UNCERTAINTY AND LEARNING IN SEQUENTIAL DECISION-MAKING:
THE CASE OF CLIMATE POLICY

by

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in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

The debate over a policy response to global climate change has been and continues to be
deadlocked between 1) the view that the impacts of climate change are too uncertain and so any
policy response should be delayed until we learn more, and 2) the view that we cannot wait to
resolve the uncertainty because climate change is irreversible so we must take precautionary
measures now. The objective of this dissertation is to sort out the role of waiting for better
information in choosing an appropriate level of emissions abatement activities today under
uncertainty.

In this dissertation, we construct two-period sequential decision models to represent the choice of
a level of emissions abatement over the next decade and another choice for the remainder of this
century, both empirical models based on a climate model of intermediate complexity, and
analytical dynamic programming models. Using the analytical models, we will show that for
learning to have an influence on the decision before the learning occurs, an interaction must be
present between strategies in the two decision periods. We define an “interaction” as the
dependence of the marginal cost or marginal damage of the future decision on today’s decision.
When an interaction is present and is uncertain, the ability to learn will introduce a bias in the
optimal first period strategy, relative to the optimal strategy if the uncertainty would never be
reduced. In general, the bias from learning can be either in the direction of higher or lower
emissions, depending on the sign of the interaction and the probability distribution over damage
losses relative to abatement costs.

We demonstrate using the empirical climate decision models that the difference between optimal
emissions abatement today with and without learning is insignificant. The reason is that the
IGSM, like most other climate assessment models, omit many of the most important interactions
between emissions today and marginal costs or damages in the future. We show that by
representing possible interactions, such as induced innovation from policy constraint or the effect
of emissions growth on ocean circulation, that learning will have an influence on today’s
decision, often in the direction of lower emissions if we expect to learn. In general, the “wait-to-
learn” is not necessarily a valid argument for delaying a climate policy that constrains emissions.

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I dedicate this thesis to my nephews and niece at the time of writing: Trevor, Randy, Jason, Jamie, Sammy, and Jenna, with the hope that your generation and that of your children will learn to make wise decisions about the uncertain and complex world we live in.
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Chapter 1    Introduction

To me our knowledge of the way things work, in society or in nature, comes trailing clouds of vagueness. Vast ills have followed a belief in certainty, whether historical inevitability, grand diplomatic designs, or extreme views on economic policy. When developing policy with wide effects for an individual or a society, caution is needed because we cannot predict the consequences.

— Kenneth Arrow

1.1 The Issue: Act Now, or Wait and Learn
1.1.1 The Nature of the Climate Change Issue

If a problem could be designed that today’s institutions are least capable of addressing, it would have the characteristics of the global climate change issue: a long time horizon, large uncertainties in both the scientific basis and the potential economic costs of addressing it, uneven distribution of costs and damages, possible irreversible effects if not addressed soon enough, and collective action required for an effective response. To a varying extent, other environmental and technology policy issues share many of these same characteristics. But for climate change, these problems are magnified in every dimension by the size and scope of the issue.

The essence of the problem is that human activities over the past century have been emitting various species of chemicals, known as “greenhouse gases”, to the atmosphere that are transparent to incoming solar radiation but trap some of the outgoing infrared radiation. Most of these greenhouse gases, including water vapor and carbon dioxide, occur naturally in the earth’s atmosphere and are responsible for the relatively warm temperature of the planet, given the incoming solar radiation. The anthropogenic contribution of these gases has become significant enough to noticeably change their concentrations in the atmosphere. The increase in greenhouse gas concentrations means that less infrared radiation is allowed to escape the atmosphere and, all else being equal, the Earth receives slightly more energy than it reradiates to space. Most of the natural and man-made greenhouse gases are very long-lived, on the order of decades to centuries, so it will be a long time before the concentrations of such gases, if increased, fall again to lower levels.
The increased radiative forcing of the atmosphere could have a wide range of effects. Certainly the global average temperature will increase as a result of the added heat in the system. But the actual temperatures in any particular region could increase or decrease. High latitudes are most likely to experience large increases in temperature, and equatorial regions will have the least change. Shifting climatic patterns will likely alter rainfall patterns. Extreme climatic events such as hurricanes, floods, and droughts may increase in both frequency and severity. Sea level will rise due to the melting of glaciers and the expansion of sea-water as the ocean temperatures increase. Perhaps the greatest concerns are for possible catastrophic events. The rapid change in average temperature and rainfall in regions may prevent natural ecological systems from adapting or migrating, and this will greatly increase the already extreme stresses on many natural ecosystems. There is the possibility that the “conveyer belt” circulation in the North Atlantic Ocean, responsible for the temperate climate of Europe, could slow or cease altogether. One other possibility is that a section of the Antarctic ice sheet, weakened by warmer water temperatures, could break off and cause a catastrophic rise in sea level of several meters.

While the potential threats of climate change could be serious, they are also very uncertain. Currently, climate scientists are unable to project the consequences of warming on a regional scale with any degree of confidence. Even the amount and the rate of globally averaged warming from a given increase in greenhouse gas concentrations have a wide range of uncertainty, due to poorly understood mechanisms and feedbacks in the climate system. Scientists judge the more catastrophic outcomes to be very low probability events, but the mechanisms involved are even less well understood than global averages, making their likelihoods difficult to estimate. Equally problematic is how to value these climate changes. Sectors of the economy that are sensitive to climate are relatively few, consisting of mainly agriculture and a handful of others. Thus the market impacts, at least in the developed countries, are likely to be small. The major damages will instead be in non-market sectors, such as natural ecosystem change and species loss, which are traditionally difficult to value economically for policy purposes.

The costs of responding to climate change are also very uncertain and potentially quite large. Unlike some other environmental issues, almost everyone could be directly
affected by measures to reduce greenhouse emissions. The primary anthropogenic
greenhouse gas, CO₂, is mainly produced by the burning of fossil fuels for energy. This
energy is essential to modern economies, and few practical alternatives to fossil fuels
currently exist. Other industrial processes also produce CO₂ and other greenhouse gases.
Cattle and agricultural practices are responsible for a large portion of methane emissions,
another potent greenhouse gas. Some advocates of emissions reduction policies claim
that economic efficiency will actually improve under some policies. Others predict
economic collapse and ruin from an attempt to slow the growth in these emissions. Since
CO₂ has never been regulated in the past, there is little experience to inform which of
these views is correct.

1.1.2 The Policy Debate Over Timing of Action

The problem of how to respond to climate change is not a decision that must be
made now and set in stone for all time. The uncertainties in climate change, the long-time
scales involved, and the potential irreversible effects of decisions, all combine to make
the essential nature of the problem one of deciding what to do now given that we may
learn more about the uncertainties. The debate over what to do now about climate change
often takes a narrower form. Discussions of climate policy are typically framed as a
choice between acting now or waiting until we learn more about the problem and become
more convinced that emissions reductions are necessary. Arguments on both sides, for
waiting to learn more and for taking precautionary action now, are made again and again
in political arenas and academic forums alike.

The basic arguments of this debate can be seen in an exchange of views that took
place in the scientific literature. In 1991 (before the Framework Convention on Climate
Change was signed), Schlesinger and Jiang (1991a) published a study in Nature that
concluded that, “a delay of 10 years in initiating a 20-year transition from the IPCC
‘business as usual’ scenario to any other IPCC scenario has only a small effect on the
projected warming in 2100.” In response to this statement, EOS Transactions, the
professional newsletter of the American Geophysical Union, published a rebuttal and
ensuing dialogue between Risbey, Handel and Stone (hereafter RHS) and Schlesinger and
RHS argued that the S&J conclusion was premature, and that there are many reasons why
there may be a larger cost to waiting to reduce emissions than captured by S&J’s analysis. The four main reasons RHS elaborate are that (1) low probability/high consequence events were not accounted for in the analysis; (2) that the focus of effects should be on local/regional climate rather than on global mean temperature; (3) that socioeconomic factors such as the lifetime of energy investments are important; and, (4) most relevant here, that the uncertainty in climate change effects is unlikely to be much smaller after only a decade. S&J responded that an accelerated climate research program is called for which would address these and other unanswered issues. S&J also argued that after 10 years, the additional decade of climate observations coupled with improved theoretical understanding would indicate whether stringent emissions reductions were warranted, and that this learning and revising of policy would continue.

Consider the EOS debate described above as a decision analysis problem. Figure 1 shows the outline of a decision tree representing the choice discussed by S&J and RHS. In 1991 (the time of the articles), the choice is whether to act by reducing emissions immediately or to wait for more information. Over the subsequent 10 years, more is learned about the salient characteristics of climate change. Then in 2001 a decision is again made about the degree of emissions reductions needed, while some uncertainty may remain which will not be resolved until later. The analytical argument made by S&J is that the expected benefits, $E\{B(\cdot)\}$, of waiting are almost the same as the expected benefits of acting immediately,

$$E\{B(\text{wait})\} \approx E\{B(\text{act})\},$$

while the expected costs, $E\{C(\cdot)\}$, of acting are clearly much larger than those of waiting,
\[ E\{C(\text{wait})\} < E\{C(\text{act})\} . \]

The analytical equivalent of the RHS argument is that,

- The remaining uncertainty node should include a low probability branch that leads to catastrophic climate change;
- S\&J use the wrong attribute (global instead of local climate) in the benefit function \( B(\cdot) \);
- Uncertainties should include socioeconomic factors;
- RHS have different priors on how much learning will occur over the 10 years 1991-2001.

Although it has been almost a decade since the above debate appeared in print, the same arguments continue to be made today, with no consensus between those who favor immediate precautionary action in the form of restricting greenhouse emissions and those who favor waiting. The current political debate is cast in terms of whether or not the Kyoto Protocol (United Nations, 1997) will be ratified and enter into force. The Kyoto Protocol was negotiated at the third Conference of the Parties to the Framework Convention on Climate Change (FCCC) (United Nations, 1992) in Kyoto, Japan in 1997. This agreement would require the industrialized nations, known as Annex I countries, to reduce emissions to 5% below 1990 levels in aggregate, although specific national commitments vary widely.

As of January 2000, 84 of the 181 parties to the FCCC had signed the protocol, but only 22 parties had actually ratified it, accounting for only a few percent of world \( \text{CO}_2 \) emissions. The protocol requires ratification by at least 55 parties and the combined 1990 \( \text{CO}_2 \) emissions of these parties must constitute at least 55% of the total 1990 emissions. One implication of the latter requirement is that the Kyoto Protocol cannot enter into force without ratification by the United States. Ratification by the United States appears unlikely because of opposition by a majority of members of the U.S. Senate, the political body that must vote on ratification. For example, Senator Hagel (1999) has referred to the Kyoto Protocol as "flawed" and has stated that it, "stands no chance of being successful, or of ever being ratified by the U.S. Senate." Numerous other members of congress have made similar statements.

Reasons cited for political opposition to the protocol include the fact that developing countries are not required to reduce emissions and high estimated costs to the
U.S. economy. But underlying the political opposition is the same argument that climate change is too uncertain and that binding commitments should be delayed into the future. Congressmen and Senators including Hagel have used the uncertainty as a reason for opposing Kyoto: “And the science is still uncertain -- at best it is contradictory.” (Hagel, 1998). This view continues to be expressed by industry and lobby groups as well. As the president of the Global Climate Coalition, an industry lobbying association representing selected electric utilities, auto manufacturers, the coal and oil industries, and others, has stated, “… that the impact of climate change is too uncertain and that the risk of economic damage is still too high to justify stronger, mandatory action.” (Stevens, 1997).

On the other side of the debate, those who believe that policy action is warranted are forced to address their argument to the uncertainty in climate change as well. In 1999, the American Geophysical Union, a leading academic association for climate and other earth sciences, issued a policy statement noting that uncertainty, “will never be eliminated,” regarding the climate system, but that present understanding, “provides a compelling basis for legitimate public concern,” over the increase in greenhouse gases (Stevens, 1999). Even the Framework Convention on Climate Change addresses the act now or wait controversy. In Article 3, Paragraph 3, it states that, “Where there are threats of serious or irreversible damage, lack of full scientific certainty should not be used as a reason for postponing [precautionary] measures …” (United Nations, 1992).

1.1.3 Framing the Question for Analysis

The objective of this dissertation is to understand which of the arguments in the “act now versus wait and learn” debate are correct or even relevant. Is it true that the ability to learn will necessarily lead us to delay emissions abatement? What determines the effect of learning on current decisions under uncertainty? Acting now versus waiting is a caricature of the continuous range of available policy responses to climate change. But the fundamental question of climate change is, and for some time will be: What should we do now, given

1) the uncertainties in costs and benefits of emissions reductions,
2) that we will revise decisions over time sequentially,
3) that learning may occur,
4) that decisions may be partly irreversible, and
5) that CO₂ is a stock pollutant?

In this dissertation, we construct an analytical framework for exploring the role of learning. Consider the sequential choice of emissions restrictions as a two-period decision problem: a choice now and a choice later (perhaps after another decade, as argued by Schlesinger et al.). The effect of learning on a current decision can be examined by comparing two extreme cases:

- All uncertainty is resolved between decisions, and
- No uncertainty is resolved before the second decision.

These two cases are illustrated in Figure 1.2. The top case, a), shows the decision problem in which the level of emissions restrictions is chosen today under uncertainty, then learning resolves all the uncertainty so that a second decision in ten years is made with perfect certainty. In the lower case, b), no learning occurs between decisions, so that the choice of abatement level is made under the same degree of uncertainty in ten years as it is today.

In order to explore the relevance of the "wait and learn" argument, we are interested in comparing the level of abatement we would choose today if we expect to reduce the uncertainty, and the level of abatement we would choose if expect that we will not learn anything more. This comparison of the two extreme cases will highlight the
effect of learning upon decision. Of course, the reality is between the two extremes, and we will most likely reduce some uncertainties while other uncertainties will still remain. This analysis seeks to clarify the role of learning, and therefore compares two cases that emphasize its effect.

In order to construct the sequential choice models, we will need to model many underlying characteristics of the global economic and physical systems. Rather than use simple parameterizations, we use detailed submodels that represent the physics, chemistry and economics explicitly in order to capture feedbacks and interactions. The decision models in this dissertation use the Integrated Global System Model (IGSM) of the MIT Joint Program on the Science and Policy of Global Change (Prinn et al., 1999) to calculate the costs and impacts of different strategy choices.

Using the IGSM and descriptions of uncertainty in its key parameters, we will show the conditions under which learning will affect today’s abatement decision, and, conversely, under what conditions learning will have little or no impact on today’s decision. We will consider several interactions, which are missing from most assessment models, that increase the effect of learning.

We begin in this chapter by next reviewing the existing theoretical and empirical literature on sequential decision to see what insights exist as to whether one should take more or less action when one expects to learn later. Section 1.3 will outline the analytical method used to explore the role of learning in this dissertation. The key results and insights of the dissertation will be summarized in Section 1.4.

1.2 Results of Previous Studies

1.2.1 Theoretical Literature on Irreversibility and Learning

An examination of the theoretical literature on optimal sequential decisions under uncertainty should perhaps begin by mentioning early work by Simon (1956). Concerned with the unfeasibility of solving the most general dynamic programming formulations of decision problems under uncertainty, Simon showed that for the case of a quadratic objective function and stationary, independent uncertainty that replacing uncertain quantities with their expectations gives an identical solution to the more general problem. This work, and its extensions by Malinvaud (1969) and others rendered a wide range of optimization problems feasible to solve.
However, in the seminal works by Arrow and Fisher (1974) and Henry (1974), it was shown that the certainty equivalent approach might not be satisfactory when the decision involves irreversibility. Irreversibility means that a decision in one period constrains or reduces the number of options available in future period decisions. By constructing a simple 2-period model for choosing whether to develop or preserve a unit area of natural resource, Arrow and Fisher show that,

... if the development involves some irreversible transformation of the environment, hence a loss in perpetuity of the benefits from preservation, and if information about the costs and benefits of both alternatives realized in one period results in a change in their expected values for the next, the answer is yes – net benefits from developing the area are reduced and, broadly speaking, less of the area should be developed. (pp. 313-314).

Arrow and Fisher term this reduction in the net benefits of development "quasi-option value". Henry (1974) uses a different formulation to show essentially the same result.

These results might suggest that the irreversibility of climate change would lead to lower emissions in an initial period. But Viscusi and Zeckhauser (1976) suggest that this may not be the case. By using a Markov model of a decision with environmental consequences and representing irreversibility as a trapping state, they show that the least risky pure strategy may be dominated by a sequential strategy that allows one to learn about the probability of the irreversible effect. This result is extended by Miller and Lad (1984), who construct a 2-period Bayesian decision problem with "active learning": the amount and type of information received depends on the period 1 decision. They conclude that development decisions with a quasi-option value are not necessarily more conservative, and that depending on the problem some development may give desirable information on the costs and benefits of the irreversible effect.

The most general result indicating whether development increases or decreases with an increase in learning is found in Epstein (1980). Epstein presents a 3-period model in which learning occurs between periods 1 and 2 through the receipt of a message $Y$, which reveals partial information about the state of the world $Z$. Sufficient conditions are given that predict how the optimal period 1 decision changes if the message $Y$ is "more informative":

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Theorem 1 (Epstein). Let $x_1^*$ and $x_1^{**}$ be the solutions of [the 2-period maximization problem] given $Y$ and $Y'$ respectively, where $Y$ is more informative that $Y'$. Let $J(x_1, \xi)$ be defined as

$$J(x_1, \xi) \equiv \max_{x_2} \left\{ \sum_i \xi_i U(x_1, x_2, z_i) | x_2 \in C_2(x_1) \right\}$$

$$\xi \in S^{m-1} = \{ \xi = (\xi_1, ..., \xi_m) | \xi_i \geq 0, \sum \xi_i = 1 \}$$

If $J_{x_1}(x_1, \xi)$ is concave (convex) in $\xi$, then $x_1^* \leq (\geq) x_1^{**}$. If $J_{x_1}(x_1, \xi)$ is neither convex nor concave, then the sign of $x_1^* - x_1^{**}$ is ambiguous.

The intuition of this result is that if the marginal costs, $J_{x_1}$, are concave in the damage costs, $\xi$, then the marginal cost of the mean damage will exceed the mean marginal cost, and thus period 1 activity will be greater in the no learning case. This result is important because it shows that depending on the specific characteristics of a problem, the ability to learn may lead to more or less of an irreversible activity. Unfortunately, requiring strict concavity or convexity is overly restrictive for representing climate change, as shown by Ulph and Ulph (1997) and shown again in this dissertation in Chapter 5. Further, this theorem does not address the conditions under which learning has no effect (i.e., cases with and without learning lead to the same level of activity in period 1), or what determines whether the magnitude of a divergence is large or small.

The focus of much of the theory indicates that irreversibility is a key characteristic in determining an initial level of action when learning and future decisions are possible. Indeed, if the decision is completely reversible, there is no effect from learning, as illustrated by Chichilnisky and Heal (1993). But in the case of climate, as opposed to the simple "preserve or develop" models in the literature, it is not obvious what action constitutes an irreversible investment decision. Is continuing to allow emissions to grow and contribute to potential climate change the irreversible investment? Or is the act of reducing emissions by shifting the composition of the capital stock the irreversible decision? Or perhaps both, as has been argued by Kolstad (1996) and others. Even if there were a single irreversible decision being considered, the results from Epstein (1980) and Miller and Lad (1984) show that it is not obvious whether the
decision should err on the side of more caution or less. More information is needed on the specific problem before we can say what direction learning will bias the decision.

Nevertheless, based on the Arrow and Fisher analysis, Chichilnisky and Heal (1993) argue that climate change decisions should be biased in the direction of more preservation because climate change is likely to be irreversible. Ulph and Ulph (1997) develop a theoretical 2-period model of climate change, rather than of a generic investment decision problem, in order to challenge the Chichilnisky and Heal argument. This two-period model consists of a case with no learning between periods, and a case with complete resolution of uncertainty between periods. The model further differentiates between a completely reversible decision and completely irreversible decision. Ulph and Ulph set out to demonstrate that the irreversibility characteristics of climate change are such that one should emit more in period 1 if learning is expected. They show that Epstein’s general result does not apply in the classic textbook formulation of linear marginal costs and benefits, since the marginal cost is neither concave nor convex. Instead, they derive an alternative sufficient condition for different solutions under learning and not learning, relying on whether the irreversibility “bites” in some states of the world and not others. Irreversibility is considered to “bite” in a state of the world if the period 2 decision would optimally undo some of the period 1 decision in the equivalent reversible problem. Their result from a two-period theoretical model is that learning will always lead to higher period 1 emissions when the irreversibility condition “bites”. This theoretical result is then tested on an empirical model of climate change (discussed in Section 1.2.2).

Finally, intuition about the effect of learning in climate change often relies on the real options literature. A body of theory has developed on irreversible decisions and option value by drawing on the option pricing literature in finance (Pindyck, 1991). The intuition about investment decisions under uncertainty from this work is that when there is irreversibility, the ability to learn leads to less of the irreversible investment activity. However, the real options approach is based on the assumption that the uncertainty in a decision problem can be modeled by a Weiner process or Brownian motion, similar to a share price. As Pindyck (1991) has noted, this approach may not apply for environmental and natural resource policy decisions, since a Weiner process is not necessarily the appropriate model for the uncertainty about the climate system.
The overall findings of the theoretical literature on generic sequential decisions with irreversibility are that the period 1 decision could involve greater irreversible action under learning than without learning, or could involve less action; the sign of the relationship depends on the characteristics of the particular decision problem. The few theoretical analyses of climate change draw contradictory conclusions about the direction of the learning effect. Because these theoretical treatments make overly restrictive assumptions and because there does not appear to be a consensus, it is necessary to examine the empirical climate policy literature, which we do in the next section.

1.2.2 Sequential Decision and Learning in the Empirical Climate Literature

The empirical literature on climate change focuses on a wide variety of questions, and therefore often neglects uncertainty and/or the sequential nature of the decision (see, e.g., Parsons and Fisher-Vanden, 1997). Indeed, most studies in the climate literature are carried out with one or a few scenarios for uncertain variables, with a once-and-for-all decision at the start. Here we will restrict our review to those authors who have framed the problem as a sequential decision under uncertainty, in which learning does or does not occur. What insights have been gained about the optimal decision today depending on the learning that occurs later?

First, we caution the reader that similar formulations are often used to address a related but different question. Much of the work on sequential climate policy choice, as we describe below, is designed to address the question: What is the value of information? Value of information calculations are important contributions to the policy discussions as a way to provide bounds on the amount worth investing in further research. Learning will always affect the decision after the learning has occurred, and the outcome is always improved (e.g., lower cost) relative to a decision without learning. However, knowing the value of information does not necessarily give insight into the effect of learning on the decision before the learning occurs.

In a study that does not treat uncertainty but demonstrates the “wait” argument, Wigley, Richels, and Edmonds (1996) argue that an economically efficient policy would delay emissions reductions for several decades. This study was a response to the publication of emissions paths over time by the Intergovernmental Panel on Climate Change (Houghton et al, 1996) that stabilize atmospheric CO₂ concentrations at various
levels. Wigley et al. argue that the same concentration targets can be achieved at lower cost by allowing emissions to grow unconstrained in the first few decades, followed by more stringent cutbacks in emissions in later decades. Three reasons given for this result are that the positive marginal productivity of capital now means that fewer resources are needed if the burden is put off until later, that the capital stock for energy production is long-lived and expensive to replace prematurely, and that technical progress will over time reduce the costs of substitutes for CO$_2$-emitting technologies. Although this analysis is performed with a deterministic model under certainty, the results under uncertainty in an expected value sense are considered likely to be qualitatively unchanged.

One early example of the treatment of climate change as a sequential decision under uncertainty is Hammit, Lempert, and Schlesinger (1992). This analysis framed the problem as a 2-period decision, and assumed that all uncertainty is resolved between periods 1 and 2. They did not assess probabilities of various outcomes, and thus do not recommend an optimal period 1 policy. Instead a sensitivity test is performed to see which period 1 policy (the only choices are “moderate” and “aggressive”) performs better for which combination of uncertain outcomes. They do not test other possible assumptions about when learning could occur, and therefore do not address how the decision changes without learning.

Manne and Richels (1992, Ch. 4) give another example of framing climate policy as a sequential decision. They compare two alternate formulations of the decision problem, which they term “learn-then-act” and “act-then-learn”. The learn-then-act case refers to scenario analysis as typically performed. Learn-then-act describes the situation in which all uncertainty is resolved before any decisions are made; each decision is made under perfect information, and the optimal path of decisions is solved for each possible uncertain outcome. Act-then-learn, in contrast, refers to the situation in which the first decisions are made under uncertainty, and then sometime later (2020 in their model) uncertainty is resolved after which remaining decisions are made under perfect information. The decisions made before uncertainty is resolved under act-then-learn constitute the “hedging strategy”. Manne and Richels demonstrate with a highly simplified cost-effectiveness analysis that the hedging strategy will always be somewhere between the perfect information optimal decisions, and that the stringency depends on the probabilities of the different uncertain outcomes.
The comparison between learn-then-act and act-then-learn is a useful illustration to help people think about climate policy decision, but it does not directly address the question of how the optimal policy today depends on whether and when learning takes place in the future. In fact, this formulation is similar to the value of perfect information calculation, comparing the optimal policy if we had perfect certainty today to the optimal policy where we are uncertain today but resolve the uncertainty completely in the future. The main insight is that the abatement decision today under uncertainty, the hedging strategy, should consist of more abatement than if we were sure that climate damages were low (the “do nothing” solution), and less abatement than if we were sure that damages were large (the “do a lot now” solution). Given that we know we are uncertain today, and do not know when we will better understand the climate system, to explore the effect of learning on today’s decision would instead require comparing two “act-then-learn” models with resolution at different times, as described in Section 1.1.3.

In a later study, Manne and Richels (1995) apply the learn-then-act / act-then-learn frameworks to a cost-benefit analysis of climate policy. The model used, MERGE 2, consists of a global economic model with five regions, a detailed energy sector, reduced-form models of the carbon-cycle and temperature change, and impact valuation that distinguishes market from non-market losses. Each region maximizes its discounted utility over the time horizon of 200 years subject to an intertemporal budget constraint. In this study, they perform a sensitivity study of various assumptions and find the optimal emissions path under perfect information in each case. All cases but one, consisting of very high damages, have virtually the same optimal level of emissions in 2000 and even in 2010, and the reduction in emissions of the high damage case is small for these early periods. Then they construct an “act-then-learn” analysis with a 95% probability of their “base-case”, 5% probability of their high damage case, and complete resolution of uncertainty in 2020. To implement act-then-learn with MERGE requires that the periods up through 2020 before learning occurs use parameter settings equal to their expected value over the two scenarios, and that the strategies in those early periods are identical in the base-case and high-damage cases. The hedging strategy for 2000 and 2010 requires only very slight reductions in emissions from the base-case. This is a useful insight for clarifying the debate between extreme solutions, showing that under uncertainty the appropriate level of response is somewhere between no abatement and drastic measures.
However, this analysis does not tell us how the arguments about learning more relate to what we should do today, given that we are uncertain.

The effects of resolving uncertainty at different points in time are explored by Nordhaus (1994) and by Nordhaus and Popp (1997) using the DICE model. DICE is a forward-looking optimal growth model that solves for the savings rate and emissions constraints over time that maximize discounted utility. The model is a one-region one-good model with simple parameterizations of abatement costs, damage costs, the carbon cycle, and temperature change, all based on a review of the literature by Nordhaus (1991). A stochastic programming version of DICE is used to model uncertainty in early periods that is resolved in later periods, and, as with MERGE, strategies in periods before learning occurs are constrained to be the same across all states of the world.

The focus in these studies is on the optimal control level after uncertainty is resolved, and on the value of information. Nordhaus notes two surprising results. First, the optimal control level after resolution of uncertainty is virtually independent of when resolution occurs. Second, the optimal carbon taxes in all periods are within 2% of the expected value of the carbon taxes under perfect information, regardless of when uncertainty is resolved. Nordhaus reasons that these results are due to the fact that damages are a function of the stock of CO₂, while costs are a function of the flow of CO₂, and that the stock of CO₂ in later periods is only weakly dependent on the control rates in early periods.

The effect on the optimal control level before uncertainty is resolved is shown in figures and tables in Nordhaus (1994) (e.g., Figure 8.4), but not discussed explicitly. The optimal control level before information arrives is almost identical irrespective of when information is received. The control level is slightly more stringent when resolution comes later, but the difference is insignificant. The value of information, the main focus of Nordhaus and Popp (1997), is found to be large even though the control level before learning is independent of when learning occurs. Information allows better decisions to be made by knowing the true state of the world, as compared with when a decision must be made under uncertainty in an expected value sense.

Kolstad (1996) explicitly investigates how optimal control today changes with different assumptions about learning. Kolstad constructs a stochastic version of the DICE model that parameterizes the rate of learning, rather than assuming perfect
resolution at different time periods. In this model of learning, there is a finite set of states of nature, with a prior probability over these states. Between decisions, a "message" is received, which is a noisy signal as to the true state of nature. A parameter $\lambda \in [0,1]$ reflects the level of information in the signal, where 0 corresponds to no information and 1 corresponds to perfect information. Learning occurs at the rate $\lambda$ over two 10-year periods, after which any remaining uncertainty is held constant over the time horizon.

Kolstad asks the question: how does the first period optimal decision change with $\lambda$, the rate of learning? Consistent with the results implicit in Nordhaus, Kolstad also finds that "optimal control levels for greenhouse gases are virtually unaffected by the rate of learning." The reason given for this effect is that the optimal level of control is insensitive to the total stock of greenhouse gases; the marginal damage function is quite flat because the total damages are linear over the range relevant to the decision. The irreversibility effect, which would cause a difference in first period control between learning and no learning, occurs when it is necessary to negatively emit. "But because future emissions are so slightly influenced by today's action, there is no scenario under which it would be optimal to negatively emit in the future to correct over-emissions today." Kolstad's criterion of "negative emissions" is the same as Ulph and Ulph's (1997) requirement that irreversibility "bite," wanting to undo the period 1 emissions. By focusing on irreversibility as the sole explanation for learning to affect decision, Kolstad requires that damages be so severe that period 2 emissions are driven to zero in order for irreversibility to bind and therefore for learning to have an effect. In this dissertation, we will show that the ability to learn can affect first period decision under less extreme conditions than required by Kolstad. In contrast to greenhouse gases, Kolstad does find a significant effect of learning when investments in emissions reductions are assumed to be irreversible. As control decisions become more irreversible, emissions will be reduced less with more rapid learning.

More evidence that the possibility of learning has little effect on emissions today is given in Ulph and Ulph (1997). As discussed above (see Section 1.2.1) they develop a theoretical two-period model, and derive sufficient conditions for an irreversibility to induce less abatement when learning will occur. To test their findings, they use an empirical model based on a deterministic optimal control model by Maddison (1995). They construct a four-period model with a quadratic cost function, which is estimated
econometrically from 500 Monte Carlo runs. Uncertainty in damage cost is represented by low, median, and high values of the damage parameter. The estimated model is used to compare optimal abatement levels in each period under no learning and under complete learning between periods 1 and 2. Ulph and Ulph find that in most cases, learning does lead to less period 1 abatement in the learning case. However, in all but two cases, the difference is negligible (i.e., learning has no effect on abatement level), an outcome not explained by their theory. Further, the relationship between when learning affects abatement and when the irreversibility constraint "bites" is not consistent with their two-period model result, suggesting that, "finding a sufficient condition for the irreversibility effect to hold," in an empirical model, "is rather more complex than for the simple two-period theoretical model." Their conclusions are consistent with Koistad's (1996), finding that the damages are too small in most cases for the irreversibility of greenhouse gas emissions to be a binding constraint.

Finally, a study by Valverde (1997) also showed that the same level of abatement was optimal in the first period with and without learning. Valverde presented a series of one- and two-period models of climate policy choice with varying assumptions about the timing and degree of learning. While Manne and Richels, Nordhaus, and Kolstad use stochastic programming models with continuous range of abatement levels to choose from, Valverde constructs decision models with discrete policy options in each period. For the two-period model, the choices in the first period consist of "No Controls", "Stabilize Emissions", "AOSIS Protocol", and "Stringent Abatement". The no learning case, the case with complete resolution of uncertainty before period 2, and all partial learning cases with different levels of accuracy all have "No Controls" as the optimal first period policy. In other words, learning and not learning lead to the same period 1 control levels.

To summarize, in the climate literature on learning to date, several studies appear to address the role of learning but in fact are not the proper formulation to determine the effect of learning on the first period control strategy (Hammitt et al, 1992; Manne and Richels, 1994). Studies that have explicitly examined the effect of learning in empirical models of climate change (Nordhaus, 1994, Kolstad, 1996; Ulph and Ulph, 1997; Valverde, 1997) have found that strategy is mostly uninfluenced by whether or not learning occurs. The theoretical literature on irreversibility and learning gives no
conditions for when learning will not have an effect on strategy, or when the effect will be negligible. Therefore the explanation in these previous studies for a lack of an effect of learning rely on two seemingly contradictory elements of the models:

1) The irreversibility constraint does not bind; i.e., the damage losses are not severe enough to drive period 2 emissions to zero; and,

2) The stock nature of greenhouse gases, the fact that the existing stock decays very slowly, means that period 1 emissions have very little influence on the total stock of greenhouse gases in the atmosphere.

Note that the second reason, the stockiness of CO$_2$, should cause it to be irreversible, contrary to the first reason.

Other studies have addressed the stock nature of CO$_2$ in the context of the choice of policy instrument. Economically efficient instruments for environmental policy include (among others) price instruments, such as a carbon tax, and quantity instruments, such as emissions quotas with tradable permits. Weitzman (1974) demonstrated that under uncertainty the choice of a policy instrument should be determined by the relative slopes of marginal control costs and marginal damages. Pizer (1999) applies the Weitzman result to climate change, using a static model of a one-time decision in 2010. Pizer illustrates that the reduction in marginal damages by reducing emissions in any year is both small and flat relative to the marginal costs of reducing emissions. The comparative difference in the slopes leads to greater efficiency under uncertainty in price instruments over quantity instruments, while the relative magnitudes lead to almost no abatement being optimal in 2010 (with an optimal carbon tax of about $7/ton C) although optimal controls increase in stringency in later decades. It is worth repeating two caveats given by Pizer that would change the nature of marginal damages; if damages are not smooth but rather increase sharply at some threshold level, or if reductions in one period are not independent of reductions in other periods, then the marginal damages may not be as flat.

After a review of the different results in both the theoretical literature on irreversibility and in the empirical climate change literature on sequential choice and learning, several puzzles still remain. First, what is a theory for explaining why, in empirical models of climate change, learning has no appreciable effect on strategy? Do climate damages need to be so severe as to drive second period emissions to zero before
learning will affect strategy? Second, predictions as to the direction of the effect of learning (i.e., does it lead to more abatement or less) are inconsistent. Theories predicting that learning will lead to more abatement and theories predicting less abatement both have been developed for simple cases, as well as some studies that predict that either direction is possible depending on the characteristics of the problem. Results of empirical models are contradictory, with learning leading to more period 1 abatement in work by Nordhaus (1994) but less abatement in that by Ulph and Ulph (1997). As described below, this dissertation will demonstrate the conditions for: 1) the existence of an effect by learning on strategy, 2) the magnitude of the learning effect, and 3) the direction of the learning effect.

1.3 The Analytical Method

1.3.1 Overview

In this dissertation, we will demonstrate that waiting to reduce uncertainties is not a valid argument for delaying policy measures. To clarify the role of learning, we will construct models of sequential choice under uncertainty. A primary component of the decision model is the method of calculating outcomes for any sequential strategy and any realization of uncertain parameters. The component that maps strategies to outcomes is based on the MIT Integrated Global System Model (IGSM), a process model of intermediate complexity that explicitly represents the economics and the physics of the global system. This approach is used instead of relying on simple parameterizations, as previous studies of climate policy choice under uncertainty have done, in order to trace the results of different parameter values through to outcomes, while including the effects of feedbacks and interactions. In order to construct the decision analytic models to explore learning in later chapters, a large amount of model development is required. The description of the methodology used is given in Chapter 2, and the estimation of uncertain parameter distributions and of reduced-form versions of the IGSM are given in Chapter 3.

The heart of the analysis is in Chapters 4, 5, and 6. Chapter 4 uses the IGSM to explore the difference between first period optimal strategy with and without learning, and shows that for the models used here the first period strategy is unchanged by whether learning occurs or not. Chapter 5 gives the reasons why learning has no effect in this
model. We develop a simple analytical two-period model to demonstrate the conditions under which learning does or does not affect the first period strategy, as well the determinants of the magnitude and direction of the effect. Chapter 6 uses the IGSM to apply the intuition from Chapter 5, and shows that the interactions in the IGSM are too weak to cause a noticeable influence by learning on strategy. Several possible interactions are introduced into the model, causing learning to have an influence on policy that can lead to more or less abatement in period 1 depending on the expected damages.

Chapter 7 examines another influence, distinct from the learning effect, on first period strategy. This chapter demonstrates that if there is even a small chance that decisions in the future may not be optimal, that more abatement is warranted in the near-term. A summary of the insights of the dissertation and the implications for research and for policy are presented in Chapter 8, along with directions for future extensions to this work.

1.3.2 Preparation of the Model System

Chapters 2 and 3 develop the methodology and the reduced-form models to calculate the outcome from any strategy choice and values of uncertain parameters. This dissertation uses a model of climate change of intermediate complexity to explore the role of learning in sequential decision. Previous studies of learning have used simple parameterizations for processes such as production in the economy, the carbon-cycle, and global mean temperature change. Models such as DICE or MERGE assume restrictive functional forms that are calibrated under certainty, and the calibration parameters are used as proxies for uncertainties in the underlying system. However, the behavior of the system under uncertainty may not be properly represented by these parameterizations. In a more detailed functional model, feedbacks and interactions may amplify or damp out the effect of uncertainty in a parameter. Because the focus is on decision under uncertainty, in this dissertation we use the MIT IGSM and represent uncertainty in several key parameters.

The IGSM and other models of similar complexity and detail have not traditionally been used to study uncertainty and learning, because the computational time required for the necessary simulations is prohibitive. There is a range of methods that can
be used to characterize and propagate uncertainty through complex models. Chapter 2 reviews several of the most commonly used approaches, including crude Monte Carlo, stratified sampling, importance sampling, and linear response surface replacement. In this context, the Deterministic Equivalent Modeling Method (DEMM) is introduced, and its performance is compared to the other traditional methods with a simple example.

DEMM is used in this dissertation to estimate reduced-form models of the IGSM in order to construct sequential decision models that can be solved efficiently. Using DEMM, we will be able to characterize the response of the IGSM under uncertainty, including effects of any feedbacks or nonlinearities, without assuming a restrictive functional form. Chapter 3 describes the estimation of the reduced-form IGSM, which is used in the decision models of Chapters 4 to 7. A large model such as the IGSM has many parameters that are uncertain, and for an analysis of decision under uncertainty, a small number of parameters that have the most impact on outcomes must be identified. Chapter 3 will also describe the sensitivity testing of the IGSM and the elicitation of expert judgments to identify the key uncertain parameters and to obtain probability distributions for these parameters. Finally, to facilitate sensitivity testing of the decision models, we convert continuous probability distributions into discrete distributions.

1.3.3 Application to Irreversibility and Learning

The influence of future learning on a decision today under uncertainty is explored in Chapters 4, 5, and 6. Chapter 4 begins by exploring the influence of learning using the IGSM and its reduced form models developed in Chapter 3. We construct a two-period model of climate policy choice, in which the rate of emissions growth in each period is chosen to minimize the sum of control costs and damage costs. Two cases are considered: one in which all uncertainty is resolved between the two periods, and one in which no uncertainty is resolved. We show that, consistent with the results of Kolstad (1996), Ulph and Ulph (1997), and Valverde (1997), the level of abatement in the first period is the same in both cases. Even as the expected damages are varied over a large range, first period abatement is almost always the same in the learning and no learning cases. Parameters are also modified to further increase the strength of the stock effect, but the range of parameter values for which strategy depends on learning is still quite small.
Chapter 5 develops the intuition for when learning will influence first period strategy or not. A simple analytical model of a two-period decision is developed. By assuming simple functional forms for the costs in each period, we use dynamic programming to derive expressions for the difference between optimal period 1 strategy with learning and optimal period 1 strategy without learning. With the analytical model, we show that for a divergence between the learn and no learn strategies to exist, there must be an interaction between periods in the form of a cross-product term in the period 2 costs. In other words, the marginal cost (control or damage) in period 2 must depend on the period 1 decision. Cost functions with an interaction term present are then used to clarify the determinants of the magnitude and the direction of the divergence between strategies. The magnitude will be shown to vary as a function of the variances of the marginal cost in period 2 and of the interaction term. We will also show that for a problem where a control cost is balanced against an uncertain damage cost, that learning influences strategy in the direction of more abatement for low expected damages and in the direction of less abatement for high expected damages.

In Chapter 6, we return to the IGSM to apply the intuition from Chapter 5. We use DEMM to estimate a new reduced-form version, this time fitting to the same functional form as the analytical model of Chapter 5. The coefficients of the reduced-form model, substituted into the expressions for the influence of learning derived in Chapter 5, indicate that the relative magnitudes of the interaction to overall uncertainty in the IGSM are such that we should expect learning to have a negligible effect on strategy, just as was found in Chapter 4. In fact interactions between period 1 strategy and period 2 marginal abatement costs and marginal damages are known to be present but very weak in the IGSM.

The key role of the interaction between periods leads to the consideration of other plausible interactions that may exist in climate change, but are omitted from the IGSM and most other climate assessment models. We show three examples of such interactions: 1) a non-linear threshold effect in the damage function, 2) a dependence of ocean circulation on the rate of near-term emissions, and 3) induced innovation where period 2 abatement costs are partly dependent on the period 1 abatement level. In all three examples of stronger interactions between periods, first period abatement does differ depending on whether learning will occur or not. The strategy with learning will
entail greater abatement relative to the no-learn case for lower expected damages, and will entail less abatement relative to the no-learn case for higher expected damages.

1.4 Key Findings and Insights from this Analysis

The analysis of the role of learning in a sequential decision under uncertainty yields several insights both for climate change policy and for future research. Previous research has focused on irreversibility as the cause of future learning influencing strategy. By irreversibility, we mean that we might wish to undo tomorrow an action taken today because of what we have learned, and will not be able to because the action is irreversible. For example, greenhouse gas emissions appear irreversible because their concentrations decay very slowly. In fact, irreversibility is one special case of the phenomenon that causes learning to influence strategy.

In general, a strategy level today chosen under uncertainty when we can resolve the uncertainty may be different from the strategy level today when we can never learn anything more, but only if there is an interaction between today’s strategy and tomorrow’s strategy. We define the term “interaction” here to mean that the marginal costs or marginal damages of tomorrow’s decision are a function of the strategy today. The ability to learn more tomorrow means that different strategies will be chosen tomorrow than if we did not learn. If there is no interaction between today’s and tomorrow’s strategies, then today’s strategy is independent of tomorrow’s expected strategy and the prospect of learning will not be relevant. If, however, there is an interaction, we anticipate different future strategies with learning than without learning. Because there is a linkage through the interaction, with learning we will adjust today’s strategy to anticipate what we expect to do tomorrow.

