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Dynamical Characterization, State Estimation and Testing of Active Compressor Blades

by

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MASTER OF SCIENCE IN MECHANICAL ENGINEERING

Abstract

This thesis is part of an effort towards the development of the Active Rotor – a transonic compressor stage in which the motion of individual blades are actuated and controlled according to prescribed trajectories. This rotor will serve as a research tool for the experimental investigation of aeroelasticity in turbomachines. The active blade consists of graphite-epoxy twin spars that are piezoelectrically actuated at the blade root. High strength-to-weight foam covers the spars to give the blade its aerodynamic shape. Sensing is attained by strain gages collocated with the actuators.

Two procedures are presented for obtaining models for the dynamics of the active blade. The first procedure adopts a purely experimental approach in which the transfer function matrix of the system is empirically determined. The second approach relies on a combination of finite element modeling and experimental results to arrive at a dynamical model of the active element and its supporting signal conditioning systems. The two approaches are implemented on the active elements of the blade. Due to discrepancies between the finite element model and experimental data, the model resulting from the experimental approach is used, recognizing the need for updating and validating the finite element model.

Using the empirical model, state estimators are designed to estimate tip deflection from strain measurements at the blade root. Under the assumptions of white, Gaussian process and sensor noises, a Kalman filter is designed to provide the estimates. The deflection estimates are compared with experimental data in the time domain. Results were comparable, and disagreements are attributed to modeling approximation. Kalman filters are not optimal in the presence of modeling errors, and robust estimation is suggested as an alternative.

The development of the Active Rotor requires spin testing throughout its development stages to test the blades for structural integrity and actuation capability. An evacuated-chamber, high-speed spin testing facility is developed for that purpose. The functional requirements of the facility are presented and related to its components. The design of a rotating hub to accommodate active blades under development is presented. Issues pertaining to assembly, instrumentation, characterization and shakedown are described in detail.

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Thesis Reader: Professor Harry Asada
Title: Professor of Mechanical Engineering
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Chapter 1

Introduction

1.1 Aeroelasticity in Turbomachines

Engine high cycle fatigue (HCF) is one of the main factors limiting the development of more powerful, lighter, and higher performance gas turbine engines. Vibration of rotating bladed structures has been a main concern for designers since the inception of gas turbines for power generation or aeropropulsion. One of the causes of vibration in turbomachines is the dynamic coupling between structural and aerodynamic forces in an engine. Under certain conditions, vibration in a bladed structure rotating in a high energy flow leads to pressure perturbations in the fluid. These pressure perturbations can feedback in a positive sense into the vibration of the structure. In effect, the fluid will be providing negative damping to the structure, thereby doing work that increases its vibration levels, leading to overall system instability. This mode of instability, known as flutter, may lead to mechanical failure of the blade, or to sustained high levels of vibration and dynamic stresses, which in turn reduce the fatigue life of the structure.

Flutter may occur in situations where a vibrating structure is exposed to a fluid flow. A structure, such as an aircraft wing, may experience flutter as it vibrates in a flow of air. In turbomachines, the problem is generally much more complex than in isolated structures [8]. In the case of a solitary structure, the fluid forcing function is solely due to its own motion. However, in blade cascades in turbomachines, the forcing function on a blade is due to its own motion, as well as the motion of other blades in the cascade. Specifically, it depends on:

- *Mode shape of vibration* of individual blades. Typically the mode shapes are of bending or twisting, as shown in in figure 1-1.
• **Temporal variation of motion of blades.** Assuming that all blades vibrate with the same mode shape, they do necessarily do so in unison. The temporal variation of motion is captured by *travelling waves* along the circumference of the rotor. Travelling waves are characterized by a *harmonic number*, or number of *nodal diameters*, as shown in figure 1-2.

• **Interblade phase angle,** which is the phase relationship between the motion of a blade to other blades.

![Figure 1-1: Two modes of blade vibration, bending and twisting.](image)

This introduces a grid of interactions with several dimensions: interblade phase angle, harmonic number of travelling wave mode, and vibratory modes of individual blades. As a result, the complexity of the problem is significantly higher than the case of a solitary structure.

Current state-of-the-art design tools fail to adequately account for flutter in the early design stages of engines. This is primarily due to the lack of flutter prediction models, which in turn is due to a lack of experimental data in that area. In order to validate any proposed model for flutter prediction, there is a need for reliable experimental data. The lack of experimental data in aeromechanics is a result of the difficulty and cost associated with conducting reliable experiments. This is largely due to the harshness of the operating environment, as well as the difficulty in controlling a flutter instability once it occurs.

For a complete and detailed treatment of aeroelasticity and forced vibration in blade cascades, refer to [8] and [21].
1.2 The Active Rotor

The Active Rotor is a proposed tool for conducting experimental investigations of flutter in blade cascades. Essentially it is a rotor in which the motion of individual blades are actuated and controlled. The two basic modes of actuation considered are bending and twisting, replicating the two fundamental flutter modes in most blades. In this section, the actuation concept is described. The need for the active rotor is then motivated through a description of three potential experiments that would require such a device. Finally a description of the rotor, its geometry, features and functional requirements is presented.

1.2.1 Concept

The currently adopted concept for the Active Rotor is what is dubbed the “spar-and-shell” concept. Because the energy required to actuate a typical steel or titanium blade is very high, alternate blade materials are used. Consider a pair of graphite-epoxy spars as shown in figure 1-3(a). Piezoelectric laminates (PZT-5A\textsuperscript{1}) are bonded to the root of the spars on each face. By applying suitable potential to these laminates with correct polarity, each spar can be independently deflected. Actuation of the spars

\textsuperscript{1}According to the convention used by Morgan Matroc Corporation.
in-phase will create an overall bending deformation of the structure, whereas actuation out-of-phase will create a twisting deformation. The two modes of actuation are illustrated in figure 1-5. Sensing is attained through the use of strain gages bonded on the exposed surfaces of the piezos, thereby providing collocated actuation and sensing. The aerodynamic shape is obtained by covering the twin-spar set with high strength-to-weight ratio foam\(^2\) that is shaped and bonded to the skeleton.

![Figure 1-3: Construction of Active Blade: (a) Graphite epoxy skeleton & piezoelectric actuators (b) Skeleton covered with foam.](image)

1.2.2 Motivation

In order to motivate the utility of the Active Rotor, three potential experiments are described that will test for different aspects of flutter. These experiments either necessitate such an experimental device, or are conducted with the active rotor with significant simplicity and reduction of costs. Experiments 1

\(^2\)RohaCell 300
Figure 1-5: Two modes of actuation on a finite element model of the twin spars superimposed on the original undeformed shape. (a) bending, (b) twisting.

and 2 focus on determining the dynamical characteristics of the rotor, and may be regarded as different approaches to the same test, whereas experiment 3 studies the effects of mistuning in rotors.

**Experiment 1: Measurement of Influence Coefficients:** The effect of the motion of one blade on another is captured by influence coefficients. In general, the motion of a blade will cause pressure perturbations on other blades. The nature of these perturbations depends on the mode shape of one blade vibration, the harmonic mode number of the traveling wave of blade vibrations, the interblade phase angle, as well as the flow conditions. Generally, if the equation of motion of a blade is written in modal coordinates, the single DOF equation of motion for a particular mode is:

\[ m_i \ddot{w} + b_i \dot{w} + k_i w = f_i \]  

(1.1)

where \( m_i, b_i, k_i \) and \( f_i \) are the modal mass, damping, stiffness and generalized forcing function for blade mode \( i \). The force \( f_i \) is a summation of contributions of the effects of the motions of all the blades in the cascade, and is given by 1.2:

\[ f_i = \frac{\rho}{2} V i^2 l_c \sum_{n=0}^{N-1} C_\theta \phi \overline{q}_\theta e^{j(\omega t + i \theta)} \]  

(1.2)
where $\phi_0$ is a blade vibration mode, with an amplitude determined by the travelling wave mode $\tilde{q}_0$ and phase $\theta$. $C_\theta$ is a complex influence coefficient that relates the force generated on the blade due to the motion of another at interblade phase angle $\alpha$ for the given modes. Proper dimensions are obtained by multiplying by the dynamic pressure $\frac{\rho V_1^2}{2} l_0$, where $\rho$ is the fluid density, $V_1$ is the mean flow velocity and $l_0$ is the blade chord length.

Using the Active Rotor, the motion of some blades may be prescribed and controlled, whereas the motion of other blades are passively measured. From the measured deflection of the passive blades, the generalized forcing function $f_i$ may be backed out. Using equation 1.2, and knowing all the prescribed motions of the other blades, the influence coefficients $C_\theta$ may be calculated.

**Experiment 2: System Identification Based Approach:** The system identification based approach is more focused on investigating the effect of the aeroelastic coupling on the overall damping of the system. Using regular systems identification techniques (e.g. measuring response due to sine sweeps), the modal characteristics of a system may be obtained in terms of natural frequencies and damping ratios. To eliminate the effect of aerodynamics, the system may be identified in vacuum, and the modal parameters obtained. The experiment may be repeated in the presence of flow, and new damping ratios may be calculated. A decrement to the damping ratio would indicate a system where the fluid is doing work on the structure.

This experimental approach may also be used to verify some of the basic assumptions of current state-of-the-art flutter analysis. One example is the assumption that the aerodynamics affect the damping ratios of the structure, but leaves the natural frequencies largely unaffected [8].

**Experiment 3: Investigation of Effect of Mistuning Parameters** Mistuning is the variation of natural frequencies and mode shapes of individual blades in a rotor. Mistuning may be a result of slight material and manufacturing tolerances of the blades, or it may be intentionally induced in a rotor as a means of passive flutter control [8]. Mistuning of blades can either have desirable or adverse effects on the structure. Under some conditions, mistuning may result in the localization of energy on a single blade. In other cases mistuning may plateau sharp resonance peaks in a structure to milder levels. The experimental study of the effect of mistuning parameters on the structural response is generally costly since it requires the building of different rotors. Investigation of mistuning in blade stiffness has been a more common approach. Such studies may be implemented in the Active Rotor by applying feedback control to the piezo actuators to provide controlled changes in stiffness and natural frequencies of individual blades. Alternatively, geometric mistuning may also be studied since altering the angle of twist of individual blades becomes possible.
1.2.3 Features and Functional Requirements

The geometry of the Active Rotor is matched to that of the rotor studied in [16]. A summary of the basic features of this rotor is outlined in table 1.1. This is 33 % dynamically reduced version of the General Electric Fan C design, and is known to exhibit flutter problems as described by table 1.2. Thus, the selected geometry is an interesting one from a flutter standpoint, and will allow bench-mark comparisons with the results in [16]. For notational purposes, the selected geometry will be referred to as the “scaled Fan C”.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of stages</td>
<td>1</td>
</tr>
<tr>
<td>Tip diameter</td>
<td>53.14 [m]</td>
</tr>
<tr>
<td>Hub/tip radius</td>
<td>0.36</td>
</tr>
<tr>
<td>Tip Mach speet</td>
<td>1.52</td>
</tr>
<tr>
<td>Rotational speet</td>
<td>16,700 [rpm]</td>
</tr>
<tr>
<td>Chord length (at root)</td>
<td>6.36 [cm]</td>
</tr>
<tr>
<td>Chord length (at tip)</td>
<td>8.92 [cm]</td>
</tr>
<tr>
<td>Max. thickness to camber ratio (at root)</td>
<td>0.114</td>
</tr>
<tr>
<td>Max. thickness to camber ratio (at tip)</td>
<td>0.027</td>
</tr>
<tr>
<td>Stagger (at root)</td>
<td>6.47 [degrees]</td>
</tr>
<tr>
<td>Stagger (at tip)</td>
<td>71.11 [degrees]</td>
</tr>
<tr>
<td>Camber (at root)</td>
<td>91.10 [degrees]</td>
</tr>
<tr>
<td>Camber (at tip)</td>
<td>-1.664 [degrees]</td>
</tr>
<tr>
<td>Solidity (at root)</td>
<td>2.414</td>
</tr>
<tr>
<td>Solidity (at tip)</td>
<td>1.389</td>
</tr>
</tbody>
</table>

Table 1.1: Active Rotor blade summary

Table 1.2 shows the flutter frequencies of the scaled Fan C blade. They suggest that the actuation bandwidth of the active rotor should be in the 1 kHz band to cover both torsional and bending flutter. A reduced bandwidth of ~350 Hz may be sufficient for the study of bending flutter only.

<table>
<thead>
<tr>
<th>Operating Speed [% of Nominal]</th>
<th>Flutter Mode</th>
<th>Flutter Frequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>Torsional</td>
<td>896</td>
</tr>
<tr>
<td>70</td>
<td>Torsional</td>
<td>896</td>
</tr>
<tr>
<td>90</td>
<td>Bending</td>
<td>320</td>
</tr>
<tr>
<td>95</td>
<td>Bending</td>
<td>336</td>
</tr>
</tbody>
</table>

Table 1.2: Flutter frequencies of the Fan C blade
From a conceptual design standpoint, the amplitude of actuation is selected to be 1 degree of tip twist, and correspondingly ~1.0 mm of tip displacement in bending at the spar tips. This requirement however may turn out to be superfluous. Experimental investigations in [12] and numerical simulations in [23] have shown that an angle of twist of 0.3 degrees are sufficient creating measurable aerodynamic responses.

1.3 Development Stages of an Active Compressor Blade

The development of the Active Rotor can be grouped into four general areas of work. They are shown schematically in figure 1-6, and outlined below:

Active Blade Development

The development of an active blade is indeed the core task towards the development of a complete rotor, and perhaps the most challenging. Major issues include:

**Design, Analysis, and Manufacture of Active Compressor Blade:** Design of the blade must ensure the capacity to sustain centrifugal loads. Finite element models were developed to aid
in the design process, and studies were performed on the optimal lay-up patterns of graphite epoxy. Coupon tests for establishing the feasibility of the actuation concept were conducted, as well as studies for actuator selection and sizing. These key issues in the design and analysis of the blade are outlined in detail in [14].

The complex bilinear curve and twist in the blade geometry adds challenges to the manufacturing processes. Several manufacturing options are considered. Until the writing of this thesis, no final conclusions have been set on the optimal method of blade manufacture. Although a pair of active spars were successfully developed and used for initial experimentation, as yet a complete blade with properly shaped foam is still under development.

**Dynamical Characterization:** The central functional requirement of the active rotor is the ability of its blades to follow prescribed deflection trajectories. In order to develop suitable control laws, the dynamics of the blade (the plant) must be carefully modeled. A model is useful only if validated experimentally, and proven to predict the response of the system in hand. This dictates the necessity to conduct experiments for validation and tuning purposes.

The main parameters of interest in the deformation of the blade are the tip deflections: pitch and plunge. Chord-wise deflections are neglected, and assumed not to contribute significantly to aeroelastic effects.

**Development of Control and Estimation Laws:** Once a suitable dynamical model for the blade is developed and verified, control algorithms are required to command the blade to specified trajectories. Recall that actuation is provided by piezos, while sensing is attained by collocated strain gages. A state estimator is needed for estimating blade tip deflection (parameters of interest) given strain readings at the root. This will serve as an alternative to direct measurement of tip deflection, which is typically difficult. Furthermore, development of state estimators is a prerequisite to the development of state feedback controllers.

