A Study of Defect Formation due to Flow Instability during Mold Filling in Lost Foam Casting

by

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Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of

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ABSTRACT

In the Lost Foam Casting (LFC) process, an expanded polystyrene foam pattern is embedded in dry, unbonded sand. The superheated liquid metal flows into the mold and replaces the foam pattern by melting and vaporizing it. Some of the polystyrene decomposition products, which cannot escape from the mold cavity, are entrapped within the casting, leading to defects in the casting. Foam entrapment is a major cause of defect formation in the LFC process.

A hypothesis based on the Rayleigh-Taylor instability is advanced to explain the foam entrapment mechanism in the LFC process. Rayleigh-Taylor instability occurs when two juxtaposed fluids of different densities are accelerated normal to their interface. The interface is either stable or unstable depending on whether the acceleration is directed from the heavier to the lighter fluid or vice versa. During the LFC process, the high-density molten metal flows against the low-density gaseous or liquid polymer. When the molten metal decelerates, the metal/gas front can become unstable. As this instability grows, fingering may occur and eventually the fingers may encounter each other, thus trapping the foam decomposition products between them.

In order to confirm the above hypothesis, an experimental set-up was designed and fabricated. Instead of using molten metal, water was used for ease of experimentation and to facilitate the recording of the flow on video. The set-up provided a deceleration directed from the water side to the air side across the interface and led to a growth of the instability of the interface.

The results from the water-air system were applied to the molten metal-polystyrene system by incorporating the properties of molten metal and polystyrene in the flow and instability growth models. This analysis shows that under certain conditions the instability of the molten metal front can grow. This growth can eventually lead to foam entrapment in castings. Therefore, by avoiding certain casting orientations and conditions, which can be done by a suitable gating system design and proper venting, defect-free castings can be obtained by the LFC process.

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Chapter 1

Introduction

1.1 Lost Foam Casting and its Defects

Lost Foam Casting (LFC), also known as Evaporative Pattern Casting (EPC), expanded polystyrene molding process, and evaporative foam process, was invented in 1958 [Shroyer, 1958]. In the LFC process, an expanded polystyrene foam pattern is embedded in dry, unbonded sand. The superheated liquid metal flows into the mold and replaces the foam pattern by melting and vaporizing it.

Patterns are produced by steam molding in metal molds by techniques developed to produce expanded polystyrene packaging and insulation products. The production of patterns commences with granules of polystyrene which are pre-expanded to form beads. After aging, these beads are steam-molded to produce an expanded polystyrene pattern. The pattern can be a one-piece pattern or, as in the case of highly complex shapes, it can be a multi-piece pattern where several simpler patterns are glued together to form the complex shape. After coating with a suitable refractory wash, the pattern is embedded in dry, unbonded sand and the sand is then vibrated to produce a rigid mold. On pouring, the molten metal displaces the polystyrene to produce the shape of the casting required. After shake-out, the casting requires minimal fettling because there is no parting line. The molding sand is entirely reclaimable, with cooling and classifying the only treatments required.

The process offers several advantages: capital equipment cost is reduced, conventional coremaking machines are eliminated, skilled labor requirement is reduced, no sand binders are required, the sand is reusable, shake-out is simplified, cores are eliminated, and reduced fettling and machining are required.

In spite of its many advantages, the LFC process is not very widely used because of the many casting defects. A significant amount of work has been done on the effects of refractory coatings [Goria et al., 1986; Sun et al., 1992; Tseng and Askeland, 1992],
gating system designs [Hill et al., 1997; Lawrence et al., 1998], temperature gradients [Bennett, 1998], metal velocity [Hill et al., 1998] and foam density [Liu et al., 1997] on defect formation in LFC. Mold filling in LFC has also been studied extensively [Liu et al., 1994; Sun et al., 1995; Fu et al., 1995]. However, the mechanism of defect formation in Lost Foam Casting is yet to be determined. Liquid or gaseous polystyrene entrapped in the casting during the mold filling process is the main source of the defects in LFC.

1.2 Rayleigh-Taylor Instability

When two juxtaposed fluids of different densities are accelerated in a direction perpendicular to their interface, the interface is either stable or unstable, depending on whether the acceleration is directed from the heavier to the lighter fluid or vice versa [Taylor, 1950]. The two possibilities are shown in Figure 1.1. The regimes of growth of the instability are: linear regime, non-linear or chaotic regime, and turbulent mixing regime.

In the linear regime, the amplitude of the instability grows exponentially with time. After the amplitude of the perturbation becomes greater than 0.4 times its wavelength, chaotic growth of the instability ensues [Lewis, 1950]. Finally, turbulent mixing of the two fluids takes place.

1.3 Fold Formation Hypothesis

During the LFC process, the high-density molten metal flows against the low-density gases (polystyrene decomposition products). If the metal decelerates [Wang et al., 1997] as it flows, there is a possibility of the metal front becoming unstable. During the flow if these instabilities grow it could be detrimental to the casting. As the instability of the metal front grows fingering might occur. These fingers might encounter each other, thereby trapping polystyrene foam decomposition products between them. This is referred to as fold formation. The proposed fold formation hypothesis is shown in Figure 1.2. The degree of these phenomena and the extent of casting defects depend on the casting parameters.
Figure 1.1 Schematic showing: (a) stable case, acceleration directed from heavier to lighter fluid, and (b) unstable case, acceleration directed from lighter to heavier fluid.

Figure 1.2 Fold formation model in Lost Foam Casting.
1.4 Work Objective

In spite of the many advantages that the LFC process offers, it is not very widely used in the casting industry because of the defects associated with it. Thus, if the defect formation mechanism can be understood and explained, suitable conditions for defect-free castings can be ensured. The objective of this work, therefore, is to identify and understand the defect formation mechanisms in LFC.

1.5 Research Approach

In order to elucidate the defect formation mechanisms in LFC, a study of the LFC process and defect formation in LFC is conducted. A study of Rayleigh-Taylor instability is also conducted and based on this, a fold formation mechanism is proposed. In order to visualize and understand the growth of the Rayleigh-Taylor instability, an experimental set-up is designed and fabricated. This set-up is chosen from a number of possible set-ups, because it satisfies the criteria for instability growth, is easy to fabricate and work with, and also facilitates the recording of the flow on video. The governing equations of flow for this set-up are formulated and validated by conducting experiments. Experiments are then conducted for a range of conditions to view the growth of the instability. All of the experiments are conducted using the water-air system. The results are then applied to the aluminum-polystyrene system, for aluminum LFC. Thus, the phenomena involved in fold formation is analyzed, visualized and discussed. Some recommendations for future investigation in the area are also suggested.

1.6 Thesis Outline

This chapter gives an introduction to the project, the Lost Foam Casting process and its defects. It also introduces the Rayleigh-Taylor Instability in fluids and the hypothesis for defect formation in LFC. Chapter 2 discusses the theoretical results for the instability growth. It also specifies conditions for the flow instability for various flow orientations. Chapter 3 discusses the requirements of the experimental set-up, the
possible options for the set-up and their analyses, the imaging system, and the fabrication of the set-up. Chapter 4 gives the experimental results and their analysis. Chapter 5 discusses the implications of the experimental results for aluminum Lost Foam Casting. Chapter 6 concludes the work by summarizing it and by making recommendations for future work.
Chapter 2

Theoretical Background

2.1 Introduction

When two juxtaposed fluids of different densities are accelerated in a direction perpendicular to their interface, the interface is either stable or unstable, depending on whether the acceleration is directed from the heavier to the lighter fluid or vice versa [Taylor, 1950]. This instability is referred to as Rayleigh-Taylor instability. A considerable amount of analysis, both theoretical [Bellman and Pennington, 1954; Chandrasekhar, 1955, 1961; Hide, 1955; Mitchner, 1964; Selig, 1964; Plesset and Hsieh, 1964; Jacobs and Catton, 1988] and that involving numerical simulations [Daly, 1967 and 1969; Otto, 1972; Baker et al., 1980; Baker and Freeman, 1981; Pullin, 1982; Tryggvason, 1988] has been done to develop the theory of Rayleigh-Taylor instability. There is a complex phenomenology associated with the evolution of the instability. As mentioned earlier, this evolution can be divided into three regimes: the linear regime, the non-linear or chaotic regime, and the turbulent mixing regime. In the linear regime, small amplitude perturbations of wavelength $\lambda$ grow exponentially with time. There is a non-linear or chaotic growth of the perturbations when the amplitude of the initial perturbations grows beyond $0.4\lambda$ [Lewis, 1950]. Finally there is turbulent mixing of the two fluids.

Several factors govern the evolution of the instability. These include the interfacial tension between the two fluids ($\sigma$) [Bellman and Pennington, 1954], the density of the two fluids ($\rho_H, \rho_L$), acceleration ($a$) and the viscosity ($\mu$) [Bellman and Pennington, 1954; Menikoff et al., 1977 and 1978] of the fluids. For the case of LFC, the viscosity is neglected in the formulations because molten metal and gaseous polystyrene can essentially be considered inviscid. Let there be two juxtaposed fluids of densities $\rho_H$ and $\rho_L$, where $\rho_H > \rho_L$. The effective acceleration is defined as:

$$G = a - g$$  \hspace{1cm} (2.1)
where \( a \) is the uniform acceleration and \( g \) is the gravitational acceleration [Sharp, 1984]. When the effective acceleration is directed from the lighter fluid to the heavier fluid, the interface is unstable, as shown in Figure 2.1.

### 2.2 Flow Orientations

#### Horizontal Flow

For two juxtaposed fluids flowing in the horizontal plane, from Equation (2.1):

\[
G = a. \tag{2.1 a}
\]

There are four possible cases as shown in Figure 2.2, where \( n \) is the outward normal to the heavier fluid, \( v \) the velocity vector and \( a \) the acceleration. Thus, depending on the acceleration of the system the interface could be stable or unstable.

In LFC, the heavy fluid is the molten metal and the light fluid is polystyrene gas/liquid. Figure 2.2(a) shows the schematic of an advancing front (\( v \cdot n \) is positive) and because the acceleration is directed from the heavy to the light fluid the front is stable. Figure 2.2(b) shows the schematic of an advancing (\( v \cdot n \) is positive), but possibly unstable (\( a \cdot n \) is negative) front. Figures 2.2(c) and (d) show the schematics of receding-stable (\( v \cdot n \) is negative and \( a \cdot n \) is positive) and, possibly, unstable (\( v \cdot n \) is negative and \( a \cdot n \) is negative) fronts, respectively.

#### Vertical Flow

**Lighter fluid on top of the heavier fluid:** The heavy fluid could be advancing (\( v \cdot n \) is positive) or receding (\( v \cdot n \) is negative). If the effective acceleration is directed from the lighter to the heavier fluid the interface is possibly unstable, and \( G \cdot n \) is negative. If the effective acceleration is directed from the heavier to the lighter fluid, then the interface is stable and \( G \cdot n \) is positive, as shown in Figures 2.3(a) and 2.3(c). Figures 2.3(b) and 2.3(d) show that the interface may or may not be unstable depending on the magnitude of \( a \).
Figure 2.1 Schematic showing the unstable interface.

