Particle Displacement Measurement Using Optical Diffraction

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B.S., Precision Instruments and Mechanology
Tsinghua University, 1999

Submitted to the Department of Mechanical Engineering and
the Department of Electrical Engineering and Computer Science
in Partial Fulfillment of the Requirements for the Degrees of
Master of Science in Mechanical Engineering
and
Master of Science in Electrical Engineering and Computer Science

at the
Massachusetts Institute of Technology
June 2002

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ABSTRACT

Both three-dimensional (3D) imaging using spot pattern projection and Particle Image Velocimetry (PIV) can be reduced to the task of detecting particle image displacement. In this thesis, a pseudo-correlation algorithm based on optical diffraction is proposed to measure spot displacement fast and accurately. When subtracting two consecutive images of a roughly Gaussian-shaped displaced spot, the normalized subtraction intensity peak height is directly proportional to the spot displacement. The peak height to displacement calibration curve is specifically defined by the optical parameters of the imaging system. Experiment observations show that the system calibration curve is highly smooth and sensitive to the spot displacement at sub-pixel level. Real-time processing is possible with only order of image size arithmetic operations. Measurement results of 3D objects and simulated flow fields prove the feasibility of the proposed diffraction method. In addition, two other algorithms, which recover 3D shape by measuring local period of the projected fringe pattern, are presented.
Acknowledgments

First thanks must go to my thesis advisor, Prof. Douglas P. Hart, who intrigued me into this exciting and enormous field of imaging, and then gave me the freedom to explore on my own while pulling me back when I got strayed. Doug's ability to rapidly assess the worth of ideas is amazing. His vision and enthusiasm for science and technology are admirable.

I have been extraordinarily lucky to be part of a research group that is more like a warm family. Federico Frigerio, Ryan Jones and Dr. Carlos Hidrovo, their intelligence and happy nature help make my academic endeavor so enjoyable. My special thanks to Dr. Janos Rohály, without whose guidance this thesis would be far less possible.

I thank fate for bringing me to MIT. There are so many wonderful things that could be said about MIT — the people, the lack of hierarchy, the open doors, the many seminars, and the tolerance of diversity. All these keep on stimulating me.

I would like to thank Prof. Chunyong Yin and Prof. Lijiang Zeng. After my graduation from Tsinghua Univ., they still reach out to care for my professional career. I think of them as lifetime mentors and role models.

My parents and Prof. Irving Singer, who also treats me like his own daughter, are very special to me. Their unconditional confidence in me encourages me to dare what seems formidable.

And for all my friends who have supported me in one way or the other, I wish to express my sincerest appreciation.

Finally I would like to dedicate this thesis to my fiancé Randy, whose existence is a continual miracle to me.
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Chapter 1

Introduction

Three-dimensional (3D) imaging is a broad imaging field, which aims at rendering a 3D spatial model of the object or scenery. Range information is critical in various kinds of applications, such as product inspection, process monitoring [1], security screening [2], remote guidance and control, home entertainment [3], medical diagnostics [4-5], bio-imaging and microstructure analysis [6], etc. Due to all these popular demands, numerous 3D imaging techniques have been proposed and polished during the past three decades, which could be categorized based on the sensor type (contact or non-contact), measurement range (macroscopic or microscopic), accuracy (qualitative or quantitative), and speed (real-time or stationary).

The motivation of this thesis can be traced back to the need of non-contact 3D measurement of the human body, e.g., face, limbs or torso, for medical diagnostic purposes. Our ultimate goal is to develop a fast, real-time measurement system, which could reach sub-pixel accuracy in the image plane. The system should be robust yet low-cost.

1.1 Optical Profilometry Techniques

There are a large number of existing non-contact profilometry methods, which fall into several broad categories, such as time in flight, laser scanning, interferometry, Moiré grating, holography, focus/defocus and stereo vision [7], etc. Some of the methods like laser scanning are highly accurate (1 part in 10,000), but slow and expensive [8]. Other methods like focus/defocus can hardly reach video-rate imaging when adequate accuracy is required [9]. There is always a tradeoff between speed and accuracy in 3D imaging.
After analyzing these categories, stereo vision is our preferred method for the human facial expression application that we have in mind. The desired relative accuracy is around 1 part in 1,000.

1.2 Stereo Vision

Stereo vision 3D imaging imitates the mechanism of human visual system. Each one of our eyes captures the same scene from a slightly different angle. These different perspectives lead to relative displacement of objects in the two monocular views. When the two images arrive simultaneously in the brain, they are united into one 3D picture. The mind combines the two images by matching up the similar features and interpreting the small differences into depth perception. This way, object is perceived as solid in three spatial dimensions — width, height and depth. Similarly in 3D imaging, usually there are two cameras at different viewing positions, and computer algorithms compare the two inputs to generate depth information.

![Figure 1.1: Illumination methods for stereo vision, (a) white light, (b) stripe pattern, (c) spot pattern.](image)

There are two main sub-categories within stereo vision based on the illumination method. The first one is white light illumination (Figure 1.1a), which is useful when objects has lots of surface texture, such as edges, curves and spots like in this motherboard shown. However, when the object is smooth, for example, in Figure 1.1b and c, the object is a smooth white diffusive curved surface. If white light illumination is still used, the computer can hardly tell from a virtually blank image what the object is. Thus the second
category is the structured light illumination, which can add texture features onto the object surface. In Figure 1.1b, stripe pattern is projected onto the curved surface, and Figure 1.1c, white spot pattern projected onto the same surface.

Projecting a periodic pattern onto an object and observing the deformation has been a common principle for various 3D profilometry techniques. For fringe projection, popular methods are phase detection [10-15] and Fourier frequency analysis [16-17]. These techniques usually need to change one system parameter during the measurement procedures, e.g., the object position or projection pattern, and also a series of images need to be taken in order to improve accuracy. As a result, speed is greatly impeded. Another disadvantage of the Fourier fringe analysis is that Fourier transformation is computationally expensive. For spot pattern projection, spatial resolution and accuracy are often sacrificed to improve accuracy [18-19].

A field of view method is developed in this thesis using fringe projection, which reveals depth information by measuring the spatial period change rather than converting to the Fourier domain. This method is extremely fast, but its accuracy is limited is no spatial averaging is allowed. While studying the field of view method, it gave inspiration to the diffraction method where the spot pattern projection is employed. This diffraction method is also very fast, and at the same time, gives sub-pixel accuracy and high spatial resolution as desired. Both of these two methods are not covered in literature yet.

1.3 Thesis Organization
The original task of this thesis is to identify novel theories and algorithms for 3D imaging applications. Several unsuccessful attempts were made at the beginning, including pure white light illumination and some Fourier analysis approaches. The field of view method was the first breakthrough, which could measure textureless surface at high speed. However, since it requires a big compromise between the accuracy and spatial resolution, this method seems to be less significant and consequently will be explained at the end of the thesis. The second explored diffraction method, which holds the potential of being both fast and accurate, will constitute the major part of this thesis.
Chapter 2 gives a review of the optical diffraction theory and explains how it could be utilized in measuring particle displacement. This diffraction method can be applied in a wide range of applications where particle disparity is the measured quantity. In Chapter 3, it is first implemented in 3D imaging. Additionally in Chapter 4, the diffraction method is tested for feasibility in Particle Image Velocimetry (PIV) applications. Chapter 5 describes the proposed field of view method as well as its implementation in 3D imaging. Finally, Chapter 6 draws some conclusions and future work is discussed.
Chapter 2

Diffraction Method

2.1 Fundamentals of the Diffraction Theory

The diffraction effect is inevitable due to the wave nature of electromagnetic signals, which include visible, infrared and ultraviolet light, X rays, ultrasound etc. Light will bend at obstructions as the way water wave does. If the light source is coherent and the obstruction size is comparable with the wavelength of light, we will observe bright and dark diffraction rings in the Fraunhofer image plane [20-21]. The optical diffraction phenomenon has been extensively used in many micro tomography techniques to study material structures in a molecular scale [22].

In traditional optical imaging fields, like the photography, because the camera aperture size is much larger than the wavelength and incoherent illumination is applied, we can no longer observe the diffraction rings. Instead, there is a blurring effect even when the camera is best focused. Since our eyes' sensitivity to intensity change is much less acute than a CCD camera, what a human sees as a sharp image is actually blurred at edges. Figure 2.1a is a best-focused image of a fringe pattern object plane. White stripes are saturated at a gray level of 255. The transition from the white to black stripe is actually gradual. Figure 2.1b illustrates the central cross-sectional intensity distribution of a best-focused spot image. Again, it doesn't have a window function shape as perceived by the eyes.

Since our eyes prefer a sharper image, in most photographic applications people have always been avoiding and ignoring the optical diffraction effect. Objects are carefully
positioned at the focal plane in order to get a sharp image. Digital filters could also be applied during post-processing to further sharpen the image until the best visual effect is achieved. In scientific image processing, detailed transitional edge information is often discarded in bimodal images in order to save memory storage space and increase the processing speed [23]. This thesis proposes that a complete set of the blurred edge information could be helpful in measuring the particle displacement.

2.2 Diffraction Method

For an imaging system with a circular aperture, the image of a point source is called an Airy pattern [21]. The image intensity distribution can be written as

\[ I(r) = \left( \frac{\pi w^2}{\lambda z} \right)^2 \left[ 2 \frac{J_1(kwr/z)}{kwr/z} \right]^2, \]  

(2.1)
where \( r \) is a radius coordinate in the image plane, \( w \) is the radius of the aperture, \( \lambda \) is the wavelength of illumination used, \( z \) is the image plane distance to aperture, \( k \) is termed as wave number and \( k = 2\pi/\lambda \), and \( J_i \) is the Bessel function of the first kind. The diameter of the central lobe is given by \( D = 1.22 \frac{2z}{w} \). For white light illumination and photographic applications, the range of visible light is much smaller compared with \( w \). In this case, an averaged wavelength, \( \lambda_0 = 540\text{nm} \) can be used in the place of \( \lambda \). Equation (2.1) can be closely approximated by a Gaussian function [24], as defined in (2.2),

\[
I(r) = \left( \frac{\pi w^2}{\lambda z} \right) \exp\left[ -\frac{r^2}{2\sigma^2} \right], \tag{2.2}
\]

where \( \sigma = \frac{\lambda z}{\sqrt{2\pi}w} \). Since exponential function is much more manageable than the Bessel function, this Gaussian approximation brings mathematical and computational simplicity.

