Fully-Plastic Open-Bend and Back-Bend Fracture Specimens

by

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Abstract

Crack growth in a low constraint plane strain environment exists in many structures such as penetrating cracks in a pressure vessel. A new fracture mechanics specimen, the back-bend specimen, has been developed to study this loading. The back-bend specimen gives a low triaxiality, plane strain test through the use of a three-point or four-point loading, thereby resulting in significantly lower load capacities as compared to direct tension tests. Both back-bend and open-bend specimens, machined from A572 Gr.50 steel plates, were tested as Charpy-sized impact specimens at various temperatures, and as large-size, slow-bend specimens at room temperature. The back-bend specimen shows increased ductility over the open-bend specimen due to a lower state of constraint (or smaller stress triaxiality) in the back-bend specimen, as compared to the open-bend specimen, in spite of the smaller slip line angle. This increased ductility was shown by a 75°C lower transition temperature and a 90% increase in $CT_{OD}$ before crack initiation for the back-bend loading, as compared to the open-bend loading. Also, an average $CTOA$ of $20°$ during crack growth for the back-bend loading was calculated from fracture surface topography data utilizing a method unique to the back-bend specimen. This value for the back-bend specimen substantially exceeds the $CTOA$ for the open-bend specimen, as is shown qualitatively in the fracture surface topography maps. Load versus displacement data were collected during the tests. Post-test data consisted of sliced crack profiles and laser profilometer measurements on the fracture surfaces.

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Chapter 1

Introduction

1.1 Motivation

There exists a need to study cracks loaded in a low constraint, small stress triaxiality, tensile environment. This type of loading exists in many engineering structures such as internal cracks in a pressure vessel. Other applications to engineering structures are discussed in the next section. The extent of stress triaxiality, defined as the ratio of the normal stress across the dominant active slip lines to the equivalent stress at that point, affects the ductility of a material because of the strong effect of the hydrostatic tensile stress on the void growth rate. Therefore, higher stress triaxialities result in lower amounts of ductility. For cleavage, the stress triaxiality is defined as the maximum principal stress to the yield strength of the material. Higher values of stress triaxiality create a shift in the transition temperature of the material, due to the larger value of the maximum principal stress increasing the probability of cleavage, as will be shown later in this thesis. Therefore, a good test procedure is needed to study the behavior of a cracked material under a low constraint, small stress triaxiality, tensile loading.

Most tests performed to obtain the fracture behavior of a material under a tensile stress environment are performed in a "brute force" type method by just pulling on each end of the specimen. Tests performed in this manner can run into many problems. First, this type of testing can require large load capacities. Therefore,
tests run on large cross-sections of materials require very expensive testing equipment. Secondly, tension tests performed under dead loading are unstable upon reaching their peak load. This phenomenon is discussed in many engineering texts as in [1]. Testing under constant displacement and other loadings can also run into instabilities which prevent stable crack growth. Strictly, it is the ratio of “machine” stiffness (or compliance) to that of the specimen which determines stability under displacement control, not the value of the load itself. However, the stored energy in the “machine” is proportional to the square of the load. Therefore by achieving lower loads, the instability would not be as significant, and crack growth in the specimen could be better controlled.

One way that lower loads can be achieved is by the use of a bend loading. Material tests that utilize a bend loading have always been advantageous. The advantage of the bend setup is due to the low loads that can be achieved, utilizing large moment arms that are inherent to the specimens. Additional material can be welded or otherwise fastened to the ends of bend specimens to further increase the mechanical advantage that is achieved in a bend loading. This may require a specimen of larger length than that required in a direct tension test. The extent of the length depends on the mechanical advantage that is required to lower the loads to a more acceptable level. Although this is an advantageous method for testing materials, the traditional open-bend test, shown in Figure 1-1(a), has a high state of stress triaxiality ahead of its crack tip. Therefore, this type of loading cannot be accurately used to describe the behavior of some large structures which have a lower state of stress triaxiality ahead of their crack tips. Therefore, there is a need for a low constraint bend specimen which would more accurately reproduce some of the loading environments that exist in engineering structures. A specimen that meets these requirements was considered by Prof. F. A. McClintock and has been named the back-bend specimen. A schematic of the back-bend specimen is shown in Figure 1-1(b). This specimen has been tested and analyzed compared to the traditional open-bend test, and the experimental results, analysis, and conclusions are presented here.
1.2 Applications

The back-bend specimen provides a useful method for studying the fracture behavior under a low constraint tensile loading. These conditions exist in many real-life engineering structures. Long part-through cracks or penetrating cracks in a plate of material loaded in tension create a low constraint, plane strain environment. These penetrating cracks can be seen in such structures as pressure vessels, ships, and most anywhere else where substantially long/wide plates are employed. A cross-section of a pressure vessel with a penetrating crack is shown schematically in Figure 1-2. The back-bend test is also useful in studying the conditions that arise in materials under impact or collision environments. Examples of these include ship collisions, structures subject to earthquakes, etc.

1.3 Organization of the Present Work

Chapter Two describes the mechanics and the general test setup of the open-bend and back-bend test specimens. To characterize the behavior of these specimens, a few fracture mechanics methods and parameters are discussed. Also, detail is given as to the stress fields ahead of the crack tips in these two types of bend loadings. These different stress fields affect some of the material properties, such as the transition temperature and other ductility measures, as will be seen in later chapters. Concluding this chapter is a demonstration of the mechanical advantage of using the back-bend loading versus a traditional tension test loading.

Chapter Three details the impact testing done on Charpy-size specimens loaded in open-bend and back-bend modes. This chapter will give the transition temperature behavior of the two specimens and discuss the differences in the results. Also, the transition temperature differences between specimens prepared with a fatigue pre-cracked tip and an electron-discharge machined crack tip are discussed.

Chapter Four details the slow-bend testing of the open-bend and back-bend specimens. The slow-bend tests were done on larger specimens, 25.4mm by 25.4mm by
203.2 mm. Companion specimens were loaded to various levels of imposed deformation and then unloaded. Load versus displacement data was collected, sliced crack profiles were examined, fractographs of the crack front at various positions were taken, and high magnification, 100X to 200X, scanning electron microscope (SEM) pictures of the fracture surfaces were taken. The sliced profiles were used to collect crack tip opening displacement and crack growth data at the centerline of the specimen. This data was used to generate a plot of the opening displacement at the initial crack tip, \( CT_{0}OD \), versus the change in crack length, \( \Delta a \). The differences and shapes of these plots for the open-bend and back-bend specimens are discussed. The high magnification SEM pictures show the micro-mechanisms of fracture for the different loading modes. Also, the 3-D edge effects seen in the fractographs are mentioned briefly.

Chapter Five shows data of the heights of the fracture surfaces of the open-bend and back-bend specimens. This data was collected using a non-contact laser profilometer. The results are presented as a 3-D surface topography map. Data taken down the centerline of the fracture surface was utilized to recreate 2-D crack profiles at various stages of the crack growth. This technique for the back-bend specimen gave the crack tip opening angle, \( CTOA \), the angle of the crack growth, \( \Theta_c \), the end-to-end bend angle, \( \psi \), and the initial crack tip opening displacement, \( CT_{0}OD \), as a function of the crack growth, \( \Delta a \). These 3-D topographic maps also give a good representation of the roughness of the fracture surfaces of the material, A572 Gr.50 steel, used in this testing.

Chapter Six summarizes the differences in ductility between the open-bend and back-bend specimens due to the difference in constraint between the two bend loadings. Also, this chapter gives suggestions for further work in this area including material selection, specimen preparation, test setup, data collection, analysis of results, and finite element modeling.
Figure 1-1: Schematic of bend loadings on similar specimens. (a) Traditional open-bend specimen with slip lines. (b) Back-bend specimen with dashed lines representing shim material.
Figure 1-2: Penetrating crack in a pressure vessel.
Chapter 2

Mechanics of Open-Bend and Back-Bend Test Specimens

2.1 Specimen Geometries and Loadings

As mentioned in the preceding chapter, many engineering structures have components that are under a state of plane strain tension. Therefore, many crack test geometries have been considered to help understand the behavior of different materials under differing loading conditions. A few of these specimens are shown in Figure 2-1.

The face-cracked or edge-cracked tensile fracture specimen is shown in Figure 2-1(a). The width of the specimen is designated as \( w \), and the remaining ligament is designated as \( l \). When loaded by extension to fully-plastic yield conditions, this specimen will have two slip lines emanating from its crack tip. Both of these slip lines are oriented at 45° to the plane of the crack. These slip lines emanate at the crack tip and conclude at the back or free surface. This slip line field is the same as would be seen in an engineering structure under plane strain tensile loading conditions. However, as noted in Chapter 1, this type of specimen can become highly unstable at max load. Although the high loads are a problem, the “instability” is somewhat mitigated by the high crack growth ductility (large CTOA).

The next specimen is the middle-cracked tensile specimen, which is shown in Figure 2-1(b). This specimen is basically two face-cracked tensile specimens connected
together. Keeping both ligaments equal in size to the ligament on the face-cracked specimen will require that the middle-cracked specimen be twice the width of the face-cracked specimen. Also, this specimen has two sets of slip lines emanating from both ends of the crack and concluding on both free surfaces. Each pair of slip lines is identical to those in the face-cracked specimen. One of the major disadvantages of the middle-cracked specimen is that it requires twice the load of the face-cracked specimen, when the ligaments on the middle-cracked specimen are equal in size to the ligament on the face-cracked specimen. Also, with two “equal” cracks, in practice, one is always “more equal” than the other, with cracking more prominent at one crack front than the other.

Finally, the back-bend specimen is shown in Figure 2-1(c). The back-bend specimen has two sets of slip lines, similar to the middle-cracked specimen. However, the slip lines emanating from the right side crack tip are in tension, whereas the slip lines emanating from the left side crack tip are in compression. Both sets of slip lines are still oriented 45° from the crack plane. Actually, the left side “crack-tip” is more accurately a “last point of contact” than a crack tip. The back-bend specimen is created with a crack of initial length, $a_o = w - l_o$. This initial crack size must be greater than half the width of the specimen. This important dimensional restriction is shown by the following relation:

$$\frac{a_o}{w} > 0.5.$$  \hspace{1cm} (2.1)

This is required to achieve a state of tension on the right and compression on the left, which balance one another for pure bending. Once the specimen is loaded in the back-bend mode, the faces on the back side will come in contact. Since some starter notches that are machined leave a considerable gap, shim material, designated by the dashed lines in Figure 2-1(c), may be required to help close the gap. Once the specimen has been loaded such that the ligament is in full plasticity, the amount of contact between the back faces should be of size equal to the length of the remaining ligament. This is a good approximation for the situation of a non-hardening limit
load. This is shown schematically in Figure 2-1(c), where the amount of contact between the back crack faces is designated by \( l \). This evolving dimension, \( l \), is also the length of the remaining specimen ligament.

