Development and Testing of Microscale Silicon Heat Exchangers for the MIT Micro Gas-Turbine Engine

by

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B.S. Mechanical Engineering (1998)

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Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering.

at the

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Abstract

Micro-scale silicon heat exchangers were designed, modeled using conventional heat transfer and fluid mechanics correlations, fabricated in the MIT Microtechnologies Laboratory, and tested as part of the MIT micro-gas turbine engine project.

Heat transfer and pressure drop experiments were carried out to measure the performance of these heat exchangers, and to validate the predictions of the model. The tested heat exchanger was 2 centimeters by 3 centimeters by 4 mm in size, and housed 220 individual heat transfer passages (110 in each direction). The heat transfer passages were rectangular in cross-section, with lengths of 7mm, widths between 100 and 150 micrometers, and depths of about 190 micrometers.

In actual microengine application, the heat exchangers might experience temperatures as high as 1400K. In the tests conducted for this project, the temperatures were limited to 350K. Results demonstrated a heat transfer rate of about 0.5 watts. Under the test conditions, this corresponds to a heat exchanger effectiveness of about 52%, which is within the experimental margin of error of the 50.5% value predicted by the model. These results indicate that the addition of a recuperative heat exchanger to the MIT micro gas turbine engine could provide a benefit to the fuel efficiency of the engine cycle.

Thesis Supervisor: John G. Brisson
Title: Professor of Mechanical Engineering
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List of Symbols

- $q$: Heat or energy transfer
- $\dot{m}$: Mass flow rate
- $c_p$: Specific heat at constant pressure
- $C$: $\dot{m} c_p$
- $T$: Temperature
- $\varepsilon$: Effectiveness
- $P$: Pressure
- $PR$: Pressure Ratio = $P_{out}/P_{in}$
- $f$: Friction Factor
- $D_h$: Hydraulic Diameter
- $\rho$: Density
- $v$: Velocity
- $\mu$: Kinematic Viscosity
- $\mu m$: Micrometers
- $Re$: Reynold’s Number
- $A$: Area
- $w$: Width
- $H$: Height
- $t$: Channel wall thickness
- $Num$: Number of Channels
- $NTU$: Number of Transfer Units
- $\lambda$: Kroeger’s deterioration coefficient
- $h$: Convection Coefficient
- $k$: Thermal Conductivity
- $r$: Spatial variable - radius
- $\phi$: Pressure Multiplier
- $W$: Watts
- $m$: Meter
- $K$: Degree Kelvin
- $P$: Power
- $V$: Voltage
- $I$: Current
- $\beta$: Bulk radiation coefficient
- $\sigma$: Boltzmann’s constant
- $F$: View factor
- $e$: Emissivity
- $P$: Perimeter
- $x$: Spatial variable – length, distance
1.0 Introduction

The fabrication techniques used to manufacture integrated circuits have seen new application as a means for creating miniature mechanical and electromechanical devices out of silicon. These processes can achieve tolerances that are much smaller than what is currently attainable using conventional techniques. This precision has given birth to a whole new breed of mechanical and electromechanical devices which are an order of magnitude smaller than any of their antecedents.

Work is currently being done at MIT to develop a miniature gas turbine engine. The engine is fabricated entirely out of silicon using microfabrication methods. All of the engine’s 3-dimensional features, including the combustion chamber, compressor blades, and turbine blades, are etched directly into silicon. Functional microbearings have already been built and spun up to 1.5 million RPM (design speed is 2.25 million RPM), and the combustion chamber has been shown to support continuous burning of hydrogen fuel [1]. Eventually running on hydrocarbon fuels, these devices may be used as jet engines for micro air vehicles and as portable electric power sources in place of batteries.

The design, fabrication, and testing of small-scale heat exchangers is a part of the microengine project. First generation engines will be tested without these miniature heat exchangers, but their inclusion in later generations of microturbines could significantly improve the overall fuel efficiency of the engine cycle.

Although there is a growing collection of work on the subject of miniature heat exchangers, this project is characterized by several new aspects. The heat exchangers under investigation here have feature sizes of ten micrometers or less, which are considerably smaller than those found in previously published studies. Additionally, the flow temperatures are measured directly within the heat exchangers themselves, near the inlets and outlets to the actual heat transfer passages; other investigations have inferred the temperatures using less-direct techniques. Finally, this investigation is closely tied to a practical application, the MIT micro gas turbine engine. As a result, once its
performance has been evaluated, the anticipated benefit of utilizing a heat exchanger with the system can be determined quickly.

1.1 Counterflow Heat Exchangers

In essence, a heat exchanger is designed to allow the exchange of heat energy between two fluids at different temperatures. In the case of the heat exchangers being considered for the microengine project, the intent is to extract heat from one gas flow and transfer it to another gas flow. Typically, this is accomplished by running the two flows in close proximity to one another, such that heat energy can conduct through a wall separating them.

A counterflow heat exchanger is characterized by the two flows running parallel to each other, but in opposite directions. Consequently, the two flows enter the heat exchanger at opposite ends. If the product of the mass flow rate and the specific heat is the same for each flow, the flows are said to be balanced. With perfect heat transfer and zero losses, each flow in the balanced case exits the heat exchanger at the same that he other one enters at. That is, the exit temperature of each flow would correspond to the inlet temperature of the other flow. Any real heat exchanger cannot live up to the ideal, however, and so there is a figure of merit used to quantify the performance of a heat exchanger as compared to this ideal. This figure of merit is the effectiveness, which is described as the ratio of the heat that is actually transferred from one flow to the other flow, to the maximum amount of heat that could be transferred if the system were perfect.

FIGURE 1.1 – Counterflow Heat Exchanger Design [2] – Two streams are passed in close proximity to each other, running in opposite directions. Thermal energy from the hotter stream transfers to the cooler stream.
This maximum is associated with the ideal case, in which the temperature changes from one flow’s inlet temperature to the inlet temperature of the other flow:

\[
q_{\text{max}} = \dot{m}c_p(T_3 - T_1)
\]

where \( T_1 \) is the inlet temperature of the colder flow, \( T_3 \) is the inlet temperature of the hotter flow, and both the mass flow and specific heat value correspond to the whichever flow has the smaller value of the product \( \dot{m}c_p \); this product may also be written \( C_{\text{min}} \).

The effectiveness, assuming constant \( c_p \), is defined as:

\[
\varepsilon = \frac{q_{\text{actual}}}{q_{\text{max}}} = \frac{|\dot{m}c_p(T_{\text{outlet}} - T_{\text{inlet}})|_{\text{either stream}}}{C_{\text{min}}(T_3 - T_1)}
\]

where \( C_{\text{min}} \) is \((\dot{m}c_p)_{\text{hot flow}}\) or \((\dot{m}c_p)_{\text{cold flow}}\), whichever is smaller. The flow with a smaller value of \( \dot{m}c_p \) will experience the greater temperature change per unit of heat transferred than a flow with a larger value of \( \dot{m}c_p \); therefore, the flow characterized by \( C_{\text{min}} \) could reach its final temperature, equal to the inlet temperature of the other flow, prior to the end of the heat exchanger. In this situation there would be no additional heat transferred in the remaining length of the heat exchanger. Since no additional temperature change will take place, this value represents the maximum heat transfer possible.

The effectiveness, as a ratio, is a dimensionless number between 0 and 1. An effectiveness of zero corresponds to a heat exchanger which does not convey any energy from one stream to the other. An effectiveness of one corresponds to an ideal heat exchanger, in which the maximum heat transfer possible is achieved.

The other figure of merit that is useful when describing the performance of a heat exchanger is the pressure drop. When used in conjunction with other components, the pressure losses in the heat exchanger can have a dramatic impact on the overall performance of a system. Some applications, such as the microengine, are sensitive to even very small pressure drops.

These pressure drops can also be expressed as ratios of the outlet pressure to the inlet pressure.

\[
PR = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} - \Delta P}{P_{\text{in}}}
\]

This Pressure Ratio, like the effectiveness, is a dimensionless number between 0 and 1, with 1 being an ideal case with no pressure drop. Therefore, a 0.95 or 95% pressure ratio corresponds to a 0.05 or 5% pressure drop.
1.2 Practical Applications

Although the emphasis of this project has been on recuperator designs intended for use in a power cycle, heat exchangers can be used in a variety of other applications. Any system in which a fluid needs to be heated might possibly benefit from a heat exchanger, provided that there is an available source of heat somewhere else in the system. Rather than supply additional heat to raise the flow temperature, recovering excess heat from another area in the system can result in reduced energy costs. Other applications which make use of heat exchangers include space heating, air-conditioning, refrigeration, chemical processing, and automotive cooling.

As mentioned previously, the intent in the MIT Microengine project is to produce a functioning microscale silicon gas turbine engine that is contained in a 2-centimeter square package. The use of a recuperative heat exchanger is a common practice in gas turbine engine cycles. By making use of the energy in the high temperature exhaust gases, the overall fuel efficiency of the system can be improved.

Proposed uses for this engine are numerous. Among the proposals are: as a jet engine for a six-inch-long semi-autonomous reconnaissance airplane for use by the army, and as a portable electrical power source capable of producing 50 Watts by burning a hydrocarbon fuel [3].

One of the added benefits of devices this size is that they have very little mass. The mass is proportional to the size of the engine cubed, while surface area is only proportional to the size of the device squared. Therefore the smaller then engine is, the greater the ratio of surface area to mass. High values of heat transfer per unit mass can therefore be effected in miniaturized designs.

Conventional aircraft engines do not use recuperators, since their addition would significantly increase the mass of the plane. A silicon microrecuperator, on the other hand, is extremely light, and can be fabricated as part of the engine design itself. The size and weight of the added recuperator are both small, and do not preclude its inclusion in a micro air vehicle or other lightweight application.

1.3 Application to the Microengine

A schematic diagram of the gas turbine Brayton Cycle is shown in Figure 1.2. Incoming air passes through the compressor (1→2), where its pressure is increased. This high pressure air enters the combustor (2→3), at which point fuel is added and the mixture is
burned. The heat addition from the burning fuel increases the internal energy of the air; expansion of the hot air through the turbine blades (3→4) causes the shaft to rotate. Finally, the gas, now at relatively low pressure, is vented to the atmosphere (4→1). Shaft power not used by the compressor can be applied to a generator in power systems; alternatively, the high-enthalpy gas can be vented through the aft end of the engine, providing the thrust necessary in jet-engine applications.

The same ideal process can be represented on a T-s diagram, as shown in Figure 1.3. Incoming air is isentropically compressed from point 1 to point 2. Heat is added from point 2 to point 3, which represents the burning of fuel in the combustion chamber. The hot gases are then passed through a turbine to extract the power needed to run the compressor; this is represented by the isentropic expansion from point 3 to point 4. Finally, the hot exhaust gases are vented to the atmosphere, rejecting heat from point 4 to point 1. These hot exhaust gases can be used to supply more power to the turbine and operate a generator system, or they can be vectored for thrust.

The exhaust gas is vented at a temperature that is elevated relative to the environment. This higher-temperature gas contains thermal energy that is discarded by the engine, but which could be used to increase the engine efficiency. This thermal energy is reclaimed.

**FIGURE 1.2 – Schematic Diagram of an Unrecovered Gas Turbine Engine.** Inlet air is compressed (1→2), fuel is added and the resulting mixture is burned in the combustor (2→3), the hot gases are expanded through the turbine (3→4) and finally vented to the atmosphere (4→1).
FIGURE 1.3 – T-s Diagram of an Unrecoerated Brayton (Gas Turbine Engine) Cycle. Gas is inentropically compressed from point 1 to point 2, heat is added (through combustion) from point 2 to point 3, the gas is isentropically expanded through the turbine from point 3 to point 4, and finally exhausted to the atmosphere (point 4 to point 1).

by the recuperator by taking the exhaust gases and passing them in close proximity to the compressed air before it enters the combustion chamber. Energy from the exhaust stream is transferred into the compressed gas flow, raising its temperature. When this preheated compressed gas stream enters the combustion chamber, less heat needs to be added to bring it up to combustion temperature; consequently less fuel needs to be burned by the engine.

Figure 1.4 shows a diagram of a recuperated gas turbine system. The system looks similar to the unrecoerated cycle, with the addition of the heat exchanger between the compressor and the combustor (between points 2 and 3). Also depicted in the diagram is a schematic representation of the heat exchanger itself, showing the two streams passing by each other. A typical temperature profile that might be found in the system is also shown. Note that the inlet temperature of the exhaust gas stream enters the heat exchanger at a much higher temperature than the pre-combustor gas stream does; however, at the exits, the pre-combustor gas stream exiting the recuperator has a higher temperature than the exiting exhaust stream.
FIGURE 1.4 – Schematic Diagram of a Recuperated Gas Turbine Engine. The hot exhaust gas entering the recuperator heats the cooler compressed gas flow. After exiting the recuperator, the compressed gas has a higher temperature than the exhaust flow.

Figure 1.5 shows the T-s diagram of this recuperated Brayton cycle. The exhaust gas is passed through the recuperator, which extracts heat from the flow prior to venting it to the atmosphere. This cooling takes the exhaust gas from point 4 to point y, reducing the amount of waste energy discarded to the environment. Simultaneously, the removed heat energy is added to the compressed gas flow prior to entering the combustion chamber, which brings the gas from point 2 to point x. As a result, the gas temperature only needs to be raised from point x to point 3, which requires less fuel.

By transferring this heat energy, the recuperator has the potential to benefit the overall cycle fuel efficiency. However, in order to have a net positive impact, the recuperator cannot introduce a major pressure drop to the gas flows. Any pressure drop in the recuperator will reduce the combustion and turbine efficiencies, possibly harming the performance of the engine more than the heat transfer improves it.
FIGURE 1.5 – T-s Diagram of a Recuperated Brayton (Gas Turbine Engine) Cycle. After compression, the gas is heated from point 2 to point x while passing through the recuperator, and then heated from point x to point 3 in the combustor. Similarly, after expansion, the gas cools from point 4 to point y in the recuperator (by transferring its energy to the pre-combustor flow), and then exhausts to the atmosphere.

Determining whether a particular recuperator design will improve the engine's performance or not requires the use of cycle analysis data. The MIT Microengine Group has already developed models which predict the performance of the engine based on gas conditions throughout the engine; using the recuperator performance model, the gas conditions entering the combustor can be predicted, and the effects of these changes on the overall cycle can be analyzed. By comparing the relative impact of various recuperator designs, a geometry can be selected which provides the greatest benefit to the engine cycle.

Table 1.1 shows the performance of the unrecuperated microengine, and the potential for performance improvement with the addition of a recuperator [4]. While the thrust (or power) is decreased when a recuperator is added, the amount of fuel needed by the system is reduced by up to 75%.
TABLE 1.1 – Operating characteristics of the unrecuperated microengine and an ideally recuperated microengine. The addition of a recuperator decreases the overall useful output (thrust or power) of the engine, but simultaneously reduces the amount of fuel needed by the engine.

In its original conception, shown in Figure 1.6, the recuperator was located on the aft end of the microengine. The static engine structure is shaded gray in the figure, the rotating compressor and turbine spool is hatched, and the recuperator is shown shaded black. The compressed air, prior to entering the combustor, is distributed into radial passages in the recuperator. This air then flows radially towards the center of the engine, before doubling back to enter the combustor. After passing through the turbine, the hot exhaust gases enter the recuperator as the second flow, traveling radially outwards from the center of the recuperator. The exhaust gases transfer heat energy to the pre-combustor gases as

![Design Sketch for the Recuperated Microengine](image)

FIGURE 1.6 – Design Sketch for the Recuperated Microengine. The Recuperator (colored black) is attached to the aft end of the engine. Radial flow passages alternate pre-combustor compressed gas and exhaust gas flows.
the flows pass alongside each other in alternating channels. This design utilizes the fact that the compressed air flow is already annularly distributed, and can be easily delivered into a series of axisymmetric heat transfer passages. Most of the work done for this project was performed with the assumption that the final recuperator geometry would be similar to this design.

1.4 Contributions of this Thesis

This thesis makes four significant contributions to the study of microscale heat exchangers. The first is proof of principle of the feasibility of fabricating silicon recuperators with geometries like the ones considered here. Heat exchangers with feature dimensions of this scale, and composed entirely of diffusion-bonded layers of silicon, are a relatively new concept. Significant work was needed in order to demonstrate that such a recuperator could be built. The fabrication processes, challenges, and results are presented in a level of detail which could easily be adapted to similar heat exchanger geometries.

The second contribution is the development of first-order modeling tools which can be used to anticipate the performance of this type of recuperator. The microengine design is evolving rapidly, and a recuperator geometry designed for use with the current layout might not be the optimum design in the future; having a modeling tool such as this facilitates the development of recuperators to be used with future generations of the microengine. Given information about a heat exchanger's geometry, the model calculates the effectiveness and pressure drop that can be expected in the device; this information can then be used to predict the impact on the engine cycle. The models take into account losses due to axial conduction along the direction of the flows, a phenomenon which is typically negligible in macroscale systems.

The third contribution is in the development and assembly of an experimental system which can be used to evaluate a microscale heat exchanger’s effectiveness and pressure drop. Issues such as instrumentation and packaging have been addressed, and the solution that has produced demonstrable results is described. The results of heat transfer experiments are presented, which validated the modeling effort.

The fourth contribution is in the description of parallel projects that were undertaken during the course of this project. Several novel concepts that might find application in microrecuperator designs were investigated, including various techniques for incorporating low thermal conductivity materials into micro heat exchangers. These
parallel technologies provided a good deal of useful information, which in some cases was directly applicable to the microscale heat exchanger project.

