Design, Analysis and Control of an Autonomous Conveyance Module for Well Exploration

by

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Bachelor of Science in Mechanical Engineering, 1999
Massachusetts Institute of Technology

submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of
Master of Science in Mechanical Engineering

at the
Massachusetts Institute of Technology

February 2001

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Abstract

Sophisticated sensors have been developed by the oil industry to provide accurate measurements of downhole physical characteristics. Traditionally, a module containing the sensors required for a given measurement job or “logging job” is lowered inside the well attached to the surface by a cable that provides power and support (wireline). The operation relies on gravity to convey the sensors to the section inside the well where measurements are needed. However, gravity does not help when the section to be attained is horizontal or at a relatively high inclination. As a preliminary response to this problem, several companies have developed tools called “tractors” which are motorized push-pull or wheeled vehicles that are attached to the front of the sensor modules and use the power provided by the wireline to pull the sensor pack inside the oil well. This preliminary solution seems to work appropriately for some horizontal and inclined wells provided the travel distance is not too large since the friction of the cable against the well casing becomes too large to overcome as its length increases. Considering that many horizontal and inclined wells in the market extend to lengths as large as 40,000 feet, considerable portions of live wells are still unattainable for measurements. Therefore, there is still a need for a solution that allows the complete exploration of horizontal wells.

Since the wireline or cable is the main problem when attempting to reach deep inside inclined and horizontal wells, this thesis proposes the development of an autonomous conveyance module for the sensors. However, an autonomous vehicle needs to carry its own power supply instead of relying on the continuous power available through the wireline. In addition, space constraints limit the amount of the supply (batteries) that can be carried downhole. Therefore, power efficiency is a first priority in the design of a module capable of traversing the required long travel distances. In general, traditional propulsion systems are very energy-inefficient since friction and heat dissipation, among others, take a good percentage of the energy used for motion. Consequently, an energy-efficient propulsion system is required in order to make the autonomous logging tool concept more feasible.

This thesis presents the theoretical analysis, design and control of a novel fluid propulsion system. The proposed solution uses the momentum of the fluid flow with a deployable, non-backdriveable mechanism in order to minimize battery usage and achieve controlled motions. A dynamical model of the physical behavior is presented and unknown model parameters are identified and validated through experimentation. The nonlinear dynamics of a module prototype are analyzed. Finally, using feedback linearization, and robust control techniques, appropriate control laws are derived in order to achieve satisfactory motion performance.

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Acknowledgments

I would like to thank the Schlumberger-Doll Research Center for allowing me the time and resources to perform this research. My sincerest gratitude goes to Bruce Boyle, Dr. Raghu Mahdavan, and Dr. Olivier Sindt of the Conveyance Technologies program for their guidance and support, which made this work possible.

I would also like to thank my thesis supervisor, Professor Samir Nayfeh for carefully reading this manuscript and helping make it coherent.
Finally, I would like to thank my family for their continuous encouragement.
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Chapter 1: Introduction

1.1 Background

In the oil industry, production logging is the process of evaluating a well by determining the properties of its rock formations and the existent fluids (e.g. oil, water, and gas). A sonde, which can contain one or several sensors, is attached to the end of a cable or “wireline” and lowered inside the well as shown in figure 1.1. Electrical, acoustical and radioactive properties of the formations and their fluids are measured by remote sensing as the sonde is brought back up the hole at a constant speed. These measurements provide the operator of the well with detailed information on the nature and behavior of fluids in the drilled hole or “borehole” during production or injection.

Production logging emerged in the 1930s with the introduction of the temperature log. By the late 1950s and early 1960s, many different production logging techniques were being used. The problem was that each technique required a separate trip into the well. The 1970s saw tools that combined several measurement techniques, meaning a more efficient single run in the well. Improvements have continued through the 1980s to the present day with better sensors and deployment methods. The latest tools use new technologies to completely measure production data around the borehole. Many sensors can be combined into one tool and recorded simultaneously to measure fluid entries and exits, standing liquid levels, bottom hole flowing and shut-in pressures, pressure losses in the tubing, and the integrity of the gravel pack and hardware assemblies. Since the measurements are made simultaneously, their correlation is less affected by any well instability that might cause downhole conditions to vary over a period of time. The sensors available for logging include: thermometers, fluid sampling, flowmeters, manometers, noise probes, calipers, radioactive tracers, water flow logging, gravel pack logging, phase velocity logging, three-phase holdup, and down hole video cameras. Finally, a production logging tool (PLT) string will always include a depth control device since the correlation between the measurements and the depth at which they were taken is crucial for their clear interpretation.
Downhole tools must also withstand the extreme well environment. Temperatures inside the borehole can range between -40 and 200 degrees Celsius, and the pressures vary from 0 to 15000 Psi (gage). In addition, flow regimes can be single or multiphase and with widely ranging densities and viscosities. Typical wells extend to depths of 10000 feet (vertically), and some horizontal or inclined branches detach at various heights. These branches can in turn extend for several thousand feet. Finally, the inside diameter of the well casing generally ranges between 1.2 and 6.2 inches and some obstacles like joints, tubing nipples, perforations, distorted or partially collapsed tubing, and some sections with sand and gravel deposits are also typically found in the borehole [17]. Obviously, these conditions vary among wells, nevertheless logging tools must be designed to withstand, fit and navigate through most of them so they can service the majority of the market.
1.2 Problem Statement

Logging measurements are a crucial part of the production process of a well because of the insight they give. Basically, their information helps determine the economic viability of exploiting a well. As explained in the previous section, PLTs are attached to the wireline and then lowered inside the well to attain the depths at which measurements are required. When the well to be logged is vertical, or nearly vertical, the weight of the tools and the weight of the cable are sufficient to overcome any friction against the borehole wall, as well as any resistance due to fluid drag. Therefore, it is relatively easy to lower any type of tool inside wells with inclinations between 0 and 75 degrees relative to the vertical axis. However, in recent years there has been a shift in the industry and more and more wells are drilled wherein at least a portion of the borehole is horizontal or at a large angle relative to the vertical axis. In this type of well gravity does not help convey the tools attached to the wireline. Hence, conveying PLTs to the inclined parts of the well becomes more complicated if not impossible. In this case, a solution is to push the logging tools with a thick-walled steel pipe denominated “drill string” used in rotary drilling [17]. However, this process is time consuming, expensive, and can damage the logging tools as well. Alternatively, some companies have developed wheeled or push-pull propulsion modules, commonly denominated as “tractors”. These modules are attached to the front of the sensor pack, and powered by the wireline. Hence, they help convey the sensor pack inside the borehole when gravity can not be used. Even so, after a couple of thousand feet of travel in a fairly inclined or horizontal part of the well the friction between the wireline cable and the well casing becomes too large to overcome. Hence, deep sections of these wells are still impossible to attain with these methods.

For an optimal logging job, assuming the sensor pack works properly, there are essentially two functional requirements. First, good motion control of the logging tool has to be achieved. The logging tool has to be able to attain any point inside the well, which translates into having control over its depth. Also, to perform good measurements the logging tool must have good control of its displacement rate as well. This motion control must be achieved regardless of the state of the downhole environment. Secondly, the PLT should not get stuck easily inside the well since this would jeopardize production.

As previously discussed, using the weight of the tools to achieve their motion does not always work, and tethered propulsion modules do not provide large ranges of motion because of friction problems.
Hence, different design parameters should be used to address the problem of moving the logging tools inside the well. An alternative method of conveyance should address any functional requirements not satisfied by the current methods and perform at least at a comparable level in settings were previous methods worked. A propulsion system, integrated to the PLT string, provides motion independently of the inclination of the well. Furthermore, in this modality a cable is not needed to control the rate of motion of the tools, and actually restricts the motion by causing friction problems. Hence, the use of the wireline services can be avoided, and an “autonomous logging tool” can be used to navigate inside the wells and perform the required measurements without being attached to the surface. However, by eliminating the wireline one also eliminates its advantages. From a technical prospective there are three advantages when using a wireline in the conveyance of tools. First, additional energy can be provided from the surface to downhole tools whenever needed. Secondly, in the event of an accident were the tools become jammed inside the well, the wireline can be used to pull them free. Finally, in vertical wells, the wireline is used to control the rate at which the PLTs move.

Hence, for an autonomous tool to be practical it must overcome three problems: power consumption, retrieval from downhole, and motion control. The problem of retrieval of tools from downhole, commonly referred as fishing, is not new in the industry and some techniques for this have been refined. Essentially, the process consists of using a cable, not necessarily the wireline, to catch and pull free the trapped tools. Presently, in most downhole tools, the housing enclosing the connectors at the downhole end of the wireline has a neck or reduced diameter designed to be caught securely during "fishing" operations. This feature can also be incorporated into tools not using the wireline. Therefore, we will center our study only to the problems of power consumption and motion control.

Downhole tools are electromechanical devices, hence an autonomous version requires batteries to supply power. However, current battery technology is not advanced enough to provide sufficient energy for a round trip inside a well when using traditional propulsion systems. Even lithium batteries, which have the highest energy density among all batteries, can not provide as much energy per volume as gasoline or other fuels [6]. Therefore, power supplies using batteries need to be recharged often, which is a wide spread problem in mobile robotics. Consequently, power requirements need to be kept at minimum if single runs inside the wellbore are to be made without the necessity of recharging the power supply in the middle of a
“mission”. Failing to do so would require the implementation of some sort of recharging stations along the well casing, so the autonomous module can complete a trip, bringing more complexity to the project.

Wheeled devises, propellers, and inchworm mechanisms are all propulsion devices that can be used effectively to achieve motion inside a pipe filled with fluids. However, the problem when using these propulsion mechanisms is their low energy-to-motion performance ratios. Among all, wheeled mechanisms are the most efficient if designed correctly. Nevertheless, not enough energy can be carried on board a reasonably seized autonomous device to achieve a round trip since distances inside a well are prohibitive. A possible solution, whose implementation is the subject of this thesis, is the use of the flow existent inside a well as means of an alternative propulsion force. A design is presented in which the fluid momentum is used to achieve motion with minimal internal power consumption. Such a device can be combined with traditional wheeled modules to achieve the propulsion necessary for an autonomous logging tool, providing motion control with an alternative low-power-consumption module.

1.3 Efficient propulsion system

For our purposes, let us define a "propulsion mechanism" to be a device that achieves the displacement of the piece of equipment in which it is contained. In our case, the motion needs to be achieved through fluids. The main problem with traditional propulsion mechanisms is that a good percentage of the energy that should be devoted to motion is actually lost through friction and heat dissipation (no motor or actuator is \( \%100 \) efficient). In addition, mechanisms such as wheeled devices or propellers require constant activity of the propulsion apparatus to achieve motion, thereby consuming energy all the time. Given our constraints, an ideal propulsion mechanism would be one that can achieve controlled motions using very little or no energy.

Under normal operating conditions, only four forces act on objects inside an oil well: drag, buoyancy, friction, and gravity. The drag is caused by the existent fluid flow inside the well and it acts in the direction of flow. This flow could be of single or multiphase nature (e.g. pure oil, water or combinations with gas). The buoyancy is the effect of the difference in densities between the tool string and its environment. Frictional forces arise because in order to perform accurate measurements, logging tools have to be aligned with the wellbore axis thus, spring loaded devices called centralizers are always in contact with the
borehole. Also, viscous friction is due to fluid interaction with the tools. Finally, downhole tools are usually made out of high strength alloys in order to withstand the harsh environments of wells, and their lengths together with their weights can vary a lot.

Among these forces, the weight of the tools is usually the largest because logging tools are in general streamlined and open to the fluid environment to keep drag forces and buoyancy minimal. Additionally, the centralizing parts that press against the well casing have wheeled joints so friction is minimized. However, given the flow ranges present in the wellbore, if drag effects are maximized, they can eventually overcome the weight of the tools plus any friction with the wellbore, and "lift" the entire tool string. If a device is designed so as to control accurately the magnitude of drag forces exerted on the tool string then it will be possible to achieve a fairly accurate motion control.

In principle, a mechanism that controls the drag should consume less power than traditional propulsion systems since it does not need to overcome external forces at all times, like wheeled devices, but instead uses their momentum to its advantage. As an analogy, one can think of the difference of energy consumption between a parachute and an elevator: both mechanisms can bring objects safely to ground, however the parachute does not need any energy to operate except for the one used to deploy it. If we can control this mechanism such as to accomplish precise motions and velocities, trajectory control could be achieved with minimal use of energy in comparison to traditional propulsion devices.

Therefore, we want to design and control a mechanism that uses the momentum provided by fluid flow to achieve axial displacement inside well pipes. If we want to use the momentum provided by fluid flow, the simplest design concept is to create an obstruction in the pipe thereby generating a pressure differential across the obstruction. The force resulting from that pressure differential can then be used for motion control.

1.4 Thesis overview

The objective of this research is to solve the problem of designing and controlling an obstruction mechanism for an autonomous logging tool. This mechanism should achieve "controlled motions" up and down stream inside segments of oil wells with low power consumption. In Chapter 2, we provide a study of the downhole dynamic behavior of a tool with a generic obstruction mechanism. Key physical relationships
are presented relating the geometry of the obstruction to its performance as part of the propulsion mechanism. The main result of this section is a generalized equation of motion for centralized objects moving inside pipe sections that we use for simulations (e.g. external dynamics). In Chapter 3, we describe the mechanical design of the proposed prototype, which is the main contribution of this thesis. Based on insights from the dynamic behavior analysis, an obstruction shape is chosen and a mechanism that implements it is devised. The methodology for material and component selection is explained accordingly. In Chapter 4, we explain how the unknown model parameters, for the obstruction shape chosen, are identified through experimentation. The results are presented along with a description of the experimental procedure. The dynamics of the electromechanical propulsion module apparatus are presented in Chapter 5. In Chapter 6, the results from the analysis of the obstruction apparatus dynamics and the module external dynamics are combined into a complete model of the module behavior, which is of nonlinear nature. The state representation of this model is used for nonlinear control design. An analysis of the module stability in open and closed loop is presented along with an appropriate implementation of control laws. The use of feedback linearization and robust control techniques for the implementation of the control laws is explained in detail, and simulations of the module trajectory tracking are presented. The hardware architecture required for the module is also laid out. Finally, in Chapter 7 we present the conclusions, a summary of the contributions of this thesis, and recommended future work. All the machine drawings, some component fabrication methodologies, the control algorithms used for the simulations, and some mathematical background for the control analysis can be found in appendices A, B, C, and D respectively.
2.1 Dynamic behavior model

The simplest way to exploit the momentum provided by fluid flow is to create an obstruction in the pipe flow. Figure 2.1 is a diagram of the basic physical elements involved in the implementation of an obstruction in the fluid flow inside a well. Fluid arrives at the obstruction with an average velocity \( v_1 \), and creates a pressure \( P_1 \) that acts underneath the obstruction. The average velocity of the fluid as it leaves the obstruction is \( v_2 \), and a pressure \( P_2 \) acts on top of the obstruction. The radius of the tool is shown as \( R_1 \), the radius of the obstruction is \( R_2 \), and the radius of the pipe is denoted as \( R \).

![Figure 2.1: Fluid flow obstruction concept.](image-url)
The obstruction behavior can be analyzed as a pipe flow problem. For the fluid dynamics analysis we use a cylindrical control volume underneath the obstruction, of fixed length \( l \) and radius \( R \), attached to the tool as it moves. Let us define the area \( A_1 \) to be the area between the well I.D and the tool O.D, and the area \( A_2 \) to be the area between the well I.D and the obstruction O.D, hence assuming that both the tool and the obstruction used have circular projected areas,

\[
A_1 = \pi \left( R^2 - R_1^2 \right)
\]

(2.1)

\[
A_2 = \pi \left( R^2 - R_2^2 \right)
\]

(2.2)

This assumption is quite normal for downhole hardware since the medium itself (the well casing) is of cylindrical nature thus in order to achieve good mobility most hardware is designed in this fashion.