**Development of a Spin-Testing Facility:** Figure 1-6 shows that spin testing is required throughout the development stages of the rotor. This necessitates the development of a high speed spin-testing facility that will fulfill such testing needs. The spin-test facility must be capable of providing evacuated environments to isolate structural and aeroelastic dynamics. Although this facility was developed with the active rotor in mind, it may be used in its own right as a general spin-testing facility for other purposes.
Integration of Multiple Blades Into a Complete Rotor

Once a reliable procedure for the development of an active blade is set, multiple blades may be reproduced and integrated into a complete rotor. Issues to be considered at this stage include the design of a suitable rotating hub to accommodate the cascade, and the proper wiring of sensors and actuators. Due to the limited number of channels available for communication with the rotor in a rotating frame of reference, some ideas have been suggested with regard to on board multiplexing of the signals to provide an expansion of the communication channels.

Shakedown in the MIT Blowdown Compressor

Shakedown is to be performed in the MIT blowdown compressor, where the rotor is briefly exposed to aerodynamic loading. This step is necessary to ensure that the rotor will survive aerodynamic loading.

Off-site Experimentation

Full off-site experimentation for flutter will require more extensive facilities. Suggested locations include NASA Glenn Research Center or Wright-Patterson facilities. Upon arriving at this stage, the experiments outlined in section 1.2.2 may be conducted.

1.4 Organization

This thesis is part of the effort towards the development of an active blade. Specifically, it focuses on three aspects:

- Chapter 2 focuses on the dynamical characterization of the active blades and obtaining models suitable for estimation and control. Two approaches are presented, with the advantages and disadvantages of each are discussed. Since active blades are under development, the procedures are implemented on the active structure of the blade, the twin spar system, which was available at the time. The first approach is a purely empirical approach in which the transfer functions of the blades are empirically determined. The experimental setup is described, and results are presented. Transfer functions are then fit to the data in a least squares sense. Model order reduction using Hankel Singular Values is presented and implemented, resulting in a model of reasonable order from an implementation perspective. The second approach depends on finite element modeling of the blade to obtain its dynamical characteristics. The model order is reduced, and the actuation effects of the piezos is modeled, and the dynamics of supporting signal conditioning amplifiers are identified experimentally, and fit to a second order system. The results of this approach were then
compared to experimental results. Appendix A compliments this chapter, and documents all the transfer functions measured and fit.

- Chapter 3 discusses the design of state estimators for estimating blade tip deflection from root strain gage measurements. Such estimates are required since direct measurement of tip deflection for a complete rotor is experimentally infeasible. Kalman filters are demonstrated via simulation. The estimates are compared to experimental data.

- Chapter 4 discusses the development of a Spin Testing Facility for testing of active blade. The Chapter discusses the functional requirements, and relates them to the sub-systems and components of the facility. The design of a rotating hub to accommodate candidate blade concepts for testing is presented, including results of finite element simulation for stress analysis under centrifugal loading. Instrumentation for rig condition monitoring is outlined. Precautionary measures with regard to rig assembly are presented based on experience gained during the development of the rig. Finally, results pertaining to thermal characterization of the rig to determine safe operation durations is presented.

- Chapter 5 concludes this thesis, and presents recommendations for future work.
Chapter 2

Dynamical Characterization of an Active Compressor Blade

The central functional requirement of the Active Rotor is the ability to dynamically deform its blades according to prescribed trajectories. To do so, piezoelectric actuators apply strain loading at the root of the blade, whereas strain gages provide collocated sensing. Before being able to make use out of the active structure, its dynamics must be studied and understood. This chapter focuses on a procedure for dynamical characterization of an active blade, and obtaining low order models that characterize the behavior of the Active Blade.

Due to the bilinear curvature, twist and taper of the blade geometry, analytical modeling is infeasible. Instead, two alternate approaches are presented. The first approach, as outlined in section 2.1, is a purely experimental approach in which the transfer functions are empirically determined. The second approach relies on a combination of finite element modeling and experimental results to arrive at a dynamical model of the blade and its supporting signal conditioning systems. This is outlined in section 2.2. Each approach has its advantages and disadvantages, and will be discussed in the sections that follow.

The goal of Chapter 3 is to design state observers for the blade, to estimate its deflection state given root strain gage readings. The parameters of interest from an aeroelasticity perspective are the two degrees of freedom of tip deflection: pitch and plunge. Obtaining a dynamical model for the blade is essential to design of model-based state estimators. Furthermore, the models generated will serve as a basis for designing controllers that will command the blade to move according to desired trajectories.

It is important to mention at this stage that at the writing of this thesis, the development of a
complete Active Blade has not been completed. Instead, a pair of curved, twisted, fully packaged and wired active spars are available without any foam. This structure is used as the test article for the studies presented here and in Chapter 3. The results will vary significantly with the addition of foam since it will alter the dynamical characteristics of the system under consideration. The foam will not only add mass, stiffness and hysteretic damping to the structure, but it will also couple the two spars together, which was not the case considered here. Nevertheless, developing the identification and estimation procedures at this early stage provides useful information that will be applicable to the full blade. Therefore the purpose of this chapter and of chapter 3 is to develop characterization and state estimation procedures that are applicable to the twin spar set, with the intention that such procedures will be replicated when the Active Blade is complete.

**Description of Active Spars**

Before getting into the identification procedures, it is worthwhile at this stage to give a detailed description of the system under consideration. Figure 2-1 shows the construction of an active spar. Recall that a single blade will have two of such spars, one close to the leading edge, and another close to the trailing edge. Additionally, the two spars do not have identical geometries as illustrated in figure 1-3(a).

![Figure 2-1: Schematic detailing active spar construction.](image)

Epoxy layers bond the inner surfaces of the piezo actuators to the spars, and the strain gages to the outer surfaces of the piezos. A potential difference is applied across the inner and outer surfaces.

---

1 The scope of this thesis does not include the development nor manufacture of the Active Blade.
Because the piezos are oriented in opposite directions (as indicated by the arrows), they work together producing maximum structure deflection. Had the piezos been oriented in the same sense, they would work against each other, producing zero net effect.

Modes of Actuation

The two basic modes of actuation of the blade are bending and twisting. A simple approximate way to achieve these modes is to provide both the leading and trailing edge piezos a common potential for bending, and opposite potentials for twisting. However, since the two spars have slightly different geometries, the phase difference from voltage applied on actuators to tip deflection is not necessarily the same for both spars. Therefore the structure is regarded as a multi-input, multi-output (MIMO) system in which the effect of each input is analyzed separately.

Model Bandwidth Requirements

Table 1.2 shows that the maximum flutter frequency of the active rotor under consideration is 896 Hz. Consequently, the bandwidth requirement set for the rotor is 1 kHz. However, it is also evident from the table that a bandwidth of 400 Hz would be sufficient to address the bending flutter problem. These figures dictate the bandwidth of validity of the dynamical models to be obtained, which depends on the flutter mode studied. As will be shown in section 2.1.1, the experimental approach to determining system transfer functions requires laser vibrometers to measure blade tip displacement. The available vibrometers are limited to a bandwidth of 400 Hz. Therefore, all models obtained by that method can be used to investigate only the bending flutter problem. Higher bandwidths would require the use of the higher bandwidth instrumentation that are not currently available. Alternatively, the second approach, the finite element approach, may be used to yield models that are valid up to higher frequencies.

Comparison of Two Modeling Approaches

The experimental approach to system identification relies on exciting the structure with broadband input up to the desired bandwidth, and measuring the transfer functions from all inputs to all outputs. The inputs in this system are command signal voltages that are amplified and applied to the piezo actuators. The outputs are strain gage readings and tip displacement measurements. This approach has several advantages:

- Directly captures the dynamics of the structure under investigation.
- Accounts for the dynamics of actuators, amplifiers and signal conditioning equipment since they are lumped with the system being identified.
• Captures imperfections in the system that are otherwise hard to model, e.g. imperfections of bonding of actuators or sensors to the structure.

The disadvantages of the experimental approach include:

• Resulting model bandwidth is limited by instrumentation bandwidth.

• No insight is obtained on how the response of the system will vary by changing some of its features.
  For each system, a new set of experiments must be conducted.

• The method can only be implemented easily on a bench top test. Identification under rotation would require special blade tip deflection measurement instrumentation.

On the other hand, the finite element model based approach relies on obtaining the dynamical characteristics (natural frequencies and mode shapes) from a finite element model. This information is then augmented by models of amplifiers and signal conditioning equipment. Finally, the entire setup is put in state space form. The resulting model, having a large order due to the nature of finite element modeling, is then approximated by a reduced order model. This approach, though being more involved and less direct than the first one, has several advantages:

• Obtains estimates for system response beyond bandwidth limitations of sensors.

• Predicts model response at all points of structure, as opposed to only points of measurement of strain at roots and displacement at tip.

• Provides for the analysis of the effects of structural modification. Since the active blade is under development, modifications are prone to happen. For example, the leading and trailing edges may need strengthening by adding fiberglass lining. Such modifications may be included in the FEM model, the effect of which may be simulated and predicted.

• Allows for investigation of gains and losses in observability and controllability of the system due to addition/removal of sensors/actuators. For example, consider a situation when an added strain gage is to be placed on the blade to obtain more accurate sensing of the deformation. Optimal locations for the sensors may be chosen on the model to yield maximum gain in observability.

• Predicts model tip response under centrifugal loads. Centrifugal stiffening effects alter the dynamics of the system, the effect of which can be studied through the model.

The main disadvantage of this approach is that tuning the finite element model to match actual system characteristics may be cumbersome, and results may be inaccurate. Careful modeling of boundary
condition and material properties is essential. Furthermore the presence of manufacturing imperfections are hard to capture.

In the sections to follow, each approach is presented in more detail, and the results are compared. As will be seen, the two results do not match to a certain extent, and possible remedies are discussed.

2.1 Experimental Determination of Transfer Functions

The experimental approach to characterizing system dynamics depends on exciting the structure with broad band excitation and measuring the response at select points. For the twin spars, there are two inputs and six output sensors as explained by table 2.1. Strain gages \( s_1 \) and \( s_3 \) are on opposite sides of the leading edge spar, and therefore measure the same strain, but with opposite signs (180° phase shift). Thus an average value for the strain at the leading edge is obtained by \( \varepsilon_1 = \frac{1}{2}(s_1 - s_3) \). Similarly, the same calculation is applied to \( \varepsilon_2 \), the average strain value at the root of the trailing edge. In summary, the system has two inputs (\( u_1 \) and \( u_2 \)), and four outputs (\( \varepsilon_1, \varepsilon_2, z_1, z_2 \)), which are determined by six sensors (\( s_1, s_2, s_3, s_4, z_1, z_2 \)).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Designation</th>
<th>Transducer/Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation at LE</td>
<td>( u_1 )</td>
<td>Piezo actuator</td>
</tr>
<tr>
<td>Actuation at TE</td>
<td>( u_2 )</td>
<td>Piezo actuator</td>
</tr>
<tr>
<td>Root strain at LE suction side</td>
<td>( s_1 )</td>
<td>Strain gage</td>
</tr>
<tr>
<td>Root strain at LE pressure side</td>
<td>( s_2 )</td>
<td>Strain gage</td>
</tr>
<tr>
<td>Root strain at TE suction side</td>
<td>( s_3 )</td>
<td>Strain gage</td>
</tr>
<tr>
<td>Root strain at TE pressure side</td>
<td>( s_4 )</td>
<td>Strain gage</td>
</tr>
<tr>
<td>Average root strain at LE</td>
<td>( \varepsilon_1 = \frac{1}{2}(s_1 - s_3) )</td>
<td>Calculated</td>
</tr>
<tr>
<td>Average root strain at TE</td>
<td>( \varepsilon_2 = \frac{1}{2}(s_2 - s_4) )</td>
<td>Calculated</td>
</tr>
<tr>
<td>Tip displacement at LE</td>
<td>( z_1 )</td>
<td>Laser displacement vibrometer</td>
</tr>
<tr>
<td>Tip displacement at TE</td>
<td>( z_2 )</td>
<td>Laser displacement vibrometer</td>
</tr>
</tbody>
</table>

Table 2.1: Inputs, outputs, and measured data of Active Blade.

2.1.1 Experimental Setup

Figure 2-2 shows the setup used to run system identification experiments. A signal generator provides an excitation sinusoidal sweep command signal. This signal is amplified via an audio power amplifier,
and its voltage is further raised by transformers. The piezo actuators excite the structure which in turn deforms. Strain gages measure the root strain and a laser displacement transducer measures tip deflections. Strain gage amplifiers implement a Wheatstone bridge circuit to accurately measure the small changes in resistance associated with the deformation of the strain gages. The frequency response of each of the outputs due to the input signal is computed by a dynamic signal analyzer. The analyzer averages out sample response until a suitable coherence spectrum is achieved.

![Diagram of experimental setup](image)

**Figure 2-2: Experimental setup for system identification.**

It is important to emphasize that the "command signal" is that generated by the function generator (on the order of 1 V<sub>pp</sub>), whereas the "piezo voltage" is that applied on the piezo, or the amplified version of the command signal (on the order of 250 V<sub>pp</sub>). Transfer functions are obtained with respect to the command signal as input to the system. This is done to take into account the dynamics of the amplifier, and have it included in the system response. Furthermore, from a controller implementation point of view, the output of the controller will be that command signal, and therefore should be taken as input to the plant.
Capacitive Effect of Epoxy Layers

The correct choice of which piezo leads to ground is very important. Figure 2-1 shows that the outer surfaces of the piezos are grounded. If the converse is true, then the upper surfaces would experience a fluctuating potential on the order of $250 \, \text{V}_{pp}$, while the strain gages will remain at a virtually constant potential. This creates a situation where two conductors (the outer piezo surface and the strain gage) are at a potential difference, separated by a dielectric layer (epoxy). This, by definition, is a capacitor, the presence of which highly contaminates strain gage readings. Figure 2-3 shows a sample transfer function obtained with the presence of such capacitive effect. Two things are immediately noticed: i) the dereverberant part of the transfer function$^3$ is increasing at a slope of $+20\,\text{dB/decade}$, and ii) the poles and zeros are highly attenuated. These trends can be explained by the presence of this capacitive effect which introduces a differentiation element, and hence a $20 \, \text{dB/decade}$ slope. The most effective way to eliminate such effects is to ground the outer surfaces of the piezos, and thereby eliminating the potential difference, and the entire capacitive effect.

![Transfer Function](image)

Figure 2-3: Transfer function contaminated by the capacitive effect. Notice the slope of the dereverberant transfer function is $20\,\text{dB/decade} = 1$ order of magnitude/decade.

