Development of a Method for Controlled Cartilage Deformation

by

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B.S. Engineering - Trinity College, Hartford, CT, 1998

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MASTER OF SCIENCE IN MECHANICAL ENGINEERING AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Development of a Method for Controlled Cartilage Deformation

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Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering

Abstract

Hip dysplasia affects as much as 2.8% of infants born, and left untreated accounts for as much as 76% of all cases of hip joint osteoarthritis. Identification of hip dysplasia is fairly arbitrary since there is not a defined procedure for diagnosis. Since hyaline cartilage is slow to regenerate, tissue engineering alternatives have not yet been perfected, and arthroplasty failure occurs 10 to 15 years following implantation, one of the best treatment options in younger and active patients with abnormal hip geometry remains surgical intervention. The development of an objective function, whereby a surgeon could straightforwardly prescribe the appropriate reconstruction technique, could be an important step to better understanding hip dysplasia and its role in the development of OA. This thesis proposes a reconsideration of the terms used to quantify and describe hip abnormality. A hypothesis for an objective function for surgical planning, and the development of the testing apparatus and protocol for the validation of this hypothesis are presented.

Thesis Supervisor: Brian D. Snyder
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“Why, anybody can have a brain. That's a very mediocre commodity. Every pusillanimous creature that crawls on the Earth or slinks through slimy seas has a brain. Back where I come from, we have universities of great learning, where men go to become great thinkers. And when they come out, they think deep thoughts and with no more brains than you have. But they have one thing that you haven't got: a diploma.”

-The Wizard of Oz
First and foremost, I would like to thank the Whitaker Foundation who provided the funding, encouragement and respect I required while pursuing this degree. Also, I am grateful to Orthofix, Inc. and OEC Medical Systems, Inc. who contributed instrumentation for this project.

I'd like to express my appreciation to Drs. Brian Snyder, S. Daniel Kwak, and Derek Rowell whose constant patience and optimism helped me finish this thesis. I am grateful for the faculty of almost every department at Trinity College, who inspired me with the hours upon hours of “extra-curricular” intellectual discussion. It was here that I learned how to really think, how to value different ideas, how to respect my own ideas, and above all that dreams are fantastic. Also, to my colleagues at Trinity, MIT and the OBL, with whom I spent many nights covering chalkboards with proofs of Schroedinger’s equations, questioning existence, and discussing the real centerfolds that turned us on – the two page crossword puzzle in “World of Puzzles.”

This is dedicated to my grandparents, who are four of the most giving people in the world, and blindly support any venture I embark upon with every actin filament composing their hearts. And to my family who allowed me to watch hours upon hours of Mr. Wizard and 3-2-1 Contact. Finally, I’d like to thank my best friend, David Stewart. Without whose laughs, Red Sox tickets, and piggyback rides during my stretch with crutches, I would not have the smile I do today. Thank you for the lifetime of happiness you have already brought me, and for giving me a lifetime of happiness to look forward to.
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CHAPTER

Introduction

1

1.1 Background

J. Anthony Herring writes, "[Developmental Dysplasia of the hip (DDH)] remains one of the most difficult disorders to understand and treat in all of orthopaedics. Many pitfalls of management have been identified, and yet an individual patient may not respond to the usual treatment for reasons not understood. [1]." Herring's confusion is well supported throughout the orthopedic community.

What is hip dysplasia? The terminology of the Developmental Dysplasia of the Hip (DDH) is often arbitrary, and the vocabulary can be used haphazardly. DDH is sometimes used synonymously with Congenital Dislocation of the Hip (CDH), while some deem these separate disorders altogether. Similarly, dysplasia, which implies a deficiency in the shape of the acetabulum, is frequently interchanged with dislocation, which describes the location of the femoral in relationship to the acetabulum. The following definition will be used for the purposes of this thesis: DDH is an abnormality of the hip that is characterized by an instability or dislocation of the femoral head relative to the acetabulum. This includes dislocation (the complete loss of contact between the femoral head and the acetabulum), subluxation (not total dislocation, but some loss of contact between the
medial portion of the acetabulum and the femoral head), and geometric anomalies (any incongruency in size or shape between the femoral head and acetabulum). DDH, hip dysplasia, and hip instability are considered the same for the principles in this thesis.

The human hip is characterized as a ball-and-socket joint that permits rotation as well as movement in all spatial planes. Legal [2] refers to the hip joint “an imperfect evolutionary solution to the problem of bipedal gait.” He continues to explain that for the proper functioning of any joint, there must be a perfect balance between “the stresses acting on the joint and the ability of the joint to withstand those stresses.” When an imbalance is present and is left untreated, hip dysplasia almost certainly leads to osteoarthritis of the hip [8-11]. Untreated hip dysplasia accounts for as much as 76% of all cases of osteoarthritis in the hip [3].

The prevalence of hip instability appears in 1 to 21.8 of every 1000 infants born [1, 4, 5]. Clinical symptoms of DDH in walking children may include hip pain, limp, restricted abduction, stiffness, shortening of the thigh and increased lumbar lordosis. When the
disease is diagnosed during early development (before 6-9 months of age), infants are treated with bracing techniques with an 88% - 95.6% success rate [5-7]. In general, treatment options include closed reduction (using a brace or traction) in infants, open reduction in toddlers, redirectional femoral and acetabular osteotomies in children and young adults, and finally total joint replacement in older patients with established osteoarthritis.

Osteoarthritis (OA) is a degenerative disease of articular cartilage that occurs in 21 million Americans [12]. Biochemical, metabolic, histological and biomechanical changes to the joint cartilage, synovial fluid and subchondral bone are all associated with OA [13]. The exact pathogenesis of OA is unknown but has been induced experimentally by both biochemical and mechanical manipulations [14-16]. If the exact pathogenesis of OA were clear, one could theoretically design an effective treatment. However, because multiple etiologies have been postulated and the regeneration of hyaline cartilage is slow, treatment options are limited and remain empirical.

With tissue engineering alternatives not yet perfected, and arthroplasty failure occurring 10 to 15 years following implantation, one of the best recommended options in younger and active patients with abnormal hip geometry remains surgical intervention. There are many techniques to perform osteotomies in the hip but the premise always remains the same: to optimize the distribution of load by minimizing pressure within the joint and restoring joint “stability.” These goals rely on the assumption that mechanical factors are responsible for the development of OA in an untreated joint and the apparent joint healing following a successful surgery. While retrospective analysis of patients by radiographs and clinical exam have demonstrated improvement in hip function, there could
be other factors involved in the surgical intervention: reducing or increasing the moment arm of the surrounding muscles and ligaments, tightening or loosening the joint capsule, changing the blood supply, etc. Therefore the question remains whether these current surgical goals are correct and are the only factors affecting joint health postoperatively. Specifically, what is the appropriate objective function for surgical intervention that will prevent or ameliorate the development of osteoarthritis?

The development of an objective function, whereby a surgeon could straightforwardly prescribe the appropriate reconstruction technique, could be an important step to better understanding DDH and its role in the development of OA. To experimentally validate this function a method is needed to simulate the mechanical anomalies presented in a dysplastic hip using a simple model that can be subjected to parametric variations.

1.2 Objectives

The objectives of this thesis are threefold:

1) To develop a tool to quantify the severity of DDH using standard radiographic measurements and to reexamine these measurements in an effort to better describe joint abnormality.

2) To propose a hypothesis for a theoretical objective function for surgical planning of osteotomy
3) To develop the tools and protocol needed to experimenally derive an objective function for the surgical planning of osteotomies about the hip, minimizing the occurrence of OA. To develop a method to validate the legitimacy of the proposed objective function.

1.3 Overview

This thesis presents a set of tools that can be used in several aspects of the diagnosis and treatment of the hip dysplasia. Figure 1.1 diagrams the progression of DDH to OA and the relationship of each chapter of this thesis to these stages of development.

Chapter 2: The current tools used to quantify hip dysplasia and collect quantitative factors used in hip dysplasia include radiologic measurements. A semi-automated tool to calculate a series of radiologic measurements from manually digitized points is presented in this chapter. These measurements represent the current factors used clinically to measure the severity of hip dysplasia. Chapter 2 discusses the downfall of these parameters and suggests the consideration of new parameters for the quantification of DDH.

Chapter 3: A hypothesis for an objective function for surgical planning of osteotomy is presented, along with supportive theory.

Chapter 4: An in situ method for describing cartilage deformation when a mechanical displacement is induced to a joint is offered. This technique will be used for the testing of the theories described in Chapter 2.

Chapter 5: A suggested protocol for the validation of the objective function presented in Chapter 3 is described, as well as further uses for the system developed in Chapter 4.

Appendix B: To carry out the experiments in Chapter 5, a surgical tool for calculating the center of rotation of a joint is needed. The tool described herein can be used real-time in
the operating room for surgical guidance of metal pins into the center of rotation of a joint.

The accuracy and precision of the tool is discussed.

Figure 1.1 Flow Chart of OA progression and treatment, with the thesis chapter involvement in parenthesis.
An instability of the hip occurs in as many as 21.8 of every 1000 births [3-5], and left untreated accounts for as much as 76% of all cases of hip joint osteoarthritis [2]. Identification of hip dysplasia is fairly arbitrary since there is not a defined protocol for diagnosis. Recognition of the disease is based upon physical examination and a variety of radiographic parameters. A tool has been developed that can be used to calculate a number of standard two-dimensional measurements used clinically to diagnose and quantify developmental dysplasia of the hip. A redefinition of clinical terms and their associated measurements is proposed that will better quantify the dysplastic hips for diagnosis and aid in treatment decisions.

2.1 Motivation

Standard x-rays are two-dimensional projections of complex three-dimensional anatomy. Physicians and radiologists generally make diagnostic measurements for hip dysplasia via estimation or directly on patients’ x-ray film. These parameters describe joint
shape, coverage, congruency of fit, and stability of the joint. An alternative method for evaluation of parameters that is theoretically more precise, more time efficient, and allows for easier collection and comparison of data from a set of patients, has been developed. A MATLAB based program has been created where the physician can manually digitize plain radiographs to calculate a variety of useful parameters for characterizing hip dysplasia. The shortcomings of these parameters are analyzed and a set of parameters that may better describe the disease is presented.

2.2 Introduction

The diagnosis of hip dysplasia in the children has been accomplished first through the use of physical examination. Newborn hips are maneuvered and palpated in adduction and abduction to dislocate and reduce the joint and classify the instability [5]. Following infancy, limp, leg length differences, and asymmetry in range of motion are used as indicators for the disease. Secondly, imaging modalities are commonly employed. Plain radiographs have been used for decades, while ultrasound, arthrogram, MRI and CT are more current diagnostic tools. Many radiographic parameters have been defined for the diagnosis, classification and evaluation of the severity level of DDH. Since each parameter generally describes only one characteristic of the condition, a few parameters are often used in concert.

Typical radiographic parameters include (but are not limited to): lateral center edge angle, acetabular index of depth to width, femoral head extrusion index, acetabular index, lateral subluxation, superior subluxation, and peak-to-edge distance (Figures 2.2 and 2.3).
A few of these parameters are based on a reference line drawn through the inferior edge of the anatomic teardrops (Hilgenreiner's line) (Figure 2.2). Lateral center-edge angle (Wiberg's center edge angle) is the angle formed by drawing a "line from the center of the femoral head to the lateral edge of the acetabular roof and a line drawn superiorly from the center of the femoral head and parallel to the longitudinal axis of the body [14] (Figure 2.2)." Center-edge angle is a measurement of the femoral head-acetabular relationship in the frontal plain, describing the degree of lateral coverage of the femoral head. Acetabular index of depth to width is simply the ratio between the depth and width of the acetabulum. This ratio gives the physician an indication of the sphericity of the weight bearing area. Femoral-head extrusion index is defined as the ratio between the horizontal length of the portion of the femoral head lateral to the edge of the acetabulum to the total horizontal width of the femoral head (in Figure 2.3: A/[A+B] x 100%). Femoral-head extrusion index

Figure 2.2: Schematic diagram of the full pelvis describing radiographic parameters used in the diagnosis of hip dysplasia.
is a measurement of the femoral head uncovering – the percent of the femoral head that is not contained within the acetabulum. The acetabular index, which measures the angle of the acetabular roof, (also called acetabular angle or Hilgenreiner’s angle) is the angle formed by the intersection of the line from the roof of the acetabulum (the sourcil or weight-bearing zone of the acetabulum) with Hilgenreiner’s line (Figure 2.2). Lateral subluxation is the horizontal distance from the lateral edge of the teardrop to the most medial portion of the femoral head (L in Figure 2.3). Superior subluxation is the vertical distance from the inferior acetabular ischium to the most inferior part of the femoral head (V in Figure 2.3). The subluxation measurements quantify the position of the femoral head within the acetabulum. Finally, peak-to-edge distance, a measurement of the acetabular size, is the distance parallel to Hilgenreiner’s line from the apex of the acetabulum to the acetabular rim (D in Figure 2.3).
Physicians and radiologists generally make these measurements via estimation or directly on the patients' x-ray film using a wax marker, ruler and protractor. A computer program was developed whereby images can be manually digitized to quantify the two dimensional radiographic parameters listed above. This program is intended for physicians interested in diagnosis and/or quantification of hip dysplasia and researchers interested in comparing imaging modalities or for the tracking of surgical outcomes or disease pathogenesis. It is proposed that this system will provide a more efficient, objective, and precise method for the evaluation of hip dysplasia.

There is a need to quantify DDH for diagnosis, yet the diagnostic limits have not been established and remain unclear. Measurement of parameters is fairly arbitrary and depends upon the judgment, tools, experience, and bias of the observer. Many parameters assume joints should be circular or spherical and have a definite center, when this may not be the case. Repeatability of patient positioning, scaling issues, different imaging modalities and variations of patient size all make individual images difficult to compare. This method is a tool for studying currently accepted parameters, and brings about ideas for the reconsideration of measurements that have been used for decades.

2.3 Methods

A program was written in MATLAB (The MathWorks, Natick, Massachusetts) (Appendix A) to read image files and prompt the user to digitize bony joint anatomy. Figure 2.4 shows an example of a patient x-ray with the left acetabulum and femoral head digitized. The program circle fits the acetabulum and femoral head using a least square
algorithm to calculating the radius, center and root mean square error of the digitized points on the bony surfaces to the calculated circle. The circle is drawn onto the image and the user can verify that the calculation best represents the joint anatomy. If accepted, the user is prompted to digitize additional joint anatomy (anatomic “teardrop,” acetabular rim, lateral, proximal and inferior edges of the femur, and acetabular apex and depth). From this information, the following parameters can be calculated: lateral center edge angle, acetabular index of depth to width, femoral head extrusion index, acetabular index, lateral subluxation, superior subluxation, and peak-to-edge distance. All values are printed on the screen and also saved into a text file.

Prior to testing the patient images, the circle fitting function using least squares fitting in MATLAB was tested for varying digitized arc length and input noise level of the digitized points. It was expected that between 25% and 50% segment of a circle was digitized when studying the patient data. Noise was created at 5 levels using random number function to create perturbations of the points of an ideal circle to be fit. This test was performed to verify that slight variation in manual digitization, and segment data will still allow the fitting algorithm to be accurate within acceptable limits.

