, and satellites don't fly at 250 km because atmospheric drag would cause them to reenter in a financially unacceptably short time. Even if they could, a single probe cannot resolve both time and space variations independently.

Remotely measured results, uncorrelated with "ground truth", may often be consistent with several existing theories. This appears to be the case in ionospheric heating experiments, where the main (and often the only) diagnostic is an incoherent scatter radar, or ISR, located on the ground, viewing a heated volume 250 km away. What are certainly complex, three-dimensional, time-dependent phenomena are sampled along a single line of sight. Along this line, time and distance resolution must be sacrificed to preserve acceptable signal to noise ratios. In the case of ionospheric heating experiments, the available line of sight (vertically upward) is not the most desirable for comparison to theoretical predictions requiring information along the magnetic field lines. The end result is a set of measurements that, no matter how carefully taken and cleverly interpreted, could mean several different things with equal certainty. Under these circumstances, it makes sense to try to answer in a laboratory context questions about the nature of the remotely observed signals.
1.2 The Ionosphere

Let us start by describing the setting. The earth's ionosphere is a layer of weakly ionized gas in the upper atmosphere, extending roughly from 80 - 2000 km in altitude (see Kelley (1989), Rishbeth and Garriott (1969), or Budden (1988) for a complete introduction). The main source of ionization is UV radiation from the sun; however, other sources include energetic particles from the magnetosphere and solar wind, cosmic rays, and meteor trails. The charged particle density is vertically (radially, in spherical coordinates) stratified, with a maximum of up to $10^{12}$ m$^{-3}$ occurring around 250 - 400 km, depending on time of day, year, etc. At night, when the sun's radiation is no longer present, recombination reduces the charged particle density and the altitude of the maximum decreases. The recombination rate is slow enough, however, that there is usually a substantial plasma left by morning. At mid to low latitudes the nighttime ionosphere is also maintained through charge exchange with plasmaspheric ions that were stored there during the day. A profile of plasma density (really backscattered radar power), taken with the Arecibo 430 MHz ISR on 22 July 1997 at 21:11:02 local time, is shown in Figure 1-1.

The ionosphere is immersed in the magnetic field of the earth, and is commonly divided into zones of latitude, defined by ranges of the magnetic field dip angle—equatorial (around 0°), mid-latitude (~45°), and high-latitude (~90°). It is further divided into the altitude bands D, E, and F, which extend roughly from below 90 km, 90 - 150 km, and 150 - 500 km, respectively. The experiments performed at Arecibo, Puerto Rico, took place in the mid-latitude nighttime F region ionosphere. At this location, the magnetic field has magnitude $3.7 \times 10^{-5}$ T, dip angle 47.5°, declination 11.3° west, and is essentially constant over the time and space scales of interest.
Figure 1-1. Backscattered radar power profile observed with the Arecibo 430 MHz ISR, 21:11:02 LT, 22 July 1997.

The neutral density is also vertically stratified and decreases exponentially with altitude, from approximately $10^{18}$ m$^{-3}$ at 110 km, through about $10^{14}$ m$^{-3}$ at 300 km, to $10^{12}$ m$^{-3}$ at 500 km. In the altitude range of interest, the dominant ion species in the F region is singly-ionized atomic oxygen, O$^+$, and the dominant neutral species is atomic oxygen, O, with molecular nitrogen present as well. The neutral temperature increases rapidly above 100 km to values that are variable, but often above 1000 K. The electron and ion temperatures near the nighttime F peak are almost equal, with $T_e \approx T_i \approx 1000$ K typically. For fairly low altitudes at night the electron temperature is approximately equal to the neutral temperature (Kelley (1989), p. 463).
Other parameters, such as collision frequencies, can be estimated from those listed above—formulae are given in Table 2-3 in the following chapter.

1.3 Ionospheric Modification

1.3.1 History

The first ionospheric modification by radio waves was observed soon after the Luxembourg high power broadcasting station came into operation in 1933. Tellegen (1933) wrote, "For the first time on April 10 of this year it was observed at Eindhoven, Holland, that when a radio-receiver was tuned to Beromunster (460 m) the modulation of the Luxembourg station (1190 m) could be heard on the background to such an intensity that during the weak passages of the programme of Beromunster the programme of Luxembourg was heard with annoying strength.". The "Luxembourg effect" was later attributed to the heating of ionospheric electrons by the waves from the Luxembourg transmitter (Bailey and Martyn (1934)).

The first successful and deliberate ionospheric modification experiments came many years later, when the Institute for Telecommunications Sciences of the United States Department of Commerce completed the high power antenna array at Platteville, Colorado, in 1970 (see Utlaut (1970) and companion papers). The intention was to alter the electron temperature in a small region and observe the ionospheric response. The response could then be used to study properties of the natural ionosphere that were inaccessible by other means (i.e., heating and cooling processes, chemical reaction rates, etc.). Since 1970, several more transmitters have been constructed around the world and many new and unexpected phenomena have been observed (for reviews see e.g. Duncan and Gordon (1982), Fejer et al. (1985), and Robinson (1989)). These facilities are called "HF heating" facilities, since the operating frequency is within the band 3 - 30 MHz and the intended result is the localized heating of the ionosphere. Several objectives have joined the original
mission of perturbative observation of the natural ionosphere. These include performing basic plasma physics experiments in the ionosphere (effectively unbounded and long-lived), production of artificial communications channels, and control or modification of naturally occurring ionospheric plasma turbulence.

1.3.2 Arecibo Heating Facility

The most recent heating facility at Arecibo, Puerto Rico, was commissioned in the early 1980s and upgraded in the mid-1990s (see Fejer et al. (1985) and Isham et al. (2000)). It consisted of HF transmitters (3 - 12 MHz, up to 400 kW CW, and occasionally 600 kW) that drove a vertical launching antenna array with approximately 23 dB gain over isotropic and half-power beamwidth of approximately 18°. Some simple calculations are instructive here. The Effective Radiated Power, or ERP, is the product of transmitter power and antenna gain,

\[ \text{ERP} = P_T G. \]  

(1.1)

The ERP is the power that would be transmitted by an isotropic radiator, assuming that the fields in the antenna pattern are representative of the entire 4\pi sr. It is a measure of the electric field strength produced by a transmitter/antenna combination. In fact, the relationship between ERP and electric field can easily be derived. The magnitude of the time-averaged Poynting flux is

\[ \mathcal{S} \equiv |\langle \mathbf{S}(t) \rangle| = \frac{\text{ERP}}{4\pi R^2}, \]  

(1.2)

where \( R \) is the distance from the source. The Poynting flux far from the source \((kR \gg 1)\) in a vacuum is related to the electric field through

\[ \mathcal{S} = \left| \left\langle \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right\rangle \right| = \frac{|\mathbf{E}|^2}{2\eta_0}. \]  

(1.3)
where $\eta_0$ is the impedance of free space ($377 \, \Omega$). Thus, neglecting absorption and refraction, the electric field magnitude in the beam is

$$|E| = \frac{\sqrt{\eta_0 \text{ERP}}}{2\pi R}.$$  

(1.4)

Using these relations, with 400 kW of transmitted power, the Arecibo HF beam had 80 MW ERP, and at 300 km altitude produced an electric field of approximately 0.23 V/m and an upward flux of 0.07 mW/m$^2$ in a patch of sky 95 km in diameter. This value of electric field is comparable to the characteristic plasma field, $E_p$, needed to cause large thermal perturbations in the plasma ($E_p$ is defined in Gurevich (1978)). In fact, the Arecibo transmitter was designed specifically to provide electric fields of this magnitude (Robinson (1989)).

Diagnostics available near the heating site included the Arecibo ISR (approximately 17 km southwest of the heating facility) and the ionosonde located at Ramey Air Force Base (about 50 km to the west). The results of the Arecibo ISR are of central importance to later sections of this thesis, and its operation is described in more detail in Section 1.4. Ionosondes transmit a fairly broad, frequency swept beam vertically and monitor the delay of the return echoes. In this way the bottom side density profile can be deduced (see Hutchinson (1987), pp. 125-128). As well, one can usually glean indications of the level of density fluctuations, or spread-F (see e.g. Georges (1970)).

1.3.3 Ionospheric Propagation

After leaving a transmitting antenna, radio energy propagates along the group velocity path, or ray. The ionosphere, a birefringent medium, splits the ray into two different paths corresponding to the ordinary and extraordinary polarizations (see e.g. Budden (1988), pp. 254-294). The ordinary ray is deflected poleward as it propagates, and for a sufficiently dense ionosphere becomes perpendicular to the magnetic field and reflects. The extraordi-
nary ray deflects toward the equator, and for vertical incidence, is nearly parallel to the magnetic field at reflection. The differential equations for ray propagation are given in Budden (1988), pp. 403-409 and Stix (1992), p. 85, and are easily solved numerically. The result of one such calculation is shown in Figure 1-2 (see Appendix B for ray tracing code).

\[ X = \frac{\omega_{pe}^2}{\omega^2} \]

**Figure 1-2.** Meridional ray paths over Arecibo for transmitted frequency 3.175 MHz, in a standard Chapman layer with peak density $1.3 \times 10^{11}$ m$^{-3}$ located at 250 km, scale height 40 km, and solar zenith angle 0. Ordinary rays are shown as solid red. Extraordinary rays are shown as dashed blue.
In this calculation, a Chapman layer (see Chapman (1931)) was used for the density profile, with peak density $1.3 \times 10^{11} \text{ m}^{-3}$ located at 250 km altitude, scale height 40 km, and solar zenith angle $0^\circ$. The transmitted frequency was 3.175 MHz. The ordinary mode reflects near the peak of the Chapman layer, and the rays display the "Spitze" described by Budden (1988), p. 266. The extraordinary rays reflect somewhat lower in altitude. In both cases, the ray at vertical incidence retraces its own path after reflection.

Near the reflection layers, standing wave patterns develop as the group velocity vanishes and the energy flux remains constant. This produces a "swelling" of the electric field near reflection—the wave must have larger local field energy to propagate the same flux (Budden (1988), p. 200). One-dimensional analytic solutions of the electric field amplitude near reflection can be obtained for certain forms of electron density variation, yielding electric field enhancement factors of about 10 (see Budden (1988), p. 200). For a linear density variation (i.e., considering only the first two terms of a Taylor expansion), the resulting electric field is (Kruer (1988), p. 34)

$$E(\eta) = 2\sqrt{\pi}\left(\frac{\omega L_n}{c}\right)^{1/6}E_0\exp(i\phi) Ai(\eta),$$

(1.5)

where $L_n$ is the density scale length, $E_0$ is the incident vacuum electric field, $\phi$ is a phase factor, $Ai(\eta)$ is the Airy integral function, and $\eta$ is defined as

$$\eta \equiv \left(\frac{\omega^2}{c^2 L_n}\right)^{1/3}(z - L_n).$$

(1.6)

The quantity $|E(z)|/|E_0|$ is plotted in Figure 1-3, for two different pump frequencies in a scale length of 70 km. The peaks in the Airy function are the altitudes at which one expects large nonlinear effects to occur. For the 5.1 MHz pump wave in Figure 1-3, the
first three peaks occur approximately 187, 595, and 882 m below the reflection altitude. At these peaks, the field enhancement $|E(z)|/|E_0|$ is approximately 8.4, 6.6, and 6.0, respectively. A standing wave in vacuum would yield a field enhancement of 2, for comparison.

![Figure 1-3. Standing wave pattern in electric field amplitude near reflection. A density scale length of 70 km has been used, and heating frequencies of 3.175 MHz (solid red) and 5.1 MHz (dashed blue) are shown.](image)

The above treatment is one-dimensional. As Figure 1-2 shows, however, the rays diverge along their entire path—at least a two-dimensional problem. In addition, the presence of “ripples” in the reflection layer may introduce additional complexity. Hence, estimates based on equation (1.5) should be viewed as order-of-magnitude only.
Energy is absorbed all along the path by collisions of the oscillating electrons with the background neutral particles and ions (Walker (1979) and Gurevich (1978)). As the wave moves into the F-region, however, collisions reduce in frequency and "anomalous" processes grow in importance. As in other branches of plasma physics, the invocation of "anomalous" processes signifies that nonlinear wave-wave and wave-particle interactions are strongly suspected to be responsible for increased absorption, but a generally successful, quantitative theory is as yet absent. We use the term here to mean all processes other than collisional absorption. Duncan and Gordon (1982) conclude that "approximately half of the incident HF [radio wave] energy is absorbed in the ionosphere, with nearly equal contributions from anomalous absorption and deviative [collisional] absorption." It is the anomalous absorption that most strongly pertains to this dissertation, and most of the next several sections is dedicated its elucidation.

1.3.4 Results of Ionospheric Modification

Robinson (1989) gives an impressive list of the plasma phenomena that are excited during ionospheric modification: "large scale plasma temperature and density changes, generation of small scale field aligned irregularities, induced anomalous absorption of high frequency (HF) radio signals, artificially induced diffusive HF scatter (spread-F), artificially stimulated scintillations, artificially enhanced air glow and particle acceleration, enhanced ion and plasma lines in incoherent radar spectra, nonlinear radio wave reflectivity, stimulated HF emissions, and cross-modulation." Note that the names of these phenomena reflect the nature of the observations and not necessarily the nature of the physics. This reflects the fact that there is still ambiguity in the physical interpretation. In fact, all of these may not be independent effects, but manifestations of the same physical process on different diagnostic outputs. Perhaps only a small number of these will be observed during a single campaign, depending upon what diagnostics are available.
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It is not the intent of the present work to examine all of these effects. Rather, we focus our attention on recent experiments aimed at studying a single result of HF heating: the enhanced plasma line in ISR spectra. The "plasma line" of an ISR spectrum, defined in detail in the next section, indicates the level and frequency spread of longitudinal electron fluctuations propagating along the line of sight of the radar. Plasma lines increase by several orders of magnitude in the presence of HF heating. Interpretations of these "enhanced" plasma lines usually fall into the category of wave-wave interactions, and are central to understanding how anomalous heating occurs. Readers interested in other effects of ionospheric modification are referred to Robinson (1989), and references therein.

1.4 Enhanced Plasma Lines

1.4.1 Incoherent Scatter Radar

The ISR at Arecibo sends out a train of pulses having carrier frequency 430 MHz and receives the radiation scattered from the ionosphere. From the time delay of the received pulses one can calculate the altitude of the scatterer, and from the shape and intensity of the received spectrum one can determine electron density, electron and ion temperatures, and line of sight drift velocities (Evans (1969)). This section is intended to provide an introduction to the theory of ISR operation sufficient to understand and interpret the output.

Most of the transmitted power from an ISR continues into space and is lost. A small fraction of the power is scattered by fluctuations in the refractive index of the ionospheric medium and returns to the receiver. Index fluctuations are the result of plasma waves, thermal level fluctuations (small amplitude, damped plasma waves), and the discrete charged particles themselves. The shape of the returned spectrum is determined by the relative sizes of the scattering wavelength, $\lambda$ (determined by the incident wavelength and the
receiving geometry), and the Debye length, $\lambda_{De}$, of the plasma. There are no correlations in the plasma on length scales smaller than $\lambda_{De}$, and so for $\lambda \ll \lambda_{De}$ the spectrum is the result of the incoherent addition of the power scattered from individual electrons. There is a broad range of Doppler shifts corresponding to the electron velocity distribution of the plasma. For $\lambda \gg \lambda_{De}$ electron motions are correlated with the ions, and the spectral power is concentrated in a much more narrow region whose spectral width is determined by the ion velocity distribution (see Hutchinson (1987), p. 258 and Evans (1969)).

From a knowledge of the electron and ion distribution functions and the plasma dielectric tensor, one can derive an expression for the coherently scattered power per unit solid angle, per unit frequency (see Hutchinson (1987), p. 260 or Salpeter (1960) for a derivation):

$$\frac{d^2P_s}{d\Omega_s d\omega_s} = \frac{e^2P_i}{2\pi A} |\mathbf{\Pi} \cdot \hat{e}|^2 n_e V S(k, \omega),$$  \hspace{1cm} (1.7)

where $r_e$ is the classical electron radius, $P_i$ is the incident power across the scattering volume, $A$ is the area of the scattering volume perpendicular to the incident wave vector, the term in the absolute value is a polarization factor, $n_e$ is the mean background plasma density, $V$ is the scattering volume, and $S(k,\omega)$ is the scattering form factor, given by

$$S(k, \omega) = \frac{2\pi}{kn_e} \left\{ 1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right\}^2 f_{ek}(\frac{\omega}{k}) + \left\{ \frac{\chi_e}{1 + \chi_e + \chi_i} \right\}^2 f_{ik}(\frac{\omega}{k}).$$  \hspace{1cm} (1.8)

The electron and ion velocity distribution functions along the $k$-direction are given by $f_{ek}$ and $f_{ik}$, respectively. The plasma susceptibilities are given by $\chi_e$ and $\chi_i$.

Here, $k$ and $\omega$ are the magnitudes of the scattering wave vector and frequency, respectively, where

$$k = k_s - k_i$$  \hspace{1cm} (1.9)
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and

\[ \omega = \omega_s - \omega_i. \]  

(1.10)

The “s” subscripts refer to the scattered waves (received), and the “i” subscripts refer to the incident waves. For direct back scatter as at the Arecibo observatory,

\[ k = -2k_i, \]  

(1.11)

and the scattering wavelength is exactly half the transmitted wavelength (about 35 cm for the 430 MHz radar at Arecibo).

For Maxwellian distributions one can simplify equation (1.8) to the following:

\[
S(k, \omega) = \frac{\sqrt{2\pi}}{\omega_{pe} \alpha} \left[ 1 + \frac{1}{\alpha^2} \exp\left( \frac{m_i T_e}{m_e T_i} \right) \exp(-\xi^2) \right] \exp\left( \frac{m_i T_e \xi}{m_e T_i} \right) \left[ 1 + \frac{1}{\alpha^2} \left[ W(\xi) + \frac{T_e}{T_i} W\left( \frac{m_i T_e \xi}{m_e T_i} \right) \right] \right]^2
\]

(1.12)

where

\[
W(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{\xi^2} d\xi + i\sqrt{\pi} \xi e^{-\xi^2},
\]

(1.13)

\[ \alpha = k \lambda_{De}, \]

(1.14)

\[ \lambda_{De} = \frac{\nu_{Te}}{\omega_{pe}} \text{ (Debye length),} \]

(1.15)

\[ \omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}} \text{ (plasma frequency),} \]

(1.16)

\[ \nu_{Te} = \sqrt{\frac{k T_e}{m_e}} \text{ (thermal velocity),} \]

(1.17)

and

30
\[ \xi = \frac{f}{\Delta f_e} = \frac{\omega}{\sqrt{2k} v_{Te}}. \] (1.18)

Note that \( \xi \) is just the frequency shift, measured in units of the Doppler shift produced by an electron moving toward the radar at the most probable thermal speed. Using (1.12) in (1.7), one can plot the scattered power spectrum, measured in units of \( \Delta f_e \). This is done in Figure 1-4 for \( O^+ \) ions with \( T_e = T_i \), and in Figure 1-5 for \( T_e = 2T_i \).

\[ \frac{\pi^{0.5}}{n_e r_e^2} \frac{d^2 P_s}{d\Omega_s d\xi} \]

\[ O^+ \]

\[ \alpha = 10 \quad 2 \quad 1 \quad 0.33 \quad 0.2 \]

Normalized Doppler shift \( \frac{f}{\Delta f_e} = \xi \)

**Figure 1-4.** Scattered power spectrum for various values of the parameter \( \alpha \). For \( \alpha \) greater than 10, the spectrum does not change significantly. The ion is assumed to be \( O^+ \), with \( T_e = T_i \).

In both these cases, increasing the parameter \( \alpha \) causes the spectrum to approach the Gaussian shape of incoherent electron scattering. For small \( \alpha \) the spectrum exhibits two peaks, one near the ion acoustic frequency (\( \xi \approx 5.8 \times 10^{-3} \), the "ion line"), and the other
Figure 1-5. Same as previous, with $T_e = 2T_i$.

near the Langmuir wave frequency ($\xi \equiv 1.4$, the "plasma line"). Increasing the ratio of $T_e/T_i$ reduces the damping of the ion acoustic waves and the ion line becomes more "humped." For very small $\alpha$ essentially all of the echo power is concentrated in the ion line.

During a typical modification experiment at Arecibo, the ISR beam is aimed slightly off vertical to intersect the vertical of the HF transmitter (~17 km away) at about 300 km altitude. Only direct back scatter is measured, and so the system is termed monostatic. The radar beam width is about 0.16°, and so the scattering volume at 300 km is about 840 m in diameter—a small fraction of the diameter of the heated volume (50 - 100 km). The Debye length in the F-region is about 1 cm, giving an $\alpha$ of about 0.18—well into the range where
coherent effects dominate the spectrum. The designation "Incoherent Scatter Radar" is then somewhat of a misnomer, and as Evans (1969) notes, it would be better termed "Thomson Scatter Radar", or TSR. However, ISR has stuck, and we will continue the tradition as well.

In the absence of modification, ISR measurements typically are able to detect only the ion line, the plasma line being much lower in amplitude and broader. Two effects can change this situation. The presence of non-thermal electrons (generated by energetic photons, for example) can generate much larger fluctuation levels. These are routinely noted in daytime experiments and have been given the name Photoelectron Enhanced Plasma Lines, or PEPLs (Yngvesson and Perkins (1968)). Second, ionospheric modification by high power radio waves produces greatly enhanced ion and plasma lines by generating coherent waves in the plasma. The scattered radiation from these coherent waves is orders of magnitude above the thermal level of equation (1.12). Measurements of the HF-enhanced plasma line, or HFPL, form the basis of comparison to our laboratory experiments.

1.4.2 ISR Measurements During HF Modification

During high power HF modification of the ionosphere, the backscattered radar spectrum changes significantly. The ion line acquires two sharply defined maxima at the ion acoustic frequency, and the plasma line increases several orders in magnitude. These effects are the result of the HF excitation of coherent waves in the plasma. A complete radar back scatter spectrum during HF modification is depicted in Figure 1-6 (reproduced from Shonwen and Kim (1978)). The spectrum extends slightly more than the natural Langmuir wave frequency to either side of the transmitted radar frequency—about 10 MHz wide in total. Three bands are evident: the downshifted plasma line (near \( f_{\text{radar}} - f_{\text{HF}} \)), the ion line (near \( f_{\text{radar}} \)), and the upshifted plasma line (near \( f_{\text{radar}} + f_{\text{HF}} \)). Sampling
rates usually limit experiments to about 250 kHz bandwidth, which means that the experimenter chooses to look at one of these three regions (in fact, Figure 1-6 is an amalgam of three different spectra). The thermal ion line shown in Figure 1-6 looks very similar to the calculated spectrum of Figures 1-4 and 1-5, allowing for the log- and linear scale differences, while the HF-enhanced line is about two times as large, with very pronounced peaks at the ion acoustic wave frequency. The thermal plasma lines are not shown in Figure 1-6, and probably were not detectable. The HF-enhanced plasma lines show several features:
1. A sharply defined peak at exactly the transmitted HF frequency (marked “Growing Mode” in Figure 1-6).

2. One or more sharply defined peaks at intervals of a few kHz, shifted toward the radar frequency from the transmitted frequency (“Decay Mode”).

3. A bump approximately 30 kHz wide, shifted toward the radar frequency from the transmitted frequency (“Broad Bump”).

A fourth feature, not shown in Figure 1-6, is often observed: a bump shifted away from the radar frequency from the transmitted frequency 10 - 80 kHz, with magnitude generally smaller than the other plasma line features (see Figure 1-7 for an example).

![Figure 1-7. Upshifted plasma line during HF modification at Arecibo.](image)
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In an ionosphere possessing a linear variation of plasma frequency with altitude, \( f_p(z) \), with broadband (in \( k \) and hence \( \omega \)) Langmuir waves present at every height, the plasma line spectral power is localized to a straight line in \((f, z)\) space through the point \((f_{HF}, z_m)\) with slope \( L_{f_p}/f_{HF} \), where \( f_{HF} \) is the HF frequency and \( L_{f_p} \) is the plasma frequency scale height for the ionosphere. The altitude \( z_m \) is the radar matching height, where the natural Langmuir wave frequency detectable by the radar is equal to \( f_{HF} \):

\[
f_{HF} = f_{ek} = \sqrt{f_{pe}^2 + 3k^2v_{Te}^2 + \Omega_e^2\sin^2\Theta_{dip}}
\]

The radar "sees" only one particular value of \( k \) (determining the value of \( k \) and \( \Theta_{dip} \)), and so the value of \( f_{pe} \) (and hence \( z_m \)) is fixed. Summing the power returned from a range of altitudes produces a broad bump in the spectrum that is indistinguishable from a broad bump produced at a single altitude. It it thus desirable to resolve the altitude of the returned signals, as well as the frequency and the time into the HF heating pulse.

Simultaneous resolution of altitude, time, and frequency to arbitrary precision is not possible in general. Time and frequency resolution vary inversely, and the integration time cannot be reduced below the minimum set by signal to noise ratio constraints. The measurements of Figures 1-6 and 1-7 (and indeed most spectra reported in the literature) are height-integrated, or averaged over a certain range of altitudes. In early measurements this was not expected to be a problem, because it was thought that plasma waves excited by the HF transmitter were produced in only a narrow layer a few hundred meters in thickness. However, later height-resolved measurements showed that the plasma line returns were originating from an altitude higher than that anticipated by theory (Muldrew and Showen (1977)). This result spurred development of several improved methods of ISR use, culminating in the work of Sulzer and Fejer (1994). Some of their measurements are reproduced
in Figure 1-8, and form the starting point for our further discussions. Results obtained several years later using the same technique have recently been reported (Isham et al. (2000)). Where appropriate, differences in the newer work are noted.

The results presented in Figure 1-8 were obtained using a very low duty cycle HF pulse train, in a quiet, nighttime ionosphere. The pulses were considered to be independent of one another and repeatable. Using a repetitive sampling technique, the experimenters were able to resolve frequency to 1 kHz, altitude to 150 m, and time to 1 ms (see Sulzer and Fejer (1994) for a complete discussion of the measurement technique and results). Only the downshifted plasma line is shown, with frequency shift on the horizontal axis. Twelve plots are shown, each representing power (black intensity) as a function of altitude (vertical axis) and frequency, in a particular time interval. In intervals where the HF power was on, a vertical bar is present to the right of the plot. There are 10 altitude intervals in each plot, each 150 m in extent.

In the first time interval only a very weak, narrow feature (marked A) was observed at a frequency shift of 0 kHz and an altitude 150 m above the bottom of the plot. Feature A is about 1 km below the reflection altitude, \( z_0 \), where \( f_{HF} = f_p(z_0) \), and presumably marks the point where the natural Langmuir wave frequency detectable by the radar is equal to the HF frequency, the matching altitude \( z_m \). Feature A increases in intensity as the HF pulse continues, although it is somewhat overshadowed by the plot clutter in times 3 through 6 (an artifact of the measurement process). After the HF pulse, the feature persists several milliseconds until about time 11. The "growing mode" (feature 1) of the height-integrated spectrum of Figure 1-6 can perhaps be identified with feature A.

Two more features appear in the second time interval—a narrow one at a frequency shift of about -78 kHz (B), and a broad feature near 0 kHz (C). Both features originate from the second- and third-highest altitude bins, about 1.2 km above \( z_m \), at the reflection
Figure 1-8. Height- and time-resolved plasma line measurements, from Sulzer and Fejer (1994). Their caption reads, "Radar back scatter spectra of the downshifted plasma line for 5.1 MHz 60 MW equivalent radiated powers HF transmissions cycled 5 ms on 995 ms off. Twelve subsets of spectra 1 ms apart are shown. Each subset consists of 10 spectra for heights 150 m apart; the lowest shaded stripe in each subset corresponds to the lowest height. The darkness of the shading represents the spectral intensity on the indicated logarithmic calibration scale. The presence of a vertical line to the right of any subset of spectra indicates that during part of the time interval of 1 ms duration represented by that subset HF pumping occurred."
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altitude $z_0$. Feature $B$ increases in intensity throughout the heater pulse and persists for several milliseconds after the pulse ends. In times 3 through 11, $B$ appears to extend down a line connecting the original spot of time 2 with feature $A$. This is completely consistent with radar observations of freely propagating Langmuir waves, as discussed on page 36. This feature can be identified with the outshifted bump in the height-integrated spectrum of Figure 1-7.

Feature $C$ also increases in intensity throughout the HF pulse, but does not spread in altitude. After the end of the pulse, $C$ dies away as quickly as can be measured, much more rapidly than the other two features. Feature $C$ may be the height-resolved version of features 1, 2, and 3 defined on page 35 (and referring to Figure 1-6).

In the more recent work of Isham et al. (2000), spectral features $A$ and $C$ are not substantially different than those of Figure 1-8. However, near the end of the heater pulse feature $B$ was observed to extend over 1 km below feature $A$.

None of these features is temporally steady over longer heating times (higher duty cycles). Amplitudes can change by a factor of ten in fractions of a second (Showen and Kim (1978)). Additional variation is produced by changes in the mean ionospheric profiles and the production of background fluctuations of various scale sizes during extended HF modification (a "preconditioned" ionosphere). One result consistently (but not universally) observed is that feature $B$ vanishes after about 10 - 20 ms of heating, coincident with the strengthening and broadening of feature $A$ (Sulzer and Fejer (1994) and Cheung et al. (1992), but see also Vilece (1992) for CW observations).

1.4.3 Wave-Wave Interactions

Before discussing interpretations of HFPLs, it is helpful to consider the processes by which a powerful electromagnetic wave can excite electrostatic waves in a plasma and the evolution of those waves, once excited. When a wave in a plasma has a sufficiently large
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amplitude, the nonlinearity of the medium couples the wave to other possible waves. A variety of processes can result: decay-type instabilities, where the energy of a single large wave is transferred to "daughter" waves; coalescence (or "up-conversion"), where two or more large waves merge to form another wave; modulational instabilities, where the envelop of a large-amplitude wave becomes unstable and pinches off; wave scattering; and particle scattering (or "nonlinear Landau damping"). These processes have been reviewed in detail in the literature (see Melrose (1986), ch. 6 & 7, Liu and Tripathi (1986), and Kaup et al. (1979), among many others).

For an ordinary-mode electromagnetic wave incident on a magnetized plasma, at a location near reflection, production of electrostatic waves is thought to be dominated by the parametric decay instability and/or the oscillating two-stream instability (Krueer (1988), p. 154). Scattering of the electromagnetic wave from existing density fluctuations may also produce Langmuir waves (Krueer (1988), p. 58). Once Langmuir waves are produced, they can grow and interact with existing turbulence to produce a great variety of phenomena. Some of these include the decay of a Langmuir wave into another Langmuir wave and a lower hybrid wave (Kuo and Lee (1999)), Langmuir wave collapse (Zakharov (1972)), and nonlinear scattering from existing fluctuations (Kuo and Lee (1992)). These processes are briefly reviewed in the following subsections.

1.4.3.1 Parametric Decay and Oscillating Two-Stream Instabilities

A light wave of sufficient amplitude can simultaneously excite ion density fluctuations and electron plasma waves, coupled through the ponderomotive force. For zero frequency density perturbations, the process is called the oscillating two-stream instability, or OTSI. For propagating density perturbations (ion acoustic waves) the process is called the ion acoustic decay or parametric decay instability (PDI). Note that there are many possible parametric instabilities; without other qualifiers, "PDI" refers to the ion acoustic decay.
Both instabilities are strongest at resonance, when frequency and wave number matching conditions are satisfied. For the PDI, these are

$$\omega_0 = \omega_e + \omega_i$$  \hspace{1cm} (1.20)

and

$$k_0 = k_e + k_i,$$  \hspace{1cm} (1.21)

where $\omega_0$ ($k_0$) is the launched frequency (wavenumber), $\omega_e$ ($k_e$) is the electron plasma wave frequency (wave number), and $\omega_i$ ($k_i$) is the ion acoustic wave frequency (wave number). Near reflection, $k_0$ vanishes, and so

$$k_e = -k_i.$$  \hspace{1cm} (1.22)

For the OTSI, there are really four waves involved: a light wave, a zero-frequency ion perturbation, and two oppositely-directed Langmuir waves (a standing wave). The matching conditions for this case are (Kuo and Lee (1999))

$$\omega_0 = \omega_{e1} + \omega_i^* = \omega_{e2} - \omega_i$$  \hspace{1cm} (1.23)

and

$$k_0 = k_{e1} + k_i = k_{e2} - k_i = 0$$  \hspace{1cm} (1.24)

where $\omega_0$ ($k_0$) is the launched frequency (wavenumber), $\omega_{e1,2}$ ($k_{e1,2}$) are the electron plasma wave frequencies (wave numbers), and $\omega_i$ ($k_i$) is the ion perturbation frequency (wave number).

Schematically, the PDI and OTSI are shown on a plot of the Langmuir wave and ion acoustic wave dispersion relations in Figure 1-9. The ion mass has been reduced to 100$m_e$ for clarity. Also shown on this plot is an instability related to the PDI, in which a Langmuir wave decays into another Langmuir wave and an ion acoustic wave, which can be excited by the daughter waves of the light wave decay. Several sequential decays are
Figure 1-9. Schematic of the PDI, OTSI, and PDI cascade sequence overlayed by the dispersion relations for Langmuir and ion acoustic waves. The ion mass has been reduced to 100 $m_e$, and the ratio of $T_e$ to $T_i$ is 3. The numerals in parentheses indicate the decay number for the ion acoustic waves corresponding to the Langmuir waves shown above.

shown, indicating the trend toward zero wave number. The OTSI, on the other hand, excites Langmuir waves slightly above the pump frequency, which have larger wave numbers.

The PDI and OTSI share a common dispersion relation that can be derived from the two-fluid equations (see e.g. Krueer (1988), p. 67):

$$(\omega^2 + i\omega v_i - k^2 \epsilon_s^2) \left[ \left( \omega + i\frac{v_e}{2} \right)^2 - \delta^2 \right] + \omega^2 p_i \frac{k^2 v_o^2 \delta}{4 \omega_0} = 0,$$

(1.25)

where
\[ c_s = \sqrt{\frac{kT_e + \gamma_i kT_i}{m_i}} \] (sound speed), \hspace{1cm} (1.26)

\[ \delta = \omega_0 - \omega_{ek} = \omega_0 - \sqrt{k^2 \omega_{pe}^2 + 3 v_T^2 e k^2} \] (frequency mismatch), \hspace{1cm} (1.27)

\[ v_{os} = \frac{eE_0}{m_e \omega_0} \] (electron quiver velocity), \hspace{1cm} (1.28)

and \( v_i \) and \( v_e \) are the ion and electron collision rates (or Landau damping rates), \( \omega \) is the perturbation frequency, \( k \) is the perturbation wave number, and \( \omega_0 \) is the launched wave frequency. The quantity \( \gamma_i \) is the ratio of specific heats for the ions, and can be taken to be about 3 (adiabatic limit).

Now let us explore the roots of equation (1.25), using parameters typical of ionospheric modification experiments in Puerto Rico. First, it is helpful to recast equation (1.25) in normalized coordinates. Let

\[ h \equiv \frac{\omega}{\omega_0} = f + ig, \quad a_{i,e} \equiv \frac{v_{i,e}}{\omega_0}, \quad b \equiv \frac{\delta}{\omega_0}, \quad d \equiv \frac{kc_s}{\omega_0}, \]

\[ \alpha \equiv \frac{\omega_{pe}}{\omega_0}, \quad \beta \equiv \frac{v_{os}}{v_T e}, \quad \tau \equiv \frac{T_e}{T_i}, \text{ and } \mu \equiv \frac{m_e}{m_i}. \hspace{1cm} (1.29) \]

Then, equation (1.25) may be written as a quartic in \( h \):

\[ h^4 + i(a_e + a_i)h^3 - \left[ \frac{\mu}{3} \left( (1 - b)^2 - \alpha^2 \right) + b^2 + a_e a_i + \frac{a_e^2}{4} \right] h^2 \]

\[ -i \left[ \frac{\mu}{3} a_e ((1 - b)^2 - \alpha^2) + a_i \left( b^2 + \frac{a_e^2}{4} \right) \right] h \]

\[ + \frac{\mu}{3} \left[ (1 - b)^2 - \alpha^2 \right] \left[ \frac{a_e^2}{4} + \frac{b \alpha^2 \beta^2}{4} \right] = 0 \]

with the wave number determined from

\[ d^2 = \frac{\mu(\tau + \gamma_i)}{3\tau} \left[ (1 - b)^2 - \alpha^2 \right]. \hspace{1cm} (1.31) \]
Some insight can be gained by neglecting the highest-order terms in equation (1.30), and then calculating field thresholds and maximum growth rates. When this is done, one finds the estimates shown in Table 1-1.

<table>
<thead>
<tr>
<th>Threshold field, $\beta_{TH}^2$</th>
<th>PDI</th>
<th>OTSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{TH}^2$ = $\frac{4a_e a_i}{\alpha^2 \sqrt{1 - \alpha^2 N_1}} \sqrt{\frac{3(\tau + \gamma_i)}{\mu(\tau + \gamma_i)}}$</td>
<td>$\alpha^2 \frac{\beta^2}{8} - \frac{a_e}{2}$</td>
<td></td>
</tr>
<tr>
<td>Max growth rate, $\gamma_m = \frac{\gamma_m}{\omega_0}$</td>
<td>$\frac{\alpha \beta}{4} \left[ (1 - \alpha^2) \frac{\mu \tau}{3(\tau + \gamma_i)} \right]^{1/4}$</td>
<td></td>
</tr>
<tr>
<td>Frequency detuning at max growth rate, $b_m = \frac{\delta_m}{\omega_0}$</td>
<td>$\sqrt{\frac{\mu(\tau + \gamma_i)}{3\tau}} (1 - \alpha^2)$</td>
<td></td>
</tr>
<tr>
<td>Wave number at max growth rate, $d_m = \frac{k m e}{\omega_0}$</td>
<td>$\sqrt{\frac{\mu(\tau + \gamma_i)}{3\tau}} \left[ \left( 1 + \frac{\beta^2 \alpha^2}{8} \right)^2 - \alpha^2 \right]$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1-1. Threshold fields and maximum growth rates for the PDI and OTSI.

The threshold field for the PDI is quite a bit smaller than that for the OTSI, because of the small factor $a_i$. However, the growth rate for the OTSI goes as $\beta^2$, and so for large enough $\beta$ the OTSI dominates. The PDI growth rate vanishes at $\alpha = 1$ (reflection), leaving only the OTSI. The frequency detuning at maximum growth rate of the PDI equals the wave number, in these normalized coordinates—corresponding to resonance with the ion acoustic mode. The wave number for the OTSI is just slightly larger. Both wave numbers go to zero at reflection ($\alpha = 1$). As seen below, the range of unstable wavelengths around this maximum value is quite small for both instabilities—i.e., for a given altitude, the unstable wave number spectrum is narrow.
For a given experiment, the collision/damping rates, mass ratio, temperature ratio, and ionic specific heat ratio may be considered constant. Then the coefficients in equation (1.30) vary only with $b$ (frequency detuning or wave number), $\alpha$ (altitude), and $\beta$ (incident power). The root-finding routines built in to MATLAB may be used to chart out the frequencies and growth rates of equation (1.30).

To orient ourselves, it is helpful to determine the value of $\alpha$ for several physically interesting locations. At reflection, $\alpha = 1$. For a pump frequency of 5.1 MHz, the matching altitude $\alpha$ is approximately 0.983. Thus we have approximate bounds on the magnitude of interesting $\alpha$'s. In between these two values are several Airy maxima of the standing wave pattern—the locations where one expects the strongest nonlinear interactions. The $\alpha$'s of the first three Airy maxima are shown in Figure 1-10, for various ionospheric density scale lengths. For $L_n = 70$ km, the first Airy maxima is located at approximately $\alpha = 0.9987$.

For the parameters $a_e = 1.7 \times 10^{-5}$, $a_i = 1.2 \times 10^{-8}$ (collisional), $a_i = 3.9 \times 10^{-4}$ (ion Landau damping), $1/\mu = 16 \times 1836$, $\tau = 1$, and $\gamma_i = 3$, the roots of equation (1.30) are shown for $\alpha = 0.99$ and several $\beta$ in Figure 1-11. Typical power levels used in ionospheric experiments yield $\beta = 0.06 - 0.08$. Positive growth rate occurs in two regions, one identified with the PDI, and the other with the OTSI. In the OTSI region, the real frequency is zero and the roots are purely imaginary. In the PDI region, the real part of the frequency is centered on the linear ion acoustic dispersion relation, with some amount of detuning that increases with increasing incident power. In this plot and those that follow, the collisional ion damping rate is used for the OTSI range (zero phase velocity means no Landau damping), while the higher ion Landau damping rate is used for the PDI range, where the instability produces a propagating wave. At the altitude shown in Figure 1-11, the growth rate of the PDI is slightly larger than that of the OTSI at low $\beta$ (incident power), but at higher $\beta$ the OTSI dominates.
Dispersion relation roots for several altitudes are plotted in Figure 1-12 versus the perturbation wavenumber. For a given altitude ($\alpha$), the region of unstable wave number is quite limited, enlarging with increasing heater power. As $\alpha$ increases, the wave number must decrease, following the parabolic dispersion relation shown earlier in Figure 1-9. The PDI and OTSI coexist at every altitude except exactly at reflection: at $\alpha = 1$, PDI cannot take place and only OTSI remains. The real part of the PDI frequency is also shown here and approximately satisfies the ion acoustic dispersion relation.

A numerical analysis of the full quartic equation, using ionospheric parameters, yields the maximum growth rates shown in Figure 1-13. Thresholds are found where these curves pass through zero. Both the PDI and the OTSI are far above threshold in iono-
Figure 1-11. Dispersion relation roots for $\alpha = 0.99$ and $\beta = 0.05, 0.10, \text{ and } 0.15$, with other parameters typical of ionospheric heating. Two regions of instability obtain, one identified with the PDI, and the other with the OTSI. The real part of the frequency of the PDI is centered on the linear ion acoustic dispersion relation, while that of the OTSI is zero.

spheric heating, with PDI dominating at lower values of $\alpha$ and $\beta$. At $\beta = 0.05$, the crossover point is at about $\alpha = 0.996$. At $\beta = 0.10$, the value is much lower, about $\alpha = 0.988$. Since the altitude range of interest covers from $0.983 < \alpha < 1$, the OTSI tends to dominate over the upper portion of the reflection region, with the PDI dominant at lower altitudes and lower power levels.

Equation (1.25) was derived in a homogeneous, unmagnetized plasma. Its use may be extended to magnetized plasmas in the sense that ordinary polarization possesses an electric field aligned with the background magnetic field, and so preferentially excites plasma
Figure 1-12. Dispersion relation roots for various $\alpha$, with $\beta = 0.10$.

waves whose $k$ is aligned with $B_0$. It must be recognized, however, that obliquely propagating waves are also generated at lower amplitudes, in a small cone around the magnetic field. This is explored further in Section 1.4.3(c) below.

Plasma inhomogeneities introduce additional effects. The addition of a density gradient to the above analysis limits the interaction to a finite region of space (near $\alpha = 1$ on the density profile). Waves produced in the interaction zone can then propagate away, taking their energy with them. The required threshold field is then raised from the homogeneous
case (Perkins and Flick (1971)). In addition, the PDI becomes a convective instability, rather than absolute. The OTSI remains absolute. For a strong gradient, the threshold field is

\[
\beta_{TH}^2 \equiv \frac{2}{k_n L_n}
\]

(1.32)

where \(L_n\) is the density scale length and \(k_n\) is the wave number parallel to the electric field of the pump (and hence, the background magnetic field). A typical value for the ionosphere is \(L_n = 70 \text{ km}\), giving a gradient threshold of about \(\beta_{TH} = 0.002\). This is still well below the applied field, and the above conclusions still hold.
Plasma inhomogeneity in the form of turbulence also modifies these instabilities (Williams et al. (1979)). Turbulence alters the phase matching between the three interacting waves, partially destroying resonant coupling. This reduces and broadens the distribution of growth rates. Also, waves that would not couple in the absence of turbulence can be driven slightly unstable as the turbulence induces random phase matching (Williams et al. (1979)). The strength and spectrum of the turbulence determines the degree of growth rate modification. For the ionospheric case, especially at mid-latitudes, the initial nighttime plasma is very quiet. However, after extended bombardment by powerful radio waves, a large amount of energy resides in turbulent fluctuations, and the character of the instabilities changes.