$^3$The backbone of the transfer function without the “peaks” and “valleys” corresponding to poles and zeros.
The twin spar test article available at the time of testing did not allow for complete freedom of choice of which leads to ground. This is because a common connection is hardwired to the inner surfaces of the four piezo. Thus it is not possible to excite both the leading edge actuators and the trailing edge actuators with different signals simultaneously. Consequently not all transfer functions could be measured directly, and some had to be inferred from the other results. Table 2.2 outlines:

<table>
<thead>
<tr>
<th>Output\Input</th>
<th>(u_1) LE Actuation Only</th>
<th>(u_2) TE Actuation Only</th>
<th>(u_1 + u_2) In-Phase Actuation</th>
<th>(u_1 - u_2) Out-of-Phase Actuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_1)</td>
<td>Measured</td>
<td>Inferred</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
<tr>
<td>(\varepsilon_2)</td>
<td>Inferred</td>
<td>Measured</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
<tr>
<td>(\varepsilon_3)</td>
<td>Measured</td>
<td>Inferred</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
<tr>
<td>(\varepsilon_4)</td>
<td>Inferred</td>
<td>Measured</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
<tr>
<td>(\varepsilon_1)</td>
<td>Measured</td>
<td>Measured</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
<tr>
<td>(\varepsilon_2)</td>
<td>Measured</td>
<td>Measured</td>
<td>Measured</td>
<td>Inferred</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of measured and inferred transfer functions

**Important Considerations**

The experimental setup in hand involves large actuation voltages, and sensing voltages on the order of millivolts, all packaged in a very compact space. To aggravate the situation further, the electric motor of the Spin Test Facility produces a significant electromagnetic field that may easily cause interference with the measured signals. Thus great care in setting up the electrical connections is essential to obtain good quality results. The following is a highlight of some good practices that must be followed:

- Electrical grounds of the piezo actuators and strain gages should remain isolated. During the course of experimentation, the potential of the two grounds may change, and the presence of a conductor with a small but finite resistance will cause ground loops that contaminate the readings of the strain gages.

- Outer sides of the piezos must be electrically grounded to eliminate the capacitive effects mentioned above.

- All wires leading to strain gages must be paired and twisted to eliminate electromagnetic interference.

- All wires in the rotor must be shielded, especially those leaving the slip ring and are subject to interference patterns due to the operation of the motor.
Equipment Setup

To excite the structure and measure its response, the equipment were set as described by table 2.3:

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function Generator</td>
<td>Mode of operation</td>
<td>Sine sweep</td>
</tr>
<tr>
<td></td>
<td>Frequency range</td>
<td>50 Hz to 1200 Hz</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td>1 Vpp</td>
</tr>
<tr>
<td></td>
<td>Sweep time</td>
<td>4 seconds</td>
</tr>
<tr>
<td></td>
<td>Sweep mode</td>
<td>Logarithmic</td>
</tr>
<tr>
<td>Audio Amplifier</td>
<td>Mode</td>
<td>2 channel, stereo</td>
</tr>
<tr>
<td></td>
<td>Gain setting</td>
<td>Maximum</td>
</tr>
<tr>
<td></td>
<td>High pass filter</td>
<td>40 Hz</td>
</tr>
<tr>
<td>Strain Gage Amplifier</td>
<td>Bridge mode</td>
<td>Quarter bridge</td>
</tr>
<tr>
<td></td>
<td>Active filter</td>
<td>10 kHz, low pass</td>
</tr>
<tr>
<td></td>
<td>Gain</td>
<td>300 με/V</td>
</tr>
<tr>
<td>Laser Vibrometer</td>
<td>Gain</td>
<td>1,623 μm/V</td>
</tr>
<tr>
<td></td>
<td>Mode</td>
<td>Fast response</td>
</tr>
<tr>
<td>Spectrum Analyzer</td>
<td>Mode</td>
<td>2 Channel</td>
</tr>
<tr>
<td></td>
<td>Measured data</td>
<td>Frequency response</td>
</tr>
<tr>
<td></td>
<td>Window mode</td>
<td>Coherence spectrum</td>
</tr>
<tr>
<td></td>
<td>Frequency range</td>
<td>Hanning</td>
</tr>
<tr>
<td></td>
<td>Frequency resolution</td>
<td>1.6 kHz</td>
</tr>
<tr>
<td></td>
<td>Averaging</td>
<td>800 lines</td>
</tr>
<tr>
<td></td>
<td>Channel setup</td>
<td>200 averages</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Grounded, auto-range</td>
</tr>
</tbody>
</table>

Table 2.3: Instrumentation setup parameters.

Since the signal analyzer available is a two channel analyzer, experiments were conducted in sequence, measuring one SISO transfer function at a time in accordance with table 2.2. Data are then transferred from the analyzer for further processing.

2.1.2 Results

All measured transfer functions are plotted in Appendix A. Only two are presented here to aid in the discussion. Figure 2-4 shows a plot of the transfer function between a command signal applied to the leading edge actuators and root strain at the leading edge for a frequency range from 0 to 1,200 Hz. The following observations are made:

- Two poles dominate the response at frequencies of 114 Hz and 322 Hz, while other less obvious poles are at approximately 279 Hz, 640 Hz and 710 Hz. Zeros cancel the poles at frequencies of 124 Hz, 293 Hz, 342 Hz, 645 Hz and 732 Hz. Thus, poles and zeros of the system interlace in a manner as expected in collocated actuation and sensing [4].
Figure 2-4: Measured transfer function from command signal applied at leading edge piezos to root strain at leading edge.

- The dereverberant transfer function is fairly constant over the entire frequency range. This is an expected trend in structures where the input is a strain actuator, and the measured output is also strain. Different actuator/sensor combinations yield different roll-off frequencies of the dereverberant transfer function.

- The phase difference between the suction side strain gage and the pressure side strain gage is an almost constant $180^\circ$. This is expected when measuring strain at two different sides of a cantilever beam. Imperfections are attributed to the twist and curvature in spar shape, as well as to the fact that strain gages are located at slightly different positions axially with respect to the spar base.

- Coherence is a measure of the quality of correlations obtained. Values are almost perfect for the entire frequency range, with the exception at zero locations. This is because the response is very small at those locations, making the signal to noise ratio small.
Figure 2-5 shows the transfer functions from a command signal applied to the leading edge actuators to tip deflection of the leading and trailing edge spars.

- Notice the sharp drop in coherence levels at the 400 Hz cut-off mark. This is the bandwidth of the laser displacement vibrometer, and will be the frequency range considered.

- As with figure 2-4, there are two dominant poles in the response of the system at 105 Hz and 296 Hz. Poles and zeros are not as closely spaced as in the case of figure 2-4 because actuation and sensing are not collocated.

- The response of the leading edge tip is larger than that of the trailing edge. This is because the coupling between the two spars is small. The presence of foam is expected to increase the coupling significantly, and thereby increasing the response of the trailing edge to level comparable with those of the leading edge.
2.1.3 Fitting Transfer Functions to Experimental Results

Once the transfer functions are obtained in numerical form, it is necessary to convert them to a functional form, which may then be converted into state space form for analysis and design of state estimators and controllers. The system in hand is a two input, four output system (the output being strain and tip displacement at the leading and trailing edges). Fitting a transfer function to a single-input, single-output (SISO) system is a relatively straightforward task. It consists of selecting the order of the numerator and denominator polynomials of sufficient orders to account for the dynamics of interest, and fitting the coefficients in a least squares manner. For a MIMO case, an additional constraint is that all transfer functions must have the same poles. From a physical standpoint, the poles are the same since the structure remains unchanged. This condition may be realized by setting the denominator polynomials to be equal for all transfer functions.

Additionally, the order of the denominator polynomials must be of a sufficiently low order to be implemented easily in a system. The target goal was to ensure that the model would have an order less than or equal to 20.

The strategy adopted is as follows:

1. **Identify the poles of each individual transfer function.** By inspection of the experimental data, the frequencies corresponding to the poles and zeros of the transfer functions are identified. This however does not fully identify the poles or zeros; rather their imaginary parts are determined. The real part may be estimated by measuring the half power bandwidth of the resonance peak, and thereby determining its quality factor, and hence damping ratio and the real part. An easier approach would be to fit the transfer function as a SISO system using a least squares or other numerical fit. **MATLAB**'s `invfreqs` function, in the Signal Processing Toolbox, was used to produce such a numerical fit. An additional advantage of using `invfreqs` is that it may be set up to guarantee that the resulting SISO system is a stable system, having all of its poles in the open left-half of the s-plane. The complex poles and zeros of the resulting system that are within the desired bandwidth of the system (400 Hz in this case) are then identified.

2. **Form a set of poles for the entire system.** Using the method described above, independent poles are identified for each transfer function. In fact, none of the eight transfer functions show all the poles that exist in the frequency range of interest. This is a result of pole-zero cancellations.

Due to slight numerical errors, a pole will not appear in every transfer function with exactly the same numerical value. Thus, the values of the poles must be averaged out over the test data.

---

4 This function can only be used to fit SISO systems.
The resulting system has a total of 18 poles, forming 9 complex conjugate pairs. This is a reasonable number from an implementation point of view. Moreover, it is expected that with the addition of foam to the structure identified, the number of states required to cover this frequency band will decrease. In the present status, the coupling between the two spars is very small, making the system act as two almost uncoupled systems.

3. For each transfer function, generate zeros that cancel out the poles that do not show. This is done by generating a polynomial that has roots that cancel out unwanted poles in a given function.

4. Apply a least squares fit for the free parameters of the transfer function. It is desired to fit a transfer function from input $i$ to output $j$ in the form of:

$$\frac{B_{ij}(s)}{A(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{n_b} s^{n_b}}{a_0 + a_1 s + a_2 s^2 + \ldots + a_{n_a} s^{n_a}} = G(s)$$

(2.1)

But since the poles of the function were computed earlier in step 2, and since zeros will be explicitly placed in the function in step 3, a more convenient way of writing 2.1 is:

$$\frac{B_{ij}(s)}{A(s)} = \frac{(b_0 + b_1 s + b_2 s^2 + \ldots + b_{n_b-n_z} s^{n_b-n_z}) (z_1 - s) (z_2 - s) \cdots (z_{n_z} - s)}{(p_1 - s) (p_2 - s) \cdots (p_{n_a} - s)} = G(s)$$

(2.2)

Where $p_1, p_2, \ldots , p_{n_a}$ are the poles previously identified, and $z_1, z_2, \ldots , z_{n_z}$ are zeros chosen to cancel out the poles that do not appear in a particular transfer function. Rearranging gives:

$$\frac{(b_0 + b_1 s + b_2 s^2 + \ldots + b_{n_b-n_z} s^{n_b-n_z}) B_z(s)}{A(s) G(s)} = \frac{A(s) G(s)}{B_z(s)}$$

(2.3)

where $B_z(s) = (z_1 - s) (z_2 - s) \cdots (z_{n_z} - s)$. Substituting for $s = i\omega$,

$$b_0 + b_1 i\omega + b_2 i\omega^2 + \ldots + b_{n_b-n_z} i\omega^{n_b-n_z} = \frac{A(i\omega) G(i\omega)}{B_z(i\omega)}$$

(2.4)

Equation 2.4 written for each of the frequencies desired. Assuming there are $k$ frequency points
results in,

\[
\begin{bmatrix}
1 & i\omega_1 & (i\omega_1)^2 & \ldots & (i\omega_1)^{n_b-n_z} \\
1 & i\omega_2 & (i\omega_2)^2 & \ldots & (i\omega_2)^{n_b-n_z} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & i\omega_k & (i\omega_k)^2 & \ldots & (i\omega_k)^{n_b-n_z}
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
\vdots \\
b_{n_b-n_z}
\end{bmatrix}
= 
\begin{bmatrix}
A(i\omega_1)G_1 \\
B_x(i\omega_1) \\
A(i\omega_2)G_2 \\
B_x(i\omega_2) \\
\vdots \\
A(i\omega_k)G_k \\
B_x(i\omega_k)
\end{bmatrix}
\]

\[Mc = v \quad (2.5)\]

\[c = (M'M)^{-1} M'v \quad (2.6)\]

where the definitions of \(M\), \(c\) and \(v\) in 2.5 are evident. Equation 2.6 gives the optimal coefficients vector \(c\) in a least squares sense. In many cases, it is desirable to add a frequency weighting function to give a better curve fit. The relative weights at each frequency can be added as diagonal elements of the weighting matrix \(S\), and equation 2.6 is then modified to:

\[c = (M'SM)^{-1} SM'v\]

In order to guarantee that the coefficients \(b_0, b_1, \ldots, b_{n_b-n_z}\) are real, the following is used instead:

\[\text{Re}(M'SM)c_R = \text{Re}(SM'v)\]

\[c_R = (\text{Re}(M'SM))^{-1} \text{Re}(M'Sv)\]

The resulting polynomial coefficient vector \(c_R = [b_0, b_1, \ldots, b_{n_b-n_z}]'\) is then convolved with the coefficients of \(B_z(s)\) to produce a numerator polynomial \(B_{ij}(s)\) that contains the originally placed zeros as well as new ones generated by the curve fit.

Note that in the above method, the zeros were explicitly placed in the numerator of the transfer function. From a theoretical standpoint, this is not necessary since the optimal fit of the polynomial coefficients should create zeros that cancel out the poles that do not appear in any particular transfer function. Numerically, however, inserting the zeros decreases the maximum power to which an \((i\omega)\) vector is raised from \(n_b\) to \(n_b - n_a - 1\), and thereby decreases round off errors.

5. **Integrate in a complete MIMO system**  Finally, the resulting transfer functions are integrated in a 2-input x 4-output transfer function matrix, with each of its elements having a denominator \(A(s)\):
6. Convert system to state-space form  A direct conversion of the system above to state-space form yields a system of order $8 \times n_a$. By removing uncontrollable and unobservable states, the minimal realization of the system has $2 \times n_a$ states. The system order is further reduced using a Hankel Singular Values based model order reduction technique that is outlined in section 2.2.3. The final system is of order 20.

2.1.4 Results

The results of the transfer function fits are shown in Appendix A.2. The fit sufficiently captures the trends of the measured transfer functions. It is noticed however, that the zeros of the fit functions are generally more damped than those of the experimental data. The errors in the vicinity of the zeros appear to be large on the log scale. Numerically however, those errors are quite small because the value of the function is small in those locations, and are therefore not heavily penalized in the least squares fit. One possible way of mitigating this is to place a much heavier weighting factors at the frequencies of the zeros, and thereby forcing the curves to fit better in those regions.

2.2 The Finite Element Approach

The finite element modeling approach relies on a finite element model to obtain the dynamical characteristics of a structure (its modes and natural frequencies). Since the open loop system considered is a cascade of dynamical systems as shown in figure 2-6, other blocks must be included in the system and are either modeled or identified experimentally.

The finite element model in hand contains approximately 3,600 nodes, corresponding to 21,600 degrees of freedom or 42,200 states. Nominally, the derived dynamical model would have this number of states, in addition to other states arising from other blocks in figure 2-6.

