SUMMARY REPORT ON HEAVY WATER, NATURAL URANIUM LATTICE RESEARCH

by

I. Kaplan, D. D. Lanning, A. E. Profio, and T. J. Thompson

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UC-34 Physics
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30.-100. Internal Distribution
This report summarizes the methods and results on 1.0-inch diameter, natural uranium metal rods in heavy water, measured in the M.I.T. exponential facility.
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I. INTRODUCTION: SCOPE AND OBJECTIVES

In 1959, the Nuclear Engineering Department of the Massachusetts Institute of Technology, with the support of the United States Atomic Energy Commission, undertook a program of research on the physics of lattices of metallic uranium rods in heavy water. The general objective of this lattice project is to carry on experimental and theoretical investigations of the nuclear properties of subcritical lattices of partially enriched uranium rods in heavy water. The first series of enriched rods is to contain $^{235}\text{U}$ at concentrations between one and two percent, and to have diameters from 0.25 to 0.75 inch. The ratio of the volumes of heavy water and uranium is to be varied, with values in the range 10 to 90. Initial studies have been completed with natural uranium rods, 1.0 inch in diameter, in triangular lattices with spacings of 4.5, 5.0, and 5.75 inches, respectively. These lattices have been used to test the accuracy of the experimental methods which have been adopted. Natural uranium, heavy water lattices have been studied extensively at several laboratories and many results are available with which the M.I.T. data could be compared. These comparisons have served to establish the validity of the methods used at M.I.T., and have also provided new information concerning such lattices. Work on lattices of partially enriched, uranium-metal rods is in progress, and will be followed by studies of lattices of enriched uranium-oxide rods in heavy water.

The main purpose of the research is to add to our understanding of fundamental reactor physics, rather than to concentrate on the design problems of a particular reactor or reactor type. The emphasis is, therefore, on a broad, deep, and flexible program of research. This kind of research program seems
most suitable for a university laboratory, and has influenced the choice of the problems to be attacked as well as the facilities used.

The measurements which have been undertaken involve the use of both steady-state and pulsed neutron techniques. The steady-state measurements include those of the following quantities: macroscopic radial and axial, thermal and epithermal neutron flux traverses (from which the critical buckling can be obtained); parameters related to the capture of thermal and resonance neutrons, including the ratio of captures in $^{238}\text{U}$ to fissions in $^{235}\text{U}$ in the fuel rod, the average $^{238}\text{U}$ cadmium ratio in the fuel rod, and quantities related to the effective resonance integral of the rods; activation ratios related to fission rates at epithermal energies, including the ratio of fissions of $^{238}\text{U}$ to fissions of $^{235}\text{U}$, and the ratio of epithermal fissions of $^{235}\text{U}$ to thermal fissions of $^{235}\text{U}$; intracell thermal and epithermal neutron flux distributions for different neutron detectors. The main subcritical assembly is a tank fed by thermal neutrons from the thermal column of the Massachusetts Institute of Technology Reactor (MITR). In addition, a small exponential assembly or "miniature lattice," is being used to study some of the quantities mentioned above as well as the theory of subcritical assemblies. Measurements are also under way on a single rod, or a few rods, in a tank of moderator.

The steady-state experiments also include measurements in the moderator, or in lattices, containing control rod materials. Control rod reactivity measurements, as well as more general reactivity and lifetime measurements, will be made with a pulsed neutron source. Finally, work is being started on measurements of the neutron energy spectrum with detectors, and work is planned on time-of-flight measurements of the neutron energy spectrum in the moderator.

The experimental research outlined above has two objectives:

(1) to improve existing methods and develop new ones where possible;
to apply the methods to different lattices. The emphasis is on precise, theoretically interpretable experiments. The theoretical program, carried on parallel to the experimental work, also has two objectives: (1) to relate the experimental results to existing theory; (2) to extend the theory where possible.

The purpose of the present report is to summarize the work on the facilities, on the methods used, and on the results obtained with the three lattices of 1.0-inch diameter, natural uranium rods in heavy water.
II. FACILITIES

A. THE SUBCRITICAL ASSEMBLY

The major experimental facility is the subcritical assembly located at the thermal column of the M.I.T. heavy water moderated and cooled, enriched uranium fueled research reactor. The reactor has been described earlier (1, 2). The choice of a subcritical, or exponential, assembly was made for several reasons: (1) the availability of the MITR as a strong source of thermal neutrons; (2) the expectation that as much, or nearly as much, adequate information could be obtained in a subcritical assembly as in a critical facility; (3) the relatively greater simplicity and ease of operation of a subcritical assembly; (4) the question of the possible hazards of a critical facility on the campus of a university in a large city. The use of a subcritical assembly also offered an important educational advantage over a critical facility in that graduate students could work more intimately, and with greater responsibility, with a subcritical assembly because of the possible hazards of a critical assembly.

The use of the thermal column of the MITR to supply neutrons to a tank containing a subcritical assembly of uranium rods raised certain problems which will be discussed in considerable detail because they may be met often in the future use of research reactors in connection with experiments in reactor physics. In planning the experiments, it was decided that the rods should be suspended vertically in the heavy water to avoid bowing of the rods and to facilitate the changing of both the lattice and of detectors in the lattice. If the source neutrons enter at the bottom of the tank containing the lattice, the neutron flux decreases exponentially along the direction of the rods, that is, along the vertical
direction. The exponential experiments can be interpreted conveniently and the results compared with those of other workers. The neutrons from the thermal column of the MITR were available, however, in a horizontal beam so that the problem arose of converting the horizontal beam into a vertical beam.

The problem and the solution adopted are shown in Fig. 1. The solution involves the use of a graphite-lined cavity, or "hohlraum," to change the direction of the neutron beam. Neutrons from the thermal column traverse the cavity and are scattered by the graphite walls. Some of the neutrons in the cavity diffuse upward through the top graphite wall and form a vertical neutron beam to the exponential tank located above the cavity. The use of a cavity for various purposes had been suggested independently by several workers (3, 4, 5, 6), but the first detailed experimental and theoretical investigation was made in connection with the M.I.T. lattice project; the details are given in Ref. 7, based on a doctoral thesis by J. T. Madell.

1. Experimental Study of the Cavity Assembly

The geometrical arrangement and the wall material of the cavity assembly were changed as part of the experimental program, but all of these changes may be described as variations on a basic cavity assembly. This basic assembly will be described first, and then the variations will be discussed. Figure 2 is a vertical section of the basic assembly (Assembly II). The floor, side walls, back wall, and top of the cavity are made up of 4-inch by 4-inch stringers of reactor grade graphite. The top of the cavity, called the "pedestal," is 16 by 72 by 72 inches and is supported over the cavity by a honeycomb structure 2 inches thick. The latter consists of very thin aluminum foil in a form similar to that of the walls of a honeycomb, held together by resin and sandwiched between two 0.064-inch thick sheets of aluminum. Most of the space between the aluminum sheets is occupied by air, so that the honeycomb structure is quite transparent to neutrons while still able to support the graphite pedestal.
Heavy concrete shielding blocks, the first two layers of which were in place during the experimental study of the flux distribution on the surface of the cavity, surround the cavity assembly on all sides.

In an effort to find a satisfactory cavity assembly, seven different assemblies were studied. Assembly I had boral and cadmium on the walls between the thermal column and the cavity. Assembly II was obtained by adding an 8-inch thick graphite "frame" in this space, as shown in Fig. 1. In Assembly III, the graphite "frame" was modified by an overhanging "tooth" arrangement, which will be discussed later. In Assembly IV, a graphite floor, 8 inches thick, was extended into the shielding and thermal column door spaces, and 4 inches of graphite were added to the sides and back of the assembly. Assemblies V, VI, and VII were all characterized by a rearrangement of the floor of the cavity. Graphite was added to the floor of the basic cavity to make a stepped, inclined surface, Assembly VII, as shown in Fig. 3. If the stepped arrangement is considered to be smoothed out, a 45° inclined plane is obtained, indicated by the dotted line in the figure. Assembly VI is the same as Assembly VII but with the "tooth" removed. Removal of the graphite frame then gives Assembly V.