The Reynolds Transport Theorem applied to the conservation of mass inside our control volume can be written as:

\[
\frac{d}{dt} \left\{ \iiint_{c.v} \rho dV \right\} + \iiint_{c.s} \rho (\vec{v}_{rel} \cdot \hat{n}) \, dA = 0
\]

where \( \vec{v}_{rel} = \vec{v}_{absolute} - \vec{v}_{c.v} \), and \( \rho \) is the density of the fluid media. Assuming the fluid inside the well is incompressible, the above equation simplifies to:

\[
\sum (A \vec{v}_{rel})_{in} = \sum (A \vec{v}_{rel})_{out}
\]

The tool and the control volume travel at a velocity \( v_{c,v} = \dot{z} \) (positive upward). The absolute velocity of the fluid coming into the control volume at \( v_{absolute} \) is \( v_1 \), therefore the relative velocity of the fluid coming into the C.V. is \( v_{rel} = v_1 - \dot{z} \). Alternatively, let us define the relative velocity leaving the control volume as \( v_{rel} = v_2 - \dot{z} = v_{rel2} \). Therefore, from the conservation of mass across the control volume we find that the relative velocity of the fluid coming out of the obstruction can be expressed in terms of the rest of the variables as:
\[ v_{rel2} = \frac{(v_1 - \dot{z})A_1}{A_2} \]

(2.3)

We now apply the Reynolds Transport Theorem (RTT) to the conservation of linear momentum on our control volume and get:

\[ \frac{d}{dt} \int_{C'} \int_{\Sigma'} \vec{v} \rho dV + \int_{C'} \int_{\Sigma'} \vec{V}_{rel} \vec{v}_{rel} \cdot d\vec{S} = \sum \vec{F}_x + \sum \vec{F}_B \]

The forces, \( F_x \), exerted on the control surface are mainly the pressure and viscous forces, and the only force, \( F_B \), acting at the control volume center is the gravity force. We purposely neglect any viscous stresses on the control volume since their magnitude is very small compared to the forces due to pressure. Therefore, for our accelerating control volume, the conservation of momentum can be written as,

\[ \frac{d}{dt} \int_{C'} \int_{\Sigma'} \vec{v}_{rel} \rho dV - \rho v_{rel2}^2 A_1 + \rho v_{rel2}^2 A_2 = \Delta P \frac{\pi R^2}{4} \left( \ddot{z} + g \right) \left( \rho_1 \pi R_1^2 L_1 - \rho_2 \pi R_2^2 L_2 + \rho l A_1 \right) \]

where \( \rho_1 \) and \( L_1 \) are the density and length of the tool providing the obstruction, \( \Delta P \) is the pressure differential across the obstruction (\( \Delta P = P_1 - P_2 \)), \( g \) is the gravitational acceleration, and \( \ddot{z} \) is the acceleration of the control volume. The term \( v_{rel2} \) represents the average z-component of velocity of all the fluid, relative to the control volume [3][4][5]. Thus,

\[ \frac{d}{dt} \int_{C'} \int_{\Sigma'} \vec{v}_{rel} \rho dV = \rho A_1 \frac{dV_{rel2}}{dt} = \rho A_1 l (\dot{v}_z - \dot{z}) \]

where \( \dot{v}_z \) is the average absolute acceleration of the fluid in the control volume. Solving the Navier-Stokes equation for one-dimensional incompressible flow, and then differentiating the result would yield an approximation of the actual value for the fluid acceleration. However, we can simply neglect \( \dot{v}_z \). If the velocity of the fluid flow across the control volume changes smoothly, then our assumption is justifiable. Otherwise, the result of this assumption would be to simply underestimate the momentum of the fluid in the control volume. The impact in the accuracy of our model will not be substantial unless the mass of the fluid in the control volume is significantly bigger than the mass of the tool which is not the case here. The
density of downhole tools is generally bigger than that of the fluids (oil and gas mixtures). If we let
\[ m' = \rho_1 \pi R_1^2 L_1 - \rho_2 \pi R_2^2 L_2, \]
denote the equivalent mass of the module (mass minus buoyant effects) we get
\[ \ddot{m}' = \rho v_{rel}^2 A_1 - \rho v_{rel}^2 A_2 + \Delta P \pi R^2 - \rho g A_1 I - m' g \]

Equation 2.4 describes the motion of the module inside a pipe. In order to solve it we need to know the values of the fluid absolute inlet velocity \( v_1 \), density \( \rho \), and the differential pressure \( \Delta P \). The values of the fluid velocity and density are a priori unknown but bounded. Typical flow velocities inside an oil well are between 0.1 and 10 \( (m/s) \), and the densities of the medium range between 500 and 1500 \( (kg/m^3) \).

We show in Chapter 6 that control laws can handle parametric uncertainties in the model provided that we have bounds for them. However, the value of the differential pressure is harder to bound directly since it depends on the obstruction effects. Hence it is necessary to attempt an approximation. If we apply the RTT to consider the conservation of energy in our accelerating control volume we would get an equation similar to 2.4. However, a simpler view can provide a straightforward and useful relation. At steady state we need not include the effects of the control volume acceleration in the energy equation thus we can use the simpler relation,
\[ \frac{d}{dt} \int_{C,V} e \rho dV + \int_{C,S} e \rho v_{rel} \cdot dS = \dot{Q} - \sum W \]
where \( e \) is the total energy of the control volume per unit mass, \( \dot{Q} \) is the heat supplied to the system by the surroundings (we will assume none) and \( \sum W \) is the sum of all the “shaft” work done by the system. Furthermore, for steady flow along a streamline between points 1 and 2 the above equation can be simplified to the extended Bernoulli equation (or mechanical energy equation):
\[ \pm w_i - \frac{2}{\rho} \frac{dP}{\rho} - gh_{loss} = \left( \frac{v_{rel}^2}{2} + gz \right)_1 - \left( \frac{v_{rel}^2}{2} + gz \right)_2 \]
where $\pm w$ is the work input/output of the moving fluid due to pumps or turbines (in this case none), and $gh_{\text{loss}}$ is an energy dissipation term due to heat losses, frictional losses, and form losses. Evaluating the above equation for our control volume we get:

$$P_1 + \frac{1}{2} \rho (v_1 - \dot{z})^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g z_2 + \frac{1}{2} \rho (v_1 - \dot{z})^2 K_i$$

with $z_2$ and $z_1$ being the heights of the top and bottom of the control volume inside the pipe with respect to an inertial reference frame. The loss coefficient $K_i$ is associated with the obstruction in the pipe and is defined as: $(K_i)_x = (h_{\text{loss}})_x \frac{2g}{v_x}$. Rearranging the terms of the above equation we get an expression for the pressure differential across the obstruction:

$$\Delta P = \frac{1}{2} \rho v_2^2 + \rho g l + \frac{1}{2} \rho (v_1 - \dot{z})^2 (K_i - 1)$$

Equation 2.5 is clearly only an approximation of the existent differential pressure since we are neglecting the effects of the accelerating control volume, and the flow around the obstruction will certainly not be steady. Nevertheless, equation 2.5 allows us to provide a model structure for $\Delta P$ with parameters for which we have bounds such as $v_x$ and $\rho$. The loss coefficient $K_i$ is unknown, and its value is highly dependent on the shape of the obstruction. Therefore, $K_i$ should be experimentally identified for the obstruction profile we choose in our design.

Combining equations 2.3, 2.4, and 2.5 we can obtain the equation of motion of the obstruction tool inside the well. Factoring the terms we can write the equation of motion as:

$$\ddot{z} = a_2 \dot{z}^2 + a_1 \dot{z} + a_0$$

(2.6)
where:

\[
a_2 = \rho \left( \frac{(K_i-1)\pi R^2}{2} + A_i - \frac{A_i^2}{A_2} + \frac{A_2^2}{2A_2^2} \pi R^2 \right) / m' \\
a_1 = -2\nu_1 \rho \left( \frac{(K_i-1)\pi R^2}{2} + A_i - \frac{A_i^2}{A_2} + \frac{A_2^2}{2A_2^2} \pi R^2 \right) / m' \\
a_0 = \left( \nu_1^2 \rho \left( \frac{(K_i-1)\pi R^2}{2} + A_i - \frac{A_i^2}{A_2} + \frac{A_2^2}{2A_2^2} \pi R^2 \right) + \rho g(l(R_i^2 - m'g) / m' \right) / m'
\]

Equation 2.6 is a nonlinear second order differential equation best known as the Riccati equation.

Equation 2.6 can be solved using a change of variables such as: \( y = \dot{z} \), which reduces the order of the equation that can now be written as:

\[
\frac{dy}{a_2y^2 + a_1y + a_0} = dt
\]

and can be solved by integration. If we take \( s_1 \) and \( s_2 \) to be the roots of \( a_2y^2 + a_1y + a_0 = 0 \) then we can find two sets of solutions:

If \( s_1 = s_2 \), then:

\[
y = \frac{1}{C - a_2t} + s_1
\]

where \( C \) is a constant that can be found from the initial conditions. If at \( t = 0 \), \( y(t) = y_0 \) then:

\[
C = \frac{1}{y_0 - s_1}, \text{ and if we change the variables back we obtain:}
\]

\[
\dot{z} = \frac{(\dot{z}_0 - s_1)}{1 - a_2t(\dot{z}_0 - s_1)} + s_1
\]

(2.7)

If, on the other hand, \( s_1 \neq s_2 \), then:

\[
y = \frac{s_1 - s_2Ce^{a_2(s_1-s_2)t}}{1 - Ce^{a_2(s_1-s_2)t}}
\]

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Analogously, if at $t = 0$, $y(t) = y_0$ then: $C = \frac{y_0 - s_1}{y_0 - s_2}$ and if we change the variables back we obtain:

$$\dot{z} = s_2 (\dot{z}_0 - s_2) - s_2 (\dot{z}_0 - s_1) e^{\alpha_1 (s_2 - s_1) t}$$

(2.8)

For a set of given design parameters, we can determine the module velocity inside the well using equations 2.7 or 2.8 provided we know the value of the loss coefficient brought by the obstruction. The module displacement can then be found by integration and considering any initial conditions. Therefore, the next step in our analysis is to establish a model for the loss coefficient so it can be identified for a given design.

\textit{2.2 Loss Coefficient Model}

In order to have a complete dynamic model of the behavior of the tool inside the well we must identify the loss coefficient brought by the shape used as the obstruction mechanism. Loss coefficients can seldom be identified analytically; instead they have to be determined experimentally [1]. However, before doing any experimentation we must have a model of the parameter dependence of such a loss coefficient.

Using dimensional analysis we can find that the dimensionless groups present in the physical implementation of the obstruction are: the Reynolds number, $Re$, of the control volume, the ratio of the areas of the top and bottom boundaries of the control volume, $\frac{A_1}{A_2}$, the roughness ratio, $\frac{\varepsilon}{L_o}$ (where $L_o$ is the obstruction's characteristic length, and $\varepsilon$ is the obstruction roughness), and the drag coefficient, $C_D$. Therefore,

$$K_i = f\left(Re, \frac{A_1}{A_2}, \frac{\varepsilon}{L_o}, C_D\right)$$

The Reynolds number for the control volume is given by:

$$Re = \frac{\rho (v_1 - \dot{z}) (R_2 - R_1)}{\mu}$$
From the module dynamics described in equation 2.6 we know that each change in the module's projected area, due to a change in $R_2$, will bring a proportional change in the module (C.V.) velocity. Specifically, as $R_2$ increases the C.V. velocity $\dot{z}$ increases and vice-versa. Therefore, the terms $(v_i - \dot{z})$ and $(R_2 - R_1)$ grow proportionally and in opposite directions. Consequently, the Reynolds number will be proportional to $R_2$ and hence to the ratio $\frac{A_i}{A_2}$. Since in our experiments we will operate in the same fluid medium, the values of the density and viscosity of the medium can be assimilated by model constants. Therefore, for simplicity we will omit a direct reference to the Reynolds number in the model. Moreover, the roughness ratio and drag coefficient can also be assimilated by model constants for a given design. Hence, for a given module design and for a given fluid medium, the loss coefficient can be assumed to be only a function of the ratio of the areas of the control volume boundaries through which fluid flows. This relationship could take any form, (linear, quadratic, etc). So as a first approach we selected a polynomial relation of second order, such as:

$$K_i = d \left( \frac{A_i}{A_2} \right)^2 + e \frac{A_i}{A_2} + f$$

(2.9)

where the parameters $d$, $e$, and $f$ depend on the obstruction profile, and the properties of the fluid medium. These parameters should be determined experimentally [1].

As it turned out our model selection proved to be a good representation of the physical phenomena involved in the loss due to the obstruction. A detailed description and results of the identification experiments for the profile chosen in our prototype are presented in Chapter 4.
3.1 Design Specifications

In order to design a tool for downhole applications it is first necessary to familiarize us with the medium. As we did in Chapter 1, we present a brief summary of the common characteristics found downhole.

3.1.1 Environment

The downhole environment is particularly severe. Equipment (machinery and instrumentation) must withstand extreme temperatures that range between -40 and 200 degrees Celsius inside the borehole. In addition downhole pressures can vary from 0 to 15000 Psi (gage). This is particularly challenging for the circuitry and actuators used in downhole tools, and oil service companies must frequently manufacture their own since commercial equipment often can not meet these requirements.

Fluid flow rates also vary widely both in speed and direction, with flow rates as large as 10 \( \text{m/s} \) in some wells. Flow regimes can be single or multiphase and with a broad range of densities and viscosities.

3.1.2 Geometry

Depending on the type of well and its purpose, well casing’s inside diameter can range between 1.2 and 6.2 inches. Typical well depths surpass 10000 feet (vertically), and some horizontal or inclined branches detach at various heights and might extend for several thousands feet as well. Finally, a well’s casing is not always smooth. The typical obstacles found downhole include joints, tubing nipples, perforations, distorted or partially collapsed tubing, and some sections with sand and gravel deposits. These disturbances pose a challenge for the accurate motion of any tool inside a well.

Obviously, these conditions may vary among wells, nevertheless the obstruction module should be designed to operate satisfactorily in most them.
3.2 Obstruction Mechanism

As we explained in Chapter 2 we want to design an obstruction mechanism that would allow the module to control its motion. For this, the main objective of the obstruction is to create a differential pressure that provides the force required for motion. From the theoretical analysis in Chapter 2, and more specifically from equation 2.5, we know that the differential pressure created by the obstruction depends on parameters such as the density, \( \rho \), of the fluid media, the velocities at the boundaries of the control volume, \( v_1 \) and \( v_2 \), the velocity of the tool, \( \dot{z} \), and the obstruction's loss coefficient, \( K_l \). The only parameter that we can control through our design is the loss coefficient since it depends partly on the geometry of the obstruction, and for a given set of parameters, the greater the loss coefficient brought by the obstruction the greater the resultant differential pressure will be. Therefore, an optimal obstruction shape would be the one that achieves the highest loss coefficient for a given projected area since this will maximize the propulsion force per flow rate per opening.

In the process of finding the optimal obstruction shape several ideas were considered. Almost every downhole tool is axially symmetric, hence that leaves two parameters in the design space: projected area and profile of the tool. From the loss coefficient model, equation 2.9, it can be inferred that in order to maximize the loss coefficient we should maximize its projected area. It is obvious that in order to maximize the projected area of the obstruction for a given opening this one should be circular (for motion inside a pipe). However, the profile that the obstruction shape should have is not evident. In a strict sense, three options can be pursued: The profile can be streamlined, flat, or concave.

We can argue that when the clearance between the obstruction and the pipe I.D is big enough, then the loss coefficient is closely related to the drag coefficient of the obstruction shape. This of course does not hold when the clearance becomes small since then the obstruction behaves more like a piston and its drag coefficient does not tell us much. However, the tool needs to perform well at all clearances, and since at small clearances the profile shape does not play an important role then we should choose our profile based on good performance at big clearances. From drag coefficient tables we can find that concave shapes have the highest drag coefficients, and of course streamlined shapes have the lowest \(^1\) \(^2\) \(^3\). Furthermore, between concave shapes it seems that hollow spherical profiles and cones with an angle of 90 degrees
achieve the highest drag coefficients. Therefore, we decided to design an obstruction with a circular projected area and a concave profile.

The next step in our design was to implement this concave shape since its profile and projected area needed to be adjustable for motion control. In simple terms, we needed to achieve a collapsible obstruction. When the obstruction is fully closed it should match the cylindrical shape of the downhole tool that carries it. In addition, it is desired that a prototype design have a fast deploy response, be single actuator driven, and have relatively simple deploying mechanism. Perhaps the most common mechanical apparatus used to implement deployable structures is the linkage mechanism [16]. Its implementation is relatively simple and structurally sound, thus making it highly desirable for our application. Therefore, we chose to use linkages for the deployment operation.

3.2.1 Linkage Mechanism

![Linkage Mechanism Diagram]

Figure 3.1: Linear actuator driven "umbrella" mechanism.
An umbrella type mechanism was chosen to achieve the obstruction. The concept is pretty simple, linear motion, $X_2$, provided by a linear actuator, opens and closes a number of linkages that pivot on the tool circumference. Figure 3.1 depicts two linkage arms of such mechanism. Each linkage arm is composed of two pieces. An L-shaped link pivots about its 90 degree corner, a second link attached to a base drives it, and a linear actuator drives the base.