1.4.3(b) Electromagnetic Wave Scattering on Ion Density Fluctuations

Ion density fluctuations in the heated plasma can efficiently couple a radio wave into a Langmuir wave (Krueer (1988), p. 58). For a static modulation in the background density

\[ n = n_0 + \Delta n \cos kx, \]  

(1.33)

the electric field of the driven Langmuir wave is

\[ E = \frac{\omega_{pe}^2 \Delta n E_0 \cos kx}{\omega^2 n_0 \varepsilon(k, \omega)}, \]  

(1.34)

where

\[ \varepsilon(k, \omega) = 1 - \frac{\omega_{pe}^2 + 3k^2
u_T^2}{\omega^2} + i \frac{\nu_e}{\omega}. \]  

(1.35)

The pump (radio) field is \( E_0 \) and \( \omega \) is the pump frequency. The energy damping rate is then given by

\[ \frac{\nu^*}{\omega} = \frac{1}{2} \left( \frac{\Delta n}{n_{cr}} \right)^2 \frac{\Omega \{ \varepsilon(k, \omega) \}}{|\varepsilon(k, \omega)|^2} \]  

(1.36)
The electric field of the excited Langmuir wave can grow very large when $\varepsilon$ approaches zero (resonance). The Langmuir wave can then drive daughter waves at $2k$, $3k$, etc. (see Kruer (1988), p. 59).

Again, for the ionospheric case, density fluctuations are expected to be very small in the mid-latitude ionosphere, and this method of coupling energy to the plasma is not expected to play a large role. However, in laboratory plasmas, density fluctuations are much larger, and we must consider this effect (see Chapter 4).

1.4.3(c) Langmuir Decay Instability

It is possible, in a magnetized plasma, for a large-amplitude Langmuir wave to decay parametrically into another Langmuir wave and a lower hybrid wave (Kuo and Lee (1999)). We denote this the Langmuir decay instability, or LDI for brevity.

The LDI only occurs in magnetized plasma, and only in cases where obliquely propagating Langmuir waves (also called upper hybrid waves, if the angle of propagation is large enough) are present with large amplitude. Although the source of the pump Langmuir wave need not be specified, we focus on the OTSI since it has the dominant growth rate near reflection altitude.

When oblique propagation is allowed, the dispersion relation for the OTSI becomes (Kuo and Lee (1999))

$$
(\gamma^2 + \Omega_i^2)(\gamma^2 + k^2c_s^2)\left[\left(\gamma + \frac{\nu_e}{2}\right)^2 + \delta^2\right] - \Omega_i^2k_{\perp}^2c_s^2\left[\left(\gamma + \frac{\nu_e}{2}\right)^2 + \delta^2\right]
$$

$$
+ (\gamma^2 + \Omega_i^2\cos^2\theta)\omega_{pi}^2 \frac{k^2\nu_{os}^2\cos^2\theta}{\omega_0} \frac{\delta}{\omega_0} = 0
$$

where $\omega = i\gamma$, $\Omega_i$ is the ion cyclotron frequency, $k_{\perp}$ is the perturbation wavenumber component perpendicular to the magnetic field, $\theta$ is the angle between the wave vector and the magnetic field, $\nu_{os}$ is the quiver velocity in the field of the pump Langmuir wave, and the frequency mismatch is defined.
\[ \delta = \omega_0 - \omega_{ek} = \omega_0 - \sqrt{\omega_{pe}^2 + 3k^2v_{Te}^2 + \Omega_e^2 \sin^2 \theta}. \]  

(1.38)

Here, \( \omega_{ek} \) is the frequency of a naturally-propagating Langmuir wave at an angle \( \theta \) to the magnetic field, and \( \Omega_e \) is the electron cyclotron frequency (this assumes that \( \omega_{pe}^2 \gg \Omega_e^2 \) is satisfied). All other quantities have the same definitions as previously.

Equation (1.37) is a 6th order polynomial in \( \gamma \), which can be solved for various parameter combinations in MATLAB, as for the parallel case of Section 1.4.3(a). First, note that the dispersion relation for magnetized Langmuir waves (equation (1.38)) is simply that of the unmagnetized Langmuir waves, shifted up by an amount approximately proportional to the angle of propagation. Thus, at a given altitude, there is an angle beyond which the OTSI is kinematically disallowed. This is easily found by solving \( \omega_{ek}(k, \theta) = \omega_{ek}(0, \theta_M) = \omega_0 \), giving a maximum angle of

\[ \theta_M = \arcsin \left( \frac{1 - \alpha^2}{\Omega_e^2 / \omega_0^2} \right). \]  

(1.39)

The quantity \( \alpha \) is defined as in equation (1.29). Very large power levels drive instabilities at angles larger than this. This relation is plotted in Figure 1-14. Near the first Airy maximum, the allowed propagation angles fall within about 14°, while at the radar matching height, the allowed angles are over 60°.

With this in mind, one can assume weak growth and determine threshold fields, as in the parallel case. This done, the ratio of oblique to parallel threshold goes like the secant of the angle of propagation,

\[ \frac{\beta_{TH}^2(\theta)}{\beta_{TH}^2(0^\circ)} = \frac{1}{\cos^2 \theta}. \]  

(1.40)
Figure 1-14. Maximum allowed angle of propagation for the oblique OTSI in the ionosphere. The radar matching location and the first Airy maximum are also shown for reference.

This is not a large change from the parallel case for angles within a fairly large cone around the magnetic field (note that the unstable wave number changes, however). Therefore, oblique Langmuir waves are expected to be generated by the OTSI, out to approximately the limit of equation (1.39). Indeed, numerical solution of the full 6th-order equation for representative parameter values yields the maximum growth rate shown in Figure 1-15. The altitude used here is approximately that of the first Airy maximum, at which point the maximum allowed angle from equation (1.39) is about 14°. The growth rate is quite uniform at angles out to about 12°, then drops dramatically as the allowed limit is approached. At this pump intensity, the threshold angle is about 15.3°, very close to the kinematic limit.
As mentioned previously, the range of unstable wave number is quite limited for a particular altitude. This can be seen in Figure 1-16, where the maximum, minimum, and maximally unstable wave numbers are plotted versus angle for two different altitudes. For a given angle of propagation, the unstable wave number is essentially fixed. Thus, the unstable waves are well characterized for a given altitude. At the lower altitude, the angular limit increases dramatically. For comparison, the wave number measured by the radar is magnitude 18 m\(^{-1}\), at an angle of about 45° to the magnetic field.

As these OTSI-produced Langmuir waves grow in amplitude, they become pump waves for other nonlinear processes. One such process is a parametric decay into another Langmuir wave and a lower hybrid wave (the LDI). These daughter waves can in turn grow to large enough amplitudes to excite another generation of waves. This process may continue until the amplitude of the daughter waves saturates at a value below the threshold.
for the next generation. Coupled-mode equations and the dispersion relation are derived in Kuo and Lee (1999). Here, we present only the threshold that results (from Kuo and Lee (1999)):

\[
\beta_{TH}^2 = 8a_em \left( 1 - \frac{m^2}{\mu^2} \right) \frac{\tau + \gamma_i}{\tau} \frac{\Delta \omega_i^2}{\omega_0^2} \frac{1}{\omega_0^2 \omega_3 d^2 \sin^2(2\theta)},
\]

(1.41)

where

\[
m \equiv \frac{\Omega_i}{\omega_0} \quad \text{(normalized ion cyclotron frequency)},
\]

(1.42)

\[
\frac{\Delta \omega_i^2}{\omega_0^2} = 2N \frac{\omega_3}{\omega_0} + a_e + \frac{m^2}{\mu^2} \sin^2 \theta + 3d^2 \frac{\tau}{\mu (\tau + \gamma_i)} \quad \text{(detuning)},
\]

(1.43)
\[
\frac{\omega_{30}}{\omega_0} = \sqrt{\frac{2d^2 \sin^2 \theta + \frac{m^2}{\mu}}{\sqrt{2d^2 \sin^2 \theta + \frac{m^2}{\mu}}}} - \frac{a_em}{\sqrt{2d^2 \sin^2 \theta + \frac{m^2}{\mu}}} \quad (\text{daughter LHW frequency}).
\] (1.44)

The number of the cascade is \( N \), and the power parameter \( \beta \) is defined on the field of the Langmuir pump wave (rather than the radio wave as above). All other quantities have been defined previously.

The normalized wavenumber, \( d \), is a known function of \( \theta, \alpha \), and \( \beta \) (see Figure 1-16). Thus, the lower hybrid wave frequency \( \omega_{30} \) is also a known function. For a given decay number \( N \), the frequency detuning \( \Delta \omega \) is also known. Then, the threshold can be calculated directly from equation (1.41).

The threshold field is plotted in Figure 1-17 for three different cascade numbers at two altitudes. The threshold is smallest for the first decay, and increases monotonically with \( N \). At a fixed altitude (\( \alpha \)), the threshold decreases from a large value at \( \theta = 0 \) to a minimum at some angle, and then increases again as the maximum allowed angle of propagation is reached. The minimum thresholds are lower at lower altitudes—on the order of \( \beta = 0.5 - 1.0 \) (10 - 20 V/m in dimensional units) at \( \alpha = 0.9934 \). At the first Airy maximum, the threshold is much larger, on the order of \( \beta = 1.5 - 3 \) (30 - 60 V/m).

Taking into consideration the swelling effect, the field produced at 300 km by a 60 MW heater is about 1.7 V/m. The OTSI-produced Langmuir waves may be expected to excite at about twice this amplitude, or 3.4 V/m. This is a factor of 3 - 9 too small to excite the first LDI cascade (depending on altitude). Considering that the transmitted power goes as the square of the electric field, the power transmitted is a factor of 9 - 80 too small to excite the LDI. Thus, if we have included enough physics in our analysis, the LDI should not be excited in ionospheric heating experiments, by a very clear margin.
**Introduction**

![Graph showing threshold oscillation velocity for the LDI versus angle of propagation of the pump Langmuir wave. Two altitudes are shown, the upper typical of the first Airy maximum. The parameter $N$ is the decay number.](image)

**Figure 1-17.** Threshold oscillation velocity for the LDI versus angle of propagation of the pump Langmuir wave. Two altitudes are shown, the upper typical of the first Airy maximum. The parameter $N$ is the decay number.

1.4.3(d) Nonlinear Scattering on Lower Hybrid Waves

A sort of inverse process to the LDI is the nonlinear scattering of Langmuir waves on lower hybrid waves (Kuo and Lee (1992)). In this process, large-amplitude Langmuir waves coalesce, or “beat”, with independently produced lower hybrid waves to give more Langmuir waves. As in the LDI, the source of Langmuir waves need not be specified, although the PDI and OTSI are prime candidates in ionospheric heating.

As in other three-wave interactions, frequency and wave number matching is satisfied:

$$\omega_{l1} \pm \omega_s = \omega_{l2}$$  \hspace{1cm} (1.45)

and
\[ k_{l1} \pm k_s = k_{l2}, \]  

where the subscripts "l1", "s", and "l2" refer to the incident Langmuir wave, the scattering lower hybrid wave, and the scattered Langmuir wave, respectively. If the incident Langmuir wave propagates nearly parallel to the magnetic field, and if the lower hybrid wave frequency is large enough, the downshifted scattered wave is nonresonant. The upshifted scattered wave can satisfy the natural Langmuir wave dispersion relation, however, because of the increased frequency of obliquely propagating modes (see equation (1.38)).

The amplitudes of the three waves are related in the resonant case by

\[ \beta_{l2} \equiv \frac{d_{l1} \cos \theta_{l2}}{a_e} \sqrt{\frac{\tau}{\tau + \gamma_i}} \sqrt{\frac{\omega_s^2}{\omega_{LHR}^2}} - 1 \beta_{l1} \beta_s, \]  

where

\[ \beta_m \equiv \frac{v_{om}}{v_{Te}} = \frac{eF_m}{m_e\omega_m v_{Te}} \]  

and \( m = l1, s, \) or \( l2. \) The incident wave number is normalized to \( d_{l1} \) as in equation (1.29), and \( \theta_{l2} \) is the angle of propagation of the scattered Langmuir wave. For a spectrum of lower hybrid waves \((\beta_s(\omega_s, k_s))\), there will be a spectrum of scattered Langmuir waves, whose power is distributed by equation (1.47). It is possible for the scattered Langmuir wave to have a much larger amplitude than the scattering lower hybrid wave. Although lower hybrid waves are not detectable by the Arecibo radar, the scattered Langmuir waves may be.

1.4.3(e) Langmuir Wave Collapse

For a large-amplitude Langmuir wave, two influential nonlinear effects are the ponderomotive force on ions and the nonlinear current due to density inhomogeneities for electrons. Including these effects in the fluid equations, one obtains a set of coupled differential equations (the "Zakharov equations") describing the self-consistent evolution
of the low-frequency density, $n_s(x, t)$, and the slowly-varying envelope of the high-frequency Langmuir waves, $\mathcal{E}(x, t)$ (see Zakharov (1972) and Nicholson (1983), p. 179). In one dimension these are

\[ 2i\omega_p e \frac{\partial \mathcal{E}}{\partial t} + 3\nu_e^2 \frac{\partial^2 \mathcal{E}}{\partial x^2} = \omega_e^2 \frac{n_e}{n_0} \mathcal{E} \quad (1.49) \]

and

\[ \left( \frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2} \right) n_s = \frac{\varepsilon_0}{4m_i} \frac{\partial^2}{\partial x^2} |\mathcal{E}|^2. \quad (1.50) \]

Here, $n_0$ is the unperturbed density, $c_s$ is the sound speed, and $t$ is the "slow" timescale of the ion acoustic waves. The left hand sides of these equations give the linear dispersion relations for Langmuir waves and ion acoustic waves, respectively. The right hand sides are the nonlinear terms that have been incorporated in this treatment: the modification of the plasma frequency because of the density perturbations, and the ponderomotive force. This set of equations can be shown to include the PDI and OTSI (unmagnetized) in the small-amplitude limit (Nicholson (1983), p. 181).

The simplest solution to these equations in one dimension is a stationary "envelope soliton," or localized packet of Langmuir waves, inside of a density "caviton," or density depletion, produced by the ponderomotive force of the high frequency waves. For two and three dimensions, cavitons are produced as well, but are unstable to collapse, a process where the dimensions shrink while the density depletion and electric field increase. In the absence of collisionless damping, cavitons would collapse to a singularity. Characteristic initial dimensions of cavitons are tens of Debye lengths. Other than the derivation of the governing equations and the simple one dimensional solution, analytical results are sparse, and behavior of the solutions must be inferred from specific cases that have been solved numerically. A recent review is given in Robinson (1997).
Chapter 1

1.4.3(f) Nonlinear Saturation

Back up for a moment, note that the previous several sections outlined three production mechanisms for Langmuir waves (PDI, OTSI, and nonlinear coupling on density fluctuations), and several other nonlinear phenomena that exhibit themselves if Langmuir waves grow large enough (PDI of large Langmuir waves, the LDI, scattering, and Langmuir collapse). Thresholds and growth rates for each process can be derived fairly easily. The results can be compared to ionospheric heating parameters to ascertain whether each individual process is stable or unstable. This is a first step toward interpreting ionospheric measurements—the presence or absence of specific instabilities. For heating parameters far above threshold, however, we are really concerned with the nonlinear development and/or saturation of the entire wave field—a much more difficult problem to analyze.

In many heating scenarios, evolution of the wave field is thought to lead to a saturated state where further temporal development ceases, at least in the mean. Physical reasoning suggests that an instability cannot enjoy growth without bound—at some point other nonlinearities apply that stop further growth. Heuristic arguments alone, however, cannot determine whether the final state is a stationary equilibrium or a stochastically oscillating one. The character of the saturated state must often be determined by a numerical simulation of a set of differential equations, hopefully including all relevant physics. There are a great variety of possible saturated states, and perhaps a greater variety of paths to reach those states. A successful description must place an experiment in the correct saturated state, or on the correct path to saturation if saturation has not been reached.

For very large incident powers ($\beta \geq 1$), the saturated state is simply trapping of all the plasma electrons in the most unstable wave (Krue (1988), p. 102). In this state, the electric field maximizes at the "wave breaking" amplitude, given by
\[ \beta_{\text{max}}^2 = \alpha^2 \left[ \beta_p^2 + 2\sqrt{3} \beta_p - \frac{8}{3^{3/4}} \beta_p^{3/2} - 1 \right] \]  

(1.51)

for a warm plasma, where

\[ \beta_p \equiv \frac{v_{p\parallel}}{v_{Te}} = \frac{\omega}{k_{\parallel} v_{Te}} \]  

(1.52)

and

\[ \beta_{\text{max}} \equiv \frac{v_{os,\text{max}}}{v_{Te}} \]  

(1.53)

Using the most unstable wave number from the OTSI analysis above, one finds that at the radar matching height in the ionosphere the Langmuir wave phase velocity is \( \beta_p \equiv 9.4 \) and the wave breaking field is \( \beta_{\text{max}} \equiv 4.3 \). This is about 70 times larger than the incident radio electric field, and it is doubtful that strong trapping would be the saturation mechanism in ionospheric heating.

For weaker pump fields, Langmuir waves saturate at amplitudes far below the wave breaking level through excitation of secondary parametric instabilities. Initially unstable waves grow large enough to become "pumps" in their own right, producing Langmuir waves at smaller wave number. These may in turn grow and excite more "daughter" waves, resulting in a Langmuir wave "cascade." Energy flows from the radio pump wave into each successive generation of cascade waves (Langmuir and ion acoustic). The cascade waves are damped, either by collisions or Landau damping, resulting in the transfer of radio wave energy into particle energy (anomalous absorption). However, for plasmas with low collisionality, the dominant damping mechanism is Landau interaction with resonant particles. The collisionless damping rate decreases with decreasing wave number. Thus it is possible to form a cascade of Langmuir waves to very small \( k \) without providing the necessary dissipation to balance the power input and give steady state. Under these
conditions, wave energy accumulates in the portion of the spectrum near zero $k$, forming a \textit{Langmuir-Bose condensate} (Zakharov (1984)). This is called the Langmuir paradox—in a turbulent plasma, wave energy accumulates at large phase velocity, where dissipation is negligible.

As noted previously, wave energy does not pile up indefinitely—some dissipation mechanism eventually becomes active and limits growth. The Langmuir-Bose condensate is unstable to modulational instabilities (same family as the OTSI) at large enough amplitudes (Robinson (1997)). Modulational instabilities transfer energy to larger wave number, where Landau damping is effective.

One saturated state of modulational instabilities is called Strong Langmuir Turbulence, or SLT (Robinson (1997) and references therein). In this scenario, a certain portion of the heated plasma volume is occupied by collapsing cavitons (see Section 1.4.3(e)). Individual cavitons collapse spatially until Landau or transit-time damping sets in, again resulting in a transfer of power from the radio pump wave to plasma particles. As pump power is increased, more and more of the heated plasma volume consists of collapsing cavitons.

The saturation state that obtains in a particular experiment is dependent on the pump power, the collision rates, the thermal energy of the plasma, and the proximity to the reflection altitude. Small changes in parameters may induce large changes in qualitative behavior—these are nonlinear systems with many degrees of freedom. It is not surprising, then, that there is great variance in observations of ionospheric heating, and great debates regarding the interpretation of those observations.

1.4.4 Interpretation of Measurements
Interpretations of ionospheric modification results all try to answer the same question: what is the nonlinear evolution of a plasma exposed to an oscillating electric field near the plasma frequency? The Vlasov-Maxwell set of equations presumably governs the entire
evolution, with appropriate collision operators in place. As in hydrodynamics, though, the full nonlinear solution to the governing equations is unknown, and one must resort either to a simplified set of equations or to numerical solutions, or both. The challenge is to reduce the full set of governing equations to an analytically or numerically tractable set that adequately describes all the relevant physics.

Given a proposed set of reduced equations, it is generally impossible to show that all relevant physics have been included. Direct comparisons with experimental results may corroborate claims of correctness but never provide absolute certitude. With these thoughts in mind, there have developed two different theoretical frameworks addressing the nonlinear evolution of plasma frequency heating: weak Langmuir turbulence (WLT) and strong Langmuir turbulence (SLT). These two frameworks are described in the following subsections.

Before discussing WLT and SLT, it is helpful to note that both theories begin by invoking the PDI, OTSI, and/or nonlinear scattering into Langmuir waves. WLT and SLT differ in the description of the evolution of the Langmuir waves, once produced. To orient the reader to the interpretation problem, Figure 1-18 shows two views of the region near ordinary mode reflection in \((f, z)\) space. The top plot shows the frequency band near the transmitted radio frequency of 5.1 MHz and the altitude range from 2 km below to 0.5 km above the reflection height. The altitudes of the first three Airy maxima are shown as dashed horizontal lines. The plasma frequency is shown as the upper solid diagonal line. The lower diagonal line is the natural Langmuir wave frequency observable by the Arecibo radar \((k = 18 \text{ m}^{-1}, \theta = 45^\circ \text{ to } B)\). The altitude at which this line crosses the pump frequency is called the matching height. Thick gray lines to either side of the pump frequency mark the locations of instability for the PDI and OTSI. The strongest electric field occurs at the first Airy maximum, followed successively by the second and third, etc.
Figure 1-18. Region near reflection, for $L_n = 40$ km and $f_0 = 5.1$ MHz. Thick gray lines indicate locus of instability for the PDI and OTSI. Horizontal dashed lines in top frame show the locations of reflection, matching, and the first three Airy maxima. Loci of dominant radar returns are shown in bottom frame as hollow circles. Both frames show the same scenario, but are split to avoid detail overload.
Introduction

These are the locations one expects the largest growth rates, and hence, the largest radar returns. In the bottom frame, the approximate locations of measured radar returns are shown as hollow circles (c.f. Figure 1-8). Note that the radar echoes do not coincide with regions of instability, except at a few points. Interpretations of the data must explain this discrepancy. In addition, the radar return spectra are not constant in time, but change as the heater pulse progresses. Successful theories must explain this temporal development as well.

1.4.4(a) Weak Turbulence

In WLT, wave amplitudes are assumed to be small enough that nonlinear effects can be considered perturbatively. Waves satisfy their linear dispersion relations and interact through nonlinear coupling terms that depend upon proximity to resonance (similar to equations (1.9) and (1.10)) and wave polarization. The random phase approximation is used, allowing a hierarchy of statistical wave kinetic equations to be developed that describe the evolution of wave amplitudes (see Sagdeev and Galeev (1969)).

Weak turbulence analyses generally apply when the bandwidth of the turbulence is large and when amplitudes are small. Waves lose their coherence quickly in such a situation, justifying the incoherent, or random phase approximation.

Kinetic, or random phase, versions of the PDI and nonlinear scattering processes exist and have generally the same effects as in the fluid treatment above. However, the OTSI and other modulational instabilities are stabilized in the random phase limit—coherent interactions are required for their existence (Robinson (1997)). Thus, in a low collisionality plasma, there is no resolution to the Langmuir paradox in weak turbulence theory. Only with sufficient collisional damping may a strictly weak turbulence cascade exist in steady state.
Application of WLT to ionospheric modification is exemplified by the papers of Fejer and Kuo (1973), Kruer and Valeo (1973), and Perkins et al. (1974). These are all numerical solutions of sets of wave kinetic equations derived using weak turbulence theory. The weak turbulence approach makes several assumptions (see Kruer and Valeo (1973)):

1. Weak HF, or "pump" field amplitude. This is quantified by requiring that the PDI growth rate be much less than the lowest real frequency in the problem, or

\[
\frac{1}{2} \frac{e_0 E_0^2}{nkT} \ll 16 \frac{\omega_{ac}}{\omega_{pe}},
\]

where \( \omega_{ac} \) is the ion acoustic wave frequency and \( E_0 \) is the pump wave amplitude.

2. Heavily damped ion acoustic waves, or

\[ T_e \equiv T_i. \]  

This allows one to neglect ion wave turbulence, keeping track of only the power transfer between the electromagnetic pump and the Langmuir waves.

3. Langmuir waves suffer only collisional damping due to electron-ion collisions.

Thus we have the limit

\[ k\lambda_{De} \ll 1. \]

This allows one to neglect quasilinear diffusion of the distribution function by the plasma waves.

Assuming these restrictions apply, one can calculate the form of the saturated power spectrum for a given pump power and angle to the magnetic field. Fejer and Kuo (1973) present several such results from a homogeneous ionosphere, and indicate that for pump powers representative of ionospheric modification and for angles smaller than about 20°, the power spectrum possesses several peaks downshifted from the pump frequency, the
first by $-\omega_{ac}$. Successive peaks are separated by $-2\omega_{ac}$. A weaker feature, symmetric about the HF frequency, is also expected and termed the “below threshold” line. Of course, the ISR at Arecibo observes only those fluctuations propagating at about 45° to the magnetic field. Additional effects must be invoked to explain the observations, such as refraction in or scattering from field-aligned irregularities. Refraction produces naturally propagating Langmuir waves which should appear along the line in $(f, z)$ space mentioned on page 36. Scattering on existing density fluctuations does not necessarily produce naturally propagating waves, although non-resonant waves are more highly damped.

The original formulation of WLT explains the height-integrated features 1, 2, and 3 as PDI cascades. However, the height-resolved measurements of Figure 1-8 clearly indicate that these spectral features (lumped into feature C) come from an altitude near reflection and not near matching, as supposed by WLT. These are obviously not Langmuir waves freely propagating in the mean background density—indeed, the locus of radar echoes in Figure 1-18 (bottom panel) extends below the local plasma frequency by 35 kHz. There must necessarily be density irregularities in the minima of which the waves are propagating. Muldrew (1978) and Muldrew and Showen (1977) proposed that these “ducts” were formed as a result of a thermal instability in the heated region, and had dimensions of about 20 m perpendicular to the magnetic field. Kelley et al. (1995) measured irregularities in the heated region with a sounding rocket, and found typical scale sizes of 10 m and relative depletions of 3-9%. More recently Lee et al. (1998b) showed convincing evidence for large sheet-like irregularities with dimensions of order hundreds of meters. Models for the formation of these irregularities are based on thermal instabilities (see e.g. Lee and Kuo (1985)), but the short rise-time of feature C perhaps indicates that the irregularities responsible for the early-time results are either indigenous, or produced by unrelated means.
As an addition to WLT theory, the LDI of Section 1.4.3(c) has been proposed to account for the broad feature C (Kuo and Lee (1999)). As mentioned earlier, however, the threshold field required to excite this instability is not present in current ionospheric modification experiments.

The original WLT formulation also requires some modification to properly explain feature B. For HF modification of a quiescent plasma, modes upshifted from \( f_{HF} \) can only be explained using four-wave interactions, of which there are none that would produce the observed frequency shift. However, if finite-amplitude low-frequency waves exist in the ionosphere before heating commences, the PDI produced Langmuir waves can "beat" or "coalesce" to produce these upshifted modes (see Kuo and Lee (1992)). The frequency of the upshifted modes varies from the pump frequency up to approximately 80 kHz higher than the pump frequency. Broadband lower hybrid waves produced by lightning, for example, could supply the needed fluctuations. Short-scale fluctuations produced directly by the heater wave may serve the same function at later heating times (see Kuo et al. (1983)). Since the PDI is operative over a large range of altitudes, the extent of feature B over several kilometers (see Isham et al. (2000)) is not surprising, assuming a fairly uniform level of lower hybrid turbulence exists over the same altitude range.

Observations of lower hybrid waves near ionospheric heating sites have not been performed as yet, but are planned. The recent acquisition of the Ionospheric Research Integrated System (IRIS) will enable radar measurements perpendicular to the magnetic field, and should settle many outstanding questions regarding the WLT interpretation of ionospheric heating.
1.4.4(b) Strong Turbulence

Zakharov (1984) described strong turbulence as "simply any turbulence which is not weak." Perturbative expansions cannot be used, since wave amplitudes can be large enough that successive powers enter to the same order-in magnitude. The waves themselves are modified by the nonlinearities, and so do not in general satisfy linear dispersion relations. Coherence between turbulent structures becomes important, and so the random phase approximation becomes invalid. Analytical progress is difficult.

Use of the term "SLT" generally refers to a numerical solution of the Zakharov equations (1.49) and (1.50), although it is not clear that the effects of strong turbulence are completely embodied in them. Numerical simulations of the driven Zakharov equations (a slightly different set than (1.49) and (1.50)) show cavitons forming and collapsing in a continuous cycle (see DuBois et al. (1988), (1990) and (1993), and Cheung et al. (1992)). After initial formation, cavitons collapse until Landau damping becomes large ("burn-out"). Cavitons then reform in what remains of the previous density depletions and the process continues. During the collapse process, Langmuir waves are "radiated" into the plasma outside of the cavitons. These are called "free modes," because they are not trapped within density depletions.

One can generate power spectra from the numerical simulations that correspond to what would be observed by a radar with a particular k. The above references have done this, and the similarity to ionospheric results is striking. A broad "caviton continuum" is observed in the simulations near the first Airy maximum (corresponding to our feature C). This feature is almost isotropic in k-space, and so would be directly observable by the ISR. The "free modes" appear several tens of kilohertz above the pump frequency (feature
at the same altitudes as the broad continuum. In addition, since the Zakharov equations include PDI, feature A is interpreted to be a weak version of a PDI cascade, exactly the same interpretation that comes out of WLT.

However, the extent of feature B over several kilometers of altitude is difficult to reconcile with SLT—one expects the free mode to appear at the same altitudes (and only those altitudes) as the caviton continuum (feature C). It can easily be shown that Langmuir waves damp after several tens of meters in the ionospheric plasma, and so propagation cannot explain the extent of the radar returns. Alternatively, caviton continua may be present at lower altitudes as well, but not be observable by current radar systems. This seems unlikely, given the large aperture of the Arecibo dish. A satisfactory explanation in terms of SLT has not yet been proposed.

In addition to the above difficulties, numerical solutions are limited in size by memory and processor time constraints. Most of the results so far obtained are for times early in the HF modification process—tens of milliseconds. For longer heating times, simulations are not practical as yet. In addition, the ambient magnetic field is often neglected in SLT calculations, although some authors have attempted (see references in Robinson (1997)). Wave modes possible in a magnetized plasma are therefore ignored in most treatments of SLT—perhaps a serious shortcoming.

1.4.4(c) Summary and Questions

Table 1-2 summarizes the theoretical interpretations of the various observed features and includes a short description of the interpretation of each feature for each theory. Several outstanding questions remain for each theory. First, SLT theories predict that feature B should be observed at the same altitudes as feature C. The results clearly show, however, that feature B exists independently of feature C for a significant altitude range. The absent feature C could be below the noise level of the receiving instruments—there is no way to
**Table 1-2. Interpretations of WLT and SLT.**

<table>
<thead>
<tr>
<th>Feature</th>
<th>WLT</th>
<th>SLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Below threshold OTSI line, evolving into a PDI cascade as the HF pulse progresses.</td>
<td>Same as WLT.</td>
</tr>
<tr>
<td>B</td>
<td>Freely propagating Langmuir waves, resonantly up-converted from the PDI-produced Langmuir waves and indigenous lower hybrid waves (or other field-aligned irregularities).</td>
<td>Freely propagating Langmuir waves, radiated by collapsing density cavitons and their corresponding Langmuir envelop solitons.</td>
</tr>
<tr>
<td>C</td>
<td>PDI cascade in field-aligned density depletions.</td>
<td>Caviton continuum. Goes through cycle of nucleation, collapse, and burnout.</td>
</tr>
</tbody>
</table>

tell for sure. Second, neither theory can explain the disappearance of feature B after several tens of milliseconds of heating, or the coincident strengthening and broadening of feature A. Third, WLT postulates background conditions that have not been verified: density irregularities in which feature C is thought to exist, and lower hybrid waves from which feature B scatters. Fourth, SLT simulations applicable to long heating times are sparse, and WLT numerical calculations for long heating times do not predict the observed time-varying spectral features.

The bottom line is that the jury is out—it is impossible to answer these questions with absolute certainty with ground-based remote sensing. Theoretically, the nonlinear mechanisms that must be responsible for the observed results are either not well understood or not well corroborated experimentally. The measurements themselves are not complete enough to provide an unambiguous picture of what actually occurred. This is not an indictment of the work that has been performed to date; rather, it is a testament to the complexity and analytical difficulty of high power plasma frequency heating. To what extent can results from our laboratory plasma shed light on the unexplained radar returns? It is the purpose of this dissertation to explore this question.
1.5 Outline

Prerequisites to a detailed examination of results are an understanding of the laboratory plasma and how measurements were made there, and an understanding of the ways in which the laboratory is similar to the ionosphere. The next two chapters address these requirements. Chapter 2 describes the various systems of the VTF used to contain, produce, and heat plasma. Diagnostics used in heating experiments are detailed here, along with theories of detection. Chapter 2 also includes a brief review of other relevant laboratory plasma frequency heating experiments.

Chapter 3 is devoted to the issue of similarity in plasmas, specifically of the ionospheric and the VTF plasmas. A method for deriving relevant dimensionless parameters is introduced, along with a discussion of boundary conditions. The two plasmas are then directly compared, resulting in a set of connections and limitations on the equivalence of results.

Chapter 4 then presents a detailed analysis of the laboratory results and a comparison with ionospheric modification data. Mechanisms proposed for ionospheric heating are then examined in light of the present laboratory results.

Finally, Chapter 5 summarizes the conclusions and lists some directions future work might take.
Chapter 2

Apparatus

"Imagine capturing the Northern Lights... in a magnetic bottle and investigating the beautiful celestial phenomena in the laboratory." —MIT Tech Talk, March 6, 1991.

The laboratory results presented in this dissertation were obtained from a machine called the Versatile Toroidal Facility, or VTF. The major motivation for the construction of the VTF was the creation of a model ionosphere within a laboratory setting. Laboratory "simulation" generally solves the diagnostic accessibility and availability problems found in the ionosphere — probes can be used to give in situ measurements and plasmas can be examined repeatedly in a convenient and controlled setting.

This chapter is intended to introduce the VTF machine and related systems, describe the background plasma configuration, and give some idea of what the resulting data looks like. Section 2.1 describes the plasma production and containment systems of the VTF, including vacuum and magnetic field configuration. The acquisition, transport, and storage of data is discussed in Section 2.2. Section 2.3 introduces the mechanical and electrical design of the sensors used to diagnose background conditions and wave activity within the plasma. Section 2.4 is devoted to sensor analysis, given its own section because of its importance in experiments like these. Section 2.5 outlines the plasma configurations used for these experiments and gives a list of typical parameters. To help place the current
experiments in the larger body of experimental plasma physics, Section 2.6 contains a brief review of laboratory plasma frequency heating experiments. Finally, Section 2.7 gives a quick look at some typical data obtained during plasma frequency heating.

2.1 Versatile Toroidal Facility

The VTF is a large toroidal plasma device with provisions for several magnetic field configurations (hence “versatile”). This machine is described in detail elsewhere (see e.g. Duraski (1991), Yoo (1991), Moriarty (1996), and Egedal et al. (2000)), so a brief summary should be sufficient here.

2.1.1 Chamber

The main vacuum chamber has a major radius of 0.94 m and is rectangular in poloidal section with height 1.07 m and width 0.65 m. It consists of two halves electrically insulated from one another. The halves are bolted together and sealed using two o-rings, differentially pumped. Sixteen side ports extend radially outwards some 0.51 m to allow mounting of radial diagnostics. Sixteen top and sixteen bottom ports, recessed only 4.4 cm, allow mounting of vertical diagnostics. The VTF is shown in top view in Figure 2-1 and in poloidal section in Figure 2-2. Ports are identified by toroidal position in degrees and vertical location, e.g. the vacuum system is mounted on the $200^\circ$ side port.

Ports are closed with port covers, some aluminum and some stainless steel, sealed with single o-rings in grooves. Half-nipples welded to the port covers allow mounting of diagnostics and RF injection antennae. For these experiments, two thermionic emitters were mounted on the floor of the chamber, and custom o-ring seals were used to provide cooling water and large electrical current feedthroughs. A collector plate mounted at the top of the chamber provided a sacrificial anode for the electron streams from the thermionic emitters. The two halves of the chamber were electrically connected on one side during these experiments, and the entire structure was grounded.
Figure 2-2. Poloidal section at 60° of the VTF. Thermionic emitter North Top is mounted on the bottom of the chamber. A collector plate extends down from the top of the chamber to provide a sacrificial anode for the electron streams of the emitters.

2.1.2 Vacuum System

Vacuum is maintained by two turbomolecular pumps in series with two roughing pumps. Base pressures in these experiments were typically 5x10^-7 torr. During operation the desired gas was leaked through a controllable metering valve to provide a stable operating pressure of 5 - 15x10^-5 torr, or 1.7 - 5x10^18 particles/m^3 neutral density, giving 0.3 - 1.0% base impurity levels. The gas most often used in these experiments was molecular hydrogen (H_2), although argon (Ar) was used occasionally as well.
Figure 2-1. Top view of the VTF. Vacuum chamber is shown in gray, toroidal field coils are shown in orange, and microwave antennae are marked in blue.
2.1.3 Magnetic Field

All the experiments described here used a dominantly toroidal magnetic field with a small amount of vertical field superposed. Eighteen four-turn copper magnets form the main toroidal field (TF). These were powered by two six-pulse rectifiers in parallel, each capable of 7000 A, giving a magnetic field in the range 0 - 2000 G. To measure the current, voltage across a 50 mV/8000 A shunt resistor was input to a Burr-Brown isolation amplifier with a gain of approximately 10. A 35 Hz low-pass filter was added to the output of the amplifier to average the fluctuating current produced by the switching process. The inductance of the TF coils is not large enough to completely attenuate the fluctuating current produced by the rectifiers. A fairly constant fluctuation level of about 600 A (peak-to-peak) at 360 Hz prevailed throughout these experiments. This amounts to a relative level of 10% with a DC current of 6000 A. The direction of the TF current is up in inboard side of the coils and down in the outboard side, giving a counter-clockwise toroidal field, as viewed from above (see Figure 2-1).

Two two-turn “Parail” coils form a Helmholtz pair aligned with the major axis of the machine. These provide up to 20 G of vertical field (VF). They are powered by a single twelve-pulse transformer-coupled rectifier, capable of 2000 A. Current is measured using a water-cooled shunt resistor. The direction of the field is down for positive current. Thus, the net magnetic field, including TF and VF, spirals down from the top of the chamber to the bottom of the chamber in a counter-clockwise direction as viewed from the top. The vertical pitch of the spiral varies linearly with major radius because the toroidal magnetic field varies inversely with major radius.

During a “shot”, or run cycle, both magnetic fields are activated approximately 2 s before the plasma, minimizing the effects of startup transients.
2.1.4 Thermionic Emitters

Thermionic emitters were mounted on the bottom port covers at 60° and 240°. These were designated "north top" and "north middle", respectively (the names have nothing to do with physical location). Each emitter consisted of several lanthanum hexaboride (LaB₆) rings friction-fit to a carbon rod approximately 10 mm in diameter (Figure 2-3).

![Diagram of emitter body](image)

**Figure 2-3.** Schematic of emitter body.

The rings were 19 mm in diameter, and the total length of the stack was 13 cm, giving an area normal to the magnetic field of approximately 25 cm². The carbon rod was friction-fit at both ends to holes in carbon end blocks. The end blocks were in turn bolted to copper support plates, which were bolted to water-cooled copper posts. Also mounted to the sup-
port plates were stainless steel back shields that prevent the backward emission of electrons from the assembly. During a shot, approximately 350 A were passed through each emitter, resistively heating the carbon rod and ceramic rings. At the end of about 60 s the rings were white-hot. The whole assembly was then biased 360 V negative with respect to the chamber. If the surface of the rings was clean and the correct neutral pressure was present in the chamber, an arc was created from the emitters up the magnetic field lines towards the collector plate at the top of the chamber. This arc was the background plasma for most of the experiments presented in this dissertation (see Section 2.5 for a more complete description of the equilibrium configuration). Emitted current varied greatly, from 0 - 200 A per emitter. Typical values for well-conditioned emitters were 30 A per emitter.

2.1.5 Magnetron

Microwave power at 2.45 GHz was injected from the 60° side port through a pyramidal waveguide horn. The opening dimensions of the horn were approximately 25 x 30 cm on the large end, tapering from standard WR-284 waveguide. These dimensions result in approximately 15 dBi gain over isotropic (see Love (1984)). A ceramic vacuum break followed by several meters of waveguide connected the horn to a 3 kW magnetron source, isolated by a three port circulator with a water-cooled dummy load. The polarization of the wave entering the machine could be changed by rotating the horn in the port (during a vacuum break). Most of these experiments used ordinary-mode polarization, with electric field approximately aligned with the magnetic field.

2.1.6 Control

All power supply timing, including magnetic field coils and magnetron, was controlled by an Allen-Bradley Programmable Logic Controller, or PLC. The PLC was controlled remotely by a personal computer. This system also provided the starting trigger for the data acquisition sequence and the scan and reset signals for the scanning Langmuir probe (see Section 2.3.1). A typical shot sequence was as follows:
Chapter 2

[T = -10 s]. Shot cycle begins. No power supplies are on.

[T = -5 s]. Warning klaxon sounds in cell. Langmuir probe resets to out position.

[T = 0 s]. Thermionic emitter filament begins to heat.

[T = 60 s]. Vertical field turns on.

[T = 61 s]. Toroidal field turns on.

[T = 62 s]. Thermionic emitter arc supply turns on. Conditions being good, an arc is produced with current limited by the power supply settings and/or emitter surface conditions and/or neutral gas pressure.

[T = 62.5 s]. Langmuir probe begins to scan radially inward. Magnetron turns on. Data acquisition sequence triggered.

[T = 65 s]. All power supplies switch off. Transfer of data to control room computers begins.

This sequence was modified as needed for the particular experiment being run.

2.2 Data Acquisition

All data were acquired in digital form and stored on the hard drive of a personal computer (PC) in the VTF control room. Mounted in the motherboard of the PC was a GPIB controller board in charge of the information flow along an extended GPIB bus, which allowed simultaneous connection of a large number of devices. Actual signal sampling was performed by several measuring instruments located near the VTF and described in the following subsections. These instruments communicated with and were controlled by the control room PC over the GPIB bus.

Data acquisition software was written in National Instruments’ LabView, a high level programming language (see Appendix B). These LabView routines utilized low level commands included in the GPIB standard to control the digitizing instruments. A graphi-
Apparatus

cal user environment was created for each instrument, allowing control of most instrument functions remotely, using the mouse and keyboard of the control room PC. The data was displayed on screen as it was stored, giving the investigators rapid feedback on the current plasma shot. The LabView routines could also display data already stored on disk. For most of the instruments used here, the following sequence of events occurred for every plasma shot (assuming the digitizing hardware was set up properly):

1. Data acquisition sequence initiated by investigator from control room PC.
2. Digitizing instruments armed and waiting for stop (or start) trigger.
3. Data acquisition trigger provided by PLC (at time specified on control room PC controlling the PLC). Sampling begins.
4. When instrument memory is full, sit and wait for readout by PC.
5. Data transferred to control room PC, one instrument at a time. When all available instruments were connected to the GPIB bus, this process took approximately 30 seconds.

All post-processing of the data was performed using Mathworks’ MATLAB environment (see Appendix B).

2.2.1 CAMAC

At the heart of a CAMAC system, an industrial standard architecture designed for data acquisition and signal conditioning, is a “crate” that provides power and limited communication pathways for modules that plug into an array of slots. Modules can be digitizers, memory, filters, communication interfaces, etc.

The VTF system used two CAMAC crates located in RF shielded cabinets. In each crate was a LeCroy 8901A GPIB module, allowing communication with the GPIB system. Also, each crate held a LeCroy 8212 digitizer, one with 4 x 32 kSample memory modules ("physics") and one with 2 x 32 ksample memories ("engineering"). The 8212s were 12-
bit, simultaneous sampling digitizers with up to 32 channels and up to 40 kSamples/s sampling rates. The physics module was set up to provide 4 bipolar (±5 V) channels, sampled at 10 ksamples/s, with 32 ksamples per channel, while the engineering module was set up for 32 unipolar (0 - 10 V) channels, sampled at 200 samples/s, with 2 ksamples per channel. The physics channels typically measured scanning Langmuir probe signals (Section 2.3.1), while engineering channels were used for power supply voltages and currents.

2.2.2 Oscilloscope

A LeCroy LT 344 WaveRunner digital storage oscilloscope was the workhorse for high-frequency signals. The LT 344 had four BNC-input channels, each of which had 250 ksamples of associated memory, at 8 bits per sample. The channels could be simultaneously sampled at up to 500 Msamples/sec. Four independent input amplifiers had a bandwidth of 500 MHz, and could be switched to provide a variety of gains. Low-pass anti-aliasing filters at 20 MHz or 200 MHz were provided. Input impedance could be switch-selected to 50 Ω, 1 MΩ DC-coupled, or 1 MΩ AC-coupled. Most of the present experiments used 50 Ω inputs with sensitivities of 10 - 50 mV/div (8 divisions full-scale, limited by digitizer saturation).

The LT 344 had a variety of programmable triggering options, including an external TTL signal, input signal level, or more complex logical operations. Most of these experiments used an external trigger provided by the PLC. Other trigger sources are noted where applicable.

The LabView data storage/viewing program for the scope included the option of displaying an estimate of the power spectral density of the measured sequences, useful the wave studies presented later.
2.2.3 Spectrum Analyzer

For measurement of signals at radio and microwave frequencies, an HP 8592L spectrum analyzer was used. This instrument gave a calibrated frequency spectrum for signals in the range of 9 kHz - 22 GHz (includes all frequencies of interest here).