The magnitude and the spatial distribution of the thermal neutron flux were measured on the walls of the seven cavity assemblies and on the wall of the thermal column that acts as the neutron source. Since the neutrons which enter the exponential tank must first pass through the honeycomb structure and the pedestal, the magnitude and distribution of the flux in the pedestal provided a basis for choosing the most suitable cavity assembly. Activation measurements were used to obtain the neutron density. A large number of foil activations was needed to map the flux distribution in the seven assemblies, and it was found convenient to use copper foils, bare and cadmium-covered. The relative activities of the copper foils were converted to absolute flux values by irradiating a cobalt foil and a copper foil in the same flux and determining the conversion factor between the absolute flux and the
relative activity. The absolute disintegration rate of the cobalt foil was measured by coincidence counting and, from the absolute disintegration rate, the absolute flux which irradiated the cobalt foil was determined. The conversion factor was obtained by comparing this absolute flux to the value of the relative activity of the copper foil irradiated in the same flux.

The details of the complete flux mapping are given in Ref. 7. The results for the flux distribution on the honeycomb, from the thermal column edge to the back wall, are shown in Fig. 4. In all cases, the absolute magnitude of the flux is in the neighborhood of $10^8$ to $10^9$ neutrons per cm$^2$ per sec per MW of reactor power, indicating that the flux entering the exponential tank would be satisfactory in magnitude. The average flux on the honeycomb was approximately one-third of that leaving the source face. Calculations for the case when the cavity is filled with graphite showed that the average magnitude of the flux at the honeycomb is about an order of magnitude greater when the cavity is used than it would be if the cavity were filled with graphite. The cavity assemblies supplied neutrons with distributions given approximately by a cosine in the transverse direction. The assemblies with partially filled cavities (V, VI, VII) had fluxes greater by a factor of about 1.7 than those of the parallelopipedal cavities. The latter, however, offered the possibility that experiments could be made in the cavity itself, saved a great deal on graphite, and had adequate flux levels.

The pedestal had the function of shaping the flux for the buckling measurements. However, the other measurements benefit more from a large flux than from a perfect flux shape, and the magnitude at the top of the pedestal was at the lower limit of the range of values desired at the base of the exponential tank. Further experiments were, therefore, undertaken (8), which resulted in the final choice of a graphite pedestal, 72 inches square by 16 inches high, with a central hole 48 inches wide under the tank containing the subcritical assembly (Fig. 5). Although Assembly III would have been preferable with a solid pedestal, the use of a hollow
pedestal permitted flux shaping by removing some graphite at the front edge, as shown in Fig. 1, and Assembly II with this final form of the pedestal was chosen. The flux distribution at the tank bottom is very close to a double cosine and with a magnitude of about $4 \times 10^8$ n/cm$^2$-sec per MW of reactor power (the MITR now operates at 2 MW and will eventually operate at 5 MW).

2. Theoretical Treatment of the Cavity Assembly

A theoretical method was also developed for the calculation of the flux on the surface of the graphite-lined cavity. The method was developed to exploit the analogy to radiant heat transfer. As would be expected from the geometrical arrangement, the calculations are complicated and required the use of a high-speed digital computer. The method will be outlined here; the details are given in Ref. 7. The calculation is straightforward and can be summarized briefly. Let $r$ define the position of a point on the surface of the wall bounding the cavity; let $J(r)$ be the neutron current (n/cm$^2$ sec) incident on the surface at the point $r$, and let $A(r)$, (cm$^2$), be the area on which the current is incident. The number of neutrons incident per second on $A(r)$ is:

$$J(r)A(r) = \int_{\text{all surfaces}} J(r')K(r,r')dA(r') + \int_{\text{source surface}} S(r')L(r,r')dA(r'),$$

(2.1)

where

- $r'$ is a point on the surface of the cavity from which neutrons contribute to $J(r)A(r)$,
- $J(r')$ is the neutron current incident on the surface at $r'$,
- $A(r')$ is the area at $r'$,
- $K(r,r')$ is a kernel defining the contribution that the neutron current $J(r')$ makes to $J(r)$; when $r$ and $r'$ are on the same plane surface, $K(r,r') = 0$,
L(r, r') is a kernel defining the contribution that the source S(r') makes to J(r); when r and r' are on the same plane surface, L(r, r') = 0.

S(r') is the surface, distributed neutron source (n/cm² sec) entering the cavity at r'.

It is evident intuitively that the kernels are complicated and that the integrals would be difficult to evaluate. An approximate method was used in which the integrals were replaced by summations:

\[ J_1 A_1 = \sum_j J_j A_j K_{j1} + \sum_j S_j A_j L_{j1}, \] (2.2)

where \( A_j \) and \( A_1 \) are small sub-areas on which the neutron currents \( J_j \) and \( J_1 \), respectively, are incident, and \( K_{j1} \) and \( L_{j1} \) define the contributions from \( A_j \) to \( A_1 \). It is assumed that the incident currents \( J_j \) and \( J_1 \) are constant over the small sub-areas \( A_j \) and \( A_1 \), respectively, and that the currents incident on the sub-areas adjoining \( A_j \) do not contribute to the current re-entering the cavity through \( A_{1j} \).

The kernel \( K_{j1} \) is separated into two factors, \( K_{j1} = \beta_j F_{j1} \); \( \beta_j \) is the albedo, or the probability that a neutron incident on the sub-area, \( A_j \), will re-enter the cavity; \( F_{j1} \) is the "view-factor" which defines the fraction of the neutrons leaving \( A_j \) that reach \( A_1 \). For the source surface, \( L_{j1} = F_{j1} \), and equation (2.2) becomes:

\[ J_1 A_1 = \sum_j \beta_j J_j A_j F_{j1} + \sum_j S_j A_j F_{j1}, \] (2.3)

Equation (2.3) is the approximate form of Eq. (2.1) used to obtain the flux distribution on the surfaces of the cavity. An equation of this type can be written for each sub-area of the cavity surface. Expressions were derived for the various view factors.
from which values could be obtained for the different sub-areas. A Monte Carlo code was written for the albedo; it had the following purposes:

(a) to obtain the average value of the albedo for each of the sub-areas on the cavity surface,
(b) to investigate the validity of expressing the albedo of a sub-area as a function of the current incident only on that sub-area,
(c) to obtain the angular distribution of the current re-entering the cavity,
(d) to study the effect of the angular distribution of the incident current.

The appropriate values of the $\beta_j$, $F_{ji}$ and $S_j$ were substituted into the set of equations (2.3), and the values of the $J_i$ were computed. Since there were often more than 100 equations in a set, the calculations were made on an IBM-709 computer. The value of the flux for each sub-area was then obtained from the appropriate current:

$$\phi_i = J_i (1+\beta_i) ,$$

and a flux map of each wall of the cavity was made. The sizes of the sub-areas were chosen to make the variation within each sub-area small enough so that the summation in Eqs. (2.2) and (2.3) is a good approximation to the integration.

The theoretical method was tested by calculating the flux distributions for the different cavity assemblies. Some typical results are shown in Fig. 6; experimental and theoretical values of the flux for 6-inch square sub-areas on the honeycomb of Assembly II are plotted. The circles represent the measured values of the flux and the horizontal lines represent the theoretical values; the curve is the flux distribution obtained by connecting the centers of the horizontal lines. The agreement between the theoretical and experimental results is good. Similar results were
The circles represent the experimental values, $\phi_{abs}$, and the lines represent the theoretical values, $\phi_{3(I,J)}$. 

- **6" off Centerline**
- **18" off Centerline**
- **30" off Centerline**
obtained for all the assemblies, the respective values differing by five per cent or less in nearly all cases. The fact that the theoretical method gave satisfactory results for a number of assemblies, some of which were quite dissimilar, indicates that the method should be useful for a wide variety of possible cavities.

3. Other Subcritical Assembly Features

The fuel rods may be loaded into tanks of different diameters, chosen so that a convenient axial relaxation length will be obtained with the predicted buckling for the assembly. The bottom of the tank is thin aluminum, but is supported by a honeycomb panel similar to the one used to support the graphite pedestal. The rods are located accurately by a thin grid plate at the bottom, and supported by double girders at the top. The girders can be rearranged to provide different spacings. A special center girder with a removable center grid plate allows just three rods to be removed for foil loading in the intracell flux plotting experiments.

The fuel girders are supported by the larger tank shown in Fig. 1. This tank is not changed with the lattices. The space between the inner and outer tanks may be used for reflected lattice experiments in the future. The outer tank is equipped with a rotating lid which in turn has a smaller eccentric rotating lid mounted on it. A port in the eccentric lid can be located over any part of the lattice, allowing fuel removal through a plastic glovebox and bag. When the whole lattice is to be removed for breakdown and reassembly, the tank is dried by a hot gas circulating system and the lids are then removed. In normal operation, the lids are sealed and a dry nitrogen blanket covers the free surfaces. These precautions have resulted in no noticeable degradation in isotopic purity over about two years of operation.