Figure 2.2 Schematics showing: (a) advancing, stable front, (b) advancing, (possibly) unstable front, (c) receding, stable and (d) receding, (possibly) unstable front.
Figure 2.3  Schematics showing: (a) advancing, stable front, (b) advancing, stable/unstable front, (c) receding stable front (d) receding stable/unstable front.
Figure 2.4  Schematics showing the four possible cases with the heavy fluid on top.
Heavier fluid on top of the lighter fluid: The four possible configurations are shown in Figure 2.4. The front could be stable or unstable in all four cases depending on the magnitude of the acceleration, \( a \).

Stratified Media in Gravitational Field

In the case [Chandrasekhar, 1961] of no flow (\( v = 0 \)), the interface acceleration is zero, \( a = 0 \). However, the system is in a gravitational field, as shown in Figure 2.5. Figure 2.5(a) shows the case where the interface is stable. As \( a = 0 \), therefore from Equation (2.1):

\[
G = -g
\]

(2.2)

and:

\[
G \cdot n = (-g) \cdot n
\]

(2.3)

so:

\[
G \cdot n = g k
\]

(2.4)

where \( k \) is the unit vector in the positive \( z \) direction. \( G \cdot n \) is positive, i.e., the effective acceleration is directed from the heavier to the lighter fluid and therefore the interface is stable. Figure 2.5(b) shows the case where the interface is unstable. Again, from Equation (2.3):

\[
G \cdot n = -g k,
\]

(2.5)

therefore the interface is unstable. If \( g = 0 \), both the configurations are stable. Thus \( g \) is the driving force for the instability.

2.3 Perturbations Growth, Linear Analysis

The following analysis is valid for the linear regime of the growth of the Rayleigh-Taylor instability, i.e., for \( \eta \leq 0.4 \lambda \), where \( \eta \) is the amplitude of the disturbance and \( \lambda \) the wavelength. The analysis also assumes that the fluids are inviscid, i.e., \( \mu = 0 \).
Figure 2.5 Stratified media in gravitational field: (a) lighter fluid on top and (b) heavier fluid on top.
The amplitude of the disturbance at any time $t$ is given as [Sharp, 1984]:

$$\eta(t) = \eta(0) \cosh(\alpha t)$$  \hspace{1cm} (2.6)

where $\eta(0)$ is the amplitude of the disturbance at time $t = 0$ and $\alpha$ is the growth factor.

The growth factor $\alpha$ is given as:

$$\alpha = \left[ \frac{GK(\rho_H - \rho_L)}{(\rho_H + \rho_L)} - \frac{\sigma}{(\rho_H + \rho_L)} K^3 \right]^{1/2} \hspace{1cm} (2.7)$$

and:

$$K = \frac{2\pi}{\lambda} \hspace{1cm} (2.8)$$

where $G$ is the magnitude of the effective acceleration, $K$ the wave number, $\rho_H$ the density of the heavy fluid (molten metal in the case of LFC), $\rho_L$ the density of the lighter fluid (polystyrene gas/liquid in the case of LFC) and $\sigma$ the interfacial tension.

There is a critical wavelength, $\lambda_0$. If the wavelength of the disturbance is less than $\lambda_0$, there is no growth of the instability. When $\lambda = \lambda_0$, $\alpha = 0$ and thus:

$$GK(\rho_H - \rho_L) - \frac{\sigma K^3}{(\rho_H + \rho_L)} = 0 \hspace{1cm} (2.9)$$

Since $K = 2\pi/\lambda_0$:

$$\lambda_0 = 2\pi \left[ \frac{\sigma}{G(\rho_H - \rho_L)} \right]^{1/2} \hspace{1cm} (2.10)$$

There is also a fastest growing or most unstable wavelength, $\lambda_M$. This is the wavelength of the fastest growing perturbation and can be found by differentiating $\alpha$ with respect to the wave number $K$ and equating to zero to obtain:

$$\lambda_M = 2\pi \left[ \frac{3\sigma}{G(\rho_H - \rho_L)} \right]^{1/2} \hspace{1cm} (2.11)$$

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From Equations (2.10) and (2.11):

$$\frac{\lambda_M}{\lambda_0} = \sqrt{3} = 1.732$$  \hfill (2.12)

Combining Equations (2.7), (2.8) and (2.11) the greatest growth factor, $\alpha_{\text{max}}$, corresponding to the fastest growing perturbation may be written as:

$$\alpha_{\text{max}} = \left( \frac{4}{27} \right)^{1/4} \left[ \frac{G^3 (\rho_H - \rho_L)^3}{(\rho_H + \rho_L)^2 \sigma} \right]^{1/4}$$  \hfill (2.13)

Thus, if two fluids are juxtaposed and accelerated such that the acceleration is directed from the lighter to the heavier fluid, then knowing the effective acceleration, $G$, the densities of the two fluids ($\rho_H, \rho_L$) and the interfacial tension, $\sigma$, the growth-factor, $\alpha_{\text{max}}$, corresponding to the fastest growing perturbation can be determined by Equation (2.13). Together with Equation (2.6) $\alpha_{\text{max}}$ can be used to predict the amplitude of the disturbance as a function of time. ($1/\alpha_{\text{max}}$) also gives an idea about the time-scale of the instability involved: the higher the value of $\alpha_{\text{max}}$, the smaller the time-scale of the instability will be. Thus, using Equation (2.13) the time-scales involved in different casting configurations can be readily determined.
Chapter 3

Design and Fabrication of the Experimental Set-up

According to the defect formation hypothesis, a large number of defects in LFC are polystyrene decomposition products that are entrapped within the casting. When liquid metal flows and decelerates against polystyrene gas/liquid, the interface becomes unstable. As the instability continues to grow, the fingers encounter each other, thereby trapping polystyrene decomposition products between them. Thus, the entire defect formation hypothesis is based on the Rayleigh-Taylor instability.

In order to validate the hypothesis and to visualize the growth of the instability, conditions that promote the growth of Rayleigh-Taylor instability must be created. Thus, suitable acceleration or deceleration needs to be provided to the system for the interfacial instability to grow. In addition to the acceleration or deceleration, the densities of the two fluids should be such that a reasonably high growth factor $\alpha$ is obtained. Table 3.1 lists the physical properties of some selected fluids [Brandes and Brook, 1992; Frisch and Saunders, 1973]. It is apparent from Equation (2.7) that a higher value of $\alpha$ requires that $\rho_H$ be significantly greater than $\rho_L$. Also, the lower the interfacial tension ($\sigma$) the higher the value of $\alpha$ will be.

The aluminum-polystyrene system is the one that is of interest in aluminum LFC. However, experimenting with this system would require dealing with high temperatures because of the high melting temperature of aluminum (933 K). This also means that recording the flow on the video requires a high-temperature glass. Other possible combinations of the two-fluid system are: water-air system and Wood’s metal-air system. Wood’s metal is a low melting-point alloy; it melts at approximately 343 K. Thus, it is reasonably easier to handle and image the flow. The interfacial tension ($\sigma$) of the water-air system is lower than that of the Wood’s metal-air system. Also, handling water is much easier than handling Wood’s metal. Thus, the water-air system was chosen for experimentation.

The choice of using a vertical or a horizontal set-up was still to be made. In the
<table>
<thead>
<tr>
<th></th>
<th>Melting Temperature, K</th>
<th>Density, kg/m$^3$</th>
<th>Surface Tension, N/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>-</td>
<td>1.29</td>
<td>-</td>
</tr>
<tr>
<td>Water</td>
<td>273</td>
<td>1000</td>
<td>0.072</td>
</tr>
<tr>
<td>Wood’s Metal*</td>
<td>343</td>
<td>9600</td>
<td>0.480</td>
</tr>
<tr>
<td>Aluminum</td>
<td>933</td>
<td>2385</td>
<td>0.914</td>
</tr>
<tr>
<td>Polystyrene (L)</td>
<td>463</td>
<td>1200</td>
<td>0.030</td>
</tr>
</tbody>
</table>

* Composition: Bi – 42.5 %, Pb – 37.7 %, Sn – 11.3 %, Cd – 8.5 %.
case of a vertical set-up, the effective acceleration is the difference between the uniform acceleration of the system and the acceleration due to gravity. The time-scales involved in a vertical set-up are very small (of the order of milliseconds). A considerable amount of experimental work has already been done using vertical set-ups [Lewis, 1950; Emmons et al., 1960; Duff et al., 1962; Cole and Tankin, 1973; Ratafia, 1973; Popil et al., 1979; Jacobs and Catton, 1988]. The effective accelerations involved have been 10-70 g. Also, in these studies the denser fluid has been accelerated by the lighter fluid. In LFC none of these conditions prevail. The accelerations involved are much smaller and the heavy fluid, i.e., aluminum, decelerates against the lighter fluid, i.e., the polystyrene decomposition products. In the case of a horizontal set-up, the acceleration due to gravity is not involved and thus the effective acceleration is the acceleration of the system itself. It is of the order of 0.1g which is comparable to that encountered by the molten metal front in LFC. Consequently, the time-scales in a horizontal set-up are much greater than those in a vertical set-up. The larger time-scales facilitate the recording and analysis of instability growth conveniently. Thus, the horizontal set-up was adopted for the experimental investigation.

Once the water-air system and the horizontal set-up had been chosen, another choice remained to be made. In order to aid the growth of the Rayleigh-Taylor instability, the effective acceleration must be directed from the air side to the water side. This admits two possibilities: the air accelerates the water-air interface, or the water flows and decelerates against the air. These two possibilities are shown in Figure 3.1, where $n$ is the outward normal to the interface, and $v$ and $a$ are the velocity and acceleration vectors, respectively. However, because in LFC, the molten metal flows and decelerates against the polystyrene gas/liquid, the second case, i.e., Figure 3.1(b), is the one that is closer to the actual LFC process and was therefore chosen.

### 3.1 Diverging Mold

Based on the above criteria, a number of different mold designs are possible for the experimental set-up. Two such designs are examined. One possibility for achieving a near-constant deceleration of the front, (i.e., a near-constant acceleration directed from
Figure 3.1 Schematic showing: (a) air accelerating the water, and (b) water decelerating against air.

Figure 3.2 Schematic of the diverging mold.
the air side to the water side) is to maintain a constant flow rate into a mold with a
diverging profile, Figure 3.2. As water flows into the mold and the cross-sectional area
of the mold increases, the velocity of the front decreases due to continuity. Thus, the
mold profile must be such that the expansion in the area provides a near-constant
deceleration. Water enters the mold at the inlet with a flow rate $Q$ that remains constant
with time. The inlet corresponds to $x = 0$ and the width of the mold at the inlet is $y_0$. The
thickness of the mold, perpendicular to the plane of the paper, is $b$ and is constant through
the length and the breadth of the mold. Let the width of the mold be $y(x)$ at some $x$ and $v$
the velocity of flow. At $x = 0$, $v = v_0$. The acceleration, $a$, of the water front is given by:

$$a = \frac{d^2 x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$$

Rearranging, integrating and using the inlet conditions, that at $x = 0$, $v = v_0$:

$$v = \sqrt{v_0^2 + 2ax}$$

From continuity of flow:

$$Q = v_0 y_0 b = y(x) v(x) b$$

and from Equation (3.2):

$$y(x) = \frac{v_0 y_0}{\sqrt{v_0^2 + 2ax}}$$

Dividing throughout by $v_0$:

$$y(x) = y_0 \frac{1}{\sqrt{1 + \left(\frac{2ax}{v_0^2}\right)}}$$

Let $\xi$ be a dimensionless length defined as:

$$\xi = \frac{v_0^2}{2a}$$

Thus:

$$y(x) = y_0 \frac{1}{\sqrt{1 + \frac{x}{\xi}}}$$
Equation (3.6) is valid when the fluid is accelerating; when the fluid is decelerating, as in the case of these experiments, then $a$ is negative and thus if $\xi$ is a positive number:

$$y(x) = y_0 \frac{1}{\sqrt{1 - \frac{x}{\xi}}} \quad (3.7)$$

or:

$$\frac{y(x)}{y_0} = \frac{1}{\sqrt{1 - \frac{x}{\xi}}} \quad (3.8)$$

The above expressions are valid for the case when the cross-section of the mold is rectangular, with a constant depth $b$. Similar results can be derived for other cross-sections. Equation (3.7) for a square and for a circular cross-section becomes:

$$y(x) = \frac{y_0}{\left(1 - \frac{x}{\xi}\right)^{1/4}} \quad (3.9)$$

where $y$ is the side for the square cross-section or the radius for the circular cross-section. Table 3.2 shows the variation of $y/y_0$ with $x/\xi$, which is plotted in Figure 3.3.