The approach can thus be summarized as follows:

\[
Y(s) = \begin{bmatrix}
\frac{B_{11}(s)}{A(s)} & \frac{B_{12}(s)}{A(s)} \\
\frac{B_{21}(s)}{A(s)} & \frac{B_{22}(s)}{A(s)} \\
\frac{B_{31}(s)}{A(s)} & \frac{B_{32}(s)}{A(s)} \\
\frac{B_{41}(s)}{A(s)} & \frac{B_{42}(s)}{A(s)} \\
\end{bmatrix} U(s)
\]
Figure 2-6: Cascade of dynamical systems. Note that loading effects between the structure and actuator are indicated by a two way arrow.

- Obtaining model modal parameters (natural frequencies and modal vectors) from FEM analysis.
- Modeling of piezo actuation, and the electro-mechanical coupling provided by the actuators.
- Obtaining models of amplifiers and signal conditioning instrumentation.
- Cascading the system in state space form, and reducing model order.
- Comparing results of model with experimental data for validation purposes.

Once the model has been validated, it may be used to investigate situations not physically tested previously, such as actuation under rotation and modifications to the blade.

### 2.2.1 Basic Model

The equation of motion of the structure in modal coordinates is

\[ M\ddot{q} + B\dot{q} + Kq = Q(t) \]  \hspace{1cm} (2.7)

where \( M, B \) and \( K \) are mass, damping and stiffness matrices respectively (in modal coordinates), \( q \) is the modal coordinate and \( Q \) is the generalized forcing function. Writing the equation in state space form in terms of two first order differential equations:

\[
\begin{bmatrix}
\dot{q} \\
\ddot{q}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}B
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} +
\begin{bmatrix}
0 \\
-M^{-1}
\end{bmatrix} Q(t)
\]  \hspace{1cm} (2.8)
Writing the equation in terms of unity modal mass, and describing the input in terms of voltage applied to the piezo:

\[
\begin{align*}
\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -\omega_r^2 & -2\zeta\omega_r \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + BV(t) \\
y &= \begin{bmatrix} \Phi \\ \Phi \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix}
\end{align*}
\]

where \(\omega_r\) is a diagonal matrix containing the natural frequencies and \(\Phi\) is the mass normalized modal matrix. Note that the form of the B matrix is not defined as yet.

The finite element model implemented on NASTRAN/PATRAN\(^5\) generates the eigenvalues \(\omega_r\) as well as the corresponding modal matrix \(\Phi\). Figure 2-7 shows the first 9 modes of the structure. The first four natural frequencies were 81.3 Hz, 91.5 Hz, 218.5 Hz and 222.5 Hz. These figures are not in agreement with the natural frequencies obtained experimentally.

Note that damping was not accounted for in the finite element model. Values for the modal damping ratios were be inserted for each mode. Nominal values of 2% were selected.

### 2.2.2 Modeling of Piezoelectric Actuation

The effect of the piezoelectric actuator on the structure needs to be carefully modeled. Crawley [5], [6] and Hagood [11] developed methods to capture the effect of the piezo on the structure. The purpose of this section is to show how the voltage applied on a piezo affects the structure, or alternatively, arriving at what constitutes the B matrix in 2.9. The foregoing analysis is entirely based on Crawley[6], and is presented here for completeness.

Consider a cantilever beam (a spar) with piezoelectric actuators bonded at the root of the beam as shown in figure 2-8. A bonding, or shear layer, transmits loads applied by the piezo to the structure. Analysis is performed on the differential element of width \(dx\) shown in figure 2-8. This system contains three basic elements, the actuator, the structure, and the shear layer (bonding layer) between the other two. Their combined mechanics are governed by:

- Strain-deflection relationships.

- Constitutive relationships and material properties.

\(\text{---}^5\text{The development and updating of the finite element model used is not part of this thesis. For details, refer to [20].}\)
Figure 2-7: First nine modes of twin spar system. Notice that the system acts to a large extent as a pair of loosely coupled beams.
- Equilibrium constraints between the three layers.

The above relationships result in eight equations, that are solved, integrated, and inserted in the
equation of motion of the structure in modal coordinates as a forcing function.

Figure 2-8: Coordinate setup for piezo actuation.

To aid in the analysis, three coordinate systems are utilized. \( x \) is the position coordinate as measured
from the base of the spar. \( \bar{x} \) is the coordinate from the base of the spar, normalized by the spar length \( l \).
\( \bar{\bar{x}} \) is a piezo-centered coordinate that is normalized by half the piezo length, \( m \). Thus the piezo extends
from \( \bar{\bar{x}} = -1 \) to \( \bar{\bar{x}} = 1 \).

Starting with the strain-displacement relationships of the actuator, beam and shear layer:

\[
\varepsilon_c = \frac{du_c}{dx} \quad \text{(2.11)}
\]
\[
\varepsilon_B^s = \frac{du_B}{dx} \quad \text{(2.12)}
\]
\[
\gamma = \frac{u_c - u_B}{t_s} \quad \text{(2.13)}
\]

where \( \varepsilon_c \) is the strain in actuator, \( \varepsilon_B^s \) is the surface strain of the structure, \( \gamma \) is the shear strain in shear
layer, \( u_c \) and \( u_B \) are the displacements at the actuator and surface of the structure, and \( t_s \) is the shear
layer thickness. The equilibrium equations are:

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where $\sigma_c$ is the axial stress in the actuator, $\sigma_B^s$ is the surface axial stress in the structure, $\tau$ is the transmitted shear force across the shear layer, $\alpha$ is a beam coefficient ($\alpha = 6$) for Bernoulli-Euler beam, and $t_c$ and $t_B$ are the thicknesses of the actuator and beam respectively. The constitutive relations for the actuator, beam and shear layer are:

$$
\sigma_c = E_c \left( \varepsilon_c - \frac{d_{31}V}{t_c} \right) = E_c (\varepsilon_c - \Lambda) \tag{2.16}
$$

$$
\sigma_B^s = E_B \varepsilon_B^s \tag{2.17}
$$

$$
\tau = G\gamma \tag{2.18}
$$

where $E_c$ and $E_B$ are Young’s moduli for the actuator and structure, $G$ is the modulus of rigidity of the shear layer, $d_{31}$ is the piezoelectric constant and $V$ is the applied voltage. Note that $\Lambda$ is the piezoelectric strain, i.e. the strain induced in a free piezo due to an electric field $V/t_c$. Thus it is the maximum possible strain induced by the piezo. The actual strain values depend on the loading effect of the structure on the actuator. Solving the above eight equations simultaneously yields the following two coupled fourth order differential equations:

$$
\varepsilon_B^{s(4)} - \Gamma \varepsilon_B^{s(2)} = 0 \tag{2.19}
$$

$$
\varepsilon_C^{s(4)} - \Gamma \varepsilon_C^{s(2)} = 0 \tag{2.20}
$$

where

$$
\Gamma = \frac{\tilde{G} \theta_s}{\bar{t}_s^2} \left( \frac{\psi + \alpha}{\psi} \right) \tag{2.21}
$$

$$
\psi = \frac{E_B t_B}{E_C t_c} = \bar{E} \theta_B \tag{2.22}
$$

The parameter $\Gamma$ is the non-dimensional shear lag parameter which is a measure of effectiveness of the shear transfer. The larger $\Gamma$ is, the better the shear transfer between the actuator and the structure. A perfect bonding layer corresponds to infinite $\Gamma$. The parameter $\psi$ is a measure of the maximum fraction
of the actuator piezoelectric strain $\Lambda$ that can be transmitted to the structure. The extreme case of $\psi = 0$ is when the structure is very thin and compliant compared to the actuator.

Applying the boundary conditions

\begin{align*}
\text{at } \bar{x} &= 1; \varepsilon_c = \Lambda; \varepsilon^*_B = \varepsilon^{*+}_B \\
\text{at } \bar{x} &= -1; \varepsilon_c = \Lambda; \varepsilon^*_B = \varepsilon^{*-}_B
\end{align*}

The solution becomes:

\begin{align*}
\varepsilon_c &= B_1 + B_2 \bar{x} + \left( \frac{\psi}{\alpha} \right) B_3 \sinh \Gamma \bar{x} + \left( \frac{-\psi}{\alpha} \right) B_3 \cosh \Gamma \bar{x} \\
\varepsilon^*_B &= B_1 + B_2 \bar{x} + B_3 \sinh \Gamma \bar{x} + B_3 \cosh \Gamma \bar{x}
\end{align*}

(2.23) (2.24)

where

\begin{align*}
B_1 &= \frac{\psi + \alpha}{\psi} \left\{ \frac{\varepsilon^+_B + \varepsilon^-_B}{2} + \frac{\alpha \Lambda}{\psi} \right\} \\
B_2 &= \frac{\psi + \alpha}{\psi} \left\{ \frac{\varepsilon^+_B - \varepsilon^-_B}{2} \right\} \\
B_3 &= \frac{\psi}{(\psi + \alpha) \sinh \Gamma} \left\{ \frac{\varepsilon^+_B - \varepsilon^-_B}{2} \right\} \\
B_4 &= \frac{\psi}{(\psi + \alpha) \cosh \Gamma} \left\{ \frac{\varepsilon^+_B - \varepsilon^-_B}{2} - \Lambda \right\}
\end{align*}

(2.25) (2.26) (2.27) (2.28)

where the + and − indicate the points immediately to the right and left of the piezo actuator respectively. Now that the strains are obtained, it is desired to obtain the forcing function $Q$ on the structure. Recall the equation of the motion in modal coordinates (equation 2.7) is

\[ M\ddot{q} + B\dot{q} + Kq = Q \]

but according to Bernoulli-Euler beam theory the forcing function $Q$ and the strain $\varepsilon^*_B$ are:
\[ Q = \int_{\tilde{a} - \tilde{m}}^{\tilde{a} + \tilde{m}} \tau bt_B \phi \, d\tilde{x} \quad (2.29) \]
\[ \varepsilon_B^S = -\frac{t_B}{2} \frac{\partial^2 \phi}{\partial x^2} q \quad (2.30) \]

where \( \phi \) is the mode shape and the tilde (\( \sim \)) indicates normalization by the beam length \( l \). Equation 2.29 says that the modal forcing function \( Q \) is the integrated shear force along the length of the actuator. In effect,

\[
Q = -\frac{Gbt_B^2}{4t_s \Gamma^2} \left[ \phi'''' + \phi''' + I_c + \frac{\phi'''' + \phi'''}{\sinh \Gamma} I_s \right] q - \frac{Gbt_B}{t_s \Gamma} \left( \frac{I_s}{\cosh \Gamma} \right) \Lambda \quad (2.31)
\]

\[
Q = -K_{\text{piezo}} q + Q_v \Lambda \quad (2.32)
\]

where the definitions of \( K_{\text{piezo}} \) and \( Q_v \) are as shown, and

\[
I_c = \int_{\tilde{a} - \tilde{m}}^{\tilde{a} + \tilde{m}} \cosh \left( \frac{2\Gamma}{\tau} (\tilde{x} - \tilde{a}) \right) \phi' (\tilde{x}) \, d\tilde{x} \quad (2.33)
\]
\[
I_s = \int_{\tilde{a} - \tilde{m}}^{\tilde{a} + \tilde{m}} \sinh \left( \frac{2\Gamma}{\tau} (\tilde{x} - \tilde{a}) \right) \phi' (\tilde{x}) \, d\tilde{x} \quad (2.34)
\]

Inserting 2.32 as a forcing function in the equation of motion of structure in modal coordinates gives:

\[
M \ddot{q} + B \dot{q} + K q = -K_{\text{piezo}} q + Q_v \Lambda \quad (2.35)
\]
\[
M \ddot{q} + B \dot{q} + (K + K_{\text{piezo}}) q = Q_v \Lambda
\]
\[
= Q_v \frac{d_{31} V}{t_3}
\]

Equation 2.35 shows that the effect of the piezo actuator on the structure can be summarized as a stiffening effect as captured by \( K_{\text{piezo}} \), and an electromechanical coupling effect as captured by \( Q_v \). Thus, the effect of the actuator on the system dynamics is captured.

Equations 2.33 and 2.34 require the integration along the center line of the beam. Since the mode shape of the spar in the situation in hand is obtained at the discrete nodes as given by the finite element model, a cubic spline was fit along those nodes, and integration was performed along the fit spline.
In summary, the aim of the previous analysis is to show that the effect of the piezo actuator on the structure is captured by the two parameters $K_{\text{piezo}}$ and $Q_v$ for each mode. $K_{\text{piezo}}$ modifies the stiffness characteristics of the structure due to the actuators own stiffness. As a result, the natural frequencies of the structure change, and the $A$ matrix in the state space representation of the system is modified. The $Q_v$ term defines the $B$ matrix, since the inputs in this case are the voltages applied to the piezos.

### 2.2.3 Finite Element Model Order Reduction

The motivation behind model order reduction is to approximate a high order system by a lower order one that will produce input-output characteristics that are as close as possible to the original system. This greatly simplifies systems to orders amenable to state space design. This reduction of system complexity is usually associated with the introduction of approximation errors in the input/output mapping. It is therefore up to the system designer to reduce the system to levels of complexity that maintain the important behavior of the system, while keeping its order to reasonable levels.

#### Principle

The main technique utilized relies on the use of Hankel Singular Values (HSV) to rank the system's controllable and observable modes according to their contribution to its input-output mapping. The modes contributing least (in a 2-norm sense) may be truncated with minimal impact on the accuracy of predicting system response. Like all model order reduction techniques, there is a compromise between the number of modes truncated and the accuracy of the reduced order model.

To describe the basic principle of model order reduction using HSV, consider a dynamical system realized in state space form by:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

(2.36)  

(2.37)

where $x$ is the state vector of the system of dimensions $n \times 1$, $u$ is the input vector of dimensions $m \times 1$, $y$ is the output vector of dimensions $p \times 1$, and $A$, $B$, and $C$ are matrices of dimensions $n \times n$, $n \times m$, $p \times n$ respectively. Note that the feed-through term $D$ is ignored since it has no effect on the procedure described. The input affects the states of the system as described by equation 2.36 whereas the output is affected by the states as described by equation 2.37. It is assumed that this realization is minimal, i.e. the system is reachable and observable.
A measure of the energy content of the output signal is its 2-norm, $\|y\|_2$, defined by:

$$\|y\|_2 = \int_0^\infty y'(t)y(t)dt \tag{2.38}$$

The zero-input response (ZIR) of the system due to initial conditions on the states, $x_0 = x(0)$ is given by $y_{ZIR}(t) = Ce^{At}x_0$. Thus,

$$\|y\|_2 = x'_0 \left( \int_0^\infty e^{A't}C'Ce^{At}dt \right) x_0 \tag{2.39}$$

$$= x'_0 W_ox_0 \tag{2.40}$$

where $W_o$ is the observability gramian of the system, which captures the transfer of signal energy from the state $x$ to the output $y$.