In order to achieve a closed and circular projected area, curved conical sections in the form of petals are attached to each external pivoting linkage bar. Hence, a variable pitch conical shape can be achieved provided each petal rides on its neighbor’s side. A first design iteration was implemented using LEGO parts because of ease, low cost, and fast turn over. Figure 3.2 shows the outcome.

Figure 3.2: LEGO prototype of umbrella mechanism
A total of four linkage arms are used in this bench level prototype. A LEGO MINDSTORMS controller brick is used to control the linear actuator driving the linkage system. Flexible cardboard "petals" are mounted on each pivoting arm proving the idea of having a series of petals riding on each other’s side to achieve a closed profile.

For purposes of experimentation we fixed a geometrical constraint on the prototype to be built. The tool’s closed diameter should be no bigger than 2.5 inches since this is representative of most logging tools. The LEGO prototype gave a lot of insight into what should be incorporated in a more realistic prototype tool. First of all, the size constraint in the diameter of the tool restricts the number of linkage arms that can be used in order to achieve the umbrella mechanism. For the diameter chosen, the maximum number of linkage arms that can be packaged reasonably is six. Figure 3.3 portrays a final concept drawing of the umbrella tool, shown here without petals. The angle between each linkage arm is 60 degrees and each L-shaped pivoting bar is 1.5 inches long and 0.25 inches wide. In order to reduce the material and equipment requirements, we decide to test the prototype in a moderate environment using water at ambient temperature. Therefore, the final prototype was built using aluminum for the outer shell and stainless steel and brass for the linkage arms. All aluminum parts were anodized to achieve higher underwater durability. This greatly reduced the cost of manufacture since real downhole tools require specialized alloys, which are more expensive than the materials we used. Figure 3.4 displays the assembled linkage mechanism. Detailed machine drawings can be found in Appendix A.
3.2.2 Petals

The final problem to be addressed was to design petals flexible enough to accommodate the change in radius of curvature due to the opening and closing of the umbrella. When the umbrella is closed, the radius of curvature of each petal should be 1.25 \textit{in} (tool radius), and at maximum opening the petals should bend to reach a radius of curvature close to the radius of the pipe where the tool is on. We decided to test the prototype in a 6 \textit{in} I.D pipe, which fixed the requirements for the petals. Some alloys are flexible enough to be used as material for the petals but since petals ride on top of each other there might be problems with friction and abrasion if metallic parts are used. Therefore, we decided to use carbon fiber composites for the petals since they have comparable strength and in addition carbon fibers auto lubricate on sliding contacts.

The axial strengths of a composite matrix vary depending on the angle at which the matrix has been laid, the number of fiber layers used, and the resin used to bond the layers. The specifications for our petals were high radial stiffness and enough tangential stiffness to withstand differential pressure while still allowing the bending of petals without fracture. Different matrix angles and bonding compounds achieve fairly varied combinations of radial and tangential stiffness. We experimented using polyurethane and epoxy resins with different hardness along with several combinations of matrix layers and angles, and we found that for our design the optimal matrix angle to be used is close to 10 degrees with respect to the
longitudinal axis of the petals. A medium hardness epoxy was used with only three layers of carbon fiber. In addition, a stainless steel insert was placed in the longitudinal axis of each petal to increase strength and minimize fracture problems due to the placement of mounting holes on the petals. The petals used in the prototype are displayed in figure 3.5. Each petal is 3.5 in long and has a major circumference of 3.25 in and a minor circumference of 1.3 in. Appendix B contains the details of the composite petal fabrication.

Figure 3.5: Composite petals

3.2.3 Linear Actuator

Using the model developed in chapter 2 we estimated the pressure ranges that the umbrella mechanism would need to overcome while deploying inside a 6 in. I.D pipe with flows ranging from 0 to 10 (m/s) since these values are common in normal well operating conditions. It was determined that a maximum of 100 lb is the force that the umbrella would need to overcome when closing. Under this restriction and the imposed size limitations we choose a linear actuator from TS Products Inc [18]. The linear actuator is composed of a 1724 MicroMo brushless DC motor, a high-speed reduction gearbox, and a drivescrew. The maximum load that the actuator can overcome is 50 lb while still in servo window, at a velocity of 25 mils/s. The actuator will stall at 87 lb. This clearly would not meet the requirements for high pressures but would suffice to test the prototype in a fair range of downhole pressures. In addition, the actuator is only 1 inch in diameter and 7 inches long, and has a 1-inch stroke, which perfectly fits inside our tool geometrical constraints. Figure 3.6 displays the linear actuator mounted on top of the umbrella mechanism.
The actuator needs to be protected against the environment, hence it was mounted inside a sealed housing and pressure seals were used at the connection with the umbrella mechanism. Drawings of the mounting parts and the sealed housing can be found in appendix A.

3.3 Position feedback module

Every logging tool has position sensors to monitor and record its position inside the well since this is crucial for correct interpretation of the logging measurements. The method most commonly used involves a casing collar locator (CCL) which, as its name stands for, is a sensor used to locate casing collars and other features of downhole hardware since these can be used as future depth references. There are two types that are available commercially: magnetic and mechanical CCLs.
Magnetic casing collar locators use a system of two permanent magnets, which produce characteristic magnetic fields. A deformation of either of the magnetic fields, produced by a gap between casing joints, or tubing hardware, is detected by a winding with high permeability core. The resulting electromagnetic imbalance is transmitted to the surface and depth correlated and can thus be recognized in the future as a downhole hardware arrangement [17]. Mechanical collar locators use feelers or finger mechanisms that produce signals that are sent to the surface whenever the feelers cross pipe connections or other downhole irregularities [17]. For the purpose of testing the obstruction module, the use of CCL technology would not be possible since it requires extra equipment. Therefore, a mechanical position feedback module was designed. Its operating principle is simple: A passive wheel is spring loaded against the tube wall so that as the module moves the wheel drives a zero-backlash belt. The belt connects to a high precision miter gear arrangement which drives a shaft connected, through pressure seals, to an internal encoder. Figure 3.7 displays the module design, which was also built from aluminum with a special cavity to house an optical incremental encoder. Detailed machine drawings can be found in Appendix A.
3.4 Complete Tool

The complete obstruction module is composed of the umbrella mechanism, the linear actuator housing and the position feedback module. These three sections are mounted between two centralizers when testing their dynamic behavior. The main goal of the project is to achieve an autonomous logging tool module that would perform adequately in a real well. However, for practical reasons the module was tested in a simulated well, and the hardware and software for the obstruction module were tested using a cable connection to the outside of a simulated well since these speed the debugging process. Figure 3.8 displays the three module parts in their normal arrangement, next to them are portrayed the special underwater cable connection for the tool and the section that houses the connection cable.

![Complete Tool Arrangement](image)

The next step in our analysis is to identify experimentally the fluid losses brought by the obstruction shape chosen in our design.
4.1 Experimental Setup

In order to identify experimentally the loss coefficient of the obstruction shape chosen we decided to perform static tests using a model of the obstruction tool. A simulated well section was designed and built in real scale in order to replicate downhole velocities and pressures. Multiphase flows are difficult to implement experimentally since extra hardware and controls are needed, and commercial pumps can handle mainly low viscosity liquids, therefore, the fluid media used for testing was single-phase water. The obstruction tool model was built at full scale with the same materials used for the real prototype in order to keep geometrical and physical proportions during the experiments. A description of the hardware used for the experiments and the criteria used for testing follows.

4.1.1 Tool Model

Prior to the fabrication of the umbrella prototype, a model of the real obstruction tool was built from solid aluminum in order to test its hydrodynamic properties. The model consisted of a shape similar to the umbrella mechanism mounted between two static centralizers as displayed in figure 4.1. Detailed machine drawings of the model can be found in Appendix A.

Figure 4.1: Tool model concept drawing.
In order to test the umbrella shape, different aluminum cones were mounted and tested on the model. A total of four cones were tested, each one having a different combination of projected area and depth. Figure 4.2 displays two of the cones used for testing.

![Figure 4.2: Aluminum Cones used to test the "umbrella" shape properties.](image)

### 4.1.2 Flowloop

A diagram of the simulated well is portrayed in figure 4.3, the system is simply a flowloop composed of a 20 ft long straight section of clear cast acrylic tubing connected to a reservoir and a centrifugal pump. The pump is used to drain water from the reservoir plus circulate and control the fluid flow inside the clear tubing section. The entire structure is secured against a solid wall. Copper tubing is used to connect the pump inlet and outlet to the top and bottom of the straight section.
4.1.3 Pump, Sensors and Hardware

The pump used in the flowloop is a centrifugal pump capable of delivering mass flow rates of 600 GPM at 140 ft of head (using water). The impeller is driven by a triphase, 30 Watt AC motor. The pump is mounted on an inertial base in order to damp any vibrations during operation. Figure 4.4 shows the pump and the inertial base.
A programmable controller regulates the pump operations. Consequently, accurate step, ramp and sinusoidal flow inputs can be applied to the loop. For experimental purposes it is desired to measure accurately the flow rates inside the loop. Several choices of flow meters are available commercially. The most commonly used in industry are mechanical flowmeters or spinners and electromagnetic flowmeters. Spinners have one or several propeller blades that are put in contact with the flow, as the flow "spins" the blades they generate small voltages via dynamos. The voltages generated can be calibrated to the flow velocities and the output can be used to determine the flow rates. However, this kind of flow meter is not very accurate and disturbs the flow profile as well.

On the other hand, electromagnetic flow meters are very accurate and work under a principle that does not disturb the flow being measured. An electromagnetic flowmeter is essentially a piece of piping with an internal isolating lining. Two electromagnetic coils are located outside the flow tube, diametrically opposed to each other and protected by a carbon steel housing. Two electrodes, inserted into the flow tube, are positioned "flush" with the internal diameter of the tube and perpendicular to the coils. A pulsed DC voltage energizes the coils and a magnetic field is generated across the flow tube section. According to Faraday's law, when this magnetic field is "cut" by a conductive liquid flowing through the meter, a voltage is generated in the liquid. This voltage is directly proportional to the liquid flow velocity, and therefore to the actual volumetric flow rate of the liquid. Provided the liquids being tested have some conductivity the small voltages generated can be measured, and flow rates can be estimated with an accuracy of 0.25% or better.

![Figure 4.5: Electromagnetic Flowmeter.](image-url)
An electromagnetic flowmeter was installed downstream from the pump outlet as shown in figure 4.5. Care was taken to install the flowmeter more than 5 pipe diameters away from the pump outlet in order to have a fully developed flow for measurement. The tool model was submerged inside the long straight section of clear tubing and secured to a load cell mounted at the top T-junction in order to perform the static tests. The load cell mounting is shown in figure 4.6. The load cell used is a universal (tension/compression) hermetically sealed cell rated for 1000 lb. This type of load cell, and in general any axial load cells, are very susceptible and can be very easily damaged by any bending or torsion applied. Therefore, special care was taken when mounting the model and a special feature was designed to take any bending moments applied to the cell while the setup was moved.

A reservoir is part of the flowloop to facilitate cleaning and replacement of the liquids. The reservoir has a capacity of 800 gallons, is made of nylon, and is supported by four casters to ease its manipulation; figure 4.7 portrays how it was mounted to the flowloop.

Figure 4.6: Load Cell mounted on top of T-junction.
Finally, the readings (voltage outputs) from the electromagnetic flowmeter and the load cell are monitored using digital multimeters interfaced to a computer running LabView. A LabView program is used to filter the signals and convert them to the desired units for calculations. Figures 4.8 and 4.9 display the data acquisition equipment sitting next to the flowmeter and a front view of the entire flowloop.
4.2 Experiments

As we explained briefly at the beginning of the chapter, the experiments performed were aimed to identify the loss coefficient brought by the umbrella mechanism. A tool model was submerged inside the straight section of clear plastic tubing and attached by a rigid connection to a universal load cell. The load cell was in turn rigidly mounted to the top T-junction so it would measure any force applied to the model inside the flowloop. Solid conical fixtures were used in the model in order to simulate the umbrella mechanism and a different conical geometry was tested in each experiment to simulate different umbrella openings and depths.

For a given experiment, with the model rigidly mounted inside the loop, a series of increasing flow rates step inputs are administered using the pump controller. The forces felt by the model and the flowmeter readings are continuously recorded using the data acquisition hardware. Once the components attributed to the weight and the buoyancy of the model are subtracted, the only force present is that due to the differential pressure across the obstruction. Figures 4.10 and 4.11 display the model being submerged inside the test rig and a view of the model once inside.
Figure 4.10: Model being submerged inside the flowloop.

Figure 4.11: Close up of model inside the loop.
The physics of a static configuration can be derived using the equations we have for the external
dynamics. Combining the energy equation and the conservation of mass relations (equations 2.3 and 2.5)
derived for the control volume analysis in Chapter 2 we find that for a static test,

\[
\Delta P = \frac{\rho}{2}v_1^2 \left( \frac{A_1^2}{A_2^2} + K_i - 1 \right) + \rho g l
\]

(4.1)

The differential pressure across the obstruction can be obtained from the force measured by the load
cell. The force, \( F \), measured by the load cell and the differential pressure across the obstruction are related
by the expression:

\[
\Delta P \text{(Tool projected area)} - m''g = F
\]

(4.2)

where the term \( m''g \) represents the effects of the weight and buoyancy of the tool model. Therefore, once
\( F \) is identified experimentally we can use equations 4.1 and 4.2 to solve for the correspondent loss
coefficient \( K_i \). Furthermore, from the theoretical analysis in chapter 2 it was determined that a suitable
model for the loss coefficient is given by:

\[
K_i = d \left( \frac{A_1}{A_2} \right)^2 + e \frac{A_1}{A_2} + f
\]

Since we can experimentally identify a loss coefficient \( K_i \) for a given \( A_2 \) and \( A_1 \), the parameters \( d \), \( e \),
and \( f \) can also be identified. Once the loss coefficient model has been identified it can be used to estimate
the loss coefficient \( K_i \) of any given obstruction geometry.

4.3 Results

Four conical shapes, which modeled possible configurations of the umbrella mechanism, were tested in
the flowloop. Table 4.1 summarizes the characteristics of each cone. We define the angle \( \alpha \) to be the
bisection angle of the cone's profile (see Appendix A for detailed dimensioning).
Table 4.1: Characteristics of conical fixtures.

<table>
<thead>
<tr>
<th>Cone #</th>
<th>Projected Area Radius [in]</th>
<th>$\alpha$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.75</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>2.75</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>2.25</td>
<td>15</td>
</tr>
</tbody>
</table>

As it was mentioned before, for each cone several experiments were performed in which a series of flow step inputs were applied and the correspondent force felt by the model was recorded. Figures 4.12 and 4.13 show typical experimental results, in this case for cones 1 and 2. The initial variations in the velocity readings are attributed to the flowmeter hardware, since at flow rates close to zero the accuracy of the meter readings diminishes significantly.

We note that typically for every flow step input there is a correspondent step increase in the force (or load) felt by the tool. Furthermore, this relationship is not linear. We can speculate from the preliminary results that the load felt by the tool seems to grow exponentially as the flow increases. This presumption is confirmed when the load brought by each cone is plotted against the velocity used to generate it. Figure 4.14 displays this relation for the four cones and when the tool is inserted in the loop without one (for calibration purposes). As it is seen in figure 4.14, it is somewhat surprising that the cone's angle does not influence its behavior as much as its projected area. It can be seen that cones 1 and 2 behave almost identically and cones 3 and 4 as well.
Figure 4.12: Experimental results for Cone 1.

Figure 4.13: Experimental results for Cone 2.
The data retrieved from the experiments was used with equations 4.1 and 4.2 to identify the parameters in the loss coefficient model. Therefore, for the geometry of the hardware used and under all the assumptions stated in Chapter 2, we get:

\[ d = 5.42, \quad e = -11.2, \quad \text{and} \quad f = 11.28 \]

Thus,

\[ K_1 = 5.42 \left( \frac{A_1}{A_2} \right)^2 - 11.2 \frac{A_1}{A_2} + 11.28 \]

(4.3)

In order to validate equation 4.3 we use it in conjunction with equations 4.1 and 4.2 to plot a theoretical (or estimated) load-flow-rate behavior for a given cone. The predictions for cones 3 and 4 are plotted together with actual experimental values in figures 4.15 and 4.16. We can see that the model does a decent job of estimating the behavior of a cone, with errors less than 1%, hence, we can conclude that the model in equation 4.3 is appropriate for purposes of simulation and or control of an obstruction mechanism dynamic behavior.
Figure 4.15: Validation of loss coefficient model using experimental data from cone 3.