Thus, if the deceleration required is known, the mold profile required for that deceleration can be obtained. However, as Figure 3.3 shows, $y/y_0$ varies very slowly initially and at an extremely fast rate when $x/\xi$ approaches unity. The maximum length of the mold is equal to $\xi$. Of course, the mold length needs to be just short of $\xi$ because $y/y_0$ shoots to infinity as $x/\xi$ tends to unity.

### 3.2 Constant Cross-Sectional Area Mold

Another way to achieve deceleration is to have a mold of constant cross-sectional area but to vary the flow rate into the mold. The change in velocity of the front is obtained due to a change in the flow rate and not due to a changing mold profile. In the previous case, a constant flow rate was required to be maintained into the mold which could be done, for example, by connecting the mold to a reservoir with a very large cross-
Table 3.2 Variation of $y/y_0$ with $x/\xi$.

<table>
<thead>
<tr>
<th>$x/\xi$</th>
<th>$y/y_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>1.05</td>
</tr>
<tr>
<td>0.2</td>
<td>1.12</td>
</tr>
<tr>
<td>0.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.4</td>
<td>1.29</td>
</tr>
<tr>
<td>0.5</td>
<td>1.41</td>
</tr>
<tr>
<td>0.6</td>
<td>1.58</td>
</tr>
<tr>
<td>0.7</td>
<td>1.83</td>
</tr>
<tr>
<td>0.8</td>
<td>2.23</td>
</tr>
<tr>
<td>0.9</td>
<td>3.16</td>
</tr>
<tr>
<td>1.0</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Figure 3.3 Variation of $y/y_0$ with $x/\xi$. 
sectional area compared with that of the mold. In the present case, however, a mechanism is required to constantly vary the flow rate. This can be conveniently achieved by having a constantly dropping hydraulic head in the reservoir. A combination of the head drop and the head losses due to viscous dissipation in the connecting tube and the mold result in an effective deceleration for the water front in the mold.

Figure 3.5 shows the reservoir, the connecting tube and the mold, where \( A_1, A_2 \) and \( A_4 \) are the cross-sectional areas of the reservoir, the connecting tube and the mold, respectively. Let the length of the connecting tube be \( l \). The base of the mold and of the reservoir are at the same level, which is assumed to be the datum. \( h_i \) is the initial height of the water in the reservoir and \( h(t) \) is the height of the water in the reservoir at any time \( t \). In what follows, a gradual change in area, wherever the area changes, is assumed. Applying the unsteady Bernoulli’s equation between point 1 on the surface of the water in the tank and point 4 on the water-air interface in the mold, [Fay, 1994]:

\[
\frac{4}{dt} \frac{dv}{ds} + \frac{v_4^2}{2} + \frac{p_4}{\rho} + g z_4 = \frac{v_1^2}{2} + \frac{p_1}{\rho} + g z_1 - g h_i
\]  

where \( v \) is the velocity, \( p \) the pressure, \( \rho \) the density of water, \( z \) the elevation from the datum, \( g \) the acceleration due to gravity, and \( h_i \) the head loss. The pressure \( p_1 \) is equal to \( p_4 \), both being the atmospheric pressure. Also, \( z_4 \) corresponds to the datum and is therefore zero. Thus, Equation (3.11) reduces to:

\[
\frac{4}{dt} \frac{dv}{ds} + \frac{v_4^2}{2} = \frac{v_1^2}{2} + g z_1 - g h_i
\]  

(3.12)

The integral on the left hand side of Equation (3.12) can be expanded as:

\[
\frac{4}{dt} \frac{dv}{ds} = \frac{dv_1}{dt} h(t) + \frac{dv_2}{dt} l_r + \frac{dv_4}{dt} x
\]  

(3.13)

where \( v_1 \) is the velocity in the reservoir, \( v_2 \) in the connecting tube and \( v_4 \) in the mold. \( x \) is the distance covered by water in the mold at any time \( t \). The head loss is given by, [Fay, 1994]:

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Figure 3.4  Mold with uniform cross-sectional area.

Figure 3.5  Schematic showing the (a) top view and (b) front view of reservoir, connecting tube and the mold.
\[ h_I = k \frac{v^2}{2g} \]  

(3.14)

where:

\[ k = \frac{f}{d} \]  

(3.15)

Here \( l \) is the length and \( d \) is the diameter of the conduit (mold, connecting tube or the reservoir) whose head loss is being calculated, and \( f \) is the Darcy friction factor, which can be obtained from the Moody diagram for a range of Reynolds numbers [Fay, 1994].

The total head loss is the sum of the following losses: loss in the reservoir, loss in the connecting tube, loss in the mold, loss due to sudden area change between the reservoir and the connecting tube, and loss due to sudden area change between the connecting tube and the mold. Thus, the total head loss is:

\[ h_I = f_r \frac{v_r^2}{2g} + f_t \frac{v_t^2}{2g} + f_m \frac{x v_m^2}{2g} + k_1 \frac{v_a^2}{2g} + k_2 \frac{v_b^2}{2g} \]  

(3.16)

where \( f_r, f_t \) and \( f_m \) are the Darcy friction factors for flow in the reservoir, connecting tube and the mold, respectively, \( d_r, d_t \) and \( d_m \) are the diameters of the reservoir, connecting tube and the mold, respectively. \( v_r, v_t \) and \( v_m \) are the velocities of flow in the reservoir, connecting tube and the mold, respectively. \( v_a \) and \( v_b \) are given by:

\[ \bar{v}_a = \frac{v_r + v_t}{2} \quad \text{and} \quad \bar{v}_b = \frac{v_r + v_m}{2} \]  

(3.17)

For the range of Reynolds numbers in which the experiments were conducted (8,000 to 40,000), the Darcy friction factor can be taken as almost constant and equal to 0.03. If the cross-section of the mold is rectangular, \( d_m \) is the hydraulic diameter and is defined as:

\[ d_m = \frac{4(\text{cross-sectional area of the mold})}{\text{perimeter of the mold}} \]  

(3.18)

From volume conservation:

\[ h(t) = h_I - \frac{A_4}{A_1} x \]
i.e.:

\[ [h_i - h(t)]A_1 = A_4x \]  

(3.19)

and continuity further implies that:

\[ v_1A_1 = v_2A_2 = v_4A_4 \]  

(3.20)

where the subscripts 1, 2 and 4 refer to the reservoir, the connecting tube and the mold, respectively.  \( k_1 \) and \( k_2 \) are defined as:

\[ k_1 = 0.4 \left( 1 - \frac{A_2}{A_1} \right) \quad \text{if} \ A_2 \leq A_1 \quad \text{and} \quad k_1 = \left( 1 - \frac{A_1}{A_2} \right)^2 \quad \text{if} \ A_1 \leq A_2 \]

and:

\[ k_2 = 0.4 \left( 1 - \frac{A_4}{A_2} \right) \quad \text{if} \ A_4 \leq A_2 \quad \text{and} \quad k_2 = \left( 1 - \frac{A_2}{A_4} \right)^2 \quad \text{if} \ A_2 \leq A_4 \]  

(3.21)

In the experiments conducted, \( A_i \) is always greater than or equal to \( A_2 \) and thus:

\[ k_1 = 0.4 \left( 1 - \frac{A_2}{A_1} \right) \quad \text{if} \ A_2 \leq A_1 \]  

(3.22)

Thus, Equation (3.16) becomes:

\[ h_1 = \frac{v_2^2}{2g} \frac{f h(t)}{d_r} \left( \frac{A_4}{A_1} \right)^2 + l_1 \frac{A_4}{d_i} \left( \frac{A_4}{A_2} \right)^2 + x \frac{k_1}{d_m} + v_4^2 \frac{k_1}{8g} \left( \frac{A_4}{A_1} + \frac{A_4}{A_2} \right)^2 + v_2^2 \frac{k_2}{8g} \left( \frac{A_4}{A_1} + 1 \right)^2 \]  

(3.23)

Equation (3.13) can be written as:

\[ \frac{4}{d_r} \frac{dv}{dt} = \frac{dv_4}{dt} \left\{ x \left[ 1 - \left( \frac{A_4}{A_1} \right)^2 \right] + \frac{A_4}{A_1} \frac{h_1}{l_1} \right\} \]  

(3.24)
Thus, Equation (3.12) becomes:

\[
\frac{d^2 x}{dt^2} = \frac{1}{2} \left( \frac{dx}{dt} \right)^2 \left[ \left( \frac{A_4}{A_1} \right)^2 - 1 \right] - \frac{f}{d_r} \left( \frac{A_4}{A_1} \right)^2 \left( h_i - \frac{A_4}{A_1} x \right) + f \frac{l_t}{d_m} \left( \frac{A_4}{A_2} \right)^2 + f \frac{x^2}{d_m} \left[ 1 - \left( \frac{A_4}{A_1} \right)^2 \right] + \frac{A_4}{A_1} h_i + \frac{A_4}{A_2} l_t
\]

Equation (3.25) gives the expression for the acceleration of the water front in the mold. Several combinations of the areas \(A_1, A_2, \) and \(A_4\) are possible: (i) \(A_1 \equiv A_2 \equiv A_4;\) (ii) \(A_1 \gg\) \(A_2\) and \(A_2 \equiv A_4;\) (iii) \(A_1 = A_2\) and \(A_2 \ll A_4;\) (iv) \(A_2 = A_4\) and \(A_1 \ll A_2.\) However, case (iii) is not feasible because \(h\) would have to be extremely large in that case. For the same reason case (iv) is also not feasible. Thus, the first two area combinations can be used, and \(h_i\) and \(l_t\) can also be varied to provide different decelerations to the water front.

Equation (3.25) is a non-linear differential equation and thus, has been solved numerically. The Runge-Kutta-Nyström method [Kreyszig, 1993] has been used for solving the equation for different experimental parameters. The details of the computation are shown in Table A.1 in Appendix A. The Matlab code used for the numerical simulations is attached in Appendix B. When Equation (3.25) is solved for cases (i) and (ii) there is a deceleration of the water front in case (i), while the flow is steady, i.e., has a uniform velocity in case (ii). Increasing the length of the tube for a fixed \(h_i\) decreases the magnitude of the deceleration in case (i). However, an increase in \(h_i\) for a fixed \(l_t\) increases the magnitude of the deceleration. Figure 3.6 shows the variation of the distance covered, the velocity and the acceleration of the water front, with time, for a particular combination of \(h_i, l_t, A_1, A_2,\) and \(A_4,\) which shows the general trend for cases in which \(A_1 \equiv A_2 \equiv A_4.\) Figure 3.7 shows the variation of the distance covered,
Figure 3.6 The variation of (a) distance, (b) velocity and (c) acceleration with time. \( A_1 = A_2 = 506 \text{ mm}^2, A_4 = 387 \text{ mm}^2, h_i = 1 \text{ m}, l_i = 0.4 \text{ m}. \)
the velocity and the acceleration of the water front for another combination of \( h_i, l_i, A_1, A_2, \) and \( A_4, \) which shows the general trend for cases in which \( A_1 \gg A_2 \) and \( A_2 \equiv A_4. \) Figure 3.8 compares the acceleration provided to the water front as a function of time in the two cases.