The back-bend specimen is similar to other bend loadings in that it can be loaded in either three-point or four-point bending. As in other bend tests, four-point loading may be more desirable since it creates a section of pure bending, devoid of shear loads. However, most impact tests are carried out in three-point bending. Referring to Figure 2-1(c), the ligament and contact length are referred to as \( l \). For three-point bending, refer to the contact length as \( l_c \) and the ligament as \( l \). One difference that can arise in the three-point bend test, as compared to the four-point bend test, of the back-bend specimen is the contact length between the back faces of the specimen. The contact length, \( l_c \), between the back crack faces in three-point bending can be less than the length of the ligament, \( l \). This is due to the local pressure, \( p \), acting along the roller contact length, \( d_c \), as shown schematically in Figure 2-2. As has been shown in many engineering texts, pressure applied equally in all three directions, hydrostatic stress, can affect the value of a particular stress component at which the material begins to yield. For load balance, the following is required:

\[
\sigma_{YS}l_c + \frac{d_c}{2} = \sigma_{YS}l. \tag{2.2}
\]

This results in the following equation for \( l_c \).

\[
l_c = l - \left( \frac{p}{\sigma_{YS}} \right) \frac{d_c}{2}. \tag{2.3}
\]

Therefore when \( p \) and \( d_c \) are greater than zero, \( l_c \) must be less than \( l \).

Before beginning testing on this new back-bend specimen, a search was conducted to see if a specimen of this type had ever been tested. The closest resemblance to the back-bend specimen that was found was the work done by H. M. Schnadt. Schnadt conducted many impact tests studying the brittle behavior of steels. One of the specimens that he used was called the Schnadt Specimen, Figure 2-3. A hardened rod was inserted in the radial slot, and the specimen was bent about this rod. Even
though it may seem that the specimen is being bent “backwards” about the hardened rod, the crack emanates from the notch on the tensile free surface. Therefore, the Schnadt Specimen more closely resembles the open-bend test. Also, the slip line field of this specimen differs greatly from that which exists in the back-bend specimen. The Schnadt Specimen is discussed briefly by Parker [2] and by Boyd [3]. A more detailed discussion of the Schnadt Specimen can be found in [4] and [5].

2.2 Fracture Mechanics Characterization Methods and Parameters

Fracture mechanics methods analyze specimens and engineering structures and develop quantitative characterizations of crack tip stress-strain fields that can be applied similarly to both the laboratory specimen and real-life engineering structure. From the stress-strain fields that are developed, some crack-tip driving parameters are extracted that help to describe the critical conditions that exist which lead to crack growth. Critical values of these crack-tip driving parameters are inferred from tests on laboratory specimens. These critical values of the crack-tip driving parameters are applied to similar crack-tip conditions in engineering structures and can provide an accurate prediction of the fracture behavior of the structure.

All of the following methods search for a set of crack-tip driving parameters which characterize the stress and strain fields surrounding the crack tip. Most solutions are represented by a series expansion. In practice these series are truncated after one, two or three terms, thereby providing a good and useful estimate of the stress and strain fields surrounding the crack tip. These methods must be used with caution, as each has many implicit and explicit assumptions.

2.2.1 Linear Elastic Fracture Mechanics

Linear Elastic Fracture Mechanics, LEFM, is the most widely known of these methods. In Mode I loading, opening normal to the plane of the crack, the stress and strain
fields in an annular elastic region surrounding the crack tip are uniquely characterized by the stress intensity factor, $K_I$. The stress intensity factor, $K_I$, is part of the first term in the Williams expansion [6] which equates the stress ahead of the crack tip as a function of the radius from the crack tip, the geometry of the specimen and loading of the specimen. Truncating all terms after the first, the Williams expansion gives

$$\sigma_{ij}(r, \theta) = \left(\frac{K_I}{\sqrt{2\pi r}}\right)f_{ij}(\theta),$$

(2.4)

where $\sigma_{ij}$ is the stress tensor at a position $(r, \theta)$ from the crack tip, $\theta$ is the angle from the crack plane, and $f_{ij}$ are dimensionless functions of $\theta$. This stress field depends on the cylindrical coordinates, $r, \theta$, centered at the crack tip. As radial distance $r \to 0$, the stress approaches infinity. This creates a plastic zone inside the region dominated by $K_I$. According to [7], a good estimate of the radial extent of this plastic zone is given by

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{YS}}\right)^2,$$

(2.5)

where $\sigma_{YS}$ is the yield strength of the material. One requirement of the use of LEFM is that the region characterized by $K_I$ is large compared to the size of the plastic zone given by Eq. 2.5, but small compared to the crack length and remaining ligament. Also, the thickness of the specimen must be large enough to give a state of plane strain. These requirements are set forth by the ASTM standard [8] which states

$$in-plane, out-of-plane dimensions \geq 2.5\left(\frac{K_I}{\sigma_{YS}}\right)^2.$$  

(2.6)

Due to these stringent requirements for small laboratory-sized specimens of high toughness materials, other methods of characterization are desired.

### 2.2.2 Elastic-Plastic Fracture Mechanics

Linear Elastic Fracture Mechanics is valid only when the plastic zone size, $r_y$, is small compared to the specimen ligament, $l$. Furthermore, for plane strain conditions, the thickness, $B$, must be large compared to the plastic zone size, $r_y$. Therefore,
when the plastic zone size is larger than the above limits set forth by LEFM, elastic-plastic or non-linear fracture mechanics must be used. Similar to the asymptotic series developed by Williams [6] for LEFM, the Hutchinson-Rice-Rosengren, HRR, singularity fields, [9] and [10], describe the stress and strain fields ahead of the crack tip for power-law hardening nonlinear stress-strain behavior. The main crack-tip driving (scaling) parameter used in the HRR fields is the $J$-integral. The $J$-integral is a path-independent integral taken around the crack tip. In the case of linear elastic conditions, the $J$-integral is related to the stress intensity factor, $K_I$. For plane strain conditions that relationship is given by

$$J = \frac{K_I^2}{E} (1 - \nu^2),$$

(2.7)

where $E$ is the Young’s modulus and $\nu$ is the Poisson’s ratio of the material. The $J$-integral is a difficult parameter to measure directly from experiments. For certain test geometries, (e.g., deeply-cracked open-bending specimens), there exists means to relate $J$ to features of overall load/displacement relations, [7]. However, a relationship between $J$ and the crack-tip opening displacement, $CTOD$, exists and is given by

$$CTOD = d_n \frac{J}{\sigma_{YS}},$$

(2.8)

where $d_n$ is a dimensionless parameter. The value of $d_n$ has been studied extensively by Shih [11] and many others, and it is estimated to range from 0.3 to 0.8.

However, elastic-plastic fracture mechanics is limited to an amount of crack growth, under a monotonically increasing load, that is small compared to the region dominated by the HRR singularity. Also, the region where finite strain effects dominate must also be small compared to the HRR field, [12].

One-parameter fracture mechanics is limited to geometries and loadings which ensure similar levels of crack tip constraint. To characterize crack-tip fields of differing constraints, more terms in the above series solutions must be included. Therefore, in addition to $K_I$ and $J$, the $T$-stress is used to help characterize the stress fields surrounding crack tips of differing constraints, [7]. The effect of the crack-tip con-
straint parameterized by the $T$-stress on cracking characterized by $J$ or $CTOD$ has been studied by Hancock, Reuter, and Parks [13]. In addition to $J-T$ formulations, the hydrostatic stress parameter, $Q$, can be used in conjunction with $J$. Larger values of $T$ and $Q$ represent higher states of crack-tip stress triaxiality, and vice versa. Constraint effects on cracking using $J-Q$ formulations have been studied by many, including Dodds, Shih, and Anderson [14].

2.2.3 Slip-Line Fracture Mechanics

Since elastic-plastic fracture mechanics is limited to relatively small amounts of plastic strains and crack growth, another method for characterizing the fracture of a specimen is required. One method that meets these needs is Slip-Line Fracture Mechanics, SLFM. SLFM is currently being developed by Prof. F. A. McClintock [15]. SLFM is based on the limiting case of a plane strain, rigid-plastic, isotropic material without strain hardening. In SLFM, the crack-tip fields are characterized by three crack-tip driving parameters: the slip line direction $\theta_s$, the normal stress across the slip plane $\sigma_n$, and the increment in relative displacement across the slip plane $du_s$. Analogous to $K_i$ and $J$, these three parameters which characterize the crack tip field are a function of loading and geometry:

$$\begin{bmatrix}
\theta_s \\
\sigma_n \\
du_s
\end{bmatrix} = f(far-field geometry, loading). \quad (2.9)$$

The parameters $\theta_s$ and $\sigma_n$ are fixed for a given geometry and loading, so a key quantity for crack initiation is the slip displacement, $u_s$. An alternative measure for initiation is the crack tip opening displacement, $CTOD$. The $CTOD$ is more readily measured from experiments and provides a good measure for crack initiation. According to McClintock’s analysis [15], the crack tip opening angle, $CTOA$, replaces the $CTOD$ for continuing crack growth. Therefore, SLFM suggests examining the $CTOD$ for fully plastic crack initiation and the $CTOA$ for continued crack growth.
2.3 Stress Field and Slip Lines for Open-Bend and Back-Bend Crack Tips

As shown in the previous section, knowledge of the slip line fields associated with a crack tip can be utilized in slip-line fracture mechanics, SLFM, to predict the far-field displacements and rotations. The orientation of the slip lines and the normal stress across them depend on not only the geometry of the specimen, but also on the type of loading applied. This is shown in this section where two specimens of identical geometry are loaded with opposite moments, resulting in two entirely different slip line fields. For details on the slip line fields for a variety of different geometries and loadings, the treatise by McClintock [16] and the text by McClintock and Argon [17] can be reviewed.

2.3.1 Open-Bend Specimen

The slip line field for the open-bend specimen is known as the Green and Hundy field, [18], and is shown in Figure 2-4(a). A schematic of this field near the crack tip is shown in Figure 2-4(c). As shown in the schematic and presented in [16], the slip lines emanate from the crack tip at \( \theta_s = \pm 72^\circ \) from the plane of the crack. The stress normal to the slip lines, \( \sigma_n \), is normalized with respect to the plane strain yield strength, \( 2k \), where \( k \) is the flow strength of the material in shear. The normalized value of \( \sigma_n \) is

\[
\frac{\sigma_n}{2k} = 1.543. \tag{2.10}
\]

The value of the normal stress shown in Eq. 2.10 results in a relatively high state of stress triaxiality at the crack tip. This state of high stress triaxiality results in lower ductility in the specimen due to the effect of the hydrostatic tensile stress on the void growth rate, [16]. Therefore, the open-bend loading mode is considered a relatively high constraint loading.
2.3.2 Back-Bend Specimen

The slip line field for the back-bend specimen is shown in Figure 2-4(b). A schematic of this field near the crack tip is shown in Figure 2-4(d). The slip lines for the back-bend specimen emanate from the crack tip at \( \theta_s = \pm 45^\circ \) from the plane of the crack. The stress normal to the slip lines, \( \sigma_n \), is again normalized with respect to the plane strain yield strength, \( 2k \). The normalized value of \( \sigma_n \) for the back-bend specimen is

\[
\frac{\sigma_n}{2k} = 0.5. \tag{2.11}
\]

As compared to the value of the normal stress in Eq. 2.10, the value of \( \sigma_n \) in Eq. 2.11 is much lower. This shows that the state of stress triaxiality is much lower for the back-bend specimen as compared to the open-bend specimen. This lower state of stress triaxiality results in higher ductility in the specimen as compared to the open-bend specimen. Therefore, the back-bend loading mode is a low constraint loading.