Suppose there is an ideal Gaussian distribution \( y_1 \) with normalized intensities, and another distribution \( y_2 \), obtained by shifting \( y_1 \) in x-direction by an amount, \( d \) (Figure 2.2a). Here we first assume that there is no CCD pixel integration error. By subtracting these two curves, the resulting positive and negative peak heights are direct indication of the displacement \( d \) (Figure 2.2b). Here \( d \) is normalized relative to the spot diameter. The first half of subtraction curve is the overlapping region. Within this highly sensitive region, \( d \) is uniquely determined by measuring the subtraction curve peak intensity. The theoretical relationship between the subtraction curve peak height and displacement as shown in Figure 2.2b is obtained by assuming that the Optical Transfer Function (OTF) of the imaging system can be accurately modeled as a Gaussian function, which is not usually the case in a real optical imaging system. A spot image may be elliptical and tilted due to optical distortion and aberration. As a result, calibration is necessary to reveal the true OTF. The calibration process will be discussed in details in Chapter 3.

After considering CCD quantization error, the maximum theoretical measurement accuracy of \( d \) is about 1/256 of the spot size. For example, if a spot size is about five pixels as shown in Figure 2.2, then a five-pixel displacement range is measured with 256
gray levels. Thus, sub-pixel accuracy is reached. Another alternative is to add up two intensity distributions (Figure 2.3). However, in practice, adding two images doubles system noise. Consequently, for single-exposure applications the subtraction method is preferred, while the summation method may be implemented in double-exposure applications. This thesis will only focus on the subtraction method. The processing algorithm is very fast since all processing is performed in image plane; no Fourier transformation or iterative correlation is necessary. A simple subtraction of two images and a search for peak height will relate to particle displacement. As a result, real-time processing is possible. The number of particles limits the spatial resolution.

![Figure 2.2:](a) Subtraction of two Gaussian-shaped distributions with a displacement of $d$, (b) peak heights and $d$ relationship.

![Figure 2.3:](a) Summation of two Gaussian-shaped distributions with a displacement of $d$, (b) positive peak height and $d$ relationship.
2.3 Assumptions

Three major assumptions are made in the above diffraction method.

- The spot size is assumed to be uniform across the object plane.
- OTF is also assumed to be uniform across the object plane, which indicates that the imaging system is shift-invariant.
- The shifted spot has the same intensity distribution as the original one.

In adverse and dynamic measurement conditions, these assumptions are not necessarily held true. Chapter 3 and 4 will explain some proper modifications to the diffraction method that need to be made in order to maintain the measurement accuracy.

2.4 Image Processing Techniques

2.4.1 Gaussian Filtering

A normalized Gaussian filter as shown in Equation (2.3) can be implemented on all images to emphasize the Gaussian shape of the image spot over random optical and electrical noise,

\[ h = c \cdot \exp\left(\frac{-x^2 + y^2}{2\theta^2}\right) \]  

(2.3)

where \( c \) is a normalizing factor. The sum of all the elements in the filter is equal to 1 as to make sure the total light energy of the image is conserved. The filter size can be 5×5 or 3×3 pixels, depending on the spot size and noise level. In Figure 2.4, after implementing the Gaussian filter \( (\theta = 2) \), energy is distributed over a doubled spot size. As a result, the intensity dynamic range is sacrificed to increase the edge details, which is essential in accurately determining the subtraction curve peak height. Optimum \( \theta \) value is obtained empirically. However, the processing speed is significantly reduced due to this Gaussian filtering. If a large spot size of 8-10 pixels is practical for the image, optical blurring is preferable over software blurring, which could be realized by slightly defocusing the object or increasing the F-number (i.e., decreasing the size of aperture) of the imaging system.
2.4.2 Normalization

All of the image spots don’t have the same brightness because of the illumination variation across the field of view. Normalization of the peak intensity makes sure each spot image pair can follow a universal calibration curve of the imaging system, like the one in Figure 2.2b. There are two ways of normalizing spot intensities. When the intensity variation from frame to frame is small, e.g., around two gray scales, the spot peak intensities in frame A are used as the reference, i.e., the spot pair is normalized by this quantity. When the intensity variation is large, e.g., about twenty gray levels, averaged peak intensities are chosen as the reference. If the variation is larger than a pre-selected threshold, e.g., two gray levels, local intensity equalization should be applied in order to validate that spot pair.

2.4.3 Peak Searching

After subtraction of two frames, local maximum (or minimum) is searched for the peak height. If the spot size is large (around 10 pixels), an integer peak intensity value will be accurate enough to approximate the true peak height. However, the CCD pixelization
error should be considered if the spot size is only 5-6 pixels. One possible solution is by using one-dimensional 4th order curve fitting to find the exact peak height. In Figure 2.5, the stars represent the normalized integer intensity values of the neighboring pixels in a horizontal direction. The solid lines are the five-point fitted curves around the pixel peaks. True peaks are detected by measuring the maximum (or minimum) of these two lines.

The most accurate way to find the peak height is by two-dimensional curve fitting. A surface is fitted to a 3x3 or 5x5 matrix around the pixel peak in a least-square sense [25]. But experimental observation shows that the accuracy improvement after 2D fitting is only about 10% over its 1D counterpart, while the processing time is an order of magnitude longer. Consequently, 2D surface fitting is not recommended.

2.5 Discussion

2.5.1 Time Derivative Method

This subtraction method can be categorized as a time derivative method (Equation 2.4), since two consecutive images are subtracted from each other, pixel by pixel, as shown in Figure 2.6.

\[
\frac{dE_{m,n}}{dt} = \frac{E_{m,n}^{t+1} - E_{m,n}^{t}}{\Delta t}
\]  

(2.4)
Since such a gradient method relies on the first derivative, it will be more susceptible to noise than other averaging methods or combinations of the gradient and averaging methods, when the noise level is high. But when noise level is much less significant, there will be very little difference between a gradient-based method and other more complicated algorithms. To conclude, this proposed diffraction method works best under low noise situations. In our 3D imaging case, the projected pattern is a binary image. If we define the signal to noise ratio (SNR) in the image plane as maximum intensity over background noise, the SNR is usually very high (> 3). However, when image quality is bad, it’s better to develop other combined algorithms.

### 2.5.2 Sensitivity of Displacement Measurement

The diameter $D$ of a normalized intensity Gaussian spot $y = \exp(-cx^2)$ can be defined by its width at the $e^{-2}$ intensity (Figure 2.7). If $D$ is normalized to 1, we obtain $c = 8$.

Next we will study the mathematic expression for the subtraction curve. Suppose the two displaced spots (Figure 2.2a) are $y_1 = \exp(-8x^2)$ and $y_2 = \exp(-10x^2)$.
$y_2 = \exp(-8 \cdot (x - d)^2)$. Then their subtraction function is $f = y_1 - y_2$. In order to find the peak position $x_0$, we take the first derivative of $f$ and set it to zero, and thus get the equation $\exp(-8d^2) \cdot \exp(-16dx) \cdot (x - d) = x$, or its equivalent logarithmic form

$$\ln \frac{x - d}{x} = 16dx + 8d^2. \quad (2.5)$$

Unfortunately, this nonlinear equation doesn’t have an analytical solution for $x_0(d)$, and consequently we cannot derive an analytical expression of the subtraction peak height to displacement function $I_p(d)[26]$. Instead, $I_p(d)$ as well as its first derivative have to be solved numerically as shown in Figure 2.8. We can notice that $\frac{dI_p}{dd}$ drops below 0.1 when $d > 0.77$ (diameter),

The subtraction curve can be modeled approximately as an exponential function:

$$I_p = 1 - \exp(-c_1 d^{c_2}). \quad (2.6)$$

Least squares optimization shows that a selection of $c_1 = 5.53$ and $c_2 = 1.35$ gives a good approximation (Figure 2.9).

**Figure 2.8:** Numerical solution and its first derivative of the positive peak subtraction curve.

**Figure 2.9:** Numerical solution and exponential model ($c_1 = 5.53$ and $c_2 = 1.35$) of the positive peak subtraction curve.

### 2.5.3 Inaccuracy Due to Overlapping
If a particle has an overlapping neighbor, its own subtraction intensity distribution will be distorted by the other particle (Figure 2.10). The worst case happens when the directions of the spacing $\Delta$ and displacement $d$ are parallel. Figure 2.11 illustrates how the subtraction peak is distorted by another particle, in the case when the two neighboring particles are horizontally aligned and displacement is leftward. Here subtraction plane is normalized by the increased local maximum in the image plane. The degree of distortion is influenced both by $\Delta$ and $d$, but mostly by $\Delta$. When $\Delta$ is small, the neighboring spot pair appear as one big spot in the image plane, which is no longer Gaussian-shaped (Figure 2.12). When $\Delta > 0.5$, we start to observe the two spots. In this leftward shift, the negative peak intensity is severely influenced by the neighboring particle. For example, when $d = 1$ and $\Delta = 1$, the negative hill is completely wiped out of the subtraction plane by the other particle. So it’s better to use only the positive peak intensity for measurement in this overlapping situation (Figure 2.13). If we define an invalid detection as when its error is larger than $10^{-3}$ diameter, the corresponding spacing range of $\Delta \in [0, 0.9]$ (diameter) will produce spurious vectors.