1.5 Thesis Organization

This thesis is divided up into five major sections beginning with Chapter 1, the Introduction. Chapter 2 presents an in-depth discussion of the counterflow heat exchangers that were designed and fabricated, and the criteria by which they were developed. Additionally, a discussion of the analytical and numerical models that were developed to evaluate the recuperators' performance will be presented. This discussion will highlight the important details and characteristics of the device.

Chapter 3 provides an overview of the fabrication process used to produce these silicon heat exchangers. Details about the dimensions, preparations, and actual etching process are provided.

Chapter 4 describes the experimental setup that was designed and built in order to test the recuperators. There are a number of sections in this chapter, which cover the full range of important issues that were considered: these include the basic concept, experimental accuracy, experimental component performance, the control and data acquisition system, the flexibility of the system, packaging, and instrumentation.

Chapter 5 presents the experimental and computational results of the recuperator testing. Effectiveness and pressure drop data for actual recuperators will be presented, as will a comparison to the results of the computational models. Important considerations for future recuperator designs will also be highlighted. This chapter also validates the model on the basis of the experimental results.

Chapter 6 describes some of the parallel work that was undertaken during the course of this research. Work done to develop a technique for fabricating silicon dioxide structures is highlighted.

Chapter 7 presents a summary of the thesis and suggestions for future work.
2.0 Modeling and Design

This chapter describes the efforts made to develop a model that could, given a specific geometry, accurately predict or reproduce the performance of a microscale heat exchanger. Each microscale device fabricated in the clean room represents a large investment in time and materials. By analyzing a design using the models, the overall amount of labor and machine time needed to determine the final layout of a heat exchanger can be reduced. The chapter also describes some of the analyses that were used to define the feature sizes eventually used in the recuperator designs.

The model discussed here can be used as a design tool in future generations of the microengine. Since the specific details of the microengine are rapidly evolving at the present time, it is impossible to select the final design for the recuperator. At any time during the engine’s development, however, these models can be used to design a recuperator that will provide the maximum benefit to the cycle.

2.1 Modeling and Design Overview

Based on the material characteristics, the geometry of the recuperator, and the performance of the compressor, the model should accurately predict the heat transfer and pressure drop within the recuperator. With that information, the temperatures and pressures of the gas flows exiting the recuperator can be predicted; those gas flows become the combustor inlet flows. This information is then used to determine the overall combustor and turbine efficiencies, and therefore the performance of the overall microengine.

The model predictions can be used to indicate how the recuperator design might be changed to improve the overall cycle efficiency. Information from experiments may be used to further refine the model.
Kohata performed a series of analyses to determine what form the first generation of recuperators should take in order to achieve the goals of the microengine project [5]. During the course of this investigation, several orientations and configurations were considered and evaluated. Comparative models of their performances indicated which designs were functionally superior and should be pursued further.

Based on Kohata's work, the radial channel configuration recuperator, shown in Figure 2.1, was selected as the focus of this project. In the radial channel design one gas stream enters the recuperator centrally and travels radially outwards through channels with a rectangular cross section. The other flow enters the recuperator at the outer radius and travels radially inwards towards the center. Each radial channel has its flow traveling in the opposite direction of its two neighbors; viewed from the top, the channels alternate flow direction around the circumference of the design.

In actual microengine application, a radial flow recuperator such as this could be added as a single additional wafer layer to the aft end of the engine. Air exiting the compressor could be delivered into half of the passages in the recuperator, while the exiting high-temperature exhaust gases could travel along the remaining passages in the opposite direction. By having the flow channels alternate which flow is being carried, heat transfer can occur between the two flows, cooling the exhaust flow and heating the compressed air prior to entering the combustor.

The radial flow design introduces some interesting geometric considerations; specifically, the changing cross sectional flow area along the radius complicates the modeling process. In order to simplify the initial model and verify that the results were reasonable, a linear recuperator configuration was also designed. The linear design simulates all of the important characteristics of the radial design; however, the cross sectional area does not change as a function of radius. By first fabricating and experimentally evaluating the simpler linear design, the validity of the model could be ascertained. The absence of geometric complications between experiment and the model simplifies comparison of the two. Understanding any deviations from the expected results would therefore be simplified as well.

The overall design of the recuperator was constrained by four objectives; maximizing the heat transfer, minimizing the pressure drop, remaining within the limitations of fabrication, and limiting the overall size of the device to that of the current microengine design. The latter constraint requires that the footprint of the recuperator fit inside that of the engine, simplifying any packaging and mounting efforts. Typically, improving the heat transfer increases the pressure drop, and improving the pressure drop increases the heat transfer. Furthermore, in each design decision, there are limitations as to whether or
not a feature with the desired characteristics can be fabricated using the available equipment and techniques.

2.2 Linear Design Modeling

The pressure drop was modeled based on the straightforward pipe-flow equation [6]:

\[ \Delta P = f \frac{L}{D_h} \frac{1}{2} \rho \left( \frac{V}{h} \right)^2 \]

where \( f \) is a friction factor, and the hydraulic diameter corresponds to the channel being considered. For the mass flow rates and the geometric sizes being considered, the Reynolds number

\[ \text{Re}_{D_h} = \frac{\nu D_h \rho}{\mu} \]

is less than 2500, indicating that the flow is laminar at all points. Substituting the Darcy laminar internal flow correlation for the friction factor

\[ f = \frac{64}{\text{Re}_{D_h}} \]

substituting the velocity with

\[ \nu = \frac{\dot{m}}{A_c} = \frac{\dot{m}}{\rho h w} \]

and finally by substituting the hydraulic diameter with

\[ D_h = \frac{4 w H}{2(w + H)} \]

where \( w \) refers to the width of the passage and \( H \) its height, the pressure drop is equal to

\[ \Delta P = \frac{8 \mu L \dot{m} (w + H)^2}{\rho H^3 w^3} \]
Figure 2.2 shows the local geometry of the linear recuperator heat transfer passages. All of the variables depicted correspond to those used in the equations. Note that the mass flow listed in Equations 2.4 and 2.6 is the mass flow per channel – this corresponds to the total mass flow divided by the total number of channels in each direction.

Since the recuperator must eventually operate as part of the microengine, the fluid (gas) properties are predetermined; that is, the density, and viscosity are all constrained by the compressor and turbine exit flow properties. The pressure drop in the recuperator can be tailored through the selection of the geometry; it can be reduced by decreasing the channel length, or by increasing the height or width of the channels. The length of the channels is limited by the recuperator’s radius, and the height is limited by the fabrication capabilities. The width of the passages can be varied by selecting the number of channels in each direction; increasing the number of channels decreases the width of each channel, which decreases the mass flow per channel. The average passage width, as a function of the number of channels, obeys the equation:

\[
\text{Channel Width} = w = \frac{w_{\text{total recup}} - 2t_{\text{outside wall}} - (2N-1)t_w}{2N}.
\]  

\(2.7\)

FIGURE 2.2 – Cutaway Diagram of Recuperator Passages. \(t_w\) is the wall thickness, \(t_b\) is the thickness of the top and bottom coverplates, \(h\) is the channel height, \(l\) is the channel length, \(w_{lp}\) is the width of the low-pressure gas flow channels, and \(w_{hp}\) is the width of the high-pressure gas flow channels.
This indicates that the average width of each channel is equal to the total width of the recuperator \( w_t \), minus the thickness of the outside wall \( t_{\text{outside wall}} \), divided by the number of channels \( N \) in each direction. This equation holds true only if all channel widths are to be the same size. In the case of the tested recuperators, however, the pressure drop in each channel was to be identical. Therefore, the low-pressure channels needed to be wider than the high pressure ones, with the factor of difference between them being equal to the ratio of their pressures. Equation 2.7 is multiplied by

\[
\frac{P_{\text{high}}}{P_{\text{low}}} \left( \frac{P_{\text{high}}}{P_{\text{low}}} \right)^{2.7} \quad 2.8a
\]

to obtain the low pressure channel width, and by

\[
1 + \frac{P_{\text{high}}}{P_{\text{low}}} \quad 2.8b
\]

to obtain the high pressure channel width. Therefore, the low pressure channel width, \( \xi \), becomes equal to

\[
\xi = \frac{P_{\text{high}}}{P_{\text{low}}} \left( \frac{P_{\text{high}}}{P_{\text{low}}} \right)^{2} \frac{w_{\text{total recup}} - 2t_{\text{outside wall}} - (2N - 1)t_w}{2N} \quad 2.9a
\]

and the high pressure channel width, \( \zeta \), becomes

\[
\zeta = \frac{1}{1 + \left( \frac{P_{\text{high}}}{P_{\text{low}}} \right)^{2}} \frac{w_{\text{total recup}} - 2t_{\text{outside wall}} - (2N - 1)t_w}{2N} \quad 2.9b
\]

To model the heat transfer, standard heat exchanger correlations were used as a starting point. However, an important consideration in the development of this model is the fact that silicon is a material with a relatively high thermal conductivity. This fact, coupled with the small length scales associated with the recuperator, can produce an effect typically negligible in macroscale heat exchangers; that is, the ineffectiveness due to axial conduction. In this device, heat being transferred between the two streams can easily conduct along the walls in the direction of flow. Due to the limits on fabrication, the cross-sectional area of silicon structure compared to that of flow passage is quite large. This communication blurs the overall temperature profile of the wall along its length towards the average value, which produces less overall transfer of heat between the flows.
Kroeger performed a detailed analytical investigation of this effect, and presented equations that account for the ineffectiveness due to this phenomenon [7]. The results were originally intended for application to extremely high-effectiveness systems designs such as cryogenic heat exchangers, in which even small sources of ineffectiveness are important. They can also be applied to structures such as silicon heat exchangers, where axial conduction is important even in designs of moderate effectiveness. The resulting device ineffectiveness with balanced flow ($C_{\min} = C_{\max}$), given here without derivation, is

$$i = \frac{1}{1 + NTU \frac{1 + \lambda \sqrt{\lambda NTU / (1 + \lambda NTU)}}{1 + \lambda NTU}}$$

2.10

where NTU refers to the number of transfer units and

$$\lambda = \frac{k_{Si} A_c}{l (mc_p)_{min}}$$

2.11

The cross sectional area, $A_c$, refers to the cross sectional area of actual heat exchanger structure material (in this case silicon) through which axial conduction can occur. From the equation for heat conduction

$$q = \frac{k A_c}{L} \Delta T,$$

2.12

it is clear that the axial conduction in the silicon is much larger than that through the air flow; the thermal conductivity of the gas is about three orders of magnitude less than that of the silicon, and the cross sectional flow area and the cross-sectional area of silicon available for axial conduction are comparable in magnitude. Clearly, the axial conduction in the silicon is the dominant contributor to the overall recuperator effectiveness, and the axial conduction through the air can be neglected.

For modeling purposes, the recuperator was assumed to have balanced flow. One reason for this assumption is that the same gas stream is used in both flows of the recuperator. Assuming that there are no leaks in the system between the compressor outlet flow and the final exhaust to the atmosphere, the mass flow in each recuperator stream will be the same. The specific heat only changes by at most 10% in the temperature ranges considered. This deviation was assumed to be low enough to neglect the change altogether. Even more significantly for the purposes of modeling, balanced flow is the worst case scenario for axial conduction losses, as explained by Kroeger. This is due to the fact that with balanced flow designs, the temperature difference between the fluids is further decreased. This decreased temperature difference reduces the amount of heat transferred between the two streams. Imbalanced flow heat exchangers suffer from the same effects, but to a lesser degree since the decrease in the temperature difference is proportionally less. Figure 2.3 shows this schematically.
FIGURE 2.3 – A schematic representation indicating the deterioration of heat exchanger performance as a result of axial conduction through the walls [7]. (a) A balanced flow heat exchanger, wherein the decrease in the temperature difference is large compared to the difference itself, and the heat transfer is reduced significantly. (b) An imbalanced flow heat exchanger; the decrease in the temperature difference is smaller compared to the temperature difference itself.

Note that this ineffectiveness equation (Equation 2.10) is simply a modified version of the standard equation

\[ i = 1 - \varepsilon = \frac{1}{1 + NTU}, \]

with the NTU term multiplied by what Kroeger calls a “deterioration” coefficient that is in itself a function of the NTUs. Clearly, in cases in which there is no axial heat conduction (\(\lambda_0\) goes to zero), Equation 2.13 simplifies to Equation 2.10. Finally, the NTUs is calculated from

\[ NTU = \frac{UA}{C_{\text{min}}} \]

and

\[ UA = \frac{1}{\left( \frac{1}{h_1 A_{\text{wall}}} + \frac{t_{\text{Si wall}}}{k_{\text{Si wall}}} + \frac{1}{h_2 A_{\text{wall}}} \right)}. \]

The wall areas described here refer to the area of the channel wall separating the two flows, or the channel depth times the length of the channel. The convection coefficients are estimated with the equation

\[ h = \frac{Nu k}{D_n}. \]
Since the flow is laminar (see Equation 2.2), a Nusselt number of 4.36 is used, corresponding to the uniform surface heat flux in laminar flow. A balanced flow recuperator will have a constant temperature defect between the two streams along its entire length, and the fluid properties are not changing drastically within the temperature range being considered. Therefore a constant heat transfer coefficient is a reasonable assumption.

2.3 Radial Design Modeling

For the radial flow design, the models need to account for the fact that the cross sectional area changes as a function of radius. The equations used for the linear system were adapted in order to reflect this. All radius-dependent dimensions were written as functions of the radius, and the equations were integrated over the length of the channels. Therefore, the pressure drop was calculated by integrating Equation 2.6 along the length of the channel.

\[ \Delta P = \frac{8 \mu m (w(r) + h)^2}{\rho h^3 w(r)^3} \]  
\[ 2.17 \]

The passage width function \( w(r) \) was derived from the total area available for gas flow:

\[ w(r) = \phi \left( \frac{2 \pi r - (2N - 1)t_w}{2N} \right) \]  
\[ 2.18 \]

where

\[ \phi = \frac{P_{\text{high}}}{P_{\text{low}}} \]  
\[ 2.19a \]

for the low pressure channels and

\[ \phi = \frac{1}{1 + \left( \frac{P_{\text{high}}}{P_{\text{low}}} \right)} \]  
\[ 2.19b \]

for the high pressure channels. The heat transfer equations were treated similarly to account for the radial variations in the circular geometry.

2.4 Correlation Validation

One vital assumption that is being made in these models is that macroscale correlations can be applied to these microscale gas flows and geometries. Demonstrating that the gas
flows is not behaving as a rarified gas would validate this assumption. Therefore the gas flows in question were examined to see if any Knudsen effects would be present.

Knudsen Effects occur when the relevant geometric feature dimensions being considered are of the same scale as the mean free path of the gas flow molecules [8]. If this is the case, the conventional continuum heat transfer and fluid mechanics models no longer apply. In the case of these recuperators, the question is whether or not the mean free path of the nitrogen gas is comparable in length to the passage widths or depths.

The mean free path of the gas molecules was calculated using the formula

\[ \lambda = 1.26 \frac{\mu}{\rho \sqrt{RT}} \]  

In the worst case scenario, characterized by low (1 atm) pressure and high (1200K) temperatures, the mean free path was on the order of 0.5 μm. The smallest relevant dimension in these recuperators was a passage width of 45 μm, which is almost 100 times larger than the mean free path. Based on this result, it is reasonable to assume that the gas flows are in the continuum limit.

2.5 Model Overview

Based on the equations used to model the recuperator, it is clear that increasing the depth of the channels benefits both the pressure drop (by increasing the cross sectional flow area, the pressure drop is decreased) and the heat transfer (a higher wall provides more surface area for heat transfer). Increasing the passage width, on the other hand, improves the pressure drop, but reduces the overall heat transfer (with wider passages, less channels fit in the recuperator footprint, reducing the number of channel walls, which reduces the total amount of heat transfer area). As mentioned before, structural considerations had to be investigated to ensure that the design would survive during operation. Furthermore, there are also limitations on what can be successfully fabricated; etching an array of high thin passage walls becomes increasingly difficult as the walls become higher and more closely packed.

Figure 2.2 clearly depicts each of the dimensions in the recuperator geometry that can be tailored to improve the overall device performance.

On the basis of the models, the effects of each geometric feature on device performance can be ascertained and plotted as trends. An initial baseline design was selected, and its performance was predicted. A series of design graphs were then constructed, each of which displayed the effect that the changing of one feature dimension had on the overall
recuperator performance. The baseline design characteristics are shown in Table 2.1; the labels in parentheses refer to the dimensions shown in Figure 2.2. All changes are referenced to this baseline design.

The mass flow rate used in the baseline recuperator design is half that of the microengine (see Table 1.1). Initial calculations indicated that the pressure drop in a single-layer high-effectiveness recuperator design would always be prohibitively high, unless the channels could be etched to a depth in excess of 600 micrometers. This goal is, at present, unattainable due to fabrication constraints. Therefore, at least two layers of recuperator channels would be needed in actual microengine application. Additional layers would each function identically to the first. As a result, the mass flow per recuperator level would be half of the total, which is represented in the operating conditions shown here.