Figure 4.16: Validation of loss coefficient model using experimental data from cone 4.
5.1 Kinematics and static analysis of linkage mechanism

A schematic of the module's internal mechanism is depicted in figure 5.1. As explained on Chapter 3, a linear actuator drives the umbrella mechanism. A voltage applied to the linear actuator produces a linear displacement, which is converted into a radial increment of the obstruction's projected area thanks to the linkage mechanism. The main elements to be modeled are the linkage kinematics, the linear actuator dynamics, and their coupling with external forces.

![Figure 5.1: Umbrella-Actuator mechanism.](image)

To start the analysis of the obstruction mechanism dynamic behavior, we need to identify the forces that are transmitted to the actuator from the interaction of the obstruction mechanism with external forces.
Therefore, we must find first the forces acting on the linkage mechanism. The free body diagram of the L-shaped portion of the linkage mechanism is portrayed in figure 5.2. Each petal is attached to one of these linkages (totaling six) so these linkages are subjected to a fraction of the differential pressure $\Delta P$ acting underneath the petals. For simplicity, we will assume that the differential pressure results in a uniformly distributed force that creates a moment about pivot 1 and as a result a reaction force $N_1$ in pivot 2. In fact the pressure differential across a petal might vary creating an inhomogeneous force distribution, which would be difficult to calculate. However, given that the module is symmetrical and centralized, a uniform force distribution remains a safe assumption.

![Figure 5.2: L-shaped linkage FBD.](image)

Taking moments about pivot 1 we find that the reaction force on each pivot 2, of the six present in our design, is given by:

$$N_1 = \frac{\Delta P \left(3\pi (R_2^2 - R_1^2)\right) + 8\pi R_1 (L p \cos(\alpha))^3}{72 b \sin(\beta + \theta)}$$

(5.1)

Figure 5.3 portrays the free body diagram of the intermediate linkage. For this linkage the reaction forces at pivots 2 and 3 are identical. Hence,
\[
N_1 = N_2
\]  
(5.2)

Figure 5.3: Intermediate linkage FBD.

Figure 5.4 displays the free body diagram of the base to which all the intermediate linkages are attached; it also acts as the connection to the linear actuator.

Since a total of six linkages are connected to the base, the resulting force \( F_2 \) is given by:

\[
F_2 = 6N_2 \cos(\theta)
\]  
(5.3)

Finally, combining equations 5.1, 5.2, and 5.3 we find that:

\[
F_2 = \frac{\Delta P \cos(\theta)\left[3\pi(R_2^2 - R_1^2) + 8\pi R_1 (Lp \cos(\alpha))^2\right]}{12b \sin(\beta + \theta)}
\]  
(5.4)
From a static point of view, the force $F_2$ is the force that the linear actuator needs to provide in order to keep the linkage mechanism at a desired open angle in the presence of a pressure differential $\Delta P$ across the obstruction module.

Some basic relationships can be developed from trigonometric relationships in the linkage system portrayed in figure 5.5. Since $\beta = \frac{\pi}{2} - \alpha$:

\[
\sin \beta = \cos \alpha
\]
\[
\cos \beta = \sin \alpha
\]

(5.5a)

Also,

\[
\sin \alpha = \frac{R_2 - R_1}{L_p}
\]
\[
\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}
\]

(5.5b)
Finally, since \( b = c \):

\[
\begin{align*}
\sin \theta &= 1 - \sin \beta = 1 - \cos \alpha = 1 - \sqrt{1 - \left( \frac{R_2 - R_1}{L_p} \right)^2} \\
\cos \theta &= \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left( 1 - \sqrt{1 - \left( \frac{R_2 - R_1}{L_p} \right)^2} \right)^2}
\end{align*}
\]

(5.5c)

Therefore, using equations 5.5 we can rewrite equation 5.4 in terms of \( R_2 \).

\[
F_2 = \frac{\Delta P \left[ 1 - \left( \frac{R_2 - R_1}{L_p} \right)^2 \right] \left[ 1 - \sqrt{1 - \left( \frac{R_2 - R_1}{L_p} \right)^2} \right] \left[ 3\pi \left( R_2^2 - R_1^2 \right) \right]^2 + 8\pi R_1 \left( \frac{R_2 - R_1}{L_p} \right) \left[ 1 - \left( \frac{R_2 - R_1}{L_p} \right)^2 \right]^3}{12b \left( 1 - \left( \frac{R_2 - R_1}{L_p} \right)^2 \right) \left[ 1 - \sqrt{1 - \left( \frac{R_2 - R_1}{L_p} \right)^2} \right] + \left( \frac{R_2 - R_1}{L_p} \right) \left[ 1 - \sqrt{1 - \left( \frac{R_2 - R_1}{L_p} \right)^2} \right]} \]
\]

The above expression, with exception of the differential pressure term, can be accurately approximated by a third degree polynomial in \( R_2 \),

\[
F_2(R_2) = \Delta P \left( K_1 R_2^3 + K_2 R_2^2 + K_3 R_2 + K_4 \right)
\]

(5.6)

where the constants \( K_1, K_2, K_3, \) and \( K_4 \) can be found numerically (values for all model parameters can be found in the simulation algorithms in Appendix C).

The next step in the analysis is to study the kinematics of the linkage mechanism. We are interested in finding the relationship between the input velocity \( V_i \) provided by the linear actuator and the corresponding rate of change in the radius of the obstruction \( \dot{R}_2 \). From figure 5.5, we can realize that the velocity of pivot 2 with respect to the inertial reference frame \( (\alpha, r, z) \) is given by:

\[
V_z = V_i + c \dot{\theta}
\]
And also by:

\[ V_2 = b\beta = b\alpha \]

Therefore:

\[ V_i + c\theta = b\alpha \]  

(5.7)

Equation 5.7 is close to the expression we are looking for, in the sense that it expresses a relationship between the input velocity and the angular velocity of the petal opening, which is linked to \( \dot{R}_2 \). Hence, we need to substitute all the elements in 5.7, except for \( V_i \), by terms containing \( \dot{R}_2 \). Using equations 5.5 we can deduce the following relationships:

\[
\dot{\alpha} = \frac{\dot{R}_2}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}}
\]

\[
\dot{\theta} = \frac{\dot{R}_2 \left(\frac{R_2 - R_1}{L_p}\right)}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}} \left(1 - \frac{1}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}}\right)
\]

Using the above relationships we can rewrite equation 5.7 as

\[
V_i = \frac{\dot{R}_2}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}} \left(b - \frac{c \left(\frac{R_2 - R_1}{L_p}\right)}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}} \left(1 - \frac{1}{\sqrt{1 - \left(\frac{R_2 - R_1}{L_p}\right)^2}}\right)\right)
\]
This new expression, although useful in content, is rather complex. Luckily, the right-hand side of the expression can be accurately approximated by a much simpler expression,

\[ V_i = \dot{R}_2 \left( K_5 R_2 + K_6 \right) \]

and thus,

\[ \dot{V}_i = \dot{R}_2 \left( K_5 R_2 + K_6 \right) + K_5 \dot{R}_2^2 \]

(5.8)

Where the constants \( K_5 \), and \( K_6 \) can be found numerically (values for all model parameters can be found in the Appendix C). The next step towards the understanding of the obstruction mechanism dynamics is to study the actuator dynamics and the way it interacts with the external forces applied to the module.

### 5.2 Actuator dynamics

![Figure 5.6: Linear actuator components.](image)

The linear actuator used in the obstruction module is depicted in figure 5.6. The motor, an ironless-core DC-motor, has a torque constant \( K_t \), back EMF constant \( K_e \), electrical resistance and armature inductance, \( R_u, L_a \) respectively. The damping (assumed viscous) due to the inner bearings, and the motor inertia are denoted \( b_m \), and \( I_m \) respectively. The transduction equations for the motor gearbox are,

\[ \omega_m = N \omega_l \]
where $N$ is the gear box ratio. In a similar fashion, the equations for the lead screw are,

$$
\omega_i = 2\pi p V_i
$$

$$
T_i = \frac{F_i}{2\pi p \varepsilon}
$$

where $p$ is the lead screw pitch, and $\varepsilon$ denotes its efficiency.

Since the actuator drives directly the linkage mechanism, the angular momentum principle applied to the motor yields the following equation

$$
K_i i_a - \frac{F_2}{N^2 \pi p \varepsilon} - \left( b_m + \frac{b}{N^2} \right) \omega_m - T_{static} \text{sgn}(\omega_m) = \left( I_m + \frac{I_{load}}{N^2} \right) \frac{d\omega_m}{dt}
$$

(5.9)

The viscous damping $b$ is due to the interaction of parts with the o-ring seals placed between the actuator housing and the linkage mechanism. As we mentioned in Chapter 3, the actuator housing is isolated from the fluid media so the shaft that transmits the actuator motion to the umbrella mechanism has to go through a seal. The term $I_{load}$ represents the payload inertia. The load $F_2$ is due to presence of the differential pressure across the obstruction as derived in equation 5.6. Finally, $T_{static}$ is the torque required to get the umbrella mechanism moving initially. This static friction torque is due to the fact that the actuator used is "not backdrivable" to a certain limit (the combination of gear reduction and friction at the lead screw significantly opposes externally driven motion). This means that, although initially the actuator has to overcome an extra force to get the umbrella moving, the umbrella mechanism can maintain its position against the flow without actuator force.

Next, the application of Kirchhoff's voltage law to the motor electric circuit yields:

$$
e_a = L_a \frac{di_a}{dt} + R_a i_a + K_e \omega_m
$$

(5.10)
However, for the actuator used in the prototype the electrical time constant is negligible in comparison
to the mechanical time constant of the system, or in other words the dynamics of the hardware are much
slower than the dynamics of the actuator electric circuit. Therefore, we can assume the events in the
electrical domain are “instantaneous” and neglect the inductance term in equation 5.10. Thus,

$$e_a = R_a i_a + K_c \omega_m$$

We can write equations 5.9 and the modified equation 5.10 in terms of the actuator output velocity $V_i$ and
combine them in to a single equation of motion,

$$\frac{R_a N \pi p}{K_t} \left( I_m + \frac{I_{load}}{N^2} \right) \frac{dV_i}{dt} + \left( b_m + \frac{b}{N} \right) \frac{R_a N}{K_t} K_c N + K_c N \right) 2 \pi p V_i + \frac{F_2 R_a}{N \pi p eK_t} + \frac{R_a T_{static} \text{ sgn}(V_i)}{K_t} = e_a$$

(5.11)

Equation 5.11 is already capable of describing the dynamic behavior of the obstruction mechanism. It
describes the dynamic relationship between the input voltage $e_a$ and the actuator output velocity $V_i$.

However, it would be more useful to know the relationship between the input voltage and the
 corresponding change in the obstruction projected area since the motion of the module inside the well is
defined with respect to the latter parameter as we concluded in Chapter 2. Therefore, we can combine
equations 5.11 and 5.8 to get a more useful input-output relationship. First, we rearrange equation 5.11 as
follows,

$$\frac{dV_i}{dt} = \frac{e_a K_i}{I_{eq} R_a N \pi p} - \left( b_m + \frac{b}{N} \right) \frac{1}{I_{eq}} + \frac{K_c K_i}{I_{eq} R_a} V_i - \frac{F_2}{I_{eq} N^2 \pi^2 p^2 e} - \frac{T_{static} \text{ sgn}(V_i)}{I_{eq} N \pi p}$$

with $I_{eq} = \left( I_m + \frac{I_{load}}{N^2} \right)$, and for simplicity we can replace all the unvarying terms by constants yielding,

$$\frac{dV_i}{dt} = e_a K_7 - K_8 V_i - K_9 F_2 - K_{10} T_{static} \text{ sgn}(V_i)$$

Now we can proceed to use the relations in 5.8 to get the expression

$$\ddot{R}_2 = -\frac{K_3 R_2^2}{(K_3 R_2 + K_6)} - K_4 \dot{R}_2 - \frac{K_5 F_2}{(K_3 R_2 + K_6)} - \frac{K_{10} T_{static} \text{ sgn}(R_2)}{(K_3 R_2 + K_6)} + \frac{e_a K_7}{(K_3 R_2 + K_6)}$$
Finally, if we define,

\[ P_1(R_2) = \frac{K_5}{(K_s R_2 + K_6)} \quad \text{and} \quad P_2(R_2) = \frac{K_0}{(K_s R_2 + K_6)} \]

\[ P_3(R_2) = \frac{K_10}{(K_s R_2 + K_6)} \quad \text{and} \quad P_4(R_2) = \frac{K_7}{(K_s R_2 + K_6)} \]

we obtain a more compact expression,

\[ \ddot{R}_2 = -P_1 \dot{R}_2^2 - K_s \dot{R}_2 - P_2 F_2 - P_3 T_{\text{static}} \text{sgn}(\dot{R}_2) + P_4 e_a \]  

Equation 5.12 governs the dynamics of the obstruction radius in terms of the input voltage and the corresponding change in obstruction radius taking into account the external forces brought about by this change (e.g. the force \( F_2 \) due to the pressure differential across the obstruction).

In this section the dynamics of the obstruction mechanism have been defined, and since the external module dynamics have already been studied in Chapter 2 it only remains to combine these two results into one model that would describe the overall dynamics of the umbrella module. We can see already that equation 5.12 is a nonlinear differential equation thus our complete model will be nonlinear as well. The next chapter will deal with the complete model formulation, stability analysis and control techniques to be implemented.
6.1 Plant model

In Chapters 2 and 5 we studied the module external and internal dynamic behavior. For control purposes we want to have a complete model of the plant. Such a model would relate the module velocity \( \dot{z} \) inside the well to a given voltage input \( e_a \) applied to the linear actuator. Therefore, the first objective in this section is to achieve a state representation of the overall module dynamics.

From equations 2.6 and 2.9 we obtain the following expression for the external dynamics of the module

\[
\ddot{z} = \frac{\rho (v_i - \dot{z})^2}{m'} \left( A_1 - \frac{A_1^2}{A_2} \left( \frac{d + 1}{2} \left( \frac{A_1}{A_2} \right)^2 \pi R^2 + \frac{e A_1}{2 A_2} \pi R^2 + \frac{(f - 1)}{2} \pi R^2 \right) + \frac{\rho g l \pi R_i^2}{m'} - g \right)
\]

The parameters, \( l, d, e, \) and \( f \) are constant, and we will assume for the moment that the density \( \rho \) of the fluid media remains constant. Thus, \( m' \) remains constant as well. Using,

\[
A_1 = \pi \left( R_i^2 - R_2^2 \right), \quad A_2 = \pi \left( R_i^2 - R_2^2 \right)
\]

we can expand expression 6.1 and rewrite it as,

\[
\ddot{z} = (v_i - \dot{z})^2 \left( \frac{K_{11} R_2^4 + K_{12} R_2^2 + K_{13}}{K_{14} R_2^4 + K_{15} R_2^2 + K_{16}} \right) - K_{17}
\]

where the constants \( K_j \) are found after the expansion. If we define a new polynomial such as,

\[
P_5(R_2) = \frac{K_{11} R_2^4 + K_{12} R_2^2 + K_{13}}{K_{14} R_2^4 + K_{15} R_2^2 + K_{16}}
\]

55
we obtain the more compact expression

\[
\ddot{z} = P_5(v_1 - \dot{z})^2 - K_{17}
\]  

(6.2)

Equation 6.2 describes the external dynamics of the module. Now, the obstruction mechanism dynamics are described by equation 5.12,

\[
\ddot{R}_2 = -P_1\dot{R}_2^2 - K_8\dot{R}_2 - P_2F_2 - P_3T_{\text{static}} \operatorname{sgn}(\dot{R}_2) + P_4 e_a
\]

But, from equation 5.6 we know that \(F_2(R_2) = \Delta P \left( K_1 R_2^3 + K_2 R_2^2 + K_3 R_2 + K_4 \right) \). Furthermore, using equations 2.3, 2.5, and 2.9 we get an expression for the differential pressure,

\[
\Delta P = \rho(v_1 - \dot{z})^2 \left( \frac{d + 1}{2} \left( \frac{A_1}{A_2} \right)^2 + \frac{e}{2} \frac{A_1}{A_2} + \frac{(f - 1)}{2} \right) + \rho gl
\]