Spectrum analyzers such as this one work by converting successive portions of the input spectrum to an intermediate frequency, which is filtered with a user-specified bandwidth, called the resolution bandwidth. The signal level at the intermediate frequency is then displayed versus signal frequency. One “sweep” of the measuring window across the input spectrum takes several tens of milliseconds.

The data output to the GPIB bus by the 8592L were groups of 401 samples of the measured spectrum at uniformly-spaced frequency intervals. Due to lack of sufficient on-board memory, measured spectra were transferred to the control room PC as they were acquired, rather than waiting for readout after the shot cycle.

In addition to measurements performed during plasma shots, this instrument was invaluable in setting proper signal levels in the microwave down-convertor system (Section 2.3.3).

2.3 Sensors: Design

For the plasmas created in the VTF, thermal energy densities were low enough in most of the plasma volume to allow immersed probes, properly designed, to survive. Immersed Langmuir probes can give a measure of electron density, electron temperature, and (roughly) plasma potential (Hutchinson (1987), pp. 50-86). Such probes can be constructed in arrays or scanned to provide spatial information. Properly constructed and impedance matched probes can also give a measure of the electric field fluctuations and/or the plasma density fluctuations over a range of frequencies set by the measurement network. Proper design can result in low plasma perturbation and high probe survivability.
(using small size and heat-resistant materials). Besides allowing in situ measurements, the low energy densities (and lack of large plasma currents) characteristic of VTF plasmas resulted in a background magnetic field that was not modified significantly by the presence of the plasma. Magnetic fields could thus be calculated from Biot-Savart integrals and the known boundary conditions (geometry and applied currents).

These experiments were primarily directed at observing wave activity in the plasma at fairly high frequencies. A scanning double probe and associated measurement and isolation networks were designed and built to fulfill this need. Background plasma density and temperature were diagnosed using a scanning Langmuir probe. This section covers the mechanical and electrical design of these systems, beginning with the scanning Langmuir probe (Section 2.3.1). Section 2.3.2 describes the design of the double probe, and Section 2.3.3 describes the measurement networks that were connected to the double probe to obtain fluctuation measurements in a variety of frequency bands. Sensor analysis and modelling are deferred until Section 2.4. The background magnetic field was simply calculated from boundary conditions (see Section 2.5).

2.3.1 Scanning Langmuir Probe

A scanning Langmuir probe was used to obtain the electron density and temperature and plasma potential in a line running radially along the midplane of the machine at 260° (see Figure 2-1). The probe tip was spherical, constructed of stainless steel, and 1.65 mm in diameter (see Figure 2-4). A 0.76 mm diameter stainless wire, covered along its entire length with a 2.5 mm diameter alumina tube, supported the tip and carried electrical current. Approximately 1.8 mm of the support wire was exposed to the plasma, which combined with the tip area yielded a total area of 12.3 mm² and a projected area (perpendicular to the magnetic field, two sides included) of 7.0 mm². Outside the probe wire, a second alumina tube 9.5 mm in diameter ran inside a 12.7 mm diameter stainless
Figure 2-4. Mechanical design of the scanning Langmuir probe.

steel tube, which provided the main structural support for the long assembly. The stainless tube was welded to a double-sided vacuum flange at the rear end. A flexible welded steel bellows placed over the center tube allowed radial motion while preserving vacuum integrity. The small diameter bellows end fittings served as support bearings for the stainless tube. An electrical vacuum feedthrough (unbalanced BNC type) was mounted at the rear of the probe body, allowing bias voltages to be applied to the tip.

The probe body was supported externally at three locations by a series of aluminum plates sliding on four hardened and ground steel rods. A digitally driven stepper motor, linked with a small chain to the outermost sliding plate, provided motion actuation. The
chain also drove a potentiometer, with which position was sensed, and supported a lead counterweight. Probe motion was controlled remotely by the PLC, as mentioned in Section 2.1.6. For the experiments presented in this dissertation, the tip was able to scan from 119.4 to 67.6 cm in radius in approximately 2 sec. Probe position could be calculated from the measured potentiometer slide voltage as

\[ R = 46.9 + 16.90 \, V, \]  

(2.1)

where \( R \) is major radius in cm and \( V \) is potentiometer voltage in V.

Bias voltage was applied and measured through an isolation network that has been described elsewhere (LaBombard and Lipschultz (1986)). Briefly, the circuit was driven by external amplifiers, and contained a small shunt resistor whose voltage drop was amplified to give a measure of the emitted probe current. A scaled and filtered version of this, along with a scaled and filtered version of the applied bias voltage, was output to the CAMAC digitizers. Thus, three digitizer channels were devoted to the scanning Langmuir probe: one each for position, voltage, and current.

2.3.2 Double Probe

Recall from Chapter 1 that we are interested in the formation and nonlinear interaction of electrostatic, or longitudinal, waves in the plasma. These are primarily Langmuir waves and ion acoustic waves in the ionosphere, although in the VTF one needs to add bounded plasma modes such as the Trivelpiece-Gould type to the list (for introduction see Krall and Trivelpiece (1986), pp. 167-177). Electrostatic waves in plasmas are easily sensed by immersed metal probes (see Chen (1965)). The use of two identical tips spaced a known distance apart allows one to determine the wavenumber as well as the frequency spectrum in a turbulent plasma (see Beall et al (1982) and Section 2.4).
With this in mind, we designed a wave diagnostic consisting of two cylindrical probe tips spaced \( b = 0.5 \text{ mm} \) apart, which allowed a minimum wavelength of 1 mm to be measured without spatial aliasing (\( |k|_{max} \approx 6.3 \text{ mm}^{-1} \), see Figure 2-5). The diameter of the tips was \( d_w = 0.28 \text{ mm} \), and the length of the parallel region was \( L = 13.5 \text{ mm} \). The analysis of how these tips responded to plasma wave fields is left until Section 2.4. The two tips were connected to two lengths of coaxial line and the whole assembly was immersed in the plasma. Because of the high heat loads, the entire assembly needed to be made of materials that retained their rigidity at high temperatures. The two tips were made of tungsten wire, which also formed the center conductors of the coaxial lines. For the coaxial dielectric we used two concentric alumina ceramic tubes, the inner having inner diameter \( \sim 0.33 \text{ mm} \) (0.013") and outer diameter 0.79 mm (0.031"), and the outer hav-
ing inner and outer diameters 0.79 mm (0.031”) and 1.60 mm (0.063”), respectively. Stainless steel tubing was used for the outer conductors (thin-walled 14 gauge 316 SS, 2.11 mm (0.083”) outer diameter), which were laser welded together at 1 cm intervals along their entire length. Covering this entire assembly (except for the tips) was a 6.6 mm (0.26”) OD alumina ceramic tube, to provide a refractory, dielectric plasma-facing surface.

This combination of inner conductor, outer conductor, and dielectric stack yielded an impedance of approximately 34 Ω, close to the value of 50 Ω required for optimum RF power transfer. However, the small cross section of the line makes this portion of coax fairly resistive, and so the length of this high temperature coaxial line was kept to a minimum, about 29 cm (the lossiness of the alumina dielectric (ε = 9.34 and σ = 3.3×10^{−12} \ \frac{S}{m} , according to CRC Handbook of Chemistry and Physics (1995), pp. 12-51 and 12-187) contributes negligibly compared to the small conducting area through which currents dissipate). Signals at 2.5 GHz propagating over this length experience a loss of about 1.5 dB. Lower frequency signals experience still lower loss. To connect to standard low-loss semi-rigid line, the two high-temperature lines contained a 45° bend near the end, onto which SMA connectors (modified for connection to the custom high-temperature coax) were soldered. The tip assembly was then entirely self-contained, and could be mounted and unmounted from the rest of the probe structure without major disassembly. Figure 2-6 is a photo collage of the various pieces of the removable tip assembly without the outer ceramic insulator.

The tips were supported on the end of a 12.7 mm diameter stainless steel tube, inside of which ran two lengths of UT-141 low-loss semi-rigid coax (see again Figure 2-6). The coaxial connections were covered with a vented aluminum cup, which also provided structural support for the tip assembly. The stainless tube was supported on two ceramic bear-
Figure 2-6. Plate of removable up resemble. Clockwise from upper left turn, an up up, an up, and an up turn made to some yard more center. The complete turn of stream into shown center of flexible lines and a part of a metal and center. The up resembles a metal pipe that had been cut for a trifocal canal. Station B can be seen in other states.
ings (vacuum compatible), the inner allowing both rotary and linear motion, and the outer allowing only rotary motion. The outer bearing was fixed to a translating stage, and so the stainless tube and attached tips could be scanned radially from a location inside the side port to well inside the thermionically produced plasma (from approximately 95 - 133 cm major radius). The tips could also be rotated about the longitudinal axis through a range of approximately 700°. To facilitate this rotary motion, the outer ends of the UT-141 were connected by short lengths (25 cm) of flexible coax (RG-174) to SMA vacuum feedthroughs. The entire double probe assembly is depicted in Figure 2-7.

The translating stage was a Velmex Unislide model MB4036P5J with a digitally controlled stepper motor drive. The Unislide was mounted to an aluminum I-beam, which also supported the front bearing housing. The double probe was installed horizontally on the side port at 60°, directly above the microwave pyramidal horn antenna. As the translating stage moved, the probe tips swept out a radial, horizontal line approximately 71.2 cm above the chamber floor, or 57.8 cm above the thermionic filaments, or 17.8 cm above the microwave antenna centerline which lies in the chamber midplane. The scanning rate was fairly slow—about 5 cm per second, and so during experimental campaigns the probe was fixed during plasma shots and moved to new locations between shots. A LabView program was written to control a multifunction data acquisition/control card (National Instruments PCI-1200), which in turn provided control signals to the linear motor drive circuit and counted the motor pulses. The linear motion could then be controlled interactively from the control room personal computers (see Appendix B for program names and locations).

Rotary actuation was provided by a geared-down, belt coupled AC motor. Rotary position was monitored by measuring the voltage drop across the slide of a corotating precision potentiometer. Another LabView program was written to interactively monitor and control the rotary position from the control room (see Appendix B).
Figure 2-7. Double probe assembly in longitudinal section.
The microwave signal path to the rear of the probe, including all connections, is shown in cartoon form in Figure 2-8. Each connection invariably introduces some mismatch, and so the number of these were kept to a minimum. In addition, wherever possible, impedance matched components rated to at least the operating frequency of 2.5 GHz were used. On the air side of the vacuum feedthroughs, identical 3 m lengths of low-loss, high frequency coaxial line (Urflex UFA210A) connected the probe to the isolating and measurement networks (described in the following section).

2.3.3 Double Probe Measurement Networks

The purposes of the measurement networks were to protect the sensitive measuring instruments (isolation) and, if necessary, to convert the signal to a form that was appropriate for measurement (conversion). Since electrical components have only a limited useful frequency range, the spectrum was divided into bands of interest. In these experiments there were two bands, each of which had its own isolation and conversion networks:

1. the "lower hybrid" band, which included 0 - 200 MHz, and which we denote the low frequency, or LF, band (not to be confused with the common definition of LF
which includes 30 kHz - 3 MHz).

2. the "plasma frequency" band, which included 2.3 - 2.6 GHz, and which we denote the high frequency, or HF, band (again, not to be confused with the common definition of HF).

Two measuring instruments were used: an HP 8592L spectrum analyzer and a LeCroy LT344 digital oscilloscope (see Section 2.2 for full descriptions). Both instruments had bandwidth covering our LF range, and so only isolation was required. The isolation circuit used for signals in this band is presented in Figure 2-9.

![Isolation circuit](image)

**Figure 2-9.** Isolation circuit used for signals in the band 0 - 200 MHz (LF). The two grounds are not the same in general. Dashed boxes represent modular components.

Here, the signal source is represented by some Thevenin equivalent $V_S$ and $Z_S$ at the far left. The coaxial line is represented by a transmission line with characteristic impedance $Z_0$ (in our case, 50 Ω), grounded to the chamber at a point in the middle of the line. The measurement instrument at the right end of the circuit ("spectrum analyzer") was either the LT 344 (with termination set at 50 Ω), or the 8592L. Both of these instruments ground the outer conductors of their coaxial inputs. Isolation was provided by a 1 mF capacitive break on the inner conductor, followed by a shunt RF transformer (MiniCircuits FTB 1-1 or FTB 1-6) between the outer and the inner conductors. This arrangement passed the band from 1 - 400 MHz. An attenuator (represented as two series resistors and a shunt
resistor) placed between the transmission line and the isolation capacitor facilitated broadband impedance matching (see Section 2.4.2(c). A $6 \text{ dB}$ attenuator was most often used, for which the values of the resistors are $R_1 = 16.6\Omega$ and $R_2 = 66.9\Omega$. Other values of attenuation yield different resistor values. When the LT 344 scope was used, its internal 200 MHz anti-aliasing low-pass filter was activated as well.

Two LF isolation networks were used—one for each probe tip. Each network possessed a flat passband to within the resolution of the spectrum analyzer (tested using a precision swept RF source and the 8592L).

In the HF band, the spectrum analyzer was capable of directly measuring signals, and so only isolation was required. The smaller bandwidth of the scope made additional conversion networks necessary (see below). Figure 2-10 shows the isolation network used at high frequencies. Again, the source is represented as a Thevenin equivalent at the far left of the schematic. The "spectrum analyzer" at the far right represents either the 8592L or the microwave down-converter (see below), both of which present a nominal $50 \Omega$ resistive load. Isolation is provided by precisely matched capacitive breaks on the inner and
outer conductors, passing frequencies above about 500 MHz (Microlab/FXR HR-52N). Again, an attenuator was placed between the transmission line and the isolator to facilitate impedance matching.

The LT 344 scope can sample at a maximum of 500 MSamples/s, giving a theoretical bandwidth of 250 MHz. Its input anti-aliasing filter attenuates frequencies above about 200 MHz. Thus, the HF band needed to be shifted down in frequency before being sampled by the scope. A microwave down-convertor was constructed to fulfil this requirement, shown in schematic form in Figure 2-11. At the heart of the convertor were two level 13 frequency mixers (MiniCircuits ZEM-4300MH), shown as purple circles. The mixer local oscillator drive was provided by a remote high-power FET amplifier (in orange). The local oscillator signal was generated by an Alfred Model 650 source, capable of output in the 2 - 4 GHz range (this was usually set at 2.35 GHz). Where appropriate, DC breaks (labelled HR-52N) were inserted to prevent ground loops. Attenuators were used to adjust the signal levels (shown in parentheses) appropriately. The inputs marked $X_{in}$ and $Y_{in}$ began at the rear of the probe, just after the vacuum RF feedthroughs. Fairly large input attenuation was used to keep the RF signals from overdriving the mixers. These values were arrived at by level testing with a calibrated spectrum analyzer. The outputs of the mixers were then fed through low-frequency isolation circuits similar to those pictured in Figure 2-9 (without the attenuator—a series capacitor followed by a shunt RF transformer), and then into the LT 344 inputs. With the fast-sampling scope, it was then possible to digitize a 250 MHz band starting at the local oscillator frequency—usually 2.35 - 2.60 GHz.

The outputs of the down-convertors were compared to each other and that of the spectrum analyzer for both monochromatic inputs and noise. The spectra agreed quite well in general, with some small level differences between mixers. The down-convertor/scope
Figure 2-11. Schematic of microwave down-converter.

combination was clearly superior to the spectrum analyzer during plasma runs—the entire spectrum could be measured in the time it took to fill up the scope’s memory, about 500 μs. The spectrum analyzer sweep time over this frequency range was about 80 ms.
2.4 Sensors: Analysis

The purpose of this section is to establish a quantitative description of the measurement process. Analyses of the sensors presented in the previous section fall naturally into two classes: the response of the probe tips to plasma conditions (both DC and fluctuating variables), and the propagation of the signals thus produced through the measurement/isolation networks to the instruments that perform the actual sampling. To this end, Sections 2.4.1 and 2.4.2 outline probe analysis and response to plasma waves and the transmission line systems used to guide signals to appropriate measurement devices.

2.4.1 Probe Tip Response

Immersed probes unavoidably perturb the plasma. Measurements taken using probes reflect the perturbed plasma conditions, and must be related analytically to unperturbed conditions. In these experiments, probes were used for DC measurements (as standard Langmuir probes), and for fluctuation measurements in two frequency bands, one near the ion plasma frequency (LF), and one near the electron plasma frequency (HF). It is useful to divide the analysis into frequency bands as well; the following three subsections apply to bands of increasing frequency. Another aspect of probe response is direction and wave-number sensitivity. This topic is given its own subsection following the HF analysis.

2.4.1(a) Tip Response to DC Fields

Let us begin by outlining the standard DC Langmuir probe analysis. Because later sections build on and modify the results of the standard analysis, we present slightly more of the analysis than would otherwise be required (for full discussion, see Chen (1965) and Hutchinson (1987), pp. 50-86).

A floating probe placed in a plasma rapidly charges negative with respect to the plasma, for \( T_e \geq T_i \), because of the much larger flux of electrons crossing a given surface. The probe reaches equilibrium at the floating potential, \( V_f \), which is usually about \( 3kT_e/e \) below the farfield plasma potential, \( V_p \). If a voltage less than the floating poten-
tial is externally imposed on the probe tip, electrons are increasingly repelled until only ions impact the surface (ion saturation). Assuming a Maxwellian electron velocity distribution for the unperturbed plasma, the value of the ion current emitted by the probe in the ion saturation region is given by

$$I_{si} = -eA_p n_\infty \exp \left( -\frac{1}{2} \sqrt{\frac{\kappa T_e}{m_i}} \right),$$

(2.2)

where $A_p$ is the area of the probe, $n_\infty$ is the farfield plasma density, and we have assumed singly-charged ions.

This value is nearly constant for tip biases

$$V_0 - V_p \leq \frac{\kappa T_e}{2e}$$

(2.3)

(the Bohm sheath criterion). In this range of applied voltages, most of the voltage drop from the probe to the plasma occurs in a thin layer adjacent to the probe surface, called the sheath. Sheath thicknesses are several Debye lengths, $\lambda_{De}$, where

$$\lambda_{De} \equiv \sqrt{\frac{\varepsilon_0 \kappa T_e}{e^2 n_e}} = \frac{v_{Te}}{\omega_{pe}}.$$  

(2.4)

Values of tip voltage greater than the limit of equation (2.3) produce a plasma that is quasi neutral right up to the probe surface.

As the probe tip bias is swept positively from the ion saturation region, more and more of the high-energy portion of the electron velocity distribution is collected. For Maxwellian distributions, the emitted electron current is given by

$$I_e = eA_p n_\infty \sqrt{\frac{\kappa T_e}{2\pi m_e}} \exp \left[ \frac{e(V_0 - V_p)}{\kappa T_e} \right].$$

(2.5)

where we have neglected the difference between probe surface area and sheath area. The total current emitted by the probe is then just
\[ I_{tot} = -eA_p n_\infty \sqrt{\frac{kT_e}{m_i}} \left\{ \exp\left( -\frac{1}{2} \right) - \frac{1}{2} \sqrt{\frac{2m_i}{\pi m_e}} \exp\left[ \frac{e(V_0 - V_p)}{kT_e} \right] \right\}. \] (2.6)

This equation contains three unknowns: \( n_\infty, T_e, \) and \( V_p. \) During experiments, the applied voltage, \( V_0, \) was swept and the probe current was measured. The three unknowns could then be determined by a least-squares fit of equation (2.6) to the \( I-V \) data, if a Maxwellian electron distribution was assumed.

For probe biases much greater than the plasma potential, all ions are repelled, and an increasing fraction of the electrons are collected, until the electron current reaches a constant value (electron saturation). Because of the large mobility of electrons, the electron saturation current is very large—large enough to vaporize the probes in the VTF. To keep probes within an acceptable operating temperature, then, one must operate at tip biases more negative than \( V_p. \) This is not a serious constraint; the electron temperature can be determined from the high-energy portion of the distribution only.

The assumption of a Maxwellian velocity distribution can be checked by plotting the \( I-V \) curve in log-linear coordinates. A straight line indicates a Maxwellian distribution, with the temperature given by the inverse slope. An \( I-V \) curve with two sections with different slopes indicates an electron distribution function with two different populations with different temperatures. To see this, note that the electron current emitted corresponds to that portion of the distribution that possessed enough energy far away from the probe to overcome the potential rise,

\[ I_e = eA_p \int_{-\sqrt{\frac{2eV_0}{m_e}}}^{\infty} n_\infty f_\infty(v_\infty) dv_\infty. \] (2.7)
where the subscript $\infty$ denotes quantities far away from the probe. Taking the derivative of equation (2.7) yields

$$\frac{dl_e}{dV_0} = \frac{e^2 A_p}{m_e} f_\infty \left( -\frac{2eV_0 \kappa T}{m_e} \right),$$  

(2.8)

which, for a Maxwellian is

$$\frac{dl_e}{dV_0} = \frac{e^2 A_p}{m_e} \frac{n_\infty}{\sqrt{2\pi \nu T_e}} \exp\left( \frac{eV_0}{\kappa T_e} \right) = \frac{I_e}{\kappa T_e/e}. \quad (2.9)$$

A strongly non-exponential $I$-$V$ curve perhaps indicates that one should use a more appropriate distribution in the probe analysis.

The analysis presented thus far assumed an isotropic plasma (no magnetic field). For a plasma in a strong magnetic field, particles are constrained to follow field lines, and so cannot approach the probe surface at arbitrary angles. Collected current is correspondingly reduced. At intermediate values of magnetic field (representative of the present situation) the electrons may be highly magnetized ($r_{Le} \ll a$, where $a$ is a typical probe dimension) while the ions are unmagnetized ($r_{Li} \gg a$). Under these conditions, the electron current varies exponentially and the temperature can be determined as before, and the ion current still follows equation (2.2) (see Hutchinson (1987), p. 66 for a full discussion). Thus, the analysis for the unmagnetized case should continue to hold.

2.4.1(b) Tip Response to Low Frequency Fields

Equation (2.6) of the previous section defines the nonlinear impedance of a Langmuir probe—that is, the relation between the terminal variables $I_{tot}$ and $V_0$. It is valid for frequencies low enough that the ions and electrons can both respond in ways that are consistent with the derivation. One might thus expect that the highest applicable frequency would be the ion plasma frequency. This is true only in certain ranges of tip bias.
The DC probe bias sets the operating point on the nonlinear $I$-$V$ characteristic, about which time-varying fields oscillate. We consider only operating points in the transition region (between ion and electron saturation), since all our measurements are taken in this range. For treatment of the ion and electron saturation regions, see Chen (1965) and references therein. In the transition region, ion behavior changes very little over wide ranges of tip bias—ions enter the sheath at the sound velocity, regardless of applied voltage. The frequency response is then set by the rate at which electrons can adjust to changes in the electric fields—generally much higher than the ion plasma frequency. The data of Takayama et al. (1960) indicate that the frequency response in the transition region may extend to near the electron plasma frequency. We are interested in the LF band (defined on page 92), which extends up to approximately one tenth the electron plasma frequency—well within the probe response range.

To find the operating point used in the present case, note that the circuit shown in Figure 2-9 ties the tip to chamber ground through a 50 $\Omega$ impedance (assuming minimal mismatch in the isolation network). This does not determine the DC operating point of the probe directly; however, the current and voltage at the probe tip are related through equation (2.6), and the current through the 50 $\Omega$ resistor determines the voltage drop across it. The intersection of these two conditions determines the operating point—i.e., the operating point $V_{op}$ is the solution of the equation

$$I_{si} \left[ 1 - \frac{m_i}{\sqrt{2\pi}m_e} \exp \left( \frac{1}{2} \right) \exp \left( \frac{V_{op} - V_p}{kT_e/e} \right) \right] = -\frac{V_{op}}{R}, \quad (2.10)$$

where $R = 50$ $\Omega$, $I_{si}$ is given by equation (2.2) and is less than zero, and all voltages are referenced to chamber ground. For typical plasma parameters, the solutions for hydrogen and argon plasmas are shown in Table 2-1. Values of $V_{op}$ are close to chamber ground, as would be expected noting the small value of $R$ and the much larger plasma-tip impedance.
<table>
<thead>
<tr>
<th></th>
<th>Hydrogen</th>
<th>Argon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$</td>
<td>10 eV</td>
<td>10 eV</td>
</tr>
<tr>
<td>$V_p$</td>
<td>+25 V</td>
<td>+25 V</td>
</tr>
<tr>
<td>$n_{\infty}$</td>
<td>$7 \times 10^{16}$ m$^{-3}$</td>
<td>$7 \times 10^{16}$ m$^{-3}$</td>
</tr>
<tr>
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<td>$1.2 \times 10^{-5}$ m$^2$</td>
</tr>
<tr>
<td>$I_{si}$</td>
<td>-2.5 mA</td>
<td>-0.4 mA</td>
</tr>
<tr>
<td>$V_{op}$</td>
<td>-0.16 V</td>
<td>-0.26 V</td>
</tr>
<tr>
<td>$I_{op}$</td>
<td>3.2 mA</td>
<td>5.3 mA</td>
</tr>
</tbody>
</table>

Table 2-1. DC operating point solutions for hydrogen and argon plasmas.

(see equation (2.19) below). The operating point is more positive than the floating potential by approximately 5 - 10 V, and so the probe draws significantly more electron current than a floating probe. The operating point current emitted is denoted by $I_{op}$.

Let us consider fluctuations in density, probe voltage, plasma potential, and hence emitted current, and ignore fluctuations in temperature. Writing each fluctuating quantity as the sum of a constant and a time-varying part, we have

\[ n_{\infty} = n_0 + n_1(t), \quad (2.11) \]

\[ V_0 = V_{op} + v_0(t), \quad (2.12) \]

\[ V_p = V_{p0} + v_p(t), \quad (2.13) \]

and

\[ I = I_{op} + i(t), \quad (2.14) \]

with $n_0$, $V_{op}$, $V_{p0}$, and $I_{op}$ constants. Taylor expanding the right-hand side ($R(t)$) of equation (2.10),

\[ R(t) = R|_{n_0, V_{op}, V_{p0}} + \left. \frac{\partial R}{\partial n_{\infty}} \right|_{n_0} n_1(t) + \left. \frac{\partial R}{\partial V_0} \right|_{V_{op}} v_0(t) + \left. \frac{\partial R}{\partial V_p} \right|_{V_{p0}} v_p(t) + \text{2nd order}, \quad (2.15) \]
or

\[
R(t) = -\frac{V_{op}}{R} - \frac{v_0(t)}{R}
\]

including all orders. To first order, the left-hand side \( (L(t)) \) becomes

\[
L(t) \equiv I_{op} + I_{op} \frac{n_1(t)}{n_0} + \frac{I_{op} - I_{si}}{\kappa T_e/e} [v_0(t) - v_p(t)].
\]

Equating (2.16) and (2.17), one can solve for \( v_0(t) \):

\[
v_0(t) = v_p(t) - \frac{R}{R + R_p} \frac{I_{op} n_1(t) RR_p}{R + R_p},
\]

where the tip-to-plasma impedance is defined as

\[
R_p \equiv \frac{\kappa T_e/e}{I_{op} - I_{si}}.
\]

The plasma-probe system can thus be modeled as a fluctuating voltage source, \( v_p(t) \), with impedance \( R_p \) in parallel with a current source of magnitude \(-n_1 I_{op}/n_0\) (see Figure 2-12).

**Figure 2-12.** Circuit model suggested by the linearized probe response relation.

Plasma voltage fluctuations are reduced by a division across \( R_p \) and \( R \). For the values of Table 2-1, the value of \( R_p \) is approximately 1750 \( \Omega \), producing a factor of 35 reduction in
the voltage at the probe tip (or about -30 dB in power). For enlightening discussions of the voltage division by a floating probe and one means of correcting for this effect, see Riddolls (1999).

Two modifications to equation (2.18) should be noted. First, the term containing $n_1(t)$ is likely to diminish in importance as the frequency is increased much beyond the ion plasma frequency. It vanishes altogether for floating probes, where $I_{op}$ is zero. This does not overly affect our measurements, since the fast electron response is still present in the term containing $v_p(t)$. Second, capacitive effects become increasingly important at high frequencies. Several authors have modeled the capacitance presented by the density depletion and sharp spatial gradient of the probe sheath (see e.g. Crawford and Grard (1966), Crawford (1965), and Harp and Crawford (1964)). The results were of the form

$$C_s = \alpha \frac{\epsilon_0 A_p}{\lambda_{De}},$$

(2.20)

where $\alpha$ is a constant of order unity that contains dependence on probe geometry. In other words, the probe sheath acts like a capacitor with free space dielectric and spacing on the order of a few Debye lengths. Using typical numbers for our plasmas, $C_s \approx 0.5 \text{ pF}$. If we include this capacitance in parallel with the tip-plasma resistance $R_p$, a frequency-dependent tip-plasma impedance results:

$$Z_p = \frac{R_p}{1 + j\omega R_p C_s}.$$  

(2.21)
Values of $Z_p$ at several frequencies are shown in Table 2-2. The addition of $C_s$ does not seriously change the magnitude of the impedance in the LF band, although a small phase shift is introduced ($\sim 32^\circ$ at 120 MHz). In the HF band (described in the next section), the sheath capacitance drastically reduces the tip-plasma impedance, and introduces a phase shift on the order of $65^\circ$.

It is instructive to note the effects of the second-order terms in equation (2.10). Taylor expanding as before and keeping terms up to second order, the right-hand side is unchanged from equation (2.16) and the left-hand side becomes

$$L(t) \equiv I_{op} + I_{op} \frac{n_1(t)}{n_0} \left( I_{op} - I_{si} \right) \left[ \frac{v_0(t) - v_p(t)}{\kappa T_e/e} + \frac{1}{2} \frac{v_0^2(t) - v_p^2(t)}{(\kappa T_e/e)^2} - \frac{v_0(t)v_p(t)}{(\kappa T_e/e)^2} + \frac{n_1(t)v_0(t) - v_p(t)}{n_0 \kappa T_e/e} \right] .$$

(2.22)

For perturbation quantities with zero mean, averaging causes the second term to vanish, as well as the first term inside the brackets (the first-order terms). Averaging the second-order terms produces some nonzero component that is a function of the relative phases of the perturbed quantities. In other words, measurements at DC include a rectified component of fluctuating variables, produced by the nonlinear probe characteristic. Several authors have investigated effects of sheath rectification, including both large signal analyses (Boschi and Magistrelli (1963)) and correlated small-signal analyses (Crawford
(1963)). The determination of density and temperature from the DC characteristic is unaffected by rectification, but the floating potential is shifted significantly negative for modest fluctuation levels. In other words, plasma potential cannot be identified with any certainty in a fluctuating plasma (in which the fluctuation amplitude is unknown).

The nonlinear terms in equation (2.22) produce another important effect: fluctuations at one frequency can produce measured power at other frequencies. This is just a more general version of sheath rectification, with the analysis extended to frequencies other than DC. Fourier transforming (2.22) gives

\[
\hat{L}(\omega) = 2\pi I_{op} \delta(\omega) + I_{op} \frac{\hat{n}_1(\omega)}{n_0} + (I_{op} - I_{st}) \left[ \frac{\hat{v}_0(\omega) - \hat{v}_p(\omega)}{\kappa T_e/e} \right] + \frac{1}{4\pi} \frac{\hat{v}_0(\omega) \otimes \hat{v}_0(\omega) - \hat{v}_p(\omega) \otimes \hat{v}_p(\omega)}{(\kappa T_e/e)^2} + \frac{1}{2\pi} \frac{\hat{n}_1(\omega) \otimes \hat{v}_0(\omega) - \hat{n}_1(\omega) \otimes \hat{v}_p(\omega)}{n_0 \kappa T_e/e} - \frac{1}{2\pi} \frac{\hat{v}_0(\omega) \otimes \hat{v}_p(\omega)}{(\kappa T_e/e)^2},
\]

where the Fourier-transform pair is

\[
\hat{g}(\omega) = \int_{-\infty}^{\infty} g(t)e^{j\omega t}dt
\]

\[
g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(\omega)e^{-j\omega t}dt
\]

and the convolution is defined

\[
g_1(t) \otimes g_2(t) = \int_{-\infty}^{\infty} g_1(x)g_2(t-x)dx.
\]

The first-order terms give rise to spectra at the excitation frequencies. The second-order terms are convolutions in frequency space and give rise to sum and difference frequencies in the power spectrum. The measured spectrum thus possesses peaks at the excitation frequencies (actual plasma fluctuations) and at sum and difference frequencies.
(probe-produced fluctuations). For small plasma fluctuations, the second-order terms are smaller than the first-order terms and one can ignore this effect. In the presence of large amplitude fluctuations, however, the frequency-shifted spectral components can be larger than the first-order components.

2.4.1(c) Tip Response to High Frequency Fields

The studies mentioned in the previous section applied to fluctuations at frequencies much less than the local plasma frequency. However, measurements of sheath rectification indicate that near and above the plasma frequency, rectification ceases—electrons are no longer governed by the thermal equilibrium Boltzmann factor (Takayama et al. (1960)). High frequencies also magnify the effects of capacitance. In fact, at microwave frequencies the response of a small probe is dominated by capacitive coupling, as is shown below.

In this section we obtain the response of a double-tip electrostatic probe to a plasma wave. Our analysis is drawn mainly from Torvén et al. (1995). We consider a probe formed of two long parallel wires connected to the center conductors of a pair of coaxial transmission lines, as shown previously in Figure 2-5. The probe wires have length $L$, diameter $d_w$, and separation $b$. They are inserted in a plasma wavefield with dimensions larger than $L$. The outer conductors of the transmission lines are welded together at the tip. If $L$ is made large enough we may neglect the perturbation of the outer conductors on the wavefield and model the probe as two parallel infinite cylinders.

The field between the cylinders is a superposition of the undisturbed wavefield, $E_{w}$, the field produced by the surface charges on the cylinders, $E_q$, and the field produced by modification of the space charge distribution, $E_p$. The potential difference between the tips is obtained by integrating along a line separating the tips:

$$V_2 - V_1 = -\int_{1}^{2} (E_w + E_q + E_p) \cdot ds$$

(2.27)
If the tip separation $b$ is small relative to the $x$-wavelength of the electric field (as it must be to avoid spatial aliasing), the wave electric field is essentially constant near the probe wires (call it $E_0$). The probe wires themselves are good conductors and so are equipotentials. The wires carry charges (per unit length) $\pm q$, the magnitude depending on the excitation and the load circuit. To begin, we consider a vacuum electric field and neglect the space charge effects of the plasma. The electric potential then satisfies the Laplace equation in the region outside the probe wires. The complete solution is found in Appendix A. The result is

$$
\phi = \frac{q}{2\pi \varepsilon_0} \xi - \frac{\alpha E_0 \sinh \xi}{\cosh \xi + \cos \theta} + 2\alpha E_0 \sum_{n=1}^{\infty} (-1)^n \exp(-n \xi_0) \frac{\sinh(n \xi)}{\sinh(n \xi_0)} \cos(n \theta), \quad (2.28)
$$

where

$$
\alpha = \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{d_w}{2}\right)^2}, \quad (2.29)
$$

$$
\xi = \text{atanh} \left( \frac{2\alpha x}{\alpha^2 + x^2 + y^2} \right), \quad (2.30)
$$

$$
\xi_0 = \text{acosh} \left( \frac{b}{d_w} \right), \quad (2.31)
$$

$$
\theta = \text{atan} \left( \frac{2\alpha y}{\alpha^2 - x^2 - y^2} \right). \quad (2.32)
$$

This solution gives the potential difference between the tips as

$$
\phi_2 - \phi_1 = E_0 b \sqrt{1 - \left(\frac{d_w}{b}\right)^2} - \frac{q \text{acosh} \left( \frac{b}{d_w} \right)}{\pi \varepsilon_0}, \quad (2.33)
$$

where we identify the capacitance per unit length between the tips to be
\[ C_0 = \frac{\pi \varepsilon_0}{\text{acosh}\left(\frac{b}{d_w}\right)} \]  

Substituting the parameters of Figure 2-5 leads to \( C_0 \) of approximately 23.5 pF/m, or a tip-to-tip capacitance of about 0.32 pF.

Equation (2.33) is the integral of the first two terms in the integrand of equation (2.27). The third integral, due to space charge modifications, we denote \( V_p \). If the probe tip spacing is small (a few Debye lengths), and the frequency is high (at or above the plasma frequency), we expect \( V_p \) to be small: electrons cannot flow quickly enough to shield the electric field.

The action of a plasma wave on the tips consists of two parts: the wave electric field drives capacitive displacement currents \( I_d \) between the tips, and wave particle currents \( I_p \) impact the probe wires (but not the outer conductors). Let us begin by treating the system without particle currents, and then estimate the relative magnitudes of \( I_d \) and \( I_p \).

We can model the system of plasma wave, tips, and coax as the circuit shown in Figure 2-13. Here, \( R \) is the nominal impedance of the coaxial line, if the transmission circuit is properly impedance matched (see Sections 2.4.2 and 4.6.2(b)). The tip-to-tip capacitance is represented by \( C_{12} \) and the tip-to-shield capacitance is \( C_s \) (assumed equal and much smaller than \( C_{12} \) for both tips). The action of the wave electric field is introduced as the emf \( U_0 \) between the tips. Comparing Figure 2-13 and equation (2.33), we can identify the quantity \( U_0 \) as

\[ U_0 = E_0 b \sqrt{1 - \left(\frac{d_w}{b}\right)^2}. \]  

(2.35)
The capacitance between the tips, $C_{12}$, is most simply calculated as $C_{12} = LC_0$, where $C_0$ was given in equation (2.34). This gives errors of a few percent with sufficiently long $L$, according to Torvén et al (1995). We ignore the small effect of $C_s$ in this analysis, only noting its existence to show that an isolated probe tip can indeed pick up signals from electrostatic waves.

Figure 2-13. Idealized probe tip circuit, neglecting particle currents.

Fourier transforming, the current through the capacitor is

$$\hat{I}_d = \hat{U}_0 \left[ \frac{j \omega C_{12}}{1 + 2j \omega RC_{12}} \right],$$

where the "hat" denotes a Fourier transformed quantity. This current we shall denote the "displacement current", referring to the sloshing of charge back and forth between the two reservoirs in the tip volumes. It is unrelated to the displacement current in plasma waves, except for the fact that it is excited by the wave electric field. If we assume that the probe wires are approximately short-circuited by the transmission line impedance ($\omega^2 C_{12}^2 R^2 \ll 1$, a condition fairly well satisfied here), the displacement current in the capacitor $C_{12}$ is
\[ I_d \equiv C_0 L \frac{dU_0}{dt} = \frac{\pi \varepsilon_0 L}{\text{acosh} \left( \frac{b}{d_w} \right)} b \sqrt{1 - \left( \frac{d_w}{b} \right)^2} \frac{dE_0}{dt}. \] (2.37)

This displacement current is driven through the end impedances of both transmission lines, nominally 2 \cdot R, causing a signal to propagate down the lines. Thus, the entire tip response circuit is localized to a small region around the tips—currents return through the impedances presented by the coaxial lines at the tip. Also note that the same current \( I_d \) flows through both coaxial lines. This means that in the absence of plasma particle currents, a capacitive double probe cannot be used to give phase-lag measurements at high frequencies—the excitation of each coaxial line is exactly the same. This also indicates that just one of the coaxial signals is required for measurement (the other can be used to give simultaneous measurements at LF, for example).

To estimate the relative magnitudes of \( I_d \) and \( I_p \), note that the oscillating current in an electrostatic wave is related to the electric field by

\[ J_1 = -j \omega \varepsilon_0 \chi_L \dot{E}_1 = j \omega \varepsilon_0 \dot{E}_1, \] (2.38)

where \( \chi_L \) is the longitudinal susceptibility (-1 for electrostatic waves), and the subscript "1" refers to small oscillating component. This relationship holds for a field-free, homogeneous equilibrium, which does not exist in general near the probe tips. However, if the probe tips were DC-biased to the plasma potential, this expression would apply exactly. Lower probe potential (near floating, for example) only reduces the magnitude of the electron particle current by a Boltzmann factor. In other words, an estimate using equation (2.38) gives an upper bound on the particle current. Now, \( I_p \) is the net current flowing between the tips, so it can be approximately related to the wave current by Taylor expanding in \( x \) and Fourier transforming:
\[ I_p = A_p \left[ J_1 \left( \frac{b}{2} \right) - J_1 \left( \frac{b}{2} \right) \right] \equiv -A_p b \frac{dJ_1}{dx} \bigg|_0 \leftrightarrow -j k A_p b J_1, \]  

(2.39)

where \( A_p \) is the probe area presented to the wave current, \( A_p \equiv \pi d_w L \). Substituting from equation (2.38) and a Fourier-transformed equation (2.37),

\[ \left| \frac{\dot{J}_p}{\dot{J}_d} \right| \equiv k d_w \text{acosh} \left( \frac{b}{d_w} \right). \]  

(2.40)

Substituting typical values for the present setup gives a displacement current larger than the particle current by a factor of \( \sim 2.5 \). Considering the conservative nature of this estimate, we treat the probe tip response in the HF band as purely capacitive displacement current, as shown in the equivalent circuit of Figure 2-13.

Note that the frequency dependence in \( \dot{J}_p \) and \( \dot{J}_d \) has cancelled, leaving a result that is independent of frequency. Thus, this analysis also applies to the LF band, with the result that in both the HF and the LF band, the double probe responds directly to electric field fluctuations. In addition, to a fair approximation, the displacement current dominates the particle current over both bands. Hence, an estimate of the absolute electric field can be made using equation (2.37), neglecting the effects of particle currents on the total tip response.

2.4.1(d) Direction/Wavenumber Sensitivity

This section briefly describes the use of a fixed probe pair (recall Figure 2-5) to extract information about the wavenumber power spectrum (magnitude and direction).

The undisturbed wave electric field may be oriented at arbitrary angles with respect to the direction of separation of the probe wires (say, \( x \)). In the plane normal to the probe wires (the \( x-y \) plane), components of electric field in the \( y \)-direction do not contribute to the potential difference between the probe wires. Only the \( x \)-component (call it \( E_0 \)) produces a potential difference. Since the length \( L \) is very large compared to the separation \( b \),
components of electric field in the direction of the probe wires are shorted by the wires, especially if the correlation length of the fluctuation is short. The probe response to isotropic three-dimensional excitation is then dominated by the x-component. For anisotropic turbulence, the probe still gives maximal response when the direction of tip separation is aligned with the anisotropy. Recording spectra at various angles should produce a maximum at a certain angle, which can be interpreted as the "dominant" direction of propagation for electrostatic waves. Thus, a double probe can give some directional information, but cannot give the kind of selectivity afforded by a scattering experiment.

The magnitude of the wavenumber spectrum can be determined using the method of Beall et al. (1982). First, some definitions. Consider a one-dimensional, stationary, homogeneous random field $\phi(x, t)$. Fourier transforming over a time interval $T$ and within a region of length $L$,

$$\Phi(x, \omega) = \int_{-T/2}^{T/2} \phi(x, t) \exp(j\omega t) dt$$

(2.41)

and

$$\Phi(k, \omega) = \int_{-L/2}^{L/2} \Phi(x, \omega) \exp(-j k x) dx.$$  

(2.42)

The spectral density is defined as

$$S(k, \omega) \equiv \lim_{L,T \to \infty} \frac{1}{LT} \langle |\Phi(k, \omega)|^2 \rangle$$

(2.43)

and the cross spectral density is

$$H(\chi, \omega) \equiv \lim_{T \to \infty} \frac{1}{T} \langle \Phi^*(x, \omega) \Phi(x + \chi, \omega) \rangle,$$

(2.44)

where the angle brackets denote ensemble averages.
The quantity \( S(k, \omega) \) is the holy grail of plasma wave diagnostics. The frequency spectral density can be obtained by integrating over \( k \):

\[
S(\omega) = \int_{-\infty}^{\infty} S(k, \omega) \, dk.
\]  

(2.45)

The first moment is the "statistical dispersion relation":

\[
\bar{k}(\omega) = \int k \frac{S(k, \omega)}{S(\omega)} \, dk.
\]

(2.46)

and the second moment is the "wavenumber spectral width":

\[
\sigma^2_k(\omega) = \int [k - \bar{k}(\omega)]^2 \frac{S(k, \omega)}{S(\omega)} \, dk.
\]

(2.47)

For waves following a deterministic dispersion relation, a contour plot of \( S(k, \omega) \) displays the dispersion relation.