The interaction must be in terms of the marginal costs of tomorrow’s decision. For stock problems such as the accumulation of greenhouse gases, today’s choice of emissions levels influences the total costs tomorrow, because most of the greenhouse gases emitted today will still be in the atmosphere tomorrow causing increased radiative forcing and raising the temperature. But this stream of damages into the future does not in itself affect tomorrow’s decision. Only if the marginal damages have changed does today’s emissions affect tomorrow’s choice of emissions.
In this dissertation, we construct a two-period model of climate policy choice, in which the rate of emissions increase is chosen first for next decade (roughly the Kyoto Protocol time-frame) and then again for the rest of this century. Using the MIT IGSM we find, just as others have, that near-term emissions are the same whether or not we resolve uncertainty after a decade. In light of the role of an interaction between periods as determining the influence by learning on strategy, this result is not surprising. The interactions in the IGSM, as in most other climate assessment models, are few and weak. There is a small interaction due to the vintaging of capital, which prevents a portion of capital from being shifted between sectors. There is also a dependence of the rate of ocean carbon uptake on how quickly emissions grow. But these effects are very weak. Whether uncertainty is resolved later or not, the representation of climate change in the IGSM leads to the same choice of emissions abatement now.

There are other possible interactions that may exist in the real world, relevant to climate change and climate policy but not represented in the IGSM or other climate assessment models. One such interaction would exist if we believed that there was a threshold level of greenhouse gas concentrations at which damages increase rapidly, as language in the Framework Convention on Climate Change implies may exist. Another possible interaction is the potential for rapid growth in concentrations to influence the rate of the ocean circulation, particularly the thermohaline circulation of the North Atlantic Ocean. A third example is the possibility that the rate of improvement in technologies that reduce energy- or carbon-intensity is partly dependent on the stringency of climate change policy over the next few decades.

In this dissertation, we show that when such potential interactions are included in the model, that the optimal abatement now will differ depending on whether we will learn more or not. The major implication for research on climate assessment for policy is that it is critical to include such interactions in assessment models, and treat the uncertainty in their magnitude. The major focus of research should include the interactions mentioned:

- How strong might induced innovation be, and how do we represent this effect properly in economic models?
- How does ocean circulation really behave, and how does it depend on transient changes in temperature and precipitation?
These effects and other mechanisms by which emissions today could affect marginal costs or marginal damages need to be studied and included in integrated assessments. The most important measure of the "value of information" is whether it changes the decision we would make now. The inclusion or omission of interaction effects alters the optimal level of abatement today far more than uncertainty in other parameters of the models, and therefore has a significant impact on whether we obtain the correct insights for policy from these models.

More broadly, we attempt to clarify the debate over whether, in the context of uncertainty and irreversibility, learning will lead to more or less abatement activity. The ability to learn may bias period 1 abatement in either direction. In the climate change issue there are irreversibilities and interactions in both directions. Considering only abatement costs, there is the concern that resources may be wasted on unnecessary investments in emissions reductions. This will lead to less abatement now if we will learn in the future. Considering only damage losses, there is the risk that increasing emissions of long-lived greenhouse gases could lead to catastrophic damages. This aspect will introduce a bias toward more abatement today if we will learn. The combined result of these effects that act in opposite directions, logically, must depend on the relative magnitudes and the probability distributions of the uncertain control costs and damage losses. There are conditions under which learning would cause a decrease in current abatement levels, but this is not always the case.

Regarding the level of greenhouse gas emissions abatement activity that is warranted today under current uncertainties, several results from the models presented in this dissertation point in the direction of a non-zero carbon price, although perhaps not as stringent as the reductions called for in the Kyoto Protocol. Any of several interaction effects that may exist, including induced innovation effects and a potential slowdown in the thermohaline circulation, cause the models to select a strategy that begins to slow the growth in emissions as optimal, especially if we expect to learn more. Further, if we cannot be sure that future governments will implement stringent emissions reductions in the future, than a further increase in current abatement levels appear desirable.

Finally, we return to the "act now or wait and learn?" debate. In fact, the ability to learn might mean that we should do more abatement now than if we thought we couldn't learn. Or, if there aren't strong interactions, whether we learn more or not is
irrelevant for the choice of abatement – it is simply based on expected costs and expected benefits. What is behind the "wait and learn" view? Learning is not a valid argument for delaying abatement activity. The "wait and learn" proponents simply believe that the expected damages are low while expected abatement costs are high. Conversely, the "precautionary action now" view is based on a belief in high expected damages and low expected abatement costs. Learning is not the reason to wait, or to not wait.

Most interactions lead to some abatement today over a wide range of parameter values. So does the possibility of hysteresis. Although the likelihood of Kyoto being implemented is in doubt, some level of serious policy in the near-term appears to be warranted in the near-term because of the uncertainty and the likelihood that we will learn more over time. Putting a real price on carbon, even if it is a low price for now, will create incentives that will spur innovation and leave the most options open to be able to respond to what we may in fact learn about climate change in the future.

The findings and implications discussed here are treated in more detail in Chapter 8, along with several areas for future improvements in sequential decision under uncertainty and learning applied to climate change.
Chapter 2  Methodologies for Uncertainty Analysis of Complex Models

It's hard to make predictions, especially about the future
- Yogi Berra

2.1 Introduction

To study sequential decision under uncertainty with learning, the uncertainty in all relevant outcomes must first be characterized. In this chapter, we begin by examining the methods that are available for characterizing uncertainty in model responses.

There are three major classes of uncertainty in models:

- Parametric uncertainty
- Model or structural uncertainty
- Surprise / indeterminacy

Parametric uncertainty refers to the uncertainty in model responses that result from uncertainty in the values of parameters or inputs of the model. Model uncertainty is the uncertainty in responses resulting from different possible model formulations that can be used to represent the system of interest. Model uncertainty might result from uncertainty whether a functional relationship is represented correctly (e.g., linear or quadratic), or from uncertainty whether key processes within the real system are omitted from the model. Surprise or indeterminacy refers to the inescapable fact that not every important factor in a problem can be anticipated ahead of time. A classic example of surprise is the effect of chloroflorocarbons (CFCs) on the stratospheric ozone, an outcome not anticipated or tested for in the otherwise rigorous health and safety tests of this compound by developer Dupont Corp in the 1930s. Surprise, by its very nature, cannot be formally treated within quantitative modeling techniques; it requires sensitivity testing and a degree of skepticism in any results, even from probabilistic analyses. Model uncertainty is also extremely difficult to treat in a rigorous and systematic fashion, since this would require a specification of all alternative models and their relative probabilities. This is difficult for even simple models, and most complex models contain hundreds of functional relationships, each of which has structural uncertainty. Of the above three
categories, the one type of uncertainty that has well-developed analytical methods is parametric uncertainty. For the remainder of this chapter and of this dissertation, we will focus exclusively on parameter uncertainty.

The procedure for implementing a parametric uncertainty analysis of a model consists of three generic stages:

1) Fit probability distributions to describe the uncertainty in the model parameters,
2) Calculate the corresponding probability distribution of model responses,
3) Analyze the response distributions for useful insights and calculate quantities to facilitate communication of the results.

In this chapter, we begin by exploring methodologies for the second step, propagating uncertainty in inputs to uncertainty in outputs. The first and third steps will be addressed in Chapter 3 in the application of an uncertainty analysis to a climate policy integrated assessment model. The question posed in this chapter is: given a complex policy model that requires large amounts of computation time for a single simulation and consisting of many functional relations, what methods exist to propagate uncertainty in inputs to obtain uncertainty in outputs, and what are their relative advantages and disadvantages?

2.1.1 Analytical Approaches to Uncertainty Propagation

There are two main approaches to uncertainty propagation:

- Analytical methods
- Random sampling methods

Analytical methods offer solutions to the uncertainty propagation problem for some sub-classes of models without requiring time-consuming simulation. The analytical approaches rely on knowledge of the specific model equation(s), and the use of this knowledge to directly calculate the probability distribution of the response. Linear models (i.e., sums of uncertain quantities) are one such sub-class. If the parameters have normal (Gaussian) probability distributions, then the response also will be normally distributed.

For example, if

\[ y = a_1 x_1 + a_2 x_2; \quad x_1 \sim N(\mu_1, \sigma_1^2) \quad x_2 \sim N(\mu_2, \sigma_2^2), \]

and \( x_1 \) and \( x_2 \) are statistically independent and the \( a_i \) are constant coefficients, then
\[ y \sim N(a_1 \mu_1 + a_2 \mu_2, a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2). \]

Similarly, models that consist of products of powers of uncertain quantities:

\[ y = \prod_{i=1}^{n} x_i^{a_i} \]

can be converted to a sum by a log transformation:

\[ \ln(y) = \sum_{i=1}^{n} a_i \ln(x_i) \]

Unfortunately, most models in practice are not simply sums or products, and not all parameters fit a normal distribution.

A more practical analytical approach is the method of moments, which is based on Taylor series approximations of the model response. Although the method of moments does not calculate the precise probability distribution of the response, it provides a way of calculating the mean and variance of the response, given the means and variances of the parameters. The main requirement to implement the method of moments is the ability to derive the partial derivatives of the response with respect to each parameter. This can either be done analytically or empirically by slightly perturbing each parameter to the model slightly.

Analytical derivation of a Taylor series expansion and other analytical approaches usually are not practical with typical policy models. Often there is no explicit functional form that can be represented, such as for finite-element or finite-difference models. In addition, the Taylor series approach only gives the mean and variance of the response. A Taylor series expansion can be used in conjunction with random sampling approaches to give more information on the probability distribution of the result, as will be shown later in this chapter. In general, analytical methods are of extremely limited practical use on most models because we cannot write down the explicit function. For propagating uncertainty through a model while treating it as a "black-box", we must turn to random sampling methods.

2.1.2 Random Sampling Approaches to Uncertainty Propagation: Monte Carlo

For any model with uncertain parameters, an approximation of the probability distribution of the model response, as well as approximations of the moments of this distribution, can be obtained using a class of methods known as Monte Carlo simulation.
(Hammersley and Handscomb, 1964; Kalos and Whitlock, 1986). The essential idea behind Monte Carlo methods is to estimate a quantity through a series of statistically independent trials. These methods are applied to two classes of problems: direct simulation of probabilistic systems, and estimation of deterministic problems that are difficult to calculate directly (i.e., evaluating integrals). The application and dissemination of Monte Carlo methods are primarily attributed to work by Fermi, Von Neumann, and Ulam in the 1940's (Von Neumann, 1951; Ulam, 1976; Hammersley and Handscomb, 1964).

The simplest form of these methods is Crude Monte Carlo. It is best demonstrated through a simple example. Suppose we want to calculate the value of the integral:

$$\theta = \int_{0}^{1} f(x)dx \quad (2.1)$$

We can draw \( n \) independent random numbers \( \xi_1, \xi_2, \ldots, \xi_n \) between 0 and 1. Then an unbiased estimator of the integral \( \theta \) is the sum:

$$\hat{f} = \frac{1}{n} \sum_{i=1}^{n} f(\xi_i) \quad (2.2)$$

Further, it can be shown that the variance of this estimate is \( \sigma^2 / n \). Thus if we need a more precise estimate, we can simply sample additional random numbers. The larger the sample size \( n \), the smaller the variance of the estimate.

We can generalize this approach to uncertainty propagation through a model. The steps in performing a Crude Monte Carlo are:

1) Generate \( n \) sample input sets from the parameter probability distributions,
2) Run the computational model for each of the \( n \) input sets, and save the response values,
3) Use the resulting \( n \) response samples to calculate mean, variance, other statistical summaries, and even an empirical probability distribution of the response,
4) If the variance of the estimate of the statistical quantities is too large, generate additional samples and repeat.

Crude Monte Carlo is simple to implement on any model, and as more random samples are drawn the obtained moments and probability distribution converge to the
"true" values. The primary disadvantage to crude Monte Carlo is that, in practice, the
number of samples needed to obtain reasonable estimates are often on the order of many
thousands. For assessment models that require hours, days, or even weeks for a single
simulation, Crude Monte Carlo becomes infeasible.

Fortunately, many variations on the Monte Carlo method have been developed
that require far fewer samples for reasonable estimates. Each of these approaches has
advantages and disadvantages, depending on the particular model being analyzed and on
what types of information about uncertainty are most desired. The remainder of this
chapter will review several of the most commonly used methods for uncertainty
propagation, and will compare them in the context of a simple example. Section 2 will
describe four commonly used methods - stratified sampling, importance sampling, linear
response surface replacement, and Taylor series expansion - and will also introduce the
Deterministic Equivalent Modeling Method (DEMM), which is a relatively recent
addition to this class of methods. Section 3 will then compare the performance of these
methods in the context of a simple logistic model of population growth.

2.2 Efficient Methods of Uncertainty Propagation

We have seen in Section 1 that the simplest approach to uncertainty propagation,
crude Monte Carlo, becomes infeasible for models that require large amounts of
computation time. Fortunately, many different methods have been developed that require
significantly fewer samples required for a given level of variance or error in the estimate.
There are essentially two fundamental ideas behind all of these methods:
• Be more intelligent about what samples are used, and
• Approximate the model with a simpler version that requires less computation time.
Efficient uncertainty propagation methods all rely on one or both of these strategies.

2.2.1 Efficient Sampling Methods

When crude Monte Carlo is applied to a model with \( m \) uncertain parameters, it is
equivalent to estimating an integral in \( m \)-dimensional space. As the dimensionality
grows, it becomes more difficult to accurately represent this space with random sampling.
One way to ensure proportionally representative sampling is to break up each of the input
distributions into subintervals or strata, and then randomly sample from each strata. This general approach is called stratified sampling.

There are many different ways of choosing the relative sizes of each strata. The most commonly used for uncertainty propagation is Latin Hypercube Sampling (LHS) (McKay et al., 1979). In Latin Hypercube Sampling, each input distribution is broken up into $n$ segments of equal probability. Figure 2.1 shows how samples might be selected from a parameter distribution that is distributed normally as $x \sim N(50,20^2)$. If 10 samples are desired, the Latin Hypercube technique is to divide the distribution into 10 equi-probable segments and then randomly sample one point from each segment. The LHS sample points are shown as the round dots. On contrast, 10 points chosen at random from the distribution as a whole (crude Monte Carlo), shown as ‘+’s in the figure, provide a much less accurate representation of the mean and spread of the true distribution.

After random samples are generated from each strata of each input, the input set for one simulation is formed by randomly selecting one sample for each of the uncertain parameters. This process can be modified to induce a desired level of statistical correlation (or independence) across parameters by constraining how sample sets are formed across parameters (Iman and Conover, 1982).

Latin Hypercube Sampling is an efficient means to estimate the mean and variance of a response. This is because LHS can represent the mean and variance of the
input distributions with far fewer samples than crude Monte Carlo. However, as will be shown in the examples in this chapter, LHS is much less effective at estimating the details of the shape of non-normally distributed random variables or tail probabilities. If a precise estimate of the probability of exceeding an extreme value of the response is desired, other methods are preferable. One example of these latter methods that we will present here is “Importance Sampling” (Clark, 1961; Hammersley and Handscomb, 1964).

The idea of importance sampling can be shown in terms of Monte Carlo integration of equation (2.1). We can reduce the variance of an estimator of this integral by scaling by another function $g(x)$:

$$G = \int \frac{f(x)}{g(x)} g(x) dx$$

where $g(x)$ is some probability density function, i.e.,

$$g(x) \geq 0, \quad \int g(x) dx = 1.$$  

It can be shown that the choice of $g(x)$ that minimizes variance is

$$g(x) = \frac{f(x)}{G}.$$ 

The idea is to choose a $g(x)$ as close as possible to $f(x)$ so that the ratio is constant, thus making the integral easier to estimate. For Monte Carlo integration, a Taylor series expansion of $f(x)$ can be used as the $g(x)$.

For uncertainty propagation, the idea is the same. The inputs are re-scaled by some other probability density function in order to sample more points from the relevant portion of the input distributions, and sample less from the less relevant portions. The problem with a large computational model is that we do not know the exact form of the function. This makes the formal choice of a scaling pdf difficult. However, even when this is not possible, importance sampling can be implemented through a form of stratified sampling. Suppose we want to estimate the probability $p$ that a response $y$ exceeds some given threshold value,

$$p = \Pr\{y \geq y_0\}.$$  

If $p$ is small (e.g., less than 5%), many samples will be required to get a precise estimate using either crude Monte Carlo or Latin Hypercube. But if the number of uncertain parameters is small, and we know roughly what segments of the joint parameter
distributions produce a y value near or above the threshold, we can sample only from those portions of the distribution, and then rescale the estimate to its true probability. Importance sampling used in this manner will also be demonstrated in the next section.

In addition to stratified sampling and importance sampling, there are other variance reduction techniques for Monte Carlo, including control variates, antithetic variates, and use of expected values to reduce variance. In addition, another approach that is used to improve the efficiency of estimates is to use an alternative method for the generation of random numbers, which is the basis of all computational Monte Carlo algorithms. Rather than using the pseudo-random number generation routines, random samples can be generated using a quasi-random technique (Hammersley and Handscomb, 1964). Quasi-random sequences will fill up n-dimensional space more uniformly than quasi-random techniques. As a result, crude Monte Carlo using quasi-random numbers can exhibit a reduction in variance as fast as 1/n, as compared to the rate of 1/√n for pseudo-random numbers. These other approaches will not be discussed further here.

2.2.2 Model Replacement Methods

The main idea behind methods such as stratified sampling and importance sampling is to reduce the variance in an estimate through the choice of the sample points. The model is then evaluated at those sample points, and the results are used to estimate characteristics of the response distribution. If each evaluation of the model requires a large amount of computation time, then even sample sizes on the order of a few hundred, often sufficient for precise estimates using Latin Hypercube, can become impractical for some models.

The alternative is to reduce the time required for each simulation. The easiest way to achieve this is to approximate the original model with a reduced-form model that requires much less computation time, for example a polynomial fit of the model. If such a reasonable approximation can be found, then even crude Monte Carlo can be applied since the number of simulations is no longer a constraint. The error in the estimate of uncertainty becomes limited by deviations of the approximation from the "true" model, rather than by variance in the estimate due to sampling. A second class of methods for uncertainty propagation is based on this idea of model approximation.
There are essentially three key issues in approximating a model with a reduced-form approximation:
1) The choice of the functional form of the approximation,
2) The method of solving for weighting coefficients, and
3) The choice of sample points at which to evaluate the "true" model.
Each method in this class uses different approaches to one or more of these issues.

One of the simplest methods to use is the same Taylor series expansion that was described under the analytical methods. We can think of a model with an uncertain response \( y \) and uncertainty parameters \( x_1, x_2, \ldots, x_n \) as a function:

\[
y = f(x_1, x_2, \ldots, x_n).
\]  
(2.3)

Then around some central reference point \( y^0 = f(x_1^0, x_2^0, \ldots, x_n^0) \) we can approximate deviations from this central value with the Taylor series:

\[
y - y^0 = \sum_{i=1}^{n} (x_i - x_i^0) \frac{\partial y}{\partial x_i} \bigg|_{x^0} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_i^0)(x_j - x_j^0) \frac{\partial^2 y}{\partial x_i \partial x_j} \bigg|_{x^0} + \\
\frac{1}{3!} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (x_i - x_i^0)(x_j - x_j^0)(x_k - x_k^0) \frac{\partial^3 y}{\partial x_i \partial x_j \partial x_k} \bigg|_{x^0} + \ldots
\]

Even if we cannot evaluate the partial derivatives analytically, we can estimate the values of these derivatives at this reference point by running the model with a slightly perturbed value of each input parameter. Thus if there are \( n \) uncertain parameters, we can solve for only the first order terms with \( n+1 \) simulations, one for each input parameter, plus the simulation at the reference values. Second order terms will require a total of \( n^2 + n + 1 \) simulations, and so on. Once the partial derivatives at the reference point are calculated, crude Monte Carlo can be applied to the approximation with large numbers of samples to obtain estimates of the response distribution.

The Taylor series approach can sometimes provide a reasonable approximation of the uncertainty in a response while only requiring runs of the actual model a few times. However, the Taylor series method has some definite disadvantages. The main drawback to Taylor series is that it is inherently a local approximation at a particular point in the parameter \( n \)-space. If the model behaves very differently in regions away from the reference point, the Taylor series will be a poor approximation. Also, as has been shown
elsewhere, the accuracy of the estimated probability distribution can be very sensitive to
the choice of the reference point (Iman and Helton, 1988).

Other approaches to approximate a model for uncertainty propagation attempt to
span the range of the parameter space in deriving a reduced-form model, and thus
produce a global approximation. The most commonly used method is the linear response
surface replacement method (Box and Draper, 1986; Downing et al., 1985). Response
surface methods use statistical regression techniques to capture the main effects of each
uncertain parameter on the response variable. Thus an approximation to the model
represented by equation (2.3) is
\[ \hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n. \] (2.4)
The coefficients \( \beta \) can then be found using a least-squares approach. For a linear model
such as (2.4), \( n+1 \) simulations of the model are required to calculate the coefficients. In
addition to the linear terms, a response surface may also include second-order terms and
cross-products in the approximation as well.

The choice of sample points to evaluate for the response surface method usually
relies on factorial or fractional factorial design. For each uncertain parameter, a small
number of levels are identified. Most common are 2-level designs, or \( 2^k \) factorial design,
in which a low and high value for each parameter is chosen. The samples are then
formed by evaluating every possible permutation of low and high values. For example,
for two parameters \( x_1 \) and \( x_2 \), the four sample sets would be:
\[ \{(x_1^L, x_2^L), (x_1^H, x_2^H), (x_1^L, x_2^H), (x_1^H, x_2^H)\} \]
When the number of terms in the expansion grows too large, designs exist for
selecting a subset of permutations to span the space while keeping the number of samples
small; these are known as \( 2^{k-p} \) fractional factorial designs (Box and Hunter, 1978).

Although response surface methods have been used with some success for
uncertainty analysis (e.g., Cox, 1977; Nguyen, 1980), in some cases they have been
shown to give less accurate estimates than other methods such as Latin Hypercube (Iman
and Helton, 1988). Depending on the degree of non-linearity of the model response, a
first or second order approximation may or may not capture the behavior of the response
over the full ranges of uncertainty. Also, the conventional approach of a two-level design
may not represent all the important sub-regions within the \( n \)-dimensional parameter space
that affect the distribution of the response.

50
One other approach to uncertainty propagation through model approximation will be briefly mentioned here. Rather than use an arbitrary polynomial or regression fit to the response, a computationally intensive model can be replaced with a reduced-scale model that is itself an explicit (although much simpler) model of the process or system. Such a reduced-scale model can then be statistically calibrated to correspond to the larger model by fitting to a small sample of simulations of the larger model. An example of using this approach to calibrate a two-box climate model to a fully dynamic 2-D climate model can be found in Valverde (1997). One advantage of the reduced-scale model approach is that the parameters of the approximation are represent conceptual elements of the modeled system, as opposed to the sometimes non-intuitive polynomial coefficients. Disadvantages include the added effort required to construct and calibrate the reduced-scale model (as compared with simply solving for weighting coefficients as required in other methods), and the fact that not all models necessarily have an analogous reduced-scale model.

2.2.3 The Deterministic Equivalent Modeling Method

As described above, all existing methods have some drawbacks. Latin Hypercube can accurately estimate the mean with a small number of samples, but sometimes requires many runs to capture other aspects of the distribution. Conversely, importance sampling is able to accurately capture the area under one tail of a distribution, but offers no help with estimating the mean and variance of the overall distribution. The traditional model replacement methods may or may not be effective depending on whether the approximation’s functional form and the choice of points for fitting capture the relevant behavior of the response over the range of uncertainty.

Here we introduce a relatively recent addition to uncertainty propagation methods, the Deterministic Equivalent Modeling Method (DEM)\(^1\) (Tatang, et al., 1997; Webster et al., 1996). DEMM seeks to characterize the probabilistic response of the uncertain model output as an expansion in orthogonal polynomials. If such an expansion can be found that accurately mimics the probabilistic behavior of the model, then a Monte Carlo procedure can easily be applied to the expansion to derive the approximate

\(^1\text{This method is also sometimes referred to as the Probabilistic Collocation Method (PCM).}\)
probability density function of the uncertain output. In this sense, DEMM is a
descendent of previous model replacement methods.

Although any numerical computer model is itself deterministic, by positing
uncertainty in a model parameter, the model's outputs become uncertain and thus can be
thought of as a random variable. One useful representation for a random variable is an
expansion of some family of orthogonal polynomials $B_n(x)$ with weighting coefficients $a_i$:

$$y = a_0B_0 + a_1B_1(x) + a_2B_2(x) + ... + a_NB_N(x)$$

where $x$ is also a random variable of known distribution. Any family of orthogonal
polynomials can be used, including Legendre, Laguerre, or Hermite. This expansion is
sometimes referred to as a polynomial chaos expansion (Weiner, 1938).

For example, suppose random variable $y$ is some function of an underlying
Gaussian distribution. Then one can represent random variable $y$ as a truncated
expansion of the Hermite polynomials, which are polynomials of a unit Gaussian random
variable $\xi$:

$$y = a_0H_0 + a_1H_1 + a_2H_2 + a_3H_3$$

$$= a_0 + a_1\xi + a_2(\xi^2 - 1) + a_3(\xi^3 - 3\xi)$$

(2.5)

This expansion is 3rd order in $\xi$, and requires that we find values of the coefficients
$a_0, ..., a_3$ to represent the random variable $y$.

Our goal is to find an expansion of this form that makes use of information about
the independent random variable $x$. Three steps are required:

1. Choose the orthogonal polynomials $B_n(x)$ to use as basis functions,
2. Choose the method for solving for the weighting coefficients $a_i$,
3. Choose the values for uncertain parameter $x$ at which to evaluate $y$ to solve
   for $a_i$.

These are the same essential steps as for any other method that replaces the model with
an approximation. DEMM differs from the traditional approaches in all three steps.

We first address the choice of the basis functions. Since a model output $y$ is some
function of its uncertain input parameter $x$, we can use information about the probability
density of $x$ to choose basis functions for the expansion. We need not necessarily be
limited to traditional orthogonal families such as Hermite polynomials. For the general case, we can derive the set of orthogonal polynomials weighted by the density function of the parameter, according to the definition of orthogonal polynomials:

\[
\int \frac{P(x)H_i(x)H_j(x)dx}{x} = C_i \delta_{ij}
\]  

(2.6)

where \( \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \)

\( H_i(x) \) and \( H_j(x) \) are orthogonal polynomial functions of \( x \) of order \( i \) and \( j \), \( P(x) \) is some weighting function, and \( C_i \) is some constant\(^1\). In other words, the integral of the product of two orthogonal polynomials of different order is always \( 0 \). By using the probability density function of an input as the weighting function \( P(x) \), a set of orthogonal polynomials can be derived recursively.\(^3\)

Having derived the basis functions for the expansion, we next approach the method for estimating the weighting coefficients, \( a \). There is a class of methods designed for solving this problem known as the methods of weighted residuals (MWR) (Villadsen and Michelsen, 1978). The residual at any realization \( x_j \) of the random variable \( x \), for some approximation \( \hat{y}(x) \) of the function \( y(x) \) is simply the difference:

\[
R_N(\tilde{a}, x_j) = y(x_j) - \hat{y}(\tilde{a}, x_j)
\]

where \( R_N(\tilde{a}, x_j) \) is the residual for an \( N \)-term expansion with weighting coefficients \( \tilde{a} = \{a_1, a_2, ..., a_N\} \).

In general, MWR solves for \( N \) coefficients by solving the \( N \) relations:

\[
\int_0^1 R_N(\tilde{a}, x)W_j(x)dx = 0, \quad j = 1, 2, ..., N
\]  

(2.7)

Alternative schemes for MWR differ by the choice of the form of the weighting function \( W_j(x) \). Commonly used schemes include the least squares method, which chooses \( W_j(x) \) to be \( \frac{\partial R_N}{\partial a_j} \), or Galerkin's method, which chooses \( W_j(x) \) to be the derivatives of

---

\(^1\)This constant is usually 1, and thus omitted, when the polynomials are normalized.

\(^3\)The zero\(^0\) order polynomial is always assumed to equal one.
the approximation \( \frac{\partial y_N}{\partial a_j} \). The difficulty with these schemes is that they require the explicit analytical form of the model in order to solve for the weighting coefficients.

Because our goal is to approximate the uncertainty in a model output for any model, however complex, a method that allows the model to be treated as a "black-box" is preferable. This leads us to choose the collocation method, which uses the dirac delta function as the weighting function:

\[
W_j(x) = \delta(x - x_j), \quad j = 1, 2, ..., N.
\]

Since the integral of a function multiplied by a delta function is just the function evaluated at that point, solving (3) is equivalent to solving:

\[
R_N(\tilde{a}, x_j) = 0, \quad j = 1, 2, ..., N \tag{2.8}
\]

In other words, we simply solve for the set of \( a_j \) such that the approximation is exactly equal to the model at \( N \) points, and thus only require the model solution at \( N \) points and not the explicit model equations.

The final step in determining the polynomial chaos expansion to approximate the random variable is to choose the points \( x_j \) at which we evaluate the "true" model \( y(x) \), in order to solve for the \( a_j \) using (2.8). For this step, we borrow from the technique of Gaussian Quadrature, which uses the summation of orthogonal polynomials multiplied by weighting coefficients to approximate the solution of an integral. In Gaussian Quadrature, the optimal choice of abscissas at which to evaluate the function being integrated are the \( N \) roots of the \( N^{th} \) order orthogonal polynomial \( B_N(x) \) (Press, et al., 1992). Similarly in DEMM, to solve for the \( N \) coefficients in the expansion

\[
a_0 + a_1 B_1(x) + ... + a_{N-1} B_{N-1}(x),
\]

we use the residual evaluated at the \( N \) roots of \( B_N(x) \).

For multiple uncertain parameters, \( N \) roots are generated for each parameter to use as possible sample values. However, not all possible permutations of the \( N \) values for each parameter will necessarily be needed, depending on the number of terms in the expansion. Rather than combine sample values randomly, as in Latin Hypercube, we can use the probability density functions of the parameters to order the \( N \) possible values by
likelihood. Then sample sets are formed by choosing permutations in decreasing order of joint probability, until the required number of sets have been formed.

DEMM cannot find a sufficiently accurate approximation in every case. In particular, discontinuities in the response surface result in poor approximations. The approximation must be checked against model results at values of the uncertain inputs other than those used to solve for the coefficients. An optimal choice of points to check the approximation against the model are based on the roots of the next higher orthogonal polynomial than the one used to find points to solve at. The roots of the next higher order will always interleave the lower order roots (Press, et al., 1992), and so these will test the approximation at a maximal distance from the fit values while still spanning the highest probability regions. Also, if the expansion of order \(N\) is poor, we already have the values needed to find the expansion of order \(N+1\). Once the expansion for the probabilistic model response is solved and found to be reasonably accurate, the approximate probability density function of the response can be derived by applying crude Monte Carlo simulation to this expansion.

For example, suppose we attempt to approximate the model response pdf with a third order expansion of two uncertain parameters. The expansion, with all cross-products up to third order included, will have 10 terms, and therefore 10 coefficients to solve for. Using the roots of the fourth-order orthogonal polynomial, each parameter will have 4 sample values spread across the high probability region of its distribution. Since only 10 values are needed to calculate the expansion coefficients, we choose the 10 pairs of values with the highest joint likelihood from among the 16 possible permutations. Other sample values are needed to check the error in the approximation. For this we use the roots of the fifth-order orthogonal polynomial for each parameter, and form the 12 sample pairs with the highest likelihood. The total number of simulations of the model required will be \(10 + 12 = 22\) runs.

DEMM has been used successfully to explore the uncertainty in a variety of scientific, engineering, and economic modeling applications (e.g., Tatang, 1995; Pan, et al., 1998; Calbo, et al., 1998; Pun, 1998; Webster, 1996; Webster and Sokolov, 2000). For many models, DEMM estimates multiple characteristics of the response distribution more efficiently than either modified sampling or traditional response surface approximation methods. DEMM's approach of representing the pdf of the uncertain
response as an expansion of underlying pdfs, and of using probabilistic information in choosing the sample points for fitting the expansion, enable more efficient approximation of the overall response distribution relative to other methods. The next section will demonstrate DEMM in the context of a simple model.

2.3 Example: Logistic Population Growth Model

In order to understand the relative merits of different methods for uncertainty propagation, in this section we compare their performance on a simple model. We will show that DEMM converges to a precise estimate of many characteristics of the distribution of a model response more quickly than other methods. A few other comparative examples of uncertainty analysis methods exist in the literature (Iman and Helton, 1988; Cox, 1977; McKay et al., 1979), but none of these include DEMM as an alternative. We will use a simple logistic model of population growth to demonstrate the tradeoffs among these techniques.

2.3.1 The Population Growth Model

Models of population growth often use a logistic function to represent exponential growth in the short term, followed by a gradual slowing and eventual stabilization in the longer term. One such model to project global population over time, beginning with a base year of 1985, is:

\[ \text{pop}(t) = \text{pop}(1985) e^{\alpha(t-1985)} \]  

where \( \text{pop}(t) \) is the global population in year \( t \), \( \alpha \) represents the growth acceleration rate, and \( \beta \) represents the rate of deceleration to a stable population level. This population model is used within the EPPA model (Yang, et al., 1996), the economic component of the MIT Integrated Global Systems Model of global climate change (Prinn, et al., 1999), which will be described in Chapter 3.

The reference values for \( \alpha \) and \( \beta \) are calibrated based on United Nations forecasts for global population (United Nations, 1996), and project a level of 11,902 by 2100. But the underlying assumptions about such populations projections are uncertain, including changes in fertility rates, mortality rates, and aging in different socio-political regions of
Table 2.1: United Nations Population Projections and Corresponding Model Parameters

<table>
<thead>
<tr>
<th></th>
<th>Medium-Low</th>
<th>Reference</th>
<th>Medium-High</th>
</tr>
</thead>
<tbody>
<tr>
<td>2025 Projection</td>
<td>7591</td>
<td>9074</td>
<td>9.445</td>
</tr>
<tr>
<td>(millions)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2100 Projection</td>
<td>6415</td>
<td>11,902</td>
<td>17,592</td>
</tr>
<tr>
<td>(millions)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3733</td>
<td>1.068</td>
<td>1.6822</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.4728</td>
<td>0.0163</td>
<td>0.01276</td>
</tr>
</tbody>
</table>


the world. The United Nations projects population for alternative assumptions about these underlying trends. Table 2.1 shows three such projections and the values for $\alpha$ and $\beta$ that allow the model in (2.9) to reproduce this behavior.

In order to explore uncertainty propagation through this model, the high and low projections are used to fit probability distributions for the parameters $\alpha$ and $\beta$. The pdfs shown in Figure 2.2 are beta distributions, fit under the assumption that the low projection is the 0.05 fractile and the high projection is the 0.95 fractile. We also assume for the simplicity of this example that $\alpha$ and $\beta$ are independent.

Figure 2.2: Probability Distributions for Model Parameters

![Probability Distributions for Model Parameters](image)
2.3.2 Uncertainty Analysis of Population Growth

An uncertainty analysis of the population growth model of equation (2.9) can of course be performed using crude Monte Carlo with very large numbers of random samples from the two input distributions. Figure 2.3 shows the probability density function for the population projection in the year 2100 using 50,000 samples. However, suppose that each simulation for one sample pair of $\alpha$ and $\beta$ required a large amount of computation time. This is often the case with models for which we wish to analyze uncertainty. We will apply several of the previously described uncertainty propagation methods. We assume for purposes of comparison that the distribution of population in 2100 obtained through a crude Monte Carlo of 50,000 samples represents the "true" distribution.

The goal of this analysis is to characterize uncertainty in the model response $pop(2100)$. Before we compare different methods, it is important to remember that different uncertainty analyses may differ in the characteristics that need to be estimated accurately. There are several different possible objectives of an uncertainty analysis of a response, including:

- Estimate of the mean or expected value,
- Estimate of the variance or standard deviation,
- Estimate of a specific quantile of the distribution (e.g., 0.95 fractile),

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- Estimate of the probability of exceeding some threshold value
- The best overall approximation of the probability density or cumulative distribution function.

Some analyses may only require one or two of the above estimates, while others may want good estimates of all these characteristics. We will demonstrate that depending on the objectives of the analysis, different methods will be preferable.

If we can simulate the population growth model of equation (2.9) only a small number of times (e.g., less than 100 samples), there are several different methods from which to choose. We will apply:

- Crude Monte Carlo,
- Latin Hypercube Sampling,
- Importance Sampling,
- Linear Response Surface Replacement, and
- Deterministic Equivalent Modeling Method.

The crude Monte Carlo is only included as a benchmark for the comparison of the rates of convergence of an estimate. Importance sampling is only used to estimate the probability of exceeding an extreme threshold value. Suppose that the statistics of the distribution of population in 2100 that we wish to estimate are 1) the mean, 2) the standard deviation, 3) the 0.95 fractile value, 4) the probability that population will exceed 20 billion in 2100, and 5) the cumulative distribution function (cdf).

We will apply each of the methods at several different sample sizes. Since all methods require random sampling, 10 independent trials at each sample size are performed to assess the methods average performance at that sample size. The error of each estimate is measured by the average sum of squared deviations from the "true" value, normalized as a percentage of the "true" value:

$$ERR = \sqrt{\frac{\sum_{i=1}^{10} (\hat{\theta}_i - \theta)^2}{10}}$$

where $\hat{\theta}_i$ is the estimate of the quantity $\theta$ from the $i^{th}$ trial. The sum of squared errors over 10 trials roughly captures both deviations in the mean estimate from the truth, as
well as variance in the estimate. The "true" value is the estimate produced from 5 trials of crude Monte Carlo of 10,000 samples each.

Crude Monte Carlo is implemented by simply randomly sampling \( n \) pairs of values for \( \alpha \) and \( \beta \) from the distributions shown in Figure 2.2. Latin Hypercube samples of size \( n \) are formed by dividing the input distributions into \( n \) strata of equal probability, randomly selecting one point in each strata, and then randomly combining these \( n \) values of \( \alpha \) and \( \beta \) into pairs. The linear response surface estimates the output with the following four-term function:

\[
y = a_0 + a_1 \alpha + a_2 \beta + a_3 \alpha \beta
\]

The coefficients \( a \) are fit using least-squares with 4 (the minimum), 9, and 21 samples. Each sample set is formed using a factorial design of two, three, and five levels, respectively. To implement DEMM, the parameter distributions (Figure 2.2) are used to derive orthogonal polynomials, and the response pdf is approximated with expansions up to first, second, third, and fourth order terms. As described in the previous section, these expansions require samples from the model of sizes 8, 14, 22, and 32, respectively. Both the linear response surface and DEMM are used to obtain estimates by solving for coefficients only once for each sample size, and then subjected to 10 independent trials of crude Monte Carlo with 10,000 samples. Finally, importance sampling, used only to estimate the probability of exceeding 20 billion, is implemented by sampling randomly from only the upper 20% of both the \( \alpha \) and the \( \beta \) distributions. The resulting samples of population are used to estimate the proportion that exceed 20 billion, and the estimate is then scaled by 0.04 to represent the true proportion within the full distribution. This process relied upon previous samples that determined that no possible combination of \( \alpha \) and \( \beta \) from the lower 80% would exceed 20 billion.

Figure 2.4 shows the error for each of these methods at different sample sizes in estimating the mean, standard deviation, 95\(^{th}\) percentile, and the probability of exceeding 20 billion. As expected, crude Monte Carlo always exhibits slower convergence than other methods. In general, all methods tend to converge (i.e., errors approach zero) as the sample size increases. Where errors do not decrease monotonically with increasing
sample size, it is due to the randomness of the 10 trials; with many more trials at each of the sample sizes, we expect that all estimates would converge monotonically.

For estimating the mean of the distribution in 2.4a, DEMM and Latin Hypercube are roughly equivalent in efficiency, and both perform significantly better than the linear response surface. For estimating the variance and the 95% percentile (Figure 2.4b & c), however, errors in DEMM estimates decrease much more rapidly than Latin Hypercube, but Latin Hypercube still performs slightly better than the linear response surface.

*These sample sizes include the simulations necessary to check the fit.*
The probability of exceeding 20 billion (Figure 2.4d) is a particularly difficult estimate for all of these methods; the "true" value is calculated as 0.24%. The linear response surface had errors so large that it is omitted from the figure. For this extreme probability, Latin Hypercube has no advantages over crude Monte Carlo. The most efficient method for this estimate is importance sampling. Errors in the DEMM estimate are smaller than errors from LHS or Monte Carlo, but larger than errors from importance sampling. Although importance sampling was not used to estimate the 95% fractile, it would most likely have smaller errors than DEMM at all sample sizes.

In addition to estimating specific moments or characteristics of the distribution, the goal of the analysis may be to get a good overall estimate of the cumulative distribution function. Figure 2.5 shows the cdf for all methods (except importance sampling). Each estimated cdf is obtained from a single trial of the method, using 50 samples for LHS, 32 samples for DEMM, and 21 samples for the linear response surface. These estimated cdfs are compared with the cdf obtained from crude Monte Carlo with 50,000 samples. The first aspect of the figure worth noting is that none of the estimated cdfs are unreasonable. The largest errors are, not surprisingly, in the tails of the distribution, especially for LHS and the linear response surface. For a rough approximation, any of the methods will work for this model. A much more non-linear model would be a better test for differentiating among methods. We can compute the
error of each estimated cdf by again averaging the sum of squared errors over each
fractile in 0.05 steps. The errors for each method, as a percentage of the median value
are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Error Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Response Surface</td>
<td>14%</td>
</tr>
<tr>
<td>Latin Hyprcube</td>
<td>11%</td>
</tr>
<tr>
<td>DEMM</td>
<td>4%</td>
</tr>
</tbody>
</table>

2.4 Future Directions for Uncertainty Propagation Methods

Despite the fact that we have only presented one simple example in this chapter,
we can draw some tentative conclusions about choosing among methods. The main idea
is that no method is always superior to others. The choice of a method for uncertainty
analysis depends on several issues, including:

- Characteristics of the model (e.g., does the response exhibit bifurcations),
- What aspects of the response distribution need to be estimated,

Methods that replace the model with an approximation, including DEMM, often
produce extremely poor estimates of responses that are discontinuous, for example
whether the response bifurcates. Importance sampling may also be difficult to implement
if one is not able to identify a small number of sub-regions within the parameter space
that may produce the threshold response. The objective of an uncertainty analysis is very
important to choosing a methodology. If only reasonable estimates of the mean and
variance of a response are needed, Latin Hypercube may be as good, or in some cases
even better, than DEMM. If the primary objective is to estimate a tail probability, as is
often the case in risk analyses, importance sampling is probably the best choice. If the
analysis requires a reasonable estimate of the entire probability distribution, DEMM is
often the best method to use.

The simple example of population growth in this chapter is not meant to represent
all possible models that may require uncertainty analysis. Further comparison across
methods requires different types of models to demonstrate when one method is preferable
to another. The number of uncertain parameters in an analysis will also affect the relative
performance of uncertainty methods. Finally, there are other methods for estimating
uncertainty in a model response not discussed here, including methods based on Bayesian
inference (e.g., Currin, et al, 1991; Sacks et al, 1989; Bowman, et al, 1993). These methods should also be compared with the traditional approaches reviewed here.