These differing levels of constraint can become quite important in analyzing designs of structures. Designs in which resistance to cracking are based on the high constraint open-bend tests lead to conservative results. However, if the actual structure has a much lower state of constraint as compared to the laboratory tests used in your analysis, your predictions may be too conservative, and the actual behavior of the structure could be highly uncertain. Therefore, the constraint of the laboratory test specimens should be similar to the constraint of the actual structure to provide more accurate predictions. After accurately predicting structural behavior, appropriate factors of safety can then be applied to the design.

2.4 Mechanical Advantage of Back-Bend Loading vs. Traditional Low Constraint Tension Tests

The mechanical advantage that can be achieved in the back-bend specimen over traditional low constraint tension-loading tests depends on the size of the moment
arm employed in the back-bend specimen. Extra length in the specimen can be used to create larger moment arms. This mechanical advantage provides a method for lowering the load capacities required, thereby allowing tests on larger ligaments. Specimen dimensions are shown schematically in Figure 2-1. The moment arm, $s$, for three-point loading is shown schematically in Figure 2-5, whereas the moment arm, $s$, for four-point loading is shown schematically in Figure 2-6. The thickness of each specimen perpendicular to the plane of the page is denoted by $B$. The initial equations solve for $P_{lim}$, which is the limit load achieved by each respective specimen. Finally, the ratio of the limit load in the back-bend specimen to the limit load in each of the traditional tension tests is taken to show the reduction in load capacity required.

2.4.1 Back-Bend Loading Compared to Face-Crack in Tension

The limit load achieved in the back-bend specimen is given by

$$P_{lim,bb} = \frac{2}{\sqrt{3}} \sigma_{TS}lB \frac{2(w-l)}{s}. \tag{2.12}$$

Similarly, the limit load achieved in the face-crack specimen is given by

$$P_{lim,fc} = \frac{2}{\sqrt{3}} \sigma_{TS}lB. \tag{2.13}$$

Taking the ratio of $P_{lim,bb}$ to $P_{lim,fc}$ results in the following load reduction

$$\frac{P_{lim,bb}}{P_{lim,fc}} = \frac{2(w-l)}{s} = \frac{2(1 - \frac{l}{w})}{\frac{s}{w}}. \tag{2.14}$$

A typical value for $l/w$ is 0.25. The value for $s/w$ that was used in the slow bend testing of the back-bend specimen, discussed in Chapter Four, was 2.5. This value for $s/w$ is easily achieved and can easily be made higher. Substituting the above values for $l/w$ and $s/w$ into Eq. 2.14, results in a ratio of 0.6. Therefore, for these
dimensional values, the back-bend specimen only requires 60% of the fully-plastic load capacity that is required by the face-crack specimen. Also, remember that the value for $s/w$ can be made much higher, thereby further decreasing the ratio in Eq. 2.14 and subsequent load capacity required by the back-bend specimen. Alternatively, at fixed machine load capacity, the mechanical advantage permits testing of larger specimens, of dimensions approaching those of many structural applications.

2.4.2 Back-Bend Loading Compared to Middle-Crack in Tension

The limit load for the back-bend specimen was given in the preceding section by Eq. 2.12. Whereas, the limit load achieved in the middle-crack tensile specimen is given by

$$P_{lim,mc} = 2 \frac{2}{\sqrt{3}} \sigma_{TS} l B.$$  \hspace{1cm} (2.15)

Taking the ratio of $P_{lim,bb}$ to $P_{lim,mc}$ results in the following load reduction

$$\frac{P_{lim,bb}}{P_{lim,mc}} = \frac{(w - l)}{s} = \frac{(1 - \frac{l}{w})}{s/w}.$$  \hspace{1cm} (2.16)

Substituting the values for $\frac{l}{w}$ and $\frac{s}{w}$ used in the previous section into Eq. 2.16, results in a ratio of 0.3. Therefore for these dimensional values, the back-bend specimen only requires 30% of the fully-plastic load capacity that is required by the middle-crack specimen. Again, remember that the value for $\frac{s}{w}$ can be made much higher, thereby further decreasing the ratio in Eq. 2.16 and subsequent load capacity required by the back-bend specimen.
Figure 2-1: Fully-plastic low constraint plane strain specimens. (a) Face-crack. (b) Middle-crack. (c) Back-bend.
Figure 2-2: Local contact of three-point bend point with backside crack faces. (a) Schematic of three-point bend contact with backside crack faces. (b) Enlarged schematic showing effect of local bend point on traction between the backside crack faces.
Figure 2-3: Schnadt specimen.
Figure 2-4: Slip line fields ahead of a crack tip. (a) Schematic of open-bend specimen with slip lines. (b) Schematic of back-bend specimen with slip lines. (c) Near crack tip schematic of open-bend slip line field. (d) Near crack tip schematic of back-bend slip line field.
Figure 2-5: Moment arm in three-point back-bend test.

Figure 2-6: Moment arm in four-point back-bend test.
Chapter 3

Loading Mode Effects on Transition Temperature

3.1 Material and Dimensions

The material selected for the open-bend and back-bend testing was A572 Gr.50 steel. This material was selected because it is a structural steel that is commonly employed in engineering structures. The material properties and chemical composition, given in the material data sheet provided by the vendor, for this material are shown in Table 3.1. Checks on the mechanical properties were done via ASTM standard tests. The experimental results from these tests are provided in Appendix A. In addition, hardness tests, performed on the batch of steel purchased, indicated a Rockwell B hardness of 83. According to a Rockwell hardness conversion chart, a Rockwell B hardness of 83 correlates with a tensile strength of 552MPa, which is extremely close to the value of 558 MPa given in the material data sheet.

The steel was purchased from Levinson Steel company. The plate ordered was one inch thick, since this is a common plate thickness for A572 Gr.50. The overall size of the plate ordered was $9\frac{3}{8}in$ by $8\frac{1}{2}in$. Twelve plates of the above dimensions were flame cut from the parent plate. Due to the flame cut, one quarter of an inch of material was removed from each edge by milling. This material was removed to eliminate any change in material properties as a result of the flame cut.
The 9$\frac{3}{8}$ in dimension on the plates of A572 Gr.50 is in the rolling direction. Specimens can be cut out of the plate in numerous orientations. Each specimen orientation studies crack growth of different behaviors, depending on the material anisotropy. A crack orientation is usually characterized by two letters. The first letter denotes the direction normal to the plane of the crack. The second letter denotes the direction of crack growth. A typical coordinate system for plates is shown in Figure 3-1. L is the longitudinal, or rolling direction, S is the short transverse or plate thickness direction, and T is the transverse or width direction. It was decided that the TS crack orientation is the most common crack orientation for plates in engineering structures under low constraint plane strain tensile conditions. Therefore, all specimens in this current work were specified to be machined from the plate with a TS crack orientation as shown in Figure 3-1.

Upon receipt of the material data sheet for the A572 Gr.50, it was evident that this steel was imported from outside the United States. Standard industrial practice in some countries does not currently use some of the purification processes that are employed in steel mills in the United States and other countries. One of the problems is due to manganese sulfide, MnS, inclusions. At hot-rolling temperatures, these sulfide inclusions are soft and, upon being rolled, flatten out like pancakes or lamellae. When a crack encounters these lamellae of sulfide, the bond between the steel and the sulfide lamellae easily breaks, creating a large crack-like defect. Since the sulfide inclusions are flattened out in the rolling plane, cracks that advance through the thickness of the plate encounter these lamellae normal to their own (crack) plane, thereby creating delaminations normal to the crack plane. These delaminations normal to the crack plane create a very rough fracture surface. Also, the amount of phosphorus (0.024% by weight), in this batch of steel, is above normal standards.

To prevent the adverse effects of these manganese sulfide lamellae, steel processing procedures in the United States and other countries have been modified. One of the methods currently being employed is to treat the steel with calcium in the melt. The calcium, in comparison to manganese, preferentially reacts with the sulfur to create hard spherical inclusions. These hard inclusions do not flatten out during the
rolling process, thereby eliminating the manganese sulfide lamellae. The formation and calcium treatment of these sulfide inclusions has been studied extensively, [19], [20], and [21].

Upon initial testing of the A572 Gr.50 steel that was acquired, it was believed that this material contained these manganese sulfide lamellae, due to the roughness in the fracture surfaces in the form of delaminations in the rolling plane. Pictures (825X magnification) of the microstructure of this material are included in Appendix A, Figure A-6. Since material of this nature is still currently being used in many engineering structures, it was decided to carry out testing on this material, but a more pedigreed steel should also be tested in the near future. Proper material selection is discussed further in Chapter 6.2.

The open-bend and back-bend testing was done on specimens of two overall sizes. Impact testing, which is the topic of this chapter, was done on specimens of Charpy-size. The impact testing was performed to study the brittle to ductile transition of A572 Gr.50 steel for two different states of crack tip stress triaxiality. Detailed machine drawings are shown in Figure 3-2. These specimens were machined out of the one inch thick plate of A572 Gr. 50 steel described in the preceding paragraphs. The 0.02mm wide slot in Figure 3-2 was electron-discharge machined, EDM, with a TS crack orientation specified. All other machining was done by a mill.

3.2 Test Procedure and Equipment

Sixty specimens were machined according to Figure 3-2. Of these sixty specimens, thirty were fatigue pre-cracked to create a sharper crack tip than that of the EDM slot. The fatigue pre-cracking procedure was conducted according to ASTM standard E-23 [22]. The pre-cracking was conducted in an Instron testing machine in three-point open-bending. The testing machine applied a load control fatigue waveform at 10Hz. As the crack grew, the load range was shifted down to keep the same maximum $\Delta K$ range, 60% of $K_{IC}$, specified by [22]. The ratio of minimum to maximum stress used was $+0.1$. The fatigue pre-crack for each of these thirty specimens was grown
Two pieces of equipment were used for the impact testing. First, a temperature chamber is required to bring the specimens to the desired temperatures to fill out an entire transition curve. The temperature chamber used in this impact testing was a Sun Electronic Systems Model EC1x, Figure 3-3(a). This temperature chamber employs a gas medium, air, to change the temperature of the specimens placed inside. According to the ASTM standard E23 [22], a metallic impact specimen of overall dimensions depicted in Figure 3-2 must stay in the temperature chamber thirty minutes when the medium for heat transfer is a gas. When the medium for heat transfer is a liquid, [22] requires that the specimen stay in the constant temperature liquid bath for only five minutes. Once the entire specimen has reached the desired temperature, it is removed from the temperature chamber and placed in the Tinius Olsen Impact Testing Machine, Figure 3-3(b). The beginning of the EDM slot of the specimen is placed away from the hammer for a three-point open-bend mode and toward the hammer for a three-point back-bend mode. After the specimen is placed in the impact testing machine, the hammer of the testing machine is immediately released. The toughness of the specimen is measured according to the potential energy loss in the impact hammer. To ensure that the temperature of the specimen has not changed appreciably prior to testing, ASTM standard E23 [22] requires that the length of time from the removal of the specimen from the temperature chamber to the time that the impact hammer strikes the specimen be less than five seconds. Due to the extremely small width of the EDM slot, 0.02mm, shim material was not required to help close the gap between the backside crack faces.