To estimate \( S(k, \omega) \) from two fixed probe tips, let us first define a quantity called the local wavenumber, \( K(x, \omega) \), analogous to "instantaneous frequency" in the time domain. We can represent \( \Phi(x, \omega) \) as

\[
\Phi(x, \omega) = a(x, \omega) \exp[j \theta(x, \omega)].
\]

(2.48)

where

\[
a(x, \omega) = |\Phi(x, \omega)|
\]

(2.49)

and

\[
\theta(x, \omega) = \frac{\text{atanh} \{ \Phi(x, \omega) \}}{\text{Re} \{ \Phi(x, \omega) \}}.
\]

(2.50)

Then,

\[
K(x, \omega) = \frac{\partial}{\partial x} \theta(x, \omega).
\]

(2.51)

This can be estimated from two fixed tips as
$$K(x, \omega) = \frac{\theta(x_2, \omega) - \theta(x_1, \omega)}{x_2 - x_1}$$  \hspace{1cm} (2.52)

with

$$x \equiv \frac{x_2 + x_1}{2}. \hspace{1cm} (2.53)$$

Considering the plasma turbulence to be a superposition of many wave packets, with the amplitude and wavenumber of each wave packet slowly varying, it can be shown that the local spectral density \( S_i(K, \omega) \) is the same as the conventional spectral density \( S(k, \omega) \) (Beall et al. (1982)).

To obtain an estimate of \( S_i(K, \omega) \), the following digital algorithm is used:

1. Take long streams of simultaneous digital samples from both tips. This gives the quantities \( \phi(x_1, t) \) and \( \phi(x_2, t) \).

2. Divide the data streams into many short pieces and perform a fast-Fourier transform (FFT) on each piece. This gives a set samples indexed by \( i \): \( \Phi^{(i)}(x_1, \omega) \) and \( \Phi^{(i)}(x_2, \omega) \).

3. Compute the sample cross spectral density,

$$H^{(i)}(x_p, \omega) = \Phi^{(i)*}(x_1, \omega)\Phi^{(i)}(x_2, \omega) = C^{(i)}(\omega) + jQ^{(i)}(\omega).$$  \hspace{1cm} (2.54)

4. Compute the sample local wavenumber,

$$K^{(i)}(\omega) = \frac{1}{x_p} \text{atan} \left[ \frac{Q^{(i)}(\omega)}{C^{(i)}(\omega)} \right].$$  \hspace{1cm} (2.55)

5. Add weighted samples to the correct bins to obtain the local wavenumber and frequency spectrum,

$$S_i(K, \omega) = \frac{1}{M} \sum_{i=1}^{M} I_{[0, \Delta k]} [K - K^{(i)}(\omega)] \frac{1}{2} [S^{(i)}(\omega) + S_2^{(i)}(\omega)].$$  \hspace{1cm} (2.56)
where $I_{[0,\Delta k]}$ is a binning function and $S_1^{(i)}(\omega)$ and $S_2^{(i)}(\omega)$ are the frequency spectral densities at tips 1 and 2.

Note that this algorithm is quite similar to averaged periodogram methods of spectral estimation (see Kay and Marple (1981)). Where wavenumber spectra are shown in this dissertation, the preceding algorithm was used in the form of a MATLAB mfile (see Appendix B).

It should be noted that there are other methods of obtaining $S(k,\omega)$, chiefly by using arrays of probes or repetitive sampling using tips whose separation can be varied (see Harker and Ilic (1974) and Ilic and Harker (1975)). The method of Beall et al. (1982) allows the mechanical design to be very simple, and does not depend on plasma conditions being identical from shot to shot.

2.4.2 Signal Propagation

At high frequencies, propagation effects become more and more consequential to the overall quality of measurements. Langmuir waves propagate at or above the plasma frequency, which is on the order of several GHz in the VTF. At these frequencies, wavelengths are often shorter than the device sizes, necessitating a full electromagnetic analysis. Fortunately there exists the simple formalism of transmission line analysis, usually included in full detail in undergraduate electromagnetic texts. This section provides an introduction sufficient to enable explanation of the concepts and effects of impedance mismatch, a graphic example of which is discussed in Chapter 4.

2.4.2(a) Transmission Lines

A transmission line is a structure, generally of uniform cross section, that transmits energy or signals from one place to another. A piece of coaxial cable is a primary example. Signals travel as electromagnetic waves, with both electric and magnetic fields oriented
transverse to the direction of propagation (TEM waves). For any TEM line, one can define a voltage, \( V(z) \), and a current, \( I(z) \), that are generally related by the transmission line equations,

\[
\frac{dV(z)}{dz} = -j\omega LI(z),
\]

and

\[
\frac{dI(z)}{dz} = -j\omega CV(z),
\]

where \( z \) is the coordinate in the direction of propagation, \( L \) is the inductance per unit length, \( C \) is the capacitance per unit length, and \( \omega \) is the angular frequency of the wave (see e.g. Staelin et al. (1994)). Differentiating and substituting, one can obtain the voltage and current wave equations

\[
\frac{d^2 V(z)}{dz^2} = -\omega^2 LCV(z)
\]

\[
\frac{d^2 I(z)}{dz^2} = -\omega^2 LCI(z).
\]

The velocity of propagation is \((LC)^{-1/2}\). The most general solution to these is

\[
V(z) = V_+ e^{-jkz} + V_- e^{jkz},
\]

where \( V_+ \) and \( V_- \) are complex amplitudes of forward- and backward-traveling waves, and \( k \) is the propagation constant, \( k^2 = LC\omega^2 \). The corresponding solution for the current is

\[
I(z) = \frac{1}{Z_0} (V_+ e^{-jkz} - V_- e^{jkz}),
\]

where \( Z_0 = \sqrt{\frac{L}{C}} \) is the characteristic impedance of the transmission line. At any point on the line the voltage and current are the sum of forward- and backward-traveling waves.
2.4.2(b) Transmission Line Systems

Let us consider a simple transmission line system consisting of a source, a transmission line, and a load (see Figure 2-14). The origin is defined to be the load position. At any point \( z \) along the line, the voltage and current are given by

\[
V(z) = V_+(e^{-jkz} + \Gamma_L e^{jkz})
\]

and

\[
I(z) = \frac{V_+}{Z_0}(e^{-jkz} - \Gamma_L e^{jkz}),
\]

where

\[
\Gamma_L \equiv \frac{V_-}{V_+}
\]

is the load reflection coefficient. The equivalent impedance (looking right in Figure 2-14) is
Apparatus

\[ Z(z) \equiv \frac{V(z)}{I(z)}. \]  \hspace{1cm} (2.66)

At the load, then, the impedance is related to \( \Gamma_L \) as

\[ Z_L = Z(0) = \frac{V(0)}{I(0)} = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}, \]  \hspace{1cm} (2.67)

or equivalently,

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}. \]  \hspace{1cm} (2.68)

The general form of the impedance is then

\[ Z(z) = Z_0 \left( \frac{Z_L - jZ_0 \tan k\phi}{Z_0 - jZ_L \tan k\phi} \right). \]  \hspace{1cm} (2.69)

Thus, the impedance presented to the source varies with the length of the transmission line and the load impedance. At any position \( z \) we can also define a complex reflection coefficient,

\[ \Gamma(z) \equiv \frac{V_e z}{V_s} e^{-jkz} = \Gamma_L e^{2jkz}. \]  \hspace{1cm} (2.70)

There is a one to one relationship between impedance and reflection coefficient:

\[ Z(z) = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)}. \]  \hspace{1cm} (2.71)

This relationship can be plotted graphically as an overlay of the complex \( Z \)-plane and the complex \( \Gamma \)-plane. This form is known as the Smith chart, and can be used for graphical estimation of impedance and reflection transformations.

Several results of the relations mentioned above should be mentioned here. First, if the line impedance \( Z_0 \) is the same as the load impedance \( Z_L \), the reflection coefficient \( \Gamma_L \) vanishes. Thus, at any point looking to the right in Figure 2-14, the impedance is just the line impedance. In other words, the transmission line collapses down to just a load of \( Z_0 \) no
matter where the observation takes place. This is the desirable case for the measurement
systems presented here because it is frequency-independent. Second, a mismatch of line
and load impedances produces a standing wave pattern on the transmission line for a sin-
gle frequency excitation, or equivalently, a frequency-varying voltage division between
the source impedance and the line. The magnitude of the voltage at any point \( z \) is given by
\[
|V(z)| = |V_+| |1 + \Gamma(z)|. \tag{2.72}
\]
For a single frequency excitation, this is an oscillatory pattern in \( z \) with positive maxima
and minima. The magnitude of the mismatch is often characterized by the voltage standing
wave ratio, or VSWR, defined as
\[
VSWR \equiv \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{1 + |\Gamma(z)|}{1 - |\Gamma(z)|}. \tag{2.73}
\]
Third, note that more complex systems can be analyzed starting at the load and moving
back toward the source, transforming impedances and reflection coefficients at every inter-
face.

2.4.2(c) Broadband "Matching"

The physical world is not ideal: every component in a transmission line system cannot
be expected to have the same nominal impedance. Luckily, various circuits that "match"
two unmatched components have been devised (e.g., stub tuners). Unfortunately, these
schemes are limited to a single frequency, much like optical quarter-wave layers.

Rather than attempting to use reactive tuning, one can simply live with the mismatch
and try to attenuate the effects as much as possible. For example, if the load impedance
were different than the line impedance in Figure 2-14, one could insert a matched attenua-
tor between the line and the load (attenuators are generally easily matched over broad fre-
quency ranges, unlike almost all other components). The wave traveling from the source
would be attenuated once, while the reflected wave would be attenuated twice. The
reflected wave would then travel back to the source, where it would be reflected again from the source-line mismatch. The reflected wave would then pass through the attenuator once again on its way to the load. In effect, this method merely provides a termination, rather than a matching of components. Inserting attenuators provides some degree of isolation between components. Low signal levels can be partially compensated by appropriate amplification.

2.5 Plasma Configuration

The VTF plasmas were produced by a hot lanthanum hexaboride (LaB₆) cathode, discharging to the anode formed by the inside wall of the chamber and the collector plate at the top of the chamber (see Figures 2-1, 2-2, and 2-3). This configuration is known as a thermionic plasma discharge (TPD). The VTF plasmas were embedded in a more complex magnetic field than usual for a plasma diode—a helix. Section 2.5.1 describes the plasma in detail sufficient for subsequent analyses of wave activity. Typical plasma parameters are calculated and listed in Section 2.5.2.

2.5.1 Thermionic Diode

In a TPD, electrical current is driven through a plasma-filled cavity by an external circuit. The cathode is made of or coated with a low-work-function material such as tungsten or LaB₆, which when heated emits electrons at a rate given by the Richardson law for the particular material (Richardson (1914); see also Reiser (1994)):

\[ J = AT^2 e^{\frac{W}{kT}}, \]  

(2.74)

where \( T \) is the cathode temperature, \( W \) is the cathode work function, and \( A \) is a constant. In our case, \( T = 1100 \) °C and the values for LaB₆ are \( W = 2.4eV \) and \( A = 40\frac{A}{cm^2K^2} \) (see Ahmed and Broers (1972) and Moriarty (1996)). The current density can be on the order of 1 - 100 A/cm². The emitted electrons are accelerated through the cathode-anode gap by
the applied voltage. As they pass through the gap the electrons collide with neutral gas molecules, ionizing them and forming a plasma. The plasmas thus produced can have densities much greater than the density of the cathode-emitted electrons.

There exists no consistent analytical theory describing the dynamics of the TPD, and so descriptions must be rather empirical. However, recent particle-in-cell numerical simulations have begun to shed some light on the microdynamics of such discharges (see e.g. Bauer and Schamel (1992), Greiner et al. (1995), and Klinger et al. (1995)).

TPDs exhibit several distinct operating modes, categorized in the pioneering work of Malter et al. (1951). For small applied voltages, the anode glow mode (AGM) obtains, the current is low, and the bulk of the plasma is "cathodic," meaning that the potential remains near the cathode potential for much of the plasma extent. Most of the potential drop (and hence ionization) occurs near the anode. As the applied voltage is increased, several unstable modes appear (for descriptions see Malter et al. (1951) and Klostermann et al. (1994)). At large applied voltages, the discharge enters the temperature-limited mode (TLM), where the current is large (only limited by equation (2.74)) and most of the potential drop occurs across a thin cathode sheath. The potential then stays constant over most of the plasma, at a value slightly greater than the anode potential. Ionization takes place throughout the bulk of the plasma. TLM plasmas tend to be near-thermal, not counting the fast electrons from the cathode, and fairly quiet in terms of density fluctuations (Klinger et al. (1995)). For low fill gas pressures, the ionization collision mean free path is of the order of the device size. This is the mode of operation of the VTF plasmas.

Most thermionic diodes presented in the literature are linear machines, although some are cylindrical. The VTF is toroidal, with a helical magnetic field spiraling toroidally from the top to the bottom of the chamber. Electrons emitted from the hot cathodes at the bottom of the chamber follow two helices—Larmor motion around the field lines with very
small radius (see next section), and guiding center motion along the helical field lines from the bottom of the chamber to the top, with radius of approximately 1 m and helical pitch that varies with radius, from approximately 9 cm at the inboard to 12 cm at the cutboard of the cathode. The locus of emitted electrons is then be a thin strip having the cross section of the cathode, extending in a twisting helix from the cathode to the collector plate. Ignoring particle drifts for the moment, one can calculate this locus by charting the cathode cross section along field lines (see Appendix B for Mathematica code). The 60° poloidal cross section of such a calculation is shown in Figure 2-15 for typical magnetic fields. The VTF plasma is produced by two hot cathodes; the emission patches due to both cathodes are shown. The arrows indicate the direction and magnitude of the magnetic field produced by the Parail coils (nominally vertical). Vertical field flux lines are also superposed. The chamber walls are marked in black, and the collector plate is represented by the shaded gray rectangle at the top. The navy blue horizontal line indicates the possible positions of the double probe tips.

The helical emission "rungs" are the plasma source region. Outside this region, the plasma diffuses across the field until the density goes to zero at the chamber walls. The plasma in the source region is quite three-dimensional, especially as viewed radially from the port where the microwaves are launched. However, outside the source region where diffusion dominates, any transverse variation in density rapidly decreases as the source is left behind. The plasma density near the edge and up to perhaps 5 cm from the emission rungs is fairly one-dimensional, with the only variation occurring radially. As long as the launched microwaves reflect in the diffusive edge plasma, we can treat the process as dominantly one-dimensional (two-dimensional if we include toroidal effects).
Emission rung locations for $I_y = 1300$ A, $I_\phi = 5700$ A in 60° (R,Z)–plane, ignoring drifts

![Diagram showing emission rung locations](image)

**Figure 2-15.** Poloidal cross section of thermionic diode plasma at the double probe location, showing enhanced current patches and double probe tip swath (ignores particle drift motion).

The hot electrons streaming from the cathode are subject to grad-B and curvature drifts in the toroidal geometry of the VTF. For the field geometry present, electrons drift downward at a uniform rate of about $7.3 \times 10^3$ m/s. Including this drift velocity in the calculation of emission rung locations yields Figure 2-16, in which the rungs are shifted downward somewhat with respect to those of Figure 2-15.

Note from Figure 2-15 that the scanning Langmuir probe cuts across several rungs of the emission helix. These show up as peaks in the ion saturation current. Let us compare the location of the measured peaks with the calculated rung locations (including drifts). A typical ion saturation current profile is shown in Figure 2-17. Peak locations are 96.1 cm.
Emission rung locations for $I_v = 1300$ A, $I_\phi = 5700$ A in 60° ($R,Z$)–plane, including drifts

![Diagram showing emission rung locations](image)

**Figure 2-16.** Poloidal cross section of thermionic diode plasma at the double probe location, showing enhanced current patches and double probe tip swath (includes grad-B and curvature drift motion).

101.1 cm, and 106.2 cm in major radius. Figure 2-18 shows the results of a trajectory calculation at the scanning Langmuir probe location (260°). The calculation shows that peaks should be observed at 94.1 cm, 100.0 cm and 107.2 cm, indicating good agreement with the measured values. However, this is not the entire story. The measured peak locations show some dependence on arc current, as shown in Figure 2-19 for several representative shots. Large arc current tends to push the peaks radially outward and together. In fact, at very large currents (~100 A), the peaks coalesce into one central source region, as shown in Figure 2-20 (incidentally, this profile includes the region adjacent to the outer wall, which is not shown in Figure 2-17). Future work might address this background
plasma configuration in more detail. For now, note that with typical plasma parameters in hydrogen and a probe area of 12.3 mm², 2.45 GHz ordinary-mode microwaves would be reflected at a value of saturation current of about 2.1 mA, or at about 112 cm in the profile of Figure 2-17 (for the profile of Figure 2-20, the reflection layer is near 113 cm). This is well away from the three-dimensional source region. All of our ionospheric heating simulation experiments took place in the diffusive edge plasma, near the reflection layer of the incident microwaves, and so we ignore the plasma source region with all its complexities.
Emission rung locations for $I_v = 1300$ A, $I_{\phi} = 5700$ A in $260^\circ$ $(R, Z)$-plane, including drifts

Figure 2-18. Poloidal cross section of thermionic diode plasma at the scanning Langmuir probe location, showing enhanced current patches and probe tip swath (includes grad-B and curvature drift motion).

To summarize: the VTF plasma is produced by thermionically emitted electrons that are accelerated in a thin cathode sheath region to 300 eV. These electrons follow complex three-dimensional trajectories up the field lines from the bottom to the top of the chamber, ionizing the fill gas in the process. Plasma diffuses outward from the complex source region, forming a two-dimensional profile of near-thermal plasma near the microwave reflection layer.

2.5.2 VTF Plasma Parameters

Quantities measured or calculated directly from measurements include electron density, electron temperature, floating potential, plasma potential, and magnetic field. From these, one can derive quantities of interest in wave studies: electron and ion plasma fre-
Figure 2-19. Dependence of peak locations on arc current.

Figure 2-20. Density profile for large thermionic arc current (~100 A). Note single source region near R = 102 cm. An earlier probe design was used with area of approximately 75 mm².
quencies, cyclotron frequencies, hybrid frequencies, Debye lengths, Larmor radii, thermal velocities, sound velocity, and scale lengths. The length of this list poses a challenge of presentation. Here, we simply show profiles of selected quantities for typical shots, along with a listing of parameters in table form.

Profiles of measured quantities are shown in Figure 2-21, for typical operating condi-

![Figure 2-21. Profiles of measured quantities in VTF plasma.](image)

tions (the upper frame is the same shot as shown previously in Figure 2-17). Density was calculated from the ion saturation current formula (equation (2.2)), and is inaccurate within the emission rungs, where beam electrons have enough energy to overcome the potential barrier presented by the probe bias. Temperature was deduced from the exponen-
tial portion of the probe characteristic. Magnetic field was calculated assuming toroidal symmetry from the known coil geometry and the measured current. Ion temperature is unknown, but is most likely much smaller than the electron temperature. Similar plasmas have ion temperatures of fractions of an electron volt (Klinger et al. (1995)).

Profiles of derived frequencies are shown in Figures 2-22 and 2-23. Figure 2-22 contains high frequencies, followed by low frequency profiles in Figure 2-23. Frequency definitions are given below in Table 2-3. The vertical dashed lines labeled “M” in Figures 2-21, 2-22, and 2-23 mark a typical probe tip location used in previous VTF investigations (Moriarty (1996)), and is placed here for future reference.

Finally, Table 2-3 lists VTF plasma parameter values. These apply to the simulation region unless otherwise noted. Where appropriate, parameters are defined in column three.
Figure 2-23. Profiles of low frequencies in VTF.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_e$</td>
<td>$7.4 \times 10^{16}$ m$^{-3}$</td>
<td>Electron density. Value at o-mode cutoff.</td>
</tr>
<tr>
<td>$n_i$</td>
<td>$7.4 \times 10^{16}$ m$^{-3}$</td>
<td>Ion density. Equals $n_e$ for $Z = 1$.</td>
</tr>
<tr>
<td>$n_b$</td>
<td>$7.3 \times 10^{15}$ m$^{-3}$</td>
<td>Beam electron density, source region. Calculate from the known velocity, arc current, and filament area, $n_b = \frac{I_{arc}}{eA_f v_b}$.</td>
</tr>
<tr>
<td>$n_{H_2}$</td>
<td>$5.0 \times 10^{18}$ m$^{-3}$</td>
<td>Neutral density using hydrogen fill. Pressure measured using ion gauge, $p = n k T$.</td>
</tr>
<tr>
<td>$n_{Ar}$</td>
<td>$2.0 \times 10^{18}$ m$^{-3}$</td>
<td>Neutral density using argon fill. Pressure measured using ion gauge, $p = n k T$.</td>
</tr>
<tr>
<td>$n_{N_2}$</td>
<td>$1.3 \times 10^{16}$ m$^{-3}$</td>
<td>Neutral density at base pressure, 80% fraction. Pressure measured using ion gauge, $p = n k T$.</td>
</tr>
<tr>
<td>$n_{O_2}$</td>
<td>$3.3 \times 10^{15}$ m$^{-3}$</td>
<td>Neutral density at base pressure, 20% fraction. Pressure measured using ion gauge, $p = n k T$.</td>
</tr>
</tbody>
</table>

Table 2-3. VTF plasma parameters in simulation region.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$</td>
<td>5.9 eV</td>
<td>Plasma electron temperature. From Langmuir probe characteristic.</td>
</tr>
<tr>
<td>$T_i$</td>
<td>$\leq 0.5$ eV</td>
<td>Ion temperature. Estimated from similar experiments (Klinger et al. (1995)).</td>
</tr>
<tr>
<td>$B_0$</td>
<td>0.068 T</td>
<td>Magnetic field magnitude. Langmuir trace locates $R$ for cutoff; then $B_0 = \frac{\mu_0 NI_{TF}}{2\pi R}$.</td>
</tr>
<tr>
<td>$v_b$</td>
<td>$1.03 \times 10^7$ m/s</td>
<td>Beam electron velocity. Electrons accelerated in thin cathode sheath to approximately the cathode-anode voltage drop, 300 eV.</td>
</tr>
<tr>
<td>$v_{Te}$</td>
<td>$1.02 \times 10^6$ m/s</td>
<td>Electron thermal velocity. $v_{Te} = \frac{\sqrt{\kappa T_e}}{m_e}$.</td>
</tr>
<tr>
<td>$v_{Ti}$</td>
<td>$6.9 \times 10^3$ m/s (H) $1.1 \times 10^3$ m/s (Ar)</td>
<td>Ion thermal velocity. $v_{Ti} = \sqrt{\frac{\kappa T_i}{m_i}}$.</td>
</tr>
<tr>
<td>$r_{Le}$</td>
<td>85 $\mu$m</td>
<td>Electron Larmor radius. $r_{Le} = \frac{v_{Te}}{\omega_{ce}}$.</td>
</tr>
<tr>
<td>$r_{Li}$</td>
<td>1.1 mm (H) $6.7$ mm (Ar)</td>
<td>Ion Larmor radius. $r_{Li} = \frac{v_{Ti}}{\omega_{ci}}$.</td>
</tr>
<tr>
<td>$c_s$</td>
<td>$2.7 \times 10^4$ m/s (H) $4.2 \times 10^3$ m/s (Ar)</td>
<td>Sound speed. $c_s = \sqrt{\frac{\kappa(T_e + \gamma_i T_i)}{m_i}}$. $\gamma_i \equiv 3$.</td>
</tr>
<tr>
<td>$\lambda_{De}$</td>
<td>66 $\mu$m</td>
<td>Electron Debye length. $\lambda_{De} = \frac{v_{Te}}{\omega_{pe}} = \sqrt{\frac{e_0 \kappa T_e}{e^2 n_e}}$.</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>12.2 cm</td>
<td>Free-space wavelength of magnetron-produced microwave.</td>
</tr>
<tr>
<td>$d_w$</td>
<td>0.28 mm</td>
<td>Diameter of double probe wire tips.</td>
</tr>
<tr>
<td>$L$</td>
<td>14 mm</td>
<td>Length of double probe wire tips.</td>
</tr>
<tr>
<td>$A_p$</td>
<td>$7.0 \text{ mm}^2 \text{ projected}$ $12.3 \text{ mm}^2 \text{ total}$</td>
<td>Area of scanning Langmuir probe.</td>
</tr>
</tbody>
</table>

Table 2-3. VTF plasma parameters in simulation region.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{plasma}}$</td>
<td>0.076 Pa</td>
<td>Plasma pressure. $p \equiv n_e \kappa T_e + n_i \kappa T_i$.</td>
</tr>
<tr>
<td>$P_{\text{neutral}}$</td>
<td>0.02 Pa (H₂) 0.008 Pa (Ar)</td>
<td>Neutral pressure. Measured with ion gauge.</td>
</tr>
<tr>
<td>$f_{pe}$</td>
<td>2.45 GHz</td>
<td>Electron plasma frequency. $f_{pe} = \frac{1}{2\pi N} \frac{e^2 n_e}{\varepsilon_0 m_e}$. Same as microwave frequency at o-mode cutoff.</td>
</tr>
<tr>
<td>$f_{pi}$</td>
<td>57 MHz (H) 9.0 MHz (Ar)</td>
<td>Ion plasma frequency. $f_{pi} = \frac{1}{2\pi N} \frac{(Ze)^2 n_i}{\varepsilon_0 m_i}$.</td>
</tr>
<tr>
<td>$f_{ce}$</td>
<td>1.91 GHz</td>
<td>Electron cyclotron frequency. $f_{ce} = \frac{1}{2\pi} \frac{e B_0}{m_e}$.</td>
</tr>
<tr>
<td>$f_{ci}$</td>
<td>1.0 MHz (H) 26 kHz (Ar)</td>
<td>Ion cyclotron frequency. $f_{ci} = \frac{1}{2\pi} \frac{ZeB_0}{m_i}$.</td>
</tr>
<tr>
<td>$f_{uh}$</td>
<td>3.1 GHz</td>
<td>Upper hybrid frequency. $f_{uh} = \sqrt{f_{pe}^2 + f_{ce}^2}$.</td>
</tr>
<tr>
<td>$f_{hr}$</td>
<td>35 MHz (H) 5.5 MHz (Ar)</td>
<td>Lower hybrid resonant frequency. $f_{hr} = \frac{f_{ce} f_{ci} \sqrt{f_{pe}^2 + f_{ce}^2 f_{ci}}}{f_{pi}^2 + f_{ce} f_{ci}}$.</td>
</tr>
<tr>
<td>$f_0$</td>
<td>2.45 GHz</td>
<td>Magnetron frequency.</td>
</tr>
<tr>
<td>$\chi_i, \text{H}$</td>
<td>13.60 eV</td>
<td>Ionization potential for atomic hydrogen. <em>(CRC Handbook of Chemistry and Physics (1995), p. 10-213).</em></td>
</tr>
<tr>
<td>$\chi_i, \text{H}_2$</td>
<td>15.43 eV</td>
<td>Ionization potential for molecular hydrogen. <em>(CRC Handbook of Chemistry and Physics (1995), p. 10-213).</em></td>
</tr>
<tr>
<td>$\chi_i, \text{Ar}$</td>
<td>15.76 eV</td>
<td>Ionization potential for Argon. <em>(CRC Handbook of Chemistry and Physics (1995), p. 10-210).</em></td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>$5.72 \times 10^{-21} \text{ m}^2$ (H₂) $2.15 \times 10^{-20} \text{ m}^2$ (Ar)</td>
<td>Electron impact ionization cross section (total). 300 eV electrons assumed. <em>(Atomic Data for Controlled Fusion Research (1977), pp. C.4.8 and C.4.16).</em></td>
</tr>
</tbody>
</table>

Table 2-3. VTF plasma parameters in simulation region.
<table>
<thead>
<tr>
<th><strong>Parameter</strong></th>
<th><strong>Value</strong></th>
<th><strong>Description/Source</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_i$</td>
<td>$35 \text{ m (H}_2\text{)}$ $23 \text{ m (Ar)}$</td>
<td>Ionization mean free path. $\lambda_i = \frac{1}{n_i \sigma_i}$. This is the length required for a one e-folding reduction in the number of fast emitted electrons in the source region (due to ionizing collision losses).</td>
</tr>
<tr>
<td>$\ln \Lambda$</td>
<td>$15.6/13.6$</td>
<td>Coulomb logarithm. $\Lambda = \frac{\lambda_{De}}{b_{min}}, b_{min} \equiv \frac{e^2}{12\pi \epsilon_0 \kappa T_e}$. First number for 300 eV beam electrons in source region. Second number for 5.9 eV plasma electrons in simulation region.</td>
</tr>
<tr>
<td>$\tau_s/\lambda_s$</td>
<td>$86 \mu s/88 \text{C m}$</td>
<td>Time/mean free path for beam slowing. $\tau_s \equiv -U \left( \frac{\partial U}{\partial t} \right)^{-1} = \frac{m_1^2 U_1^3}{4\pi n_2 \left( q_1 q_2 / 4\pi \epsilon_0 \right)^2 \left( 2 + m_1 / m_2 \right) \ln \Lambda}$, where $U_1$ is the beam speed, and &quot;1&quot; and &quot;2&quot; refer to beam and target particles, respectively. This represents the slowing of the fast emitted electrons in the plasma source region (Krall and Trivelpiece (1986), p. 302).</td>
</tr>
<tr>
<td>$\tau_d/\lambda_d$</td>
<td>$43 \mu s/442 \text{ m}$</td>
<td>Time/mean free path for beam spreading. $\tau_d \equiv U^2 \left( \frac{\partial U}{\partial t} \right)_\perp^{-1} = \frac{m_1^2 U_1^3}{16\pi n_2 \left( q_1 q_2 / 4\pi \epsilon_0 \right)^2 \ln \Lambda}$, where $U_1$ is the beam speed, and &quot;1&quot; and &quot;2&quot; refer to beam and target particles, respectively. This represents the spreading of the fast emitted electrons in the plasma source region (Krall and Trivelpiece (1986), p. 304).</td>
</tr>
<tr>
<td>$\nu_{en}$</td>
<td>$2.0 \text{ MHz (H}_2\text{)}$ $0.82 \text{ MHz (Ar)}$</td>
<td>Electron-neutral collision frequency. $\nu_{en} = n_n \sigma_s v_T e$, with $\sigma_s = 4 \times 10^{-19} \text{ m}^2$. This number applies in the simulation region (Krall and Trivelpiece (1986), p. 321).</td>
</tr>
<tr>
<td>$\nu_{ei}$</td>
<td>$150 \text{ kHz}$</td>
<td>Electron-ion collision frequency (momentum). $\nu_{ei}^{(p)} = \left( \frac{Ze^2}{4\pi \epsilon_0} \right)^2 \frac{4\pi}{3 \sqrt{3} m_e^{1/2} (\kappa T_e)^{3/2}} n_i \ln \Lambda$ (from treatment of Rose and Clark (1961), pp. 158-179).</td>
</tr>
<tr>
<td>$\nu_{ee}$</td>
<td>$150 \text{ kHz}$</td>
<td>Electron-electron collision frequency (momentum). $\nu_{ee}^{(p)} = \frac{1}{Z^2 n_i} \nu_{ei}^{(p)}$ (from treatment of Rose and Clark (1961), pp. 158-179).</td>
</tr>
</tbody>
</table>

Table 2-3. VTF plasma parameters in simulation region.
### Table 2-3. VTF plasma parameters in simulation region.

#### Parameter | Value | Description/Source
--- | --- | ---
\(\nu_{\text{in}}\) | 17 kHz (H) 1.1 kHz (Ar) | Ion-neutral collision frequency. \(\nu_{\text{in}} = n_n \sigma_n \nu_{Ti}\), with \(\sigma_n = 5 \times 10^{-19} \text{ m}^2\). This number applies in the simulation region (Krall and Trivelpiece (1986), p. 321).

\(\nu_{\text{ie}}\) | 81 Hz (H) 2.0 Hz (Ar) | Ion-electron collision frequency (momentum). \(\nu_{\text{ie}}^{(p)} = \frac{m_e n_e v_{ei}^{(p)}}{m_i n_i} \) (from treatment of Rose and Clark (1961), pp. 158-179).

\(\nu_{\text{ii}}\) | 140 kHz (H) 22 kHz (Ar) | Ion-ion collision frequency (momentum). \(\nu_{\text{ee}}^{(p)} = Z^2 \left(\frac{T_e}{T_i}\right)^{3/2} \left(\frac{m_e}{m_i}\right)^{1/2} v_{ei}^{(p)} \) (from treatment of Rose and Clark (1961), pp. 158-179).

\(E_0\) | 5.4 kV/m | Electric field amplitude of incident microwave. \(E_0 = \frac{2 \eta_0 P}{\sqrt{A_{\text{horn}}}}\), where \(\eta_0\) is the impedance of free space (377 Ω), \(P\) is power launched, and \(A_{\text{horn}}\) is the horn antenna area at the exit.

\(L_n\) | 10 cm | Density scale length. \(L_n = \left(\frac{1}{n \partial n / \partial R}\right)^{-1}\).

\(L_B\) | 115 cm | Magnetic field scale length. \(L_B = \left(\frac{1}{B \partial B / \partial R}\right)^{-1}\).

\(L_{Te}\) | 20 cm | Temperature scale length. \(L_{Te} = \left(\frac{1}{T_e \partial T_e / \partial R}\right)^{-1}\).

**2.6 Review of Laboratory Plasma Frequency Heating**

Before moving on, it is helpful to place the current work in the setting of other documented laboratory experiments. Distinguishing characteristics of the present experiment are as follows:

1. the heating electromagnetic wave is launched from outside a spatially varying plasma
2. ordinary incident polarization is used
3. the plasma is weakly magnetized, with $\omega_{ce} < \omega_{pe}$
4. the incident power is far above instability threshold for PDI and OTSI
5. the plasma possesses a sharp density gradient, with density scale length of order incident wavelength
6. plasma wave turbulence produced by the heating wave is diagnosed.

With the exception of 5, these characteristics were chosen to model the heating of the earth's ionosphere by powerful radio waves (see Chapter 3). The use of a microwave-heated laboratory plasma to model ionospheric heating experiments is quite unusual. To the author's knowledge, aside from prior VTF experiments (Moriarty (1996), Lee et al. (1997), Lee et al. (1998c)) the only other such experiment is described in Cros et al. (1991). The plasma used by Cros et al. (1991) was unmagnetized, and the dominant coupling to plasma waves was through resonance absorption—a situation more comparable to laser plasma heating.

Plasma frequency heating has been studied in a variety of contexts, including anomalous absorption (Dreicer et al. (1971), Dreicer et al. (1973), Porkoláb et al. (1976)), instability threshold measurements (Eubank (1971), Porkoláb et al. (1976)), resonance absorption (Cros et al. (1990), Nishida et al. (1990)), plasma wave trapping and collapse (Kim et al. (1974), Eggleston et al. (1982), Gol'tsman et al. (1985), Bauer et al. (1990), Sergeichev and Sychev (1990)), laser plasma interactions (Shearer et al. (1972), Fabre and Stenz (1974), Manes et al. (1977), Bach et al. (1983), Perry et al. (1989), Mizuno et al. (1994), Labaune et al. (1998)), and modeling of laser plasma interactions (Mizuno et al. (1986)). Of these, only Dreicer et al. (1971), Dreicer et al. (1973), Eubank (1971), Porkoláb et al. (1976), and Sergeichev and Sychev (1990) used a magnetized plasma. Wave polarizations are fundamentally different with and without a magnetic field, and phenomena such as resonance absorption are absent in magnetized plasma (Freidberg et al. (1972)). In addition, the character of the electrostatic dispersion relation changes at $\omega_{ce}$
= \omega_{pe}: the Langmuir wave dispersion relation connects with that of the upper hybrid wave for finite angles when \omega_{ce} < \omega_{pe}, while for \omega_{ce} > \omega_{pe} Langmuir waves smoothly connect to lower hybrid waves at finite angle. The two experiments by Dreicer et al. used strong magnetic fields to confine the plasma (\omega_{ce} \gg \omega_{pe}), similar to many laboratory fusion experiments but very different from the ionosphere, which has a weak magnetic field. The experiment by Eubank (1971) was of a preliminary nature and essentially did nothing more than verify the gradient threshold theory of Perkins and Flick (1971). We are left with two experiments possessing parameters similar to those of the present one.

In the experiment of Porkoláb et al. (1976), a 10.4 GHz, collimated beam was launched perpendicular to the magnetic field of a linear plasma device. The plasma density gradient scale length was of the same order or less than the free-space microwave wavelength—a steep gradient. The magnetic field strength was such that \omega_{ce}/\omega_{pe} \leq 0.5, and the electron/ion temperature ratio was approximately 50. High-frequency probes measured the wave spectra, which indicated the occurrence of the PDI (see Chapter 1). The low frequency spectrum extended up to the ion plasma frequency (f_{pi}) at low and intermediate power levels, while for high powers the spectrum extended somewhat beyond f_{pi}. Instability thresholds were measured and compared with inhomogeneous linear theory (equation (1.32)). Theory gave the correct order of magnitude, but the functional variation on wavelength did not match the experiment for most cases. Spatial interferometry was used to determine the dispersion relation of the decay waves at power levels somewhat above threshold. The low frequency waves were indeed found to be ion acoustic, as expected in the PDI. Spatial interferometry could not be used at high frequency (wavelengths became comparable to probe dimensions) or high power (the plasma became strongly turbulent). In addition, electron and ion heating rates were studied with a retarding potential analyzer. At low power, electrons were heated from 5 eV to 6.5 eV, and at
high power, main body electrons were heated to 23 eV with a high velocity tail of 41 eV. Ions also gained significant energy, including a 15 V drift and parallel energy up to 60 eV. Both electron and ion heating were localized to regions where the PDI waves had large amplitudes. The heating rates were found to be about 20 times faster than expected from collisional damping alone.

The experiment of Sergeichev and Sychev (1990) specifically examined the excitation of strong turbulence in a microwave field. Their plasma was the decay of a pulsed linear beam-plasma discharge, and had $\omega_{ce}/\omega_{pe} = 0.047$ and $T_e/T_i = 10$. Microwaves were injected from the cut off end of a waveguide perpendicular to the magnetic field. Density profile scale lengths were not given, and the effects of inhomogeneity were ignored in the analysis, but the column diameter cannot have been very much larger than the free-space wavelength of the microwaves. The periphery of the plasma was lined with microwave absorbers, in a successful attempt to localize the microwave field. At points removed along the plasma column from the waveguide, electrons accelerated back towards the waveguide were measured with a shielded Langmuir probe. For such accelerated electrons, time delays versus the microwave turn-on were measured and found to increase with distance from the waveguide. The time delay of accelerated electrons was interpreted to be due to the finite group velocity of decay waves produced by the powerful pump wave. A second diagnostic, a microwave antenna probe, was used to measure bursts of microwave radiation in the periphery of the plasma column. The beginning and end of burst series corresponded with the rising and falling edges of the microwave pump power. Bursts were assumed to come from sources having small dimensions ($\leq 100\lambda_{De}$) and burst durations were on the order of $10/\omega_{pi}$. Although the results were interpreted in terms of SLT (see Chapter 1), other mechanisms were not ruled out. Indeed, the effects of PDI turbulence were not mentioned at all.
Two observations may enlarge the number of relevant experiments. First, if the threshold of the LDI (see Chapter 1) is not exceeded, if scattering from perpendicular density fluctuations can be ignored, and if non-parallel propagation is ignored, the case of ordinary-mode heating of a magnetized plasma may be fairly similar to transverse electric (TE or s)-polarized heating of an unmagnetized plasma. In this case, the results of TE-polarized laser plasma heating experiments would transfer. Many laser plasma experiments use transverse magnetic (TM or p) polarization, however, which allows linear conversion of electromagnetic to Langmuir waves at the critical density (Freidberg et al. (1972)). In addition, most laboratory experiments designed to examine caviton formation use the TM polarization for efficient generation of Langmuir waves—fundamentally different than ionospheric heating. Second, if one is interested purely in the evolution of Langmuir turbulence, the initial production or drive mechanism may be unimportant. In this case, one may consider turbulence generated by beams (Wong and Cheung (1984), Cheung and Wong (1985)), double layers (Gunell et al. (1996), Theisen and Carpenter (1996) & (2000)), or resonance absorption (see earlier references) as well as that generated by pump-wave instabilities and/or scattering (see Chapter 1). However, the nonlinear evolution of Langmuir turbulence may be sensitively dependent upon the form of the source, and it is not obvious that alternate generation mechanisms give the same results.

The above references used relatively small linear devices, in which refraction effects are important as mentioned in Porkoláb et al. (1976). The present experiment uses a large toroidal plasma with a nearly uniform 1 m vertical extent. The collimated microwaves launched from the horn antenna then see a nearly flat plasma that is several wavelengths in the transverse directions. The setup of Sergeichev and Sychev (1990) made no attempt to collimate the incident wave; the use of a horn reduces the effects of the fringing fields on the edges of the heated volume.
Aside from these experimental configuration issues, the main differences between experiments are in the type and quantity of diagnostic instruments. The main advances of the present work are the construction of the scannable double probe and the microwave down-convertor. In combination with the fast oscilloscope, these diagnostics allow the simultaneous, time-resolved measurement of low and high frequency fluctuations, in addition to the estimation of the turbulent wavenumber spectra. Absolute electric field measurements are also possible, in principle, using the analysis of Section 2.4.1(c). None of these capabilities were existent in either prior VTF work or in the two relevant experiments described above. In addition, the motorization and remote PC control of the scannable probe facilitated measurements at many more locations than were practicable in prior VTF experiments.

One additional note is in order, regarding the placement of the measuring probe tip in prior VTF work (Moriarty (1996), Lee et al. (1997), Lee et al. (1998c)). Most results were presented for the case when the tip radius was well outside the simulation zone of the present experiments, in a region where the plasma frequency was 2 - 5 times greater than the microwave frequency. In such a region, the incident microwave should have been heavily cut off, and the PDI and OTSI should not have been operative at all. Full discussion of this issue is left until Chapter 4.

Finally, Table 2-4 summarizes the characteristics of several cited experiments in relation to the present work. Gray shading indicates a lesser degree of relevance. Characteristics of ionospheric modification experiments are also shown for comparison.

2.7 Sample Results

Typical results obtained in VTF plasma frequency heating experiments are shown in Figure 2-24, for the HF band (defined on page 93). Frequency follows the horizontal axis,
<table>
<thead>
<tr>
<th>experiment</th>
<th>$\frac{\omega_{ce}}{\omega_{pe}}$</th>
<th>$\frac{L_n}{\lambda_o}$</th>
<th>$\frac{\beta}{\beta_{TH}}$</th>
<th>ES wave energy source</th>
<th>Wave diagnostics</th>
</tr>
</thead>
<tbody>
<tr>
<td>VTF, current (values given for simulation zone)</td>
<td>~0.78</td>
<td>~0.83</td>
<td>» 1</td>
<td>Remotely launched EM wave with collimating horn antenna</td>
<td>Scannable double probe &amp; microwave receiver —time resolved —simultaneous low and high frequency —turbulent $k$-spectra —absolute $E$</td>
</tr>
<tr>
<td>VTF, Moriarty (1996) (values estimated from known parameters)</td>
<td>0.25-0.5 $\omega_{pe} \geq 2 \omega_o$</td>
<td>~0.83</td>
<td>» 1</td>
<td>Remotely launched EM wave with collimating horn antenna</td>
<td>—Dipole antenna (3 cm) —Single-tipped high frequency Langmuir probe (not located at same point)</td>
</tr>
<tr>
<td>Porkoláb et al. (1976)</td>
<td>&lt;0.5</td>
<td>$\leq 1$</td>
<td>$\sim 1$ to » 1</td>
<td>Remotely launched EM wave with collimating horn antenna</td>
<td>—Spatial interferometer (low power) —High frequency probes —Retarding potential analyzer —Diamagnetic loop</td>
</tr>
<tr>
<td>Sergeichev and Sychev (1990)</td>
<td>~0.047</td>
<td>~1</td>
<td>» 1</td>
<td>Remotely launched EM wave, no collimation, absorbing boundary</td>
<td>—Unidirectional Langmuir probe —Single-tipped high frequency probe</td>
</tr>
<tr>
<td>Cros et al. (1991)</td>
<td>0</td>
<td>2 - 12</td>
<td>» 1</td>
<td>Remotely launched EM wave with collimating horn antenna</td>
<td>—Dipole probe —Langmuir probe</td>
</tr>
<tr>
<td>Kim et al. (1974)</td>
<td>0</td>
<td>0.24</td>
<td>&gt; 1</td>
<td>EQS field inside capacitor</td>
<td>—Resonant dipole probe —Low density electron beam</td>
</tr>
<tr>
<td>Wong and Cheung (1984)</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>Remotely launched EM wave</td>
<td>Scannable double probe</td>
</tr>
<tr>
<td>Mizuno et al. (1986)</td>
<td>0</td>
<td>&lt; 0.13</td>
<td>» 1</td>
<td>Internal field of waveguide</td>
<td>High frequency probes, energy analyzer</td>
</tr>
<tr>
<td>Laser plasma (e.g. Labaune et al. (1998))</td>
<td>0</td>
<td>~1000</td>
<td>» 1</td>
<td>Laser beam or SRS EPWs</td>
<td>Coherent scattering (laser)</td>
</tr>
<tr>
<td>Ionospheric modification</td>
<td>0.19</td>
<td>1190</td>
<td>» 1</td>
<td>Remotely launched EM wave with collimating antenna</td>
<td>Monostatic radar, $\theta = 45^\circ$, $k\lambda_{De} \approx 0.18$ —single $k$ —space and time resolved</td>
</tr>
</tbody>
</table>

Table 2-4. Comparison of laboratory plasma frequency heating experiments. Shaded rows are less relevant.
Figure 2-24. Typical results of VTF plasma frequency heating. HF band.

and power (in dB) is shown on the vertical axis. The central peak is the injected microwave signal. Two curves are shown; the upper curve is a power spectral estimate of data taken during a typical heating shot. The lower curve was taken in an interval without the thermionic diode plasma present, and near the microwave frequency is a good approximation to the forward power spectrum of the magnetron. Any signal above this curve has been produced by plasma fluctuations (not counting the sharp interference spikes).