For the remainder of this dissertation, DEMM will be used as the method for uncertainty propagation. The focus of this dissertation is on sequential policy choice for global climate change. In climate change, it is important to have good approximations of both the mean and variance of a response, and also the probability of extreme events. Therefore, a good overall approximation of response distributions is desired, and DEMM is an efficient means of obtaining such approximations. Chapter 3 will present the uncertainty in many different responses from an assessment model of climate change, describing both abatement costs and damage impacts. These uncertainties will then be used in Chapters 4-7 to explore sequential policy choice under uncertainty.
Chapter 3  Uncertainty Analysis of an Integrated Assessment Model

Do not expect to arrive at certainty in every subject which you pursue. There are a hundred things wherein we mortals . . . must be content with probability, where our best light and reasoning will reach no farther.

- Isaac Watts

Previous studies of uncertainty and sequential decision in climate policy have used simple parameterizations of the climate and economy (e.g., Nordhaus, 1994; Morgan and Dowlatabadi, 1996). This dissertation will instead use a climate/economy model of intermediate complexity in order to represent the many feedbacks and nonlinearities of the global system. The estimation of uncertainty in model responses is performed by using the Deterministic Equivalent Modeling Method (see Chapter 2), which preserves the feedbacks and nonlinearities in a less computationally demanding representation. This chapter describes the model, and shows the uncertainty in some sample model responses in the form of continuous distributions, as well as how these distributions are obtained. Then, in preparation for the sequential decision analyses in Chapters 4-7, we develop reduced-form representations of the model in terms of sequential strategy variables. Discrete representations of the uncertainty in model parameters are also developed from the continuous distributions.

Section 3.1 will briefly describe the climate assessment model used. Section 3.2 will show the results of sensitivity testing of parameters to determine which are the most important to treat in an uncertainty analysis. The issue of subjective assessment of probability distributions is discussed in Section 3.3, and the parameter distributions used in the uncertainty analysis are given. The uncertainties in emissions, policy costs, and temperature change, are described in Section 3.4 in the form of continuous distributions. A damage function that calculates economic losses from temperature increase is added to the modeling framework, and is described in Section 3.5. Finally, Section 3.6 will show the construction of reduced-form models to calculate the outcome for any sequential strategy and value of uncertain parameters. The conversion of elicited parameter distributions from continuous to discrete is also given in Section 3.6.
3.1 Overview of MIT Integrated Global Systems Model

To explore sequential choice of climate policy, there are many different processes, systems, and feedbacks that need to be represented in a model. The model must be able to project emissions of CO₂, other greenhouse gases, and aerosols for a reference case without policy, and also project emissions under some constraint or carbon tax. The resulting economic losses to different nations or regions from an emissions constraint must also be calculated. Physical and chemical processes in the atmosphere, ocean, and biosphere must be modeled to calculate the concentrations that result from the emissions, the changes in radiative forcing of the atmosphere from these concentrations, and the changes in the climate system such as temperature change, precipitation, and sea level rise over time from the increased forcing. Finally, models are needed that can translate these effects into regional climate change, ecological impacts, and ultimately some form of economic losses from climate change.

A model that can represent most or all of the above processes and linkages is referred to as an “integrated assessment model” of climate change. Many integrated assessment models have been developed that vary widely in the types of models used, the complexity of the component models, and the breadth of disciplines represented (Parson and Fisher-Vanden, 1997). The integrated assessment model used in this dissertation is the MIT Integrated Global System Model [IGSM] (Prinn et al., 1999), augmented with a damage function related to change in global mean temperature. This section will briefly describe the main components of this model. Figure 3.1 shows the main components of the system, and some of the quantities that are passed between components and/or used as results (for a full description, including components and feedbacks not discussed here, see Prinn et al [1999]). The main components of the IGSM itself include:

- The Emissions Prediction and Policy Analysis (EPPA) model – projects emissions of radiatively important gases and the costs by region of any emissions policy;
- The 2-D Ocean Carbon Sink Model (OCSM) – represents carbon fluxes between ocean and atmosphere to determine CO₂ concentrations from the projected emissions;
- The 2-D Climate and Chemistry model – dynamically simulates zonally averaged climate over time, with fully interactive atmospheric chemistry;
and precipitation to calculate changes in the net primary production and net ecosystems production for natural (unmanaged) ecosystems.

The uncertainty analysis presented in this chapter does not attempt to treat all components of the IGSM, nor does it treat every uncertain parameter or uncertain output. The analysis shown here propagates uncertainty in a subset of parameters that appear to have the strongest impact on uncertainty on costs and on climate indicators. Uncertainty in impacts is obtained in terms of global mean temperature change. A damage function, not a part of the MIT IGSM, is added to the modeling framework to facilitate the comparison of outcomes in the decision models. This damage function is described in Section 3.5. Model components and outputs not treated in the uncertainty analysis are indicated in Figure 3.1 by dashed lines, including ecosystems impacts, precipitation, and non-CO$_2$ greenhouse gases and aerosols. Also, in order to keep the computation time tractable for analyzing the climate model, the interactive chemistry is turned off, which
reduces the time required for each simulation by a factor of three. As described below, the calculations here will treat uncertainty in CO$_2$ only, so the errors from excluding the interactive chemistry are likely to be small. Future analyses will treat other greenhouse gases and aerosols, and may require the explicit chemistry component (see Chapter 8). The model components (solid boxes) and uncertain outcomes (bold type) treated in the uncertainty analysis are described below.

3.1.1 Emissions Prediction and Policy Analysis (EPPA) Model

The EPPA model, version 2.3$^1$ (Yang et al., 1996; Jacoby et al., 1997) is the economic component of the MIT Integrated Global System Model. EPPA is a recursive-dynamic, computable general equilibrium (CGE) model. It is derived from the GREEN model of the OECD (Burniaux et al., 1992). EPPA is divided into 12 geopolitical-economic regions. Six regions represent the Annex I countries as defined by the Framework Convention and that must reduce emissions under the Kyoto Protocol: the United States. (USA), the European Community (EEC), Japan (JPN), the remainder of the OECD as of 1992 (OOE) which includes Canada, Australia, New Zealand, and most of Scandinavia, the Former Soviet Union (FSU), and Central and Eastern Europe (EET). The remaining six regions are Brazil (BRA), China (CHN), India (IND), the dynamic Asian economies (DAE)$^2$, the energy exporting countries in the Middle East as well as Mexico and Indonesia (EEX), and the remaining countries (ROW).

Each region has 8 production sectors (agriculture, energy intensive industries, other industries, crude oil, refined oil, gas, coal, and electricity), three future technology production sectors (carbon-free electric backstop$^3$, carbon-intensive liquid fuel backstop from shale oil$^4$, and hydrogen fuels), and four consumption sectors (food and beverage,

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$^1$The results in this paper use version 2.3 of EPPA, which has a number of modifications from version 1.7, used in (Jacoby, et al., 1997). The modifications include the addition of a third backstop technology (hydrogen fuels), different nesting of production functions between electric and non-electric energy, modified elasticities of substitution between factors, and the addition of a materials input to the backstop production functions.

$^2$e.g., Taiwan, Singapore, Malaysia.

$^3$“Backstop” is an economic term used to mean an alternative substitute for a scarce resource. The alternative is originally too costly to use, but as the conventional resource becomes more expensive, the alternative is eventually substituted, which prevents the price of the conventional resource from rising infinitely.

$^4$The carbon-intensive oil backstop (i.e., tars, oil sands, oil shale) is assumed to be present only in USA, FSU, EEX, and OOE.
transport and communication, energy, and other goods and services). The model accounts for trade between regions, and can track the imports and exports from each region. In each region, there is a representative consumer that maximizes its utility based on a bundle of the consumption goods.

As with most CGE formulations, EPPA Version 2.3 is a static model. Based on the previous period solution and prescribed growth in some exogenous parameters, EPPA recursively solves for equilibrium in each time period in 5 year intervals, starting with the calibration year of 1985, and ending with 2100. For each time period, the model produces as output the amount produced by each sector in each region and the relative prices. By using carbon emission coefficients, the CO₂ emissions are also calculated. The cost of a policy to a region can be measured as the change in consumption between a no-policy scenario and a scenario in which a policy is implemented. EPPA can model a variety of policy instruments, including carbon taxes, energy taxes, and quantitative restrictions, with or without allowing tradable permit schemes.

EPPA also models limited mobility of capital across sectors. In each period, savings is invested as new capital stock distributed across the production sectors. In addition, some of the remaining capital in each sector after depreciation can also be reallocated to more productive sectors. However, a portion of the capital in each sector in each year is vintaged: this capital cannot be reallocated to a different sector. This vintaging of some of the capital stock will turn out to have implications for sequential decision and learning in Chapter 6.

3.1.2 2-D Ocean Carbon Sink Model

The 2-D Ocean Carbon Sink Model (Holian, 1998) parameterizes the physics and chemistry that determine the absorption of CO₂ from the atmosphere. The oceanic structure consists of a mixed layer attached to 10 layers of an eddy-diffusive deep ocean. The zonally averaged horizontal latitude structure corresponds to the 2-D Climate Model (see below), spaced at 7.826 degrees. The flux of CO₂ into the mixed layer is calculated at each latitude as a function of the partial pressure of CO₂ in the mixed layer and the piston velocity driven by wind speed. The partial pressure is in turn determined by modeling the chemistry of total dissolved inorganic carbon, the sum of aqueous CO₃²⁻, carbonate, and bicarbonate ions, which is highly dependent on temperature, salinity, and
alkalinity. The deep ocean mixing of total dissolved inorganic carbon is represented as a diffusion-only transport process, and depends on the vertical eddy diffusion coefficient \( K_z \). This same approach is used for heat in the 2-D climate model, and the diffusion coefficients are assumed to be perfectly correlated in the two models.

3.1.3 MIT 2-D Climate Model

The atmospheric component of the MIT 2-D (zonal averaged) climate model (Sokolov and Stone, 1998) is a modification of the GISS 2D dynamical-statistical model (Yao and Stone, 1987; Stone and Yao, 1987 and 1990) developed from the GISS GCM (Hansen et al., 1983). The model includes parameterizations of all the main physical atmospheric processes. The MIT 2-D model allows up to four different kinds of surface in the same grid cell: land, land ice, open ocean, and sea ice. Surface characteristics (e.g., temperature, albedo, etc.) as well as turbulent and radiative fluxes are calculated separately for each kind of surface. The MIT 2-D model is capable of reproducing many of the non-linear interactions simulated by atmospheric GCMs.

Two primary uncertain parameters of the MIT 2-D model are the climate sensitivity and the vertical diffusion coefficient that represents uptake of heat by the deep ocean. In simulations of transient climate change the heat uptake by the deep ocean is parameterized by diffusive mixing of the mixed layer temperature’s deviation from its present-day values. Zonally averaged values of diffusion coefficients calculated from measurements of the tritium dispersal in the ocean are hereafter referred to as the “standard” values. These values for the diffusion coefficients vary from about 0.5 cm\(^2\)/s in the equatorial region to about 5 cm\(^2\)/s in high latitudes with a global average, denoted as \( K_z \), equal to 2.5 cm\(^2\)/s. In this model, vertical diffusion is used to represent all processes responsible for heat penetration into the deep ocean, and so these values represent “effective” diffusion coefficients which are significantly larger than those used to represent subgrid scale mixing in oceanic GCMs.

The versions of the MIT 2D model with different climate sensitivities are obtained by inserting an additional cloud feedback. Changes in different climate variables produced by different versions of the MIT 2D model, in terms of both globally averaged values and latitudinal distributions, are consistent with the results of GCMs with different climate sensitivities (Sokolov and Stone, 1998; Prinn et al., 1999). The MIT 2D model
also simulates seasonal differences in those changes, such as enhanced surface warming in high latitudes of the winter hemisphere.

The ability of the 2D model to reproduce different climate feedbacks is also extremely important in simulating a transient response to changing forcing. Different feedbacks come into play at different times. While water vapor and cloud feedbacks are active from the very beginning, sea-ice feedback becomes critical only after warming reaches some level. The strengths of different feedbacks change with time (see, for example, Murphy 1995). One feedback that will prove important later is the uptake of heat and carbon by the deep ocean. As discussed above, the absorption of carbon by the ocean is temperature dependent. As the surface layer of the ocean warms, less carbon will be absorbed. Since the additional carbon remaining the atmosphere will contribute to additional warming, this becomes a feedback. This feedback will have an impact on the role of learning in Chapter 6.

### 3.2 Sensitivity Analysis of Model Parameters

The first step in uncertainty analysis of a large (i.e., many variables and parameters) model such as the MIT IGSM is to determine the subset of parameters that have the most impact on model responses, and therefore warrant detailed study. This selection can be based on a preliminary screening sensitivity analysis of many parameters of the model, or the selection may be based on the modeler’s expertise and judgment. The two uncertain parameters of the 2-D climate model chosen for study, climate sensitivity and oceanic uptake, are based on the judgment of the model experts, primarily because these two parameters enable the MIT climate model to reproduce the results of several 3-D general circulation models. The one uncertain parameter treated in the ocean carbon sink model, deep-ocean mixing, was shown by Holian (1998) to account for over 90% of the variance in the carbon sink in an uncertainty analysis of several model parameters. Furthermore, within the MIT IGSM the oceanic uptake of heat in the climate model and the deep-ocean mixing in the ocean carbon sink model are assumed to be perfectly correlated, and thus can be represented with a single underlying distribution.
### Figure 3.2: Sensitivity of Cumulative CO₂ Emissions (Reference)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference Value</th>
<th>Sensitivity</th>
<th>1.2</th>
<th>0.9</th>
<th>0.6</th>
<th>1.1</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elas. of Subst. (L,KEF)</td>
<td>0.8</td>
<td>+20%</td>
<td>-20%</td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resource Extract Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elas. of Subst. (Fossil Fuels)</td>
<td>0.8</td>
<td>-20%</td>
<td>+20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor Productivity Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resource Base/Yet-to-find</td>
<td>-20%</td>
<td>+20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population Growth Accel.</td>
<td>-20%</td>
<td>+20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AEEI</td>
<td>1.2</td>
<td></td>
<td></td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricity Backstop Price</td>
<td>12</td>
<td></td>
<td></td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elas. of Subst. (E,KF)</td>
<td>0.8</td>
<td></td>
<td></td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population Growth Deceleration</td>
<td>-20%</td>
<td>+20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shale Backstop Price</td>
<td>1.7</td>
<td></td>
<td></td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elas. of Subst. (Non-Elec,Elec)</td>
<td>0.6</td>
<td></td>
<td></td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cumulative CO₂ Emissions (GtC) 1990-2100

Selection of uncertain parameters of the EPPA model for study are, in contrast, chosen based on a screening sensitivity analysis. The EPPA model has many different parameters to which either emissions or costs of emissions constraints may be sensitive. Such parameters include those determining population growth rates, labor productivity growth rates, the degree of substitutability between factors of production, the rates of discovery and extraction of fossil fuel resources, and the costs of future energy substitutes. For each parameter, the model is run at a high and a low value, keeping all other parameters at reference values. The resulting changes in several different model responses are then compared. One dilemma in performing a screening sensitivity analysis is how one chooses a high and low value for each parameter. Technically, the high and low values should be the same degree of probability or likelihood above and below the reference across parameters to ensure a fair comparison. But since the goal of the sensitivity analysis is to choose the parameters for which we will construct a probability distribution, this becomes a “Catch-22”. For this analysis, the high and low values were chosen to be 20% above and below the reference value for each parameter.

Some results of the sensitivity analysis of EPPA parameters are shown in the form of tornado diagrams in Figures 3.2-3.4. Figure 3.2 shows the sensitivity of cumulative CO₂ emissions over the period 1990-2100 for the reference case (no emissions.
Figure 3.3: Sensitivity of Welfare Loss to USA (Kyoto Protocol)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elec Backstop Price</td>
<td>12 - 18</td>
</tr>
<tr>
<td>Resource Extract Rate</td>
<td>+20% -20%</td>
</tr>
<tr>
<td>Shale Backstop Price</td>
<td>1.7 - 1.1</td>
</tr>
<tr>
<td>Resource Base/Yet-to-find</td>
<td>-20% +20%</td>
</tr>
<tr>
<td>Elas(Subst)</td>
<td>0.8 - 1.2</td>
</tr>
<tr>
<td>Elas(E,KF)</td>
<td>0.8 - 0.6</td>
</tr>
<tr>
<td>Elas(L,KEF)</td>
<td>0.8 - 1.2</td>
</tr>
<tr>
<td>AEEI</td>
<td>1.2 - 0.9</td>
</tr>
<tr>
<td>Population Growth Acceleration</td>
<td>-20% +20%</td>
</tr>
<tr>
<td>Elas(Non-Elec,Elec)</td>
<td>0.6 - 0.4</td>
</tr>
<tr>
<td>Labor Productivity Growth</td>
<td>-20% +20%</td>
</tr>
<tr>
<td>Population Growth Deceleration</td>
<td>-20% +20%</td>
</tr>
</tbody>
</table>

% Welfare Loss to USA

Cumulative CO₂ emissions are chosen as one criterion because the uncertainty in climate impacts and damages depend on the cumulative emissions rather than the emissions in any one year. The parameter that most affects cumulative emissions is the elasticity of substitution of labor for other factors in the production of an output good. The next four most influential parameters on emissions are two parameters that determine the rate of extraction of fossil resources, the substitutability between coal, oil, and gas in production, and the rate of increase in labor productivity.

The parameters that most influence emissions are not necessarily the same as those that most influence the costs of a policy. To compare the sensitivity of costs, implementation of the Kyoto Protocol is simulated in EPPA (Babiker et al., 1999), and the costs to each region in the year 2010 are examined. Figures 3.3 and 3.4 show the tornado diagrams for two regions, USA and EEC. The five most influential parameters are the same for these two regions, although the order differs. Three of these parameters were also important for emissions: the resource extraction rates and the substitutability of fossil fuels. In addition, two other parameters can cause a large variation in costs: the cost of a carbon-free electricity substitute, and the cost of a synthetic carbon-liquid fuels...
resource (shale). The sensitivities of the costs for other regions (not shown) are very similar.

Based on the results of this analysis, five parameters were chosen that are responsible for the greatest variation in both emissions and costs. The resource extraction parameters were omitted from this study because it was instead decided to completely revise the resource sub-model within EPPA, in part due to the demonstrated sensitivity of results shown by this analysis. The revised resource sub-model is documented in (Babiker et al., 1999), and its parameters will be treated in a future uncertainty analysis. The parameters chosen for detailed uncertainty propagation are:

- Elasticity of substitution between labor and other factors in production of goods and services,
- Elasticity of substitution between oil, coal, and natural gas as energy inputs to production,
- Labor productivity growth rate,
- Initial price of carbon-free electricity backstop, and
- Initial price of carbon synthetic fuels backstop.
3.3 Eliciting Subjective Probabilities

An analysis of parametric uncertainty requires that an assumed probability distribution for each parameter. In this section, the assumed distributions will be given for the parameters in this study, and the methods by which these distributions were constructed will be described.

3.3.1 Issues in Expert Elicitation

There are several different ways that a probability distribution can be obtained for an uncertain model parameter. One source for constructing a distribution is a data set of measurements for the uncertain quantity. For example, initial conditions such as ocean surface temperatures may be sampled many times, and such a data set can be used to fit a distribution. Even if the raw data is not available, there may be estimates of the mean and standard error of the quantity in the literature. Often statistical estimates can be found for such quantities as elasticities of substitution.

Unfortunately, data or estimates for uncertain quantities represented by model parameters are rarely available or applicable. Often the uncertain parameter is the projection into the future of some rate or level of an activity. Although estimates of past rates can provide partial guidance, there is no way to measure future outcomes. Even when data or estimates exist, they may not be applicable to the model. Many econometric studies make very restrictive assumptions, or measure a very small sub-population within the large-scale system being modeled. As a result, the estimate may be inconsistent with the model structure and cannot be used. Finally, some parameters are representations of entire processes or sub-systems that cannot be modeled explicitly. These parameters therefore do not correspond to any quantity that can be measured or estimated directly.

When no data or estimates exist, the only source for a probability distribution is subjective opinion, usually of an expert. While this is often the only recourse to obtain probability information, it is problematic due to well-known biases in people’s ability to make probabilistic judgments (Tversky and Kahneman, 1974; Morgan and Henrion, 1990). Studies by Tversky and Kahneman (1974) have identified three major types of cognitive heuristics used in forming judgments about probability, representativeness, availability, and anchoring and adjustment, which all typically lead to biases and
overconfidence. These effects have been shown in expert as well as in non-expert situations (Henrion and Fischhoff, 1986).

Several different protocols have been developed for eliciting subjective judgment about probabilities (Morgan and Henrion, 1990; Spetzler and Holstein, 1975; Wallsten and Whitfield, 1986). These approaches have several common elements. Low-probability extreme values are elicited first in order to reduce the anchoring and adjustment effect. The subject is asked for the detailed reasoning and assumptions behind each judgment in order to aid the thinking process. Often the assessor will ask the subject for a scenario that could produce a more extreme value than given, which sometimes prompts re-evaluation of previous responses and may result in longer tails and hopefully less overconfidence. Finally, redundant information, such as eliciting both probabilities and fractile values, are obtained to check for coherence, and if a distribution is not coherent the subject is presented with the conflicts and asked to revise judgements. The procedure used in eliciting the parameter distributions for this study incorporated all of these strategies.

Beyond the problems of subjective judgement, an additional difficulty is how to use judgments from multiple experts who disagree. The distributions for the economic parameters were elicited from five experts, and distributions for climate sensitivity are available for sixteen experts (Morgan and Keith, 1995), all of which are different from the others. In general, there are three main approaches when faced with differing expert opinions: 1) Propagate each expert's distribution separately; 2) Require the experts to create a single consensus distribution (Dalkey, 1967); 3) Combine the expert opinions in some way (Genest and Zidek, 1986; Winkler, 1986).

Debates in the literature reveal no consensus on how to deal with this choice in the context of integrated assessment models. For example, Titus and Narayanan (1995; 1996) obtained probability distributions for about 20 parameters from three sets of experts on global climate, polar precipitation, and glacial processes to estimate the probability distribution for future sea level rise, assuming that each reviewer's assessment was equally probable. Keith (1996), Morgan (1997), and Pate-Cornell (1996; 1997) objected to this approach for two reasons. First, they objected to combining expert judgments at all. They argued that analyses of each expert's opinion would be more useful, and that models and hypotheses can be combined but not expert opinions. Titus
responded that these arguments might have some validity in a study such as Morgan and Keith (1995), where individual factors are being analyzed one-at-a-time. In a case where different groups of experts only provide assessments of some of the parameters of a model, no single expert is providing the parameters necessary to run the model; hence their recommendation could not possibly be carried out without requiring people to express opinions on parameters about which they had no knowledge. Alternatively, one would have to present the results of every pairing of reviewers, in which case their study would have to include 256 different resulting distributions. Moreover, the civil engineers and local planners to which their study was directed needed a single “bottom-line” estimate representing the combined expertise of the scientific community, Titus and Narayanan argued (see also, Titus, 1997). Casman, Morgan, and Dowlatabadi (1999) also note the practical limitations to the strict Bayesian approach of specifying all possible hypotheses with axiomatically correct priors in commenting that, “... a prescription that one’s analytical formulation should grow in complexity and computational intensity as one knows less and less about the problem, will not pass the laugh test in real-world policy circles.”

A second objection, raised by Keith (1996) and Pate-Cornell (1996, 1997), is that it is arbitrary to weight reviewer opinions equally, because the fraction of experts holding an opinion is unlikely to be equal to the probability of the hypothesis being true. Titus agreed that equal weighting was arbitrary, but in the absence of more information is straightforward, transparent, and fair. The resulting probability distribution should be interpreted as the results of the combined wisdom of the reviewers that participated, not as a representation of absolute truth.

The existing literature (e.g., Dalkey, 1967; Genest and Zidek, 1986; Keith, 1996; Moss and Schneider, 1997; Morgan, 1997; Pate-Cornell, 1996, 1997; Titus, 1997; Titus and Narayanan, 1995, 1996; Winkler, 1986) is not entirely clear about whether this debate truly involves alternative paradigms, or whether different situations and study objectives simply require different approaches. The objectives of the uncertainty analysis in this chapter are 1) to illustrate the range of uncertainty in an integrated assessment model, and 2) to provide reference distributions of outcomes for the sequential decision analysis in the following chapters, which will be subjected to extensive sensitivity testing. Thus, rather than propagate every set of expert judgments separately, each parameter is
given one reference distribution, created by combining all experts with equal weight. The details of one of these assessments, and the disagreement among the experts, are given below.

3.3.2 Elicited Distributions for the EPPA Model

Here, for each of the five parameters selected for study in Section 3.2, we will briefly describe the function within the EPPA model and give the elicited probability distribution. The opinions of individual experts before being combined will be shown for the elasticity of substitution between labor and other production factors. However, individual expert opinions on the other parameters will not be shown here.

Elasticity of Substitution between Labor and Other Factors

Production of the output good/service for each production sector in the EPPA model is modeled using a nested constant elasticity of substitution (CES) representation. A CES function of output produced from two input factors \( A \) and \( B \) is of the form:

\[
x = \left[ b_A A^\rho + b_B B^\rho \right]^{\frac{\rho}{\sigma}} \quad \rho = 1 - \frac{1}{\sigma}
\]

where \( \sigma \) is the elasticity of substitution between the two factors \( A \) and \( B \). Larger values of \( \sigma (\sigma > 1) \) represent greater ease of substitution of \( A \) for \( B \), while smaller values of \( \sigma (\sigma < 1) \) represent less possibility of substitution.

Figure 3.5 shows the nested production structure assumed in each sector. Labor is combined with an aggregate of other factors. This aggregate is in turn made up of a capital/fixed-factor aggregate and an energy aggregate, and so on. In any period, the endowments of primary factors, labor (L), capital (K), and fixed factors (FF) representing land or resources, are fixed. To optimize welfare, the model can adjust the level of output of each sector, and the amount of production factors allocated across sectors used to produce that output, subject to constraints on the total amounts. Thus the ease of substituting between factors, represented by the \( \sigma \) at each level in the nesting, has a strong effect on both the overall optimized welfare and on the individual levels of output from each sector.
The elasticity at the top level of the nesting, denoted as $\sigma_{LKEF}$, determines the ease of substitution between labor and the aggregate bundle of capital, energy and fixed factor. This parameter was shown to have a major influence on carbon emissions by the sensitivity study described in Section 3.2. Five economists participated in the elicitation of this and the other economic parameter distributions\(^5\). After initial discussion among the experts, they decided that the distribution of the elasticity differs according to the specific production sector. The sectors were split into three sub-groups: 1) Fossil Energy Sectors (Coal, Oil, Gas, Refined Oil and Electricity); 2) Energy Intensive and Other Industries; and 3) Agriculture. Separate probability distributions of $\sigma_{LKEF}$ were then elicited for each of these three groups of sectors. The opinions of the individual experts are shown in Figure 3.6. As seen in the figure, there was a wide range of dispersion in both the median values and in the width of the possible range of values across the experts. Nevertheless, in general labor is believed to be much less substitutable for other factors in the fossil fuel sectors, and most substitutable in agriculture.

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\(^5\) The five experts were all MIT participants in the modeling group that developed the EPPA model.
Figure 3.6: Subjective Distributions for \( \sigma_{x,EF} \) from Five Experts

The probability distributions of the individual experts were then combined into a single reference distribution for each group of sectors. The reference distributions are shown in Figure 3.7 for the three groups. The three distributions were then fit to a common underlying beta distribution with shape parameters \( \alpha = 1.646 \) and \( \beta = 1.622 \). The elasticity of substitution for each sector is drawn from this distribution and then scaled and shifted appropriately.

Also shown in Figure 3.7 is the reference value for the elasticity of substitution of labor for other factors that was assumed for all sectors prior to this elicitation. The
careful thinking required by the elicitation resulted in significant changes in the reference assumptions for this parameter. Such changes were incorporated into the revision of EPPA that has produced version 3.0 (Babiker et al, 1999).

**Elasticity of Substitution between Fossil Fuels**

A second parameter that was shown to strongly influence both emissions and costs is the elasticity of substitution between fossil fuel inputs to production. This elasticity, denoted $\sigma_{EN}$ in Figure 3.5, determines the how easily coal, natural gas, crude oil, and refined oil can be substituted for one another as inputs into the production process.

For this parameter, distributions were also separately elicited and constructed according to sector. The sub-groupings are different than for the elasticity of labor substitution. It was generally believed that fossil fuels are less interchangeable for agriculture, very easily interchangeable in the electricity and energy intensive sectors, and moderately interchangeable for all other sectors.

The combined distributions for each sector group are shown in Figure 3.8. The uncertainty in $\sigma_{EN}$ is fit to a single beta distribution with shape parameters alpha $= 7.97$ and beta $= 13.77$. Each time a value is sampled from this distribution, it is scaled and shifted appropriately to obtain the elasticity for each sector. In Figure 3.8, we again show the original reference value for all sectors prior to the elicitation exercise. The new reference value is only significantly changed for the agriculture sector.
Labor Productivity Growth Rate

An important driver in the growth of the regional economies over time is the amount of "effective" labor supply available. The effective labor supply is the product of the population and the level of labor productivity. The labor productivity level is determined by:

\[ P_l(t) = e^{\alpha_{l,r}(1-e^{-\beta_{l,r}})} \]

\[ \alpha_{l,r} = \frac{\ln(1 + g_{0,r})}{1 - e^{-\beta_{l,r}}} \]

\[ \beta_{l,r} = \ln\left[ \frac{\ln(1 + g_{0,r})}{\ln(1 + g_{n,r})} \right] / Hz \]

where \( g_{0,r} \) is the initial labor productivity growth rate for region \( r \) from calibration data, \( g_{n,r} \) is the assumed labor productivity growth rate in the last time period (2100), and \( Hz \) is the time horizon which is 115 years for 1985 to 2100. Thus, a driving assumption is the growth rate in 2100.

The reference assumption is that all OECD regions (USA, JPN, EEC, and OOE) as well as EEX all converge to a rate of 1.0% per annum by 2100, and that all other regions converge to a rate of 2.0% per annum. By making \( g_{n,r} \) uncertain, the labor productivity growth rate will change in all periods, and have an impact on the effective supply of labor over time.
Figure 3.9: Reference Distribution for Labor Productivity Growth Rate

The mode of the distribution is again chosen to reproduce the reference assumption. The lowest possible assumed rates in 2100 are 0.5% and 1.0% for the OECD and non-OECD respectively; the highest possible rates are 1.5% and 3.0% respectively. This uncertainty is represented by a multiplicative factor applied to $g_{n,r}$ which ranges from 0.5 to 1.5 with a median value of 1.0. This is modeled by a beta distribution with parameters $\alpha=1.5$, $\beta=1.5$, as depicted in Figure 3.9.

Initial Price of Carbon-Free Electricity Backstop

In addition to the eight conventional production sectors in EPPA, there are three future technology production sectors that may enter anytime after 2000: 1) a synthetic liquid fuel substitute for refined oil produced from shale, tar sands, or heavy oils, 2) a hydrogen fuel source, and 3) a carbon-free alternative technology which produces electricity. The hydrogen fuel source is very expensive and only enters the solution under carbon restrictions far more stringent than anything studied here. The carbon liquid will be described below. Here we briefly describe the uncertainty in the carbon-free electricity technology.

The carbon-free electric backstop is not a particular technology, but rather a generic representative of a collection of technologies, including solar, advanced nuclear, and wind. It is modeled as being available in unlimited supply, requiring only capital, labor, and material inputs, the prices of which determine the cost of producing electricity

83
from the backstop. The technology produces electricity that is perfectly substitutable for that produced by the conventional electricity sector.

The fundamental uncertainties about the backstop are when and how quickly this technology would become available. However, the 1985 cost is currently used as a proxy for characterizing this uncertainty. The lower the initial year price, the sooner the backstop will become economically desirable; higher prices cause the backstop to take longer to become available. The parameter is therefore the initial year price for the carbon-free electric backstop.

The price of the carbon-free electric backstop is calibrated by the price in \( \epsilon/kWh \) of a unit of the output in the initial period (1985). This is used to set the relative quantities of the labor and capital inputs needed, given their 1985 prices. After the initial calibration, the price is determined from the changing prices of capital and labor, which evolve endogenously in the model. When the price of the backstop in a region falls below that of conventional electricity (the price of which tends to rise over time in the model as oil and gas are depleted), the backstop begins producing in the economy.

The nominal 1985 price for the backstop used is 15\( \epsilon/kWh \). Retaining this as a most likely value, the extreme endpoints for the distribution were assessed as 3\( \epsilon/kWh \) at the low end and 27\( \epsilon/kWh \) at the high end. These values are chosen for their effect on the time required for the backstop to become available, and do not reflect the true prices in 1985. The probability density function was fit to a beta with the same shape parameters.
as the PDF of labor productivity growth. The resulting distribution is shown in Figure 3.10.

**Initial Price of Synthetic Carbon Liquid Fuels Resource**

Another uncertainty in future technology is in the potential source for liquid carbon fuels as current grade petroleum becomes depleted. The liquid fuels are the primary energy source for transportation systems, the demand for these fuels may well continue to increase. Depending on the assumptions about oil reserves and future discovery rates (not treated as uncertain in this analysis), the crude oil needed for refining may become scarce. However, this does not mean that refined oil would disappear. Other resources, not quite cost-effective today, could become the next raw materials for producing carbon-based synthetic liquid fuels. Such resources include tar sands, shale, and heavy oil deposits.

Thus another important uncertainty is in the cost of these currently uneconomic resources. The cost of the shale oils relative to higher grade crude oil will determine both when refined oil begins to be produced from shale and the world oil price. Furthermore, the production of refined oil from these heavy oil and coal resources tend to produce more carbon emissions than the equivalent output from crude oil, as a result of the energy intensity\(^6\) of the gasification process (Harlan, 1982). As seen in the sensitivity analysis of Section 3.2, however, the primary influence of the uncertainty in the synthetic fuel price is on welfare levels and welfare losses under a policy.

As with the electric backstop, we use the price of the resource in the initial model period as a proxy for setting the rate at which this resource becomes competitive. The combined distribution from the experts is shown in Figure 3.11, in terms of 1998 US dollars per barrel of oil. This price is used to calculate the relative cost penalty over crude oil in the initial model period. The relative price then evolves endogenously from this initial level as 1) depletion of crude oil resources increases the rents and therefore the price of oil from the conventional sector, and 2) growth in labor productivity and investment in capital lower the cost of producing synthetic fuel from this essentially infinite resource.

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\(^6\) Recent data from development of Tar Sands in South America indicate that the energy intensity may not be significantly more than that of crude oil.
3.3.3 Elicited Distributions for the 2-D Climate Model

In the case of climate sensitivity, we draw upon an elicitation of 16 climate science experts by Morgan and Keith (1995). As a standard distribution for climate sensitivity, we combine the 16 distributions from Morgan and Keith (1995) with equal weight on each expert to create a standard distribution, and then construct alternative distributions to explore the sensitivity of the results. We choose to combine the distributions to better represent the current range of opinion about uncertainty than any one expert's opinion would, and to use equal weighting for its simplicity and transparency. To consider the extent to which results are affected by the choice of weight, we construct alternative distributions to explore the sensitivity of the results. Finally, while 5 of the 16 experts interviewed by Morgan and Keith had negative tails on their distributions for climate sensitivity (i.e., the possibility of cooling from doubled CO₂), we truncate our distribution at zero. A climate sensitivity that is negative, as well as a very large sensitivity due to some "surprise", would require a change in the state of the climate system, such as a collapse of the thermohaline circulation. The climate model used here is not capable of reproducing such a change, and recent simulations with AOGCMs suggest that a significant change in the state of the ocean is unlikely in the one hundred-year time frame that is the focus of our analysis. We choose a beta distribution with shape parameters α=2.85, β=14.0 scaled over the range 0.0 to 15.0 to represent the distribution of climate sensitivity, shown in Figure 3.12a, along with the IPCC low, best guess, and
The probability assessments of the rate of heat uptake by the deep ocean are based on the values of $K_r$, needed to match the behavior of different AOGCMs (Sokolov and Stone, 1998). After consultation with Professor Peter Stone of MIT (pers. comm., 1997) the following probability distribution has been constructed. The "standard" values of diffusion coefficients ($K_r=2.5 \text{ cm}^2/\text{s}$) obtained from observation of tritium mixing into the deep ocean were chosen to be the median of the probability distribution for the rate of heat uptake by the deep ocean. The values of diffusion coefficients needed to match the transient behavior of different AOGCMs range from $K_r=0 \text{ cm}^2/\text{s}$ for the NCAR W&M AOGCM to $K_r=25 \text{ cm}^2/\text{s}$ for the MPI model. Since both of those values do not seem to be highly probable, $K_r=0.5 \text{ cm}^2/\text{s}$ and $K_r=12.5 \text{ cm}^2/\text{s}$ were chosen as 0.05 and 0.95 fractiles respectively. The probability density function for the coefficient of heat uptake $K_r$ is given in Figure 3.12b. The probability distribution is actually constructed not for $K_r$ but for the square root of $K_r$. The square root is used because the depth of penetration of a temperature change into the deep ocean, which together with climate sensitivity defines the model response time, is proportional to the square root of the diffusion coefficient (see, for example, Hansen et al., 1985). The probability density function is a beta distribution, with shape parameters $\alpha=2.72$, $\beta=12.2$ over a range of 0.0 to 10.0 (0.0 to

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7 A study by Forest et al (1999), not available at the time of this analysis, has further constrained the joint probability of climate sensitivity and ocean uptake by using a statistical detection algorithm to match observed climate changes of the past century.
100.0 cm/s). Figure 3.12b also gives the values of the coefficient for heat uptake that allow the MIT model to reproduce the behavior of several AOGCMs.

Estimates of uncertainty in the rate of oceanic heat uptake based on observations are not available. However, IPCC (1996) does give an uncertainty range for the oceanic carbon uptake averaged over 1980s (2.0±0.8 PgC/year). In the MIT model, heat uptake and carbon uptake by the deep ocean are treated similarly: the coefficients for vertical diffusion of carbon are assumed to be proportional to those for heat (Sokolov et al., 1998; Holian, 1998; Prinn et al., 1999). The values of oceanic carbon uptake averaged for the last five years of simulations with an off-line version of the oceanic carbon model, forced by the observed changes in atmospheric CO₂ from 1865 to 1984, are 1.11, 1.94 and 3.3 PgC/year, for diffusion coefficients with \( K_v \) equal to 0.5, 2.5 and 12.5 cm²/s, respectively. For \( K_v = 0.8 \) and \( K_v = 7.2 \) (±\( \sigma \) of our distribution), the corresponding values of carbon uptake are 1.30 and 2.65 PgC/year. The ocean carbon uptake obtained in the simulation with diffusion coefficients corresponding to the median value of the proposed probability distribution is very close to the middle of the IPCC range, and the IPCC range is matched by values of diffusion coefficients lying between \( \sigma \) and 2\( \sigma \). Thus, the assumed distribution for the rate of carbon uptake is consistent with the IPCC estimate of uncertainty in oceanic carbon uptake.8

Finally, regarding the relationship between climate sensitivity and the rate of heat uptake, we assume independence between these two parameters. Climate sensitivity for most AGCMs is obtained from simulations with a mixed layer ocean model (i.e., with no deep ocean). The diffusion and mixing processes represented by the uptake parameter are unrelated to the cloud and aerosol processes represented by climate sensitivity. Therefore, climate sensitivity and the rate of oceanic uptake are considered to be uncorrelated in this study. However, Forest et al (2000) have recently shown that in order to reproduce observed climate changes of the past century, some interdependence between sensitivity and uptake is required. Improved uncertainty analysis using

---

8 The uncertainty in oceanic carbon uptake considered here is only that associated with uncertainty in the rate of carbon mixing into the deep ocean. Holian (1998) has shown that there is an additional uncertainty in carbon uptake associated with uncertainties in parameterizations of oceanic chemistry and carbon flux between atmosphere and ocean. According to his results, the total uncertainty in carbon uptake is twice as large as that associated with the uncertainty in the rate of oceanic mixing.
constrained distributions from detection studies such as Forest et al. (2000), are the subject of future work (see Chapter 8).

3.4 Continuous Distributions of IGSM Responses

In the decision models that will be developed in Chapter 4, distributions of uncertain parameters will be approximated by discrete distributions. To illustrate the behavior of the IGSM under uncertainty, however, in this section we will show continuous distributions for a few sample model responses. Section 3.5 will describe the damage function used to value the impacts of climate change in economic terms. In Section 3.6 we will develop the reduced form models of the IGSM that can be directly incorporated into the decision models of the next chapter.

3.4.1 Propagation of Uncertainty through the EPPA Model

Using the Deterministic Equivalent Modeling Method (DEMM), discussed in Chapter 2, the five uncertain economic parameters described in Section 3.3 are propagated through the EPPA model. The EPPA model projects carbon emissions and welfare losses from emissions constraint policies. The probability distributions of emissions and costs can be obtained for any emissions policy. In this section, we present a few results from two policy cases:

- Reference: no emissions constraints;
- Kyoto Protocol, No Trading: Kyoto Protocol commitments are imposed on the six Annex I regions beginning in 2010 and held constant out to 2100, and no constraints are imposed on Non-Annex I;

EPPA projects CO₂ emissions as a consequence of using coal, oil, and natural gas as energy inputs into the economy. As the economies of each of the twelve regions continue to grow from the increase in population, labor productivity, and investment, the demand for the energy increases, and so do CO₂ emissions. Figure 3.13 shows a trajectory of CO₂ emissions over time for the reference case (no emissions constraints) for reference assumptions for all parameters. Global emissions grow from a level of roughly 7 gigatons of carbon (GtC) today to about 22 GtC by the year 2100.

This projection of emissions is sensitive to assumptions about the rate of labor productivity growth over the century, the ability to substitute among production factors.
and in future technologies that may at some point become economically competitive. We can propagate the probability distributions for parameters described in Section 3.3 through the model, and obtain resulting probability distributions for emissions in each year. Figure 3.14 shows the probability distributions for emissions in three different years under the reference case, 2010, 2050, and 2100. While the mean or expected value for emissions is increasing over time, the variance or uncertainty also increases over time. By accounting for just the subset of uncertainties treated here, with 95% probability the CO₂ emissions in 2100 could be anywhere between 12 GtC and 40 GtC (between half of the reference projection to twice that).

The Kyoto Protocol in reality specifies emissions reduction objectives for a set of countries (Annex I) to achieve within the time period 2008-2012. No commitments beyond these years have been even discussed. In simulating such a policy with an integrated assessment model, it is necessary to make some assumption about the constraints in later years over the model's time horizon. Here we assume for purposes of illustration that the Kyoto commitments for 2010 are held constant up through 2100. Thus Annex I emissions will stay at the same level, while non-Annex I countries are allowed to increase emissions without restriction.

Figure 3.13 shows the trajectory for global CO₂ emissions under this simulation of the Kyoto Protocol. All the growth in emissions occurs in developing countries. Under the Kyoto commitments, global emissions only grow to about 12 GtC instead of 22 GtC for the unconstrained case. How do the same uncertainties in growth, substitutability, and
future technology affect emissions under a policy? The constrained regions will always emit up to their limit, and try to maximize welfare under this constraint. Thus the emissions will not be uncertain from these regions, but the loss of welfare from the policy will be uncertain. The unconstrained regions, on the other hand, will still have uncertainty in their emissions. Thus the impact of the policy on emissions will have some variance. Figure 3.15 shows the probability distributions of cumulative CO$_2$ emissions over the period 1990-2100 for both the reference case and the Kyoto case. Despite the large uncertainty in the total CO$_2$ emissions over the next century in both cases, the Kyoto constraints over a century would reduce emissions. The mean reduction
by the Kyoto case is 25% of cumulative emissions. A very low emissions world (lower 95% bound) would experience a reduction of 20% of cumulative emissions from this policy, while a very high emissions world (upper 95% bound) would have a 28% reduction in emissions.