Figure 2.4 shows the effects of changing the passage height on the effectiveness and pressure ratio of the linear recuperator design. This graph was produced by changing the passage height parameter in the model for the linear recuperator. The resulting effectiveness and pressure ratio values were then plotted as a function of the changing passage height. Clearly, maximizing the passage depth is crucial to the performance of the device. With increasing channel depth, both the pressure ratio and the effectiveness improve. Also shown on the graph is a vertical line corresponding to the current fabrication limit, and a horizontal line corresponding to the minimum pressure ratio

<table>
<thead>
<tr>
<th>Mass flow rate</th>
<th>0.18 g/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow 1 Inlet Temp.</td>
<td>600 K</td>
</tr>
<tr>
<td>Flow 2 Inlet Temp.</td>
<td>900 K</td>
</tr>
<tr>
<td>Flow 1 Inlet Press.</td>
<td>1.8 atm</td>
</tr>
<tr>
<td>Flow 2 Inlet Press.</td>
<td>1.2 atm</td>
</tr>
<tr>
<td>Channel Wall Thick.</td>
<td>10 µm</td>
</tr>
<tr>
<td>Top/Bottom Thick.</td>
<td>100 µm</td>
</tr>
<tr>
<td>Channel Depth</td>
<td>500 µm</td>
</tr>
<tr>
<td>Small Passage Width</td>
<td>60 µm</td>
</tr>
<tr>
<td>Number of Passages</td>
<td>187</td>
</tr>
</tbody>
</table>

TABLE 2.1 – The baseline conditions used for comparing recuperator features. These operating conditions and the geometric feature dimensions were selected as being achievable. The labels in parentheses refer to the dimensions shown in Figure 2.2. The pressure drop and effectiveness resulting from a recuperator with these characteristics were then used as the basis of comparison for potential changes to the design.
needed to benefit the microengine cycle. Since the design space is currently beyond the
capabilities of fabrication, the dual-layered recuperator design mentioned above is
needed. Although it adds another conduction path between the flow inlets and outlets, it
effectively doubles the available channel depth. Also shown is a graph of the
effectiveness and pressure ratio as a function of the number of channels in the recuperator
design (see Figure 2.5). This graph was also generated using the computer model to
arrive at pressure ratios and effectiveness ratings for each geometry.

FIGURE 2.4 – A graph of the recuperator effectiveness and pressure ratio versus
the depth of the heat transfer passages. The horizontal line is the minimum
pressure ratio needed to benefit the engine cycle, and the vertical line is the current
limit of fabrication.

FIGURE 2.5 – A graph of the recuperator effectiveness and pressure ratio versus
The axial conduction through the silicon was a large contributor to the ineffectiveness of these designs, and therefore the focus of a great deal of effort. This can be seen in graphs of the effectiveness versus the thickness of the top and bottom coverplates to the heat transfer passages. These coverplates are the greatest contributor to axial conduction, since they represent the largest fraction of the silicon cross-sectional area along the direction of flow. The extent of this effect can be seen in Figure 2.6.

A graph showing the effect of \( t_w \) is not included. Although making the passage walls thinner improves the heat conduction from one stream to the other, and reduces the cross sectional area for axial conduction in the direction of flows, these effects are small. The thermal resistance through the walls is already much smaller than the convective resistance in the gas streams, so thinning the walls further doesn’t significantly improve the heat transfer. Furthermore, the effect of reducing the cross sectional area is negligible, since the coverplates are already so large by comparison.

![Graph showing Pressure Ratio, Effectiveness vs. Coverplate Thickness \( t_b \)](image)

**FIGURE 2.6** – A graph of the effectiveness and pressure ratio as a function of the thickness of the top and bottom coverplates to the heat transfer passages.

### 2.6 Pressure Considerations

In addition to optimizing the performance of the engine, physical constraints imposed by the operating conditions had to be observed. For example, while the wall thickness had to be as thin as possible in order to maximize the heat transfer across the wall and to minimize the cross sectional area for heat transfer in the direction of flow, they also had to be large enough to sustain the pressure differences across them without rupturing. Furthermore, the limitations of fabrication had to be taken into account.
A first-order structural analysis indicated that a 10-micron thick wall 500 microns tall could easily contain the high pressure gas without failing. This analysis considered the wall as a plate with fixed end condition at the top and bottom, and an evenly distributed pressure load acting along the surface. Using standard beam calculations [9], the maximum force on the beam is equal to

\[ V_{\text{max}} = \frac{1}{2} w L , \]  \hspace{1cm} (2.21)

where \( L \) is the length of the beam (or, in this case, the height of the wall) and \( w \) is the force per unit length. The corresponding maximum moment experienced is equal to

\[ M_{\text{max}} = \frac{w L^2}{8} , \]  \hspace{1cm} (2.22)

and the corresponding maximum stress is

\[ \sigma_{\text{max}} = \frac{|M|_{\text{max}} c}{I} . \]  \hspace{1cm} (2.23)

Given the anticipated pressure conditions to be found in the recuperator (2-4 atmospheres on the high pressure side, ~1 atmosphere on the low pressure side), the stress in the wall is about 100 kPa. This is less than 0.5% of the maximum allowable stress, which was calculated as

\[ \sigma_{\text{all}} = \frac{1}{2} S_y = 27.5 \text{ MPa} . \]  \hspace{1cm} (2.24)

In summary, the pressure differential across the 10 \( \mu \text{m} \) wall produces stresses well below the maximum allowable stresses. Although the maximum allowable stress decreases with increasing temperature, the value listed above was calculated at the average of the two stream inlet temperatures.

Under pressurized conditions, the walls can be expected to bow slightly due to the pressure differential. At a pressure differential of 1 atmosphere across the wall, the maximum calculated deflection was about 1.5 micrometers. The minimum passage width found in any of the recuperators is 40 micrometers; the change in passage width due to bowing was therefore considered negligible.

### 2.7 Structural Considerations

In selecting the feature dimensions, the threat of buckling in the walls of the microengine-compatible radial design recuperator also had to be studied. In microengine applications, the exhaust gases will heat up the center region of the recuperator design, while the outside edges are kept cool by the ambient environment and the pre-combustor gas flow. Any thermal expansion of the thin engine walls will be constrained by the outside edges, inducing buckling forces in the walls.
Several analyses were performed on the heat transfer walls to investigate this concern. The first analysis considered the walls as thin (10 micron) rectangular structures that were free standing and derived no benefit from being affixed to the top and bottom coverplates. This model is shown in Figure 2.7a. The temperature conditions that were applied in this analysis were consistent with a high-effectiveness device performance, corresponding to a $\Delta T$ of 600-800 K along the length of the walls. The stress seen by the beam would then be

$$\sigma = \frac{P}{A} = -E \alpha \Delta T \sim 351 \text{ MPa.}$$

2.25

where $\alpha$ is the coefficient of thermal expansion. Under these conditions, this value is over 1000 times larger than the critical buckling stress, determined by

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \sim 250 \text{ kPa.}$$

2.26

However, the system constraints used here are not representative of the actual recuperator design. In an effort to more accurately model the actual effective geometry, the heat transfer channels were then modeled as thin-walled hollow boxes (see Figure 2.7b).

Under these conditions, the critical buckling force is slightly more than the applied stress. An even more accurate axisymmetric model is shown in Figure 2.7c, with a constrained top and bottom to simulate the support provided by the rest of the engine. In this model, the applied stresses were estimated at about half of the critical buckling stresses, indicating that the recuperator would survive the thermal variations along its radius.

Figure 2.7 - (a) initial buckling analysis, considering walls independently (b) secondary buckling analysis, considering walls as thin hollow boxes (c) final buckling analysis, considering walls as linked I-beam structures.
As an added level of insurance, the experimental devices were designed so that the hot gases could be located along the outer radii of the recuperators, and the cooler gases in the central regions; this arrangement allows for the free expansion of the recuperator, thus eliminating buckling concerns altogether.

In retrospect, it is doubtful that the recuperators will ever see a thermal variation along their length as large as this model anticipated. The large top and bottom plates will conduct heat easily, and blur the profile of the heat transfer walls. Furthermore, in microengine application, the engine itself will act as a structural support. Nevertheless, it was worthwhile performing the analysis in order to demonstrate that the test designs would not fail due to buckling during experimentation.

2.8 Cycle Implications

The models indicated that it was possible to have designs characterized by extremely high heat transfer rates, or extremely low pressure drops, but not both in the size and structural constraints being considered. The final designs have combinations of pressure drops and heat transfer rates that optimize the performance of the engine cycle.

The effects of adding a recuperator with a given performance could be predicted using cycle analysis data provided by other members of the MIT microengine team [10]. As shown in Figure 1.4, the gas properties at the compressor and turbine exits correspond to the conditions at the recuperator inlets. Given the recuperator’s effectiveness and pressure drop, the recuperator’s outlet conditions can be predicted. The outlet conditions correspond to the combustor inlet and exhaust flow conditions. This data is then used to evaluate the combustor efficiency and the thrust or power that the engine produces. These new combustor inlet and exhaust properties may change the compressor and turbine exit conditions, so an iterative process must be used. The result is an understanding of the effect of adding a recuperator with a given performance to the engine.

Figure 2.8 shows this information in graphical format. The x-axis represents the power that can be extracted from the engine shaft, and the y-axis represents the power specific fuel consumption. Plotted directly on the graph is a map of recuperator performance; the numbers on the left side of the map are recuperator effectiveness, and those along the top are the pressure ratio (defined as the ratio of the outlet pressure to the inlet pressure of either of the recuperator gas streams). The point labeled A corresponds to an
unrecuperated microengine – the effectiveness is nearly zero (no heat transfer from the
exhaust gases to the pre-combustor gas flow), and no pressure drop associated with that
heat transfer (pressure ratio equals 1.0).

Any combination of effectiveness and pressure ratio that lies below the horizontal line
passing through point A improves the efficiency of the engine. In each case, the total
power output decreases, but more importantly the power specific fuel consumption
decreases. That is to say, the engine produces less power, but the fuel it is expending per
unit of power goes down.

Figure 2.9 gives the same information with respect to the thrust that the engine could
produce.

For example, assume that an unrecuperated engine produces 13.8 watts of power (this
 corresponds to the anticipated performance of the current engine as designed). This
engine is burning 1.2 grams of fuel per hour to provide this output. Now consider the
addition of a recuperator that has an effectiveness of 0.7 and a pressure ratio of 0.98. One
of these recuperated engines produces 11.3 watts of power, but for only half of the fuel,
0.6 grams per hour (see Figure 2.8). By using two recuperated engines, 22.6 watts of
power can be produced for the same amount of fuel required by the unrecuperated engine
to produce half as much thrust.
### 2.9 Modeling Results and Design

The proposed recuperator designs had to function in an experimental setup. That is, it had to be designed with flow distribution and collection regions for each of the two flows. In actual microengine application, these functions can be performed in the engine itself, whereas in the experimental device all of the plumbing has to be self-contained in the recuperator package. Since silicon etching is limited to two-dimensional features "extruded" into the wafer, minimizing the complexity of the system was extremely important.

The resulting recuperator design, shown in Figure 2.10, consists of three individual layers which are bonded together; the top cover level (wafer #1) through which the gases enter and exit the device, a second layer (wafer #2) which seals the top of the heat transfer passages and defines the flow distribution and collection regions for one of the flows, and a third level (wafer #3) which contains the actual heat transfer walls, the bottom of the heat transfer passages, and the flow distribution and collection regions for the other flow. Figure 2.10 also shows an example gas flow path in each direction.
The top layer (wafer #1) contains six holes that are etched completely through the 450-micron thick wafer, into which Kovar tubes will be sealed to convey the gases to and from the recuperator. The high-pressure flow enters through a single hole, passes through the heat transfer channels, and exits through the other single hole. The low pressure flow enters through the pair of holes opposite the high-pressure flow entrance, passes through the heat transfer channels, and exits at the last pair of holes.

There are two holes at the low-pressure inlet and outlet in order to minimize the pressure drop in the experimental setup. The low-pressure gas flow is also the one that enters the recuperator at a higher temperature (simulating the gas turbine exhaust flow), and is consequently at a significantly lower density that the high-pressure flow. Since the density is inversely proportional to the flow velocity, the $\frac{1}{2}\rho v^2$ coefficient in the pressure drop increases as the density decreases. To ensure that the pressure drops in the Kovar tubes leading to and from the recuperator were not excessively large, the flow area for this gas stream was doubled through the use of an additional inlet and outlet tube.

The top wafer level also has a number of long thin etches that divide it into 4 rectangles, each housing one inlet or outlet flow set. These etches provide thermal isolation, by eliminating any thermal communication between the flow distribution and collection plenums. By maximizing the thermal isolation in all of the areas outside of the actual heat transfer passages, the measured effectiveness more accurately represents the performance of the recuperator designs, and mitigates the effects of headering on the measurements.

The second wafer level (wafer #2) contains four separate plenae. The two innermost plenae consist of large open regions, and then a series of small thin holes that pass through the wafer to the third layer. Each of these holes empties into one end of the long thin box passages whose walls are the heat transfer surfaces. The two outermost regions are open to the third layer, and allow the gas flow to distribute around the outsides of the box passages. The bottom surface of the second wafer level is bonded to the top of the heat transfer passages, sealing them from each other.

The third wafer level (wafer #3) consists of two flow distribution and collection plenae, and the actual heat transfer channels. A close-up of Wafer #3 is shown in Figure 2.11. The two plenae are located at the outer edges of the recuperator die, and allow the gas flow to pass around the heat transfer boxes. The entire central region of the die is made up of a series of long thin deep box structures, inside which the other flow travels.
FIGURE 2.10 – Drawing of the Three-Layer Recuperator Design. The arrowed lines represent two typical gas flow paths entering the system and traveling in opposite directions through the heat transfer channels. Wafer #1 is a cover layer through which the gases flow. Wafer #2 distributes gas flow into and collects gas flow from the heat transfer passages, and seals the top of the actual heat transfer passages. Wafer #3 contains the actual heat transfer passages, and the flow distribution/collection regions for the second gas flow.

Using the model and cycle analysis data, specific final dimensions were selected for both the linear and radial design heat exchangers. Since the four-inch wafers used to fabricate the recuperators can each accommodate multiple recuperator dies, several geometric variations could be produced simultaneously. By using recuperators with different geometries, discrepancies between the experiment and the model can be more easily understood and corrected.

The dimensions that were finally chosen for the four linear designs are shown in Table 2.2. The selected dimensions for the three radial designs are given in Table 2.3. Note that the effectiveness and pressure ratios listed in Table 2.2 and 2.3 were evaluated using the same operating conditions as those given in Table 2.1.
One flow travels inside the box structures
One flow travels outside of the box structures

FIGURE 2.11 – Close-Up View of the Heat Transfer Passages, showing the box structure that accomplishes the necessary flow separation. One gas flow enters into one end of a long thin box structure from above, and exits upward at the other end of the box. The other flow travels in the other direction along the outside of the boxes, beginning at one end and ending at the other. Heat transfer occurs through the thin walls of the boxes. The two arrows shown represent typical gas flow paths.

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Table 2.2 – Design Specifications for the linear recuperator designs produced on each wafer. Each complete wafer produced provides 2 dies each of recuperators #1, #2, #3, and one of #4.
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Table 2.3 – Design Specifications for the radial recuperator designs produced on each wafer. Each complete wafer produced provides 2 dies each of recuperators #1, 2, and 3.

As shown in Table 2.3, it is estimated that a silicon recuperator with an effectiveness of 0.55 and a pressure drop of 3% can be obtained using design number 2 on the radial wafer. Referring to the cycle analysis graphs, this recuperator performance would provide an overall improvement in the engine cycle; a 33% reduction in the power specific fuel consumption, or a 15% reduction in the thrust specific fuel consumption.
3.0 Recuperator Fabrication

Demonstrating that recuperators with the proposed geometries could be successfully fabricated was an important aspect of this thesis. In addition to providing proof of concept, fabricated devices could be experimentally tested to validate the model. In this section the fabrication process is discussed, along with the challenges encountered while making these devices.

3.1 Fabrication Process

The silicon recuperators were fabricated using techniques derived for the integrated circuit industry. Although it is not an ideal material choice for many reasons, there is already a great deal of fabrication experience for working with silicon at this scale. It is for this familiarity that silicon is used in the fabrication of these devices.

One of the traditional limitations to fabricating mechanical structures in silicon is that geometries are typically restricted to extruded two-dimensional shapes. Silicon fabrication involves patterning the wafer surface with a desired shape and etching straight into the wafer around these shapes. The recuperator, like other devices in the microengine project, makes use of multiple wafer levels. Although each wafer contains only two-dimensional extruded shapes, three-dimensional structures can be constructed by stacking and bonding multiple wafers to each other.

The initial step in producing the recuperator is selecting an appropriate geometry that fulfills the device requirements, yet can still be fabricated. Specifically, the geometry needed to provide high-effectiveness heat transfer with a small pressure drop, while maintaining feature sizes that could be successfully etched. This process of selecting the geometry has already been described in Chapter 2. Since the experimental designs are tested independent of the microengine, all of the necessary plumbing and headering had to be designed into the silicon device itself. In actual microengine application, flow channels in the engine proper can be used to distribute the flow appropriately.
Once the design was selected, the necessary fabrication steps were defined. These were the individual steps that are followed in the clean room to ultimately produce the intended design. Arturo Ayon and Xin Zhang at the MIT Microtechnologies Laboratory (MTL) were the personnel responsible for actually producing the finished devices. Extensive dialogue with each of them helped to formulate and refine the final recuperator design. The fabrication steps decided on for the recuperator are depicted in Figure 3.1.

Although there are only three wafer levels to the recuperator (see Figure 2.10), fabrication of the device requires the use of five masks. Only one mask is needed for the first level, which has the inlet and outlet plumbing holes. The second and third levels each require two masks.

In the case of the second level, the first mask defines the large flow distribution and collection regions. The second mask defines a series of small holes which actually deliver or receive the flow from each of the heat transfer box passages in the third level.

In the third wafer level, one of the masks defined only the narrow heat transfer passages. The second mask etches the heat transfer passages and the large flow distribution/collection regions at either end. Both of these masks are needed in order to produce smooth wafer features of uniform depth, as described later in this chapter.

Several different recuperators of the same basic design were fabricated, each with different passage width dimensions. This variation could be useful in determining the effects of feature sizes on the overall recuperator performance. Rather than produce a separate set of masks for each recuperator variation, there were several different designs on each mask. Minimizing the number of masks reduced the cost outlay necessary to produce the designs.