### 3.3 Experimental Set-up

On the basis of an examination of the two possible options in Sections (3.1) and (3.2), a choice was made to use one of them. The set-up described in Section (3.2) is better than the one in Section (3.1) for two reasons. First, the cross-sectional area of the mold is constant all along the length of the mold in the set-up described in Section (3.2). In the diverging mold, however, the water-air interface that is to be observed is itself being modified constantly, due to the expansion in the cross-sectional area. Thus, the constant cross-sectional area mold is better suited to observe any instability of the interface. Second, it may not be possible to accurately machine the profile of the diverging mold due to the extremely slow variation of \( y/y_0 \) with \( x/\xi \) initially and the extremely fast variation of \( y/y_0 \) with \( x/\xi \) when \( x/\xi \) tends to unity.

Thus, the constant cross-sectional area mold was chosen as the one to be used in the experiments. The schematic diagram of the experimental set-up (without the imaging system) is shown in Figure 3.9. Figure 3.10 shows a photograph of the entire experimental set-up, including the imaging system. The mold itself was formed by two long glass plates (1.4 m long, 0.1 m wide and 6.35 mm thick) that were separated by a rubber cord between them. This length of the mold provided sufficient time for the instability to grow and to be observed. The width of the mold was fixed after gradually increasing it, starting from 25.4 mm, so that at least one full wave could be seen. The thickness was chosen to facilitate the maintenance of a vertical water front. The glass plates were treated to seal the microscopic pores on their surfaces with a super-slick, non-stick fluid. This made the glass plates non-wetting for water and thus allowed the instability to grow freely, according to the Rayleigh-Taylor instability theory. The rubber cord ran along the edge of the glass plates. So, the glass plates formed the top and the bottom of the mold and the rubber cord formed the sides. By varying the dimensions of
Figure 3.7 The variation of (a) distance, (b) velocity and (c) acceleration with time. 
$A_1 = 413 \times 10^2 \text{ mm}^2$, $A_2 = 126 \text{ mm}^2$, $A_4 = 387 \text{ mm}^2$, $h_i = 0.2 \text{ m}$ and $l_i = 0.4 \text{ m}$. 
Figure 3.8 A comparison of the acceleration of the water front as a function of time for case (i) $A_1 = A_2 = 506 \text{ mm}^2$, $A_4 = 387 \text{ mm}^2$, $h_i = 1 \text{ m}$, $l_i = 0.4 \text{ m}$ (dashed line) and case (ii) $A_1 = 413 \times 10^2 \text{ mm}^2$, $A_2 = 126 \text{ mm}^2$, $A_4 = 387 \text{ mm}^2$, $h_i = 0.2 \text{ m}$, $l_i = 0.4 \text{ m}$ (solid line).
Figure 3.9 Schematic diagram of the experimental set-up.
Figure 3.10 Photograph of the apparatus with the imaging system.
the rubber cord the mold depth could be varied. Two holes were drilled in the bottom glass plate near the two ends. These formed the entry and exit ports, to and from the mold. The bottom glass plate was made to rest on an aluminum base plate, which was of the same length and breadth as that of the glass plate to provide support to the glass plates. The entire mold was lightly clamped together. The aluminum base plate was also drilled at the two ends. The entry and exit connections to the mold were made through the aluminum base plate. A Tygon tube connected the mold to the reservoir through pipe-fittings. In the experiments conducted, \( A_1 = A_2 \) and thus, the connecting tube itself formed the "reservoir". A container was placed under the exit on the other side of the mold to collect any water that might flow out of the mold.

3.4 Image Acquisition System

In order to analyze the growth of the Rayleigh-Taylor instability in the experiments, it was necessary to photograph the flow front at a few locations along the length of the mold. Since the flow velocities were high, up to 1.5 m/s, high-speed photography was employed. A ‘Redlake Imaging S Series Motionscope’ high-speed camera system was used for this purpose. The particular system used was 8000S and the specifications are shown in Table D.1 in Appendix D.

As shown in Figure 3.10, the camera was mounted on a tripod stand. The camera was connected to the Motionscope, which was the control and display unit for the entire imaging system. The Motionscope has a monitor that displays the image that the camera captures. It can record images up to 8000 frames/second and play them at any frame rate. In most cases the recording was done at 500 frames/second and played back at 3 frames/second. The Motionscope was connected to a video cassette recorder to enable recording of the flow on the video tape. The video recorder was in turn connected to a computer that was used to capture frames from the recording and print them for analysis. When light in the laboratory was insufficient, an extra lamp was used and in some cases indirect lighting was employed to prevent a blinding reflection from the aluminum base into the camera.
Chapter 4

Experimental Results and Analysis

Experiments were conducted to visualize and analyze the instability growth. As a first step, experiments to validate Equation (3.25) for the acceleration or deceleration of the water front in the mold were conducted. Once confirmed, Equation (3.25) was then used to select the various dimensions of the experimental set-up to give different decelerations to the water front. Experiments were then conducted at different decelerations and the front was photographed at a few locations along the length of the mold to analyze the instability growth. Thereafter, using the Rayleigh-Taylor instability theory, discussed in Chapter 2, the growth parameters, as well as the length- and time-scales of the instability, were calculated for the water-air system and for the molten metal-polystyrene system. In light of these results, various casting orientations were then examined for defect formation due to the Rayleigh-Taylor instability.

4.1 Flow Analysis for the Experimental Set-up

To verify Equation (3.25), experiments were conducted with different experimental configurations, i.e., different combinations of the areas $A_1$, $A_2$ and $A_4$, different lengths of the connecting tube, $l_i$, and different values of the initial hydraulic head $h_i$. Initially water filled the connecting tube, which also served as a reservoir, up to a height $h_i$ above the datum, the mold level. To start with, there was no water in the mold. As soon as the water was allowed to flow, it flowed into the mold and its height in the tube fell constantly. This is shown in Figure 4.1. Figure 4.1(a) shows the configuration at time $t = 0$, while Figure 4.1(b) shows the configuration at a later time $t$. The times at which water reached specified points along the length of the mold were recorded. These were the experimental times for the interface to reach different distances along the mold. Equation (3.25) was solved numerically and the distance traveled by the
Figure 4.1 Schematics showing the configuration at (a) time $t = 0$, before the water starts flowing and (b) a later time $t$, when the water has flowed into the mold.
water front as a function of time was obtained. These were the theoretical times for the water to flow different distances in the mold. The theoretical and experimental times were then compared.

A number of experiments were conducted with different experimental configurations. Table 4.1 lists the configurations used for different experimental runs. The plots showing the theoretically obtained distance versus time curves for each experimental configuration, and the average and the spread of the experimental times to cover specified distances are shown in Figures 4.2 through 4.5. In all cases, an extremely good match between the experimentally obtained times and those obtained from Equation (3.25) was obtained. This match confirmed the validity of Equation (3.25).

4.2 Results for the Rayleigh-Taylor Instability Experiments

Using Equation (3.25), the parameters of the experimental set-up were varied to provide different decelerations to the water front. The front was then photographed at a few locations along the length of the mold to analyze the growth of the instability. Thus, specific combinations of the values of the variables \((A_1, A_2, A_4, l, h, d, d_t, d_m)\) were chosen to provide different decelerations to the water front. Each of the combinations provided a different deceleration to the water front. In each case, Equation (3.25) gave the deceleration of the water front as a function of time. Since the deceleration of the water front was not constant, the average deceleration of the front was found during the time the front decelerated. The experiment was then run and the water front was photographed at two locations. The first photograph was taken at a time \(t_1\), which was after the time \(t_0\), \(t_0\) being the time when the velocity was maximum and the water front began to decelerate. The second photograph was taken at a later time \(t_2\), \((t_2 > t_1)\) at which point the front had already been decelerating for some time. The experimental photographs show the wavelength, \(\lambda\), and the amplitude of any disturbance of the interface, \(\eta\).

With the average deceleration obtained from Equation (3.25), and the experimentally obtained wavelength and amplitudes, the growth factor \(\alpha\) was obtained.
Table 4.1 Configurations of the various experimental runs.

<table>
<thead>
<tr>
<th>#</th>
<th>$A_1$ (mm$^2$)</th>
<th>$A_2$ (mm$^2$)</th>
<th>$A_4$ (mm$^2$)</th>
<th>$h_1$ (m)</th>
<th>$l_1$ (m)</th>
<th>$d_t$ (mm)</th>
<th>$d_m$ (mm)</th>
<th>$d_r$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>126</td>
<td>126</td>
<td>387</td>
<td>1.5</td>
<td>0.3</td>
<td>12.7</td>
<td>9.53</td>
<td>12.7</td>
</tr>
<tr>
<td>2.</td>
<td>506</td>
<td>506</td>
<td>387</td>
<td>0.75</td>
<td>0.85</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
<tr>
<td>3.</td>
<td>506</td>
<td>506</td>
<td>387</td>
<td>0.5</td>
<td>0.4</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
<tr>
<td>4.</td>
<td>506</td>
<td>506</td>
<td>387</td>
<td>1.0</td>
<td>0.4</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
</tbody>
</table>
Figure 4.2 The theoretical distance vs. time curve and the average (*) and the range (solid line) of the experimental times for experiment # 1.

Figure 4.3 The theoretical distance vs. time curve and the average (*) and the range (solid line) of the experimental times for experiment # 2.
Figure 4.4 The theoretical distance vs. time curve and the average (*) and the range (solid line) of the experimental times for experiment # 3.

Figure 4.5 The theoretical distance vs. time curve and the average (*) and the range (solid line) of the experimental times for experiment # 4.
both experimentally and theoretically. The growth factors thus obtained were then compared.

The analysis for each combination of experimental parameters was made as follows:

- The experiment was run and the flow front was photographed at two points: (i) at a time \( t_1 \), which was after the time \( t_0 \), \( t_0 \) being the time when the front began to decelerate and (ii) at a later time \( t_2 \), where \( t_2 > t_1 \).
- The average deceleration of the water front was found, averaged between the times \( t_0 \) and \( t_2 \), from the solution of the Equation (3.25).
- From the photographs, twice the amplitudes of the (the distance between the crest and the trough) disturbance at time \( t_1 \), \( 2\eta(t_1) \), and time \( t_2 \), \( 2\eta(t_2) \), were measured. The wavelength, \( \lambda \), of the disturbance was also measured from the photograph.
- Using the wavelength, \( \lambda \), and the average deceleration found above, the value of the growth factor, \( \alpha \), was calculated from Equation (2.7). This is the theoretically calculated value of the growth factor, \( \alpha_{th} \).
- Using the values, \( 2\eta(t_1) \) and \( 2\eta(t_2) \), the experimental value of the growth factor, \( \alpha_{exp} \) was found. From Equation (2.6) it follows that:

\[
\frac{2\eta(t_2)}{2\eta(t_1)} = \cosh[\alpha(t_2 - t_0)] / \cosh[\alpha(t_1 - t_0)]
\]

Substitution was used to find the value of \( \alpha \) which satisfies the above equation.