**Figure 2.10:** Two particles with a spacing $\Delta$ have a uniform displacement $d$ within time $\Delta t$.

**Figure 2.11:** Positive and negative peak heights when displacement $d$ changes from 0 to 1 (diameter) and the spacing $\Delta$ of the interfering particle changes from 0 to 2 (diameter).
Figure 2.12: Intensity distributions in the image plane and subtraction plane when \( d = 0.4 \): \( A = 0.3 \) (left) and \( A = 0.5 \) (right).

Figure 2.13: Measured displacement \( d \) based on distorted positive peak intensity.

2.5.4 Particle Spacing Distribution

In this section, we discuss particle spacing distribution at any given image density. Image density \( \rho_i \) can be defined to be the average numbers of particles per unit area [27]. Here we normalize area relative to particle diameter.

\[
\rho_i = \frac{K}{N^2},
\]

(2.7)

where \( K \) is the number of particles in \( N^2 \) (\( \text{diameter}^2 \)) image plane area. For example, \( \rho_i = 1 \) corresponds to a seeding density in which all particles can be viewed as solid spheres and are compactly packed. In the case of a typical image density, 10 particles per 32x32
pixels interrogation window, if the particle diameter is 4 pixels, we will have $\rho_f = \frac{10}{8^2} = 0.1563$.

Particle spacing refers to the distance between each particle and its closest neighbor. We can randomly place a large number of particles in a sizeable area whose edge effect (particles sitting on the edge) can be safely ignored, and then analyze the spacing between each particle. Figure 2.14 illustrates the statistics of three sample sets when $\rho_f = 0.5$, $N = 40$ and thus $K = 800$. Each sample data, e.g., at $\Delta = 0.5$ (diameter), represents the percentage of particles that has a minimum distance from its neighbor between (0.4 0.5].

We can see from Figure 2.14 that the particle spacing distribution resembles a Rayleigh distribution, which is reasonable. If each particle’s positions, $x$ and $y$, are randomly distributed variables, then the variable $R = \sqrt{x^2 + y^2}$ is distributed in accordance to the Rayleigh’s law [28]. The probability density function $p_R(x)$ of Rayleigh distribution $R(\mu)$ is:

$$p_R(x) = \begin{cases} \frac{x}{\mu^2} \exp\left(-\frac{x^2}{2\mu^2}\right), & x \geq 0 \\ 0, & x < 0. \end{cases} \quad (2.8)$$
By averaging the three sample sets and calculating the mean spacing \( \Delta_{\text{mean}} \), we have \( \Delta_{\text{mean}} = \sqrt{\frac{\pi}{2}} \mu = 0.7644 \). Solving for \( \mu \) gives \( \mu = 0.61 \). \( R(0.61) \) shows to be a good model in Figure 2.15. In order to study the relationship between \( \rho_t \) and \( \mu \), statistical analysis is performed with different \( \rho_t \) values (Table 2.1). Fifth degree polynomial curve fitting provides a good approximation of the \( \rho_t(\mu) \) function (Figure 2.16). Since when \( \rho_t > 1.1 \), most particle spacing will fall below 0.6 (diameter), such dense seeding situations will not be discussed here.

Table 2.1: Statistical analysis data

<table>
<thead>
<tr>
<th>( \rho_t ) (diameter)</th>
<th>( N )</th>
<th>Mean spacing ( \Delta_{\text{mean}} ) (diameter)</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>100</td>
<td>1.655</td>
<td>1.321</td>
</tr>
<tr>
<td>0.2</td>
<td>40</td>
<td>1.205</td>
<td>0.962</td>
</tr>
<tr>
<td>0.3</td>
<td>40</td>
<td>0.995</td>
<td>0.794</td>
</tr>
<tr>
<td>0.4</td>
<td>40</td>
<td>0.845</td>
<td>0.674</td>
</tr>
<tr>
<td>0.5</td>
<td>40</td>
<td>0.764</td>
<td>0.610</td>
</tr>
<tr>
<td>0.6</td>
<td>40</td>
<td>0.702</td>
<td>0.560</td>
</tr>
<tr>
<td>0.7</td>
<td>40</td>
<td>0.662</td>
<td>0.528</td>
</tr>
<tr>
<td>0.9</td>
<td>40</td>
<td>0.616</td>
<td>0.491</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>0.578</td>
<td>0.462</td>
</tr>
<tr>
<td>1.1</td>
<td>30</td>
<td>0.532</td>
<td>0.424</td>
</tr>
</tbody>
</table>

2.5.5 Optimum Seeding Density

The number of valid vectors per unit image area \( V \) can be expressed as

\[
V = \frac{\rho_t R_{\text{correct}}}{D^2},
\]

where \( \rho_t \) is the image seeding density, \( D \) is the particle diameter in pixels and \( R_{\text{correct}} \) is the probability that particle spacing is outside the restricted region. Here \( D \) is set to be 4. If the restricted region is set to be \([0, 0.9]\) as explained in 2.5.2, then the optimum seeding density is about 0.5 (Figure 2.17). Due to the possible random noise in real imaging applications, a more stringent invalid spacing criterion of \([0, 1]\) is chosen, and the corresponding optimum \( \rho_t \) is about 0.34. The maximum valid-vector yield is \( V_{\text{max}} = 0.008/\text{pixel}^2 \), which is equal to one valid vector per 11.3x11.3 pixel interrogation.
window. This is about 5 times better than what is typically done with cross-correlation based PIV (32×32 pixels).

**Figure 2.17:** Number of particles that give valid displacement detections when invalid spacing region is restricted to (left) [0, 0.9] and (right) [0, 1].
Chapter 3

Application in Three-Dimensional Imaging

The previous chapter explained how to measure the spot displacement. In this chapter, a modified stereo vision system is built to capture the 3D information [29-32]. By placing a step-motor controlled off-axis rotating aperture in the lens, the depth information is coded into the spot displacement in the image plane. An illumination system projects binary spot pattern onto the object while a CCD camera captures video-rate images for processing.

3.1 Hardware Design

3.1.1 Principle

Suppose within a time interval $\Delta t$, the aperture position is rotated by $180^\circ$ and we capture two images at these two aperture positions. If the object point source is in the focal plane, the two image spots in the image plane will completely overlap (Figure 3.1a). However, when the object is out-of-focus, there will be a disparity of $d$ in the image plane (Figure 3.1b). The depth position $Z$ and disparity $d$ follows the relationship

$$Z = \left( \frac{1}{L} + \frac{d}{2 R f L} \right)^{-1},$$

where $L$ is the focal plane position, $f$ is the focal length of the camera objective, $R$ is the radius of the off-axis aperture position, and $d$ is the blurred diameter of the image spot rotation circle when the aperture rotates. The size of the aperture only influences the image spot size and brightness, not disparity.
There are several major advantages of using an imaging system with a single lens and off-axis rotating aperture, rather than the traditional two cameras stereo vision system. First, the equipment cost is significantly less and system simplicity improves by using only one camera and one objective. Second, the alignment problem associated with two-camera systems is avoided. Third, every two images at two known aperture positions can generate one depth map. As the aperture rotates, an infinite number of measurements could continuously improve the measurement accuracy and update the object position.
3.1.2 Rotating Aperture Design

![Figure 3.2: Centered iris position of a commercial lens.](image1)

![Figure 3.3: Modified camera lens with a rotating off-axis aperture.](image2)

Usually, a commercial lens has a centered adjustable iris, which controls the aperture size of the imaging system (Figure 3.2). In our system, in order to create the artificial spot disparity in the image plane of a defocused object point, a fixed-size off-axis aperture is placed at the back of the lens, while the original iris is fully opened (Figure 3.3). A stepping motor controls the rotation of this aperture. A mechanical device encapsulates the off-axis aperture and step motor, and connects the lens with the CCD camera. Figure 3.4 illustrates the relative positions among the lens, off-axis aperture and CCD.

![Figure 3.4: Relative positions among the lens, off-axis aperture and CCD.](image3)

In our specific design, the additional off-axis aperture is placed outside the commercial lens, rather than inside the lens at the iris position. This choice is made based on the consideration of easy designing of the mechanical rotating device. However, placing the
rotating aperture at the original iris position of the commercial lens may result in much less optical aberration, but at the cost of complicated lens design.

3.1.3 Projection System

The function of a projection system is to add strong features onto a rather textureless surface. There are many possible hardware solutions for a working projection system, three of which are discussed in the following:

- Computer LCD projector

We used a computer LCD digital color projector (Toshiba G5) to project black and white spot patterns onto the object. Since a commercial computer projector is designed for conference room demonstrations, both of its projection area and distance are quite large, about 2m×2m and 3~4m respectively. In contrast, our 3D imaging application requires a much smaller projection area of 0.2×0.2m and a shorter projection distance less than 1.5m. So a close-up lens is necessary to achieve a high density and resolution in the projected pattern. We use a lens with an effective focal length \( f = 600 \text{mm} \) \((Diameter = 95 \text{mm})\). Additionally, a small aperture \((Diameter \approx 8 \text{mm})\) is placed in front of the projector in order to increase the depth of focus of the projected pattern. However, this aperture will block away a large amount of the illumination power. Figure 3.5 demonstrates the layout of the computer projector system. The advantage of such a system is that we can easily and dynamically change the projection pattern.

![Figure 3.5: Layout of the computer projector system.](image-url)
The computer projector illumination design has two major disadvantages. First, it gives a small depth of focus (less than 10mm). Second, it has a large divergence angle, which means that the projected spot size changes significantly within the depth of focus. Choosing laser as the light source can overcome these difficulties. Figure 3.6 illustrates the setup of a laser projection system. The output from a He-Ne laser (Novette 15080 by Uniphase Inc., 0.7W) is first expanded by a 20× beam-expander, which is composed of two lenses. The focal length of one lens is twenty times of the other one. The distance between the two lenses is the combination of the two focal lengths. Then the expanded beam passes through a pattern generator, which is a piece of glass with holographic or interference spot pattern on it.