The linear recuperator masks house 7 recuperator dies. Among these dies are four different geometric variations. There are 3 different radial variations on the six dies on each radial recuperator mask, two of each. Each design has a different number of heat transfer passages, and therefore different passage widths. This range of geometries provides a broad base of variables that can provide information about the effects of feature geometries on the performance of the heat exchanger.

After the geometries were selected, CAD drawings of each mask had to be made. These drawings could then be converted into .DXF files, and outsourced to a company that specializes in making photolithographic masks. The company used for the devices in this thesis was Advance Reproductions, in Andover, MA. Once the masks were produced they were given to Arturo Ayon and Xin Zhang to begin fabrication of the recuperators.
Figure 3.1 – The fabrication steps for the three wafer levels of the recuperator
The actual fabrication of the top wafer, wafer #1, is straightforward. The single mask is used to etch through the entire thickness of the silicon. The features are all relatively large and etched uniformly. No complications arose in this process.

Fabrication of wafer #2 was more problematic. In a typical silicon-based device, relatively small areas are etched away, leaving a large percentage of the silicon substrate remaining. The second level of the recuperator, on the other hand, seeks to do just the opposite, by removing most of the area on the wafer.

Not only is the idea to etch large surface areas away to leave only thin walls standing, but also to etch as deeply as possible. This process is depicted in Figure 3.2. The deeper the regions are etched (dimension h), the thinner the bottom silicon substrate becomes (dimension \( t_b \)). Since the bottom of the second and third wafers cover the heat transfer passages, etching the features as deeply as possible minimizes the cross-sectional area available for axial heat conduction.

**FIGURE 3.2** – Diagram depicting the deep etches and the coverplates. The dotted lines define the unetched wafer 3 surface. The deeper the flow channels are etched (h) into the wafer, the thinner the remaining silicon (\( t_b \)) beneath the channels.
As a result, the first few level two wafers that were etched cracked prior to bonding. The sharp corners of the flow distribution regions acted as stress concentration points. The combination of 90° corners and the thin remaining silicon substrate resulted in a design that was hard to successfully produce. Figure 3.3 depicts a level 2 wafer that had become contaminated during fabrication. The sharp corners and large etch areas can be clearly seen on the wafer. Once a wafer cracked, processing on that wafer was halted. Typically, cracking occurred when the wafer was nearly complete, as the remaining silicon became thinner. At this point, most of the necessary time and effort had already been invested in the wafer.

In order to circumvent this problem, a second version of the level two wafer mask was designed and produced. This new version reduced the surface area that would be etched away while still retaining the critical features of the level. Additionally, corners from different die on the wafer were offset so that they were no longer collinear. Wafers made using this mask demonstrated a much higher likelihood of surviving the etching process, so further alterations to the mask were considered unnecessary. In the event that the wafers still suffered from frequent breakage, there were plans to design radial fillets into the corners on a new mask, but this plan was never implemented.

As can be seen in Figure 3.3, not all of the features would finish etching at the same time. Although some of the flow distribution plenum regions are etched completely though (A), other, such as (B) and (C), are not completed yet. In cases like this, etchant could pass through holes like (A) and begin to pit the wafer’s backside. In order to prevent this, most wafers had a protective layer of oxide deposited on the rear surface so that uneven etch rates would not jeopardize the smooth surfaces needed for bonding.

The third wafer level (wafer #3) contains the actual heat transfer passages. The heat transfer passages are defined by rows of long, thin, deep silicon walls with narrow spacing, and the rest of the wafer is characterized by large areas being etched uniformly. This disparity introduced some challenges in the production of this level. Due to the closely-packed nature of the silicon walls, the etching process is greatly retarded in this region; the chemical etchant diffuses into and out of the small gaps slowly. If all of the features were etched simultaneously, the large regions would etch at a much higher rate, and actually etch completely through the wafer long before the heat transfer reached the desired depth. Figure 3.4 shows the two-mask process that was used to address this concern. One mask is designed to only expose the heat-transfer passage wall features, so they can be preferentially etched. The second mask exposes the entire recuperator level,
FIGURE 3.3 – A contaminated level-2 wafer. Clearly shown are the multiple corners which act as stress-concentration points. These corners resulted in a number of cracked wafers. The picture also demonstrates the challenge associated with the varying etch rate across the wafer; the area marked (a) corresponds to a flow distribution plenum which is completely etched through. Both (b) and (c) should look the same as (a), but they have not yet etched completely through. Further etching can degrade the features in areas that are already completed.

FIGURE 3.4 – A schematic diagram depicting the two-mask process used to etch the heat transfer passages. (a) The unetched wafer. (b) Initial etching of only the heat transfer passages using the first mask. (c) Etching of both heat transfer passages (which etch slowly) and flow distribution regions (which etch quickly) simultaneously using the second mask. (d) The finished wafer, with the heat transfer passages and the flow distribution regions etched to the same depth.
consisting of both the heat transfer level and the flow distribution regions. Ideally, the etching of the larger regions will catch up to the heat transfer passages at the desired depth. In this case, there will be a smooth transition from the flow distribution regions into the heat transfer channels at the designed depth. This is basically a matter of timing the two etches correctly.

Another challenge was to etch the wafer as deeply as possible, ideally as much as 400 microns, while leaving tall thin 10 μm wide structures standing. In one of the wafers the vertical etch became reentrant, resulting in a whole section of the heat transfer passage walls falling over. This wafer is shown in Figure 3.5, with the long thin passage walls laying haphazardly on the wafer surface. This situation was thereafter avoided by adjusting the etching recipe and the settings on the etching machine.

Once all three wafer levels were fabricated, each die was examined under an optical microscope. The wall thickness and passage widths were measured to within half a micron, and the depth of the passages was measured to within 5 microns. This information is then used in the models to predict the anticipated performance of each recuperator. Table 3.1 gives all of the geometric information about the completed recuperators.

Figure 3.5 – An improperly etched level-3 wafer. Reentrant etching undercut the heat transfer passage walls, allowing them to collapse. The irregular thin light stripes visible on the surface are actually these separated walls lying haphazardly.
After examination, the three wafer levels were bonded together. All three wafers are aligned, during which process they adhere to each other through electrostatic attraction. The three-wafer stack is then placed in a high temperature annealing furnace at 1100°C, where diffusion bonding occurs; the molecules from adjacent wafers are able to diffuse into each other, and the boundary between the two essentially disappears.

Infrared cameras are then used to determine how well the wafers are bonded together. Since silicon is nearly transparent to infrared light, pictures taken in this fashion reveal considerable information about the internal structure of the device. Areas that are not cleanly bonded together show interference patterns in the infrared photograph, due to the trapped thin pockets of air that are present at the unbonded boundary. The infrared photographs of builds 3 and 4 are shown in Figures 3.6a and b, respectively. Note that the various designs can be distinguished; designs with smaller passage widths are packed together more closely, and the heat transfer regions appear darker in the infrared photograph.

Finally, after bonding, the individual recuperator dies are separated from each other through the process of diesawing. An adhesive sheet is affixed to one side of the wafer, providing stability and support during cutting. A water-cooled fine blade and visual microscope assembly are used to cut the wafer precisely. Each recuperator, once separated from the rest of the wafer, is washed with solvent to remove the adhesive sheet and any oil.

![FIGURE 3.6 – (a) The infrared photograph of recuperator build number 3, and (b) the infrared photograph of recuperator build number 4. Regions that show interference fringes indicate poor bonding.](image)
Four builds of the linear recuperators were completed. The first two builds each had low yields; of the seven die, only four bonded well enough to seal the flows from the environment and each other. The third stack produced a yield of 100%. The fourth stack yielded 5 useful recuperator dies. Table 3.1 compiles the information corresponding to all of the recuperators that were successfully built. The geometric values were all measured values, while the effectiveness and pressure ratio values are predictions based on the model. Figure 3.7 shows two pictures of completed silicon recuperators. 3.7a shows a recuperator by itself, and 3.7b shows the recuperator alongside a standard US Postal Stamp for comparison. Figure 3.8 contains several scanning electron microscope images, showing the silicon features in each of the three wafer levels of the recuperator.

The heat transfer channels were never etched to the desired depth as a result of fabrication constraints. Ideally, the heat transfer passages would have been 400 to 500 microns deep. However, in each build the channels were only about 200 microns deep. Channel depth was limited by two factors: (1) the inability to control the width of the high-aspect-ratio walls with the accuracy needed to prevent undercutting, and (2) the length of time required to etch the channels deeper became prohibitive. Efforts to etch the heat transfer passages deeper resulted in collapsed walls due to reentrant etching, or required excessively long time to perform the etch. Additionally, further etching thins the silicon layer underneath the walls, which makes the wafer more fragile. As mentioned before, this was one of the most problematic aspects of the second wafer level, and attempts were made to avoid its recurrence in the third wafer level. For the purposes of the experimental testing, it was determined that depths of around 200 microns were acceptable. Although the performance would suffer from the limited depth, the information obtained would be sufficient to verify the model and establish a good understanding of the recuperator's performance.

Figure 3.7 – (a) a completed silicon recuperator prior to packaging (b) a size comparison between the recuperator (approximately 2cm x 3 cm) and a standard USPS Stamp.
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<td>81</td>
<td>55</td>
<td>9</td>
<td>~260</td>
<td>182-193</td>
<td>7</td>
<td>66</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 3.1 – The measured feature sizes and anticipated performances of fabricated recuperators
Figure 3.8 - Scanning Electron Microscope (SEM) pictures of recuperator details. (1) and (2) Wafer 1 - Flow inlet and outlet holes (3) Wafer 2 - Flow collection and distribution regions; the holes that open down into the heat transfer box passages are clearly seen. (4) The heat transfer box passages. One flow travels within the boxes, the other around the outside of the boxes. The heat transfer passage walls shown here are 10 μm wide and 200 μm deep.

In actual microengine application, there are several possible solutions that could be used to further increase the depth of the channels. One potential method would entail etching two wafers to this proven 200 micron depth, and then mating them face to face. This would double the heat transfer passage depth, improving both the pressure drop and the effectiveness. Another possible strategy is to aggressively address the fabrication process. By optimizing the deep-etching characteristics, a greater depth may be attainable without sacrificing reasonable etch times. Finally, a third strategy is to increase the number of recuperation wafers to be added to the engine. Increasing the number of parallel passages reduces the mass flow per channel, improving both the heat transfer and the pressure drop. This last technique is actually a very important one to the recuperator program; unless fabrication methods improve such that 10 micron walls can
be etched 600 or more microns deep, a multiple recuperator level design will be required in order to fulfill the stringent pressure drop requirements of the engine.

Additionally, the coverplates were never fabricated as thin as desired, due to the problems with fracturing mentioned above. The relatively thick coverplates constitute a large fraction of the overall cross sectional area through which heat can conduct. This axial heat conduction is one of the primary contributors to the overall ineffectiveness of the recuperators – about 20-30% of the total ineffectiveness is due to conduction through the coverplates alone. Minimizing the thickness of these surfaces without breaking the wafers is another area in which further fabrication development is needed.
4.0 Experimentation

The experimental setup used to test the performance of the heat exchangers was designed to be versatile and accurate. The effectiveness can be calculated using either of two different methods, and the inlet temperatures and pressures to both flows of the recuperator can be controlled to provide a broad range of operating conditions. A computerized data acquisition system records the temperatures and pressures at each of the recuperator's inlets and outlets, and continuously calculates the device's performance.

FIGURE 4.1 - The Experimental Setup
4.1 Layout

The same flow is used for both of the recuperator gas streams, which simplifies the plumbing and minimizes the instrumentation requirements. A schematic representation of the experimental setup is shown in Figure 4.2. Nitrogen enters the experimental setup and passes through an MKS pressure controller, which regulates the inlet pressure into the recuperator. Subsequently, the gas enters a tungsten-element Osram-Sylvania electric heater, which preheats the gas flow to a preset temperature. This heated gas flow enters the recuperator and passes through one set of flow channels. Both the pressure and the temperature of the gas entering the recuperator are set by the user.

After exiting the recuperator, the nitrogen gas is plumbed through a second heater, which is used to control the gas temperature entering the recuperator's second set of flow channels. After passing through the second set of heat transfer passages, the air stream exits the recuperator and passes through a cold-water heat exchanger. This heat exchanger uses city water to reduce the gas temperature, preventing damage to the downstream mass flow controller. After passing through the mass flow controller, the gas is finally vented to the atmosphere.

![Schematic Diagram of the Experimental Apparatus](image-url)
In order to provide the greatest amount of versatility possible, the system provides two different means of measuring the recuperator effectiveness. Both are derived from the definition of effectiveness,

\[ \varepsilon = \frac{q_{\text{transferred}}}{q_{\text{maximum}}} \]  

In the first method, \( q_{\text{transferred}} \) is calculated as the gain in energy of the cooler of the two gas streams

\[ q_{\text{transferred}} = \dot{m}_c \int_{T_1}^{T_2} c_p(T) dT \]  

or, assuming \( c_p \) is constant across the temperature change,

\[ q_{\text{transferred}} = \dot{m}_c \Delta T \] 

The maximum amount of heat that could conceivably be transferred between the two streams is

\[ q_{\text{max}} = (\dot{m}_c p)_{\text{min}} (T_3 - T_1) \] 

where \( (\dot{m}_c p)_{\text{min}} \) is the smaller of the two streams' products [11]. If the product \( (\dot{m}_c p)_{\text{min}} \) is assumed to be nearly identical for the two streams, then the equation simplifies to

\[ \varepsilon = \frac{T_2 - T_1}{T_3 - T_1} \] 

Alternatively, the effectiveness of the recuperator can be related to the steady state power being supplied to the second heater, which lies in series between the recuperator's first stream exit and its second stream inlet. In an ideal recuperator, the first stream exit temperature would be identical to the second stream inlet temperature; that is, all available heat energy would have been transferred. A real recuperator is not ideal, however, and has some ineffectiveness associated with it. Therefore there is a temperature deficit between the first stream exit temperature and the second stream inlet temperature. The energy required to heat the gas flow up to the preset second stream inlet temperature is directly related to the ineffectiveness of the heat exchanger. That added energy can be measured from the voltage across and current through the heater, and used to deduce the ineffectiveness of the recuperator.

The equation for the heat exchanger effectiveness is

\[ \varepsilon = \frac{\dot{m}_c p (T_2 - T_1)}{\dot{m}_c p (T_3 - T_1)} \] 

which can also be written as

\[ \varepsilon = \frac{\dot{m}_c p (T_2 - T_1)}{\dot{m}_c p (T_3 - T_2) + \dot{m}_c p (T_2 - T_1)} \]
By drawing the control volume shown in Figure 4.3 around the second heater and the tubing between the first stream outlet and the second stream inlet, \( \dot{m}c_p (T_3 - T_2) \) can be written as \( Q_{\text{loss}} - Q_i \), where \( Q_i \) is calculated from the power \( VI \) going into the heater and \( Q_{\text{loss}} \) is the sum of the thermal losses from the tubing and heater. Thus,

\[
\varepsilon = \frac{\dot{m}c_p (T_2 - T_1)}{\dot{m}c_p (T_2 - T_1) + VI - Q_{\text{loss}}}.
\]

Several measures were taken to ensure that the measured effectiveness of the recuperators was as accurate as possible. These methods included eliminating convection losses from the equations and quantifying the conduction and radiation losses in the system.

As shown in Figure 2.2, the recuperator, both heaters, and all of the intermediary plumbing were enclosed in a vacuum can during testing. Running the experiments in a vacuum eliminates convective losses from both the heaters and the recuperator, thus removing an uncertainty from the tests. From Lafferty [8], the thermal conductivity of air decreases nearly linearly with pressures below 1 centimeter of mercury (and is nearly independent of pressure above one centimeter Hg). Over the temperature range from 300K to 900K, the thermal conductivity of atmospheric-pressure air varies between 0.0259 and 0.0597 W/m-K. With this thermal conductivity and an average recuperator temperature of 600 K, the heat transfer due to convection is estimated to be on the order of 6 Watts. Compared to the expected 30 Watts of heat transfer within the recuperator, this convection loss is significant.

![FIGURE 4.3 – Schematic Representation of the Control Volume Space for Heater 2 Effectiveness Measurement](image-url)
The roughing pump used with the experimental setup can produce vacuum pressures of 50 millibars. At this pressure, the thermal conductivity drops to between 0.0001295 and 0.0002985 W/m-K, or less than 0.5% of what it was at one atmosphere. Since the convection losses are proportional to the thermal conductivity of the fluid, the convective heat transfer coefficient will also decrease by this same amount. At this pressure, the losses due to convection are negligibly small compared to the heat transfer being effected in the recuperator (0.1%), and are dominated by the radiation and conduction losses.

Additionally, conduction losses though the solid parts of the system are quite small. All of the heated components are connected to the vacuum flange via long tubes with small cross sectional area. Even with the high temperatures at the recuperator and ambient temperatures outside of the vacuum flange, the conduction through metal tubes out of the system is much less than 1 Watt.

Radiation dominates as the primary source of thermal losses in the system. Even at moderately high temperatures such as 500K, the conduction losses will be less than 5% of the radiation losses, and the convective losses will be much smaller than that. At higher temperatures, the contributions of convection and conduction become even less significant, since the heat loss due to radiation is proportional to the fourth power of the temperatures involved.