Also prior to testing, the precision and interobserver repeatability of the program was determined using MRI mid-slice data from two patients used in the x-ray study. The radius of the right and left femur and acetabulum, acetabular index, center edge angle and femoral head extrusion index were measured using the MATLAB algorithms. Each image was measured four times, alternating patients in an attempt to eliminate the observer remembering the previously digitized path, and digitize based only on joint geometry.
Four patients (3 males and one female), ranging in ages from 4 years 8 months to 8 years 3 months (average 6 years 10 months) undergoing osteotomies for diagnosed severe bilateral hip dysplasia were chosen for this study. X-rays in the neutral and the frog lateral (abduction) positions were included for each patient, one patient’s data at two different points in time (five months apart) in the neutral and frog lateral positions were used (for a total of 10 hips at two positions). Patient X-ray data was scanned using a high-resolution x-ray scanner (Eskoscan 2540, Purup-Eskofot, Inc. Kennesaw, Georgia). One observer performed data collection and the average of 3 repeated measurements was calculated for each patient and position.

2.4 Results

From the testing of the least squares fitting function, there was no significant change in the curve by changing the number of points to be fit. For small imposed noise levels and arc lengths covering 50% or more of the circle there was no significant change in the expected radius and center results. With no noise, error is on the order of $10^{-6}$ for arc length to 25% of the arc and 10% of the degrees (i.e. 9 points) on the arc fitted. With noise level created by altering the y-coordinate by a randomly generated percentage, and 25% of the arc digitized, the error of the fit curve compared to the ideal curve, the error was determined on the order of $10^{-2}$. Large noise levels and arc lengths of 25% or less in most cases expectedly proved to have a larger error in calculation of the expected results. From these results, the algorithm has been judged to be accurate for these purposes.

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From the MRI precision study, there was a fairly good precision between femoral radius and the three radiographic parameters measured. The average data are presented in Table 2.2. Standard deviations were 2.8%, 11.2%, 9.9%, and 7.2% of the average for the femoral radius, acetabular index, center edge angle and femoral head extrusion index, respectively. The acetabular radius was not in such agreement between trials. The bony femoral head on the MRI images is an easy landmark to define. The acetabulum becomes difficult to differentiate around the medial portion in the MRI images, therefore only a few points could be digitized through this region and the circle fitting become less accurate as the digitized segment becomes shorter and more of a straight line.

Results of the patient x-ray data are shown in Table 2.2. Though is meaningless to compare anatomy of children of different sexes and ages, the radii results were fairly close over the patient set. The femoral head radius averaged over all of the patients at both positions was \(18.07 \pm 3.98\) mm. The acetabular radius was calculated to be \(31.35 \pm 7.53\) mm for the same population. In general, both femoral and acetabular radius increased with age. Radiographic parameters had fairly good accuracy within the three trials for each patient and position. However, between positions, patients, and opposite hips standard

<table>
<thead>
<tr>
<th>Femoral Head Radius</th>
<th>Acetabular Radius</th>
<th>LCEA</th>
<th>FHEI</th>
<th>AI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient #3 Left</td>
<td>11.3537275</td>
<td>56.204345</td>
<td>22.6084</td>
<td>89.2109</td>
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<tr>
<td>Right</td>
<td>14.28695</td>
<td>49.4254225</td>
<td>10.2807</td>
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</tr>
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<td>Patient #4 Left</td>
<td>9.16556675</td>
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<td>102.7586</td>
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<tr>
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<td>45.5</td>
<td>56.2869</td>
<td>142.130975</td>
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Table 2.2 Results of MRI mid-slice digitization for radiographic parameters.
<table>
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<tr>
<th>Patient</th>
<th>Age</th>
<th>Side</th>
<th>Fem Rad (mm)</th>
<th>Fem Err</th>
<th>Acet Rad (mm)</th>
<th>Acet Err</th>
<th>LCEA (deg)</th>
<th>ADTW</th>
<th>FHEI</th>
<th>AI (deg)</th>
<th>LSUB (mm)</th>
<th>SSUB (mm)</th>
<th>P2ED (mm)</th>
</tr>
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<tbody>
<tr>
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<td>8y3m</td>
<td>Left</td>
<td>21.67</td>
<td>0.05</td>
<td>34.69</td>
<td>0.05</td>
<td>55.35</td>
<td>16.87</td>
<td>94.51</td>
<td>64.77</td>
<td>27.32</td>
<td>18.63</td>
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<td></td>
<td></td>
<td>Right</td>
<td>20.87</td>
<td>0.07</td>
<td>32.72</td>
<td>0.03</td>
<td>21.32</td>
<td>7.71</td>
<td>71.79</td>
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<td>23.38</td>
<td>28.06</td>
<td>6.55</td>
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<td>31.53</td>
<td>0.05</td>
<td>22.14</td>
<td>13.04</td>
<td>67.93</td>
<td>61.73</td>
<td>19.49</td>
<td>29.94</td>
<td>9.26</td>
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<td>10.03</td>
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<tr>
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<td>50.58</td>
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<td>0.09</td>
<td>46.14</td>
<td>0.05</td>
<td>12.62</td>
<td>14.07</td>
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<td>18.17</td>
<td>26.28</td>
<td>7.97</td>
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Table 2.2 Results of the x-ray digitization for 4 patients averaged over 3 trials. Where Fem Rad = femoral radius; Fem Err = root mean square error of the circle fitting of the femur; Acet Rad = acetabular radius; Acet Err = root mean square error of the circle fitting of the acetabulum; LCEA = lateral center edge angle; ADTW = acetabular depth to width; FHEI = femoral head extrusion index; AI = acetabular index; LSUB = lateral subluxation; SSUB = superior subluxation; P2ED = peak-to-edge distance.

Figure 2.4 Plain radiograph of Patient 1a in neutral position, with the left acetabulum femoral head digitized for curve fitting.
deviations were large.

A comparison of the mid-slice MRI data to the X-ray data for two of the patients studied showed an average 24% error in the calculation of femoral and acetabular radii, center edge angle, femoral head extrusion index and acetabular index. The difference could be attributed to the difficulty in differentiating structures in the MRI images, human error arising from manual digitization, and error from manual digitization of the x-ray scaling. Also even though x-rays are acceptable for measurement in the clinical community, x-rays are in fact shadow images and may depend upon the radiation beam and the beam source distance from the imaged object (termed the fan beam effect). Standard deviations were lowest in center edge angle and femoral radius (2.82 and 1.78 respectively) and highest in the femoral head extrusion index (17.92) and acetabular index (17.55).

2.5 Discussion

Radiographic differences will naturally occur since the severity of the DDH for each hip may not be identical, not to mention inherent differences in the sex, age and size of patients. The measured values of lateral center edge angle, acetabular index and lateral subluxation for most of the hips in the study do not fit within the normal ranges given in the literature [14, 15] which indicates hip abnormality. To judge the relative “accuracy” of these measurements, a comparison the program generated results with physician measured data via the traditional ruler and protractor method would be needed. The term “accuracy” seems trivial since the measurements of these parameters are highly subject to the bias of the observer. These measurements are used to quantify the severity of DDH and support
the treatment decisions made. However, the terms do little to characterize the abnormality of the joint, and may be redundant or irrelevant. For example, because patient are different size, absolute measures such as lateral subluxation, superior subluxation, and peak-to-edge distance can not be used to compare patient populations and therefore a practical "normal" range cannot be formed. Acetabular index does little to describe the "dysplasticness" of a joint since this measurement merely describes the angle from one side of the weight bearing zone to the other. There is no incorporation of the geometry of the entire acetabulum in acetabular index. Lateral center edge angle and femoral head extrusion index are good measures of the femoral head coverage, however these parameters alone are not indicative of the entire spectrum of maladies encountered with abnormal hip geometry.

One of the biggest downfalls of current radiographic parameters is the two-dimensionalization of a three-dimensional problem. As the presence of MRI and CT machines in hospitals across the world continues to grow, so to does the opportunity for three-dimensional measurements. The quantification of the parameters within a new set of definitions and measurements, incorporating three-dimensional imaging technology, may be used to better assess treatment options and better define the nature of the joint instability. It has been suggested to categorize the joint measurements under the following terminology: congruency, sphericity, stability, contact area, and containment.

Congruency is defined as the relationship between the curve of the lines formed by measuring the distance from the center of the femoral head to the surface of the acetabulum for every angle around the femoral head. Congruency quantifies the lack of mismatch between radii of curvature for the femur and acetabulum. The calculation of congruency can be made in one of two ways when the center point of the femoral head is
previously defined, or manually prescribed. One possible method would involve first calculating the distance from every previously digitized point on the femoral head and the acetabular bony surfaces to the femoral head center followed by calculating the angle the digitized points create with an arbitrary reference line that intersects the femoral head center. The other method would involve the user digitizing points on the two bony surfaces at a series of determined angles, thereby allowing a direct comparison of the distance and angle between the femoral head and acetabulum (as compared to a curve fit plot in the prior mentioned method). A lack of congruency in the joint would influence contact area and can be a result of a lack of sphericity.

Sphericity is defined as a measure of “how far off” the actual morphology (digitized points) of the acetabulum and femoral head are to a least squares circle or sphere fit. Using least squares fitting techniques, the center and radius of the acetabulum and femoral head can be calculated using user described digitized points. The level of sphericity can then be described by root-mean-square error of points to the “ideal” curve or by non-linear regression techniques. The root mean square error for the circle fit in the current study found that the hips included in this study can be circle fit fairly well. Sphericity is expected to be compromised in patients with Perthes’ disease, slipped capital femoral epiphysis, epiphyseal dysplasia and advanced OA. Sphericity is an important parameter for an ideal ball and socket joint: as sphericity in both the femoral head and acetabulum increases, so too does the overall joint efficiency.

Stability is defined as the translation of the center of the femoral head from neutral to both abducted and adducted positions with respect to the center of the acetabulum. Stability is calculated using the coordinates of the center of the acetabulum and the femoral
head from the least squares fit calculation for each of three hip position (neutral, abductions and adduction). The center point of the acetabulum is subtracted from the center point of the femur for each position and the magnitude and direction from the resulting femoral head center position of the abduction of adduction as compared to the neutral position center. Stability quantifies how much motion the femoral head undergoes through abduction and adduction, and would seem to be most related to the containment of the joint.

Contact area is defined as the arc length or area for which the femoral head is in contact with the acetabulum. Contact area can be calculated directly from the MRI images, or by determining a standard distance from the bony surfaces that describes contact (perhaps by knowing cartilage thickness) and mathematically calculating whether or not the femoral head and acetabulum are within this distance. Current surgical planing techniques to minimize pressure in the joint optimize the total surface contact area. Total contact area is related to containment and congruency.

Containment can be defined by a series of clinically defined measures previously described from two-dimensional images: lateral center edge angle, acetabular index, femoral head extrusion index, and peak-to-edge distance. Containment is highly related to joint congruency.

The hip joint is most efficient when the acetabular and femoral head geometry are closely matched (congruency). When the joints are congruent the greatest contact area can occur, which distributes the joint loads more evenly over the surface of the joint maintaining better joint health [2, 16]. A mismatch of the radii of curvature could lead to point contact and high pressures in only one portion of the cartilage. From an engineering
perspective the optimal contact area occurs when joints surfaces are spherical and the femoral head is completely contained within the acetabular cup, leading to the highest level of stability; anatomically the surface of the femoral head is only 50% contained by the acetabulum in the normal hip. The information from this set of measurements is not limited to diagnosis, but can be extended to surgical planning. For example, if both the femoral head and the acetabulum appear spherical, but are not congruent, it would be assumed that there is a size difference within the joint, and the acetabulum may have to be reconstructed to fit the size of the femoral head. Similarly, if the femoral head appears to be spherical, while the acetabulum is not, an osteotomy may have to adjust in the same fashion. However, if both sides of the joint are spherical and congruent, and there is an instability allowing some subluxation, the surgeon may have to redirect the femoral head or acetabulum to increase the containment.

In an attempt to draw relationships between the parameters used to describe hip dysplasia, the average radiographic parameters for each hip in the neutral position of each patient were plotted against each other and fit to a straight line. The $R^2$ values of the line fit are shown in Table 2.3. Lateral center edge angle and femoral head extrusion index correlated the most with the other parameters (average $R^2$ for both is around 0.41), and

<table>
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<th>AI</th>
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Table 2.3. Table of the $R^2$ values from the line fit of radiographic parameters for the 10 hips at neutral position. Where LCEA = lateral center edge angle; ADTW = acetabular depth to width; FHEI = femoral head extrusion index; AI = acetabular index; LSUB = lateral subluxation; SSUB = superior subluxation; P2ED = peak-to-edge distance.
correlated well together ($R^2 = 0.87$). None of the parameters were a good predictor of lateral subluxation (average $R^2 = 0.029$). When radiographic parameters were compared with the defined parameters of sphericity and stability for the 10 hips, there was no correlation (average $R^2 = 0.07$).

2.6 Conclusion

A tool that can be used for the quantification of accepted clinical measurements for the diagnosis of developmental dysplasia of the hip was developed. This tool has been tested with the plain radiographs of 4 patients diagnosed with severe bilateral hip dysplasia and undergoing surgical intervention for the disease. Measured parameters were feasible within the realm of published literature values. It was also suggested to establish a new categorization of terms and measurements used in the diagnosis and measurement of hip instability. These terms may provide an improved insight in the quantification and description of the disease and aid in the recommendation of therapy.

2.7 Future Use

This tool can be used in both a clinical and research setting. Clinically, physicians and radiologists may find this tool useful in the diagnosis of DDH, as well as an outcome measurement following treatment of the disease. Currently this program is being used in research for the radiographic quantification of instability in young adults exhibiting the first signs of early osteoarthritis. The results are being compared to biochemical and
imaging techniques being tested for the diagnosis of early cartilage degeneration. This tool can be used for further comparison of imaging modalities for the diagnosis of hip dysplasia (plain radiographs, ultrasound, arthrogram, computed tomography, and magnetic resonance imaging) and for the correlation of two- and three-dimensional parameters. Furthermore, this tool can assist in the analysis of new measurements that may better aid in the diagnosis of DDH as suggested above.
Chapter 3: Hypothesis for Objective Function for Osteotomy

The relationship of the femoral head to the acetabulum is a fundamental concept in the treatment of hip dysplasia. In Chapter 2 the current two-dimensional parameters used to quantify hip anatomy were examined and new categorization of terms was suggested. A discussion of the current concepts used in planning osteomies and a hypothesis for the re-evaluation of these notions is proposed in this chapter.

3.1 Theory and Hypothesis

Criticism of current radiographic parameters for the quantification of hip dysplasia has developed primarily because of the lack of descriptiveness in the measurements. Additional critique of radiography as a method for characterizing anatomic geometry is the static two-dimensionalization of a dynamic three-dimensional problem. Rab [1] explains that “assessment of the femoral head [in relationship to the acetabulum] by a single anteroposterior roentgenogram of the hip is detrimental to this description because it tends to mislead the orthopedist during a single standing X-ray with the overall function of
the hip during normal gait and activity levels.” Medical imaging, literally, paints a picture of the anatomic problem, and allows physicians to become aware that a problem exists. However, it is proposed here that once a geometric anomaly has been recognized, medical imaging should not be sole provider of information for the surgical planning of to correct this disorder.