Therefore, we can expand the above expression for the force \( F_2 \), and rewrite it as:

\[
F_2(R_2) = (v_1 - \dot{z})^2 \left( \frac{K_{18} R_2^4 + K_{19} R_2^2 + K_{20}}{K_{21} R_2^4 + K_{22} R_2^2 + K_{23}} + K_{24} \right) \left( K_1 R_2^3 + K_2 R_2^2 + K_3 R_2 + K_4 \right)
\]

Defining,

\[ P_6(R_2) = K_1 R_2^3 + K_2 R_2^2 + K_3 R_2 + K_4 \]

\[ P_7(R_2) = \frac{K_{18} R_2^4 + K_{19} R_2^2 + K_{20}}{K_{21} R_2^4 + K_{22} R_2^2 + K_{23}} + K_{24} \]

\[ P_8(R_2) = P_2(R_2) P_6(R_2) P_7(R_2) \]

and, \( P_8(R_2) = P_2(R_2) P_6(R_2) \)

we get an equation governing the internal dynamics:

\[
\ddot{R}_2 = -P_1\dot{R}_2^2 - K_8\dot{R}_2 - P_8(v_1 - \dot{z})^2 - P_9K_{24} - P_3T_{\text{static}} \operatorname{sgn}(\dot{R}_2) + P_4 e_a
\]

(6.3)

If we define our state vector as \( x = [R_2 \ \dot{R}_2 \ \dot{z}]^T \), we can use equations 6.2 and 6.3 to form a state representation of the module dynamics of the form, \( \dot{x} = f(x, e_a, v_1) \). The system has one input, \( e_a \), which
is the voltage applied to the linear actuator. The fluid flow velocity, \( v_1 \), acts as a disturbance that can be measured. The complete state space model is:

\[
\dot{x} = \begin{bmatrix}
-x_2^2 + K_8 x_2 - P_s (v_1 - x_4)^2 - P_9 K_{24} - P_3 T_{\text{static}} \text{sgn}(x_2) \\
0 \\
-x_4^2 + P_5 (v_1 - x_4)^2 - K_1 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
P_4 \\
0 \\
0 \\
\end{bmatrix} e_a 
\]

(6.4a)

and for our purposes, the output equation is:

\[ y = x_3 \]

(6.4b)

The model described by 6.4 is nonlinear and of fourth order. An attempt to linearize the system wouldn't make any sense since we can not assume a small range of operation for any of the dynamics involved, therefore any linearization would destroy completely the model's capability to describe the physical phenomena going on. As an example, in equation 6.3, we can not assume small changes in \( R_2 \) about an operating value because that is simply not true, in the real module we would actually like to have this value change over a wide range. The same goes for the module velocity \( \dot{z} \) in equation 6.2. Therefore, we must proceed in our analysis using nonlinear analysis and control techniques. We first analyze the open loop stability of the system and then propose appropriate control schemes. Finally, we present the closed loop module behavior in simulations.

6.2 Open loop stability analysis

Traditionally, linear systems of the form \( \dot{x} = Ax \), are defined as time variant (LTV) or time invariant (LTI) depending on whether the matrix \( A \) varies with time or not. For non-linear systems "autonomous" and "non-autonomous" replace these adjectives. A non-linear system \( \dot{x} = f(x,t) \) is said to be "autonomous" if \( f \) does not depend explicitly on time, and "non-autonomous" otherwise. It should be clear however that "autonomous" as well as time invariant systems are idealized notions since all physical
systems are non-autonomous. Nevertheless, in practice, system parameters often change very slowly so in some cases such idealizations are justifiable. In our case, as a first approach we will assume that the values of the disturbance velocity $V_1$ and the density $\rho$ of the fluid media remain constant and known. Therefore, the plant can be defined as autonomous since, apart from the state variables, all other parameters involved in 6.4 are "constant". It is interesting to note as well that when the obstruction mechanism is not moving the static friction torque $T_{\text{static}}$ is large enough to maintain the opening of the umbrella against the flow, hence no input voltage is required.

We study first the open loop stability of the umbrella module. We are interested in analyzing stability for two instances. First, it is desired that the umbrella module "float" at a particular height for a predetermined amount of time, hence we are interested in the stability of the model in 6.4 around its static equilibrium points. Alternatively, we would like to make the output $y(t)$ track a desired trajectory $y_d(t)$ while keeping the whole state bounded. For both instances, it is desired to have at least stability in the sense of Lyapunov. For the nominal motion problem it is further desirable to achieve global asymptotic stability. Mathematically, a state $x^*$ is an equilibrium point for the system $\dot{x} = f(x)$ if it satisfies,

$$0 = f(x^*)$$

(6.5)

which physically means that once the system attains the state $x^*$ it remains at it for all future time. For the umbrella module, the equilibrium points are denoted by,

$$x^* = [R_2^*, \dot{R}_2^*, z^*, \dot{z}^*]^T$$

(6.6)

For the first case, when the umbrella module is in static equilibrium (floating at a certain height), the module has zero velocity, thus $\dot{z}^* = 0$, and the obstruction radius should be constant so $\dot{R}_2^* = 0$. In principle, the state $z^*$ could take any constant value since the module can be in equilibrium at any height. One can think of an infinitely long well where force balance between the module weight and the force due to the differential pressure across the obstruction can happen anywhere (in the reality, when the module is
close to the pipe boundaries equilibrium might not be feasible). The obstruction radius, \( R_2^* \), is physically bounded since it can not be bigger than the pipe internal radius \( R \), and it can not be smaller than the tool radius \( R_1 \). Also, at equilibrium, the umbrella module holds its petal opening, thus \( \dot{R}_2^* = 0 \). Therefore, from 6.5, the state \( R_2^* \) is found by solving the system of equations,

\[
\begin{align*}
-P_g(R_2^*)v_1^2 - P_g(R_2^*)K_{24} &= 0 \\
v_1^2 P_g(R_2^*) - K_{17} &= 0
\end{align*}
\]

subject to the physical constraints described above. For the equilibrium point of interest we will introduce a new variable,

\[
\gamma = x - x^*
\]

and substituting, \( x = \gamma - x^* \) into the state equations 6.4 we obtain an equivalent state representation in which the equilibrium point is at the origin of the state-space. The new set of equations obtained is of the form

\[
\dot{\gamma} = f(\gamma + x^*)
\]

so we can study instead the stability of 6.8 in the neighborhood of the origin of the state space. Let \( B_R \) denote a region in the state space defined by \( \| \gamma \| < R \). The equilibrium state \( \gamma = 0 \) is said to be stable if, for any \( R > 0 \), there exists an \( r > 0 \), such that if \( \| \gamma(0) \| < r \), then \( \| \gamma(t) \| < R \) for all \( t \geq 0 \).

Essentially, this means that if the system is stable its trajectories in the state space can be kept arbitrarily close to the equilibrium point by starting sufficiently close to it, otherwise the system is unstable. Specifically, a value \( r(R) \) can be found such that starting within the region \( B_r \) at time 0 guarantees that the state will stay within \( B_R \) thereafter [9].

For the system in 6.8, where the equilibrium point is at the origin of the state pace, if we slightly perturb \( \gamma_1 \) (the equivalent \( R_2 \)) the force balance would be lost. This is easily proved since we can realize
from equation 5.6 that by changing the value of $R_2$, the value of the force $F_2$ (force due to differential pressure) changes,

$$F_2(y_2^* + \delta y_2^*) = (v_i - 2)(P_1(y_2^* + \delta y_2^*) + K_{24})^2 + K_p(y_2^* + \delta y_2^*)$$

So if $\delta y_2^* > 0$, then

$$F_2(y_2^* + \delta y_2^*) > F_2(y_2^*)$$

Alternatively, if $\delta y_2^* < 0$, then

$$F_2(y_2^* + \delta y_2^*) < F_2(y_2^*)$$

Once the force balance is lost, $\gamma_4$ (the equivalent module velocity $\dot{z}$) will not be zero since the tool would either start to fall or move up in the pipe depending on the direction of the perturbation. As a result, $\gamma_3$ (the equivalent module height $z$) would not be constant anymore, it would increase or decrease "indefinitely" (or until the well ends). Therefore, we can conclude that for this case the system is unstable because starting from a region close to the equilibrium point (after perturbation) the state trajectories do not stay arbitrarily close to the origin (the module moves indefinitely).

For the second case, when output $y(t)$ tracks a desired trajectory $y_d(t)$, the stability of the module motion can be transformed into an equivalent stability problem around an equilibrium point. Let $x^*(t)$ be the nominal motion trajectory corresponding to an initial condition $x_0 = x^*(0)$. In order to study the stability of the motion, we can perturb the initial condition such that $x(0) = x_0 + \delta x_0$, and study the associated variation of motion error defined by,

$$e(t) = x(t) - x^*(t)$$

Then $e(t)$ satisfies the non-autonomous differential equation,

$$\dot{e} = f(x^* + e, t) - f(x^*, t) = g(e, t)$$

(6.9)
with initial condition $e(0) = \delta x_0$. The dynamic system defined in 6.9 has an equilibrium point at the origin of the state space since $g(0, t) = 0$ (e.g. no motion error) and the new state vector is $e$. Therefore, instead of studying the stability of the original system's nominal motion we can analyze the stability of the perturbation dynamics defined in 6.9 around its equilibrium point [9].

For non-autonomous systems, an equilibrium point is stable at $t_0$ if for any $R > 0$, a positive scalar $r(R, t_0)$ exists such that if $\|e(t_0)\| < r$, then $\|e(t)\| < R$ for any time $t > t_0$. It should be noted that the only difference between the definition of stability for autonomous systems and non-autonomous systems is the fact that the boundary of the region $B$, depends on the initial time $t_0$.

Once again, in this case, instability is not hard to prove. The new state vector being analyzed is the vector, $e$, associated with the motion error of our old state vector $x$. In order to simplify our analysis let us just consider a perturbation in the obstruction radius. If we first analyze intuitively what happens with $x^*_1$ (radius $R_2$ of obstruction at $t_0$) when perturbed,

$$e_1(t) = x_1(t) - x^*_1(t)$$

The umbrella module is not backdrivable. Thus, $x_1$ will be constant for all $t$, and since $x^*_1$ was constant as well we can conclude that $e_1$ would be constant or bounded for all $t$. Now the module velocity, $x_4$, will have a value different than $x_4^*$. The new velocity would be bigger or smaller depending on the direction of the perturbation in $x_1$, but their difference will remain constant at steady state (since the module does not accelerate indefinitely). Therefore,

$$e_4(t) = x_4(t) - x_4^*(t)$$

will converge towards a constant value. This is rather encouraging as a start. However, when we turn our attention to the new state $e_3$, which describes the position error due to the perturbation,

$$e_3(t) = x_3(t) - x_3^*(t)$$
we notice that the value of \(x_3\) will diverge from \(x_3^*\) as time passes since the corresponding module velocities would be different. Hence, the state \(e_3\) will not stay arbitrarily close to the equilibrium point, regardless of the time \(t_0\) at which the perturbation occurs. Therefore, having at least one state "unbounded", the system in 6.9 can not be stable.

It was shown that the open loop system is unstable in the two configurations of interest for the module operation. Our analysis is based on the assumption that the original system is autonomous (e.g. the disturbance velocity \(v_1\) and the density \(\rho\) of the fluid media remain constant and known). Nevertheless, our results can be extended to the more realistic scenario, where \(v_1\) and \(\rho\) are measurable but with time changing values, since their variations would do nothing but hinder stability. These results indicate the need for the implementation of an appropriate control scheme.

**6.3 Control implementation and objectives**

In order to decrease the complexity of the control implementation we will use feedback linearization techniques to simplify the state representation of the system. Note that feedback linearization is conceptually different from Lyapunov linearization (Jacobian linearization). In feedback linearization we "simplify" the state equations by using the feedback through the control input to cancel all nonlinearities and then implement the control law with an equivalent input [9] [10]. This concept is analogous to classic dynamics analysis in that we profit from the fact that the complexity of a system might depend on the "reference frame" from which we are trying to analyze it. Accordingly, by appropriately changing our reference frame we can "simplify" the state representation. And by doing this the dynamics of the system are not altered since exact state transformations are used instead of the linear approximations used in Lyapunov linearization.

There are two main objectives for the umbrella mechanism operation. As we mentioned before, the first requirement is for the module to maintain constant position inside the well (e.g. float at a predefined height for a specific amount of time). The second requirement is for the module to track a predefined trajectory. In essence, satisfying both objectives can be reduced to a single problem if we notice that a
requirement of constant position is essentially tracking an output \( y_d = Q \), where \( Q \) is a constant defining the required height. Therefore, by achieving a good tracking control we can actually satisfy both requirements. In addition, an implicit control objective is to minimize the control force, since the umbrella mechanism was conceptually developed to provide an energy-efficient counterpart to wheeled and other types of propulsion devices. This requirement is partly addressed by the mechanical design of the apparatus. It was already mentioned in the previous chapter that no control force is actually needed in order to maintain the position of the umbrella mechanism petals. This is translated into significant savings in energy since moving the umbrella mechanism should represent less than half of the module actions during operation (significant time would be spent "cruising" the lengthy sections of the wells). However, the control algorithms should implement functions to cut power while the module is in "cruising" mode.

6.3.1 Input-Output linearization

We want to achieve good tracking control; therefore the closed loop system should be asymptotically (or exponentially) stable. The idea of canceling the nonlinearities of the system and imposing desired linear dynamics can be applied to nonlinear systems whose state representation can be converted to the companion form, which is the case for the system in 6.4. Therefore, we can proceed with feedback linearization. In our case, a noticeable difficulty is that the output is not directly related to the input. Hence, it is not obvious how to implement an input that would control appropriately the tracking behavior of the output. Input-output linearization consists of finding such a relationship by differentiation of the output equation until the relationship to the input is direct. Subsequently, the input should be chosen so as to cancel all nonlinearities. The number of differentiations required for the input to appear explicitly is called the relative degree of the system. The relative degree of a system can not be bigger than the system's order. If the relative degree of the system is the same as its order then the linearization of the system is completed and a control law that would stabilize the plant can be directly implemented. However, if the degree is less than the system order, the new linear state representation must be completed using some of the old states which become "internal dynamics" since they are not controlled directly by the control input. Thus, for the new control scheme to be satisfactory, such "internal dynamics" must be stable [9] [10].
In this section some mathematical tools and notation from differential geometry, essentially Lie derivatives, will be used in order to derive the input-output linearization. The definitions of the terms used here, in the context of nonlinear control, can be found in Appendix D. The system in 6.4 is of the form,

\[ \dot{x} = f(x) + g(x)u \]
\[ y = h(x) \]

were \( h(x) \) is a scalar function, and \( f(x) \), and \( g(x) \) are vector fields. The process of differentiation of the output equation is equivalent to finding the Lie derivatives of \( h \) with respect to \( f \) and \( g \): \( L_f h \) and \( L_g h \) respectively. Therefore, we must compute

\[ y^{(i)} = L_f^i h(x) + L_g L_f^{i-1} h(x) u \]

until for some integer \( r \), we have

\[ L_g L_f^{r-1} h(x) u \neq 0 \]

In our case, \( r = 4 \) since,

\[ y = x_3 \]
\[ \dot{y} = x_4 \]
\[ \ddot{y} = P_5 (x_4 - v_1)^2 - K_{17} \]
\[ \dddot{y} = \frac{\partial P_5}{\partial x_1} x_2 (x_4 - v_1)^2 + 2P_5 (x_4 - v_1)^2 - K_{17} \] \( (x_4 - v_1) \)

and we can distinguish already that in the next differentiation step the term \( \dot{x}_2 \) appears, which if replaced using 6.4 will bring the term \( e_a \). Therefore, if we apply the control law

\[ e_a = \frac{1}{L_g L_f^3} \left( - L_f^4 h + v \right) \]

(6.12)

to the last differentiation step given by
we obtain the linear relationship

\[ y^{(4)} = \nu \]

(6.14)

were \( \nu \) is an equivalent control input. In our case we do not end up with "internal dynamics" and with the system linearized the next step is to choose an appropriate equivalent control input.