The facility is equipped with a moderator dump system, together with an ion exchanger, filling and circulating pump, and electric heaters. A signal arm type safety blade is included for the enriched lattices. The safety blade and the dump are activated
by a safety system consisting of two ion channels with level and period scrams, as well as by the usual manual scrams. Interlocks are provided, including entry interlocks on high radiation level in the room.

Further details of the facility may be found in Ref. 9.

B. THE SMALL SUBCRITICAL ASSEMBLY

In addition to the large subcritical facility, a small subcritical assembly has also proven to be useful, both for the exploration of the theory of such assemblies and for making certain measurements (10). A small subcritical assembly or "miniature lattice" is contained in a thin-walled aluminum tank in the form of a right circular cylinder, 21 inches high and 20 inches in diameter, with a removable base plate of 1/2-inch thick aluminum. The tank wall and top are made of 1/16-inch aluminum sheet. The tank is equipped with a wheeled stand, as well as lines and valves to allow filling or draining from a 3/4-inch line. Different lattices can be studied provided appropriate grid plates are available. In an experiment, the assembly is positioned under the thermal neutron beam port in the Medical Therapy Facility of the MITR. Fast and slow neutron shields are placed around it and the entire assembly, with the shielding, is lifted on a hoist to a position about three feet below the beam port. Figure 7 shows the assembly in the experimental position.

Owing to the small size and ease of use of the facility, a wide variety of experiments can be made at small cost. An irradiation time of two hours is adequate for many measurements with the reactor at a power level of 1.8 MW. At this power level, the flux is nearly flat across the top of the assembly, with a magnitude of about $6 \times 10^8$ neutrons/cm$^2$ sec at the center which fell slowly to about $4 \times 10^8$ neutrons/cm$^2$ sec at the boron carbide shield. The assembly is allowed to sit for about 90 minutes after irradiation to reduce the exposure during handling; it is then removed, the moderator drained, and foils counted.
C. PULSED-NEUTRON FACILITIES

Two pulsed-neutron sources are available to the lattice research project. The first is a 150 kev Cockcroft-Walton accelerator, made by the Texas Nuclear Corporation, which can be operated with either a titanium deuteride or a titanium tritide target. With the former target, about $5 \times 10^8$ neutrons per second are produced during a pulse, at an average energy of 2.5 Mev; with the tritium target, up to $10^{11}$ neutrons per second are produced, at an average energy of 14 Mev. The accelerator is equipped with a versatile beam deflection pulsing system. Pulse lengths from $10^{-8}$ sec to more than $10^{-3}$ sec can be selected, at repetition rates from 10 to $10^6$ pulses per sec. The accelerator can also be pulsed in a single shot, or repetitively at less than 10 pulses per sec if driven by an external pulse generator. The accelerator has been used with appropriate time analyzer to investigate slowing down in hydrogenous moderators (11), and the diffusion of thermal neutrons in $D_2O$ (12). Further work on thermalization is planned as well as studies of the reactivity of subcritical lattices with and without control rod materials.

Preparations are also being made for the use of a second pulsed-neutron source - a Kaman tube which uses a pulsed HV supply discussed in greater detail, together with other pulsed-neutron work, in Ref. 13.
A. THE MATERIAL BUCKLING

1. Methods

One of the main purposes of subcritical experiments is, of course, the determination of the material buckling of lattices from radial and axial thermal neutron flux traverses. A comprehensive program of experiments has been carried out to show that such measurements can be made with satisfactory precision. The details are given in Ref. 8, based on a doctoral thesis by P. F. Palmedo; a summary of the methods and results will be given here.

In the natural uranium lattices, axial and radial flux traverses were made with gold foils 0.010 inch thick and, in most cases, 1/4 inch in diameter. Gold was chosen as the foil material for several reasons: it is readily available in pure form and easily handled; there is only one stable gold nuclide, Au$^{197}$, to undergo activation; the thermal activation cross section of 96 barns is conveniently high so that the irradiation times can be short; the activity produced has a conveniently long half-life, 2.7 days, so that the foils need not be counted hastily after irradiation. The foils were punched from the same sheet of material, weighed, and placed in groups with each group containing foils with no more than one per cent variation in weight. The beta activity of the foils was counted with a Geiger-Muller gasflow counter used in connection with a Baird-Atomic sample changer. The analytical treatment of the experimental data was made efficient, complete, and rapid by the development of a set of codes for an IBM-709 computer.

The first set of experiments was performed with the exponential tank filled with D$_2$O, and were designed to serve three purposes:
(1) to examine the adequacy of the spatial distribution of the thermal neutron flux entering the exponential tank, that is, to see if the flux could be characterized by a single $J_0$ Bessel function over a considerable vertical distance in the tank;

(2) to compare and test experimental and analytical techniques;

(3) to investigate the epithermal component of the flux entering the tank.

The experiments gave the following results:

1. Above a height of 55 cm in the tank, the flux can be considered exponential in nature.

2. The radial flux distribution at three different heights, 99 cm, 113 cm, 131 cm, could be described by a single function of the form $J_0(\alpha r)$; higher harmonics contributed less than one percent to the radial flux.

3. Values of the gold cadmium ratio in the tank were in the range 1000 to 3000, indicating that the epicadmium component of the source, or background, would not be a serious problem.

The measured value of the radial buckling with the tank filled by $D_2O$ (99.8%) was found to be $14.70 \pm 0.19 \text{ m}^{-2}$. If it is assumed that the radius of the tank is, indeed, 24 inches, or 61.0 cm, and if the extrapolation distance is taken as $0.71 \lambda_{tr}$ with $\lambda_{tr} = 2.65 \text{ cm}$ for the $D_2O$ used, then the value of $a^2$ calculated from the expression:

$$a^2 = \left( \frac{2.405}{R + 0.71 \lambda_{tr}} \right)^2$$

would be 14.67 cm$^{-2}$, in good agreement with the measured value. The calculated value was not used in any actual determination of the material buckling because the measured radius of the tank was found to increase slightly (about 1/4 inch) with height near the top of the tank; only experimental values of the radial buckling were used.
The initial lattice experiments and their analysis had two general purposes: (1) to ensure that certain assumptions inherent in the method of determining the buckling are valid under the conditions of the measurements; (2) that satisfactory precision could be obtained in the measurements. Two fundamental assumptions are usually made: (1) that the macroscopic flux distribution is separable from the intracellular flux distribution; (2) that the spatial distribution is separable from the energy distribution.

The first assumption may be expressed by writing the radial flux distribution in the form:

\[ \phi(r) = A J_0(\alpha r)f_\infty(r), \]  

(3.2)

where \( f_\infty(r) \) represents the flux distribution in one cell of an infinite assembly of cells. In terms of the measurement of the buckling, this assumption takes the form operationally of assuming that it is only necessary to make measurements of the macroscopic flux distribution at equivalent points in lattice cells. Attempts to test the validity of the assumption in the past have given conflicting results. In one experiment (14), the effect of the macroscopic distribution on measurements of the intracellular flux distribution were studied, by measuring the latter in a cell at the center of the lattice and in a cell 6.7 inches from the tank wall in a 4-foot diameter tank. Each measured value of the flux was divided by the appropriate value of the \( J_0 \) term describing the macroscopic distribution. The resulting microscopic distribution measured in the outer cell was "only slightly distorted" from that measured in the central cell. On the other hand, recent studies (15) of the question of the separability of macroscopic and intracellular flux distributions have indicated detectable non-separability effects in graphite-moderated lattices.

The analysis of the experimental data was made with the aid of several computer codes. As an example, the basic code (\( J_0 \) code) for the radial flux distribution performs the following operations:
1. Experimental values of the flux at various radial positions are calculated from measured foil activities by correcting for (a) decay during and before counting, (b) the measured counter background, (c) the counter dead time measured as a function of count rate, and (d) the perpendicular distance of the chord along which the measurements are made from the lattice center.

2. The values of the corrected foil activities for each radial position are printed out for comparison and then averaged to obtain a flux at that point. A multiplicative correction factor may then be applied to the flux at any point to correct for foil weight, intracellular flux distribution, etc. The values of the flux are normalized, that nearest the center being set equal to 1.0.