For example, the radiation loss at 600K would be on the order of 7 Watts, which is significant compared to the 30 Watts of expected heat transfer within the recuperator. It is therefore necessary to precisely account for the radiation losses; by developing a correlation, the heat transfer results can be corrected for the effects of radiation. This correlation was derived from data provided by a series of baseline experiments. A generic heat exchanger was installed in the system, and several tests were made; the inlet gas flows were heated to a wide range of temperatures, and heat transfer data was acquired for each case. This was accomplished by considering the same control volume shown in Figure 4.3. Energy transfer into the control volume comes from two sources; the initial incoming gas flow, with:

\[ q = \dot{m}c_p T_2 \]  

and the power going into the second heater,

\[ P = VI. \]  

The only modes for the transfer of energy out of the control volume are the exiting gas flow

\[ q = \dot{m}c_p T_3 \]
and the radiation losses to the environment (assuming convection and conduction to be negligible). Since the net transfer of heat at steady state must equal zero, the radiation losses are equal to the sum of the other three components.

\[ q_{\text{radiation loss}} = \dot{m} c_p (T_2 - T_3) + V I \]  

(Note that the radiation losses in the recuperator alone can also be determined experimentally, being derived from the flow inlet and outlet temperatures:

\[ q_{\text{radiation loss}} = \dot{m} c_p T_1 - \dot{m} c_p T_2 + \dot{m} c_p T_3 - \dot{m} c_p T_4 \]  

The radiation losses for the control volume were then plotted as functions of the second heater outlet temperature. Since the radiation term is a function of temperature to the fourth power, the dependence on the lower heater outlet temperature was much weaker, and for the purpose of deriving a radiation correlation the lower temperature was ignored.

The result was an equation for a radiation coefficient \( \beta \), which was defined as a composite variable that incorporated the view factor \( F \), the emissivity \( \varepsilon \) as a function of temperature, Planck's constant \( c_\epsilon \), the radiative heat transfer area \( A \), and a temperature term corresponding to

\[ \frac{(T_3^4 - T_{\text{ambient}}^4)}{(T_3 - T_{\text{ambient}})} \]  

The resulting heat loss coefficient takes the form

\[ \beta = \sigma \varepsilon A F \frac{(T_3^4 - T_{\text{ambient}}^4)}{(T_3 - T_{\text{ambient}})} \]  

so that the radiative heat transfer as a function of the heater 2 outlet temperature is

\[ q(T) = \sigma \varepsilon A F (T_3^4 - T_{\text{ambient}}^4) = \beta (T_3 - T_{\text{ambient}}) \]  

Figure 4.4 shows a graph of the experimental data resulting from these radiation tests. Based on the preliminary data, the bulk radiation coefficient \( \beta \) is best correlated as

\[ \beta = -0.0084 T_3 + 12.668 \]  

Using this term and the outlet temperature of the second heater, the radiation loss term in Equation 4.8 can be determined. In reality, this correlation also accounts for the small conduction and convection losses that are present in the system, but which are dominated by the radiation terms. With this equation, the pure effectiveness of the recuperator itself, calculated as if all of the losses in the system were equal to zero, can be measured.

For any given set of operating conditions, the method of calculating the recuperator effectiveness that produced a lower experimental error was used. Under certain operating conditions, the accuracy of the effectiveness calculated using Equation 4.6 is quite poor, since the error associated with the thermocouples is large at high temperatures. By using
the heater-input method, the error can be reduced. For example, in situations in which there is a small temperature difference between the two inlet flows, or in low-effectiveness designs, the errors associated with the calculation will be large due to the relatively high errors associated with each of the temperature readings. Since the heater-deficit method requires fewer temperature measurements, the error associated with its use will be considerably lower. It should be noted that under most of the operating condition considered, the differences in error between the two methods are small.

4.3 Pressure Measurements

In addition to the effectiveness of the recuperator, the change in gas pressure along the heat transfer passages was also measured. As described in Chapter 2, the pressure drop in the recuperator is extremely important in determining the impact of the recuperator on the engine cycle. In fact, by examining the effects of recuperator performance on the TSFC and PSFC of the engine, it is clear that even a small pressure drop can significantly reduce the overall performance of the engine, both in terms of its output and its fuel efficiency. Pressure measurements are taken in the flow distribution and collection regions at the inlets and outlets to the heat transfer passages. The pressure taps themselves consist of 0.030" (0.762 mm) OD stainless steel tubes that are nested inside of the flow inlet and outlet plumbing. The taps are long enough to extend through the flange of the vacuum can; in this fashion, the sensitive circuitry of the pressure transducers is removed from the high temperatures vacuum environment inside the can.
Furthermore, this setup allows the Endevco pressure transducers to be located external to the vacuum chamber; accessing and adjusting the system can be done without having to worry about venting or re-establishing the vacuum.

Two of the pressure transducers provide absolute pressure readings, and the other two provide differential pressure readings. The two differential pressure transducers are used to minimize the experimental error in calculating the pressure drop. In situations where the pressure drop is small, using the difference between two absolute readings can produce a large error. This is especially true when the error of each transducer is large compared to the difference between the two readings; in that case, it is difficult to determine even the sign of the pressure change. Each recuperator gas stream has an absolute pressure transducer at one end of the heat transfer passages, and a differential unit gives the difference in pressure at the other end. The resulting pressure reading is more accurate than would be possible using two absolute measuring units.

4.4 Experimental Error

Extensive effort was made to minimize the experimental error associated with measuring the recuperator’s effectiveness and pressure drop. As mentioned previously, two methods of calculating the recuperator effectiveness were derived. Additionally, differential pressure transducers were used to reduce the error in calculating the pressure drop.

Table 4.1 shows the error associated with each of the components of the system. The error for the derived effectiveness and pressure drop values can also be determined using these numbers.

<table>
<thead>
<tr>
<th>Device</th>
<th>Experimental Error, w</th>
</tr>
</thead>
<tbody>
<tr>
<td>K Style Thermocouple</td>
<td>1.0 % of reading</td>
</tr>
<tr>
<td>Mass Flow Meter</td>
<td>1.0 % of max. flow value</td>
</tr>
<tr>
<td>Voltmeter</td>
<td>0.1 % of reading</td>
</tr>
<tr>
<td>Ammeter</td>
<td>0.1 % of reading</td>
</tr>
<tr>
<td>Pressure Transducers</td>
<td>1.0 % of max. pressure</td>
</tr>
</tbody>
</table>

Table 4.1 – The measurement error of each device in the experimental setup.
Several preliminary runs were made with a stainless steel heat exchanger. This heat exchanger was used to test the experimental setup, and establish that it performed as expected. These tests provided useful data for evaluating the typical experimental errors in the system. On the basis of one of these trials, the effectiveness of the test heat exchanger was calculated to be 0.325. However, the two different methods that can be used to calculate the heat exchanger effectiveness produce different experimental errors; the first method, which uses the equation

\[ \varepsilon = \frac{mc_p(T_2 - T_1)}{mc_p(T_3 - T_1)} \]  

produced an experimental error of \( \pm 0.049 \). The second method, where the effectiveness is related to the energy going into the second heater by the equation

\[ \varepsilon = \frac{mc_p(T_2 - T_1)}{mc_p(T_2 - T_1) + VI - q_{loss}} \]  

gave an experimental error of \( \pm 0.037 \).

Clearly, for this given set of operating conditions, it makes more sense to use the heater-deficit method of determining the effectiveness, since that method produces the smaller error (0.325±0.037 versus 0.325±0.049).

### 4.5 Packaging

Recuperators built for experimentation need to be packaged so that they can be easily connected to the macroscale system used to test them. Traditional packaging techniques proved to be inadequate. Since the coefficients of thermal expansion of solder and braze are quite different than that of silicon, using either of these materials tends to crack the recuperators; once the hot metals start to cool, the stresses introduced by differential expansion shatter the silicon. Epoxies, another means of sealing tubing to the recuperators, are usually incapable of surviving high temperatures, and those that do rarely remain leaktight. While investigating some high-temperature epoxy sealants, differential thermal expansions again became an issue; when heated, the sealants caused breakage in the silicon devices. The current recuperator packaging scheme is derived from the experiences of other groups in the microengine project. In this design, Kovar tubes are sealed directly into the silicon devices.

Kovar is a metal that was developed for the electronics packaging industry; it has a coefficient of thermal expansion very close to that of glass and silicon, and melted glass wets well to it. Kovar wire sealed into a glass plug can act as an insulated electrical lead. When heated, the similar thermal expansions prevent the glass from cracking. Furthermore, the glass-Kovar seals can be used in airtight or vacuum applications.
For the purpose of packaging these silicon devices, a glass pre-form in the shape of a torus is used. The pre-form is made from a powder of Corning 3200 glass, which is pressed into a mold and fused into a solid piece. The pre-form is placed around the Kovar and is in contact with the silicon, as shown in Figure 4.5. This assembly – the silicon device, the Kovar tube situated in the device, and the glass perform around the Kovar – is placed in a high temperature furnace with a carbon fixture to supply pressure between the pieces. The glass softens and wets to both the silicon and the Kovar. The assembly is removed from the furnace and cooled. The glass hardens, sealing the two pieces together.

The mounting of the recuperator into the experimental setup was problematic. The intent was to minimize the thermal communication between gas flows. The designs initially attempted all had the inlet and outlet flow tubes standing independently of each other, so that there could be no thermal linkage between them.

These free-standing Kovar tubes were then to be nested inside stainless steel tubes on the experimental setup. Unfortunately, the process of mating the two ends of the tubes, which did not always line up perfectly, induced stresses at the silicon-to-Kovar glass seal. Even small forces on the Kovar tubes caused the glass seals to fracture; the tubes acted as long cantilever beams, and small forces applied to the end of the tubes produced large moments at the silicon surface. The induced moments tended to either delaminate the seal from the silicon, or actually fracture a section of the silicon altogether. Figure 4.6 shows examples of fractured seals.

FIGURE 4.5 – Schematic Diagram of the Kovar-to-Silicon Sealing Process
In a few instances, the Kovar tubes were successfully sheathed into their stainless steel counterparts. However, additional difficulties arose in sealing the Kovar-to-stainless steel connection. Attempts to braze the junction again resulted in damage to the recuperators. Thermal expansion stresses induced by localized heating from the torch caused the seals, and in one case the recuperator itself, to fracture.

High temperature epoxies were also used in an attempt to seal the nested tubes together. These epoxies, as mentioned before, typically do not function at the high temperatures needed for this application. Furthermore, many of the high temperature epoxies do not retain their sealing qualities at high temperatures.

Nevertheless, a high-temperature low-viscosity epoxy was found which was capable of remaining airtight up to temperatures of about 500 K. Although this is much lower than the experimental setup was originally designed for, this temperature range was acceptable for the purposes of some initial low-temperature testing.

The tubes were sealed together, and a series of pressure drop measurements were taken and recorded. Then the heaters were then turned on, with the intent of analyzing the heat transfer performance of the mounted recuperator. Unfortunately, the thermal expansion of the heaters during their quick temperature rise exerted forces on the Kovar tubes, causing the glass seals to delaminate and leak. Calculations confirmed that the thermal expansion of the heaters was enough to cause the observed fracturing.

Miniature bellows were purchased to eliminate this problem. The measured stiffness of these bellows was low enough that they were capable of absorbing the heaters’ deflections without applying forces to the Kovar tubes that would break the glass seals.
Unfortunately, connecting the Kovar tubes to the bellows still required the use of the high
temperature epoxy. The epoxy had a low viscosity, and flowed into the tubing during
sealing, plugging the Kovar tubes completely.

Finally, a proven sealing method was adapted and implemented. This method had
already been demonstrated in other areas of the MIT microengine project; the microscale
combustion chambers and microscale rockets used designs similar to the one
implemented successfully with the recuperator program. In this method, shown at the top
of Figure 4.7, the silicon device is sealed to the Kovar tubes as described before. A braze
paste is then applied to the ends of the Kovar tubes, which are inserted into a stainless
steel mounting block. A second pass through the furnace melts the braze paste and seals
the Kovar tubes to the mounting block.

The resulting structure has a number of advantages. The stainless steel plate is rigid
evenough to be handled without conveying any forces to the delicate recuperator
connections. Once affixed to the mounting block, the recuperator never needs to be
directly handled at all. The stainless steel plate also protects the recuperator from
damage once installed in the system. Incidental contact with the setup, which previously
would have cracked the silicon, is no longer a concern.

In the experimental setup the mounting block and recuperator assembly is mated to
another stainless steel block. This is shown in Figure 4.7. This second block is
permanently affixed to the experimental setup, and has grooves cut into its face for o-ring
seals. The two blocks are bolted together with metal o-ring seals between them,
connecting a recuperator to the test rig without conveying any forces to the fragile
recuperator.

In other areas of the MIT microengine project, the block-to-block sealing has been
accomplished by bolting the two blocks together with rubber O-rings seals between them.
However, the gases entering the recuperator (and therefore passing through the stainless
steel blocks) are at elevated temperatures (up to 900K), far higher than rubber o-rings can
tolerate. Metal (nickel-plated stainless steel) 3/8” diameter o-rings were designed and
fabricated by Ultra-Seal, Inc. These are placed between the two stainless steel blocks,
and clamped with a large force applied through a ring of bolts.

One of the concerns about using a stainless steel block is that the thermal expansion of
the steel is different from that of the silicon. When heated the stainless steel will expand
more than the recuperator, and the Kovar tubes will undergo stress loads. Conceivably,
the expansions might be different enough that the spreading of the Kovar tubes actually fractures the silicon recuperator. To address this the Kovar tubes were analyzed as long beams, as was the recuperator. Calculations for the force on the silicon resulting from the differential thermal expansion indicated that the entire assembly would survive heating within the intended range.

Another concern about using a stainless steel mounting block for this purpose was that it acts as a large thermal link between the gas flow tubes. Heat from the high-temperature flow can flow through the stainless steel to the cold temperature tubes. If that heat is then transferred into the cold flow gas downstream of the thermocouple, the measured value of the gas temperature entering the recuperator will be wrong. This will subsequently produce inaccurate effectiveness calculations.

In order to eliminate this problem, the finning behavior of the Kovar tubes was first analyzed. Figure 4.8 depicts the finning model schematically. The system was modeled with each Kovar tube acting as a fin, and the stainless steel block between the tubes representing a thermal connection between the two tubes. Some heat invariably convects from the high temperature flow into the stainless steel block, and then conducts over to
FIGURE 4.8 – The system model used to analyze the finning effect of the Kovar tubes

the low temperature flow tube. Once at the low temperature flow tube, it can convect into the low temperature gas stream or conduct up the Kovar tube. The intent is to determine the maximum distance (in the x-direction in Figure 4.8) that heat from the stainless steel block will conduct up the Kovar tube. After that point, there will be no heat addition from the stainless steel block into the gas stream. A thermocouple located beyond that maximum distance will give the flow temperature, including any heat addition from the block. These temperature measurements, corrected for the thermal losses described in section 4.2, will provide accurate recuperator effectiveness data.

The worst case scenario, in which the addition or removal of heat does not change the gas temperature as it travels through the tube, is assumed. The standard fin correlation is

\[
\frac{\theta}{\theta_b} = e^{-mx}
\]

where \( \theta \) refers to \((T_x-T_{gas})\), \( \theta_b \) refers to \((T_0-T_{gas})\), and

\[
m = \frac{hP}{kA_c}
\]

The heat transfer from the fins to the gas flow is convective in nature, and therefore proportional to the local temperature of the fin. The heat transfer from the fin to the gas is given by the standard convection equation:

\[
q = h A (T_{fin\,local} - T_{gas\,local})
\]
Therefore, the heat transfer itself is proportional to an exponential function of x,

\[ \frac{q_x}{q_{x=0}} \propto \exp \left( \frac{hP}{\sqrt{k_A}} \right)^x. \]

Figure 4.9 shows the temperature ratio \( \theta/\theta_0 \) as a function of x, the distance along the fin away from the stainless steel plate.

Based on this information, it is clear that after 3 centimeters of length, the Kovar tube is no longer behaving as a fin. Any heat energy that is going to be transferred to the gas stream has already done so by that point. As long as the thermocouple is reading the gas stream temperature in the recuperator at least 3 centimeters from the stainless steel mounting block, the measured value corresponds to the local gas temperature, and there will be negligible heat addition to the gas flow after that point.

The mounting scheme was consequently designed so that slightly more than 3 centimeters of Kovar tubing separates the stainless steel block and the recuperator. The thermocouples take the local temperature measurements at the flow distribution and collection regions of the recuperator, beyond the limit of the effects of the conduction in the stainless steel block. The first attempt to assemble a completely packaged recuperator met with partial success. Although the glass sealed to the Kovar and silicon well, the

![Fin Effectiveness of Kovar Tube](image)

**FIGURE 4.9** – The temperature ratio \( \theta/\theta_0 = (T_x - T_{gas})/(T_0 - T_{gas}) \) as a function of x, the position on the tube measured from the stainless steel mounting block. At a distance of 3 centimeters, the tube is no longer acting like a fin. The assumptions listed correspond to the experimental conditions used to test the recuperators.
metal braze did not wet to the stainless steel plate. The resulting structure was not leaktight. The remaining stainless steel mounting blocks were nickel-plated prior to brazing. All subsequent braze operations worked extremely well. Figure 4.10 shows the difference between the plated and non-plated braze sealing. Figure 4.11 shows two fully packaged recuperators.

FIGURE 4.10 – Braze wetting on the stainless steel block. (a) An unplated stainless steel block produces unwetted braze joints (b) A nickel-plated stainless steel block demonstrates excellent braze wetting

FIGURE 4.11 – Fully packaged recuperators. The silicon device is sealed to Kovar tubing with glass. The Kovar tubes are then brazed to a stainless steel plate, which can then be bolted onto the experimental setup.
4.6 Instrumentation

One of the persistent challenges in experimentally testing microscale devices is that of obtaining accurate and useful measurements. With passages that are on the order of 50 micrometers across, inserting even the smallest of thermocouples into the actual flow stream becomes impossible. Furthermore, with packaging being as difficult as it is, each additional insert into the recuperator introduces additional sealing challenges that could reduce the yield of usable recuperators. Instrumentation methods that are traditionally very successful in macroscale applications are clearly inadequate for use in the microscale.