Both lower and upper sidebands may be observed in Figure 2-24. The most prominent is the lower sideband, which extends downward from the microwave frequency approximately 100 MHz. This feature appears quite similar to features A and C noted in Chapter 1, and identified as either a parametric decay cascade or a caviton continuum, depending
upon the theoretical viewpoint. The upper sideband in the VTF data is less pronounced. However, there is still some power present, compared with the magnetron forward power spectrum below. This feature may be the VTF analog of the feature B of the ionospheric data.

The correspondence of these observations with ionospheric results reinforce the notion that VTF results can shed light on ionospheric physics. Before beginning a detailed comparison, however, we must address the issue of scaling. This is taken up in Chapter 3, followed by a detailed presentation and discussion of VTF data in Chapter 4.
Chapter 3

Similarity

"One reason for the neglect of the principle [of similitude] may be that, at any rate in its applications to particular cases, it does not much interest mathematicians."
—Lord Rayleigh (1915).

Undertaking a model experiment requires examination of the similarity of the model and the prototype. It is obvious that one cannot make a full-size ionosphere in the laboratory, and so the length scale must be reduced considerably. This requires large changes in other parameters as well, which one can perhaps estimate purely on the basis of intuition. There are, however, well established methods of dealing with scaling problems (which are quite powerful, despite being mathematically uninteresting). The purpose of this chapter is to forge robust links between the VTF heating experiment and the ionospheric one, in preparation for the detailed examination of results to follow in Chapter 4.

There are a large number of problems in physics to which the solution is known only empirically. These include problems whose governing equations are known, but whose solutions are intractable mathematically. Despite lack of a specific analytic solution, information about the general character of the solution may be derived from consideration of the parameters and variables known to be important to the problem and the governing equations (if they exist). This is called fractional analysis. In scaling problems, the solution is unknown, but fractional analysis can yield enough information about the solution to proceed with a reasonable degree of similitude.
Fractional analysis includes the technique of *dimensional analysis*, which begins from a list of all possible variables upon which the solution to a particular problem depends, and distills these into a smaller list of dimensionless parameters (see Buckingham (1914)). From this type of analysis, one can sometimes deduce the form of the solution—quite a powerful tool indeed. It has been applied to plasmas by Kadomtsev (1975), specifically to tokamak scaling. We do not discuss it further, choosing instead to employ a second type of fractional analysis that extracts information directly from the set of governing equations.

Analysis of the governing equations directly provides a set of dimensionless parameters that, when matched among different experiments, guarantee that the same physics are sampled. Section 3.1 applies this type of analysis to plasma frequency heating, and Section 3.2 then presents a comparison of VTF and ionospheric parameters.

Model experiments that exactly match all dimensionless parameters with the prototype problem are called *exact simulations*. When the list of parameters becomes large, however, matching becomes extremely difficult or impossible. In this case, the principle of *limited simulation* may be used to demonstrate qualitative agreement, as outlined in Section 3.3.

### 3.1 Fractional Analysis of the Governing Equations

Recent monographs on the fractional analysis of governing equations include Sedov (1993) and Kline (1986), from which the term “fractional analysis” is borrowed, and to which the interested reader is referred for a more complete discussion.

Let us begin by stating a postulate that readily appeals to our intuition:

If two systems obey the same set of governing equations and conditions and if the values of all parameters in these equations and conditions are made the same, then the two systems must exhibit similar behavior provided only that a unique solution to the set of equations and conditions exists (Kline, 1986).
Thus, to compare two different systems (e.g., ionosphere and VTF), we must have a common set of governing equations, boundary/initial conditions, and parameters. It seems obvious that although the absolute values of the parameters (density, temperature, etc.) are not at all the same between the two systems, there may exist some common "normalized" equations that are common to the two systems.

A method for obtaining such equations is to normalize each variable in the equations by an appropriate (constant) scale for the problem. One then obtains a set of unit-free or dimensionless equations that are identical to the original equations except for constant multiplicative factors in each term. These dimensionless factors are called the parameters of the problem. If one chooses similar normalizations for the two systems, equal values of the new non-dimensional variables are called homologous points. It is clear that the solutions of the equations differ only if the parameters or the normalized boundary conditions take on different values. Restated another way: equal parameter values and boundary conditions are sufficient to guarantee similar behavior for systems that follow the same set of governing equations.

Let us refresh our mental picture of the physical process of ionospheric plasma frequency heating: a powerful electromagnetic wave is launched from vacuum into a magnetized plasma whose density increases with distance from the wave source. The wave polarization is such that when it enters the plasma it corresponds to the ordinary mode of propagation. At critical plasma density, the wave reflects and returns out of the plasma. The forward and reverse-propagating waves form a standing wave pattern in which the electric field oscillates back and forth at stationary positions (nodes and anti-nodes). Near reflection the electric field of the wave is parallel to the background magnetic field. If the electric field is large enough, nonlinear plasma instabilities (unspecified) are excited. The wave dissipates due to both collisional damping and "anomalous" nonlinear wave-wave
and wave-particle interactions (processes considered unknown here). We are interested in recreating these processes in the laboratory. The ionospheric interactions are known to occur in a narrow layer near reflection, and so we may, if necessary, restrict the similarity analysis to such a thin layer as well ("local" similarity).

A fairly general set of equations describing a plasma immersed in an electromagnetic field consists of Boltzmann equations for each particle species \( s \), along with self-consistently coupled Maxwell’s equations:

\[
\frac{\partial f_s}{\partial t} + v \cdot \frac{\partial f_s}{\partial r} + \frac{q_s}{m_s} [E + v \times B] \cdot \frac{\partial f_s}{\partial v} = \frac{\partial f_s}{\partial t} \bigg|_c , \tag{3.1}
\]

\[
\nabla \times B = \mu_0 \sum_s q_s \int v f_s d\mathbf{v} + \frac{1}{c^2} \frac{\partial E}{\partial t} , \tag{3.2}
\]

\[
\nabla \cdot B = 0, \tag{3.3}
\]

\[
\nabla \times E = \frac{\partial B}{\partial t} \tag{3.4}
\]

and

\[
\nabla \cdot \varepsilon_0 E = \sum_s q_s \int f_s d\mathbf{v}, \tag{3.5}
\]

where \( \frac{\partial}{\partial r} \) and \( \frac{\partial}{\partial v} \) denote the del operator in configuration space and velocity space, respectively.

The collision term on the right-hand-side of equation (3.1) can take several forms. For Coulomb collisions, a good approximation is the Fokker-Planck operator (see e.g., Krall and Trivelpiece (1986), p. 300),

\[
\frac{\partial f_s}{\partial t} \bigg|_c = -\sum_r \frac{\partial}{\partial v} \left[ \frac{2\pi m_s}{q_s q_r} \frac{2}{4\pi} \int \frac{u^2 (u - uu)}{u^2} \left[ \frac{f_s(v)}{m_s} \frac{\partial}{\partial v} f_r(v) - \frac{f_r(v)}{m_s} \frac{\partial}{\partial v} f_s(v) \right] dv \right] \tag{3.6}
\]
where \( u = v' - v \), \( \Lambda_{rs} \) is the Coulomb logarithm, and we have used the Landau form. For weakly ionized gases in which collisions with neutrals dominate, the Krook model can be used (Bhatnagar et al. (1954)):

\[
\frac{\partial f_s}{\partial t} \bigg|_c = \frac{f_s(r, v, t) - f_{s0}}{\tau_s},
\]

(3.7)

where \( f_{s0} \) is the equilibrium or neutral distribution, and \( \tau_s \) is the characteristic time of relaxation to \( f_{s0} \). The Krook model can also be used as an approximation to the full Fokker-Planck operator when there are no large gradients in velocity space. In either case, the collision term causes a relaxation toward a Maxwellian distribution on a slow time scale.

The set of equations (3.1)-(3.7) encompass all the physical processes thus far proposed to account for the results of ionospheric heating—including all of the plasma waves of interest, wave-wave and wave-particle interactions, and collisional and collisionless damping. It describes nonlinear and linear processes in every stage of development and is not limited to small wave amplitudes. Most explanations use simpler, more specific sets of equations—both the Zakharov equations and the perturbative wave-interaction description are derived from two-fluid equations, which in turn can be derived from the above set. Assuming that this set includes all the effects in which we are interested, let us proceed with the nondimensionalization.

For each variable we define a corresponding dimensionless variable ("*"):

\[
t = t^*t_0, \quad r = r^*R_0, \quad v = v^*U_s, \quad B = B^*B_0, \quad f_s = f_s^*\frac{n_0}{u_3}, \quad E = E^*E_0,
\]

(3.8)

where \( t_0 \) is a characteristic (fast) timescale, \( R_0 \) is a characteristic length, \( U_s \) is a characteristic velocity, \( B_0 \) is a characteristic magnetic field, \( n_0 \) is a characteristic particle density, \( \bar{u} \) is a characteristic random velocity (defining extent of the distribution function in velocity
space), and $E_0$ is a characteristic electric field. All of the normalization factors are constants, and may be chosen as befits the situation. Different choices of these constants yield different, but equally valid, dimensionless parameters.

Substituting the new variables into the governing equations yields the following set:

$\frac{R_0}{U_s t_0} \frac{\partial f_s}{\partial t^*} + \mathbf{v}^* \cdot \frac{\partial f_s}{\partial \mathbf{r}^*} + \frac{q_s R_0 B_0}{m_s U_s} \left[ \frac{E_0}{U_s B_0} \mathbf{E}^* + \mathbf{v}^* \times \mathbf{B}^* \right] \cdot \frac{\partial f_s}{\partial \mathbf{v}^*} = \frac{R_0}{U_s \tau_{s0}} \frac{\partial f_s}{\partial t^*}$ \hspace{1cm} (3.9)

$\nabla^* \times \mathbf{B}^* = \sum_s \frac{\mu_0 R_0 q_s U_s n_0}{B_0} \int \mathbf{v}^* f_s^* d\mathbf{v}^* + \frac{R_0 E_0}{c^2 B_0 t_0} \frac{\partial E^*}{\partial t^*}$ \hspace{1cm} (3.10)

$\nabla^* \cdot \mathbf{B}^* = 0$ \hspace{1cm} (3.11)

$\nabla^* \times \mathbf{E}^* = -\frac{R_0 B_0}{E_0 t_0} \frac{\partial \mathbf{B}^*}{\partial t^*}$ \hspace{1cm} (3.12)

$\nabla^* \cdot \mathbf{E}^* = \sum_s \frac{R_0 q_s n_0}{\epsilon_0 E_0} \int f_s^* d\mathbf{v}^*$. \hspace{1cm} (3.13)

We have assumed a slow collisional time scale $\tau_{s0}$ for the collision operator.

From the set (3.9)-(3.13) the following 14 dimensionless parameters can be identified (a subscript $s$ indicates separate parameters for electrons and ions; only one species of ion is considered):

$A_{s0} = \frac{R_0}{U_s t_0}$ \hspace{2cm} $A_{s4} = \frac{\mu_0 q_s n_0 U_s R_0}{B_0}$

$A_{s1} = \frac{q_s R_0 B_0}{m_s U_s}$ \hspace{2cm} $A_5 = \frac{R_0 E_0}{c^2 t_0 B_0}$

$A_{s2} = \frac{E_0}{U_s B_0}$ \hspace{2cm} $A_6 = \frac{R_0 B_0}{t_0 E_0}$

$A_{s3} = \frac{R_0}{U_s \tau_{s0}}$ \hspace{2cm} $A_{s7} = \frac{q_s n_0 R_0}{\epsilon_0 E_0}$

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Not all parameters are independent. In fact, nine parameters are sufficient to characterize the solution to this set of equations.

Returning for a moment to the physical problem at hand, we are dealing with plasmas that are initially at rest (no flows), and so a sensible choice of characteristic velocity is the initial root-mean-square random velocity for a single direction, \( \bar{u} \). This varies with position in the plasma, but we are concerned only with the plasma very near the reflection layer, over which we can take \( \bar{u} \) to be a constant. Therefore, let us set \( U_s = \bar{u} \). Also, if we assume that the initial distributions are Maxwellian, we can define temperatures, \( T_s \). Then, \( \bar{u} = \nu T_s = \sqrt{\kappa T_s / m_s} \). A characteristic time for an RF heating problem is the period of the pump wave, \( t_0 = \frac{2\pi}{\omega_0} \). Other times could perhaps be defined, such as the time for an electron at \( \nu T_s \) to traverse \( R_0 \), or the growth time for instabilities. For now, let us choose the first mentioned time scale. We take the collisional time scales to be the inverse of the highest relevant collision frequency in the vicinity of the reflection layer. For the length scale, we choose a physically interesting distance for heating experiments. A number of choices could be made, including the free-space wavelength of the heating beam, the width of the first Airy maximum in a linear gradient of the appropriate scale length (defines the thickness of the layer of plasma we are most interested in), the mean Debye length in the region of interest (which defines the minimum Langmuir wavelength), or the Larmor radius of one of the particles. We choose \( R_0 \) to be the distance over which the oscillating electric field strongly affects the plasma particles—a finite length in both the ionosphere and the VTF.
With these specifications we can put the parameters $A_{xx}$ in forms that are more recognizable:

\[
A_{e0} = \frac{R_0}{v_{Te} t_0} \\
A_{i0} = \frac{v_{Te}}{v_{Ti}} A_{e0} \\
A_{s1} = \text{sgn}(q_s) \frac{R_0}{r_{Ls}} \\
A_{s2} = \frac{r_{Ls}}{\lambda_{De}} \frac{\varepsilon_0 E_0^2}{n_0 \kappa T_s} \\
A_{e4} = -2\pi \frac{r_{Le}}{\lambda_{De}} \frac{\omega_0 e v_{Te}^2}{c^2 A_{e0}} \\
A_{i4} = \frac{v_{Ti}}{v_{Te}} |A_{e4}| \\
A_{s4} = \frac{r_{Le}}{\lambda_{De}} \frac{\varepsilon_0 E_0^2}{n_0 \kappa T_e} c^2 \\
A_{s5} = \frac{\lambda_{De}}{r_{Le}} \frac{1}{\varepsilon_0 E_0^2} \frac{1}{n_0 \kappa T_e} \\
A_{s7} = \text{sgn}(q_s) \frac{R_0}{\lambda_{Ds}} \frac{1}{\varepsilon_0 E_0^2} \frac{1}{n_0 \kappa T_s}
\]

Note that the Larmor radii and the Debye lengths are defined either as "layer-averaged" quantities or evaluated at a specific location, and are assumed to be slowly-varying over the region of interest. Written in this form, we can pick out the list of independent parameters shown in Table 3-1. The right column provides a simple explanation of each parameter. There are nine parameters, followed by two more groups (that could be formed from the nine) that are also useful to know. Below these is one more parameter introduced by boundary condition normalization (see below). Inspection shows that matching of these ten ratios ensures matching of the larger set $A_{xx}$. If all of these are matched between the laboratory and space, the simulation is exact.
<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\omega_{pe}}{\omega_0}$</td>
<td>Ratio of the plasma frequency to the launched wave frequency. This increases as the launched wave moves into the plasma. Near the reflection layer, this is near 1.</td>
</tr>
<tr>
<td>$\frac{\omega_{pe}}{\omega_{ce}} = \frac{r_{Le}}{\lambda_{De}}$</td>
<td>Ratio of the electron plasma to cyclotron frequency. The dispersion relations for plasma waves are different for $&gt; 1$ or $&lt; 1$.</td>
</tr>
<tr>
<td>$\frac{R_0}{r_{Le}}$</td>
<td>Ratio of the characteristic system size to the Larmor radius of the electrons. If this is very small, electrons appear unmagnetized; if very large, highly magnetized.</td>
</tr>
<tr>
<td>$\frac{v_{as}}{v_{Te}} = \frac{eE_0}{m_e\omega_0v_{Te}}$</td>
<td>Ratio of the oscillation velocity of electrons in the launched wave field to the electron thermal velocity. This is a measure of the applied power. As it increases, nonlinear phenomena may be expected to play an increased role.</td>
</tr>
<tr>
<td>$\frac{v_e}{\omega_0}$</td>
<td>Ratio of electron collision frequency (or damping rate) to the launched wave frequency. Gives indication of the importance of collisions (or damping).</td>
</tr>
<tr>
<td>$\frac{v_i}{\omega_0}$</td>
<td>Ratio of ion collision frequency (or damping rate) to the launched wave frequency. Gives indication of the importance of collisions (or damping).</td>
</tr>
<tr>
<td>$\frac{v_{Te}}{c}$</td>
<td>Ratio of electron thermal velocity to speed of light—always $&lt; 1$, and most of the time $&lt;&lt; 1$.</td>
</tr>
<tr>
<td>$\frac{T_e}{T_i}$</td>
<td>Ratio of electron and ion temperatures. Important in considering ion acoustic wave damping, for example.</td>
</tr>
<tr>
<td>$\frac{m_i}{m_e}$</td>
<td>Ratio of ion and electron masses. Since electron mass is fixed (non relativistic), this is determined by the ion mass.</td>
</tr>
<tr>
<td>$\frac{e_0 E_0^2}{n_0 k T_s}$</td>
<td>Ratio of electric-energy density to thermal energy density. As this parameter is increased, nonlinear phenomena may be expected to play increased roles.</td>
</tr>
<tr>
<td>$\frac{R_0}{v_{Te} t_0}$</td>
<td>Ratio of electron transit time to launched wave period. Number of wave periods in the time it takes an electron to transit the heated region.</td>
</tr>
<tr>
<td>$\frac{\lambda_0}{L_n}$</td>
<td>Ratio of free-space launched wavelength to density scale length. This is introduced by boundary conditions.</td>
</tr>
</tbody>
</table>

Table 3-1. Nondimensional parameters of plasma frequency heating.
However, all ten parameters cannot, in general, be matched simultaneously. To see this, note that matching the ratio of electron thermal velocity to the speed of light requires the same electron temperature in the lab and in space. With the other parameters, this requires that the ion temperature be the same as well, along with the ion mass. This is very difficult to accomplish in practice. However, the ratio $\frac{v_0 e}{c^2} \ll 1$ for both the laboratory plasma and the ionosphere, and so one might choose to relax this constraint. This type of justification is taken up again in Section 3.3.

Matching parameters is only half the solution. Boundary conditions also play a key role in the determination of the solution to the set of partial differential equations. For our set of governing equations ((3.1)-(3.7)), let us consider the unperturbed distribution functions $f_{s0}(r, v)$ as initial conditions and the background magnetic field $B_0(r)$ as a boundary condition, and given. In addition, the pump wave must be launched from vacuum (a source-type boundary condition) and decay in amplitude as $r \to \infty$.

Specifying the initial distribution functions $f_{s0}(r, v)$ gives initial profiles of particle density and temperature which determine where the launched wave reflects and the thickness of the Airy maxima. Strictly speaking, the exact shape of the profiles must match to satisfy geometric similarity—i.e., the distributions of the ionosphere and the VTF must differ by only a scale factor of their arguments. However, if the region under consideration is limited to the layer near reflection, then we require only that the density profile must monotonically increase through the critical value and some distance either side. The density scale length, along with the incident wavelength, then sets the thickness of the standing wave maxima (see Budden (1988)). Density variations in the direction transverse to the incidence angle are minimal in the ionosphere, at least on the scale of the heater spot size (50 km). In the VTF, the plasma is toroidal, and the geometry is not exactly planar. However, over the width of the microwave horn antenna, the layer position varies by less
than 1.5 cm. Thus, the geometry can be approximated as one-dimensional. Assuming the density profile is monotonically increasing through the region of critical density, this initial condition can be matched by introducing the ratio of the incident free space wavelength to the density scale length, \( \frac{\lambda_0}{L_n} \), as an additional similarity criterion.

The magnetic field of the ionosphere varies on a much larger scale than the heating experiment, and so the field magnitude and direction are effectively constant in the experiment. In the VTF, \( \mathbf{B} \sim \frac{1}{R} \), but the scale length is still much longer than the scale length of the density profile (see Table 2-3), and so can be regarded as constant over the reflection region. The direction of \( \mathbf{B} \) determines the angle at which guiding center motions intersect the reflection layer. In the ionosphere over Arecibo, this angle is approximately 45°, and so the length of the interaction region is approximately \( \sqrt{2} \) times the layer thickness. In the VTF, the angle is 0° (parallel), and so the length of the interaction region is approximately given by the width of the microwave horn antenna. In both cases the electric field of the wave is parallel to the magnetic field at reflection—in the ionosphere, the density scale length is many wavelengths in extent, and the wave refracts; in the VTF the wave is launched with \( \mathbf{E} \parallel \mathbf{B}_0 \).

In specifying the pump wave, it is helpful to realize that the launching details are not so very important, as long as an oscillating electric field is produced near reflection and the direction of the field is parallel to the background magnetic field. This is satisfied in both the VTF and the ionosphere for different reasons, as mentioned above. Therefore, only the magnitude of the launched field matters, and this is already accounted for in our list of parameters (see Table 3-1).

Thus, consideration of boundary conditions in this case introduces one new dimensionless parameter to be matched, with the understanding that the scope of the simulation is limited to the region near ordinary mode reflection. Geometric similarity is not satisfied,
even on a very local scale, but effects of this are not expected to play a large role. Note that
in both experiments, measurements are taken near the center of the heated volume in a
spot much smaller than the heated volume itself.

Another boundary condition that has been neglected until now is the presence in the
experimental volume of waves that have originated elsewhere and have propagated to the
volume of interest. We have ignored these in the foregoing analysis, but their presence has
been postulated as an explanation for one feature of the ionospheric observations (see
Chapter 1). It is difficult to include externally-produced waves in a simulation—their pres-
ence has not been verified in the ionosphere; their source has not been determined for cer-
tain. We shall ignore their presence for the purposes of this chapter.

Finally, note that our choice of normalizations is not unique—other choices may be
more appropriate for different physical settings. For example, Baranov (1969) applied this
type of analysis to the same set of governing equations for laboratory simulation of the
solar wind-magnetosphere interaction. His results included a different set of parameters
appropriate for low-frequency, planet-scale plasma physics. Conner and Taylor (1977)
applied a similar analysis to the same set of governing equations and derived the general
form of heat flux scaling in tokamaks.

### 3.2 Evaluation of Dimensionless Parameters

Recall that the characteristic length scale, $R_0$, was chosen to be the distance over
which the oscillating electric field strongly affects the plasma particles. For the Puerto
Rican ionosphere this is $\sqrt{2}$ times the width of the first Airy maximum (since the mag-
netic field lines pierce the reflection layer at an angle). The width can be estimated from
the data of Sulzer and Fejer (1994) to be approximately 200 m, giving $R_0 = 300$ m (this
also agrees with the calculation of Figure 1-3). For the VTF, $R_0$ is simply the width of the
microwave beam at the reflection layer, since the magnetic field is parallel to the layer. This is estimated to be approximately 40 cm. As electrons move along magnetic field lines, they experience an oscillating electric field over a time $\Delta t = \frac{R_0}{v_{Te}}$. Values of ionospheric $E_0$ are estimated including the standing wave “swelling” effect, increasing the vacuum value by a factor of 8.4. In the VTF, very little swelling is expected due to the short density scale length, and $E_0$ is estimated using average power flux exiting the area of the microwave horn (including a factor of 2 for the standing wave). These and other dimensional parameters used in the evaluation of the nondimensional parameters are listed in Table 3-2. Ionospheric values are taken from Rishbeth and Garriott (1969). VTF values are taken from Table 2-3.

<table>
<thead>
<tr>
<th>parameter</th>
<th>VTF</th>
<th>ionosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
<td>40 cm</td>
<td>300 m</td>
</tr>
<tr>
<td>$T_s$</td>
<td>$5.9 \text{ eV (e)}$ $0.5 \text{ eV (H)}$</td>
<td>$980 \text{ K (0.084 eV)}$</td>
</tr>
<tr>
<td>$v_{Ts}$</td>
<td>$1.0 \times 10^6 \text{ m/s (e)}$ $6.9 \times 10^3 \text{ m/s (H)}$</td>
<td>$1.2 \times 10^2 \text{ m/s (e)}$ $710 \text{ m/s (O)}$</td>
</tr>
<tr>
<td>$t_0$</td>
<td>0.39 $\mu$s</td>
<td>2.5 ms</td>
</tr>
<tr>
<td>$B_0$</td>
<td>0.068 T</td>
<td>35 $\mu$T</td>
</tr>
<tr>
<td>$q_s$</td>
<td>$\pm 1.6 \times 10^{-19} \text{ C}$ (singly-charged ions)</td>
<td>$\pm 1.6 \times 10^{-19} \text{ C}$ (singly-charged ions)</td>
</tr>
<tr>
<td>$m_s$</td>
<td>$9.1 \times 10^{-31} \text{ kg (e)}$ $1.67 \times 10^{-27} \text{ kg (H)}$</td>
<td>$9.1 \times 10^{-31} \text{ kg (e)}$ $2.67 \times 10^{-26} \text{ kg (O)}$</td>
</tr>
<tr>
<td>$E_0$</td>
<td>10.8 kV/m</td>
<td>1.7 V/m (60 MW ERP) $1.4 \text{ V/m (40 MW ERP)}$</td>
</tr>
<tr>
<td>$\tau_{s0}$</td>
<td>0.5 $\mu$s ($1/v_{en}$) $59 \mu$s ($1/v_{in}$)</td>
<td>1.8 ms ($1/v_{ei}$) $2.5 \text{ s (1/v}_{in}$)</td>
</tr>
<tr>
<td>$n_0$</td>
<td>$7.4 \times 10^{16} \text{ m}^{-3}$ (critical for $f_0=2.45 \text{ GHz}$)</td>
<td>$3.2 \times 10^{11} \text{ m}^{-3}$ (critical for $f_0=5.1 \text{ MHz}$)</td>
</tr>
</tbody>
</table>

*Table 3-2. Dimensional parameters of the VTF and ionospheric plasma frequency heating experiments.*
Chapter 3

The ten dimensionless parameters derived in the previous section are evaluated in Table 3-3 for both the VTF and ionospheric plasma frequency heating experiments. Some variation in these parameters can be expected as experimental conditions vary, but these numbers give a flavor of the similarity of the two plasmas.

<table>
<thead>
<tr>
<th>parameter</th>
<th>VTF</th>
<th>ionosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\omega_{pe}}{\omega_0}$</td>
<td>0.8 - 1</td>
<td>0.93 - 1</td>
</tr>
<tr>
<td>$\frac{\omega_{pe}}{\omega_{ce}}$</td>
<td>1.3</td>
<td>5.3</td>
</tr>
<tr>
<td>$\frac{R_0}{r_{Le}}$</td>
<td>4.7x10^3</td>
<td>15x10^3</td>
</tr>
<tr>
<td>$\frac{v_{os}}{v_{Te}}$</td>
<td>0.12</td>
<td>0.077 (60 MW ERP) 0.063 (40 MW ERP)</td>
</tr>
<tr>
<td>$\frac{v_e}{\omega_0}$</td>
<td>1.3 x 10^{-4}</td>
<td>1.7 x 10^{-5}</td>
</tr>
<tr>
<td>$\frac{v_i}{\omega_0}$</td>
<td>1.1 x 10^{-6} (includes collisions only)</td>
<td>1.2 x 10^{-8} (includes collisions only)</td>
</tr>
<tr>
<td>$\frac{v_{Te}}{c}$</td>
<td>3.4 x 10^{-3}</td>
<td>4.0 x 10^{-4}</td>
</tr>
<tr>
<td>$\frac{T_e}{T_i}$</td>
<td>12</td>
<td>~3 initially, ~3 after extended heating</td>
</tr>
<tr>
<td>$\frac{m_i}{m_e}$</td>
<td>1836 (H) 73400 (Ar)</td>
<td>29360 (O)</td>
</tr>
<tr>
<td>$\frac{\lambda_0}{L_n}$</td>
<td>1.2</td>
<td>8.4 x 10^{-4}</td>
</tr>
</tbody>
</table>

Table 3-3. Nondimensional parameters of the VTF and ionospheric plasma frequency heating experiments. Parameters are evaluated near the reflection layer.
Comparison of the values shown in Table 3-3 shows that the VTF heating experiment is not exactly similar to ionospheric heating, but is very close in many parameters. The first parameter is just the ratio of plasma to pump frequency, and determines where on the density profile the experiment takes place. Most of the interesting interactions occur very near reflection, and so this parameter is approximately one in both experiments. The second parameter is the ratio of plasma to cyclotron frequencies, or the ratio of Larmor radius to Debye length, and is close to but greater than unity in both systems. Wave propagation changes character as this parameter crosses from less than to greater than unity. Since it is rather large in the ionospheric case, many theoretical investigations make the approximation that its inverse, \( \frac{\lambda_{De}}{r_{Le}} \approx \frac{\omega_{ce}}{\omega_{pe}} \ll 1 \), not well justified in the VTF plasma. However, the character of the solution does not change significantly unless \( |\omega_{ce}| > \omega_{pe} \). Additionally, a value very close to unity indicates that cyclotron resonance effects may enter, since the pump wave frequency is the same as the plasma frequency at reflection. In the VTF, this ratio must be low enough to push the cyclotron resonance inboard a number of e-foldings past the reflection layer (satisfied, see Figure 2-21). The third parameter gives the number of Larmor radii in the system size—a very large number in both systems, for both electrons and ions. In combination with the previous group, this parameter also shows that the systems are very many Debye lengths in extent.

The fourth parameter is the ratio of electron oscillation velocity in the wavefield to the thermal velocity, call it \( \beta \) (not to be confused with the usual definition). The electric field of the pump wave provides the driving force of the problem—if \( \beta = 0 \), the distribution functions remain at their initial values. As \( \beta \) is increased, one enters the domain where WLT theory applies (see page 66). If \( \beta \) is increased still further, WLT ceases to describe the physical processes and one must use analyses that include strong turbulence effects. Zakharov (1972) defines the threshold for strong turbulence to be \( \beta \geq \frac{(k\lambda_{De})^4}{m_e/m_i} \), where \( k \) is
the wavenumber, indicating that the long-wavelength part of the $k$-spectrum experiences strong turbulence effects before the rest. The value of $\beta$ determines the character of the turbulence, strong or weak, and is critical to the success of the simulation. The VTF value is about 50% larger than the ionospheric value. A factor of two in $\beta$ translates into a factor of only 1.2 in unstable $k$. This is not a serious deviation, but it does make an SLT-type analysis more applicable to the laboratory plasma.

The fifth and sixth groups measure the collisionality of each species. In each case, particles experience many wave periods per collision. These ratios partially determine the thresholds and growth rates for the nonlinear processes of interest in plasma frequency heating—PDI, OTSI, and LDI (see Section 1.4.3). In practice, the Landau damping rate may be substituted for the collision frequency, giving a much larger number for the case when the temperature ratio is near unity.

The seventh parameter, the ratio of electron thermal velocity to the speed of light, is much less than one for both systems. The values differ by an order of magnitude, however, since the VTF plasma is much hotter than the ionosphere. Discussion of this is continued in the next section, where the concept of limited simulation is introduced.

The eighth parameter is the ratio of the electron and ion temperatures. This parameter determines the level of Landau damping of ion acoustic waves, and is important in both dominant theoretical interpretations of plasma frequency heating. In the VTF, this is much greater than unity, and ion Landau damping is small. In the ionosphere, this parameter is initially one, giving strong Landau damping of ion acoustic waves. As the ionosphere is heated, this ratio increases to about three, giving smaller Landau damping, but still not negligible as in the VTF experiments. The question here is whether this difference alters the essential results significantly. Strong ion wave damping should not affect the OTSI, which has a purely growing ion mode. On the other hand, the PDI loses its resonance char-
characteristic as the ion damping is increased, changing into what is called "nonlinear ion Landau damping", a process where a high frequency wave loses its energy to another high frequency wave and an ion acoustic "quasi-mode". This may indeed change the character of the measured turbulence, as the ion acoustic resonance is washed out by strong damping. Additionally, this may imply that weak turbulence analysis is more applicable to ionospheric heating than to VTF heating, and conversely, that strong turbulence analyses may be required to describe VTF heating physics.

To match the ninth parameter, one must use the same ion species in both experiments. However, this ratio merely provides the degree of separation of electron and ion time scales, which does not overly concern us. The two gases used in the VTF experiments span the ionospheric ion mass.

The tenth parameter arose from the normalization of boundary conditions, and measures the steepness of the density gradient in terms of the pump free space wavelength. The values are very different in the two experiments. This requires that the simulation be limited to just the region near reflection, since wave propagation outside this region is qualitatively different for the two cases. In addition, the very steep gradient that obtains in the VTF plasma introduces additional loss mechanisms, and the wave-wave interactions become gradient limited (see Section 1.4.3(a)).

3.3 Exact and Limited Simulation

In the previous sections the requirements for exact simulation have been identified, and it has been shown that it is difficult or impossible to satisfy all of these simultaneously. This forces a reexamination of the desired results of a laboratory simulation. Exact simulation is certainly the most ambitious goal, and one to be striven for if possible.
Chapter 3

If the nondimensional problem parameters deviate by a small amount from exact matching, the qualitative nature of the solution does not change, provided the solution is "structurally stable"—not extremely sensitive to small changes in parameters (Ryutov et al. (1999)). We have already identified one "structural stability" boundary, where $|\omega_{ce}| = \omega_{pe}$, and have shown that both experiments are well separated from this value. Another set of boundaries exist in the magnitudes of the collision ratios and the parameter $\beta$—increasing the electric field from zero causes various instability thresholds to be crossed, the placement of which is determined by the wave damping rates. When the electric field is far above threshold, as in both experiments, sensitivity to the exact value of $\beta$ is not expected to be high, although substantial differences might affect the character of the solution. For example, different routes to fully developed turbulence are possible, and are determined by the magnitude of $\beta$ (see e.g. Krueer (1988), p. 106).

The parameter $\nu_{T_e}/c$ varies by one order of magnitude between the VTF and ionosphere, and it is not very convincing to use the "structurally stable" argument. However, in both systems the parameter is much less than unity, and intuition hints that the sensitivity of the solution to this parameter is very low. This type of discrepancy led Podgorny and Sagdeev (1970) to introduce the principle of limited simulation.

"The impossibility of exact simulation does not mean at all that the characteristics of phenomena in space cannot be reproduced in the laboratory," wrote Podgorny. "To reproduce some particular cosmic phenomenon in the laboratory, there is no need to follow the similarity laws accurately. It suffices to satisfy the conditions under which the investigated phenomenon takes place." Mathematically, the principle of limited simulation states that parameters which characterize the course of a particular phenomenon, which are of order unity, must be matched. Parameters many orders of magnitude greater or less then unity in
one experiment must be correspondingly greater or less than unity in the other, although the precise order of magnitude need not be the same. This ensures qualitative agreement of the solution to a first approximation.

Note, however, that limited simulation is possible only when one has some knowledge of the solution—i.e., the parameters most important to the characterization of the solution. Much has been learned already about plasma frequency heating, facilitating the use of the principle of limited simulation. For example, most theories of ionospheric heating invoke the electrostatic approximation to describe the developing plasma turbulence. This decouples the electric and magnetic fields, effectively removing equation (3.2) from the system. This in turn removes the parameter $\nu_T/c$ from the list of required matches. We do not expect the mass ratio to drastically affect our observations, other than changing the frequency range of excited waves, and so matching of this parameter can be relaxed as well. The large value of the parameter $\lambda_0/L_n$ introduces a new energy loss mechanism in the VTF plasma, raising instability thresholds and changing the character of certain instabilities from absolute to convective (see Section 1.4.3). We expect the electric field to be well above threshold in spite of this, and we tentatively neglect this parameter as well. The remaining parameters match satisfactorily between the VTF and the ionosphere, with the exception of the temperature ratio. The temperature ratio changes during extended ionospheric heating, from one to about three. Thus, the present VTF results may be more comparable to long-time ionospheric heating than to the initial-time results most commonly reported in the literature.

### 3.4 Remarks

To summarize: fractional analysis of the governing equations resulted in a set of ten dimensionless parameters that determine the character of the physics of plasma frequency heating. Two geometrically similar experiments with matched values of these parameters
are guaranteed to possess the same physics. However, all ten parameters cannot be matched simultaneously, and geometric similarity is not satisfied between the VTF and ionospheric plasma frequency heating experiments.

One can think of the set of ten dimensionless groups as the coordinates in a 10-dimensional parameter space. The solution to the heating problem is characterized by a single point in this space. However, the qualitative behavior of the system can be similar in some region around this point. In fact, the entire 10-dimensional volume may be divided into subvolumes of qualitatively similar behavior. For example, one axis is the heating power parameter, $\beta$. Near the origin of this axis, no instabilities exist. As $\beta$ increases, one enters a different subvolume where the PDI and/or OTSI are excited weakly. With further increases in $\beta$, the system traverses a number of behavior subvolumes until, at very large $\beta$, the entire particle population is trapped in the electric field of the wave (see Krueer (1988), p. 106). The principle of limited simulation is simply the idea that as long as the model system resides in the same subvolume as the prototype, the behavior should be qualitatively similar. However, the determination of the boundaries of the subvolume is not trivial, and often one must resort to heuristic arguments, as we have done here. Finally, the notion of hitting a small volume in 10-space emphasizes the difficulty of a plasma simulation.

Using the principle of limited simulation, several offending parameters were dropped from the list, leaving a set of seven parameters, six of which have comparable magnitude in the VTF and the ionosphere. Small variations in these parameters are tolerable if the solution is "structurally stable"—i.e., if a boundary in 10-space is not crossed. However, structural stability is not assured. The great disparity of existing ionospheric results and the difficulty in reliably reproducing results may be a consequence of observing behavior
in neighboring subvolumes of 10-space. Although care has been taken to identify boundaries (e.g., field thresholds), linear and quasi-linear analysis can only go so far. Where the system is strongly nonlinear, boundaries in parameter space are blurry.

The bottom line of our analysis is that out of ten parameters, nine either match satisfactorily or can be neglected to first approximation. Additionally, the analysis of this chapter has highlighted the following differences between the two experiments:

1. The disparate values of $T_e/T_i$ may produce a solution of slightly different character, through determination of ion Landau damping. The extent of the character difference is not clear a priori. It may be noted that extended ionospheric heating can increase this ratio to about three, greatly reducing the damping rate, and that the nature of the observed spectra in ionospheric experiments also changes character during extended heating (Cheung et al. (1992) and Sulzer and Fejer (1994)). Larger values of $T_e/T_i$ tend toward plasmas that cannot be described with WLT analysis. Thus, the VTF experiments may be slightly more amenable to SLT-type analyses.

2. Geometric similarity is not satisfied even locally, because of the differing angles between the background magnetic fields and the density gradients of the two plasmas. Also, the normalized density scale lengths in the VTF and the ionosphere are greatly different. Restriction of the scope of the simulation to the region very near ordinary-mode reflection reduces the effects of the geometric disparities, since in both cases the pump electric field is parallel to the magnetic field at reflection. What happens outside this layer does not appear to be important to the present investigation, and in fact cannot be simulated in the VTF. Accordingly, the results of the following chapter only cover this region, marked the "simulation zone" in Figures 2-21 - 2-23.
3. Background turbulence cannot be accounted for in similarity analyses, since it has not been diagnosed in the ionospheric case. Background turbulence can easily change the character of the solution in each plasma, as has been pointed out in Kuo and Lee (1992).

Insofar as none of these differences influences the solution overmuch, we have a reasonable expectation of similar physics in ionospheric and VTF heating experiments.

One note on the timescales: the rise time of the spectral features noted in Figure 1-8 is 1-2 ms. Assuming that the growth times scale with the pump frequency, the corresponding time in the VTF should be 2-4 μs—about 1000-2000 samples of the fast oscilloscope (see Chapter 2). Spectral features that appear and disappear on this time scale should be just observable in the VTF.

Finally, we have the needed links to tie the two experiments together and provide a degree of transportability to the results. This has great scientific potential, since once a connection is made between two plasmas, results can flow in both directions—the VTF can elucidate behavior of the ionosphere and vice versa.
Chapter 4

Results

A first look at VTF RF heating results was given at the end of Chapter 2, where a strong resemblance to ionospheric results was noted. Chapter 3 further showed that the RF-heated outer VTF plasma was quite similar to the ionospheric heated layer, and that qualitatively similar physics may be expected in the two plasmas. We present here the detailed results of plasma frequency heating experiments performed in the VTF.

The background plasma was the outer portion of the VTF thermionic plasma discharge (TPD). Ordinary mode microwaves (RF) were launched perpendicular to the background magnetic field, from the low field side, into the increasing density profile of the TPD. The goals of the experiments were to characterize the turbulence produced by the RF injection and to compare results with similar ionospheric experiments. It was also hoped that ionospheric results could be extended and elucidated through these model experiments—that VTF results could shed light on the processes of ionospheric heating, some of which are still not well understood.

A general feature of ionospheric heating that was noted in Chapter 1 is that about half the incident power is absorbed, much more than can be explained by collisional damping. A similar observation has been made in VTF experiments, in which only 10% of the launched power was reflected back to the horn antenna (Riddolls (2000)). A simple-
minded estimate of the return power fraction, \( F \), accounting only for antenna pattern divergence and ignoring absorption, gives \( F \approx 0.30 \). Since very little power is transmitted through the dense core plasma,

\[
P_{\text{incident}} = P_{\text{reflected}} + P_{\text{absorbed}}
\]  

(4.1)

What is measured is the quantity \( F \cdot P_{\text{reflected}} / P_{\text{absorbed}} \approx 0.10 \). Solving for the fractional reflection (reflectivity) and fractional absorption (absorptivity),

\[
f_R = P_{\text{reflected}} / P_{\text{incident}} \approx 0.33
\]  

(4.2)

\[
f_A = P_{\text{absorbed}} / P_{\text{incident}} \approx 0.67
\]  

(4.3)

Now, the power returning to the horn antenna is in large portion made up of waves that have been reflected once—since the horn itself is a good absorber. Thus, despite the possibility of multiple reflections in the metal chamber cavity, the single-pass absorption is still quite large—about 67%. This result is rather surprising, considering that the cyclotron resonance is well inside the plasma frequency cutoff on both sides of the density maximum (see Figure 2-22). The fractional absorption due to collisional damping may be estimated as (see Krue (1988), p. 50)

\[
f_{A, \text{coll}} = 1 - \exp\left(-\frac{8v_e L_n}{3c}\right) \approx 0.002,
\]  

(4.4)

where \( v_e \) is a representative electron collision frequency and \( L_n \) is the density scale length. As in the ionospheric case, then, wave absorption must be taking place in large part due to nonlinear effects—wave-wave and wave-particle interactions.
Results

The effects of extended plasma frequency heating in the ionosphere are dramatic—electron temperature triples and large-scale density "bubbles" form and move away along field lines (Robinson (1989) and Lee et al. (1998a)). In VTF experiments RF heating noticeably increases the brightness of the discharge, as observed with a plasma-viewing CCD camera. In addition, the thermionic arc current increases 20 - 30% during RF injection—an indication that heating significantly perturbs the bulk quantities of the plasma. Indeed, electron temperature measurements taken before and during RF heating indicate a 30% increase during RF injection (Riddolls (2000)).

Absorption measurements are integrated, "zero-dimensional" indications of nonlinear processes that occur during plasma heating. The details of the absorption process can be elucidated by studies of the background and heated wave spectra. This is the approach taken in the following sections. Section 4.1 begins by characterizing the background turbulence of the TPD in the absence of RF injection. Section 4.2 then follows with a description of RF-heated turbulence spectra. Time-resolved spectra are presented in Section 4.3, followed by k-spectra in Section 4.5. In light of the present work, the results of previous VTF experiments are discussed in Section 4.6. A comparison with ionospheric experimental results is given in Section 4.7. Finally, a synthesis of the previous sections and a comparison to theory is begun in Section 4.8. In an attempt to separate the tasks of data presentation and physical interpretation, the first four sections intentionally omit any mention of physics. These sections encapsulate the most important data, and will remain valid even if the interpretations of the succeeding sections are superseded.