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In consideration of the eye safety, white light illumination is preferred. The optics of a slide projector could be carefully designed to guarantee convergence of the projection pattern within the depth of field of the camera objective. The light source can either use an incandescent bulb, white LED or flashlight. The only disadvantage of a slide projector is that the projection pattern is fixed for each measurement.

3.1.4 Overall Setup
Figure 3.7 illustrates the final experimental setup. We use a Nikkor 35mm fixed focal length lens and a DALSTAR CA-D4 CCD monochrome camera manufactured by DALSA Inc. It has a resolution of 1024×1024 pixels and 12µm square pixel size with 100% fill factor. The graphics card is Viper Digital Board manufactured by Coreco Inc.

3.2 Experiment Observation

Based on this unique off-axis aperture imaging system, we can examine several interesting phenomena. Phase detection is one of them, which is a common practice for 3D measurement using the fringe projection. For example, if we subtract the same rows of the two images taken at different aperture positions (Figure 3.8), we will observe the beat phenomenon. The beat frequency and beat amplitude vary with the depth. There are many techniques to extract the phase (depth) information from beat phenomena. Inspired by the traditional phase detection methods, we propose a possible phase measurement method that could be integrated into our rotating aperture system. It diverges significantly from the traditional methods since we no longer decompose the signal into Fourier series, and herein may be better categorized as correlation-based method.

Figure 3.9 is an example of two images of a flat object plane at two horizontal aperture positions 90° and 270°, respectively. If an object is defocused, its image rotates around a circle in image plane while the aperture rotates. The diameter of this circle is proportional to the depth. The horizontal image shift in Figure 3.9 indicates the circle diameter (depth). Here white stripes are saturated while dark stripes have a roughly Gaussian-shaped intensity distribution (Figure 3.10). The skewed shape of dark stripes is due to the asymmetric aperture design. We can subtract each image pair at a series of depth positions and search for the peak height. Figure 3.11 illustrates how peak height changes with depth. If the white stripes are not saturated, the possible measurement range will
expand further. From Figure 3.11, we can see $z = 10\text{mm}$ is approximately the actual focal position.

**Figure 3.8:** Examples of beat phenomena of three objects: (a) flat plane, (b) cylinder, (c) cube.
Figure 3.9: Two images of a flat object plane at two horizontal aperture positions of 90° and 270° (z = 20mm).

Figure 3.10: Intensity distribution of two images at 90° and 270°, and their subtraction (z = 10mm).

Figure 3.11: Peak height changes with depth.
Next we measure a calibration target with a regular spot pattern at a series of depth positions (-25mm ~ 25mm). We track a specific spot that has a roughly Gaussian shape, then subtract two images at $0^\circ$ and $20^\circ$, measure both positive and negative peak intensities in the subtraction plane and normalize it with the peak intensity in a single image. Figure 3.12 illustrates how normalized peak intensities vary with depth. There is an optimum degree separation ($20^\circ$ in this case) between the image pair for each system setup, which will give us the largest measurement range and lowest noise level.

![Figure 3.12: Normalized peak intensities vary with depth.](image)

For real measurements, it is important to adjust the illumination pattern as to maintain sufficient spot size and contrast level. Gaussian filtering of the entire image may help eliminate the optical distortions. Gaussian curve fitting of the spot will reveal more accurate peak height.

Alternatively we can sum up two images and measure the maximum intensity in the summation plane (Figure 3.13). The relationship between the peak height and depth is very irregular (Figure 3.14), not as the smooth theoretical result shown in Figure 2.3b. The reason for the loss of correspondence is that a dip at the summation curve peak appears as two spots separate. Limited spatial resolution of the CCD hinders the recovery of the real peak intensity. Besides, the summation method is much more sensitive to noise.
than the subtraction method. Based on these arguments, the summation method is abandoned in this thesis.

![Figure 3.13: Summation plane of two images at 0° and 20°.](image)

![Figure 3.14: Peak height of summation plane to depth plot.](image)

### 3.3 Calibration

The theoretical relationship between the subtraction curve peak height and displacement as shown in Figure 2.2b is obtained by assuming that the OTF of the imaging system can be accurately modeled as a Gaussian function, which is not usually the case in practice. A spot image may be elliptical and tilted due to optical distortions and aberrations. Also, the magnification ratio is not uniform across the image plane. As a result, calibration is necessary to reveal the true OTF. In order to have accurate measurement results over the entire field of view, a shift-variant model of the imaging system should apply, which means that the OTF changes over the image plane.

### 3.3.1 Calibration Setup

A regular spot pattern is used as the object for system calibration (Figure 3.15). The object plane is perpendicular to the optical axis, and is shifted longitudinally in the $z$-direction by a micrometer (about ±2 microns in accuracy). The total calibration range of the object is 100mm with a step size of 2mm. The reference plane of $z = 0$ is roughly placed at the focal plane. As $z$ increases, the object moves further away from both the focal plane and the lens. For example, in our case the focal plane is approximately...
500mm away from the camera, then the measurement range is set at 500~600mm from the camera. The principle is to place the object as close to the focal plane as possible because the image spot size will expand dramatically when the object is moved out of the camera’s depth of field. It will also be a good practice to place the measurement range across the focal plane, i.e., 450mm ~ 550mm. There isn’t an ambiguity problem, since if a spot pair in front of the focal plane shifts leftwards, another spot pair behind the focal plane will then shift rightwards.

Since every two images at two known aperture positions will disclose the depth information, a smaller angle of 30° between two apertures is chosen rather than 180°, in order to guarantee the spot pair overlap with each other within the measurement range. Figure 3.16 shows two spot pairs at two different depth locations, \( z_1 \) and \( z_2 \), which corresponds to the two large rotation diameters, \( d_1 \) and \( d_2 \). By choosing a small angle \( \theta \), the spot pairs remain in contact. The small disparities, \( D_1 \) and \( D_2 \), are proportional to the depth.

An 18x18 cm object field is observed. The image spot size is about 6-7 pixels when the object is close to the focal plane. Two images are taken at each depth \( z \), with the aperture positions at 0° and 30°, respectively. Then these two images are subtracted from each other and the positive and negative peak intensities are measured. For example, Figure
3.17a illustrates the spot displacement by zooming in to a single spot, when \( z = 40\text{mm} \) and the aperture rotates from 0° to 30°. Figure 3.17b shows the corresponding normalized subtraction intensity plane of this spot pair.

Figure 3.17: (a) Two images of a spot pair when \( z = 40\text{mm} \), and the aperture positions at 0° and 30°, respectively, (b) corresponding normalized subtraction intensity plane.

The selection of the rotation angle is based on the sensitivity of the calibration curves. For example, a rough local calibration analysis at rotation angles of 20° and 30° is shown in Figure 3.18. Here the image plane is divided into 32×32 interrogation regions. \( J \) and \( K \) are index numbers, where \( J = 1 \) and \( K = 1 \) refer to the upper-left corner of the image.

Figure 3.18: Calibration curves at rotation angles of 20° and 30°, respectively.
plane. As the object moves away from the focal plane, the spot pair becomes more and more separated and the image spot blurred. As a result, measurement results saturate at large depth position in both (a) and (b). The standard deviations (STD) in depth at rotation angles 20° and 30° are 1.58mm and 1.42mm, respectively. The 0°-30° pair has a higher accuracy since it spreads over a larger intensity range. Generally, due to strong aberrations and specific object surface properties, both 0°-20° and 0°-30° pairs are measured and the one with better global accuracy is chosen.

3.3.2 Processing

Two-dimensional curve fitting in the subtraction plane can reveal a more accurate peak height than simply using the integer pixel value. In Figure 3.19b, a 2D surface curve fitting is implemented using 3x3 sample points to reach the 2nd order fit in both horizontal and vertical directions. The improvement in accuracy is about 17%.

![Figure 3.19: Calibration curves at a rotation angle of 30°: (a) only integer pixel peak value is used, (b) 2D curve fitting implemented.](image)

Local equalization is the method that forces the same interrogation windows in two images have the same maximum intensity. In our system, since the aperture is small and off-centered, the strong astigmatism in the outer region of the image plane should be compensated. In Figure 3.20a, the positive and negative calibration curves in the upper-right corner are not symmetric as a result of the significant intensity variation between
two aperture positions. After implementing local equalization (only using integer pixel peak value), we generally achieve better curve shape and higher or same accuracy around the edge of the image plane (Figure 3.20b). No equalization is needed for the center region of the image plane since the intensity variation there is hardly noticeable.

![Figure 3.20: Calibration curves at upper-left corner of image plane: (a) not compensated, (b) local equalization implemented.](image)

Figure 3.21 is an example of the final calibration curves for a depth range of 100mm. Measurement results highly agree with the 3rd degree fitted calibration curves, which has a fluctuating noise level of 0.004 over the 0-1 normalized intensity scale, or 0.4mm in depth \( z \), correspondingly. The first 50mm range has a lower noise level of 0.003. This random noise is caused by a combination of CCD quantization error and pixelization. By using both positive and negative curves, the displacement measurement accuracy is improved. Calibration curves saturate as two spots become separated from each other. Also, as the object plane moves out of the depth

![Figure 3.21: Example of positive and negative peak calibration curves (\( J = 17, K = 17 \)), where dots represent measurement error relative to calibration curves.](image)
of field, the blurred spot diameter increases significantly. Consequently, the displacement
to spot diameter relationship no longer remains linear in this region. Since the noise level
is constant, measurement accuracy will drop as the object moves further away from the
focal plane. The above phenomena set the limit of the measurement range.