3. The values of the flux are then fitted, according to the least squares criterion, to the theoretical distribution given by
   \[ \phi(r) = AJ_o[a(r-c)] \]  
   (3.3)
where the "best" values of \( A \), the normalization factor, and \( a^2 \), the radial buckling, are determined in the fitting process. Since the equation is not linear in \( a \), an iterational scheme must be used, with the value of \( c \) taken as an input datum.

4. The fitted and experimental values of the flux at each point are printed out, together with the fitted values of \( a^2 \), \( A \), and their probable errors.

5. The fitting process is repeated for all experimental points except the outermost until some preset number of points has been used in the fit.

6. The process is repeated for other values of \( c \). Although this code was designed primarily to obtain values of the radial buckling, \( a^2 \), from radial flux distributions, it can be used to examine edge effects (by refitting, with points dropped), gross distribution shifts (by refitting with different values of \( c \)), microscopic effects (by using periodic correction factors), and so on.

A second code was designed to analyze a measured radial flux distribution for the presence of higher harmonics. The
experimental flux is fitted by least squares to the equation:

\[ \phi(r) = A_1 J_0(a_1 r) + A_2 J_0(a_2 r) + A_3 J_0(a_3 R), \]

where

\[ a_1 = \frac{2.4048}{R'}, \quad a_2 = \frac{5.5201}{R'}, \quad a_3 = \frac{8.654}{R'}, \]

and

\[ R' = \text{the extrapolated radius}. \]

In this case, \( R' \) is an input datum and can be estimated by using the \( J_0 \) code. The harmonic code solves for the "best" values of \( A_1, A_2, \) and \( A_3 \), and prints them out, along with values of the experimental and fitted values of the flux.

A third code used to analyze radial flux distributions was designed to look for possible reflector (room) effects. The outer surface of the tank was lined with a sheet of cadmium, 0.020 inch thick, which could prevent room-reflection of thermal neutrons. It would not absorb fast neutrons, and the possibility that fast neutrons might escape from the lattice, be reflected back into it, and then slowed down might cause some perturbation of the \( J_0 \) distribution. The measured flux is fitted to the formula:

\[ \phi(r) = A_1 J_0(a_1 r) + A_2 J_0(a_2 r), \quad (3.4) \]

where \( A_1, A_2, a_1, a_2 \) are determined in the iterative, least-squares fitting processes.

Care was taken in the use of the above codes to analyze a flux distribution that might involve both harmonic and reflector effects, since each code takes into account one effect, under the assumption that the other effect is absent. First, the codes were applied to radial traverses made at different heights in the lattice. The contributions of the higher harmonics should decrease with height while the reflector effect should not vary much. Thus, the reflector effect, as determined by the code, should (it is hoped) approach some constant value. Second, the codes were applied to fewer points by dropping points successively from the outer end of the distribution.
The harmonic contribution, as determined by the code, should vary as outer points are dropped only if a reflector effect is involved. As fewer points are used, the contribution of the reflector effect should decrease and the harmonic contribution, as determined by the code, should approach a constant value.

Analogous codes were also used for the analysis of the axial flux distribution; they could be used together with the codes for the radial distribution for other tests of harmonic and reflector effects.

While the set of computer codes was used to test the spatial "purity" of the fitted flux distributions, the assumption of energy-space separability was tested in two other ways. The bare gold foils used in the experiments have a relatively high response to epithermal neutrons, particularly at the 4.9 ev resonance. If the neutron energy spectrum shifts appreciably near the edge of the lattice, different values of the measured radial buckling should be obtained as fewer points are used in the fit. A second, and more sensitive, test can be made by measuring the radial distribution with cadmium-covered foils. If separability holds, the value of $a^2$ determined by fitting the data from the cadmium-covered foils should be the same as that obtained from the data obtained with bare foils. This effect can also be studied by examining the cadmium ratio as a function of radial position, with a region of constant cadmium ratio taken to indicate a region of energy-space separability. The difficulty with this last method is that the cadmium ratio is, at best, a crude index of the neutron energy spectrum, and the cadmium ratio may be constant even though the energy spectrum may change significantly. Indications of this last effect have recently been found in other research at M.I.T. (16).

2. Results of the Buckling Measurements

The experiments for which results will be reported were made on cylindrical fuel rods of natural uranium metal, 1.010 ± 0.005 inches in diameter and 60 inches long, canned in Type 1100 aluminum tubes,
0.028 inch thick. Three lattices were studied, in which the rods were arranged in triangular arrays with spacings of 4.5, 5.0, and 5.75 inches, respectively; the ratio of the volume of moderator to the volume of uranium had the values 21.0, 26.4, and 35.6, respectively.

A typical example of a radial flux traverse is shown in Fig. 8 for the lattice with the 4.5-inch spacing. The experimental points are well represented by the fitted $J_0$ distribution, and no reflector effect can be seen. A more sensitive test for a reflector effect can be made by examining the residuals, that is, the values of the experimental flux minus the fitted flux, as a function of radial or chordal position; if there is a reflector effect, the residuals should tend to become more positive near the outside of the lattice. A typical graph of the residuals, taken from a traverse in the lattice with the 4.5-inch spacing, is shown in Fig. 9. Points from both sides of the center are included; the outermost points recorded were obtained in the outer cell of the lattice. The random distribution of the residuals attests to the adequacy of a $J_0$ distribution for characterizing the radial flux.

Values of the cadmium ratio, defined as the ratio of the activity of a bare gold foil to that of a cadmium-covered gold foil are plotted in Fig. 10. The average value for 0.010-inch thick gold foils is 10.5 with a standard deviation of 0.10. The value at the outermost point, 10.8, is three standard deviations from the average, which may indicate a significant shift. The effect, if any, is small, since a 3 per cent shift in the cadmium ratio corresponds to a change in the epicadmium flux of only about 0.3 per cent.

The macroscopic radial flux distribution was corrected for the intracellular flux variation when more than one foil was exposed in a single cell. The effect of the correction was to leave the value of the radial buckling unchanged, but to reduce the probable error and the magnitudes of the residuals. Some typical results are shown in Table 3.1, together with the effects of some other variations in the analysis. Different weightings were used in the fitting process
1.0175 \frac{J_0(2.4048/63.96)}{r}

(FITTED DISTRIBUTION)

EXPERIMENTAL POINTS, STANDARD DEVIATIONS INDICATED BY SIZE OF CIRCLES

CENTER OF OUTER CELL OF LATTICE

r, RADIAL DISTANCE FROM CENTER, cm
RELATIVE POSITIONS OF FUEL RODS

DIRECTION OF TRAVERSE

CENTRAL ROD

STANDARD DEVIATIONS INDICATED ARE ESTIMATES BASED ON COUNTING STATISTICS ETC.

Distance along traverse chord, cm

Rcd

11.0

10.5

10.0

10.0 20 30 40 50 60

10 20 30 40 50 60

10.0 11.0
TABLE 3.1
Fitted Radial Buckling for the 5.75-Inch Lattice
Under Various Analytical Conditions

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Corrected for Intracell Flux Distribution?</th>
<th>Weighting Exponent</th>
<th>Number of Points Used in Fit</th>
<th>Radial Buckling $a^2 (m^{-2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4101 a</td>
<td>No</td>
<td>0</td>
<td>23</td>
<td>$14.29 \pm 0.23$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td></td>
<td>21</td>
<td>$14.31 \pm 0.25$</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>19</td>
<td>$14.34 \pm 0.28$</td>
</tr>
<tr>
<td>4102 a</td>
<td>No</td>
<td>$-2$</td>
<td>23</td>
<td>$14.22 \pm 0.18$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td></td>
<td>21</td>
<td>$14.32 \pm 0.21$</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>19</td>
<td>$14.48 \pm 0.25$</td>
</tr>
<tr>
<td>4103 a</td>
<td>Yes</td>
<td>0</td>
<td>23</td>
<td>$14.30 \pm 0.06$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td></td>
<td>21</td>
<td>$14.31 \pm 0.06$</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>19</td>
<td>$14.30 \pm 0.06$</td>
</tr>
<tr>
<td>4104 a</td>
<td>Yes</td>
<td>$-2$</td>
<td>23</td>
<td>$14.29 \pm 0.09$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td></td>
<td>21</td>
<td>$14.34 \pm 0.08$</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td></td>
<td>19</td>
<td>$14.31 \pm 0.08$</td>
</tr>
</tbody>
</table>
in another test of the \( J_\circ \) fit. With all points having unit weighting, the points near the center of the lattice are accentuated; if a weighting inversely proportional to the square of the flux is used, the outer points are weighted much more heavily. If the flux distribution differs from the \( J_\circ \) function, the value of the radial buckling derived in the two cases should differ. It is evident from the results that the weighting had no significant effect.