4.1 Background Turbulence

Before examining the turbulence produced during a heating campaign, it is helpful to look at the indigenous fluctuation spectrum. Indeed, "preconditioning" of the ionosphere is thought to be instrumental to the particular nonlinear process that develops during heat-
ing, although it is not well-diagnosed in general (DuBois et al. (1993)). In the TPD of the VTF, a certain turbulence level is inherent to the plasma configuration. This section describes the background turbulence in the VTF, in bands of increasing frequency.

4.1.1 Low Frequency Background Turbulence

All quantities vary across the section of the VTF, and so spectra must be examined at a variety of radii. Figure 4-1 shows estimates of the power spectral density (PSD) sensed by one tip of the double probe at several radial locations taken on July 28, 1999. Each radial location represents a separate shot. The magnetic field was kept constant, with $I_{TF} = 5200$ A and $I_{VF} = 1300$ A from shot to shot, and the thermionic arc current varied from 40 - 55 A (fairly constant during each shot). The probe tip separation vector, $\mathbf{d}$, was oriented approximately perpendicular to the magnetic field, or $\theta \equiv 90^\circ$, where $\theta$ is the angle between $\mathbf{d}$ and $\mathbf{B}$. This and successive figures should be compared with Figures 2-22 and 2-23, which give profiles of characteristic frequencies.

Two features of the spectra are immediately evident: a band of noise extends from near DC (at about -15 dB) to about 35 MHz (about -50 dB). This shall be called the "VLF" band (not to be confused with the standard band from 3 - 30 kHz). The VLF feature becomes more and more prominent as the tips move into the plasma. It does not simply rise out of the noise floor with decreasing $R$, but the slope of the spectrum also changes, from about -1.5 dB/MHz at the outside of the plasma to somewhat less than -1 dB/MHz at 110 cm major radius. The VLF feature is strikingly reproducible, given comparable thermionic arc current, as seen in succeeding plots.

The second feature is most notable at the inner radii, and has a local maximum near 50 MHz. It extends from about 35 MHz out to 80 or 90 MHz—covering the band of frequencies near the local lower hybrid frequency. Hence, we denote it by "LH" in the interest of simple nomenclature, although identification as a lower hybrid mode has not yet been per-
formed. The LH peak is absent at the outer radii, and rises 23 dB above the noise floor at its maximum near \( R = 110 \) cm. Although quite prominent in Figure 4-1, the LH peak can be almost totally obscured by the VLF feature at smaller radii. Figure 4-2 is another view of the same data, perhaps more amenable to examination. Note that the radii of interest for heating simulations are between the outer wall at 126 cm and approximately 115 cm major radius, where critical density for ordinary mode reflection is reached. Through most of the simulation zone, the LH feature has a low amplitude.

**Figure 4-1.** Background turbulence: LF band, for 28 July 1999 run day (shots 1, 5, 7, 10). NT and NM emitters, \( I_{arc} = 40 - 55 \) A total, \( I_{TF} = 5200 \) A, \( I_{VF} = 1300 \) A, gas = \( \text{H}_2 \), \( p = 1.52 \times 10^{-4} \) torr, \( \theta = 92^\circ \).
Figure 4-2. Background turbulence: LF band (same setup as previous).

In contrast to the VLF feature, the LH bump has a non-trivial variation with small parameter changes. For example, Figure 4-3 superposes spectra from two different run days, between which there were only three small differences: the neutral pressure was 14% higher, the vertical field current was 15% larger, and the probe tips were oriented parallel instead of perpendicular to the magnetic field. One of the spectra shows the presence of the LH feature as in Figure 4-1, and in one spectrum the LH feature is completely absent. The VLF portions of the spectra are strikingly similar. Increasing the neutral pres-
Figure 4-3. Comparison of 28 July 1999, shot 7, and 11 August 1999, shot 17. In both cases, the tips were positioned at $R = 115 \text{ cm}$.

sure increases the collision frequency, but should not cause a dramatic change in behavior unless a nonlinear threshold is spanned by the parameter changes. Increasing the vertical field changes the position of the thermionic emission rungs by a few centimeters, but these measurements are taken at a radius (115 cm) where the three-dimensional effects of the source region are small. The remaining parameter, probe tip angle, only enters through the capacitive pickup between the two probe tips. The tip to tip impedance should be large (at 50 MHz, $|Z_{12}| \approx 10 \text{ k}\Omega$) so that the signal on one tip should not couple extensively to the other tip. However, the direct capacitive pickup outlined in Section 2.4.1(c) could be coming into play even at these low frequencies. On the log scale of our PSD estimate, this may
be sufficient to produce the dramatic difference between the two shots of Figure 4-3, and may indicate that the background LH turbulence is anisotropic, with preferential propagation in the perpendicular direction.

The sharp spikes in the previous plots are most likely interference from local radio sources, and should be ignored.

All the shots presented so far used hydrogen as the fill gas. When argon was used instead, the qualitative behavior of the LF spectrum remained the same as with hydrogen, with VLF and LH features in evidence. However, the power in the VLF band was much lower in the argon shots and had a different shape, and the LH peak seemed to be shifted higher in frequency (relative to hydrogen shots). One example is shown in Figure 4-4. The LH peak is located at approximately 80 MHz. The flat band between 20 - 45 MHz is the instrument noise floor.

4.1.2 High Frequency Background Turbulence

The HF band in the absence of RF is shown in Figure 4-5. There are no plasma features—this is simply a plot of the instrument noise floor. In particular, there are no features near the plasma frequency. The sharp spikes near the local oscillator frequency (DC on this plot) are probably the result of interference entering the IF stage of the frequency down-convertor.

Between the HF and LF bands, there were no spectral features in the shots where the thermionic arc current was greater than about 10 A (includes all the cases of the present study). Lower current shots (1 - 8 A) have pronounced broadband fluctuation spectra that extend up to a large fraction of the electron cyclotron frequency. The interested reader is referred to Moriarty (1996). All of the experiments presented here had much larger arc currents, and did not possess this feature.
4.2 RF-Heated Turbulence

During RF heating, the turbulence power spectra change conspicuously. The spectra presented in this section are averaged over 500 µs, starting at a time when both the TPD and the RF heating were well established (10 ms after RF turn-on). Thus, these results represent the long-time, steady-state, saturated condition of the nonlinear evolution.

The dramatic changes in the spectra during application of RF power do not necessarily indicate a production mechanism directly from the RF pump wave—RF power could perhaps affect another process in such a way as to change the measured spectra. In other words, unless we have a very good idea of the production mechanism, the measurements
Figure 4-5. Background turbulence: HF band (11 August 1999, shot 17). NT and NM emitters, $I_{arc} = 60$ A total, $I_{TF} = 5200$ A, $I_{VP} = 1500$ A, gas = H$_2$, $p = 1.74 \times 10^{-4}$ torr, $R_t = 115$ cm, $\theta = 4^\circ$.

themselves do not indicate causality, only correlation. Hence the title "RF-Heated," rather than "RF-Produced." This issue is taken up again in Section 4.8 following the data presentation.

4.2.1 Low Frequency RF-Heated Turbulence

Power spectrum estimates for the LF band are shown in Figures 4-6 and 4-7 for several radii, all other parameters remaining nominally constant in each group (the total arc current varied from 35-60 A). The angle of the probe tip separation vector to the magnetic field was approximately $92^\circ$ in Figure 4-6 and $4^\circ$ in Figure 4-7. The VLF and LH features appear, as in Figure 4-1. The behavior of the VLF feature is qualitatively unchanged from the unheated case over much of its band—it decreases in amplitude from DC to about 35 MHz, and tends to increase with decreasing radius. Spectra with and without RF
Figure 4-6. RF-heated turbulence in the LF band, 28 July 1999 (shots 4, 6, 9, 11, and 13). RF on, NT and NM emitters, \( I_{arc} = 40 - 55 \) A total, \( I_{TF} = 5200 \) A, \( I_{VP} = 1300 \) A, gas = \( \text{H}_2 \), \( p = 1.52 \times 10^{-4} \) torr, \( \theta = 92^\circ \).

present are nearly indistinguishable (see Figure 4-12 below for an arresting comparison). However, in the range 0 - 10 MHz, a series of peaks may be noted in Figure 4-7 (where \( \theta = 4^\circ \)). This is shown in enlarged view in Figure 4-8, where all radii have been superposed. The arrows indicate the position of peaks, which are found at the two outermost radii at 2.3, 4.4, 6.5, and 9.4 MHz. As the probe moved into the plasma, the peak at 4.4 MHz disappeared, leaving only peaks at 2.3, 6.5, and 9.4 MHz. The feature at 27 MHz is most likely interference. In measurements taken at \( \theta = 92^\circ \) with or without RF, the largest peak at 2.3 MHz still exists, but the other peaks are absent. This portion of the spec-
Figure 4-7. RF-heated turbulence in the LF band, 11 August 1999 (shots 9, 11 - 16, and 18). RF on, NT and NM emitters, $I_{arc} = 35 - 60$ A total, $I_{TF} = 5200$ A, $I_{VF} = 1500$ A, gas = $H_2$, $p = 1.74 \times 10^{-4}$ torr, $\theta = 4^\circ$.

The LH feature is much larger during RF heating, appearing at all radii and generally increasing in magnitude as radius decreases. In addition, the LH feature is detected at all angles with respect to the magnetic field. The location of the LH peak shifts to lower frequencies as radius decreases, as may be seen in Figures 4-9 and 4-10, which are just alternate representations of the two plots already shown. Peak locations are plotted in Figure 4-11, showing frequencies between 50 and 90 MHz in all the cases discussed so far. On the
Figure 4-8. Enlarged view of VLF spectrum of Figures 4-7 and 4-10. Arrows denote position of peaks in spectrum. All radii are superposed; inner radii have higher power levels.

high frequency side of the LH peak, the LH feature extends up to 200 MHz at the more innermost radii. Some thermionic arc current dependence was observed, but there was no clear relation in the range of interest (35 - 60 A). The dominant dependencies were on radius and presence of RF. At 100 MHz, the RF-heated LH levels were 8 - 20 dB above the noise floor, and 7 - 18 dB above the background turbulence level.

A graphic example of the effect of RF on the LH spectrum is given in Figure 4-12. In the VLF range, the two spectra are nearly indistinguishable. At about 35 MHz, however, there is a marked departure of the two curves, showing the increased LH feature during RF injection. The RF-produced LH bump extends up to about 200 MHz, where the two curves coincide with the instrument noise floor. The sharp spikes at ~130 MHz and ~160 MHz
Figure 4-9. Radial dependence of LF spectra during RF heating, 28 July 1999 (shots 1, 5, 7, and 10). RF on, NT and NM emitters, \( I_{arc} = 40 - 55 \) A total, \( I_{RF} = 5200 \) A, \( I_{VF} = 1300 \) A, gas = H\(_2\), \( p = 1.52 \times 10^{-4} \) torr, \( \theta = 92^\circ \).

are most likely interference. The effect of the RF on the LH feature decreases as radius decreases—where the background turbulence level is already high, adding RF has less effect. For example, comparing Figure 4-2 and Figure 4-9 at \( R = 110 \) cm, the RF only increases the LH peak value slightly, while at higher frequencies, the effect is more dramatic (an increase of 7 dB at 100 MHz).
For shots where argon was used as the fill gas instead of hydrogen, application of RF also increased the magnitude of the LH bump, leaving the VLF feature basically unchanged. Again, in argon, the LH peak seemed to be shifted upward in frequency, with a value of about 100 MHz at \( R = 115 \) cm. This may be compared to the values noted for hydrogen in Figure 4-11, \(-60\) MHz at \( R = 115 \) cm.
4.2.2 High Frequency RF-Heated Turbulence

In the band near the microwave frequency only thermal-level waves are present in the absence of RF injection, and only the noise floor is observable. With RF, the main spectral feature is a large, narrow peak at the RF frequency (near 2.45 GHz). Figures 4-13 and 4-14 show spectra covering the HF band for several radii, with $\theta = 92^\circ$. Figures 4-15 and 4-16 show similar spectra for $\theta = 4^\circ$, with better spatial resolution near the edge. In both sets of plots, the RF peak is largest at the outside of the plasma and decreases as the probe tips move into the plasma. In the center where the density is very large (not shown here), the microwave peak is reduced from the edge value by up to 50 dB (a factor of 100,000—fairly well cut off).
Figure 4-12. Effect of RF on the LF spectrum. Shots 17 and 18 of 11 August 1999 are shown. In both cases, $R = 115$ cm, NT and NM emitters were on, $I_{arc} = 60$ A total, $I_{TF} = 5200$ A, $I_{VF} = 1500$ A, $gas = H_2$, $p = 1.74 \times 10^{-4}$ torr, $\theta = 4^\circ$.

On both the upshifted and downshifted side of the RF peak, plasma wave sidebands may be observed at levels far below the peak value. These are asymmetric about the pump frequency, with more power on the downshifted side. The sidebands decrease in magnitude and change character slightly (some slope change) as the tips move into the plasma. At the outermost radii in Figure 4-13, the low frequency sidebands start at -47 to -49 dB near the pump frequency, decreasing to the noise floor of about -60 dB in 70 MHz. The upshifted sidebands start at comparable levels near the pump frequency, and decrease much more quickly to the noise floor in about 40 MHz. The shape of the upshifted and downshifted sidebands is different—the low-frequency portion is convex upwards and the high-frequency portion is concave.
Figure 4-13. RF-heated turbulence in the HF band, 28 July 1999 (shots 4, 6, 9, 11, and 13). RF on, NT and NM emitters, $I_{\text{arc}} = 40 - 55$ A total, $I_T = 5200$ A, $I_{\text{out}} = 1300$ A, gas = H_2, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$.

Prominent upshifted maxima were not observed in these long-time results, in contrast to earlier measurements in VTF experiments. Rather, the upshifted sideband decreases monotonically from the pump frequency to the noise floor. This issue is treated in more detail in Section 4.6.
Figure 4-14. Superposed view of the data of Figure 4-13.

It should be noted that the spectra in Figures 4-13 through 4-16 were obtained with the microwave down-convertor described in Chapter 2. In Figures 4-13 and 4-14, the local oscillator frequency was measured independently and added during post-processing to the intermediate frequency to give actual frequency on the x-axis. In Figures 4-15 and 4-16, the local oscillator frequency was set and measured before the experiments to a value very near 2.35 GHz. Thus, DC in this plot corresponds to 2.35 GHz. Also note that the features near 2.35 GHz in both plots are artifacts of the frequency mixing process and should be ignored. The sharp lines at $f_{LO} + 130$ MHz and $f_{LO} + 160$ MHz are probably interference, entering through the IF stage of the mixers.
Figure 4-15. RF-heated turbulence in the HF band, 11 August 1999 (shots 9, 11 - 16, and 18). RF on, NT and NM emitters, $I_{arc} = 35 - 60$ A total, $I_{te} = 5200$ A, $I_{vf}$ = 1500 A, gas = $H_2$, $p = 1.74 \times 10^{-4}$ torr, $\theta =$ 4°.

4.3 Time-Resolved Spectra

Since the primary spectral measurements were calculated from long data streams recorded by the fast oscilloscope, time variation of spectral features could be resolved using FFTs with length much shorter than the data length. Most of these data streams were 250000 points long, sampled at 500 MSamples/s. Thus, using a 1024 point FFT and overlapping the segments by half, a time resolution of about 1 µs can be obtained.
Figure 4-16. Superposed view of the data of Figure 4-15.

Figure 4-17 shows spectrograms of the HF and LF bands, recorded simultaneously using the fast oscilloscope. One tip of the double probe was connected through the HF isolation network to the microwave down-convertor, whose output was in turn sampled by the scope. The other tip was connected through the LF isolation network to a separate channel of the scope. All scope channels were sampled simultaneously. Power spectral density estimates for this same shot, averaged over the entire data stream length, were shown previously in Figures 4-10 and 4-15. Sampling started 10 ms after RF turn-on. Time is shown on the horizontal axis and frequency on the vertical axis. Pixel colors indicate PSD level, as shown by the color bar to the right of each plot. The upper panel shows the HF spectrogram and the lower panel shows the LF band. The injected RF appears as a dark, nearly horizontal line 110 MHz above the local oscillator frequency (or about 2.46 GHz). The
Figure 4-17. Spectrograms of double probe signals in the HF and LF bands for shot 12, 11 August 1996. RF on, NT and NM emitters, $I_{m} = 55$ A total, $I_{TR} = 5200$ A, $I_{TP} = 1500$ A, gas = H$_2$, $p = 1.74 \times 10^{-4}$ torr, $\theta = 4^\circ$, $R = 124$ cm.
upper and lower HF sidebands show up as gray patches above and below the RF peak. In the LF band, the LH feature appears between 40 and 110 MHz and the VLF feature is the dark region near DC.

Immediately apparent is the fact that both the LF and HF spectra are "bursty"—not steady in time. In the LF band, bursts of large amplitude and width 2 - 12 μs are followed by periods of small amplitude in a non-periodic fashion. During certain time periods the PSD approaches the noise floor across the entire band, and in other periods the PSD possesses large amplitude all the way out to 200 MHz. The PSD in the entire HF band fluctuates wildly as well, including the frequency bins containing the injected RF waves. The power level of the RF peak fluctuates by up to 40 dB, and other portions of the spectrum experience similar variations. One way to present this variation is shown in Figure 4-18, where the minimum, maximum, and mean values are shown for each frequency bin. This is very similar to the output of a spectrum analyzer whose traces display "max hold", "min hold", and "trace average." In both bands the difference between the maximum and minimum curves is on the order of 40 dB—a fairly large difference.

In contrast to the edge position of Figure 4-17, spectrograms taken at 115 cm major radius are shown in Figure 4-19. The width and mean period between bursts are much larger at 115 cm—in fact, much of the rapid fluctuation in the VLF range seems to have disappeared, replaced by ~ 100 μs long bursts separated by ~ 10 μs quiet periods. In the HF band as well, bursts of noise have lengthened and separated. The RF peak power varies by up to 40 dB, as at 124 cm radius.

Figure 4-20 shows a spectrogram of the LF band where argon was used as the fill gas, at the same tip radius as the previous plot (R = 115 cm). It differs notably from the hydrogen case, especially in the VLF range, where the amplitude is much smaller. In the LH range, the bursts have similar width to the bursts in hydrogen, although the separation is
Figure 4-18. Minimum, maximum, and mean value of each frequency bin of the spectrograms of Figure 4-17. These curves are very similar to what would be displayed on a (very fast) spectrum analyzer whose traces were set to "min hold", "max hold", and "trace average."
Figure 4-19. Spectrograms of double probe signals in the HF and LF bands for shot 18, 11 August 1999. RF on. NT and NM emitters. \( I_{EC} = 60 \) A total, \( I_{TF} = 5200 \) A, \( I_{VF} = 1500 \) A. gas = H\(_2\), \( p = 1.74 \times 10^{-4} \) torr, \( \theta = 4^\circ \), \( R = 115 \) cm.
larger. Also, the bursts seem to be swept in frequency, starting from near 70 MHz and broadening as time progresses to 50 - 200 MHz in extent—a kind of turbulent chirp. The argon dataset is much smaller than the hydrogen one, and so it is not known whether the chirping behavior was an isolated event or not.

![LF spectrogram](image)

*Figure 4-20. Spectrogram of double probe signal in the LF band, argon fill gas. Shot 10, 13 May 1999. NT emitter on, \( I_{arc} = 20 \) A, \( I_{TF} = 5200 \) A, \( I_{VF} = 1400 \) A, gas = Ar, \( p = 6.0 \times 10^{-5} \) torr, \( R = 115 \) cm.*

At still smaller radii, LF bursts merge into a quasi-uniform noise spectrum, while the HF signal decreases dramatically in magnitude due to the plasma frequency cut off, as shown in Figure 4-21.
Figure 4-21. Spectrograms of double probe signals in the HF and LF bands for shot 13, 28 July 1999. RF on, NT and NM emitters. $I_{arc} = 50$ A total, $I_{TF} = 5200$ A, $I_{VE} = 1300$ A, gas = H$_2$, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$, $R = 105$ cm.
In all these spectrograms, short periods of no fluctuation appear as white vertical lines, and indicate where the oscilloscope digitizers were saturated. Interference peaks show up as thin horizontal lines.

Thus far, we have presented only results where the injected RF has been on for a relatively long time (10 ms, usually). From the plots so far shown, it is not possible to determine whether the spectral bursts are produced by the RF or are preexisting in the beam plasma system. In Figures 4-22 and 4-23, however, the microwave was activated in the middle of the measurement window, enabling comparison of the spectrogram before and after RF injection. In both cases, only the noise floor was present in the HF band before RF injection, as expected. In the LF band, only the VLF feature exists, the LH bump being absent until application of RF power. Examining the VLF range, it is apparent that no short bursts appear before RF turn-on—there may be longer-period fluctuations that cannot be plotted in 200 µs. Commensurate with RF power, short bursts are revealed, having length and separation similar to those in the long-time spectrograms of similar radii. At inner radii where the RF peak is very small, LF spectrograms show long, 100 µs fluctuations with and without RF injection.

Since the short bursts are apparently the result of RF injection (directly or indirectly), one might question whether the observed fluctuations are the result of variations in the injected RF power. To answer this question, the injected power of the magnetron was measured with a crystal detector connected to the forward power tap of a waveguide directional coupler on the RF input waveguide. The detector voltage was sampled by a third scope channel, simultaneous with the two channels used for the HF and LF bands. The RF peak power calculated from the spectrogram is compared to the detector voltage in Figure 4-24, during a time period including microwave turn-on. The detector voltage trace shows the RF power rise time to be approximately 100 µs. After turn-on, the power fluctuates
Results

![HF spectrogram](image)

![LF spectrogram](image)

Figure 4-22. Spectrograms of double probe signals in the HF and LF bands for shot 20, 26 July 1999. RF on, NT and NM emitters, $l_{arb} = 50$ A total, $l_{TF} = 5200$ A, $l_{VP} = 1300$ A, gas = H$_2$, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$, $R = 125$ cm.
Figure 4-23. Spectrograms of double probe signals in the HF and LF bands for shot 1, 27 July 1999. RF on, NT and NM emitters, $I_{arc} = 50$ A total, $I_{TF} = 5200$ A, $I_{VF} = 1300$ A, gas = $H_2$, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$, $R = 125$ cm.
Figure 4-24. RF turn-on, measured with a crystal detector on the forward power tap of a waveguide directional coupler and with one tip of the double probe in combination with the microwave down-converter and fast oscilloscope. Shot 35, 26 July 1999. RF on, NT and NM emitters, $I_{arc} = 45$ A total, $I_{TF} = 5200$ A, $I_{VP} = 1300$ A, $gas = H_2$, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$, $R = 125$ cm.
approximately 10%. The bottom plot shows the RF peak power from the double probe tip during the same time period. The probe measurement fluctuates by 30 dB—a factor of 1000. Certainly then, the probe-measured fluctuations are not the result of injected RF power fluctuations.

It may be noted from the previous plots that the HF and LF spectrograms show a marked anticorrelation at the outer radii (the reader is encouraged to view the page at an angle to verify this). To be somewhat more quantitative, the correlation coefficient \( \rho_{\text{LF, HF}} \) between the HF and LF spectrograms was calculated, for a band of frequencies including most of the fluctuation power (10 - 90 MHz for LF, 40 - 120 MHz from the local oscillator frequency for HF). Here, the correlation coefficient is defined

\[
\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y},
\]

where \( \sigma_{xy} \) is the covariance and \( \sigma_x \) and \( \sigma_y \) are the standard deviations of variables \( x \) and \( y \). The result is plotted versus radius for two run days in Figure 4-25. As noted using the page-tilting method, there is indeed a large negative correlation at the outer radii that vanishes as the probe tips move into the plasma. At the innermost radii, a positive correlation exists, although the data is somewhat more sketchy there. What the anticorrelation means is discussed in Section 4.8.

Yet another feature can be gleaned from the time-resolved spectra: the presence or absence of an upshifted plasma sideband in the HF spectrum. As mentioned in Chapter 1, ionospheric heating experiments indicate the presence of an upshifted electrostatic feature during early heating that disappears for longer-term application of RF power. Also noted in Section 3.4, the duration of these upshifted features in the VTF, should they appear, should be on the order of 1 \( \mu \)s. The turn-on time of the microwave is about 100 \( \mu \)s. This
Results

Figure 4-25. Correlation of HF and LF spectrograms as a function of probe tip radius. For each data point, the bands 10 - 90 MHz and 2390 - 2470 MHz were frequency-averaged and cross-correlated.

fact alone may preclude measurement of rapidly developing spectral features, but let us try it anyway. Already we have seen spectrograms of RF turn-on in Figures 4-22 and 4-23. In both these cases, upshifted noise bands appear and disappear, well separated from the main microwave peak. However, the turn-on of the microwave is quite cluttered in spectrogram space, and is often accompanied by sidebands above and below the microwave peak frequency that disappear at later times (these two plots were among the least cluttered). It is not clear whether these features are produced in the magnetron and launched along with the principal frequency, or whether they are plasma wave turbulence excited by the RF injection. If they are produced in the magnetron, one would expect them to be rather sharply defined in frequency—resonant modes of the magnetron cavities. The rather
broad features noted in Figures 4-22 and 4-23 are more indicative of plasma turbulence. Another even clearer example of an initial upshifted feature is given in Figure 4-26 (indicated by the arrow). This feature persists for only $\sim 20 \mu s$ at RF startup, and does not reappear.

4.4 Absolute Electric Field Measurement

As discussed in Section 2.4, the magnitude of the absolute electric field can be estimated from the double probe signal, both in the HF and much of the LF bands. The ratio of tip displacement current (from which the electric field is estimated) to plasma particle current is at least 2.5, so that a rough estimate can be gleaned simply by equating the measured current to the expression for displacement current in equation (2.37). This yields

$$\left| \hat{E}_0 \right| = \frac{\text{acosh} \left( \frac{b}{d_w} \right) \left| \tilde{I}_d \right|}{\pi \varepsilon_0 \omega L b \sqrt{1 - \left( \frac{d_w}{b} \right)^2}}$$

(4.6)

where $\left| \tilde{I}_d \right|$ is the magnitude of the displacement current between the probe tips. A detailed measurement of absolute field strength would allow for a scale factor, and would calibrate in a known field. Here, we simply calculate the raw field estimate and compare to the time-integrated and time resolved spectra shown earlier.

A calibrated spectrum analyzer (HP 8592L) measures the amount of power in the resolution bandwidth. This is equal to half the mean-squared current times the line resistance,

$$P(\omega) \Delta \omega_{rbw} = \frac{\left| \tilde{I}_d \right|^2 R}{2}$$

(4.7)

Combining these two equations gives
Figure 4.26. Spectrograms of double probe signals in the HF and LF bands for shot 35, 26 July 1999. RF on, NT and NM emitters, $I_{arc} = 45$ A total, $I_{TF} = 5200$ A, $I_{VF} = 1300$ A, gas = H$_2$, $p = 1.52 \times 10^{-4}$ torr, $\theta = 92^\circ$, $R = 125$ cm.
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\[ |\hat{E}_0| = \frac{\text{acosh} \left(\frac{b}{d_w}\right)}{\pi \varepsilon_0 \omega L b} \left[1 - \left(\frac{d_w}{b}\right)^2\right]^{1/2} \frac{2P(\omega)\Delta \omega_{rbw}}{R} \]

(4.8)

Near the pump frequency, a typical RF signal level measured with the double probe tips at the outside of the plasma is 18 dBm. Plugging into equation (4.8), one finds

\[ |\hat{E}_0| = 23.8 \text{ kV/m} \]

(4.9)

This is about twice as large as the simple estimate of Table 3-2. The reasons for this are not clear immediately. It could be that a calibrated probe would yield the correct value. Or, the time-averaged electric field at the probe location could actually be twice as large as expected, due to some field concentration mechanism (Langmuir collapse or self-focusing, for example).

The “wings” of the pump spectrum, which are excited plasma waves, have power levels up to -40 dB relative to the central pump wave peak, or about -22 dBm. Using the same calculation as before, the electric field magnitude is about 240 V/m. At the lower frequencies near the LH peak, a typical signal level is -45 dBm. The above calculation yields field levels of about 550 V/m—pretty large. If the probe calibration is high by a factor of two, as perhaps indicated by the pump measurement, then the actual field values would decrease to around 120 V/m (for the HF plasma waves) and 275 V/m (for the LH peak).

All of these calculations have been performed using data from a calibrated spectrum analyzer. One can extend these results to the time resolved measurements by applying appropriate scale factors. In other words, averaging over the whole data stream should give a value analogous to the output of the spectrum analyzer. Power levels in smaller intervals can be scaled accordingly, to give a flavor of the electric field variation during burst events. For example, using variations similar to those in Figure 4-18, the RF pump
wave electric field varied from 1.3 to 130 kV/m, that of the HF plasma wave varied from 13 to 1300 V/m, and the LH peak electric field varied from 31 to 3100 V/m. All of these vary by a factor of 100, with the time-averaged value lying toward the lower end of the range. The bursts that we have measured are thus periods of intense field concentration, and are not simply perturbations about some dominant mean value. Interpretations that do not account for or explain this time variation omit essential physical processes.

4.5 Wavenumber Spectra

The use of a probe with two closely spaced tips permits estimation of wavenumber as well as frequency spectra in a turbulent plasma (see Section 2.4.1(d)). Using the digital analysis method of Beall et al. (1982) yields $S_f(K, \omega)$, the power spectrum in wavenumber and frequency space. A plot of $S_f(K, \omega)$ is shown in Figure 4-27, with local wavenumber $K$ on the horizontal axis, frequency on the vertical axis, and power level shown in color. First, note that there are no well-defined, coherent modes, which would show up as sharp lines in this plot. The spectrum is turbulent, having a wide range of wave numbers at a given frequency. Despite this, one can note that the turbulence is confined to small wave numbers in the VLF range—most of the spectral power is concentrated within $|K| < 1 \text{ mm}^{-1}$. In the band 30 - 140 MHz, the $K$-spectrum has broadened and shifted, including roughly the range -6 < $K$ < 2 mm$^{-1}$. At the highest frequencies, the $K$-spectrum is nearly flat (no preferred wavenumber). Thus, typical wavelengths in the VLF range are greater than about 6 mm, while typical wavelengths in the LH range can be as small as 1 mm (the electron Debye length is about 0.1 mm). Other plasma shots produce quite similar results, although the shift of the mean $K$ away from 0 is not generally noted in lower current shots. It may be an indication of a dominant propagation direction (up or down in this case).
Figure 4-27. Local spectral density $S_f(K, \omega)$, for shot 26, 28 June 1989. RI on. 
N1 and N1 emitters. $I_{e_r} = 85$ A total. $I_{H} = 5800$ A. $I_{e_f} = 1500$ A. gas = H$_2$. $p = 1.52 \times 10^{-4}$ torr. $\theta = 92^\circ$. $R = 125$ cm

Because of the sensitivity of the double probe to electric field, the response of the probe maximizes when the separation vector is aligned with the dominant propagation direction. Figure 4-28 shows spectra from three shots, nearly identical except for the probe orientation. In the range 30 - 150 MHz, the highest response occurs when the tips are perpendicular to the magnetic field, followed by 45°, and finally 0° (parallel). Parallel response is down by ~7 dB from the perpendicular case. This indicates that the electric field (and if electrostatic, the propagation direction) of the turbulence in this range is dominantly perpendicular to the magnetic field. Combined with the observation of short wavelengths above, this gives a strong indication that the LH turbulence in the VTF TPD is primarily electrostatic, propagating perpendicular to the background magnetic field.
A similar analysis of the HF sidebands indicates a preferred propagation direction parallel to the magnetic field.

### 4.6 Comparison to Previous VTF Experiments

The first plasma frequency heating experiments in the VTF were performed in the fall of 1995 and are documented in Moriarty (1996), Lee et al. (1997), and Lee et al. (1998c). The background plasma was the same TPD as used in the present experiments, and the magnetic field configuration was quite similar as well. In these experiments, a 3 cm dipole antenna placed just over the microwave horn antenna was used to make spectral measure-
ments the HF frequency range. Measurements of the LF frequency range were made with a single-tip probe at a different toroidal location. Measurements were performed at a very limited set of radii, mainly because the probe tips had to be readjusted manually.

Results in the LF band were very similar to the time-integrated spectra of Figures 4-6 and 4-7 of the current work. In the HF band, the dominant feature was the pump microwave signal, with smaller features up- and down-shifted from this. Some samples from the Lee et al. (1998c) are shown in Figures 4-29 and 4-30. The sidebands appear dramatically

![Graph of power vs. frequency](image)

**Figure 4-29.** Results of microwave measurement. Caption of Lee et al. (1998c) reads, “Spectrum of microwave-excited Langmuir waves, measured by a dipole probe at an angle of 70° with respect to the magnetic field, in the presence of lower hybrid waves.”

different from those of Figures 4-13 and 4-15. The most noticeable difference is the appearance of a hump with a peak about 10 dB above the noise floor, upshifted from the pump frequency by about 55 MHz. The width of this feature is approximately 30 MHz. Also apparent in Figure 4-30 are humps downshifted in frequency from the pump by 25 MHz and 90 MHz. The pump signal is approximately 25 dB larger than the largest side-
Figure 4-30. Results of microwave measurement. Caption of Lee et al. (1998c) reads, "Spectrum of microwave-excited Langmuir waves, measured by a dipole probe oriented along the magnetic field, in the presence of lower hybrid waves.

band signal in both these plots. These humps were taken as verification of the nonlinear scattering process thought to occur in the WLT interpretation of ionospheric heating measurements (see Section 1.4.4), and the present work was initiated to explore this process further.

As Figures 4-13 to 4-16 show, broad spectral humps up- or down-shifted from the pump frequency were not measured in the present work. Some transient upshifted power was observed at RF turn-on (see page 198), but the duration of the upshifted power was not large enough to register on the spectrum analyzer used in the 1995 experiments (minimum sweep time ~20 ms).

The reason for these differing results may never be known with absolute certitude—the dipole probe, with its associated modular signal conditioning components and coaxial lines, no longer exists. Nonetheless, it is useful to explore several possible explanations. To begin, one can choose between two avenues of thought:

(1) the spectral humps of the 1995 data were the result of excitation by actual plasma
waves. Explanations for the absence of the humps in the present work then focus on differences in plasma conditions.

(2) the spectral humps of the 1995 data were not the result of plasma waves. In this case, explanations focus on differences in experimental setup.

4.6.1 Hypothesis 1: Plasma Waves Excited Humps

It is possible that plasma waves whose power spectrum possessed humps generated the dipole probe signal seen in Figures 4-29 and 4-30. This is the original interpretation, and is still a good explanation for the results. From this point, there are two logical possibilities: either the plasma of the present work does not have the same wave spectrum (humpy), or the present plasma does indeed have the same wave spectrum but the measurement setup somehow masks the presence of the humps.

4.6.1(a) Wave Spectra Not Similar

Differences in the power spectra of two experiments using the same machine, similar magnetic configurations, similar plasma conditions, and the same pump wave should not be large. Indeed, spatial resolution is obtained on the assumption that variations are small from shot to shot. Despite this, there is a small but finite possibility that small parameter changes between the two experiments caused the crossing of a boundary in parameter space delimiting two distinct modes of plasma behavior.

In the present experiments, the base vacuum pressure was nearly an order of magnitude smaller than in earlier work (5 x 10^-7 torr versus 2 x 10^-6 torr). The operating pressure remained about the same, however, resulting in a smaller impurity ion fraction in the present plasma. This would tend to reduce the importance of impurity ion modes in the overall wave spectrum, which might eliminate the humps. The spacing of the humps from the pump frequency should equal the frequency of the low frequency scattering mode, if three-wave coalescence is responsible for the humps (as proposed in Kuo and Lee (1992)).
Thus, we are concerned with modes having frequency near 55 MHz (c.f. Figure 4-29). For high-mass impurity ions, the lower hybrid frequency is much smaller than this in the VTF. Thus, this is an unlikely candidate explanation.

Another parameter that varied consistently between the two generations of experiments was the thermionic arc current. About 55 A (total) in the present experiments, in previous work the total was 80 - 120 A. This change was apparently enough to change the two-dimensional density profile in the poloidal $(R,Z)$ plane (see Figures 2-19 and 2-20) from a single peak source region to a multiple-peak source. The larger current probably raised the peak plasma density in the chamber as well. However, the pump wave frequency was the same in both experiments, and therefore so was the critical density. An increase in the peak density would only shift the resonance layer outward in radius. Also, the shape of the source region should not affect the simulation zone at the edge of the plasma. Aside from density profile issues, the larger current of the past experiments might have generated a higher level of lower hybrid turbulence in the background plasma. This does not seem to be the case, however, given that turbulence levels in the LF band seem actually to be smaller in the older work. Therefore, variation of the arc current does not seem to be able to explain the disparate results.

Because plasma parameters vary with radius (as well as between experiments), probe tip placement is very crucial. In previous work, as mentioned above, the available radii were severely limited. In addition, most experiments were performed at or about a radius of 100.7 cm. This has been marked on Figures 2-21, 2-22, and 2-23 as an “M” and a vertical dashed line. On the density profile of Figure 2-20 this radius is actually on the inboard side of the plasma source region—several centimeters removed from the simulation zone (where $f_0 \gtrapprox f_{pe}$). Even allowing for a 50% uncertainty in the density measured by the Langmuir probe, the simulation zone is still removed from the radius chosen in the older
work. This might have had several effects, including generation of cyclotron resonance sidebands such as electron Bernstein modes or even parametric generation of whistler modes. In all probability, the placement of the tip precluded measurement of ionospheric-relevant physics.

4.6.1(b) Masked Humps

Another explanation of the vanishing humps involves some kind of flaw in the present measurement setup that masks the humps of the plasma wave spectra en route to the fast scope. Such a flaw must allow certain portions of the spectrum to pass, while attenuating other portions, in such a way that the measured spectrum varies smoothly with frequency and from shot to shot. While it is indeed possible to add bumps to spectra by introducing nonuniformities to the probe transmission line system (see Section 4.6.2), it seems quite unlikely that bumps could be selectively removed, leaving a smoothly monotonic spectrum. Additionally, the present measurement networks were tested thoroughly for mismatch and flat power transfer capability before use. In fact, it was during probe mismatch testing that the hypothesis of the following section was formed.

4.6.2 Hypothesis 2: Humps Not From Plasma

Given that the present investigation did not observe prominent humps in the power spectrum, it is natural to question whether there was a feature of the measurement setup used in the 1995 experiments that was able to produce such spectral features from a plasma excitation that did not possess them. The old measurement setup no longer exists in its complete form, and so we can only hypothesize and show the results of analysis and testing with the current setup.

4.6.2(a) Setup

To the best of our knowledge, the measurement setup was as depicted in Figure 4-31. In a microwave measurement system, impedance matching is critical to obtaining unambiguous results. Along a single length of coaxial line, impedance can be expected to
remain fairly constant—within manufacturing tolerances, unless a portion of the line has been damaged or bent too sharply. Boundaries between components, however, always introduce some mismatch. Therefore, it is always desirable to keep the number of transitions to a minimum. Note that the setup in Figure 4-31 had eight boundaries, including tip-to-vacuum and the connector-connector boundaries, and nineteen boundaries, if one includes the transitions from components to connectors as well. With so many connections, the quality of each connection becomes very important to the overall performance of the measurement system. Also, several connections in a short space can combine to form a significant mismatch, even if their individual mismatches are low.

Several line and connector types exist, only some of which are useful at microwave frequencies. For example, SMA connectors are generally specified up to 26 GHz, N-types have a somewhat lower frequency limit, and BNC types are used only for signals in the
hundreds of MHz. This frequency specification is derived from several factors, including impedance matching, power-handling capability, RF leakage, and attenuation. Note that the system in Figure 4-31 used four BNC connectors (only good to a few hundred MHz) and a long length of RG-58 coaxial line, which has very high attenuation at microwave frequencies.

4.6.2(b) Mismatch Hypothesis

Our hypothesis is as follows: an impedance mismatch existed at the rear of the probe in the 1995 setup, near the vacuum feedthrough (see Figure 4-31). This could have had several causes: improperly welded vacuum feedthrough, damaged connector (there were several in this area), use of BNC connector/cable outside of design frequency band, improper connector coupling, or melted dielectric, to name a few. The effects of such a mismatch are detailed in the following sections.

4.6.2(c) Explanation

Let us model the mismatched measurement system as sketched in Figure 4-32 (compare with Figure 4-31). We model the plasma as a voltage source with some source impedance $Z_0$.

![Figure 4-32. Transmission line model of mismatched probe circuit.](image)

which is surely not 50 $\Omega$; a half-wave dipole antenna in free space has an impedance of 73 $\Omega$; in a plasma, a dipole antenna can assume nearly any impedance, see e.g. Balmain (1964), Balmain (1969), or Bhat and Rao (1973)). This is connected to a 1.59 m length of transmission line having impedance $Z_0 = 50 \Omega$ and dielectric constant $\varepsilon = 2.03$ (from spec
sheet of UT-85 semi-rigid line). The end of this line is shunted with the mismatch impedance, \( Z_m \). A 3 m, \( Z_0 = 50 \Omega \) line is connected in parallel with this, and ends in the spectrum analyzer impedance, \( Z_L = 50 \Omega \). Thus \( A = 3 \text{ m} \) and \( B = 4.59 \text{ m} \) in Figure 4-32.

Our task is to estimate the amount of power dissipated in \( Z_L \) as a function of frequency. We begin at the load, \( z = 0 \), and move toward the source. Using equation (2.68), \( \Gamma_L = 0 \), since \( Z_L = Z_0 \). From equation (2.70) it follows immediately that the complex reflection coefficient is 0 for all \( z > -A \), and the complex impedance is then \( Z_0 \) for all \( z > -A \) (from equation (2.71)). Note that all properly matched transmission lines "collapse" like this to the characteristic impedance of the line—the right-looking impedance ceases to depend upon length, for a monochromatic excitation, or frequency, for a broadband excitation.

At \( z = -A \), the impedance is

\[
Z(-A^\ast) = Z_0 \parallel Z_m = \frac{Z_0Z_m}{Z_0 + Z_m}.
\]

(4.10)

The reflection coefficient at the same point is then

\[
\Gamma(-A^\ast) = -\frac{Z_0}{Z_0 + 2Z_m}.
\]

(4.11)

In the region \( -A > z > -B \), the reflection coefficient is (see equation (2.70))

\[
\Gamma(z) = -\frac{Z_0}{Z_0 + 2Z_m}e^{2jk(z + A^\ast)}.
\]

(4.12)

From equation (2.71),

\[
Z(-B) = \frac{1 - \frac{Z_0}{Z_0 + 2Z_m}e^{2jk(A-B)}}{1 + \frac{Z_0}{Z_0 + 2Z_m}e^{2jk(A-B)}} = \frac{Z_0 + 2Z_m - Z_0e^{2jk(A-B)}}{Z_0 + 2Z_m + Z_0e^{2jk(A-B)}}.
\]

(4.13)
A voltage division gives the voltage on the line at \( z = -B \):

\[
V(-B) = V_s \frac{Z(-B)}{Z(-B) + Z_s}.
\]

(4.14)

From this we can calculate \( V_+ \),

\[
V_+ = \frac{V(-B)}{e^{jk(B-A)} + \Gamma(-A)e^{-jk(B-A)}}.
\]

(4.15)

The power dissipated in \( Z_L \) is then just

\[
P_{Z_L} = \frac{1}{2} Re \left\{ \frac{|V(-A)|^2}{Z_L^*} \right\} = \frac{1}{2Z_L} Re \{ |V_+|^2 [1 + \Gamma(-A)]^2 \},
\]

(4.16)

assuming \( Z_L \) is real. Plugging in from above,

\[
P_{Z_L} = \frac{|V_s|^2 |Z(-B)|^2}{2Z_L[Z_s + Z(-B)]^2} \cdot \frac{|1 + \Gamma(-A)|^2}{1 + |\Gamma(-A)|^2 + 2Re \{ \Gamma(-A)e^{-2jk(B-A)} \}}.
\]

(4.17)

Normalizing to the total power delivered by the source,

\[
\bar{P}_{Z_L} = \frac{P_{Z_L}}{P_s} = \frac{|Z(-B)|^2}{Z_L Re \{ Z_s + Z(-B) \}} \cdot \frac{|1 + \Gamma(-A)|^2}{1 + |\Gamma(-A)|^2 + 2Re \{ \Gamma(-A)e^{-2jk(B-A)} \}}.
\]

(4.18)

The frequency dependence in this expression is contained in the exponentials in \( Z(-B) \) (see equation (4.13)) and the denominator.