Therefore, many innovative solutions to obtain data from microscale devices have been devised. Successful techniques have included the use of infrared imaging (silicon is nearly transparent to infrared light), on-chip instrumentation, and the creative use of miniaturized conventional measurement devices.

Fortunately, no measurements from within the recuperator itself are needed. Specifically, it is important to obtain temperature and pressure data only at the inlets and outlets to the recuperator heat transfer passages; these measurements are then used to calculate the recuperator effectiveness and pressure drop using the methods described earlier in the chapter. The mass flow rate and the power required by the second heater can both be measured outside of the vacuum test chamber.

In order to obtain the pressures immediately before and after the heat transfer channels in the recuperator, miniature pressure taps are inserted into the flow distribution and collections regions. This is achieved using the method shown in Figure 4.12. The pressure taps, 0.030” (0.762 mm) in diameter, are inserted through the flow tubing; that is, within the Kovar tubes connecting the recuperator to the test rig are thin pressure taps that extend into the silicon device itself. Nested within these pressure taps are even smaller K-style thermocouples, 0.010” (0.254 mm) in diameter, produced by General Measurements, Inc.

Figure 4.13 shows the method used to extract the pressure taps and thermocouple from the flow tubing. The main flow separates from the pressure tap about 6 inches (15.24 cm) away from the recuperator. The extracted pressure tap is connected to a small sealed volume whose pressure is monitored by pressure transducers. The thermocouples nested within the pressure taps are extracted from these pressure chambers, so that the temperature reading at the head of the thermocouple can be measured. In this fashion both the pressure and temperature can be measured at each of the inlets and outlets within the recuperator.
Kovar Tubing

Pressure Tap

0.010" K-Style Thermocouple

FIGURE 4.12 – Diagram of Nested Instrumentation – Flow channel, pressure tap, and thermocouple

Recuperator

Thermocouple Junction

Flow Path

Pressure Tap

Pressure Transducer

Thermocouple Lead

FIGURE 4.13 – The measurement extraction scheme. Pressure taps are separated from the flow tubing, and terminate in a sealed chamber. The thermocouple is then separated out of the pressure chamber.
4.7 Data Acquisition

In order to streamline the analysis of the recuperators’ performance, a computerized data acquisition system was designed and implemented. Twelve different readings are taken and two control signals are applied simultaneously. The twelve acquired measurements are as follows:

- 4 flow temperature readings, one at each of the recuperator’s inlets and outlets
- 4 pressure readings, one at each of the recuperator’s inlets and outlets
- 1 pressure reading from the pressure controller, located upstream of the recuperator
- 1 mass flow reading from the mass flow controller, downstream of the recuperator
- 1 voltage reading from the second heater
- 1 current reading from the second heater

Each of the two output signals controls one of the two heaters; a PID control loop compares the flow temperature to the setpoint temperature, and determines what correction, if any, needs to be made to the heater output temperature. This correction is converted into a voltage that is applied to the heater controllers. The feedback loop used to control the heaters is shown in Figure 4.14.

All of the signals are channeled through a National Instruments 5B backplane with the appropriate signal conditioning modules, and then into an IBM compatible computer. The computer is running National Instruments Labview software, which is running a control loop designed specifically for this experiment. From the virtual environment of the computer screen, the user can set the temperatures of each of the two recuperator inlet flows, and the sampling rate at which data is acquired. For the experimental results presented here the sampling rate used was 3 Hertz.

![Figure 4.14 - Control feedback loop used to control the flow heaters.](image-url)
The software program also incorporates additional features; the user is prompted to choose a filename for each separate data acquisition run to be saved to, there are controls and emergency shutoffs for each of the two heaters, and there is a switch for initiating or terminating the collection of data. Note that the inlet pressure and the mass flow rate are also set by the user, though not through the computer system – those settings are made using controllers located under the computer table.

The collected data is converted to the appropriate units in real time, and is immediately saved to the filename specified. Furthermore, the collected information is portrayed graphically on the computer screen in real time. Graphs of the four temperatures, the four pressures, the calculated effectiveness, and the pressure ratio are continuously updated, providing a continuous visual indication of the behavior of the system. Digital readouts provide the same information, as well as the values of the mass flow and pressure controller output. The Labview program used to run the data acquisition system is shown in Figure 4.15. A sample of the output screen is shown in Figure 4.16.

![Labview diagram of the data acquisition program used to acquire and write the data. A typical sampling frequency used for this thesis was 3 Hertz.](image)
4.8 System Flexibility

The various components in the system were selected in part for their flexibility. The heaters utilize a tungsten coil filament which is capable of maintaining an extremely high temperature, and the mass flow and pressure controllers were chosen to provide a broad range of operating conditions for testing.

Table 4.2 shows the range of operating conditions that can be applied to the inlet gas flows. These provide a large range of possible inlet conditions into the recuperator.

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>RANGE OF VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Flow Temperature (K)</td>
<td>R.T. - 1100</td>
</tr>
<tr>
<td>2nd Flow Temperature (K)</td>
<td>R.T. - 1100</td>
</tr>
<tr>
<td>Mass Flow Rate (g/s)</td>
<td>0 to 0.281</td>
</tr>
<tr>
<td>Inlet Pressure (atm)</td>
<td>1 - 2.5</td>
</tr>
</tbody>
</table>

TABLE 4.2 – The flexibility of the experimental system, showing the ranges for the inlet conditions for the gas flows entering the recuperator.
4.9 Experimental Procedure

Following is a checklist for use with the data acquisition system. The checklist assumes that all components are unpowered at the beginning of the test run.

START
1) Open Cold-Water Heat Exchanger Valve
2) Open Nitrogen Flow Line
3) Open System Valves
4) Set Desired Mass Flow Rate
5) Set Desired Initial Inlet Pressure
6) Open Labview
7) Set Desired Inlet Temperatures
8) Run Labview DAQ Program
9) Select Filename to Save Data to
10) Turn on Pressure Controller
11) Turn on Mass Flow Controller
12) Turn Power On to Heaters
13) Turn on Heaters in Labview Program
14) Wait for Steady State

SHUTDOWN
1) Turn Heaters Off in Labview Program
2) Turn off Power to Heaters
3) Stop Data Acquisition Program
4) Let System Come Down to Room Temperature
5) Turn Off Pressure Controller
6) Turn Off Mass Flow Controller
7) Close System Valves
8) Close Nitrogen Feed Valve
9) Shut Off Cold Water Valve

4.10 Verification of Experimental Setup

A series of validation tests were made using the simple parallel-tube stainless steel counterflow heat exchanger. In addition to establishing the functionality of the experimental setup, this process was also valuable for the information it provided regarding the radiation losses in the system predicted in Section 4.2.
The heat exchanger used in these validation tests consisted of two 1-meter lengths of 1/8” (3.175 mm) OD, 1/16” (1.588 mm) ID stainless steel tubing silver-soldered together along their entire length and then coiled to fit inside the space of the vacuum can. The silver braze used was a Cadmium-free material with a melting temperature of 950K. This design was robust enough that it wouldn’t break, even with rough handling, and it was straightforward enough to model accurately. Although this heat exchanger is dissimilar to the geometry of the silicon recuperators, the validation of the apparatus is independent of the device being tested. After the bulk heat loss coefficient is applied to the measured value of the heat exchanger effectiveness, the result should match the calculated value. A good fit indicates that the apparatus is working as expected.

Figure 4.17 shows the resulting test data for the stainless steel heat exchanger used to validate the experimental apparatus. The straight line represents the expected recuperator effectiveness based on standard heat transfer correlations. The four diamond-shaped data points shown depict four separate trials of raw heat exchange data. The four circular data points correspond to the same data, but compensated for the heat losses in the experimental apparatus (see Section 4.2). These new data points match well with the calculated values, indicating that the correlation being applied is reasonable, and that the experimental setup functions as expected.

FIGURE 4.17 – Effectiveness data for the heat exchanger used to test the experimental setup; the measured effectiveness is given for each of four runs. The data represented by diamonds corresponds to the unadjusted effectiveness. The data represented by circles corresponds to the effectiveness corrected for the heat loss in the experimental setup itself, external to the recuperator. The straight line shows the predicted effectiveness of the heat exchanger based on standard heat transfer correlations.
5.0 Experimental Results

Experimental data was collected from two silicon recuperators. One of the recuperators provided information about the pressure drop within the device, and the other recuperator provided data about the heat transfer performance of the device. The two inlet pressures for the pressure-drop measurements were 2 atmospheres and about 1.2 atmospheres. The inlet temperatures for the heat transfer data ranged from 300K to about 350K. Higher temperature data was not collected, since both recuperators fractured due to thermal stresses.

5.1 Preliminary Pressure Drop Results

The pressure drop in the recuperator was measured using a linear design #4 recuperator, which has 109 heat transfer passages. The mass flow of air was 0.04 grams per second, both the flows were at room temperature, and the inlet pressure into the compressed gas stream was about 2 atmospheres. The four measurements were taken over the course of an hour. Figure 5.1 depicts the four measured values, and compares them to the expected value of the pressure ratio in the recuperator, which is shown as the dotted line. This expected pressure ratio value of 0.956 (4.4% pressure drop) was calculated using the linear recuperator model described in Chapter 2.

The measured value of the compressed gas pressure drop is very close to the calculated value. The measured value for the exhaust gas, however, is about 7% below the calculated value. One possible explanation for the exhaust gas deviation is that some of the epoxy used as sealant in this test might have wicked into the Kovar tubing and partially blocked the flow. Another possible explanation was discovered after the recuperator fractured. The Kovar inlet and outlet gas flow tubes exhibited significant flow restriction at the ends that were inside the recuperator.
FIGURE 5.1 - Pressure drop results obtained using linear recuperator design #4 from build number 2. The passages were etched 200 microns deep, and the mass flow was 0.04 grams per second.

The recuperator design has through-holes though the first wafer level. The Kovar inlet and outlet flow tubes were sealed into these holes. However, the Kovar tubes had to be situated so that they did not rest flush against the silicon structure; if they were, the gas flow would be blocked. Figure 5.2a shows a schematic of a flush Kovar tube, demonstrating the blockage that occurs.

Ideally, the Kovar tube could be situated precisely within the silicon structure, leaving a gap where the air flow can pass unimpeded from the tube into the recuperator (Figure 5.2b). In reality, however, placing the Kovar tube precisely is extremely difficult.

In order to address this concern, Thunderline-Z, the vendor responsible for packaging the Kovar to the silicon structure, cut 4 slits into each tube. The Kovar tube rests flush against the silicon structure, eliminating the need for blind positioning. However, the air flow can pass through the slits. Figure 5.3a shows this schematically.

As can be seen in Figure 5.3b, the exhaust inlets and outlets have two of these slitted tubes side by side. In actuality, the percent of Kovar tube removed with the slits is about 50%, meaning that there is still a considerable amount of blockage. Furthermore, any air leaving the second Kovar tube must pass through the slits in its tube, and the slits of the other adjacent tube, before entering the flow distribution plenum. The pressure measurement is taken near the end of the Kovar tube, but prior to the pressure drop associated with the flow blockage. It is possible that the additional pressure drop in the exhaust gas stream is a consequence of this situation.
a.

FIGURE 5.2 – (a) A Kovar tube flush against the silicon structure blocks the gas flow from entering (or leaving) the recuperator (b) A gap between the end of the Kovar tube and the silicon structure allows the gas to flow freely into and out of the recuperator.

b.

FIGURE 5.3 – (a) Schematic representation of the flow slits in the Kovar tubes. Air flows through the Kovar tube, and then radially out into the recuperator through slits cut into the tube (b) Close-up of a fractured recuperator, showing the slits in the Kovar tubes for air flow.

One indicator that there is some blockage accounting for the pressure drop is the reduced mass flow rate. At the design inlet pressure, there was only 0.04 grams per second of air flowing through the system, as compared to the intended flow of 0.18 grams per second. Somewhere in the flow path is a large resistance to the gas flow that is not accounted for in the calculations for the pressure drop through the tubes, plenums, and heat transfer passages.
5.2 Preliminary Heat Transfer Results

As described in Chapter 4, the effectiveness of the device could be calculated in two different ways. For the experimental results described here, the effectiveness was calculated using the temperature deficit method – at the flow conditions being measured here, this method produced the lower experimental error. The design tested was a linear recuperator #4 design.

Figure 5.4 shows the measured values of the effectiveness. The value of the effectiveness calculated from the computer model, 50.5%, is also shown on the graph as a dotted line. The experimental error of effectiveness measured using the temperature deficit method mentioned above is calculated to be around ±3%; a sample error bar is depicted on the graph.

![Effectiveness vs. Time](image)

**FIGURE 5.4** – The measured effectiveness of the tested recuperator for the time range from minute 55 to minute 65. The dotted line represent the calculated effectiveness produced by the model. The error for the effectiveness measurement is ±0.03.
The data shows the measured effectiveness from minute 55 to minute 65, during which time the system is demonstrating steady state behavior. The four temperature measurements for the entire trial can be seen in Figure 5.5. The entire test lasted about 100 minutes from beginning to end. The test has been separated into several regions of different behavior.

The portion of the test denoted as “C” is characterized by a poor electrical contact to one of the thermocouples. When the circuit was not closed, the thermocouple output reads 1560 K. This reading, when read by the computer system, turns the corresponding heater off since that value is above the setpoint value established by the user. Consequently, the system temperatures decrease during the period of poor electrical contact. Once the electrical contact was fixed, the system came to steady state at the preset inlet temperature, 340K. This can be seen in section “E”, which includes the time span shown in Figure 5.4. Note that one of the thermocouples, the bottom one on the graph, exhibits a slow temperature rise; this is due to the fact that it is measuring the temperature of the

![Temperature vs. Time graph](image)

**FIGURE 5.5** – The measured reading at each of the four thermocouples used in the experimental test. The entire test ran for about 100 minutes; regions of differing system behavior are separated and each is denoted by a letter.
first steam inlet gas flow, which was not being heated during the test. The slow temperature rise corresponds to the conduction of heat from the hot mounting block into the flow tubing and then into the gas stream.

After allowing the effectiveness to come to its steady state value at the hot flow inlet temperature of 340K, the heater setting was increased to 360K. The region labeled F shows the system’s behavior as a result of this input. The temperature quickly starts to rise with the new input, and then starts to level off as it approaches the preset temperature.

At the boundary between regions “F” and “G”, two of the temperatures experience a sharp drop, while a third experiences a sudden rise. This is indicative of the recuperator having fractured during the heating process. Indeed, at this point the mass flow reading becomes a negative number, as ambient air is pulled through the exhaust tubing and into the vacuum chamber.

5.3 Further Heat Transfer Results

In addition to the steady state value of region E, the time response of the silicon heat exchanger was considered to determine whether additional data could be obtained from the experimental run. If the time constant associated with the transfer of heat energy from the gas flow to the silicon recuperator is sufficiently small, then the collected data can be considered as a series of smaller steady-state steps. Each of the small steady-state steps can then be analyzed as discrete points of heat transfer data.

The Biot number can be used to determine whether the conduction in the recuperator represents a significant thermal resistance in the system compared to the convective resistance, or if it can be neglected altogether. If the Biot number much smaller than 1, the silicon recuperator could be considered nearly isothermal at all times, and the time constant would be a function of the convective resistance only. To determine the time constant for the system, the Biot number is calculated using the equation

\[ Bi = \frac{h L}{k} \approx 0.2 \]

where \( h \) is the heat transfer coefficient from the gas flow to the silicon recuperator, \( k \) is the thermal conductivity of the silicon, and \( L \) is the length scale associated with the conduction length. For a conservative estimate, the half distance of the longest dimension is used.
The calculated Biot number is of order unity, indicating that the internal resistance of the silicon is not negligible compared to that of the gas flow convection. The time constant can therefore be estimated using the equation for heat propagation in a semi-infinite slab:

$$\tau = \frac{1}{2} \frac{L}{\alpha} = \frac{1}{2} \frac{\rho c_p L^2}{k} \approx 1.89 \text{ seconds}$$

95% of any change in the heat transfer rate will have occurred throughout the system after 3 time constants, or 5.66 seconds, has elapsed. Since this time value is so small compared to the time scale of the experiment, regions in which the gas temperatures are changing slowly may be accurately modeled as steady-state periods. The area labeled B in Figure 5.5 contains data that can be analyzed using this method. Figure 5.6 shows this area in closer detail.

The measured values of the effectiveness in this time range lie close to the calculated value. The margin of error is around 2-3% for this region of the test. The majority of the

![Effectiveness vs. Time](image)

FIGURE 5.6 – Steady state heat transfer performance from minute 74 of the test to minute 78.5 of the test. Note that the x-axis units are in 0.5 minute increments, or about 30 seconds. The calculated effectiveness of the heat exchanger is depicted as a dotted line. The experimental error of the data is about ±0.025.
data points lie within the margin of experimental error, indicating that the model reflects
the performance of the actual device well. This furthermore suggests that standard
macroscale heat transfer and fluid mechanics correlation can be successfully applied to
microscale systems that do not experience Knudsen effects.
6.0 Parallel Investigations

As a parallel endeavor, techniques were investigated for utilizing other materials in the construction of recuperator structures. Several short-loop experiments were run to determine the feasibility of fabricating heat transfer passage walls out of silicon and silicon dioxide or silicon nitride. Structures made from these materials benefit from their low thermal conductivity.

Several arrays of silicon pillars were etched on a wafer. The pillars were closely spaced in one direction, and widely spaced in the perpendicular direction. By exposing the pillars to an oxygen-rich environment, an oxide film is grown on the pillars. With the proper spacing and an appropriate film thickness, the deposition fills the gaps between pillars along the closely-spaced direction. Eventually, the deposition layers from two adjacent pillars will meet and grow together, closing the gap between them. A schematic of this process is shown in Figure 6.1.