The radial buckling was also found to be independent of the axial height at which the measurements were made, and of the azimuth.

To test the assumption of the separability of macroscopic and intracellular flux distributions, the following experiment was performed. In the lattices with 5-inch and 5.75-inch spacings, flux traverses were made near rows of fuel rods to obtain both distributions. The residuals shown in Fig. 11 are typical of the results obtained. To study the separability, the experimental points were corrected for the fitted \( J_\circ \) distribution. Then, for each cell, a quantity \( \Delta \), related to the difference between the flux near the fuel rods and the flux in the moderator, was determined,

\[
\Delta = \frac{\delta_f - \bar{\delta}_m}{\phi_0}.
\] (3.5)

The values of \( \delta \) are given in Fig. 11, where \( \delta_f \) is the residual (difference between the experimental and fitted \( J_\circ \) flux values) at a point near a fuel rod, and \( \bar{\delta}_m \) is the average of the residual at adjacent points in the moderator; \( \phi_0 \) is the flux at the center of the lattice and acts as a normalization factor. If the intracellular flux distribution is not affected by the macroscopic lattice distribution, \( \Delta \) should be a constant. If the macroscopic and intracellular distributions are not separable, \( \Delta \) should vary as a function of position. Figure 12 is a graph of \( \Delta \) as a function of radial position for the lattice with the 5.75-inch spacing. Each rod represents an average of six measurements made along the six radial directions determined by the first ring of six rods around the central rod of the
RELATIVE POSITIONS OF FUEL RODS

POINTS NOT CORRECTED FOR MICROSCOPIC DISTRIBUTION

POINTS CORRECTED

DISTANCE ALONG EXPERIMENTAL CHORD, cm
DISTANCE FROM LATTICE CENTER IN LATTICE UNITS
lattice. It is evident that there is a small deviation from exact separability: in terms of the macroscopic distribution, there is a variation of about 2 per cent in the intracellular distribution between the central and the outer cell. The radial variation of the cadmium ratio for the same lattice is shown in Fig. 13, and differs from that for the lattice with 4.5-inch spacing as can be seen by comparison with Fig. 10.

**TABLE 3.2**

Measured Radial Buckling of the Lattice with 4.5-Inch Spacing

<table>
<thead>
<tr>
<th>Run Number</th>
<th>Radial Buckling ( (a^2 : m^{-2}) )</th>
<th>Detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>14.14</td>
<td>1/4-inch diameter, bare, gold foils</td>
</tr>
<tr>
<td>23</td>
<td>14.04</td>
<td>1/2-inch diameter, bare, gold foils</td>
</tr>
<tr>
<td>25</td>
<td>14.23</td>
<td>1/2-inch, cadmium-covered, gold foils</td>
</tr>
<tr>
<td>26</td>
<td>14.16</td>
<td>1/4-inch, bare, gold foils</td>
</tr>
<tr>
<td>50</td>
<td>14.10</td>
<td>cadmium-covered, dilute, uranium foils</td>
</tr>
</tbody>
</table>

Average (of runs 20, 23, 26) = 14.11 ± 0.06 m\(^{-2}\)

The results of a test of the space-energy separability of the radial flux distribution are given in Table 3.2. The bare and cadmium-covered gold foil, and the uranium foils gave the same results. As mentioned earlier, this test is, at best, a very rough one. In another study (17), axial and radial flux traverses were made in the 5.75-inch lattice with lutetium foils. The absorption cross section of lutetium at low energies is strongly non-1/v, but the macroscopic flux traverses obtained with lutetium agreed very well with those obtained with gold and gave the same values of the bucklings. Further studies of the possible dependence of the buckling on neutron energy are being made at M.I.T.
The axial flux distribution was examined to determine the region of the exponential tank in which the formula,

$$\phi(z) = A \sinh \gamma(h' - z),$$

(3.6)
is valid. Measurements with bare and cadmium-covered gold foils, and with cadmium-covered foils of uranium-aluminum alloy showed that the three sets of measurements gave the same axial distribution between 40 cm and 120 cm from the bottom of the tank; the value of the cadmium ratio was constant in the region. The procedure of refitting the data points of a traverse with successively fewer points was also used, with the result that the fitted value of $\gamma^2$ quickly approached a constant value as the topmost points were dropped. The "best" value of the extrapolated height was also determined, for example, by varying $h'$ and calculating the residuals; the value of $h'$ that resulted in the most nearly random distribution of residuals was chosen. In all cases, the same values of the axial buckling (and also of the radial buckling) were obtained with bare and cadmium-covered gold foils, indicating that, at least with this rough spectral index, the buckling is independent of energy.

The values obtained for the radial, axial, and material bucklings of the three lattices studied are listed in Table 3.3.

**TABLE 3.3**

Measured Values of the Material Buckling for Three Lattices of 1.010-Inch Diameter Natural Uranium Rods in 99.75 Per Cent $D_2O$

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>Volume Ratio ($V_{D_2O}/V_u$)</th>
<th>Radial Buckling ($a^2: m^{-2}$)</th>
<th>Axial Buckling ($\gamma^2: m^{-2}$)</th>
<th>Material Buckling ($B^2: m^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>21.0</td>
<td>14.11 ± 0.06</td>
<td>5.63 ± 0.08</td>
<td>8.48 ± 0.10</td>
</tr>
<tr>
<td>5.0</td>
<td>26.4</td>
<td>14.12 ± 0.06</td>
<td>5.47 ± 0.08</td>
<td>8.65 ± 0.10</td>
</tr>
<tr>
<td>5.75</td>
<td>35.6</td>
<td>14.20 ± 0.05</td>
<td>6.05 ± 0.05</td>
<td>8.15 ± 0.08</td>
</tr>
</tbody>
</table>
Each value represents an average of the results of three to five separate measurements; thus, the best value obtained in Run 41 (cf. Table 3.1) was one of five which resulted in the value, 14.20 ± 0.05 m\(^{-2}\), given in Table 3.2 for the radial buckling for the 5.75-inch lattice. The values of the probable errors listed are considered to be conservative and may be overestimates. The probable errors calculated from the standard deviations of the average values of \(a^2\) and \(\gamma^2\) used to obtain \(B_m^2\) would be 0.05 m\(^{-2}\) and 0.03 m\(^{-2}\) for the 5.0 and 5.75-inch lattices, respectively. The smaller value in the latter case is due to the fact that the methods were being improved during the course of the experiments.

The values obtained for the material buckling are plotted in Fig. 14 together with values obtained at other laboratories in recent years. The three new points fall satisfactorily on the "international" curve, indicating that the experimental methods should be trustworthy when applied to lattices with enriched uranium rods.

B. INTRACELLULAR FLUX TRAVERSES AND THE THERMAL UTILIZATION

Intracellular flux distributions were measured with bare and cadmium-covered gold, lutetium, and europium foils in the lattices discussed in the section on the material buckling. Gold was used, as a 1/\(v\)-absorber, to determine the relative neutron density; \(^{176}\text{Lu}\) has a neutron capture resonance at 0.142 ev so that its activity depends strongly on the energy spectrum of the neutrons; \(^{151}\text{Eu}\) has a thermal neutron absorption cross section varying nearly as 1/\(v^2\) and a strong resonance at 0.46 ev. The use of the three types of detector made possible integral measurements of the variation of the neutron energy spectrum with position within a cell. For example, effective absorptions cross sections and effective neutron temperatures can be derived by using Westcott's method. The experimental distributions were also used
to derive values of disadvantage factors and the thermal utilization which could be compared with theoretical results. The details of the work are given in Ref. 17, based on a doctoral thesis by P. S. Brown; a brief summary will be given here.