### 4.6.2(d) Numerical Example

To provide a concrete example, let us assume the parameters of the experimental setup of Lee et al. (1998c): \( Z_0 = Z_L = 50\Omega \), \( A = 3 \text{ m} \), \( B = 4.59 \text{ m} \), and the relative dielectric constant \( \varepsilon = 2.03 \). The source impedance \( Z_s \) is not known exactly, but we estimate it to be large. Linearizing about the floating potential of a Langmuir probe gives the small-signal impedance...
\[ Z_p = \frac{T_e}{eI_{si}}, \]  

which for an 8 eV plasma with typical probe dimensions is approximately 8000 Ω. This is valid at fairly low frequencies, but it illustrates the typical size of the impedance. The exact value is not important to the qualitative behavior of the solution. Let us choose \( Z_d = 1000 \) Ω. Also, note that

\[ k = \frac{2\pi f}{c} \sqrt{\varepsilon}. \]

Figure 4-33 shows the power delivered to the spectrum analyzer for a “white” plasma excitation signal. Several values of \( Z_m \) are shown, all real for this example, corresponding to a resistive mismatch. Other mismatch models could be used. For example, a lumped-element capacitive mismatch would have a frequency-dependent impedance that approaches a short at high frequencies. The exact mismatch model does not change the basic behavior of the solution: periodic spectral humps. The flat line produced by \( Z_m \to \infty \) corresponds to the matched case.

The basic structure of the measured spectrum is a series of maxima and minima spaced equally in frequency. The first peak is located at approximately 33 MHz, and other peaks occur at approximately 66 MHz intervals. Near the heater frequency of 2.45 GHz the pattern retains its overall shape and spacing, but is quite sensitive to the parameters \( B-A \) and \( \varepsilon \)—the cumulative effect of adding small differences over many cycles. Figure 4-34 displays this sensitivity to the length \( B \), all other parameters fixed, with \( Z_m = 25 \) Ω. A variation of 1 cm produces a very different peak location.

For plasma excitation spectra that are not white, the patterns shown in Figure 4-33 and Figure 4-34 must be multiplied by the actual excitation spectrum, giving a qualitatively different measured spectrum; i.e., for a monotonic excitation spectrum, the measured
Figure 4-33. Normalized power delivered to the spectrum analyzer for various values of $Z_m$, with a white excitation spectrum. The line marked $\infty$ corresponds to perfect matching.

spectrum is nonmonotonic. To demonstrate this effect, Figure 4-35 displays a sample (analytical) excitation spectrum and the corresponding measured spectrum for the case of $Z_0 = Z_L = 50 \Omega$, $A = 3 \text{ m}$, $B = 4.59 \text{ m}$, $\varepsilon = 2.03$, $Z_s = 1000 \ \Omega$, and $Z_m = 25 \ \Omega$.

The upper curve in Figure 4-35 is the (hypothetical) spectrum that would be measured by a perfectly matched probe. The lower curve is what would be measured by a mismatched probe, given the (hypothetical) excitation above. Several peaks appear in the mismatched spectrum that are not present in the actual excitation. Here, the noise floor was set at -65 dB, but if the noise floor were 5 dB higher, only the upshifted peak would be visible.
4.6.2(e) Present Setup

For comparison with Figure 4-31, the setup of the present experiments is outlined in Figure 4-36. In this schematic, only one of the two coaxial circuits is shown—in the actual probe, there are two of everything shown, from identical tips to vacuum feedthroughs.

Differences from the measurement setup of Lee et al. (1997) include the use of parallel-wire tips, double signal paths, high-temperature stainless-steel/ceramic/tungsten coaxial line in the immersion region, larger diameter semi-rigid coaxial line, short lengths of flexible RG-174 to allow rotational movement, longer overall probe, use of low-loss, microwave rated Utiflex line to connect probe and measurement stand, and the use of a 6
Figure 4-35. Hypothetically measured spectra of perfectly matched and mismatched probes for identical excitations. The matched case is upper. The mismatched lower case displays several peaks not present in the actual excitation spectrum.

dB terminating attenuator to facilitate broadband “matching” (see Section 2.4.2(c)). Like the setup of Lee et al. (1997), this setup has quite a number of connections and boundaries; however, all of the connectors were rated for microwave use. Wherever possible, low-loss cable was selected. As stated previously, the measurements taken using this setup did not show prominent upshifted peaks or non-monotonic “PDI” spectra.

4.6.2(f) Partial Experimental Check

As a partial check of the mismatch hypothesis, a 50 Ω load on a tee was inserted at the rear of the present probe setup, giving \( Z_m = 50 \Omega \) (Figure 4-37 shows intentionally mismatched setup). Figure 4-38 shows a comparison of actual data taken during two different
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Figure 4-36. The measurement setup used in the present experiments. Only one of the two identical signal paths is shown. The lettered blocks represent connectors; the letters indicate the type of coaxial connector: S = SMA, N = N-type.

plasma shots, one intentionally mismatched and one with no intentional mismatch. The resulting spectra are qualitatively different—the mismatched spectra show a non-monotonic “PDI” spectrum and up- and down-shifted peaks, similar to those observed with the 1995 setup. Without the mismatch, the spectra decay monotonically from the microwave frequency to the noise floor of the spectrum analyzer. In the mismatched case, the distance between transmission peaks is about 55 MHz, a smaller shift than what probably existed in the 1995 setup (the present probe is somewhat longer than the 1995 probe). Also, observing near the heater frequency compounds the effects of even small length differences, as
shown in Figure 4-34. Therefore, one would not expect to reproduce the results of Lee et al. (1997) in their entirety. However, this simple test showed that a mismatch at the rear of the probe can dramatically alter the appearance of measured spectra.

4.6.2(g) Remarks on Hypothesis 2

The measured spectra reported by Lee et al. (1997) and Lee et al. (1998c) are similar to spectra calculated using a simple impedance mismatch argument and a monotonic sample excitation spectrum (see Figure 4-35). A mismatch intentionally placed in the present probe measurement circuit also partially reproduced the results observed by Lee et al. (1997) (see Figure 4-38). This supports suspicions that there was an impedance mismatch in the 1995 measurement circuit.
Interpreting a spectrum from a mismatched measurement circuit can be quite difficult, even if the mismatch is fairly well characterized. The measured spectrum changes qualitatively for small changes in probe length and coaxial dielectric constant. Peaks produced by the mismatch may or may not be visible above the noise floor. Spectral symmetries are not preserved—excitation spectra that are symmetric about the pump frequency may be transformed into asymmetric measured spectra, and vice versa. However, a probe mismatch does not place spectral power in a frequency band where there is no excitation—the presence of a peak indicates that plasma fluctuations do exist in that frequency range, but the detailed line shape, including frequency shift of the peak, should be ignored.
4.6.3 Probability Assessment

Several possible explanations have been presented in the last few subsections, and it is not possible to determine with absolute certitude which is responsible for the vanishing spectral humps. We can speak only of probabilities and likelihoods. Assignment of probability scores is a rather subjective task. In the interest of definiteness, the views of the author are presented in the following list, in order of decreasing probability: probe mismatch, tip placement, and arc current. The other mentioned explanations are not very likely, and were only mentioned for the sake of completeness.

In light of the results of the foregoing sections, the verification of the anti-Stokes scattering process in a laboratory setting becomes rather uncertain. A truly convincing demonstration of the process would be repeatable and would include tests of the measurement system, thus ruling out the explanation of Section 4.6.2. Finally, note that the explanations of the previous few sections were included as an honest assessment of the validity of past and present work. The interpretation of the results of any experiment must be regarded as an ongoing process—a learning process.

4.7 Comparison to Ionospheric Experimental Data

Before discussing the outputs of the two experiments, it is helpful to note that similarity of the physics of ionospheric and VTF plasma frequency heating does not necessarily guarantee the same output—the diagnostics in the two cases are different. In the ionospheric case, the Thompson scatter radar selects a single \( k \), at about 45° to the magnetic field line (vertically upward from the radar transmitter). In the VTF, the double probe is most sensitive to waves whose \( k \) is aligned with the tip separation vector \( d \). However, waves in other directions are still picked up by the probe, and direction sensitivity is obtained by taking measurements at a variety of tip angles.
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Despite differences in diagnostics, the outputs of VTF and ionospheric experiments are remarkably similar in the frequency band near the pump frequency: both upshifted and downshifted sidebands occur, with more power on the downshifted side. The shape of the spectra are similar, with sidebands decreasing monotonically from the pump frequency to the noise floor in a width of several times the ion acoustic frequency.

The upshifted peak observed in most ionospheric experiments to last only a few milliseconds past pump turn-on (but see Vilece (1992) for CW observations), may have been observed also in these VTF experiments. In shots where the microwave turn-on was fairly clean (rare), some upshifted power was observed at ~90 MHz from the microwave frequency that lasted 20 μs. However, the magnetron used in these experiments had a turn-on time of about 100 μs, and so this may have precluded a more repeatable reproduction of this feature. Also, background turbulence seems to play a role in the observation of this feature in ionospheric experiments, and most results are presented for a "quiet", or un-pre-conditioned, ionosphere. This feature is not observed at extended heating times (tens of ms). In the VTF, spectral power in the HF band varies wildly throughout the duration of applied RF power—perhaps more comparable to extended heating campaigns than initial-time ones.

The time variability of the HF spectra in VTF heating experiments was rather surprising—even large density fluctuations would not, by themselves, produce this result. However, this feature was also noted in the ionospheric experiments of Showen and Kim (1978), where a variation in sideband power of about 1 order of magnitude was noted, with spectral features maintaining their relative amplitudes during fluctuations. No correlation to launched power was observed, and the conclusion was that a time-varying pump
electric field near reflection caused these sideband fluctuations. In the present VTF experiments, variations of three orders of magnitude were observed, with spectral features retaining their relative amplitudes (hence the vertical “bursts” in the spectrograms).

A note on simulation boundary conditions may be in order here. Interpretations of ionospheric results often postulate properties of the background ionosphere to explain measured results. Examples of these are postulated lower hybrid waves to explain the upshifted Langmuir waves in WLT, and the requirement of a “quiet” ionosphere for comparison to SLT simulations. Neither of these properties has been measured in a convincing manner. Both enter the set of governing equations as boundary conditions—the initial plasma state. A proper laboratory simulation would possess all the same (scaled) boundary conditions as the ionospheric case. However, in the absence of ionospheric measurements, simulation is difficult. One can only present the background measurements from the laboratory plasma and leave more detailed comparison until ionospheric measurements can be made.

Features of extended ionospheric heating not mentioned above, such as field-aligned “sheets” or rising “bubbles”, are large-scale effects that are outside the parameter range of the present simulation. Only effects contained within the heated layer near reflection may be simulated with any confidence in the laboratory plasma. This includes short-wave-length electrostatic turbulence, and perhaps density fluctuations of somewhat larger scale, but does not include features larger than the heated volume.

4.8 Discussion

Discussions of physics have been intentionally suppressed up until this point, in the interest of objective data presentation. This section presents connections between the data of earlier sections and the theoretical foundations of Chapter 1. The first issue is wave
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identification, discussed in Section 4.8.1. Section 4.8.2 considers a related issue: the identification and origin of the spectral bursts observed in time-resolved spectra. A comparison to heating theories is begun in Section 4.8.3, followed by a final synthesis in Section 4.8.4.

4.8.1 Wave Identification

The measurements of the present work consist, in large part, of fluctuation spectra measured at various points, time intervals, and frequency bands within the plasma. By themselves, spectral measurements cannot conclusively identify wave modes. We have supplemented spectral measurements with wavenumber spectra and direction sensing, where possible. The combination of these data with a theoretical knowledge of plasma waves results in a short list of possibilities. Proof positive of the operative mode is often difficult to obtain; hence, we simply list the possibilities and point out the most likely candidates. It should be noted that in a strongly turbulent plasma, the term “wave modes” loses significance as waves cease to follow their linear dispersion relations, and identification in terms of one or another “mode” becomes futile.

In the VLF range (0 - 35 MHz), the principal feature was a broad, turbulent spectrum that decreased nearly monotonically from DC. The magnitude increased with decreasing radius (corresponding to increasing plasma density and magnetic field). Application of RF did not seem to change the spectrum significantly, although small peaks were noted at 2.3, 6.5, and 9.4 MHz during RF heating when d was parallel to B. Changing the fill gas from hydrogen to argon reduced the magnitude of these fluctuations dramatically. Wavenumber measurements indicated that most of the fluctuation power had wavelength longer than about 6 mm, with no preferred direction. This frequency range is much lower than the ion plasma frequency (about 57 MHz for hydrogen), and higher than the ion cyclotron frequency (about 1 MHz for hydrogen).
Ion acoustic waves are the most likely candidate here, although one can imagine other possibilities, such as the lower branch of the Trivelpiece-Gould (T-G) dispersion relation for bounded plasmas, or ion Bernstein waves at harmonics of the local cyclotron frequency. Peaks at harmonics of the ion cyclotron frequency have not been observed, probably ruling out ion Bernstein waves. The drastic reduction in spectral power with a 40x increase in ion mass is consistent with ion acoustic waves, whose frequency, for a given \( k \), is inversely proportional to the square root of the ion mass. The source of the VLF waves is probably located in the plasma source region of the TPD, where fast electrons, large velocity shears, and beam-plasma instabilities exist, since VLF spectra increase as the plasma source region is approached. The electrons in the source region have a non-thermal velocity distribution and move much faster than the ion acoustic phase velocity, and so could produce ion acoustic waves via Cerenkov radiation. The same mechanism could apply to T-G modes, since they also are slow waves.

In the LH portion of the LF band (30 - 150 MHz), fluctuations were nearly absent at outer radii in the background plasma, although a peak of increasing magnitude appeared as radius decreased. Application of RF produced a similar-looking peak at all radii but generally increasing with decreasing radius, with a maximum that varied from about 50 MHz (at \( R = 100 \) cm) to about 90 MHz (at \( R = 126 \) cm). Changing the fill gas from hydrogen to argon resulted in a 67% increase in peak frequency and somewhat more occupied bandwidth. Wavenumber spectra indicated the presence of wavelengths as small as 1 mm, with most of the power having wavelength larger than about 2 mm. Measurements taken at a variety of angles indicated a preferred propagation direction perpendicular to the magnetic field.
The LH range includes the ion plasma frequency for hydrogen (about 57 MHz), but not that of argon (about 9 MHz). Wave possibilities include lower hybrid waves, ion plasma waves (below $f_{pi}$), whistler waves, and lower T-G waves. The short wavelengths suggest electrostatic modes, but all of the above choices are nearly electrostatic anyway (considering whistlers on their resonance cone). Lower hybrid waves propagate nearly perpendicular to the applied field at and above the lower hybrid resonant frequency, which is about 35 MHz for hydrogen (5.5 MHz for argon). Ion plasma waves are really the high frequency piece of the ion acoustic dispersion relation, and are highly damped ($k\lambda_{De} \sim 1$). The resonance cone of whistler waves in this frequency range, for relevant parameter choices, makes an angle of about 87° with the magnetic field (phase velocity perpendicular, group velocity along the field). T-G modes propagate along the magnetic field by definition—the spatial Fourier transform is taken in only one direction (see Krall and Trivelpiece (1986), p. 167). Since the measurements indicate dominantly perpendicular propagation, the list of possible LH candidates may be trimmed to lower hybrid waves and electrostatic whistlers.

Consider first the background plasma (no RF). Whistler instabilities typically require a loss-cone type distribution function—the opposite of that present in the VTF (see e.g. Kennel and Petschek (1966)). However, Cerenkov radiation from fast electrons may effectively produce whistler waves, particularly if the particles are bunched (see e.g. Farrell et al. (1988) and Gough et al. (1998)). In this case, the radiation is coherent and can be several orders of magnitude larger than the incoherent case. Beam experiments in space observe whistler wave spectra that extend up to the electron cyclotron frequency (Farrell et al. (1989)). The VTF spectra, on the other hand, have a much smaller extent—up to perhaps three times the ion plasma frequency. Although this does not rule out whistler waves, it makes lower hybrid waves a more likely candidate, since their frequency is limited to a
small range above the lower hybrid resonance frequency. Several theories have been proposed for the excitation of lower hybrid waves in a sheared velocity field (see e.g. Keskinen and Huba (1983)). Even in the absence of shear, hot electrons excite waves through Cerenkov radiation when the velocity of the hot particles exceeds the parallel phase velocity of the waves. In the plasma source region of the VTF TPD, the 300 eV electrons excite waves having parallel wave numbers greater than about 40 m\(^{-1}\) at 60 MHz. An additional lower hybrid excitation mechanism may be operative if a powerful whistler spectrum is present as well (Lee and Kuo (1984)). The LH waves are most likely lower hybrid waves then, as the abbreviation has been suggesting all along.

The source of the RF-heated LH turbulence in the outer plasma could be one of two possibilities: either the RF directly creates it through some decay or excitation process, or the RF changes some parameter of the background plasma in such a way as to enhance an instability in this frequency range. One possible decay process would be the parametric decay of an electromagnetic wave into upper hybrid and lower hybrid waves. The polarization of the launched microwaves is exactly wrong to excite hybrid waves, however, and no corresponding upper hybrid decay waves have been noted in the HF spectra (these would have to be shifted by the lower hybrid wave frequency from the pump wave—not observed). Indirect effects of RF are more likely, such as an increase in the edge electron temperature, reducing the damping of waves excited in the electron beam region of the plasma. Or, the application of RF could result in the formation of an energetic tail on the electron distribution function, which could in turn excite waves via Cerenkov resonance.

Since the simulation zone is some distance removed from the region of strong shear, waves generated in the plasma source region would have to propagate outward to reach the probe tips. Spatial damping would then explain the observed background LH feature’s variation with radius—as waves propagate from the core to the edge of the plasma, the
amplitude decreases. Adding RF reduces the damping, and the waves propagate further outwards. This picture is further enforced by the lower panel of Figure 4-11, showing peak LH power versus radius. Without RF, the peak power tends to drop at a rate of about 0.5 dB/cm. With RF, the curve is much flatter out to about $R = 120$ cm, after which it drops at a much higher rate ($\sim 3$ dB/cm).

The application of RF power tends to increase electron temperature in the edge region. This can either increase or decrease the damping rate, depending upon the ratio of parallel phase velocity to the thermal velocity of the plasma. For electron Landau damping of cold lower hybrid waves, the spatial damping rate goes like

$$k_i \equiv \sqrt{\pi k \zeta_{0e}^3} \exp(-\zeta_{0e}^2)$$

(4.21)

where

$$\zeta_{0e} \equiv \frac{\omega}{k_{||}v_{te}}.$$  

(4.22)

This decreases with increasing temperature for parallel wave numbers

$$k_{||} > 0.816 \frac{\omega}{v_{te}}.$$  

(4.23)

At a frequency of 60 MHz in a 5.9 eV plasma, this requires

$$k_{||} > 300 \text{ m}^{-1},$$  

(4.24)

which is satisfied in the VTF TPD. Thus, increasing electron temperature at the edge decreases the electron Landau damping rate of cold lower hybrid waves having parallel wave numbers larger than equation (4.24)—the same lower hybrid waves that are excited by the hot electrons in the source region.
In the HF frequency range (2.3 - 2.6 GHz), the bands of noise below and above the microwave frequency may be either Langmuir waves or upper hybrid waves. These are really two limits of the same electrostatic dispersion relation—the perpendicular limit yields upper hybrid waves, and the parallel limit gives Langmuir waves (for the case $\omega_{pe} > \omega_{ce}$). In reality, the angle of propagation need not be limited, and is set by the excitation mechanism and any subsequent scattering events that occur. The polarization of the launched microwaves is along the magnetic field, and this is also the dominant propagation direction of excited waves (see Section 1.4.3(c)). We therefore consider the HF sidebands to be Langmuir waves, with the understanding that a continuum of propagation angles is present.

To summarize: from 0 - 30 MHz, electrostatic turbulence exists in the VTF TPD, independent of applied RF power. This is most likely ion acoustic in nature, produced by the beam-plasma system, although other possibilities have not been ruled out. The LH feature from 30 - 150 MHz in the background (no RF heating) plasma is most likely lower hybrid turbulence produced by the beam discharge. The RF dependence of the LH feature can be explained as a reduction in damping due to increased electron temperature at the edge, but other LH production mechanisms are possible. The sidebands near the pump frequency are magnetized Langmuir turbulence.

4.8.2 Spectral Bursts

Before beginning a discussion of the spectral bursts, it may be noted that other researchers have observed Langmuir bursts in slightly different experimental configurations. As mentioned in Chapter 2, Sergeichev and Sychev (1990) observed short bursts of RF emission during magnetized plasma frequency heating, and Gol'tsman et al. (1984) observed Langmuir wave bursts during microwave heating of an unmagnetized plasma. Langmuir wave bursts have also been observed in anode double layer experiments (Gunell
et al. (1996), Theisen and Carpenter (1996) and (2000)). All of these investigators interpreted their results in terms of SLT theory: envelope solitons that simultaneously collapse and propagate are observed as bursts of Langmuir wave energy at a fixed point in space.

Since the magnitudes of the time-averaged HF spectra decrease with decreasing radius (or increasing density), and since the magnitudes of the LF spectra have the opposite trend, it is logical to suppose that the anticorrelated bursts at the plasma edge are simply the result of alternating regions of high and low density moving past the probe tips (in space or time). High frequency electric field fluctuations might then concentrate in the low-density patches, resulting in the spectral bursts of the time-resolved measurements and the >30 dB variations in the peak amplitude.

If these postulated regions of high and low density are the result of a surface wave, the burst periods indicate wave frequencies of tens to hundreds of kHz. This is much smaller than the ion cyclotron and plasma frequencies, and covers the expected range of drift wave frequencies in the VTF. Drift waves propagate up or down the outer layer of plasma in the VTF, and could possibly generate density variations of several percent. It remains to be seen whether such a density variation could produce a factor of 100 change in fluctuation electric field, as is observed.

In addition, the bursts appear to be linked to the application of RF power. Bursts are only observed during RF heating. Their effect is seen most strongly at the edge of the plasma where the pump electric field is the strongest. The mean length and separation of the bursts increases with decreasing radius. In short, both the character and the presence of the bursts depends upon pump wave magnitude—indication that this is a nonlinear effect of RF heating.
Of the two main theories of plasma frequency heating, SLT is the only one that includes a prediction of time-varying density patches. As mentioned in Chapter 1, numerical simulations of driven Langmuir turbulence indicate the presence of localized density depressions which trap Langmuir waves (Robinson (1997)). The trapped Langmuir waves increase in intensity due to the energy supplied by the driver (pump field). The increased electric field produces a ponderomotive force which further reduces the density in the depressions, which trap more Langmuir waves. The depressions then reduce in physical extent while decreasing their density—collapse. At a size of roughly 20 Debye lengths, collapse is arrested by Landau and transit-time damping, and the wave energy is transferred to plasma particles. The electric field is much reduced by the damping, and can no longer support the large density deficit. The depression relaxes back to large spatial extent and a density near the background value. In 3D simulations, this cycle is observed to repeat itself in a period given approximately by (Robinson (1997))

\[ T = 40 \langle W \rangle^{-1} \omega_p^{-1}, \]  

where

\[ W \equiv \frac{\mathcal{E}_0 \varepsilon_{\text{rms}}^2}{2 n \kappa T_e}. \]  

For numbers representative of the simulation zone in the VTF plasma (see Tables 2-3 and 3-2), equation (4.25) gives a mean period of about 0.4 \( \mu \)s. As the tip moves into the plasma, the density increases and electric field decreases; both changes increase the cycle period, as observed. For example, if the density doubles and the electric field reduces to half its value, the period becomes about 2.3 \( \mu \)s. These numbers overlap the range of burst periods noted earlier for outer radii (2 - 12 \( \mu \)s), and follow the same trend with tip radius.
The anti-correlation of HF and LF bursts at the plasma edge can also be explained using the caviton model. The HF bursts may be Langmuir waves trapped in cavitons. Near the end of an HF burst, the (postulated) caviton damps on particles, perhaps producing energetic electrons. If these heated electrons exceed the parallel phase velocity of lower hybrid waves, then lower hybrid waves are excited, starting near the end of each HF burst. This corresponds to what is observed near the plasma edge.

An alternative explanation for the appearance of lower hybrid bursts is the LDI (see Section 1.4.3(c), page 51). The threshold field for the LDI is a function of angle of propagation and background plasma parameters, and may be calculated for the VTF in the same way as for the ionospheric case (see Section 1.4.3(c)). For $\alpha = 0.85$, $\theta = 21^\circ$, $m = 4.23 \times 10^{-4}$, $1/\mu = 1836$, $\tau = 11.8$, $\gamma_i = 2.5$, and $a_e = 1.3 \times 10^{-4}$, the normalized threshold field is $\beta_{TH} \equiv 0.43$, or $E_{TH} \equiv 38$ kV/m. This is larger than the expected value (10.8 kV/m) and the measured mean value (23.8 kV/m), but is well in the range of the total variation of electric field in a burst (1.3 - 130 kV/m, see Section 4.4). Thus, the LDI would not be operative in the initial stages of a burst, when the electric field is smaller than threshold. As the caviton develops, the Langmuir wave electric field increases, until at some point either collisionless damping sets in or the threshold for LDI is crossed. If the LDI threshold is exceeded before significant damping arises, the production of lower hybrid waves may become a dominant saturation mechanism, superseding and/or supplementing collisionless damping.

It is not possible at this time to rule out other possibilities for the bursts, such as the drift wave turbulence mentioned above. However, it is difficult to imagine how large field variations come about without the action of some instability. Collapsing cavitons seem to
be a satisfactory explanation of these variations. In addition, the burst period variation with radius and the consistency with the results of SLT simulations makes the caviton cycle an attractive interpretation.

One final note: Langmuir probe measurements of density and temperature in the VTF TPD have always been difficult because of the large fluctuation level in the data. Efforts to resolve these fluctuations in time have been unsuccessful. Our microwave receiver in combination with the fast scope have revealed that the RF-heated plasma contains bursts of microsecond duration. A voltage sweep in less than 1 μs would be required to resolve temperature variations, currently beyond the capability of our electronics.

4.8.3 Comparison to Theories of Heating

Both main theories of plasma frequency heating start with the production of Langmuir waves via the OTSI and the PDI. For the parameters of Table 2-3 and 3-2, with α = 0.85, the thresholds for the OTSI and the PDI in the VTF are $\beta_{TH} \equiv 0.027$ and $\beta_{TH} \equiv 3.5 \times 10^{-4}$, respectively. The expected field ($\beta = 0.12$) is far above each of these. Accounting for the large density gradient yields higher thresholds (see equation (1.32)) of $\beta_{TH} \equiv 0.07 - 0.10$, which should still be exceeded by the VTF pump wave.

Another means of producing Langmuir waves is through scattering on background ion density fluctuations (Section 1.4.3(b)). The large VLF feature in the VTF spectra is most likely beam-generated ion acoustic turbulence, and scattering on these fluctuations could directly produce Langmuir waves, even in the absence of parametric instabilities. This stands in contrast to ionospheric heating, where the background plasma is very quiet (at least in middle latitudes at night). Background fluctuations cannot be accounted for in the similarity analysis of Chapter 3, and their presence may limit the validity of the laboratory simulation.
Results

Once Langmuir waves are present, the two theories of heating diverge in their approaches. Weak turbulence analyses use the random-phase approximation to develop systems of wave kinetic equations that describe the turbulence microdynamics (see Section 1.4.4(a)). Strong turbulence analyses reduce the governing equations to a much simpler set that is easier to solve numerically (see Section 1.4.4(b)). In either case, theoretical predictions are the result of extensive numerical calculations, and are relevant only to the particular set of plasma parameters input. None of these numerical calculations is specific to VTF heating (this is a good area for future work). In such a situation, one can only perform qualitative comparisons.

Time-averaged measurements in the band near the pump frequency look quite similar to predictions of both WLT and SLT: fluctuation power decreases monotonically to the noise floor on the downshifted side of the RF pump. WLT interprets this feature as a PDI cascade spectrum, and SLT theorists believe it is the spectral manifestation of cavitons. We conclude that the fluctuations are Langmuir waves in Section 4.8.1, in agreement with both WLT and SLT. The distinguishing factor between the two theories is really the spatial configuration: are the fluctuations the result of a large number of incoherently interacting Langmuir waves (WLT), or are they coherently developing, trapped waves (SLT)? It is difficult to say, simply from inspection of time-averaged spectra. The bursty behavior observed in the time-resolved measurements certainly supports SLT (in some form) rather than WLT.

On the frequency upshifted side of the pump, the present time-averaged spectra show no indication of a hump, as opposed to ionospheric measurements that do possess a hump (see Figure 1-7). SLT simulations manage to reproduce this “free mode” peak, and its production is supposed to be linked to the caviton collapse cycle (although the exact mechanism is not clear). The postulated addition of lower hybrid waves in the heated ionospheric
volume enables WLT analysis to account for this "anti-Stokes" peak through a nonlinear scattering mechanism (see Section 1.4.3(d)). One can estimate the expected power in the upshifted peak using equation (1.47) and the absolute electric field measurements of Section 4.4. For $E_{l1} = 120$ V/m, $E_{lhw} = 275$ V/m, $k_{l1} = 1000$ m$^{-1}$, and $\theta_{l2} = 45^\circ$, the electric field of the "anti-Stokes" peak should have been about 200 V/m. The measured "anti-Stokes" peak in the VTF should have been larger than the PDI cascade—quite observable above the noise floor of the measurement network. The conspicuous absence of such a peak implies that the nonlinear scattering on lower hybrid waves is not operative in the VTF heating experiments. Although no quantitative theory exists for the "free mode" of SLT, its absence in the VTF experiments casts doubt on the generality of the numerical results obtained thus far.

At low frequencies, neither the original formulation of WLT nor SLT predicts the production of lower hybrid waves. If Langmuir waves are present with sufficient intensity, however, the threshold of the LDI may be exceeded, generating lower hybrid waves parametrically. The required threshold is far too large in ionospheric heating situations, without some field concentration mechanism. This is also the case in VTF, where the required threshold is about four times the expected pump field. Our time-resolved measurements indicate that the threshold for the LDI may be exceeded during short Langmuir bursts, however. Collapsing envelope solitons could very well be the operative field concentration mechanism in the VTF, producing the Langmuir bursts and precipitating the phase-delayed lower hybrid bursts. A similar set of events may occur in ionospheric heating experiments as well, but field-aligned modes such as lower hybrid waves cannot be observed with radars presently deployed. The new MIT IRIS system, however, will be one of the first radars to observe perpendicular to the magnetic field during heating campaigns, possibly uncovering the role of hybrid waves in magnetized plasma frequency heating.
4.8.4 Synthesis

Based on the results presented in this chapter, the following picture of VTF plasma frequency heating emerges. Before RF turn-on in the background TPD plasma, the dominant turbulence appears to be ion acoustic in nature. In addition, at positions very near the thermionic emission “rungs” a spectral feature in the lower hybrid band rises from the noise floor. Near the plasma frequency, no fluctuations are observed. Commensurate with RF turn-on, a large lower hybrid feature appears at the edge of the plasma. The ion acoustic turbulence is largely unchanged by the RF. In the band near the plasma frequency, Langmuir waves appear both below and above the applied RF frequency. Neither the lower hybrid nor the Langmuir waves are steady in time, but occur in short bursts, phase delayed from one another. The magnitude of the wave electric field varies by a factor of 100 during a burst cycle, and can reach values that are about 10 times what is expected. Evidently, some process is active that concentrates the field in the bursts. Isolated spectral peaks with frequency upshifted from the pump are not observed in long-time heating. In early heating, upshifted peaks are sometimes observed that last ~20 μs from the onset of RF injection, although they may be features of the generator turn-on.

One possible mechanism for field concentration is the caviton collapse cycle of strong turbulence theory. The lower Langmuir wave sideband is also consistent with the “caviton continuum” of SLT, but the “free mode” predicted by SLT is not observed in VTF experiments. Additionally, the lower hybrid bursts that appear to be intimately connected with the high frequency turbulence in the VTF have no SLT counterpart. In fact, most SLT calculations neglect the magnetic field entirely. Therefore, it is clear that although the collapse cycle may explain the observed high frequency bursts, the theory is rather incomplete.
Weak turbulence theories have difficulties completely explaining the observations also. The lower Langmuir wave sideband is largely consistent with a cascading PDI spectrum, but the upshifted “anti-Stokes” waves predicted by WLT-type analyses are not observed in the VTF experiments. In fact, use of absolute electric field measurements shows that, according to existing theory, anti-Stokes waves should have been observable with our setup. Also, the turbulent bursting cannot be explained with WLT. The appearance of lower hybrid waves with RF turn-on could be a result of the LDI, but expected field thresholds are much too high to admit the LDI.

The shortcomings of SLT and WLT can be mitigated to some extent by combining elements of both approaches. For example, if the caviton collapse cycle is operative in VTF heating, it could be manifested as the observed Langmuir bursts. Instead of damping solely on particles, the Langmuir solitons may grow large enough to excite the LDI. This would handily explain the phase-delayed appearance of lower hybrid wave bursts after the onset of Langmuir wave bursts. The cycle may then be modified enough to prevent the excitation of free Langmuir waves.

With regard to the simulation of ionospheric plasma frequency heating, it is clear that some main results have been reproduced in the VTF, such as the downshifted Langmuir wave sideband, large absorption, and electron heating. Other features of ionospheric heating have not been produced in the VTF experiments, such as the lack of an upshifted “anti-Stokes” or “free mode” peak. The bottom line is that although the outputs of the two experiments are quite similar in some respects, there are differences.

Some features of VTF experiments have not yet been observed in ionospheric campaigns: namely the appearance of lower hybrid waves during RF heating. Current ionospheric diagnostics do not permit the observation of lower hybrid waves, but similar results may be observed upon deployment of the new IRIS radar. Also, spectral bursts
Results

have been measured that agree with the SLT picture of ionospheric heating. These have not been measured in ionospheric experiments, and have chiefly been inferred from comparison to numerical simulations. Thus, the present measurements may represent an extension of the ionospheric heating database through laboratory simulation.
Chapter 5

Conclusions

This dissertation has covered many topics, and this final chapter is intended to bring the most important points together into one place. Section 5.1 briefly reiterates the main results, followed by a list of conclusions in Section 5.2. Directions for future work are suggested in Section 5.3.

5.1 Summary of Results

The purpose of the present work was to explore the extent to which plasma heating experiments in the VTF could simulate ionospheric heating experiments. To accomplish this, several subjects had to be addressed. In Chapter 1, the ionospheric experiment and diagnostics were presented. Available theories of plasma frequency heating were also described, with the conclusion that the remote sensing techniques used in ionospheric heating experiments cannot provide sufficient information to resolve outstanding theoretical questions. It is thus reasonable and desirable to explore the possibility of laboratory simulation.

Chapter 2 introduced the laboratory experiment used for the simulation, the VTF, including diagnostics and theories of operation. A 2.45 GHz RF pump wave was launched perpendicular to the density gradient of an overdense, magnetized plasma. To measure the electrostatic waves produced by the RF pump at low and high frequency, a parallel tip double probe was designed and built. A remotely controlled traverse mechanism was built that allowed the tips to rotate through approximately 700° in the (Z, Φ) plane and scan
radially across the outer portion of the plasma. Two independent signal paths connected the tips to diagnostics at the rear of the probe. The use of a double probe allowed estimation of wavenumber spectra. Additionally, analysis of the probe showed that the closely spaced tips possessed a capacitance sufficient to allow direct, absolute sensing of electric fields. A microwave receiver was also designed and built, allowing simultaneous, time-resolved measurements of low and high frequency signals. A literature search revealed only two comparable published experiments, neither of which measured time-resolved spectra or absolute electric fields. Previous experiments on the VTF were of a preliminary nature, and were performed in slightly different plasma conditions.

To forge links between the ionospheric and VTF experiments, Chapter 3 derived similarity rules from the governing equations. Ten dimensionless parameters resulted, which if matched would guarantee that the same physics are sampled in both experiments. Unfortunately, it was also shown that it is impossible to match all ten parameters exactly. The principle of limited simulation was introduced, allowing the neglect of several parameters that do not agree. Seven parameters remain, six of which match satisfactorily. Matching of boundary conditions effectively reduced the scope of the simulation to the thin layer near ordinary mode reflection. Overall, the two heating experiments were quite similar, with three possible differences: the ratio of electron to ion temperature (determines ion wave damping), geometric configuration, and background turbulence (not characterized in the ionosphere).

Finally, the results of the present experiments were presented in Chapter 4. Background turbulence (no applied RF) was measured at a variety of radii in the VTF. This occurred predominantly in the ion acoustic band from 0 - 35 MHz. Some lower hybrid tur-
bulence (35 - 150 MHz) was observed as well, occurring only near the plasma source region. Wavenumber measurements indicated that typical ion acoustic wavelengths were greater than about 6 mm, or $k\lambda_{De} \leq 0.069$.

Application of the RF pump wave produced Langmuir wave sidebands both above and below the pump frequency. In time-averaged measurements, these sidebands extended monotonically from the RF peak to the noise floor on either side, and appeared to propagate parallel to the applied magnetic field. Time-resolved measurements occasionally showed transient peaks upshifted from the pump frequency by about 90 MHz and lasting only 20 $\mu$s from RF turn-on. Previously recorded steady state upshifted peaks remain a mystery, but were most likely the result of an impedance mismatch in the measurement setup. Inserting an intentional mismatch in the present measurement setup partially reproduced the earlier results.

Application of RF also caused the lower hybrid turbulence to increase 10 - 20 dB above the background case and extend all the way to the edge of the plasma. The lower hybrid turbulence was found to propagate primarily perpendicular to the applied magnetic field, with wavelengths as short as 1 mm ($k\lambda_{De} \leq 0.41$).

Time-resolved measurements revealed that the wave spectra at both high and low frequency were not steady in time, but were grouped in a series of bursts. The bursts were aperiodic, and the duration and mean period lengthened with increasing plasma density. The HF and LF bursts were anticorrelated at the edge of the plasma, and became more and more uncorrelated as the probe tips moved into the plasma. The period of the bursts varied from 2 - 12 $\mu$s at the edge of the plasma to $\sim$100 $\mu$s at inner radii.

Estimation of absolute electric field yielded some surprising results: the time-averaged RF pump field was about 23.8 kV/m—more than twice the expected field. For a shot with this average magnitude, the instantaneous pump field varied from 1.3 kV/m to 130 kV/m.
The electric field of the electrostatic waves varied a similar amount: for the downshifted Langmuir waves, $13 < E < 1300 \text{ V/m}$; for the lower hybrid waves, $31 < E < 3100 \text{ V/m}$. The injected RF power varied less than 10% over a shot.

To produce this kind of variation, some means of concentrating the electric field must be at play. A likely candidate is the caviton collapse cycle of strong turbulence theory, in which Langmuir waves trapped in density depressions cause the depressions to deepen and contract, increasing the electric field. The period of a caviton collapse cycle in threedimensional numerical simulations, for similar plasma parameters, was calculated to be $0.4 - 2.3$ μs, increasing with either increased density or decreased electric field. Thus, both the period and the expected parameter variation of caviton collapse cycles approximately matched those of the VTF bursts. Other means of concentrating the electric field have not been ruled out, however. Surface waves are undoubtedly present in the VTF plasma, and may be able to produce some field variation. However, a factor of 100 variation is observed—difficult to explain with a surface wave.

The bursts of lower hybrid radiation are a new observation. Neither SLT theory nor weak turbulence theory can explain their presence. Two lower hybrid production scenarios were presented here. In the first, lower hybrid waves are produced in the core of the plasma but are damped before they can propagate to the edge of the unheated plasma. Application of RF increases the electron temperature, in turn decreasing the collisionless damping rate of the shorter parallel wavelength waves. Electron heating may occur in bursts, especially if caviton collapse is operative. In the second scenario, lower hybrid waves are produced parametrically through the LDI. The threshold field of the LDI is not exceeded until the electric field is larger than about four times the expected pump field. This does not take place until the latter portion of the Langmuir wave bursts, resulting in the anti-correlation of Langmuir and lower hybrid bursts.
Conclusions

The principal ionospheric heating results were reproduced in the VTF experiments: monotonically decreasing HF sidebands, order-of-magnitude power variation, large absorption, and electron heating. In addition, background characteristics sometimes postulated during interpretation of ionospheric data were actually measured here: lower hybrid turbulence extending over a wide range of frequency and radii, and bursts of fluctuation power. However, the upshifted Langmuir wave peaks observed in the ionosphere and known as "free modes" or "anti-Stokes" Langmuir waves were not observed in the VTF. In addition, absolute electric field measurements were used to determine that, if "anti-Stokes" theory were operative, upshifted Langmuir waves should have been observed. The absence of any upshifted peak thus indicates that neither the nonlinear scattering theory of WLT, nor SLT theory as presently recorded in the literature, can accurately describe the physics of heating in the VTF.

5.2 Conclusions

The intrinsic ambiguity of remote sensing in ionospheric heating experiments motivated this laboratory simulation. In this and any model experiment, one must determine the extent to which the model is similar to the prototype before embarking on comparisons of measurements. The similarity analysis of the present work indicated that the VTF experiments should reproduce the physics of ionospheric heating fairly well, except for three problem areas. First, the ratio of $T_e/T_i$ is much larger in VTF plasmas than in the ionosphere, reducing the collisionless damping rate and allowing ion turbulence to evolve along with plasma wave turbulence. Second, geometric similarity is not and cannot be satisfied. This limits the scope of the simulation to the region very near ordinary mode reflection. Larger-scale effects of ionospheric heating, such as rising bubbles or kilometer-scale sheets, cannot be adequately modeled in the VTF plasma. Third, background turbulence cannot be included in our similarity analysis, largely because ionospheric background tur-
bulence remains undiagnosed. The evolution of plasma frequency heating is known to sensitively depend on the initial plasma state, and this important effect must be left out of our analysis.

Recognizing these limitations, the experimental phase of this work set out to show how the results of ionospheric heating could be elucidated or even extended by means of laboratory experiments. The main results of ionospheric heating were reproduced in the VTF: monotonically decreasing HF sidebands, order-of-magnitude power variation, large absorption, and electron heating. Coupled with the similarity analysis, this gives credence to the claim that similar physics were operative in the two experiments.

A major difference from ionospheric results was the lack of an upshifted Langmuir wave peak in the heated spectrum, as observed during the first several milliseconds of ionospheric heating. Both main theoretical treatments, WLT and SLT, predict the presence of upshifted Langmuir waves in ionospheric heating. The nonlinear scattering theory of Kuo and Lee (1992), with absolute electric field measurements as input, predicts a large upshifted peak in the VTF experiments also. Such a prominent upshifted peak is not observed in the present VTF experiments. Earlier VTF results remain mysterious; they were probably the result of transmission line impedance mismatch.

Two new results, not measurable in current ionospheric experiments, are the appearance of lower hybrid waves with the onset of RF and the time-resolved observation that both the Langmuir and the lower hybrid turbulence occur in bursts. These bursts have period and variation with density that would be expected of collapsing cavitons, a feature of the SLT theory of plasma frequency heating. During a burst period, the electric field varies by a factor of 100, with an average that is more than twice the expected value. This large variation is typical of collapsing cavitons. However, SLT theory has not predicted lower hybrid wave production, as most published accounts assume zero magnetic field.
Conclusions

A consistent explanation of the present observations is obtained by combining the collapsing caviton concept of SLT with the parametric production of lower hybrid waves (typical of the WLT approach). In such a picture, the high frequency electric field concentrates in collapsing density cavities, and increases until either collisionless damping or the LDI drains enough energy to arrest collapse. The process repeats itself, resulting in the observed anti-correlated bursts at low and high frequency.

5.3 Future Work

As this thesis progressed, it became apparent that the entire problem of plasma frequency heating in the VTF could not be solved in this set of experiments, and much remained to be done. Many paths of inquiry were left unexplored in the interest of timely completion. A brief list of these follows, along with suggestions for improved measurements.

1. A wave kinetic analysis of anti-Stokes modes would yield an estimate of the equilibrium spectrum in both the ionosphere and the VTF. This would certainly not be beyond current computer capabilities, and would bolster the theory of upshifted Langmuir waves tremendously. Included in this type of analysis would be the resulting ion acoustic and lower hybrid spectra—both of which would provide more data for comparison with experiment.

2. A PIC simulation of the VTF thermionic plasma discharge would give a much clearer picture of the background plasma dynamics, and would assist investigations of electron bunching and coherent Cerenkov radiation. The freely available bounded-plasma PIC codes from the University of California at Berkeley would be a good start.

3. Background plasma characterization is a big issue when attempting laboratory
simulations. Electron and ion distribution functions at several positions could be measured with a suitably designed retarding potential analyzer.

4. It may be very interesting to perform plasma frequency heating experiments in the recently available cusp magnetic field on the VTF. This configuration may well turn out to be comparable to polar ionospheric heating, or even to pump waves launched from spacecraft immersed in the ionosphere.

5. Finally, suggestions for improved diagnostic capabilities: a double probe with parallel grid tips spaced close together may yield larger signals than the present wire tips. Also, a variable tip separation or a large array of tips would allow more detailed wavenumber spectra to be obtained. Finally, the use of level 27 mixers would allow more dynamic range in the spectral estimations of the HF band. This would require quite a lot of local oscillator power, however—an expensive proposition.