There are several alternative measures of the cost of a policy. When an emissions constraint is imposed on a region, aggregate output may be reduced, production is shifted between sectors, and factors are shifted within sectors in order to maximize welfare under the new constraint (Jacoby et al., 1997). Thus both the aggregate consumption level and the composition of the consumption bundle will change. The loss in welfare associated with the reduced level of consumption is one possible measure of cost. The left side of Figure 3.16 shows the probability distribution for the costs of Kyoto to the USA in 2010, measured in terms of welfare loss. It is assumed that there is no trade in emissions permits for the case shown.

Another measure is the shadow price of carbon. This price represents the value of the resources shifted to reduce the last ton of carbon to just meet the constraint, or in other words the marginal cost. Thus, with no transaction costs, a carbon tax of this amount would theoretically achieve the same emissions levels as the constraint. The right side of Figure 3.16 shows the costs of Kyoto to the USA in terms of the carbon price in 1985 dollars. Thus one could summarize the expected value of the costs of this policy either as a loss in aggregate consumption of about 2%, or as equivalent to the imposition of a $150/ton carbon tax. The decision models will use the percentage loss in welfare as a measure of abatement cost under any sequential policy. In this form the abatement costs will be directly comparable to the losses as measured by the damage function, which are also in percentage terms (See Section 3.5).

The uncertainty in the costs of the Kyoto Protocol are quite uncertain in this model, given the distributions of the five uncertain parameters. Figure 3.17 shows the abatement costs in the form of carbon prices to the Annex I regions. The former Soviet Union (FSU) does not have a distribution because under the full range of uncertainty, the Kyoto Protocol targets never become binding because of the economic collapse of the region in the 1990's. We show the costs here for each region separately, although the decision models of the next chapter will treat Annex I as a single aggregate, in order to develop insights for a single decision-maker.
3.4.2 Propagation of Uncertainty through the 2-D Climate Model

The uncertainty in emissions described in the previous section results in uncertainty in CO₂ concentrations and in climate impacts such as temperature change. In addition, the ocean carbon sink and climate models have parameters that are also uncertain. In this section, the emissions uncertainty for the reference or no policy case is propagated through the climate model to obtain a description of the uncertainty in impacts, measured in terms of global mean temperature.
Figure 3.18: Uncertainty in CO₂ Concentrations

The ocean carbon sink model determines atmospheric CO₂ concentrations from an emissions trajectory. The main uncertain parameter for this model is the deep-ocean mixing parameter (Figure 3.12). DEMM is used to estimate the uncertainty in CO₂ concentrations in each decade resulting from uncertainty in the five economic parameters and the deep-ocean mixing parameter. Figure 3.18 shows the mean and 90% bounds on the path of CO₂ concentrations over time.

The propagation of this uncertainty through the climate model requires an intermediate step to be computationally feasible. A straightforward approach for estimating uncertainty in projected climate change would be to perform simulations that sample directly from the distributions for economic parameters, climate sensitivity, and the rate of oceanic heat and carbon uptake. This approach is not efficient for two reasons. First, it would require fitting the IGSM with a polynomial of seven variables (five economic and two climate parameters\(^1\)), while the climate-chemistry model only treats emissions as a single input conceptually. In using DEMM to produce an approximation of a more complex model, the number of input parameters and the order of the polynomial fit are the determinants of the number of runs required of the original model, and thus of the computational expense. Second, the climate-chemistry model needed to calculate CO₂ concentration from emissions is about three times more expensive computationally than

\(^1\) In the IGSM, the coefficient for vertical diffusion of carbon is assumed to be proportional to that for heat (i.e., perfectly correlated) (Sokolov et al., 1998; Holian, 1998), so there is no additional uncertainty associated with the rate of the oceanic carbon uptake.
just the climate model used in the previous section. For the analysis described here, the straightforward approach was considered not tractable in terms of the amount of computation time required. Using greater computational resources, more detailed uncertainty analysis is planned for the future that will include the atmospheric chemistry in order to treat other trace greenhouse gases and aerosols (Chapter 8).

An indirect parameterization is used to make propagation through the climate model feasible. The results of previously performed simulations with the climate-chemistry model have shown that CO₂ concentration is mainly determined by CO₂ emissions and the rate of oceanic carbon uptake and is not sensitive to predicted changes in atmospheric temperature and humidity, and thus to climate sensitivity (Prinn et al., 1999). Therefore, if increases in atmospheric CO₂ concentration can be described by a unique function of the whole range of possible emissions and rates of carbon uptake, then the necessary simulations can be performed with just the climate model, forced by the prescribed changes in atmospheric CO₂, and the computational cost of creating DEMM approximations will be greatly reduced. As long as this function depends on less than five (the number of uncertain parameters treated in the EPPA model) parameters, it will also decrease the total number of input parameters and, as a result, decrease the number of runs with the real model needed to obtain approximations.

The projected CO₂ concentration is determined not just by oceanic carbon uptake, but also by the carbon uptake in the terrestrial ecosystem. While present-day values of both oceanic and terrestrial uptake are uncertain, the total carbon uptake is fairly well known from the data for carbon emissions and the increase in atmospheric CO₂ concentration for the recent past. Values of the terrestrial carbon uptake used in the simulations with different values of vertical diffusion coefficients were calculated as the difference between the prescribed total uptake (4.1 PgC/year) and the value of oceanic uptake averaged over the last five years of the off-line simulation. As a result, total carbon uptake is essentially the same during the initial stages of all simulations, regardless of the rate of oceanic uptake. Because the relative uncertainty in emissions is small for the first few years, and because of the constraint on total carbon uptake, there is little difference between projected CO₂ concentrations for different values of input parameters until the year 2010 (see Prinn et al., 1999, Fig. 28; Sokolov et al., 1998).
A variety of functional forms with one or two parameters were tested to see which could best reproduce a variety of concentration paths over the whole range of economic parameters and the rate of carbon uptake. It was found that changes in atmospheric CO$_2$ concentrations for the years 2010-2100 are accurately approximated by an annually compounded percentage increase from an initial value. The values for CO$_2$ concentration in 1985-2009 use the corresponding concentrations resulting from reference values for all uncertain parameters. Thus the procedure to find a single number to construct an equivalent concentration path for a given set of EPPA parameter values and rate of oceanic carbon uptake is:

1. Use emissions metamodel to find an emission path over time for given set of economic parameters;

2. Use the oceanic carbon model to find the concentration path for the given emission path and the given rate of oceanic carbon uptake;

3. Find the percentage rate $\alpha$ which solves:

$$\min_{\alpha} \sum_{i=1}^{90} \left[ \left( C_i - \hat{C}_{i-1}(1 + \alpha) \right)^2 \right] : \hat{C}_0 = C_0 = C_{2010}$$

where $C_i$ is the concentration for year $i$ and $\hat{C}_i$ is the approximation based on exponential growth at rate $\alpha$.

With this approach, the distributions for the three economic uncertainties and the rate of oceanic carbon uptake can be propagated through the emissions metamodel, oceanic carbon model, and optimal fit procedure to yield a distribution of CO$_2$ rates of increase that reflects those uncertainties. This distribution can then be used as a third uncertain input into the MIT 2-D climate model (Webster and Sokolov, 2000).

Using the above procedure as a “true” model (see Chapter 2), a third order polynomial approximation is used to create a metamodel that directly calculates a percentage rate of CO$_2$ increase as a function of the six uncertain inputs. Performing 10,000 Monte Carlo simulations of this metamodel with sampling from the four input distributions yields a derived probability distribution for the percentage rate of increase of CO$_2$, shown in Figure 3.19.

The uncertainty in responses from the 2-D climate model are estimated by DEMM from three uncertain parameter distributions:
Figure 3.19: Uncertainty in CO₂ Expressed as Exponential Rate of Increase

- Rate of CO₂ Increase (Figure 3.19)
- Climate Sensitivity (Figure 3.12)
- Heat Uptake by Deep Ocean (Figure 3.12)

The uptake of heat in the climate model and the uptake of carbon in the ocean carbon sink model are assumed to be perfectly correlated. Therefore the rate of CO₂ increase, which is partly a product of the deep-ocean uncertainty, is not independent of the heat uptake uncertainty. Using the 10,000 Monte Carlo simulations of the ocean sink model, the partial correlation coefficient between deep-ocean heat uptake and CO₂ increase is calculated to be −0.60. Monte Carlo simulations of the climate model uses random samples designed to ensure this value of correlation between the two parameters. As an illustration of the uncertainty in the climate impacts of the reference case, we present the distribution of global mean temperature change in 2100 in Figure 3.20. In the next chapter we will consider sequential decision with uncertainty in climate impacts. Figure 3.21 shows the resulting distributions for temperature change from only uncertainty in economic parameters (dotted line) as compared with that resulting from uncertainty in climate sensitivity and uptake of heat and carbon. The majority of the variance in temperature change results from uncertainty in the ocean carbon sink and climate model parameters. Uncertainty in impacts in the decision models of Chapter 4 will therefore neglect uncertainty in emissions that exist even when constraining Annex I. The uncertainty in economic parameters will be used when we consider decisions under uncertain abatement costs in Chapter 6.
To construct a decision model, we need some means of modeling preferences over outcomes, which consist of both a stream of abatement costs and of global mean temperature change (or other climate indicators). The next section will describe the damage function that translates temperature impacts into economic losses.
3.5 Damage Functions

Up to this point, this chapter has described models for projecting economic costs of restricting CO₂ emissions and physical indicators of the impacts of emissions, such as global mean temperature change. Beginning in Chapter 4, we will develop models of optimal sequential choice under uncertainty, and use these models to investigate the role of learning. But in order to choose one set of strategies as “optimal”, we require a basis for comparing the costs of reducing emissions with the benefits of avoiding damages. A utility function for the decision-maker must be constructed that, explicitly or implicitly, values the tradeoffs between foregone consumption and climate damages. The MIT IGSM does not have a component that performs such a valuation of impacts, but we can add a damage function for the valuation step (see Figure 3.1). In this section we describe the damage function that is used to calculate the value of avoided climate damage in economic terms.

There are different approaches to capturing the tradeoffs between costs and impacts within a single utility function. One approach, commonly used in decision analysis, is to construct a multi-attribute utility (MAU) function (Keeney and Raiffa, 1993). The MAU approach allows different dimensions of an outcome to each have its own measure or “attribute” in its own units. For example, a utility function for climate change might have two attributes: consumption losses due to abatement and global mean temperature change. To construct the multi-attribute utility function, the analyst must elicit the explicit tradeoff weightings between attributes.

A second approach is to model the economic losses that occur as a result of a given amount of climate change, either in dollars or in percentage of gross product. This approach, known as the “damage function” approach, calculates the resulting benefits of a policy in the same units as the costs and thus allows the utility function to be defined over a single attribute. In the climate change literature, the damage function approach was first used by Nordhaus (1994) in his Dynamic Integrated Climate-Economy (DICE) model. The vast majority of climate change studies since then have used either the Nordhaus damage function or some variant (e.g., Kolstad, 1996; Lempert et al., 1996; Peck and Teisberg, 1992; Pizer, 1999). In this dissertation, we use the Nordhaus damage function for consistency and comparability with other integrated assessment studies of climate policy.
The Nordhaus damage function estimates the percentage loss of gross world product as a function of the global mean temperature change. The damage function takes the form:

\[ d(t) = \eta[\Delta T(t)]^\pi \]  \hspace{1cm} (3.1)

where \( d(t) \) is the fraction of world product lost due to climate damages in year \( t \), and \( \Delta T(t) \) is the increase in global mean temperature from preindustrial levels. We refer to \( \eta \) as the damage coefficient and \( \pi \) as the damage exponent.

Nordhaus (1994) bases his estimate for the damage coefficient on his study of the economic impacts of warming. His study of the impacts of a 3°C rise in global mean temperature for the United States estimates that the impacts in market sectors would be only 0.25% of gross domestic product (Nordhaus, 1991). Since the majority of impacts are believed to be in non-market sectors, he increases the total estimated loss to 1% of GDP. Finally, based on estimated shares of the relevant market and non-market sectors in various countries and the projected changes in these shares, the global estimated loss from a 3°C rise in temperature is 1.33%.

Damages from climate change are generally believed to be non-linear in temperature change, and Nordhaus assumes that damages are quadratic in temperature. Thus, with the added assumption of zero damage from zero increase in temperature, Nordhaus’ reference damage function is:

\[ d(t) = 0.0133 \left( \frac{\Delta T(t)}{3} \right)^2 \]

The parameters of this damage function are extremely uncertain. These parameters are based on future projections that cannot be made with much confidence, including the regional impacts of climate change around the globe, the effects of these changes on different market and non-market activities, and the share of these activities in different nations’ economies. Varying estimates exist for a “best guess” of the percentage economic loss from a doubling of CO₂. Table 3.1, taken from the IPCC (Bruce et al., 1996), shows the range of estimates for the US only. Most researchers of the climate issue believe there to be a wide range of uncertainty around these estimates. Furthermore, there is disagreement between experts on what a reasonable range of uncertainty in damage costs is. Nordhaus (1994b) conducted a survey of 19 experts, including natural
scientists, environmental economists, and other social scientists. He elicited probability estimates of the losses to the global economy due to three different climate change scenarios: a 3°C warming by 2090, a 6°C warming by 2175, and a 6°C warming by 2090. Probability information was elicited in the form of both fractiles (0.1, 0.5, and 0.9) and of the probability in each scenario of losses exceeding 25% of world product.

Nordhaus (1994) used his expert assessment results to construct a probability distribution for the climate damage loss from a doubling of CO₂. However, Nordhaus "trims" his data by removing the three highest and three lowest estimates from the experts. Roughgarden and Schneider (1999) present a distribution that is more representative of the full range of expert opinion. They fit Weibull distributions to each expert’s assessed fractiles, and aggregate to one distribution. The cumulative density function for the Roughgarden and Schneider aggregate is shown in Figure 3.22.

Several previous studies of uncertainty have compared the relative importance of the two parameters of the damage function, η and π, to each other and to other parameters in causing variance in optimal emissions control rates (Nordhaus, 1994; Nordhaus and Popp, 1997; Roughgarden and Schneider, 1999). These studies conclude that, over the range of temperature increases typically studied, the uncertainty in the damage coefficient η causes greater uncertainty in outcomes and optimal choice than the damage exponent π. Based on these findings, in this thesis we represent the uncertainty in the valuation of climate damages by treating only the uncertainty in the damage coefficient and assume that π = 2.

The distribution in Figure 3.22 is assumed to represent the uncertainty in the parameter η in equation (3.1). The sequential decision models will use a discrete distribution as an approximation of the continuous distribution. The derivation of the discrete distribution is described in Section 3.6.2.
3.6 Reduced-Form Models for use in Decision Analysis

This chapter has thus far described the MIT Integrated Global System Model and the uncertainty in key model responses under two sample policies, reference (no policy) and the Kyoto Protocol. The main objective of this dissertation is to explore the implications of learning on emissions abatement choice in a multi-stage decision framework. Chapters 4-7 will present several models of sequential decision under uncertainty, implemented in the form of decision trees and influence diagrams. Because of the complexity and computation time required, the full IGSM cannot be used for calculating the costs and impacts for each path through a decision tree. Instead, reduced-form models are used that approximate the behavior of the IGSM.

In this section, we describe the estimation using DEMM of reduced-form models that can be used within the sequential decision models. We also convert the continuous distributions of the uncertain parameters described above into discrete distributions, which simplify the sensitivity analysis of the decision models.

3.6.1 Reduced-Form Models for a Two-Stage Decision Problem

The probability distributions of responses presented in Section 3.4 are generated from polynomial chaos expansions, which approximate the probabilistic response of the output variable as a function of the uncertainty in the parameters (see Chapter 2). These
reduced-form models produced by DEMM, while primarily developed for propagating uncertainty, can also be used as a reduced-form approximation of the original model (Calbo et al., 1998; Mayer et al., 2000). Here we describe the use of reduced-form models, constructed by applying DEMM to components of the IGSM, for performing the calculations necessary for two-stage models of decision under uncertainty.

The conceptual framework for the decision models to be used in the analysis of sequential choice and learning will be described in detail in Chapter 4, but we briefly review its key elements here, because they motivate the design of the reduced-form models. With the focus on the implications of learning for sequential policy choice, we construct decision models with two periods as the simplest possible representation of multiple stage decisions. The first model decision stage is defined as the decade 2010-2020, and the second decision stage is 2020-2100.

The decision models applied here all assume a single decision-maker representing an aggregate of the Annex I countries. We assume that only Annex I countries abate emissions, and that emissions from the non-Annex I countries grow unconstrained. To construct the reduced form models, we need simple choice variables for each of the two decision periods that are applied equally to all Annex I regions. We define the control strategy for each period to be the maximum allowable annual rate of increase in emissions. The rate of increase is chosen for its flexibility, since it allows for slowed emissions growth, stabilization, or emissions reductions.

In addition to a maximum allowable rate of increase, the first period strategy also simultaneously specifies to what degree the reductions called for in the Kyoto Protocol will be met. This allows an optimal first period strategy choice in any experiment to be compared with the stringency of the Kyoto agreement. Since the most stringent emissions reductions by 2010 currently under discussion are the Kyoto levels, these emission levels are calibrated to be one extreme of the range of possible strategies, and the other extreme is no emissions constraints at all.

We define the strategy variables for the two decision periods as Policy2010 and Policy2020, respectively. Both strategies are defined over a continuous range, as shown in Table 3.2. The strategy for the first period can range from emissions stabilization (at Kyoto Protocol levels) to unconstrained growth, and applies to 2010 and 2015. The
Table 3.2: Possible Strategies in each Decision Period

<table>
<thead>
<tr>
<th>Decision Period</th>
<th>Strategy Variable</th>
<th>Years</th>
<th>Most Stringent Constraint</th>
<th>Least Stringent Constraint (No Limits on Emissions Growth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Policy2010</td>
<td>2010-2019</td>
<td>0%/Year</td>
<td>1.4%/Year</td>
</tr>
<tr>
<td>2</td>
<td>Policy2020</td>
<td>2020-2100</td>
<td>-0.8%/Year</td>
<td>1.2%/Year</td>
</tr>
</tbody>
</table>

second period strategy ranges between a 0.8%/year continuous reduction in emissions over 80 years to unconstrained growth, and applies to 2020, 2025, ..., 2100.

The two strategy variables Policy2010 and Policy2020 provide a compact means of specifying a wide range of emissions paths over time. We can use DEMM to construct reduced-form models to produce the needed abatement costs and damage costs as functions of Policy2010 and Policy2020. The decision models of subsequent chapters will use total cost – the sum of discounted abatement costs and damage costs over time – as the disutility function to be minimized. The procedure for calculating the total costs for a particular strategy and for particular values of uncertain parameters, using the reduced-form models, involves six steps:

1. Calculate CO₂ emissions and abatement costs
2. Calculate CO₂ concentrations
3. Convert concentrations time-series into a rate of increase
4. Calculate global mean temperature change over time
5. Apply damage function to get damage costs
6. Discount and combine abatement costs and damage costs

Each step is described in detail below.

Step 1: Calculating CO₂ emissions and abatement costs for a sequential strategy choice

The first step is to use reduced-form models to calculate global CO₂ emissions and abatement costs to each region over time. The reduced-form models were obtained by treating the two strategy variables as pseudo-random variables:

\[
\text{Policy2010} \sim \text{Uniform}(0, 1.4) \\
\text{Policy2020} \sim \text{Uniform}(-0.8, 1.2)
\]

Using these distributions, a polynomial chaos expansion is obtained for

- Global CO₂ Emissions(\(t\)) where \(t = 2000, 2005, 2010, \ldots, 2100\)
- Welfare Loss \((r, t)\) where \(r = \text{USA, EEC, JPN, OOE, FSU, and EET},\) and 
  \[t = 2000, 2005, 2010, \ldots, 2100\]

The uncertain parameters of the EPPA model are not used for these expansions, and are kept fixed at their reference values. We treat these parameters as certain in this estimations because in Chapter 4 we assume certainty in abatement costs, and as shown in Section 3.4 the uncertainty in emissions can be neglected with only a minimal reduction in variance of global mean temperature increase. EPPA parameters will be treated as uncertain in Chapter 6, when we introduce uncertainty in abatement costs into the decision models.

Each response is approximated by a fifth-order expansion in orthogonal polynomials, which are functions of the two strategy variables. Note that the basis functions (orthogonal polynomials) are the same for every model response, and only the coefficients differ. Fifth-order was required to reduce the errors in all the response variables to less than 1\% of the mean value. An example of one such expansion for CO\(_2\) emissions in 2050 is:

\[
\begin{align*}
\text{Emissions}(2050) &= 12.00 - 0.2643 \text{ Policy2010} + 0.1639 \text{ Policy2010}^2 \\
&\quad - 0.04027 \text{ Policy2010}^3 + 0.005483 \text{ Policy2010}^4 - 0.0002868 \text{ Policy2010}^5 \\
&\quad + 0.1170 \text{ Policy2020} - 0.05971 \text{ Policy2020}^2 + 0.007408 \text{ Policy2020}^3 \\
&\quad - 0.002932 \text{ Policy2020}^4 + 0.0003306 \text{ Policy2020}^5 \\
&\quad - 0.06523 \text{ Policy2010 Policy2020} + 0.03110 \text{ Policy2010}^2 \text{ Policy2020} \\
&\quad - 0.002626 \text{ Policy2010}^3 \text{ Policy2020} + 0.09473 \text{ Policy2010 Policy2020}^2 \\
&\quad - 0.02118 \text{ Policy2010}^2 \text{ Policy2020}^2 + 0.001313 \text{ Policy2010}^3 \text{ Policy2020}^3 \\
&\quad - 0.004622 \text{ Policy2010 Policy2020}^3 + 0.0006603 \text{ Policy2010}^2 \text{ Policy2020}^3 
\end{align*}
\] (3.2)

This approximation is shown with terms expanded and simplified for ease of readability. The approximations for all other responses have the same form as equation (3.2) but different coefficients.

For emissions, it is feasible to estimate global emissions as a function of the collective Annex I sequential control strategy, rather than estimating emissions for each region separately. Thus the expansion in equation (3.2) is for global emissions. However, abatement costs under any strategy exhibit different behavior by region (see Section 3.4). Abatement costs are estimated separately for each Annex I region and then
aggregated to obtain the total Annex I costs. Because this analysis considers only a decision maker, all regional costs must be eventually aggregated.

Figure 3.23 shows the accuracy of two different expansions: global CO$_2$ emissions in 2050 and the welfare measure for the USA in 2050. Both graphs are scatter plots of actual results from the EPPA model as compared with results from the DEMM expansion for 19 different strategy pairs (Policy2010, Policy2020). A perfect approximation would have all points on the 45° line. The expansions shown, typical of all the response approximations, have very little error.

Thus the first step in calculating the costs of a two-stage strategy is to use this set of expansions to calculate emissions and abatement costs in five-year intervals.

**Step 2: Calculating CO$_2$ concentrations for a strategy choice**

The concentrations of CO$_2$ are outputs of the Ocean Carbon Sink Model (see Section 3.1). CO$_2$ concentrations over time are determined by the emissions of CO$_2$ (results from the EPPA model) and the (uncertain) deep-ocean mixing parameter, $K_s$. We have constructed reduced-form models of CO$_2$ concentrations in ten-year intervals as functions of the two policy variables as in step 1 and the ocean diffusion parameter:

$$\text{CO}_2 \text{ Concentration}(t) = f(\text{Policy2010}, \text{Policy2020}, K_s)$$

$t = 2010, 2020, 2030, \ldots, 2100$
The approximations of concentration are of the same form as equation (3.2), except that a fourth-order expansion was sufficient to reduce the errors to less than 0.1% of the mean for all responses.

After the approximations are used to calculate the concentrations of CO₂ in ten-year intervals, these values are linearly interpolated to obtain yearly concentrations from 2010 to 2100 as a function of strategy choice.

Step 3: Convert concentrations time series into a yearly percentage rate of increase

In preparation for calculating global mean temperature impacts, the next step is to use the procedure described in Section 3.4.2 for converting a concentration time-series into an annual rate of increase. For the strategy pair (Policy2010, Policy2020) and the value of ocean uptake, Kᵣ, for which we have calculated emissions and concentrations in Steps 1 and 2, we solve for an equivalent rate α to minimize:

\[
\text{Min}_{\alpha} \sum_{i=1}^{90} \left( \left(C_i - \hat{C}_i \cdot (1+ \alpha) \right) \right)^2 : \hat{C}_0 = C_0 = C_{2010}
\]  

(3.3)

Step 4: Calculate Global Mean Temperature Change

The reduced-form climate model (Section 3.4.2) approximates decadal averages of global mean temperature change as a function of the (uncertain) climate sensitivity (sens), (uncertain) oceanic uptake (Kᵣ), and the rate of increase of CO₂ concentrations (α):

\[\Delta T(t) = f(\text{sens}, Kᵣ, \alpha) \quad t = 2000, 2010, \ldots, 2100\]

Approximations with reasonably small errors were obtained using a third-order expansion in the three parameters. For example, the expansion for the decadal average temperature change at 2100, in expanded and simplified form, is:

\[\Delta T(2100) = -0.8527 + 0.2109 \text{ Sens} - 0.06622 \text{ Sens}^2 + 0.003152 \text{ Sens}^3\]
\[+ 0.2800 Kᵣ - 0.02271 Kᵣ^2 - 0.003618 Kᵣ^3\]
\[+ 279.3 \alpha - 63637 \alpha^2 + 4.715 \times 10^4 \alpha^3\]

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\[ -44.57 \alpha K + 244.7 \alpha Sens - 0.1312 K Sens \\
+ 1448 \beta^2 K + 8.210 \alpha K + 10353 \alpha^2 Sens + 0.01330 K Sens^2 \\
- 6.569 \alpha Sens^2 + 0.005825 K Sens^2 + 9.628 \alpha K Sens \]

The value for \( \alpha \) calculated in step 3 (which is derived via equation (3.3) from a particular pair (Policy2010, Policy2020)), along with the values for climate sensitivity and oceanic uptake, are used to obtain the global mean temperature change in ten-year intervals. The decadal averages are then linearly interpolated to obtain yearly estimates for global mean temperature change.

**Step 5: Apply Nordhaus Damage Function**

The temperature change time-series calculated in step 4 is then used to determine economic damages. The Nordhaus damage function, described in Section 3.5, is applied to the temperature change data to obtain the percentage losses to welfare in each region and year. Using a form of equation (3.1) with quadratic dependence on temperature change, and uncertain overall damage coefficient \( \eta \), we solve for:

\[ d(t) = \eta[\Delta T(t)]^2 \]

The fractional losses due to climate damage are multiplied by the projected welfare in the absence of policy to obtain the annual damage costs in dollars. These costs are then discounted back to the year 2000 at a reference discount rate of 3% to obtain total damage costs for each region.

**Step 6: Combine abatement and damage costs**

The abatement costs in five-year intervals for each region are linearly interpolated to obtain yearly abatement costs for each region. The annual abatement costs are discounted at the reference rate of 3% back to 2000 for each region. Because the decision-maker’s objective is to minimize aggregate total cost for Annex I, we sum the abatement costs for all regions, and sum the damage costs for all regions.

\[ abate \ costs = \sum_r abate \ costs(r) \]

\[ damage \ costs = \sum_r damage \ costs(r) \]

\[ r = USA, EEC, JPN, OOE, FSU, and EET \]
Finally, abatement costs and damage costs are combined to obtain total costs to Annex I. This total cost is the disutility measure to be minimized in the decision models of subsequent chapters.

3.6.2 Development of discrete distributions for uncertain parameters

This chapter has presented probability distributions for several important uncertain parameters in the assessment model. The decision trees in later chapters can be implemented with these uncertainties represented as continuous distributions, using Monte Carlo simulation in finding the optimal choice. However, we will instead approximate the continuous distributions with discrete distributions for the uncertain parameters for three reasons:

- Ease of performing sensitivity analysis,
- Ability to effectively approximate continuous distributions with discrete distributions, and
- Ease of implementation of decision model.

The probability distributions for parameters are obtained from expert elicitation, and so are subject to all the biases of subjective judgment about probability (see the discussion in Section 3.3.1). Also, for almost all parameters, there is wide disagreement between experts. It is crucial therefore to subject all results from decision models to sensitivity testing of the assumed distributions. Moreover, an additional objective is to find what one must believe about the uncertainty in climate damages and abatement costs in order for some level of abatement to be optimal in the near-term. A discrete distribution with a small number of values is easiest to test for sensitivity by varying the corresponding probabilities of each value. A sensitivity test of a continuous distribution can be performed by varying some parameters of the distributions (e.g., mean and variance), but this is more difficult both to implement and to present the results.

Secondly, for roughly symmetric distributions such as the parameter distributions in this study, there is an effective method for approximating the continuous distribution with a discrete distribution with minimal loss of information. The extended Pearson-Tukey method (Keefer and Bodily, 1983) is commonly used by decision analysts. Finally, a decision model with discrete uncertainties and discrete strategy choices is easier to implement. The number of calculations and variables in the procedure outlined
Table 3.3: Pearson-Tukey Approximation for a Continuous Distribution

<table>
<thead>
<tr>
<th>Uncertain Value</th>
<th>Fractile from Continuous Distribution</th>
<th>Probability of Discrete Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.05</td>
<td>.185</td>
</tr>
<tr>
<td>2</td>
<td>.50</td>
<td>.63</td>
</tr>
<tr>
<td>3</td>
<td>.95</td>
<td>.185</td>
</tr>
</tbody>
</table>

above in Section 3.6.1 would be difficult to represent within the decision analysis software that is used for constructing and solving the decision models (TreeAge, 1999). Instead, the calculations are performed offline for a predetermined set of two-stage strategies and uncertain parameters, and the results are saved in a lookup table for use by the decision model.

The Pearson-Tukey approximation uses three fractile values from the continuous distributions, and assigns probabilities to these discrete values as shown in Table 3.3. Using this approach, we choose three values for each uncertain parameter. For example, to represent uncertainty in climate damages we include climate sensitivity, ocean uptake, and the damage function coefficient. This results in 27 different outcomes for each two-stage strategy. However, we can further reduce the dimensionality of the decision problem without significant loss of information. The climate change decision problem is one in which median values for the various uncertainties do not warrant any emissions abatement in the near-term (Wigley et al, 1996; Valverde et al, 1999; Kolstad, 1996). Because of the focus of the analysis is on when first period abatement becomes optimal, separately representing a low value and a median value for climate sensitivity, for example, adds no information to the analysis. The optimal strategies are always the same for the median and low values for climate sensitivity and damage coefficient, and for the median and high values for ocean uptake. Thus, without significantly altering the results, we can reduce the discrete distributions to two values: a median value and a high (or low) value. Consistent with the Pearson-Tukey approximation, we assign reference probabilities of 0.8 and 0.2 to the median and extreme values, respectively. These probabilities become the base from which sensitivity analysis is performed on all the decision models.

The discrete distributions for uncertain damage parameters are shown in Table 3.4. These distributions are used in all decision models with uncertain damages and
Table 3.4: Discrete Distributions for Decision Models

<table>
<thead>
<tr>
<th></th>
<th>Branch 1 (P=0.8)</th>
<th>Branch 2 (P=0.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate Sensitivity (°C)</td>
<td>2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Oceanic Uptake (cm²/s)</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Damage Cost Coefficient (%)</td>
<td>.02</td>
<td>.16</td>
</tr>
</tbody>
</table>

certainty in abatement costs in Chapters 4-7. Reduced-form models that also include uncertainty in abatement costs will be described in Chapter 6.
Chapter 4  The Role of Learning in Sequential Decision: An Example Using the IGSM

_Risk - If one has to jump a stream and knows how wide it is, he will not jump. If he doesn't know how wide it is, he'll jump and six times out of ten he'll make it._

- Persian Proverb

The political debate on a response to the threat of global climate change has been deadlocked between the view that action should be delayed until the climate system is better understood and the view that the risks of waiting are too great and that emissions abatement is needed now. The time scale of climate change necessarily implies that any policy action taken now can and probably will be revised repeatedly in the future as we learn more, and as political and economic conditions change. Because of this sequential nature of climate policy, the tools of decision analysis have been applied in the attempt to yield insights into what level of emissions abatement should be taken now given that we may learn more in the future. Most analyses to date of sequential climate policy under uncertainty appear to support the argument that delaying emissions reductions until more is understood about climate is preferable to expensive emissions reductions that may prove unnecessary (Wigley et al., 1996; Manne and Richels, 1995; Nordhaus, 1994; Valverde et al., 1999).

The MIT IGSM, described in Chapter 3, represents feedbacks and nonlinear processes in both the economic and the climate submodels. Previous analyses of sequential choice under uncertainty have used much simpler parameterizations (e.g., Kolstad, 1996; Manne and Richels, 1995). The detailed functional relationships in the IGSM allow the propagation of uncertainty across submodels while accounting for nonlinear dependencies between uncertain parameters. Using the IGSM as an underlying model that captures the complex feedbacks and interactions within economies and between the atmosphere, ocean, and biosphere, we explore the effect of learning on a sequential choice over emissions constraints under uncertainty in multiple parameters.

The main question addressed in this chapter is: "Is waiting for information a valid argument for a no-abatement strategy in the near-term?" A sequential decision framework is used to explore the effect that learning has upon the optimal choice of a
near-term strategy, when the strategy can be revised later. As will be shown, a decision-analytic framing of climate policy using the IGSM almost always leads to the same optimal first period strategy, independent of whether or not uncertainty is resolved before the next decision time.

In order to explore the implications of learning and sequential decision for near-term emissions reductions, many simplifications are made. Decision-makers about climate policy in reality include negotiators for the 160+ nations that have signed the FCCC, their national governments, local authorities, firms, NGOs, and individuals. This chapter develops insights for decision making under uncertainty for a single decision-maker. The implications of decision under uncertainty when there are several parties to a negotiation are not addressed in this dissertation, but possible approaches for extending the analysis here to treat this issue are outlined in Chapter 8. For simplicity, we also limit the decision possibilities to a choice over the level of emissions abatement. In addition to abatement, other elements of a complete climate policy include research into the behavior of the climate system, the costs and impacts of climate change, and possible mitigation options, the development of adaptation strategies for the climate change that will occur, and geoengineering solutions (Manne and Richels, 1992).

This chapter is laid out as follows. Section 4.1 will develop decision models to represent the choice of near-term reductions when the decision can be revised. The models consist of two periods, and include uncertainty in the damages from climate change. Two alternative cases are considered:

i) Complete resolution of uncertainty before the next decision period, and

ii) No resolution of uncertainty until after all decisions are made.

Section 4.2 will compare the solutions for these two cases under a wide range of assumptions. For abatement costs and damages derived from the MIT IGSM, the optimal first period strategy will be shown to be the same in both cases for almost any assumption about the uncertain climate damages. Previous studies (Kolstad, 1996, Nordhaus, 1994) have conjectured that the independence of near-term strategy from learning is related to the stock nature of greenhouse gases. Alternative decision models that vary the magnitude of the “stock effect” are also developed, and the near-term strategy is shown to be still mostly uninfluenced by the timing of the resolution of uncertainty. Although the decision models presented here use abatement costs from an economic model without
perfect foresight, Section 4.3 describes why the same results would be expected from the use of a forward-looking optimal growth model.

The explanation for the lack of influence of learning on first-period strategy is left for Chapters 5 and 6. In Chapter 5, we will develop an analytical two-period decision model and use it to demonstrate that strategy will only depend on learning if the period 1 strategy affects the marginal costs of period 2. We will show in Chapter 6 that the learning has no effect because such interactions are to weak in the IGSM. But first, in this chapter, we must construct the decision models and show how the first period depends (or doesn’t) on learning.

4.1 Framing the Decision Problem

In the previous chapter, DEMM was used to derive reduced form models of various components of an integrated assessment model of climate change in order to quantify the uncertainty in system responses. Having derived these reduced form models and quantified the uncertainty, we can now turn to the task of constructing a model of the decision about climate policy. Recall from Chapter 1 that to choose a policy response to climate change, we want to choose a policy today given that

- Outcomes are uncertain,
- The policy can be revised over time,
- Effects of decisions may be irreversible,
- Some uncertainty may reduced, and
- Greenhouse gases are a stock pollutant.

We now develop a decision model that can capture all of these characteristics of climate change policy choice.

Decision analytic methods, decision trees and influence diagrams, are used to represent the decision problem. These methods allow the explicit representation of sequential choice of strategies over multiple decision periods and of different degrees of uncertainty at each decision point. Using backward induction as the solution method, these types of models solve for the strategy at the first decision point that maximizes or minimizes the expected utility of the decision-maker. This utility is measured by a function that represents the modeled decision-maker’s preferences over outcomes. The elements of a decision model that must be specified are:
1) The decision periods,
2) The utility function to be maximized or minimized,
3) The set of strategies available to the decision-maker in each period,
4) The uncertain quantities and their probabilistic description at each decision point, and
5) The means of calculating the outcomes as a function of strategy choices and the values of uncertain quantities.

The agreements, policies, and choices regarding human activities that emit greenhouse gases will continuously be revised over the next century. Changes in climate-related policies will occur because of new information from research, observations of climate and climatic events, changing political and economic circumstances, and perhaps other reasons as well. To investigate the effect of what we may learn later upon choices we make now, we need to represent a minimum of two distinct decision periods. By including two or more periods in a model, the initial strategy may be revised at least once, allowing a response to new information if received. Although including more than two periods would give more detail and more flexibility in the timing of actions, it is unlikely that more than two periods would significantly alter the role of learning between periods on the optimal strategy in the period before the learning (Intriligator, 1971). However, a three period example is worth exploring and is included in the list of future extensions of this work (Chapter 8).

The current political response to climate change is the Kyoto Protocol, which specifies emission level targets for Annex I countries to achieve within the 2008-2012 time period (see Chapter 1). Although the Kyoto Protocol has been signed it has not been ratified by enough nations to enter into force, and its implementation and achievement of its goals are currently in doubt. We design the first period in our model to allow the re-examination of abatement levels for 2010 that appear optimal in a cost-benefit sense. Due to the inertia in energy systems and other major greenhouse gas emitting industries, abatement before 2010 is not likely to be significant. Similarly, a decade is assumed to be the minimum time required for new information to be generated by research and observations and reacted to by increasing or decreasing the stringency of emissions targets. Thus the two periods in this decision model are defined as:
Period 2: 2020-2100.

The strategy is chosen at the start of each period, and then held constant over that period.

The next element to be defined is the utility function to measure preferences over outcomes. Before we can determine the decision-maker's preferences, we must identify who the decision-maker is. In reality there are many decision-makers that affect greenhouse gas emitting human activities, each with different individual preferences. The decision model developed here is for a single decision maker. An extension of this analysis to multiple decision-makers with differing preferences is discussed in Chapter 8. The industrialized Annex I nations are required by the Framework Convention to take the lead in addressing climate change, and commit to near-term emissions reductions under the Kyoto Protocol. Therefore we define the unitary decision-maker within this model as a representative for the aggregate of Annex I nations.

Ideally, we wish to model a utility function that defines relative preferences over several attributes of any outcome. Concerns about climate change arise from a wide range of possible undesirable effects, including species loss, damage to natural ecosystems, increased frequency and severity of floods, droughts, and hurricanes, losses to coastal activities due to sea level rise, and agricultural losses. Unfortunately, the ability of current assessment models to predict the specific physical and biological impacts of climate change is severely limited, as is the ability to use detailed data by effect and region. Instead, global mean temperature change is often used as a proxy measure for the overall magnitude of the sum of such effects.

One form for a utility function is to construct an explicit multi-attribute model of preferences, complete with tradeoffs across attributes. One example would be a function over two attributes: consumption loss due to abatement activities (units in $) and global mean temperature increase (units in degrees C). An alternative form is to perform an economic value calculation on the temperature change with a damage function (see Section 3.5), and then specify a single attribute utility function in dollar units. The multi-attribute approach is more explicit about tradeoffs and allows different risk preferences to be defined over each attribute, but requires the elicitation of someone's preferences. The elicitation is difficult for a fictional aggregate decision-maker such as "Annex I". The damage function approach obscures the tradeoffs between economic losses and non-
market environmental effects, but has been more commonly used in many other analyses of climate policy. For consistency with the literature, we use the damage function approach here, subjected to thorough sensitivity analysis across a wide range of valuation. The possible advantages of the multi-attribute approach are discussed in Chapter 8.

For any chosen strategy and any realization of uncertain quantities, the outcome is a stream of consumption losses over time due to abatement activity, and a stream of economic losses due to climate change, as measured by global mean temperature rise and valued by the damage function. Both of these streams are discounted back to a single cost at a reference discount rate of 3%. The sum of abatement costs and damage costs in dollars is the attribute over which preferences are measured. The objective of this model is to minimize the expectation of the total costs. We assume for the default models that the decision-maker is risk-neutral. Finally, we note that the abatement costs are obtained from a recursive dynamic general equilibrium model (see Chapter 3), which has no knowledge of future prices. The implications of the choice of underlying economic model are discussed in Section 4.3.

For each of the two decision periods, the set of strategies from which the decision-maker chooses must be specified. This model represents choice over levels of emissions abatement only; other possible complementary policies of research, adaptation, and geoengineering are not modeled here. Furthermore, as mentioned above, because the focus here is on the "waiting" option today, only Annex I nations constrain emissions in this model. The less developed nations increase their emissions of greenhouse gases unrestricted. The analysis is easily extended to cases where non-Annex I also constrains emissions, either now or in some future period, but the insights gained here about the role of learning would not change. Because the decision-maker is "Annex I", the strategy choice must be applied equally across the Annex I regions. These models represent choices of CO₂ abatement only. Other trace greenhouse gases, such as CH₄ and N₂O are assumed to increase in all regions at reference rates of increase (Prinn et al., 1999), and are unaffected by emissions constraints. Reductions in non-CO₂ greenhouse gases could be added to this model at additional cost in computation, and this extension is discussed in Chapter 8.
Table 4.1: Strategy Choices in each Period

<table>
<thead>
<tr>
<th>Decision Period</th>
<th>Strategy Variable</th>
<th>Years</th>
<th>Most Stringent Constraint</th>
<th>Least Stringent Constraint (No Limits on Emissions Growth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Policy2010</td>
<td>2010-2019</td>
<td>0%/Year</td>
<td>1.4%/Year</td>
</tr>
<tr>
<td>2</td>
<td>Policy2020</td>
<td>2020-2010</td>
<td>-0.8%/Year</td>
<td>1.2%/Year</td>
</tr>
</tbody>
</table>

Table 4.2: Emission Targets for 2010 as a Function of Strategy Level

<table>
<thead>
<tr>
<th>Policy2010</th>
<th>2010 Emissions Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100% Kyoto</td>
</tr>
<tr>
<td>0.2%</td>
<td>85% Kyoto</td>
</tr>
<tr>
<td>0.4%</td>
<td>70% Kyoto</td>
</tr>
<tr>
<td>0.6%</td>
<td>55% Kyoto</td>
</tr>
<tr>
<td>0.8%</td>
<td>40% Kyoto</td>
</tr>
<tr>
<td>1.0%</td>
<td>25% Kyoto</td>
</tr>
<tr>
<td>1.2%</td>
<td>10% Kyoto</td>
</tr>
<tr>
<td>1.4%</td>
<td>Reference (No Controls)</td>
</tr>
</tbody>
</table>

The first period strategy is a choice of how much of the Kyoto Protocol emissions reduction targets will be met. Thereafter, the strategy in each period also specifies how emissions are allowed to grow or how quickly they are reduced. The strategies are specified as the maximum allowable growth (or decrease) in emissions as a percentage of the previous year’s emissions. The ranges of possible rates of increase/decrease in emissions are given in Table 4.1. The period 1 strategy may be any rate between 0% per year (stabilization of emissions) to an allowable increase of 1.4% per year (which does not constrain the emissions of any region). The period 2 strategy is a rate between −0.8% per year (a continuous decrease in emissions) and 1.2% per year (no constraint on emissions). The period 1 strategy also determines the level of emissions in 2010. Table 4.2 shows the corresponding emissions limits in 2010 as a function of the rate chosen. This two-component definition of the first period strategy is to facilitate the direct comparison of the period 1 strategy choice with the Kyoto Protocol emission levels for different cases of the decision models.