In order to implement good tracking control a simple pole placement methodology can be used. Let us define \( \mu = [y, \dot{y}, \ddot{y}, \dddot{y}]^T \) to be the new state vector, and \( \mu_d = [y_d, \dot{y}_d, \ddot{y}_d, \dddot{y}_d]^T \) to be the vector of desired state trajectories. The tracking error vector is then given by

\[ \tilde{\mu}(t) = \mu(t) - \mu_d(t) \]

(6.15)

If we use the equivalent control law

\[ \nu = y_d^{(4)} - q_0 \dddot{\mu} - q_1 \dddot{\mu}_1 - q_2 \dddot{\mu}_2 - q_3 \dddot{\mu}_3 - q_4 \dddot{\mu}_4 \]

(6.16)

in equation 6.14 we obtain the tracking dynamics

\[ \dddot{\mu}_5 + q_3 \dddot{\mu}_4 + q_2 \dddot{\mu}_2 + q_1 \dddot{\mu}_1 + q_0 \dddot{\mu}_1 = 0 \]

(6.17)

Therefore, choosing the constants \( q_i \) such that the roots of the characteristic equation of 6.17 are strictly in the left-hand plane guaranties that the whole new state remains bounded and the tracking error \( \tilde{\mu} \) converges to zero exponentially. A drawback of the pole placement strategy is that its implementation is carried out in a trial and error fashion and robustness is difficult to guarantee [7] [8] [12].

Further disadvantages of feedback linearization are that its implementation relies on the accuracy of the model used. Moreover, all parameters in the model should be known. For the model in 6.4, the only unknown parameter is the density of the fluid, but its value is bounded. Nevertheless, given the presence of disturbances such as the velocity of the fluid, a more robust technique should be applied.
6.3.2 Robust Control

Robust control techniques address the determination of the control laws so that parametric and modeling uncertainties do not affect the desired characteristics of the system response. In the case of the obstruction module we implement a sliding controller to guarantee "perfect" tracking. The model in 6.4 is not directly utilizable thus we use the feedback-linearized model from 6.13. If we let \( L_f^4 h(x) = \alpha(x) \), and \( L_k L_f^3 h(x) = \beta(x) \), we can rewrite our model as,

\[
y^{(4)} = \alpha(x) + \beta(x) e
\]

(6.18)

The sliding surface \( s = 0 \) is defined by,

\[
s(x,t) = \tilde{y} + 3\lambda \tilde{y} + 3\lambda^2 \tilde{y} + \lambda^3 \tilde{y} = 0
\]

(6.19)

where \( \tilde{y} = y - y_d \), and \( \lambda \) is a positive constant that determines the rate of convergence towards the sliding surface. Essentially, the sliding variable \( s \) is a weighted sum of the output errors. The sliding surface is then by definition the surface defined by all the system states for which the weighted sum or output errors is equal to zero [9]. The best estimate for the control input is the one for which \( \hat{s} = 0 \). Thus, form 6.18 and 6.19 we get,

\[
\dot{e}_u = \left( -\alpha + y^{(4)} d - 3\lambda \tilde{y} + 3\lambda^2 \tilde{y} - \lambda^3 \tilde{y} \right) \hat{B}^{-1}
\]

(6.20)

and the control law is given by: \( e_u = \hat{e}_u - k \text{sgn}(s) \) where \( k \) must satisfy the lyapunov-like function,

\[
\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s|
\]

(6.21)

in order to guarantee convergence towards \( s = 0 \) (perfect tracking). Therefore, solving 6.21 we get,

\[
k = C(A + \eta) + (C - 1) |\hat{e}_u|
\]

66
with \( C = \left( \frac{\beta_{\text{max}}}{\beta_{\text{min}}} \right)^{\frac{1}{2}} \).

A drawback of sliding controllers is the excessive control activity caused by the chatter in the control law computation [9]. This can be solved by smoothing the control discontinuity using a boundary layer of thickness \( \phi \). The smoothed control law is given by,

\[
e_s = \hat{e}_a - \bar{k} \text{sat}(\gamma \phi)
\]

(6.22)

with \( \bar{k} = k - \dot{\phi} \).

The idea of the sliding controller is similar to feedback-linearization in the sense that we are canceling the plant nonlinearities with estimates, and then we want to make sure that the dynamics of the remaining tracking errors (sliding surface) are stable.

Using the smoothed switching controller given by 6.22 we succeeded in making the plant track two types of trajectories in the face of constant, and time varying disturbances. First we provide the plant with desired trajectories that will make the module maintain a fixed position inside the pipe. The second kind of trajectories was aimed at making the plant achieve a constant velocity. The choice of \( \lambda \) for the controller was based on the fact that we wanted a small time constant for the tracking errors so we used values between 800 and 1000.

The simulation results are portrayed in figures 6.1 through 6.6. Figure 6.1 displays the response of all four states in the presence of a constant disturbance \( v_1 = 1 \text{ (m/s)} \), and with no control force applied to the module.
Figure 6.1: Open loop system behavior with constant disturbance.

The tracking errors and control input of the system when required to track

\[ y_d = 0 \]  

(6.23)

with the constant disturbance are portrayed in figure 6.2. Tracking of 6.23 makes the module maintain a constant position inside the pipe, which is achieved reasonably well since the tracking errors converge to zero as time passes. This is evident for state \( x_4 \), the module's velocity. The tracking error for the module position, \( x_1 \), seems to grow but in fact stays bounded with a value never exceeding of \( 3 \times 10^{-5} \) (m).
Figure 6.2: Tracking errors and Control activity when maintaining constant position, against a constant disturbance.

Figure 6.3 displays the same information when the system is made to track

\[ y_d = 0.05t - \frac{0.05\pi}{2} \sin \left( \frac{\pi t}{2} \right), \quad 0 \leq t < 2 \]

\[ y_d = 0.1t - 0.1, \quad 2 \leq t \]

(6.24)

The trajectory in 6.24 forces the system to maintain a constant velocity in the steady state. The tracking errors converge very quickly (less than 2 seconds) towards negligible values.
For the simulations displayed in figures 6.2 and 6.3, the initial state vector used was, $x(t = 0) = [0.05, 0, 0, 0]$. This was done to avoid excessive control requirements during the transients. In both simulations we can appreciate that the values of the control voltage do not exceed 3 volts during transients, and tend toward zero at the steady state. Actually, given the tracking requirements imposed by the control law the trajectories are forced to converge towards the desired value. Hence, if initially there is a large difference between both values the amount of “force” required to bring the trajectories closer to the desired values is large. In some instances this initial “force” might surpass the available energy of the actuator. This can be solved by two approaches. The first approach, being the one we undertook in the simulations, is to make sure the initial state vector facilitates the convergence of the trajectories. This of course sounds a little “dishonest” but since we have a model of the module dynamic behavior the responses of different initial state vectors can be determined straightforwardly in open loop. Therefore, initial state vectors can be a priori computed so energy is not wasted during transients. Alternatively, we can slow down the speed of convergence demanded by the control law (using smaller values for $\lambda$) but this can prolong the transients for too long.
Figure 6.4 displays the response of all fours states in the presence of a time varying disturbance

\[ v_1 = \frac{6(1-e^{-t}) + \sin(3t)}{24} + 0.1 \]  

(6.21)

The simulations results for position and velocity control against 6.21 are displayed in figures 6.5 and 6.6 respectively. The initial state vector used was, \( x(t=0) = [0.07, 0, 0, 0] \). Again the tracking errors converge rapidly towards negligible values within few seconds, or stay bounded. Furthermore, the control activity remains reasonably low.

Figure 6.4: Open loop system behavior with time varying disturbance.
Figure 6.5: Tracking errors and Control activity when maintaining constant position, against a time varying disturbance.

Figure 6.6: Tracking errors and Control activity when maintaining constant velocity, against a time varying disturbance.
The simulation results are rather promising since they display the main characteristics sought in the control objectives. In the presence of constant and time varying disturbances, the model is able to track satisfactorily the two types of trajectories required for position and velocity control. Finally, the predicted control activity remains low thus also fulfilling the energy efficiency requirement.

The last point to be made in the model performance is its dependence on the fluid flow. The module depends on the available differential pressure across its obstruction in order to move. This is partly controlled by the opening of the umbrella, which is influenced by the control laws. However, the available differential pressure also depends on the fluid flow. Therefore, not all desired trajectories can be achieved. During operation, the model, along with measurements of the fluid flow, need to be used to determine a desired trajectory that is feasible and a correspondent initial state vector that can minimize control force.

The Matlab code for the simulations, containing the values of all the system parameters, can be found in Appendix C.

6.4 Prototype Hardware implementation

In order to physically implement the control strategies from the previous section we had to decide on an appropriate platform. A desirable hardware platform should run a behavior program based on the control laws and command the linear actuator driving the umbrella module using appropriate sensor information. The module program can be run from a small (16-bit) microprocessor like a Motorola 6811 with the sensors interfaced through analog-to-digital converters and a digital-to-analog converter interfacing the linear actuator. There are several application boards commercially available that can be used as platforms for our device, however size constraints are a significant problem. Most commercial boards are laid out in rectangular shapes far wider than what can be fitted inside the umbrella module whose outside diameter is only 2.5 inches. Custom boards have longer manufacturing lead times and are expensive if ordered in small quantities. Therefore, in order to test our prototype we decided to use a commercially available board.

In the experimental setup, the module needs to be submerged inside the flowloop, since the board cannot be mounted inside the prototype it is placed outside the flowloop and interfaced with the module through an underwater cable. The controller board is interfaced serially (e.g. RS-232 connection) to a host
computer through which the behavior programs can be modified. Nevertheless, the board uses a power supply similar to what a real autonomous module would require. The power is split between the logic circuitry, which requires 9 volts, and the actuator that can require up to 24 volts. Although this approach appears to defeat the idea of developing an autonomous module it proved to be more practical in terms of debugging. The commercial board used is the Rug Warrior Pro controller board from AK Peters [19]. The board uses a Motorola MC68HC11A1 microcontroller clocked by a crystal oscillator with a frequency of 8 MHz.

The operating system and the behavior program are stored in external memory, which has a battery backup circuit protecting its contents when power is turned off. Programs can be written both in assembly language or Interactive C code (IC), whose syntax is very similar to ANSI C, and downloaded to the board. A limited amount of information can be displayed in real time through the board’s LCD, which comes in handy while debugging the behavior programs. Figure 6.7 displays the hardware architecture used in the module controller board. The Rug Warrior Pro controller board was designed for mobile robotic applications and as such it has several input and output channels. However, for our purposes we only use 2 analog inputs, which come from the shaft encoder at the position feedback module, and the flowmeter readings. The only outputs used are a motor driver that controls the linear actuator with pulse width modulation (PWM) and the LCD display.
The board is interfaced to the umbrella module and programs can be successfully run to operate the opening of the petals. Nevertheless, the accuracy of the readings from the position feedback module is extremely noisy and inaccurate. There are two main problems with this module. First, the encoder used has a low resolution. The encoder consists of a photo reflector cell that encodes values from a patterned disk. This disk is attached to a shaft that is moved by the external passive wheel, and it only has 64 marks per revolution. In addition, even though the external passive wheel is pressed against the pipe inner surface at all times, slippage occurs, which contributes to inaccurate displacement readings.

Since a good position sensor is essential for feedback and the implementation of the state observer in the control algorithms, these could not be tested experimentally. A better position sensor needs to be used for the successful testing of the module, nevertheless the minimal space available inside the umbrella module limits the kind of encoders that can be used thus the position feedback module might need to be redesigned.
Chapter 7: Conclusions and future work

7.1 Summary of contributions

The conceptual and mechanical design of a novel propulsion module for well logging jobs was presented. The basic physical principles of operation were analyzed, modeled and tested in simulations. A working prototype of the propulsion module was built implementing the key features of the conceptual design. The mechanical obstruction was implemented using an innovative combination of a linkage mechanism and composite petals.

A complete dynamical model of the system was developed to a level required for control implementation. Finally, nonlinear control techniques were used to achieve an appropriate and straightforward implementation of the control laws despite the complexity of the module dynamics.

The simulations of the model behavior inside an idealized well offered encouraging results since the system successfully maintained position and velocity in the presence of constant and time varying disturbances. The prototype was tested inside a flowloop proving that all systems are operational. However, proper testing of trajectory tracking was not feasible due to the limited resolution of the position feedback module.

The prototype uses a tethered configuration. All the control algorithms can be implemented using a small Motorola 6811 microcontroller board with split power supply that was adapted for the umbrella module. In its current configuration the board requires 9 volts to power its logic, and can regulate up to 24 volts for the linear actuator. This configuration can easily be reengineered to fit inside the module, thus resulting in a completely autonomous system.
7.2 Future work

The next stage in this project is to validate the simulation results with experiments. The flowloop built for the static testing of the umbrella module can be used for position control experiments. However, this flowloop might be too short in length for the velocity control experiments.

The advantage of using a sliding controller for the control laws is its robustness against parametric and modeling uncertainties. Therefore, even if the model of the umbrella dynamic behavior is not accurate, convergence towards the desired trajectories can still be guaranteed. However, this advantage has a limit, in that the convergence might require high control activity. Furthermore, as described in Chapter 6, there is a need for a new and more accurate position feedback module in order to successfully test the control algorithms.

Once the model is tested in a simulated well, the next step should be to develop a prototype fit for testing in a real oil well. This would entail mainly the use of different materials in the manufacture of the prototype since the design is rugged enough to survive oil well environments. In addition, a linear actuator with a larger force rating than the one currently used is recommended to increase the range of effectiveness of the module. The control algorithms can be easily changed so as to reflect the difference in fluid media, so testing could be done in multi-phase flow regimes.
References


[18]. TS Products, Inc., 5550-2 North McGuire Road, Post Falls, ID, USA 83854-6506.

[19]. AK Peters, Ltd., 63 South Avenue Natick, MA 01760-4626.
Appendix A: Machine Drawings

This appendix contains the detailed drawings of all the parts of the obstruction module concept model used for the experimental part of system identification, and the umbrella module prototype. The drawings were produced with ProENGINEER.
Schlumberger-Oil Research

Drag Navigation Module

Date: 18-Oct-99

DES: Valdivia

Ang: Valdivia

Model: CSDR

SECTION FF-FF

Core

15.00°

0.0625

1.00

114.39°

1.00

174.39°

30.00

0.375

45° x 0.0625

10.00

1.00

2.00

4.00

15.00

4.00

6.00

5.00

3.00

2.25

174.39°

174.39°
SECTION A-A
SEE DETAIL H

SECTION E-E
x4 1/4-20 UNC thread min 0.5in

-45° X .125

MATERIAL SUGGESTED: Aluminum

SCALE 1:500

Same on both ends

1-8 UNC thread min 1.5in typ

Same on both ends

MATERIAL SUGGESTED: Aluminum

SCALE 1:500

Same on both ends
1. Sketch of a cylindrical object with dimensions and features.

- Diameter: 2.50
- Diameter: 1.00
- Length: 10.00
- 1/8 UNC thread min 0.8 in
- 0.8 thread relief

2. Sketch of a rectangular object with dimensions and features.

- Width: 2.00
- Height: 4.50
- 1/8 UNC thread min 1.5 in


- Width: 2.00
- Height: 4.50

4. Material Suggested: Aluminum

5. Scale: 1:500

6. Design and Engineering Notes:
   - 1:8 UNC thread min 0.5 in
   - 5/40 UNC thread min 0.5 in
   - Ø0.8 thread relief

7. Schematic Diagram:

   - See Detail A

8. Date: 21-Sep-99

9. Created by: Valdivia

10. Drawing Number: CSDR

11. Revision: v1.1

12. Project: Schlumberger-Doll Research

13. Skirt Module top
MATERIAL SUGGESTED: Aluminum
**SECTION M-M**

- $\Phi 1.500 \pm 0.005$
- $-0.000$

- $1.25$
- $2.45$

**SECTION N-N**

- $x2 1/4-20$ UNC drill & tap thru
- $45^\circ \times .125$

**DETAIL P**

- SCALE 2.000

**MATERIAL SUGGESTED:** Aluminum

---

**Schlumberger-Well Research**

**Skit Module Bottom**

**DATE:** 27-Sep-99

**DESIGN:** Veldino

**PROJ. ENG.:**

**SDEL 2000**

**SDEL 2000**
Original length of bar should be approx. 30.2 in.

MATERIAL SUGGESTED: 1/2x3/8 rectangular Steel stock

SEE DETAIL D
MATERIAL SUGGESTED: Flat steel stock

Some on both ends

DETAIL G
SCALE: 2:000

Schlumberger-Doll Research

Skirt Module Arc

Date: 27-Sep-99
Des Eng: Valdavia
Wtr Eng:
Proj Eng:

C S D R 96

\( \text{SEE DETAIL G} \)
1-8 UNC thread min 0.8in

∅0.8 thread relief

MATERIAL SUGGESTED: Aluminum

SCHLUMBERGER-DOLL Research

Nose Cone

ANGLE PROJECTION

Date: 27-Sep-95

CSBR CONE

DRAFT COC

Volvili
Eng: Valdivia
Proj Eng: Valdivia

Make: 0.01
Print: 0.001

CONTRACT AND TRADE SECRET

SCHLUMBERGER-DOLL RESEARCH CORP.