The Matlab code for this is attached in Appendix C.

- The two growth factors, \( \alpha_{th} \) and \( \alpha_{exp} \) were then compared.

The results are summarized below. The first experiment was done with accelerating flow and the second one with steady flow. In both cases no growth of the instability was observed, as expected. In the third experiment the experimental parameters were chosen to provide a deceleration less than that required to make the instability grow. The next three experiments were conducted with the experimental parameters chosen to provide a deceleration sufficient enough to make the instability grow. The experimental parameters are shown in Tables 4.2, 4.3, 4.4, 4.5, 4.7 and 4.9 for experiments 1 through 6.
respectively. The plots showing the distance covered, velocity and acceleration as a function of time are shown in Figures 4.7, 4.9, 4.11, 4.13 and 4.15 for experiments 2, 3, 4, 5 and 6, respectively. The photographs of the interface are shown in Figures 4.6, 4.8, 4.10, 4.12, 4.14 and 4.16 for experiments 1 through 6, respectively. The analysis, using the steps described above, is shown in Tables 4.6, 4.8 and 4.10 for experiments 4, 5 and 6 respectively, where \( d \) refers to the average deceleration of the interface, \( \alpha_{th} \) and \( \alpha_{exp} \) refer to the theoretical and experimental values of the growth factor, respectively, and the percentage difference is between these theoretical and experimental values of the growth factor.

Figure 4.6 shows the interface at two times, \( t_1 \) and \( t_2 \), for accelerating flow. It can be seen that the interface has a smoothly curved profile at time \( t_1 = 0.7s \) and maintains this smooth profile until a later time \( t_2 = 2.16s \). This is as expected, for the flow is accelerating. Experiment \# 2 involved steady flow. This can be seen from Figure 4.7 (c), which shows the acceleration of the interface as a function of time. Figures 4.8 (a) and 4.8 (b), again show the expected result, i.e., no growth of any instability. In experiment \# 3, the experimental parameters were chosen to provide a deceleration. However, this deceleration was insufficient to make the instability grow as is seen in Figures 4.10 (a) and 4.10 (b), which show the interface at times \( t_1 = 0.2s \) and \( t_2 = 0.7s \).

In experiment \# 4, the experimental parameters were so chosen as to provide a deceleration large enough for the instability to grow. This is seen in Figures 4.12 (a) and 4.12 (b), which show photographs of the interface taken at times \( t_1 = 0.39s \) and \( t_2 = 0.55s \), respectively. The initial instability at time \( t_1 \) grows into one with a larger amplitude at time \( t_2 \). Table 4.6 lists the amplitudes and the theoretical and experimental values of the growth factor \( \alpha \). The two differ by about 3.91 percent. Compared with experiment \# 4, the magnitude of the deceleration is higher in experiment \# 5. Correspondingly, the instability of the interface grows at a faster rate. The interface, photographed at times \( t_1 = 0.4s \) and \( t_2 = 0.6s \), is shown in Figures 4.14 (a) and 4.14 (b), respectively. The theoretical and experimental values of the growth factors are close, and are higher than that of experiment \# 4. The corresponding values of the growth factor are even higher in experiment \# 6, corresponding to higher deceleration. The interface, photographed at times \( t_1 = 0.4s \) and \( t_2 = 0.64s \), is shown in Figures 4.16 (a) and 4.16 (b),
Table 4.2 The experimental parameters for experiment # 1.

<table>
<thead>
<tr>
<th>$A_1 (\text{mm}^2)$</th>
<th>$A_2 (\text{mm}^2)$</th>
<th>$A_4 (\text{mm}^2)$</th>
<th>$h_i (\text{m})$</th>
<th>$l_i (\text{m})$</th>
<th>$d_i (\text{mm})$</th>
<th>$d_m (\text{mm})$</th>
<th>$d_r (\text{mm})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$413 \times 10^2$</td>
<td>126</td>
<td>387</td>
<td>0.14</td>
<td>0.4</td>
<td>12.7</td>
<td>9.53</td>
<td>191</td>
</tr>
</tbody>
</table>

Figure 4.6 The interface at times (a) $t_1 = 0.7s$ and (b) $t_2 = 2.16s$ in experiment # 1 (Accelerating flow).
Table 4.3 The experimental parameters for experiment # 2.

<table>
<thead>
<tr>
<th>$A_1$ (mm$^2$)</th>
<th>$A_2$ (mm$^2$)</th>
<th>$A_4$ (mm$^2$)</th>
<th>$h_1$ (m)</th>
<th>$l_1$ (m)</th>
<th>$d_1$ (mm)</th>
<th>$d_m$ (mm)</th>
<th>$d_f$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$413 \times 10^2$</td>
<td>126</td>
<td>387</td>
<td>0.15</td>
<td>0.4</td>
<td>12.7</td>
<td>9.53</td>
<td>191</td>
</tr>
</tbody>
</table>

Figure 4.7 The (a) distance covered, (b) velocity and (c) acceleration of the water front as a function of time in experiment # 2.
Figure 4.8  The interface at times (a) $t_1 = 0.5s$ and (b) $t_2 = 2s$ in experiment # 2.
Table 4.4 The experimental parameters for experiment #3.

<table>
<thead>
<tr>
<th>$A_1$(mm$^2$)</th>
<th>$A_2$(mm$^2$)</th>
<th>$A_4$(mm$^2$)</th>
<th>$h_i$(m)</th>
<th>$l_t$(m)</th>
<th>$d_r$(mm)</th>
<th>$d_m$(mm)</th>
<th>$d_s$(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
<td>126</td>
<td>387</td>
<td>1.5</td>
<td>0.3</td>
<td>12.7</td>
<td>9.53</td>
<td>12.7</td>
</tr>
</tbody>
</table>

Figure 4.9 The (a) distance covered, (b) velocity and (c) acceleration of the water front as a function of time in experiment #3.
Figure 4.10  The interface at times (a) $t_1 = 0.2s$ and (b) $t_2 = 0.7s$ in experiment #3.
Table 4.5 The experimental parameters for experiment # 4.

<table>
<thead>
<tr>
<th>A₁ (mm²)</th>
<th>A₂ (mm²)</th>
<th>A₄ (mm²)</th>
<th>h (m)</th>
<th>l (m)</th>
<th>d₁ (mm)</th>
<th>d₉ (mm)</th>
<th>d₄ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>506</td>
<td>506</td>
<td>387</td>
<td>0.5</td>
<td>0.4</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Figure 4.11 The (a) distance covered, (b) velocity and (c) acceleration of the water front as a function of time in experiment # 4.
**Figure 4.12** The interface at times (a) $t_1 = 0.39s$ and (b) $t_2 = 0.55s$ in experiment #4

**Table 4.6** The theoretical and experimental values of the growth factor, $\alpha$, for experiment #4.

<table>
<thead>
<tr>
<th>$t_0$(s)</th>
<th>$t_1$(s)</th>
<th>$t_2$(s)</th>
<th>$d$(m/s$^2$)</th>
<th>$2\eta(t_1)$(mm)</th>
<th>$2\eta(t_2)$(mm)</th>
<th>$\lambda$(m)</th>
<th>$\alpha_{th}$(s$^{-1}$)</th>
<th>$\alpha_{exp}$(s$^{-1}$)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.39</td>
<td>0.55</td>
<td>1.2</td>
<td>25.0</td>
<td>28.75</td>
<td>0.05</td>
<td>2.81</td>
<td>2.92</td>
<td>3.91</td>
</tr>
</tbody>
</table>
Table 4.7 The experimental parameters for experiment # 5.

<table>
<thead>
<tr>
<th>( A_1 (\text{mm}^2) )</th>
<th>( A_2 (\text{mm}^2) )</th>
<th>( A_4 (\text{mm}^2) )</th>
<th>( h (\text{m}) )</th>
<th>( l (\text{m}) )</th>
<th>( d (\text{mm}) )</th>
<th>( d_m (\text{mm}) )</th>
<th>( d_t (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>506</td>
<td>506</td>
<td>387</td>
<td>0.75</td>
<td>0.4</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Figure 4.13 The (a) distance covered, (b) velocity and (c) acceleration of the water front as a function of time in experiment # 5.
Figure 4.14  The interface at times (a) \( t_1 = 0.4s \) and (b) \( t_2 = 0.6s \) in experiment # 5.

Table 4.8 The theoretical and calculated values of the growth factor, \( \alpha \), for experiment # 5.

<table>
<thead>
<tr>
<th>( t_0(s) )</th>
<th>( t_1(s) )</th>
<th>( t_2(s) )</th>
<th>( d(m/s^2) )</th>
<th>( 2\eta(t_1)(mm) )</th>
<th>( 2\eta(t_2)(mm) )</th>
<th>( \lambda(m) )</th>
<th>( \alpha_{th}(s^{-1}) )</th>
<th>( \alpha_{exp}(s^{-1}) )</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37</td>
<td>0.4</td>
<td>0.6</td>
<td>1.42</td>
<td>21.25</td>
<td>38.75</td>
<td>0.05</td>
<td>5.96</td>
<td>5.35</td>
<td>10.27</td>
</tr>
</tbody>
</table>
Table 4.9 The experimental parameters for experiment # 6.

<table>
<thead>
<tr>
<th>A_1(mm^2)</th>
<th>A_2(mm^2)</th>
<th>A_4(mm^2)</th>
<th>h_(l)(m)</th>
<th>l_(l)(m)</th>
<th>d_(l)(mm)</th>
<th>d_m(mm)</th>
<th>d_r(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>506</td>
<td>506</td>
<td>387</td>
<td>1.0</td>
<td>0.4</td>
<td>25.4</td>
<td>9.53</td>
<td>25.4</td>
</tr>
</tbody>
</table>

Figure 4.15 The (a) distance covered, (b) velocity and (c) acceleration of the water front as a function of time in experiment # 6.
Figure 4.16 The interface at times (a) $t_1 = 0.4s$ and (b) $t_2 = 0.64s$ in experiment # 6.

Table 4.10 The theoretical and experimental values of the growth factor, $\alpha$, for experiment # 6.

<table>
<thead>
<tr>
<th>$t_0$(s)</th>
<th>$t_1$(s)</th>
<th>$t_2$(s)</th>
<th>$d$(m/s$^2$)</th>
<th>$2\eta(t_1)$(mm)</th>
<th>$2\eta(t_2)$(mm)</th>
<th>$\lambda$(m)</th>
<th>$\alpha_{th}$(s$^{-1}$)</th>
<th>$\alpha_{exp}$(s$^{-1}$)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.39</td>
<td>0.4</td>
<td>0.64</td>
<td>1.6</td>
<td>12.5</td>
<td>33.75</td>
<td>0.05</td>
<td>7.63</td>
<td>6.61</td>
<td>13.19</td>
</tr>
</tbody>
</table>
respectively.