FIGURE 6.1 – The two-step process for fabrication hybrid silicon-silicon dioxide structures. (a) A framework of silicon pillars, close-spaced in one direction, is made. (b) An oxide film is deposited on the framework, eventually closing the gap between close-spaced pillars.
These structures are attractive from the point of view of the recuperator. In cross section, they appear as long walls of silicon dioxide with only small isolated islands of silicon embedded within them. The high thermal-conductivity silicon framework can be fabricated with tight tolerances, and the low thermal-conductivity silicon dioxide structures can be built directly onto that framework. The resulting wall is a structure with a low overall thermal conductivity. This would reduce the ineffectiveness due to axial conduction along the direction of the flows.

For the purposes of this investigation, 36 different arrays were produced on each wafer, each array being 1cm x 1cm square. Each array had different geometric characteristics, so that the most important features and dimensions of this process could be determined. Variables included the diameter and shape of the pillars, and the distances between adjacent pillars in both the closely-space and widely-spaced directions.

The deposition process worked well in most cases (see Figures 6.2a and 6.2b). The oxide films on adjacent pillars grew together and filled the space between them. In situations in which the pillars were tightly packed in both directions, the oxygen-rich air was not able to flow freely between them, especially as the films grew thicker. Consequently, the deposition was hindered. Figure 6.2c shows a close-packed square array of pillars in which the films did not grow together as a result of the spacing.

FIGURE 6.2 – Scanning electron microscope photographs of three different hybrid silicon/silicon-dioxide structures. (a) 20 μm square posts with 5 μm spacing (b) 30 μm square posts with 5 μm spacing (c) 5 μm square posts with 17.5 μm spacing.
Unfortunately, the residual stresses from the deposition caused cracking during the annealing process. Depositions that were flawless prior to annealing fractured and split under the high temperature conditions. Examples of the cracking can be seen on the outer edges of Figures 6.2a and 6.2b. The resulting structures would be unable to maintain separation between flows on opposing sides of the wall. Various techniques have been proposed for reducing the residual stresses, but these have not been pursued in light of some of the additional challenges associated with hybrid silicon-silicon dioxide or silicon-silicon nitride structures fabricated in this manner.

One such challenge is preparing the top surface of the fragile hybrid walls for bonding. Another is that silicon's softening temperature is 900K, and that of silicon dioxide’s, although higher at 1075 K, is still below that of the exhaust gas temperatures. Producing a structure made of silicon dioxide does not address concerns about the structural integrity of such a structure in a high temperature environment.

Further investigation into hybrid silicon-silicon dioxide or silicon-silicon nitride structures were forgone in favor of more promising options. Corderite, for example, is a high temperature ceramic material with a low thermal conductivity and a coefficient of thermal expansion that closely matches that of silicon. It has been successfully bonded to silicon and silicon dioxide surfaces. Furthermore, it has been demonstrated that laser-etching techniques may be capable of producing the desired recuperator feature sizes and geometries. In light of these factors, further efforts on hybrid structures of the sort described in this chapter were suspended.
7.0 Summary and Conclusions

The results presented in this thesis demonstrate the feasibility of designing and fabricating recuperators with microscale features for use in conjunction in a microscale Brayton cycle application. Preliminary testing of the actual device indicates that there are no insurmountable barriers in the performance of the recuperator that prohibit its eventual integration with the MIT microengine. Furthermore, the recuperator performance is within expected ranges, and further investigation could refine the models to provide a closer match between the predictions and the testing results.

Although silicon is a poor material choice for this application, the modeling tools and experimental methods developed to test silicon devices can be applied to recuperators made from other materials. This chapter will summarize the findings of the current research efforts, discuss some of the problems and challenges encountered, and suggest areas of interest and importance for future work.

7.1 Modeling Summary

A computer model which can be used to predict the recuperator’s performance was designed using a combination of analytical methods and standard heat transfer correlations. The model is specific to one of two general heat exchanger designs: a linear counterflow recuperator with an array of alternating flow passages, and a radial counterflow designs with a series of alternating radially-oriented flow passages. Given specific geometric information, data about the incoming gas flows, and the recuperator material, the effectiveness and the corresponding pressure drop of the design is predicted.

This information can then be used in conjunction with MIT microengine cycle analysis data to determine the benefit derived from incorporating the recuperator design into the microengine. Furthermore, by varying the inputs, the model was used to determine the critical features and dimensions of the heat exchanger.
Additional modeling efforts were made to ensure that a design would survive the structural and thermal requirements of its operation. For example, the minimum heat transfer passage wall thickness needed to survive the pressure differential across it was calculated, and the entire recuperator was analyzed in order to ensure that it could bear the buckling stresses caused by thermal expansion.

Several linear and radial recuperator designs were finally proposed, each of which was within the limitations of fabrication, and had performances that were comparable to those required for application in a microengine cycle.

7.2 Fabrication

Selected recuperator designs were fabricated out of silicon using techniques derived from methods used in the computer chip production industry. The recuperator geometry was imported into a CAD drawing, which was provided to an outside vendor capable of producing photolithographic masks with the feature sizes of interest for this project. These masks were then provided to the MIT Microtechnologies Laboratory, where the actual fabrication took place.

The recuperator designs were each composed of three wafer levels that were individually etched and then bonded together to form the 3-dimensional flow geometries necessary to achieve the desired performances. Fabrication of one of the levels was straightforward. The other two levels each posed a number of challenges that had to be addressed during the course of fabrication. These were successfully surmounted, and about 15 complete recuperators were eventually produced.

Several compromises were made during the fabrication process in order to produce devices in the required time frame. Channel depths of 500 μm were originally anticipated, but actual etched flow passages never exceeded 20 μm in depth due to the excessive time and development that would have been required to attain the original goal. Likewise, the thickness of the coverplates was about 2.5 times thicker than designed, which contributed to the axial conduction losses apparent in the system. Thinning these features down to the original 100 μm specification introduced a dramatic decrease in the yield of the recuperators, and was therefore forgone in this project.
7.3 Experimental Apparatus

An experimental apparatus to be used to test the fabricated silicon recuperators was designed and assembled at MIT. The system provided for a wide variety of gas flow inlet conditions, simulating a broad range of microengine operating conditions. Mass flow, inlet pressure and inlet temperatures were all controlled remotely, and could be changed during the course of an experiment. A computerized data acquisition system monitored four temperatures, four pressures, the mass flow, and the power required by the heater, and recorded the information in real time.

Careful consideration was taken to ensure that the experimental accuracy was as high as possible. The recuperator being tested was enclosed in a vacuum can in order to remove convection as a source of heat loss. The radiation losses were qualified, and the conduction losses were calculated as being far less than the radiation losses, and were therefore neglected. The experimental instruments were chosen in part because of their accuracy, and in part due to the range of conditions over which they could operate.

The experimental apparatus was itself tested using a stainless steel heat exchanger. All components worked as expected. The measured values of the heat exchanger's effectiveness and pressure drop were equal to the predicted values calculated using standard heat transfer and fluid mechanics calculations, within the margins of error.

7.4 Experimental Testing

A great deal of work went into the development of methods and techniques for actually testing the fabricated recuperators. Successfully packaging a recuperator was problematic, and several attempts were made before a viable scheme was developed. Once achieved, two linear recuperator designs were tested. The first provided pressure drop information from the device, and the second provided information about the heat transfer performance of the recuperator designs.

The pressure drop tests indicated an exhaust-side pressure drop slightly higher than was originally anticipated within the recuperators. The calculated pressure drop lies outside the margin of error of the measured value. Investigation has indicated several possible reasons for this discrepancy. The first possibility is that the flow distribution and flow collection regions are not modeled accurately. These plenae were modeled simply as a series of right hand flow turns, when in actuality the flow might be considerably more complicated than that in these regions. A second possibility is that the conditions at the ends of the Kovar inlet and outlet flow tubes are producing a higher pressure drop than
expected. A third possibility is that some of the liquid epoxy used to seal the Kovar tubes to the stainless steel apparatus tubing may have wicked into the flow passage and introduced some blockage in the gas stream. Further investigation into the causes of this higher pressure drop are warranted.

The heat transfer tests produced results that were in keeping with anticipated and predicted values. The test data lies within the margin of error from the calculated value, indicating that the heat transfer modeling equations are in close agreement with the actual physical system.

7.5 Parallel Investigation

Silicon dioxide was deposited on a framework of silicon posts in order to create low thermal conductivity walls. The deposition grew around the silicon posts, eventually closing the gap between close-spaced columns. This test proved that it is possible to create three-dimensional silicon dioxide structures by growing a film on a framework of silicon.

Several challenges are associated with the practical application of these hybrid silicon-silicon dioxide structures. During the annealing process, thermal stresses tended to crack the walls. Additionally, any further surface preparation for bonding the silicon dioxide surface to another wafer might result in cracking. Some possible solutions to these challenges were considered, but were not pursued during the course of this project.

7.6 Challenges, Recommendations, and Future Work

There are a number of challenges that can be addressed in further micro-recuperator investigation.

Since the heat exchanger is designed to transfer energy from the hot exhaust gas stream to the cooler pre-combustor gas stream, local structure temperatures will approach the temperatures of each of the gas flows. The exhaust gas flow is expected to be around $1400 \text{ K}$, which is higher than the softening temperature of silicon. This is an issue of paramount concern to the successful integration of a recuperator with the microengine. Research into the use of refractory materials such as silicon carbide and cordierite for the recuperator structure need to be pursued.
The failures experienced in the experimental testing should also be addressed. Additional testing is required in order to refine the computer models used to predict the recuperator’s performance. Current experience indicates that the recuperators will consistently fracture at the flow inlet and outlet holes; this may be due to stress concentrations in the area, or microscopic cracks that are introduced during the silicon-Kovar sealing process. Clearly, the plumbing design of the recuperator should be revisited in order to ensure that the test devices will not fracture under high temperatures and elevated gas pressured.

In fact, a redesign of the entire flow inlet and outlet regions could provide a great benefit to further experimentation. The current scheme requires that the Kovar inlet and outlet tubes have slits machined into them in order to prevent blockage to the flow. However, the exhaust pressure drop measurements indicate that the current scheme may not be adequate. Instead of machining the Kovar, the first and second wafers might be redesigned with a countersunk hole which would position the Kovar tube precisely and still allow for unhindered flow. An example system of this type is shown in Figure 7.1.

The current limitations on the fabrication of the recuperators should also be examined. In particular, maximizing the depth of the heat transfer passages and minimizing the thickness of the coverplates will provide tremendous benefit to the engine’s performance. Although deeper passages could be simulated by stacking multiple recuperator wafers, the added complexity of bonding the entire microengine becomes prohibitive. Additionally, each added wafer level adds another coverplate layer, which is the prime contributor to the axial conduction losses. Thinning the coverplates is itself absolutely necessary to reduce the losses due to axial conduction; without thinner coverplates the effectiveness of this type of recuperator cannot exceed 70%. By optimizing the fabrication processes, it may be possible to extend the channel depths and thin the coverplates, improving the overall recuperator performance significantly.

![FIGURE 7.1](image_url)  
**FIGURE 7.1** – A schematic representation of a possible Kovar tube positioning technique. A countersunk hole (a) allows the gas to flow unimpeded into the plenum, while situating the Kovar in place precisely to be sealed (b).
7.7 Conclusions

The microrecuperator project demonstrated that the addition of a recuperator to the MIT microengine project can provide a real and substantial improvement to the engine's performance. Using silicon as the recuperator material, computer models indicated that the power specific fuel consumption can be improved by at least 33%, and the thrust specific fuel consumption by nearly 20%. Additional improvements can be accomplished with the use of other materials. An experimental system to test the performance of the recuperators was designed and built with the goals of maximizing its flexibility and minimizing its experimental error. Furthermore, heat exchangers with microscale features were successfully fabricated, packaged, and subjected to preliminary testing. The results from these preliminary test indicate that the heat exchangers will indeed perform as expected, and with further development may successfully be applied to microscale MEMS systems such as the MIT microengine.
## Appendix A
### Linear Recuperator Computer Models

#### ALPHA_RECT_I

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<table>
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<td>750</td>
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</table>

<table>
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<th>R (1/kJ/kgK)</th>
<th>mu (kg/m/s)</th>
<th>k (SI) (W/mK)</th>
</tr>
</thead>
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<td>3.35E-05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>rho (kg/m³)</th>
<th>nu (m/²s)</th>
<th>U (m/s)</th>
<th>ReD</th>
<th>Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>compressor</td>
<td>low</td>
<td>1.03E+00</td>
<td>3.22E-06</td>
<td>3.18E+01</td>
</tr>
<tr>
<td>exhaust</td>
<td>low</td>
<td>4.46E+01</td>
<td>7.46E-06</td>
<td>4.63E+01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>2.04</td>
<td>3.44</td>
<td>6.74</td>
<td>3.66</td>
<td>4.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>COMPRRESSOR</th>
<th>L/D</th>
<th>AR</th>
<th>f</th>
<th>t/AR</th>
<th>f, Blasius</th>
<th>t, Conventional</th>
<th>f, Petyi et al.</th>
<th>f, Peng et al.</th>
<th>f, Copeland</th>
<th>f, Copeland</th>
<th>f, White</th>
<th>f, (used)</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>61.05</td>
<td>8.59</td>
<td>0.116</td>
<td>0.098</td>
<td>0.599</td>
<td>1.030</td>
<td>3.120</td>
<td>0.194</td>
<td>0.496</td>
<td>0.780</td>
<td>0.599</td>
<td></td>
</tr>
<tr>
<td>EXHAUST</td>
<td>42.822</td>
<td>5.729</td>
<td>0.175</td>
<td>0.100</td>
<td>0.631</td>
<td>1.084</td>
<td>3.450</td>
<td>0.204</td>
<td>0.522</td>
<td>0.772</td>
<td>0.631</td>
<td></td>
</tr>
</tbody>
</table>

G = 0.8133

94
<table>
<thead>
<tr>
<th></th>
<th>$h_c$ (W/m²K)</th>
<th>$h_{ex}$ (W/m²K)</th>
<th>$mCp$ (J/kgK)</th>
<th>$UA$ (W/K)</th>
<th>NTU</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>1859.4</td>
<td>1304.2</td>
<td>1.22E-03</td>
<td>5.90E-03</td>
<td>4.845</td>
</tr>
<tr>
<td>Wall Ac (m²)</td>
<td>Cap Ac (m²)</td>
<td>lambda</td>
<td>CALCULATIONS</td>
<td>e</td>
<td>l</td>
</tr>
<tr>
<td>low</td>
<td>5.50E-06</td>
<td>1.80E-08</td>
<td>2.33E-01</td>
<td>7.26E-01</td>
<td>5.50E-01</td>
</tr>
<tr>
<td>q(W)</td>
<td>Tout,Comp (K)</td>
<td>Tout,Ex (K)</td>
<td>#REF!</td>
<td>618.1</td>
<td>681.9</td>
</tr>
<tr>
<td>low</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>deltaP (I) (Pa)</td>
<td>rho(ent)</td>
<td>rho (exit)</td>
<td>Ventry (m/s)</td>
<td>Vexit (m/s)</td>
</tr>
<tr>
<td>COMPRESS</td>
<td>low</td>
<td>17624</td>
<td>1.516</td>
<td>0.699</td>
<td>20.6</td>
</tr>
<tr>
<td>EXHAUST</td>
<td>low</td>
<td>13007</td>
<td>0.449</td>
<td>0.559</td>
<td>46.3</td>
</tr>
</tbody>
</table>
Appendix B
Radial Recuperator Computer Models