Figure 4.1 shows the emissions paths over time that result from three sample 2-period strategies that could be chosen. The solid line shows emissions from a choice of 1.4% in the first period and 1.2% in the second period, which allows emissions to grow unconstrained (i.e., this is the reference case). The dashed line shows the emissions when the first period strategy is 0% and the second period strategy is 0.6%. Here Annex I
emissions are reduced to the level committed to in the Kyoto Protocol in 2010, held at this level at 2015 (0% growth), and from 2020 on emissions are allowed to grow at a rate of 0.6% per year. The dotted line shows emissions when the period 1 strategy is 0.4% and the period 2 strategy is 0%. Here the Annex I emissions in 2010 are reduced by 70% of the difference between the reference emissions level and the Kyoto commitment level. Note that a slight increase is allowed between 2010 and 2015 as well. Then from 2020 onward, emissions remain stabilized at this level (0% growth).

The uncertain quantities that affect the outcomes must also be specified. Initially we focus on the uncertainty that affects the impacts of climate change. Three parameters of the underlying models that have a strong influence on the ultimate damage costs are the climate sensitivity, the oceanic uptake of heat and carbon, and the damage valuation or coefficient in the damage function. Each of these parameters is described in Chapter 3. Continuous distributions for parameters elicited from experts are represented by two-point discrete distributions, and the development of these discrete distributions is described in Section 3.6.2. The distributions of the uncertain quantities are shown in Table 4.3. The relative probabilities of these values are varied from 0 to 1 in sensitivity.

<table>
<thead>
<tr>
<th>Table 4.3: Distributions for Uncertain Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Climate Sensitivity (°C)</td>
</tr>
<tr>
<td>Oceanic Uptake (cm³/s)</td>
</tr>
<tr>
<td>Damage Cost Coefficient (%)</td>
</tr>
</tbody>
</table>
testing of all results below. Also, for reasons described in Chapter 3, we assume that these three uncertain quantities are probabilistically independent.

The three uncertainties considered affect the damage costs, but not the abatement costs. Despite the large uncertainty in abatement costs of any particular policy, these costs will initially be considered certain at their mean or reference values, as obtained from EPPA and its equivalent reduced form models. In Chapter 6, the effect of uncertainty in abatement costs will be considered. Initially, however, we begin by asking what is the effect of learning about the level and value of climate damages, which is the focus of most debates about uncertainty in climate change (see Chapter 1).

The distributions in Table 4.3 describe what is known about the uncertain quantities when the period 1 strategy is chosen. Some of this uncertainty may be reduced or even resolved before the second period decision must be made. In order to understand the influence of the timing of the resolution of uncertainty on the first period's strategy, we will compare two extreme cases:

1) "Complete Learning": in which the true value of all uncertain quantities become known with certainty before the period 2 strategy is chosen, and
2) "No Learning": in which the beliefs about the uncertain quantities in period 2 are unchanged from period 1, and these beliefs are described by the distributions in Table 4.3.

These two cases bound the amount of reduction in uncertainty possible between the two periods. If the optimal strategy in period 1 has some dependence on the amount of learning that occurs, then the maximum difference in the optimal level of abatement will be the difference between these two cases.

There are several ways in which uncertainty can be reduced (Kolstad, 1996). One way is active learning, in which the evolution of the climate and economy are observed and beliefs are updated. A second is purchased learning, where improved information is purchased with an explicit cost modeled (e.g., R&D). A third type of learning is

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1 While the economic valuation of climate damages are clearly independent of the physical uncertainties in the climate system, Forest et al (2000) have shown that a correlation between climate sensitivity and oceanic heat uptake exists when climate model runs are statistically matched to observations.
autonomous or exogenous learning, in which uncertainty is resolved simply with the passage of time. The models here represent autonomous learning only².

The functional relationships between strategy choice, uncertain parameters, and outcomes are represented by the set of reduced-form models derived from the MIT IGSM, as described in Section 3.6.1. Figure 4.2 shows the influence diagram for the “Complete Learning” decision model. The rectangular nodes represent the decisions over emissions growth rates. The first period decision is “Emissions Rate 2010-2019” and the second period decision is “Emissions Rate 2020-2100,” as indicated by the direction of the arc. The oval-shaped uncertainty nodes represent the uncertain quantities: “Climate Sensitivity”, “Damage Valuation”, and “Oceanic Uptake”. The arcs between the uncertainty nodes and the second period decision indicate that the true values become

---

² Preliminary explorations of a dependence of learning on strategy indicated that this dependence does not change the qualitative results of this chapter and of Chapter 6. However, the influence of strategy on learning warrants further study, and is discussed in Chapter 8.
known with certainty before the strategy must be chosen. The arcs from the uncertainty to the “Damage Costs” outcome node indicate that these quantities influence the damage costs. The uncertain quantities have no effect on the abatement costs. Both decisions influence the two attributes “Damage Costs” and “Abatement Costs” (these represent the costs over both periods), which are combined to obtain “Total Cost”. The total cost is the disutility function to be minimized.

The “No Learning” decision model is shown in Figure 4.3. This influence diagram is identical to that to the Complete Learning case with only one exception. The arcs between the uncertainty nodes and the second period decision node are missing. In this influence diagram, the true values of the uncertain quantities do not become known until after the second period strategy has been chosen.

In the next section, we explore the effect of learning on the optimal strategy in period 1. We will use the models to find the strategy that minimizes expected total costs in each of these two cases, and compare the two strategies.

4.2 Exploring the Effects of Learning

4.2.1 Optimal Strategy with Complete Resolution of Uncertainty

As time passes, research may reduce some of the uncertainty about the magnitude of climate damages. The state of knowledge under which future decisions will be made may be different than that of today’s decisions. What difference does the rate of learning make for today’s optimal decision? Should we undertake more abatement in period 1 if we expect to learn, or should we undertake less abatement? The uncertainty in learning can be represented either as an uncertainty in the rate of learning (when will we learn) or equivalently in the amount of learning that occurs by some fixed time period (how much will we learn). In order to examine this question, let us compare the two extreme cases of Complete Learning and No Learning. The real world is likely to fall somewhere between these two cases, so we will try to understand the role of learning by examining the difference between the extremes. How much period 1 abatement is optimal in these two cases and how do they compare?

First we examine the Complete Learning case, which assumes that the uncertainty in climate sensitivity, oceanic uptake, and damage valuation will be completely resolved by the time the 2020 decision is made. The learning is represented in Figure 4.2 by the
Table 4.4: Probabilistic Strategies for Complete Learning

<table>
<thead>
<tr>
<th>Emissions Rate 2010-2019</th>
<th>Emissions Rate 2020-2100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy</td>
<td>Probability</td>
</tr>
<tr>
<td>1.4%</td>
<td>1.0</td>
</tr>
<tr>
<td>1.2%</td>
<td>0</td>
</tr>
<tr>
<td>1.0%</td>
<td>0</td>
</tr>
<tr>
<td>0.8%</td>
<td>0</td>
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<td></td>
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</tbody>
</table>

Figure 4.4: Optimal Annex I Emissions with and without Learning

![Graph showing optimal Annex I CO₂ emissions with and without learning.]

information arcs from the uncertainty nodes to the second period decision node, which indicate that these quantities become known before the decision is made. The solution of this model gives an optimal decision for period 1, the 2010 emissions rate. For the discrete distributions described above, the optimal choice is to leave emissions unconstrained (1.4% growth) during 2010-2020. The second period decision varies depending on which possible state of the world obtains, but the optimal decision in all states entail some constraint on emissions growth, ranging from a stabilization of emissions (0%) to slightly constrained growth (0.8%/year). The probabilities of choosing each strategy under reference assumptions are given in Table 4.4. The resulting Annex I emissions from choosing the optimal two-period strategy are shown in Figure 4.4. The dashed lines show the emissions paths that result in the complete learning case. Depending on what the true states of damage valuation, climate sensitivity, and ocean
uptake are revealed to be, one of the four emissions paths (the solid line is a possible choice under learning as well) will be chosen.

Thus at reference values for parameters, the optimal first period decision for this model is to leave emissions unconstrained. However, experts disagree as to the probability distributions of the uncertain parameters (Section 3.3). To see how the optimal strategy depends on the beliefs about the uncertain quantities, the results must be tested for sensitivity to the assumptions of the model. Figure 4.5 shows a tornado diagram for the No Learning decision model described in Figure 4.3. In this figure, the probability of the high value (the "P=0.2" column in Table 4.3) is varied from 0 to 1.0 for each of the three uncertainties, and the resulting expected value of the optimal strategy path is compared. For the assumed discrete distributions, the uncertainty in damage valuation has the largest impact on expected value, while the impact of uncertainty in climate sensitivity and ocean uptake is significantly less.

We perform sensitivity tests on the optimal strategy by varying the probability that damage valuation is high \((Prob\{ \eta = 16\% \})\), which has reference probability 0.2). Varying this probability roughly captures the effect of assuming different distributions for the valuation uncertainty. The right side of Figure 4.6 shows the total costs as this probability is varied from 0 to 1, in the case where uncertainty is expected to be resolved. For all values of this probability, a strategy of unconstrained emissions in period 1 (i.e., 1.4%/year growth) always results in the lowest costs. For any assumption about the probability of high damage, the period 2 strategy varies depending on the revealed true state of the damage valuation, climate sensitivity, and ocean uptake (see Table 4.4 for
reference results). The magnitude of costs also varies significantly with this probability, from 1% to more than 5% in terms of the net present value of welfare. The optimal choice of no emissions constraint in period 1 appears to be robust.

In a three-way sensitivity of all uncertainties it can be optimal to constrain emissions in period 1, but only if the probabilities of high climate sensitivity, slow oceanic uptake, and high damage valuation are simultaneously quite high. Figure 4.7 shows a slice of this sensitivity surface. To limit the graph to two dimensions, we set a fixed high probability (0.9 instead of the reference value of 0.2) of slow oceanic uptake, and therefore very long carbon lifetime in the atmosphere. The graphs display the optimal first period strategy for every possible pairing of the other two uncertain variables, the probability of high damage valuation and the probability of high climate sensitivity. The optimal strategy for the case where learning is expected is shown on the right side of Figure 4.7. In the upper right corner of the graph, corresponding to very high probabilities of high damage valuation and high climate sensitivity, the optimal strategy in the first period are growth rates in emissions of 1.0% or 1.2%. The dots in both graphs indicate the reference probabilities of high damage valuation and high climate sensitivity, 0.2 and 0.2, respectively. Note that even for this much higher probability of a slow
ocean, the reference values of the other two uncertainties are far from the region where some first period abatement is optimal.

4.2.2 Optimal Strategy with No Resolution of Uncertainty

It is perhaps not surprising that an optimal solution involves no abatement in the first period when the uncertainties will be completely resolved by the second period. Given this ability to learn and the fact that greenhouse gas concentrations grow slowly, it seems wise to simply wait for ten years, at which time the appropriate emissions rate can be set based on the severity of climate change, rather than undertake expensive reductions that may later prove unnecessary. But what if no learning at all occurs? If we will not know any more by period 2 than we do in period 1, what then is the optimal emissions growth rate in the first period? To properly perform this comparison, a decision model is constructed that still consists of two decision periods, but both decisions are made under the same uncertainties, as illustrated in the influence diagram in Figure 4.3. The arcs from the uncertainty nodes in Figure 4.3 now only point to “Damage Costs”, indicating that these elements are still uncertain when the “2020 Decision” is chosen.

Under the reference probabilities of Table 4.3, the solution of the decision model with no resolution of uncertainty is:
Figure 4.8: Optimal Second Period Strategy with No Learning

**Period 1**: Unconstrained emissions growth (1.4%/year), and

**Period 2**: 0.6%/year emissions growth rate (half the unconstrained growth rate). The Annex I CO₂ emissions path that results from this strategy is shown as the solid line in Figure 4.4.

If the probability of high damage valuation is varied between 0 and 1, a strategy of unconstrained emissions remains optimal in period 1 for all values (left side of Figure 4.6). Thus the optimal period 1 decision is the same whether learning completely eliminates uncertainty by period 2 or no learning occurs at all. It is optimal to delay action regardless of whether learning occurs.

Note that although no learning occurs, a second decision is still made in 2020, and in general the strategy in the second period differs from that of the first period. Thus the optimal strategy under no learning is not to leave emissions unconstrained after 2020. Because the second period strategy is chosen under uncertainty, it too is sensitive to the assumed probabilities of high damage outcomes. Figure 4.8 shows the optimal strategies in the second period as the probability of high damage costs varies. For most possible assumptions the reference rate of 0.6%/year growth in emissions is optimal over 2020-2100, but if high damage is very unlikely the optimal rate becomes 0.8%/year, and if high damages are very likely the optimal rate drops to 0.2%/year (substantially slowed growth in emissions). In this case the unconstrained growth rate of 1.2% is never an optimal strategy in the second period, for any belief about the valuation of climate damages.

Other formulations of the climate decision problem have appeared in the literature that are not appropriate for the assessment of the effect of learning on near-term decision
in a sequential decision context, but are sometimes confused with it. One common formulation of a no learning case allows only a single, once-and-for-all decision (e.g., Valverde, 1997\(^3\)), rather than two or more sequential decisions. This one-period model may be constructed without learning (all uncertainties resolved after the decision is made), but this approach does not capture the essential characteristic of the decision problem. The comparison of a one-period no-learning formulation to the Complete Learning case described above conflates the gains of learning with the increased flexibility of allowing sequential decision.

A second common formulation is the so-called Learn-Then-Act approach of Manne and Richels (1992, Ch. 2; 1995). Learn-Then-Act refers to a two-period decision model, in which all uncertainties are resolved before the first period. Thus both first and second period strategies are chosen with perfect information. Manne and Richels use this formulation to contrast with the “Act-Then-Learn” approach, which is equivalent to the Complete Learning case above. By comparing Act-Then-Learn with Learn-Then-Act, Manne and Richels illustrate the difference between choice under uncertainty and choice under perfect certainty. This is one approach to addressing the question of the value of information.

The value of information for a decision problem is an important and useful calculation, but it is a different question than the one being asked in this dissertation. Here our focus is not on the value of information, but rather on what is the optimal decision now given that we are uncertain now, and may or may not learn in time for the next decision. The fact that the optimal first period decision is independent of learning does not imply anything about the value of information. If learning occurs, the expected costs will not be the same as they are without learning by period 2. We will always be at least as well off, and almost always better off, with learning than without. But the converse is also true: a high value of information about uncertainty in a decision problem does not imply that the first period strategy is necessarily different with and without learning. Strategy may or may not differ as a function of learning depending on other conditions, as will be demonstrated in Chapter 5.

\(^3\) Valverde (1997) also goes on to compare this formulation with several two-period formulations equivalent to the No Learning and Complete Learning cases.
Learn-Then-Act, which assumes that all uncertainty is already resolved, provides a measure of the value of information if it could be obtained immediately. We can also calculate the value of perfect information that required a decade to obtain. Consider again the graphs in Figure 4.6, which show that the optimal first period strategy is the same with and without learning for any probability of high damage valuation from 0 to 1. But notice that the actual expected cost under that optimal decision is *not* the same with learning and without. For the reference probability of high damage of 0.2, the expected costs (abatement plus damages) without learning are 1.650% of the sum of welfare for the Annex I aggregate over 2010-2100, discounted at 3%. The equivalent expected costs with Complete Learning before the second period decision is 1.616%. Although this may appear to be a small difference, it is significant in absolute terms. The value of perfect information by 2020 is 0.034% of the reference level of discounted sum of welfare of $524 trillion (discounted to year 2000 US $), or $178 billion. This number is of a similar order of magnitude of other studies of the value of information in climate change policy (e.g., Nordhaus and Popp, 1997).

Returning to the question of period 1 strategy with and without learning before period 2, to find any difference in the optimal choice as a function of learning, we must look at an extreme corner of a 3-way sensitivity analysis where the high damage outcome is given a very high probability. Figure 4.7 shows a slice of the three-way sensitivity surface for both the Complete Learning and the No Learning cases, with the probability of slow ocean uptake fixed at 0.90. Comparing the two graphs in Figure 4.7, there are regions in which the ability to learn will result in a lower growth rate than is optimal without learning. For example, for a probability of high damage valuation of 0.7 and a probability of high climate sensitivity of 0.9 it is optimal to leave emissions unconstrained if uncertainty will not be resolved, but to restrict growth to 1.2%/year if uncertainty will be resolved. (Insights into the reason that emissions are constrained more when learning will occur are developed in Chapter 5). However these regions are quite small: they appear only where there is a very high probability of high damage valuation, a very high probability of high climate sensitivity, and a very high probability of slow ocean mixing. For the vast majority of assumptions about the parameter distributions, the optimal period 1 decision will be the same regardless of whether learning will occur or not.
At first, this result may seem surprising. One might expect some abatement in the no learning case, since waiting to learn is often an argument for no abatement in the near-term. However, this result is consistent with other studies of sequential climate policy decision, as reviewed in Chapter 1. Kolstad (1996) and Ulph and Ulph (1997) explicitly find that the optimal period 1 decision is virtually independent of whether learning occurs or not, and similar results are implicit in Nordhaus (1994), Manne and Richels (1995), and Valverde (1997).

4.2.3 The Influence of Different Magnitudes of the “Stock” Effect

The next question to ask then is why the near-term optimal decision is independent of whether (or when) learning occurs. Nordhaus (1994) noted the different but related effect that the control rate after the resolution of uncertainty was independent of when uncertainty was resolved. The independence of the control rate before resolution was also present in Nordhaus’ results, but not discussed explicitly (see Figure 8.4 in Nordhaus, 1994). Nordhaus explained these puzzling results as resulting from a low dependence of the stock of greenhouse gases on the early period control rates. Kolstad (1996) finds the same lack of influence of learning on the first period decision as shown above. Kolstad shows that the marginal damage is very insensitive to the total stock of greenhouse gases in the initial periods, and that this condition holds for several alternative non-linear damage functions. Because CO₂ is a stock pollutant, according to Kolstad, learning only matters if one would want to have a net absorption of carbon in period 2 to undo the emissions of the previous period. “But because future emissions are so slightly influenced by today’s actions, there is no scenario under which it would be optimal to negatively emit in the future to correct over-emissions today” (Kolstad, 1996).

The stock effect (flat marginal damages and steep marginal costs) is prominent in both the Nordhaus and Kolstad explanations. However, as both Kolstad (1996) and Ulph and Ulph (1997) point out, the critical feature for learning to matter is to have irreversibility, which is a characteristic of stock problems. For a flow-type pollutant that dissipates quickly (e.g., noise pollution), there is no irreversibility since the impact of one period’s decisions is gone by the next period. So, by this argument, if CO₂ were to have a shorter lifetime than assumed in the models here and therefore to reduce the “stockiness” of the problem, the influence of learning should only further weaken. But this leads to the
hypothesis that increasing the stock effect might increase the influence of learning. The impact of the initial period’s emissions on future damages is small, but with the models used here we can artificially increase the impact and see what happens.

The two decision models presented above represent the decade 2010-2019 as the first period and 2020-2100 as the second period. One reason that the first period has such a negligible effect on future damages is that one decade’s emissions are small relative to the pre-existing stock of CO₂ plus 9 more decades of emissions in the second period. Discrete time periods are a useful simplification for analysis, but real time is continuous and policies for climate change may be revised any number of times over the next century. A two-period model is the simplest way to represent the notion of doing something “now” and then perhaps deciding to change course “later”. But the division of the next century into two periods is arbitrary⁴. Suppose now that period 1 represents a forty-year interval from 2010-2049 and period 2 represents 2050-2100. Such a choice can reasonably be justified by considering that once an international policy regime commits to a particular course, it may be slow to change.

A first period of 40 years will have a much more significant effect on the total stock of CO₂ in the second period. Two decision models are again constructed, one in which uncertainty is resolved between periods and one in which uncertainty is resolved only after both decisions. Figure 4.9 shows the two-dimensional sensitivity/strategy diagram, with the probability of slow ocean uptake at its reference value of 0.2. For every possible value for the probabilities of high damage valuation and high climate sensitivity, the optimal strategy is shown. Just as before, the same strategy is optimal in the first period in both cases for the vast majority of possible distributions. The solutions diverge for only a small set of parameter values that indicate very strong belief in both high damage valuation and high climate sensitivity, and which justify a more stringent policy in the case where no learning is expected. An interesting difference from the model with a decade-long first period is that, whereas unrestricted emissions are optimal for most distributions if we consider a first period of only one decade (2010-2019), the optimal decision for a forty-year initial period is to slightly constrain the growth in

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⁴ The time horizon of 100 years for climate change assessment is equally arbitrary, mainly resulting from a compromise between the natural time-scales of climate models (several centuries) and economic models (one or two decades).
emissions at a rate of 1.2%/year. The level of stringency is sensitive to the length of time represented by the initial period, but still is not sensitive to whether learning is expected.

Another way to increase the impact of period 1 emissions on future damages is to make the emitted CO₂ remain in the atmosphere longer before it is absorbed by the ocean. By setting a slower rate of uptake by the ocean than in the reference case, the lifetime of CO₂ in the atmosphere will become longer and increase the “stockiness” of CO₂. The rate of ocean uptake is set to the extremely low value of $K_s = 0.1$, which corresponds to an atmospheric lifetime of CO₂ of about 140 years. Figure 4.10 shows the strategy diagram for the sensitivity analysis of the probability of high damage valuation under the slow ocean assumption. These decision models use the reference definition of the periods as 2010-2019 and 2020-2100. The left side of the figure shows that when no learning is expected, the optimal strategy is to slightly constrain emissions growth to 1.2%/year only if the probability of high damage valuation exceeds 70%, and otherwise not to constrain emissions at all. For the complete learning case, shown on the right side of the figure, the constraint of 1.2%/year is optimal if the likelihood of high damage is more than 50%, otherwise no abatement is preferred. In contrast, for the reference version of the decision models with uncertain but more rapid ocean uptake, the optimal strategy is unconstrained emissions (1.4%/year) for all probabilities of high damage valuation (Figure 4.6). The slower ocean here takes up less carbon dioxide and less heat from the atmosphere, and when high damage costs are likely, constraints on emissions in period 1 become justified.
Figure 4.10: Optimal Period 1 Decision for Long CO₂ Lifetime (140 Years)

Here there is a definite zone of divergence between the no learning and complete learning cases. A probability of high damage costs between 50% and 70% lead to different strategies. However, this zone of divergence is still relatively small, as is the increase in stringency. Also, the direction of the divergence effect is opposite to that of the previous example: more stringent abatement will be taken when uncertainty is expected to be resolved than when it is not!

What is it about the climate change problem or the way it is represented in assessment models that makes assumptions about learning appear mostly irrelevant in choosing a near-term response to climate change? Under what conditions will this result continue to hold? What is the intuition behind the conditions where learning seems to result in more abatement, where it calls for less abatement, and when it results in the same level as with no learning? To answer these questions, we need to formulate a simpler analytical model and explore the effect of learning on decisions, which is the topic of the next section. Chapter 5 will present such an analytical model, and will clarify the effect of learning on decision. The intuition from the analytical model will then be applied to the models shown in this chapter, and the reason behind the lack of a learning effect will be explained.

4.3 Myopic vs. Forward-looking Economic Models

Analyses of optimal sequential decision, such as those presented in this chapter, are a subclass of normative analysis methodologies that are designed to solve for the level
of activity that maximizes net benefits or minimizes costs. For climate policy, one essential function of any model is to provide a means of calculating economic losses (or welfare losses) from a given level of carbon constraint. The welfare losses are determined in the context of economic growth, trade between regions, and technologies that allow for substitution among factors. The decision models presented in this chapter use a recursive dynamic general equilibrium model that has no foresight, and its reduced-form equivalent for calculating the corresponding welfare loss for any choice of emissions restriction. In this section, we briefly discuss why the results of Section 4.2 are likely to be robust to the type of underlying economic model used.

An important assumption that varies across different economic model types is how expectations about the future are handled. The expectations or foresight about future prices and decisions will affect the how allocations are made both across time and across sectors. Allocations of income to savings/investment and the setting of endogenous control variables such as emissions constraints are especially likely to be affected by expectations about the future. Some models, such as EPPA have no foresight, and optimize only current decisions about resource allocations to maximize welfare in the current period. Other economic models are constructed with perfect foresight (e.g., Manne and Richels, 1995; Nordhaus, 1994). One type of economic model that has perfect foresight is a Ramsey optimal growth model (Ramsey, 1928; Koopmans, 1967), which calculates the optimal allocation of savings and consumption over all time periods in order to maximize the net present value of welfare over the time horizon. Other economic models may have imperfect foresight, such as models with lagged expectations or myopic CGE models that are coupled to optimal growth models to drive the investment and capital formation behavior (Yang, 1999).

Because assumptions about foresight can potentially alter the timing of strategic decisions, we must consider how different assumptions might affect the results of this chapter regarding the role of learning and the value of delaying abatement. There are two ways in which having perfect foresight could potentially alter the optimal policy choice:

1. Feedback in the amount of investment each period, and
2. Direct effect of foresight on strategy choice.

We consider each of these in turn.
Savings decisions may slightly diverge between a forward-looking model and a myopic model. Myopic models typically use some estimates of aggregate investment behavior, such as the marginal propensity to save. This approach fixes the fraction of income that is saved, although the total amount will vary and is determined endogenously. The primary driver behind savings behavior in a forward-looking model is the pure rate of time preference used as a discount rate in the welfare function. In addition, by choosing the optimal investment in all periods simultaneously, a forward-looking model can adjust investment to account for any time varying trends such as shifts in relative prices. Because all models use parameters that are calibrated to historical investment and growth rates, the projections from both types of models are likely to be quite similar in practice. While the investment levels over time in forward-looking and myopic models will not be identical, in most cases the differences are likely to be small.

The second concern is whether the use of a forward-looking model would alter the optimal choice of emissions constraints from those of a decision tree that uses a model without foresight to calculate abatement costs. Many forward-looking models have been used to investigate sequential choice under uncertainty and the effect of learning (e.g., Nordhaus and Popp, 1997; Manne and Richels, 1995; Kolstad, 1996). When a forward-looking dynamic model is used for estimating economic losses, it is often used as the framework for the sequential policy choice as well. In addition to allocation of consumption over time and allocation of productive factors across sectors, the optimal abatement level in each period becomes another choice variable for the optimization. To simulate choice under uncertainty, stochastic programming is used (Kall and Wallace, 1994). With this method, the forward-looking model is required to choose abatement levels in the periods before uncertainty is resolved that is equal across states of the world. After uncertainty is resolved the optimal abatement level is found for the realized parameter values, which will differ across states of the world.

By contrast, a static recursive model cannot choose a policy level. Emissions constraints are imposed exogenously. This kind of model is useful for producing cost estimates of different levels of abatement activity. To perform optimal sequential choice under uncertainty, cost estimates for alternative levels of control, along with the corresponding estimates of the avoided damage losses, are used within a dynamic
programming framework such as a decision tree or influence diagram. What are the differences between these two approaches?

In general these two approaches correspond to the two alternative methods for specifying a dynamic optimization problem. The two general approaches are:

- Optimal control theory
- Dynamic programming

An optimal control problem solves for the path that maximizes or minimizes the objective function using a Lagrangian or variational calculus approach. A dynamic programming problem is solved through backward induction, finding the best strategy at the current point assuming that all subsequent strategies will be optimally chosen.

Optimal control theory is the basis for solving forward-looking stochastic programming models. Dynamic programming is the solution method for a decision tree model, where the cost estimates might come from a myopic macro-economic model. In general, the same problem posed in terms of either solution method will yield identical results (Chiang, 1992). Thus, as long as underlying assumptions such as production functions, damage functions, growth rates, etc. are the same, we expect the same optimal choice of emissions abatement over time whether it is obtained from a forward-looking optimizing model or from a decision tree with costs from a static model.

Finally, there is additional evidence from the literature that models of sequential climate policy under uncertainty, employing forward-looking approaches, yield the same solutions as those presented here. Kolstad (1996) used a stochastic version of the DICE model (Nordhaus, 1994) to explore the effect of learning on near-term abatement decisions (see Section 1.5). Using this model, which is forward-looking, Kolstad obtains qualitatively identical results to our models: no abatement is optimal in early periods, and learning about damages appears to have no effect on the first period abatement decision. A similar study by Ulph and Ulph (1997) using a forward-looking model based on Maddison (1995) also find the same lack of an effect of learning on first period strategy choice.

In summary, the findings of this chapter, that optimal first period emissions constraints are virtually unaffected by whether uncertainty will be resolved or not before the next decision, are not likely to be dependent on the use of a myopic model to calculate abatement costs. The alternative approaches of optimal control and dynamic
programming should lead to the same optimal strategy sequence over time for the same problem. And while one expects small differences in savings-consumption allocations between a forward-looking and a myopic model, the fact that models are calibrated to produce similar growth patterns imply that resulting differences in absolute welfare will not be significant, and differences of welfare loss from a policy should be even smaller.
Chapter 5 Interaction Effects, Learning, and Sequential Choice

*Indecision is like a stepchild: if he does not wash his hands, he is called dirty, if he does, he is wasting water.*

_African Proverb_

Chapter 4 presented models of climate policy choice that frame the problem as a sequential decision under uncertainty. Using reduced-form models of the MIT Integrated Global Systems Model to calculate the costs and damages for any choice of emissions constraints, the same strategy (no emissions constraints) is optimal for the first period under reference conditions regardless of whether or not uncertainty will be resolved before the next decision. Further, varying over a wide range of assumptions about the uncertainty in damages, near-term strategy remains mostly uninfluenced by assumptions about learning. Even after an increase in the impact of the first period decision on the total stock of greenhouse gases, the effect of learning on first period strategy remained small.

Why does the resolution of uncertainty appear to not affect near-term optimal strategy? Under what conditions does strategy depend on whether learning is expected or not? To clarify the role of learning in sequential decision, this chapter will develop a simple analytical model of a dynamic-programming two-period decision problem. Using this model, we will demonstrate that learning affects the optimal period 1 strategy only if there is an _interaction_ present between the period 1 strategy and the total cost (abatement plus damages) of the period 2 strategy. The term “interaction” is used here to mean that the _marginal_ abatement or _marginal_ damage costs in the second period are a function of what was done in the first period. One example of this kind of interaction, which will be explored in detail in Chapter 6, occurs where the period 1 strategy changes the rate of technological innovation, and thereby lowers the marginal cost of abatement in period 2.

Because greenhouse gases are stock pollutants, emissions in period 1 will influence the costs in period 2 in two distinct ways. In any case, the damage cost in period 2 will be affected by the stock pollutant left from the previous period, because the
pollutant decays slowly. Much of the CO₂ emitted in period 1 will still be present in the atmosphere in period 2 and thus will contribute to the total damage costs in period 2. This effect may not, however, affect the marginal abatement or marginal damage costs of period 2 emissions. A second effect occurs if the damage function is nonlinear. If the damage costs are a nonlinear function of the stock, then a higher level of the stock at the start of period 2, as a result of the period 1 strategy, will also result in a different marginal damage than if the stock were lower. This shift in the marginal damages is one possible interaction that can cause the period 1 strategy to depend on whether learning occurs.

In this chapter, we use several variants of an analytical two-period model to demonstrate the conditions that determine:

- The existence of an effect on first period strategy by learning,
- The magnitude of the influence on strategy by learning, and
- The direction of the effect (i.e., does learning lead to more or less abatement).

Section 5.1 lays out the basic framework for the two-period dynamic programming model, without assuming any particular functional form. We derive the generic solutions to the optimal period 1 strategy with two different assumptions about learning: 1) no resolution before the period 2 decision; and 2) uncertainty is completely resolved and the period 2 decision is made under perfect certainty about the state of the world. The key distinction between the optimal strategies in these two cases is the fact that without learning there is only one strategy for all states of the world that is chosen in period 2, based on expected values, while with learning a different period 2 strategy is chosen for each state of the world.

In order to get more insight into the difference between strategies with and without learning, a specific functional form for the objective function is required. In Section 5.2, we show that for a general second-order objective function without an interaction between the period 1 strategy and the period 2 strategy (in the form of a cross-product), the optimal period 1 strategies with and without learning are identical. In Section 5.3, we modify the objective function to include an interaction in the form of a linear cross-product of strategy levels, and show that in this case, the optimal period 1 strategy with learning is not the same as the optimal strategy without learning. The key
to causing a divergence between strategies in these two cases is the presence of an interaction.

With the understanding that an interaction between strategies in periods 1 and 2 is a necessary condition for a divergence, we next want to understand the conditions that determine the magnitude of this divergence and its direction. The issue of the magnitude is addressed in Section 5.4, where we show that it is not sufficient to have an interaction for the effect of learning on strategy to be significant. The magnitude of the divergence between first period strategies with and without learning will be shown to be a function of the variance of the interaction relative to the variance of other parameters in the objective function.

The objective functions for the formulations in Sections 5.2-5.4 are defined in terms of total net costs (abatement plus damage costs) to be minimized, or equivalently as net benefits to be maximized. We do not distinguish between control costs and damage costs because the derivations in these sections would have more terms and would therefore be more difficult for the reader to interpret. The relationships that will be demonstrated in terms of total cost are analogous to relationships that can be derived by keeping abatement and damage costs separate. However, in order to fully clarify the issue of the direction of the learning effect, Section 5.5 uses an objective function that separates abatement costs from damage costs. In this section, we will treat abatement costs as certain, and focus on the effect of learning about uncertain damage costs.

It is necessary to distinguish control costs from damage costs to explore the direction of the bias from learning because it is the balancing between control costs and damages that determines the direction. We will show in Section 5.5 that learning can lead either to higher emissions or to lower emissions, relative to the optimal period 1 emissions without learning. The direction is a function of the distribution of uncertain damages relative to abatement costs. When the expected damages are significant relative to control costs, stringent abatement will occur in the absence of learning. Therefore if learning occurs and the damage state is lower than expected, there will have been wasted capital on unnecessary emissions reductions. The expectation of this outcome when learning will occur will bias emissions upward in period 1. Conversely, if expected damage costs are relatively low compared with control costs, but we learn in period 2 that the damage state is much higher, we will regret not having reduced emissions further. In
this situation, optimal first period emissions with learning are lower than in the no learning case.

Once we have clarified the role of the interaction between periods in causing learning to influence period 1 strategy, and the conditions that determine the strength and direction of that influence, these insights will be used in Chapter 6 to explain the lack of influence of learning in models using the IGSM. We will show that although there are weak interactions in the IGSM, the relative variance is too small for learning to have a noticeable effect on period 1 strategy. We will then develop alternative formulations that include interactions that might be present in the real world but are omitted from the IGSM, and show that period 1 strategy depends on whether learning will occur.

5.1 A Dynamic Programming Formulation

In this section, we define a simple theoretical model that can be used to explore the implications of learning on the first period emissions decision. The choice of a climate change policy under uncertainty is defined as a dynamic programming problem, with 2 periods. Define

\[ t = 1, 2 \]

\[ X_t = \text{the set of all possible emissions levels that can be chosen in period } t \]

\[ x_t = \text{the level of allowed emissions chosen in period } t, \text{ chosen from the set } X_t \]

\[ \theta = \text{the severity of damage costs from climate change} \]

\[ C_1(x_1, \theta) = \text{abatement costs and damage costs in period 1} \]

\[ C_2(x_1, x_2, \theta) = \text{abatement costs and damage costs in period 2} \]

and let \( E_{\theta} \{ \} \) denote the expectation with respect to the marginal distribution of \( \theta \).

In each period \( t \), a level of allowed emissions \( x_t \) is chosen. Each period has a total cost function \( C_t(\cdot) \) stated in terms of emissions level, which includes both abatement costs and damages from the accumulated stock of carbon. The uncertainty in the damages from climate change is represented by different states of the world \( \theta \) that may obtain, and so the damage costs (and therefore the total costs) are also a function of \( \theta \).
Define the value functions:

\[ V_1(x_1, x_2, \theta) = \text{the sum of abatement costs and damage costs over both periods} \]

given emissions level \( x_t \) at \( t = 1 \) and emissions level \( x_{2t} \) at \( t = 2 \) and \( \tilde{\theta} = \theta \),

\[ V_2(x_1, x_2, \theta) = \text{the abatement and damage costs at } t = 2 \text{ if emissions level } x_2 \text{ is} \]

chosen in light of \( \tilde{\theta} = \theta \), given that emissions level at \( t = 1 \) was \( x_t \).

At \( t = 2 \), we choose emissions level \( x_2 \) so as to minimize the sum of abatement and damage costs given that period 1 emissions level was \( x_1 \) and that the severity of climate change damage costs is \( \theta \). As in the empirical models of Chapter 4, we consider two cases here. In the "learning" case, the true value of \( \theta \) becomes known with certainty at the start of period 2. In this case, the emissions level \( x_2 \) is chosen with certainty about \( \theta \), and the value function for period 2 is:

\[ V_2(x_1, x_2, \theta) = \min_{x_2 \in X_2} [C_2(x_1, x_2, \theta)] \quad (5.1a) \]

In the second case, referred to as "no learning", the uncertainty in \( \theta \) in period 2 remains identical to that of period 1. The emissions level \( x_2 \) must be chosen under uncertainty in \( \theta \), and the value function for period 2 is:

\[ V_2(x_1, x_2, \theta) = \min_{x_2 \in X_2} [E_\theta \{C_2(x_1, x_2, \theta)\}] \quad (5.1b) \]

Our objective is to choose the period 1 emissions level \( x_1 \) to minimize the expectation of the sum of costs over both periods. Namely,

\[ V_1(x_1, \tilde{\theta}) = \min_{x_1 \in X_1} [E_\theta \{C_1(x_1, \tilde{\theta}) + V_2(x_1, x_2(\tilde{\theta}), \tilde{\theta})\}] \]

More compactly, we can define

\[ \bar{V}_2^*(x_1) = E_\theta \{V_2(x_1, x_2(\tilde{\theta}), \tilde{\theta})\} \]

where \( \bar{V}_2 \) means the expectation of the value function with respect to \( \theta \), and \( V_2^*(x_1) \) indicates that for any choice of \( x_1 \), the second period strategy \( x_2 \) is chosen optimally. Similarly, the expectation of period 1 costs are denoted as:

\[ \bar{C}_1(x_1) = E_\theta \{C_1(x_1, \tilde{\theta})\}. \]

Then the objective function for this dynamic program becomes

\[ \bar{V}_1^*(x_1) = \min_{x_1 \in X_1} \bar{V}_1(x_1) = \min_{x_1 \in X_1} [\bar{C}_1(x_1) + \bar{V}_2^*(x_1)] \quad (5.2) \]
There are several important characteristics of this abstract model worth highlighting. First, the stock nature of the problem is represented by the dependency of \( C_2 \), the cost in the second period, on \( x_1 \), the decision made in the first period. In multi-period decisions about stock pollutants, capital stock, or other quantities that accumulate over time, the costs and/or benefits in any period are partly a function of decisions made in previous periods. Analogously, the current period's decisions will have cost/benefit impacts in future periods. This formulation is in contrast to flow-type problems in which the implications of each period's decision are felt in that period only, and costs have no relation to what has previously occurred (e.g., noise pollution).

The second element of the model to note is the difference between the learning case and the no learning case. In the no learning case, there is only a single choice of \( x \), that must minimize the mean or expected costs across all possible states of the world, since we don't yet know which one is the true state. In contrast, in the learning case, many different choices of \( x \) can be made, one for each possible state of the world \( \theta \) that minimizes costs in that world. Of course, even when there is learning, from the perspective of period 1 the outcome is still an expected value over all possible states.

The question we ask is: how does the first period's emissions level decision change if learning does or does not occur? If we denote the optimal period 1 strategy in the case where learning occurs as \( x_1^L \) and the optimal strategy without learning as \( x_1^N \), we wish to know whether \( x_1^L = x_1^N \) or \( x_1^L < x_1^N \) or \( x_1^L > x_1^N \)? Always, or under certain conditions? And if so, under what conditions do these relationships hold?

5.2 A Specific Functional Form with no Cross-Period Interaction

To fully explore the influence of learning on the first period strategy, we need to make assumptions about the form of the cost function. In this section, we will consider the objective function to represent total costs (abatement plus damage) to be minimized. In later sections we will distinguish between control costs and damage costs. Here, we begin by assuming that the period 2 cost function has no cross-products in the period 1 and period 2 strategies.

First, let us make the simple assumptions that the first period costs, given a state of the world \( \theta \), are linear,
\[ C_1(x_1, \theta) = a(\theta)x_1 + b(\theta), \quad (5.3) \]

and that the second period costs are a simple quadratic function of both periods' decisions

\[ C_2(x_1, x_2, \theta) = c(\theta)x_2^2 + d(\theta)x_2 + e(\theta)x_1^2 + f(\theta)x_1 + g(\theta). \quad (5.4) \]

In this model, the stock nature of the problem is represented by the terms \( e(\theta)x_1^2 \) and \( f(\theta)x_1 \) in the second period cost function. The decision made in the first period will influence costs in the second period. \(^1\)

First, consider this model for the case where uncertainty is resolved. At \( t = 2 \) the cost minimizing level of emissions level is:

\[ \frac{\partial C_2}{\partial x_2} = 2c(\theta)x_2 + d(\theta) = 0, \]

\[ x_2^* = -\frac{d(\theta)}{2c(\theta)}. \]

This choice of \( x_2 \) is a minimum only if \( c(\theta) > 0 \).