The goal of any osteotomy about the hip is to optimize the distribution of load by minimizing the pressure within the joint and restoring joint “stability.” This principle relies on the theory that mechanical factors are responsible for the development of osteoarthritis (OA) in an abnormal joint and that the removal of these factors causes the apparent joint healing following a successful surgery. The internal stress and pressure distributions within the hip joint have been examined by a number of researchers [2-6]. Surgical planning and guidance tools have been developed to optimize pressure distribution within a joint for surgery [1]. Again, criticism arises from these evaluations.

In situ pressure measurements have been made in a number of ways, including the introduction of various pressure transducers and miniature piezoresistive transducers to the joint, Fuji Pressensor film, pressure sensitive strips of high resolution matrix-based resistors, and computational modeling. With each of these measurements come weaknesses that reduce the validity of the results. These measurement techniques are extremely invasive and compromise the integrity of the joint. Soft tissues associated with the joint are excised and distorted and joint anatomy is altered with the implantation of sensors into surrounding bone or cartilage or the introduction of a film into the joint space, both of which cause changes in joint mechanical properties, behavior and relationships.
Computational modeling does not compromise the joint, however because of its theoretical nature, requires a validation of the results to be acceptable.

Another difficulty with the characterization of pressure within the joint is the complex nature of cartilage. Cartilage is a complex composite hydrated matrix. The solid components of the matrix are primarily collagen and proteoglycans. Proteoglycans are composed of a core protein with side chains ending in glycoaminoglycan (GAG). GAG chains are negatively charged and provide the matrix with the majority of its fixed charge. The fluid component of cartilage is predominantly water. Nutrients, ions and chondrocytes (cells) can be found in small amounts within cartilage. The collagen matrix appears to provide the tensile support for the material. However, when in compression, the tendency of the negatively charged proteoglycans to repel each other provides the majority of the material properties. When loaded, the speed at which the fluid component of the matrix can flow out of the intricacy of the matrix supplies the time dependent properties of cartilage. The intrinsic complexity of the material makes it difficult for engineers to evaluate both empirically and analytically. Even the characterization of the material in mechanical terms has not been unanimously accepted. Cartilage has been described as biphasic, triphasic, poroelastic and viscoelastic and modeling is often simplified to be linear elastic. Since the nature of cartilage is not yet fully understood, assumptions of minimizing pressure becomes complex at the tissue level.

A change in paradigm is recommended to better tackle the complexity of the in vivo mechanical environment of a joint from a surgical planning perspective. The movement from a reliance on pressure and stress to the examination of displacements and deformation would provide simpler grounds for measurement. Displacement space can be measured in...
vivo with medical imaging techniques, including Magnetic Resonance Imaging (MRI) and Computed Tomography (CT). From a modeling perspective, boundary conditions can always be measured in displacement space.

One option for modeling the joint when the positions of joint anatomy are known is through the application of a simple model. Suppose two joint surfaces are modeled as rigid bodies and the initial position of each surface is known. When motion in the system occurs and one rigid body overlaps the other, the assumption is that the amount of overlap of the modeled systems represents the amount of deformation in the actual joint. Because of cartilage material properties compared to those of bone in the joint, the deformation is assumed to be occurring in the cartilage. Presuming models such as this are valid, a shift in purpose can occur in the planning of osteotomy of the hip and a new objective function can be devised.

The hypothesis becomes that the objective function for the surgical planning of osteotomy about the hip is the minimizing of cartilage deformation. A protocol for the validation and testing this hypothesis follow in Chapters 4 and 5. Once the hypothesis has been tested and proved, a surgical planning program can be developed using a simplified model, such as the one above, for the positional recommendation of reconstruction in the joint to optimize cartilage deformation.
Current methods to test cartilage material properties and cartilage mechanics at a known strain level are carried out by examining cartilage explants. Designing a system whereby a known displacement can be imposed and maintained in vivo would allow researchers to more accurately examine joint health in a physiologic setting. A system has been developed whereby a known displacement can be imposed on an external fixation system that is attached to a sagitally sectioned joint. The actual cartilage displacement is measured and the relationship between the input displacement to the fixator and the measured cartilage displacement is calculated. This relationship can then be used for to impose a known displacement to an in vivo joint.

4.1 Motivation:

It has been shown that mechanical factors play a role in the pathogenesis of osteoarthritis [1-10]. Previous animal studies where osteoarthritis is induced in joints have
involved the application of external forces. The mechanical conditions at the joint are left unknown or assumed. In order to examine the effects a measurable mechanical parameter (displacement) has directly on cartilage health, a technique must be developed whereby a repeatable known displacement can be applied to cartilage in vivo. This technique can then be applied for the validation of the hypothesis presented in Chapter 3.

4.2 Introduction:

According to the Arthritis Foundation roughly 21 million Americans suffer from osteoarthritis, a degenerative disease of joint articular cartilage that leads to joint stiffness and pain [11]. Articular cartilage is a tissue lining the surfaces of joints and is composed of a low-density collagen matrix and a proteoglycan and protein gel. Because of the complexity of the material, the exact pathogenesis of articular cartilage degradation has yet to be resolved. A number of studies establish that mechanical factors lead to the onslaught of the disease and that mechanical properties of the material are altered as the disease transgresses. For example, an increased strain in the tissue may lead to tissue degradation, similarly, tissue degradation is characterized by a thinning of the cartilage.

Researchers have mechanically induced osteoarthritis experimentally in animal models for decades via injury to the cartilage surface [1], altering of load bearing to a joint though ligament transection or osteotomy [2-6], immobilization [7, 8] and repetitive impulse loading [9, 10]. By and large the previous studies inducing osteoarthritic changes through mechanical means have assumed over-pressurization of the cartilage tissue to be
the dependent variable in the cartilage health. However, current methods for *in vivo* joint pressure measurements tend to be invasive, inaccurate and relatively unreliable. Alternatively, a change in cartilage thickness is not only a measurable parameter, but can be related to the initiation of degeneration through the relationship of strain to pressure changes.

A majority of work with displacement control has been focused on cartilage explants with a few studies controlling displacement changes on cadaveric material. The effects of displacement changes on *in vivo* cartilage health in animal models have been mentioned in joint distraction studies [12] and in limb lengthening work [13]. Because surgical decisions, such as limb lengthening, immobilization, distraction, and osteotomy, tend to change the strain a joint experiences with normal activity, cartilage health can be affected. The behavior of the cartilage, the extent to which damage can occur, or healing can be initiated with response to strain changes is an interesting factor to examine from the point of view of osteoarthritis progression.

Previous studies of *in situ* cartilage deformation have used modern imaging techniques such as computed tomography arthrogram (CT arthrogram), ultrasound, and most recently magnetic resonance imaging (MRI). In comparison to computed tomography (CT) - the clinical standard of medical imaging techniques, MRI provides the ability to view soft tissues, such as cartilage, and provide some information on tissue chemical composition [14-17]. Though MRI can adequately measure cartilage deformation in static loading cases [18], real time measurement of cartilage deformation is not yet possible with MRI since cartilage response is time dependant and MRI imaging takes time.

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Cooper, J.A., MSME Thesis
A system has been developed that can be used to impose a displacement on an intact joint cartilage in an animal model. By fixing a slightly altered Orthofix Articulated MiniRail Fixator (Figure 4.1a) to a rabbit elbow joint, cartilage deformation can be imposed to the joint through surgical pins implanted into the humerus and ulnar olecranon. Because the input displacement may not be completely experienced at the joint cartilage level, a relationship between the \textit{in situ} cartilage displacement, and an externally measured displacement must be developed. This study validates the use of external fixation and a material testing system to deform cartilage in a cross sectioned animal model joint. An optical tracking system and an extensometer are used to monitor the displacement of the \textit{in situ} subchondral bone. Using this system a correlation between the magnitude of cartilage displacement observed (from the optical tracking and extensometer data) to the input to the external fixator can be made. Using this relationship, experimental studies can be performed whereby a similar apparatus on an intact joint can be displaced and the cartilage displacement can be calculated.

4.3 Materials & Methods:

Adult New Zealand white rabbits were obtained from a study that is not thought to have orthopedic implications, immediately following sacrifice. The left forearms from humerus to carpals were excised and skinned for testing (humerus, radius, ulna and associated soft tissues). Guide holes were drilled into the humerus and ulna to fit the
Figure 4.1: Experimental test setup for the calculation of cartilage displacement. (A) Schematics of test setup, where F represents the input function from the Instron. (B) Photograph of a rabbit elbow joint fixed in the Instron system with extensometer attached to the surgical pins.

An external fixation device, and the joint was frozen overnight in a -20°C freezer at 90° flexion. The frozen joint was cut in half sagittaly using a vertical band saw and was allowed to thaw wrapped in 0.9% saline-soaked gauze. A metacarpal external fixator was fixed to the joint using surgical pins (M111 Metacarpal Fixator; DM420 Threaded Wire 70/15 1.6mm; Orthofix, Italy). The external fixation system and joint was rigidly fixed into an Instron 8115 system and a 2200N-load cell, as shown schematically in Figure 4.1. A uniaxial load was applied through the external fixator system to the pins that had been inserted in the bone.

An Instron extensometer was fixed to the lower anterior humeral pin and medial ulnar pin at approximately 90° flexion. The fixation to the load cell as well as the extensometer
Figure 4.2. Cross-sectioned joint with reflective tape markers labeled with numerical assignments used in data processing.

Fixation to the pins is assumed to be rigid. Reflective tape markers approximately 1 mm square (3M) were affixed to the subchondral bone surface (3 markers were placed on the humeral side, and 3 markers on the ulnar side, as pictured in Figure 4.2) and one marker was placed on the Instron actuator. The load cell, actuator, and extensometer were interfaced to PC software to collect data using commercial data collection software (AqcKnowledge, Biopac Inc.), and PCreflex (Qualisys, Inc. Glastonbury, CT) was used to track the reflective markers during testing. Both data collection systems were calibrated independently and prior to testing. Periodically throughout testing, saline was injected into the joint using a 1-cc syringe to keep the joint surface hydrated. During the first test, the joint was cycled for 5 minutes with a trapezoidal loading regime (2.5 mm-displacement from the initial position, 0.1 mm/sec ramp, 0.008 Hz). In a subsequent test, we cycled the joint to impose a 1.5-mm displacement to the cartilage from the initial starting position at 0.15 Hz with a trapezoidal loading function for 30 minutes. During testing the marker, load cell, actuator and extensometer data were recorded at 5 data points/second. The test was repeated for 3 joints. The data was imported into Microsoft Excel for data analysis.
4.4 Results:

Overall, the results for the first test with 2.5-mm imposed displacement with a relatively

Figure 4.3: Displacement (mm) vs. Time (s) of the Instron actuator during the first test (2.5 mm; 0.1 mm/sec).

Figure 4.4: Load cell data (N) vs. Time (s) during the first test (2.5 mm; 0.1 mm/sec).
Figure 4.5: The first loading cycle of the first test (2.5 mm; 0.1 mm/sec). The left y-axis represents the load cell data (N), the right y-axis is the difference between reflective markers (4 and 5 from Figure 4.2) on either side of the cartilage surfaces (mm), and the x-axis is time (s).

low strain rate gave expected results. Figure 4.3 represents the displacement regime from the Instron actuator output with time, which shows an accurate portrayal of the intended displacement. The PCReflex and Instron data strongly correlate to each other, as well as the intended displacement. Figure 4.4 represents the load cell data during the test for the same time scale as the displacement data in Figure 4.3. The load is as expected: with an imposed displacement, the cartilage viscoelastic creep properties are exhibited, during the unloading phase, some viscoelastic response is seen, conceivably from soft tissue creep during a lift off phase. A similar response is seen through the PCReflex marker data. Figure 4 shows first displacement cycle load data along with the displacement data for the first cycle of the difference between markers (4 and 5) on opposite sides of the joint. The three curves represent the load response after 5, 15 and 25 minutes of loading. of the subchondral bone surface (assumed to be equal to the cartilage deformation). The load and
displacement curve respond mutually, as anticipated. The total displacement seen in the cartilage is approximately 0.1 mm for a system input of 2.5 mm in this model joint. The second study, showing the longer term cycling again shows seeming expected results. With time the load is decreased with viscoelastic changes in the cartilage. Figure 4.6 shows the load cell data over the entire testing time, and a closer examination of the load data at three time periods (5, 15 and 25 minutes after the start of loading). All of the data in the graphs represents the testing of one elbow. Joints from other animals showed similar response, with approximately 10% input displacement seen at the cartilage level.

The information about the differences in cartilage displacement along the subchondral bone surface throughout the joint could give interesting insight to the in vivo cartilage properties, or the mechanical response to the loading configuration. There was some “lift-off” between cycles, that is, the cartilage surfaces were not always in contact over the entire tissue surface during loading and/or the relaxation phase. This may be due to the slippage of pins or other system components, the natural joint anatomy and/or the viscoelastic properties of the cartilage and other soft tissue. The loading appeared to occur...
by first an initial cartilage deformation followed by a deformation of the rest of the system, in which the motion of the humerus in the direction of the imposed displacement was shifted from the ulnar surface of the joint to the radius cartilage surface. This could be because of a moment produced by applying the displacement through the surgical pins. There is some rotation of the joint that is seen by the three dimensional PCReflex data, possibly due to slippage within the system, and expectedly a result of the true joint geometry, which acts like a screw mechanism. Also, during the second test, the tissue was not injected with saline; the dehydration of the tissue during testing may have amplified the viscoelastic changes over time.

4.5 Proposed System:

This tool leads to the development of an in vivo controlled displacement system that can be attached to a live animal model (Figure 4.7). This system uses similar principles as the

![Figure 4.7 Proposed in vivo controlled displacement device.](image-url)
Instron controlled system employing the Orthofix external fixator as the core of the system. The system will be scaled for a rabbit elbow, and will be entirely non-ferrous to allow for MR imaging throughout studies involving this system.

The off-the-shelf Orthofix fixator design will be slightly altered, as mentioned above, to better fit the requirements of the study. The following modifications have been incorporated into the design of the system to be used in a rabbit elbow joint model (as seen in Figure 4.8):

![Figure 4.8 Orthofix M111 Mini-Rail Fixator with custom modifications](image)

1) In order to meet the non-ferrous design constraint to allow for MRI compatibility the hinge joint, screws, and pins will be replaced with titanium or aluminum parts.

2) The slider mechanism will be rigidly fixed to the screw shaft.

3) The screw shaft of the fixator will be altered such that motion in the shaft is purely linear. This includes removing set pins securing the screw mechanism to the frame of the external fixator on the hinge joint side. The removal of these pins permits motion of the shaft in more than one plane, therefore a 1.0 mm x 6.00 mm track will be milled.
into the side of the fixator and small pins will serve as bearings restricting motion to be purely unidirectional.