3.3 Transition Temperature Results

For the transition temperature testing, four different types of specimens were tested. All sixty specimens were machined according to Figure 3-2. Thirty of these were fatigue pre-cracked. Sixteen of the fatigue pre-cracked specimens were tested in a back-bend mode, and sixteen of the specimens without a fatigue pre-crack were also
tested in a back-bend mode. The remaining fourteen with a pre-crack and fourteen without a pre-crack were tested in an open-bend mode.

3.3.1 Transition Temperature Results for Open-Bend Loadings

The results for the testing done on the specimens impacted in an open-bend mode are shown in Figure 3-4. The impact toughness in Joules is plotted against the temperature in °C. The transition between lower shelf behavior and upper shelf behavior occurs between −30°C and 10°C, with −10°C being the mean value. Specimens on the lower shelf failed by cleavage, showing their lack of ductility at these temperatures. Whereas, specimens on the upper shelf displayed their ductility by failing in a hole growth and tearing mechanism.

Also, Figure 3-4 shows relatively no difference between the specimens that had only an EDM notch, versus those that were fatigue pre-cracked. The only noticeable difference that can be seen is on the lower shelf, where the pre-cracked specimens displayed about 1 Joule less toughness than the specimens with just an EDM notch. This behavior was also noticed experimentally by Böhme [23]. In his impact bend tests on German reactor pressure vessel steel, Böhme saw no apparent difference in the transition toughness behavior of an EDM notch versus a fatigue pre-crack. The only difference displayed by Böhme’s results was a smaller lower shelf toughness value for pre-cracked specimens as compared to the lower shelf toughness of specimens with just an EDM notch.

3.3.2 Transition Temperature Results for Back-Bend Loadings

The results for the testing done on the specimens impacted in the back-bend mode are shown in Figure 3-5. The impact toughness in Joules is again plotted against the temperature in °C. The transition between lower shelf behavior and upper shelf behavior occurs between −100°C and −70°C, with −85°C being the mean value.
Specimens on the lower shelf failed by cleavage which remained on the original crack plane. Whereas, specimens on the upper shelf failed by a hole growth and tearing mechanism on a plane rotated approximately ±45° from the original crack plane. All specimens that failed on the upper shelf exhibited this orientation of the failure plane, which coincides with one of the two slip planes for the back-bend mode shown in Figure 2-4. Also, no specimens failed in a back-bend mode with toughness value between about 40 and 95 Joules. Therefore, the back-bend transition curve, Figure 3-5, does not display the smooth transition between lower shelf and upper shelf behavior displayed in the open-bend transition curve, Figure 3-4.

The transition data for the back-bend mode, Figure 3-5, displays no noticeable difference between the pre-cracked specimens and the specimens with just an EDM notch. Even the lower shelf behavior of these two different notch preparations displays no apparent trends as compared to the data for the open-bend tests, Figure 3-4.

3.3.3 Shift in Transition Temperature Due to Difference in Loading Modes

Combining the results for both bend modes gives the curves in Figure 3-6. The data shows that the transition temperature for the back-bend impacted specimens is approximately 75°C lower than the transition temperature for the open-bend impacted specimens. Qualitatively, this is expected and is conceptualized by the Davidenkov diagram, which is included in many texts, such as McClintock and Argon [17]. The Davidenkov diagram explains that the mechanism controlling cleavage is the maximum principal stress:

\[ \sigma_{\text{max}} = \sigma_1 = MY(\ldots) \]  

\[ Y = \dot{Y}(\text{Temperature, StrainRate}(\dot{\varepsilon}), \text{Alloy}, \ldots) \]  

\( M \) is a geometric factor representing the extent of the stress triaxiality. While \( \sigma_{\text{max}} \) is the driving stress, there exists a stress, \( \sigma_{\text{cleavage}} \), at which cleavage will take place. \( \sigma_{\text{cleavage}} \) is a material property while \( \sigma_{\text{max}} \) is due to the present loading condition.
In the present testing, the large stress triaxiality present in the open-bend loading causes specimens loaded in this manner to have a higher value for the geometric factor, $M$. This creates a higher value, as compared to the back-bend specimen, for $\sigma_{\text{max}}$ in the open-bend specimen for a set value of $Y$. As the temperature rises, the value for the yield stress, $Y$, decreases, eventually causing $\sigma_{\text{max}}$ to drop below the critical value for cleavage, $\sigma_{\text{cleavage}}$. Since the value for $M$ is lower for the back-bend specimen, as compared to the open-bend specimen, the temperature at which the back-bend specimen no longer reaches the critical stress for cleavage is lower than the temperature at which the open-bend specimen no longer reaches the critical stress for cleavage. Therefore, the transition temperature for the back-bend specimen is lower than that for the open-bend specimen when all other variables are constant. Consequently, high constraint crack tips create more brittle behavior resulting in a higher transition temperature as compared to lower constraint crack tips. This phenomenon has been studied by many including Gao, Shih, Tvergaard, and Needleman [24] and Ruggieri, Dodds, and Wallin [25]. This analysis is satisfactory for a macro root radius (notches). Whereas for a sharp crack, a length scale must be considered to account for size of the defect zone ahead of the crack tip as discussed in [12].

Finally, the specimens that were impacted in a back-bend mode showed substantially larger toughness values as compared to the open-bend specimens. This is expected because the specimens loaded in a back-bend mode rotate back on their EDM slot. Therefore, an initial segment of the EDM slot is closed and more area or material is involved in the loading as compared to the open-bend loading. This is also shown in the next chapter, where the maximum loads for two identical specimens differ by an order of magnitude due to the two different bending modes applied. Estimates of the area under the load versus displacement curves for the open-bend and back-bend slow-bend tests displayed in the next chapter are 50 J and 300 J, respectively. This correlates to a 600% increase in work in the back-bend specimen as compared to the open-bend specimen. This is also shown in Figure 3-6 where the back-bend upper shelf toughness is approximately 120 J as compared to approximately 20 J for the open-bend specimens. This is again a 600% increase in work. Although this difference
in toughness is present, the brittle to ductile transition behavior of the material as a function of temperature and crack tip constraint is still easily observable from the data.
Table 3.1: Material properties and chemical composition of A572 Gr.50 used in current testing.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Yield Strength</th>
<th>353 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mill specification)</td>
<td>Tensile Strength</td>
<td>558 MPa</td>
</tr>
<tr>
<td></td>
<td>Elongation Percentage</td>
<td>26.00%</td>
</tr>
<tr>
<td>Chemical Composition</td>
<td>C</td>
<td>0.19%</td>
</tr>
<tr>
<td>(weight percent)</td>
<td>Fe</td>
<td>98.39%</td>
</tr>
<tr>
<td></td>
<td>Mn</td>
<td>1.04%</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>0.024%</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>0.026%</td>
</tr>
<tr>
<td></td>
<td>Si</td>
<td>0.33%</td>
</tr>
</tbody>
</table>
Figure 3-1: Nomenclature for crack orientations in a rolled plate.

Figure 3-2: Machine drawing of specimens used in impact testing.
Figure 3-3: Test equipment. (a) Sun Electronic Systems Model EC1x temperature chamber. (b) Tinius Olsen impact testing machine.
Figure 3-4: Transition curves for A572 Gr.50 specimens impacted in an open-bend mode.
Figure 3-5: Transition curves for A572 Gr.50 specimens impacted in a back-bend mode.
Figure 3-6: Transition curves for A572 Gr.50 depicting shift in transition temperature for different crack tip constraints.
Chapter 4

Slow-Bend Tests on Large Specimens in Open-Bend and Back-Bend Loading Modes

4.1 Material and Dimensions

The material selected for the slow-bend testing discussed in this chapter is the same A572 Gr.50 steel detailed in Section 3.1 and further in Appendix A. The specimens used in the slow-bend testing were machined from the one inch thick plates of A572 Gr.50 steel according to Figure 4-1. The Chevron notch was machined to facilitate fatigue pre-crack initiation. Again, the specimens were specified to be machined from the plate with a TS crack orientation as shown in Figure 3-1. The 0.062 in. wide slot with the Chevron notch was electronically discharge machined, EDM. All other machining was carried out by a mill.

4.2 Test Procedure and Equipment

Prior to the slow-bend testing, all specimens were fatigue pre-cracked. The fatigue pre-cracking procedure was conducted according to ASTM standard E-399 [8]. The pre-cracking was done in four-point open-bending by an Instron Model 8501 testing
machine. The testing machine applied a load control waveform at 10Hz. As the crack grew, the load range was shifted down to keep the same maximum $\Delta K$ range, 60% of $K_{ic}$, specified by [8]. The ratio of minimum to maximum stress used was +0.1. The fatigue pre-cracks were grown to approximately 75% of the width of the specimen. This gives a total initial crack length of 0.75 in. (19mm).

The slow-bend testing was carried out according to the setup depicted in Figure 4-2. This setup includes stainless steel shim material, 0.062 in. thick, used to close the 0.062 in. gap shown in Figure 4-1. The shim material has a Rockwell C hardness of 40. This correlates to a tensile strength of the shim material of approximately 1250 MPa. As stated in Chapter 3, the A572 Gr.50 steel that was purchased had a Rockwell B hardness of 83, correlating to a tensile strength of approximately 552 MPa. This means that the shim is significantly harder than the A572 Gr.50 steel being tested; therefore, the shim can be treated as a rigid body.

The testing machine used was an Instron Model 8501 hydraulic testing machine. The testing machine and included test setup is shown in Figure 4-3(a). A closer view of the test specimen in the four-point bend setup is pictured in Figure 4-3(b).

The load capacity of the testing machine utilized is 100kN. For a specimen with a 6.4mm by 25.4mm ligament and an ultimate tensile strength of 558 MPa tested in direct plane strain tension, the load capacity required would exceed 100kN, the capacity of the testing machine employed. However, due to the mechanical advantage of the back-bend specimen, this size ligament can be tested with the current testing machine. The mechanical advantage of the back-bend specimen was discussed in Section 2.4. The dimensions in Figure 4-1 and Figure 4-2 and a pre-crack of 75% of the specimen width were used in the analysis in Section 2.4. The analysis showed that the ratio of the max load required by back-bend specimen to the max load required by a face-crack specimen was 0.6, Eq. 2.14. The max load required by the back-bend specimen with the above dimensions is estimated by Eq. 2.12 to be approximately 62kN, which is less than the 100kN load capacity of the Instron testing machine.
4.3 Experimental Results

Due to the fact that the back-bend specimen closes the gap between the backside crack faces, many experimental data-collecting techniques such as mounting a crack mouth opening gauge can not be utilized. The data that was collected during the slow-bend testing is the load as a function of the crosshead displacement. Although no further data was collected during the test, many forensic techniques can be used to gain further knowledge from the test. Some of these include marking the crack front, sectioning the specimen, fracture surface photographs, height measurements on the fracture surfaces, etc.