Given the emissions level that will be chosen in the second period, we now solve for the cost minimizing level of emissions in the first period. The total cost for a level of first period emissions \( x_1 \) and for a state of the world \( \theta \) is:

\[ V_i(x_i, \theta) = E_\theta \left\{ a(\theta)x_i + b(\theta) - \frac{d^2(\theta)}{4c(\theta)} \right\} + e(\theta)x_1^2 + f(\theta)x_1 + g(\theta) \]

If we denote \( E_\theta \{ a(\theta) \} \) as \( \bar{a} \), and analogously for other coefficients, we can rewrite total costs as:

\[ V_i(x_i, \theta) = \bar{a}x_i + \bar{b} - E_\theta \left\{ \frac{d^2(\theta)}{4c(\theta)} \right\} + \bar{e}x_1^2 + \bar{f}x_1 + \bar{g} \]

The derivative of the objective function with respect to \( x_1 \) is

\[ \frac{\partial V_i}{\partial x_1} = \bar{a} + 2\bar{e}x_1 + \bar{f} = 0 \]

\(^1\) These terms do not capture any change in the marginal damages that may occur with a non-linear damage function. In this formulation the total period 2 damage is a function of period 1 strategy, but the marginal damage is not. The dependence of marginal damage on first period strategy is a different effect, and is treated in the next example in Section 5.3.
If $\bar{e} > 0$, then the cost minimizing emissions level in the first period is:

$$x_1^* = \frac{-\bar{a} + \bar{f}}{2\bar{e}}$$  \hspace{1cm} (5.5)

When no learning occurs, the emissions level for period 2 is chosen without knowledge of $\theta$. In this case, we solve for one value of the second period decision $x_2$ that minimizes the expectation of costs under uncertainty:

$$\frac{\partial C_2}{\partial x_2} = 2\bar{c}x_2 + \bar{d} = 0$$

As long as $\bar{c} > 0$, the emissions level in period 2 that minimizes expected costs is:

$$x_2^* = \frac{-\bar{d}}{2\bar{c}}$$

Given the strategy that will be chosen in period 2, the total costs to be minimized in the no learning case become:

$$V_1(x_1, \theta) = \bar{a}x_1 + \bar{b} - \frac{\bar{d}^2}{4\bar{c}} + \bar{e}x_1^2 + \bar{f}x_1 + \bar{g}$$

The derivative of this objective function is

$$\frac{\partial V_1}{\partial x_1} = \bar{a} + 2\bar{e}x_1 + \bar{f} = 0$$

Therefore the optimal decision in period 1 without learning is also

$$x_1^{**} = \frac{-\bar{a} + \bar{f}}{2\bar{e}}$$  \hspace{1cm} (5.6)

Comparing equations (5.5) and (5.6), we can see that for this example the two solutions are identical: $x_1^L = x_1^N$. The reason the solutions are identical is that, although the period 1 decision does affect the total costs in period 2 (damages from the remaining stock), it does not interact in any way with the period 2 decision through an influence on marginal cost. This result leads to the following proposition.

**Proposition 1.**

For any two-period sequential decision under uncertainty represented by equations (5.1) and (5.2), and where the solution to (5.1a) where learning does not occur is denoted by $x_1^N$ and the solution to (5.1b) is denoted by $x_1^L$,  

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If \( \frac{\partial^2 C_2}{\partial x_1 \partial x_2} = 0 \) then \( x_1^L = x_1^N \).

The proof of this proposition is given in Appendix A.

The intuition for this result is that if today's decision does not change the costs tomorrow of tomorrow's decision, then the two choices are independent, and learning or not learning (which does influence tomorrow's decision) is irrelevant for today's decision. In other words, if our best decision tomorrow does not depend on what we did today, then either we will learn and make some choice A tomorrow or we will not learn and make some choice B tomorrow but the shift of assumption about learning does not change what we do today. In that case, today's decision is merely made on the basis of costs and benefits today, plus any remaining costs and benefits that continue to accrue in future periods as a result of today's decision (the non-interacting stock effects).

5.3 A Model with Interaction across Periods

The previous example showed that if the cost function for period 2 has no interaction terms, then the period 1 optimal choice is unaffected by assumptions about learning. What happens in the more general case when there is an interaction term?

Assume the same linear cost function for period 1 as in the previous example,

\[
C_1(x_1, \theta) = a(\theta)x_1 + b(\theta),
\]

but define the period 2 cost function with a quadratic term in the second period strategy and a single linear interaction term between the two decisions:

\[
C_2(x_1, x_2, \theta) = c(\theta)x_1^2 + d(\theta)x_1x_2
\]

What might this interaction term represent? As noted earlier, it could represent non-linearity in the damage function. If a larger stock of CO\(_2\) from higher emissions in period 1 changes the marginal damages in period 2, then this change in marginal damages will show up in this cross-term. Other interactions may also be represented by this term. Depending on the sign of the cross-term coefficient, it represents the fact that decisions in different time periods can act as substitutes (\(d>0\)) or as complements (\(d<0\)). Decisions act as substitutes when an increase in the period 1 activity level, \(x_1\), results in an additional increase in the marginal cost of choosing activity level \(x_2\) in period 2. Generally substitution exists in problems where there is some finite resource that can be
used across both periods; using more of the resource in period 1 leaves less of the resource or results in increasingly expensive alternatives for the second period decision. We call this type of interaction a “save your effort” situation. If action today makes action tomorrow more costly, we might do less today in order to do more tomorrow (e.g., preserve a depletable resource).

Decisions act as complements when an increase in activity in the first period causes the per-unit cost of period 2 action to decrease. Complementary situations typically exist when first period action constitutes some form of investment that reduces future costs. We call this complementary type of interaction a “head start” effect. If we anticipate having to do a lot tomorrow, we will want to begin doing some today. In Section 5.3, we will explore different possible interactions that may exist specific to climate change policy decisions. Here we will first derive a general expression to describe the difference in optimal first period strategy depending on whether learning occurs or not.

Note that other terms from the previous version of this cost function (equation (5.4)) have been dropped in equation (5.8). In particular, the linear and quadratic terms in $x_i$ that represent the non-interacting stock effect from period 1 decisions are omitted. We could include these terms, and they would increase the contribution of period 1 emissions to period 2 total costs. But because they do not affect the marginal cost in period 2, they would only contribute to the component of the solution that is common to both learning and no learning. To keep the derivations as simple as possible, we begin with this simple two-term cost function for period 2. Also note that although this function only includes a single linear cross-term, it can be thought of as an approximation of a function with a more general form. Suppose the period 2 cost function has other higher order terms as well, such as $x_i^2 x_j$ or $x_i x_j^2$. The more complicated function can still always be approximated to first-order by a function with independent terms and a single linear interaction term as in the example here.

We begin with the case where uncertainty is resolved before the second period strategy is chosen. At $t = 2$, the derivative of the cost function is:

$$\frac{\partial V_2}{\partial x_2} = 2c(\theta)x_2 + d(\theta)x_1 = 0$$

This leads to the optimal second period strategy of
\[ x_2^{*L}(\theta) = -\frac{d(\theta)}{2c(\theta)} x_1 \]

Assuming that the optimal strategy is chosen in period 2, the value function in period 1 becomes:

\[ V_1(x_1, \theta) = \bar{a}x_1 + \bar{b} - E_\theta \left\{ \frac{d^2(\theta)x_1^2}{4c(\theta)} \right\}. \]

The optimal emissions level for the first period is then

\[ x_1^{*L} = \frac{2\bar{a}}{E_\theta \left\{ \frac{d^2(\theta)}{c(\theta)} \right\}} \quad (5.9) \]

When uncertainty is not resolved by \( t = 2 \), the second period strategy is chosen to minimize the expectation of costs:

\[ \frac{\partial V_2}{\partial x_2} = 2\bar{c}x_2 + \bar{a}x_1 = 0 \]

which leads to an optimal second period strategy of:

\[ x_2^{*N} = -\frac{\bar{d}}{2\bar{c}} x_1 \]

At \( t = 1 \) the value function to be minimized becomes

\[ V_1(x_1, \theta) = \bar{a}x_1 + \bar{b} - \frac{\bar{d}^2}{4\bar{c}} x_1^2 \]

The cost-minimizing period 1 emissions level when uncertainty will not be resolved by the second period is therefore:

\[ x_1^{*N} = \frac{2\bar{a}\bar{c}}{\bar{d}^2} \quad (5.10) \]

Compare equations (5.9) and (5.10). For this cost function, the solutions under learning and under no learning are different. The introduction of an interaction term between \( x_t \) and \( x_t \) in the second period's cost function causes the first period strategy to differ depending on whether learning will occur. Given the presence of an interaction, the next two questions are:

1) When there is a divergence between solutions, what does the magnitude of the divergence depend upon?
2) Under what conditions does learning lead to more action than not learning, and under what conditions does learning lead to less action?

Before we address each of these questions, we re-derive the expressions for optimal strategy with and without learning for a special case of this model. Instead of the general continuous distributions for the uncertain cost coefficients, consider the discrete distribution of states of the world $\theta$, with only two possible states:

$$\theta = \{\text{low, high}\}.$$ 

Denote the cost coefficients as 

$$a(\text{low}) = a_L; \quad a(\text{high}) = a_H.$$ 

We also define

$$P \equiv \Pr(\theta = \text{high}),$$

the probability of being in the high damage state of the world. Thus the expectation with respect to $\theta$ can be written:

$$\bar{a} = E_{\theta}\{a(\theta)\} = Pa_H + (1 - P)a_L.$$

For this simple case with only two discrete states, it can be shown that the optimal period 1 strategies become

$$x_t^N = 2\bar{a}\left(\frac{\bar{c}}{d'}\right)$$ (5.11)

$$x_t^L = 2\bar{a}\left(\frac{c_Lc_H}{c_Ld_H^2P + c_Hd_L^2(1-P)}\right)$$ (5.12)

for the cases without and with learning, respectively. Note that only the last term differs between (5.11) and (5.12). For this problem, the optimal period 1 decision is determined by the average or expected period 1 marginal cost $\bar{a} = E\{a\}$, and then scaled by a term representing the second period marginal costs. Below we will use these expressions to demonstrate the conditions that result in a large divergence in strategies with and without learning.

5.4 Magnitude of the Learning Effect

In this section we will show that the magnitude of the divergence between optimal strategy under learning and under no learning is determined by the variance, or
magnitude of uncertainty, in the cost coefficients. To explore this relation, we consider
two cases: $c$ constant and $d$ uncertain, and vice versa. Consider first the role of
uncertainty in the interaction term, assuming for the moment that there is no uncertainty
in the quadratic cost coefficient $c(\theta)$; i.e., assume that $c_L = c_H = \bar{c} = c$. Then the
solution for learning, given in equation (5.12), simplifies to:

$$x_1^L(\theta) = \frac{2\bar{c}}{E\{d^2(\theta)\}}.$$ 

In this case, we can now subtract this from equation (5.11), the strategy with no learning,
and express the difference between the strategies under learning and no learning as:

$$x_1^N - x_1^L = \frac{2\bar{c}}{(E\{d\})^2} - \frac{2\bar{c}}{E\{d^2\}}.$$ 

It can be shown that the difference between the two solutions is proportional to a function
of the variance of the interaction effect $d(\theta)$:

$$x_1^N - x_1^L \propto \frac{1}{d^2} - \frac{1}{VAR(d) + \bar{d}^2}.$$ 

As the variance of $d$ approaches zero, and the uncertainty in the interaction effect gets
small, the difference between solutions approaches zero:

$$VAR(d) \to 0, \quad x_1^N - x_1^L \to 0$$

Conversely, as the variance of $d$ grows infinitely large, the divergence between the
optimal strategies approach a limit defined in terms of the expectation of marginal cost
terms:

$$VAR(d) \to \infty, \quad x_1^N - x_1^L \to \frac{2\bar{c}}{\bar{d}^2}.$$ 

Given the presence of an interaction, uncertainty in other components of the costs
also causes a divergence between the solutions with and without learning. The
uncertainties in all coefficients that appear in the marginal cost of the second period (see
equations (5.7) and (5.8)) contribute to the divergence between solutions. In this
example, the parameter $c(\theta)$, which represents the slope of the marginal cost in period 2,
also contributes.

Next, assume that $d$ is certain ($d_L = d_H = d$), but that $c$ is uncertain. Then the
difference between the optimal strategies simplifies to:
\[ x_i^N - x_i^L = \frac{2a}{d} [P(1-P)(c_H - c_L)^2] \]

In words, the difference between the optimal level of emissions in the first period with and without learning is directly proportional to the variance in the uncertain marginal cost of emissions in period 2:

\[ x_i^N - x_i^L \propto \text{VAR}(c) \]

The more uncertain the costs of different period 2 strategy choices, the more the interaction will lead to a different strategy, relative to without learning, in period 1 in order to anticipate the extreme responses that might occur after learning.

In the general case when an interaction is present, there will often be uncertainty both in the marginal cost \( c \) and in the interaction between periods \( d \). It is not the case that larger variance in both parameters necessarily contributes to a larger divergence between the strategy with learning and the strategy without learning. In fact, the largest divergence in strategies will occur when one parameter has a much larger variance than the other, as we will demonstrate next.

Suppose now that \( c \) is uncertain with the distribution \( \{ c_H = 100; c_L = 10 \} \), and let \( d \) be uncertain as well, with an expected value of roughly the same order of magnitude: \( \bar{d} = 20 \). For simplicity, let \( d \) have symmetric uncertainty such that

\[ d_H = \bar{d} + \Delta \quad d_L = \bar{d} - \Delta \]

We also assume that the high and low damage states are equally likely; i.e., the probability of the high damage state is:

\[ P = 0.5 \]

How does the bias in the period 1 strategy introduced by learning change as a function of \( \Delta \)? Does it monotonically increase in \( \Delta \)? The answer, shown in Figure 5.1, is no. The figure shows the ratio of the strategy without learning to the strategy with learning, \( x^N/x^L \). When this ratio is close to 1, the difference between strategies is insignificant. Note that if there is no uncertainty in \( d \) (\( \Delta = 0 \)), the no-learn strategy abates 3 times as much as the strategy with learning. Increasing the uncertainty in this parameter up to \( \Delta = 16 \) will decrease the divergence between strategies until there is no effect of learning. If the uncertainty is increased beyond this point, the strategies again diverge with the difference growing larger as the variance in \( d \) is increased.
We can solve to find an expression for the minimum of the ratio $x^N/x^L$. Taking the ratio of equations (5.11) and (5.12), we get:

$$\frac{x_1^N}{x_1^L} = \frac{E[c] \left[ c_L d_H^2 P + c_H d_L^2 (1 - P) \right]}{c_L c_H E[d]^2}.$$  (5.14)

If we rewrite this expression in terms of $\Delta$ by replacing $d_H$ and $d_L$ with the identities in equation (5.13), we can take the derivative and solve for the value of $\Delta$ that gives a minimum. The value that minimizes the difference between strategies is:

$$\Delta_{\text{min}} = \left( \frac{c_H - c_L}{c_H + c_L} \right) \bar{d}.$$  (5.15)

But because we have defined the uncertainty in $d$ as,

$$\Delta = \frac{d_H - d_L}{2},$$

and because with $P = 0.5$, the expectation of $c$ is,

$$\bar{c} = \frac{c_H + c_L}{2},$$

so we can rearrange equation (5.15) to show that the divergence between first period strategies with and without learning are minimized when:

$$\frac{d_H - d_L}{d} = \frac{c_H - c_L}{\bar{c}}.$$  (5.16)
Figure 5.2: Effect of Learning on Strategy Depends on Variance of both Cost Parameters

This relationship in equation (5.16) indicates that if the uncertainty or variance of the interaction (d) relative to its expected value (i.e., in percentage terms), is equal to the variance in the marginal cost (c) relative to its mean value, then there will be essentially no difference between the strategies with and without learning.

Equation (5.16) represents that the fact that there is an offsetting effect between the relative variances in these two parameters with regard to the divergence between strategies. Figure 5.2 shows three qualitatively different situations that can arise. If there is very little uncertainty in the parameter c, then the divergence between strategies will monotonically increase with the uncertainty in d, as shown panel a. This is the same case derived above by assuming no uncertainty in c. As the uncertainty in c becomes very large, then the largest divergence occurs with little or no uncertainty in d, and increasing the uncertainty in d will actually decrease the effect of learning (panel c). Panel b in Figure 5.2 shows the case for moderate levels of uncertainty in c. In this case, the maximum effect of learning occurs either with very little uncertainty in d, or with very large uncertainty in d. The same behavior can be observed by fixing the level of uncertainty in d, and varying the uncertainty in c to find where the greatest influence of learning occurs.

In general, if there is an interaction across periods, the difference in the strategies increases with the variance of the uncertain marginal cost coefficients. The intuition behind this relationship is that the optimal first period strategy varies as a function of the anticipated second period decision. The second period decision under no learning will be made based on expected values of the uncertainty, and the optimal first period decision will account for this anticipated period 2 decision also in terms of expectations. The
second period strategy under learning will entail either more or less activity than that under no learning, depending on which state of the world obtains. Thus the anticipation of the second period decision in choosing the optimal first period strategy must account for the more variable or divergent strategies that may be chosen later. Therefore, the difference between optimal first period strategy with and without learning is a function of the variance in the marginal cost.

The lack of an influence of learning on strategy when the relative range of uncertainty is the same, as illustrated in Figure 5.2, can be understood as follows. When the marginal costs of the period 2 strategy are very uncertain, but we are more sure about how we can anticipate responses to learning (knowing that the interaction is "head start" or "save your effort"), then there will be a large divergence between the learn and no-learn strategies. Similarly, if we are relatively sure about tomorrow's cost and strategy choice, but not sure how to anticipate it with action today (don't know interaction type), there will again be a divergence. But if we are equally unsure of what tomorrow's costs are and of what we should do today to anticipate any particular choice, then these two effects essentially cancel each other out, leaving the optimal period 1 strategy with learning no different than the no-learn optimal strategy.

5.5 Direction of the Learning Effect

Our next main objective is to understand whether learning will result in more or less action when an interaction is present. Here we will show that whether learning results in a higher or a lower level of action depends of the probability distribution across states, and on the direction of the interaction effect. First, we will show that in a problem where cost increases monotonically with strategy level, learning will always bias first period strategy in the same direction. Then we will use cost functions for a more general decision in which abatement costs are balanced against damage costs, and show that learning can bias strategy up or down.

Consider the decision with cost functions (5.7) and (5.8) with all coefficients greater than zero. For this decision, the marginal costs increase monotonically with increasing activity. Suppose these functions represent damage losses (ignoring abatement costs) as a function of emissions level \( (x_1 \text{ and } x_2) \). As discussed above, the
argument to delay abatement because learning is expected implies that less abatement will be optimal under learning when solutions diverge. To investigate the direction of the learning effect, we can examine the ratio of the two optimal strategies, \( \frac{x_1^N}{x_1^L} \), from equation (5.10). When this ratio is greater than 1, then emissions will be greater when there is no learning; if the ratio is less than 1, then learning will result in higher emissions. It can be shown that for any values of the coefficients \( c(\theta) \) and \( d(\theta) \) that the first period strategy level (e.g., emissions) are higher in the no learning case. For example, Figure 5.3 shows the ratio of optimal period 1 strategies as we vary the probability of the high cost state for the following parameters:

\[
\begin{align*}
c_L &= 10 & c_h &= 100 \\
d_L &= 20 & d_h &= 50
\end{align*}
\]

At the extremes, representing certainty about the true costs, the strategy is uninfluenced by learning since there is nothing to learn about. Otherwise when there is uncertainty, the period 1 strategy entails a higher level of activity without learning, relative to the strategy when learning will occur (i.e., ratio is always greater than one).

Learning always leads to a lower strategy level \( x \), because costs grow monotonically. There is a downside to doing too much, but no equivalent downside to doing too little. Thus the irreversibility and uncertainty leads to a lower level of the irreversible activity if learning and correction are possible later. If we consider the costs
as damage losses and $x$, as emissions, this is equivalent to the Arrow and Fisher (1974) result that learning leads to lower emissions with learning. If we consider the costs as representing only control costs and $x$, as emissions level, this is equivalent to the investment under uncertainty models of Pindyck (1991) in which learning will lead to higher emissions (i.e., less abatement with learning).

Real problems such as a decision about climate policy involve both abatement and damage costs that change in opposite directions with the emissions level. The irreversibility in both damages and control costs causes two effects of learning in opposing directions. In this more general case, learning can bias emissions up or down depending on the relative magnitudes of control costs and damage costs.

The dominant direction of the learning bias can be explained in terms of two elements of the decision: 1) the anticipation of the period 2 strategy through the interaction, and 2) the relative regret over outcomes after learning occurs. Section 5.3 showed that, in the presence of an interaction, the optimal first period decision is the product of two components: the marginal costs of period 1 (independent of learning), and a function of the marginal costs of period 2 (dependent on learning). The second component represents the anticipation of the period 2 strategy, and the appropriate response, in terms of the interaction, in period 1 to that anticipated period 2 strategy choice.

When a period 1 strategy is chosen under uncertainty, and then the uncertainty is resolved in period 2, some regret over the period 1 choice is inevitable\(^2\). Consider the left side of Figure 5.4, which shows the optimal emissions level chosen in each period with and without learning. The emissions levels chosen when learning will not occur are shown as the solid lines. These emissions are chosen based on expected values under the uncertainty, and consist of lower emissions in period 2 than in period 1. Suppose in period 2 we learn that the damages from climate change are less severe than expected. Then we will choose a higher level of emissions in period 2 than we did under no learning (the upper dashed line in period 2). Because of the interaction, we will wish that we had anticipated this higher emissions level in period 1, and chosen higher emissions in period 1. We will regret having spent more on abatement cost in period 1 than turned out

\(^2\) Unless of course the revealed true state is exactly equal to the expectation under uncertainty.
to be necessary. Now, suppose in period 2 we learn that the damages are more severe than the expectation. In this case, we will lower emissions further in period 2 than we would have without learning (lower dashed line in period 2). Because of the interaction, we will wish we had anticipated this lower emissions strategy by emitting less in period 1. We will regret not having taken enough precaution in the face of uncertain climate damage.

Whether learning leads to a bias towards higher or lower emissions, relative to no learning, depends on which regret is larger. Figure 5.4a illustrates a situation in which the amount that period 2 emissions are lowered for the high damage state is more than the amount that emissions are increased for the low damage state. In this case, the anticipation of these two possible strategies leads to a decrease in period 1 emissions relative to the no learning case. We call this situation the “precautionary case,” in which the dominant concern is over not having reduced emissions enough.

The converse of this situation is shown in Figure 5.4b. Here, the amount by which emissions are increased if we learn that damages are low is greater than the amount by which we decrease emissions if we learn that damages are high. The anticipation of the much higher period 2 emissions in the low damage state creates a bias towards higher emissions in period 1. We call this the “sunk cost” situation, where we are more worried about having over-controoled emissions at the expense of not investing as much in other parts of the economy.
The difference between the precautionary case and the sunk cost case can be illustrated with the shape of the period 2 marginal costs. Consider the cost functions in Figure 5.5 as representing damage costs minus abatement costs, so that a higher marginal cost leads to lower emissions in period 2. Figure 5.5a shows the case where the marginal cost is convex. Without learning, the second period emissions level is chosen based on the expectation of the damage state $E(\theta)$, and its corresponding marginal cost $MC(E(\theta))$. If learning occurs, the period 2 emissions level will be set based on either $MC(\theta_h)$ or $MC(\theta_l)$ depending on which state was revealed to be true. From the point of view of period 1, the marginal period 2 cost under learning is the expectation of these two levels, as read off of the dashed line and indicated by $E[MC]$.

For a convex cost curve, the expected marginal cost in the learning case will be greater than the marginal cost for the expected damage state:

$$E[MC] > MC(E(\theta)).$$

This asymmetry reflects the fact that the increase in marginal costs in the high damage state is larger than the decrease in costs in the low damage state. The convexity causes a bias toward lower first period emissions when learning will occur (Figure 5.4a). The convex marginal costs cause the influence of learning to be in the direction of the precautionary case.

The sunk cost case, conversely, occurs when the period 2 marginal costs are concave, as shown in Figure 5.5b. Because of the concavity of costs, the marginal cost of
the expected damage (no learning) will be greater than the expected marginal cost after learning:

\[ E\{MC\} < MC(E\{\theta\}) \, . \]

In other words, the decrease in cost in the low damage state is greater than the cost increase in the high damage state. In this case, the period 1 emissions will be higher with learning than without. This illustration of the direction of the learning effect determined by the concavity or convexity of the marginal cost curve, shown in Figure 5.6, is exactly equivalent to Theorem 1 of Epstein (1980) (see Chapter 1).

We now modify the analytical model in order to demonstrate that learning can bias first period emissions up or down. The cost functions of Sections 5.2 – 5.4 represented net (damage plus abatement) costs. Here, we will be explicit about the damage costs (increasing in emissions; i.e., in \( x \)) separate from the abatement costs (decreasing in emissions). We will retain the cost coefficients from equations (5.5) and (5.6) to represent damage costs, and add one linear term (decreasing in \( x \)) in each period to represent abatement costs:

\[ C_1(x_1, \theta) = a(\theta)x_1 + b(\theta) - fx_1, \tag{5.17} \]

\[ C_2(x_1, x_2, \theta) = c(\theta)x_1^2 + d(\theta)x_1x_2 - ex_2 \tag{5.18} \]

For simplicity, we assume that abatement costs (\( f \) and \( e \)) are certain. This model can easily be generalized to have uncertain abatement costs as well, but learning about damages would not necessarily reduce uncertainty in those terms.

Solving for the optimal first period strategies with and without learning for this decision problem, we obtain the following expressions:

\[ x^*_N = \frac{\bar{a} - f + \frac{ed}{2\bar{c}}}{\frac{d^2}{2\bar{c}}} \tag{5.19} \]

\[ x^*_L = \frac{\bar{a} - f + E_\theta \left\{ \frac{ed(\theta)}{2c(\theta)} \right\}}{E_\theta \left\{ \frac{d^2(\theta)}{2c(\theta)} \right\}} \tag{5.20} \]

Using the expressions in equations (5.19) and (5.20), we will demonstrate with some simple empirical cases that learning can either lead to higher or lower emissions in period
Table 5.1: Three Different Distributions of Uncertain Damage Costs and Interactions with Certain Abatement Cost

<table>
<thead>
<tr>
<th></th>
<th>Damage State</th>
<th>Abatement Cost (c)</th>
<th>Marginal Cost (d)</th>
<th>Interaction (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Low</td>
<td>500</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>500</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Case 2</td>
<td>Low</td>
<td>500</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>500</td>
<td>1000</td>
<td>100</td>
</tr>
<tr>
<td>Case 3</td>
<td>Low</td>
<td>500</td>
<td>100</td>
<td>-100</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>500</td>
<td>1000</td>
<td>-20</td>
</tr>
</tbody>
</table>

Figure 5.6: Difference Between Strategies with and without Learning

1. Three different sets of parameter assumptions are shown in Table 5.1. We assume that the period 1 cost (equation 5.16) consists of a linear abatement cost with no damage cost:

   \[ a = 0, \quad f = 10. \]

We treat abatement costs in both periods as certain, and we assume that the probability of the high damage state is 0.5, to maximize the uncertainty in these discrete distributions. Below we examine the effect of learning in each case.

Case 1 in Table 5.1 has an expected damage cost that is small relative to the abatement cost. The ratio between the period 1 strategies is shown in Figure 5.6a. The ratio is always greater than one, meaning that period 1 emissions without learning are higher than period 1 emissions with learning. The effect of learning in Case 1 is the in direction of the precautionary case, shown in Figure 5.4a. The relatively low
expected marginal damages causes the losses in the high damage state to be greater. This asymmetry in outcomes after learning is anticipated by lowering emissions in period 1.

In Case 2 of Table 5.1, we now increase the expected damage losses by increasing the marginal damage cost $c$ for the high damage state from 200 to 1000, while all other parameter values remain the same as in Case 1. The ratio of optimal period 1 strategies for this case is shown in Figure 5.6b. The ratio here is always less than one, so the first period emissions with learning are higher than the emissions without learning. The higher expected damages cause the greater loss after learning to occur in the low damage state, because of overly stringent constraints on period 1 emissions. This is the sunk cost case, shown in Figure 5.4b, where less period 1 abatement is optimal if we will learn.

Even when marginal costs are linear (i.e., neither concave nor convex) as in this formulation, learning can cause emissions to be higher or lower in period 1. The direction of the learning effect is also affected by the skewness of the probability distribution over damage states. When the distribution is skewed towards low damages and has a low expected damage (Case 1), the greater downside to learning is when the damage state is high, so period 1 emissions are higher. When the distribution is skewed towards higher damage and has a high expected damage (Case 2), then the greater downside to learning is when the damage state is low, and period 1 emissions are lower with learning.
The other determinant of the direction of the learning effect is the sign of the interaction effect. The results above all assumed interactions (d) that are increasing in emissions. As described in Section 5.3, this type of interaction represents the “head start” effect or a complementary relationship between strategies. Lower period 1 emissions are optimal in anticipation of lower period 2 emissions. If the interaction is negative, the “save your effort” effect, then the direction of the influence on period 1 strategy by learning is reversed. As an example, consider Case 3 in Table 5.1. All parameters are the same as Case 2 except for the interaction terms, which are both less than zero. As shown in Figure 5.7, period 1 emissions with learning are always lower than without learning for Case 3 (the ratio of strategies is greater than one). Whereas Case 2 exhibited the “sunk cost” effect and emissions were higher with learning, the same distribution over damage states here leads to a “precautionary” effect. The probability distribution of uncertain damage states can create an asymmetry in which one response to learning will have greater losses, but it is the sign of the interaction that determines the direction of the response to the asymmetry.

5.6 Summary

In summary, what have we learned from constructing an abstract two-period problem and solving with and without learning? First, there must be some interaction between the decisions in both periods that affects second period total costs. Without such an interaction, where the period 1 strategy alters the marginal costs of the period 2 strategy, the problem decomposes to independent one-period decision problems. In the presence of an interaction, learning introduces a bias in the period 1 strategy that diverges from the strategy in the no learning case.

Second, the magnitude of this bias grows with the variance of the uncertain parameters. Increasing the expected value of any parameter will not change the divergence between strategies if the relative variance is constant in percentage terms. The reason for this is that the difference in strategies with and without learning are a function of the losses that occur if we learn that the true state is at an extreme of the distribution. If the relative uncertainties in the marginal period 2 costs and in the interaction effect are roughly equal, the divergence in strategies will be insignificant. If
Table 5.2: Comparison of Optimal Period 1 Emissions with and without Learning

<table>
<thead>
<tr>
<th>Expected Damage State</th>
<th>No Interaction</th>
<th>Interaction (−) &quot;Save Effort&quot;</th>
<th>Interaction (+) &quot;Head Start&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very High</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L = x_i^N$</td>
</tr>
<tr>
<td>High</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L &lt; x_i^N$</td>
<td>$x_i^L &gt; x_i^N$</td>
</tr>
<tr>
<td>Low</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L &gt; x_i^N$</td>
<td>$x_i^L &lt; x_i^N$</td>
</tr>
<tr>
<td>Very Low</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L = x_i^N$</td>
<td>$x_i^L = x_i^N$</td>
</tr>
</tbody>
</table>

However, either uncertainty is much larger than the other, the period 1 strategy with learning may be quite different than the strategy that would be chosen without learning.

Third, when the strategies differ depending on whether learning occurs, the direction of the effect depends in part on the probability distribution over the damage states, and how large expected damage costs are relative to abatement costs. When expected damages are high, we will abate more if we cannot learn; this is the "precautionary" case. If expected damages are low, we will abate emissions more if we do expect to learn; this is the "sunk cost" case. The direction of the learning effect is also determined by the sign of the interaction. The above results hold when the interaction effect is positive in emissions, the "head start" interaction. If the interaction is negative in emissions, the "save your effort" effect, then the results are exactly the reverse: High expected damages lead to higher emissions in period 1 with learning, and low expected damages lead to lower period 1 emissions with learning.

The qualitatively different effect of learning on first period strategy in Cases 1 and 2 were demonstrated by changing one value in a discrete two-point distribution of damage states. A similar change in the distribution of damage states can be made by varying the relative probabilities of the states. In Chapter 6, we will use this technique of varying the probability in sensitivity tests to show when we are in a precautionary situation and when we are in a sunk cost situation.

The conditions that determine the effect of learning on strategy are summarized in Table 5.2. Without an interaction, the strategy with learning will be the same as the strategy without learning. Similarly, if the expected damages are too low or too high relative to abatement costs, there will be no difference between strategies because with or without learning the strategy will be the extreme of the feasible set (i.e., a corner
solution). With a Head Start interaction, high expected damages bias emissions up with learning (sunk cost) while low expected damages bias emissions down with learning (precautionary). The Save Your Effort interaction has the opposite effect.

Further work on different formulations of the analytical models presented in this chapter is warranted. It would be helpful to derive the conditions that determine the magnitude and direction of the divergence between strategies for more general cost functions, including other terms excluded in the examples here (e.g., linear, quadratic), as well as higher order terms that will induce convexity or concavity in the marginal costs. It is also desirable to derive expressions for these conditions using more general distributions of damages than the two-point discrete Bernoulli distributions used here. While the relationships and insights developed here will prove helpful in Chapter 6 in explaining the role of learning in an empirical model using the IGS, more general results may prove these insights to be more robust and applicable to a wide range of decision problems.
Chapter 6  Analysis of Interaction Effects in the IGSM and in Alternative Formulations

*The policy of being too cautious is the greatest risk of all.*
- Jawaharlal Nehru

Chapter 4 presented two-period sequential decision models for climate policy using a reduced-form version of the MIT IGSM to calculate outcomes. Using these models, we showed that the optimal period 1 emissions rate in the case where all uncertainty in climate damages were resolved before period 2 is almost always the same as the period 1 emissions rate when the uncertainty is not resolved. In Chapter 5, we used an analytical two-period dynamic programming model to show the conditions under which first period strategy is affected by learning between periods. In this chapter, we return to the empirical climate decision models of Chapter 4, and apply the insights of Chapter 5 to explain why learning does not affect strategy in this model.

Other studies have previously shown the same lack of influence of learning on first period emissions. (Kolstad, 1996; Ulph and Ulph, 1997). As described in Chapter 1, the reasons given for the lack of influence by learning are that the irreversibility constraint on greenhouse gas emissions is not binding (Ulph and Ulph, 1997), and that the stock pollutant nature of greenhouse gases causes first period emissions to have little effect on marginal damages. By framing the discussion in terms of the presence or absence of an interaction between the strategy in period 1 and the strategy in period 2, rather than in terms of stock pollutant or not, we will demonstrate that other assumptions in the models besides the linearity of damages can help explain the lack of influence from learning.

Framing the issue in terms of interaction, we will show that sequential decision analyses of climate policy tend to be insensitive to assumptions about future learning because several possible interaction effects that could be included are either insignificant in magnitude or altogether omitted from typical integrated assessment models. Moreover, it would be incorrect to conclude on the basis of such models that the potential for learning about the costs and impacts of climate change are irrelevant for the choice of a near-term policy. In fact, there are possible influences of near-term policy on long-term
marginal costs. The most obvious is the presence of extreme non-linearity, and in particular a threshold effect, in the damage function. But there are other interactions as well. For abatement costs, the degree of innovation and technical change (leading to lower cost abatement options in the future) may partly depend on the stringency of current policy. Regarding damages, it is possible (although now considered a low probability) that rapidly rising emissions could result in a change in the circulation of the ocean, which in turn would slow the ocean’s ability to absorb CO₂ and require more stringent emissions reductions later to reach some concentration target. Both of these phenomena tend to be omitted from most climate assessment models.

When we represent these interaction effects by inducing simple correlations in decision models, the optimal strategy does become dependent on whether learning will occur. The ability to learn does not always lead to less abatement in the initial period, however, as many have argued in the context of the climate issue (see Chapter 1). The fact that we may learn more in the future may actually lead to more stringent abatement in the near-term, and is not necessarily a valid argument for waiting. In general, whether expected future learning leads to more or less current abatement will depend on how high the expected damages are. If expected damages are not too high then it is optimal to abate more now if we expect to reduce uncertainty than if we do not expect to learn. If expected damages are higher, then it becomes optimal to undertake more abatement when one cannot learn. This will be true whether current emissions influence marginal costs through increasing technical progress or marginal damages by causing some kind of state change in the climate system.

Section 6.1 will use reduced-form models to demonstrate that the interactions required for learning to influence strategy are not significant enough to have an effect. The reduced-form models used here are in a simpler form than the models developed in Section 3.6 and used in the decision models of Chapter 4. These simpler reduced-form models are fit to the same functional form as the cost functions of the analytical model of Chapter 5 to facilitate the application of the insights from the analytical model. The coefficients of the reduced-form models will demonstrate numerically that the uncertainty in interaction in these models is insignificant, and that this is the reason why the first period strategy does not depend on learning. Similar estimates are performed for decision
models that have uncertain abatement costs, and the interactions here will also be shown to be small.

Although the magnitude of the interactions typically captured by integrated assessment models are relatively small, other more significant interactions are omitted. Such interactions include the impact of emissions rate on the probability of a state change in ocean circulation, and the effect of near-term policies upon the rate of development of technological options for the future. The potential existence of a non-linear threshold effect in the damage costs would also result in a large interaction between the period 1 strategy and period 2 marginal damages. Section 6.2 will present alternative formulations of the decision models to include each of these possible interactions. With the alternative formulations, we demonstrate that if any such interactions are included in the decision models, the optimal first period policy will depend on whether learning is expected.

6.1 Reduced-Form IGSM to Highlight Interaction Effects

6.1.1 Climate Decision Models with Uncertain Damage Costs

In this section, we will now apply the insights from the analytical model of Chapter 5 to explain the puzzling lack of an effect by learning in the decision models of Chapter 4. To do this, we will replace the reduced-form models that are used to calculate outcomes from strategy choice and uncertain quantities with approximations formulated to allow the exploration in the IGSM context of the insights developed in Chapter 5. The new approximations calculate the total cost directly as explicit quadratic functions of the strategy variables. This will enable the substitution of the cost coefficients estimated from the IGSM into the expressions derived in Chapter 5.

The decision models presented in Chapter 4 determine the total cost for any set of strategies and values of uncertain parameters by using a large set of polynomial chaos expansions that approximate the behavior of different components of the Integrated Global System Model. The sequence of calculations followed to obtain total cost for one strategy and one state of the world, described in Section 3.6.1, consists of six steps:

1. Calculate CO₂ emissions and abatement costs
2. Calculate CO₂ concentrations
3. Convert concentrations time-series into a rate of increase
4. Calculate global mean temperature change over time
5. Apply damage function to get damage costs
6. Discount and combine abatement costs and damage costs

Each of these variables has a separate approximation for each point in time (e.g., emissions in 2025). Abatement costs are approximated for each region and point in time. The approximation for each intermediate variable is either a third-, fourth-, or fifth-order polynomial expansion. This set of approximations, used in steps 1-6 above, constitutes an implicit model of the form:

$$TotalCost = f(Policy2010, Policy2020, ClimateSensitivity, OceanUptake, DamageCoefficient)$$

This model is implicit in the sense that there is not a function that takes values for Policy2010, etc., and calculates the total cost in a single step with one equation. Because of this, there is no one coefficient that can be examined to determine, for example, the size of the interaction of Policy2010 with Policy2020. Here, we will estimate a new reduced-form model that is explicit, calculating the total cost in each period with a single equation. We will use the same functional form as the analytical model of Chapter 5, including a single linear cross-product in the two periods' emissions.

For simplicity and consistency with the analytical model of Chapter 5, we will only treat one uncertain parameter: the damage function coefficient. As shown in Chapter 4, total expected costs are most sensitive to the uncertainty in damage valuation (see Figure 4.5). We will use the same two-point discrete distribution as in Chapter 4, (low = 0.02; high = 0.16). As in Chapter 5, we will estimate two sets of coefficients for the cost function, one for the low damage state and one for the high damage state. We estimate the following four outcomes with polynomial chaos expansions:

- Abatement Cost in Period 1 (2010-2019),
- Abatement Cost in Period 2 (2020-2100),
- Damage Cost in Period 1, and
- Damage Cost in Period 2.

Each of these costs is a discounted sum to all of the Annex I regions combined. Two separate sets of approximations, one for each value of the damage coefficient, are estimated, for a total of eight expansions. Each expansion is a second-order polynomial in the two strategy variables and includes a single cross-term; i.e. of the form:
\[\text{Cost}(\text{Policy2010}, \text{Policy2020}, \text{DamageState}) = \]
\[a_0 + a, \text{Policy2010} + a, \text{Policy2010}^2 + a, \text{Policy2020} + a, \text{Policy2020}^2 + a, \text{Policy2010 Policy2020}\]

More terms could be added to the expansion, and would further improve the accuracy of the approximation, but second-order produces a reasonable fit as will be shown below and the coefficients are directly comparable to the coefficients of the analytical model in Chapter 5.

The Deterministic Equivalent Modeling Method is used to estimate these reduced-form models, just as for the reduced-form models estimated in Section 3.6 and used in Chapter 4. Recall from Chapter 2 that rather than use a minimum least squares method, DEMM uses the collocation approach, which requires a residual of zero between the expansion and the model at all sample points, to solve for the coefficients of the expansion. To fit each approximation of the form (6.1) requires solving for the six coefficients \([a_0, \ldots, a_5]\), and therefore six sample strategy pairs and their resulting cost from the IGSM are needed to solve the linear system of equations. The six sample pairs of \((\text{Policy2010, Policy2020})\) used to estimate the cost function approximations are shown in Figure 6.1. The strategies indicated by the "●" symbols are imposed as an emissions constraint for a simulation of the full integrated global system model. The resulting abatement costs and damage costs are discounted and combined for all Annex I regions. The resulting total costs for the six strategy sets are used to calculate the coefficients of the expansion \([a_0, \ldots, a_5]\) in equation (6.1)).

To check the accuracy of the new reduced-form approximation, an additional eight strategy pairs are used in simulations of the full IGSM, and the resulting abatement and damage costs for the two periods are compared with the estimates produced by the new approximations. The strategy pairs used for error checking as indicated by the "+" symbols in Figure 6.1.

The reduced-form model for damage cost in period 2 is:
\[DC_2(x_1, x_2, H) = 11881 + 136x_1 + 8.7x_1^2 + 359x_2 + 34.1x_2^2 + 18.1x_1x_2\]  
(6.2)
for the high damage state. The variables \(x_1\) and \(x_2\) here represent the strategy variables \(\text{Policy2010}\) and \(\text{Policy2020}\), respectively. The damage cost function in equation (6.2) has probability \(P_n\) (probability of the high damage state).
The approximation for damage cost for period 2 in the low damage state is:

\[ DC_2(x_1, x_2, L) = 1485 + 17.0x_1 + 1.09x_1^2 + 45.0x_2 + 4.26x_2^2 + 2.26x_1x_2 \quad (6.3) \]

Equation (6.3) has probability 1 - \( P_p \) (probability of the low damage state).

Figure 6.2 illustrates the accuracy of the approximations of damage costs for periods 1 and 2 when the damage coefficient is assumed to be low. Points that fall on the 45\(^o\) line represent negligible error. Although there are some noticeable deviations for damage costs in period 1, these errors constitute less than 1% of the mean value (in terms of sum
Figure 6.3: Accuracy of Abatement Cost Approximations (Equation (6.4))

a) Annex I discounted abatement costs (3%) for 2010-2019
b) Annex I discounted abatement costs (3%) for 2020-2000

The second period damage cost estimate is even more accurate, with errors of less than 0.1%. The approximations of damage cost when the damage coefficient is assumed to be high are of comparable accuracy.