COPYRIGHT. CONFIDENTIAL. DISCLOSE. STOR. 21, 1994
MATERIAL SUGGESTED: Aluminum

UNLESS OTHERWISE SPECIFIED SIZES ARE IN INCHES TOLERANCES: ± .006" MATERIAL: FREE FROM ENVIRONMENTAL CONTAMINATION

Date: 27-Sep-95

Design: Valdivo

Model No.: Proj Leg:
SECTION B-B

MATERIAL SUGGESTED: Aluminum

UNLESS OTHERWISE SPECIFIED
DIMENSIONS GIVEN IN INCHES

30-Sep-99

DESIGNER: Valeria

SCHLUMBERGER-DOLL RESEARCH

RING
MATERIAL SUGGESTED: Aluminum

02.50
45° X .125 typ

01.00
1.25
1-8 UNC thread min 0.8 in (typ)

0.8 thread relief (typ)

2.25
4.00
15.00

0.8

1.25

1.50 ± 0.000
0.005

45° X .125

1.00

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26-Sep-99
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DATE: 26 Sep 99
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1.25
1-8 UNC thread min 0.8 in (typ)

0.8 thread relief (typ)

2.25
4.00
15.00

0.8

1.25

1.50 ± 0.000
0.005

45° X .125

1.00

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1-8 UNC thread min 0.8 in (typ)

0.8 thread relief (typ)

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1.50 ± 0.000
0.005

45° X .125

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0.8 thread relief (typ)

2.25
4.00
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1.25

1.50 ± 0.000
0.005

45° X .125

1.00

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0.8 thread relief (typ)

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4.00
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0.8

1.25

1.50 ± 0.000
0.005

45° X .125

1.00

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DATE: 26 Sep 99
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DI SCLOSE.

APPROVAL FROM

INTERPRETATION AND INTERPRETATION

By:

DATE: 27-Nov-99

DRAWER: Valdivia

Drawn by: Valdivia

Scale: 1/8

Scale for: 1/8

Schlumberger-Doll Research

Drag module assembly

CSDR FULL-OPEN

5000

1/8

1

0

2

3

4
SUGGESTED MATERIAL: Aluminum

QUANTITY: 1

SECTION DD-DD

SEE DETAIL HOLES

SECTION FF-FF

DETAIL HOLES

SCALE 1.500

UNLESS OTHERWISE SPECIFIED
DIM ARE IN INCHES
TOL ON ANGLE ± .002
2 PL ± .001 3 PL ± .001
INTERPRET DIM AND TOL PER
ASME Y14.5M - 1994

ANGLE PROJECTION

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OR USE WITHOUT PRIOR WRITTEN
APPROVAL FROM SCHLUMBERGER

21-Nov-99
Valdivia
Valdivia

Schlumberger-Doll Research

Bottom

A  S  D  R  BOTTOM
0.333
1 1
10-32 UNF drill & tap 0.5 in max, 0.4 in min.

1/4-20 UNC drill & tap 0.25 in max 0.22 in min.

Quantity: 1
Material: Aluminum
SUGGESTED MATERIAL: Stainless Steel

QUANTITY: 6

Schlumberger-Doll Research

Finger

18-Nov-99
Valdivia
Valdivia

A S D R FINGER
1000 1 1

UNLESS OTHERWISE SPECIFIED
DIM ARE IN INCHES
TOL ON ANGLE ±
2 PL ± 0.01 3 PL ± 0.001
INTERPRET DIM AND TOL PER
ASME Y14.5M - 1994

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ANGLE PROJECTION
SECTION AA-AA

8-32 drill & tap min .25 in max .3 in

SECTION HH-HH

SUGGESTED MATERIAL: Brass

UNLESS OTHERWISE SPECIFIED DIM ARE IN INCHES TOL ON ANGLE ± 2 PL ± 0.01 3 PL ± 0.001 INTERPRET DIM AND TOL PER ASME Y14.5M - 1994

INNER-RIB

Schlumberger-Doll Research

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Vadivia

19-Nov-99

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Inner-rib

Valdivia

Valdivia
SUGGESTED MATERIAL: Stainless Steel  QUANTITY: 6

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19-Nov-99
Valdivia
Valdivia

Ligament

A     S    D    R     LIGAMENT
2000

1       1
SUGGESTED MATERIAL: Aluminum

QUANTITY: 1

SECTION BB-BB

Drill & tap thru

4 x 10-32 UNF drill & tap thru

1 1/2-8 UNF drill & tap 1.25 in max

MOTOR HOUSING A

Schlumberger-Oil Research

22-05-09

Date

Motor Housing A

CSDR

Project: Motor Housing A

PAGE 2 OF 3
Quantity: 1
Material: Aluminum

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DIM ARE IN INCHES
TOL ON ANGLE ±
2 PL ± 0.01  3 PL ± 0.001
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17-Dec-99
Valdivia
Valdivia

Schlumberger-Doll Research

Mounting Plate

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**DIMENSIONS**

- **Detail Hole**
  - Diameter: 0.50
  - Tolerance: 0.001

- **Piston**
  - Material: Aluminum
  - Quantity: 1

**Section Details**

- **Section AA-AA**
  - Dimension: 0.25

- **Section BB-BB**
  - Dimension: 0.280

**Notes**

- Angle Projection
- Suggested Material: Aluminum

**Schlumberger-Doll Research**

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SUGGESTED MATERIAL: Stainless Steel  QUANTITY: 6

UNLESS OTHERWISE SPECIFIED DIAMETERS IN INCHES LENGTHS IN FEET

DATE: 23-Nov-93
DESIGN ENGINEER: Valdiviez
PREPARED BY: VALDIVIEZ
CHECKED BY: VALDIVIEZ

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ANGLE PROJECTION

SECTION DD-DD

QUANTITY: 6

120.

.25 flat bottom

.3125+0.001 sym

-.000

x2 10-32 UNF drill & tap 0.5 in min

.25

.75

3.25

1.00

.75

5.25

60.0°

R1.125

R.75

.401 REF

.911 REF

.785 REF

.375 REF

.105 REF

.125 REF

.455 REF

.050 REF

.025 REF

.018 REF

.015 REF

.012 REF

.010 REF

.008 REF

.004 REF

.002 REF

.001 REF

SUGGESTED MATERIAL: Stainless Steel  QUANTITY: 6
SUGGESTED MATERIAL: Aluminum QUANTITY: 1

UNLESS OTHERWISE SPECIFIED
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TOL ON ANGLE ±
2 PL ± 0.01
3 PL ± 0.001
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ANGLE PROJECTION

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19-Nov-99
Valdivia
Valdivia

Schlumberger-Doll Research

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Quantity: 1
Material: aluminum

Schlumberger-Doll Research

Tube

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UNLESS OTHERWISE SPECIFIED
DIM ARE IN INCHES
TOL ON ANGLE ±
2 PL ± 0.01 3 PL ± 0.001
INTERPRET DIM AND TOL PER
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Date: 24-Jan-01
Dwn by: Valdivia
Des Eng: Valdivia
Mfg Eng:
Proj Eng:

Schlumberger-Doll Research
Encoder Module

Encoder Module

Scale: 0.500
UNLESS OTHERWISE SPECIFIED
DIM ARE IN INCHES
TOL ON ANGLE ±
2 PL ± 0.01
3 PL ± 0.001
INTERPRET DIM AND TOL PER
ASME Y14.5M - 1994

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APPROVAL FROM SCHLUMBERGER

Quantity: 2
Material: Aluminum

Schlumberger-Doll Research

Date: 11-Jan-00

Design: Valdivia

Drawing: Valdivia

Project: Valdivia

Scale: 1:1000

Sheet 1 of 1
Material: Aluminum

UNLESS OTHERWISE SPECIFIED
DIM ARE IN INCHES
TOL ON ANGLE ±
2 PL ± 0.01 3 PL ± 0.001
INTERPRET DIM AND TOL PER
ASME Y14.5M - 1994

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APPROVAL FROM SCHLUMBERGER

Date: 11-Jan-00

Dwn by: Valdivia
Des Eng: Valdivia
Mfg Eng: Valdivia
Proj Eng: Valdivia

Schlumberger-Doll Research
rod3

Quantity: 1

ANGLE PROJECTION

2000
Appendix B: Composite Petals Fabrication

As we discussed in Chapter 3, it is desired to have petals strong enough in the radial direction to withstand the force created by the differential pressure across them. In addition, it is required for the petals to have a flexible curvature so they can easily adapt to different openings of the umbrella. Therefore, the petals are made using composite fibers since the matrix strength in the two main axes can be manipulated by varying the relative orientation and number of fiber layers.

The final petal design uses three layers of carbon fiber with a stainless steel insert in between. Figure B.1 displays the first stage of the petal fabrication. The first two layers of fiber are oriented at an angle $\alpha = 10^\circ$ from the radial axis $1$, and the stainless steel insert with two holes for petal mounting is placed along the radial axis.

![Figure B.1: Petal Layers](image)

Each layer is covered with a medium harness epoxy resin and the excess resin is removed with a roller. The process is repeated each time a new layer is added. Figure B.2 shows a side view of the petal layers on
top of the Teflon surface used when applying the epoxy resin. The roller used can be seen in the background.

Figure B.2: Epoxy application

Once all the layers are well soaked in epoxy (a third layer goes on top of the setup in figure B.2) the composite matrix is set inside a vacuum oven at 21 degrees Celsius for 30 minutes to eliminate the air bubbles created inside the matrix while applying the resin. Figure B.3 shows the fiber matrix being placed inside the vacuum oven. The temperature on the vacuum oven was kept low avoiding premature curing of the resin. Once the air bubbles from the matrix are extracted this one is set in a forming die consisting of an aluminum cylinder of the same radius as the umbrella module and a matching half cylinder. The composite matrix is clamped in between the dies and the entire setup is place inside an oven at 90 degrees Celsius for 4 hours to allow complete curing of the epoxy resin. Teflon layers are used in between the composite matrix and the dies to avoid the resin from bonding with the dies. Figure B.4 shows the forming die inside the oven.
Figure B.3: Fiber layers are put into a vacuum oven.

Figure B.4: Fiber layers clamped on dies are placed inside an oven.
Once the matrix is cured, it is removed from the forming die. The shape of the petal is cut and two mounting holes are drilled. The resulting petals are shown in figure B.5.
Appendix C: Simulation Algorithms

The matlab code used for the simulations in chapter 6 is presented in this appendix.

clear all;

% integration time

t0=0;     % initial time
tf=5;     % final time

% initial conditions
x0=[0.07 0 0 0 0];

% integration
options=odeset('AbsTol',1e-4);
[t,x]=ode45('sliding3',[t0 tf],x0,options);

global s;

% Disturbance values (fluid flow)
v1=0.1+((1-exp(-t))+((sin(3*t))/6))/4;
% v1=1;

% desired trajectories to maintain constant position
yd=0;
dyd=0;
dddy=0;
ddddy=0;
dddddy=0;

% desired trajectories to maintain constant velocity
m=0.05;
n=0.05;
o=pi/2;

if 0<=t & t<2
    yd=(m*t).*(n*sin(o*t))/o;
    dyd=m*(n*cos(o*t));
    dddy=n^2.*o*sin(o*t);
    ddddy=0;
    dddddy=0;
else
    yd=0.1*t;0.1;
    dyd=0.1;
    dddy=0;
end
```matlab
% dddyd=0;
% dddydyd=0;
%end

% Printing results
T=t0:0.1:tf;

figure(1)
plot(t,x(:,1),'k',t,x(:,2),'k--',t,x(:,3),'k--',t,x(:,4),'k',t,v1,'k+');
xlabel('time (sec)');
axis([0 5 -0.5 1.5]);
legend('X1 (m)','X2 (m/s)','X3 (m)','X4 (m/s)','v1 (m/s)');
grid;

figure(2)
subplot(3,1,3),plot(t,x(:,3)-yd,'bx');
grid;
ylabel('Tracking error X3');
xlabel('time (sec)');
subplot(3,1,2),plot(t,x(:,4)-dyd,'bx');
grid;
ylabel('Tracking error X4');
xlabel('time (sec)');
subplot(3,1,1),plot(t,x(:,5),'bx');
grid;
ylabel('Control input (V)');
xlabel('time (sec)');

figure(3)
subplot(2,1,2),plot(t,x(:,3),'k',t,yd,'r-');
grid;
xlabel('time (sec)');
legend('X3 (m)','yd (m)');

subplot(2,1,1),plot(t,x(:,4),'k',t,dyd,'r-');
grid;
xlabel('time (sec)');
legend('X4 (m)','dyd (m)');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 Function: sliding3.m
% computes the values of the states by integration (observer)
```
% calls control_sld3.m to get value of control voltage
% written by: Pablo Valdivia y Alvarado 12/06/2000

function xdot=sliding3(t,x)

% States
x1=x(1);
x2=x(2);
x3=x(3);
x4=x(4);
time=t;

% Model parameters
b=0.0127;
Lp=0.0762;
Lt=1.65;
R1=0.03175;
R2=x1;
Q=(R2-R1)/Lp;
Q1=sqrt(1-(Q^2));
Q2=sqrt(1-(1-Q1^2));
Im=1.0e-7; Ra=16; bm=1.27e-5; K=0.0142; N=66; p=1275.14; La=0.4e-3;
E=0.8;
l=0.5; M=142; R1=0.03175; R=0.0762; Ts=1*(K/Ra); d=5.42; e=-11.2;
f=11.28;
I=1e-6; Imin=0; Imax=1e-5; Ihat=5e-6; Ibigs=5e-6;
rho=1000; rhomin=500; rhomax=1500; rhohat=1000; RHO=500; rhotool=2700;

% Disturbance values
v1=0.1+((1-exp(-time))+((sin(3*time))/6))/4;
% vi=1;

% Kij parameters used in model polynomials
k0=72.59248732755159;
k1=0.25645497047501;
k2=-0.25419986451983;
k3=0.00638498879676;
k4=-0.00002897112642;
k5=-0.14;
k6=0.017;

k7=(K/(Ra*N^2*pi*p))/((I/NA2)+Im);
k7hat=(K/(Ra*N^2*pi*p))/((Ihat/NA2)+Im);
k7max=(K/(Ra*N^2*pi*p))/((Imax/NA2)+Im);
k7min=(K/(Ra*N^2*pi*p))/((Imin/NA2)+Im);
k8=(bm+(K^2/Ra))/((I/NA2)+Im);
k8hat=(bm+(K^2/Ra))/((Ihat/NA2)+Im);
k8max=(bm+(K^2/Ra))/((Imax/NA2)+Im);
k8min=(bm+(K^2/Ra))/((Imin/NA2)+Im);
k9=(1/(N^2*pi^2*p))*((p^2)*E))/((I/NA2)+Im);
k9hat=(1/(N^2*pi^2*p))*((p^2)*E))/((Ihat/NA2)+Im);
k9max=(1/(N^2*pi^2*p))*((p^2)*E))/((Imax/NA2)+Im);
k9min=(1/(N^2*pi^2*p))*((p^2)*E))/((Imin/NA2)+Im);
k10=(1/(N^2*pi^2*p))/((I/NA2)+Im);
k10hat=(1/(N^2*pi^2*p))/((Ihat/NA2)+Im);
k10max=(1/(N^2*pi^2*p))/((Imax/NA2)+Im);
k10min=(1/(N^2*pi^2*p))/((Imin/NA2)+Im);
\[ k_{11} = \rho \pi \left( (R^2 - (R1A2)) \times ((f+1)/2) - (R1A2) \right); \]
\[ k_{12} = \rho \pi \left( ((R^2) - (R1A2)) \times ((f+1)/2) - (R1A2) \right) - (2 \times (R^4) \times ((f-1)/2)) ; \]
\[ k_{13} = \rho \pi \left( ((R^2) - (R1A2)) \times ((R^2) - (R1A2)) \times ((1-d)/2) + (R^4) \times ((d+1+e)/2) + (R^6) \times ((f-1)/2) \right); \]
\[ k_{14} = \left( \text{rottool} - \rho \right) \times \pi \times (R1A2) \times Lt; \]
\[ k_{15} = 2 \times (R^2) \times \left( \text{rottool} - \rho \right) \times \pi \times (R1A2) \times Lt; \]
\[ k_{16} = (R^4) \times \left( \text{rottool} - \rho \right) \times \pi \times (R1A2) \times Lt; \]
\[ k_{17} = \left( \text{rottool} - \rho \right) \times (R1A2) \times Lt; \]
\[ k_{18} = \rho \times \left( (f-1)/2 \right); \]
\[ k_{19} = \rho \times \left( (e/2) \times (R^2 - (R1A2)) \times (2 \times (R^2) \times ((f-1)/2)) \right); \]
\[ k_{20} = \rho \times \left( (f-1)/2 \right) \times (R^4) \times \pi \times (R1A2) \times Lt; \]
\[ k_{21} = 1; \]
\[ k_{22} = -2 \times (R^2); \]
\[ k_{23} = R^4; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ k_{24} = \rho \times 9.8 \times 1; \]
\[ % input \]
\[ u = \text{control_slid3}(t,x); \]
\[ \% polynomials \]
\[ p_1 = k_5 / ((k_5 \times x_1) + k_6); \]
\[ p_2 = k_9 / ((k_5 \times x_1) + k_6); \]
\[ p_3 = k_10 / ((k_5 \times x_1) + k_6); \]
\[ p_4 = k_7 / ((k_5 \times x_1) + k_6); \]
\[ p_5 = k_{11} \times x_1^4 + (k_{12} \times x_1^2) + k_{13} / ((k_{14} \times x_1^4) + (k_{15} \times x_1^2) + k_{16}); \]
\[ p_6 = k_{20} \times x_1^4 + (k_{21} \times x_1^2) + k_{20} / ((k_{22} \times x_1^4) + (k_{23} \times x_1^2) + k_{23}); \]
\[ p_7 = (k_{18} \times x_1^4) + (k_{19} \times x_1^2) + k_{20}; \]
\[ p_8 = p_2 \times p_6 \times p_7; \]
\[ p_9 = p_2 \times p_6; \]
\[ % plant equations \]
\[ \text{if} \ x_2 > 0 \]
\[ \quad \text{sgnx2} = 1; \]
\[ \text{elseif} \ x_2 = 0 \]
\[ \quad \text{sgnx2} = 0; \]
\[ \text{else} \]
\[ \quad \text{sgnx2} = -1; \]
\[ \text{end} \]
x1dot=x2;
x2dot=(-p1*x2^2)-(k8*x2)*p8*(v1-x4)^2)-(p9*k24)-(p3*Ts*sgnx2)+p4*u;
x3dot=x4;
x4dot=p5*((v1-x4)^2)*k17;