Additionally, the energy content of state $x$ is defined by the 2-norm $\|x\|_2$,

$$\|x\|_2 = \int_0^\infty x'(t)x(t)dt \tag{2.41}$$

The zero-state response (ZSR) of the system due to a unit impulse $u_0\delta(t)$ is given by $x(t) = e^{At}Bu_0$. Thus,

$$\|x_{ZSR}\|_2 = u'_0 \left( \int_0^\infty B'e^{A't}e^{At}Bdt \right) u_0 \tag{2.42}$$

$$= u'_0 \left( \int_0^\infty e^{At}BB'e^{A't}dt \right) u_0 \tag{2.43}$$

$$= u'_0 W_c u_0 \tag{2.44}$$

where $W_c$ is the controllability gramian of the system, which captures the transfer of signal energy from the input $u$ to the state $x$.

The product of $W_c$ and $W_o$ captures the transfer of signal energy from input $u$ to output $y$. A balanced state space realization of the system is one in which $W_c$ and $W_o$ are equal and diagonal. The diagonal elements of this product are known as the Hankel Singular Values (HSV) of the system.

$$HSV = \text{diag}(W_oW_c)$$
Thus, in the balanced state space realization of the system, the modes that contribute most to the input output characteristics of the system are easily identified: they are the ones that correspond to the largest HSV of the system. Therefore, the modes to be eliminated from the system are those that correspond to the smallest HSVs since they are the ones that contribute least to the input output mapping [7].

**Note on Implementation** To perform the procedure outlined above, it is assumed that the starting model is a full order model. This however is virtually impossible, since the full order model has an order of 40,000 states (due to the large number of degrees of freedom provided in the finite element model). A more realistic approach would be to take a sufficiently large number of modes (i.e. 80 modes), and take that to be the “full order model”. This model contains all the modes with natural frequencies up to 7.2 kHz, and may therefore be safely taken as an actual replication of the system for the bandwidth mentioned. This 80 states model is then reduced by the procedure outlined above to 12 states.

The results of the this procedure are shown in section 2.2.5. Before discussing them, the model is completed by taking into account the dynamics of the power amplifier.

### 2.2.4 Identification of Power Amplifier

To complete the model, the power amplifier must be modeled to account for its dynamics. The amplifier was identified under loading conditions using a sine sweep, and taking the transfer function between its input and output. Figure 2-9 shows the obtained transfer function, and a second order fit to it.

The second order fit had the transfer function:

\[ G_{Amp}(s) = \frac{136.9s^2}{s^2 + 120s + 3500} \]
2.2.5 Results

The model order reduction procedure described above is implemented to the dynamical model derived from the finite element model. Note that this model had 80 states to start with. Figure 2-10 shows a plot of the value of the diagonal elements of the $W_c$ and $W_o$ for the system, and the HSVs. From the figure, it is clear that the first 16 modes have an appreciable value of their corresponding HSV of the system. Remaining modes have near zero HSV's, and are therefore truncated.

Figure 2-11 shows a plot of the response of the system as predicted by the full order model and the reduced order models for one particular transfer function. Results show that the reduced order model almost perfectly matches with the full order model up to a frequency range of 800 Hz. However, this response is substantially different from that observed in experiments.

The difference between the finite element model prediction and the experimental results is primarily a result of differences in the natural frequencies predicted. This can be attributed to several factors:

- Inaccuracy in modeling clamping boundary conditions on the spar.

- Imperfections in actual spar due to manufacturing errors.

- Discrepancies between listed material properties and actual ones.
Figure 2-10: Diagonal elements of $W_o$ and $W_o$ and the Hankel Singular Values for the 80 modes of the system.

Figure 2-11: Full order model and reduced order model transfer functions as predicted by FEM based approach.
2.3 Summary and Conclusions

This chapter outlined two approaches used for obtaining dynamical models of the Active Blade, an experimental approach and a finite element model based approach. Both methods were implemented. The data resulting from experimental system identification is fit to a low order model with sufficient accuracy to perform state estimation. The model resulting from the finite element approach did not match the data, and future work needs to be done to update the model to reflect experimental data.

The model resulting from the experimental approach is used for designing state estimators in Chapter 3. This model accurately matches the data, and is considered valid. Ultimately, when modifications to the Active Blade are necessary, or when actuation under rotation is to be investigated, the finite element model must be updated to reflect the experimental data accurately.
Chapter 3

State Estimation

3.1 Introduction

State estimation is an important task in the operation of the Active Rotor. As mentioned in Chapter 1, root strain of the spars is measured, whereas the parameter of interest from an aeroelasticity point of view is tip deflection. The state estimator is used to obtain the unmeasured output of interest.

During experimentation with the Active Rotor, some blades will be actively controlled, while others will deform passively due to the forces exerted by the fluid. In both cases, state estimation is required. For the passive blades, knowing the state of the blade will enable the reconstruction of the outputs of interest. For active blades, the state estimate is essential to implement control laws to command the desired outputs. By the separation principle, estimators and controllers may each be optimally designed individually to yield an overall optimal system response [25].

The models obtained in Chapter 2 are all converted into state space form. Only the state vectors are estimated. All outputs of interest can be derived from those estimates.

One way to characterize blade tip deflection is in terms of pitch and plunge. Alternatively, the displacement of any two points that are sufficiently spaced chordwise on the tip would convey the same information. During operation of the Active Rotor, tip deflection cannot be measured directly for all the blades in the rotor, primarily due to the difficulty in making such measurements for a large number of blades with sufficient bandwidth. Instead, model-based state observers are used to estimate the blade's response due to the applied inputs, and correct for the deviation due to noise, disturbance, and modeling errors. Readings from the strain gauges, which are readily available, are used to make that correction.

The model in hand is a multi-input, multi-output system (MIMO). The system is largely decoupled since the spars are almost independent. One possible approach to dealing with such systems is to ignore
the coupling terms, and treat the system as a set of single-input, single-output (SISO) systems [9]. This approach may work well if state estimation of the twin spar system is the objective, but is likely to fail once foam is added due to its coupling effect. Therefore this analysis will adopt a MIMO approach to state estimation.

Estimation must take into account the presence of noise and disturbances in the system. On one hand, sensor noise is random error introduced to the measurements by the sensor, and are not present in the system output. In the Active Rotor, sensor noise is that present in the strain gage instrumentation. On the other hand, process noise is the uncontrolled disturbance that the system is physically subject to due to influence from the environment. This can be a result of machine vibration (for example, due to unbalance of the rotor), or aerodynamic disturbances on the blades (in case of aerodynamic testing).

This chapter discusses the design of Kalman filters. The principles of Kalman filtering are first introduced, and the equations are tailored to suit the problem of the active blade. The model obtained from experimental identification procedure as outlined in Chapter 2 is utilized. Simulation based on the model is then presented, showing the guidelines that were used to develop the estimator. The results are then compared with experimental bench top results, where tip deflection measurements are compared to the output of the estimators.

3.2 Principles

3.2.1 Model Setup

The starting point in the analysis is the empirically based model obtained in Chapter 2. The inputs to the plant are the command signals, $u$, from the signal generator (or controller eventually, if the blade is being controlled). The output $y$ consists of strain at the root of each spar, $\varepsilon$, and the tip deflection $z$. Recall that during operation of the blades, only the root strain $\varepsilon$ is measured. The system may be described according to the following state space model:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

This description however assumes full knowledge of the input applied to the system, and does not take into consideration any form of sensor or process noise. In order to write the model in a more general way, and in order to distinguish between measured output and unmeasured output, equations 3.1 and 3.2 are written as:
\[
\dot{x} = Ax + Bu + \Gamma_1 w \\
y = \begin{bmatrix} \varepsilon_v \\ z \end{bmatrix} = Cx + Du + \Gamma_2 w + v \\
= \begin{bmatrix} C_\varepsilon \\ C_z \end{bmatrix} x + \begin{bmatrix} D_\varepsilon \\ D_z \end{bmatrix} y + \begin{bmatrix} \Gamma_{2z} \\ \Gamma_{2z} \end{bmatrix} w + \begin{bmatrix} v_\varepsilon \\ 0 \end{bmatrix}
\] (3.3) (3.4)

The input \( u \) is a 2 x 1 vector with elements \( \{u_1, u_2\}' \) being the input command signals to the piezos, \( \varepsilon \) is a 2 x 1 vector with elements \( \{\varepsilon_1, \varepsilon_2\}' \) being the root strain measurements, and \( z \) is a 2 x 1 vector with elements \( \{z_1, z_2\}' \) being the tip deflections. The scripts 1 and 2 refer to leading edge and trailing edge respectively. Recall that the state vector \( x \) is an 20 x 1 vector. \( w \) and \( v \) are process and signal noise vectors respectively that are not known beforehand. During operation of the active rotor, the measurements available are \( \varepsilon_v \), and the command input \( u \). Note that sensor noise is added only to the measured signals \( \varepsilon_v \), but not to the unmeasured output \( y \).

The definition of the matrices \( \Gamma_1 \) and \( \Gamma_2 \) depends on how the process noise \( w \) is constructed. In one representation, \( w \) may be chosen as additive noise on the command signals \( u \). In this case, \( \Gamma_1 = B \) and \( \Gamma_2 = D \). Under the assumption that the pair \((A, B)\) is controllable, the process noise vector is guaranteed to affect all the states of the system. Under this representation, the block diagram of the system would be that of figure 3-1.

![Figure 3-1: Basic system block diagram including sensor and process noise.](image)

In another representation, the matrices \( \Gamma_1 \) and \( \Gamma_2 \) are chose as \( I \) and \( 0 \) respectively. In this case, the process noise would enter the state vector directly, and would show in the output equation only through the states. In this case, the block diagram of the system would be that of figure 3-2. This is the chosen method of implementation. The resulting estimators were found to track the strain readings faster, and adjust accordingly.

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Figure 3-2: Block diagram of system with process noise adding directly to state vector.

Consider a dynamical system represented by the block diagram in figure 3-2. Our objective is to design a model based estimator that will produce an estimate of the unmeasured output \( z \) given knowledge of the input applied \( u \) and the measurements \( e \), in the presence of noise. The optimal steady state estimation for this problem can be written as:

\[
\begin{align*}
\dot{x} &= Ax + Bu + L(e - \varepsilon_e) \\
\hat{e} &= C_1\dot{x} + D_1u \\
\hat{z} &= C_2\dot{x} + D_2u
\end{align*}
\]  

(3.5)  
(3.6)  
(3.7)

where the hat (\(^\wedge\)) denotes that the quantity is an estimate, and \( L \) is the estimator gain matrix. It can be seen from equation 3.5 that without the \( L(e - \varepsilon_e) \) term, the estimator would simply simulate the response of the system due to known input, ignoring the presence of noise. The state estimate is partly driven by the error term, \( (\hat{e} - \varepsilon) \), and the observer gain matrix \( L \) corrects for the difference between actual state and their estimates. A necessary condition is that the pair \((A, C_1)\) is observable so that the state vector can be reconstructed from the measurements \( e \).
Different estimator design algorithms are based on different assumptions and optimality criteria, and thus resulting in different gain matrices $L$. One of the simplest methods of designing estimators is that of pole-placement. However, for a $20^{th}$ order system, pole-placement becomes numerically fragile and the placed eigenvalues of the system end up far from their intended locations in the $s$-plane.

### 3.2.2 Kalman Filters

The Kalman filter is chosen as the state estimator. The assumptions behind the Kalman filter are:

- The dynamics of the system are exactly modeled.
- The disturbance and noise signals are random with Gaussian distribution and zero mean.
- The signals are stationary. That is, their statistical properties do not vary with time.
- The frequency content of noise and disturbance is uniform. If colored frequency content is to be modeled, a coloring filter may be augmented to the plant, keeping the white noise input assumption in place.
- The covariances of $w$ and $v$ are known. The covariance is defined by:

$$
\text{Cov}(w) = \int_0^T w(t)w'(t)dt
$$

The derivation of the Kalman filter can be found in many texts, including [1] and [9]. The main idea behind optimal state estimation is defining a cost functional that is positive definite with respect to estimation errors. The simplest such functional is 2-norm of the error,

$$
J = E \left[ \int_0^\infty e'(t)e(t)dt \right]
$$

where $E$ is the expectation operator, and $e$ is the error in the state vector defined as $(x - \hat{x})$. The Kalman filter gain matrix $L$ is the matrix that minimizes $J$ subject to the assumptions above. The solution for $L$ is given by:

$$
L = PC'(R_v)^{-1}
$$

where $P$ is the state error covariance matrix that satisfies the continuous algebraic Riccati equation:
\[ \dot{P} = AP + PA' + R_w - PC'(R_v)^{-1}CP \] (3.10)

The steady state solution of the Riccati equation is when \( \dot{P} = 0 \), thus

\[ 0 = AP + PA' + R_w - PC'(R_v)^{-1}CP \] (3.11)

It can be shown that the Kalman filter minimizes the 2-norm of the state estimation error. Invoking Parseval’s equality shows that this is equivalent to minimizing the integral of the singular values of transfer function from disturbances to estimate errors \( G(s) \). Thus, the Kalman filter minimizes the \( H_2 \) norm of \( G(s) \) [1].

### 3.3 Application to the Active Rotor

In this section, the application of Kalman filter to the Active Rotor is presented. A discussion on the inclusion of sensor and process noise to the system is presented, followed by comparison of Kalman filter estimates with experimental results.

#### 3.3.1 Sensor and Process Noise

Generally it is easier to measure sensor noise than to measure process noise. The steady state “jitter” of the sensors can be measured, and the covariance associated can be computed. Figure 3-4 shows plots of steady state noise. Recall that the measurements \( \varepsilon_1 = \frac{1}{2} (s_1 - s_3) \), and \( \varepsilon_2 = \frac{1}{2} (s_2 - s_4) \), with the subscripts as defined in table 2.1. It can be seen clearly that the strain values \( \varepsilon_2 \) are generally higher than \( \varepsilon_1 \). This reflects in the sensor noise covariance matrix:

\[ R_v = \begin{bmatrix} 4.3102 & 0 \\ 0 & 12.0045 \end{bmatrix} \] (3.12)

The strain gage sensors are supposedly identical sensors, installed in the same way, and subject to the same operating conditions. However, it is apparent from figure 3-4 that the noise covariances are different. This may be attributed to imperfections in soldering and wire routing. It is clear from the figure that the noise is not white, and that some frequencies override the readings. This may result in
covariances that are higher than that due to the white noise content in the signal. However, as will be seen, since process noise is not known beforehand, and will be varied as a free parameter to yield the best estimates. Knowledge of the exact covariance of the noise estimates is not necessary. It is the ratio of process to sensor noise that is of importance.

![Strain 1 Noise](image1)

![Strain 2 Noise](image2)

**Figure 3-4:** Strain gage noise.

Generally it is more difficult to obtain an estimate for process noise because it cannot be measured directly as with the case sensor noise. Furthermore, even when spectral estimates are available, it is not known how this noise affects the system. In other words the $\Gamma_1$ and the $\Gamma_2$ matrices are not obviously defined.

The process noise for the active blade is modeled as additive noise as shown in figure 3-1. This modelling setup assumes that all process noise can be represented as disturbances to the input applied by the piezos. Under this assumption, $\Gamma_1 = B$ and $\Gamma_2 = D$.