3.3.3 Shift-Variant Model

The optical system we use should be modeled as shift-variant, because the aperture is
placed off-axis and the aperture size is small compared with the lens diameter. By
examining various parts of the image plane, we notice the shape of the calibration curves
changes gradually due to non-uniformity of the OTF. In Figure 3.22, the entire image
plane is divided into 32×32 smaller windows, and each window’s specific curves are
calculated. Nine equally spaced sub-windows at the center and periphery of the image
plane are shown in the figure. For precise measurement, the gradual change in the
calibration curves cannot be ignored, and thus specific curves are assigned to different
local regions.

![Figure 3.22: Calibration curves of nine equally spaced sub-windows. The outer eight windows are at the periphery of image plane, while the center one is at the center.](image-url)
3.4 Sample Experiment

In order to test the feasibility of the above diffraction method, a laser-printed regular pattern of uniform white spots over a black background is used to simulate the laser speckle projection and is attached onto a smooth curved surface. Here the regular pattern is chosen because we would like to compare and possibly combine the diffraction method with the traditional triangulation method for stereo vision in the future. Random pattern will also be a good choice. In that case, cross-correlation algorithms could be integrated with the proposed method. Figure 3.23 illustrates the two images of the object taken at two aperture positions of $0^\circ$ and $30^\circ$, respectively. Due to the small off-axis aperture (off-axis shift = 4mm, F-number = 7), we can notice strong illumination variation from frame to frame at the edge of the image plane. So local intensity equalization is necessary.

![Figure 3.23: Two images of a curved surface taken at two aperture positions of $0^\circ$ and $30^\circ$, respectively.](image)

Figure 3.24 shows the final measurement result of the curved surface, which has a depth range from 0 to 80mm. Measurement accuracy deteriorates in the outer region of the cylinder as discussed before, where the depth is larger than 50mm. Assigning averaged calibration curves to each local window causes the segmentation effect of the measured 3D surface.
Figure 3.24: Measured 3D shape of a curved surface: (a) side view, (b) 3D model.
Chapter 4

Application in PIV

4.1 Introduction to PIV

4.1.1 Background

The revolutionary quantitative visualization techniques of flows seeded with tracer particles were introduced a couple of decades ago [24], which brought tremendous freedom to people’s understanding of fluid flows. Since then, these techniques have been constantly improved with the advances in imaging and computing technologies. Two main techniques of them are called Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV). They differ in the tracer density used. In PTV, particles are sparse enough so that each particle can be identified and tracked without ambiguity. However, seeding density in PIV is much higher and particles start to overlap with each other, so that not all particles can be identified unambiguously [24]. The border between PIV and PTV is somewhat vague and empirical, so they are often broadly referred to as PIV.

In a typical PIV experiment setup [33], the flow is seeded with small particles or bubbles, which are assumed to exactly follow the flow’s movement, if not interfering with it. The output from a pulsed laser passes through a cylindrical lens and forms a thin laser sheet (Figure 4.1), which illuminates a fixed plane in the flow field. For single-exposure applications, the camera is synchronized with the pulsed laser and each image records the position of the particles at one instance. The following image captures the position of approximately the same particle set after known time interval $\Delta t$. After correct identification of particle pairs between two images and measurement of the particle
displacement, the local fluid velocity is determined. Then the entire measured flow field is compared with the predictions made with computational fluid dynamics computer models, or gives rise to new models. In Figure 4.1, two images are overlapped together to demonstrate the relative displacement.

![Figure 4.1: Typical PIV experiment setup.](image)

The exploration of new theories to detect the particle image displacement is never-ending, along with the efforts to advance in currently existing methods. The criterion for evaluating a new algorithm is based on its speed and/or accuracy, since a fast processing speed can enable real-time online inspection and control, while accurate velocity measurement is essential to study fine structures in a flow field. This chapter evaluates the diffraction method in the particle displacement application, which processes the complete set of the spot intensity distribution information in the image plane without iteration. It holds the potential of calculating the accurate displacement amount at ultra-fast speed.

### 4.1.2 Problems

How to improve the velocity measurement accuracy and processing speed are two of the main themes in both PIV and PTV. Sub-pixel accuracy is desired by today's PIV applications [34], especially when exploring the small-scale structures in turbulent flows. For most correlation-based algorithms, a large number of iterations and error corrections
have to be implemented in order to achieve sub-pixel accuracy and extremely high spatial resolution. As a result, data acquisition and processing are often separated. During the first stage, a seeded flow is illuminated by a laser sheet in a 2D visualization system and high-speed cameras record a certain amount of time of the flow activity. Digitized images are then post-processed to reach best accuracy, which may take up to several minutes to calculate one velocity vector field, depending the processing algorithm used. For some applications, feedback is essential during the experiment and thus real-time processing is required. For example, wind tunnel design tests can be more efficient if the model shape or orientation can be continuously modified according to real-time flow measurement results [35]. However, the accuracy and spatial resolution will suffer at video-rate processing. There is always a trade-off between accuracy and speed in PIV. Efforts have been made to search for new particle displacement measurement theories without using computationally expensive 2D correlation, which may substantially improve processing speed and accuracy at the same time.

Diffraction phenomenon has been a major principle for many tomography techniques. It could be applied in PIV as well to measure particle displacement. Due to the limiting size of any lens apertures, the optical diffraction effect is inevitable in photogrammetric applications such as PIV. As a result, the image of a point source is blurred into the shape of a sinc function (central segment), since the Optical Transfer Function (OTF) of an imaging system is basically sinc, if the distortion and noise can be safely neglected. Sinc functions can often be well approximated as Gaussian functions in image processing, for mathematical and computational simplicity. This Gaussian character of the intensity distribution has been extensively utilized to locate the image spot centroid or determine the spot size. In order to save memory storage space and speed up processing, very often only part of the entire spot intensity distribution information is processed, which is necessary for Gaussian peak fitting, and the detailed edge intensity distribution is discarded as redundant information [36]. In this chapter, the proposed diffraction method is implemented in measuring particle displacement, which takes advantage of the entire Gaussian-shaped intensity map to reveal the accurate displacement amount at ultra-fast speed.
4.2 Measurement Principle

The measurement principle of particle displacement is the same as discussed in Section 2.2. Since now we are measuring the lateral movement, the off-axis aperture is no longer needed, which encodes longitudinal displacement into lateral shift. In PIV, the camera aperture is fully opened and centered.

There are several important experimental parameters in PIV. The seeding density determines the spatial resolution of measured the flow velocity field. But it should not be high enough to alter fluid properties. In this thesis, currently studied seeding density indicates that a majority of particles are identifiable, around 10 particles per 32x32 interrogation area. Overlapping particles often has large intensity variation from frame to frame. So the number of particles limits the spatial resolution. Optimized separation time between the illumination pulses can ensure that the displaced particle pair overlaps with each other.

Two major assumptions are made in the above diffraction method. First, the tracer particle size is assumed uniform across the flow field. Second, the shifted particle has the same peak intensity as the original one. This assumption is reasonable when the displacement is less than one spot size. From experimental observations, integer peak intensity change is less than 2 out of 256 gray levels within particle pairs. However, this assumption should be checked when the displacement is several times of the spot size and out-of-plane movement is significant. Single-camera PIV can only measure the X and Y flow velocity components in the illuminated plane. If a particle has a significant longitudinal velocity, it may disappear in the next frame or its brightness and image spot size are reduced.

4.3 Calibration

4.3.1 Calibration Setup
A regular spot pattern is used as the object for system calibration (Figure 4.2). The object plane is perpendicular to the optical axis, and is shifted transversely in the x-direction by a micrometer (about ±2 microns in accuracy). The camera and lens are the same as in the 3D imaging setup, except that the off-axis aperture is removed. The total transverse displacement of the object is 1mm with a step size of 10 microns.

An 18x18 cm object field is observed. The spot size is around 3-4 pixels. The image taken at $x = 0\text{mm}$ is used as a reference image. Then each image at different $x$ positions is subtracted from this reference image. For example, Figure 4.3a illustrates the spot displacement between two images at $x = 0\text{mm}$ and $x = 0.4\text{mm}$, respectively, by zooming in to a single spot. Figure 4.3b shows the corresponding normalized subtraction intensity plane of this spot pair. As the displacement $d$ becomes larger than about 0.8mm, the two spots no longer overlap with each other, as shown in Figure 4.4 when $d = 1\text{mm}$. The magnification ratio is that 1mm in the object plane corresponds to approximately 6 pixels in the image plane.

![Figure 4.2: PIV calibration setup.](image)

![Figure 4.3: (a) Combined images of $x = 0\text{mm}$ and $x = 0.4\text{mm}$, (b) corresponding normalized subtraction intensity plane.](image)
4.3.2 Processing

Because of the small spot size, Gaussian filtering over the entire image is necessary to expand the spot diameter and smooth out quantization error. Figure 4.5 illustrates the improvement in calibration accuracy after the Gaussian filtering ($\theta = 2$). The STD of Figure 4.5a relative to the 3rd degree fitted curve is 0.096 pixels in the image plane, while the STD of Figure 4.5b is 0.053 pixels. Larger $\theta$ won’t improve more in accuracy. We usually set $\theta = 1$–2. One-dimensional curve fitting routine in the subtraction plane helps find the exact peak height, rather than simply using integer pixel intensity readings (Figure 4.6a). After implementing 1D fourth degree curve fitting, the STD in Figure 4.6b is only 0.028 pixels. 2D curve fitting in this case will further improve the accuracy a little bit, but is too computationally expensive.
There is a problem with the Matlab curve-fitting function: *polyfit*. If the $x$-coordinate of data set is uniform while $y$-coordinate data set irregular, Matlab can generate a smooth fit (Figure 4.7a). However if the data set is rotated by $90^\circ$ and now $x$ is irregular while $y$ uniform, there will be significant discrepancy in the fitted curve (Figure 4.7b). Here displacement $d$, which is the resultant from the measurement, is uniform. Both cases use a 3rd degree curve fitting. Higher order curve fitting won’t help in the latter case. The Figure 4.7b case poses an ill-conditioned problem for the Vandermonde matrix to solve in the *polyfit* algorithm.