4) The portion of the original fixator that was on the opposite side of the slider mechanism will be altered so that the surgical pins can be placed into the ulnar olecranon of the rabbit joint. This fixture has been designed so that the holes for the clamping mechanism are separated by 10 mm on a line 5-mm from the center of the hinge joint.

5) Finally, the end of the screw shaft on the opposite side of the hinge joint has been threaded so that the screw mechanism can easily be connected to the rest of the system. A linear motion positioning stage will be used to drive the Orthofix external fixation device instead of the Instron actuator. Linear stages with the capabilities needed for this study have vertical carrying capacity of around 1110 N, which according the validation study and Figure 4.6 should be more than sufficient for the force required in the rabbit elbow model. The stages generally move to a maximum displacement of approximately 10 cm with accuracy and repeatability on the order of 5 microns, which will provide the precision desired. Furthermore, linear stage can be externally controlled and monitored. Inline with the linear stage will be a piezoelectric load cell that will allow for load data and can be used for feedback if the testing protocol warrants the need. A 5 mm gauge length extensometer will be affixed on one side to the inferrior humeral surgical pin and on the other to a connector between both of the pins inserted into the olecranon.

The extensometer, load cell and linear positioning stage will all be interfaced to a personal computer through a data acquisition board. Custom written software will be
created in LABView to both control the system and collect data during testing. This testing rig will then be attached to a live animal model for *in vivo* testing.

4.6 Discussion & Conclusions:

A technique has been used to make a calculation of the relationship between the displacement at the cartilage surfaces when a displacement is applied to the joint using the Orthofix External Fixation system. The relationship between the input and measured displacement at the cartilage surfaces can be modeled as linear. Over 90% of the input displacement is not seen at the cartilage surface, however, a displacement can be imposed to the joint of the order of magnitude desired to induce osteoarthritis. Possible contributions to this loss could be deformation between the pin fixation system, deformation between the pin bone interface, deformation of the bone tissue in compression and/or motion in other planes due to a moment arm created by the external fixation system. From visual observation of the test, the greatest percentage of the deformation loss appeared not to be in the plastic deformation of the bone, which may cause complication in the study, instead, it seemed to occur in the deformation of the surgical pins. The longer term cycling of the joint showed that an intended displacement can be closely maintained over a period of time, with expected viscoelastic tissue changes. The main errors that could compromise this study's accuracy to predict the *in vivo* displacement is the invasion of the normal joint. Using the half space joint one can only assume that the cartilage will deform similarly in the intact joint, regardless of the fact that the bone and cartilage material has...
been removed, normal joint hydration, as well as joint pressure, and joint soft tissue support had been compromised by invasion the joint.

4.7 Future work:

Validation of this technique can be performed using magnetic resonance imaging (MRI) to compare the measured \textit{in vivo} cartilage deformation of the intact joint with the predicted value. All of the equipment required would need to be MRI compatible and a high resolution MRI magnet would have to be used for this sort of validation. Because MRI imaging takes time, deformation would have to be measured quasi-statically, therefore likely measuring equilibrium values instead of real time dynamics. Future work with this tool could include investigations such as the derivation of a threshold displacement at which cartilage degeneration is initiated and the implications of cartilage differences throughout the subchondral bone surface and the pathogenesis of osteoarthritis in those areas.
The ability to achieve controlled cartilage deformation in vivo provides a number of opportunities for research endeavors. This technique eliminates the criticisms of cartilage plug research, since cartilage remains in vivo during loading. Contact surfaces remain true joint surfaces as opposed to metal platens often used in plug studies. Cartilage maintains physiologic functioning and nourishment permit natural progression and healing of diseases. A study for an objective function for surgical planning of osteotomies is presented, as well as a discussion of other possible opportunities for this tool.

5.1 Objective Function for Surgical Planning of Osteotomies

5.1.1. Introduction

Osteoarthritis (OA) is a disease characterized by progressive degeneration of the cartilage of the diarthrodial joints, that affects over 21 million people in the United States. Because of the prevalence of OA, there continues to be an ongoing research effort to understand the pathogenesis of the disease and evaluation of treatment protocols. Cartilage
degradation has been attributed to both biomechanical and biochemical etiologies, but it has not been determined which factor contributes to the initial pathogenesis of the disease. Cartilage healing or regeneration appears to be limited. While tissue engineering and gene therapy technologies are being developed for hyaline cartilage, current treatment options for early arthritis are limited to redirectional osteotomy, based on the assumption that there is a mechanical basis for disease progression. In older patients, total joint arthroplasty has generally proven successful, however because of prosthetic loosening and polyethelene wear, joint replacement is not a practical option for younger active patients.

As many as 90% of middle aged adults can attribute congenital etiology of hip dysplasia for their coxarthrosis (osteoarthritis of the hip) [1, 2]. Congenital hip dysplasia is the most common cause of coxarthrosis in young and middle aged adults. If the acetabulum is deficient anteriorly and laterally there will be an increased pressure in the hip joint and occasionally mechanical instability of the hip joint. It is widely assumed that surgical intervention can delay or possibly prevent OA in joints with abnormal or incongruent geometry. The premise underlying these surgical techniques remains the same: “optimize” the distribution of load, minimize pressure within a given joint, and restore joint stability. However the mechanisms of changing joint shear and normal traction loads have not been investigated to provide surgeons with predictable outcomes for preventing or delaying the development of arthritis. Also, there is some suggestion that the surgery itself may not only alter the mechanical load distribution, but also affect changes in vascularity, denervate the joint, and/or adjust ligamentous loads, which could play a role in the apparent changes in joint health following surgery. A clear experimentally derived
objective function, based on optimizing the mechanical behavior and an understanding of
the pathogenesis of OA, would be beneficial for pre-operative planning and computer-
assisted surgery.

5.1.2 Specific Aims

This project is broken down into three specific aims:

- **Aim 1**: What effect does shear traction have on OA?

  While the exact pathogenesis of OA is unknown, studies have shown that abnormally
  high contact pressure (large normal surface traction) induces OA. In hip dysplasia, the
  geometry of the acetabulum and femoral head are often incongruent, which allows for
  sliding or dislocation of the joint, called instability. A simple illustration of the
  cartilage contact area, and therefore pressure changes, as the femoral head slides in the
  acetabulum to dislocation is shown in Appendix C. During osteotomies, surgeons try to
  reduce contact stress by altering joint load and to increase joint stability. Whereas
  small shear motion as seen in continuous passive motion has proven to be beneficial to
  cartilage health, the role of large shear traction combined with large normal traction in
  OA is unknown and may be detrimental to cartilage health [3-6]. If sliding within the
  joint does not play a role in OA progression, surgeons would not have to limit both
  factors during surgical intervention. Aim 1 can be investigated by examining the
  effects of large normal traction vs. large normal traction with shear traction.
Hypothesis: OA progresses faster when both large normal and shear traction forces are applied on the cartilage surface than when only large normal traction force is applied.

• **Aim 2: Does loading rate affect the progression of OA?**

There may be other mechanical factors that cause OA apart from abnormally high surface traction. Since cartilage has been shown to be a viscoelastic material, loading rate may be one of these factors [4, 7-9]. This aim could be tested by studying the effects of loading at two different rates.

_Hypothesis: For a given peak stress value, different rates of loading causes different rates of OA progression._

• **Aim 3: Does the progression of OA change if excessive normal traction is removed?**

The direct effect of surgically changing mechanical loading of a joint on cartilage health is unknown. Some studies have shown that OA induced by immobilizing a joint reverses when the joint is again mobilized [10-14]. By removing large normal traction without surgical intervention, this aim investigates the best possible surgical outcome from a surgery.

_Hypothesis: OA progresses slower if excessive normal traction is removed when compared to the joint continually loaded with excessive normal traction._

The overall purpose of this study is to investigate and identify a mechanical function that optimizes osteotomies, such that orthopedic surgeons have a clear set of goals to be used in operative planning. Also to validate the objective function hypothesis proposed in Chapter 3.
5.1.3 Methods

For this study, skeletally mature rabbits, randomly assigned to 5 groups, could be used. Groups are designed to test the following variables: control, high compressive load, high compressive loading plus shear traction, variable loading rate, and the recovery after removal of high compressive loading. The rabbit elbow joint will serve as the model since the proximal ulna and radius are fused together at the elbow in the rabbit forming hinge joint with a near constant center of rotation. Titanium pins will be surgically inserted parallel to the axis of rotation through the radius and ulna and the humerus of the rabbit elbow approximately 1 cm from the center of rotation of the elbow joint. The center of rotation of the joint will be found using the application created in Appendix B. Pins will be surgically inserted at the beginning of the experiment and will remain in the animal throughout the testing. A known cartilage deformation will be applied at a specified loading rate to the specimen. Joint loading will be applied at two load magnitudes a) low normal deformation (approximately 150 μm) and b) high normal deformation (approximately 250 μm). Animals will be loaded for 3000 cycles per day at 1Hz frequency (or 2 Hz frequency for the loading rate group), five days a week, for six weeks. During loading of the elbow the animals will need to be anesthetized using a combination mixture IM of Butorphanol tartrate at 0.2 mg/kg, Midazolam at 0.2 mg/kg, and Xylazine at 0.2 mg/kg. During testing, animals will be placed in restraint bags, but will otherwise be free to move within their caging units. Each study groups is described in Table 5.1.
<table>
<thead>
<tr>
<th>Testing Group</th>
<th>Elbow A</th>
<th>Elbow B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>Low compressive load</td>
<td>No pins or load</td>
</tr>
<tr>
<td>High Compressive Load</td>
<td>Low compressive load</td>
<td>High compressive load</td>
</tr>
<tr>
<td>High Compressive Load + Shear</td>
<td>High compressive load</td>
<td>High compressive load + shear</td>
</tr>
<tr>
<td>Loading Rate</td>
<td>High compressive load</td>
<td>High compressive load with high loading rate</td>
</tr>
<tr>
<td>Removal of high load</td>
<td>High compressive load</td>
<td>High compressive load (3 wks.) + Low compressive load (3 wks.)</td>
</tr>
</tbody>
</table>

Table 5.1 Testing groups for objective function study

OA development could be monitored progressively in each animal using MRI and synovial fluid markers at 0, 1, 3 and 6 weeks after the start of the loading regime. Animals can be anesthetized with an initial 50:50 mixture of Ketamine/Midazolam 1cc/20 kg IM, followed by an IV infusion of Propofol 2 mg/kg, during MRI evaluation. MRI measurements will include glucosaminoglycan (GAG) content of elbow cartilage using MRI tools, and cartilage thickness measurements from reconstructed 3D images. Before MRI imaging, each animal will be injected intravenously with Gd-DTPA, a contrast medium that has no known side effects when injected into either animals or humans. Synovial fluid will be assayed to identify breakdown products of cartilage matrix – the earliest markers of cartilage degradation. Synovial fluid will be obtained by first injecting 2 mL of saline into the elbow joint. The joint will be moved through its range of motion for 1-2 minutes to ensure thorough mixing of the saline with synovial fluid and then will be aspirated from the joint. After the six week testing period, animals will be sacrificed. Quantitative histology of cartilage and subchondral bone, measurements of chondrocyte
apoptosis, and mechanical testing of cartilage material properties will be performed for each animal.

Results will be analyzed using paired t-tests within each test group, and repeated measures analysis for changes glycoaminoglycan content and cartilage thickness for each elbow. Mechanical variables are controlled without confounding surgical variability allowing analysis of the effect of shear traction, loading rate on cartilage degradation and whether reversal of excessive normal traction loads and/or shear traction abates degradation of cartilage. The results of this study will provide an experimental basis for the derivation of an appropriate objective function to be optimized in surgical planning of redirectional osteotomies about the hip.

5.2 Other directions

There is a good deal of cartilage plug research that could take advantage of this in vivo strategy, specifically in the study of cartilage metabolism with the pathogenesis of OA. With a more complete understanding of the cartilage matrix metabolism response to mechanical injury, scientists can develop improved repair strategies and a better understanding of the pathogenesis of osteoarthritis. For example, in explant studies, collagen and proteoglycan turnover can provide scientists with a better understanding of biomechanical/biochemical pathways for the management of extracellular matrix metabolism. This could provide information on biochemical regulators or mechanical trigger that could initiate (or prevent) osteoarthritis. Similarly, a the analysis of apoptosis
(programmed cell death) in the chondrocyte population of the cartilage tissue could provide information on averting cartilage deterioration.
References

Chapter 1


Chapter 2

Chapter 3


Chapter 4


Chapter 5


function xray_digit()

% XRAY-DIGIT
% WRITTEN FOR THE QUANTIFICATION OF HIP
% DYSPLASIA USING PELVIC PLAIN RADIOGRAPHS
% WRITTEN BY JENNIFER A. COOPER 14 JULY 1999
% ORTHOPEDIC BIOMECHANICS LABORATORY
% BETH ISRAEL DEACONESS MEDICAL CENTER &
% MASSACHUSETTS INSTITUTE OF TECHNOLOGY
%

clear
cic
colormap(gray)
close all

% --------------------- OPEN THE IMAGE
filename = input('IMAGE NAME <filename>: ','s');
img = imread(filename, 'bmp');
imshow(img);
iaxis = axis;
axis(iaxis);
hold('on');

% --------------------- DECLARING MENU
menu = char('<0> CANCEL ','1> EXIT PROGRAM ','2> CIRCLE FIT ','3> CALCULATE INDICES ','4> ANGLE/LENGTH ','5> ADJUST GRAPHICS ','6> MARKER ','7> REMOVE LAST POINT ','8> RESET ALL ');