Process and sensor noise values are assumed to be stationary, that is, their covariance matrices $R_v$ and $R_w$ do not vary with time. Consequently, a steady state Kalman filter is used. From an implementation perspective, the computational expense of solving the unsteady state Riccati equation (Equation 3.10)
is high, making its real time implementation difficult. Thus the steady state estimator is computed once and used throughout.

Depending on the ratio of sensor to process noise, the gains of the Kalman filter vary. The the factor \( \alpha \) is introduced. \( \alpha \) multiplies out the maximum covariance in the sensor noise covariance matrix. Thus \( R_w = \alpha R_{e(2,2)} I \). When \( \alpha = 0 \), process noise is zero. As \( \alpha \to \infty \), the process noise is much higher, making the sensor noise comparatively very small.

In the case where \( \alpha = 0 \), it is assumed that the response of the plant is entirely due to known inputs. Bearing in mind that Kalman filters assume that plant models are perfect, the estimator reduces to simulating the system based on the known inputs. No matter how small the noise levels are on measurement, they are discarded since they would introduce error to the simulation. This is done by setting the \( L \) gain matrix to 0. As \( \alpha \) increases, the Kalman filter places more weight on the measurements relative to the known inputs.

### 3.3.2 Comparison with Experimental Results

To compare the output of Kalman filters with experimental data, the system identification set-up described in Chapter 2 is utilized to acquire time domain data. Measurements included input, strain gage readings, and laser vibrometer tip deflection. The response of the estimator is tested for three categories of inputs. They are illustrated in figure 3-5. In the first case the input is a sinusoidal function applied. The response captures the transients, and settles down to the steady state response. The second case is when a steady state response due to sinusoidal input is achieved, but a disturbance is induced to the system. The disturbance is induced by applying an external impulsive load to the tips of the spars spars. The third case is when only an impulsive disturbance in applied with no sinusoidal input.

The first set of time domain data are sine waves starting from zero initial conditions. With this data, the estimator is tested for the transient as well as steady state response due to the input. Note that in this case, process noise is almost negligible. In the second set, sine input functions are also maintained, but an external disturbance is induced to the system by plucking the tip of the spars. The purpose is to see how the estimator will react to such external disturbances. The third set of data are for pure disturbances, without any input voltage applied at the piezos.

In order to choose a suitable value of \( \alpha \) (from \( \alpha = 10^{-4} \) to \( \alpha = 10^4 \)) that would yield accurate estimates, different filters are constructed for a range of values of \( \alpha \), and their performance is examined.

For the first case, sinusoidal excitations were applied to the leading edge piezo, with \( \varepsilon_1, \varepsilon_2, \) and \( z_1 \) as measured responses. These experimental results are compared to Kalman filter predictions for values

\[z_1 \] Since only one laser vibrometer was available, only the leading edge tip deflection was measured.
of $\alpha$ ranging from $10^{-4}$ to $10^4$. Figure 3-6 shows the resulting root mean square errors for the three measured variables. The following remarks are made:

- Estimation errors are highly dependent on excitation frequency.

- Errors in strain measurements decrease with increasing $\alpha$ asymptotically. That is, up to a limit of $\alpha = 100$, the gain in accuracy in strain estimates due to increasing $\alpha$ is negligible. Note that a very large $\alpha$ will cause the estimates to track sensor noise, which is undesirable. Therefore qualitative judgement of sensor noise rejection must be applied to choose $\alpha$.

- The change in error in tip deflection readings is minimally affected by $\alpha$.

- Errors in phase are present in the tip deflection estimates. These are attributed to approximations resulting from the least squares fit discussed in Chapter 3.

Figure 3-7 shows the resulting root mean square errors for the three measured variables for the case when pure disturbances are applied to the structure, with no input to the piezo. The figures show the case when the disturbance is applied at the leading edge (labelled “disturbance 1”), and when the disturbance is applied at the trailing edge (labelled “disturbance 2”). The trends noted with regard to figure 3-6 also apply here.

Thus the value of $\alpha$ does not affect the errors in tip deflection estimates. However, increasing $\alpha$ would decrease the errors in strain estimates. At a value of $\alpha = 1$, the errors in strain are sufficiently small (less than 10 microstrain). This value is sufficiently small since it is below the covariance computed in 3.12. The estimator chosen was designed with $\alpha = 1$.

Figures ??, 3-9 and 3-10 compare the response of the filter to that of the experiment in time domain. In all three cases, it can be seen that the filter adjusts the state estimate to match the strain gage readings. Tip deflection matches to a lesser degree. This is attributed to approximation errors between the model and the actual system, especially in the transfer functions involving tip deflection. It can also be seen that in figures 3-9 and 3-10, the estimator responds slowly to the disturbances induced by the external impulse.

The Kalman filter is not expected to respond quickly to impulse transients since it is based on the steady state algebraic Riccati equation. Better response may be obtained using time varying Kalman filters. That however would require knowledge of the variation of error covariances with time. This information is typically not available in this application.
3.4 Conclusions

This chapter presents Kalman filters as estimators for blade tip deflection. Kalman filters provide optimal state estimates under the assumptions of white, Gaussian process and sensor noise of known covariances. The filters provided tip deflection estimates that were comparable to experimental results.

The filter adjusted the states of the system to track strain gage readings. This did not lead to tracking tip deflection equally well. This is attributed to modelling errors, especially with the transfer function relating tip deflection to the input. To mitigate this problem, robust estimation techniques may be used. Appelby [1] presents a derivation of robust state estimators. Minimax filters provide robustness to noise modeling errors. The assumption of white noise is relaxed, and the filters may be used for noise of any spectral content. Furthermore, robustness to plant modelling may be achieved through $H_\infty$ optimization and $\mu$-synthesis.
Figure 3-5: Three types of test cases: Pure sinusoidal input, Sinusoidal input with disturbance, No input with disturbance. Disturbances are applied externally and not shown in the plots.
Figure 3-6: Variation of errors in estimates with $\alpha$ in response to sinusoidal excitation at various frequencies.
Errors in Strain 1 Estimate

Errors in Strain 2 Estimate

Errors in Tip Deflection Estimate

Figure 3-7: Variation of errors in estimates with $\alpha$ in response to a pure disturbance.
Figure 3-8: Comparison between Kalman filter estimate and experiment for a sinusoidal excitation with disturbance, $\alpha = 1$. Measured and estimate curves coincide in the strain plots.
In-Phase Excitation, Disturbance at LE

Figure 3-9: Comparison between Kalman filter estimate and experiment for a sinusoidal excitation.
\( \alpha = 1 \). Measured and estimate curves coincide in the strain plots.
Figure 3-10: Comparison between Kalman filter estimate and experiment for a pure disturbance, $\alpha = 1$. Measured and estimate curves coincide in the strain plots.
Chapter 4

The Spin Test Facility

As outlined in Chapter 1, spin testing is essential throughout the development stages of the Active Rotor. This chapter provides a complete description of the Spin Test Facility\(^1\). It is introduced in a general overview, followed by a detailed discussion of its main subsystems and components. The design of one of the main components of the rig, the rotating hub, is then presented. Issues pertaining to assembly are discussed. Finally, shake-down of the rig and its characterization is presented.

4.1 Overview

This section presents an overview of the Spin Test Facility. The functional requirements are presented first to motivate the main subsystems and components.

4.1.1 Functional Requirements

The main functional requirements of the Spin Test Facility are determined by the Active Rotor, although it may be used for other spinning applications.

Driving Rotor at Required Speeds: Table 1-1 shows that the required tip speed for the Active Rotor is Mach 1.52, which corresponds to a rotational speed of 16,700 rpm. At the conceptual design stage, the requirement set for the Spin Test Facility is the capacity to sustain a speed of 20,000 rpm. Although this figure is higher than the operational speed of the Active Rotor, it provides a margin of safety, and gives the facility a higher speed capacity that may be used for other tasks.

\(^1\)The terms "rig" and "facility" will also be used synonymously with "Spin Test Facility".
Providing Vacuum Environment: In order to isolate structural dynamics from the effects of aerodynamical loading in the Active Rotor, experimentation under vacuum is necessary. This has the added advantage of substantially relaxing the power requirements of the drive system, making spin testing easier. Experience has shown that vacuum levels of 100 to 300 mTorr are sufficient to eliminate any noticeable aerodynamic forces on the rotating structure [18]. The allowable time for the system to reduce pressure to the given levels is set to 20 minutes.

Communication with Rotating Blade: Communication with the rotating structure is essential to provide command signals to the piezo actuators and to read deformation sensing signals from the strain gages. Ultimately, 26 blades with 2 degrees-of-freedom would require at least 52 channels of communication to provide command signals, and at least that number to provide sensing if individual control of each blade is desired. Under actual experimental situations, the requirements may be significantly decreased by actuating only a smaller number of blades, or by constraining the degrees-of-freedom of the entire rotor.

Condition Monitoring: High-speed rotating rigs must be constantly monitored to ensure reliable and healthy operation. Quantities such as overall vibration levels and temperatures of critical components serve as good indicators of the operating condition of the facility.

Safety: The facility will be used to test articles under development in high g-fields. These articles are prone to mechanical failure. Therefore safety is an essential requirement, and the ability to contain and absorb such failures is imperative.

4.1.2 Overall Description

Figure 4-1 shows an overview of the Spin Test Facility, and figure 4-2 provides a close-up view of its main assembly. To give a sense of dimensions, the diameter of the vacuum chamber is approximately 1 m and the height of the rig from top to bottom is also 1 m. With reference to the two figures, a basic description of the facility follows:

- A high-speed motor (20,000 rpm) drives the main shaft via a double spline connection. Two sets of ball bearing provide support for the shaft. A rotating hub is mounted on the top face of the main shaft to accommodate the rotating blades under test.

- The rotor is enclosed in a chamber which is evacuated by a vacuum pumping system connected at the vacuum port. At all mating surfaces, viton o-rings seal the vacuum from atmospheric pressure.
There are two locations of interface between a stationary frame of reference and a rotating frame of reference. To maintain the desired pressure difference across such interfaces, graphite face seals are utilized. The face seals remain stationary and rub against highly polished rotating mating rings, creating the sealing interface.

- Communication with the sensors and actuators of the Active Blade is attained by wiring the blades through a 26 channel slip ring. Wires are routed from the blade along the rotating hub, to the wire distribution hub, through a connector to the slip-ring, which in turn connects to the stationary frame of reference. The slip-ring consists of a rotating shaft containing coin-silver rings that rotate against graphite brushes. Leaf springs ensure that the brushes maintain contact with the rings at all times. The wires leaving the slip ring are then connected to a data acquisition and control system for reading measurements and providing control signals.

- Vibration, rotation speed and temperature sensors are connected at the bearing housing and spline housing of the facility for condition monitoring. The readouts of those sensors, as well as the motor control panel are mounted on a control panel external to the Spin Test Facility.

- For safety purposes, a three inch thick cast iron containment ring is present in the vacuum chamber to absorb test articles that may fly radially in case of failure. Additionally, during high-speed operation, all operators leave the room, and monitoring and control of the rig is conducted from an external control room.
Figure 4-2: Close up view of the main assembly of the Spin Test Facility.
Pictures of the Spin Test Facility are shown in figures 4-3, 4-4, 4-7, 4-8, 4-10 and 4-12. Table 4.1 relates the subsystems to the functional requirements of the test rig.

Figure 4-3: Vacuum chamber.

Figure 4-4: View from top of vacuum chamber with slip-ring and rotating hub removed.
<table>
<thead>
<tr>
<th>Primary Function</th>
<th>Sub-Function</th>
<th>Component(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driving rotor to required speed</td>
<td>Power</td>
<td>Motor</td>
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<td></td>
<td>Control</td>
<td>Controller</td>
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<tr>
<td></td>
<td>Alignment</td>
<td>Spline connector</td>
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<td></td>
<td>Support</td>
<td>Bearings</td>
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<td></td>
<td>Lubrication</td>
<td>Oil-mist system</td>
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<td></td>
<td>Cooling</td>
<td>Oil-Free pressurized air supply</td>
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<td></td>
<td></td>
<td>Water connections to motor</td>
</tr>
<tr>
<td>Providing and maintaining vacuum</td>
<td>Pumping</td>
<td>Vacuum pump</td>
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<tr>
<td></td>
<td>Pressure measurement</td>
<td>Pressure sensor</td>
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<tr>
<td></td>
<td>Connections</td>
<td>Pressure readout</td>
</tr>
<tr>
<td></td>
<td>Sealing parts</td>
<td>Vacuum tubing, connectors and valve</td>
</tr>
<tr>
<td></td>
<td>Sealing rotating and stationary parts</td>
<td>O-rings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Face seals</td>
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<tr>
<td></td>
<td></td>
<td>Mating rings</td>
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<tr>
<td>Rig monitoring</td>
<td>Vibration</td>
<td>Accelerometers</td>
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<tr>
<td></td>
<td>Temperature</td>
<td>Conditioning-amplifier</td>
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<tr>
<td></td>
<td>Rotational Speed</td>
<td>RMS readout</td>
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<td></td>
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<td>Thermocouples</td>
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<td>Readout</td>
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<td>Proximity probe</td>
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<td>Conditioning circuit</td>
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<td></td>
<td></td>
<td>Counter</td>
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<tr>
<td>Communication</td>
<td>Communication with rotating frame</td>
<td>Slip-ring</td>
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<tr>
<td></td>
<td>Routing</td>
<td>Wiring</td>
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<td></td>
<td>Reading and storage</td>
<td>Wire distribution hub</td>
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<tr>
<td></td>
<td></td>
<td>Data acquisition system</td>
</tr>
<tr>
<td>Safety</td>
<td></td>
<td>Containment ring</td>
</tr>
</tbody>
</table>

Table 4.1: Spin Test Facility summary
4.2 Description of Main Components and Subsystems

4.2.1 Drive Train

Figure 4-5 shows that the motor drives the main shaft via a double spline connection. This provides tolerance for slight misalignment between the two shafts, and thereby decreasing the load on the bearings of the motor and rig.

![Exploded view of drive train](image)

Figure 4-5: Exploded view of drive train.

The shaft is supported by an upper pair of duplex angular contact ball bearings and a lower angular contact ball bearing. The duplex bearing pair are arranged in tandem to provide a large thrust carrying capacity in the upward direction. It is estimated that when a complete Active Rotor is installed and tested under full aerodynamic loads, a thrust of 1.333 kN will be generated. Torrington Fafnir bearings 2MMV99110-WN-CR-DUL and 2MMV99107-WN-CR-DUL were used for the upper and lower sets respectively. The bearing \( L_1 \) life\(^2 \) is 3.9 hours for the duplex pair and 52.5 hours for the lower bearing, assuming operation at full operating speed. This is a considerable period of time given that an actual test under full aerodynamical loading is expected to last for durations on the order of one minute.

Axial preload is necessary to ensure that all balls in the bearing are in full contact with the races such that no skidding occurs. The bearings require a minimum preload of 600 N combined. There are two factors that contribute to the preload in the bearings:

\(^2\)The \( L_1 \) life of a bearing is the expected time of operation (usually in hours) that 99% of bearings of a particular type will survive under given loading conditions.
• 24 helical compression springs\(^3\) placed in the spring cage. Each spring has a stiffness of 392 N/m and provides a force of 25 N at the compression length provided by the geometry of surrounding parts. Note that this barely provides the required loading of the bearings.