**Figure 4.6**: (a) 1D curve fitting to find the exact peak, (b) calibration curves after implementing 1D curve fitting.

**Figure 4.7**: Calibration data set and fitted curves by Matlab: (a) horizontal, (b) vertical.
This curve fitting error in the calibration data causes artificial waviness in the measurement result. The failure of the curve fitting is due to the nature of the data set. After rotation, \( x \) is not uniform, sparser data in the lower region and denser data in the upper region. There will be even stronger error in curve fitting by interpolating the data set to make the \( x \)-coordinate uniformly spaced.

The solution to avoid the curve fitting error is not to fit the vertical data set at all. Only fit the horizontal data set, and then use Newton's Method to solve for the displacement given any measured subtraction peak height [37].

Figure 4.8 is an example of the final calibration curves for a displacement ranging from 0 to 1\( \text{mm} \). Measurement results highly agree with the 3\textsuperscript{rd} degree fitted calibration curves, which has a fluctuating noise level of 0.003 over the 0-1 normalized intensity scale. This random noise is caused by a combination of CCD quantization error and pixelization. By using both positive and negative curves, the displacement measurement accuracy is improved. Calibration curves saturate as the two spots become completely separated from each other (\( d > 0.8 \text{mm} \)). Since the noise level is constant, measurement accuracy will drop as the displacement increases. By examining various parts of the image plane, the shape of the calibration curves changes gradually due to non-uniformity of the OTF. In Figure 4.9, the entire image plane is divided into 42\( \times \)42 smaller windows, and each
window’s specific curves are calculated. Nine equally spaced sub-windows at the center and periphery of the image plane are shown in the figure. For precise measurement, the slight change in the calibration curves cannot be ignored, and thus specific curves are assigned to different local regions. Compared to Figure 3.22, the variation of the calibration curves across the image plane is relatively smaller.

![Figure 4.9: Calibration curves of nine equally spaced sub-windows. The outer eight windows are at the periphery of image plane, while the center one is at the center.](image)

### 4.4 Measurement Results
Printed patterns of white random spots over a black background, instead of the real laser sheet illuminated flow field, are used as the objects to test the feasibility of the above diffraction method.

#### 4.4.1 Uniform Displacement Test Flow
The entire flow field is arranged to have an arbitrary uniform translational displacement. Two images are taken and each particle pair within a sub-window gives a local displacement measurement result by looking up local calibration curves. Figure 4.10 illustrates the uniformity of 42×42 local displacement measurement results when the
test flow has a known displacement \( d = 0.4 \text{mm} \). Specific fluctuation errors for various displacement amounts are listed in Table 4.1. As discussed before, measurement error increases as the slope of the calibration curve flattens. Best accuracy of 10 microns (or 0.06 pixels in image plane) is achieved when the displacement is less than 0.5 particle diameter. If relative accuracy is defined as absolute accuracy over particle diameter (4.5 pixels), it is less than 1/70 in this case.

Table 4.1: Measurement results and uncertainty range at different displacement \( d \) using the diffraction method.

<table>
<thead>
<tr>
<th>( d ) (mm/diameter)</th>
<th>Error (±10^{-3} \text{ mm} / ±10^{-3} \text{ diameter})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/0</td>
<td>4/5.3</td>
</tr>
<tr>
<td>0.1/0.13</td>
<td>4/5.3</td>
</tr>
<tr>
<td>0.2/0.27</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.3/0.4</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.4/0.53</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.5/0.67</td>
<td>8/11</td>
</tr>
<tr>
<td>0.6/0.8</td>
<td>12/16</td>
</tr>
<tr>
<td>0.7/0.93</td>
<td>16/21</td>
</tr>
<tr>
<td>0.8/1.07</td>
<td>34/45</td>
</tr>
</tbody>
</table>

Compared to the Gaussian fit method which could also solve this particle displacement problem, the proposed diffraction method has a higher accuracy when the displacement is less than half the spot size. Three-point Gaussian curve-fitting is implemented to find the
particle centroids in the above uniform displacement test flow. A typical Gaussian curve has the form

\[ y = c \exp\left(-\frac{(x-a)^2}{2b^2}\right), \]  

(4.1)

where \( y \) is the intensity, \( x \) is the pixel position, \( a \) is the exact center position, \( b \) is the spot size, and \( c \) is the exact peak intensity. Given three pixels around the spot center, we can solve for \( a \) (\( b \) and \( c \)). OTF change across the image plane is also considered. Its uncertainty range at different displacement \( d \) is listed in Table 4.2. Figure 4.11 compares the uncertainty ranges of both methods. The error of the diffraction method is one-half of the Gaussian fit for small displacements.

**Table 4.2:** Measurement results and uncertainty range at different displacement \( d \) using the Gaussian fit method.

<table>
<thead>
<tr>
<th>( d ) (mm/diameter)</th>
<th>Error (±10^{-3} \text{ mm / 10^{-3} diameter})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0/0</td>
<td>9/12</td>
</tr>
<tr>
<td>0.1/0.13</td>
<td>8/11</td>
</tr>
<tr>
<td>0.2/0.27</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.3/0.4</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.4/0.53</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.5/0.67</td>
<td>6/8</td>
</tr>
<tr>
<td>0.6/0.8</td>
<td>5/6.7</td>
</tr>
<tr>
<td>0.7/0.93</td>
<td>6/8</td>
</tr>
<tr>
<td>0.8/1.07</td>
<td>7/9.3</td>
</tr>
<tr>
<td>0.9/1.2</td>
<td>6/8</td>
</tr>
<tr>
<td>1/1.33</td>
<td>7/9.3</td>
</tr>
</tbody>
</table>

**Figure 4.11:** Uncertainty ranges of both diffraction method and Gaussian fit method for uniform displacement test flow.
4.4.2 Swirling Test Flow

An object plane with a random spot pattern is shifted in space within a small time interval as a simulated swirling flow, whose local velocity rises exponentially with the radius relative to the center of the image plane. Figure 4.12 shows a single exposure image frame pair. At the outer region of flow field, the displacement is as large as 2mm in the object space, which is several times the particle diameter. Here the diffraction method is no longer applicable by itself. In order to expand its application in flow fields whose maximum displacement is larger than one particle diameter, a traditional FFT based coarse cross-correlation software (Insight 3.1 by TSI Inc.) is first used to get a rough estimate of the local displacement up to an integer pixel accuracy. The interrogation window size is 32x32 with 50% overlapping. The larger maximum particle displacement, the smaller interrogation window size should be in order to achieve an integer pixel accuracy locally. Then one image is shifted by this amount so that it can now overlap with the second image. Finally the diffraction method is applied to acquire sub-pixel accuracy. Figure 4.13 illustrates the measured displacement field using both cross-correlation and the diffraction method.

![Image Frame A](image1.png) ![Image Frame B](image2.png)

Figure 4.12: Single exposure image frame pair of the swirling test flow.
**Figure 4.13:** Measurement result of a swirling flow by first using coarse cross-correlation for rough estimation, and then the proposed algorithm for the sub-pixel accuracy.

### 4.4.3 PIV Custom-Made Standard Images

The Visualization Society of Japan promotes the standardization of various PIV algorithms. Standard images are synthesized images based on the random noisy nature of real flow field. So they provide an objective measure of an algorithm’s performance. Two consecutive PIV custom-made standard images (Figure 4.14) are used to evaluate the proposed diffraction method (http://piv.vsi.or.jp/piv/java/tmp2/1919/index.html). The

**Figure 4.14:** Single exposure image frame pair of custom-made standard images.
image size is 256×256 pixels with a maximum particle displacement of 4.5 pixels. The particle size is uniformly 8 pixels. A total of 640 particles are placed in each image, which corresponds to an average seeding density of 10 particles in each 32×32 interrogation area.

Gaussian filtering is not necessary here for the image pre-processing since the particle size is large enough. Considering the particle intensity variation from frame to frame (due to out-of-plane displacement), local equalization is implemented to force each particle pair to have the same peak intensity within two frames. In order to take advantage of the highly sensitive, pseudo-linear region where the particle displacement is less than one pixel, a FFT based cross-correlation with a 32×32 interrogation area and zero overlapping is first calculated. Then one spot is shifted by this estimated amount and finally subtraction processing is carried out. The processing speed is larger than 500 vectors/sec with MATLAB. Figure 4.15 illustrates the relationship between the real displacement of particles given by the standard information and the measured displacement using the diffraction method. There are two main error sources. One situation is when the particle is very dark, for example, if its peak intensity is 5 out of 256 gray levels and only 2×2 CCD pixels have readout of signal. In this case, the signal-to-noise ratio (SNR) is very low. Another situation is when the particles overlap in one image. As a result, the

![Figure 4.15: Diagram of the real displacement and measured displacement using the diffraction method.](image)
neighboring pair distorts the shape of one particle pair in the subtraction plane, and the subtraction peak search is no longer accurate. This phenomenon limits the possible highest seeding density.