% nonlinear observer

% xdot=[x1dot x2dot x3dot x4dot]';

xdot=[x1dot; x2dot; x3dot; x4dot; u];

%%%%%%

%%%%%%% % Function control_slid3
%%%%%%% % computes required control voltage from state values using sliding control laws
%%%%%%% % written by: Pablo Valdivia y Alvarado 12/06/2000

function u=control_slid3(t,x)

x1=x(1);
x2=x(2);
x3=x(3);
x4=x(4);
time=t;

global s;

if x2>0
    sgnx2=1;
elseif x2==0
    sgnx2=0;
else
    sgnx2=-1;
end

% Model paramters
b=0.0127;
Lp=0.0762;
Lt=1.65;
R1=0.03175;
R2=x1;
Q=(R2-R1)/Lp;
Q1=sqrt(1-(Q^2));
Q2=sqrt(1-((1-Q1)^2));
Im=1.01e-7; Ra=16; bm=1.27e-5; K=0.0142; N=66; p=1275.14; La=0.4e-3; E=0.8;
I=0.5; M=142; R1=0.03175; R=0.0762; Ts=1*(K/Ra); d=5.42; e=-11.2; f=11.28;
I=1e-6; Imin=0; Imax=1e-5; Ihat=5e-6; Ibig=5e-6;
rho=1000; rhomin=500; rhomax=1500; rhohat=1000; RHO=500; rhotool=2700;

% Disturbance values
v1=0.1+((1-exp(-time))+(sin(3*time))/6))/4;
% Kij parameters used in model polynomials
k0=72.59248732755159;
k1=0.25645497047501;
k2=-0.25419986451983;
k3=0.00638498879676;
k4=-0.00002897112642;
k5=0.017;
k6=0.017;
k7=(K/(Ra*N*2*pi*p))/((I/NA2)+Im);
k7hat=(K/(Ra*N*2*pi*p))/((Ihat/NA2)+Im);
k7max=(K/(Ra*N*2*pi*p))/((Imax/NA2)+Im);
k7min=(K/(Ra*N*2*pi*p))/((Imin/NA2)+Im);
k8=(bm+(KA2/Ra))/((I/NA2)+Im);
k8hat=(bm+(KA2/Ra))/((Ihat/NA2)+Im);
k8=(bm+(KA2/Ra))/((I/big/NA2)+Im);
k8max=(K/(Ra*N*2*pi*p))/((Imax/NA2)+Im);
k8min=(K/(Ra*N*2*pi*p))/((Imin/NA2)+Im);
k9= (1/((NA2)*2*(piA2)*(pA2)*E))/((I/NA2)+Im);
k9hat= (1/((NA2)*2*(piA2)*(pA2)*E))/((Ihat/NA2)+Im);
k9=(bm+(KA2/Ra))/((I/big/NA2)+Im);
k10= (1/(N*2*pi*p))/((I/NA2)+Im);
k10hat= (1/(N*2*pi*p))/((Ihat/NA2)+Im);
k10=(1/(N*2*pi*p))/((I/big/NA2)+Im);
k11=rho*pi*((RA2)*((f+1)/2)-(R1A2));
k12=rho*pi*((RA2)- (R1A2))*(-(RA2)*(1+(e/2)))-(R1A2)- (2*(RA4)*((f-1)/2));
k13=rho*pi*((RA2)- (R1A2))*((RA2)-(R1A2)*(1-d)/2)+(RA4)*((d+1+e)/2)+(RA6)*((f-1)/2));
k14=(rhotool-rho)*pi*(R1A2)*Lt;
k15=2*(RA2)*((rhotool-rho)*pi*(R1A2)*Lt;
k16=(RA4)*((rhotool-rho)*pi*(R1A2)*Lt;
k17= ((rho*9.8*1*(R1A2))/((rhotool-rho)*(R1A2)*Lt))9.8;
k17hat= (rhohat*9.8*1*(R1A2))/((rhotool-rhohat)*(R1A2)*Lt))9.8;
k17=(RHO*9.8*1*(R1A2))/((rhotool-RHO)*(R1A2)*Lt))9.8;
k18=(rho*(f-1)/2);
k18hat=(rhohat*(f-1)/2);
k18=(RHO*(f-1)/2);
k19=rho* ((e/2)*((RA2)- (R1A2)))- (2*(RA2)*((f-1)/2));
k19hat= rhohat*(-((e/2)*((RA2)- (R1A2)))- (2*(RA2)*((f-1)/2)));k19=RHO* ((e/2)*((RA2)- (R1A2)))- (2*(RA2)*((f-1)/2));
k20= rho* (((f-1)/2)* (RA4))+(e/2)*((RA4)-((R1A2)* (RA2)))+((d+1)/2)* (RA4)- 2*(RA2)* (R1A2))+ (RA4)));
k20hat= rhohat* (((f-1)/2)* (RA4))+(e/2)*((RA4)-((R1A2)* (RA2)))+((d+1)/2)* (RA4)- 2*(RA2)* (R1A2))+ (RA4)));
k20=RHO* (((f-1)/2)* (RA4))+(e/2)*((RA4)-((R1A2)* (RA2)))+((d+1)/2)* (RA4)- 2*(RA2)* (R1A2))+ (RA4)));
k21=1; k22= -2*(RA2); k23=RA4;
k24=rho*9.8*1;
k24hat= rhohat*9.8*1;
K24=RHO*9.8*1;

% polynomials

p1=k5/((k5*x1)+k6);
p2=k9/((k5*x1)+k6);
p2hat=k9hat/((k5*x1)+k6);
P2=K9/((k5*x1)+k6);
p3=k10/((k5*x1)+k6);
p3hat=k10hat/((k5*x1)+k6);
P3=K10/((k5*x1)+k6);
p4=k7/((k5*x1)+k6);
p4hat=k7hat/((k5*x1)+k6);
p4max=k7max/((k5*x1)+k6);
p4min=k7min/((k5*x1)+k6);
P4=K7/((k5*x1)+k6);
p5=((k11*x1A4)+(k12*x1A2)+k13)/((k14*x1A4)+(k15*x1A2)+k16);
p6=(Q2*pi*(R2A2)-(R1A2))*(0.1+4*b*((Q1*Q2)+(Q*(1-Q1))));
p7=((k18*x1A4)+(k19*x1A2)+k20)/((k21*x1A4)+(k22*x1A2)+k23);
p7hat=((k18hat*x1A4)+(k19hat*x1A2)+k20hat)/((k21*x1A4)+(k22*x1A2)+k23);
p8=p2*p6*p7;
p8hat=p2hat*p6*p7hat;
P8=P2*p6*P7;
p9=p2*p6;
p9hat=p2hat*p6;
P9=P2*p6;

dp5=(2*x1*(-k13*k15+k12*k16-2*(k13*k14-k11*k16)+(x1A2)- (k12+k14-
k11*k15)+(x1A4)))/(k16+k15*(x1A2)+k14*(x1A4))^2);

ddp5=(2*(-k13*k15+k12*k16+2*(k13+k14-k11+k16)+(x1A2)- (k12+k14-
k11*k15)+(x1A4)))/(k16+k15*(x1A2)+k14*(x1A4))^2);

% derivatives of y

y=x3;
dy=x4;
ddy=dp5*(x4-v1)^2+k17;
dddy=dp5*x2*(x4-v1)^2+2*dp5*(x5*((x4-v1)^2)-k17)*(x4-v1);

h=ddp5*x2A2* (x4-v1)^2+dp5*((x4-v1)^2)*(-p1*x2A2*k8*x2-p8*(v1-x4)^2-
p9*k24*p3*Ts*sgnx2)+4*dp5*x2*(p5*(x4-v1)^2-k17)*(x4-
v1)+2*dp5*p5*x2*(x4-v1)^2+4*p5A2*(p5*(x4-v1)-k17)*(x4-
v1)+2*dp5*(x5*(x4-v1)^2-k17)^2;

hhat=ddp5*x2A2* (x4-v1)^2+dp5*((x4-v1)^2)*(-p1*x2A2*k8hat*x2-p8hat*(v1-
x4)^2-p9hat*k24hat*Ts*sgnx2)+4*dp5*x2*(p5*(x4-v1)^2-k17hat)*(x4-
v1)+2*dp5*p5*x2*(x4-v1)^2+4*p5A2*(p5*(x4-v1)-k17hat)*(x4-
v1)^2+2*dp5*(p5*(x4-v1)^2-k17hat)^2;
\[ H = d dp 5 * x 2 ^ 2 * ( x 4 - v 1 ) ^ 2 + d dp 5 * ( ( x 4 - v 1 ) ^ 2 ) * ( - p 1 * x 2 ^ 2 - K 8 * x 2 - P 8 * ( v 1 - x 4 ) ^ 2 - P 9 * K 4 * T s * s g n x 2 ) + 4 * d p 5 * x 2 ^ 2 * ( p 5 * ( x 4 - v 1 ) ^ 2 - K 1 7 ) * ( x 4 - v 1 ) + 2 * d p 5 * p 5 * x 2 ^ 2 * ( x 4 - v 1 ) ^ 2 + 4 * p 5 ^ 2 * ( p 5 * ( x 4 - v 1 ) - K 1 7 ) * ( x 4 - v 1 ) ^ 2 + 2 * p 5 * ( p 5 * ( x 4 - v 1 ) ^ 2 - K 1 7 ) ^ 2; \]

\[ g = d p 5 * ( x 4 - v 1 ) ^ 2 * p 4 ; \]
\[ g h a t = d p 5 * ( x 4 - v 1 ) ^ 2 * p 4 h a t ; \]
\[ G = d p 5 * ( x 4 - v 1 ) ^ 2 * P 4 ; \]

\% desired trajectories to maintain constant position
\[ y d = 0; \]
\[ d y d = 0; \]
\[ d d y d = 0; \]
\[ d d d y d = 0; \]
\[ d d d d y d = 0; \]

\% desired trajectories to maintain constant velocity
\[ m = 0.05; \]
\[ n = 0.05; \]
\[ o = \pi / 2; \]
\% if 0 <= t < 2
\% \quad y d = ( m * t ) - ( n * s i n ( o * t ) ) / o ;
\% \quad d y d = m - n * c o s ( o * t );
\% \quad d d y d = n * o * s i n ( o * t );
\% \quad d d d y d = n * ( o ^ 2 ) * c o s ( o * t ) ;
\% else
\% \quad y d = 0.1 * t - 0.1 ;
\% \quad d y d = 0.1 ;
\% \quad d d y d = 0 ;
\% \quad d d d y d = 0 ;
\% end

\% tracking errors
\[ y t i l d a = y - y d ; \]
\[ d y t i l d a = d y - d y d ; \]
\[ d d y t i l d a = d d y - d d y d ; \]
\[ d d d y t i l d a = d d d y - d d d y d ; \]

\% sliding surface
\% \lambda = 1 0 0 0 0 ;
\% lambda = 1 0 0 0 ;
\[ s = d d d y t i l d a + 3 * l a m b d a * d d y t i l d a + 3 * l a m b d a ^ 2 * d y t i l d a + l a m b d a ^ 3 * y t i l d a ; \]

if \( s > 0 \)
\quad \text{sgns} = 1;
elseif \( s = 0 \)
\quad \text{sgns} = 0;
else
\quad \text{sgns} = -1;
end
\% constant boundary layer
\% \phi = 3 ;
\% phi = 0.1 ;
\% phidot = 0 ;
if abs(s/\phi)\leq 1
    sat=(s/\phi);
else
    sat=s\text{gn}s;
end

% control law

\% \text{etha}=1;
\text{etha}=100;
\% Beta = p4*d5*(x4∗v1)\^2;
v1max=2;
v1min=0.1;
x4max=1.5;
x4min=-0.1;
x1max=R;
x1min=R1;
dp5max=(2∗x1max*(-k13*k15+k12*k16-2*(k13*k14-k11*k16)*(x1max\^2)
\text{-}
(k12*k14-k11*k15)*(x1max\^4))/((k16+k15*(x1max\^2)+k14*(x1max\^4))^2);
dp5min=(2∗x1min*(-k13*k15+k12*k16-2*(k13*k14-k11*k16)*(x1min\^2)
\text{-}
(k12*k14-k11*k15)*(x1min\^4))/((k16+k15*(x1min\^2)+k14*(x1min\^4))^2);

\% Beta = sqrt((p4max*dp5*(x4-v1max)^2)/(p4min*dp5*(x4-v1min)^2));
Beta=sqrt((p4max*dp5max*(x4max-v1max)^2)/(p4min*dp5min*(x4min-v1min)^2));
\text{uhat}=(\hat{\text{hhat}}+ddddyd∗3*\lambda m\text{dbad}∗ddytilda∗3*\lambda m\text{dbad}^2∗dbytd\text{ilda}
\text{-}
\lambda m\text{dbad}^3∗dbytd\text{ilda});
\% K\text{control}=1.0*(\text{Beta}^*(H-\text{etha})+(\text{Beta}-1)*\text{abs}(\text{uhat}));
\% u=(\text{uhat}∗K\text{control}∗s\text{gn}s)/(p4*hat∗dp5*(x4∗v1)^2);
\% u=0;

% smoothing control law
K\text{controlbar}=K\text{control}∗p\text{hidot};
u=(\text{uhat}∗K\text{controlbar}∗s\text{at})/(p4∗dp5∗(x4∗v1)^2);
Appendix D: Lie Derivatives

This section contains the formal definition of the Lie derivatives used in the derivations of chapter 6. This concept is borrowed from differential geometry but is useful in nonlinear control analysis.

First, in differential geometry a vector function $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is called a vector field since to every vector function $f$ corresponds a field of vectors in an n-dimensional space. For our analysis we are only interested in smooth vector fields which require that the function $f(x)$ has continuous partial derivatives of any required order.

**Gradient of a scalar function:** Given a smooth scalar function $g(x)$ of the state $x$, the gradient of $g$ is denoted by $\nabla g$

$$\nabla g = \frac{\partial g}{\partial x}$$

**Lie derivative:** Given a scalar function $g(x)$ and a vector field $f(x)$, we define a new scalar function $L_f g$, called the Lie derivative of $g$ with respect to $f$ as,

$$L_f g = \nabla g \cdot f$$

Therefore, the Lie derivative $L_f g$ is simply the directional derivative of $g$ along the direction of the vector $f$.