One additional note: the use of the VTF as a laboratory simulation device is not limited to ionospheric heating. Experiments are already underway to simulate magnetic reconnection (Egedal et al. (2000)), and many other uses could also be envisioned. The old machine has met and surpassed its grand goals published by the MIT Tech Talk at its commissioning, “Imagine capturing the Northern Lights in a bottle.”
Appendix A

Solution of Laplace Equation for Double Probe

This is an expanded version of the solution given by Torvén et al. (1995). We consider the probe to be two long parallel wires, and hence two dimensional. The separation between centers is $b$ and the diameter of the wires is $d_w$. If the tip separation $b$ is small relative to the wavelength of the electric field (as it must be to avoid spatial aliasing), the wave electric field is essentially constant near the probe wires. The probe wires themselves are good conductors and so are equipotentials. The wires carry charges (per unit length) $\pm q$, the magnitude depending both on the excitation and the load circuit.

To begin, we consider a vacuum electric field (and neglect the space charge effects of the plasma) in the two-dimensional region $D$ bounded by the probe wires, as shown in Figure A-1.

![Figure A-1](image)

Figure A-1. Region $D$ (where Laplace's equation holds) is shaded; the probe wires are shown as white circles in cross section, with boundary conditions as shown. Point $P$ is the point of observation of the field.
Laplace's equation holds in $D$: $\nabla^2 \phi = 0$, or

$$
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0
$$

(A.1)

where $\phi$ is the electric potential. As $r \to \infty$, $\phi \to -E_0x$ (just the undisturbed wave potential). At the wire surfaces the potential is constant but unknown. These boundary conditions are enough to give us a unique solution.

To speed the solution process, note that the general solution for the potential is a superposition of several functions satisfying equation (A.1) whose sum satisfies all the boundary conditions. One solution is the potential far away from the probe tips,

$$
\phi_{\infty} = -E_0x.
$$

(A.2)

Another is the potential due to two opposite line charges at $(x, y) = (\pm a, 0)$,

$$
\phi_i = -\frac{q}{2\pi \varepsilon_0} \ln \frac{r_1}{r_0} + \frac{q}{2\pi \varepsilon_0} \ln \frac{r_2}{r_0} = \frac{q}{2\pi \varepsilon_0} \ln \frac{r_2}{r_1},
$$

(A.3)

where the line charge $q$ sits at $x = a$ and $-q$ sits at $x = -a$, and $r_i$ is the distance from the observation point to the charge $i$. Thus,

$$
r_1 = \sqrt{(x-a)^2 + y^2}
$$

(A.4)

and

$$
r_2 = \sqrt{(x+a)^2 + y^2},
$$

(A.5)

giving

$$
\phi_i = \frac{q}{2\pi \varepsilon_0} \ln \frac{x^2 + y^2 + 2ax + a^2}{x^2 + y^2 - 2ax + a^2}.
$$

(A.6)

On equipotential surfaces, $\phi_i = U = \text{constant}$. This gives
Solution of Laplace Equation for Double Probe

\[ \frac{r_2}{r_1} = \exp \left[ \frac{2\pi \varepsilon_0 U}{q} \right] = k = \text{constant} \quad (A.7) \]

or

\[ \left( x - \frac{a(k^2 + 1)}{k^2 - 1} \right)^2 + y^2 = \frac{4k^2a^2}{(k^2 - 1)^2}. \quad (A.8) \]

These are circles of radius \( R \) and center \((x_c, y_c)\), where

\[ R = \frac{2ka}{k^2 - 1} = \sqrt{l^2 - a^2}, \quad (A.9) \]

where \( l \equiv \frac{a(k^2 + 1)}{k^2 - 1} = kR - a, \quad (A.10) \]

and

\[ (x_c, y_c) = (\pm l, 0). \quad (A.11) \]

Far away from the probe tips, \( \phi_l \) vanishes. Our conducting cylinders can replace two equipotentials, giving the same overall field. However, the addition of \( \phi_l \) and \( \phi_{\infty} \) destroys the circular equipotentials of equation (A.3), necessitating the addition of a series solution.

Because the boundary shapes do not resemble the coordinate grids of any simple coordinate system, it is helpful to change variables such that the boundary shapes become simpler in the new coordinates. This is called conformal mapping (for a complete introduction to the subject, see Greenberg (1978), p. 291). Regarding the variables \( x \) and \( y \) to define the complex plane \( z \), any function \( w(z) \) that is analytic and has \( w'(z) \neq 0 \) throughout \( D \) defines a new complex plane in which Laplace' equation still holds. A judicious choice of \( w(z) \) yields simpler boundary conditions. In our case, we let \( z = x + iy \) and \( w = 2 \text{atanh}(z/\alpha) \). Using the definitions of the inverse hyperbolic tangent and the complex logarithm,
\[ w = \ln \frac{1 + z/\alpha}{1 - z/\alpha} = \ln \left| \frac{1 + z/\alpha}{1 - z/\alpha} \right| + i \arg \left( \frac{1 + z/\alpha}{1 - z/\alpha} \right). \] (A.12)

Now let us define
\[ \xi \equiv \Re \{ w \} = \ln \left| \frac{1 + z/\alpha}{1 - z/\alpha} \right| \] (A.13)

and
\[ \theta \equiv \Im \{ w \} = \arg \left( \frac{1 + z/\alpha}{1 - z/\alpha} \right). \] (A.14)

After a little algebra, we find that
\[ \xi = \operatorname{atanh} \left[ \frac{2\alpha x}{\alpha^2 + x^2 + y^2} \right] \] (A.15)

and
\[ \theta = \operatorname{atan} \left[ \frac{2\alpha y}{\alpha^2 - x^2 - y^2} \right]. \] (A.16)

Note that lines of constant \( \xi \) are circles in \((x, y)\) space with radius \( \alpha / |\sinh \xi| \) and center \((\alpha \coth \xi, 0)\). Let us relate these quantities to the \( l, k, R, U, \) and \( a \) of earlier. Let one of our cylinders have radius \( R \) at center \( l \) with charge \( q \). Then from (A.7) and (A.9),
\[ U = \frac{q}{2\pi \varepsilon_0} \ln \left[ l + \sqrt{l^2 - R^2} \right]. \] (A.17)

Also, if the value of \( \xi \) at the surface of the cylinder is \( \xi_0 \),
\[ R = \frac{\alpha}{\sinh \xi_0} \] (A.18)

and
\[ l = \alpha \coth \xi_0, \] (A.19)
giving
\[ \xi_0 = \text{acosh} \left( \frac{l}{R} \right) \]  
(A.20)

and

\[ \alpha = \sqrt{l^2 - R^2} = a. \]  
(A.21)

Then, comparing (A.15) with (A.6) and using (A.21), the conformal mapping of the
line charge solution to \((\xi, \theta)\) space is simply

\[ \phi_l = \frac{q}{2\pi\varepsilon_0} \xi. \]  
(A.22)

The wave solution at infinity must be mapped also. Using the relations (A.15) and
(A.16) we can solve for \(x\):

\[ x = \frac{\alpha \sinh \xi}{\cosh \xi + \cos \theta}. \]  
(A.23)

Thus,

\[ \phi_\infty = -E_0 x = -\frac{\alpha E_0 \sinh \xi}{\cosh \xi + \cos \theta}. \]  
(A.24)

The boundary conditions map as shown in Figure A-2. On the cylinder surfaces

\[ \phi = \phi_j = \text{constant}, \]  

Figure A-2. Conformal mapping domains and boundary conditions.
\[ \phi_1 = -\phi_2 = U = \frac{q}{2\pi\varepsilon_0} \ln \left[ \frac{l + \sqrt{l^2 - R^2}}{R} \right]. \quad (A.25) \]

The lines \( \theta = \pm \pi \) both map to the \( x \)-axis, one axis of symmetry. Thus,

\[ \phi(\theta = \pi) = \phi(\theta = -\pi) \quad (A.26) \]

and

\[ \left. \frac{\partial \phi}{\partial \theta} \right|_{\theta = \pm \pi} = 0. \quad (A.27) \]

We have four boundary conditions and should be able to solve.

Now we can find the series solution. Note that we have mapped region \( D \) to a rectangular region in \( (\xi, \theta) \) space. Laplace’ equation in such a region admits separable solutions made up of terms that look like sin and cos in one coordinate and sinh and cosh in the other. Since we must have periodicity in \( \theta \), we choose sin- and cos-type terms for the \( \theta \)-direction. Also, since the solution must be symmetric about the \( x \)-axis (even in \( y \)) it must also be even in \( \theta \) (choose cos). In the \( \xi \)-direction, the solution must be odd to satisfy the first two boundary conditions (A.25). The solution to (A.1) is then

\[ \phi = \frac{q}{2\pi\varepsilon_0} \xi - \frac{\alpha E_0 \sinh \xi}{\cosh \xi + \cos \theta} + \sum_{n = 1}^{\infty} A_n \cos n\theta \frac{\sinh n\xi}{\sinh n\xi_0}, \quad (A.28) \]

with \( A_n \) undetermined constants. Apply boundary conditions to find \( A_n \). At \( \xi = \pm \xi_0 \),

\[ \phi(\xi = \pm \xi_0) = \pm \phi_1 = \pm \left[ \frac{q\xi_0}{2\pi\varepsilon_0} - \frac{\alpha E_0 \sinh \xi_0}{\cosh \xi_0 + \cos \theta} + \sum_{n = 1}^{\infty} A_n \cos n\theta \right] \quad (A.29) \]

\[ \Rightarrow \sum_{n = 1}^{\infty} A_n \cos n\theta = \phi_1 - \frac{q\xi_0}{2\pi\varepsilon_0} + \frac{\alpha E_0 \sinh \xi_0}{\cosh \xi_0 + \cos \theta}. \quad (A.30) \]

Multiply by \( \cos m\theta \), integrate, and solve for \( A_n \):
\[ A_n = (-1)^n 2\alpha E_0 \exp(-n\xi_0). \]  

Thus,

\[ \phi = \frac{q}{2\pi\varepsilon_0} \xi - \frac{\alpha E_0 \sinh \xi}{\cosh \xi + \cos \theta} + 2\alpha E_0 \sum_{n=1}^{\infty} (-1)^n \exp(-n\xi_0) \frac{\sinh(n\xi)}{\sinh(n\xi_0)} \cos(n\theta). \]  

In terms of the variables of the problem,

\[ \alpha = \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{d_w}{2}\right)^2}, \]  

\[ \xi = \text{atanh}\left(\frac{2\alpha x}{\alpha^2 + x^2 + y^2}\right), \]  

\[ \xi_0 = \text{acosh}\left(\frac{b}{d_w}\right), \]  

\[ \theta = \text{atan}\left(\frac{2\alpha y}{\alpha^2 - x^2 - y^2}\right). \]  

The potential difference between the tips is then

\[ \phi_2 - \phi_1 = E_0 b \sqrt{1 - \left(\frac{d_w}{b}\right)^2} - \frac{q \text{acosh}\left(\frac{b}{d_w}\right)}{\pi\varepsilon_0}. \]
Appendix B

Software

This Appendix is included for the benefit of individuals who may want to delve into the archived dataset, or who want a starting point for setting up new experiments. File formats are given, and routines for reading files are listed. Some analysis routines are listed as well, including Mathematica routines for calculating ray trajectories in the ionosphere and magnetic field trajectories in the VTF. The listings shown here are only a few of the total that are required to run the experiment and process the data. These should be sufficient to allow the stored data to be read and results to be verified.

B.1 CAMAC Software

Routines were written in National Instruments’ LabView to control and read out several types of CAMAC modules over a LeCroy 8901A GPIB interface. These routines all accomplished the same tasks: set up the CAMAC module for data acquisition, poll the module for status, transfer the contents of CAMAC memory to control room PC memory, write the data to disk, and display some form of the data for quick feedback during experimental campaigns.

LabView is a graphical programming environment and is not very amenable to program listings. Hence, we simply list the icons and short descriptions of a few of the major components. A copy of all the programs may be found on the VTF control room PCs.

LC_8212_EngineeringData.vi—Runs the LeCroy 8212 digitizer. Default options appropriate for the particular digitizer used for engineering signals. Allows user to
set sampling rate, post-trigger samples, trigger source, storage path, and file name from one front panel. Also plots data from current or past shots.

LC_8212_PhysicsData.vi—Runs the LeCroy 8212 digitizer. Default options appropriate for the particular digitizer used for physics signals. Allows user to set sampling rate, post-trigger samples, trigger source, storage path, and file name from one front panel. Also plots data from current or past shots.

The file format for all the CAMAC data acquisition files is ASCII, with floating point values already scaled correctly. The first entry the sampling period in seconds, followed by the data stream, each value in mks units appropriate to the quantity being measured (e.g., arc current is in amps, arc voltage in volts).

B.2 Probe Control Software

Double probe position and orientation were controlled remotely by a PC running LabView. In the PC was a National Instruments PCI-1200 multi-purpose data acquisition/con-
control card, which provided analog and digital measurement capability, as well as digital output lines. The PCI-1200 was operated by the LabView routines listed below.

C:\users\nate\DAQ_stuff\Linear_motion.vi

Linear_motion.vi—Allows the user to interactively control the linear position of the probe using mouse controls and visual position feedback.

C:\users\nate\DAQ_stuff\Rotary_motion.vi

Rotary_motion.vi—Allows the user to interactively control the angular orientation of the probe tips using mouse controls and visual feedback.

B.3 Digital Oscilloscope Software

A LabView program called LC_LT344_simple.vi controlled the LeCroy LT344 scope over a GPIB connection, and wrote the data to binary files. The file format is the same as the format used internally to the scope, so that data stored on disk could conceivably be loaded back into the scope and displayed on its screen. This format is detailed in the Programming Manual for the LT344, and is a binary format, with a large header that records the entire state of the scope, as well as the data in memory. To read these files as the first step of further analysis, a MATLAB m-file called scope.m was written.

Listing of scope.m

function [data,time]=scope(prefix,shot);
%  [data,time]=scope(prefix,shot);

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% build filename
fname=[prefix,'_',sprintf('%02d',shot),'.dat'];

% open binary file for reading
[fid,message]=fopen(fname,'r','b');
if fid==-1
    error(message)
end

% read header information
a=fread(fid,40,'uchar'); % read first 40 bytes
k=findstr(char(a),'WAVEDESC')-1; % find start of WAVEDESC block
fseek(fid,k+36,-1); % point to WAVE_DESCRIPTOR
wdl=fread(fid,1,'int32'); % read length of WAVE_DESCRIPTOR
fseek(fid,k+40,-1); % point to USER_TEXT
usel=fread(fid,1,'int32'); % read length of user text
fseek(fid,k+48,-1); % point to TRIGTIME_ARRAY
trigl=fread(fid,1,'int32'); % read length of trigger time array
fseek(fid,k+52,-1); % point to RIS_TIME_ARRAY
risl=fread(fid,1,'int32'); % read length of RIS time array
fseek(fid,k+60,-1); % point to WAVE_ARRAY_1
wal=fread(fid,1,'int32'); % read length of data array
fseek(fid,k+156,-1); % point to VERTICAL_GAIN
vgain=fread(fid,1,'float32'); % read vertical gain
fseek(fid,k+160,-1); % point to VERTICAL_OFFSET
voff=fread(fid,1,'float32'); % read vertical offset
fseek(fid,k+176,-1); % point to HORIZ_INTERVAL
dt=fread(fid,1,'float32'); % read sampling interval
fseek(fid,k+180,-1); % point to HORIZ_OFFSET
t0=fread(fid,1,'float64'); % read trigger offset

offset=k+wdl+usel+trigl+risl;

% read data array
fseek(fid,offset,-1); % point to beginning of data
raw=fread(fid,wal,'int8'); % read raw data
data=vgain*raw-voff; % data=raw;

% make time array
time=[t0:dt:(wal-1)*dt+t0];

% close binary file
fclose(fid);

End of program listing
B.4 Spectrum Analyzer Software

A LabView program called HP8592_VTFapp.vi controlled the HP 8592L spectrum analyzer over a GPIB connection, and wrote the data to text files. The file format has a header that records the state of the spectrum analyzer, as well as possibly multiple trace data. To read these files before final plotting, a MATLAB m-file called HP8592.m was written.

Listing of HP8592.m

function [f,data,ref,atten,rbw,vbw,st]=HP8592(fname);
%  [f,data,ref,atten,rbw,vbw,st]=HP8592(fname);
%
% This function imports spectra from files made with the LabView routine called “hp8592 VTFapp.vi”. Parameters are
%  f frequencies
%  data power data corresponding to frequencies
%  ref reference level, dBm
%  atten attenuation, dB
%  rbw resolution bandwidth
%  vbw video bandwidth
%  st sweep time
%
fid=fopen(fname);
  junk=fscanf(fid,’%f’,inf);
close(fid);

f0=junk(1);
df=junk(2);
ref=junk(3);
atten=junk(4);
rbw=junk(5);
vbw=junk(6);
st=junk(7);
data=reshape(junk(8:length(junk)),length(junk(8:length(junk))))/401,401);
data=data’;
f=[f0:df:400*df+f0]’;

End of program listing
B.5 MATLAB Analysis Software

Routines for calculating power spectrum estimates and spectrograms come with MATLAB's signal processing toolbox, and so we only need to cover the calculation of \( S(k, \omega) \) here. This was accomplished using the following MATLAB routine, called \texttt{fastk_mem.m}. This routine is passed the two time series (one from each probe tip), as well as the sampling period, the number of desired frequency bins, and the number of desired wavenumber bins. The outputs are passed on to a plotting routine.

\textit{Listing of fastk_mem.m}

```matlab
function [Skw,s1hat,Khat,Kmag,Simagahat,S1hat,S2hat,Hhat,f,K,minK,maxK,dK,sep] = fastk_mem(s1,s2,dt,N,nK,shot);

% [Skw,s1hat,Khat,Kmag,Simagahat,S1hat,S2hat,Hhat,f,K,minK,maxK,dK,sep] = fastk_mem(s1,s2,dt,N,nK,shot);
% This function attempts to find the K-spectrum of data taken with a probe pair.
% s1 and s2 are time sequences from each tip.
% dt is the sampling interval.
% N = record length = 2*number of frequency bins (power of 2)
% nK = number of K bins (should be odd)
% shot is the shot number (for plot labeling)
%

sdate=setdate; %sets the date from the path string

% Number of pieces of data to look at
M=floor(length(s1)/N);

% probe separation
sep=0.5/1000; %set at 1/2 mm

% Maximum and minimum K possible given sep
maxK=pi/(sep);
minK=-pi/(sep);
dK=(maxK-minK)/(nK-1);

% Initialize Skw
Skw=zeros(N,nK);

% check to make sure s1 and s2 are column vectors.
[rows,cols] = size(s1);
if rows==1
    s1=s1';
    s2=s2';
end
```

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%remove mean
s1=s1-mean(s1);
s2=s2-mean(s2);

%scale time series to be roughly the same amplitude
slrms=sqrt(sum(s1.^2));
s2rms=sqrt(sum(s2.^2));
s2=s2*slrms/s2rms;

warning off
for j=1:M
    fs1(:,j)=1/N*fft(hanning(N).*s1((j-1)*N+1:j*N));
    fs2(:,j)=1/N*fft(hanning(N).*s2((j-1)*N+1:j*N));
    S1(:,j)=(abs(fs1(:,j))).^2; %Magnitude squared.
    S2(:,j)=(abs(fs2(:,j))).^2;
    H(:,j)=conj(fs1(:,j)).*fs2(:,j);
    C(:,j)=real(H(:,j));
    Q(:,j)=imag(H(:,j));

    %Calculate the sample K vector using 4-quadrant atan.
    K(:,j)=atan2(Q(:,j),C(:,j))/sep;
    %Make vector of indices, shifted from zero.
    ki=floor(K(:,j)/dK)+ceil(nK/2);
    %Check for the case C=Q=0. Replace with K=0.
    if find(isnan(ki))
        ki(find(isnan(ki)))=floor(nK/2)+1;
    end
    %Calculate index. Note that Skw is 2D; use single-index method.
    ind=N*(ki-1)+[1:N]';
    %Not every point in Skw will be touched. Only those selected by ki.
    %The values are weighted by S(w).
    Skw(ind)=Skw(ind)+0.5*(S1(:,j)+S2(:,j));
end

%Normalize: divide by number of data slices.
Skw=1/M*Skw;
S1hat=mean(S1,2);
S2hat=mean(S2,2);
Hhat=mean(H,2);

%Compute the conditional spectrum estimate
s1hat=Skw./(0.5*(S1hat+S2hat)*ones(1,nK));

%Compute first moment (statistical dispersion relation)
Khat=cumsum((ones(N,1)*[minK:dK:maxK]).*s1hat,2);
%Take last column of above matrix
Khat=Khat(:,nK)/1000; %in mm^-1

%Compute first moment magnitude(magnitude dispersion relation)
Kmag=cumsum(abs(ones(N,1)*[minK:dK:maxK]).*s1hat,2);
%Take last column of above matrix
Kmag=Kmag(:,nK)/1000; %in mm^-1
\textbf{Appendix B}

\begin{verbatim}
%Compute second moment (spectral broadening)
Sigmahat=cumsum(((ones(N,1)*[minK:dK:maxK])-Khat*ones(1,nK)).^2.*slhat,2);
%Take square root of last column of above matrix
Sigmahat=((Sigmahat(:,nK)).^0.5)/1000;  %in mm^-1
%Integrate over frequency to get S1k
S1k=mean(Skw(1:floor(N/2),:),1);

%Define frequency vector
f=[0:1/(N*dt):1/(2*dt) - 1/(N*dt)];

warning on

%Scale f and K
f=f/1e6;  %put into MHz
K=[minK:dK:maxK]/1000;  %put into mm^-1

if narginout==0
    %call post-plotting routine
    Kplots(Skw,slhat,Khat,Kmag,Sigmahat,S1hat,S2hat,Hhat,f,K,N,nK,sep,shot);
end
\end{verbatim}

\textit{End of program listing}

\textbf{B.6 Mathematica Analysis Software}

Two Mathematica notebooks may be of interest: a ray-tracing program designed for ionospheric HF radio wave propagation, and a magnetic field analysis program for the VTF machine. These are listed on the following pages.
A ray-tracing study of o- and x-mode propagation over Arecibo

N.E. Dalrymple
4 March 2000

Define indices of refraction according to Appleton–Hartree (cold plasma dispersion relation). Note that \( r \) and \( n \) are vectors.

\[
\text{nor}(r, n) := \left\{ \begin{array}{ll}
1 - & \\
\sqrt{\frac{X(r) (1 - X(r))}{\frac{1}{2} Y.Y \sin^2 (\Theta(Y, n)) + \sqrt{\frac{1}{4} (Y.Y)^2 \sin^4 (\Theta(Y, n)) + \cos^2 (\Theta(Y, n))} Y.Y (1 - X(r))^2 - X(r) + 1}}
\end{array} \right.
\]
\[\text{nex}(r, n) := \frac{1 - \sqrt{X(r)(1 - X(r))}}{-\frac{1}{2} Y Y \sin^2(\Theta(Y, n)) - \sqrt{\frac{1}{4} (Y Y)^2 \sin^4(\Theta(Y, n)) + \cos^2(\Theta(Y, n)) Y Y (1 - X(r))^2} - X(r) + 1}\]

Now, define the function \(G\) for both ordinary and extraordinary cases. Note that by definition, \(G \equiv 1\).

\[\text{Gor}(r, n) := \frac{\sqrt{n.n}}{\text{nor}(r, n)}\]

\[\text{Gex}(r, n) := \frac{\sqrt{n.n}}{\text{nex}(r, n)}\]

Let's check to make sure \(G\) is what we think it is.

\[\text{Gor}[z \{s\}, n]\]

\[\sqrt{n.n} \frac{1 - \frac{1}{2} Y Y \sin[\Theta(Y, n)]^2 + \sqrt{\frac{1}{4} (Y Y)^2 \sin[\Theta(Y, n)]^4 + \cos[\Theta(Y, n)]^2 Y Y (1 - X[z\{s\}])^2} - X[z\{s\}]}{1 - \frac{1}{2} Y Y \sin[\Theta(Y, n)]^2 - \sqrt{\frac{1}{4} (Y Y)^2 \sin[\Theta(Y, n)]^4 + \cos[\Theta(Y, n)]^2 Y Y (1 - X[z\{s\}])^2} - X[z\{s\}]}\]

\[\text{Gex}[z \{s\}, n]\]

\[\sqrt{n.n} \frac{1 - \frac{1}{2} Y Y \sin[\Theta(Y, n)]^2 + \sqrt{\frac{1}{4} (Y Y)^2 \sin[\Theta(Y, n)]^4 + \cos[\Theta(Y, n)]^2 Y Y (1 - X[z\{s\}])^2} - X[z\{s\}]}{1 - \frac{1}{2} Y Y \sin[\Theta(Y, n)]^2 - \sqrt{\frac{1}{4} (Y Y)^2 \sin[\Theta(Y, n)]^4 + \cos[\Theta(Y, n)]^2 Y Y (1 - X[z\{s\}])^2} - X[z\{s\}]}\]

The angle \(\Theta\) is between the wave normal and the antiparallel to the magnetic field, \(Y\).

\[\Theta(Y, n) := \cos^{-1}\left(\frac{Y.n}{\sqrt{Y.Y.n.n}}\right)\]

Define the vector \(Y\). This is suitable for Arecibo heating.

\[Y = 0.3334 \{-\cos[47.5 \text{ Degree}], 0, \sin[47.5 \text{ Degree}]\}\]

\[\{-0.225242, 0, 0.245808\}\]
Now, define the plasma density profile. Let's choose a Chapman layer.

\[ ne(r) := ne0 \cdot \frac{1}{2} \left( \frac{a - e}{h} - ne0(e^{-\frac{a - e}{h}} - 1) \right) \]

Define the parameter \( X \).

\[ X(r) := \frac{e^2 \cdot ne(r)}{e0 \cdot me \cdot (2 \pi f)^2} \]

Solar zenith angle.

\[ \chi = 0; \]

Height of maximum plasma density, in km (approximately).

\[ z0 = 250; \]

Scale height of the ionosphere (km).

\[ H = 40; \]

Maximum plasma density \((m^{-3})\).

\[ ne0 = 130000 \cdot 10^4; \]

Physical constants, MKS units.

\[ e = \frac{1.602}{10^{19}}; \]

\[ e0 = \frac{8.854}{10^{12}}; \]

\[ me = \frac{0.91}{10^{30}}; \]

Heater frequency. This is a weak ionosphere case.

\[ f = 3.175 \cdot 10^6; \]
There are two window points, one northward and one southward from vertical in the magnetic meridian plane. The incident angles (from vertical) are given by plus or minus $\theta_c$.

$$
\theta_c = \frac{\text{ArcSin} \left[ \sqrt{\frac{\text{Sin}[42.5\text{ Degree}]}{\sqrt{T.T+1}}} \right]}{1\text{ Degree}}
$$

19.7441
To avoid the singularity at reflection in the meridian plane, we’ll use a very small deviation from that plane into the y–direction, defined $\beta$.

$$\beta = 0.0015$$

Now, we are ready to run a calculation of the ray paths. The following produces solutions for the ordinary–mode at evenly spaced incident angles in (or nearly in) the meridian plane.

Define min, max, and step of angles launched (in degrees), and the distance parameter smax.

$$\theta_{\text{min}} = -9 \text{ Degree;}$$

$$\theta_{\text{max}} = 9 \text{ Degree;}$$

$$\theta_{\text{step}} = 4.5 \text{ Degree;}$$

$$\text{smax} = 450;$$

```math
\text{ordsol} = \text{Table}[\text{NDSolve}[
\{x'[s] == D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], nx[s]],
y'[s] == D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], ny[s]],
z'[s] == D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], nz[s]],
x''[s] == -D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], x[s]],
y''[s] == -D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], y[s]],
z''[s] == -D[\text{Gor}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], z[s]],
x[0] == 0, y[0] == 0, z[0] == 0, nx[0] == \sin[a],
ny[0] == \beta, nz[0] == \sqrt{1 - (nx[0]^2 + ny[0]^2)}),
\{x[s], y[s], z[s], nx[s], ny[s], nz[s],},
\{s, 0, smax}, \{s, \theta_{\text{min}}, \theta_{\text{max}}, \theta_{\text{step}}\}];
```

The following produces solutions for the extraordinary–mode at evenly spaced incident angles in (or nearly in) the meridian plane.

```math
\text{extsol} = \text{Table}[\text{NDSolve}[
\{x'[s] == D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], nx[s]],
y'[s] == D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], ny[s]],
z'[s] == D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}], nz[s]],
x''[s] == -D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], x[s]],
y''[s] == -D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], y[s]],
z''[s] == -D[\text{Gex}[x[s], y[s], z[s]], \{nx[s], ny[s], nz[s]\}],], z[s]],
x[0] == 0, y[0] == 0, z[0] == 0, nx[0] == \sin[a],
ny[0] == \beta, nz[0] == \sqrt{1 - (nx[0]^2 + ny[0]^2)}),
\{x[s], y[s], z[s], nx[s], ny[s], nz[s],},
\{s, 0, smax}, \{s, \theta_{\text{min}}, \theta_{\text{max}}, \theta_{\text{step}}\}];
```

Define plot ranges.

$$px_{\text{min}} = -60;$$

$$px_{\text{max}} = 75;$$

$$py_{\text{max}} = 250;$$
pasp = pymax / (pxmax - xmin)

50
27

psoordmax = s /. Table[FindRoot[Evaluate[z[s] /. ordSol[[a, 1]]] == 210, 
{s, xmin - 50, xmin - 60}], {a, 1, Dimensions[ordSol][[1]]}]
(226.509, 223.379, 222.341, 223.337, 226.559)

ordPlot = 
Show[Table[ParametricPlot[Evaluate[{z[s], z[s]} /. ordSol[[a, 1]]],
{s, 0, psoordmax[a]}], PlotRange -> {{xmin, xmax}, (0, pymax)},
AspectRatio -> pasp, DisplayFunction -> Identity, PlotStyle -> Red],
{a, 1, Dimensions[ordSol][[1]]}],
DisplayFunction -> $DisplayFunction]
- Graphics -

psexxtmax = s /. Table[FindRoot[Evaluate[z[s] /. extSol[[a, 1]]] == 180,
{s, xmin - 120, xmin - 130}], {a, 1, Dimensions[extSol][[1]]}]
(211.601, 208.921, 208.034, 208.92, 211.598)

extPlot = 
Show[Table[ParametricPlot[Evaluate[{z[s], z[s]} /. extSol[[a, 1]]],
{s, 0, psexxtmax[a]}], PlotRange -> {{xmin, xmax}, (0, pymax)},
AspectRatio -> pasp, DisplayFunction -> Identity, 
PlotStyle -> {Blue, Dashing[{0.03, 0.02}]], {a, 1, 
Dimensions[extSol][[1]]}], DisplayFunction -> $DisplayFunction]
- Graphics -

rays = Show[ordPlot, extPlot], Axes -> (None, None)]
- Graphics -

layer = ParametricPlot[{X[[0, 0, z]], z},
{z, 0, pymax}, PlotRange -> {{0, 1.2}, (0, pymax)},
AspectRatio -> 3, Axes -> (None, None), PlotStyle -> Orange]
- Graphics -
both = 
Show[Graphics[{Rectangle[{xmin - xmax/3, 0}, {xmax, ymax}], layer],
Rectangle[{xmin, 0}, {xmax, ymax}, rays], Text["O", {62, 210}],
Text["X", {50, 185}], Text["B", {-70, 160}], Green,
Arrow[{-140, 220}, {-70, 220 - 70 Tan[47.5 Degree]}]],
AspectRatio -> Automatic, Frame -> True,
FrameLabel -> {FromCharacterCode[Table[32, {n, 1, 40}]] <>
"Northward Distance (km)", "Altitude (km)"},
"X = \frac{\omega p^2}{\omega^2}" <> FromCharacterCode[Table[32, {n, 1, 75}]], ""},
RotateLabel -> False, FrameTicks -> {(-50, 0, 50), Automatic,
{{xmin - xmax/3, "0"}, {xmax - xmax/3 + 0.2, "1"}}, None}]

\[ X = \frac{\omega p^2}{\omega^2} \]

Altitude (km)

Northward Distance (km)

Display["-/thesis/mathematica/RayPlot.eps", both, "EPS"]

- Graphics -
Magnetic field configuration of beam plasma in VTF

Nathan Dalrymple
25 May 2000

Load field plotting routine and plot colors:

```
<< Graphics'Colors`
<< Graphics'PlotField'
Off[General::spell1]

TextStyle = {FontSize -> 12, FontFamily -> "Times"}
FontSize -> 12, FontFamily -> Times}
```

Calculate Biot–Savart integrals:

\[
\text{Bradial} = \text{FullSimplify}\left[ \frac{\mu_0 i}{4\pi} \int_0^3 \frac{\text{Cos}[u]}{(z^2 + r^2 + a^2 - 2 a \text{Cos}[u])^{3/2}} \text{du} \right] \\
\left( i \frac{z}{(a^2 + r^2 + z^2)} \text{EllipticE}\left[ -\frac{4 r}{(a - r)^2 + z^2} \right] - \frac{((a + r)^2 + z^2)}{2 \pi r \sqrt{(a - r)^2 + z^2} ((a + r)^2 + z^2)} \right) \mu_0
\]

\[
\text{Bvertical} = \text{FullSimplify}\left[ \frac{\mu_0 i}{4\pi} \int_0^3 \frac{a - r \text{Cos}[u]}{(z^2 + r^2 + a^2 - 2 a \text{Cos}[u])^{3/2}} \text{du} \right] \\
\left( i \frac{(a^2 - r^2 - z^2)}{(a^2 + r^2 + z^2)} \text{EllipticE}\left[ -\frac{4 r}{(a - r)^2 + z^2} \right] + \frac{((a + r)^2 + z^2)}{2 \pi r \sqrt{(a - r)^2 + z^2} ((a + r)^2 + z^2)} \right) \mu_0
\]

Define magnetic field components as functions of z and r, with i and a as parameters.

\[
\text{Br}[z_, r_, i_, a_] := \\
- \left( i \frac{z}{(a^2 + r^2 + z^2)} \text{EllipticE}\left[ -\frac{4 r}{(a - r)^2 + z^2} \right] - \frac{((a + r)^2 + z^2)}{2 \pi r \sqrt{(a - r)^2 + z^2} ((a + r)^2 + z^2)} \right) \mu_0
\]
\[ Bz[x_, r_, i_, a_] := \]
\[ \left[ \frac{4ar}{(a-r)^2 + z^2} \right] \text{EllipticK}\left[ -\frac{4ar}{(a-r)^2 + z^2} \right] \mu_0 \right]/\left( 2\pi \sqrt{(a-r)^2 + z^2} \right) \]

Now, assign values to the parameters.

\[ \mu_0 = 4\pi 10^{-7}; \]

Parail coil radius.

\[ a = 1.816; \]

Current (2 turns times power supply current).

\[ i = 2600; \]

Vertical location of Parail coils.

\[ zloc = 1.651/2; \]

Plot\[\{\text{Evaluate}[10000 (Br[zloc, r, i, a] + Br[-zloc, r, i, a])], \]
\[ \text{Evaluate}[10000 (Bs[zloc, r, i, a] + Bs[zloc, r, i, a])], \]
\[ \{r, 0, 2\}, \text{Frame} \rightarrow \text{True}, \text{RotateLabel} \rightarrow \text{False}, \]
\[ \text{FrameLabel} \rightarrow \{"R (m)"", "B (G)"", "MidPlane Vertical and Radial Components of Parail Field, I=1300 A", ","}, \text{GridLines} \rightarrow \text{Automatic}\}\]

MidPlane Vertical and Radial Components of Parail Field, I=1300 A

- Graphics -

Plot the vector field of the two Parail coils.
Contour plot the vertical field flux surfaces.

\[ cp = \text{ContourPlot[} \]
\[ \text{NIntegrate[} 2 \pi \rho (B_y[x + zloc, \rho, i, a] + B_z[x - zloc, \rho, i, a]), \{\rho, 0, r\}], \]
\[ \{x, 0.5, 2\}, \{z, -1, 1\}, \text{ContourShading -> False, PlotPoints -> 20} \]
\[ \text{- ContourGraphics -} \]

Estimate the inner and outer radii of the emitters (in cm):

\[ Rin = 61.5 + 13.3 + 14 + 2.54 \]
\[ 91.34 \]

\[ Rout = Rin + 6 + 2.54 \]
\[ 106.58 \]

Estimate starting vertical location of emitter:

\[ Zem = - (45.7 / 2 + 30.5 - 15.9 + 2.54) \]
\[ -39.99 \]

Estimate top of chamber (actually, collector plate):

\[ Zcp = 45.7 / 2 + 30.5 + 4.4 - 6.7 - 17.8 \]
\[ 33.25 \]

Define the toroidal field:

\[ B_\phi[x, i_\phi] := - \frac{\mu_0 4 \pi 18 \cdot i_\phi}{2 \pi x} \]

Assign value to toroidal field current:

\[ i_\phi = 5700; \]

Define parallel velocity:

\[ v_{par} = \sqrt{\frac{2 \cdot 300 \cdot 1.602 \cdot 10^{-19}}{9.1 \cdot 10^{-31}}} \]
\[ 1.02775 \times 10^7 \]

Define drift velocity ratio:
\[
\tau = - \frac{9.1 \times 10^{-31} \cdot v \cdot \text{par} \cdot 2\pi}{1.602 \times 10^{-19} \cdot \mu_0 \cdot 4 \times 18 \cdot 1\phi}
\]
\[-0.000711259\]

Solve for the field line trajectories (no drifts):

\[
\text{trajec} = \text{Table}[\text{MDSSolve[}
\{x'[\phi] == x[\phi] \frac{Br[x[\phi] + zloc, x[\phi], i, a] + Br[z[\phi] - zloc, x[\phi], i, a]}{2\phi[x[\phi], i\phi]},
   z'[\phi] == x[\phi] \frac{Br[x[\phi] + zloc, x[\phi], i, a] + Br[z[\phi] - zloc, x[\phi], i, a]}{2\phi[x[\phi], i\phi]},
\}
\{x[0] == R, z[0] == Zem/100\}, \{x[\phi], z[\phi]\}, \{\phi, 0, 20\pi\}], \{R, Rin/100, Rout/100, 0.01\}];
\]

Solve for the field line trajectories (including drifts):

\[
\text{trajecD} = \text{Table}[\text{MDSSolve[}
\{x'[\phi] == x[\phi] \frac{Br[x[\phi] + zloc, x[\phi], i, a] + Br[z[\phi] - zloc, x[\phi], i, a]}{2\phi[x[\phi], i\phi]},
   z'[\phi] == x[\phi] \frac{Br[x[\phi] + zloc, x[\phi], i, a] + Br[z[\phi] - zloc, x[\phi], i, a]}{2\phi[x[\phi], i\phi]} + vr,
\}
\{x[0] == R, z[0] == Zem/100\}, \{x[\phi], z[\phi]\}, \{\phi, 0, 20\pi\}], \{R, Rin/100, Rout/100, 0.01\}];
\]

Now, choose the R,Z plane through the NT emitter and the double probe. Calculate where the trajectories pierce this plane for both the NT and NM emitters.

\[
\text{ntpts = Flatten[}
\text{Table[Flatten[\{(x[\phi], z[\phi]) /.. trajec /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 0, 9}\}], 1];
\]

\[
\text{nnmpts = Flatten[Table[}
\text{Flatten[\{(x[\phi], z[\phi]) /.. trajec /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 0, 9}\}], 1];
\]

\[
\text{ntptsD = Flatten[Table[}
\text{Flatten[\{(x[\phi], z[\phi]) /.. trajecD /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 0, 9}\}], 1];
\]

\[
\text{nnmptsD = Flatten[Table[}
\text{Flatten[\{(x[\phi], z[\phi]) /.. trajecD /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 0, 9}\}], 1];
\]

For the plane containing the scanning Langmuir probe:

\[
\text{ntptsLP = Flatten[Table[Flatten[\{(x[\phi], z[\phi]) /.. trajec /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 10/18, 9 - 10/18}\}], 1];}
\]

\[
\text{nnmptsLP = Flatten[Table[Flatten[\{(x[\phi], z[\phi]) /.. trajec /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 10/18, 9 - 10/18}\}], 1];}
\]

\[
\text{ntptsLPD = Flatten[Table[Flatten[\{(x[\phi], z[\phi]) /.. trajecD /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 10/18, 9 - 10/18}\}], 1];}
\]

\[
\text{nnmptsLPD = Flatten[Table[Flatten[\{(x[\phi], z[\phi]) /.. trajecD /.. \phi -> 2\pi \cdot \text{rot}, 1\}], \{\text{rot, 10/18, 9 - 10/18}\}], 1];}
\]
\texttt{ntplot = ListPlot[ntpts, PlotStyle -> \{Plum, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{ntplotD = ListPlot[ntptsD, PlotStyle -> \{Plum, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{ntplotLP = ListPlot[ntptsLP, PlotStyle -> \{Plum, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{ntplotLPD = ListPlot[ntptsLPD, PlotStyle -> \{Plum, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{nmplot = ListPlot[nmpts, PlotStyle -> \{Purple, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{nmplotD = ListPlot[nmptsD, PlotStyle -> \{Purple, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{nmplotLP = ListPlot[nmptsLP, PlotStyle -> \{Purple, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

\texttt{nmplotLPD = ListPlot[nmptsLPD, PlotStyle -> \{Purple, PointSize[0.015]\},
PlotRange -> \{(0.6, 1.3), \{-0.6, 0.33\}\}, Frame -> True\]}

- Graphics -

Draw the boundaries of the chamber and the coils.

\texttt{cham = Graphics[Line[\{(0.615, 1.067/2), \{0.615, -1.067/2\},
(0.615 + 0.648, -1.067/2), \{0.615 + 0.648, 0.305 - 1.067/2\},
(0.615 + 0.648 + 0.508, 0.305 - 1.067/2), \{0.615 + 0.648 + 0.508,
0.457 + 0.305 - 1.067/2\}, \{0.615 + 0.648, 0.457 + 0.305 - 1.067/2\},
(0.615 + 0.648, 1.067/2), \{0.615, 1.067/2\}\]]};

Define size of coil box.

\texttt{s = 0.04;}

\texttt{coils = Graphics[Line[
\{(a - s, zloc - s), \{a - s, zloc + s\}, \{a + s, zloc + s\}, \{a + s, zloc - s\},
\{a - s, zloc - s\}, \{a + s, zloc - s\}, \{a + s, zloc + s\}, \{a - s, zloc + s\}\}],
Line[\{(a - s, -zloc - s), \{a - s, -zloc + s\}, \{a + s, -zloc + s\},
\{a + s, -zloc - s\}, \{a - s, -zloc - s\}, \{a + s, -zloc + s\},
\{a + s, -zloc - s\}, \{a - s, -zloc + s\}\]]};
Define probe swaths.

```plaintext
probeLF = Graphics[{Thickness[0.01], ManganeseBlue,
    Line[{{0.615 + 0.648 + 0.508, 0}, {0.615 + 0.05, 0}}]}];

probeDP = Graphics[{Thickness[0.01], NavyBlue,
    Line[{{0.615 + 0.648 + 0.508, 0.178}, {0.615 + 0.35, 0.178}}]}];
```

Make a white box that covers the excess points in nmplot and ntplot.

```plaintext
coverbox = Graphics[{White, Rectangle[{0.7, 1.067/2}, {1.1, 0.59}]}];
```

Make a box that represents the collector plate.

```plaintext
cplate = Graphics[{LightSteelBlue, Rectangle[{0.7, 0.33}, {1.2, 1.067/2}]}];
```

Make a box that represents the emitter body.

```plaintext
ebody = Graphics[{SlateGray, Rectangle[{0.87, -1.067/2}, {1.11, -0.38}]}];
```
Emission rung locations for \( I_r = 1300 \, \text{A}, I_p = 5700 \, \text{A} \) in \( 60^\circ \) \((R,Z)\)-plane, ignoring drifts
wholeD = Show[abody, cplate, nmplotD, ntplotD, coverbox, cp, fp, coils, cham, probeDP,
AspectRatio -> Automatic, PlotRange -> {{0.6, 1.8}, {-0.6, 0.6}},
Frame -> True, FrameLabel -> {"R (m)", "Z (m)"},
"Emission rung locations for I_v=1300 A, I_φ=5700 A" <>
FromCharacterCode[10] <> "in 60° (R,Z)-plane, including drifts",
""}, RotateLabel -> False]

- Graphics -

Display["~/thesis/plots/beam_cross_section_60.eps", whole, "EPS"]

Display: pserr : PostScript language error:
Warning: substituting font Utopia-Regular for Times-Roman

- Graphics -

Display["~/thesis/plots/beam_cross_section_60D.eps", wholeD, "EPS"]

Display: pserr : PostScript language error:
Warning: substituting font Utopia-Regular for Times-Roman

- Graphics -
References


References


References


References
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