4.3.1 Load Versus Displacement Data

Multiple tests were performed on the specimens detailed in Figure 4-1 in both open-bend and back-bend loadings. Load versus displacement curves for those tested in open-bending are shown in Figure 4-4. These tests were stopped at various points in their loading, for reasons to be discussed in the next section. The curves are for specimens that were stopped at max load, 10% load drop, 20% load drop, 40% load drop, 60% load drop, and complete fracture surface separation. Each curve is labeled with its respective load drop percentage. The expected value of the limit load can be calculated by

\[ P_{lim,ob} = (1.222) \frac{2}{\sqrt{3}} \sigma_T s B^2 l^2 / s, \]  

(4.1)

where \( P_{lim,ob} \) is the limit load in open-bending, \( B \) is the thickness of the specimen, \( w \) is the width of the specimen, \( l \) is the remaining ligament, and \( s \) is the moment arm achieved in the test setup. Eq. 4.1 with the numerical factor of 1.222 can be found in McClintock’s treatise, Plasticity Aspects in Fracture [16]. Notice the quadratic (2nd order) dependence of the limit load, \( P_{lim,ob} \), on the ligament size, \( l \). Eq. 4.1 gives an estimated limit load for the open-bend tests of 5.2 kN. Actual values of the maximum observed load ranged from -20.0% to +8.7% from the expected value of the limit load. Estimates for the limit load assume a fully-plastic solution with a stationary crack. If the crack grows before the limit load is reached, then maximum observed load will not
be equal to the limit load estimated. Therefore, this rather broad range for the value of the maximum observed load in Figure 4-4 could be due to crack growth before the limit load is reached as a result of the random distribution of the MnS inclusions.

Figure 4-5 displays the load versus displacement curves for the specimens loaded in a back-bend mode. Again, the curves are for specimens that were stopped at max load, 10% load drop, 20% load drop, 40% load drop, 60% load drop, and complete fracture surface separation. These curves are also labeled with their respective load drop percentage. The initial portion of the curves is due to the settling of the bend rollers and to the back crack faces eliminating any gaps between themselves and the shim material. The next part of the curve is the elastic portion, with a fairly constant yield point as its endpoint. The expected value of the limit load, 62kN, was calculated using Eq. 2.12. Actual values of the maximum observed load ranged from -16.6% to +5.2% from the expected value. This deviation from the expected limit load could again be due to crack growth before limit load is reached. Other sources for this deviation could be that the specimen is not fully plane strain across the entire thickness, 3-D effects, etc.

4.3.2 Sectioning Technique Utilized to View 2-D Crack Profiles at Centerline of Specimen

One forensic technique used to acquire data from a fracture experiment is to section the specimen down its length to obtain a 2-D view of the crack profile. As noted in the preceding section, specimens were loaded in back-bending and open-bending to max load, 10% load drop, 20% load drop, 40% load drop, and 60% load drop. These specimens were sectioned according to Figure 4-6. This sectioning technique was used to view the 2-D crack profile at the centerline of these specimens loaded in both bending modes, with companion specimens deformed to multiple crack lengths.

The sectioned pieces were metallographically polished. After polishing, pictures of these 2-D crack profiles were taken using a scanning electron microscope, SEM, at fairly low magnifications, 10X-50X. The 2-D crack profiles for the specimens tested in
open-bending are shown in Figure 4-7. Figure 4-7 shows crack profiles for specimens taken to different points in the loading curve, as indicated in the caption. The 2-D crack profiles for specimens loaded in back-bending are shown in Figure 4-8. Again, these pictures are for specimens taken to various imposed deformations.

These pictures were used to measure the opening displacement of the initial crack tip, $CT_oOD$, as a function of the crack growth, $\Delta a$. These quantities were measured as shown in Figure 4-9. The data measured from the pictures of the open-bend and back-bend 2-D crack profiles is plotted in Figure 4-10. The data points labeled “open-bend” and “back-bend” are data measured from the 2-D center-plane crack profiles. These data points are also labeled with their respective load drop percentages. The data point for 60% load drop of an open-bend specimen is from a specimen machined with an incorrect crack orientation, as is shown later. To distinguish this data point from the rest, it is filled in solid. The plot shows that the lower constraint crack tip which exists in the back-bend mode displays higher ductility than the higher constraint crack tip which exists in the open-bend mode. This higher ductility is evident in the fact that the back-bend specimens deviate from the blunting line at a higher value of $CT_oOD$ than do the open-bend specimens. The blunting line denotes the alternating sliding which takes place before crack initiation, [16]. This alternating sliding determines the shape of the blunting crack given by the following equation:

$$\Delta d_{bl} = \frac{1}{2} CT_oOD$$

(4.2)

The deviation from the blunting line will be used to represent crack initiation. Therefore, the lower crack tip constraint prevalent in the back-bend specimen requires more crack tip blunting before crack initiation than the open-bend specimen, demonstrating its increased ductility. Also, the initial slope of the $CT_oOD$ versus $\Delta a$ curve for the low constraint back-bend specimens is higher than the initial slope for the higher constraint open-bend specimens. These two phenomena which display the increase in ductility due to lower constraint crack tip conditions have been studied by many including Hancock, Reuter, and Parks [13].
However, the apparent slope of the back-bend data, in Figure 4-10, tends to decrease with crack growth. This is due to the rigid rotation of crack faces toward one another in the back-bend loading. Since the crack tip opening displacement is being measured between the same points as the crack advances, the slope between successive back-bend data points gradually decreases, due to the rigid rotations. The opposite should be true for the open-bend specimens. The $CT_oOD$ vs. $\Delta a$ curve for the open-bend specimens should show an increasing slope between the data points due to the rigid rotations rotating the crack faces away from one another. However, the data begins with a small initial slope which slightly increases (as is expected) until it reaches a value of $\Delta a \approx 2\,mm$. At this point, the data dramatically increases in slope to a slope that is practically infinite. Examining the crack profiles in Figure 4-7, it is apparent, especially in Figures 4-7(b) and (d), that the crack reaches a certain length then begins to blunt and even turn the crack almost normal to the original crack plane. This phenomenon could be due to the flattened manganese sulfide inclusions with LT planes, discussed in Chapter 3.1, believed to exist in this batch of A572 Gr.50 steel.

### 4.3.3 Fracture Surface Photographs

When the specimens were sectioned according to Figure 4-6, one half was polished to the centerline to view the 2-D crack profile. The other half that remained from the sectioning was submerged in liquid nitrogen then bent in an opening mode at an impact loading rate. This method was employed to mark the crack front at its current position by terminal cleavage fracture. Pictures of crack front from overhead for an open-bend and back-bend specimen, loaded to 60% load drop, are shown in Figure 4-11.

Figure 4-11(a) shows an overhead view of one-half of an open-bend specimen fracture surface in which the specimen was loaded to 60% load drop. From bottom to top, the figure shows the Chevron notch, the pre-cracked zone, the ductile tearing zone created in open-bending, and finally the zone which failed in cleavage due to the liquid nitrogen temperatures. The feature denoted by A in Figure 4-11(a) is a local
edge effect common to open-bend specimens.

Figure 4-11(b) shows an overhead view of one-half of a back-bend specimen fracture surface in which the specimen was loaded to 60% load drop. From the bottom to top, the figure shows the Chevron notch, the pre-cracked zone, the ductile tearing zone created in back-bending, and finally the zone which failed in cleavage due to the liquid nitrogen temperatures. The feature denoted by B in Figure 4-11(b) is a local edge effect due to the plasticity effects. Plasticity requires the volume of the ligament to remain constant. Since the ligament is being strained or stretched in tension, its cross sectional area will decrease, thereby causing the edges to be drawn inwardly. The above phenomenon is also responsible for the edge being drawn inward on the back edge which is denoted by C. Also, notice the curvature on the back edge denoted by C. This is due to the fact that the crack advance varies along the width of the specimen, thereby creating different amounts of strain on the back side. Therefore, the curvature on the back face could possibly be used to help indicate the crack advance along the width of the specimen.

Figure 4-12 shows some magnified pictures of an open-bend fracture surface, in which the specimen was loaded to 60% load drop, taken by a scanning electron microscope, SEM. These pictures provide evidence of the micro-mechanisms of fracture which took place in each stage of the loading. Figure 4-12(a) shows the transition from the pre-cracking zone on the bottom to the ductile tearing zone at the top. The dark vertical lines are delaminations in the material due to the flattened manganese sulfide inclusions in the A572 Gr.50 purchased. Since the delaminations are oriented vertically, it is assumed that this specimen was incorrectly machined with a TL crack orientation and not the specified TS crack orientation. This is why Figure 4-12 shows +L as the direction of crack growth as compared to +S. Figure 4-12(b) shows the transition from the ductile tearing zone at the bottom to the cleaved zone at the top. Figure 4-13 similarly shows some 200X SEM pictures of an open-bend specimen taken to 40% load drop. This specimen was correctly machined with a TS crack orientation evident by the horizontal delaminations. As in the previous figure, part (a) shows the transition from the pre-cracking zone on the bottom to the ductile tearing zone.
at the top, and part (b) shows the transition from the ductile tearing zone at the bottom to the cleaved zone at the top.

Similarly for the back-bend loading, Figure 4-14(a) and Figure 4-15(a) show the transition from the pre-cracking zone on the bottom to the ductile tearing zone at the top for specimens loaded to 60% and 40% load drop, respectively. Whereas, Figure 4-14(b) and Figure 4-15(b) show the transition from the ductile tearing zone at the bottom to the cleaved zone at the top for specimens loaded to 60% and 40% load drop, respectively.

Notice that delaminations are more evident in the pictures of the open-bend fracture surfaces, Figure 4-12, as compared to the pictures of the back-bend fracture surfaces, Figure 4-14. This is due to the high state of stress triaxiality in the open-bend specimen as compared to the back-bend specimen. The higher state of stress triaxiality leads to more delaminations of the manganese sulfide inclusions in the open-bend specimens. Refering to Figure 4-7(e), delaminations are not apparent in the vertical direction, as compared to Figures 4-7(b), (c), and (d). This further confirms our assumption that the open-bend specimen loaded to 60% load drop was incorrectly machined with a TL crack orientation, instead of a TS crack orientation.
Figure 4-1: Machine drawing of specimens used in slow-bend testing. All dimensions are in inches.