Thus, in all cases the experimental and theoretical values of the growth factor are close, as shown in Figure 4.17. Also, steady and accelerating flows do not have any instabilities, as predicted by the linear theory. Figure 4.18 shows stable and unstable zones, in terms of the deceleration, $d$, of the water front for different widths of the mold. The experimental data point for experiment # 3 (+) falls in the stable region and those for experiments 4, 5 and 6 (*) fall in the unstable region, as expected.
Figure 4.17 The experimental and the theoretical values of the growth factor ($\alpha$) for experiment numbers 4 (*), 5 (x) and 6 (+).
Figure 4.18 The stable and unstable zones in terms of the deceleration, $d$, of the water front for different widths of the mold. The data points are for experiment # 3 (+) and experiment # 4, 5 and 6 (*).
Chapter 5

Implications for Lost Foam Casting

The Rayleigh-Taylor instability theory, as described in Chapter 2, can be used to predict the growth of instabilities, both for the water-air system and the aluminum-polystyrene system. Table 5.1 lists the critical wavelength, \( \lambda_0 \), the fastest growing wavelength, \( \lambda_M \), the growth factor, \( \alpha_{\text{max}} \), corresponding to the fastest growing wavelength, and \( (1/\alpha_{\text{max}}) \), for various values of the effective deceleration, \( G \), for the water-air system. Tables 5.2 and 5.3 list the corresponding parameters for the aluminum-polystyrene (liquid) and aluminum-polystyrene (gas) systems, respectively. The interfacial tensions for the water-air system and the aluminum-polystyrene system are 0.072 N/m and 0.914 N/m, respectively. \( \lambda_M \) is the length-scale of the instability, \( L^* \), and \( (1/\alpha_{\text{max}}) \) is the time-scale, \( \tau^* \), i.e.,

\[
L^* = \lambda_M
\]

and

\[
\tau^* = \frac{1}{\alpha_{\text{max}}}
\]

Figures 5.1, 5.3 and 5.5 show the variation of the growth factor, \( \alpha \), with the wavelength, \( \lambda \), for different values of the effective deceleration, for the water-air, aluminum-polystyrene (liquid) and aluminum-polystyrene (gas) systems, respectively. Figures 5.2, 5.4 and 5.6 show the variation of the growth factor, \( \alpha \), with the wave number, \( K \), for different values of the effective deceleration, for the water-air, aluminum-polystyrene (liquid) and aluminum-polystyrene (gas) systems, respectively. As the effective deceleration increases, the growth factor increases. This makes the instability grow faster, leading to smaller time-scales. Also, any instability with a wavelength smaller than the critical wavelength, \( \lambda_0 \), does not grow. From Tables 5.1, 5.2 and 5.3 it can be seen that for the same value of the effective deceleration the instability grows faster in the water-air system than in the aluminum-polystyrene system. This is primarily due to the fact that the interfacial tension of the water-air system is much lower.

66
### Table 5.1 Growth parameters of the water-air system.

<table>
<thead>
<tr>
<th>G, m/s²</th>
<th>( \lambda_{wp} ), mm</th>
<th>( \lambda_{mp} ), mm</th>
<th>( \alpha_{max} ), s⁻¹</th>
<th>( 1/\alpha_{max} ), s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>168.6</td>
<td>292.0</td>
<td>1.1977</td>
<td>0.8349</td>
</tr>
<tr>
<td>0.5</td>
<td>75.4</td>
<td>130.6</td>
<td>4.0047</td>
<td>0.2497</td>
</tr>
<tr>
<td>1.0</td>
<td>53.3</td>
<td>92.3</td>
<td>6.7351</td>
<td>0.1485</td>
</tr>
<tr>
<td>5.0</td>
<td>23.8</td>
<td>41.3</td>
<td>22.5200</td>
<td>0.0444</td>
</tr>
<tr>
<td>10.0</td>
<td>16.9</td>
<td>29.2</td>
<td>37.874</td>
<td>0.0264</td>
</tr>
</tbody>
</table>

### Table 5.2 Growth parameters of the aluminum-polystyrene (liquid) system.

<table>
<thead>
<tr>
<th>G, m/s²</th>
<th>( \lambda_{wp} ), mm</th>
<th>( \lambda_{mp} ), mm</th>
<th>( \alpha_{max} ), s⁻¹</th>
<th>( 1/\alpha_{max} ), s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>551.8</td>
<td>955.8</td>
<td>0.3806</td>
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</tr>
<tr>
<td>0.5</td>
<td>246.8</td>
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<td>1.2727</td>
<td>0.7858</td>
</tr>
<tr>
<td>1.0</td>
<td>174.5</td>
<td>302.2</td>
<td>2.1403</td>
<td>0.4672</td>
</tr>
<tr>
<td>5.0</td>
<td>78.0</td>
<td>135.2</td>
<td>7.1566</td>
<td>0.1397</td>
</tr>
<tr>
<td>10.0</td>
<td>55.2</td>
<td>95.6</td>
<td>12.0360</td>
<td>0.0831</td>
</tr>
</tbody>
</table>

### Table 5.3 Growth parameters of the aluminum-polystyrene (gas) system.

<table>
<thead>
<tr>
<th>G, m/s²</th>
<th>( \lambda_{wp} ), mm</th>
<th>( \lambda_{mp} ), mm</th>
<th>( \alpha_{max} ), s⁻¹</th>
<th>( 1/\alpha_{max} ), s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
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<td>0.7885</td>
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</tr>
<tr>
<td>0.5</td>
<td>173.9</td>
<td>301.3</td>
<td>2.6366</td>
<td>0.3793</td>
</tr>
<tr>
<td>1.0</td>
<td>123.0</td>
<td>213.0</td>
<td>4.4341</td>
<td>0.2255</td>
</tr>
<tr>
<td>5.0</td>
<td>55.0</td>
<td>95.3</td>
<td>14.8264</td>
<td>0.0674</td>
</tr>
<tr>
<td>10.0</td>
<td>38.9</td>
<td>67.4</td>
<td>24.9350</td>
<td>0.0401</td>
</tr>
</tbody>
</table>
**Figure 5.1** The variation of the growth factor \( \alpha \) with the wavelength \( \lambda \) for different values of the effective deceleration \( G \), for the water-air system.

**Figure 5.2** The variation of the growth factor \( \alpha \) with the wave number \( K \) for different values of the effective deceleration \( G \), for the water-air system.
Figure 5.3 The variation of the growth factor $\alpha$ with the wavelength $\lambda$ for different values of the effective deceleration $G$, for the Al-Ps (liquid) system.

Figure 5.4 The variation of the growth factor $\alpha$ with the wave number $K$ for different values of the effective deceleration $G$, for the Al-Ps (liquid) system.
Figure 5.5 The variation of the growth factor $\alpha$ with the wavelength $\lambda$ for different values of the effective deceleration $G$, for the Al-Ps (gas) system.

Figure 5.6 The variation of the growth factor $\alpha$ with the wave number $K$ for different values of the effective deceleration $G$, for the Al-Ps (gas) system.
Furthermore, for the same effective deceleration, the instability grows faster in the aluminum-polystyrene (gas) system than in the aluminum-polystyrene (liquid) system. This is because the density of liquid polystyrene is much higher than that of the gaseous polystyrene decomposition products, and is significant in comparison with the density of aluminum. In LFC, an interface is formed between molten aluminum and the gaseous polystyrene decomposition products.

Using the Rayleigh-Taylor instability theory, the various casting configurations are studied for instability growth. The possible horizontal configurations in Lost Foam Casting are: (i) advancing and accelerating metal front (Figure 5.7 (a)) and (ii) advancing metal front decelerating against the foam (Figure 5.7 (b)). When the metal front accelerates, the interface is always stable. If the metal front decelerates, it will be unstable if $|a_1| > \frac{\sigma K^3}{\rho}$, where $|a_1|$ is the deceleration of the metal front, $K$ is the wave number of the perturbation, $\sigma$ the interfacial tension and $\rho$ the density of aluminum.

There are two possible vertical configurations: (i) metal below the foam and (ii) metal above the foam. The two possible conditions in LFC when the metal is below the foam are: (a) metal flowing upwards and accelerating (Figure 5.8 (a)) and (b) metal flowing upwards and decelerating (Figure 5.8 (b)). When the metal is flowing upwards and accelerating the flow is always stable and there can be no growth of any instability. However, when the metal is flowing upwards but decelerating against the foam, the interface will be unstable if $|a_2| > 9.81 \text{ m/s}^2$ and if $|G|K > \frac{\sigma K^3}{\rho}$, where $G = a - g$ and $|a_2|$ the deceleration of the flow front. The two possible conditions in LFC when the metal is above the foam are: (i) metal flowing downwards and accelerating (Figure 5.9 (a)) and (ii) metal flowing downwards and decelerating (Figure 5.9 (b)). Case (i) is unstable if $|a_3| < 9.81 \text{ m/s}^2$ and if $|G|K > \frac{\sigma K^3}{\rho}$, and is stable otherwise. Case (ii) is unstable if $|G|K > \frac{\sigma K^3}{\rho}$ irrespective of the magnitude of the deceleration.

For the various casting configurations possible, the conditions that lead to the growth of the instability are worked out for different sizes of castings. Table 5.4 shows the casting configuration, the size of the casting and the conditions for instability. Here, the size of the casting refers to the larger dimension of the cross-section that is being examined for instability growth. Table 5.4 shows that in the case of horizontal metal flow, instabilities for small castings can grow only if the deceleration of the metal front is
Figure 5.7 Schematics showing: (a) advancing and accelerating metal front (always stable), (b) advancing and decelerating metal front (may or may not be unstable).

Figure 5.8 Schematics showing: (a) advancing and accelerating metal front (always stable), (b) advancing and decelerating metal front (may or may not be unstable).
Figure 5.9 Schematics showing: (a) advancing and accelerating metal front (may or may not be unstable), (b) advancing and decelerating metal front (may or may not be unstable).
Table 5.4 Conditions for instability growth in Al LFC.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>size of casting (mm)</th>
<th>Acceleration/ deceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Horizontal flow - metal front decelerating against the foam.</td>
<td>Minimum deceleration for instability</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>605.17</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>151.29</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>6.05</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.06</td>
</tr>
<tr>
<td>3. Horizontal flow - metal front moving with uniform velocity.</td>
<td>—</td>
<td>Always stable.</td>
</tr>
<tr>
<td>4. Metal below the foam - flowing upwards and accelerating.</td>
<td>—</td>
<td>Always stable.</td>
</tr>
<tr>
<td>5. Metal below the foam - flowing upwards and decelerating.</td>
<td>Minimum deceleration for instability</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>614.97</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>161.09</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>15.85</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>11.31</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>10.04</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>9.86</td>
</tr>
<tr>
<td>Configuration</td>
<td>6. Metal below the foam - flowing upwards with uniform velocity.</td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>-----------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>—</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always stable</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration</th>
<th>7. Metal above the foam - flowing downwards and accelerating.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum acceleration for instability</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Stable at all accelerations</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Stable at all accelerations</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>3.75</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>8.29</td>
</tr>
<tr>
<td></td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>9.56</td>
</tr>
<tr>
<td></td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>9.74</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration</th>
<th>8. Metal above the foam - flowing downwards and decelerating.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum deceleration for instability</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>595.37</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>141.49</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Unstable at all decelerations</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Unstable at all decelerations</td>
</tr>
<tr>
<td></td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>Unstable at all decelerations</td>
</tr>
<tr>
<td></td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Unstable at all decelerations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration</th>
<th>9. Metal above the foam - moving downwards with uniform velocity.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>≤ 39.2</td>
</tr>
<tr>
<td></td>
<td>Always stable</td>
</tr>
<tr>
<td></td>
<td>&gt; 39.2</td>
</tr>
<tr>
<td></td>
<td>Always unstable</td>
</tr>
</tbody>
</table>
large. However, for large castings the instabilities can grow even at very low decelerations of the metal front, but in no case does the instability grow if the flow accelerates.