% --------------------- SCALE THE IMAGE
p = zeros(2,0);
done = 0;
circfit = 0;
toscale = input('Would you like to scale the image?(0 for No, 1 for Yes): '); if toscale == 0,
    scale = 1;
elseif toscale == 1,
    disp('MOUSE INPUT: DIGITIZE END POINTS ON MARKER.');
    while ~done,
        [x,y,mouse] = ginput(1);

        switch mouse
        case 1
            p(:,(end+1)) = [x;y];
            plot(p(1,:), p(2,:), 'b*')
            if size(p,2) == 2,
                done = 1;
                actuallength = input('ACTUAL LENGTH OF MARKER (mm): ');
                pixellength = sqrt((p(1,2)-p(1,1))^2 + (p(2,2)-p(2,1))^2);
                scale = actuallength/pixellength;
            end

        case 2
            p(:,end) = [x;y];
case 2
p = p(:,1:(end-1)) ;
axistemp = axis ;
imshow(img) ;
iaxis = axis ;
axis(iaxis) ;
hold('on') ;
case 3
% end switch
end % end while
else
    disp('Invalid input. Image will not be scaled.' ,...
    'You must quit and start again to scale image.');
end

disp('DIGITIZE THE LEFT FEMORAL HEAD.');
%--------------------------(END OF SCALING IMAGE)---
%------------------------------------- MOUSE INPUT---
done = 0;
p = zeros(2,0);  % COORDINATES OF POINTS
markmark = 0 ;  % FEM AND/OR ACETAB. DIGITIZED
mark = zeros(1,4) ;  % SIZE OF EACH FEM AND/OR ACETAB.
while ~done, % MAIN WHILE
    plot(p(1,:),p(2,:), 'b+');
    figure(1)
    if markmark == 0,
        p(1,:), p(2,:), 'g+');
    elseif markmark == 1,
        plot(p(1,1:mark(l)), p(2,1:mark(l)), 'g+-');
        if mark(l)+1 <= size(p,2),
            plot(p(l,mark(l)+1:size(p,2)),p(2,mark(l)+1:size(p,2)), 'r+-');
        end
    elseif markmark == 2,
        plot(p(1,1:mark(l)), p(2,1:mark(l)), 'g+-');
        plot(p(l,mark(l)+1:mark(2)), p(2,mark(l)+1:mark(2)), 'r+-');
        if mark(2) <= size(p,2),
            plot(p(1,mark(2)+1:size(p,2)),p(2,mark(2)+1:size(p,2)), 'y+-');
        end
    elseif markmark == 3,
        plot(p(1,1:mark(l)), p(2,1:mark(l)), 'g+-');
        plot(p(l,mark(l)+1:mark(2)), p(2,mark(l)+1:mark(2)), 'r+-');
        plot(p(1,mark(2)+1:mark(3)), p(2,mark(2)+1:mark(3)), 'y+-');
        if mark(3)+1 <= size(p,2),
            plot(p(1,mark(3)+1:size(p,2)),p(2,mark(3)+1:size(p,2)), 'c+-');
        end
    else
        plot(p(1,1:mark(l)), p(2,1:mark(l)), 'g+-');
        plot(p(l,mark(l)+1:mark(2)), p(2,mark(l)+1:mark(2)), 'r+-');
        plot(p(1,mark(2)+1:mark(3)), p(2,mark(2)+1:mark(3)), 'y+-');
        plot(p(1,mark(3)+1:size(p,2)), p(1,mark(3)+1:size(p,2)), 'c+-');
    end

[x,y,mouse] = ginput(1) ;
switch mouse
%--------------------------- LEFT MOUSE BUTTON (MAIN)
case 1 % DIGITIZE POINT
    p(:,(end+1)) = [x;y];

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%------------------------ MIDDLE MOUSE BUTTON (MAIN)
  case 2 % REMOVE LAST DIGITIZED POINT
  p = p(:,1:(end-1));
  iaxis = axis;
  hold('off');
  imshow(img);
  axis(iaxis);
  hold('on');
%------------------------ RIGHT MOUSE BUTTON (MAIN)
  case 3 % SHOW MENU
  disp(menu);
  imenu = input('MENU CHOICE <0> :');
  if isempty(imenu), imenu = 0; end

  switch imenu
  case 0 % CANCEL
    disp('...........................');
  case 1 % EXIT
    if(yes_no('EXIT PROGRAM? <n> : ');
      done = 1;
      save xray_digit_data;
      folder=[char(filename) num2str(i) '.dat'];
      fid=fopen(folder,'w');
      fprintf(fid, '%4.3f %4.3f %4.3f\n', p, mark, f2ctrl, f2ctrr, loentere, ld2wr, lfehei, lacetabi, l1sub, l1supsub, l1ped, rcentere, rd2wr, rfehei, racetabi, r1sub, rs1supsub, rp2e);
      fclose(fid);
    else
      disp('...........................');
    end
  case 2 % CIRCLE FIT
    cprint(4) = 0;
    if (yes_no('POINTS SELECTED? <n> : ','n'),
      if markmark == 0, % USER HAS DIGITIZED ONE STRUCTURE
        f2l = p;
        cprint(1) = 1;
      elseif markmark == 1, % USER INDICATES TWO STRUCTURES
        f2l = p(:,1:mark(1));
        a2l = p(:,(mark(1)+1):size(p,2));
        cprint(1:2) = 1;
      elseif markmark == 2, % USER HAS DIGITIZED THREE STRUCTURES
        f2l = p(:,1:mark(1));
        a2l = p(:,(mark(1)+1):mark(2));
        a2r = p(:,(mark(2)+1):mark(3));
        f2r = p(:,(mark(2)+1):size(p,2));
        cprint(1:3) = 1;
      else % USER HAS DIGITIZED 4 OR MORE STRUCTURES
        f2l = p(:,1:mark(1));
        a2l = p(:,(mark(1)+1):mark(2));
        a2r = p(:,(mark(3)+1):mark(4));
        f2r = p(:,(mark(2)+1):mark(3));
        f2r = p(:,(mark(2)+1):mark(3));
        a2r = p(:,(mark(3)+1):size(p,2));
        cprint(1:4) = 1;
      end
      if cprint(1) == 1, % WILL FIT RIGHT FEMORAL HEAD
        hold('off');
        imshow(img);
        axis(iaxis);
        hold('on');
        [f2ctrl,f2radl,f2errl] = circle_fit(f2l);
        disp('...........................');
        disp('LEFT FEMUR: CENTER(x,y): (%0.7g, %0.7g) ...'
            ,f2ctrl(1), f2ctrl(2)));
        disp(sprintf(' RADIUS: %0.7g', f2radl));

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disp(sprintf(' ERROR: %0.7g', f2errl));

end

if cprint(2) == 1, % WILL FIT RIGHT ACETABULUM
    [a2ctrl, a2radl, a2errl] = circle_fit(a21);
    disp('');
    disp('LEFT ACETABULUM:');
    disp(sprintf(' CENTER(x,y): (%0.7g, %0.7g)', a2ctrl(1), a2ctrl(2)));
    disp(sprintf(' RADIUS: %0.7g', a2radl));
    disp(sprintf(' ERROR: %0.7g', a2errl));
    [alx, aly] = gen_circle(a2radl, 100);
    plot(alx+a2ctrl(1), aly+a2ctrl(2), 'r-');
end

if cprint(3) == 1, % WILL FIT LEFT FEMORAL HEAD
    [f2ctrr, f2radr, f2errr] = circlefit(f2r);
    disp('');
    disp('RIGHT FEMUR:');
    disp(sprintf(' CENTER(x,y): (%0.7g, %0.7g)', f2ctrr(1), f2ctrr(2)));
    disp(sprintf(' RADIUS: %0.7g', f2radr));
    disp(sprintf(' ERROR: %0.7g', f2errr));
    circfit = 1;
    [frx, fry] = gen_circle(f2radr, 100);
    plot(frx+f2ctrr(1), fry+f2ctrr(2), 'y-');
end

if cprint(4) == 1, % WILL FIT LEFT ACETABULUM
    [a2ctrr, a2radr, a2errr] = circlefit(a2r);
    disp('');
    disp('RIGHT ACETABULUM:');
    disp(sprintf(' CENTER(x,y): (%0.7g, %0.7g)', a2ctrr(1), a2ctrr(2)));
    disp(sprintf(' RADIUS: %0.7g', a2radr));
    disp(sprintf(' ERROR: %0.7g', a2errr));
    [arx, ary] = gen_circle(a2radr, 100);
    plot(arx+a2ctrr(1), ary+a2ctrr(2), 'c-');
end

if (yesno('ARE YOU HAPPY WITH BEST FIT LINE? <n>: ', 'n')),
    hold('off');
    imshow(img);
    axis(iaxis); hold('on');
    circfit = 1;
else
    p = zeros(2,0);
    markmark = 0;
    mark(size(mark,1), islice+1) = 0;
    circfit = 0;
    disp('REDIGITIZE STARTING WITH LEFT FEMORAL HEAD.');
    hold('off');
    imshow(img);
    axis(iaxis); hold('on');
end

else
    disp('.........................');
end

case 3 % CALCULATE INDICES
    if circfit == 0,
disp('YOU MUST CIRCLE FIT BEFORE CALCULATING INDICES.');}
else
    hold('off');
    imshow(img);
    axis('equal');
    hold('on');
    c = zeros(2,0);
    congdone = 0;
    disp('DIGITIZE LATERAL INFERIOR PORTION OF TEAR DROP (L THEN R).');
while -congdone,
    [x,y,mouse] = ginput(1);
    switch mouse
    case 1
        % LEFT MOUSE BUTTON
        c(:,end+1) = [x;y];
        plot(c(:,1),c(:,2),'bo');
        %----- ANATOMIC LANDMARKS --------------
        % TEARDROP (c(:,1:2))
        % ACETABULAR RIM (c(:,5:6))
        % LATERAL EDGE OF FEMUR (c(:,7:8))
        % PROXIMAL EDGE OF FEMUR (c(:,9:10))
        % ACETABULAR APEX (c(:,11:12))
        % ACETABULAR DEPTH (c(:,13,14))
        % INFERIOR PART OF FEMUR (c(:,15,16))
        %----------------------------------%
        if size(c,2) == 2, % calculate hilgenreiner's line
            h = polyfit(c(:,1:2),c(:,1:2),1);
            c(1,3:4) = [c(1,1)+250, c(1,2)-250];
            c(2,3:4) = polyval(h, c(1,3:4));
            plot(c(1,3:4), c(2,3:4), 'y-');
            disp('DIGITIZE ACETABULAR RIM EDGE (L THEN R).');
        end
        if size(c,2) == 6, % calculate center edge angle
            if h(1) == 0,
                interceptl = f2ctrl(2) + f2ctrl(1)/h(1); % line perp to hilgen
                xtemp = [-h(1)*c(2,5) - interceptl, c(2,5)]; % point on perp line
                ytemp = [-h(1)*c(2,6) - interceptl, c(2,6)];
            else
                xtemp = [f2ctrl(1), c(2,6)]; % point on perp line
                ytemp = [f2ctrl(1), c(2,6)];
            end
            lcentere = calcangle(f2ctrl(1), f2ctrl(2), xtemp(1), xtemp(2), c(1,5), c(2,5));
            rcentere = calcangle(f2ctrr(1), f2ctrr(2), ytemp(1), ytemp(2), c(1,6), c(2,6));
            plot([f2ctrr(1), rtemp(1)], [f2ctrr(2), rtemp(2)], 'g-');
            plot([f2ctrl(1), c(1,6)], [f2ctrl(2), c(1,6)], 'g-');
            plot([f2ctrr(1), f2ctrr(2), rtemp(1)], [f2ctrr(2), rtemp(2)], 'g-');
            plot([f2ctrl(1), f2ctrl(2), c(1,6)], [f2ctrl(2), c(1,6)], 'g-');
            disp('DIGITIZE LATERAL EDGE OF FEMUR (L THEN R).');
        end
        if size(c,2) == 8, % calculate the lateral subluxation
            lsub = (c(1,7) - c(1,1))*scale;
            rsub = (c(1,2) - c(1,8))*scale;
            disp('DIGITIZE PROXIMAL EDGE OF FEMUR (L THEN R).');
        end
        if size(c,2) == 10, % calculate femoral head extrusion index
            lfhei = 100*(c(1,9) - c(1,5))/(c(1,9) - c(1,7));
            rfhei = 100*(c(1,6) - c(1,10))/(c(1,8) - c(1,10));
            disp('DIGITIZE ACETABULAR APEX (L THEN R).');
        end
    end
end

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if size(c,2) == 12, % calculate peak to edge distance
lp2ed=sqrt((c(1,5)-c(1,11))^2 + (polyval(h,c(1,5))-polyval(h,c(1,11)))^2)*scale;
rp2ed = sqrt((c(1,6)-c(1,12))^2 + (polyval(h,c(1,6))-polyval(h,c(1,12)))^2)*scale;
disp('DIGITIZE ACETABULAR DEPTH (L THEN R).');
end

if size(c,2) == 14, % calculate depth to width ratio and acetabular index
  % DEPTH TO WIDTH RATIO
  law = sqrt((c(1,5) - c(1,11))^2 + (c(2,5) - c(2,11))^2);
  raw = sqrt((c(1,6) - c(1,12))^2 + (c(2,6) - c(2,12))^2);
  l2wr = (ltemp(l)*sin(ltheta*pi/180))/law;
  r2wr = (rtemp(l)*sin(rtheta*pi/180))/raw;
  % CALCULATE ACETABULAR INDEX
  lactabi = calcangle(c(1,13),c(2,13),c(1,5),c(2,5),ltemp(1),ltemp(2));
  racetabi = calcangle(c(1,14),c(2,14),c(1,6),c(2,6),rtemp(1),rtemp(2));
  plot([c(1,13),c(1,5)], [c(2,13),c(2,5)], 'c-')
  plot([c(1,13),ltemp(l)], [c(2,13),ltemp(2)], 'c-')
  plot([c(1,14),c(1,6)], [c(2,14),c(2,6)], 'c-')
  plot([c(1,12),rtemp(l)], [c(2,12),rtemp(2)], 'c-')
disp('DIGITIZE INFERIOR PORTION OF THE FEMORAL HEAD (L THEN R).');
end

if size(c,2) == 16, % calculate superior subluxation
  lsubsub = scale*(c(2,15) - c(2,11));
  rsubsub = scale*(c(2,16) - c(2,12));
  disp('LEFT HIP:');
disp(sprintf('LATERAL CENTER EDGE ANGLE(deg): %0.7g', lcentere));
disp(sprintf('ACETABULAR DEPTH TO WIDTH: %0.7g', l2wr));
disp(sprintf('FEMORAL HEAD EXTRUSION INDEX: %0.7g', lfhei));
disp(sprintf('ACETABULAR INDEX(deg): %0.7g', lactabi));
disp(sprintf('SUPERIOR SUBLUXATION(mm): %0.7g', lsubsub));
disp(sprintf('PEAK-TO-EDGE DISTANCE(mm): %0.7g', lp2ed));
disp('');
disp('RIGHT HIP:');
disp(sprintf('LATERAL CENTER EDGE ANGLE(deg): %0.7g', rcentere));
disp(sprintf('ACETABULAR DEPTH TO WIDTH: %0.7g', r2wr));
disp(sprintf('FEMORAL HEAD EXTRUSION INDEX: %0.7g', rfhei));
disp(sprintf('ACETABULAR INDEX(deg): %0.7g', racetabi));
disp(sprintf('SUPERIOR SUBLUXATION(mm): %0.7g', rsubsub));
disp(sprintf('PEAK-TO-EDGE DISTANCE(mm): %0.7g', rp2ed));
disp('');
end
end % switch
end % while
end % IF/ELSE FOR CIRCLE FIT CHECK.