Figure 4-2: Four-point bend setup used in slow-bend testing with shim material included.
Figure 4-3: Test equipment used in slow-bend testing. (a) Instron Model 8501. (b) Magnified view of four-point bend setup including specimen with shim.
Figure 4-4: Load versus displacement curves for open-bend specimens. Indicated percentages refer to fraction of post-peak load drop.
Figure 4-5: Load versus displacement curves for back-bend specimens. Indicated percentages refer to fraction of post-peak load drop.
Dashed Lines Indicate Saw Cuts

Figure 4-6: Sectioning technique utilized to view 2-D crack profiles at centerline of specimens.
Figure 4-7: 2-D crack profiles of companion open-bend specimens. (a) Max load. (b) 10% load drop. (c) 20% load drop. (d) 40% load drop. (e) 60% load drop.
Figure 4-8: 2-D crack profiles of companion back-bend specimens. (a) Max load. (b) 10% load drop. (c) 20% load drop. (d) 40% load drop. (e) 60% load drop.
Figure 4-9: Schematic of data measured from 2-D crack profiles.
Figure 4-10: Initial crack tip opening displacement, \( C_{T_{OD}} \), versus \( \Delta a \) at the centerline of open-bend and back-bend specimens.
Figure 4-11: Overhead view of fracture surfaces. (a) Crack front of open-bend specimen at 60% load drop. (b) Crack front of back-bend specimen at 60% load drop.
Figure 4-12: Magnified view of open-bend fracture surface transitions for specimen loaded to 60% load drop. Direction of cracking is in +L direction. (a) Transition from pre-cracking zone to ductile tearing zone. (b) Transition from ductile tearing zone to cleavage zone.
Figure 4-13: Magnified view of open-bend fracture surface transitions for specimen loaded to 40% load drop. Direction of cracking is in +S direction. (a) Transition from pre-cracking zone to ductile tearing zone. (b) Transition from ductile tearing zone to cleavage zone.
Figure 4-14: Magnified view of back-bend fracture surface transitions for specimen loaded to 60% load drop. Direction of cracking is in +S direction. (a) Transition from pre-cracking zone to ductile tearing zone. (b) Transition from ductile tearing zone to cleavage zone.
Figure 4-15: Magnified view of back-bend fracture surface transitions for specimen loaded to 40% load drop. Direction of cracking is in +S direction. (a) Transition from pre-cracking zone to ductile tearing zone. (b) Transition from ductile tearing zone to cleavage zone.
Chapter 5

3-D Topography Maps of Fracture Surfaces

5.1 Scanning Techniques

Height measurements as a function of position on the fracture surface can be useful in analyzing crack growth. Many scanning techniques have been developed to provide these height measurements, including contact profilometers, confocal laser scanning microscopes, and laser profilometers. A CyberOptics Model DRS-2000 laser profilometer, set up at the Idaho National Engineering Laboratories under W. R. Lloyd, was used to acquire the height measurements on the fracture surfaces of an open-bend and back-bend specimen.

The specimens were machined, according to Figure 4-1, and tested in four-point bending, as reported in Chapter 4, with a shim in the EDM notch for the back-bend tests. One specimen was fractured completely in an open-bend mode and another in a back-bend mode. Both were tested at room temperature with a loading rate of 0.5 mm/min. Both halves of each specimen were scanned using the laser profilometer. The data ($\Delta x, \Delta y = 0.25mm$) was used to plot 3-D topographical maps of the open-bend and back-bend specimen fracture surfaces, Figures 5-1 and 5-2. The z-axis on all four topographical plots was exaggerated by a factor of six to help show the height differences between the different fracture surfaces. Notice that the overall height
difference, between the lowest point on the fracture surface and the highest point, is larger for the back-bend specimen as compared to the open-bend specimen. This was expected qualitatively due to the increased ductility in the back-bend specimen, which is a result of the lower normal stress across the slip lines. The increased ductility in the back-bend specimen leads to a larger $CT_{OD}$ before crack initiation, as compared to the open-bend specimen. Similarly, a larger $CTOA$ is expected for the back-bend specimen, as compared to the open-bend specimen, during crack growth due to its increased ductility. The value for the crack tip opening angle, $CTOA$, during crack growth is one of the subjects discussed in the next section.

5.2 Technique Used to Calculate Local and Global Crack Growth Data

5.2.1 Local Crack Tip Opening Angle and Direction as a Function of Crack Growth

The contact between the backside crack faces in the back-bend specimen prevented us from utilizing many data gathering techniques during the test, such as a crack mouth opening gauge. However, the contact between the backside crack faces is the basis for a useful technique for analyzing data gathered from the scans on the fracture surface. The data to be analyzed is a line scan taken down the middle of the back-bend fracture surface. Two line scans taken from opposing back-bend fracture surfaces are plotted in Figure 5-3. The increment ($\Delta y$) between the data points is 0.05 mm. They are connected, with the shim inserted, as they would have been the instant before complete fracture.

The technique currently being discussed utilizes the fact that, to first order, the contact length between the backside crack faces equals the length of the remaining ligament, which is under plane strain tension. This is required by equilibrium. A reference line can be drawn from the last point of contact, or pivot point, between the backside crack faces to the current crack tip as shown in Figure 5-4. At this point,
the angles, $\Theta_{p,t}$ and $\Theta_{p,b}$, from this reference line to each of the local crack faces can be measured. The crack tip opening angle, $CTOA$, can then be calculated as follows:

$$CTOA = \Theta_{p,t} + \Theta_{p,b}.$$  \hfill (5.1)

This technique works not only for symmetrical crack growth, Figure 5-4, but also for asymmetrical crack growth, Figure 5-5, where the crack deviates from the original crack plane. The crack dimensions in Figure 5-5 are exaggerated to allow more clear labeling of the angles to be calculated. However for asymmetrical crack growth, note that one of the angles, $\Theta_{p,t}$ or $\Theta_{p,b}$, can be negative. The angle of the direction of the crack from the original crack plane, $\Theta_c$, can also be calculated from the known quantities by either of the following equations:

$$\Theta_c = \Theta_{p,b} - \frac{CTOA}{2}$$
$$\Theta_c = -\Theta_{p,t} + \frac{CTOA}{2}.$$  \hfill (5.2)

The deformed lengths of the ligament and the backside crack faces can be unequal due to differences, [26], in the two deformation fields, even though the initial contact length between the backside crack faces was equal to the initial length of the ligament. Although the deformed lengths of the backside crack faces and the ligament differ, the above simple analysis can still be used to obtain accurate measurements of $CTOA$ and $\Theta_c$. This is because the change in angle of the reference line in Figure 5-4 due to a different pivot point, as a result of a unequal deformed lengths, is negligible when compared to the local cracking angles. Even if the deformed length of the backside crack faces was as much as $\pm50\%$ of the deformed length of the ligament, it would only result in errors of less than 5% in the calculated angles, $\Theta_{p,t}$ and $\Theta_{p,b}$.

The line scan data, measured using the laser profilometer, was analyzed using a simple MATLAB script that is included in Appendix B. The resulting data was extremely noisy since the angles were calculated utilizing the next data point. The "noise" in the data is not due to the usual electronic noise present in output signals.
Rather, this "noise" is due to the roughness of the opposing fracture surfaces. Although many averaging algorithms exist, which will smooth the data gathered from this technique, only the next data point was used, and the noise was filtered through the use of the discrete Fourier transform. Therefore, the discrete Fourier transform of the data was taken. The information from the first three fundamental frequencies was saved, and the high frequency information was removed. Next, the inverse transform was taken to reconstruct the original data minus the high frequency noise. The information contained in the first three fundamental frequencies demonstrates the general trends of the data. Utilizing information from higher frequencies results in a noisy plot, and therefore was not included. The noisy information is due to the roughness of the fracture surfaces which is in part due to the impurities in the material. All of this numerical procedure is included in the MATLAB script located in Appendix B. In addition, the original "noisy" raw data is included in Appendix D. The resulting data was used to construct Figure 5-6 and Figure 5-7, where $\Delta a$ is the crack growth in the initial crack direction. Figure 5-6 shows that the local CTOA varies from $15^\circ$ to $32^\circ$, with a mean value of approximately $20^\circ$. These values for the CTOA for the back-bend specimen should be larger than the associated CTOA values for the open-bend specimen due to the higher ductility inherent in the lower constraint back-bend loading. The larger CTOA expected in the back-bend specimen, as compared to the open-bend specimen, can be seen qualitatively in Figure 5-1 and Figure 5-2. The larger CTOA in the back-bend specimen leads to a larger overall height difference in the fracture surface of the back-bend specimens, as compared to the open-bend specimens. The expected smaller crack tip opening angles for the open-bend specimens can also be seen in Figures 4-7(a) and (e). The angles in the pictures in Figure 4-7(b), (c), and (d) are affected by the delaminations in the LT planes, which are often a result of manganese sulfide impurities in the A572 Gr.50 steel. A technique to regenerate crack profiles from the fracture surfaces of opposing open-bend specimen halves is being developed, and is discussed further in Chapter 6.2.

Figure 5-7 shows that the crack direction varies from $-9^\circ$ to $11^\circ$, with a mean value of approximately $-1^\circ$. This shows that although the crack growth may become
asymmetrical, the average crack direction is in the direction of the initial crack growth.

5.2.2 End-to-End Bend Angle, \( \psi \), and \( CT_{0}OD \) as a Function of Crack Growth

If the differences in deformation between the backside crack faces and the ligament are not neglected, other quantities can be calculated with precision. These quantities include the end-to-end bend angle, \( \psi \), and the opening displacement of the initial crack-tip, \( CT_{0}OD \), which was discussed previously in Chapter 4. As seen in Figure 5-8 and Figure 5-9, once the crack profiles are rotated towards one another, to recreate the crack profiles at various stages, the two halves interfere with one another over the distance, \( l_{i} \). To recreate the actual ligament length, \( l_{a} \), these interfering points are slid back towards one another along the 45° slip lines, along which they originally deformed. Therefore, this actual ligament length, \( l_{a} \), is the actual contact length between the backside crack faces, due to equilibrium. In other words, when the crack tip was at the position to give interfering length, \( l_{i} \), the ligament was actually \( l_{a} > l_{i} \).

Utilizing a rigid line of data, along the middle portion of the line scan, that is unaffected by the deformations on each end of the specimen, the end-to-end bend angle of the specimen can be monitored as a function of the crack growth. The difference in angle between the rigid line of data and the reference line as the crack grows, discussed previously, is the rigid rotation or end-to-end rotation of the specimen. This technique was applied to the line scan data for the back-bend specimen, Figure 5-3, and resulted in the curve shown in Figure 5-10. The curve in Figure 5-10 was created using a second order polynomial fit on the data resulting from the analysis.

In addition to the end-to-end bend angle, the value for \( CT_{0}OD \) can be calculated as a function of crack growth and compared to the data for the 2-D crack profiles shown in Chapter 4. Summing the distance from the original crack tip to the reference line for both opposing fracture surfaces gives the value of \( CT_{0}OD \) for various crack lengths. This technique was applied to the line scan data for the back-bend specimen, Figure 5-3, and is plotted along with the data from the 2-D crack profiles of the
back-bend specimens in Figure 5-11. The data points are the same as those from Figure 4-10, whereas the continuous curve of data was generated using the present technique. As before, the curve in Figure 5-11 was generated using a second order polynomial fit on the data resulting from the analysis. The MATLAB script used to apply this technique is included in Appendix C.