In order to study the potential of the proposed diffraction method, three other common techniques for measuring the particle displacement are tested using the same two images in Figure 4.14. The first one is the Gaussian fit method in PTV, which implements the iterative Levenberg-Marquardt optimization algorithm to accurately solve the particle centroid position [38]. The second one also calculates Gaussian centroid, but only by solving two 1D closed-form Gaussian fitting equations to approximate centroid position [24]. Both of these two measurement results are compared with the real displacement of particles. The third one is the FFT based cross-correlation algorithm used to obtain rough estimates for the diffraction method. Its result is compared with the average of real particle displacement within each interrogation area. Figure 4.16 illustrates the error distributions of all four methods. 60% of correlation results have errors greater than 0.1 pixels. The two Gaussian Centroid methods and diffraction method have roughly the same accuracy. 95% of their displacement calculation results have errors no greater than 0.08 pixels. There is a slight difference for the frequency of errors, which is no greater

Figure 4.16: Displacement measurement error distributions of Levenberg-Marquardt method, Diffraction method, 1D Gaussian fit method and FFT cross-correlation.
than 0.02 pixels, with Levenberg-Marquardt highest, followed by diffraction method, then 1D Gaussian fit. The speed ranking is reversed, with 1D Gaussian fit fastest and Levenberg-Marquardt much lower. The speed of the diffraction method is comparable with 1D Gaussian fit, but it could provide a more robust particle pair identification method because each pair appears as neighboring positive and negative peaks in the subtraction plane.
Chapter 5

Field of View Method

In the study of the fringe pattern projection, we noticed the phenomenon that the field of view enlarges as the object distance increases, so more fringes squeeze into the image plane from both sides as the camera moves away from the object. In Figure 5.1 are two images of a flat plane with a printed fringe pattern. Figure 5.1b is taken when the camera is 40mm further away from the object plane than Figure 5.1a. The density of fringes serves as an indicator of the object position. The closer the object, the smaller field of view in angle, and thus the sparser fringe density. There are two ways to measure the density of fringes: one is to measure the local frequency; the other is to measure the local period of fringes.

(a) $z = -20\text{mm}$, 33 fringes in total  
(b) $z = 20\text{mm}$, 37 fringes in total

Figure 5.1: More fringes come into field of view as the camera moves away from the object.
5.1 Frequency Measurement

5.1.1 Basics of the Fourier Optics

The Fourier optics uses Fourier Transform to study the frequency components of an image [39][40]. A signal can be decomposed into a series of sine or cosine waves, no matter whether it is in the time domain (such as an electrical signal) or the spatial domain (such as an image). The frequency domain records the amplitude and phase information at each frequency of a specific composite sine wave (Figure 5.2). In this chapter, we are only concerned with the amplitude information so each frequency domain plot only represents the intensity of a specific sinusoidal wave.

![Comparison of electrical signal and optical signal in two domains.](image)

An image is a two-dimensional signal, so is its frequency domain. Figure 5.3 are two examples of images and their frequency domain plots. There are one separated 2D horizontal sinusoidal wave and one vertical sinusoidal wave in Figure 5.3a. The period and frequency has reciprocal relationship. The smaller the period, the higher the frequency will be. Figure 5.3b is an example of a more general image with multiple frequency components.
Figure 5.3: Examples of 2D images and their frequency components.

A system concept is essential in Fourier Optics as shown in Figure 5.4, where \( g(x, y) \) is the spatial domain distribution and \( G(x, y) \) frequency domain distribution. The object is the input and the image transferred to the computer is the output. A specific imaging system consists of several sub-systems, e.g., the optical system, imaging sensing element, digitizer and frame grabber, and system parameters, e.g., object plane distance, image plane distance and F-number. By modifying any one element in the imaging system, the overall system behavior in both spatial and frequency domains is changed.

Figure 5.4: System concept in Fourier Optics.
The overall system response in the frequency domain is characterized by $H(f_x, f_y)$, which is called the Optical Transfer Function (OTF). Its modulus is called the Modulation Transfer Function (MTF). From the shift-invariant system theory, input, OTF and output follow the simple multiplication relationship:

$$G_{image}(f_x, f_y) = H(f_x, f_y) \cdot G_{object}(f_x, f_y).$$

(5.1)

Figure 5.5 is an example of a square low-pass MTF applied to the example in Figure 5.1b. As a result in the output frequency domain, higher frequencies in the input are completely cut off.

Figure 5.5: Example of the effect of MTF in the frequency domain.

The overall system response in the spatial domain is characterized by $h(x, y)$, which is called the Point Spread Function (PSF). For a shift-invariant system, input, PSF and output follow the simple convolution relationship:

$$g_{image}(x', y') = h(x, y) * g_{object}(x, y)$$

(5.2)

A typical PSF tends to blur the image. For example, in Figure 5.6, a point source is blurred into an *Airy disk* after passing through a finite-diameter lens.

Figure 5.6: Effect of the PSF.
The relation between OTF and PSF is a 2D Fourier transformation:

\[ H(f_x, f_y) = FT\{h(x, y)\}. \] (5.3)

### 5.1.2 Frequency Method

The experiment hardware is still the off-axis rotating camera. We use a flat plane with a printed uniform fringe pattern as our object (Figure 5.1) and take a series of images at a fixed aperture position (e.g. 90°) at camera depth positions from \(z = -20\text{mm}\) to \(z = 20\text{mm}\) and from \(z = 30\text{mm}\) to \(z = 70\text{mm}\), where positive \(z\) indicates that the camera moves further away from the object. Then we implement 2D Fourier transform over each entire

---

**Figure 5.7**: 2D frequency plane of the image taken at \(z = 20\text{mm}\) after Fourier transformation.

**Figure 5.8**: Averaged frequency distribution at \(z = 20\text{mm}\).

**Figure 5.9**: Peak positions shift to higher frequency region as depth \(z \text{ (mm)}\) increases.
image plane (1024×1024 pixels) at each $z$ position (Figure 5.7). Since all fringes are vertical oriented, we can average along the vertical direction and obtain the intensity distribution for all horizontal spatial frequencies (Figure 5.8). We can see that the frequency peaks shift to higher frequency region as $z$ increases (Figure 5.9). Figure 5.10 illustrates the frequency peak positions to the object distance relationship. The intensity height of each peak also varies when $z$ changes (Figure 5.11). However, several other factors also influence the peak height, such as background illumination variation, object shape and surface reflectance variation. So the peak height is more prone to noise than the peak position. Therefore we cannot use it for measurement without some kind of normalization method.

**Figure 5.10:** Peak position – $z$ plot.

**Figure 5.11:** Peak height – $z$ plot.

To improve the spatial resolution of the measurement, we need to step down on the

**Figure 5.12:** 128×128 interrogation window: (a) peak shifting at different $z$ (mm), (b) peak position – $z$ (mm) plot for two windows.
interrogation window size. For a 128×128 window, the peak movement is still measurable, though we will need curve-fitting techniques to find the precise peak position. Also waviness becomes obvious (Figure 5.12). If the window size goes further down to 64×64, waviness dominates and we lose the linearity characteristic (Figure 5.13).

As the interrogation window size goes down, the necessary spatial information contained in each window decreases. The result is that the error of Fourier transformation increases. The step shape in Figure 5.12b and 5.13b is due to the pixelization of CCD and the Fourier transform quantization error.

Before the measurement of a 3D object, we first calibrate the system by every 128×128 window with 0% or 50% overlap over the entire 1024×1024 image plane. For each window we obtain a calibration curve like the one in Figure 5.12b. Then we ignore the superimposed waviness and fit a linear curve, which serves as our calibration curve for this local area. Now we are ready to use this calibration information to measure the shape of an arbitrary object. At this stage we didn’t project a fixed frequency fringe pattern onto the object surface. For simplicity, we stick a piece of paper printed with uniform fringe pattern onto object. Figure 5.14 is a tilted plane, Figure 5.16 is the sharp 90° corner of a cube and Figure 5.18 is a cylinder. Figure 5.15, 5.17 and 5.19 are their respective measured shape. The strong waviness in Figure 5.15 is the result of ignoring the waviness in the calibration curve.
Figure 5.14: A tilted plane as the object.

Figure 5.15: Measured shape of a tilted plane with (a) 0% and (b) 50% overlap using a 128x128 interrogation window size.

Figure 5.16: Image of a sharp 90° corner of a cube.

Figure 5.17: Measured shape of the sharp 90° corner, with a 128x128 window size and 50% overlap.
In Figure 5.19, we noticed the shape of the cylinder has been elongated. It should fall into the −50mm to 50 mm depth range instead of the −100mm to 200mm range, based on the experiment setup. The reason of this distortion is that the fringe density is determined by both the object position and surface orientation. If the surface is tilted from the standard parallel orientation, its fringe density will increase sharply. In Figure 5.15, 5.17 and 5.19 we mistook the density change due to local orientation as to the depth.

**Figure 5.20:** Frequency change due to angle.

**Figure 5.21:** Measured and theoretical frequency change due to orientation.
Theoretically, the fringe density increases as the reciprocal of $\cos(\theta)$ (Figure 5.20). Suppose the object surface is tilted by an angle $\theta$ from the parallel position 1 and the fringe frequency at 1 is $f_1$. Then $f_2$ at position 2 is

$$f_2 = \frac{1}{d \cos \theta} = f_1 \frac{1}{\cos \theta} \quad (5.3)$$

There's a small discrepancy between the theoretical estimation and measured frequency variation based on the orientation angle (Figure 5.21), which should be calibrated out for measurement.

Since the fringe density is determined by both the depth and angle, there is an ambiguity in determining the depth if we only take one image of each object (Figure 5.22). We cannot solve one equation (5.4) for two unknowns, $z$ and $\theta$:

$$f = (f_0 + kz) \frac{1}{\cos \theta} \quad (5.4)$$

where $f$ is the measured local frequency, $f_0$ is the frequency at a parallel reference plane, $k$ is the slope of the calibration line and $z$ is the object distance from the reference plane.

Figure 5.22: Peak position ambiguity due to both the depth and angle.
We need at least two independent equations to solve for both $z$ and $\theta$. A double-frequency fringe pattern projection (Figure 5.23) cannot solve the problem since it gives us two dependent equations for the two frequencies.