Pressure Ratio Model:

\[ h := 0.000500 \]
\[ r_1 := 0.00400 \]
\[ r_2 := 0.01000 \]
\[ n_o := 100 \]
\[ m := \frac{n_o}{0.00018} \]
\[ w_t := 0.000020 \]

\[ \text{win} := \frac{\left( (\pi \cdot r_2 - 2) - (n_o \cdot w_t) \right)}{2.5} \]

\[ \text{wout} := \frac{\left( (\pi \cdot r_2 - 2) - (n_o \cdot w_t) \right)}{2.5} \]

\[ w(r) := \text{win} + \frac{(\text{wout} - \text{win}) (r - r_1)}{r_2 - r_1} \]

\[ p(r) := \left[ (0.650 - 1.039 \cdot \frac{r - r_1}{r_2 - r_1}) \right] + 1.039 \]

\[ \mu(r) := \left[ (0.00003753 - 0.00002909 \cdot \frac{r - r_1}{r_2 - r_1}) \right] + 0.00002908 \]

\[ P(r) := 2 \cdot h + 2 \cdot w(r) \]

\[ A_1(r) := h \cdot w(r) \]

\[ V_I(r) := \frac{m}{(A_1(r) \cdot p(r))} \]

\[ D_h(r) := \frac{(4 - A_1(r))}{P(r)} \]
\[ \text{Rey}(r) := \frac{Vl(r) \cdot Dh(r) \cdot \rho(r)}{\mu(r)} \]

\[ f(r) := \frac{64}{\text{Rey}(r)} \]

\[ \frac{1}{2} \int_{r1}^{r2} \frac{f(r) \cdot \rho(r) \cdot Vl(r)^2}{Dh(r)} \, dr = 3.249 \times 10^3 \]

\[ \text{Po} := 1.8 \times 10^{13} \]

\[ \left[ \frac{-1}{2} \right] \int_{r1}^{r2} \frac{f(r) \cdot \rho(r) \cdot Vl(r)^2}{Dh(r)} \, dr \]

\[ \left[ \frac{-1}{2} \right] \int_{r1}^{r2} \frac{f(r) \cdot \rho(r) \cdot Vl(r)^2}{Dh(r)} \, dr + 1 = 0.982 \]
Heat Transfer Model:

\[ \text{mdot} := 0.00018 \]

\[ r1 := 0.004 \]

\[ r2 := 0.01 \]

\[ \text{Length} := r2 - r1 \]

\[ \rho_{\text{ex}} := 0.565 \]

\[ \rho_{\text{comp}} := 0.847 \]

\[ k_{\text{Si}} := 42.2 \]

\[ \mu := 0.0000335 \]

\[ WT := 0.000010 \]

\[ \theta := 1.7905 \frac{\pi}{180} \]

\[ h := 0.000500 \]

\[ \text{num} := 99 \]

\[ \text{PW}_{\text{ex}}(r) := \frac{1.5 (\pi \cdot r - \text{num} \cdot 2 \cdot WT)}{\text{num} \cdot 2.5} \]

\[ \text{PW}_{\text{comp}}(r) := \frac{(2 \cdot \pi \cdot r - \text{num} \cdot 2 \cdot WT)}{\text{num} \cdot 2.5} \]

\[ V_{\text{ex}}(r) := \frac{\text{mdot}}{h \cdot \text{num} \cdot \text{PW}_{\text{ex}}(r) \cdot \rho_{\text{ex}}} \]

\[ V_{\text{comp}}(r) := \frac{\text{mdot}}{h \cdot \text{num} \cdot \text{PW}_{\text{comp}}(r) \cdot \rho_{\text{comp}}} \]

\[ D_{\text{hex}}(r) := 4 \cdot \frac{(h \cdot \text{PW}_{\text{ex}}(r))}{2 (h + \text{PW}_{\text{ex}}(r))} \]

\[ D_{\text{hcp}}(r) := 4 \cdot \frac{(h \cdot \text{PW}_{\text{comp}}(r))}{2 (h + \text{PW}_{\text{comp}}(r))} \]

\[ \text{Re}_{\text{ex}}(r) := \frac{(V_{\text{ex}}(r) \cdot D_{\text{hex}}(r) \cdot \rho_{\text{ex}})}{\mu} \]

\[ \text{Re}_{\text{comp}}(r) := \frac{(V_{\text{comp}}(r) \cdot D_{\text{hcp}}(r) \cdot \rho_{\text{comp}})}{\mu} \]
\[ \begin{align*}
\text{Nu} & := 3.66 \\
k1 & := 0.0446 \\
k2 & := 0.0597 \\
k(r) & := k1 + \left[ (k2 - k1) \frac{r - r1}{\text{Length}} \right] \\
h\text{comp}(r) & := \text{Nu} \frac{k(r)}{D_{h\text{comp}}(r)} \\
h\text{ex}(r) & := \frac{k(r)}{D_{h\text{ex}}(r)} \\
cp1 & := 1075 \\
cp2 & := 1146 \\
cp(r) & := cp1 + \left[ (cp2 - cp1) \frac{r - r1}{\text{Length}} \right] \\
mcp(r) & := \frac{\text{mdot}}{\text{num}} \cdot cp(r) \\
\text{UA} & := \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} + \begin{pmatrix}
\frac{r2}{r1} & 2 \cdot h\text{comp}(r) \cdot h\text{dr} \\
\frac{r2}{r1} & 2 \cdot k(r) \cdot h\text{dr} \\
\frac{r2}{r1} & 2 \cdot h\text{ex}(r) \cdot h\text{dr} \\
\end{pmatrix}
\end{align*} \]

\[ \text{UA} \begin{pmatrix}
mcp \\
\frac{(r1 + r2)}{2} \\
\end{pmatrix} = 0.935 \]

\[ \text{NTU} := \frac{\text{UA}}{mcp} \begin{pmatrix}
mcp \frac{(r1 + r2)}{2} \\
\end{pmatrix} \]

\[ \frac{\text{NTU}}{(\text{NTU} + 1)} = 0.483 \]

\[ \text{mcp\_total} := \frac{1}{(r2 - r1)} \int_{r1}^{r2} (mcp(r)) \text{dr} \]
\( \text{mcp\_total} = 2.019 \times 10^{-3} \)

\[
\begin{align*}
\text{Ax}(r) := 2 \cdot WT \cdot h + 2 \cdot (2 \cdot WT + PW_{\text{ex}}(r) + PW_{\text{comp}}(r)) \cdot \frac{h}{5} \\
\text{k\_Si}(r) := 135576 \left[ 600 + \left[ 300 \cdot \frac{(r - r_l)}{(r_2 - r_l)} \right] \right]^{1.2036} \\
\int_{r_l}^{r_2} k_{\text{Si}}(r) dr \left[ \frac{1}{(r_2 - r_l)} \right] = 47.817 \\
\int_{r_l}^{r_2} \text{Ax}(r) dr \left[ \frac{1}{(r_2 - r_l)} \right] = 9.885 \times 10^8 \\
\text{Ax}(0.0049) = 7.22 \times 10^8 \\
\int_{r_l}^{r_2} (k_{\text{Si}}(r) \cdot \text{Ax}(r)) dr \left[ \frac{1}{(r_2 - r_l)} \right] = 4.579 \times 10^6 \\
k_{\text{Si}}(0.0084) = 42.181 \\
k_{\text{Si}}(0.0084) \cdot \text{Ax}(0.0049) = 3.045 \times 10^6 \\
\lambda := \frac{\int_{r_l}^{r_2} (k_{\text{Si}}(r) \cdot \text{Ax}(r)) dr \left[ \frac{1}{(r_2 - r_l)} \right]}{\text{mcp\_total} \cdot (r_2 - r_l)} \\
\lambda_2 := \left[ \int_{r_l}^{r_2} k_{\text{Si}}(r) dr \left[ \frac{1}{(r_2 - r_l)} \right] \int_{r_l}^{r_2} \text{Ax}(r) dr \left[ \frac{1}{(r_2 - r_l)} \right] \right]^{r_2}_{r_l} \\
\lambda = 0.378 \\
\lambda_2 = 0.39 \\
i := \frac{1}{1 + \text{NTU}} \left[ 1 + \sqrt{\frac{(\lambda \cdot \text{NTU})}{1 + \lambda \cdot \text{NTU}}} \right]
\end{align*}
\]
\[ i_2 := \frac{1}{1 + \frac{1 + \frac{(\lambda_2 \cdot NTU)}{\sqrt{1 + \lambda_2 \cdot NTU}}}{1 + \lambda_2 \cdot NTU}} \]

\( i = 0.489 \)

\( i_2 = 0.49 \)

\( 1 - i = 0.511 \)

\( 1 - i_2 = 0.51 \)
Appendix C
The Uncertainty in the Calculation of Effectiveness Using Heater Power Method

One of the equations used to calculate the recuperator effectiveness is based on the measured values for the power required by the second heater. The equation used to calculate the effectiveness is as follows:

\[ \varepsilon = \frac{m c_p (T_2 - T_1)}{m c_p (T_2 - T_1) + Q_{in} - Q_{loss}}. \]  

The experimental error for this equation is derived from the least squares method, which gives:

\[ w_\varepsilon = \left( \frac{d\varepsilon}{d m} w_m \right)^2 + \left( \frac{d\varepsilon}{d c_p} w_{c_p} \right)^2 + \left( \frac{d\varepsilon}{d T_1} w_{T_1} \right)^2 + \left( \frac{d\varepsilon}{d Q_{loss}} w_{Q_{loss}} \right)^2 + \left( \frac{d\varepsilon}{d Q_{in}} w_{Q_{in}} \right)^2 \right)^{1/2} \]

where \( T_1 \) and \( T_2 \) correspond to the values shown in Figure 4.3.

Each of the derivatives in Equation C.2 is given here, without explanation:

\[
\begin{align*}
\frac{d\varepsilon}{d Q_{loss}} &= \frac{m c_p (T_3 - T_1)}{(m c_p (T_3 - T_1) + Q_{in} - Q_{loss})^2} \tag{C.3} \\
\frac{d\varepsilon}{d Q_{in}} &= \frac{m c_p (T_3 - T_1)}{(m c_p (T_3 - T_1) + Q_{in} - Q_{loss})^2} \tag{C.4} \\
\frac{d\varepsilon}{d m} &= \frac{c_p (T_2 - T_1)}{m c_p (T_2 - T_1) + Q_{in} - Q_{loss}} - \frac{m c_p (T_2 - T_1)^2}{(m c_p (T_2 - T_1) + Q_{in} - Q_{loss})^2} \tag{C.5} \\
\frac{d\varepsilon}{d T_1} &= \frac{m c_p}{m c_p (T_2 - T_1) + Q_{in} - Q_{loss}} + \frac{m^2 c_p^2 (T_2 - T_1)}{(m c_p (T_2 - T_1) + Q_{in} - Q_{loss})^2} \tag{C.6} \\
\frac{d\varepsilon}{d T_2} &= \frac{m c_p}{m c_p (T_2 - T_1) + Q_{in} - Q_{loss}} - \frac{m^2 c_p^2 (T_2 - T_1)}{(m c_p (T_2 - T_1) + Q_{in} - Q_{loss})^2} \tag{C.7}
\end{align*}
\]
\[
\frac{d\varepsilon}{dc_p} = \frac{\dot{m}(T_2 - T_1)}{mc_p(T_2 - T_1) + Q_{in} - Q_{loss}} = \frac{\dot{m}^2 c_p (T_2 - T_1)^2}{(mc_p(T_2 - T_1) + Q_{in} - Q_{loss})^2}
\]

The term \(Q_{loss}\) which appear in some of the derivatives is itself a calculation from the equation:

\[
Q_{loss} = \dot{m}c_p(T_{in} - T_{out}) + VI.
\]

The error in \(Q_{loss}\) is calculated using the same method, which gives:

\[
w_{Q_{loss}} = \left(\frac{dQ_{loss}}{dm}w_{in}\right)^2 + \left(\frac{dQ_{loss}}{dc_p}w_{c_p}\right)^2 + \left(\frac{dQ_{loss}}{dT_{in}}w_{T_{in}}\right)^2 + \left(\frac{dQ_{loss}}{dT_{out}}w_{T_{out}}\right)^2 + \left(\frac{dQ_{loss}}{dV}w_{V}\right)^2 + \left(\frac{dQ_{loss}}{dl}w_{l}\right)^2\right)^\frac{1}{2}
\]

The derivative in Equation C.10 are shown here:

\[
\frac{dQ_{loss}}{dm} = c_p(T_{in} - T_{out}) \quad \text{C.11} \quad \frac{dQ_{loss}}{dT_{out}} = \dot{m}c_p \quad \text{C.14}
\]

\[
\frac{dQ_{loss}}{dc_p} = \dot{m}(T_{in} - T_{out}) \quad \text{C.12} \quad \frac{dQ_{loss}}{dV} = I \quad \text{C.15}
\]

\[
\frac{dQ_{loss}}{dT_{in}} = \dot{m}c_p \quad \text{C.13} \quad \frac{dQ_{loss}}{dl} = V \quad \text{C.16}
\]

Likewise, \(Q_{in}\) is calculated from

\[
Q_{in} = VI, \quad \text{C.17}
\]

which has an error of

\[
w_{Q_{in}} = \left(\frac{dQ_{in}}{dV}w_{V}\right)^2 + \left(\frac{dQ_{in}}{dl}w_{l}\right)^2 \right)^\frac{1}{2}.\]

The derivatives in that equation are:

\[
\frac{dQ_{in}}{dV} = I \quad \text{and} \quad \frac{dQ_{in}}{dl} = V. \quad \text{C.19}
\]

In all of these equations, there are only eight independent variables, assuming that the mass flow is constant throughout the system and that the specific heat is constant as well. In situations where it is suspected that one of these conditions is not met, the equations can be adjusted appropriately to reflect the changing values of the variables. However, as discussed in Chapter 2, these assumptions are reasonable for the conditions being studied here. The eight variables, along with their specified errors, are then as follows:
uncertainty, along with their errors.

Based on these values, the error associated with measurements:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reading</th>
<th>Specified Error</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$ (K)</td>
<td>450</td>
<td>1% of Reading</td>
<td>4.5</td>
</tr>
<tr>
<td>$T_2$ (K)</td>
<td>490</td>
<td>1% of Reading</td>
<td>4.9</td>
</tr>
<tr>
<td>$T_3$ (K)</td>
<td>600</td>
<td>1% of Reading</td>
<td>6.0</td>
</tr>
<tr>
<td>$T_4$ (K)</td>
<td>425</td>
<td>1% of Reading</td>
<td>4.25</td>
</tr>
<tr>
<td>Mass Flow $m$ (kg/s)</td>
<td>0.0001</td>
<td>1% of Maximum Flow</td>
<td>2.808E-06</td>
</tr>
<tr>
<td>Specific Heat $c_p$</td>
<td>1050</td>
<td>± 0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Applied Voltage V (volts)</td>
<td>10</td>
<td>0.1% of Reading</td>
<td>0.01</td>
</tr>
<tr>
<td>Applied Current I (amps)</td>
<td>11</td>
<td>0.1% of Reading</td>
<td>0.011</td>
</tr>
</tbody>
</table>

As an example computation, consider an experiment that produces the following measurements:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reading</th>
<th>Specified Error</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>11</td>
<td>0.01</td>
<td>0.11</td>
</tr>
<tr>
<td>$I$</td>
<td>10</td>
<td>0.011</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Based on these values, the error associated with $Q_{in}$ is:

<table>
<thead>
<tr>
<th>Variable $s$</th>
<th>$\frac{dQ_{in}}{ds}$</th>
<th>$w_s$</th>
<th>$\frac{dQ_{in}}{ds}$</th>
<th>$w_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>11</td>
<td>0.01</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>10</td>
<td>0.011</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

TABLE C.1 – The eight independent variables that appear in the error calculation for the recuperator effectiveness.

TABLE C.2 – Example values for each of the measurements used in the calculation of effectiveness, along with their errors.

TABLE C.3 – Example values for each of the variables used in the calculation of $Q_{in}$ uncertainty, along with their errors.
These values give an error in $Q_{in}$ as follows:

<table>
<thead>
<tr>
<th>$Q_{in}$</th>
<th>$w_{Q_{in}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>0.155563</td>
</tr>
</tbody>
</table>

**TABLE C.4 – The calculated value of $Q_{in}$ and the uncertainty in its value.**

The error associated with each of the terms in $Q_{loss}$ is:

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\frac{dQ_{loss}}{ds}$</th>
<th>$w_s$</th>
<th>$\frac{dQ_{loss}}{ds}$</th>
<th>$w_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}$</td>
<td>-115500</td>
<td>2.8083E-06</td>
<td>-0.3244</td>
<td></td>
</tr>
<tr>
<td>$c_p$</td>
<td>-0.011</td>
<td>0.5</td>
<td>-0.0055</td>
<td></td>
</tr>
<tr>
<td>$T_1$</td>
<td>0.105</td>
<td>4.5</td>
<td>0.4725</td>
<td></td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.105</td>
<td>4.9</td>
<td>0.5145</td>
<td></td>
</tr>
<tr>
<td>$T_3$</td>
<td>0.105</td>
<td>6</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>$T_4$</td>
<td>0.105</td>
<td>4.25</td>
<td>0.44625</td>
<td></td>
</tr>
<tr>
<td>$V$</td>
<td>11</td>
<td>0.01</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>10</td>
<td>0.011</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE C.5 – Example values for each of the variables used in the calculation of $Q_{loss}$ uncertainty, along with their errors.**

These values give an error in $Q_{loss}$ of 0.88941, as shown in Table C.1

<table>
<thead>
<tr>
<th>$Q_{loss}$</th>
<th>$w_{Q_{loss}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.45</td>
<td>0.88941</td>
</tr>
</tbody>
</table>

**TABLE C.6 – The calculated value of $Q_{loss}$ and the uncertainty in its value.**
The terms for the error in the effectiveness $\varepsilon$ are:

<table>
<thead>
<tr>
<th>Variable $s$</th>
<th>$\frac{d\varepsilon}{ds}$</th>
<th>$w_s$</th>
<th>$\frac{d\varepsilon}{ds}$ $w_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>2193.75</td>
<td>2.8083E-06</td>
<td>0.0001</td>
</tr>
<tr>
<td>$c_p$</td>
<td>0.00021</td>
<td>0.5</td>
<td>-0.0055</td>
</tr>
<tr>
<td>$T_1$</td>
<td>-0.0042</td>
<td>4.4</td>
<td>-0.0186</td>
</tr>
<tr>
<td>$T_2$</td>
<td>0.00422</td>
<td>6</td>
<td>0.02531</td>
</tr>
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<td>$Q_{loss}$</td>
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**TABLE C.7** – Example values for each of the variables used in the calculation of recuperator effectiveness uncertainty, along with their errors.

Finally, the resulting terms are:

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<th>$\varepsilon$</th>
<th>$w_\varepsilon$</th>
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**TABLE C.8** – The calculated value of recuperator effectiveness and the uncertainty in its value.

This indicates that the effectiveness of a recuperator that produces the measurements listed above is $0.325 \pm 0.036$, or $32.5\% \pm 3.6\%$.

Using the temperature deficit method of calculating the effectiveness of the recuperator, the error would derive from the equation

$$w_\varepsilon \left[ \left( \frac{d\varepsilon}{dT_3} w_{T_3} \right)^2 + \left( \frac{d\varepsilon}{dT_2} w_{T_2} \right)^2 + \left( \frac{d\varepsilon}{dT_1} w_{T_1} \right)^2 \right]^{\frac{1}{2}},$$

C.20

with $w_n$ being the uncertainty or error associated with each reading of the value of $n$, and the temperature readings corresponding to the values depicted in Figure 4.3.
### Appendix D

**Linear Recuperator Computerized Data Acquisition Sample Data Set**

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<th>Data Point</th>
<th>Time (min.)</th>
<th>T₁ K</th>
<th>T₂ K</th>
<th>T₃ K</th>
<th>T₄ K</th>
<th>Htr2Pwr (W)</th>
<th>m g/s</th>
<th>P₁ atm</th>
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References