The costs of abatement are assumed for the moment to be certain (i.e., \( AC(x_1, x_2, L) = AC(x_1, x_2, H) \)). The approximation of the abatement costs in period 2 is:

\[
AC_2(x_1, x_2) = 17227 - 2618x_1 + 144x_1^2 - 3007x_2 + 192x_2^2 + 159x_1x_2 \quad (6.4)
\]

Figure 6.3 shows the accuracy of the reduced-form models of abatement cost equation (6.4) and the approximation for period 1). As can be seen from the figure, for all eight strategies used for checking the accuracy of the approximations, there is little or no error. Numerically, these approximations have errors of \( 1\% \sim 2\% \) of the mean.

Because the objective of the two-period decision model is to minimize total costs, we sum the costs to obtain a single function for total period 2 costs in the high state:

\[
TC_2(x_1, x_2, H) = 27108 - 2482x_1 + 153x_1^2 - 2648x_2 + 226x_2^2 + 177x_1x_2 \quad (6.5)
\]

and in the low state:

\[
TC_2(x_1, x_2, L) = 18712 - 2601x_1 + 145x_1^2 - 2962x_2 + 196x_2^2 + 161x_1x_2 \quad (6.6)
\]

Let us now examine the estimated coefficients of these cost functions, and check for the necessary conditions, developed in Chapter 5, for learning to affect the period 1 decision. Our primary interest is to examine the magnitude and variance in the
coefficient of the interaction term in the second period cost function. The first condition is that there must be an interaction between the two periods' decisions that affects the second period's costs. The coefficients of the interaction terms in equations (6.5) and (6.6), 177 and 161, are non-zero, so our first condition appears to be satisfied. The net effect of the interaction is that an increase in period 1 emissions will increase the marginal cost of period 2 emissions. This positive interaction is the "head start" type described in Chapter 5, where abatement in period 2 is made less costly by some reduction in emissions in period 1.

The majority of this interaction effect comes from the deterministic estimate of abatement costs (the coefficient 159 in equation (6.4)). The interaction effect in the damage costs is relatively small (18.1 and 2.26 in equations (6.2) and (6.3), respectively), and as a result the uncertainty in the interaction effect is actually quite small. We can see this by ignoring the linear term in $x_i$ and applying the expressions derived previously. Assume that $P_\mu = 0.5$ to maximize the uncertainty between the two states. Using the expression in equation (5.12), the optimal period 1 strategy in the learning case will be proportional to

$$x_i^L \propto \frac{c_L c_H}{c_L d_H^2 P + c_H d_L^2 (1 - P)} = \frac{(196)(226)}{(196)(177^2)(0.5) + (226)(161^2)(0.5)} = 0.00738,$$

and in the case with no learning (using equation (5.11)) the optimal decision will be proportional to:

$$x_i^N \propto \frac{E[c]}{E[d]^2} = \frac{211}{169 \times 169} = 0.00739.$$

Although the interaction is present, the relative levels of uncertainty in the cost coefficients appear to be in a region that produces a negligible divergence between strategies. Given the uncertainty in marginal cost (196 vs. 226), we can examine the behavior of the ratio between optimal strategies as in Figures 5.1 and 5.2. Figure 6.4 shows the graph of the ratio as a function of the uncertainty in the interaction term. The region corresponding to the values of the coefficients in equations (6.5) and (6.6) is magnified in Figure 6.4a. The interaction term for high damages is 177 and for low damages is 161. If we assume that $P_\mu$ is 0.5, then the uncertainty in this term is 169±8. As shown in the figure, the difference between strategy with and without learning is
Figure 6.4: Effect of Learning on Strategy in MIT IGSM

![Diagram showing the effect of learning on strategy in MIT IGSM. The diagram compares the difference between interaction high/low and mean for two different scenarios, with a clear indication of the impact of uncertainty.]

indistinguishable at this level of uncertainty. This is the reason for the lack of influence of learning on strategy in Chapter 4.

Figure 6.4b shows how the difference between strategies will increase as the uncertainty in this interaction increases. To get a bigger effect from learning (still considering only the $x_i^2$ and the $x_i x_j$ terms of the model) either the uncertainty in the marginal cost of emissions in period 2 must be increased, or the uncertainty in the interaction between emissions level in the two periods must be increased.

Why is the uncertainty in the interaction effect too small for learning to make a difference? To understand this, we must first consider the sources of interactions in the underlying economic and climate models. The main interaction between abatement in the two periods results from abatement costs, as seen by the term $159x_i x_j$ in equation (6.4). One interaction in the economic model that will tend to increase costs is the vintaging of capital stock. In the EPPA model, a portion of the preexisting capital stock in any period is not malleable, and cannot be shifted to different sectors when relative prices shift. As a result, a lack of abatement in the first period can result in investment in new carbon-emitting capital, some of which cannot be shifted if abatement is undertaken in the second period (Jacoby and Sue Wing, 1999). Interactions that decrease costs also exist in the model. Increasing abatement in the first period will result in lower consumption and welfare levels after adjusting to the additional constraint. The costs in the second period are measured relative to a reference welfare level without policy, but
this reference level is now lower than it would have been because of the first period abatement. Thus, costs from period 2 abatement in percentage terms will be slightly less if period 1 abatement increases. Although both of these effects are present, neither is very large relative to the level of abatement costs. Comparing the coefficients in the different terms of equation (6.4), this interaction is only a small effect, and acts in the direction of increasing costs.

There are also interaction effects between period 1 emissions and period 2 marginal damages present. The interaction effects across time in the climate system are the result of various feedbacks in the oceanic carbon uptake from the atmosphere. One direction of the feedback occurs because if more carbon is emitted sooner (i.e., in the first period), the carbon concentration gradient between the atmosphere and the ocean surface is greater, which will increase the absorption of carbon by the ocean. This would mean that as period 1 emissions increase, the corresponding increase in period 2 concentrations gradually becomes smaller. On the other hand, the rate of ocean uptake will gradually slow over time due to rising surface temperatures, which will cause higher period 2 carbon concentrations in the atmosphere. The slowing of ocean uptake at higher temperatures\(^1\) becomes a feedback; higher emissions in the first period cause an increase in surface warming, which will further lower the rate of ocean uptake of CO\(_2\) and leave higher concentrations in the atmosphere\(^2\) (Holian, 1998). Because of the change in ocean circulation, higher period 1 emissions increase the marginal damage cost of period 2 emissions.

The estimate of the magnitude of these effects shows that they are also fairly minor relative to the overall damage costs. The marginal damage cost, in billions of dollars, of increasing emissions by one unit in the second period is

\[
MD_2(x_1, x_2, H) = 68.2x_2 + 359 + 18.1x_1
\]

for the high damage state (this is the derivative of equation (6.2) with respect to \(x_2\)). The impact of the interaction term \((18.1x_1)\) on the marginal damage of course depends on the

\(^1\) The solubility of CO\(_2\) in the surface ocean layer is governed by Henry’s Law, which allows the conversion between concentration and partial pressure: \([CO_2]^{\text{in}} = a_{\text{in}} \cdot pCO_2^{\text{in}}\). Henry’s coefficient \(a_{\text{in}}\) has a strong dependence on temperature.

\(^2\) The feedback mechanisms described here are distinct from an abrupt collapse of the thermohaline circulation, which will be discussed in Section 5.4. The mechanism here is only a gradual change in the rate of the vertical mixing in the ocean.
levels of \( x_i \) and \( x_j \), but in general the contribution of this term will be less than 5% of the overall marginal damage. The magnitude of the interaction term in the low damage state, 2.26 (see equation (6.3)), similarly contributes less than 5% to the marginal increase in damage costs. The net effect of the feedbacks is that an increase in period 1 emissions will increase the per-unit damage of emissions in period 2 \( (x_i, x_j) \) terms in equations (6.5) and (6.6) are positive). Since the only parameter varied is the valuation of temperature change, the uncertainty in the interaction effect (18.1 vs. 2.26) is due to the uncertainty in the valuation of the increased carbon concentration, and its subsequent warming contribution.

So with uncertainty in damage costs from the MIT IGSM, the optimal strategy in period 1 is unaffected by learning because the interactions across periods are too insignificant, in magnitude and in uncertainty. Next, we will demonstrate that the same is true for uncertainty in abatement costs as well.

6.1.2 Climate Decision Models with Uncertain Abatement Costs

The previous discussion has illustrated that even when the damage costs of climate change are uncertain, the level of near-term abatement is the same whether or not we expect to learn more about these damage costs. The assumptions about learning have little effect on strategy in these decision models because the uncertainty in the interaction between near-term and long-term decisions is small. The next question is whether the same holds true for uncertainty in the costs of emissions reductions. The costs of stringent reductions over several decades are highly uncertain, as demonstrated in Chapter 3. It is plausible that reducing the uncertainty in abatement costs would cause different actions to be optimal than if such learning did not occur.

To test this hypothesis, we first construct a set of decision models consistent with those in the previous section. These decision models have uncertainty both in damages and in abatement costs. We again consider two cases: the uncertainty in abatement costs will either be eliminated (complete learning) or will remain unchanged in period 2 (no learning). Figures 6.5 and 6.6 show the influence diagrams for the no learning and complete learning decision models, respectively. Like damage costs, we assume that abatement costs are either low or high, so there is only a single uncertainty node. The only difference between the two models is the arc from the backstop uncertainty node.
This arc points to "abatement costs" in the No Learning model, indicating that the true state is only known after the 2020 decision, and points to "Emissions Rate 2020-2100" in the Complete Learning model, indicating that the true state becomes known with certainty before the 2020 decision is made. Damages are still uncertain, but learning does not reduce this uncertainty, so the "Damage Valuation" node is resolved at the same point in both models, after all decisions have been made.

We estimated a set of reduced-form models of the IGSM in which the costs of abatement are uncertain. As with the damage costs, abatement costs have a simple discrete distribution with two possible states: low represents a world where emissions reductions are relatively inexpensive and high represents a world where emissions reductions are costly. Three uncertain parameters of the EPPA model are used to create the low and high cases: the price of the carbon-free electric backstop $B_S$, the elasticity of substitution of fossil fuels $\sigma_{ff}$, and the elasticity of substitution of labor for other factors in production $\sigma_{LKEF}$. Each of these parameters is fully described in Chapter 3. The low-cost world is represented by the parameter settings:
\[ BS = 5.0 \quad \sigma_{fr} = 0.3 \quad \sigma_{LKEF} = 0.7 \]
while the high-cost world is represented by
\[ BS = 25.0 \quad \sigma_{fr} = 0.5 \quad \sigma_{LKEF} = 0.3 \]

Both cases are estimated using DEMM to produce reduced-form models of abatement costs in the two periods. These reduce-form models are explicit second-order expansions in the strategy variables, including a linear cross-product, just as in the previous section. The approximations for abatement cost in the second period are:
\[ AC_2(x_1, x_2, L) = 8577 - 2039x_1 + 179x_1^2 - 2154x_2 + 238x_2^2 + 65x_1x_2 \] (6.7)
\[ AC_2(x_1, x_2, H) = 40133 - 5293x_1 + 253x_1^2 - 7212x_2 + 358x_2^2 + 416x_1x_2 \] (6.8)
The uncertainty in the interaction term coefficient, 65 in the low case and 416 in the high case, is more significant than it was in the experiments with uncertainty in damages (161 vs. 177 in equations (6.5) and (6.6)).

Figure 6.7 shows the optimal decisions under learning and no learning as a function of \( P_{HC} \), the probability of high abatement costs. In the No Learning case, 1.1% per year emissions growth is the lowest cost strategy for all possible values of \( P_{HC} \) (Figure 6.7a). The optimal decision for the Complete Learning case is the same, 1.1% per year, for most values of \( P_{HC} \). However, when the probability of high abatement cost is 0.1 or lower and the uncertainty will be resolved, more stringent abatement is optimal. In other words, only if we are virtually certain that abatement costs will be low (relative to the expected damages), and if we can learn whether costs are low with certainty, is it optimal
to constrain emissions further in the first period. This is the "precautionary" effect described in Chapter 5 (see Figure 5.4a). Learning results in more stringent abatement in this region because the ability to learn and correct makes period 1 abatement activity less costly than it would be without the ability to learn and correct.

In the decision models of Figures 6.5 and 6.6, the uncertainty in damage costs is assumed to be the same as it was in the models of section 6.1. The damage uncertainty is never resolved in the cases here; learning only removes the uncertainty in abatement costs. The assumed probability of high damage costs $P_h$ may still affect the optimal period 1 policy, because the strategy is chosen to balance between expected control costs and expected damage costs. Therefore, a more thorough sensitivity analysis, shown in Figure 6.8, varies both the probability of high abatement costs and the probability of high damage costs. Whether the uncertainty in abatement costs is resolved or not, for most possible distributions of the cost uncertainties the optimal period 1 decision is the same emission rate of 1.1%. When damage costs are very likely to be high and abatement costs are very likely to be low, it is optimal in both cases to undertake the more stringent emission rate of 0.8%/year. There is one region where learning results in a different decision than no learning, where high damage costs are likely (0.4 to 1.0 on the Y-axis) and high abatement costs are fairly unlikely (0.1 to 0.3 on the X-axis) (the difference between the two graphs in Figure 6.8). Only if the joint distribution of uncertain abatement and damage costs fall in this small region will more stringent action be preferred when learning is expected. It appears that there is not enough uncertainty in the
interaction across periods in these models for learning to be a major determinant of optimal emissions control.

When the solutions with and without learning do diverge, more stringent abatement will be preferred with learning. Recall from Chapter 5 that whether learning leads to more or less action depends on the probability distribution over damage states. In this case, the relevant probability distribution is that of abatement costs, since that is the uncertainty which learning reduces. Without the ability to learn, the decision-maker who will not learn is still averse to the risk of high abatement cost at these assumptions about probabilities, and will not move to a more stringent level of abatement. In contrast, the decision-maker who will learn about abatement costs with certainty is more willing to risk the unlikely event of high abatement costs on order to reduce the expected damages (which are high in this region). In terms of Table 5.2, we expect emissions to be lower with learning \((x_t^l < x_t^N)\), because the interaction is a “head start” effect (+) and the expected abatement costs (the uncertainty we learn about) are low relative to expected damages. The role of the expected resolution of uncertainty is to make the risk of over-restricting emissions less costly, because information will be available to revise stringency up or down.

In summary, this section has shown that numerical decision models for climate policy based on the IGSM have too small an interaction effect for learning to significantly affect near-term policy. This has been shown for both uncertainty in damages and uncertainty in abatement costs. If we believed that all possible interactions between near-term policy and future marginal costs were properly represented in the assessment model, we would have to conclude that whether we expect to learn more about climate change was irrelevant to choosing current emissions abatement levels. But several types of significant (and uncertain) interactions are typically omitted from climate and economic assessment models. The next section will describe a few such interactions, and show that when these interactions are included, that near-term policy may well depend on whether — and what — we will learn.
6.2 The Influence of Strong Interaction Terms

Section 6.1 showed that, using the MIT IGSM, the optimal level of abatement in the first decade is almost always the same whether or not we expect to learn more about uncertain climate damage. A simple relationship was derived in Chapter 5 that predicts the difference between learning and no learning solutions in terms of the uncertainty in the interaction between period 1 and period 2 decisions. This analytical relation provides an explanation for the lack of an influence by learning in the case of climate change. The analytical formulation also points to alternative ways to frame the problem, in which whether we expect to learn will indeed matter. Furthermore, by continuing our search for decision models in which the optimal level of abatement does depend on whether learning is anticipated, it will be necessary to include salient features of the climate change decision problem typically omitted from quantitative analysis. The inclusion of these feedbacks and processes will cause optimal near-term strategy to entail more stringent emissions constraints than is true without the feedbacks.

In this section, we will develop decision models that include other interactions that are not captured by the underlying computational models. We will show that assumptions about learning are more important when decisions in the first period actually change the probability that damages per unit emissions are higher (or lower), or the probability that abatement will cost more (or less). This is distinctly different from the sensitivity tests that have been demonstrated up to this point. Previous sections have varied the probability of high damage to compare optimal strategies with and without learning, but the assumptions about probability were kept independent of the strategy chosen in period 1. In this section we will explore the effect of introducing an explicit dependence of the probability of high damage (or abatement cost) on the first period strategy. The dependency of probability on first period strategy provides a simple and convenient conceptual representation of the feedbacks that are not present in the underlying IGSM.

One possible interaction between first period strategy and future marginal damages is if the marginal damages are nonlinear in CO$_2$ concentrations, due to the presence of a threshold level above which marginal damages increase rapidly. Another mechanism for current emissions to affect marginal damages is for the warming to cause
a dramatic shift in the patterns of ocean circulation. On the abatement cost side, we include the possibility that current policies can affect future marginal costs by influencing the rate of technological progress — a concept known as endogenous technical change. Each of these three examples of interaction, and their effect on optimal period 1 strategy, are presented below.

6.2.1 The Influence of Learning with a Threshold Damage Effect

Based only on the results of Section 6.1, it would be premature to conclude that the likelihood of learning more about the many uncertainties in climate change is irrelevant to choosing a near-term emission control policy. Although the models we have examined thus far have relatively little interaction of policies across periods, we must question whether the models include all relevant interactions. In addition to the interactions across periods that the models do capture, outlined in Section 6.1.1, there are other, perhaps more important, interactions that are not represented by the models used here.

One candidate for a stronger interaction between near-term strategy and long term marginal costs is a non-linear damage function. As discussed previously, the damage function used here and in many other models of climate policy is non-linear in temperature change, but in terms of CO₂ emissions or concentrations the damages are linear, or nearly so. Because the damage function is linear in CO₂, changes in the period 1 emissions do not change the marginal damage cost in period 2. Without a strong interaction effect, the optimal level of abatement period 1 is the same whether or not learning will occur.

However, suppose the climate damages were potentially nonlinear in CO₂ concentrations. Uncertainty about a significant non-linearity should cause the period 1 strategy with learning to diverge from the strategy without learning. Here we present an illustrative example to test our hypothesis that learning should influence strategy in this case. To exaggerate the effect, we construct a damage function with a discontinuous threshold effect.

We use the same reduced-form models of the MIT IGSM as in Chapter 4, with the same probability distributions for damage parameters. However, we add one modification to the damage function (Section 3.5). When the damage cost parameter is
Figure 6.9: High and Low Damage Costs with a Strategy of No Abatement

- If CO₂ concentrations > 650 ppm, then damage losses = 99%

Before CO₂ concentrations exceed this threshold, the damages are the same in the high state as in the calculations of Chapter 4. Also, when the damage coefficient parameter is assumed to low, damages are the same as before (i.e., no threshold in the low state).

The threshold of 650 ppm, as well as the level of damage losses resulting from this threshold, are not scientifically based and are merely intended to illustrate the effect of nonlinearity. It is worth noting, however, that Article 2 of the Framework Convention of Climate Change states that the ultimate objective of the convention is to achieve, “…stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system.” (United Nations, 1992). This goal implies the existence of a threshold level above which damages become intolerable. Whether such a threshold actually exists, and if so what that level is, continues to be debated by climate scientists and policy makers (Moss, 1995; Tol 1994, 1996).

Figure 6.9 shows the behavior of the damage function under the low damage and high damage assumptions. Panel (a) shows the path of CO₂ concentrations over time when the strategy is to leave emissions unconstrained in both periods. Under this strategy, concentrations exceed the threshold of 650 ppm between 2080 and 2090. The
resulting economic losses in each year are shown in Panel (b). For the low damage assumption, costs rise gradually over the century. For the high damage assumption, damages rise smoothly until the threshold is exceeded, and then suddenly jump to a much higher level.

Does the period 1 strategy differ depending on whether learning between periods will occur? Figure 6.10 shows the optimal period 1 abatement strategy for the case with no learning between periods (left side) and for the case with complete resolution of uncertainty between periods (right side). For many parameter values, the period 1 strategy when learning will occur is more stringent than if there is no learning. More than half of the possible values for the probability of high damage costs result in a divergence in strategies. This is in sharp contrast to the results of Chapter 4 (e.g., Figure 4.6) where the strategies diverge for only very small ranges of parameter values. The reason that learning has a much larger effect in this example is that the damages have a nonlinear jump at a threshold level of the stock pollutant. The period 1 emissions strategy can potentially change the marginal damages of emissions in period 2 by increasing the stock to close to that threshold. With this interaction, the ability to learn increases the value of abatement in the first period. The threshold effectively increases the interaction term (e.g., 18.1 in equation (6.2)) to a much higher value. Learning allows the threshold to be avoided if we learn that the damage costs are high, by selecting lower period 2 emissions. Without learning, we are less likely to be able to avoid crossing the threshold; lower emissions in period 2 are chosen on the basis of expected damage, which
may or may not avoid the threshold. Thus, without learning, early abatement effectively has lower benefits. Because the threshold is a function of concentrations, the likelihood of crossing it depends on the rate of ocean uptake, which remains uncertain at reference probabilities \( P\{fast\} = 0.8; P\{slow\} = 0.2 \). Thus even in the high damage state, the expected damage costs are still relatively low, and we observe the “precautionary” effect where learning leads to lower period 1 emissions.

Despite the language in Article 2 of the FCCC, there may or may not be a threshold level of greenhouse gas concentrations at which damages increase more rapidly. The point of this example is not about what shape a climate damage function might be, but to show that learning has a more significant effect on strategy in the presence of a threshold level for damages. While Kolstad (1996) used the fact that \( \text{CO}_2 \) is a stock pollutant as the explanation for the lack of influence by learning, it is actually the fact that only linear damage functions (in terms of the stock) without threshold effects are used in his model. This example shows that learning may have an influence for a stock pollutant if a threshold level might exist. Many other stocks may exhibit nonlinear behavior or threshold effects in the benefit function, including capital stock for production and population levels of a species.

A threshold effect in climate damages may not exist, but there are other possible sources of interactions between period 1 abatement strategy and marginal costs/damages in period 2. The next two sections will give examples of other interaction effects and show that again learning will affect the choice of period 1 abatement.

6.2.2 Influencing the Chances of a Change in Ocean Circulation

We next consider whether there are any feedbacks from emissions now that could change the marginal damages later, other than a nonlinear marginal damage function. Recall that impacts on total damages by the lingering effects of long-lived gases, which may be significant, nevertheless do not contribute to a divergent optimal strategy between learning and no learning. The emissions today must change the per-unit-emissions implications of future emissions constraint choices for learning to have an effect. A change in marginal damages would only occur if emissions today alter a physical property of the climate system, such as changing the lifetime of carbon in the atmosphere or the degree of cloud feedback on radiative forcing. In fact, just such a feedback is
already often discussed but rarely represented in assessment models: a change in the ocean circulation.

One of the main concerns of climate scientists is that increasing global temperatures could push the global climate into a fundamentally different state, from which it would not quickly return. One possible cause for such a change of state is a collapse of the thermohaline circulation of the North Atlantic Ocean, which drives deep-water formation at high latitudes (Manabe and Stouffer, 1995; Stouffer and Manabe, 1999, Schmittner and Stocker, 1999). Although such an event is considered improbable, at least some climate experts believe that, if the atmospheric concentration of CO₂ reaches double the preindustrial level, the probability of a change of state in the climate system could be as much as 0.1, and the majority of experts assess the probability as non-zero (Morgan and Keith, 1995).

In addition to the severe immediate impact on human activities, a change in ocean circulation would result in a positive feedback on the warming of the atmosphere. This feedback occurs for two reasons. First, a significant component of the absorption of carbon by the ocean occurs because the deep-water formation transports carbon from the surface layers to the deep ocean. Without the thermohaline circulation, the ocean would absorb much less carbon, effectively resulting in a much longer lifetime of carbon in the atmosphere. Each molecule of CO₂ emitted after the change of state would contribute to more radiative forcing than the same molecule emitted today. Second, the same sinking of water in high latitudes also allows the ocean to absorb more heat from the atmosphere by transporting it from the surface layers, which are closer to equilibrium in temperature with the atmosphere, to the much colder deeper layers. Thus a change in ocean circulation would cause a change in the rate of warming of the atmosphere for the same radiative forcing (Sokolov et al., 1998; Prinn et al., 1999).

It has been demonstrated using general circulation models that a rapid growth in emissions, and therefore a rapid warming, could theoretically induce a collapse of the thermohaline and/or other state change in the climate system (Stouffer and Manabe, 1999, Schmittner and Stocker, 1999). However, the mechanisms that determine ocean circulation and its interaction with climate change are not well understood. Given the current state of knowledge, it would not be absurd to posit the existence of a relationship
between the probability of a change in ocean circulation and the rate of emissions growth. The strength of such a relationship is not known and could easily be negligible or could be a significant determinant of the probability of a state change. The partial dependence of the likelihood of a change in climatic state on emissions growth is one example of an interaction between near-term policy and future marginal damages.

The decision models that have been presented so far in this chapter have assumed that the uncertainty in the damage from climate change is independent of and exogenous to policy choices. In particular, one of the three uncertain parameters related to climate damage was the rate of ocean uptake of carbon and of heat. The models in Chapter 4 assumed that the probability of slower ocean uptake is the same for all emissions control strategies that can be chosen. Here, the decision models are modified to reflect the dependence of the probability of slower ocean uptake. Figure 6.11 shows the influence diagram for the Complete Learning case where the rate of ocean uptake is now dependent on the strategy in period 1. The arc from “Emissions Rate 2010-2019” to “Oceanic Uptake” indicates that the probability that uptake is slow is now a function of the emissions rate. The reduced-form models of the IGSM have not changed; they are the same as in Chapter 4 and Section 6.1.

As in Chapter 4, the random variable ocean uptake is given a Bernoulli distribution:

\[
\text{Prob}\{K_r = 0.5\} = P_{\text{slow}} \quad \text{Prob}\{K_r = 2.5\} = 1 - P_{\text{slow}}.
\]

Whereas in Chapter 4 and Section 6.1 the probability of slow ocean uptake was

\footnote{Six of the 16 climate experts interviewed in Morgan and Keith (1995).}
independent of strategy ($P_{slow}$ constant), in this formulation the probability of the slow state $P_{slow}$ has some initial likelihood $P_0$, which is then incremented or decremented depending on what first period strategy $x_i$ is chosen. The probability of the slow state is defined as:

$$P_{slow} = P_0 + strength \left( \frac{x_i - \bar{x}}{x_{max} - x_{min}} \right)$$

where $\bar{x}$ is some reference rate of emissions growth below which the likelihood of a change in ocean state decreases and above which the likelihood increases. Since the magnitude of this effect, if it even exists, is unknown, the parameter $strength$ is included which scales from 0 to 1 to represent no dependence on strategy to a complete dependence on strategy, respectively.

For this formulation, the added dependency with a value of $strength > 0$ is effectively equivalent to increasing the coefficient of the cross-term in the period 2 cost function (e.g., the coefficient $d$ in equation (5.8)). This interaction is increasing in emissions (+), and therefore is a "head start" effect (see Table 5.2). The addition of the dependence of ocean uptake on the first period strategy should cause a divergence between the optimal strategies under No Learning and Complete Learning. Figure 6.12 shows how the optimal strategy changes as the strength of the relationship between uptake and strategy increases, assuming the reference probabilities of high damage and climate sensitivity of 0.2, and setting the initial probability of slow uptake $P_0$ to 0.2.

When the strength of the dependency is zero, these models are equivalent to the uncertain
damage models of Chapter 4, and as before the optimal strategy with and without learning is no abatement. If the strength of the dependency is very strong and the emissions rate becomes the dominant factor in the probability of slow uptake, then, not surprisingly, the most stringent policy becomes optimal, again regardless of learning. For low to moderate degrees of dependency, however, the optimal strategies with and without learning do diverge, and the direction of the divergence is not always the same. When the effect is very weak but non-zero (0.1 to 0.3), more abatement is preferred when we expect to learn between decisions. A weak effect will still have relatively low expected damages, since the slow ocean, which increases damage costs, will still have relatively low probability. The low expected damages induce the "precautionary" behavior. But if the emissions rate has a moderate (0.4 to 0.55) effect on the likelihood of slow uptake, more abatement will be preferred when uncertainty cannot be reduced. The stringer dependence results in a higher probability of slow ocean uptake, and therefore higher expected damages. Here we see the "sunk cost" effect: emissions are higher in period 1 when learning will occur.

Similar behavior can be seen by varying the probability of high damage valuation for a fixed level of weak dependence where strength = 0.2 (Figure 6.13). Near certainty that damages are valued little or valued highly (near 0.0 or near 1.0) lead to the same optimal strategy independent of learning. If the likelihood of high damage valuation is low to moderate (0.3 to 0.6), the expectation of learning leads to more stringent abatement. If the probability of high damage valuation is greater (0.6 to 0.8) the reverse
is true and the inability to learn leads to more stringent abatement. These results contrast with those for the equivalent models without any dependency (Chapter 4), which exhibit no difference in the optimal strategy for any probability of high damage valuation, and only a very small region of all three damage uncertainties varied simultaneously have divergent solutions (see Figures 4.6 and 4.7).

The difference in stringency between learning and no learning depends on the probability of high damage, just as for the analytical model of Chapter 5. The ability to learn allows the decision-maker to make the best choice in the second period under certainty, for each different outcome that might occur. Learning in the presence of an interaction effect allows the decision-maker to consider trying to decrease the likelihood of a bad outcome, and then discover the outcome of the experiment. If expected damages are low relative to abatement costs (probability of high damage is 0.3 to 0.6), the ability to learn makes it worthwhile to incur costs and experiment with some constraints in period 1. If damages turn out to be low, the constraints can be relaxed in the second period without great losses. In contrast, relative to the no-learn strategy the costs of learning that the damage state is high are greater, and thus learning induces a precautionary effect (see Figure 5.4a). But if expected damages are higher ($P_d$ is 0.6 to 0.8), then the inability to learn causes the decision-maker to hedge his bets and undertake more stringent abatement to balance the expectation of damages and costs, whereas the decision-maker who will learn can afford to undertake a little less abatement and correct later if necessary (Figure 5.4a). Relative to the decision-maker who cannot learn, the one who will learn is averse to the risk of unnecessary abatement costs for higher expected damages (sunk cost), and averse to the risk of excessive damages when expected damages are higher (precautionary).

The conditions that cause a divergence in optimal strategies with and without learning are both the presence (mean size) of an interaction, and a large degree of uncertainty in this interaction (variance). The results of Chapter 5 predict that adding uncertainty to whether the dependency exists or not should further increase the divergence between learning and not learning. We can test this by simply adding another uncertainty to the model. Assume that the probability of slow uptake is partially functionally dependent on period 1 strategy as described above ($strength = 0.2$) with probability $P_{depend}$, and is independent ($strength = 0$) with probability $1 - P_{depend}$.
Figure 6.14 shows the optimal strategies with and without learning as the probability of high damage varies. When the weak ocean dependence is certain (Figure 6.13), under 50% of the possible probability distributions of damage valuation have optimal strategies that depend on whether learning occurs. In the formulation where the dependence is uncertain, over 70% of all possible damage valuation distributions will have divergent strategies for the first period. Lower expected damage costs (0.15 to 0.5) cause tighter constraints to be optimal when we can learn and adapt, while high expected damages (0.5 to 0.85) cause tighter constraints to be preferred when learning is not possible.

Because damage valuation is only one of several uncertainties that determine damages, a two-way sensitivity test can better illustrate over what ranges of beliefs about uncertain parameters the optimal strategy depends on whether learning occurs or not. Figure 6.15 shows two-way sensitivity diagrams where we simultaneously vary both the probability of high damage valuation and the initial probability of slow uptake before the influence of policy. Roughly half of all possible joint distributions over these parameters will have different optimal strategies in the first period. Consistent with the previous results, the general trend is for more stringent policy under learning when expected damages are lower (e.g., 0.3 for high damage, 0.4 for slow ocean), and more stringent policy without learning when expected damages are high (e.g., 0.8 for high damage, 0.8 for slow ocean).
The range of emissions rates in the next few decades that would differentially impact the probability of a change in ocean circulation is not at all known. The effect may well be much weaker than in the illustrative results presented here. There are two objectives of presenting these results. First, it is a way to test the predictions from Chapter 5 about the role of the interaction effect as an explanation for when optimal strategies depend on learning. Secondly, it demonstrates that by including a feedback that is often discussed and is a major concern, one obtains very different implications for the level of emissions constraints that should be pursued now. The time frame for reaching a better understanding of the atmosphere-ocean-biosphere interactions may be much longer than one or two decades (i.e., we are most likely in the No Learning world with regard to damages). Also, the thermohaline circulation may be too poorly understood, its collapse too unlikely, and its reaction to warming rates on a much-longer time scale to warrant its inclusion in integrated assessment models. However, we will next turn to the abatement cost side, where the existence of interactions between current policy and future marginal costs are much more plausible.

6.2.3 Innovation and Low-Hanging Fruit Effects

The previous two examples have shown the effect of adding an influence of policy today on marginal damages in the future, by altering the probability of a shift in ocean circulation. Although we cannot rule out such an effect, it is considered unlikely.
particularly in the time period of integrated assessment modeling analysis. In this section, we turn to consider whether there are any such interactions on the side of control costs. Are there any feedbacks in economic systems by which an emissions constraint today could change the marginal cost of abatement in the future?

One interaction of strategy with abatement costs is the effect on innovation and technological change that first period policies are likely to have. It has been argued that in order to sustain stringent abatement later in the next century, that modest emissions reductions are needed now to stimulate the development of lower-cost alternative technologies. We will demonstrate a model that represents this effect, and show the difference that learning makes in such a model. We will also demonstrate a phenomenon that acts in reverse, where each unit of emissions reductions in the first period increases the cost of emissions reductions in the second period. B~...~ models will exhibit a much greater difference in the optimal period 1 decision depending on whether learning is expected.

Wigley, Richels, and Edmonds (1996) have argued that it is economically optimal to delay abatement for several decades. One of their primary arguments for waiting is that technical progress, occurring over time, will provide more options for low-cost abatement in the future. This view assumes that technological innovation is an exogenous process, i.e., independent of policies and market prices. Alternatively, innovation might be an endogenous phenomenon, as argued by Grubb (1997). Grubb proposes a model of the economy in which, “much technological development is induced by market circumstances – market experience leads to cost reductions, and expectations about future market opportunities determine how industries deploy their R&D efforts.” (Grubb, 1997). If technical progress is indeed at least partly endogenous, it leads to a very different policy prescription than Wigley et al. Only by undertaking some emissions abatement in the near-term, so that market prices reflect a cost to using carbon, will firms invest in R&D to develop alternative low-cost technologies.

The models presented here provide a means to examine the effect of endogenous technical change in sequential climate policy decision. Previous versions of the decision models have had relatively little interaction between the first period decision and the marginal cost of the second period decision. This is partly because the underlying economic model treats technical progress as exogenous (Yang et al., 1996). We will now
modify the decision model to include some endogenous technical change as well. The model below is not meant to be an explicit representation of endogenous technical change, and is not based on any estimates of the magnitude of the induced innovation effect. A simple relationship is used to illustrate the concept and the effects of including this assumption.

The decision models with uncertainty in abatement cost in Section 6.1.2 assumed that the probability of the low state of the world, in which inexpensive abatement options are available, was independent of the period 1 decision. The decision models here represent the endogenous technical change assumption by allowing the period 1 decision to alter the probability distribution of the uncertain abatement costs. This linkage is represented by the influence arc from the “Emissions Rate 2010-2019” node to the “Backstop Price and Substitution Possibilities” node in Figure 6.16. Note that Figures 6.6 and 6.16 are identical except for the addition of this arc in 6.16.

The probability distribution for the uncertain abatement cost is a simple Bernoulli distribution in which the high cost state occurs with probability \( P_{CH} \) and the low cost state occurs with probability \( (1 - P_{CH}) \). To include the effect of the period 1 decision on the probability distribution of cost, we simply make \( P_{CH} \) a function of the period 1 decision variable. In this formulation, we also include a means of making this functional dependence stronger or weaker, represented by the parameter strength, so that in one extreme it reduces to the uncertain cost models without interactions of section 6.1.2. Let \( P_{CH}^0 \) be the probability of high abatement cost in the absence of policy and in the absence of any induced innovation, which we will assume to be 0.5 for the moment. Next define the variable increment as the increase in the probability of high abatement cost resulting from the period 1 emissions rate:
Figure 6.17: Optimal Period 1 Strategy by Strength of Interaction

(Probability of High Abatement Cost is 0.5)

\[ P_{CH} = P_{CH}^0 + \text{increment} \]

The value of \textit{increment} is then calibrated so that when the endogenous effect dominates (\textit{strength} = 1), the highest emissions level in period 1 guarantees high abatement cost \((P_{CH} = 1.0)\) and the lowest emissions level guarantees low abatement cost \((P_{CH} = 0.0)\):

\[ \text{increment} = \left( \frac{x_1 - \text{Min}(x_1)}{\text{Max}(x_1) - \text{Min}(x_1)} - P_{CH}^0 \right) \times \text{strength} \]  

(6.9)

At values less than 1 for \textit{strength}, the increment or decrement to the likelihood of high abatement cost is much smaller. When \textit{strength} is 0, the probability of high cost is \(P_{CH}^0\) and is independent of period 1 action.

With this additional interaction between period 1’s decision and period 2’s marginal costs, we can examine whether learning will lead to a different near-term policy. For the reference assumption that the probability of high damage costs is 0.2, Figure 6.17 shows how the optimal period 1 policy changes as the dependence of the probability of high abatement cost on the period 1 decision changes. When the strength of this dependency is zero, the optimal first period policies are the same whether learning occurs or not, just as we have seen before. The left-hand side of Figure 6.17 shows that if the uncertainty in abatement costs can never be resolved, then the optimal policy is more stringent (0.8% per year emissions growth) if the strength of the dependency exceeds
0.55. If the uncertainty in costs will be completely resolved by the second period (right-hand side of 6.17), the optimal policy is 1.1% for weak or no dependency, the more stringent emissions rate of 0.8% for moderate dependency, and is the even more stringent rate of 0.6% for very strong dependency. Two aspects of these results are important. First, whether or not learning occurs, the more future costs are influenced by present decisions, the more it is optimal to undertake more abatement now (stringency of optimal policy increases as we move to the right in both graphs in 6.17). Second, we now see larger ranges of parameters for which the optimal solutions do diverge depending on whether one expects uncertainty to be resolved. In this case, when the solutions diverge, more abatement is optimal when learning is expected, and less abatement is optimal if uncertainty will continue. We observe only the precautionary effect because the expected damages are low ($P_H = 0.2$).

When a cost interaction exists because of endogenous technical change, then the solutions will depend on learning, and whether learning causes more or less stringency depends on the probability of high damages. Figure 6.18 again shows the optimal period 1 policy under no resolution and complete resolution of uncertainty, this time varying the probability of high damage costs, and assuming the strength of the induced innovation effect to be very strong ($strength = 0.9$). If the probability of high damage costs from climate change is believed to be anywhere between 0.0 and 0.55, the optimal emissions rate of 0.8% per year is independent of whether learning occurs or not. If high damage is believed to be more likely, between 0.55 and 0.75, then as above more learning results in
more stringent abatement. This is the precautionary effect, since expected damages are low relative to expected abatement costs (Figure 5.4a). However, if high damage is considered very likely (greater than 0.75), then it is optimal to abate more when uncertainty is not expected to be resolved. The higher expected damages induce a sunk cost effect (Figure 5.4b).

Compared with the models without interaction effects, there is a much larger range of probability distributions over damage costs for which the optimal policy depends on whether uncertainty will be resolved before the next decision. The divergence in solutions occurs over a larger range of parameter values because we have effectively increased the uncertainty in the magnitude of the interaction between period 1 and period 2 decisions, as predicted in Chapter 5.

The dependence of the optimal policy on learning can be observed even if the endogenous technical change effect is much weaker. While Figure 6.18 shows the optimal solutions when \( strength = 0.9 \), Figure 6.19 shows the same for when \( strength = 0.4 \). Here again the solutions differ between no learning and complete learning for probability of high damage costs anywhere between 0.35 and 0.75, still close to half of all possible probability distributions. In this case, the effect is precautionary, preferring lower emissions when learning will occur.

The optimal period 1 abatement for the endogenous technical change cases are consistently more stringent than the equivalent cases with purely exogenous technical change (Section 6.1.2). Whereas preferred abatement levels in the exogenous cases range
between 0.8%/year and 1.1%/year growth in emissions, policies as stringent as stabilization of emissions (0%) may be optimal if technical change is partly endogenous, even with the same expected costs and benefits.

There is disagreement among economists (Kennedy, 1964; Samuelson, 1965; Ahmad, 1966; Nordhaus, 1973) over whether such an induced innovation effect even exists, let alone what the magnitude of such an effect might be. In addition, there is also a possible phenomenon that would work in the opposite direction for climate change policies. Some in the climate change debate are wary of a "low-hanging fruit" effect, in which inexpensive emissions reduction options such as forest sequestration and methane emission reductions used in the near-term leave only more costly options if further abatement is warranted in later years.

The same approach as for endogenous technical change may be used here to construct a simple model of the "low-hanging fruit" effect. By merely changing the sign of increment in equation (6.9), more abatement in period 1 will increase the likelihood of high abatement costs in period 2. The results of such a model with and without learning are shown in Figure 6.20. As opposed to the induced innovation effect, the stronger the low-hanging fruit phenomenon the less desirable it is to abate emissions in the first period. Again, due to the increased uncertainty in the interaction between periods, there is a significant range over which the solutions diverge depending on learning.
6.3 Summary

At the start of Chapter 4, it was shown that the optimal first period strategy was independent of whether learning occurs between periods in a two-period model of climate policy choice under uncertainty. This result may have at first appeared counter-intuitive, especially since there is a high value of resolving uncertainty. However, as we demonstrated in Chapter 5, the independence of optimal choice from learning should in fact be expected when the cost and benefit functions are derived from typical integrated assessment models, such as the MIT IGSM. The reason is that most of these assessment models have almost no mechanisms by which period 1 emissions reductions can alter the marginal costs or benefits for period 2. With the magnitude and the uncertainty in magnitude of such feedbacks insignificant, the sequential decision problem decouples into two independent choices for each period. In this case, learning will not affect the first period strategy.

If typical integrated assessment models represented all relevant feedbacks in both the socioeconomic and the climate systems, then we could conclude that, while reducing uncertainty will enable better decisions in the future, whether we expect to learn or not should not affect our choice of emissions control policy today. But in fact, there are several important feedbacks that are known to be omitted from most assessment models. Section 6.2 illustrates three examples of these feedbacks: a nonlinear threshold effect in damages losses from climate change, a change in ocean circulation by which less abatement today could increase marginal damages in the future, and endogenous technical change by which more abatement today could decrease marginal abatement costs in the future. By introducing simple dependencies in the decision models to represent such feedbacks, the optimal near-term strategy diverges as a function of whether learning occurs. Making these feedbacks uncertain even further increases the divergence between optimal strategies. The relative stringency of abatement with learning, compared to that without learning, will depend on the beliefs about the likelihood of serious damage costs from climate change. Thus if we expect to learn more, it may optimal to constrain emissions more than if we would not learn. In addition, optimal near-term strategy is more stringent with these feedbacks included than it is without. The classical argument first raised in Chapter 1 – we should delay emissions constraints until we learn more – does not hold up as a reasonable argument under careful
analysis. The policy implications of the results of this chapter will be fully explored in Chapter 8.
Chapter 7  Political Will and Institutional Capacity

_The greatest loss of time is delay and expectation, which depend upon the future. We let go the present, which we have in our power, and look forward to that which depends upon chance, and so relinquish a certainty for an uncertainty._

- Seneca

7.1 Introduction

In Chapters 4 and 6, we have presented two-period models of sequential choice of climate policy. The IGSM-based models in Chapter 4 are consistent with most of the formal climate analysis literature, which shows that the optimal strategy in the first period is to leave emissions unconstrained under almost any beliefs about uncertainty in the climate response to emissions and the valuation of climate damages. Chapter 6 demonstrated that by including in the models interactions between first period strategy and second period marginal costs or marginal damages, that some level of abatement may be optimal in the first period. The amount of near-term abatement that minimizes costs depends both on whether the uncertainty in damages will be reduced and on the probability distribution over damage states. Thus the omission of interactions in climate assessment models affects their common result, that emissions reductions should be delayed a decade or more.