Intracellular flux traverses were made in the central cell inside a split fuel rod, and on thin aluminum foil holders strapped to the rod and extending outward into the moderator, toward the next adjacent rod (rod-to-rod traverse) as well as toward a point midway between the next two adjacent rods (rod-to-moderator traverse). Aluminum was used rather than plastic because an early experiment showed that plastic holders depressed the flux by as much as 3 per cent but aluminum by only about 0.5 per cent. The foils in the moderator were corrected for this flux depression by the aluminum. The foils in the fuel were arranged in a spiral pattern to provide maximum spacing between foils and minimum flux perturbation (calculated to be negligible). Activation measurements were made with bare foils and foils covered with 0.023-inch thick cadmium. The cadmium-covered foils were irradiated at the same time as the bare foils, but at different heights in the exponential tank to avoid perturbation of the thermal flux by the cadmium. The measured macroscopic axial flux distribution was used to correct for the differences in height. All foils were 1/8 inch in diameter. The gold foils were about 2 mils thick, and were weighed to within a fraction of a per cent. The lutetium and europium foils were very dilute and only a fraction of a mean free path thick. They were prepared by spraying glyptal suspensions of the oxides on to 0.005-inch thick aluminum backing and were calibrated accurately in a uniform thermal neutron flux on a rotating foil wheel.

The activity distributions for europium gave rise to a problem because of the presence of the europium resonance at 0.46 ev which is practically at the cadmium cutoff (usually taken to be between 0.4 ev and 0.5 ev). When the activity of the cadmium-covered europium foils is subtracted from that of the bare foils, there is
some question as to where the thermal energy cutoff should be taken, because of the absorption by cadmium of some of the neutrons with energies in the range of the europium resonance. If the 0.023-inch thick cadmium covers could be replaced by "infinitely thin" covers so that they would not absorb those europium resonance neutrons, but would still cut out all thermal neutrons below the 0.46 ev resonance, the problem would disappear. A procedure was developed to correct for the effect; it involved the study of the activity induced in cadmium-covered europium foils as a function of the thickness of the cadmium filter. The procedure is described in detail in Ref. 17, and is outlined briefly in Ref. 18; it will, therefore, not be given here.

After exposure, the foils were counted by beta- or gamma-counting, the activities were corrected for background, activity decay, foil weight or intercalibration, counter dead time, foil holder flux perturbations, the macroscopic $J_0$ flux distribution in the exponential tank, and the height at which the foils were exposed in the tank.

The measured activity distributions were compared with those computed with the THERMOS code (19). Estimates of the effective neutron temperature were made by using THERMOS as well as Westcott's effective cross sections (20). Values of the disadvantages factors and the thermal utilization were computed with THERMOS and by the method of Amouyal, Benoist, and Horowitz (21).

Measured activity distributions were normalized in the neighborhood of the cell boundary to those calculated with THERMOS. Two sets of measurements were made with each detector at each of the three lattice spacings; the results were found to agree to within about one per cent. The cell edge was chosen for the normalization because that is where the energy distribution of the thermal neutrons is closest to a Maxwellian distribution. Some typical results are shown in Figs. 15, 16, and 17; these are for the lattice with 4.5-inch spacing. In all
THERMOS COMPUTED CURVES - SUBCADMIUM

RELATIVE ACTIVITY

0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

RADIUS - INCHES

0
1.0
2.0
3.0
4.0

EPICADMIUM ACTIVITIES

- - Au ROD TO ROD
- - Au ROD TO MODERATOR
- - Lu ROD TO ROD
- - Lu ROD TO MODERATOR
THERMOS CURVE CALCULATED FOR 0.194 ev CUTOFF

EUROPIUM ACTIVITY NO. 2a
NATU - \( \text{D}_2\text{O} \)
1.010" RODS
4.5" TRIANGULAR PITCH
1.5% COUNTING STATISTICS

DATA POINTS IN THE UPPER CURVE ARE (BARE ACTIVITY) - (2.246)(CADMIUM COVERED ACTIVITY); WHERE 2.246 IS A FACTOR TO CORRECT FOR ABSORPTION OF RESONANCE NEUTRONS IN THE CADMIUM

CADMIUM COVERED ACTIVITY (70% COUNTING STATISTICS)
THERMOS CURVE CALCULATED FOR 0.632 ev CUTOFF

EUROPIUM ACTIVITY NO 2b
NATU - D₂O
1.010" RODS
4.5" TRIANGULAR PITCH
1.5% COUNTING STATISTICS

DATA POINTS IN THE UPPER CURVE ARE (BARE ACTIVITY)-(0.439)(CADMIUM COVERED ACTIVITY); WHERE 0.439 IS A CORRECTION FACTOR TO ELIMINATE THE CADMIUM COVERED Eu ACTIVITY CONTRIBUTED BY THE 0.46 ev RESONANCE

CADMIUM COVERED ACTIVITY (7.0% COUNTING STATISTICS)
three lattices studied, the shape of the experimental activity distribution in the moderator agreed very well with the shapes calculated with THERMOS. Any differences between theory and experiment should then appear in the flux distribution in the fuel. The gold distributions in the fuel showed the best agreement between theory and experiment. The experimental value for the subcadmium activity at the center of the fuel was less than one per cent higher than that given by THERMOS. Since gold is very nearly a $1/v$-absorber, these results indicate excellent prediction, by THERMOS, of the distribution of the thermal neutron density.

The results for lutetium were also in good agreement with those predicted by THERMOS. In all cases, the experimental results at the cell center lay just a few per cent higher than the theoretical results, with differences of 5 per cent for the 4.5-inch lattice, and of 2.5 per cent for the 5-inch and 5.75-inch lattices. The agreement, although not quite as good as that found for gold, is still good. When the gold and lutetium plots are compared, as in Fig. 15, it appears that the change in the ratio of lutetium activity to gold activity, with position in the cell, is predicted well by THERMOS. The change in this ratio is an indication of the degree of spectral hardening in the cell. The measured changes, from cell edge to rod center, in the activity ratio were, for the 4.5-inch, 5.0-inch, and 5.75-inch lattices: 26.2 per cent, 22.5 per cent, and 22.9 per cent, respectively. These values compare favorably with those computed with THERMOS: 21.4 per cent, 21.6 per cent, and 21.7 per cent, respectively.

For europium, two graphs are shown, corresponding to two ways of correcting for the effect of the cadmium covers, that is, by using two values of the cadmium cutoff energy, one below and one above the 0.46 ev resonance in europium. The experimental activity at the center of the fuel was higher than that predicted by THERMOS by about 4 per cent.

A detailed discussion of the estimation of effective neutron temperatures is given in Ref. 17.
The thermal neutron density disadvantage factors for the three lattices studied are compared with those computed with THERMOS in Table 3.4. The experimental values were obtained by graphical integration of the gold activity distributions over the volume of each cell region. The experimental and theoretical disadvantage factors agree well.

The method of Amouyal, Benoist, and Horowitz (ABH) was used to calculate the values of $\phi_a / \bar{\phi}_f$ (the ratio of the flux at the surface of the fuel to the average flux in the fuel), and the values of $\bar{\phi}_m / \bar{\phi}_f$ (the ratio of the average flux in the moderator to the average flux in the fuel). The effects of spectral hardening in the fuel were taken into account by using the average values of the cross sections computed with the THERMOS code for each region of the cell. It is difficult to compare the calculations made with the ABH method directly with experimental results because measurements with $1/v$-absorbers give neutron densities while the ABH calculations involve neutron fluxes. An experimentally measured neutron density must be multiplied by an appropriate average speed to convert it to a flux. To do so, it is necessary to know the degree of spectral hardening, which causes the average velocity to vary significantly from the moderator to the fuel.

### Table 3.4

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>$\bar{N}<em>{\text{mod}} / \bar{N}</em>{\text{fuel}}$ Experiment</th>
<th>THERMOS</th>
<th>$\bar{N}<em>{\text{clad}} / \bar{N}</em>{\text{fuel}}$ Experiment</th>
<th>THERMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>1.720 ± 0.017</td>
<td>1.734</td>
<td>1.254 ± 0.015</td>
<td>1.266</td>
</tr>
<tr>
<td>5.0</td>
<td>1.721 ± 0.022</td>
<td>1.760</td>
<td>1.248 ± 0.019</td>
<td>1.268</td>
</tr>
<tr>
<td>5.75</td>
<td>1.796 ± 0.012</td>
<td>1.795</td>
<td>1.247 ± 0.011</td>
<td>1.271</td>
</tr>
</tbody>
</table>
Since the neutron density distributions measured with gold agreed well with those computed with THERMOS, the flux disadvantage factor computed with THERMOS will be compared with those calculated with the ABH method. This procedure is equivalent to comparing the ABH disadvantage factors with those obtained from the experimental neutron densities transformed into fluxes by multiplication with the appropriate values of \( \bar{v} \). The results are listed in Table 3.5; it is evident that the disadvantage factors calculated by means of the two methods agree well.