Load versus displacement data can also be calculated from the above total rotation versus crack growth data. The internal moment in the specimen can be calculated as follows when the specimen has reached full plasticity:

\[ M = \frac{2}{\sqrt{3}} \sigma_{TS} B (l_o - \Delta a)[w - (l_o - \Delta a)] = \frac{P}{2}s. \]  

(5.3)

\( M \) is the moment, \( \sigma_{TS} \) is the tensile strength, \( B \) is the thickness, \( l_o \) is the original ligament length, and \( w \) is the width of the specimen. Solving for the load \( P \):

\[ P = 2 \frac{2}{\sqrt{3}} \sigma_{TS} B \frac{(l_o - \Delta a)[w - (l_o - \Delta a)]}{s}. \]  

(5.4)

The total end-to-end rotation can be used to calculate the displacement of the rollers through the following equation:

\[ Displacement = \tan(\frac{\psi}{2})s \]  

(5.5)

Figure 5-12 displays the regenerated load versus displacement curve along with the actual load versus displacement curve collected during the test. The regenerated curve only contains the last portion of the original curve because the above technique only works for fully-plastic crack growth. Whereas, the original curve contained load versus displacement data before the specimen reached full plasticity, and the original curve also contains data during crack tip blunting which occurs just prior to crack growth. The relative agreement of the two curves in Figure 5-12 proves that the technique discussed in this chapter generates fairly accurate results.
Figure 5-1: Photographs and 3-D topographical maps of both halves of an open-bend fracture specimen. Z-axis exaggerated by a factor of six.
Figure 5-2: Photographs and 3-D topographical maps of both halves of a back-bend fracture specimen. Z-axis exaggerated by a factor of six.
Figure 5-3: 2-D line scans from the centerlines of mating back-bend fracture surfaces.
Figure 5-4: Geometry for calculating CTOA as a function of crack growth.
Figure 5-5: Angles calculated for asymmetrical crack growth. Crack dimensions are exaggerated to better show angles to be calculated.
Figure 5-6: Crack tip opening angle, $CTOA$, as a function of crack growth, $\Delta a$. 

A572 Gr.50

$a_0 = 19\text{mm}$

$w = 25.4\text{mm}$

$B = 25.4\text{mm}$

Mean Value

$= 20.4^\circ$
Figure 5-7: Orientation of the crack with respect to the original crack plane, $\Theta_c$, as a function of crack growth, $\Delta a$. 
Figure 5-8: Schematic of the recreation of 2-D crack profiles for various crack lengths utilizing line scan data from back-bend specimen.
Figure 5-9: Schematic of the interference of opposing fracture surfaces when recreating 2-D crack profiles.
Figure 5-10: Total end-to-end angle of rotation, $\psi$, of the back-bend specimen as a function of crack growth, $\Delta a$. 

\[ \psi/2 \]
Figure 5-11: Initial crack tip opening displacement, $CT_{o}OD$, versus $\Delta a$ at the centerline of back-bend specimens.
Figure 5-12: Load versus displacement curves for a back-bend specimen.

Crosshead Displacement = 0.5 mm/min

A572 Gr.50

\[ a_0 = 19\text{mm} \]
\[ w = 25.4\text{ mm} \]
\[ B = 25.4\text{ mm} \]
Chapter 6

Conclusions and Future Work

6.1 Conclusions

Low constraint plane strain crack tip conditions exist in many engineering structures. The back-bend specimen provides an excellent test method to study fully-plastic conditions on specimens of large ligament sizes without requiring large load capacities. The initial testing of this specimen was done along with a traditional open-bend specimen for comparison.

It was shown that two specimens of identical geometries had two entirely different crack tip stress fields when loaded in the two different bending modes. One of the most significant differences in the crack tip stress fields of the open-bend and back-bend specimens was the difference in crack tip stress triaxialities. The normal stress across the slip lines, normalized by the plane strain yield strength for the open-bend specimen, was shown to be 1.543, compared to 0.5 for the back-bend specimen. The smaller normal stress across the slip lines gives a lower state of constraint for the back-bend specimen.

This lower state of constraint for the back-bend specimen was shown to increase its ductility as compared to the open-bend specimen. This increase in ductility was shown in three ways. First, the back-bend specimen had a lower brittle to ductile transition temperature as compared to the open-bend specimen. Second, the crack tip of the back-bend specimen exhibited more blunting before crack initiation as
compared to the open-bend specimen. This was shown by the larger values for the crack-tip opening displacement before crack initiation in the back-bend specimen as compared to the open-bend specimen. Finally, the crack-tip opening angle during crack growth for the back-bend specimen was shown to be quite significant. The mean value of 20° for the CTOA of the back-bend specimen was shown qualitatively to be greater than the CTOA, during crack growth, for the open-bend specimen. A method for determining the CTOA during crack growth for the open-bend specimen needs to be developed as is discussed in the next section on Future Work.

6.2 Future Work

The initial testing of the back-bend specimen has been detailed in this thesis. It is evident that the back-bend specimen can become an extremely useful tool in studying crack growth under low constraint plane strain tensile conditions. In addition to the current work, further development of the back-bend specimen should concentrate on the following areas: material selection, temperature and loading rate controlled tests on large specimens, fracture surface regeneration techniques, and finite element modeling.

The material selected for the current testing was a commercially-obtained A572 Gr.50 steel. It was selected because it is a medium strength structural steel. Although the material selected was fine, the batch of steel plates purchased is suspected to contain manganese sulfide impurities which created many adverse effects during the testing. Therefore, it is suggested that the material to be used for future back-bend testing be a more pedigreed material that has been properly treated for material impurities.

In addition to the temperature-controlled impact testing done on Charpy-size specimens, it is suggested that future testing should include temperature and rate controlled tests on large size specimens, Figure 4-1. New Instron testing machines with environmentally controlled chambers have become available to conduct this type of testing. Performing these types of tests would give insight not only to the temper-
ature transition behavior but also the load rate behavior and specimen size effect.

The technique used in Chapter 5 to regenerate 2-D crack profiles for the back-bend specimen was enabled by the equilibrium constraint on contact between the backside crack faces. Regenerating two-dimensional crack profiles for fracture specimens loaded in direct tension and open-bending would also be desirable. Techniques for regenerating these types of crack histories are currently being developed by many engineers and scientists including [27] and [28].

Finally, finite element modeling of the back-bend specimen would provide further insight into the mechanics of this specimen. In addition, three-dimensional models would further help to understand the size effects inherent in fracture specimens due to surface phenomenon. Also, finite element models can help to clarify the issues of small-scale yielding during build-up of contact to fully-plastic load levels.

It has been discussed that tests on high constraint specimens lead to conservative results. Whereas, tests on specimens of similar constraint would lead to more accurate predictions in behavior. Furthermore, the back-bend specimen allows tests on larger ligament sizes without requiring larger load capacities, due to its mechanical advantage. Tests can now be performed on larger ligament sizes, that approach the size of ligaments in real-life structures. Finally, $CT_0D$ and $CTOA$ data help us to more accurately describe crack initiation and crack growth, respectively, in structures. All of the above advantages combined with the current work and further development, on the back-bend specimen, would provide yet another tool for engineers to better characterize fracture in real-life structures.
Appendix A

Standard Material Test Results and Material Microstructure

The material used in the testing described in this thesis was A572 Gr.50 steel. Some material properties listed on the material data sheet for this batch of steel are shown in Table 3.1. To better characterize and understand this material a few standard material tests were performed on this steel to give some well known material properties. Also, the microstructure of this batch of A572 Gr.50 is displayed.

A.1 ASTM Standard Tension Testing

First, a couple of standard tension test samples with a nominal diameter of 6.35mm were machined from the plate of A572 Gr.50. The gage length, 25.4mm, and other specimen requirements are set forth by the ASTM Standard for tension testing [29]. These specimens were tested using a hydraulic Instron testing machine, model 8501. The data collected from the test was used to create an engineering stress-strain curve for this material, Figure A-1. This curve shows a yield strength, $\sigma_{YS}$, of around 350 MPa. This is extremely close to the yield strength of 353 MPa stated in the material data sheet for this batch of A572 Gr.50. Also, Figure A-1 shows an ultimate tensile strength, $\sigma_{TS}$, of approximately 580 MPa. This is a little high as compared to the ultimate tensile strength of 558 MPa as stated in the material data sheet, but the
difference is only 4%, which is still reasonable. In addition, the change in the nominal
diameter at large strains, due to plastic incompressibility, of the tension specimens
was measured and was used to help generate a true stress-strain curve, Figure A-2,
for this batch of A572 Gr.50 steel.

A.2 ASTM Standard Compression Testing

Compression samples with a nominal diameter of 8.89mm and a height of 12.7mm
were machined from one of the plates of A572 Gr.50 steel. These specimens were
tested in compression using the same hydraulic Instron testing machine, model 8501,
used in the tension testing. The tests were conducted according to the ASTM stan-
dard on compression testing of metallic materials, [30]. The data collected from the
tests was used to create a true stress-strain curve, Figure A-3, for this batch of A572
Gr.50 in compression. Comparing the true stress-strain curve in tension, Figure A-2,
to the true stress-strain curve for compression, Figure A-3, shows close correlation up
to a strain of approximately 0.16, the limit of the true stress-strain curve in tension.

A popular method of characterizing the strain hardening of a material is to use a
power law expression,

\[ \sigma = K \epsilon^n, \]  

(A.1)

where \( K \) is a constant and \( n \) is the power law exponent for the material. Taking the
log of both side of Eq. A.1 gives

\[ \log(\sigma) = \log(K) + n \log(\epsilon). \]  

(A.2)

Therefore, plotting the stress-strain curve using a log-log plot will give us the val-
dues of \( n \) and \( K \). A log-log plot of the compressive stress-strain curve is shown in
Figure A-4. After the elastic portion of the curve, the material seems to follow a
two stage hardening which is shown by the two different slopes on the log-log plot,
which exist after the elastic portion. The first stage which goes from the yield stress,
approximately 350 MPa, to around 600 MPa exhibits a power law exponent of 0.35.
The second stage which ranges from around 600 MPa to the end of the curve exhibits a power law exponent of 0.16.

**A.3 ASTM Standard Charpy V-Notch Testing**

Another standard test that is performed to better understand the behavior of a material is the Charpy impact test. Charpy V-Notch specimens were machined from one of the plates of A572 Gr.50 steel with a TS crack orientation. These specimens were of the standard dimensions detailed in the ASTM standard for Charpy V-Notch impact specimens, [22]. These tests were performed using a Sun Electronics Systems model EC1x temperature chamber and a Tinius Olsen impact testing machine, Figure 3-3. The results for these tests are shown in Figure A-5. The transition from brittle lower shelf behavior to ductile upper shelf behavior ranges from $-20^\circ C$ to $20^\circ C$, with a mean value of $0^\circ C$.

**A.4 Material Microstructure**

The microstructure of the batch of A572 Gr.50 steel that was purchased was obtained through the following procedure. First small 12mm cubed pieces of the steel were removed from one of the plates. These three pieces were removed from three different orientations to show the microstructure on three mutually perpendicular planes. Next, these pieces were individually mounted in phenolic by combining them with phenolic powder, heating the mixture, and pressing them together. Next, the mounted pieces of A572 Gr.50 were metallographically polished starting with a low grit paper and ending with a 0.05 micron particle solution. Finally, the polished specimens were etched to show the steel’s microstructure. An etching solution of 2% nital was suggested by the Metals Handbook, [31].

The coordinate system of a rolled steel plate, depicted in Figure 3-1, is used to help describe the orientations of these three pieces of A572 Gr.50. A picture of the microstructure of the rolling plane, perpendicular to the S direction, is shown in
Figure A-6(a). A picture of the microstructure of the plane perpendicular to the rolling direction, L, is displayed in Figure A-6(b). Finally, the microstructure of the plane perpendicular to T, long transverse direction, is pictured in Figure A-6(c).