In the case of metal being below the foam, in a vertical configuration, as in Figure 5.8, the growth of instability for small castings is possible only if the metal front decelerates at a very fast rate as it flows. For large castings the deceleration required for instability growth decreases. However, if the metal front accelerates, or decelerates at a rate less than $9.81 \text{ m/s}^2$, as it flows upwards, then in no case does the instability grow. When the metal is above the foam, in a vertical configuration, as in Figure 5.9, it should decelerate at a very fast rate for any growth of instability in case of small castings. For large castings, however, the instability grows if the front decelerates (at any rate), or even if the front accelerates, as long as the acceleration of the front is less than $9.81 \text{ m/s}^2$.

The stability and instability zones for various casting sizes in terms of the acceleration or deceleration of the metal front, for the cases mentioned in Table 5.4, are shown in Figures 5.11, 5.13, 5.15 and 5.17. Figure 5.11 is for the case of horizontal flow, and shows the minimum deceleration, $d$, required for instability growth for different sizes of castings. Figure 5.13 is for the case of vertical flow, metal below the foam, flowing upwards and decelerating. It shows the minimum deceleration, $d$, required for the instability to grow for different sizes, $s$, of the castings. Figure 5.15 is for the case of vertical flow, metal above the foam, flowing downwards and accelerating. It shows the maximum acceleration, $a$, for instability growth (if the acceleration is higher then there is no growth of the instability) for different sizes, $s$, of the castings. Figure 5.17 is for the case of vertical flow, metal above the foam, flowing downwards and decelerating. It shows the minimum deceleration, $d$, required for instability growth for different sizes, $s$, of the castings. When the metal is above the foam in a vertical configuration, flowing downwards with a uniform velocity, then there will be no growth of any instability if the size, $s$, of the casting is less than 39.2 mm. For larger castings, the instability will grow even for uniform flow downwards.

Thus, in general, the horizontal casting configuration, and the vertical casting configuration in which the metal is below the foam, are stable configurations as long as the metal front does not decelerate. However, a vertical configuration in which the metal
is above the foam is inherently unstable for castings of all sizes (other than very small castings). The best casting configurations, from the point of view of instability related defect formation, are: horizontal flow—metal accelerating against the foam, and vertical flow (metal below the foam)—metal flowing upwards and accelerating.
Figure 5.10 Metal front decelerating against the foam in the horizontal flow condition.

Figure 5.11 The stable and unstable zones in terms of the deceleration, $d$, of the metal front for different sizes, $s$, of the casting for the horizontal flow condition.
Figure 5.12 Metal below the foam, flowing upwards and decelerating.

Figure 5.13 The stable and unstable zones in terms of the deceleration, $d$, of the metal front for different sizes, $s$, of the casting for vertical flow condition (metal below the foam, flowing upwards and decelerating).
Figure 5.14 Metal above the foam, flowing downwards and accelerating.

Figure 5.15 The stable and unstable zones in terms of the acceleration, $a$, of the metal front for different sizes, $s$, of the casting for vertical flow condition (metal above the foam, flowing downwards and accelerating).
Figure 5.16 Metal above the foam, flowing downwards and decelerating.

Figure 5.17 The stable and unstable zones in terms of the deceleration, \( d \), of the metal front for different sizes, \( s \), of the casting for vertical flow condition (metal above the foam, flowing downwards and decelerating).
Chapter 6

Summary and Future Work

6.1 Summary

In this work, a hypothesis based on Rayleigh-Taylor instability was advanced for the defect formation mechanism in Lost Foam Casting (LFC). One of the major reasons for defect formation in LFC is the entrapment of foam decomposition products within the castings. The hypothesis states that any instability of the aluminum-polystyrene interface could grow, and fingers could thus be formed. The fingers could eventually encounter each other, thereby trapping the foam decomposition products between them. To confirm the hypothesis and to visualize and analyze the growth of the instability an apparatus was designed and fabricated.

A substantial amount of experimental work has already been done on the Rayleigh-Taylor instability. However, most of the studies have employed vertical set-ups to visualize and quantify the instability growth. The effective accelerations involved have been 10-70 g. Also, in these studies the denser fluid has been accelerated by the lighter fluid. In LFC, none of these conditions prevail. The accelerations involved are much smaller and the heavy fluid, i.e., aluminum, decelerates against the lighter fluid, i.e., the polystyrene decomposition products. Thus, the experimental set-up built provided smaller decelerations, of the order of 0.1 g, and was one in which the water decelerated against the air.

The experimental set-up used the falling hydraulic head and viscous losses to provide small decelerations to the flow. An equation governing the kinematics of the flow in the set-up was developed using the unsteady Bernoulli’s equation, the head loss equations and continuity. The experimental parameters could be varied to achieve different decelerations of the flow front. High-speed photography was employed to capture the image of the interface because of the high flow velocities.
Experiments were conducted for cases in which the instability would grow, i.e., water decelerating, as well as for the cases where there would be no growth of the instability, i.e., steady or accelerating flow. The results were in agreement with the linear theory for the Rayleigh-Taylor instability. The theory was then used to quantify the growth of instabilities for different conditions of flow for both the water-air and the aluminum-polystyrene systems. The length- and time-scales of the instabilities were also worked out. The results of the experiments conducted, as well as the calculations based on the theory, support the hypothesis of defect formation in LFC.

To avoid defect formation in LFC the best flow configurations would be: (i) horizontal flow—metal accelerating against the foam and (ii) vertical flow (metal below the foam)—metal flowing upwards and accelerating. Two very important factors governing defect formation in LFC are: (i) gating system design and (ii) venting. Any perturbation with a wavelength smaller than the critical wavelength does not grow. This fact can be employed when designing castings by orienting them such that the effective deceleration in each direction gives a value of the critical wavelength that is larger than the size of the casting normal to that direction. In this way, the growth of the instability can be prevented and fold formation due to Rayleigh-Taylor instability can be avoided. However, it may not always be possible to orient the castings suitably or to control the sizes. Another possibility is to alter the decelerations faced by the aluminum-polystyrene front by reducing the back pressure that the aluminum-polystyrene front faces as it flows. As the polystyrene foam decomposes, the decomposition products, if not eliminated from the casting, will exert a back pressure on the aluminum-polystyrene front and thus decelerate it. The use of vents [Fu et al., 1991] to vent out some of the foam decomposition products can help in reducing the back pressure and thus the deceleration encountered by the interface. As the deceleration decreases, the growth factor, $\alpha$, would also decrease and the critical wavelength, $\lambda_0$, would increase. Both these effects help in reducing the rate of growth of the instability. Foam density can also be controlled in order to reduce the effective deceleration. A low-density foam would generate a smaller volume of decomposition products and would thus exert a lower back pressure on the interface.
6.2 Future Work

Through this work, an attempt has been made to provide a different perspective on defect formation in Lost Foam Casting. However, because this was a first attempt to explore the role of instabilities in defect formation in LFC, a substantial amount of work must be conducted in the near future.

The experiments conducted and the instability growth calculations were based on the linear theory only. During the later stages of the instability, the non-linear theory might be better suited to analyze the problem. Thus, models based on the non-linear theory must be developed and verified through experimental work.

The experiments conducted in this study used the water-air system. Confirming experiments should be done using the aluminum-polystyrene system. Different decelerations should be provided to the aluminum-polystyrene front and the severity of defects related to the magnitude of the deceleration applied. This will help predict more accurately the growth of the instabilities and thus the length- and time-scales of defect formation in LFC. Another approach, which should be used for validating the defect formation model, is the direct imaging of the LFC process. A convenient way of imaging the process is to use x-ray imaging techniques such as tomography and laminography. The advantage of these techniques is that they are non-destructive in nature.

Finally, in addition to the solutions summarized in this work, more solutions should be generated and applied so as to reduce the defects in Lost Foam Casting and to facilitate its use for wider commercial applications.


Appendix A

Table A.1 Runge-Kutta-Nyström method

<table>
<thead>
<tr>
<th>Algorithm R-K-N(f, x₀, y₀, y₀', h, N).</th>
</tr>
</thead>
<tbody>
<tr>
<td>This algorithm computes the solution of the initial value problem</td>
</tr>
<tr>
<td>y'' = f(x, y, y'), y(x₀) = y₀, y'(x₀) = y₀' at equidistant points</td>
</tr>
<tr>
<td>x₁ = x₀ + h, x₂ = x₀ + 2h, ..., xₙ = x₀ + Nh; here f is such that this</td>
</tr>
<tr>
<td>problem has a unique solution on the interval [x₀, xₙ].</td>
</tr>
<tr>
<td>INPUT: Initial values x₀, y₀, y₀', step size h, number of steps N</td>
</tr>
<tr>
<td>OUTPUT: Approximation yₙ₊₁ to the solution y(xₙ₊₁) at</td>
</tr>
<tr>
<td>xₙ₊₁ = x₀ + (n + 1)h, where n = 0, 1, ..., N - 1</td>
</tr>
<tr>
<td>For n = 0, 1, ..., N - 1 do:</td>
</tr>
<tr>
<td>k₁ = ( \frac{1}{2} hf(xₙ, yₙ, yₙ') )</td>
</tr>
<tr>
<td>k₂ = ( \frac{1}{2} hf(xₙ + \frac{1}{2} h, yₙ + K, yₙ' + k₁) )</td>
</tr>
<tr>
<td>where K = ( \frac{1}{2} h(yₙ' + \frac{1}{2} k₁) )</td>
</tr>
<tr>
<td>k₃ = ( \frac{1}{2} hf(xₙ + \frac{1}{2} h, yₙ + k, yₙ' + k₂) )</td>
</tr>
<tr>
<td>k₄ = ( \frac{1}{2} hf(xₙ + h, yₙ + L, yₙ' + 2k₃) )</td>
</tr>
<tr>
<td>where L = h(yₙ' + k₃)</td>
</tr>
<tr>
<td>xₙ₊₁ = xₙ + h</td>
</tr>
<tr>
<td>yₙ₊₁ = yₙ + h(yₙ' + \frac{1}{3}(k₁ + k₂ + k₃))</td>
</tr>
<tr>
<td>OUTPUT xₙ₊₁, yₙ₊₁</td>
</tr>
<tr>
<td>yₙ₊₁' = yₙ' + \frac{1}{3}(k₁ + 2k₂ + 2k₃ + k₄)</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>Stop</td>
</tr>
</tbody>
</table>

x in the above algorithm corresponds to time, t, in Equation (3.25); y corresponds to the distance covered, x; y' corresponds to \( \frac{dx}{dt} \); and y'' corresponds to \( \frac{d²x}{dt²} \), in Equation (3.25). Thus, the solution of the above algorithm gives the velocity of the front at every instant of time and from that the deceleration of the front is determined.
Appendix B

Matlab simulation code for obtaining the acceleration of the flow front – for the case when the connecting tube area is greater than the mold area.