• The force due to the pressure differential amounts to 3 kN, which is significantly higher than the requirements of the bearings. The presence of this force guarantees that the bearings operate under good contact.

A “bear-hug” nut\(^4\) is required to lock the bearings in place against the spacer collar and shaft. The nut must be tightened at a torque of 65 ft lb.

### 4.2.2 Vacuum System

The vacuum system is required to evacuated the vacuum chamber of approximate volume of 10 cubic feet within a duration of 20 minutes. A Leibold Trivac D16B pump was selected because of its sufficient pumping speed of 13.4 cfm. As shown in figure 4-6, the pump is connected to the chamber via a flexible vacuum hose, valve and T-connector. All of these components add flow resistances, and were taken into consideration during pump selection. The valve is utilized to lock vacuum in the chamber, and to provide a safe method to release vacuum when operation of the rig is complete.

![Figure 4-6: Schematic of vacuum system.](image)

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\(^3\)Helical compression springs. Part number C0120-24-1000C-M from Associated Springs, Barnes Group.

4.2.3 Lubrication

Lubrication of the bearings is attained through the use of a Norgren Micro-Fog system (model # 10-015-002, with a capacity of 8-32 bearing inches\(^5\)). This system atomizes oil droplets, and applies them to the bearings via pressurized air passages. This creates a cloud of oil in the bearing housing that reaches out to all parts of the bearings, and ensures proper and efficient lubrication of the bearings.

Shell Oil Vitrea 22 with a viscosity of 20.3 centiStokes at 100 °C is recommended by bearing manufacturers, and is the lubricant of choice.

4.2.4 Instrumentation and Condition Monitoring

The following parameters need to be constantly monitored during the operation of the spin test facility:

- Rotor speed
- Vacuum level
- Overall vibration level
- Thermal condition of the rig

---

\(^5\)The unit for characterizing machine lubrication requirements is Bearing Inch. It is a function of bearing type, inner race diameter and preloading conditions. The requirements for the Spin Test Facility are 10.5 bearing inches.
Measurement of Rotor Speed: Several options were available for measurement of shaft speed. Shaft encoders are capable of determining a shaft’s angular position accurately, and hence its velocity. Their major drawback is their high cost. Another alternative is to obtain a once per revolution marker signal that can be used in conjunction with a frequency counter to measure the shaft’s speed. This is the adopted solution due to its ease of implementation and relatively low cost.

An eddy current proximity probe (Bently Nevada model 330101) was selected. An eddy current sensor is preferred over an optical sensor because it is more robust to environmental factors such as the presence of lubrication oil or foreign bodies that may contaminate optical readings. It is mounted radially in the spline housing to determine the proximity of the spline coupling shaft, where a notch was machined to give the once per revolution signal. The notch in the spline connector shaft was sized appropriately such that a significant signal is generated at maximum operating speeds. Referring to figure 4-9, the selected dimensions $d$ and $w$ were 0.8”, and 0.45” respectively. This is consistent with a shaft diameter of 1 inch rotating at 20,000 rpm. This will result in the desired 5 - 10 V pulse per turn. To alleviate forces on the bearings due to the imbalance created by a single notch, an opposing notch is machined and covered with light weight metallic conducting tape to provide the required asymmetry as seen by the eddy current sensor.

Measurement of Vacuum Chamber Pressure: Vacuum chamber pressure is measured using a
thermocouple" pressure probe with scale readings of 1 to 1000 mTorr. A thermocouple pressure probe has a hot wire with known electrical power input and known resistance. Thus, the heat generated is also known. The temperature of the wire depends on the rate of natural convection, which in turn is a function of pressure. A thermocouple measures the temperature of the hot wire, and pressure levels are deduced. Special readouts accompany the probe.

**Measurement of Vibration Levels:** The need for a vibration sensor is threefold: i) measurement of vibration levels during in-situ balancing procedures, ii) measurement of overall vibration levels for monitoring condition and health of the test rig, and iii) providing good vibration signals for machine diagnostics in case of high overall levels. The first two requirements indicate that the sensor should have the largest possible sensitivity, whereas the third requirement translates to constraints on the minimum bandwidth of the sensor. This requirement is set by considering the largest vibration frequency of interest. Consider the case of a damaged bearing at the inner race. The associated frequency is of vibration for the the duplex pair would be 3.82 kHz. In order to see a good signal, the bandwidth of the sensor must be higher than this excitation frequency, and preferably wide enough to see the harmonics of this excitation if any.

The bandwidth requirement automatically discards proximity probes and velocity meters since they are limited to 2 kHz. A Brüel & Kjær accelerometer type 4382V was selected. It is a piezoelectric accelerometer with a bandwidth of 8.4 kHz and a sensitivity of 3.1 pC/ms⁻². Associated with it is the Nexus conditioning amplifier type 2692 with a bandwidth of 20 kHz, and selected gains varying from 0.1

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6The highest frequency indicating bearing damage is the Ball Pass Frequency, Inner race (BPF1) factor which is 13.67 for the upper duplex bearings [22]. Assuming operation at a speed of 16,700 rpm, or 280 Hz, the resulting frequency of vibration is 3.82 kHz.
mV/pC to 10V/pC. Correct mounting of the accelerometer is essential to achieve reliable measurement. A 10-32 UNF threaded stud is used. This mounting method ensures the full bandwidth of the sensor, in contrast to other mounting techniques\(^7\) that put a limitation on the bandwidth of the measurement. With the selected arrangement, the vibration measurement system has the capability of measuring vibration measurements for machine diagnostics, and sensing the second harmonic of a vibration at the maximum expected frequency.

**Measurement of Bearing Temperatures:** Measurement of temperatures of critical components is essential to ensure sound operation of the Spin Test Facility. Rig and motor bearings are continuously monitored via thermocouples. J-type thermocouples are prepackaged with the motor, whereas more corrosion resistant K-type thermocouples are placed in their corresponding ports in the bearing housing. High thermal conductivity thermocouple paste is used to ensure good thermal contact.

![Diagram of lower portion of the rig with sensors installed](image)

**Figure 4-10:** View of lower portion of the rig with sensors installed.

All sensor readings were integrated in an instrument panel that accommodates readouts as well as the motor control panel as shown in figure 4-11.

\(^7\)Such as magnets, mounting wax and adhesives.
4.3 Rotating Hub Design

This section discusses the design of the rotating hub utilized in testing Active Blades. Since those blades are under development, the hub designed is an intermediate one utilized for testing the limited number of blades produced. It was specified to accommodate three blades. This is sufficient until candidate blade concepts are tested and finalized. Once a concept is agreed upon and multiple blades are manufactured, a new hub will be designed accommodating the full number of blades.

4.3.1 Functional Requirements

The functional requirements of the hub were set to:

- Ability to sustain three blades of mass 30 grams, with center of mass at a radius of 15.7 cm, for a maximum speed of 20,000 rpm. Although the actual maximum rotational speed of the Active Blade is less, this requirement was set to give a margin of safety and added functionality to the rotor. The rotating hub was designed before the completion of a blade, and it turns out that 30 grams of blade mass is an underestimate. As a result, the more massive blades cannot be rotated at the full speed. A blade mass with the same center of gravity rotating at 16,700 rpm (Active Rotor full speed) should not exceed 41 grams.

- Ability to accommodate blades with varying root geometries. During development stages, blade
root shapes are prone to change. This requirement is essential to provide flexibility to the entire test rig.

- Having provisions for in-situ balancing of the rotor. Since different blades will be tested, in-situ balancing is important to save time and costs associated with shop balancing.

### 4.3.2 Concept

As a result of the requirements set above, the concept adopted for the rotating hub is the “hub and clip” concept shown in figure 4-12. A rotating hub is sized to mate to the main shaft at the bottom. The hub contains three “T-blocks”, each accommodating a pair of clips that hold the blade in place. The radial centrifugal load of the blade is transmitted through the clips to the T-blocks, that bear the forces due to the rotation of both the blade and the pair of clips. This suggests that clip materials must be as light as possible to minimize the load on the T-blocks. This design concept is versatile enough to accommodate different blades by having to tailor only the clip set for each blade root design. Furthermore, the hub may accommodate only a single blade, with dummy counterweights placed in the vacant clips.

![Figure 4-12: Hub and clip concept.](image)
4.3.3 Analysis and Design

Once the general geometrical configuration is set, dimensions are proposed, and finite element analysis is utilized to predict maximum stresses in the components. Several dimensions were set due to boundary constraints of the design problem such as at the interface with the main shaft. Other dimensions are free parameters. Finite element models were created, and the different dimensions were iterated upon until a satisfactory configuration yielded reasonable stress levels. The dimensions that were varied are shown in figure 4-13.

![Figure 4-13: Dimensions that were varied in the hub and clip during design stage.](image)

Finite element analysis was performed on MSC/NASTRAN, whereas MSC/PATRAN was used as a pre-processor and post-processor. Parametric meshing, with manual mesh seed input was utilized to provide a high degree of control over the mesh densities at different locations of the model. Mesh densities were progressively increased until asymptotic stress levels were reached in both the clip and hub models. In order to economize in computational requirements, symmetries in the parts designed were exploited, avoiding analysis of the entire part. One half of a clip and one sixth of the hub were analyzed with appropriate symmetry boundary conditions applied. Meshes of the models are shown in figures 4-14 and 4-16.

The loads and boundary conditions applied on the hub model are as follows:

- Inertial loading due to the rotation of the parts at 20,000 rpm (333.33 Hz).
• Cyclic symmetry boundary condition at edges.

• Zero displacement boundary conditions as a result of the interface between the main rotating shaft and the rotating hub.

• Force boundary condition on the T-block to model the centrifugal loads of the blade and the two clips rotating. A Multiple Point Constraint (MPC)8 was utilized to distribute the force over the area of application in a manner similar to that of a full body contact.

The loads and boundary conditions applied on the clip model are as follows:

• Inertial loading due to rotation of the parts at 20,000 rpm (333.33 Hz).

• Mirror symmetry boundary conditions at edges.

• Zero displacement boundary conditions as a result of the interface between the rotating hub and the clip.

• Force boundary condition on the inclined faces to model the loading due to blade rotation.

Since centrifugal force boundary conditions on the hub depend on the mass of the clips, the sequence adopted is to design the clips first with the objective of minimizing their weight. The resulting centrifugal forces are then applied to the model of the hub.

Material Selection

Materials were chosen with one primary criterion in mind: yield strength-to-weight ratio. The components in hand are primarily load carrying components, and this load is a function of its own weight as well as the weight of the blade.

Table 4.2 lists relevant properties of the materials selected for the hub and clips (source [17]).

Results

Meshes of the models and their resulting solutions are shown in figures 4-14, 4-15, 4-16, and 4-17. Table 4.3 summarizes the results of the finite element models.

8MPC's are constraints applied on a set of nodes to model certain loading conditions in a finite element model. The MPC used is "rigid body". It distributes the load from a point node to an entire surface.
<table>
<thead>
<tr>
<th>Component</th>
<th>Rotating Hub</th>
<th>Retention Clips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Steel 4140 OQT 400</td>
<td>Aluminum 7001-T6</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>2,000</td>
<td>676</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>1,730</td>
<td>627</td>
</tr>
<tr>
<td>Density [kg/m³]</td>
<td>7,680</td>
<td>2,800</td>
</tr>
<tr>
<td>Modulus of Elasticity [MPa]</td>
<td>207</td>
<td>73</td>
</tr>
<tr>
<td>Poisson Ration</td>
<td>0.27</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 4.2: Mechanical properties of selected materials for rotating hub and blade retention clips.

<table>
<thead>
<tr>
<th>Component</th>
<th>Rotating Hub</th>
<th>Retention Clips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry modeled</td>
<td>One sixth of hub</td>
<td>One half of clip</td>
</tr>
<tr>
<td>Number of elements</td>
<td>2084</td>
<td>2133</td>
</tr>
<tr>
<td>Element type</td>
<td>Hexagonal - 8 node</td>
<td>Hexagonal - 8 node</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>8714</td>
<td>2133</td>
</tr>
<tr>
<td>Maximum Von Mises Stress [MPa]</td>
<td>900</td>
<td>486</td>
</tr>
<tr>
<td>Factor of Safety</td>
<td>1.92</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters of finite element models of hub and clips.

The maximum stresses for the hub occur at the fillet of the T-block due to the high moment applied by the centrifugal force generated by clips and blade. The maximum stresses in the clips were at a location that is also subject to high moments as a result of the centrifugal force of the blade. The factors of safety shown in table indicate that the hub is at a less critical stress state than the clips. This may suggest that the hub is over-designed compared to the clip. Note however that due to potential redesign of the blade, a need may arise for the hub may accommodate more massive blades and clips that would generate higher forces.
Figure 4-14: Finite element mesh of one sixth of rotating hub.

Figure 4-15: Von Mises stresses of rotating hub at full rotational speed (20,000 rpm).
Figure 4-16: Finite element mesh of one half of clip.

Figure 4-17: Von Mises stresses of clip at full rotational speed (20,000 rpm).
4.4 Rig Assembly, Balancing and Characterization

4.4.1 Assembly

This section a brief outline of the assembly procedure is described, and certain precautionary measures worth highlighting are presented.

**Bearing Assembly**  The most critical step in the assembly of the Spin Test Facility is that of inserting the bearings on the main shaft. Since the tolerances of the shaft are chosen to provide a shrink fit with the bearings, the dimensions need to be temporarily changed in order to avoid force fits that are damaging to bearings. The life and durability of a bearing is very sensitive to its handling conditions during assembly. If mishandled, they become very fragile, and if inserted properly, they become quite robust.

At different attempts, three methods were used to place bearing sets:

1. Use of liquid nitrogen to shrink the shaft and insert the bearings
2. Use of hot oil bath to expand the bearings
3. Use of conical electrical heaters to expand bearing inner race.

Only the third method proved to be successful, primarily because of the clean handling conditions made available. In the first and second method, the bearings were handled using tongs and tweezers that were impractical. That resulted in slight jamming of the bearings on the shaft and required the use of force to position them in place.

**Face Seals**  Three important considerations must be carefully observed when installing the face seals.

- Ensuring that they are placed perfectly normal to the mating ring to minimize wobbling effects.
- Ensuring that the mating ring is finely polished up to specifications (Helium light-band lap, surface finish grade 4). This produced a surface with topologies variations less than 24 micro-inches.\(^9\)
- Ensuring that the graphite surfaces are not damaged, and are well lubricated with excess oil, before operation of the rig. This minimizes the rate of wear of the graphite faces.

**Tolerances**

Throughout the assembly steps, measurement of benchmark dimensions were constantly done to ensure that mating parts did mate properly.