The pictures show no observable differences in the microstructures of these three mutually perpendicular planes. The pictures, in Figure A-6, resemble a microstructure of bainite. This means that the material was most likely rolled then immediately quenched at a fast cooling rate, thereby resulting in a microstructure of bainite. If this batch of steel had been hot-rolled and cooled slowly, then evidence of Lueder’s strain would have existed at the yield point in the stress-strain curves. Also if the A572 Gr.50 plates had been cold-rolled, the grains in the rolling plane would be stretched or aligned in the rolling direction.
Figure A-1: Engineering stress-strain curve for ASTM standard tension test on A572 Gr.50 steel.
Figure A-2: True stress-strain curve for ASTM standard tension test on A572 Gr.50 steel.
Figure A-3: True stress-strain curve for ASTM standard compression tests on A572 Gr.50 steel.
Figure A-4: Log-log plot of true stress-strain curve for ASTM standard compression tests on A572 Gr.50 steel.
Figure A-5: Charpy V-Notch transition temperature curve for A572 Gr.50 steel.
Figure A-6: Pictures of material microstructure. (a) Plane normal to S, short transverse direction. (b) Plane normal to L, rolling direction. (c) Plane normal to T, long transverse direction.
Appendix B

MATLAB Script for Calculating $CTOA$ and $\Theta_c$

% Simple MATLAB Code for Calculating $CTOA$ and $\Theta_c$
% for the Back-Bend Specimen.

% The next two lines change to directory where data is stored
% and loads the data.
cd d:/users/kbass/scans
load l201.txt

% x1 and y1 are the variables where the 2-D line scan data is stored.
x1=l201(:,2);
y1=l201(:,3);

% Height data was given in $\mu m$, so it is converted to $mm$
% for consistent units.
y1=y1*0.001;

% n = number of data points. step [mm] = increment between data points.
n=length(y1);
step=0.05;
Variable, len, is the number of points where the calculations are done.

Therefore, (len-1) times step equals the total crack growth from
initiation to complete fracture surface separation.

len=83;

Variable for \( \Theta_{p,t} \).

\[
\text{theta1} = \text{zeros(len,1)};
\]

Variable for \( \Delta a \).

\[
\text{deltaa} = \text{zeros(len,1)};
\]

This loop fills the vector for \( \Delta a \).

\[
\text{for } j=1:\text{len} \\
\text{deltaa}(j)=j*\text{step};
\]

This loop calculates \( \Theta_{p,t} \) in degrees at each point.

The numerical factor of 0.7874mm = half the thickness of the shim.

\[
\text{for } i=1:\text{len} \\
k=i-1; \\
\text{baseang}=\text{atan}((y1(n-k)-(y1(i)+0.7874))/(x1(n-k)-x1(i))); \\
\text{localang}=\text{atan}((y1(n-k)-y1(n-i))/(x1(n-k)-x1(n-i))); \\
\text{theta1}(len-k)=(\text{localang}-\text{baseang})*(180/\pi);
\]

The code is repeated here for the mating line scan, with variables
XXX1 being replaced with XXX2.

load 1202.txt

\[
x2=1202(:,2);
\]

\[
y2=1202(:,3);
\]

\[
y2=y2*0.001;
\]
n=length(y2);
step=0.05;
theta2=zeros(len,1);
for i=1:len
    k=i-1;
    baseang=atan((y2(n-k)-(y2(i)+0.7874))/(x2(n-k)-x2(i)));
    localang=atan((y2(n-k)-y2(n-i))/(x2(n-k)-x2(n-i)));
    theta2(len-k)=(localang-baseang)*(180/pi);
end

% Variable for CTOA
ctoa=theta1+theta2;
% Variable for Θc
thetac=theta1-(ctoa*0.5);

% The next two lines take the discrete Fourier transform of CTOA
% and Θc and places the discrete Fourier Transform data
% in the vectors y1 and y2, respectively.
y1=fft(ctoa,len);
y2=fft(thetac,len);

% The below loop keeps the data from the first three fundamental
% frequencies and sets the rest to zero, thereby eliminating the
% high frequency noise.
s=len-3;
for i=1:s
    y1(i+3)=0;
    y2(i+3)=0;
end
% The next two lines takes the inverse Fourier transform to
% reconstruct the data curves.
x1=ifft(y1,len);
x2=ifft(y2,len);

% The next four lines set the remaining frequency information to zero
% except for the zeroth frequency which gives the mean value.
y1(2)=0;
y1(3)=0;
y2(2)=0;
y2(3)=0;

% The inverse Fourier transform is taken on the zeroth frequency data
% to give the mean or average values.
avg1=ifft(y1,len);
avg2=ifft(y2,len);

% The remaining lines are used to generate plots of the data.
plot(deltaa,x1,deltaa,avg1)
xlabel('Delta a')
ylabel('CTOA')
axis([0 4.5 0 50])
figure
plot(deltaa,x2,deltaa,avg2)
xlabel('Delta a')
ylabel('Theta c')
axis([0 4.5 -15 15])
Appendix C

MATLAB Script for Calculating $\psi$ and $CT_0OD$

% Simple MATLAB Code for Calculating $\psi$ and $CT_0OD$
% for the Back-Bend Specimen.

% The next two lines change to directory where data is stored
% and loads the data.
cd d:/users/kbass/scans
load l201.txt

% x1 and y1 are the variables where the 2-D line scan data is stored.
x1=l201(:,2); y1=l201(:,3);
% Height data was given in $\mu$m, so it is converted to $mm$
% for consistent units.
y1=y1*0.001;
% n = number of data points. step [mm] = increment between data points.
n=length(y1);
step=0.05;

% Variable, len, is the number of points where the calculations are done.

105
Therefore, (len-1) times step equals the total crack growth from
initiation to complete fracture surface separation.

len=83;

p and q are the end points of a rigid line of data that are not
affected by deformations. This rigid line is used to help calculate
the total end to end rotation, $\psi$, of the specimen.

p=191;
q=211;

Variable used to store angles of rigid line discussed above.

rigang1=zeros(len,1);

Variable for $\Delta a$.

deltaa=zeros(len,1);

Variable for $CT_{OD}$ measurements from one-half of specimen.

ctod1=zeros(len,1);

This loop fills the vector for $\Delta a$.

for j=1:len,
    deltaa(j)=j*step;
end

The next 6 lines calculate initial value of rigang1 in radians % and ctod1 in mm.

The numerical factor of 0.7874mm = half the thickness of the shim.

refang=atan((y1(n)-(y1(1)+0.7874))/(x1(n)-x1(1)));
base=atan((y1(p)-y1(q))/(x1(p)-x1(q)));
rigang1(1)=refang-base;
ctang=atan((y1(n-len)-(y1(1)+0.7874))/(x1(n-len)-x1(1)));
ctodang=refang-ctang;
ctod1(len)=sin(ctodang)*(x1(n-len)-x1(1));
% This loop calculates rigang1 and ctod1 at each point.
for i=1:(len-1)
    actlig=(x1(n)-x1(n-i))+(tan(pi/4)*(y1(n)-y1(n-i)));
    ligsteps=round(actlig/step);
    refang=atan((y1(n-i)-(y1(ligsteps)+0.7874))/(x1(n-i)-x1(ligsteps)));
    rigang1(i+1)=refang-base;
    ctang=atan((y1(n-len)-(y1(ligsteps)+0.7874))/(x1(n-len)-x1(ligsteps)));
    ctodang=refang-ctang;
    ctod1(len-i)=tan(ctodang)*(x1(n-len)-x1(ligsteps));
end

% The next two lines adjust raw data for rigang1 to give it a
% starting value of zero.
ml=min(rigang1);
rigang1=rigang1-ml;

% The code is repeated here for the mating line scan, with variables
% XXX1 being replaced with XXX2.
load l202.txt
x2=l202(:,2);
y2=l202(:,3);
y2=-y2*0.001;
n=length(y2);
step=0.05;
len=83;
rigang2=zeros(len,1);
ctod2=zeros(len,1);
refang=atan((y2(n)-(y2(1)+0.7874))/(x2(n)-x2(1)));
base=atan((y2(p)-y2(q))/(x2(p)-x2(q)))
\[
\begin{align*}
\text{rigang2}(1) &= \text{refang-base}; \\
\text{ctang} &= \text{atan}((y2(n-len)-(y2(1)+0.7874))/(x2(n-len)-x2(1))); \\
\text{ctodang} &= \text{refang-ctang}; \\
\text{ctod2}(len) &= \sin(\text{ctodang})*(x2(n-len)-x2(1)); \quad \text{for } i=1:(len-1) \\
\text{actlig} &= (x2(n)-x2(n-i)) + (\tan(\pi/4)* (y2(n)-y2(n-i))); \\
\text{ligsteps} &= \text{round}(\text{actlig}/\text{step}); \\
\text{refang} &= \text{atan}((y2(n-i)-(y2(ligsteps)+0.7874))/(x2(n-i)-x2(ligsteps))); \\
\text{rigang2}(i+1) &= \text{refang-base}; \\
\text{ctang} &= \text{atan}((y2(n-len)-(y2(ligsteps)+0.7874))/(x2(n-len)-x2(ligsteps))); \\
\text{ctodang} &= \text{refang-ctang}; \\
\text{ctod2}(len-i) &= \tan(\text{ctodang})*(x2(n-len)-x2(ligsteps)); \\
\end{align*}
\]

end

\[m2 = \min(\text{rigang2});\]
\[\text{rigang2} = \text{rigang2-m2};\]

% The next 6 lines re-orders data in rigang1 and rigang2 to start
% from the beginning of the crack to the end and places the
% re-ordered data in rot1 and rot2.
rot1 = zeros(len, 1);
rot2 = zeros(len, 1);
for i = 1:(len)
rot1(i) = rigang1(len+1-i);
rot2(i) = rigang2(len+1-i);
end

% The next two lines sums the data from both halves
% and places it in totrot and ctood.
% The next line also converts totrot from radians to degrees.
totrot = (rot1+rot2)*180/pi;
ctood=ctod1+ctod2;

% The next four lines curve fit the data
% for a more aesthetic appearance.
p1=polyfit(deltaa,totrot,2);
f1=polyval(p1,deltaa);
p2=polyfit(deltaa,ctood,2);
f2=polyval(p2,deltaa);

% The remaining lines are used to generate plots of the data.
plot(deltaa,f1)
xlabel('Delta a')
ylabel('End to End Rotation')
figure
x11=[2.05 4.87 2.5 1.05 0.369];
y11=[1.36 1.84 1.46 0.799 0.725];
x12=[1.99 0.644 2.13 2.18 1.84];
y12=[0.803 0.361 1.67 1.08 0.384];
x13=[0 0.5];
y13=[0 1];
plot(deltaa,f2,x11,y11,'x',x12,y12,'o',x13,y13)
xlabel('Delta a')
ylabel('CTOD')
Appendix D

Raw Data of $CTOA$ and $\Theta_c$ versus $\Delta a$

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<th>$CTOA[°]$</th>
<th>$\Theta_c[°]$</th>
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Bibliography