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axis(iaxis) ;
hold('on') ;
while ~mdone,
    choice = input(['<1> LENGTH MEASUREMENT, <2> ANGLE CALCULATION, <3> EXIT, : ']) ;
    if isempty(choice), ;
    elseif choice == 1,
        disp('MARK END POINTS OF LENGTH.') ;
        m2done = 0 ;
        m = zeros(2,0) ;
        while ~m2done,
            [x,y,mouse] = ginput(1) ;
            switch mouse
                case 1 % LEFT MOUSE BUTTON
                    m(:,(end+1)) = [x;y] ;
                    plot(m(1,:),m(2,:), 'bo-') ;
                    if size(m,2) == 2,
                        dist = sqrt( (m(1,1)-m(1,2))^2 + (m(2,1)-m(2,2))^2 )*scale ;
                        m2done = 1 ;
                        disp(sprintf('DISTANCE (mm): %0.7g', dist)) ;
                    end
                end
            end
        end
    elseif choice == 2,
        disp('MARK THREE POINTS, (MARK THE VERTEX SECOND)') ;
        m2done = 0 ;
        m = zeros(2,0) ;
        while ~m2done,
            [x,y,mouse] = ginput(1) ;
            switch mouse
                case 1 % LEFT MOUSE BUTTON
                    m(:,(end+1)) = [x;y] ;
                    plot(m(1,:),m(2,:), 'bo-') ;
                    if size(m,2) == 3,
                        angl = calcangle( m(1,2), m(2,2), m(1,1), m(2,1), m(1,3), m(2,3)) ;
                        m2done = 1 ;
                        disp(sprintf('ANGLE (deg): %0.7g', angl)) ;
                    end
                end
            end
        end
    elseif choice == 3,
        mdone = 1 ;
    else,
        disp('ERROR IN MENU CHOICE.') ;
    end
end
hold('off') ;
imshow(img) ;
axis(iaxis) ;
hold('on') ;

case 5 % ADJUST GRAPHICS
    gdone = 0;
    zoom on;
    disp('ZOOM ON.') ;
    while ~gdone,
        choice = input(['<1> BRIGHTEN, <2> DARKEN, <3> EXIT, : ']) ;
        if isempty(choice), ;
        elseif choice==1, brighten(0.15) ;
        elseif choice==2, brighten(-0.15) ;
        elseif choice==3,
            zoom off;
            disp('ZOOM OFF') ;
        gdone = 1;
    else,
        disp('ERROR IN MENU CHOICE.') ;
    end

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end

case 6  % MARKER
if markmark == 0,
   disp('DIGITIZE THE LEFT ACETABULUM.');
   markmark = markmark + 1;
   mark(mark(markmark)) = size(p,2);
elseif markmark == 1,
   disp('DIGITIZE THE RIGHT FEMORAL HEAD.');
   markmark = markmark + 1;
   mark(mark(markmark)) = size(p,2);
elseif markmark == 2,
   disp('DIGITIZE THE RIGHT ACETABULUM.');
   markmark = markmark + 1;
   mark(mark(markmark)) = size(p,2);
else
   disp('MARKING IS NOT NECESSARY.');
end

case 7  % REMOVE LAST DIGITIZED POINT
p = p(:,1:(end-1));
if markmark > 0,
   if mark(mark(markmark)) > size(p,2),
      mark = mark(1:end-1);
      markmark = markmark - 1;
   end
end
iaxis = axis;
hold('off');
imshow(img);
axis(iaxis);
hold('on');

case 8  % RESET ALL
end  % END OF MENU CHOICES
end  % END OF WHILE STATEMENT (MAIN)
end

%--------------------------

function yn=yes_no(prompt,d)
% Inquiries yes/no question
% yn=yes_no(prompt,d)
% prompt: question string
% d: default string 'y' or 'n', if not given 'y' is used
% yn: if 'y' yn=1, if 'n' yn=0
% By S. Daniel Kwak Dec. 1997
if nargin==1
   yn=1;
elseif isempty(d)
   yn = 1;
elseif strcmp(d,'N')
   yn = 0;
else
   yn = 1;
end
y = input(prompt,'s');

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if isempty(y)
  yn = 1;
elseif strncmp(y,'Y',1) | strncmp(y,'y',1) | strcmp(y,'1')
  yn = 1;
elseif strncmp(y,'N',1) | strncmp(y,'n',1) | strcmp(y,'0')
  yn = 0;
end

%-------------------------------------------------------------------------

% This function calculates the distance between two lines
% (using the law of cosines) from three points in a 2-D cartesian plane.
% Written by Jennifer A. Cooper July 1999

% o c / % / / % o--------o b % a

A = sqrt((bx-cx)^2 + (by-cy)^2);  
B = sqrt((ax-cx)^2 + (ay-cy)^2);  
C = sqrt((ax-bx)^2 + (ay-by)^2);  

thetarad = acos(((B^2 + C^2 - A^2)/(2*B*C)));  
theta = thetarad*180/pi;
%-------------------------------------------------------------------------

%-------------------------------------------------------------------------

function [ctr, r, err] = circle_fit(p)

% Circle fitting using least squares method.
% Error normalized to the calculated radius.

if size(p,1)==2, error('Input matrix must be (2,n)'); end
if size(p,2)<3, error('There must be more than 2 points.'); end

options = foptions;  
[ctr0, r0] = guess_ctr(p);  
x0 = [ctr0;r0];  

warning off  
[x, options] = leastsq('circlefit_fun', x0, options, ...  
  'circlefit_grad', p);  

warning on  
ctr = x(1:2);  
r = x(3);  

ax = ctr(1);  
ay = ctr(2);  
for i = 1:size(p,2),  
  bx = p(1,i);  
  by = p(2,i);  
  erad(i) = radius(ax,ay,bx,by);  
  ediff2(i) = (erad(i) - r)^2;
end
err = sqrt(sum(ediff2))/r;  

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% function [ctr0, r0] = guessCtr(pp)
avg = mean(pp, 2);
p = pp - avg*ones(1, size(pp, 2)); % offset by avg

d = sum(p.^2);
A = [p; d];
AA = A(1:3, 1:3)*A(1:3, 1:3);
bAt = sum(A');

x = bAt/AA;
tmp = 2*x(3);
ctr0 = [-x(1)/tmp; -x(2)/tmp];
ctr0 = ctr0 + avg; % offset back by avg
r0 = sqrt((sum(x(1:2).^2) + 3*x(3))/(3*x(3)^2));

% function f=circlefitFun(x,p)
c = x(1:2);
r = x(3);
f = magVector(Mv_add_v(p, -c)) - r;

% function df = circlefit_grad(x,p)
c = x(1:2);
tmp = magVector(Mv_add_v(p, -c));
df = -[Mv_div_s(Mv_add_v(p, -c), tmp); ones(1, size(p, 2))];

% function Mw = Mv_add_v(Mv, v)
% Mw = Mv_add_v(Mv, v) returns Mw(m,n) = Mv(m,n) + v(m).
% Each column in Mv(:,1) is added by v.
% By S. Daniel Kwak Dec. 1997
% Input

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% Mv: array of column vectors
% v: single vector
% Output:
% Mw: array of column vectors

if ndims(Mv)~=2 | ndims(v)~=2 % must be dim=2
  error ('Input must be size=2 matrix');
elseif size(v,2)~=1 % must be a single column vector
  error ('Input argument v must be a column vector');
elseif size(Mv,1)~=size(v,1)
  error ('Input argument must have same number of rows');
end

A = v*ones(1,size(Mv,2));
Mw = Mv + A;

%---------------------------------------------------

function m = magvector(v)
% m = magvector(v) returns scalar magnitude of vector v.
% if v is an array of column vectors, m is scalar vector of same length.
% m = sqrt(v.v)
% By S. Daniel Kwak Dec. 1997
% Input
% v: vector
% Output
% m: scalar magnitude of m

if ndims(v) ~= 2
  error ('input variable must be 2D array');
end

m = sqrt(sum(v .* v));

%---------------------------------------------------

function Mw = Mv_div_sv(Mv,sv)
% Mw = Mv_div_sv(Mv,sv) returns Mw(m,n) = Mv(m,n) / sv(1,n).
% Each column in Mv(:,i) is divided by sv(i).
% By S. Daniel Kwak Jan. 29, 1997
% Input
% Mv: array of column vectors
% sv: array of scalar values
% Output:
% Mw: array of column vectors

if ndims(Mv)~=2 | ndims(sv)~=2 % must be dim=2
  error ('Input must be size=2 matrix');
elseif size(sv,1)~=1 % must be a single row vector
  error ('Input argument sv must be a row vector');
elseif size(Mv,2)~=size(sv,2)
  error ('Input argument must have same number of columns');
end

A = ones(size(Mv,1),1)*sv;
Mw = Mv ./ A;

%---------------------------------------------------

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APPENDIX B
Center of Rotation Finding Tool for Surgical Guidance

The understanding of the kinematics of joints in the body is an important tool for
the diagnosis of joint injury or disease and the design of joint prosthetics. The location of
the joint center of rotation is a significant factor in these judgments. In addition, knowing
the joint center of rotation can be useful for the placement of surgical devices. A system
has been developed that allows a user to manually digitize points on a video image from a
digital x-ray unit to calculate a center of rotation.

Notation
R = direction cosine matrix of planar clockwise rotation
φ = angle of rotation of angular displacement
x = initial position vector within the initial coordinate system
x' = final position vector within the initial coordinate system
X = initial position vector within the transformed coordinate system
X' = final position vector within the transformed coordinate system
C = displacement vector describing translation from an initial to transformed coordinate systems
u = the vector from the two point rigidly fixed on the body prior to translation/rotation
u' = the vector from the two point rigidly fixed on the body prior to translation/rotation
A = position vector of point A prior to translation/rotation
B = position vector of point B prior to translation/rotation
A' = position vector of point A following translation/rotation
B' = position vector of point B following translation/rotation
C_x = the displacement along the x-axis or x coordinate from the origin of the chosen coordinate system to the
center of rotation
C_y = the displacement along the y-axis or the y coordinate from the origin of the chosen coordinate system to
the center of rotation

B.1 Motivation

The fixation of an external fixation device around a joint should be placed such that
the center of rotation of the fixator is at the center of rotation of the joint. In the operating
room, surgeons estimate the joint center of rotation. Inaccurate placement of the surgical
pins for fixation could cause abnormal joint loading. It has been shown that irregular joint loading is a trigger for the initiation of osteoarthritis [1-3]. A tool was needed that could quickly and easily be used during surgery to precisely find the joint center of rotation and guide the surgeon in the placement of pins for external fixation.

B.2 Introduction

Researchers and clinicians have been interested in investigating and describing the kinematics of human joints for decades [4-6]. Though these data are often criticized for having no function, kinematic data have been used in the diagnosis of joint injury or disease, describing disease progression, and in the design of prosthetic joint replacements [7-10]. One of these kinematic parameters used in helping describe joint function is the center of rotation or axis of rotation. Specifically, it would be important to know the exact center of rotation when fitting an external fixation system to a patient joint (e.g. ankle, knee or metacarpals). If such an external fixation system is affixed to the patient off axis, the joint could be unnaturally loaded leading to growth complications or premature osteoarthritis. Keeping this in mind, a tool was developed to guide surgeons, in real time, to the center of rotation of a joint.

Reuleaux is credited as having first described the calculation of the center of rotation in the 1870s for machine design [11]. Since this time, rigid body center of rotation has been described throughout classical mechanics texts [12-15]. The instantaneous center of rotation is defined as a fixed point whereby, as a body moves through space from one
position to another, the body rotates by some angle about that fixed point. The instantaneous center of rotation can also be described as the point on a body where, as the body rotates in space, the velocity of that fixed point remains zero.

J.P. Den Hartog [14] describes students two graphical methods for calculating instantaneous center of rotation. In the first method (Figure B.1.A) Den Hartog develops the concept of mapping the transverse velocities of points arbitrarily chosen points on the body (P and Q), and from this being able to find the transverse velocity of any point on the body. Specifically the point at which the transverse velocity is zero can be found (i.e. the center of rotation). The second method is Reuleaux’s method (Figure B.1.B) where the positions of two points rigidly fixed to a body are known before and after motion in a plane. The point at which the perpendicular bisectors of the lines connecting the initial and final positions of these points meet is the center of rotation.

Lee and coworkers [16] studied the relationship of the center of rotation to the instability of the cervical spine in humans using the concepts of Reuleaux’s graphing
techniques. Though classical mechanists tend to use the graphical techniques or kinematics data, biomechanists have been calculating the rigid body center of rotation through a variety of joint analyses. Youm [17] described a least squares method of center of rotation calculation for the use of designing total joint replacements and describing joint paths. Dimnet [18] used a stored program calculator to calculate joint center of rotation in the radio-ulnar joints. Hollister and coworkers [19, 20] found the axes of rotation of various thumb joints using mechanical methods. A rod inserted into rotating body will help to visualize the motion about that axis. The rod will describe an arc unless inserted into the axis of rotation, where it will rotate only with no center of rotation. Hollister et al use this principle to create a device where a K-wire is adjusted until the axis of rotation is found. Hollister and associates carry extend this axis-finding principle for the knee and the forearm [21, 22]. Grant [7] found the instantaneous center of rotation of the temporomandibular joint by examining muscle origins and insertion points. Halvorsen and coworkers [23] describe a three dimensional technique for estimating the axis and center of rotation in the talocalcaneal joint of the foot. In addition, researchers have examined the hip [9], elbow [24] and feline cervical spines [25]. Currently, the most popular instantaneous center of rotation calculation stems from the works of Panjabi and coworkers (Reuleaux method) [26, 27] and Speigelman and Woo [28].
B.3 Center of Rotation Calculation

Throughout almost all center of rotation calculation work there is a mention of a high tendency for error [8, 10, 25-33]. Investigators have attempted to examine the source of this error and ways to eliminate the error - parametric studies of algorithms and attempts to optimize marker location in various calculation techniques have been mentioned [8, 10, 26-28, 32, 33]. Spiegelman and Woo noted the importance of the incorporating the independence of marker angle to improve the accuracy of the center of rotation calculation. Crisco goes a step further to decompose the marker angle and allow for a parametric analysis of where error is occurring in the calculations. The Crisco algorithm for center of rotation calculation is as equally accurate as Spiegelman and Woo’s when the center of rotation is located midway between the markers, however as the distance between the midline and the center of rotation increased Crisco method became more accurate. In the development of this tool, two methods of center of rotation guidance were employed for accuracy comparison: the graphical Reuleaux method, and an explicit method similar to that utilized by Crisco and coworkers.

B.3.1 Graphical

The Reuleaux method for finding the center of rotation of a rigid body is easily employed when the position of two points fixed to the surface are known before and after motion in a plane. To incorporate this into a surgical guidance program, the perpendicular bisector of the two points on the body at each time point is drawn. The placement of pins
will occur at the intersection of the perpendicular bisectors drawn from different time points. Since the guidance tool is visual for the surgeon, the coordinates of the center of rotation will be found by manual digitization, since this points is needed only for accuracy calculations. The line is drawn using the simple geometric relations in Figure B.2.