**TABLE 3.5**

Values of Flux Disadvantage Factors in Lattices of 1.0-Inch Diameter Natural Uranium Rods in Heavy Water

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>( \phi_a/\bar{\phi}_f ) ABH</th>
<th>( \phi_a/\bar{\phi}_f ) THERMOS</th>
<th>( \bar{\phi}_m/\bar{\phi}_f ) ABH</th>
<th>( \bar{\phi}_m/\bar{\phi}_f ) THERMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>1.146</td>
<td>1.548</td>
<td>1.546</td>
<td>1.570</td>
</tr>
<tr>
<td>5.0</td>
<td>1.149</td>
<td>1.160</td>
<td>1.580</td>
<td>1.594</td>
</tr>
<tr>
<td>5.75</td>
<td>1.152</td>
<td>1.163</td>
<td>1.623</td>
<td>1.625</td>
</tr>
</tbody>
</table>

Values of the thermal utilization were calculated from the experimentally determined neutron density disadvantage factors and with the THERMOS code. It was not necessary to take spectral hardening into account explicitly since all neutron cross sections, except that of \( U^{235} \), vary as \( 1/\bar{v} \), and the absorption cross section of \( U^{235} \) varies nearly as \( 1/\bar{v} \). Thus, calculations with THERMOS showed that the product \( \Sigma_a\bar{v} \) changes by less than one per cent from the moderator to the fuel. All the cross sections must, however, be evaluated at the same effective temperature. Equation 3.6 was, therefore, used to calculate the experimental values of the thermal utilization, with the values of \( \Sigma_a \), the macroscopic absorption cross sections, taken at 0.025 ev.
\[
\frac{1}{\bar{f}} = 1 + \frac{(\Sigma a V N)_{\text{mod}}}{(\Sigma a V N)_{\text{fuel}}} + \frac{(\Sigma a V N)_{\text{clad}}}{(\Sigma a V N)_{\text{fuel}}}. 
\]

Values were also calculated with THERMOS, and by means of the ABH method with THERMOS-hardened cross sections. The results are listed in Table 3.6. The experimental and theoretical values are in excellent agreement. The major contribution to the uncertainties in both sets of values is an uncertainty of 22 per cent in the absorption cross section of the heavy water which had a purity of 93.77 per cent D\textsubscript{2}O by weight. The uncertainties in f are small in absolute magnitude because the values of f are close to unity; the uncertainties are about 4 per cent in 1-f.

**TABLE 3.6**

Values of the Thermal Utilization for Lattices of 1.0-Inch Natural Uranium Rods in Heavy Water (99.77 Per Cent D\textsubscript{2}O by Weight)

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>Experimental</th>
<th>THERMOS</th>
<th>ABH</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>0.9855 ± 0.0006</td>
<td>0.9849 ± 0.0006</td>
<td>0.9851 ± 0.0006</td>
</tr>
<tr>
<td>5.0</td>
<td>0.9836 ± 0.0007</td>
<td>0.9828 ± 0.0007</td>
<td>0.9830 ± 0.0007</td>
</tr>
<tr>
<td>5.75</td>
<td>0.9799 ± 0.0010</td>
<td>0.9792 ± 0.0010</td>
<td>0.9793 ± 0.0010</td>
</tr>
</tbody>
</table>

C. MEASUREMENTS OF NEUTRON CAPTURE IN U\textsuperscript{238}: Cadmium Ratio, Conversion Ratio, Resonance Escape Probability

The investigation of neutron capture in U\textsuperscript{238} included the following experiments which are reported in Ref. 22, based on a doctoral thesis by A. Weitzberg:

1. Measurement of the average U\textsuperscript{238} cadmium ratio of the fuel rods;
2. Measurement of the average ratio of captures in U\textsuperscript{238} to fissions of U\textsuperscript{235} in the fuel rod;
(3) Measurement of the distribution of resonance neutrons in the moderator (both macroscopic and intracellular);

(4) Measurement of quantities directly related to the effective resonance integral of the rods.

The $^{238}\text{U}$ cadmium ratio, $R_{28}$, was measured by irradiating two identical 0.005-inch thick uranium foils, depleted to 15 parts of $^{235}\text{U}$ per million, in two fuel rods at equivalent lattice positions at a height of two feet above the bottom of the exponential tank. One foil was placed between two fuel slugs with thin (0.001-inch thick) aluminum catcher foils on either side to prevent the pickup of fission products from the surrounding natural uranium. The second foil was placed in a pill box of 0.020-inch thick cadmium, composed of a sleeve 0.25 inch long imbedded in the aluminum cladding and two 1.010-inch diameter discs placed on either side of the foil. Two buttons of natural uranium 0.050-inch thick were included within the cadmium covers to minimize the effect of the streaming of neutrons from the moderator through the cadmium. After a four-hour irradiation and a cooling period of four to eight hours, the two foils were counted alternately by means of a gamma-gamma coincidence counting described in detail in Appendix A of Ref. 22. To increase the amount of data obtained, both singles channels were used and the counts in each summed. The spectrometers were set to straddle the 103 kev peak of $^{239}\text{Np}$ with window widths of 30 kev. The foils were counted over a period of several days to verify that the observed radiation decayed with the proper half-life. The ratio of the two activities was taken for each pair of counts and these were then averaged to obtain the final result. Each pair of foils was matched in weight to within 0.5 per cent to obviate the need to correct for the attenuation of the gamma rays in the foils.

The experimental conversion ratio, $C^*$, defined as the ratio of the number of captures in $^{238}\text{U}$ to the number of fissions of $^{235}\text{U}$, was measured at the same positions at which the cadmium ratio measurements were made. Two foils, a 0.005-inch thick depleted foil and a 0.005-inch thick foil of uranium-aluminum alloy,
9.8 per cent by weight, of uranium enriched to 93.17 atom per cent \( {\text{U}}^{235} \), were exposed, back-to-back. Aluminum catcher foils were used to separate the foils from each other and from the surrounding rod. The foils were irradiated for four hours at a flux of about \( 5 \times 10^8 \) neutrons/cm\(^2\) and, after a wait of four to eight hours, they were counted alternately several times a day for about a week in the gamma-spectrometer system. For the depleted foil, the \( \gamma \)-rays of the 103 kev peak of \( \text{Np}^{239} \) were counted with a window width of 30 kev; for the \( \text{U}^{235} \) foil, the 143 kev fission product peak was counted. The spectrometer was calibrated by using three \( \gamma \)-ray peaks: the 103 kev peak of \( \text{Np}^{239} \) and the 70 and 280 kev peaks of \( \text{Hg}^{203} \).

The ratio of the 143 kev fission product activity to the 103 kev \( \text{Np}^{239} \) activity, \( R_L \), was calculated for each pair of counts. To normalize the relative fission product and \( \text{Np}^{239} \) activities of the test foils to a known standard, similar foils were irradiated back-to-back in a well thermalized flux and the corresponding ratio, \( R_S \), was determined. The exponential tank containing \( \text{D}_2\text{O} \), but no uranium rods, was used; the gold cadmium ratio was 2000 \( \pm \) 100. The 103 kev \( \text{Np}^{239} \) and 145 kev fission product activities were measured, at various times, in the well thermalized flux, and the ratio determined as a function of time. A power series in time was fitted to the data. With this expression, the value of \( R_S \) for the time after irradiation corresponding to each \( R_L \) data point was calculated and the ratio, \( R_S/R_L \), was determined. The values were averaged to obtain the final result, \( (R_S/R_L)_{\text{avg}} \). The reaction rates for \( \text{U}^{235} \) and \( \text{U}^{238} \) corresponding to the standard curves were then calculated from the known atom concentrations and thermal cross sections; these were used to convert the measured value of \( (R_S/R_L)_{\text{avg}} \) to the conversion ratio.

The results of the measurements of the cadmium ratio and the conversion ratio are shown in Table 3.7.