\[ A_1 = 0.000506; \quad \% \text{Area of the Reservoir} \]
\[ A_2 = 0.000506; \quad \% \text{Area of the connecting tube} \]
\[ A_4 = 0.000387; \quad \% \text{Area of the mold} \]
\[ l_t = 0.4; \quad \% \text{Length of the connecting tube} \]
\[ h_i = 1; \quad \% \text{Initial hydraulic head} \]
\[ d_t = 0.0254; \quad \% \text{Diameter of the tube} \]
\[ d_m = 0.00953; \quad \% \text{Hydraulic diameter of the mold} \]
\[ d_r = 0.0254; \quad \% \text{Diameter of the reservoir} \]
\[ f = 0.03; \quad \% \text{Darcy friction factor} \]
\[ c_1 = 0.4*(1-(A_2/A_1)); \quad \% \text{Head loss coefficient, K, for area change between the reservoir and the connecting tube} \]
\[ c_2 = 0.4*(1-(A_4/A_2)); \quad \% \text{Head loss coefficient, K, for area change between the connecting tube and the mold} \]
\[ x(1)=0; \quad \% x \text{ is the time, } y \text{ the distance traveled, } z \text{ the velocity and acc is the acceleration of the flow front} \]
\[ y(1)=0; \]
\[ z(1)=0; \]
\[ h=0.01; \]
\[ \text{sum}(1)=0; \]
\[ j=0; \]
\[ \text{for } i=1:85 \]

\[ k_1 = 0.5*h*((0.5*(z(i)+k_1)^2)*((A_4/A_1)^2)-(f/dr*)((A_4/A_1)^2)*h_i*(A_4/A_1)*y(i))-f*(lt/dt*)((A_4/A_2)^2)-f*(((A_4/A_2)^2)-f*y(i)*dm/(c_1/4)*(((A_4/A_1)+(A_4/A_2))^2)-(c_2/4)*(((A_4/A_1)+1)^2))+9.8*(h_i-(A_4/A_1)*y(i))/(y(i)*((1-((A_4/A_1)^2))+A_4*hi/A_1+A_4*lt/A_2)); \]
\[ c=0.5*h*(z(i)+0.5*k_1); \]

\[ k_2 = 0.5*h*(((0.5*((z(i)+k_1)^2)*((A_4/A_1)^2)-(f/dr*)((A_4/A_1)^2)*h_i*(A_4/A_1)*y(i)+c)-f*(lt/dt*)((A_4/A_2)^2)-f*(((A_4/A_2)^2)+f*(((y(i)+c)/dm/(c_1/4)*(((A_4/A_1)+(A_4/A_2))^2)-(c_2/4)*(((A_4/A_1)+1)^2))+9.8*(h_i-(A_4/A_1)*(y(i)+c))/(y(i)+c)*((1-((A_4/A_1)^2))+A_4*hi/A_1+A_4*lt/A_2)); \]

\[ k_3 = 0.5*h*((0.5*((z(i)+k_2)^2)*((A_4/A_1)^2)-(f/dr*)((A_4/A_1)^2)*h_i*(A_4/A_1)*(y(i)+c))-f*(lt/dt*)((A_4/A_2)^2)-f*(((y(i)+c)/dm/(c_1/4)*(((A_4/A_1)+(A_4/A_2))^2)-(c_2/4)*(((A_4/A_2)+1)^2))+9.8*(h_i-(A_4/A_1)*(y(i)+c))/(y(i)+c)*((1-((A_4/A_1)^2))+A_4*hi/A_1+A_4*lt/A_2)); \]

\[ l=h*(z(i)+k_3); \]
\[ k4 = 0.5 \cdot h \cdot ((0.5 \cdot ((z(i) + 2 \cdot k3)^2) \cdot ((A4/A1)^2) - 1 - (f/d) \cdot ((A4/A1)^2) \cdot (h - (A4/A1) \cdot (y(i) + l)) - f \cdot (lt/d) \cdot ((A4/A2)^2) \cdot f \cdot (y(i) + l) \cdot (dm - c1/4) \cdot ((A4/A1) + (A4/A2))^{2}) - (c2/4) \cdot ((A4/A2) + 1)^2)) + 9.8 \cdot (h - (A4/A1) \cdot (y(i) + l)) \cdot ((y(i) + l) \cdot (1 - ((A4/A1)^2) + A4 \cdot hi/A1 + A4 \cdot lt/A2)); \]

\[ x(i+1) = x(i) + h; \]
\[ y(i+1) = y(i) + h \cdot (z(i) + (1/3) \cdot (k1 + k2 + k3)); \]
\[ z(i+1) = z(i) + (1/3) \cdot (k1 + 2 \cdot k2 + 2 \cdot k3 + k4); \]
\[ acc(i) = ((z(i+1) - z(i))/h); \]
\[ if \ acc(i) < 0 \]
\[ \quad sum(i+1) = sum(i) + acc(i); \]
\[ \quad j = j + 1; \]
\[ else \ sum(i+1) = sum(i); \]
\[ end \]
\[ end \]
\[ acc(i+1) = acc(i) + (acc(i) - acc(i-1)); \]
\[ avacc = (sum(i+1) + acc(i+1))/(j+1); \]
\[ dtime = (j+1)/100; \]
\[ plot(x, acc); \]
Matlab simulation code for obtaining the acceleration of the flow front – for the case when the connecting tube area is less than the mold area.

A1=0.000126;  % Area of the Reservoir
A2=0.000126;  % Area of the connecting tube
A4=0.000387;  % Area of the mold
lt=0.3;  % Length of the connecting tube
hi=1.5;  % Initial hydraulic head
dt=0.0126;  % Diameter of the tube
dm=0.00953;  % Hydraulic diameter of the mold
dr=0.0126;  % Diameter of the reservoir
f=0.03;  % Darcy friction factor
c1=0.4*(1-(A2/A1));  % Head loss coefficient, K, for area change between the reservoir and the connecting tube
c2=1-(A4/A2)/dm;  % Head loss coefficient, K, for area change between the connecting tube and the mold
x(1)=0;  % x is the time, y the distance traveled, z the velocity and ace is the acceleration of the flow front
y(1)=0;
z(1)=0;
h=0.01;
sum(1)=0;
j=0;
for i=1:75

k1=0.5*h*((0.5*(z(i)^2)*((A4/A1)^2)-1-(f/dr)*((A4/A1)^2)*(hi-(A4/A1)*y(i))-f*(lt/dt)*((A4/A2)^2)-f*(y(i)/dm)-(c1/4)*(((A4/A1)+(A4/A2))^2)-(c2/4)*(((A4/A2)+1)^2)))+9.8*(hi-(A4/A1)*y(i))/((y(i)+(1-(A4/A1)^2))+A4*hi/A1+A4*lt/A2));

k2=0.5*h*((0.5*((z(i)+k1)^2)*((A4/A1)^2)-1-(f/dr)*((A4/A1)^2)*(hi-(A4/A1)*y(i)+c))-f*(lt/dt)*((A4/A2)^2)+f*(y(i)+c)/dm-(c1/4)*(((A4/A1)+(A4/A2))^2)-(c2/4)*(((A4/A2)+1)^2))+9.8*(hi-(A4/A1)*(y(i)+c))/((y(i)+c)*(1-(A4/A1)^2))+A4*hi/A1+A4*lt/A2));

k3=0.5*h*((0.5*((z(i)+k2)^2)*((A4/A1)^2)-1-(f/dr)*((A4/A1)^2)+(hi-(A4/A1)*(y(i)+c))-f*(lt/dt)*((A4/A2)^2)-f*(y(i)+c)/dm+(c1/4)*(((A4/A1)+(A4/A2)^2)-(c2/4)*(((A4/A2)+1)^2)))+9.8*(hi-(A4/A1)*(y(i)+c))/((y(i)+c)*(1-(A4/A1)^2))+A4*hi/A1+A4*lt/A2));

l=h*(z(i)+k3);

k4=0.5*h*((0.5*(z(i)+2*k3)^2)*((A4/A1)^2)-1-(f/dr)*((A4/A1)^2)+(hi-(A4/A1)*(y(i)+l))-f*(lt/dt)*((A4/A2)^2)-f*(y(i)+l)/dm+(c1/4)*(((A4/A1)+(A4/A2)^2)-(c2/4)*(((A4/A2)+1)^2)))+9.8*(hi-(A4/A1)*(y(i)+l))/((y(i)+l)*(1-(A4/A1)^2))+A4*hi/A1+A4*lt/A2));

end
\[(c2/4)(((A4/A2)+1)^2))+9.8*(hi-(A4/A1)*(y(i)+1))/((y(i)+1)*(1-(A4/A1)^2)+A4*hi/A1+A4*lt/A2));\]

\[x(i+1)=x(i)+h;\]
\[y(i+1)=y(i)+h*(z(i)+(1/3)*(k1+k2+k3));\]
\[z(i+1)=z(i)+(1/3)*(k1+2*k2+2*k3+k4);\]
\[acc(i)=((z(i+1)-z(i))/h);\]
\[\text{if acc(i)<0}\]
\[\text{sum(i+1)=sum(i)+acc(i); j=j+1;}\]
\[\text{else sum(i+1)=sum(i);}\]
\[\text{end}\]
\[\text{end}\]
\[\text{acc(i+1)=acc(i)+(acc(i)-acc(i-1));}\]
\[\text{avacc=(sum(i+1)+acc(i+1))/(j+1);}\]
\[\text{dtime=(j+1)/100;}\]
\[\text{plot(x,acc);}\]
Appendix C

Matlab code for finding the experimental value of the growth-factor ($\alpha$)

\begin{verbatim}
G=1.6;  % Magnitude of the effective acceleration
L=0.05; % Wavelength of the instability
Al=((6.283*((G/L)-(0.0028424/(L^3))))^(0.5));  % Theoretical value of the growth-factor

% Time, t_1
% Time, t_2
A1=1.25;  % Amplitude of the instability at time t_1
A2=3.375;  % Amplitude of the instability at time t_2

LL=(A1-5);
UL=(A1+5);
R=A2/A1;
E(1)=2;
S(1)=LL;
for i=2:100
    x(i)=LL+(i-2)*(UL-LL)/98.0;
    y(i)=cosh(x(i)*t2);
    z(i)=cosh(x(i)*t1);
    M(i)=y(i)/z(i);
    T(i)=abs(M(i)-R);
    if T(i)<E(i-1) E(i)=T(i);
    S(i)=x(i);
    else E(i)=E(i-1); S(i)=S(i-1);
end
end
% S(100) is the experimental value of the growth-factor
\end{verbatim}
## Appendix D

Table D.1  MotionScope  8000 S Monochrome Recording Time/ Frame Storage.

<table>
<thead>
<tr>
<th>Frame Rate</th>
<th>Resolution</th>
<th>Memory</th>
</tr>
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<tr>
<td>(frames/s)</td>
<td>(pixels)</td>
<td># of frames</td>
</tr>
<tr>
<td>50</td>
<td>480 x 420</td>
<td>2,048</td>
</tr>
<tr>
<td>50E</td>
<td>240 x 210</td>
<td>8,192</td>
</tr>
<tr>
<td>60</td>
<td>480 x 420</td>
<td>2,048</td>
</tr>
<tr>
<td>60E</td>
<td>240 x 210</td>
<td>8,192</td>
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<tr>
<td>125</td>
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<td>2,048</td>
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<tr>
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<td>8,192</td>
</tr>
<tr>
<td>250</td>
<td>480 x 420</td>
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<tr>
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</tr>
<tr>
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<td>60 x 68</td>
<td>65,5536</td>
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