Knowing the coordinates for both points fixed to the body (points A and B) the magnitude of the vector between the points can be represented as:

\[
\overrightarrow{AB} = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} \quad (B.1)
\]

Also, from the coordinates of points A and B, the angle between the vectors, \( \theta \), can be calculated as:

\[
\theta = \tan^{-1} \left( \frac{Y_2 - Y_1}{X_2 - X_1} \right) \quad (B.2)
\]

Figure 3.2 Geometric relationship for graphically mapping Reuleaux method.
From here, the coordinates of point D, which is the midpoint of line AB, can be found as in Equations B.3 and B.4.

\[ X_3 = X_1 - \frac{\overline{AB}}{2} \sin \theta \]  
\[ Y_3 = Y_1 - \frac{\overline{AB}}{2} \cos \theta \]  

(B.3)  

(B.4)

Similarly, the coordinates of point E, which is used to create the line DE - the perpendicular bisector of line AB.

\[ X_4 = X_3 - c \overline{AB} \sin \theta \]  
\[ Y_4 = Y_3 - c \overline{AB} \cos \theta \]  

(B.5)  

(B.6)

The term c, in Equations B.5 and B.6 is some constant that determined the length of the perpendicular bisector to be drawn. This calculation is carried out for each point in time for the motion through the plane. The scaling factor, c, is adjusted depending on what is needed to obtain an intersection of the bisectors for the center of rotation using Reuleaux’s principles.

**B.3.2 Explicit**

The algorithm derived in this work is done explicitly, as in Crisco et al [27]. Assume the coordinates of two points fixed to a rigid body have been identified before and after motion in a plane. From the coordinate data the center of rotation can be calculated
independent of the coordinate system. The solution is valid for both translational and rotational motion due to the freedom from the coordinate system.

Returning to elementary concepts of rigid body motion the transformation matrix of rotation between these two positions in a plane can be derived. (Note: This work is carried out in two-dimensions since radiographic images are planar. A similar method of calculation can be derived for three-dimensional space.) The direction cosine matrix, \( \mathbf{R} \), for a rigid body undergoing a counterclockwise rotation of \( \phi \) can be derived with help from Figure B.3. The axis transformation equations from the original axis to the primed axis is described by:

\[
\begin{align*}
x' &= x \cos \phi + y \sin \phi \\
y' &= -x \sin \phi + y \cos \phi
\end{align*}
\]  

(B.7) \hspace{1cm} (B.8)

Then, the relationship between the original vector, \( \mathbf{x} \), and the vector following rotation, \( \mathbf{x}' \), where \( \mathbf{R} \) is the direction cosine matrix, is described by Eq.B.9:

\[
\mathbf{x}' = \mathbf{Rx} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \mathbf{x}
\]  

(B.9).
Suppose the rigid body moves in an arbitrary coordinate system in which the body positions can be described. Regardless of the preference of the coordinate system, the rotation matrix, $\mathbf{R}$, and the rotation angle, $\phi$, will remain the same. However, as Crisco and coworkers [27] describe, "the location and orientation of the coordinate system affect both magnitude and direction of the translation vector, $\mathbf{v}$." As previously mentioned the unique choice of the origin for a coordinate system such that the magnitude of the translation vector is zero is defined as the instantaneous center of rotation.

The shift of a coordinate systems $\mathbf{X}$ and $\mathbf{X}'$ by a translation vector $\mathbf{C}$ is expressed by equations B.10 and B.11 and can be seen in Figure B.4.

$$\mathbf{X} = \mathbf{C} + \mathbf{x} \quad \text{(B.10)}$$

$$\mathbf{X}' = \mathbf{C} + \mathbf{x}' \quad \text{(B.11)}$$

![Figure B.4 Rigid body motion in a plane. Markers A and B are fixed to the body which is rotated and translated to the final position of A' and B'. The vector C is the instantaneous center of rotation.](image-url)
Substituting equations 3.10 and 3.11 into equation 3.3, the relationship becomes:

\[ X' - C = R(X - C) \]  \hspace{1cm} (B.12).

Similar to the derivation by Crisco et al [27], the location of the center of rotation in the global coordinate system, \( C \), can be solved for directly from equation B.13.

\[ X' - C = RX - RC \]  \hspace{1cm} (B.13)

\[ X' - RX = C[I - R] \]  \hspace{1cm} (B.14)

\[ C = [x_{cr} \ y_{cr}] = [I - R]^{-1}(X' - RX) \]  \hspace{1cm} (B.15)

Solving explicitly for the inverse of \([I - R]\), we find:

\[ [I - R]^{-1} = \begin{bmatrix} 1 - \cos \phi & \sin \phi \\ -\sin \phi & 1 - \cos \phi \end{bmatrix}^{-1} = \frac{1}{(1 - \cos^2 \phi) + \sin^2 \phi} \begin{bmatrix} 1 - \cos \phi & -\sin \phi \\ \sin \phi & 1 - \cos \phi \end{bmatrix} \]

\[ = \frac{1}{2(1 - \cos \phi)} \begin{bmatrix} 1 - \cos \phi & -\sin \phi \\ \sin \phi & 1 - \cos \phi \end{bmatrix} \]

From this point, if the coordinates of two points on the body prior to rotation [points \( A = (x_1, y_1) \) and \( B = (x_3, y_3) \)] and following rotation [points \( A' = (x_2, y_2) \) and \( B' = (x_4, y_4) \)] are known, the angle of rotation can be determined. Equations B.17 and B.18 give the definition of scalar (dot) product and vector (cross) product respectively, for the angle of rotation, \( \phi \),

\[ \cos \phi = \frac{u \cdot u'}{|u||u'|} \]  \hspace{1cm} (B.17)
where $u = A - B$, and $u' = A' - B'$. Expanding these expressions,

$$\cos \phi = \frac{(A - B) \cdot (A' - B')}{\|A - B\|\|A' - B'\|} = \frac{\left( \begin{array}{c} x_1 \\ y_1 \end{array} \right) - \left( \begin{array}{c} x_3 \\ y_3 \end{array} \right) \cdot \left( \begin{array}{c} x_2 \\ y_2 \end{array} \right) - \left( \begin{array}{c} x_4 \\ y_4 \end{array} \right)}{\|A - B\|\|A' - B'\|} \quad (B.19)$$

Since the points $A$ and $B$ are rigidly fixed to the body throughout the motion of the body, the distance between $A$ and $B$ and the distance between $A'$ and $B'$ must be equal, equation B.20 can be simplified:

$$\cos \phi = \frac{(x_1 - x_3)(x_2 - x_4) + (y_1 - y_3)(y_2 - y_4)}{(x_1 - x_3)^2 + (y_1 - y_3)^2} \quad (B.20)$$

In the same manner, $\sin \phi$ can be expanded:

$$\sin \phi = \frac{(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)}{(x_1 - x_3)^2 + (y_1 - y_3)^2} \quad (B.21)$$

Substituting equations B.16, B.20 and B.21 into equation B.15 and expanding the notation we find that the coordinates of the center of rotation can be solved as:

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \left[ \frac{1}{2} (x_2 - x_1 \cos \phi + y_1 \cos \phi) - \frac{\sin \phi}{2(1 - \cos \phi)} (y_2 - x_1 \sin \phi - y_1 \cos \phi) \right]$$

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \left[ \frac{\sin \phi}{2(1 - \cos \phi)} (x_2 - x_1 \cos \phi + y_1 \cos \phi) + \frac{1}{2} (y_2 - x_1 \sin \phi - y_1 \cos \phi) \right] \quad (B.22)$$

$$C_x = \frac{1}{2} (x_1 + x_2) + \frac{(y_1 - y_2) \sin \phi}{2(1 - \cos \phi)} \quad (B.23)$$

$$C_y = \frac{1}{2} (y_1 + y_2) + \frac{(x_1 - x_2) \sin \phi}{2(1 - \cos \phi)} \quad (B.24)$$
Equations B.24 and B.25 were used for the calculation of center of rotation in the program developed.

**B.4 Methods**

The video output signal (EIA RS170 60Hz) of a mobile digital x-ray unit (OEC Mini6600 Digital Mobile C-Arm, OEC Medical Systems, Inc., Salt Lake City, UT) (Figure 3.5) was connected to a personal computer (Dell Optiplex GX1, Dell Computer Corporation, Round Rock, TX). The video signal was read through a video capture card (Matrox Meteor-II, Matrox Electronic Systems, Ltd., Mooers, NY). A Visual Basic program was written to continuously view, grab and save the video.

![Figure B.5 OEC Mini6600 Digital Mobile C-Arm](image-url)
images from the x-ray unit. The code also allows the user to manually digitize points that are used to calculate the center of rotation using the algorithm above (Equations B.23 and B.24), as well as the algorithm published by Spiegelman and Woo [28]. A center of rotation marker is then overlaid on the live video image to allow the surgeon to line up alignment pins using the live video image as a visual guide.

**B.5 Error Analysis**

An effort was made to examine the precision and accuracy of the overall accuracy of the video capture image system in comparison to the true object, the manual digitization, and of the center of rotation algorithms. Two pieces of radiolucent plastic were nailed together to form a simple planar pin joint about the metal nail (Figure B.6).

![Figure B.6 Rig used for experimental testing of center finding system. The pin labeled 1 is a pin joint and therefore the experimental center of rotation. Each labeled pin (1-5) is a metal K-wire with an optical marker for both x-ray and optical tracking.](image)

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Four 0.67 mm K-wires were inserted vertically into the rig approximately 18 mm and 20 mm away from the fixed center of rotation. Affixed to the tops of each of the four K-wires was a 2mm x 2mm optical marker. A 5th K-wire 12.05 mm in length was fixed horizontally to the rig, used for scaling the optical images during the digitization process. The rig in Figure 3.5 was placed on the fluoroscopy unit (seen in Figure 3.4) and two 60 Hz motion measurement cameras (PC Reflex, Qualysis, Glastonbury, CT) were calibrated to the surface. The mean spatial accuracy of the motion measurement cameras was approximately $6.7 \times 10^{-3} \pm 7 \times 10^{-5}$ mm. The pin joint was x-rayed and optically imaged at 0, 5, 10, 15 and 20 degrees of rotation. This regime was repeated 3 times for precision analysis. The coordinates from the motion detection system were extracted using commercially included software. To obtain the marker coordinates from the x-ray images, the data was loaded into a custom written Visual Basic program that output the coordinates of the manually digitized markers and the scaling factor calculated from the horizontal K-wire. The first trial was digitized 5 times for each degree level to examine manual digitization error. The coordinates of both the x-ray and optically tracked markers were offset such that for each image, the expected center of rotation (marker 1) was at the origin. An axis transformation of 15 degrees clockwise rotation was performed uniformly on the x-ray data so that the coordinate axes of each imaging modality were approximately aligned. A plot of the resulting coordinates for each marker at each rotation can be seen in Figure B.7, notice the precision of the x-ray and optically tracked markers.
First the accuracy of the video capturing and x-ray imaging system was evaluated. The motion system data serves as the standard to compare the video analyzed in the relative spatial coordinate system. The coordinates for each marker at each rotation were averaged separately for both the x-ray and PCReflex data. For each marker at each rotation the difference between the coordinates was calculated and averaged. The average difference was calculated as $0.04 \pm 0.99$ mm, and the average absolute difference was $0.67 \pm 0.73$ mm. The standard deviation is relatively high, therefore, the error is not considered to be systematic. It is expected the difference arose from manual digitization or from the marker and camera locations in the PCReflex system coordinate calculation. The motion cameras detect the markers as pixel maps of intensity of reflected infrared light. The
coordinates are calculated as the centroid of the pixels detected for each marker. The markers were 2mm x 2mm square, and the cameras were at an odd angle to view around the C-arm device, which could cause the camera to not detect the full marker surface. With incomplete marker surface data, the calculation of the marker centroid could be inaccurate. Regardless, the accuracy of the video system falls within an acceptable range (within one diameter of the K-wire center). There was no relationship detected between the errors between marker sets, or rotation angle.

The precision of the manual digitization was analyzed. 15 total images were captured from the Mini6600 C-Arm (3 trials at 5 positions), and on each image 7 points were manually digitized (each marker, and the end points of the scaling wire) by one observer. The first trial images at each rotation point were digitized 5 times each for intra-observer repeatability. The average standard deviation between the 5 first trial measurements for all 5 angles was 0.14 mm. Since the pixel length of the images has been calculated to be .29 mm, the user variation is considered very respectable and practically unavoidable. When the same calculation was performed for all trials, including the 5 repeated first trials the average standard deviation was calculated to be 0.56 mm. This is considered to be acceptable since small motion may have occurred during testing, and put into perspective, the error is smaller than the diameter of the K-wire.

Finally, the digitized data was fed into the proposed algorithms. The Spiegelman and Woo, Crisco, and Reuleaux methods were used to calculate center of rotation for the digitized points. The results are shown in Table B.1. Assuming that the true center of

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rotation is the origin (X=0, Y=0), all of the methods appear to have a good deal of error, as expected. The Spiegelman and Reuleaux algorithms were both affected by rotation angle and by distance from the center of rotation (as shown by the difference in marker sets 2-3). The Crisco method performed least predictably for these test conditions, while the other two methods performed more as expected. The Reuleaux method was judged as the best method for this purpose. It is suggested that the markers be placed as far from the center of rotation as expected and that a large rotation angle is used for maximum accuracy.

**B.6 Discussion**

Systems similar to this are currently used in the surgical room for surgical guidance and placement. Chen et al hold a United States Patent entitled “Video-Based Surgical Targeting System. [34]” One objective of this device is pre-operative patient images are scanned in and used as three-dimensional animated guides as cameras monitor the real time surgery, this aiding the surgeon in targeting anatomy. This system is different because
does not need any previous patient data to aid the surgeon, and all calculations are made real-time to find the center of rotation.

One question that remains when performing the technique of inserting pins is whether or not the correct plane of motion has been chosen for calculation and insertion. This method assumes that the joint motion is purely in the two dimensional plane of the x-ray images, however in some joints, the motion may not be planar, or parallel to the x-ray imaging surface. This aspect of this tool error has not been investigated. One method to check both the placement of a pin in the correct axis of rotation would be similar to the technique by Hollister [19, 20] mentioned above. Once the pin has been placed into the joint, if in fact it has been correctly aligned, and the joint is rotated, the pin should only see rotational motion – or in the x-ray image, the pin would have no motion.

**B.7 Conclusions**

A computer program was created to aid surgeons in the placement of surgical pins into a joint center of rotation using video signal from an x-ray system, and a commercially available video capture card on a personal computer. As seen in center of rotation calculations throughout the literature, this algorithm has a standard tendency for error.
B.8 Future Work

Future work on this technique would be to improve the center of rotation calculation so that the propensity for error in the calculation would be decreased. Secondly, a mechanical check of the correct placement of the pin could be integrated into the current program by examining the motion of the inserted pin as the joint is rotated. The final device needed to complete this tool is a radiolucent drill that could be used to insert the surgical pins without interference in the viewing area.