![Double-frequency fringe pattern projection](image)

**Figure 5.23**: Double-frequency fringe pattern projection.

We have an approximate solution to this ambiguity problem when only one image of the single-frequency fringe pattern projection is taken. From Figure 5.21 and 5.22 we can see that the spatial frequency climbs up rapidly when $\theta$ is larger than 30°, where the frequency is higher than that solely caused by the depth increment within the measurement range. So we can make the assumption that for small local peak frequency shift, it is completely caused by the depth and we can look up $d$ on the linear calibration line while setting $\theta = 0^\circ$. However, for large frequency shift, we assume it’s completely caused by orientation, then we can solve for $\theta$ using Equation 5.4, where $z$ is the depth of the adjacent interrogation window 1. We obtain $z$ for current window 2 by Equation 5.5 (see Figure 5.24):

$$z_2 = z_1 - \frac{a}{2} \cdot \sin \theta_1 - \frac{a}{2} \cdot \sin \theta_2$$

(5.5)

where $a$ is the interrogation window size in the object space. By using these assumptions we can obtain the correct cylinder shape within the right depth range (Figure 5.25).

One exact solution to the depth-orientation ambiguity is by changing one of the imaging system parameters and taking two images at different settings. For example, we can change the object distance, image distance, lens position or focal length. For example, we
can choose to take two images at two object positions. Thus we obtain two independent equations and can solve for $z$ and $\theta$ (will be discussed in details in section 5.2).

From general experience, Fourier transform based methods fail in terms of high spatial resolution since the minimum window size required should be quite large (in our case, 128×128 pixels). And it is computational expensive to calculate the Fourier transform. Also this method is sensitive to the noise (e.g., the “bulk” in Figure 5.25). So next we switch to the spatial domain to measure the local fringe density.

### 5.2 Period Measurement

Three methods are attempted to measure the local fringe period in the spatial domain: the valley-to-valley distance method, linear intersection method and curve fitting intersection method.

#### 5.2.1 Valley-to-Valley Distance Method

We can curve-fit each valley to find its lowest point position, and then subtract the two adjacent valley positions to get $d$ (Figure 5.26). Or we can average over several periods. If we adjust the illumination intensity or exposure time carefully so that the image is not saturated, we can also measure the peak-to-peak distance.
**Figure 5.26:** Measure the valley-to-valley distance.

**Figure 5.27:** Period-depth relation when averaged over 4 or 9 periods.

**Figure 5.28:** STD to averaging period number plot.
Figure 5.27 and 5.28 show how the standard deviation decreases as the averaging period number used increases. A lowest STD (= 0.0243 pixels) occurs when averaging over 7 neighboring periods, which corresponds to a depth measurement accuracy of 1.27mm.

5.2.2 Linear Intersection Method

Alternatively, we can draw an intersection line across the fringe intensity plot (Figure 5.29). Then we interpolate the two adjacent sampling pixels across the intersection line linearly to find the position of A (in pixels). Similarly we can calculate the positions of B, C and D and obtain the period = (x_C - x_A) or (x_D - x_B). We can draw several intersection lines close to the local mean intensity of the image and each line will give a period measurement. Figure 5.30 shows the standard deviation to period number plot when there
are five intersection lines. The smallest STD is approximately 0.53mm in depth.

5.2.3 Curve Fitting Intersection Method

Alternatively, we can first fit a curve of an edge, and then evaluate the intersection point position on this curve (Figure 5.31). Both 4th or 5th order polynomial curve fitting and three-point Gaussian curve fitting give us roughly the same accuracy.

Similarly as in Figure 5.29, we can calculate the period by measuring the horizontal positions of A, B, C and D, and let $\text{period} = (x_C-x_A+x_D-x_B)/2$. Figure 5.32 is an example of the period-depth calibration curve when using 5th order polynomial curve fitting, five intersection lines, both left and right edges of the fringe and averaging over four periods. Figure 5.33 illustrates how the STD decreases as the averaging number of period

![Figure 5.31: Curve fitting of an edge: (a) Fourth order polynomial, (b) Fifth order polynomial, (c) Three-point Gaussian.](image-url)
increases. The lowest STD is 0.5mm in depth. The spatial resolution of this method is one period, which is smaller than 10 pixels in our experiment.

5.2.4 Conclusion

The valley-to-valley method has the worst accuracy, while the second and third methods have roughly the same accuracy. We choose curve fitting intersection method as our standard period measurement method. The processing settings are: 5th order polynomial curve fitting, five intersection lines, both left and right edges of fringe used and averaging

![Figure 5.32: Period-depth plot, averaging over four periods.](image)

![Figure 5.33: STD decreases as the averaging period number increases.](image)

**Figure 5.34:** (a) Image of a cube corner, and (b) the period distribution along the center horizontal cross section line.
over four periods. Figure 5.34 is an example of the above period measurement result, where (a) is an image of a cube corner, and (b) shows the period distribution along the center horizontal cross section line.

5.3 Sample Measurements

We take two images at two camera positions in order to solve for both \( z \) and \( \theta \). Figure 5.35 illustrates the system geometry.

![System geometry](image)

Figure 5.35: System geometry when moving camera by \( s \).

The two camera positions have a known displacement, \( s = 10 \text{mm} \). The virtual reference plane is moved by \( s \) accordingly between the two setups. The local periods in the two images follow the relationship:

\[
d_1 = (d_0 - kz) \cos \theta \quad (5.6)
\]
\[
d_2 = [d_0 - k(z + s)] \cos \theta \quad (5.7)
\]

where \( d_0 \) is the period on reference plane, and \( k \) is the slope of the calibration line.

Solving Equation 5.6 and 5.7 for \( z \) and \( \theta \), we have:

\[
z = \frac{d_0 - d_1 s}{k} \frac{d_1 - d_2}{d_1 - d_2} \quad (5.8)
\]

\[
\theta = \arccos \left( \frac{d_1}{d_0 - kz} \right) \quad (5.9)
\]

One important note is that \( d_1 \) and \( d_2 \) are not the local period of the same pixel in the two images, because the image of the same object point shifts toward the center of image plane as the camera moves further away from the object. We should compare \( d_1 \) and \( d_2 \) of the same image point. The depth measurement range is about 100mm and the average
object distance is about 1 meter. Since the measurement range is much smaller than the object distance, we can assume that within the measurement range all the image points shift the same amount from the center of expansion (or contraction) due to the camera movement $s$. We can calibrate the image point shift over the entire image plane by placing an object plane in the middle of the measurement range. Since we are using vertical fringe pattern we only need to calibrate the horizontal image point shift. Figure

![Image](image.png)

**Figure 5.36:** Image point shift in the horizontal direction.

5.36 is an example of how much the image points shift when the camera is moved from $z = 30\text{mm}$ to $z = 20\text{mm}$ ($s = 10\text{mm}$). Positive shift $dx$ represents a rightward displacement.

Figure 5.37a demonstrates the calculated cross-sectional period map of a protruding corner with an angle $\approx 130^\circ$. The two images are taken at camera positions $z = 20\text{mm}$ and $30\text{mm}$ (further away from the object than $z = 20\text{mm}$). Figure 5.37b is the calculated 3D shape. Figure 5.38 shows another example of a cylindrical object. There is some systematic error in Figure 5.37b and 5.38b, which should be calibrated out or eliminated by modifying Equation 5.8.

In real measurement setup, we can either use a pattern projection focused at infinity or at a finite distance. In the latter case, the projection divergence angle should also be considered.
**Figure 5.37:** (a) Period map of a protruding corner at $z=20$mm and $z=30$mm, (b) the calculated depth map.

**Figure 5.38:** (a) Period map of a cylindrical object at $z=20$mm and $z=30$mm, (b) the calculated depth map.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions
A novel diffraction method is proposed to measure particle displacement based on the fixed OTF of an imaging system. This method is ultra-fast since all processing is done in the image plane. A simple subtraction of two consecutive image planes and search for local peak height in the subtraction plane will relate to the displacement. Sub-pixel accuracy can theoretically reach at least $1/50$ of the spot size. This method works best when most particles are identifiable and illumination variation is small so that individual particle peak intensity can be assumed uniform from frame to frame. Otherwise, local equalization should be implemented. When displacement is several times of spot size, coarse cross-correlation can first be used to get a rough estimate of the displacement, and then the diffraction method can be used to obtain sub-pixel accuracy. Since the diffraction method is fast enough to achieve high-speed processing, in the future it could be integrated with other algorithms to further improve accuracy.

The feasibility of the proposed diffraction method is proved in two kinds of potential applications, namely 3D imaging and PIV. In 3D imaging first we use spot pattern projection and a rotating off-axis aperture to encode object depth information into particle displacement in image plane. Then the object shape is revealed by the diffraction method. In PIV imaging, we use a fully opened and centered aperture since we only need to measure 2D particle velocity. Consequently the optical aberrations are much smaller in PIV than our 3D imaging application.
A couple of other approaches are also studied to recover 3D shape. Frequency domain technique is a very useful tool in grasping the underlying nature of the imaging system and for problem shooting, though it is not so practical for real measurement since it is relatively slow and its spatial resolution is limited. Using direct period measurement method to render 3-D shape has not appeared in literature yet. It is also very fast and currently gives a depth accuracy of 0.5mm as presented in this thesis.

6.2 Recommendations

The diffraction method provides a simple and fast approach of processing images. However, noisy measurement data and unstable environment will require the method to be further complicated in order to handle those non-ideal situations, i.e., to increase the robustness of the method. Since this diffraction method is fast enough, it can be integrated with other processing algorithms to improve accuracy, such as the reverse hierarchical iterative algorithm developed by Prof. Hart etc. More pixel points in the subtraction plane in addition to the peak can be utilized based on some optical flow concept.

In 3D imaging, future research can be focused on the optimization of system parameters, such as the size and location of the off-axis aperture, both by trial and error and numerical simulation. By increasing the aperture size and bringing it closer to the optical axis, we can reduce the strong aberrations. Designing a projection system with small divergence angle and large depth of field will also be an important topic. In PIV imaging, it is very valuable to apply the diffraction method in some real flow experiments and study the algorithm’s ability in distinguishing out-of-plane motion from 2D velocity.

Direct period measurement method holds a lot of potential in fast recovery of 3D shape. Future work could focus on how to measure local period more accurately and look for the systematic error sources.
Reference


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