In this chapter we demonstrate another assumption to which near-term optimal strategy is very sensitive. Analyses of climate policy as an optimal sequential choice problem are framed in this way to develop insights regarding sensible policy choices for the near-term given the uncertainty and the fact that policies will change in the future. In any dynamic optimization model, one crucial assumption is that all strategies chosen after period 1 will be optimal. Thus the optimal strategy in the first period is what one _ought_ to do, _given_ that one _will_ implement the optimal strategies in all subsequent periods.

In analyses of climate change, the optimal choice is often to delay emissions reductions for a few decades, and then to reduce emissions quite rapidly later in the next century. An example is the recommendation in Wigley et al. (1996) that an optimal path to achieve stabilization of greenhouse gases consists of this strategy of “do nothing now,
do a lot later.” The decision models of Chapter 4 give a similar recommended strategy. The question we consider here is: will the optimal strategies for future periods be implemented? What happens to the conclusions if one cannot be sure?

The tools of decision analysis were designed for, and are primarily applied to, decisions by individuals or by representative agents of firms. A typical decision problem is a choice of investment strategy for the individual or firm, with a time-horizon of no more than a few decades. In these problems, the decision-maker, whether individual or firm, who revises later decisions as new information is obtained is the same as the decision-maker who makes the initial decision.

In applying decision analysis to climate change, we must ask whether the decision-maker in ten, twenty, and fifty years will be the same decision-maker as now, with the same preferences. Clearly, the international delegates to a climate change treaty, the regimes that govern the states they serve, and the political pressures and incentives upon them will all be different in the future. Herein lies a serious problem with the use of sequential decision methods such as those applied in Chapters 4 and 6. The optimal strategy that is a solution to a decision analysis is only optimal because we assume that the optimal decisions will be made in later periods given the information that will be available then. For climate change policy, we cannot assume that future generations will choose to be bound to the strategies that appear optimal to us today.

Suppose that several decades from now, uncertainty regarding climate change has been reduced and stringent constraints on greenhouse gas emissions appear justified. There is the possibility that one or more of the major emitting countries may still choose not to constrain emissions. The reasons for not undertaking reductions in the future could be a lack of political support for such a policy, perceived high costs of achieving reductions, or that the necessary institutions to implement stringent reductions on a global scale were never developed to enable constraints to be implemented efficiently. We use the term hysteresis\(^1\) to describe the situation in which a policy action is not taken despite the fact that the policy would have net economic benefits.

In this chapter, we modify the decision models of previous chapters to include the possibility of hysteresis, or non-optimal emissions reductions in the future. In Section 7.2

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\(^1\)“Hysteresis” is defined by the Webster’s dictionary as “a lag in response exhibited by a body reacting to changes in forces.”
we will describe the modified model. We demonstrate that even a very small probability that stringent reductions are not implemented in the future lead to some emissions constraints to be optimal in the near-term. Section 7.3 further extends the model to include a dependency between the probability of optimal future reductions and the first period strategy. In this case, even more stringent emissions constraints will appear optimal in the first period.

7.2 Models of Sequential Choice when Future Strategy might not be Optimal

In this section, we examine the implications of removing the assumption that the strategies chosen after period 1 will necessarily be the strategies that are optimal to today's decision-maker. There are several ways that the two-period decision models of the previous sections can be modified to represent the phenomenon of hysteresis. One approach is to assume that because the decision-maker has no control over the decisions made a generation or more in the future, they are more accurately represented as an uncertainty rather than a decision from the perspective of today's decision maker. However, a model of this form would require an estimate of the probability distributions over all possible future strategies. The approach of representing future decisions as uncertainties prevents the inclusion of some partial ability of the future decision-maker to exercise choice.

Another approach, which will be used here, is to assume that the second period decision is still a decision, but that there is some uncertainty about what options will be available to the decision-maker. Specifically, a new random event is added which determines whether more stringent emissions reductions are available to be chosen. The hysteresis phenomenon is modeled as a constraint on the set of available strategies from which to choose in the second period. It is unlikely that hysteresis will result in reducing more emissions than is optimal. The main concern is that future decision-makers will be unwilling or unable to undertake the stringent reductions that may be required by a delay in near-term abatement. Thus we assume that the constrained set of period 2 strategies excludes rapid emissions reductions. To keep the illustration simple, we consider two alternative conditions for the period 2 strategy choice:

1. Any strategy may be chosen, and
2. Constrained: only modest emissions reductions are possible.
The influence diagram in Figure 7.1 illustrates this modified decision model. The uncertainty "Capability of Stringent Abatement" conditions the set of strategies that can be chosen at the second decision "Emissions Rate 2020-2100". This uncertainty can either be treated as independent of the period 1 strategy, or the first period decision ("Emissions Rate 2010-2019") can influence the uncertainty in whether the hysteresis effect occurs. Here we begin with the simplest model, in which the probability that stringent reduction options are available is independent of the first period strategy (there is no influence arc from "Emissions Rate 2010-2019" to "Capability of Stringent Abatement").

Let the parameter \( P_{\text{stringent}} \) represent the probability that the second period strategy can be chosen from the full range of emissions levels, equivalent to the non-hysteresis model. Then with probability \( 1 - P_{\text{stringent}} \), the only possible options are no constraints or very weak constraints on emissions. Figure 7.2 shows the uncertainty between the two possible decisions in period 2 as a fragment of a decision tree.

How sensitive are the results of the non-hysteresis models to the assumption that the optimal decision can always be made later? The standard models of previous sections are equivalent to assuming that \( P_{\text{stringent}} \) is 1.0. Suppose we vary this probability to allow some chance that future reductions are not possible. Figure 7.3 shows the optimal period 1 strategies that result for different values of \( P_{\text{stringent}} \). It should not be surprising that the less likely it is that higher levels of abatement will be possible in the future, if needed, the more emissions should be constrained now. A striking feature of the graph is that it requires only a very small doubt in the future ability to reduce emissions, a 2-3\% chance that reductions are not possible, to justify some constraint on emissions now. This result
Figure 7.2: Tree Representation of Decision with Hysteresis

2010 Decision  | Hysteresis Uncertainty  | 2020 Decision
--- | --- | ---
0%  | ++  |
0.2% | ++ |
0.4% | ++ |
0.6% | ++ |
0.8% | ++ |
1.0% | ++ |
1.2% | ++ |
1.0% | ++ |
1.2% | ++ |

Can Take Stringent Action
Pstringent

Period 1 Strategy

Cannot Take Stringent Action
1 - Pstringent

Figure 7.3: Optimal Period 1 Emissions Growth by Degree of Hysteresis

is with the reference probability distributions over the uncertainty in damage-related parameters.

Contrast this with the results from Chapter 4 in which very high expected damages are required to justify any constraint in period 1 for the same model without hysteresis. Figure 7.4 shows this comparison between the standard (no-hysteresis) model and the hysteresis model with the cynical assumption that the probability of being able to
choose any level of abatement is 0.7. The no-hysteresis model never finds it optimal to constrain emissions, regardless of the probability of high damage valuation, while in the hysteresis case some constraint on emissions (1.2% annual growth rate) is optimal for almost any belief about damage valuation, and even more constraints are warranted if damages are likely to highly valued.

7.3 Models of Sequential Choice where Hysteresis is Dependent on Period 1 Strategy

Results from the decision model with hysteresis presented in Section 7.2 illustrate that if we cannot assume for sure that stringent constraints will be possible in the future even if warranted, then it becomes optimal to “hedge” against this uncertainty by undertaking some abatement now. The less one is certain that future societies will be able to restrict emissions at any need level according to the calculation, the more one should constrain now.

The above model treats the probability of hysteresis as independent of the first period strategy. However, this probability may actually be partly dependent on the strategy chosen for period 1. There are several mechanisms by which near-term policy can influence the likelihood of a hysteresis effect in the future. One mechanism is through near-term policy’s effect on the cost of abatement through price-induced innovation. This mechanism was discussed at length in Chapter 5. Another mechanism
by which near-term policy affects future likelihood of hysteresis is through the development of necessary institutional structures.

The creation of institutional mechanisms is at least as important as cost-competitive technological options. If it should prove necessary to undertake extreme reductions in carbon emissions beginning some time in the next century, institutional structures are needed to provide information, coordinate large transfers of wealth, and conduct monitoring and enforcement. Depending on whether emissions constraints are implemented now, and how the policies for constraints are designed, the institutions needed to implement large-scale reductions may or may not be developed. Such institutional capabilities currently do not exist in the international political system\(^2\). The importance of developing institutional capabilities to cope with climate change has been raised in the literature (Schmalensee, 1996; Jacoby, et al, 1998), but because of the difficulty of representing these concepts quantitatively, they are typically not treated in assessment models.

In this section, we modify the model of sequential choice with hysteresis from Section 7.2 by adding a partial dependence on first period strategy. Figure 7.5 shows the influence diagram for a decision model with hysteresis that depends on first period strategy. Now the decision node "Emissions Rate 2010-2019" has an influence arc to the uncertainty "Capability of Stringent Abatement". We modify the decision model so that the probability that the period 2 strategy set is constrained is partly a function of the

\(^2\) The negotiation of and attempt to implement the Kyoto Protocol is a step towards the development of such institutions. Unfortunately, the details of the agreement and the actual level of emissions constraints that will be implemented are very much in doubt (see Chapter 1).
period 1 strategy.

Suppose that there is some a priori probability that all abatement options are available in the second period, $P_o$. In addition, assume that implementing emissions constraints in the first period can increase this probability by some amount $\Delta P$. The more emissions are constrained, the greater is the likelihood of available stringent abatement. Finally, a parameter $H_{strength}$ is included to vary the strength of this influence of period 1 policy on future decision options. When $H_{strength}$ is zero, the probability of a hysteresis effect is independent of the first period just as above. The higher the value of $H_{strength}$, the more stringent policies in period 1 can decrease the likelihood of hysteresis.

Figure 7.6 shows the optimal solutions for this model as the assumptions about the a priori probability of hysteresis and the degree of impact of the period 1 strategy are both varied. The dominant parameter is still the initial probability that stringent abatement will be possible. The less likely that all options will be available, the more stringent near-term constraints become optimal. Within certain regions of belief, the ability to influence this probability with first period constraints does have an impact. Note that the direction of this impact varies. If the likelihood is high that stringent action will be available (0.7 – 0.8), then the stronger the impact of policy, the more stringent will be the optimal strategy. However, if the likelihood is fairly low that stringent options later will be available (0.3-0.4), then greater impact of today’s policy leads to a less stringent choice for the near-term. This effect occurs in this example because when the impact is high, not only is the additional gain in expected value from moving 1.2% to 1.0% still not
worth the additional cost, the but the added impact from having chosen 1.2% as further increased the expected benefits at the same cost, so this choice becomes optimal for even lower a priori beliefs about hysteresis.

A comparison of the optimal solutions under hysteresis with and without the dependence on first period strategy is shown in Figure 7.7. Here the dependent case assumes that emissions constraints in the near-term can cause a large increase in the likelihood that stringent options are available later ($H_{\text{strength}} = 0.8$). The optimal strategies are shown in both cases as a function of the a priori belief that stringent options will be available. This again shows that for a range of beliefs (0.7 – 0.8) about future abilities, if one also believes that policy today can increase the odds of being able to what’s needed in the future, then less doubt about future ability is required to justify near-term constraints.

The models presented are intended only as simple conceptual illustrations. The point of these examples is simply that for the issue of climate change, we cannot assume that future generations will be able or willing to follow through on policies to which the current generation commits them. For many reasons it may be optimal from the point of view of a decision-maker today to delay restrictions on emissions for a few decades and then undertake drastic restrictions in the future. However there is at least the possibility that those future generations will balk at these drastic emissions constraints when the time comes. By implementing even modest constraints today, we may be able to
decrease the likelihood that future emissions reductions will be politically or economically infeasible. One way that near-term emissions constraints can do this is through developing the international and domestic institutions that will be necessary for the sustained and serious mitigation of climate change over the next century. Whether through the implementation of the Kyoto Protocol or some other modified agreement, experience in emissions reductions is crucial to the development of an effective climate change regime.

In addition to the political implications, the illustrative models of this chapter have a methodological implication as well. Decision analytic techniques are useful in framing the climate issue in ways that emphasize the sequential nature of choice under uncertainty with learning. However, the assumptions of these methods are not always valid, and must be scrutinized before being applied. Otherwise the insights derived from analysis can be misleading or incorrect.
Chapter 8  Implications for Policy and Research

This is as true in everyday life as it is in battle: we are given one life and the decision is ours whether to wait for circumstances to make up our mind, or whether to act, and in acting, to live.

- General Omar Bradley

The objective of this dissertation has been to explore the validity of the “wait to learn” argument, the view that because the impacts of climate change and their economic valuation are so uncertain that we should wait to reduce the uncertainty before undertaking activities to reduce emissions. By constructing decision models based upon a model that represents the complexity in the underlying physics and economics, we have shown that, for a wide range of assumptions, the ability to learn about the uncertainty may lead either to the same level of abatement as would be optimal without learning, or even more stringent abatement. The influence of learning in the future on what we do today depends on how today’s decision changes the costs or benefits of tomorrow’s decision. Two primary conclusions emerge from this work, one with implications for research in the area of climate change assessment, and one with implications for climate policy as well as other science and technology policy issues.

On the research side, several potential sources of interactions, through which the choice of abatement level today might influence the costs or benefits of future decisions, are usually omitted from integrated assessment models for climate change policy. Such interactions include endogenous technical change and potential changes in ocean circulation, effects that are generally believed to be possible but are very uncertain and difficult to model. Because the omission of these uncertain potential interactions affect the basic insights from these models for policy – what should we do now? – it is crucial that research focus on including these and other such interactions that might be present in the real world.

The policy debate, as described in Chapter 1, over whether to ratify and implement the Kyoto Protocol still includes the argument that the science of climate change is too uncertain, and that we should wait until uncertainty is reduced. Waiting to learn is not a valid argument for delaying abatement. In fact, given current estimates of the range of possible climate damages, it is likely that we should be doing more if we expect to learn. Behind the positions on both sides of the “Act Now vs. Wait to Learn” debate are simply different beliefs as to the expected damage costs from climate change and expected abatement costs. The rhetoric about uncertainty and
learning has merely served to obscure and confuse the difficult choice of a measured response to climate change given what we know, and don’t know, today.

These two conclusions, along with other implications and areas for future research, are discussed in more detail in this chapter. Section 8.1 presents the implications for research and next steps for climate policy analysis. The policy implications are discussed in Section 8.2

8.1 Implications for Research

8.1.1 Include Interactions in Integrated Assessment Models

In quantitative analysis for policy, there are several different measures of the contribution to uncertainty, whether for parametric uncertainty or structural/model uncertainty. One measure is how sensitive is a key output or model response (e.g., global mean temperature change) to different assumptions. A better measure for parameter uncertainty is the variance in the distribution of the response of interest caused by propagating a probability distribution for the parameter. If the problem is structured as a sequential decision problem, the value of information calculation can be used to see the change in overall utility for different assumptions. However, none of these measures necessarily indicate whether or not the uncertainty results in a different decision. Ultimately, the most important uncertainties in a decision problem are those that lead to different decisions over the range of possible assumptions.

Integrated assessment and analysis of climate change policy is performed to provide insights into the difficult decisions that must be made in the real world, highlighting the important factors to consider and tradeoffs that exist. Analyses are performed for a number of questions, including what is the cost-effectiveness of different approaches to achieve a goal, how might non-participating parties be affected, etc. But behind many of these is the fundamental question of what is an appropriate level of emissions constraints now, given the current state of knowledge.

We have shown in this dissertation that the inclusion of several phenomena that may exist in the real world, but are omitted from the climate model, will change the qualitative insights that emerge from a modeling exercise regarding an appropriate level of abatement. Thus, future improvements for climate policy assessment models should include as a priority:

- 3-D ocean circulation and the possibility of a collapse or even slowdown of the thermohaline circulation in the North Atlantic Ocean, and
- A mechanism by which policies alter the rate of technological innovation.
The ocean component of most integrated assessment models, from the two-box model in DICE to the two-dimensional ocean in the IGSM, assume that present-day circulation patterns are unaffected by rising global mean temperature. But the thermohaline circulation of the North Atlantic Ocean, one of the primary drivers of mixing between the upper surface of the ocean with deeper layers, could be altered by changes in sea-ice caused by warming at high latitudes. If this circulation were to slow down or stop altogether because of warming, an important sink for both carbon and heat from the atmosphere would decrease and future climate impacts would be much more severe than estimated by current models. Such an event is considered to be low probability, but even at low probability its inclusion changes the implications from models for the choice of abatement level under uncertainty.

Modeling of technical change in economic models also affects the qualitative insights from an analysis. Wigley et al. (1996) argue that a long-term target can be achieved at lower cost by leaving emissions unconstrained for several decades, then undertaking more stringent abatement later. But this result occurs from their model only because the rate of technical progress that produces low-cost abatement options is independent of whether carbon emissions have a price or not. The results in this dissertation show that once the possibility that the rate of technical change depends on current policy, it suddenly becomes preferable to place some constraints on emissions today.

Other interactions may exist as well, either that decrease marginal abatement costs in the future like the induced innovation effect or that work in the opposite direction and increase future costs. For example, Chapter 6 described the "low-hanging fruit" phenomenon, in which abatement today may use all of the low-cost easily implemented reduction possibilities, leaving higher cost measures as the only options for future abatement. Other potential feedbacks between constraints on emissions now and the marginal costs or marginal damages of emissions reductions later need to be identified and if possible represented in climate assessment models.

Of course, whether these interaction effects exist, and if so what are the magnitudes of the effects, are uncertain. In fact, as we have shown here, it is the uncertainty in these phenomena that biases strategy choice under uncertainty. Uncertainty is an inherent part of the climate and economic systems, and must be treated so if modeling is to produce useful insights for real world decisions.
8.1.2 Next Steps for Modeling Uncertainty in Integrated Assessment Models

The decision models used in this dissertation are based on the MIT Integrated Global System Model, a model of intermediate complexity that includes a 2-dimensional climate model that parameterizes all main atmospheric processes coupled with interactive photochemistry, and a twelve-region eight-sector computable general equilibrium economic model. The simulations required to perform an analysis of sequential choice under uncertainty would require too much computation time to use the full IGSM. Instead, we have used the Deterministic Equivalent Modeling Method to estimate a set of reduced-form models that approximate the behavior of the IGSM under uncertainty in several of its key parameters.

In the work presented here, several uncertainties were not included in the estimation procedure, but were kept fixed at reference values to reduce the time required to estimate accurate reduced-form models. As part of continuing this research, new estimates are currently planned that will include other potentially significant sources of uncertainty. A major objective of this work is the propagation of uncertainty from a more extensive set of parameter distributions to obtain distributions of response variables.

Some of the uncertainties are in parameters of the EPPA model. The sensitivity and uncertainty analyses of the EPPA model (Chapter 3) highlight a few crucial assumptions where continued research effort is needed, including the development of future (backstop) technologies, and assumptions about fossil fuel (conventional oil) resource availability. Another improvement is to explicitly treat the other non-CO₂ greenhouse gases and aerosol emissions, especially CO, CH₄, N₂O, and SO₂. The emissions of these gases are determined by activity coefficients, which represent the amount of emissions resulting from a unit of activity in each production sector of the economy. It is particularly important to propagate the uncertainty in how these activity coefficients might change over time.

There are several improvements to the uncertainty propagation of the climate model that can be made. In addition to the climate sensitivity parameter and oceanic uptake of heat and carbon, a third influential uncertainty is the radiative forcing of aerosols. The propagation of the emissions uncertainty through the climate model should include the other important non-CO₂ greenhouse gases. Also, as mentioned in Chapter 3, a recent study by Forest et al (2000) has used climate observations from the past century to constrain the joint probability distribution for climate sensitivity and heat uptake by the deep ocean. Rather than use parameter distributions based solely on expert elicitation, we can combine the expert priors with the constraints from
observations to obtain a new joint distribution for these parameters. The representation of emissions uncertainty as an input to the climate model approximates any CO₂ emissions path as an exponential rate of increase of CO₂ concentrations. While this procedure is very accurate for propagating the uncertainty in the reference (i.e., no emissions constraints) case, errors are introduced when simulating policies, particularly more stringent policies such as concentration stabilization. To improve the accuracy of modeling the uncertainty under a range of policies, a different functional form with at least two parameters will be required for propagating emissions uncertainty. Finally, the approximations of impacts for the first few decades of a hundred year simulation tend to have larger errors because the previous estimation was performed for only one initial condition of the atmosphere and ocean. Better estimates of decadal averages of impacts, such as global mean temperature change and sea level rise, require using ensemble averages of at least two, and possibly more, initial conditions. This would reduce the noise from model variability that swamp the signal in these responses for the early decades.

In addition to estimating the probability distribution of global mean temperature change from resulting from uncertainties in parameters like climate sensitivity, other useful uncertainty studies might also make use of the 2D climate model. One experiment would be to propagate uncertainties in more detailed parameterizations of the model such as for cloud formation. Every climate model, including the MIT 2D model, has its own natural climate sensitivity, which is a function of many different feedbacks and parameterized processes. As described in Chapter 3, the 2D climate model is able to simulate different climate sensitivities by adjusting the cloud cover amount. By performing a propagation of uncertainty on the parameters of the fundamental atmospheric processes, we could obtain an alternative estimate of the uncertainty in the climate sensitivity of a model.

Another interesting experiment that is planned is to use a 3-dimensional ocean model to explore the uncertainty in circulation changes. A 3D ocean model has been developed at MIT (Kamenkovich et al., 2000) that is capable of simulating the thermohaline circulation of the North Atlantic. Under some conditions, this circulation will slow down or even halt altogether for a period of time. The propagation of uncertainty through the 2D climate model coupled with a 3D ocean might be able to estimate the probability of a slowdown in the thermohaline circulation, and to explore the feedback of this event on other climate indicators such as temperature change.
A crucial area for continuing research is the development of new tools for uncertainty propagation through computationally demanding models such as the IGSM. As discussed in Chapter 2, different methods have different advantages depending on the particular model being studied and the statistic of the output being estimated. In addition to the methods reviewed in Chapter 2, there are several other methods worth exploring and comparing to traditional methods, including Bayesian and non-linear time series approaches.

Finally, there is one other application for the methods discussed in this section to be explored further. The reduced-form models presented here can also be used for educational purposes in addition to the direct research applications of uncertainty analysis and sequential decision analysis. The reduced-form models of the IGSM used in the decision models of this dissertation have also been given a graphical user interface in order to function as a stand-alone Windows PC-based computer application, called the “Toy IGSM.” The Toy IGSM allows a user to directly change the emissions rate strategies and values of uncertain parameters and immediately view the impact on all relevant output variables, both graphically and numerically. The Toy IGSM also has a Monte Carlo interface to allow real-time uncertainty propagation, allowing the user to obtain probability distributions for any model response. This program has been used successfully as a teaching tool within the context of graduate course at MIT. Further developments with the Toy IGSM could aid in research into what type of tools are useful for the purposes of both education and decision-support for a complex scientific and economic policy issue such as climate change.

8.1.3 Next Steps for Sequential Analysis of Climate Policy

There are many ways in which the sequential decision analysis presented in this dissertation can be extended and modified that could make the findings more robust and yield insights into other aspects of climate policy. We will briefly describe several such extensions worth pursuing:

- Extending the analysis to three or more periods,
- Exploring endogenous learning effects,
- Explicitly representing multi-attribute utilities for climate policy decision instead of using damage functions, and
- Developing analysis tools for sequential decision under uncertainty for multiple stakeholders.
The sequential decision models developed in this dissertation are two-period models. This is the simplest model that can represent the sequential nature of climate policy and allow for a response to new information. Although the insights into the role of learning and its effect on strategy choice are likely to be robust to the number of periods in the model, this is one assumption that is important to investigate. Three or more periods allow multiple points where responses to new information may occur, and also give more flexibility in the timing of abatement activity.

A related question that might be addressed through a three-period model is the “boiled frog” dilemma. If a frog, in preparation for being eaten, is to be boiled and is dropped into already boiling water, it will jump out of the water immediately. If, however, the frog is placed in cool water that is then gradually heated to a boil, the frog will never jump out but will calmly remain in the water until it has died. Is this situation at all analogous to climate change? A simple two-period analysis, ignoring the interaction and learning effects presented in this dissertation, might show that abatement now is not worthwhile because the benefits are too small relative to the cost. If abatement is therefore delayed for a decade and then the same two-period analysis is repeated ten years later, will the conclusions not be the same then, and in the next decade as well? Does this type of analysis leave out something that indicates when abatement is justified before irreversible climate change has occurred? This is a puzzle worth exploring further.

There are other formulations of sequential decision models still to be tested to see if the results here are robust. The IGSM used for calculations in our models uses an economic model that is not forward-looking to guide investment and savings behavior. A useful experiment would be to compare the strategy choice in a decision model using a myopic economic model to one using a forward-looking economic model to see what differences, if any, exist. The results derived from the analytical models in Chapter 5 were based on restrictive assumptions of the functional form of the objective function in order to keep the derivations simple. Equivalent expressions for the divergence between first period strategies with and without learning should be derived for general functional forms with more terms and for higher order functions that exhibit convexity or concavity in their marginal costs.

Another set of assumptions still to be explored thoroughly relate to learning. The models here assumed exogenous learning, where the mere passage of time resolves uncertainty. Two other types of learning are purchased learning, where resolution of uncertainty depends
(probabilistically) on an amount of investment (e.g., R&D), and endogenous learning where what is learned depends on the strategies chosen. With respect to the uncertainties in abatement cost, endogenous learning may provide a better model, since this uncertainty cannot be reduced in the absence of any emissions constraint. How would this dependence of learning (whether it occurs or not) on first period strategy affect the dependence of strategy on learning (the relationship explored in this dissertation)? Finally the role of partial learning and its effect on sequential choice of abatement remains to be explored. To clarify the influence of learning on strategy choice, we focused on the comparison between the two extremes of no learning and complete resolution of uncertainty. More likely, as time we pass some uncertainties will be reduced but not completely resolved, while other uncertainties may remain or even grow larger. How much reduction in variance causes a bias in first period strategy? What is a reasonable way to model this type of process?

A different area that needs substantial improvement is the understanding and representation of the valuation of climate impacts for optimal choice analyses such the one in this dissertation. This study, as most other studies of optimal climate policy choice, has used a variation of the Nordhaus damage function (Nordhaus, 1994; see Section 3.5). We have shown that the uncertainty in both the range of possible damages and the probability distribution over that range can lead to widely varying strategy choices of emissions abatement. This damage function approach, as well as the estimates underlying its reference parameter values, has received a fair bit of criticism in the literature (e.g., Tol, 1994, 1996; Roughgarden and Schneider, 1999), but continues to be used for lack of a better alternative and for comparability with previous studies.

One criticism of the damage function approach that we would make is that it hides implicit valuation tradeoffs between economic losses due to abatement and losses from climate impacts, the majority of which are estimated to be non-market effects (Nordhaus, 1994). One alternative to the damage function approach with its highly uncertain parameters is to instead use a multi-attribute utility function (Keeney and Raiffa, 1993) for valuing outcomes in a sequential decision model. Because a damage function simply uses global mean temperature change as a proxy variable for the regional climate impacts that we are currently unable to model with any confidence, an MAU function could simply represent the utility level for different levels of global mean temperature realized by, for example, the year 2100 as one attribute. Another
Figure 8.1: A Multi-Attribute Utility Function for Sequential Climate Policy Decision

![Utility Function Diagram]

attribute could be the discounted net present value of foregone consumption due to emissions reductions. The utility of any outcome could be expressed as:

$$U(C, \Delta T) = W * U_T(\Delta T) + (1 - W) * U_C(C)$$

where $C$ represents the net present value of abatement costs, $\Delta T$ is the global mean temperature change realized by some reference year, and $W$ is the importance weight of temperature change relative to abatement costs. The notation $U_i(x)$ represents the utility function in each attribute, which allows differing degrees of risk aversion to be modeled in each attribute. An example of a multi-attribute utility (MAU) function is illustrated in Figure 8.1. The MAU shown has one particular assumption about the tradeoff weight $W$ and risk aversion in the temperature change attribute.

The advantage of an MAU approach is that it makes the tradeoffs, which are a necessary part of climate policy choices, more explicit than the damage function approach. The weight parameter $W$ can be varied in sensitivity testing to find threshold assumptions at which abatement strategies change. It is also a more flexible form, allowing risk aversion and also interactions between attributes to be represented. The main difficulty with this approach is where to get the preference weights and coefficients of risk aversion from. Whose preferences should be elicited? This is not a trivial question, but the damage function approach suffers from the same difficulty. Since the majority of damage valuation is thought to be non-market effects, an assumption is already being made about the tradeoffs between climate change and economic costs; it is simply harder to see what tradeoff is being assumed. Developing MAU representation...
for decision-maker preferences in climate policy requires careful thought, but has the potential to make future sequential decision analyses more transparent, and therefore more useful.

Lastly, a major drawback to applying sequential decision analysis to climate change policy is that decision analysis provides a set of tools for a single decision-maker while climate policy is inherently a multi-stakeholder negotiation process. There is no single decision-maker or entity that makes decisions on the basis of global welfare. One of the most challenging aspects of climate policy is the fact that so many nations are affected differently, so many nations would all need to cooperate to address the problem, and each nation has differing goals and preferences. Indeed, within each nation is a complex group of divergent and conflicting interests. How can analysis of the type developed here be made more informative for a problem that is by its very nature a multi-party negotiation?

There are a few previous studies that attempted to develop insights by representing different nations or regions with different preferences. Hammit and Adams (1996) use a game-theoretic approach to model the cooperative and non-cooperative solutions between “North” and “South.” Morgan and Dowlatabadi (1996) show the preferred abatement strategies (no abatement, emissions stabilization, or concentrations stabilization) for seven different regions and four different possible decision rules. Here, we propose an alternative approach that could be utilized. Different stakeholders can be defined with different MAU preferences (as outlined above) to represent nations or similar groups of nations, for example USA and China. Sequential decision models like the ones used in this dissertation can be used to calculate the expected utility, with respect to the joint probability distribution of damages and control costs, of different
possible first period strategies, allowing for potential future learning and revising of abatement levels. The relative preferences over strategies can be examined for pairs of stakeholders by graphing their expected utilities as a scatter plot, as in Figure 8.2. In this figure, the preferences are modeled in terms of disutility, so that the two stakeholders shown would be most satisfied with the outcome (0,0). The Pareto optimal frontier is the set of strategies along the left and bottom borders of the strategy group. From any of these strategies, the outcome for one stakeholder cannot be improved without lowering the other’s utility. Some strategies (in the upper right of the scatter cloud) are clearly inferior to all others and should not be considered in a negotiation. Further, by examining the common elements to the strategies on the Pareto frontiers, non-controversial components may be able to be identified and separated from more difficult issues. The approach, known as Multi-Attribute Tradeoff Analysis (Connors, 1999), has yet to be applied to climate change to see if any useful insights result.

8.2 Implications for Policy

The Framework Convention on Climate Change has produced the Kyoto Protocol as its first attempt at getting nations to commit to a reduction in greenhouse gases emissions. But the prospects for the protocol to actually enter into force are currently in doubt, mainly as a result of opposition by the United States. Several arguments are used against the protocol or any other actions to restrict CO₂ emissions in the U.S.; one of the primary arguments used by some in government and industry is that the scientific basis is too uncertain to justify action.

Many aspects of climate change are indeed quite uncertain. The range of projections for climate using “plausible” assumptions over the next century, assuming current emission trends continue, is quite large and include changes that would be hardly noticeable to changes that could be quite disruptive to society. There are many poorly understood aspects of the science that account for this large range of uncertainty, including the feedback effects of different types of clouds, the role of aerosol particles in the air, and interactions between the atmosphere and the ocean circulation. The regional impacts of a change in global mean temperature is even less well understood, as is the economic value we can attach to such changes. The costs of restricting or reducing the emissions of greenhouse gases are also quite uncertain. Predictions of the cost of meeting the Kyoto commitments in the U.S. have been variably predicted to be anywhere from costless to causing an economic depression. Uncertainty in the degree to which new
technologies and fuels can be developed over the next decade to replace existing technology is a major reason for the wide range of cost estimates. It is also uncertain to what extent policy instruments such as emissions permit trading would be effective at reducing the overall cost of an emissions reduction policy.

What should society do now in the face of all this uncertainty on both the cost and the impact sides? Proponents of delaying constraints on emissions now have often used justifications that the uncertainty is too great and society should wait for more information before undertaking expensive restrictions on carbon emitting activities. Similarly, the opposing view also justifies the need for immediate reductions on the great uncertainties surrounding climate change, invoking the precautionary principle to argue that the risks of waiting are too great. This view appears to imply that any improvement in knowledge about the climate system will come too late. Which of these arguments is correct? Should we in fact wait until the range of uncertainty is reduced?

The answer that emerges from the analysis in this dissertation is no, we should not necessarily wait for a reduction in uncertainty before putting some constraint on greenhouse gas emissions now. By itself, the uncertainty and irreversibility of climate change from emitting greenhouse gases should lead us to restrict emissions a little more if we expect to learn more in the future. Similarly, on its own the irreversibility of capital that is invested in emissions reductions as opposed to other parts of the economy should lead us restrict emissions less now if we expect to learn more in the future.

The question is how these two opposing effects counterbalance each other. If we believed that climate damages were most likely to be serious but that there is a small chance that it is not at all a serious problem, then in that case we would in fact want to undertake slightly less abatement, relative to what we would do if we couldn’t learn. However, this situation does not describe the current estimates of probability distributions of the costs of climate damage. Most estimates by experts tend to put the greatest likelihood on losses from climate damage being relatively low, with a smaller chance that the damage costs would be very large. In this situation, the net effect of considering the irreversibilities in both directions should actually lead us to restrict emissions more than what we would do if we couldn’t learn.

In fact, the “Act Now vs. Wait to Learn” debate turns out to be not really about learning at all. The proponents of the “wait” argument simply believe that the expected costs of restricting carbon emissions would be great while the expected damages from climate change
would be small. These beliefs, if true, do indeed make it optimal to not undertake any abatement for now, regardless of whether we will learn more in the future or not. Similarly, those who invoke the precautionary principle and argue for stringent reductions immediately signal their beliefs that the expected costs of abatement would be relatively low, while the expected damages from climate change would be high. These policy prescriptions are driven by convictions as to the relative costs and benefits of emissions reductions, and not by considerations of learning and irreversibility.

What insights can we gain from the models and the analysis with regard to the level of abatement that is warranted today, including considerations of uncertainty, learning, and irreversibility? There are several components to the determination of an “optimal” level:

- The direct costs and benefits of the abatement itself,
- How what we do today may make future abatement easier or more difficult (i.e., interactions),
- The likelihood that future governments will implement more stringent reductions that we “commit” them to, especially when choosing to delay abatement, and
- How what we do might influence what we will learn.

Most analyses of climate change have shown that little or no abatement is warranted in the next decade or two. The amount by which the total stock of greenhouse gases is reduced by constraining emissions over only the next decade is quite small. On the other hand, the costs of meeting emissions reduction targets, such as those in the Kyoto Protocol, are likely to be quite high in the short term for the United States and for many other industrialized nations. Any attempt to reduce the burning of fossil fuels in the absence of technological alternatives will have quickly rising costs with the level of reduction.

However, there are other reasons to consider some level of emissions constraints, even if the direct net benefits are small. One important role of abatement today is how it might affect future emissions reduction policies, via the interactions that have been a focus of this dissertation. As we have illustrated with simple models, there are several possible mechanisms through which a constraint on emissions today might lower the marginal cost of future emissions reductions or increase the marginal damages of greenhouse gas emissions in the future. Some have argued for delaying abatement into the future because future technologies will make the reductions less costly than they would be today. But it is not clear that these developments will take place in the absence of economic incentives. The development of new low-carbon emitting
technologies may well occur at faster rate if there is a non-zero price on carbon emissions than it would otherwise. On the damage side, the more rapidly global temperature rises due to greenhouse gas emissions, the greater the chance that the circulation of the ocean would be altered, and with it the ability of the ocean to absorb heat and CO$_2$ from the atmosphere. These interactions, of the type we call “head start”, make some level of near-term abatement desirable because of the options for future decisions that it would enhance.

The argument to delay abatement into the future because it would be less costly (under the assumptions of technological advancement mentioned above) to undertake more stringent reductions in the future rather than to begin more gradually reducing emissions now, also rests upon other assumptions about future societies ability to make these reductions. However, those future decision-makers may not be able or willing to make those stringent reductions to which we commit them. Consideration of the possibility of such a hysteresis effect adds further benefits to any abatement undertaken today.

Finally, because of the considerable uncertainty, we should consider what we will learn in the future and how to reduce the uncertainty. To reduce the uncertainty in climate impacts will require continued research into the science, coupled with observations of the atmosphere and ocean to provide more data that can help to constrain and verify models. The uncertainty in the costs of greenhouse gas abatement measures, on the other hand, will not be reduced if there is no abatement policy. The only way to begin to understand the economic effects of CO$_2$ reductions is to try some level, perhaps a very low carbon price, and observe the result. Also, if climate change is in fact a serious problem and stringent reductions will be required in the future, the institutions that will be necessary do not currently exist, and may take decades to develop. Some of this development is already occurring as a part of the FCCC process. But much of the institutional development will only occur as part of a policy process where nations are actually constraining their greenhouse emissions.

Thus, while the direct net benefits of emissions abatement over the next decade or two appears to be small or even zero, there are several indirect benefits to abatement that make some level of constraint desirable. The level of emissions constraint need not be as stringent as targets in the Kyoto Protocol. At this early stage of an international response to climate change, policies that create a small but non-zero price for carbon would be enough to get the indirect benefits described above, without risking too great an economic disruption. A price on carbon and other greenhouse emissions would send signals that would allow industry and consumers to begin to
adjust and develop new alternatives, in anticipation of the possibility that climate change is serious and that significant reductions may be needed in the future. From considerations of uncertainty, learning, and irreversibilities, this policy response is most likely to retain or create the largest set of options for future adjustments as information is gained, whatever that information will be.
Appendix A

Consider the two-period dynamic programming problem:

\[ V_1(x_1, \theta) = \min_{x_1 \in X_1} \left[ E_\theta \{ C_1(x_1, \theta) + V_2(x_1, x_2(\tilde{\theta}), \tilde{\theta}) \} \right] \]  \hspace{1cm} (A.1)

where the value function when uncertainty is completely resolved before period 2 is:

\[ V_2(x_1, x_2, \theta) = \min_{x_2 \in X_2} \left[ C_2(x_1, x_2, \theta) \right] \]  \hspace{1cm} (A.2)

and the period 2 value function when uncertainty is not resolved is:

\[ V_2(x_1, x_2, \theta) = \min_{x_2 \in X_2} \left[ E_\theta \{ C_2(x_1, x_2, \theta) \} \right] \]  \hspace{1cm} (A.3)

**Proposition 1.**

For any two-period sequential decision under uncertainty represented by equations (A.1-3), and where the solution to (A.3) where learning does not occur is denoted by \( x_1^N \) and the solution to (A.2) is denoted by \( x_1^L \), then

\[ \text{If } \frac{\partial^2 C_2}{\partial x_1 \partial x_2} = 0 \text{ then } x_1^L = x_1^N. \]

**Proof:**

The problem is to solve

Learning: \( \min_{x_1} \left[ E \{ C_1(x_1, \theta) \} + \int_{\theta} \left( \min_{x_2} \left( C_2(x_1, x_2, \theta) \right) f_\theta(\theta) d\theta \right) \right] \)  \hspace{1cm} (A.4)

No Learning: \( \min_{x_1} \left[ E \{ C_1(x_1, \theta) \} + \min_{x_2} E \{ C_2(x_1, x_2, \theta) \} \right] \)  \hspace{1cm} (A.5)

The first component of the objective function, \( E \{ C_1(x_1, \theta) \} \), is identical in both cases. Thus it is the second term which causes the divergence between \( x_1^N \) and \( x_1^L \).

Suppose that \( \frac{\partial^2 C_2}{\partial x_1 \partial x_2} = 0 \) then by definition the derivative of the cost function \( \frac{\partial C_2}{\partial x_2} \) does not depend on \( x_1 \), and therefore the optimal second period decision \( x_2^* \) does not depend on \( x_1 \). Further, we can separate the second period cost function into a sum of two independent component functions:

\[ C_2(x_1, x_2, \theta) = C_{2a}(x_1, \theta) + C_{2b}(x_2, \theta) \]  \hspace{1cm} (A.6)
Let $C_{2b}^*(\theta) = C_{2b}(x_2^*, \theta)$ be the minimized cost component that is dependent on $x_2$.

Then for the learning case, the expected second period costs reduce to the sum of two expectations:

$$
\int \left( \min_{x_2} C_2(x_1, x_2, \theta) \right) f_\theta(\theta) d\theta = \int \left( C_{2b}^*(\theta) + C_{2a}(x_1, \theta) \right) f_\theta(\theta) d\theta
$$

$$
= E\{C_{2b}^*(\theta)\} + E\{C_{2a}(x_1, \theta)\}
$$

(A.7)

In the no learning case, the second component of the objective function (A.6) simplifies to:

$$
\begin{align*}
\text{Min} E\{C_2(x_1, x_2, \theta)\} &= \text{Min}_{x_2} \left[ E\{C_{2a}(x_1, \theta)\} + E\{C_{2b}(x_2, \theta)\} \right] \\
&= E\{C_{2a}(x_1, \theta)\} + \text{Min}_{x_2} \left[ E\{C_{2b}(x_2, \theta)\} \right] \\
&= E\{C_{2a}(x_1, \theta)\} + E\{C_{2b}^{**}(\theta)\}
\end{align*}
$$

(A.8)

Note that $E\{C_{2b}^*(\theta)\}$ in (A.7) and $E\{C_{2b}^{**}(\theta)\}$ in (A.8) are not necessarily equal.

The optimization problems have now been reduced to:

Learning: \[\text{Min}_{x_1} \left[ E\{C_1(x_1, \theta)\} + E\{C_{2a}(x_1, \theta)\} + E\{C_{2b}^*(\theta)\} \right] = \text{Min}_{x_1} g(x_1, \theta)\] \hspace{1cm} (A.9)

No Learning: \[\text{Min}_{x_1} \left[ E\{C_1(x_1, \theta)\} + E\{C_{2a}(x_1, \theta) + E\{C_{2b}^{**}(\theta)\} \right] = \text{Min}_{x_1} h(x_1, \theta)\] \hspace{1cm} (A.10)

The first two components in the objective functions (A.9) and (A.10) are now identical, and the third components do not depend on $x_1$. Therefore the derivatives of the two objective functions must be equal $\frac{\partial g}{\partial x_1} = \frac{\partial h}{\partial x_1}$, and so are the solutions to both optimizations $x_1^L = x_1^N$. 

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