B.9 Center of Rotation Program Code (Visual Basic)

```vbnet
Option Explicit
Dim h, points(7, 1) As Integer
Dim IMAGE_FILE As String
Dim PointsBool As Boolean
Dim CentRotX, CentRotY, CentCriscoX, CentCriscoY,
sol1, sol2, phi, d, m, intcep, dx1, dx2, midx, midy As Double
Private Sub CmdEnd_Click() 'End the program
    End
End Sub

Private Sub Form_Load()
    PointsBool = True
    h = 0
    ' Adjust the form and display sizes.
    AdjustForm Me
    ' Size the Image Buffer and begin grab.
    ImageBuffer.SizeX = Digitizer.SizeX * Digitizer.ScaleX
    ImageBuffer.SizeY = Digitizer.SizeY * Digitizer.ScaleY
    ImageBuffer.Allocate
    Digitizer.GrabContinuous
    ' Beep
    ' Print message
    IblComment = "Click 'NEXT' to capture image."
    Shape1.Visible = False
    Shape2.Visible = False
    ImageBuffer.Halt
End Sub

Private Sub cmdNext_Click()
    Shape2.Visible = False
    ' Disable the Next button
    cmdNext.Enabled = False
    ' Stop continuous grab
    Digitizer.Halt
    ' Print message
    IblComment = "Digitize point A then B."
    DoEvents
    CentRotX = 0
    CentRotY = 0
End Sub
```
Private Sub Display_MouseDown(Button As Integer, Shift As Integer, X As Single, Y As Single)
    If PointsBool = True Then
        If cmdNext.Enabled = False Then
            points(h, 0) = X
            points(h, 1) = Y
            CurrentX = X
            CurrentY = Y
            With Shape1
                .Left = X
                .Top = Y
                .Visible = True
            End With
            'Reuleaux Method
            If h = I Then
                Call DistForm(points(0, 0), points(1, 0), points(0, 1),
                              points(1, 1))
                sol1 = points(1, 0) - points(0, 0)
                sol2 = points(1, 1) - points(0, 1)
                If sol2 = 0 Then
                    sol2 = 0.0000001
                End If
                phi = (Atn(sol1 / sol2))
                dx1 = (d / 2) * Sin(phi)
                dx2 = (d / 2) * Cos(phi)
                If sol2 < 0 Then
                    dx1 = -dx1
                    dx2 = -dx2
                End If
                midx = points(0, 0) + dx1
                midy = points(0, 1) + dx2
                Line1.X1 = points(0, 0)
                Line1.X2 = points(1, 0)
                Line1.Y1 = points(0, 1)
                Line1.Y2 = points(1, 1)
                Line1.Visible = True
                Line2.X1 = midx + 30 * dx2
                Line2.Y1 = midy + 30 * dx1
                Line2.X2 = midx - 30 * dx2
                Line2.Y2 = midy - 30 * dx1
                Line2.Visible = True
            ElseIf h = 3 Then
                Call DistForm(points(2, 0), points(3, 0), points(2, 1),
                              points(3, 1))
                sol1 = points(3, 0) - points(2, 0)
                sol2 = points(3, 1) - points(2, 1)
                If sol2 = 0 Then
                    sol2 = 0.0000001
                End If
                phi = (Atn(sol1 / sol2))
                dx1 = (d / 2) * Sin(phi)
                dx2 = (d / 2) * Cos(phi)
            End If
        End If
        cmdNext.Enabled = True
        cmdNext.SetFocus
    End If
    If h Mod 2 = 1 Then  'Save image at each timepoint
        IMAGEFILE = "image" & h \\ 2 & ".tif"
        lblComment = h
        Digitizer.Image.Save (IMAGE_FILE)
        Digitizer.GrabContinuous
        cmdNext.Enabled = True
        cmdNext.SetFocus
    End If
    If h = 7 Then
        PointsBool = False
    End If
    h = h + 1
End Sub

Public Function DistForm(Xl, X2, Yl, Y2)
    d = Sqr((X1 - X2)^2 + (Y1 - Y2)^2)
End Function

Private Sub cmdCalc_Click()
    Dim p(3, 1), j, i As Integer
    j = 0
    CentRotX = 0
    CentRotY = 0
    ReDim CentX(h), CentY(h) As Integer
    If h <= 1 Then
        lblComment = "Digitize more points before calculating."
    Else
        For i = 2 To 7 - h - 1

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' Reshape matrix for rotation algorithms.
p(0, 0) = points(0, 0)
p(2, 0) = points(1, 0)
p(1, 0) = points(0, 0)
p(3, 0) = points(i + 1, 0)
p(0, 1) = points(0, 1)
p(2, 1) = points(1, 1)
p(1, 1) = points(0, 1)
p(3, 1) = points(i + 1, 1)

Call CenterOfRot(p, CentRotX, CentRotY)
Call CriscoCenter(p, CentCriscoX, CentCriscoY)
CentX(i) = CentRotX
CentY(i) = CentRotY
j = j + 1
Next i

For j = 0 To UBound(CentX, 1)
CentRotX = CentRotX + CentX(j)
CentRotY = CentRotY + CentY(j)
Next j
CentRotX = CentRotX / (UBound(CentX) + 1)
CentRotY = CentRotY / (UBound(CentY) + 1)
lblComment = CentX(UBound(CentX, 1))

With Shape1
  .Left = CentRotX - 42
  .Top = CentRotY - 42
  .Visible = True
End With

With Shape2
  .Left = CentRotX - 125
  .Top = CentRotY - 125
  .Visible = True
End With

With Shape3
  .Left = CentCriscoX - 50
  .Top = CentCriscoY - 50
  .Visible = True
End With

lblComment = "Completed."
PointsBool = True
End If

h = 0
For i = LBound(CentY, 1) To UBound(CentY, 1)
  CentX(i) = 0
  CentY(i) = 0
Next i
For i = LBound(p, 1) To UBound(p, 1)
  p(i, 0) = 0
  p(i, 1) = 0
  j = 0
Next i

End Sub

Private Sub Display_mouseMove(Button As Integer, Shift As Integer, X As Single, Y As Single)

' Gives audio sound when mouse has entered the target circle.
MousePointer = vbCrosshair
CurrentX = X
CurrentY = Y

If CurrentX > Shape2.Left Then
  If CurrentX < Shape2.Left + 250 Then
    If CurrentY < Shape2.Top + 250 Then
      If CurrentY < Shape2.Top Then
        If CurrentY < Shape2.Top + 250 Then
          IblComment = "In boundary."
          Shape1.Visible = False
          End If
          End If
        End If
        End If
      End If
      End If
    End If
    End If
  End If
  End If
If CurrentX > CentRotX - 25 Then
  If CurrentX < CentRotX + 25 Then
    If CurrentY < CentRotY - 25 Then
      If CurrentY < CentRotY + 25 Then
        Beep
        IblComment = "Insert Pin."
        Shape1.Visible = False
        End If
        End If
      End If
      End If
    End If
    End If
  End If
  End If
End Sub
' WHERE P(3,1) REPRESENTS POINTS A, B, A', B'

Dim S, SP, T, TP, COSO, SINO, X(3), Y(3) As Double

X(0) = p(0, 0)
X(1) = p(2, 0)
X(2) = p(1, 0)
X(3) = p(3, 0)
Y(0) = p(0, 1)
Y(1) = p(2, 1)
Y(2) = p(1, 1)
Y(3) = p(3, 1)

S = X(0) - X(2)
SP = X(1) - X(3)
T = Y(0) - Y(2)
TP = Y(1) - Y(3)

COSO = (S * SP + T * TP) / (S ^ 2 + T ^ 2)
SINO = (SP * T + TP * S) / (S ^ 2 + T ^ 2)

CentCriscoX = (1 / 2) * ((X(0) + X(1)) + (Y(0) - Y(1)) * SINO
/ (2 * (1 - COSO))
CentCriscoY = (1 / 2) * ((Y(0) + Y(1)) - (X(0) - X(1)) * SINO
/ (2 * (1 - COSO))

End Sub

Public Sub CenterOfRot(p, CentRotX, CentRotY)
' CALCULATES THE CENTER OF ROTATION OF INPUT
MATRIX P,
' WHERE P(3,1) REPRESENTS POINTS A, B, A', B'

Dim S, SP, T, TP, COSO, SINO, U, V, X(3), Y(3) As Double

X(0) = p(0, 0)
X(1) = p(2, 0)
X(2) = p(1, 0)
X(3) = p(3, 0)
Y(0) = p(0, 1)
Y(1) = p(2, 1)
Y(2) = p(1, 1)
Y(3) = p(3, 1)

S = X(0) - X(2)
SP = X(1) - X(3)
T = Y(0) - Y(2)
TP = Y(1) - Y(3)

COSO = (S * SP + T * TP) / (S ^ 2 + T ^ 2)
SINO = (SP * T - TP * S) / (S ^ 2 + T ^ 2)

U = (SINO * (X(0) - X(1))) / (2 * (1 - COSO)) + (Y(0) +
Y(1)) / 2
V = (SINO * (Y(0) - Y(1))) / (2 * (1 - COSO)) + (X(0) +
X(1)) / 2

CentRotY = X(0) + (Y(1) - U) / SINO - (COSO * (Y(0) - U)) / SINO
CentRotX = Y(0) - (X(1) - V) / SINO + (COSO * (X(0) - V)) / SINO

' With FormObject.Shape1
' .Left = CentRotX - 42
' .Top = CentRotY - 42
' .Visible = True
' End With

FormObject.lblComment = CentRotX & " " & CentRotY

End Sub
B.9 References

This two dimensional model has been created to simply portray the changes in contact area that occur as a femoral head slides laterally in an acetabulum through contact to partial contact and full dislocation (no contact). The model, created in MATLAB, received an input of the femoral head radius, the acetabulum radius, and the distance between the femoral head and acetabular centers. A full circle represented the femoral head, while the acetabulum was idealized as one-third of a full circle (120 degrees). The output of the program was a graphical representation of the femoral head at three positions during the translation to dislocation, and also a plot of the contact area between the femoral head and acetabulum as a function of the translation.

The center of the acetabulum was set as the origin of the coordinate system, and the acetabulum was defined as the arc beginning at −30 degrees and ending at 90 degrees at a radius the user specifies. To create translation, the coordinates of the center of the femoral head were calculated as the distance between the acetabular and femoral head centers for each of 120 degrees. Knowing the center of the femoral head, the minimum and maximum point of contact between the circle defining the femoral head and the arc defining the acetabulum could be found. The area of overlap between these points, which is considered the contact area, was then calculated through simple integration.
The results of three scenarios are shown in Figures C.1-C.3. Figure C.1 represents the situation where the femoral head is much smaller than the acetabulum. Figure C.2 is a “normal” hip situation with a comparable femoral head radius for the specified acetabular radius. Figure C.3 uses the radii calculated in Table 2.1 – an average for a set of dysplastic patients.

![Figure C.1](image1)

Figure C.1: Output for acetabular radius = 10; femoral head radius = 4; distance between centers = 6.

![Figure C.2](image2)

Figure C.2: Output for acetabular radius = 10; femoral head radius = 8; distance between centers = 4.
function camodel()

% Cooper 10/16/98
% Program which solves for
% “contact area” between a third of a larger circle, (acetabulum) and a full smaller circle (femoral head).

ra = input('ACETABULAR RADIUS: ') ;
rf = input('FEMORAL HEAD RADIUS: ') ;
rc = input('DISTANCE BETWEEN CENTERS: ') ;

off = 0 ; % the femoral head has gone "off" the acetabulum.

for j = 0:120, % acetabulum is taken as 120 degrees of a full circle

    thetadeg = -30 + j ;
    theta(j+1) = thetadeg*pi/180 ; % matlab loves radians
    xcf(j+1) = rc*sin(theta(j+1)) ; %coordinates of femoral head center
    ycf(j+1) = rc*cos(theta(j+1)) ;
    x = solve( 'sqrt(rf^2 - (x-xcf(j+1))^2) + ycf - sqrt(ra^2 - x^2) = 0');
    xl = min(x) ;
    xr = max(x) ;
    % (After running solve for the above equation, the answers are
    % pasted in for xl and xr, since solve output a
    % symbolic equation instead of numeric.)

    xl(j+1) = 1/2/(4*ycf(j+1)^2+4*xcf(j+1)^2)*(4*xcf(j+1)^3-4*rf^2*xcf(j+1)+4*ra^2*xcf(j+1)+4*xcf(j+1)*ycf(j+1)^2+4*ycf(j+1)^2+2*ra*2*rf+2*ra^2*xcf(j+1)+2*ycf(j+1)*2+2*rf^2*xcf(j+1)+4*ycf(j+1)^2+2*ycf(j+1)^2+4*ra^2-ycf(j+1)^2+2*rf^4-4*ycf(j+1)^6)^2*(1/2));
    xr(j+1) = 1/2/(4*ycf(j+1)^2+4*xcf(j+1)^2)*(4*xcf(j+1)^3-4*rf^2*xcf(j+1)+4*ra^2*xcf(j+1)+4*xcf(j+1)*ycf(j+1)^2+4*ycf(j+1)^2+2*ra*2*rf+2*ra^2*xcf(j+1)+2*ycf(j+1)^2+2*rf^2*xcf(j+1)+2*ycf(j+1)^2+2*ycf(j+1)^2+4*ra^2-ycf(j+1)^2+2*rf^4-4*ycf(j+1)^6)^2*(1/2));

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Figure C.3: Using the data from Table 2.: output for acetabular radius = 31.35; femoral head radius = 18.07; distance between centers = 14.
if xl(j+1) < ra*cos(pi/3), xl(j+1) = ra*cos(pi/3) ; end
if ra*cos(asin(xl(j+1)/ra)) <= 0, off = 1 ; end
if off == 1, xr(j+1) = ra ; end

hold('on')

for i = 1:30:120,
    thf = (pi/180)*(-10:45);
    xfx = rf*sin(thf);
    yfy = rf*cos(thf);
    offx(1:size(xfx,2)) = xcf(i);
    offy(1:size(yfy,2)) = ycf(i);
    plot(xfx + offx, yfy + offy)
end

x = rf*sin(pi*(0:360)/180) + rc;
y = rf*cos(pi*(0:360)/180);
plot(x,y, 'r')

x = rf*sin(pi*(0:360)/180) + rc*cos(pi/4);
y = rf*cos(pi*(0:360)/180) + rc*sin(pi/4);
plot(x,y, 'b')

x = rf*sin(pi*(0:360)/180) + rc*cos(pi/2);
y = rf*cos(pi*(0:360)/180) + rc*sin(pi/2);
plot(x,y, 'g')

thfa = pi*(-30:90)/180;
x = ra*sin(thfa);
y = ra*cos(thfa);
plot(x,y)
axis equal;

cl = ((xr(j+1) - xcf(j+1))/2)*sqrt((rf^2 - (xr(j+1) - xcf(j+1))^2)) ;
c2 = (rf^2/2)*asin((xr(j+1)-xcf(j+1))/rf) ;
c3 = (xr(j+1)/2)*(sqrt((ra^2 - xr(j+1)^2))) ;
c4 = (ra^2/2)*asin(xr(j+1)/ra) ;
c5 = ((xl(j+1) - xcf(j+1))/2)*sqrt((rf^2 - (xl(j+1) - xcf(j+1))^2)) ;
c6 = (rf^2/2)*asin((xl(j+1)-xcf(j+1))/rf) ;
c7 = (xl(j+1)/2)*(sqrt((ra^2 - xl(j+1)^2))) ;
c8 = (ra^2/2)*asin(xl(j+1)/ra) ;

carea(j+1) = cl + c2 - c3 - c4 - c5 - c6 + c7 + c8 ;
end

figure(2)
plot(abs(carea), 'r-.') ;
save camodel_data