The initial conversion ratio, \( C \), for a lattice is usually defined as the ratio of the capture rate in \( \text{U}^{238} \) to the absorption rate in \( \text{U}^{235} \).
It can be obtained from $C^*$, the measured conversion ratio, or independently from $R_{28}$, the measured $U^{238}$ cadmium ratio. Thus,

$$C = \frac{C^*}{(1+\bar{a})},$$

where $\bar{a}$ is the average value of the ratio of capture to fission in the fuel. The conversion ratio $C$ is also related to the cadmium ratio by the equation,

$$C = \left(\frac{1+\rho_{28}}{1+\rho_{25}}\right) \left(\frac{\Sigma_{28}^a}{\Sigma_{25}^a}\right),$$

where $\rho_{28} = \frac{1}{R_{28}^\alpha - 1}$, $\rho_{25}$ is the ratio of epicadmium to subcadmium absorption in $U^{235}$, and the cross section ratio is that of the average values below the cadmium cutoff. The values of $\bar{a}$ and the cross section ratio were obtained by means of the THERMOS code and were found to be:

$$1 + \bar{a} = 1.174 \pm 0.010,$$

$$\frac{\Sigma_{28}^a}{\Sigma_{25}^a} = 0.574 \pm 0.006.$$ 

The values of $\delta_{25}$ were obtained independently from fission measurements on $U^{235}$ foils.

The results of the measurements are listed in Table 3.8. The results obtained for the conversion ratio by means of the two independent methods agree reasonably well. A small discrepancy, slightly greater than the estimated experimental uncertainty, appears for the tighter lattices. It was considered too small to warrant further studies in these lattices, but investigations of the discrepancy are being continued in the lattices of partially enriched uranium rods in heavy water. Recent work at the Brookhaven National Laboratory (23, 24) indicates the possible existence of certain systematic errors in the counting technique used in the determination of the cadmium
### TABLE 3.7
Values of the Cadmium Ratio and Conversion Ratio for Lattices of 1.0-Inch Diameter Natural Uranium Rods in Heavy Water

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>Ratio of Moderator Volume to Fuel Volume</th>
<th>$U^{238}$ Cadmium Ratio ($R_{28}$)</th>
<th>Ratio of Epithermal $U^{238}$ Capture to Thermal $U^{238}$ Capture ($\rho_{28} = \frac{1}{R_{28} - 1}$)</th>
<th>Conversion Ratio $C^* = \frac{U^{238} \text{Capture}}{U^{235} \text{Fission}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>21.0</td>
<td>$2.972 \pm 0.031$</td>
<td>$0.507 \pm 0.008$</td>
<td>$1.017 \pm 0.023$</td>
</tr>
<tr>
<td>5.0</td>
<td>26.4</td>
<td>$3.495 \pm 0.010$</td>
<td>$0.401 \pm 0.002$</td>
<td>$0.948 \pm 0.020$</td>
</tr>
<tr>
<td>5.75</td>
<td>35.6</td>
<td>$4.223 \pm 0.043$</td>
<td>$0.310 \pm 0.004$</td>
<td>$0.859 \pm 0.016$</td>
</tr>
</tbody>
</table>

### TABLE 3.8
Values of the Initial Conversion Ratio for Lattices of 1.0-Inch Diameter Uranium Rods in Heavy Water

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>Ratio of Moderator Volume to Uranium Volume</th>
<th>Ratio of Epicadmium Absorption to Subcadmium Absorption in $U^{235}$ ($\rho_{25}$)</th>
<th>From Measured Value of $C^*$, $\frac{U^{238} \text{Absorption}}{U^{235} \text{Fission}}$</th>
<th>From $U^{238}$ Cadmium Ratio, $R_{28}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>21.0</td>
<td>$0.0479 \pm 0.0019$</td>
<td>$0.866 \pm 0.021$</td>
<td>$0.826 \pm 0.010$</td>
</tr>
<tr>
<td>5.0</td>
<td>26.4</td>
<td>$0.0340 \pm 0.0030$</td>
<td>$0.808 \pm 0.018$</td>
<td>$0.775 \pm 0.009$</td>
</tr>
<tr>
<td>5.75</td>
<td>35.6</td>
<td>$0.0268 \pm 0.0004$</td>
<td>$0.732 \pm 0.018$</td>
<td>$0.733 \pm 0.008$</td>
</tr>
</tbody>
</table>
ratio, and this has encouraged further studies of this method at M.I.T.

The values of the cadmium ratio have been combined with results of other measurements and some calculated correction factors to obtain values of the resonance escape probability. The latter are based on a straightforward analysis of the neutron cycle (25) rather than any particular theoretical model of resonance absorption. The results are given and discussed in Ref. 22, together with the experiments on effective resonance integrals.

D. MEASUREMENTS OF FAST FISSION

Measurements have been made of quantities related to the fast fission factor, ε, which appears in the four-factor formula for $k_\infty$. Research at M.I.T. on fast fission has been concerned mainly with the measurement of $\delta_{28}$, and has included work in the following areas:

1. The development of a new method for measuring $\delta_{28}$, which involves the determination of the ratio of the 1.60 Mev γ-ray activity of La$^{140}$ in uranium foils of different U$^{235}$ concentration; this method is described in another paper in these proceedings (26), and will not be discussed in detail here;

2. The measurement of $\delta_{28}$ in natural uranium rods, 1.010 inch in diameter, in the three lattices previously discussed;

3. Studies of the effects of changes in experimental conditions on the measurement of $\delta_{28}$;

4. Measurements of the flux of neutrons with energies greater than the U$^{238}$ fission threshold, as a function of position within a fuel rod and in the moderator;

5. Measurements of $\delta_{25}$, the ratio of epicadmium to sub-cadmium fissions in U$^{235}$;

6. Studies of the fission product gamma-ray spectrum as a function of time after irradiation, for gamma rays with energies up to 2.7 Mev.
The details of the methods and results are given in Ref. 27, based on a doctoral thesis by J. R. Wolberg. The results obtained for $\delta_{28}$ and $\delta_{25}$, the 1.0-inch rods in the three lattices, and in a single rod in D$_2$O, are given in Table 3.9.

TABLE 3.9

Values of $\delta_{28}$ (Ratio of $^{238}$U Fissions to $^{235}$U Fissions) and $\delta_{25}$ (Ratio of Epicadmium $^{235}$U Fissions to Subcadmium $^{235}$U Fissions) in 1.0-Inch Diameter Uranium Rods

<table>
<thead>
<tr>
<th>Lattice Spacing (inches)</th>
<th>Ratio of Moderator Volume to Uranium Volume</th>
<th>$^{238}$U Fissions</th>
<th>$^{235}$U Fissions ((\delta_{28}))</th>
<th>Epicadmium $^{235}$U Fissions</th>
<th>Subcadmium $^{235}$U Fissions ((\delta_{25}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>21.0</td>
<td>0.0597 ± 0.0020</td>
<td>0.0479 ± 0.0019</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>26.4</td>
<td>0.0596 ± 0.0017</td>
<td>0.0340 ± 0.0030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.75</td>
<td>35.6</td>
<td>0.0583 ± 0.0012</td>
<td>0.0268 ± 0.0010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\infty)</td>
<td></td>
<td>0.0559 ± 0.0015</td>
<td>0.0086 ± 0.0024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Rod</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV. PRESENT STATUS AND FUTURE PLANS

A. EXPERIMENTAL

The work reported on the lattices of natural uranium rods represents the first phase of the M.I.T. research program. The methods discussed are being improved where possible and are presently being applied to lattices of 0.25-inch diameter rods of uranium containing 1.03 per cent U\textsuperscript{235} at different lattice spacings. In addition to the steady-state methods described, pulsed-neutron studies of reactivity have also been started. Studies are being continued, in the small subcritical assembly, on "miniature lattices" of 0.25-inch diameter rods containing 1.143 per cent U\textsuperscript{235}. Two-region exponential experiments with the slightly enriched uranium rods are under way. Investigations of neutron distributions in and around single rods of 1.0-inch diameter, natural uranium rods are in progress.

B. THEORETICAL

The theoretical methods discussed above are being applied to lattices of slightly enriched rods, extended where necessary to handle tighter lattices of small diameter rods. When data have been obtained over a wider range of enrichments, diameters, and volume ratios, an over-all study of the theory of heavy water lattices will be made.
REFERENCES


3. T. J. Thompson, personal communication, M.I.T., 1961. The use of a cavity was suggested for the conceptual design of a facility for a heavy water reactor prepared for the Bell Telephone Laboratory in 1956.