\(^9\)According to surface scans conducted by Lapmaster International.
4.4.2 Balancing

Balancing of individual parts (shaft, rotating hub and clips, wire distribution hub and slip-ring) is performed in a balancing shop to ensure high accuracy of the rotating components alone. Shop balancing of the rotating components of the rig was done in a progression of steps. Since the rotating shaft may be used with different rotating hubs (i.e. a 3 blade rotating hub and 26 blade Active Rotor hub), it was balanced by itself in a balancing shop by removing metal shavings from predesignated areas. The rotating hub (with clips in place) is then installed, and the new assembly is relaballed, ensuring that all balancing masses are removed from the rotating hub assembly. This results in an assembly that is well balanced, and leaves the initial balancing of the main shaft intact for operation with future rotors.

In-situ balancing is performed when blades are mounted in the rotating hub. Bulk counter weights are designed to match the centrifugal effect of the blade under test. Provision for very fine tuning is made by the placement of balancing holes in the rim of the wire distribution hub.

4.4.3 Thermal Characterization

The rubbing of the face seals against their mating rings produces substantial amounts of heat that may cause shaft temperatures to elevate, causing the bearings to operate under damaging temperatures and fail. Since forced cooling measures were not incorporated in the rig, there is a need to estimate the safe operating duration of the rig without exceeding the bearing temperature limits of 160 °F. Experiments were conducted, measuring a time profile at different operating speeds. Figure 4-18 shows the results for such tests.

Results show that temperatures were in continuous rise for the duration of the test. This suggests that the rate of heat dissipation is substantially lower than the rate of generation. Furthermore, the rate of increase of temperature was not proportional to the rotational speed.
Figure 4-18: Results of thermal test.
Chapter 5

Summary, Conclusions and Future Work

5.1 Summary

This work is part of an effort towards the development of the Active Rotor – a compressor stage in which the blades are individually actuated and caused to follow prescribed trajectories. This rotor will serve as a research vehicle for the experimental study of flutter in turbomachines, a dynamical instability arising from the interaction of the fluidic and structural forces. An active blade consists of a graphite-epoxy twin spar system that is covered with high strength-to-weight ratio foam to give the blade its aerodynamic shape. Piezoelectric actuators are bonded to the spar roots for actuation, whereas collocated strain gages provide sensing.

This thesis focuses on two lines of work with regard to the Active Rotor development process. The first is developing procedures for dynamical characterization of the active blade, and using the resulting models for designing state estimators. The second aspect is the development of a high-speed spin-test facility that will be used to test candidate blades under rotation throughout their development stages.

Two approaches are presented for the characterization of the Active blade to obtain models suitable for design of controllers and estimators. The first approach is purely experimental where the transfer functions of the system are experimentally determined. The second relies on a finite element model of the blade, modeling of the impact of the piezo on the structure, and identification of signal conditioning equipment. The two procedures are implemented on the active twin spar system. For the experimental approach, a model with bandwidth of validity of 400 Hz was fit to the data. The order of the models
obtained was reduced using Hankel Singular Value decomposition. This technique eliminates the least controllable/observable modes of the system, thereby reducing its order with minimal effect on the input-output mapping characteristics. The models obtained were reduced to a 20\textsuperscript{th} order system for the model obtained using the experimental approach, and a 16\textsuperscript{th} order system for the model obtained using the finite element approach. These system orders are amenable to real-time implementation of estimators and controllers.

The use of Kalman filtering to estimate blade tip deflection from root strain measurements is investigated. The Kalman filter was implemented based on the model obtained from the experimental approach. The performance is evaluated based on simulation and time domain experimental results.

A Spin Test Facility is developed to test the blade for structural integrity and actuation capability. This involved design of critical components of the rig such as the rotating hub to accommodate blades under development. A “hub and clip” concept is utilized to give flexibility to accommodate different blade root geometries. Finite element analysis is performed to ensure the structural strength of the rotating hub. A vacuum system is installed in order to enable testing the blades with no aerodynamic effects. Instrumentation for the facility includes thermal, vibration and pressure monitoring systems.

5.2 Conclusions

Dynamical Characterization of Active Blade

- Of the two approaches presented for obtaining models of the blade dynamics, the experimental approach produced results that are more accurate, and hence was used for state estimation. Frequency domain identification was implemented, which yielded Bode plots that closely matched experimental transfer functions. Time domain measurements, however, did not match that of the model with the same level of accuracy. Consequently, errors in estimates of tip deflection of the Kalman filter were noted. Time domain identification is a track worth investigation, and may yield better models for better estimates.

- Model order reduction using Hankel Singular Values was successful, yielding models that are of a sufficiently low order to perform real time control. Fitting experimental data to functional forms is highly dependent on the choice of weighting functions, and several iterations are required to achieve satisfactory results.

- Incorrect grounding can lead to a capacitive effect in strain gage readings. Careful choice of which sides of the piezos to ground is necessary. The wiring of the structure tested with its current status
did not allow for complete freedom to choose the grounds of the two blade independently. This scheme has been motivated by reducing the number of slip ring channels to be used per blade. At early testing stages (when a single blade is tested), excess channels may be used to ensure full flexibility in characterizing the blade.

- For the finite element approach to be of value in terms of accurate state estimation, several iterations need to be made to tune the model to match the experimentally measured dynamics. This may be a difficult and lengthy procedure, and may not necessarily yield accurate results. However, the finite element approach can be useful to qualitatively investigate the effect of structural modifications, as well as the addition/removal of sensors/actuators.

**State Estimation**

- Kalman filters are based on the assumption that the working model is accurate. Due to approximation errors in the model utilized, the estimates did not exactly match the results of experimentation. Hence, robust estimator design tools are suggested as alternatives if more accurate estimates are desired.

- Varying the sensor to process noise ratios as basis for the design of the filter affected the accuracy of strain estimates, but did not affect the accuracy of tip deflection as much.

**Spin Test Facility**

- Bearing assembly is the most critical task in the assembly procedure of the Spin Test Facility. Heating the bearings with conical heaters for insertion on the main shaft has proven to be the most effective method.

- Operating time of the facility is limited by the maximum temperatures the bearings can sustain. Results showed that the rate of temperature rise is not linearly related to speed. For speeds up to 6000 rpm, an operational time of 15 minutes is acceptable. For longer operational duration, explicit cooling systems for the bearings must be installed.

- The design presented for the rotating hub can sustain a load of 30 grams at a radius of rotation of 15.7 cm for a speed of 20,000 rpm. A mass of 41 grams can be spun at 16,700 rpm (full speed of Active Rotor) with the same factors of safety. The clip is the “bottle neck” in the design in terms of load carrying capacity. At 16,700 rpm, the hub can sustain a blade weight of 92 grams at the same factor of safety, assuming the clip weight remains unchanged.
A blade root geometry with a smaller root taper angle would ease in the design process if the clips, since the moments generated would be smaller. The material grades utilized have very high strength-to-weight ratio, and are among strongest in their class. Titanium is an expensive option, but may be necessary as blade weights increase and aluminum fails to provide the required strength.

5.3 Recommendations for Future Work

The following points address future work that needs to be done with regard to work on dynamics and control of the Active Blade:

- *Replication of system identification procedures presented to obtain a new model of the blade with the foam added.* Once a blade design is settled upon that was proven structurally valid, it may be identified and fit to a model as outlined in 2.1.1.

- *Updating the finite element model to reflect the blade’s dynamical response and match experimental data.* This is important for all the advantages previously mentioned in chapter 2 of having a finite element based dynamical model. Updating the model should account for any geometrical modifications made, and more accurate modeling of boundary conditions.

- *Characterization of the blade under rotation.* This involves actuation under rotation in vacuum, and measuring strain response at the root. Tip deflection may be obtained from the dynamical model based on the finite element approach. The availability of a tip deflection eddy current sensor will provide a good check for the validity of models.

- *Design and implementation of state estimators and controllers utilized to command blades to their desired trajectories.* It is expected that the dynamics of the blade will vary with rotational speed. This is due to centrifugal stiffening and blade untwist associated with high rotational speeds. Thus, a single controller/estimator may not work well over an the entire range of rotational speeds. One possible way to account for this is to use gain scheduling where different controllers are designed for specific ranges of rotational speed.

The above points are dependent on the development of a complete active blade. In order to do this, the following is necessary:

- *Development of repeatable and accurate fabrication techniques for shaping blade foam.* This is the most pressing task that must be finalized in order to build a complete active blade. Thermoforming techniques have proven inadequate to shape the foam to its high level curvature and twist. Foam
cracks under resulting high strains. Computer numerically controlled machining promises to be a feasible option. It has the advantage of shaping foam to exact geometries without inducing high strains as thermoforming does, and with higher repeatability. Furthermore, by combining this with numerical water jet cutting techniques for shaping the twin spars from a graphite-epoxy blank will make the two parts mate better and bond easier.

- **Structural testing of the blade by rotating it in the spin test facility at its full speed (16,700 rpm).** This will ensure that the blade survives maximum centrifugal loading conditions. Before subjecting the blade to its maximum rotational speed, experimentation at lower speeds are useful. A speed of 11,000 rpm corresponds to a tip speed of Mach 1, thus taking the rotor into the transonic regime. Preliminary characterization and testing of blades at 11,000 rpm is useful. Incremental stages of testing and characterization may be performed in an alternating fashion.

The above points address some of the pressing tasks towards the development of an active blade. Once the problems have been addressed, multiple blades are fabricated and integrated in a complete rotor. This leads to another regime of research in which experimental data are obtained, and focus will shift from the development of the Active Rotor as a research tool, to the investigation of the phenomenon of flutter.
Appendix A

Transfer Functions

A.1 Experimentally Determined Transfer Functions

This appendix contains all of the transfer functions that where determined experimentally as outlined in section 2.1. The following table summarizes.

<table>
<thead>
<tr>
<th>Input Excitation</th>
<th>Output(s)</th>
<th>Measured/Inferred</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$ (at leading edge only)</td>
<td>$s_1, s_3$</td>
<td>Measured</td>
<td>A-1</td>
</tr>
<tr>
<td>$u_1$ (at leading edge only)</td>
<td>$s_1$</td>
<td>Measured</td>
<td>A-2</td>
</tr>
<tr>
<td>$u_1$ (at leading edge only)</td>
<td>$z_2$</td>
<td>Measured</td>
<td>A-3</td>
</tr>
<tr>
<td>$u_2$ (at trailing edge only)</td>
<td>$s_2, s_4$</td>
<td>Measured</td>
<td>A-4</td>
</tr>
<tr>
<td>$u_1 + u_2$ (in phase)</td>
<td>$s_1, s_3$</td>
<td>Measured</td>
<td>A-5</td>
</tr>
<tr>
<td>$u_1 + u_2$ (in phase)</td>
<td>$s_2, s_4$</td>
<td>Measured</td>
<td>A-6</td>
</tr>
<tr>
<td>$u_1 + u_2$ (in phase)</td>
<td>$z_1$</td>
<td>Measured</td>
<td>A-7</td>
</tr>
<tr>
<td>$u_1 + u_2$ (in phase)</td>
<td>$z_2$</td>
<td>Measured</td>
<td>A-7</td>
</tr>
<tr>
<td>$u_1$ (at leading edge only)</td>
<td>$s_1, s_3$</td>
<td>Inferred</td>
<td>A-8</td>
</tr>
<tr>
<td>$u_2$ (at trailing edge only)</td>
<td>$s_2, s_4$</td>
<td>Inferred</td>
<td>A-9</td>
</tr>
</tbody>
</table>

Table A.1: Directly measured transfer functions and their figures.
Figure A-1: Measured transfer function from command signal applied at leading edge piezos to strain at root of leading edge (LE).
Transfer Function from Command Signal to Tip Deflection. Input at LE Piezos.

**Figure A-2:**Measured transfer function from command signal applied at leading edge piezos to tip displacement at leading edge (LE) and trailing edge (TE).
Figure A-3: Measured transfer function from command signal applied at trailing edge piezos to strain at root of trailing edge (TE).
Figure A-4: Measured transfer function from command signal applied at trailing edge piezos to tip displacement at leading edge (LE) and trailing edge (TE).
Figure A-5: Measured transfer function from command signal applied at both piezos (in phase) to root strain at leading edge (LE).
Figure A-6: Measured transfer function from command signal applied at both piezos (in phase) to root strain at trailing edge (TE).
Figure A-7: Measured transfer function from command signal applied at both piezos (in phase) to tip deflection at leading edge (LE) and trailing edge (TE).
Figure A-8: Inferred transfer function from command signal applied at leading edge piezos to root strain at trailing edge (TE).

Figure A-9: Inferred transfer function from command signal applied at trailing edge to root strain at leading edge (LE).
A.2 Transfer Function Fit to Experimental Data

The following figures compare the experimentally determined transfer functions with the model fit. Inputs $u_1$ and $u_2$ are command signals to the leading and trailing edge actuators respectively. Outputs $\varepsilon_1$ and $\varepsilon_2$ are strain response at roots of leading and trailing edges respectively. Outputs $z_1$ and $z_2$ are spar deflections at leading and trailing edges.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Command Signal to leading edge piezos ($u_1$)</td>
<td>Root strain at leading edge ($\varepsilon_1$)</td>
<td>A-10</td>
</tr>
<tr>
<td>Command Signal to leading edge piezos ($u_1$)</td>
<td>Root strain at leading edge ($\varepsilon_2$)</td>
<td>A-11</td>
</tr>
<tr>
<td>Command Signal to leading edge piezos ($u_1$)</td>
<td>Root strain at leading edge ($z_1$)</td>
<td>A-12</td>
</tr>
<tr>
<td>Command Signal to leading edge piezos ($u_1$)</td>
<td>Root strain at leading edge ($z_2$)</td>
<td>A-13</td>
</tr>
<tr>
<td>Command Signal to trailing edge piezos ($u_2$)</td>
<td>Root strain at leading edge ($\varepsilon_1$)</td>
<td>A-14</td>
</tr>
<tr>
<td>Command Signal to trailing edge piezos ($u_2$)</td>
<td>Root strain at leading edge ($\varepsilon_2$)</td>
<td>A-15</td>
</tr>
<tr>
<td>Command Signal to trailing edge piezos ($u_2$)</td>
<td>Root strain at leading edge ($z_1$)</td>
<td>A-16</td>
</tr>
<tr>
<td>Command Signal to trailing edge piezos ($u_2$)</td>
<td>Root strain at leading edge ($z_2$)</td>
<td>A-17</td>
</tr>
</tbody>
</table>

Figure A-10: Transfer function fit to measured data: Input $u_1$, output $\varepsilon_1$. 

110
Figure A-11: Transfer function fit to measured data: Input $u_1$, output $\varepsilon_2$. 

Figure A-12: Transfer function fit to measured data: Input $u_1$, output $y_1$. 

111
Figure A-13: Transfer function fit to measured data: Input $u_1$, output $y_2$.

Figure A-14: Transfer function fit to measured data: Input $u_2$, output $\epsilon_1$. 
Figure A-15: Transfer function fit to measured data: Input $u_2$, output $\epsilon_2$.

Figure A-16: Transfer function fit to measured data: Input $u_2$, output $y_1$. 
Figure A-17: Transfer function fit to measured data: Input $u_2$, output $y_2$. 
Bibliography


